An investigation of knowledge

part one of eight

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As its title page promises, the investigation is in eight parts.

At the time of the writing of this introduction, parts one, two, and three have been completed. This has demanded 1332 pages. Dense pages. Parts four and eight are nearing completion. The research for parts five, six, and seven is substantially complete and their commitment to paper as well has begun. The whole will demand some 4000 pages. Dense pages.

ii

The odds that an investigation of knowledge, being an investigation of knowledge, will be examined have never been particularly good. Today they are particularly bad.

Philosophy and history have been degenerating for two centuries. Politics has been degenerating for two centuries. Mathematics has been degenerating for one century. Physics has begun degenerating, too.

It is thus not a time of enlightenment either. No one needs enlightenment. One knows everything one cares to know. Should there be anything else, too, one's smart phone will tell one not only what one wants to know, it will tell one what to want to know. One's smart phone is the smart one.

Today the odds that this investigation of knowledge will be examined are minimal. Maybe they will be better tomorrow. Maybe philosophy, history, politics, mathematics, physics will revive. Maybe it will be again a time of enlightenment. Either way, the investigation is laboring very very hard against all that to bring about this.

iii

What the investigation has done, and continues to do, is the following.

It is an investigation of all knowledge. All knowledge acquired on earth from the beginning of the acquisition of knowledge on earth through the time of writing is collected in the course of the investigation explicitly. All mathematics with all its parts, all history with all its parts, more generally all physics with all its parts, all politics with all its parts – all ethics with all its parts, too, even if ethics has not been a part of the acquisition of knowledge – and more generally all philosophy with all its parts. Bit by bit by bit by bit, and until all the bits are collected.

Nothing in the investigation is from secondary sources, never mind tertiary, quaternary, quinquenary, and so on indefinitely. All is primary.

Mathematics is mathematics. The 3 oldest calculations of area are from the century around *c.* 3080^{BC}. This is the first mathematician. And the 3 oldest calculations of area themselves are presented, and all the successors of the first mathematician themselves are presented. The book of 87 problems, the first calculation of the diagonal of the square, at $\sqrt{2} = 1.414\,212\,963\ldots$, the discoveries of Pythagoras, Eudoxus, the treatises of Euclid, Archimedes, Apollonius, the discoveries of Hipparchus, the treatises of Ptolemy, Diophantus, Khayyam, Fibonacci, Viète, Galileo, Kepler, Descartes, Fermat, Huygens, Newton, Leibniz, Euler, Lagrange, Laplace, Fourier, Gauss, Cauchy, Lobachevsky, Hamilton, Riemann, Cantor, Frege, Peano, Hilbert, Gödel, to note only some of the most important, in each case the fount itself is presented. The treatises of Turing, too, the immediate fount of that other fount, the smart phone, themselves are presented.

History is history. If Herodotus is the father of history, the Sumerian prince list, compiled under Ibbi-Sin, prince *c.* $2037\text{-}13^{BC}$, is its grandfather. This is the first historian. And the Sumerian prince list itself is presented, and all the successors of the first historian themselves are presented. The Abydos prince list, the Assyrian prince list, the treatises of Herodotus, Thucydides, Manetho, Polybius, Livy, Eusebius, Otto of Freising, ibn Khaldun, Bruni, Guicciardini, Bacon, Spinoza, Voltaire, Hume, Gibbon, to note only some of the most important, in each case the fount itself is presented. And as history more recently has staunchly devoted itself, as Leibniz

describes this devotion of history: to the utterly useless, or in any case to the least important (*New essays*, IV, 16), the founts of this, too, Braudel, for instance, themselves are presented.

Physics is physics. The oldest observation is of Sirius, in the 7th year of Senwosret III, prince *c.* 1878-45BC. This is in the valley of the Nile. Two-odd centuries later, in the valley of the two rivers, the observation of Venus over twenty-one years, and thus the discovery of the planet, is recorded on the Venus tablet of Ammisaduqa. Ammisaduqa was prince *c.* 1646-25BC. The Venus tablet of Ammisaduqa is the first physicist. And the Venus tablet of Ammisaduqa itself is presented, and all the successors of the first physicist themselves are presented. The oldest accurate observation, the discoveries of the first philosophers, Thales, Anaximander, Anaximenes, the treatises of Parmenides, Anaxagoras, Empedocles, the discoveries of Democritus, Eudoxus, the treatises of Aristotle, Theophrastus, Euclid, Aristarchus, Archimedes, the discoveries of Hipparchus, the treatises of Ptolemy, Copernicus, Gilbert, Galileo, Kepler, Descartes, Boyle, Huygens, Newton, Leibniz, Halley, Linnaeus, Euler, Buffon, Lagrange, Herschel, Lavoisier, Laplace, Fourier, Cuvier, Ampère, Gauss, Bessel, Faraday, Carnot, Hamilton, Darwin, Schwann, Helmholtz, Mendel, Clausius, Maxwell, Planck, Minkowski, Einstein, Bohr, Schrödinger, Friedmann, Hubble, Heisenberg, Crick and Watson, Jacob and Monod, Gell-Mann, to note only some of the most important, in each case the fount itself is presented.

Politics is politics. The first prince is Narmer, prince *c.* 3000BC, in the valley of the Nile. Three-odd centuries later, the first prince in the valley of the two rivers is Enmebaragesi, prince *c.* 2680BC. This is the beginning of the state, and *ipso facto* the beginning of politics. Politics *par excellence*, in deeds. Narmer has left behind the record of his deeds. He has also left behind a palette and a macehead. Enmebaragesi has left behind the record of his deeds. He has also left behind an inscription. And the record itself of the deeds of Narmer is presented, the record itself of the deeds of Enmebaragesi, and the record itself of the deeds of all their successors who have continued to build the state. The most important are these. – The state of Akkad, built by Sargon and his successors, the first state to succeed in bringing all other states under itself. – The state of Ur, built by Ur-Nammu and his successors, the first state under a code of laws, the code of Ur-Nammu. – The middle kingdom in the valley of the Nile, built greater than the old kingdom, at its greatest under Senwosret III. – The state of Babylon, built by Hammurabi and his successors, under the code of Hammurabi. This is the time also of the first great flourishing of mathematics. – The new kingdom in the valley of the Nile, built greater than the middle kingdom, at its greatest under Amenhotep III. – The state of the Assyrian princes, who for seven centuries, beginning with Ashur-uballit, tried to bring all other states under one state, and again, and again, and succeeded. Under Tiglath-pileser III and his successors it was for one century the state of one tenth of humanity. This is the time also of the destruction of Israel and Judah, and hence of the oldest Hebrew writings. – The state of the great kings, built by Cyrus, Cambyses, and Darius, reaching from the gulf of Sidra to the Indus, for two centuries the state of one fifth of humanity. This is the time also of the rise of philosophy, and hence of great acceleration of the acquisition of knowledge. – Rome, beginning with the conquest of Veii, by Camillus. Victory followed victory, and after the victory in the second Punic war, by Fabius Maximus, Marcellus, Scipio, it succeeded, as Polybius writes: in bringing nearly the entire known earth under a single government (I, 1). Counting from the victory in the second Punic war through the reign of Theodosius, nearly the entire known earth was under a single government for six centuries. Counting from the conquest of Veii to the fall of Constantinople to the knights of the fourth crusade, one and the same state continued for sixteen centuries. – The Christian church, built by the fathers of the church, Origen, Athanasius, Augustine. Where the state of Rome had been ecumenical, the Christian church became ecumenical. At the pinnacle of its power, princes from the king of England in the west to the crusader kings in the east gave Innocent III obeisance. – The caliphate, built by Umar, Uthman, Muawiyah. At the pinnacle of its power, under al-Mansur, it reached from the Atlantic to the Indus. – Portugal, as with

prince Henry exploration began encompassing the earth, and Spain, as under Charles V exploration encompassed the earth, and the state began encompassing the earth. Hence this is the time also, again, at long last, of great acceleration of the acquisition of knowledge. – France, which Louis XIV tried to build into the dominant power on earth, and Britain, which under William III and his successors defeated Louis XIV, and became, under Pitt, the dominant power on earth. – The building of the United States, by Washington, Adams, Hamilton, the rebuilding of the state in the revolution, by Sieyès, Robespierre, Napoleon, the building of Italy, by Cavour, the building of Germany, by Bismarck, the building of the Soviet Union, by Lenin. – In each case the founts themselves are presented. – Also, twenty-six centuries after the beginning of the state, Plato first built the state in speech. Polybius writes: Plato's republic is praised highly by many philosophers; but we do not admit to the contests athletes who have not trained for them; and to bring Plato's republic to a discussion of the constitutions of Sparta, Rome, and Carthage is like bringing a statue to compete with the athletes who have trained for the contest (VI, 47). Before Polybius, Aristotle and Zeno of Citium had also each brought a statue to the contest of the athletes. And after Polybius as before, the philosophers have continued to bring statues to the contest of the athletes. Cicero, Augustine, al-Farabi, Aquinas, Marsilius, Occam, More, Grotius, Hobbes, Spinoza, Locke, Montesquieu, Rousseau, Godwin, Hegel, Marx, among the more important. The first statue already informs the athletes: until the philosophers become kings or the kings become philosophers, the states will have no respite from evil (Plato, *Republic*, V, 473c-e). And so the statues have continued to inform the athletes. The statues themselves are thus also presented.

Ethics is ethics. This begins with Socrates, one and a half centuries into the development of philosophy. The date of Socrates's final lesson, the drinking of the hemlock, is 399^{BC}. And ethics has not been a part of the acquisition of knowledge, but it has been from the first the judge of the acquisition of knowledge. Its very stern judge. Socrates examined his predecessors the physicists, and concluded: earth and air and other such things are not the true causes of things; the true cause of things is what is for the best (*Phaedo*, 97b-9b). From the first and, twenty-four-odd centuries now, to the last. Plato, Pyrrho, st Paul, Augustine, Muhammad, al-Ghazali, Luther, Berkeley, Hegel, to note only the sternest judges. Thus the teachings of Socrates themselves are also presented, and the teachings of all his successors themselves, in each case the fount.

And philosophy is philosophy. The first philosopher is Thales. Thales said that a solar eclipse will occur, a solar eclipse occurred where and when Thales said it will, in central Asia Minor on 28 May 585^{BC} right after 15:30, and this dates the first philosopher. The second philosopher is Anaximander, the third philosopher is Anaximenes, the fourth philosopher is Pythagoras. The first three philosophers have been listed above among the most important physicists, the fourth philosopher has been listed above among the most important mathematicians, and so much should suffice to show that the lists above, of some of the most important mathematicians, historians, physicists, politicians, and ethicists, are *ipso facto* a list of many of the most important philosophers. And the discoveries of the first four philosophers themselves are presented, and beginning with the fifth philosopher, the fifth philosopher is Xenophanes, the treatises of the philosophers themselves are presented, in each case the fount. The sixth philosopher, the seventh philosopher, the eighth philosopher, and so on, through the last philosopher, so far. Thus, as history has more recently devoted itself to the utterly useless, or in any case to the least important, an even greater calamity has befallen philosophy. And the founts of this, too, Wittgenstein and Heidegger, for instance, even Kuhn and Michel Foucault, themselves are presented.

And all is related to all.

The acquisition of knowledge is one development. From the 3 oldest calculations of area and Narmer through Turing and the consequences of his results, the calamity that has befallen history, quantum theory and the big bang theory, the continuing contest of the athletes as well as of the statues, the ever sterner judgment of the acquisition of knowledge by that very stern judge even

as the acquisition of knowledge has continued to accelerate, and the greatest calamity of all that has befallen philosophy, it is all one and only one development. And it is the central concern of the investigation to try and show how all is related to all, in this one and only one development, and find its causes, and ultimately cause.

iv

The second rule of the method is: to divide difficulties into parts such as will lead to their better solution (Descartes, *Method*, 2).

Accordingly, the investigation is divided into eight parts. Each of the eight parts is itself divided into a few parts. Each of these parts of parts is itself also divided into many parts.

The eight main parts should, to one degree or another, stand on its own each. Given the bulk of the whole, each of these eight parts will have to be published, if it is published, in a separate volume. The parts of these main parts should also, to a lesser degree, stand on its own each. The parts of these parts of parts will not stand on their own.

One way or another, all parts, parts of parts, and parts of parts of parts, are one another's premises and consequences. The investigation is – to borrow Darwin's description of *The origin* (15) – one long argument.

The titles of the eight main parts – listed in the following contents – are all borrowed, too. Of part one from Bacon, two from Descartes, three from Aristotle, four from Spinoza, five from Newton, six from Leibniz, seven from Planck, eight again from Descartes. The eight titles are one way to summarize the argument.

Part one begins with a survey of the barriers before the investigation of knowledge. These are also, and to begin with, barriers before the acquisition of knowledge. Nor will they readily be breached, or even only circumvented. They are a majestic system of fortifications. So the survey first identifies the essential elements of the system, the citadel, the fortress, and the outworks. It then reviews the labors at reinforcing the barriers. These begin with Protagoras who first said: man is the measure of all things (Plato, *Theaetetus*, 152a-d). They culminate, so far, in, as Hegel describes it: the restoration of spirit to the substantiality and purity of its being (*Phenomenology*, pr.). The survey concludes by assigning a cause to the barriers and the labors at reinforcing them. – The affirmative argument then begins. It is grounded in Aristotle, Ptolemy, Hume, Kant, Darwin, and Frege. The problems which knowledge poses are collected into one problem. A fundamental part of the problem is to distinguish the acquisition of knowledge and knowledge. There can be no access to knowledge but through its acquisition. And the problem of knowledge cannot be solved, if the problem of the acquisition of knowledge is not solved first. A fundamental part of the problem of the acquisition of knowledge is to collect the acquisition of knowledge in its entirety. And the problem of the acquisition of knowledge cannot be solved, if the problem of collecting the acquisition of knowledge in its entirety is not solved first. Nevertheless, long before the acquisition of knowledge is collected in its entirety, part one concludes with a preliminary solution of the problem of knowledge.

Part two presents the method. The method is not difficult to state, it is difficult to follow. This part of the investigation is thus a sustained observation of the method. The method is not a method, the method is the method. For this reason it has been more often and more vigorously scorned than followed. It was, however, followed religiously in the three generations of Bacon, Galileo, and Kepler, Descartes, Huygens and Spinoza, and Newton and Leibniz. Hence the acquisition of knowledge advanced in these three generations more and more substantially than at any other time before or after. This part of the investigation is thus an observation of how in these three generations the method was followed religiously, and hence triumphantly.

Part three establishes the ground of the collection of the acquisition of knowledge in its entirety, and hence of the solution of the problem of knowledge. Generalization is over the particulars. The particulars, over which all generalization about knowledge must be a generalization, are the predecessors of the philosophers and philosophers, the mathematicians, the historians, more generally the physicists, the politicians, and the ethicists, too. The predecessors of the philoso-

phers and philosophers are propositions. These are the particulars. And some of them are the important. And some of these are the steps of advance in the acquisition of knowledge. The ground is thus established in three steps. The first step is grounded in the first great success of the method. This is the Sumerian prince list. The Sumerian prince list is a collection of its predecessors. On this ground it is shown that all the particulars can be collected. The second step is grounded in the first philosophers who judge their predecessors. These are the first philosophers who have left behind their own words, the fifth philosopher, the seventh, and the eighth, Xenophanes, Hecataeus, and Heraclitus. On this ground it is shown that importance is a fact, and all the important can be collected. The third step is grounded in the philosopher who discovered the systematic review of our predecessors. This is Aristotle. In book I of the *Metaphysics* Aristotle collects the development of the problem of the principles of all things in its entirety, from its beginning through the time of writing. And on this ground it is shown that all the steps of advance in the acquisition of knowledge can be collected. – Next, to show how firm the ground is, and continue the collection of the acquisition of knowledge in its entirety, and hence the solution of the problem of knowledge, the development of five problems is collected in its entirety. The formulations of the problems are from Aristotle. Collected is, one, the development of the problem of the earth, its position, whether it is at rest or in motion, and its shape, from its remotest beginning through Bessel. Two, the development of the problem of the soul, and both, what it is, and what you should do about it, the physical problem of the soul, and the ethical problem of the soul, from its remotest beginning through Darwin. Three, the development of the problem of the best state, and both, the well-governed state, and the state conceived in speech, the historical problem of the best state, and the political problem of the best state, in the four-odd centuries from Tiglath-pileser III through Alexander. Four, the development of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, from its remotest beginning through Riemann. And five, the development of the problem of the principles of all things, and all three, the elements of all things, the causes of all things, and the formal principles of all things, from its remotest beginning through Clausius.

At the time of the writing of this introduction this has been done. Part one has demanded 326 pages, part two 264 pages, part three 742 pages. The following is being done.

Parts four and five are the systematic collection of the acquisition of knowledge in its entirety, and on this ground now the explicit formulation and solution of the problem of knowledge. In parts one, two, and three many of the particulars, the important among them, and the steps of advance in the acquisition of knowledge among them, have been collected with respect to the more particular problems addressed in these parts – the survey of the barriers, the formulation of the problem of knowledge, the observation of the method, the ground of the collection of the acquisition of knowledge, the five problems showing how firm the ground is – and the problem of knowledge has been addressed, explicitly and implicitly, in relation to these more particular problems. Parts four and five are the generalization. The particulars, all important among them, and all steps of advance in the acquisition of knowledge among them, are collected in their series and order, the one development which the acquisition of knowledge is, all in relation to all, and hence the problem of knowledge is formulated and solved explicitly. The particulars are the predecessors of the philosophers and philosophers, the mathematicians, the historians, more generally the physicists, the politicians, and the ethicists, too. In part four the politicians and the historians are collected. The development of the state, and the development of history, and in their dependance on one another, the dependance of the development of history on the development of the state, and conversely, the dependance of the development of the state on the development of history, is collected. In part five the mathematicians, more generally the physicists, and more generally the philosophers are collected. The development of physics, inclusive of the development of mathematics and the development of history, and more generally the development of

philosophy, inclusive of the development of physics as well as of the development of ethics, and in its dependance on the development of the state, and conversely, in the dependance of the development of the state on the development of philosophy, is collected.

More particularly, part four begins with the resolution of two difficulties. If the record has lost even 1 important predecessor of the philosophers or philosopher, nothing can be affirmed about all important predecessors of the philosophers and philosophers. It is shown that the record has lost nothing important, not 1 thing. And chronology is a *sine qua non* of the investigation of knowledge. The temporal series and order of the predecessors of the philosophers and philosophers has to be determined in all cases precisely. Chronology is fixed explicitly, and its certainty in all cases shown. – Then the development of the state, inclusive of the development of the law and the development of religion, and the development of history, inclusive of the development of geography, and in their dependance on one another, is collected, particular after particular after particular, all important among them, and all steps of advance in the acquisition of knowledge among them, Narmer through the time of writing. Along the way the problem of the beginning of the state is formulated and solved. All along causes are assigned.

Part five begins with a final calibration. The units of the acquisition of knowledge are the predecessors of the philosophers and philosophers, the predecessors of the philosophers and philosophers are propositions, the acquisition of knowledge is the winning of propositions, and the whole thing is done by machinery. – Then the development of physics, inclusive of mathematics and of history, and more generally the development of philosophy, inclusive of physics as well as of ethics, and in its dependance on the development of the state, and conversely, is collected, particular after particular after particular, all important among them, and all steps of advance in the acquisition of knowledge among them, the 3 calculations of area through the time of writing. All along causes are assigned. – And the problem of knowledge with all its parts, the beginning of the acquisition of knowledge, the collection of the acquisition in its entirety, the relation of the acquisition of knowledge and the development of philosophy, the kinds of the acquisition of knowledge, and the relation of knowledge and truth, and of knowledge and the world, is formulated explicitly and the formulated problem solved.

Part six is an assessment of the completeness of the argument, part seven an assessment of its generality.

The state begins in the valley of the Nile with Narmer, prince *c.* 3000^{BC}. It begins in the valley of the two rivers with Enmebaragesi, prince *c.* 2680^{BC}. – The princes of the valley of the Nile first ventured into the other valley. Senwosret III, prince *c.* 1878-45^{BC}, first campaigned in Asia beyond Sinai. It was then three and a half centuries before his successors ventured again into Asia. Thutmose, prince *c.* 1521-09^{BC}, conquered the Asian Mediterranean littoral north to the Euphrates. His successors in the next three centuries maintained their control over the Asian Mediterranean littoral, at first firmly, then not so firmly. Then their state declined catastrophically. – The princes of the valley of the two rivers first conquered the other valley. Tiglath-pileser III and his successors brought the entire valley of the two rivers under one state, and Esarhaddon, prince 680-68^{BC}, conquered the valley of the Nile. – First, then, the development of the state in the valley of the Nile and the development of the state in the valley of the two rivers are altogether independent of one another, for eight-odd centuries, counting from Enmebaragesi to Senwosret III, for eleven and a half centuries, counting from Narmer to Senwosret III. Next, the two converge, gradually, haltingly, still largely independent of one another, for twelve centuries, counting from Senwosret III to Esarhaddon. Finally, the two become and remain a joint development, for twenty-seven centuries now, counting since Esarhaddon. – As the development of the state, so the acquisition of knowledge. At first altogether independent in each of the two valleys, it becomes at length a joint development. – Beginning to end, it is one and only one development. Independent of one another at first, converging, becoming a joint development, these are all parts of one and only one development. And to show this is a central concern of parts one through five.

The state begins in the valleys of the Chang Jiang and Huang He not too long before 841^{BC}. Sima Qian fixed the dates of his predecessors back to the end of the reign of the Zhou prince Li in 841^{BC} (14). The earlier record, Sima Qian concluded: is confused and unreliable (13). Through the confusion, the Zhou princes can be traced back, more or less reliably, some two centuries before Li. Through the increasing confusion, the earlier Shang princes can be traced, not entirely unreliably, some two centuries further back. Further back they can be traced only entirely unreliably. – Genghis Khan, prince 1206-27^{AD}, campaigned from the Pacific in the east to the Black Sea in the west. Genghis Khan and his successors in the next two generations, Ögedei, Batu, Möngke, Kublai, and Hülegü, began unifying, first by overrunning and pillaging it, then by ruling it, the domain from the Dniester and the Euphrates in the west to the Pacific in the east. Kublai, prince 1260-94, became Yuan emperor. Polo reached China under Kublai, served Kublai, and wrote the *Description of the world*. – Da Gama reached India in 1498, Albuquerque captured Malacca in 1511, a Portuguese ship reached China in 1513, a Portuguese ship reached Japan in 1543. In the following three centuries trade grew, and became conquest. In 1839-42 Britain won the first opium war, and imposed the first of the unequal treaties on the Qing emperor. In 1853 American warships imposed a treaty on Japan. In 1868 the Meiji restoration began. In 1912 China became a republic under Sun Yat-sen. – First, then, the development of the state in the west and the development of the state in the east are altogether independent of one another, for twenty-four and a half centuries, counting from some four centuries before Li to Genghis Khan, for forty-two-odd centuries, counting from Narmer to Genghis Khan. Next, the two converge, gradually, haltingly, still largely independent of one another, for six and a half centuries, counting from Genghis Khan to Meiji. Finally, the two become and remain a joint development, for one and a half centuries now, counting since Meiji. – As the development of the state, so the acquisition of knowledge. At first altogether independent in the west and in the east, it becomes at length a joint development. – Beginning to end, it is one and only one development. Independent of one another at first, converging, becoming a joint development, these are all parts of one and only one development.

To try and show this is the problem of part six.

The acquisition of knowledge is always limited in space and time. For instance. Eudoxus first measured the earth. One century later, Aristarchus first measured the distances of the moon and the sun. Four centuries later, Ptolemy first measured the world, its radius is 19 865 earth radii. Nineteen centuries later, the big bang theory measures the world, the big bang occurred 1.4×10^{10} years ago, entailing the limit of observation at 4.6×10^{10} light years away. Thus, tabulating, in kilometers, the diameter of the world is,

before Eudoxus	no inkling,
Ptolemy	2×10^{8} km,
the big bang theory	9×10^{23} km.

And the agreement of Ptolemy and the big bang theory is very much greater than the disagreement not only in that Ptolemy's result is a very much greater advance over the earlier results than the result of the big bang theory is over his result. The agreement is very much greater than the disagreement in that the big bang theory stands on Ptolemy. The agreement of Ptolemy and Copernicus is very much greater than the disagreement, and most particularly in that the agreement is the ground of the disagreement, and hence of the great advance which Copernicus is over Ptolemy. The agreement of Copernicus and Kepler is very much greater than the disagreement, and most particularly in that the agreement is the ground of the disagreement, and hence of the great advance which Kepler is over Copernicus. In the same way Newton's great advance over Kepler. In the same way Einstein's great advance over Newton. And in the same way the foundation of the big bang theory is Friedmann's solution of the field equations of general relativity, the agreement of Einstein and Friedmann is very much greater than the disagreement, and most particularly in that the agreement is the ground of the disagreement, and hence of the great advance which Friedmann is over Einstein. – There is thus no need for anxiety that the big bang theory will be overthrown tomorrow. The big bang theory stands unshakably not because it

is the theory of the world. Not even because it has reached the beginning of the world and the end of the world, both. Because it stands on Ptolemy, and Ptolemy already stands unshakably. As Ptolemy stands on Eudoxus, and Eudoxus already stands unshakably. Nor is Eudoxus *ex nihilo*. The series Eudoxus, Aristarchus, Ptolemy, Copernicus, Kepler, Newton, Einstein, Friedmann is a series of the accumulation of conclusive results. And not only are in each step the conclusive results more, essentially and substantially, than the errors, the conclusive results are the ground, exclusively, on which in the next step the errors are corrected. Limitation in space and time is not one of the barriers before the acquisition of knowledge. On the contrary, it is in some predecessors of the philosophers and philosophers, not all, far from it, yet in some, the ground on which conclusive results are obtained. It is what the acquisition of knowledge is, the accumulation of conclusive results. And to show this is a central concern of parts one through six.

In the same way, the investigation of knowledge is always limited in space and time. The investigation of the acquisition of knowledge, and the investigation of knowledge on this ground, begins with Aristotle. And the facts on which Aristotle's investigation rests are limited. Spatially, to the domain between the Atlantic and the Indus west to east, between the northern shores of the Black Sea and the southern shores of the Red Sea north to south. Temporally, to the 4 centuries from Homer to himself. Twenty-four centuries later, the facts on which the investigation of knowledge must rest are still limited, yet reach very much further. Spatially, they encompass the entire surface of the earth, and indeed the entire solar system. Temporally, counting back to the 3 oldest calculations of area and Narmer, they extend over 51 centuries, counting back to the oldest city, at Çatal Hüyük, over 85 centuries, counting back to the oldest tools, the Oldowan tools, over some 26 000 centuries, counting back to the beginning of life on earth, and indeed in the solar system, over some 40 000 000 centuries. – And there is no need to suppose that the investigation of knowledge is *ad hoc*. Grounded in the acquisition of knowledge, the investigation of knowledge has been an attempt to acquire knowledge of the acquisition of knowledge, and hence knowledge of knowledge, and a most successful attempt, too. Aristotle, Bacon, Descartes, Spinoza, Leibniz, Hume, Kant, Frege, to note only the most important steps in this series. Limitation in space and time is not one of the barriers before the investigation of knowledge. The investigation of knowledge is the accumulation of conclusive results, too. And to show this is a central concern of parts one through six as well, and it is, one last time, the problem of part seven.

Part eight is the enumeration and review of the particulars collected in the course of the investigation. Of the attempt to collect the acquisition of knowledge in its entirety, and on this ground formulate and solve the problem of knowledge.

v

That there is a problem does not imply that there is a solution of the problem. But it does imply that the attempt to solve the problem can be made.

vi

Most writers are rendered into English from the original, the rest in each case from multiple translations.

Citation is not chapter and verse. Such became normative two millennia ago as the means of showing and testing orthodoxy fast, and still serves no better end. Citation is of the treatise, when the treatise is short, of the treatise and chapter, when the treatise is long.

The appeal is never to the words, and only incidentally to the sense. The appeal is always to the reference, always with a view to showing that the reference of all false propositions is the same, the false, and the reference of all true propositions is the same, the true.

There are and can only be two ways to search for truth, the false and the true 11
It is much better never to search for truth than to search without the method, the method must be followed as staunchly in the search for truth as the thread of Theseus in the labyrinth 331
In order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous 595
The method of history does not differ from the method of physics at all, but agrees with it entirely. 1341
The entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena . . . . . .
There will be no more need for disputes between philosophers, it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate . . . . . .
That the inhabitants of Mars, if they have the necessary intelligence, recognize the law of gravity and the principle of conservation of energy, should be self-evident, to anyone to whom this is not self-evident, on the other hand, it will forever remain an unresolved mystery that the same physics is made in the United States as in Germany . . . . . .

enumeration and review . . . . . .

There are and can only be two ways to search for truth, the false and the true

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The barriers have been reinforced, and reinforced, and reinforced 51
Yet the investigation of knowledge has advanced greatly, the acquisition of knowledge has advanced very greatly, and the problem of knowledge can be formulated explicitly 144

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vii

The investigation of knowledge should not begin by turning to its proper object. It is not clear what knowledge is. It is not even clear what is knowledge, and what is not.

On the other hand, the main reason for the lack of clarity is that there are barriers before the investigation of knowledge. They have been barriers before the investigation of knowledge as long as knowledge has been investigated, and knowledge has been investigated for twenty-four centuries now. And they have been barriers before the acquisition of knowledge longer than they have been barriers before the investigation of knowledge, and as long as knowledge has been acquired, and knowledge has been acquired for fifty-one centuries now. They have also been reinforced, and reinforced, and reinforced. And they continue to be barriers before the acquisition of knowledge, and barriers before the investigation of knowledge, and to be reinforced, and they have been barriers before the investigation of knowledge and have been reinforced so long, they have come to be taken to be not barriers before the investigation of knowledge but results from it, and the more they have been taken to be results rather than barriers the more they have been reinforced, and the more they have been reinforced the more they have been taken to be results rather than barriers.

Clearly, the investigation of knowledge should begin with the survey and assessment of the barriers.

The barriers before the investigation of knowledge are many and formidable

viii

Plato's, in the *Theaetetus*, is the first investigation of knowledge.

The importance of the *Theaetetus* should need no recommendation. That it is the first investigation of knowledge is only one of its great virtues. Another is that Plato is at his most explicit about the two centuries of development of philosophy of which he is the successor. He notes 20 predecessors and contemporaries of his, 12 of the 20 are important, and of 6 of the 12 his reports are very important. He has an anecdote about the first philosopher. He attests the discovery of irrationals other than $\sqrt{2}$. He reports on the first philosophical war. He reports that it has been found out that man is the measure of all things. And he has Socrates describe his art.

More particularly, Thales is the first philosopher, and Aristotle, in the generation after Plato, discovered this fact. Plato, however, comes to dwell in the *Theaetetus* on what the philosopher is, and what the philosopher is not, and has this to say about Thales: Thales, once, while gazing at the stars, fell into a well; a clever Thracian slave-girl was passing by and drew the moral, eager to know what was in the heaven, Thales did not know what was in his feet (173e-4a), and Plato himself draws the moral: the joke applies to all philosophers (*id.*).

Two generations after the first philosopher, Pythagoras discovered the incommensurability of the side and diagonal of the square, and thus the irrationality of $\sqrt{2}$. Plato does not note this discovery. One century-odd after Pythagoras, however, as Plato attests in the *Theaetetus*: Theodorus of Cyrene proved that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational (147d).

Some two generations after Pythagoras, one century-odd after the first philosopher, the first philosophical war was fought. The followers of Heraclitus and Parmenides fought it. Heraclitus did not himself have followers, but he died, and then there were followers of Heraclitus. Parmenides, in the next generation, did himself have followers: our Eleatic tribe (242d), as the Eleatic stranger in Plato's *Sophist* describes his allegiances. And by this time not only were there followers of Heraclitus. Plato reports in the *Theaetetus*: all his fellow Ephesians have become Heracliteans, and they are defending his doctrines so violently, you might as well be talking to a maniac if you tried to talk to one of them; and not only that; to everything the Heracliteans say the Eleatics say the opposite (179d-81a), so Plato asks: what are we to do, pulled in this tug-of-war now this way now that (*id.*)? And what we are to do, to judge by what Plato does, is not let ourselves be pulled now this way now that, but resolutely pull the right way. For philosophical wars, or tugs-of-war, have never since run out of the will to fight them.

Later in the same generation, not quite one and a half centuries after the first philosopher, Protagoras found out that man is the measure of all things. Plato reports: Protagoras said, man is the measure of all things, of those that are that they are, of those that are not that they are not; thus his *Truth* began (152a-d).

And not quite one generation after Protagoras, one and a half centuries after the first philosopher, Socrates describes his art in this way: do you not know, he asks, that I am the son of a good hefty midwife? and that I practice the same art? I am barren of wisdom myself, but those who associate with me I help discover many beautiful things within themselves; and, with god's help, I deliver them of their offspring (149a-50d).

The other 6 of the 12 important predecessors and contemporaries of his whom Plato notes in the *Theaetetus* are Homer, Hesiod, Empedocles, Melissus, Euclid of Megara, and Theaetetus.

Barren of wisdom himself, then, Socrates helps those who associate with him discover many beautiful things within themselves. In particular, he proposes to help Theaetetus discover many beautiful things within himself, and to do so by asking: what is knowledge (145e)?

It is the kind of question he has asked many times. He has asked: what is piety? in the *Euthyphro*, what is courage? in the *Laches*, what is friendship? in the *Lysis*, what is temperance? in the *Charmides*, what is beauty? in the *Greater Hippias*, what is justice? in the *Republic*. And in the *Meno* he has not only asked: what is virtue? but when Meno has offered him, in response, the virtues of a man, and of a woman, of a child, and of an old man, of a free man, and of a slave, he has mused: I seem to be in luck, I ask for one virtue and you give me a swarm of them; but even if there are all these virtues, what makes them all virtue (71d-2c)?

So he asks: what is knowledge? and when Theaetetus offers him a swarm of knowledges, in this case as in that Socrates complains: I ask for one thing and you give me many, I ask for something simple and you give me variety; but I do not ask of what one has knowledge or how many kinds of knowledge there are, I ask what knowledge itself is (145d-6e).

Unlike Meno, however, and unlike, indeed, most all of those whom in the Platonic dialogues Socrates helps discover many beautiful things within themselves, Theaetetus understands what Socrates wants. Unlike other Platonic dialogues, the *Theaetetus* does not just dwell on its question,

it answers it. In particular, the dialogue is the examination of three answers to the question, what is knowledge? and the results of the examination are affirmative not less than they are negative, even as it turns out that all three answers are wrong.

The first answer is: knowledge is perception (151e). But: this is the same as what Protagoras said, that man is the measure of all things, of those that are that they are, of those that are not that they are not (152a), and this cannot be maintained consistently. Man cannot be the measure, lest man both measured and *ad hoc*, and when Plato asks: why did not Protagoras say that pig is the measure? or baboon (161c)? the answer can only be, Protagoras could have. Or pig could have. Or baboon. Because: in geometrical proofs not every man for himself is the judge, the geometer alone is the judge (169a), and not every man for himself is the judge for the same reason not every pig for himself is the judge. Or every baboon. The reason is: men and animals alike have perceptions as soon as they are born, reasoning about these, however, only comes after a long development; hence, knowledge cannot be in experience, it can only be in reasoning about experience (186c-d), and if so: perception and knowledge cannot be the same (186e).

The second answer is: knowledge is belief, and since belief can be either true or false, knowledge is true belief (187b). But: that there is belief without knowledge, and either true belief or false, the art of lawyers and orators dramatically shows (201a). Even if not always so dramatically, every man, pig, and baboon shows it, too.

The third answer is: knowledge is true belief with the reason for it (201d). But: the reason can itself only be knowledge; and it is silly, when trying to find out what knowledge is, to say that it is true belief with the knowledge for it (210a).

Plato concludes: knowledge is not perception, not true belief, and not true belief with the reason for it (210a-b).

After two centuries of development of philosophy, then, and of great success in the acquisition of knowledge in the course of this development, the first investigation of knowledge is greatly successful itself.

As far as the success of the acquisition of knowledge is concerned, suffice it to note two developments. Pythagoras discovered the incommensurability of the side and diagonal of the square, Theodorus proved that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational, and by the time of Plato his younger contemporary Theaetetus had formulated the theory of the irrationals, such as Euclid presents it in book X of the *Elements*. Parmenides discovered that the earth is a sphere, and a century later, by the time of Plato, his younger contemporary Eudoxus had formulated the theory of the motions of the heavenly bodies in concentric spheres, such as it remained the foundation of the investigation of the heavenly bodies through and including Copernicus.

As far as the success of the first investigation of knowledge is concerned, it is, again, affirmative not less than negative. Negatively, knowledge is not perception. Further, though, affirmatively, knowledge only comes after a long development. Negatively, knowledge is not belief. Further again, though, affirmatively, knowledge and belief are exclusive of one another. For if knowledge is not belief, and knowledge is not true belief either, and knowledge is not true belief with the reason for it either, it has to be concluded, belief is not knowledge either. Negatively, man is not the measure. Once again further, though, affirmatively, in geometrical proofs the geometer alone is the measure.

These results of the *Theaetetus* have to be one and all conclusive.

Ipso facto its general result has to be conclusive, too. Whatever knowledge is, not clear as it is what knowledge is, and if there is to be a chance of making it clear what knowledge is, knowledge, on one side, has to be separated sharply from perception, belief, and the supposition that man is the measure, all on the other side jointly and inseparably. Jointly and inseparably, for not only, if knowledge is perception, man is the measure, and if knowledge is belief, man is the measure. Even if not all knowledge is perception, but perception is a kind of knowledge, man is the measure, and even if not all knowledge is belief, but belief is a kind of knowledge, man is the measure.

Which is precisely why, apparently, nearly two and a half millennia later, the investigation of

knowledge finds itself obtaining all the results which should have been rendered obsolete by the *Theaetetus* already.

Russell, Wittgenstein, Heidegger, Carnap, and Popper are not all important philosophers of the last century-odd, but they are all important philosophers of the last century-odd who have investigated knowledge. And they all affirm what the *Theaetetus* denies.

Russell argues, in 1911, that there is knowledge by acquaintance, and that that knowledge is instanced by: I hear a noise, I see the sun, I desire food (*Description*). Thus as far as perception is concerned. As far as belief is concerned, he writes, in 1912: knowledge is not a precise conception, it merges into probable opinion (*Problems*, 13), and in 1959: I still agree that knowledge is not a precise conception, but merges into probable opinion (*Ph. development*, 9). As far as man is the measure is concerned, he writes, in 1944: theory of knowledge has a certain essential subjectivity; its data are egocentric, and so are the earlier stages of its argumentation; I have not so far got beyond the earlier stages, and have therefore seemed more subjective in outlook than in fact I am (*Ment. development*).

It should have been superfluous to note that it is a *petitio principii* that its data are egocentric. Nor is it too reassuring that Russell has seemed more subjective than he is. For he is far and away less subjective than his successors.

Wittgenstein writes, in his first *magnum opus*, in 1921: theory of knowledge is the philosophy of psychology (*Treatise*, 4.1121). In his second *magnum opus*, in 1953: when the philosophers use the word knowledge, and try to grasp the essence of the thing, one must always ask, is the word so used in the language which is its home? we bring back the words from their metaphysical to their everyday use (*Investigations*, 116).

By 1921 the special theory of relativity had been formulated, the general theory of relativity had been formulated, and quantum theory was being formulated. And were theory of knowledge to presume that those theories have anything to do with knowledge, those theories were, in 1921, matters of psychology. By 1953 quantum theory had been formulated, too. All those theories, though, had in the meantime quit being matters of psychology. Were the philosophers to presume to try to grasp the essence of knowledge, they were to ask, in 1953, what the everyday use of the word knowledge is, never mind the Lorentz transformation, or the field equations, or Schrödinger's equation. Unless they were to ask what the everyday use of the words transformation and equation is? or the words Lorentz and Schrödinger? or the word field?

Heidegger writes, in his *magnum opus*, in 1927: knowing is a mode of being of this-being as being-in-the-world; it may be objected that the problem of knowledge is thus annihilated, because knowing comes along with its world; but which is the higher court that decides that there is a problem of knowledge other than the kind of the being of the knower (*Being*, 13)?

Why, Heidegger is the higher court. For which is the higher court that decides that Heidegger is not the higher court?

It may have seemed – once knowing began coming along with its world – that things cannot get worse.

Carnap writes, in 1928: the epistemological nucleus of every concrete knowledge of heteropsychological occurrence consists of a perception (*Pseudoproblems*, 4), whence: the theses of realism, that the external world is real, and of idealism, that the external world is not real, are both meaningless (10). In 1961: I have maintained these views ever since (*Structure*, pr. 2nd ed.).

Why did he not say, man is the measure? or pig? or baboon? and can the thesis that realism and idealism are both meaningless not be idealism?

Popper writes, in 1934: we do not know, we can only guess (*Logic*, 85). In 1960: what we should do is admit that all knowledge is human (*Sources*, 17). In 1967: what is relevant for epistemology is the study of the largely autonomous third world of objective knowledge (*Epistemology*, 1), namely: next to the two worlds of physical objects and of states of consciousness, the third is the world of objective contents of thought, especially of scientific and poetic thoughts and of works of art (*id.*).

It may have again seemed – once relevant for epistemology became the study of a third world,

nay, a world of the contents of thought, nay, the contents of thought of works of art – that things cannot get worse.

Things have got much worse since.

ix

If, then, not even one and a half centuries after the first philosopher: man is the measure of all things, and twenty-four centuries later: essential subjectivity, egocentric, psychology, everyday use, this-being, knowing comes along with its world, heteropsychology, realism and idealism are meaningless, we do not know, we can only guess, all knowledge is human, and epistemology is to study a third world, too, then one or the other. Man is a result from the investigation of knowledge, indeed its most fundamental result. Or, if man is not a result, yet keeps pretending to be a result, and the most fundamental result, man is a barrier before the investigation of knowledge, and the most formidable barrier.

And man is not a result.

To begin with, it should be most implausible – even as it is not – that man is the measure. So implausible, it should be absurd. Then again, Cicero said: nothing so absurd can be said as not to have been said already by one philosopher or another (*Divination*, II, 119), and said so more than two millennia ago.

It is not clear what knowledge is. Yet it should be most implausible that man is the measure whatever knowledge is. Whatever knowledge is, it has to be likely that knowledge is instanced by incommensurability, the sphericity of the earth, the Lorentz transformation, the field equations, Schrödinger's equation. If so, man is the measure of incommensurability. The measure also of the sphericity of the earth. Everyday usage is the key to the Lorentz transformation, the field equations, and Schrödinger's equation. To vary the examples, the theory of evolution was formulated two generations before the theory of relativity, and natural selection is a mode of being of this-being. The theory of sets was formulated one generation before the theory of relativity, and transfinite numbers are also a mode of being of this-being. The big bang theory was formulated one generation after quantum theory, and the epistemological nucleus of the big bang consists of a perception. Needless to add, all these, incommensurability, the sphericity of the earth, natural selection, transfinite numbers, the Lorentz transformation, the field equations, Schrödinger's equation, the big bang, show that we do not know, we can only guess.

But even if knowledge were instanced by seeing the sun and desiring food, it should be most implausible that man is the measure, everyday usage is the key, knowing is a being of being, and the epistemological nucleus is a perception. Granted that those – seeing the sun and desiring food – are matters of psychology, including heteropsychology, they cannot be matters of psychology without being matters of physiology and anatomy, they cannot be matters of psychology or physiology or anatomy without being matters of selection, and again, man is the measure of selection, usage is the key to anatomy, physiology is a being of being, and the epistemological nucleus of psychology is a perception. It should be asked, too, in the proposition, we do not know, we can only guess, what is the reference of 'we'? if it is men, is it *homo sapiens*? or also *erectus*? and *australopithecus*? or the reference is the philosophers? but if so, are the philosophers a variety of men? and again, of *sapiens*? or the reference is just Popper? but even so, is Popper a philosopher? or a man? or both? and if both, again, is philosopher a variety of man? of *sapiens*?

It should be most implausible that man is the measure, but it is not. Protagoras and the philosophers who have agreed with him, through Wittgenstein, Heidegger, Carnap, Popper, have kept giving that supposition plausibility. Decisive, however, has been not that these philosophers have kept giving that supposition plausibility. Decisive has been that Plato and the philosophers who have agreed with him have kept giving that supposition plausibility.

Theory of knowledge has a certain essential subjectivity, Russell found out. So: the earlier stages of its argumentation are egocentric, and I have not so far got beyond the earlier stages.

As it is, then, presuming that the later stages are objective, perhaps even essentially objective, Russell did not get to the argumentation of the objective. He got, however, to the argumentation

of the subjective. Worse, if the earlier stages are subjective, and the later stages are objective, then even if Russell had got to the argumentation of the later stages, or his successors got to their argumentation, the argumentation of the objective is doomed to depend on the argumentation of the subjective. Worse still, if Russell got to the argumentation of the subjective, and not further, not only the ground on which it will be presumed that the later stages in the argumentation are objective, but the ground on which it will be presumed that there is anything objective at all, cannot be too objective. It can, in fact, only be subjective.

Plato disapproves of Protagoras. But he approves of Socrates. So, man is not the measure of all things. But what Socrates does is: help those who associate with him discover many beautiful things within themselves. And the supposition that those who associate with Socrates will discover many beautiful things within themselves can ground itself in nothing but the supposition that man is the measure of all things.

It is reassuring that, to judge by the Platonic dialogues, what those who associate with Socrates tend to discover within themselves is: yes, Socrates; yes, Socrates; yes, Socrates (*passim*). It can be presumed that the many beautiful things they discover within themselves are all the same, but consent to the things Socrates teaches them. This has seldom been presumed. What has been, instead, is that they do discover the things Socrates teaches them within themselves, and if they discover these things within themselves, they can discover many beautiful things within themselves, and if they can discover many beautiful things within themselves, they can discover many other things within themselves, and if they can discover all these beautiful and other things within themselves, they can discover anything within themselves, and if those who associate with Socrates can discover anything within themselves, anyone can discover anything within oneself.

Nothing has contributed more to making man the most formidable barrier before the investigation of knowledge – and before the acquisition of knowledge, too – than the Platonic dialogues. The delusion that there are beautiful things within oneself to discover, which the dialogues foster relentlessly, is one of the reasons.

Another is that Socrates teaches me, and just as relentlessly, to know myself. Worse, he teaches me: it seems ridiculous to me to search for other things before I know myself (*Phaedrus*, 230a). The reference of 'myself' can in any case be nothing but man such as man is the measure of all things. Once again worse, however. Protagoras's proposition is still interested, in its way, in all things. Socrates's teaching is not. If I am not only to know myself, but to know myself before I search for other things, indeed, it is ridiculous to search for other things, how likely am I to ever search for other things? For how likely am I to ever know enough about that most irresistible, most fascinating, most profound, most elevated of things, myself? And yet worse, Socrates indicts his adversaries: they teach nothing but the common opinions of the rabble and this is what they call wisdom (*Republic*, VI, 493a). With all due respect to Socrates, was anyone – man, or pig, or baboon – ever wanting the opinion, and in need to be taught, that he ought to know first and foremost that most irresistible, most fascinating, most profound, most elevated of things, myself?

Still, the most powerful reason is this. Aristotle reports: Socrates abandoned physics altogether, and devoted himself to ethics (*Metaphysics*, I, 6).

It is the devotion to ethics – what is piety? what is courage? what is friendship? what is temperance? what is beauty? what is justice? what is virtue? – that gives the twin enterprises of knowing myself and of expecting to discover many beautiful things within myself not only plausibility, but respectability, and not only respectability, but philosophical respectability. The twin enterprises – they are not self-indulgence, they are philosophy. It is the devotion to ethics as well, and whether the devotion is explicit or implicit, that makes the fact that the twin enterprises rest unconditionally on the supposition that man is the measure appear either circumvented, on that other Socratic supposition of immutable justice and virtue and the like, or the more just and virtuous and the like to embrace.

The devotion to ethics, in turn, rests unconditionally on abandoning physics altogether.

Wittgenstein is representative: the world is my world (*Treatise*, 5.62). Accordingly: if a man could write a book on ethics, which was really a book on ethics, this book would destroy with an explosion all other books in the world (*Ethics*). It would destroy Euclid's books, and Archimedes's, and Ptolemy's, and Copernicus's, and Galileo's, and Kepler's, and Descartes's, and Newton's, and Leibniz's, and Laplace's, and Darwin's, and Maxwell's, and Cantor's, and Frege's, and Planck's, and Einstein's, and Bohr's, and Schrödinger's, and Hubble's, and Heisenberg's. It would destroy the books in which the discoveries of Pythagoras and Parmenides are recorded, too.

Man being what man is, Plato's dialogues have kept being so read as to keep them reinforcing the barriers before the investigation of knowledge, and most particularly man. But they do not have to be so read.

What Socrates teaches me I can ignore.

And I should ignore. Because Plato's results from the first investigation of knowledge are conclusive, and these results all by themselves show that man is not a result from the investigation of knowledge.

Knowledge only comes after a long development. Plato draws the conclusion that knowledge cannot be in experience, it can only be in reasoning about experience. It can be argued against the conclusion, and it should be. Either way, if it is argued against the conclusion, and if it is not, it is a fact that knowledge only comes after a long development – whatever knowledge is, and however the development has to be described, in terms of experience, or in terms of reasoning, or in terms of the relation of experience and reasoning, or in other terms perhaps.

Knowledge only comes after a long development. However it has to be described, and in order to establish how it has to be described, the development can be considered with respect to man, and either with respect to man individually or with respect to the kind of man, his species, his genus, his order, his class, his phylum, his kingdom. Or it can be considered with respect to not man. And however it is considered, knowledge only comes after a long development, and when it does come, it does not come to man.

To take development with respect to man individually. If Theaetetus, for instance, is such a man, Theaetetus was fifteen at the time of his conversation with Socrates. The theory of the irrationals he formulated later. Before he was fifteen he was ten, before that he was five, before that he was two, before that he was a gastrula, before that he was a blastula, before that he was a fertilized egg. Whatever knowledge is, then, if Theaetetus is a man, knowledge only came to that man after a long development. It did, if the theory of the irrationals instances knowledge, and it did, if seeing the sun and desiring food instance knowledge, and even if the fertilized egg already indulged in seeing and desiring. If the theory of the irrationals instances knowledge, knowledge does not come to man, it comes, at most, to a handful of men, to Theaetetus and those who are like him in that he formulated the theory of the irrationals, one of whom, not incidentally, Socrates is not. If seeing the sun and desiring food instance knowledge, knowledge does not come to man either, it comes to man, pig, and baboon indiscriminately.

To take development with respect to the kind of man. If seeing the sun and desiring food instance knowledge, knowledge does not come to just man, pig, and baboon. It does not come to just the class either, to mammal, it comes to other classes, too. It does not come to just the phylum either, to chordates, it comes to other phyla, too. But it does not come to just the kingdom either, to animals. For it cannot be presumed, no questions asked, that worms do not see the sun and do not desire food, it cannot be presumed, no questions asked, that jelly fish do not see the sun and do not desire food, it cannot be presumed, no questions asked, that sponges do not see the sun and do not desire food, and the relevant questions cannot be answered, or addressed, or even asked, without involving other kingdoms, positively fungi and plants. If the theory of the irrationals instances knowledge, knowledge does not come to a genus, *homo*, and it does not come to a species, and whether *habilis*, or *erectus*, or *sapiens*. It does not come – heaven forbid! – to a variety either. And to account for all possible ways in which the kind of man can be taken, it should suffice to note that the expectation that the line separating knowledge from not knowledge

will coincide with any of the lines produced by selection when it has separated one kind of organism from another, this expectation is not just epistemologically disastrous, abolish as it will epistemology as an investigation independent of biology. This expectation is grounded in nothing.

To take development with respect to not man, finally. There is the development of the earth, there is the development of the solar system, there is the development of the galaxy, and there is the development of the world, beginning with the big bang. All in all, there are 10^{11} plus galaxies in the world, with an average, presuming the Milky Way average, of 2×10^{11} plus stars in each galaxy, for a total of 2×10^{22} plus stars. Never mind the planets. Knowledge only comes after a long development, and the probability that, when it comes, it comes to man, should not be judged infinitesimal. It should be judged zero.

Another and even more fundamental result of the *Theaetetus* is that knowledge and belief are exclusive of one another.

Russell writes: knowledge is not a precise conception, it merges into probable opinion. If it merges into probable opinion, or merges into anything at all, it is not a precise conception. If so, that conception has to be discarded and replaced with a precise conception. In fact, if that conception is not precise, and to discard it and replace it with a precise conception is not what the investigation of knowledge has to be all about, the investigation of knowledge is not the investigation of knowledge. The investigation of knowledge is the investigation of probable opinion, and if the investigation of knowledge is the investigation of probable opinion, the investigation of knowledge is the investigation of anything, *ad hoc*, and at will.

Whether the conception merges into probable opinion or not, or merges into anything else or not, what the conception refers to – or has to refer to – knowledge, cannot merge into probable opinion, or into anything else. If there is knowledge, there is knowledge, there is not knowledge, and *tertium non datur*. Probable opinion is not knowledge, improbable opinion is not knowledge, absurd opinion is not knowledge, any opinion is not knowledge, all opinions are not knowledge, any and all belief is not knowledge, anything that is not knowledge is not knowledge, and all things that are not knowledge are not knowledge. Or synonymously, they are ignorance.

For instance. Heidegger's *magnum opus* is all about time. So are, however, Einstein's *opera*. And embarrassing as what Heidegger has to say about the theory of relativity should be, it cannot be nearly as embarrassing as it should be that such as he has to say about the theory of relativity he is as important as there are philosophers in the last century. Heidegger says: we shall not go here into time measurement in the theory of relativity; yet the illumination of the ontological foundations of this measurement presupposes a clarification of inner temporality from the temporality of this-being (*Being*, 80).

If what Einstein has to say about time instances knowledge, what Heidegger has to say about time instances not knowledge. Irrespective of whether the belief it instances is probable, or improbable, or absurd, or ubiquitous, or ubiquitous because absurd.

Feynman reports, in 1963: in fact, a surprisingly large number of philosophers will say, that all is relative is a consequence of Einstein and it has profound influences on our ideas; it has been demonstrated in physics, they will say, that phenomena depend on your frame of reference; so, that a person looks different from the front than from the back, is an example of the theory of relativity from the philosopher's point of view (*Lectures*, I, 16-1).

And they will say so, not because Einstein has not informed what they will say, explicitly or implicitly, but because Heidegger has informed it, explicitly or implicitly. Knowing comes along with its world, and to say that all is relative has become ubiquitous at large, and *de rigueur* in philosophy. The theory of relativity has not become ubiquitous at large, or *de rigueur* in philosophy. Which will do as well to illustrate the fact that belief may well come to all men, but knowledge may not.

As Heidegger's *magnum opus* is all about being, too, so are, even if not nearly as ostentatiously, Einstein's *opera*. And again, if what Einstein has to say about being instances knowledge, what Heidegger has to say about being

instances not knowledge.

But the most fundamental result of the *Theaetetus* is this: in geometrical proofs not every man for himself is the judge, the geometer alone is the judge.

Moreover, the result has to be generalized, and on the authority of the *Theaetetus* as well. Because, before the three answers which Socrates discusses once he begins helping Theaetetus discover many beautiful things within himself, there is, in the dialogue, another answer to the question: what is knowledge?

What is knowledge? Socrates asks, and Theaetetus replies: the things which Theodorus teaches, geometry, astronomy, music, and arithmetic, are knowledge; and crafts such as cobbling should also be knowledge (145d-6d).

Eager to help Theaetetus discover many beautiful things within himself, Socrates complains: I ask for one thing and you give me many, I ask for something simple and you give me variety; but I do not ask of what one has knowledge or how many kinds of knowledge there are, I ask what knowledge itself is.

He does not, but he should. If he wants one thing, he will only have it out of those many, if he wants something simple, he will only have it out of that variety, and if he wants what knowledge itself is, he will only have it out of what one has knowledge of and how many kinds of knowledge there are. If, that is, he wants to know rather than perceive, or believe, including truly believe, including truly believe with the reason for it.

For this answer has the following virtue which none of the other answers has. Theaetetus has found it before Socrates begins helping him discover many beautiful things within himself, and has thus not only found it without himself, not within. He has found it, in particular, not in himself, not in myself, and not even in Socrates who helps those who associate with him discover many beautiful things within themselves. He has found it in Theodorus who proved that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational. He has found it in the geometer who alone is the measure.

With respect to geometry the geometer alone is the measure, with respect to astronomy the astronomer alone is the measure, with respect to music the musician alone is the measure, with respect to arithmetic the arithmetician alone is the measure, with respect to cobbling the cobbler alone is the measure. Hence, the generalization is unavoidable. Whether geometry, astronomy, music, arithmetic, and cobbling all instance knowledge, or only some of them instance it, or none of them instances it, whatever instances knowledge, with respect to knowledge whoever has acquired it alone is the measure.

But the geometer is a man, it will be objected, and even if he is not a man, he has invariably been a man, and, in general, whoever has acquired knowledge has invariably been a man.

Even if so, not the other way around. Even if the geometer is a man, man is not a geometer. Even if the geometer has invariably been a man, man has not invariably been a geometer. In general, even if whoever has acquired knowledge has invariably been a man, man has not invariably acquired knowledge. Although he may have invariably acquired not knowledge.

And if so, that the geometer is a man – if he is a man – is relevant not more than that he is a German or an American. That the geometer has invariably been a man is relevant not more than that he has much more often been a German than an American. In general, that whoever has acquired knowledge has invariably been a man is relevant not more than that he has much more often been a German than an American.

Planck writes: that the inhabitants of Mars, if they exist, have eyes and ears like ours, we do not know; it is quite improbable; but that they, if they have the necessary intelligence, recognize the law of gravity and the principle of conservation of energy most physicists will hold as self-evident; for anyone to whom this is not self-evident, on the other hand, it will forever remain an unresolved mystery that the same physics is made in the United States as in Germany (*Physics*, 1).

Man is not a result from the investigation of knowledge. Man is the most formidable barrier before the investigation of knowledge.

x

As a barrier is harder to dislodge, or overcome, or even circumvent, when there is another barrier before it, and the more barriers before it the more so, and, however formidable the barrier, the more

formidable the barriers before it the more formidable that formidable barrier is, so man is the most formidable barrier because, a very formidable barrier by himself, he is yet guarded by many and formidable barriers before him.

Moreover, none of the barriers is either a barrier before the investigation of knowledge or a formidable barrier before the investigation of knowledge just by chance. What nature has provided is so plentiful, if the analogy would be a mountain, it should rather be mountain ranges, and indeed the Himalayas, such as they are not just the highest on earth, but are the ranges of the Great Himalayas, before these the ranges of the Lesser Himalayas, and before these the ranges of the Outer Himalayas. Nevertheless, to what nature has provided, plentiful as it is, art has been added, and more art, and ever more. So much so, the analogy should rather be a system of fortifications. There is the citadel in the middle, with its barbicans and turrets. Surrounding the citadel is the fortress, with its bastions, with their ramparts and parapets, before them the ditches, with their escarps and counterscarps, in them the ravelins, before them the glacis. Further out, providing the first defenses, are the outworks.

And if man is the citadel, the kinds of knowledge are the fortress, with its bastions, ramparts, parapets, ditches, escarps, counterscarps, ravelins, glacis.

Socrates complains: I do not ask how many kinds of knowledge there are. He should, and not only how many, he should ask which the kinds are. For, lest there were no kinds of knowledge, he will be given kinds. He will be given them tacitly, if he is not given them explicitly, and will be given them whether he wants them or not. Whether it is knowledge or anything else, no one, not even Socrates, can have a genus without having its species, implicitly if not explicitly. Socrates will not have the genus without its species, one way or another. He will not have the genus at all, if he is given the wrong species, and whether explicitly or tacitly. And tacitly, if not explicitly, he may well be given fictional species.

Nor is this all. Lest knowledge were *sui generis*, he will be given the genus of which knowledge is in turn the species, too. Whether it is knowledge or anything else, no one, not even Socrates, can have a species without having its genus either, implicitly if not explicitly. Socrates will have the genus of knowledge one way or another, tacitly, if not explicitly, and whether he wants it or not. And in this case he need not be apprehensive that he may be given a fiction, he can take it for granted that he will be. He should be apprehensive, however, that he may well be given that fiction deliberately.

Russell, Wittgenstein, Heidegger, Carnap, and Popper are all important philosophers of the last century-odd who have investigated knowledge, and none of them asks how many the kinds of knowledge are, or which they are, any more than Socrates does. By their time, in fact, the problem of the kinds of knowledge had been abandoned. Attempts to solve the problem, or even to formulate it, had not been made for nearly a century. A century later still – and for nearly two centuries now – the problem continues to be abandoned. Fictional kinds have entrenched themselves so firmly, they do not have to pretend they are real any longer. Having assumed the appearance of permanence, they pretend they have been the kinds of knowledge forever. Nor do they do it tacitly. Or cautiously, or tentatively, or discreetly. They do it explicitly. And as aggressively as scriptural kinds used to do it.

Russell, Wittgenstein, Heidegger, Carnap, and Popper do not ask which the kinds of knowledge are. They are thus each given kinds, whether he wants them or not, and each the same kinds.

There are the empirical sciences, or the natural sciences, or simply the sciences. Of their large and ever growing number physics is but one, and of their large and ever growing number none needs to be a part of physics. Distinct from the empirical sciences, or the natural sciences, or simply the sciences – and very sensitive about being distinct from them – are the social sciences, or the human sciences, or simply the humanities. History is but one of the large and ever growing number of these, and none of these needs to be a part of history either. Mathematics, neither natural nor human, apparently, is *sui generis*. Philosophy is sometimes also *sui generis*, in deference to its glorious past, perhaps. More often, though, never mind its glorious past, it is another of the human sciences, none of which needs to be a part

of philosophy either.

Science and humanity could suggest themselves as the genera of the respective species. But they should not. All is relative, and the relations of the kinds with one another are relative, or in any case to be said to be relative. One kind is as good, or in any case to be said to be as good, as another. Questions about the relations of species and genera, about genera and their genera, and especially about the highest genus, are avoided, and to be avoided.

The relation of the kinds to knowledge is thus complicated. As knowledge is human, the kinds are all, clearly, human. Just as clearly, however, the human are more human than the natural.

All this, the kinds, their relations with one another, their relation to knowledge, has been entrenched so long and so firmly, it can hardly need an illustration. Instead, the depth of the entrenchment can be illustrated.

Popper writes, in 1934, in his *magnum opus*: the problem of finding a criterion to distinguish the empirical sciences from mathematics and logic as well as from "metaphysical" systems, I call the problem of demarcation (*Logic*, 4), and: the problem of demarcation is the central problem of the theory of knowledge (*id.*).

That the empirical sciences are a kind is beyond question. The kind, for its part, does not just pose a problem, it poses the central problem of the theory of knowledge.

That the empirical sciences, or the natural sciences, or simply the sciences, are a kind may be beyond question. Yet, that this kind may not be a real kind the fact that it is the counterpart of that other kind, the social sciences, or the human sciences, or simply the humanities, should have always sufficed to show. Even as it, clearly, has not.

That the two kinds, the natural sciences and the human sciences, are not real kinds, but are fictional kinds, the division which they suppose fundamental, nature, on one side, man, on the other, should have always sufficed to show. Lest nature were not man, man were not nature.

Then again, this is the crux of the matter.

Nature is not man, man is not nature. The supposition that the division is fundamental, and hence the two kinds, did not establish themselves and then entrench themselves inadvertently. They were not established and then entrenched tacitly either, or even surreptitiously. They were established and then entrenched in open warfare – that most ruthless of warfares, philosophical warfare – and warfare, needless to note, brought to successful conclusion. All of which the provenance of the supposition, and hence of the kinds, cannot not show. For the provenance is immaculate, and immaculate in general as well as in particular. In general, it is Hegel, in particular, it is Dilthey.

In 1807, in *The phenomenology of spirit*, Hegel reports: spirit has lost its essential life; but it is conscious of the loss; turning away from the chaff, confessing that it lies in wickedness, and reviling itself for doing it, spirit demands from philosophy the restoration of the substantiality and purity of its being (pr.).

And philosophy complied. The war for the restoration of spirit to the substantiality and purity of its being had begun three generations before Hegel, Hegel fought, and won, the decisive battles, and two-odd generations after Hegel the war was successfully concluded by Dilthey.

In 1883, in the *Introduction to the spiritual sciences*, Dilthey concludes: the spiritual sciences are an independent of the natural sciences whole (I, 2), and: they can break the fetters in which the older sister has held the younger since Descartes, Spinoza, and Hobbes, who imposed on these sciences their developed in mathematics and natural science methods (19).

In fairness, then – in respect for the victors – this is what the two kinds are. The social sciences, or the human sciences, or simply the humanities, are the spiritual sciences. The natural sciences, or the empirical sciences, or simply the sciences, are the not spiritual sciences.

Before the restoration of spirit to the substantiality and purity of its being physics was a part of philosophy. Mathematics was inseparable from physics. History learned from physics and mathematics.

Physics is not a part of philosophy any longer. Mathematics is separate from physics. History does not learn from physics and mathematics.

The new kinds are not the old kinds. Catastrophically so. And it may be taken for granted that, given the new kinds, these are the real kinds.

Which the real kinds are need not be asked, whether the new kinds are the real kinds need not be asked, whether the old kinds may not be the real kinds need not be asked, and what the cause of the catastrophic transformation of the old kinds into the new is need not be asked either.

If the new kinds were brought into being by the restoration of spirit to the substantiality and purity of its being, however, the catastrophic transformation has a cause. The restoration is its cause.

And it may be taken for granted that, once spirit is restored to the substantiality and purity of its being, being of spirit is not a *contradictio in adiecto* any longer. For being of spirit is spiritual. The restoration of spirit is spiritual, its cause is spiritual, it is itself a spiritual cause, and its effects are spiritual. Indeed, if not everything is spiritual, everything that needs to be is.

Being of spirit is nevertheless a *contradictio in adiecto*. The restoration of spirit to the substantiality and purity of its being is real, not spiritual, if only because it has proved itself victorious in open, and ruthless, if philosophical, warfare. Its cause is real, not spiritual, and it is itself a real cause, not spiritual, even if some of its effects are spiritual, or what is the same, fictional.

Brought into being by the restoration of spirit to the substantiality and purity of its being, a physics that is not a part of philosophy cannot not be fictional, a mathematics that is separate from physics cannot not be fictional, a history that does not learn from physics and mathematics cannot not be fictional, and a philosophy of which physics is not a part, and mathematics is not a part, and history is not a part, can itself be nothing but fiction.

If man is the citadel, the kinds of knowledge are the fortress guarding the citadel.

Involving as it is physics, mathematics, history, and philosophy, old kinds and new kinds, real and fictional, clearly, if there is to be hope of breaching it, the fortress needs to be surveyed in more detail.

xi

To approach the problem of the kinds of knowledge, old and new, real and fictional, there can be no better way than to review the attempts to solve the problem of the kinds of knowledge.

The important attempts are five.

One generation-odd after Plato, Aristotle made the first attempt to solve the problem of the kinds of philosophy, and hence implicitly of the kinds of knowledge. It remains the most successful attempt, by far.

Not quite two generations after Aristotle, Zeno of Citium divided philosophy, and hence implicitly knowledge, into kinds differently. Zeno's division became normative, and remained normative through the early Christian centuries.

Two-odd centuries after Zeno, Varro divided the arts, and hence implicitly knowledge, into nine kinds. The division of the arts into seven of these nine kinds became normative after the early Christian centuries, and remained normative in the following millennium-odd.

Seventeen centuries after Varro, Bacon made the first attempt to solve the problem of the kinds of knowledge explicitly. One and a half centuries later, Diderot and d'Alembert presented Bacon's solution of the problem somewhat more explicitly than Bacon. And Bacon's remains the most successful attempt to solve the problem of the kinds of knowledge explicitly.

Finally, two and a half centuries after Bacon, not quite one century after Diderot and d'Alembert, Comte proposed a classification of the sciences.

These are all important attempts to solve the problem of the kinds of knowledge, explicitly or implicitly. Only two of the five are very important, however, Aristotle's, and Bacon's.

xii

Aristotle solved the problem of the kinds of philosophy in books II and VI of the *Metaphysics*.

The attempt to examine the kinds of philosophy is thus led astray before it begins, one way or another. The attempt can take the *Metaphysics*, and metaphysics, for granted, and then metaphysics, if not the *Metaphysics*, will lead it altogether astray. Or the attempt can accept that it has to address the matters of what the *Metaphysics* is, and what metaphysics is, before turning, with metaphysics out of the way, to the kinds of philosophy. For metaphysics is itself a kind, and

not a kind that will be just another kind among many. Allowed in any gathering of kinds, metaphysics will take over the gathering.

There is a virtue in this adversity, though. Where metaphysics is at issue, the question, what is real? what is not? had better be raised. Indeed, implicitly if not explicitly, this question is raised whatever is at issue. But explicitly is always better than implicitly. And this of all questions had better be raised explicitly, and in the investigation of knowledge it had better be raised early, and the earlier the better.

Aristotle, then, did not collect the treatises comprising the *Metaphysics* into a whole, and did not name that whole *Metaphysics*. Three centuries after Aristotle, Andronicus edited Aristotle's treatises – Plutarch (*Sulla*, 26) and Porphyry (*Plotinus*, 24) report – and Andronicus did both, collect the treatises comprising the *Metaphysics* into a whole, and name that whole *Metaphysics*.

The reason is straightforward. Aristotle predates the establishment of libraries. The first and greatest of them all, the Alexandrian, was established two generations after him. He thus predates also the philological and antiquarian scholarship that came into being with the establishment of libraries, whose *raison d'être* was, and remains, to edit books, comment on them, edit the editions and the commentaries, comment on the editions and the commentaries, edit the editions and the commentaries of the editions and the commentaries, comment on the editions and the commentaries of the editions and the commentaries, and so on indefinitely. Andronicus postdates that scholarship. By his time books were edited as a matter of course.

Andronicus postdates also Zeno of Citium. Zeno had divided philosophy into logic, physics, and ethics. Two and a half centuries after him, by the time of Andronicus, the division was normative, and Andronicus arranged Aristotle's treatises according to the norm. In time the arrangement of Andronicus itself became normative. It is the foundation of subsequent editions of Aristotle, including that of Bekker, in 1831-70, and in Bekker's edition Zeno's trichotomy is readily apparent. The 1 462 pages of Bekker's edition begin with 184 pages of logic, continue with 796 pages of physics, and end with 369 pages of what would be ethics.

This accounts for 1 349 pages. The remaining 113 pages comprise the *Metaphysics*, inserted after the physics and before the ethics. Presumably, having gathered the logical treatises, τὰ λογικά, the physical treatises, τὰ φυσικά, and what would be the ethical treatises, τὰ ἠθικά, Andronicus had some leftover treatises on his hands, appended these to the physical treatises, and named them just that, the treatises after the physical treatises, τὰ μετὰ τὰ φυσικά. The presumption rests, other than on those above, on the following facts. The coinage τὰ μετὰ τὰ φυσικά is not attested earlier. The fourteen books of the *Metaphysics* are a motley collection. And the treatises comprising the motley collection nevertheless have in common great generality which makes them harder than other Aristotelean treatises to fit into one part of Zeno's trichotomy rather than another.

The *Metaphysics* comprises fourteen books, which in turn comprise not less than nine distinct treatises. Books VI-X are one treatise, books III and IV are probably one treatise, and the remaining seven books are certainly a distinct treatise each. Also, book XI is certainly spurious, and book V is probably spurious.

In book I the problem of the principles of all things is solved. Book II distinguishes theory and practice. The central problem of books III-IV is the principle of contradiction. Book V is an epitome of thirty problems. The central problem of books VI-X is truth. Book XI is a collection of passages from other books. Book XII is a general survey of physics, ending with the first mover. Book XIII is an expanded version of book XIV, both arguing, against Plato and the Pythagoreans, that there is no immaterial substance.

The great generality of the treatises makes them harder to fit into Zeno's trichotomy. They all fit into it nevertheless. They are all logic, or physics. Nor need there be any ambiguity as to what is logic, and what is physics. What is logic, and what is physics, can hardly be clearer, in fact. The 184 pages of logic in the Aristotelean corpus are logic paradigmatically, and remained so more or less exclusively until Frege. The 796 pages of physics in the corpus are physics paradigmatically, and remained so predominantly, if not in

this case quite so exclusively, until Galileo and Kepler.

The treatise whose central problem is the principle of contradiction, books III-IV, is logical. The treatise whose central problem is truth, books VI-X, is logical as well. This is also the treatise which begins with the problem of the kinds of philosophy, and the solution of this problem takes the distinction of theory and practice for granted. The treatise whose concern this distinction is, book II, is on this ground, as well as on the ground that: the end of theory is truth (II, 1), also a logical treatise.

Book XII is a survey of problems which are all, including the first mover, treated in more detail in the *Physics*. The problem of the principles of all things, the concern of book I, is the central problem of books I and II of the *Physics*. Substance is a logical problem, it is discussed in the treatise on truth, in book VII of the *Metaphysics*, but sensible substance, discussed in the same treatise, in book VIII of the *Metaphysics*, is a physical problem, and so is the argument, the concern of books XIII-XIV, that, pace Plato, there is no immaterial substance.

Even the spurious treatises are logic, or physics. The collection of passages, book XI, is a collection of passages from other books of the *Metaphysics* and from the *Physics*. The epitome, book V, is an epitome of physical problems: cause (2), element (3), nature (4), and of logical problems: principle (1), substance (8), same and different (9), contradictory and contrary (10), prior and posterior (11).

It can be concluded that, if metaphysics is the *Metaphysics*, metaphysics is part logic, part physics. Ignoring the spurious treatises, it is two thirds logic, one third physics. And if so, it has to be further concluded that metaphysics is not a bad mix, not bad at all.

Would that metaphysics were nothing else.

The *Metaphysics* came into being with the labors of Andronicus, but metaphysics did not come into being with his labors. Had it been just an editorial matter, τὰ μετὰ τὰ φυσικά could have remained just the treatises after the physical treatises, presumably. But it was not just an editorial matter. The matter could not help being a philosophical matter, and τὰ μετὰ τὰ φυσικά could not help suggesting the things beyond the physical things instead. For it goes without saying that, if there are such things, the philosophers would be eager to concern themselves with those things. It goes without saying as well that, if there are no such things, the philosophers may well be even more eager to concern themselves with those things. It goes without saying, too, that, whether there are such things or not, with the advent of Christianity among them the philosophers would be eager to concern themselves with nothing else.

Whether there is metaphysics in the *Metaphysics*, and, if there is metaphysics in the *Metaphysics*, it is part logic, part physics, there has been metaphysics after the *Metaphysics*, and it has not been part logic, part physics. It has been the concern with the things beyond the physical things.

After the *Metaphysics*, as well as before it. If it is the concern with the things beyond the physical things, metaphysics may have had to wait for the *Metaphysics* to receive its name, but it need not have waited for the *Metaphysics* to come into being in order to come into being itself. And to argue that the philosophers were eager to concern themselves with the things beyond the physical things before the *Metaphysics*, and before Aristotle already, an appeal to Plato will appear to do. The Platonic dialogues abound with things beyond the physical things.

Or with attempts to find out whether there are such things.

In general, then, one or the other. There are matters of great generality, such as those investigated in the treatises collected in the *Metaphysics*, these matters are logical and physical, and either some of them logical, some of them physical, or all of them part logical, part physical. But there are also, or there are not, the things beyond the physical things. Accordingly, metaphysics can be the investigation of matters of great generality, a part of logic, or a part of physics, or a part of both. Or, whether there are things beyond the physical things or not, metaphysics can be the concern with the things beyond the physical things.

Accordingly, Plato concerned himself with the things beyond the physical things. Or, investigat-

ing the matters of great generality investigated in the treatises collected in the *Metaphysics*, or some of these matters, Plato addressed the questions also of whether these matters may not involve things beyond the physical things, and whether there are such things to begin with.

Aristotle then settled the matter. Hence, metaphysics could be part logic, part physics.

Things then went the other way, uncompromisingly so with the advent of Christianity among the philosophers. Metaphysics could only be the concern with the things beyond the physical things.

At length things went back the other way again. Bacon, Galileo, Kepler, Descartes had little use for things beyond the physical things, their successors had no use for such things at all, metaphysics began being embarrassed to be the concern with such things, and eventually even found the courage to become, once again, part logic, part physics. Things went this way for one and a half centuries, through and including Hume and Kant. Hume, in 1748, writes: all profound reasonings are what is commonly called metaphysics (*Inquiry*, 1), and: we shall consider what can reasonably be pleaded in their behalf (*id.*). Kant, in 1764, writes: the true method of metaphysics is in its foundation the same as that introduced by Newton in natural science with such benefit to it; it is to seek, by experiments, and with the help of geometry, the rules by which the phenomena of nature occur (*Clarity*, 2). Having then labored for seventeen years to make this proposition stand, upon success, in 1781, Kant reports: the task of this critique is to change the ways of metaphysics by undertaking, following the example of the geometers and the investigators of nature, its complete revolution (*Critique*, pr.). Like the revolution accomplished by: Copernicus (*id.*), not like the revolution accomplished in the early Christian centuries.

Things then went back the other way once again, and how far they have gone once again the more recent developments illustrate.

What is metaphysics? Heidegger asks, in 1929. He quotes Hegel: pure being and pure nothing is the same (3), and comments: this proposition of Hegel is correct, because being reveals itself only in the transcendence of being into the nothing (*id.*). He asks: what about the nothing (1)? replies: inaccessible to logic (2); but shown by our anxiety, the nothing nothings; it is in the being of this-being that the nothing nothings (3), and concludes: if the question of being is the fundamental question of metaphysics, the question of the nothing embraces the whole of metaphysics (*id.*).

Three years later, in 1932, Carnap undertook the *Elimination of metaphysics through logical analysis of language*. He exemplifies metaphysics with Heidegger's: the nothing nothings (5), argues that: all metaphysics is meaningless (6), and concludes with the pertinent, if *ad hominem*, question: when a metaphysician expresses his attitude toward life in words, is it not perhaps because he lacks the ability of a Mozart or Beethoven to express it in its proper medium (7)?

The honor for initiating this bout of philosophical warfare should go to Heidegger. Carnap's in 1932 is indeed a call for the elimination of metaphysics, in general, and of Heidegger, in particular. Five years earlier, however, in 1927, Heidegger's has been a call for destruction more total than that inflicted by the recent and the upcoming world wars, both. He writes: we must destroy the tradition concealing the primordial experiences in which the first determinations of being were won (*Being*, 6), and if we must destroy that tradition, in general, we certainly must destroy Carnap, in particular.

Heidegger also won the bout.

It is not only that the proposition: metaphysics is meaningless, can hardly help being an answer to the question: what is metaphysics? and thus being metaphysics. Which Carnap concedes when in a postscript, in 1957, he notes: metaphysics is used in this article for the alleged knowledge of the essence of things which transcends the empirically grounded inductive science, it does not include attempts at synthesis and generalization of the results of the sciences (*id.*). It is that it is impossible to resist the anxiety that the proposition: metaphysics is meaningless, cannot help nothinging, and thus being metaphysics. Nor does the later distinction alleviate the anxiety. The distinction: knowledge of the essence of things which transcends the empirically grounded inductive science, on one side, attempts at synthe-

sis and generalization of the results of the sciences, on the other, will lend no support to Carnap's argument unless one of the two sides is: alleged, and it is then impossible to resist the anxiety that the proposition: metaphysics is alleged knowledge, cannot help nothinging.

As to what Heidegger won, it can be affirmed without any anxiety that he won nothing.

Metaphysics is part logic, part physics. Or it is nothing.

And what is real? what is not? is a difficult question. Not nearly as difficult, though, as millennia's worth of obfuscation would make it.

The physical things are real, the not physical things, including the things beyond the physical things, are not real.

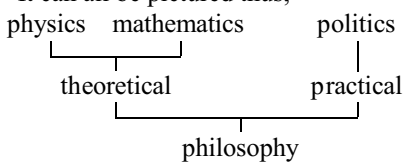
It is but a partial answer to the question. Nevertheless, however small the two parts may be, the physical things among all that is real, and the not physical things, including the things beyond the physical things, among all that is not real – and they cannot be too small – and however much more explicit it has to be made what the physical things are, and hence what the not physical things are, such as it is the answer suffices to preclude an indefinite amount of things, including most particularly among the kinds of knowledge, from pretending that the real may be not real, the not real may be real, and what is real, what is not real, may be not only not clear, but impossible to make clear.

xiii

Aristotle writes in book II of the *Metaphysics*: philosophy is rightly called knowledge of truth; for the end of the theoretical is truth, while the end of the practical is action (1). And in book VI: physics is theoretical; physics is the investigation of the movable and not separable from matter; mathematics is also theoretical; mathematics is the investigation of the not movable but also not separable from matter; and there has to be an investigation which is to find out whether there is anything not movable and separable from matter; the investigation of a higher genus, that will be first philosophy; theoretical philosophy will thus be of three kinds, physics, mathematics, and theology, for if anything is divine, that has to be; if there is no substance other than those in nature, then, physics will be the first knowledge; if there is such a substance, first philosophy will be prior to physics (1).

From the *Nicomachean ethics* it can also be gathered: every action aims at some good (I, 1); if there is a good which is desired for its own sake, it must be the chief good; knowledge of this good has to be the master art; and politics appears to be this art (2).

It can all be pictured thus,



presuming that the more particular kinds should be represented at the top, the most general kind at the bottom. Presuming also that the hypothetical investigation which is to find out whether there is a substance other than those in nature, first philosophy, or theology, does not, pending being found out, belong among the kinds.

The great virtues of Aristotle's solution of the problem of the kinds of philosophy, and hence implicitly of knowledge, begin with the fact that it is grounded in truth. The end of theoretical philosophy is truth. What is truth is clear, too. Aristotle defined truth and falsehood, and for good: what is and what is not are truth and falsehood (*Metaphysics*, IX, 10). Hence: to affirm of what is that it is not, and of what is not that it is, is false, to affirm of what is that it is, and of what is not that it is not, is true (IV, 7), for: not because we truly say that you are white you are white, but because you are white we truly say that you are white (IX, 10). And it is for good. The last important attempt to improve on Aristotle's definition, for instance, is Tarski's, in 1944, and whether he has succeeded in improving on Aristotle's definition or not, Tarski begins by proposing: we should like our definition to do justice to the intuitions adhering to the classical Aristotelean conception of truth (*Truth*, 3), and ends by proposing: I do not have any doubts that our formulation does conform to the intuitive content of that of Aristotle (17).

Grounded in truth, Aristotle's solution of the problem of the kinds is also complete. The kinds of philosophy are two, and can only be two. The

end is truth, or the end is not truth. *Tertium non datur*. If philosophy is rightly called knowledge of truth, and even if not all philosophy, but only a part of it, is rightly so called, philosophy has to be divided by dichotomy. Lest the neither true nor false came into being. Theoretical has to be not practical, and practical not theoretical. When Aristotle writes, for instance: all reasoning is practical or productive or theoretical (*Metaphysics*, VI, 1), if all reasoning is to be part of philosophy, all reasoning has to be part of either theoretical philosophy or practical philosophy. Say, theoretical reasoning be part of theoretical philosophy, practical and productive reasoning both be part of practical philosophy. Also, the end of practical philosophy is action, and action is not the only end among all the ends that are not truth, far from it. Yet this only shows that, if philosophy is to search for truth, it had better search for all of it, but if it is to search also for not truth, it had better not search for all of it. Just for the part of it that is action.

Where the kinds of the kinds are concerned, the solution of the problem of the kinds of the theoretical kind is complete explicitly. Things are movable or not movable, and separable from matter or not separable, and taking it for granted that movable and separable from matter is an impossibility – for: there can be no motion apart from what moves (*Physics*, III, 1) – physics, mathematics, and first philosophy are all possible kinds, real or hypothetical.

Where the relations of the kinds of the kinds are concerned, if there is no substance other than those in nature, physics is first. It is prior, in particular, to mathematics. Physics is the investigation of the movable and not separable from matter, mathematics of the not movable and not separable. Or in terms of matter and form: the physicist is concerned with form as it is not separable from matter (II, 2), while: the mathematician separates form from matter for the sake of analysis (*id.*). Thus, for instance: while geometry studies natural lines but not as natural, optics studies mathematical lines but as natural, not as mathematical (*id.*).

The absence of logic from Aristotle's solution has to be presumed only apparent. Concluding his logical researches, such as they were to coincide with the foundations of logic for the following twenty-two and a half centuries, Aristotle reports: in these matters it was not that something was done before us and something was not; nothing at all was done; and in order to develop them, we had to labor hard for a long time (*Refutations*, 34). Aristotle's hard labor for a long time belongs in the solution. And if so, the most purely formal of investigations, logic has to be a kind of mathematics. For which there is also the unimpeachable authority of Leibniz and Frege. Leibniz writes: Aristotle first wrote mathematically outside mathematics (to Wagner, 1696). Frege writes: the sharp separation of logic and mathematics I consider impossible (*Foundations*, intr.).

As to the investigation which is to find out whether there is a substance other than those in nature, it can be presumed that it is metaphysics, or part of metaphysics. It is one of Kant's major results in the complete revolution of metaphysics, that such a substance is an impossibility. If so, though, the hypothetical first philosophy, or theology, cannot be allowed to pretend to be a kind next to physics and mathematics. Even if it can be allowed, in consolation, given that logic is mathematics, to be part physics, part mathematics – and, more general, this is an even better mix.

Practical philosophy is all of one kind, the master art, politics. Or what is the same, the master art coincides with practical philosophy. The master art, however, has its own kinds. Aristotle writes: politics encompasses even the most highly esteemed actions, warfare, economics, rhetoric; it legislates; and even when the end is the same for man and for the city, that of the city is greater and more complete (*Nic. ethics*, I, 2). The virtue here is the subordination of ethics to politics. Ethics is a kind of politics. It is not allowed to pretend that it is a kind next to politics, let alone a kind next to physics and logic. That ethics is subordinated to politics has to be a matter of course, if, with Aristotle: a man who is not a part of a state is a beast or a god (*Politics*, I, 2). Moreover, not only is man a political animal: man is a political animal by nature (*id.*).

Practical philosophy, Aristotle writes, is: the philosophy of the human things (*Nic. ethics*, X, 9). But it cannot be of just the human things any more than it can be of just the German things or

just the American things. It can only be of the human and Martian things indiscriminately, and whether the inhabitants of Mars have eyes and ears like ours or not. Just as the inhabitants of Mars, if they have the necessary intelligence to recognize the law of gravity and the principle of conservation of energy, will have practical philosophy, which itself can only be of the Martian and human things indiscriminately.

The end is truth, or the end is not truth. The end of practical philosophy is not truth. Or what is the same, it is falsehood. But, again, it does not have to be all falsehood. It can be only such falsehood as tries to be truth. Given that truth is the ground of the division of philosophy into theoretical and practical, practical philosophy can be the attempts to apply the results of the search for truth. This can be presumed implied by its affirmative description. Practical philosophy is, negatively, and implicitly, the search for not truth. Affirmatively, and explicitly, it is the search for action.

Thus, lest there were a substance other than those in nature, nature is what physics investigates. Nature is also what mathematics investigates formally. And nature has to be also the concern of politics. Or what has to be the same, politics has to be an application of physics. This has to be a consequence of the fact that: man is a political animal by nature. Ethics, also, if it is to concern itself with man, and be a part of philosophy, rather than self-indulgence, can only be an application of physics, too. This has to be a consequence of the fact that ethics is a kind of the master kind.

Yet another great virtue of Aristotle's solution of the problem of the kinds is that it is descriptive. It is the outcome of induction, not deduction. Hence, the kinds do not have to depend on their general definitions. The specimens over which the definitions generalize, and which have to be the test of the definitions, are available for inspection in the Aristotelean corpus, as well as without.

In particular, there are the 796 pages of physics in the corpus to show what physics is. To show, for instance, that physics includes astronomy, in *On the heaven*, chemistry, in *On generation and corruption*, biology, and in *On the soul* not less than in the *History of animals*, *Parts of animals*, *Movement of animals*, *Progression of animals*, and *Generation of animals*. The 184 pages of logic can show what mathematics is. Without the corpus, Eudoxus is an older contemporary of Aristotle, Eudoxus developed the theory of proportion, the theory is presented in book V of the *Elements*, and there is no theory that can show what mathematics is better than the theory of proportion.

Also, it is the two kinds of the kind theoretical philosophy, physics and mathematics, that in *A treatise of human nature, being an attempt to introduce the experimental method of reasoning into moral subjects* Hume takes for granted in order to introduce the experimental method of reasoning into moral subjects, and in the *Critique of pure reason* Kant takes for granted in order to achieve the complete revolution of metaphysics.

On the side of practical philosophy, among the 369 pages of practical philosophy in the Aristotelean corpus there are the *Politics*, the *Rhetoric*, the *Ethics*. Without the corpus, there is the fact that, pace all philosophers who have argued, and continue to argue, that what ought to be is what is, what is is what is. And counting back no further than the father of history, the historians – including the father, Herodotus, as well as Thucydides – had been recording what is, attesting that, in fact, what is is what is, for one century by Aristotle's time.

Next to all its great virtues, Aristotle's solution of the problem of the kinds has one great vice. It is vague on the relation of philosophy and knowledge.

Philosophy is rightly called knowledge of truth.

So it may seem that all philosophy, theoretical as well as practical, is a kind of knowledge. That knowledge is the genus of philosophy. But if so, whether philosophy is the only species of knowledge, or there are other species, and if so, which they are, and in any case on what grounds will the relations of species and genus be ascertained, are the lesser difficulties. If philosophy, practical as well as theoretical, is a kind of knowledge, and knowledge is knowledge of truth, and knowledge had better be knowledge of truth, practical philosophy cannot be the search for not truth. Philoso-

phy, in fact, cannot have anything to do with not knowledge, or with not truth, and this may well be commendable, but it may not be true. Moreover, given the fact of development of philosophy, and development of all its parts, development of physics, and development of mathematics, and development of politics, if philosophy is a kind of knowledge, all this development cannot not make, catastrophically, knowledge develop, and if knowledge is knowledge of truth, and it had better be, cannot not make also, also catastrophically, truth develop.

It is thus better to presume that, the other way around, philosophy is the genus. And that the species is not knowledge, and not knowledge of truth, but the search for truth, and the acquisition of knowledge. That, rather, the acquisition of knowledge is rightly called philosophy. The acquisition of knowledge will then be a species of philosophy, and it will be, in particular, clearly, theoretical philosophy. Clearly, also, philosophy will include more than the acquisition of knowledge. It will include practical philosophy. The acquisition of knowledge, in turn, will be the genus of the species physics and mathematics.

xiv

Plato gathered his followers in the Academy *c.* 385, endowed the gathering with buildings and grounds – and a meal, even if: the common meal of the philosophers was not burdened with surfeit, a visitor reported (Plutarch, *Conversations*, VI, pr.) – and thus the Academy came into being. It turned out to be a most successful precedent. Schools of philosophers proliferated, with or without buildings and grounds, or a meal, among them the Lyceum, founded by Aristotle in 335, the Garden, founded by Epicurus in 307, and the Stoa, founded by Zeno of Citium in 301. Of all the schools these four, the Academy, the Lyceum, the Garden, and the Stoa, were the most successful – measuring success by the amount of teaching and learning the schools produced – flourishing well into the Christian centuries. Of the four the most successful was the Stoa.

And, Laertius reports: Zeno of Citium first divided philosophy into physics, ethics, and logic (VII, 39).

It is not a better solution of the problem of the kinds of philosophy.

It ignores mathematics. It can be presumed that mathematics is a kind of logic. But it will be much more eagerly presumed that mathematics is not philosophy, encouraging the philosophers to ignore mathematics. If they needed encouragement, for the solution may well be descriptive, the description gathered from the all too many philosophers who need no encouragement to ignore mathematics. One of whom, again, Aristotle is not. When he writes in the *Topics*: problems can be divided into ethical, like ought one to obey one's parents or the law if these disagree? logical, like is the knowledge of opposites the same or not? and physical, like is the world eternal or not (I, 14)? it is an incomplete illustration. Lest the kind of theoretical philosophy mathematics were wanting in problems.

Politics will be presumed a kind of ethics, encouraging the delusion that what ought to be is what is.

The fundamental division of philosophy into theoretical and practical is ignored, encouraging the confusion, and obfuscation, of theory and practice.

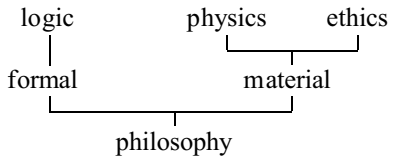
Most of all, the solution is not grounded in truth. It was, in fact, not grounded at all. Whatever defense the solution needed, similes were to provide it. Laertius reports: they say that philosophy is like an animal and logic is the bones and sinew, ethics is the flesh, physics is the soul; or philosophy is like an egg and logic is the shell, ethics is the white, physics is the yolk; or philosophy is like an orchard and logic is the fence, ethics is the fruit, physics is the earth and the tree (VII, 40).

Needless to say, if these are the possibilities, philosophy would not be an animal, or an egg. Philosophy would be an orchard. And never mind the fence, and the earth and the tree. A student of Zeno already, Ariston of Chios, said: physics is above us, logic is nothing to us, ethics alone is about us (160). Even if, as they began committing themselves ever more unwaveringly to the fruit, the philosophers would rather call it, more respectably, apparently, the end, τὸ τέλος. Cicero rendered it: *summum bonum* (*Ends*, I, 11).

Twenty and a half centuries after Zeno, Kant begins in the *Groundwork for the metaphysics of*

morals: the old Greek philosophy divided itself into three sciences, physics, ethics, and logic; this division is entirely appropriate to the nature of the matter and cannot be improved, except possibly by providing its principle so as to ensure its completeness (pr.). And Kant provides its principle: all knowledge grounded in reason is either material or formal; formal philosophy is logic; but material philosophy is twofold, because there are laws of nature and laws of freedom; and the science of the former is physics, the science of the latter ethics (*id.*).

To picture it,



It is not a good solution of the problem of the kinds of philosophy, if only because the theoretical and the practical – a division of philosophy as fundamental on Kant's authority as on Aristotle's – have to separate logic and physics, on one side, from ethics, on the other.

Hegel's *Encyclopedia of the philosophical sciences* is also tripartite: science of logic, natural philosophy, and philosophy of spirit (*id.*). The principle is, in this case, spirit.

It is because it has been ubiquitous that Zeno's solution of the problem of the kinds of philosophy, and hence implicitly of knowledge, is important.

It is for the same reason that Varro's solution of the problem of the kinds of the arts, and hence implicitly of knowledge, is important.

Two-odd centuries after Zeno, Varro is one of the earlier Roman writers. It was after his time that it became *de rigueur* for the scions of the Roman aristocracy to frequent the philosophical schools.

Varro wrote *The disciplines*, which does not survive. It can be recovered from his successors, however, from Vitruvius (VII, pr.), Gellius (X, 1; XVIII, 15), and later writers, that *The disciplines* was divided into nine books, and these concerned themselves with grammar, dialectic, rhetoric, geometry, arithmetic, astronomy, music, medicine, and architecture.

On what ground the list is composed is obscure. It is not a division of philosophy, as the absence of ethics shows. On the other hand, dialectic is logic, or a part of logic, and although next to geometry, arithmetic, and music, astronomy will be a part of mathematics, it can be presumed a part of physics as well. One way or another, whatever Varro took discipline to be, that he did not propose the list as complete the fact that he wrote history, the *Antiquities*, can show. Or the fact that he wrote agriculture, *On rustic matters*.

With later writers the nine disciplines turned out to be two too many, and rid of medicine and architecture, the remaining seven became arts.

Of these later writers the most important is Capella. Five centuries after Varro, Capella wrote the *Philology*. It came to go under the name *The wedding of Philology and Mercury*. With good reason. It is the story of the wedding of Philology and Mercury, of how the bride brought her seven maids to the wedding, and of how the maids presented themselves to the bridegroom and the assembled guests. This is how Geometry begins her presentation: I see that my Archimedes and the most learned Euclid are among the assembled philosophers, and I could call on them to present my doctrines at length; but as the occasion requires rhetoric, I will disclose them to you myself (6).

Granted that Euclid and Archimedes would not rise to an occasion requiring rhetoric, Philology's maid, excellent as her rhetoric may be, discloses of what there is in Archimedes nothing, and of what there is in Euclid but words.

The same instance will do to show that rhetoric was now the master art, or Rhetoric the master maid.

When they became arts, the seven became also, in particular, the liberal and human arts. This name: *ingenuae et humanae artes*, was also found in Cicero, quite appropriately in *On the orator* (III, 21).

Half a millennium after Capella, when learning began to revive in Christendom, and in the following centuries universities were established and began flourishing, the first three maids became the *trivium*, the other four the *quadrivium*. A millennium after Capella the sevenfold division of the arts remained normative.

The prominence of the division need not be blamed on Varro. It cannot be blamed on Capella either, he is not important enough to blame for anything. It can, however, be blamed on Augustine. Augustine is an older contemporary of Capella, and Augustine did not usher in the darkest of dark ages single-handedly, but he did do more than any other philosopher to usher it in.

xv

Of the planned six parts of the *Great instauration of the sciences* Bacon completed the first and the second. He completed the second in 1620, when he published the *New organon, or directions for the interpretation of nature*. He completed the first in 1623, when he expanded *The advancement of learning*, first published in 1605, into *On the dignity and growth of the sciences*.

The solution of the problem of the kinds of knowledge – and still the most successful attempt to solve the problem of the kinds of knowledge explicitly – Bacon formulated first in book II of *The advancement*. He is explicit: I will now attempt to make a general and faithful perambulation of learning (*id.*). He restated the solution, illustrating it also more amply, in books II-IX of *The dignity and growth*. He is again explicit: I propose to make the circuit of knowledge (II, 1).

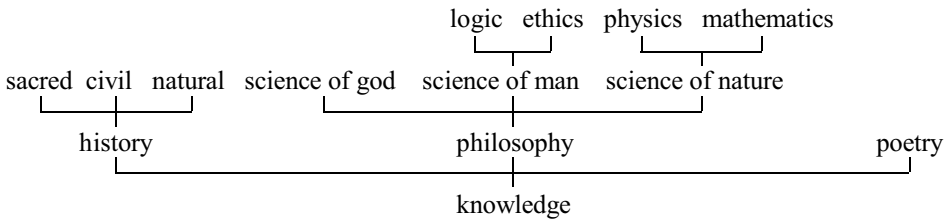
Not quite one and a half centuries later, in 1751, the first volume of the *Encyclopedia* appeared. D'Alembert wrote the *Preliminary discourse*, in which one of his tasks is to present: the tree of knowledge (*id.*), or: the world map of knowledge (*id.*). Diderot added a *Detailed explanation of the system of human knowledge*. And the tree of knowledge, or the world map of knowledge, or the system of human knowledge, d'Alembert reports: we owe to a celebrated author (*Discourse*), namely: the immortal chancellor of England Francis Bacon, whom one is tempted to regard the greatest, most general, and most eloquent of philosophers (*id.*).

Except in being somewhat more explicit, the system of human knowledge, or the tree of knowledge, or the world map of knowledge, presented by Diderot and d'Alembert does not depart from the perambulation of learning, or the circuit of knowledge, made by the greatest, most general, and most eloquent of philosophers.

It is pictured below, ignoring, for the sake of perspicuity, a large number of further subdivisions. Among the kinds of the kinds of knowledge the picture ignores those of poetry, among the kinds of the kinds of the kinds of knowledge it ignores those of sacred history, of civil history, of natural history, and of the science of god. It ignores all kinds of the kinds of the kinds of the kinds of knowledge, and all further kinds of kinds. That no virtue of the solution of the problem of the kinds of knowledge is thus ignored the following example can show. One of the kinds of natural history is history of the uniformity of nature one of whose kinds is history of the heavenly bodies, yet one of the kinds of physics is particular physics one of whose kinds is physical astronomy, yet one of the kinds of mathematics is mixed mathematics one of whose kinds is geometrical astronomy, whose kinds in turn include cosmography, whose kinds in turn include uranography.

It is a virtue of the solution that mathematics is one of the kinds of the science of nature, and that the science of nature is one of the kinds of philosophy, or synonymously science – for, with Diderot: philosophy and science are synonyms (*Explanation*). It cannot be a great virtue, though, considering how well confined to being kinds of a kind of a kind of knowledge physics and mathematics are. Neither of them, clearly, can be as important a part of knowledge as the kind of a kind, the science of god. Never mind being as important as the kind, poetry.

The great virtue of the solution is that it is a solution of the problem of the kinds, explicitly, of



knowledge.

Hence its generality. Not clear as it is what knowledge is, not clear as it is even what is knowledge, and what is not, it has to be most unlikely that there ever be a kind of knowledge which – between history and the science of god, ethics and physics, mathematics and poetry – the solution would not be able to accommodate.

Hence also its completeness. Among the subdivisions which the picture ignores are the following. The two kinds of civil history are civil history proper and literary history which in turn share the kind memoirs. Among the kinds of natural history is history of the uses of nature among whose kinds are trades and manufactures. The science of god is of two kinds, ontology, or the science of being, or general metaphysics, and pneumatology, or the science of spirit, or particular metaphysics. Ontology, or the science of being, or general metaphysics, has among its kinds revealed theology. Pneumatology, or the science of spirit, or particular metaphysics, has among its kinds angelology and demonology. Poetry, including architecture, music, painting, and sculpture, is of three kinds, narrative, dramatic, and parabolic. Clearly, whatever knowledge is, the solution has to be most unlikely to contain errors of omission. Likely as it has to be to contain errors of commission.

The solution is also descriptive, the outcome of induction, not deduction. This is in Bacon, as it is in Aristotle, a virtue, but not nearly as great a virtue in Bacon as it is in Aristotle. And it is not a virtue at all in Diderot and d'Alembert.

Bacon writes: I propose to survey the circuit of knowledge, noticing which parts lie waste, uncultivated, and abandoned by the industry of man, with a view to engage, by a faithful mapping of the deserted tracts, the energies of public and private persons in their improvement; my attention is confined to the discovery of errors, not their correction, for it is one thing to point what land lies uncultivated, another to improve husbandry (*Growth*, II, 1).

Bacon proposes to discover errors. Not to correct them. To notice which lands lie waste, uncultivated, and abandoned. Not to improve husbandry.

He proposes to do what he has to do.

The darkest of dark ages had continued for six and a half centuries. For the following five and a half light had been gradually, and ever more decisively, dispelling the darkness. Nevertheless, in Bacon's time the instauration of the sciences still lay in the future, to be explicitly undertaken first by Bacon himself, and enlightenment at its brightest still lay in the future, too, in the time of Diderot and d'Alembert. Bacon writes before Galileo, Kepler, Harvey, Snel, Hobbes, Desargues, Descartes, Cavalieri, Fermat, Guericke, Torricelli, Wallis, Grimaldi, Picard, Pascal, Cassini, Boyle, Malpighi, Huygens, Spinoza, Locke, Leeuwenhoek, Hooke, Steno, Newton, Rømer, Leibniz, Jakob Bernoulli, Halley, Stahl, Gray, Johann Bernoulli, Geoffroy, Taylor, Montesquieu, Voltaire, Maupertuis, Daniel Bernoulli, Franklin, Linnaeus, Euler, Buffon, Hume, Rousseau, d'Alembert.

Bacon's solution of the problem of the kinds of knowledge is negative. It is the perambulation of learning, or the circuit of knowledge, such as it still lay waste, uncultivated, and abandoned, still waiting for husbandry to improve. Pace d'Alembert, Bacon's solution is not the world map of knowledge, or the tree of knowledge.

D'Alembert writes: the general system of the sciences and arts is a labyrinth, a tortuous road the mind follows without knowing which way to turn (*Discourse*), nay: the road is blocked by a thousand obstacles (*id.*).

The difficulty of the problem of the kinds attests Bacon's great success in solving it, and the great importance of his solution. The difficulty can explain also why, if they had to introduce the *Encyclopedia* with the world map of knowledge, or the tree of knowledge, and in any case with the system of human knowledge, Diderot and d'Alembert adopted the one and only explicit solution of the problem of the kinds of knowledge. One and a half centuries after Bacon, however, husbandry had greatly improved. Bacon's solution is descriptive, and it is a virtue that it is descriptive, even if it is confined to the discovery of errors, not their correction, because the discovery of errors cannot not imply their correction. One and a half centuries after Bacon, however, the description is but of past errors, the errors have since been corrected, too, implicitly if

not explicitly, and it cannot be a virtue to presume that the description of past errors is but the description of past errors. Diderot and d'Alembert write after Bacon, Galileo, Kepler, Harvey, Snel, Hobbes, Desargues, Descartes, Cavalieri, Fermat, Guericke, Torricelli, Wallis, Grimaldi, Picard, Pascal, Cassini, Boyle, Malpighi, Huygens, Spinoza, Locke, Leeuwenhoek, Hooke, Steno, Newton, Rømer, Leibniz, Jakob Bernoulli, Halley, Stahl, Gray, Johann Bernoulli, Geoffroy, Taylor, Montesquieu, Voltaire, Maupertuis, Daniel Bernoulli, Franklin, Linnaeus, Euler, Buffon, Hume, Rousseau.

How greatly husbandry had improved, d'Alembert himself shows.

Bacon undertook the instauration of the sciences, and to this end to ground them in the method, in the *New organon*, the earlier of the two most important treatises on the method. Galileo, in the *Two new sciences dealing with mechanics and locomotion*, demonstrated conclusively that physics is mathematical essentially. Kepler discovered laws, the laws of the motions of the heavenly bodies, in the *New astronomy* and the *Harmonics of the world*. Descartes formulated the rules of the method, in the *Discourse on the method*, the later and more important of the two most important treatises on the method, which included also the *Geometry*, in which the foundations of analysis were laid, and with *The principles of philosophy* accomplished the synthesis of the successes of Bacon, Galileo, and Kepler. Huygens formulated the wave theory. Newton and Leibniz developed the differential and integral calculus, developing analysis further. Newton, in *The mathematical principles of natural philosophy*, solved the problems posed by Descartes, in *The principles of philosophy*, with the formulation of the laws of motion, and of gravity, and the grounding of the description of the system of the world in the laws. And d'Alembert continued the development of analysis, discovered the wave equation, and, in the *Treatise on dynamics*, reformulated Newton's solutions analytically.

Bacon confines his attention to the discovery of errors, but the discovery of errors cannot not imply their correction, and, given the hindsight of Diderot and d'Alembert, it implies it unambiguously.

Bacon writes: the justest division of human learning is that derived from the three faculties of the soul, history being relative to memory, poetry to imagination, philosophy to reason (*Growth*, II, 1).

Diderot concurs: the understanding is concerned with its perceptions in only three ways according to its three principal faculties, memory, reason, and imagination; whence results a general division of human knowledge into history, which is related to memory, philosophy, which emanates from reason, and poetry, which is born from imagination (*Explanation*).

The division is grounded in man.

The division thus may be the justest. But it may not be the true. It may not be the true division of knowledge – and not even on the supposition that knowledge is human. With Aristotle: the end of the theoretical is truth, the end of the practical is action – and what Bacon describes, and Diderot and d'Alembert affirm, is practical through and through. Memoirs are history, trades and manufactures are natural history, revealed theology is metaphysics, angelology and demonology are metaphysics, and it need not be doubted that, if this is the company they keep, physics and mathematics can themselves be but practices.

It is an error that the division is grounded in man. Bacon's solution of the problem of the kinds of knowledge is negative.

The first aphorism of the first of the two most important treatises on the method is: man, being the servant and interpreter of nature, can do and understand so much and so much only as he has in fact observed in the course of nature or has inferred from it; beyond this he knows nothing and can do nothing (*id.*).

The servant and interpreter of nature can only observe the course of nature and infer from it, and he had better observe, and infer, that the servant and interpreter of nature is himself nature. And this is the correction of the error, nature. Nature, or synonymously the world.

Grounded in man, the solution of the problem of the kinds of knowledge cannot, pace d'Alembert, be the world map of knowledge. Grounded in nature, however, it can, with d'Alembert, be. Taking it for granted that, pace Wittgenstein, the

world is not my world, and pace Heidegger, the world does not come along with the world map. The world map comes along with the world, and after much labor, exclusively.

It is an error also that the division of the kinds is practical. The division must be theoretical.

It is by reducing theory to practice, truth to action, that kinds of knowledge are a barrier before the investigation of knowledge, as well as before the acquisition of knowledge, such as this barrier protects that other, the most formidable barrier. A fortress such as it protects the citadel. The kinds just noted, for instance. If physics and mathematics are two among all those kinds, memoirs, trades and manufactures, revealed theology, angelology and demonology, and architecture and sculpture, too, how different can they be in what makes them all kinds of knowledge? can physics be different from trades and manufactures? mathematics from angelology and demonology? and certainly truth cannot be the end of physics and mathematics any more than it is the end of revealed theology.

The more the problem of the kinds of knowledge is neglected the firmer practical kinds will entrench themselves, and the firmer practical kinds entrench themselves the more brazenly will they pretend to be not just results, but the only possible results from the investigation of knowledge, as well as from the acquisition of knowledge.

The division must be theoretical. Which all by itself shows, too, that Aristotle's solution of the problem is the best, the only one to reaffirm, and the only one to labor to improve upon.

D'Alembert reports: we do not believe that our system of knowledge is the only one or the best (*Discourse*), and there can be no quarrel with him about that. But he adds: the system can be constructed by dividing knowledge into natural and revealed, or into useful and agreeable, or into speculative and practical, or into evident, certain, probable, and sensible, or into knowledge of things and knowledge of signs, and so on to infinity (*id.*), and there has to be a quarrel with him about this. Constructed in any of those ways, and so on to infinity, it can in any case, and in all cases, be said to be the system of knowledge, but it will not be in any case, or in all cases, the system of knowledge, and positively not because it will be said to be the system of knowledge. Whether it will be said to be the system of knowledge or not, the system of knowledge will be the system of knowledge only when it is constructed in the only way the system of knowledge can be constructed. The true way. D'Alembert writes: for someone who knew how to embrace it in a single point of view the universe will be but one fact and one grand truth (*id.*). And as the world so the system of knowledge. The system of knowledge is but one fact and one grand truth.

D'Alembert reports: we do not wish to resemble those naturalists, censured with good reason by a modern philosopher, who are ceaselessly occupied in dividing the productions of nature into genera and species, consuming with their labors time which would have been much better employed in the study of these productions; what would be said of an architect who, having to build an immense edifice, passed his whole life in drawing the plans for it (*id.*)?

That he became a mathematician? or a physicist? or a philosopher?

In any case, the naturalists ceaselessly occupied in dividing the productions of nature into genera and species are Tournefort and Linnaeus. The modern philosopher who censured them is Buffon.

He did not censure them with good reason, though. For Linnaeus did just that, embrace in a single point of view, not indeed the world, yet all organisms in the world. Linnaeus solved the problem of the classification of organisms.

And as one problem of the relations of genera and species has been solved – and Linnaeus solved one of the most general of the problems of the relations of genera and species that have had to be solved, and have been solved – in the same way all problems of the relations of genera and species can be, and must be, solved. In particular, Linnaeus shows how Aristotle's solution of the problem of the kinds of philosophy, and hence of knowledge, can be, and must be, improved upon.

The main difficulty before the problem of the kinds d'Alembert identifies as this: the system has to be simultaneously genealogical and encyclopedic, indicating the origin of the different kinds of knowledge as well as their connections

with one another (*id.*), but: the sciences which seem to contain the principles of all other sciences, and which must for this reason occupy the first places in the encyclopedic order, seem not to have the same rank in the genealogical order of ideas because they were not invented first (*id.*). Accordingly: we have chosen a division which appears to us the most satisfactory for the encyclopedic order of our knowledge and simultaneously for its genealogical order (*id.*).

Or, rather, practice chose the division for Bacon, and hence for Diderot and d'Alembert.

Encyclopedic, then, is the order of the kinds of knowledge indicating their connections with one another. More particularly, the encyclopedic order indicates the relations of their principles. It indicates the principles of which kind contain the principles of which kind, and the principles of which kind, in the first place, contain the principles of all other kinds.

But if so, how difficult can it be to find the principle of all other principles?

The *Encyclopedia* – the only important of the many and ever more encyclopedias – illustrates. In 17 volumes of text and 11 of pictures, the *Encyclopedia, or analytic dictionary of the sciences, arts, and trades, by a society of men of letters* appeared between 1751 and 1772 under the direction of Diderot. For volume V, in 1755, describing as well as prescribing, Diderot himself wrote the article *Encyclopedia*. He writes: if mankind were banished from the surface of the earth, the sublime spectacle of nature would be nothing but a scene of desolation; it is the presence of man that renders other things interesting; so why should we not introduce man in our work as he has been introduced in the universe? why should we not make him our central focus (*id.*)?

Why, indeed?

Genealogical is the order of the kinds of knowledge indicating their origin.

It is on Darwin's authority that Buffon did not censure Linnaeus with good reason. Darwin writes in *The origin of species*: the arrangement of species, genera, and families into what is called the natural system some authors regard as merely a scheme, or an artificial method; but the famous expression of Linnaeus that the characters do not make the genus, the genus gives the characters, seems to me to imply that some deeper bond is included in our classification than mere resemblance (14). And in *The descent of man*: Linnaeus, so remarkable for his sagacity, placed man in the same order with the quadrumana under the title primates (6).

Never mind man. Not even primate, and not even quadrumanus is the principle of all other principles.

The kinds of knowledge are not invented, they are discovered. The order of their discovery is the order of their successive development.

The genealogical order indicates the origin of the kinds. It indicates their development, their development in relation to one another, and most particularly their development from one another.

And if the arrangement of species, genera, and families is the natural system, and, with Linnaeus, not the characters make the genus, but the genus gives the characters, the development of the kinds from one another must indicate their connections with one another. It must indicate the relations of their principles, and most particularly the principles of which kind contain the principles of which kind, and the principles of which kind contain the principles of all other kinds.

The system of knowledge must be genealogical, exclusively.

Also, it is the introduction of man in the universe, when we make him our central focus, that makes the sublime spectacle of nature nothing but a scene of desolation.

xvi

Not quite one century separates Comte from Diderot and d'Alembert.

It is the century of the revolution. The buildup to 1789 was followed by the decade of the acceleration of the revolution, followed by the decade of the arrest of the revolution, followed by the half decade of the defeat of the revolution. It was defeated conclusively in 1815. And the defeat of the revolution was the defeat as well of enlightenment.

Comte published the *Course in positive philosophy*, in six volumes, from 1830 to 1842. Early on, in volume I, he reports: in studying the total development of human intelligence from its first and simplest beginning to our own time, I believe

I have discovered a great fundamental law to which the mind is subjected by invariable necessity (1.4); the law consists in the fact that each of our principal conceptions and each of the branches of knowledge passes through three theoretical stages, the theological, or fictitious, the metaphysical, or abstract, and the scientific, or positive (*id.*).

And among the many conclusions which Comte draws from the law, the first is: the classification of the positive sciences (2). The positive sciences are six: having since Descartes and Newton been the foundation of all philosophy, mathematics is at the head of the positive philosophy (2.93-6), followed by: astronomy, physics, chemistry, physiology, and social physics (2.52-66).

Philosophy and science are synonyms, physiology is biology, and the prospective science of social physics, whose development is the central concern of the *Course*, Comte renames in volume IV sociology.

Comte proposes the classification explicitly as: theoretical, and not practical (2.24). It is a virtue of the classification also that it is clear about the principle on which it stands. The order mathematics, astronomy, physics, chemistry, physiology, social physics, Comte writes: is determined by the degree of simplicity, or what is the same, of generality of phenomena (2.49); the simplest phenomena being necessarily the most general, for whatever is observed in the greatest number of cases is for this reason disengaged in the greatest degree from the circumstances peculiar to each separate case (2.50).

It would have been a virtue of the classification as well that it is the outcome of: the study of the total development of human intelligence from its first and simplest beginning to our own time, had it exemplified the proposition: no idea can be understood apart from its history (1.3).

As it is, the classification is not genealogical, it is: encyclopedic (2.52). Because: as a science progresses, the historical order of exposition becomes more and more impractical (2.32); and however perfect, a classification can never conform absolutely to the historical succession of the sciences (2.42).

Hegel, in the previous generation, studied the total development of spirit from its first and simplest beginning to our own time and discovered a great fundamental law to which spirit is subjected by invariable necessity. Marx, in the next generation, studied the total development of society from its first and simplest beginning to our own time and discovered a great fundamental law to which society is subjected by invariable necessity.

And to Comte – and with respect to the classification of the sciences, in particular – as to Hegel earlier and to Marx later, the study of the total development of human intelligence, as the study of the total development of spirit earlier and the study of the total development of society later, from its first and simplest beginning to our own time, did not suggest the study of, or even respect for, the facts.

Then again, the study of, or even respect for, the facts could but bar the way to the goal. And the goal is in Comte as spiritual as it is in Hegel, and in Augustine, and as practical as it is in Marx, and in Robespierre.

After the *Course in positive philosophy* Comte turned to the *System of positive politics, or treatise on sociology, instituting the religion of humanity*, in four volumes, from 1851 to 1854. In 1852 he provided also the *Positive catechism, or summary exposition of the universal religion, in thirteen systematic conversations between a woman and a priest of humanity*. The woman asks: why, my dear father, even if you deny all supernatural belief, do you insist on describing your universal doctrine as religion (1)? and the priest of humanity replies: because, my dear child, it is religion that orders every individual nature and wins over all individuals (*id.*); and the doctrine will not become universal until its spirit procures the essential affinity of all beliefs commanded by previous religions (*id.*).

The positive sciences – the earlier six reduced to four, and the four joined by a new one – are now these five: mathematics, physics, biology, sociology, and morality (2). Of which the priest informs his dear child while instructing it in: the dogma of the religion (*id.*).

xvii

These are all important attempts, explicit or

implicit, to solve the problem of the kinds of knowledge.

The permissiveness of the review – including Zeno's, and Varro's, and Comte's, among the important attempts – should all by itself make it probable that the review is complete. Drawing the line between the important attempts and the not important attempts explicitly, by pointing to some of the most important among the not important attempts, should make it certain.

Seven and a half centuries after Augustine, four and a half centuries before Bacon, Hugh of St Victor wrote the *Instruction, on the study of reading*. He writes: philosophy is divided into theoretical, practical, mechanical, and logical; and these four contain all knowledge (II, 1); the theoretical is divided into theology, mathematics, and physics (*id.*); the practical is divided into ethics, economics, and politics (19); the mechanical is divided into the making of fabrics, the making of arms, commerce, agriculture, hunting, medicine, and theater (20); the logical is divided into grammar and arguments (28).

The *Instruction, on the study of reading* is one of the treatises which began dispelling the darkest of dark ages decisively. Hugh of St Victor is an important philosopher, and because of the *Instruction*. But not because philosophy is theoretical, practical, mechanical, and logical.

Two and a half centuries after Hugh of St Victor, two-odd centuries before Bacon, ibn Khaldun wrote the *Introduction to history*. The *Introduction* is in six parts, and part VI is *On the various kinds of sciences*. Ibn Khaldun writes: the sciences are of two kinds; one is natural to man and he is guided to these sciences by his own reason; the other is traditional and man learns these sciences from those who discovered them (9); the first kind comprises the philosophical sciences (*id.*); the second kind comprises the sciences based on the authority of the religious law; in these sciences there is no room for reason; their foundation is the law of the koran and the sunnah, given to us by god and his messenger (*id.*). Ibn Khaldun then discusses the sciences in detail, beginning with the traditional. These are: the science of the reading of the koran and its interpretation (10); the science of the hadith (11); jurisprudence (12-3); theology (14); sufism (16); and the interpretation of dreams (17). He then turns to the natural sciences. He writes: the natural sciences, or the rational sciences, are four, logic, physics, metaphysics, and mathematics; and mathematics itself comprises four sciences, geometry, arithmetic, music, and astronomy (18). Also: arithmetic includes the craft of calculation, algebra, business arithmetic, and inheritance laws (20); geometry includes spherical figures, conic sections, mechanics, surveying, and optics (20); astronomy includes tables of the motions of the heavenly bodies (21). Turning to logic, next, ibn Khaldun describes Aristotle's logical treatises, the *Rhetoric*, and the *Poetics*, and notes: all logic is wrong according to the theologians (22). Next: physics includes the crafts of medicine and agriculture (23-5). Metaphysics follows (26), followed by: the sciences of magic and sorcery (27); the science of the secrets of letters (28); and alchemy (29). Ibn Khaldun then refutes philosophy, astrology, and alchemy: because the harm which they do to religion is very great (30). And he concludes by discussing at some length education, scholarship, grammar, poetry, and prose, including along the way the difficulties: the great number of scholarly books is a barrier to scholarship (34), and: it is difficult for someone whose language is not Arabic to acquire the sciences (43).

The central problem of the *Introduction to history* is the development of civilization. Ibn Khaldun is a very important philosopher, and because of the *Introduction*. But not because the sciences are natural and traditional. Even if there is no room for reason in the traditional, all logic is wrong according to the theologians, and the harm which philosophy, astrology, and alchemy do to religion is very great.

Two and a half centuries after ibn Khaldun, one generation after Bacon, Hobbes published the *Leviathan, or the matter, form, and power of the commonwealth ecclesiastical and civil*, in 1651. And in the *Leviathan* Hobbes has a division of: science, or the knowledge of consequences, which is called also philosophy (9). Science, or philosophy, is divided into natural philosophy and politics, or civil philosophy (*id.*). Natural philosophy is divided and subdivided eight times, to obtain: first philosophy, geometry, arithmetic,

astronomy, geography, engineering, architecture, navigation, meteorology, sciography, astrology, consequences from the qualities of liquids, consequences from the qualities of minerals, consequences from the qualities of plants, optics, music, consequences from the other senses, ethics, poetry, rhetoric, logic, and the science of just and unjust (*id.*). Politics is divided once, to obtain: consequences from the institution of commonwealths for the rights and duties of the body politic, or sovereign, and consequences from the same for the duty and right of the subject (*id.*).

Hobbes, to borrow Hume's description: introduced the experimental method of reasoning into moral subjects. Or, to borrow Dilthey's description: imposed on the spiritual sciences the developed in mathematics and natural science methods. Hobbes is a very important philosopher, and because of the *Leviathan*. But not because science, or philosophy, is natural and civil.

And not quite two centuries after Hobbes, in the generation of Comte, Ampère published the *Essay on the philosophy of the sciences, or analytic exposition of a natural classification of all human knowledge*, in 1834-43. Knowledge is: cosmological and noological; the cosmological is cosmological proper and physiological, the noological is noological proper and social; the cosmological proper is mathematical and physical, the physiological is natural and medical, the noological proper is philosophical and nootechnical, the social is ethnological and political; the mathematical is mathematical proper and physicomathematical, the physical is physical proper and geological, the natural is phytological and zoological, the medical is physicomedical and medical proper, the philosophical is philosophical proper and moral, the nootechnical is nootechnical proper and didagmatic, the ethnological is ethnological proper and historical, the political is physicosocial and ethnogetical (*id.*) – and Ampère does reiterate the dichotomy once more to obtain: 32 sciences of the first order (*id.*), once more to obtain: 64 sciences of the second order (*id.*), and yet once more to obtain: 128 sciences of the third order (*id.*).

Ampère developed electrodynamics, and discovered one of its laws, generalized into one of Maxwell's equations. Ampère is a very important philosopher. But not because of the *Essay on the philosophy of the sciences*, or because knowledge is cosmological and noological, and the sciences are 128.

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From the review of the attempts to solve the problem of the kinds of knowledge, then, the following conclusions can be drawn.

Explicitly, in Aristotle's solution the highest genus is philosophy. So it is in Zeno's solution. Of the seven of Varro's nine kinds which became normative the highest genus is the liberal and human arts. In Bacon's solution the highest genus is human knowledge, and so it is in Diderot and d'Alembert's affirmation of the solution. So it is in Comte's solution.

None of the philosophers asks explicitly which the genus is of which knowledge is the species. Hence, from the first attempt to the second still only implicitly, but from the second attempt to the third already explicitly, and beginning with the third attempt not only explicitly, but ever more aggressively, a genus has established itself. Knowledge is human. Among all of man's chattel there is also that, knowledge.

The development illustrates the fact that nothing but the world, or synonymously nature, can be *sui generis*, and nothing but the world, or synonymously nature, comes without its genus, implicitly, if not explicitly. If it is implicitly, and not explicitly, it is as likely as not to come with a fictional genus. The higher the stakes are supposed practically – that the end is action, not truth – the more likely it is to come with a fictional genus. For that very reason, that the end is not truth. And the higher the stakes are supposed practically, and the longer they are so supposed, the more likely is the fictional genus to entrench itself, and unquestionably.

Even if the genus of knowledge can be human not more than it can be German or American.

In Aristotle's solution the species of the highest genus are the theoretical and the practical. Because the end of the theoretical is truth, the end of the practical is not truth.

Of the subsequent solutions none concerns itself explicitly with truth. Accordingly, none of

them confines not truth. Accordingly, truth suffers the consequences.

In Zeno's solution the species of the highest genus are logic, physics, and ethics. Because logic is the fence, physics is the tree, ethics is the fruit. Hence, when one of the three is abandoned, it will not be the fruit. The fruit will not be abandoned even when two of the three are abandoned. And the fruit will urge that the other two be abandoned. For this fruit will not find it difficult to be a fruit without a tree, never mind a fence. It will, indeed, find it easier to be a sweeter fruit, and the sweetest fruit, without a tree, never mind a fence. As to the tree and the fence, whether they are abandoned or not, they will have to become, or to be supposed to become, fruits themselves. In terms of the abandoned Aristotelean dichotomy, to become, or to be supposed to become, practical.

In the next solution among the species of the highest genus are both geometry and rhetoric. Hence, geometry will not teach what rhetoric is, and geometry will not teach what geometry is. Rhetoric will teach what rhetoric is, and rhetoric will teach what geometry is.

In Bacon's solution the species of the highest genus are history, philosophy, and poetry. But poetry is fiction. And as ethics so fiction, and as rhetoric so fiction. History and philosophy will each have to become, or to be supposed to become, fiction. The kinds of philosophy, physics and mathematics, will each have to become, or to be supposed to become, fiction as well. Fiction will also teach what history is, and what philosophy is, and what physics is, and what mathematics is.

So it is a matter of course, too, that the verbal transformation of social physics into sociology will be of a kind of physics into a kind of not physics, and the transformation of the one kind sociology into the two kinds sociology and morality will fructify, and be fructified by, religion.

This development illustrates the fact affirmed by the famous expression of Linnaeus. The characters do not make the genus, the genus gives the characters.

The main conclusion that has to be drawn from the review is thus this. If twenty-four centuries after Aristotle Aristotle's remains the most successful attempt to solve the problem, if this most successful attempt to solve the problem has to be recovered from two brief passages, if more than four centuries after Bacon Bacon's remains the most successful attempt to solve the problem explicitly, if this most successful attempt to solve the problem explicitly is negative, if this negative attempt is liable to be taken affirmatively, and if after Comte's attempt, such as it is, there has been no attempt of any importance to solve the problem for two centuries now, then the philosophers have ignored the problem of the kinds of knowledge more or less altogether, and in the last couple of centuries altogether. The philosophers, clearly: have not wished to resemble those naturalists ceaselessly occupied in dividing the productions of nature into genera and species.

But the philosophers should have. For: the famous expression of Linnaeus implies that some deeper bond is included in the arrangement of species, genera, and families.

It is in the *Botanical philosophy*, in 1751, that Linnaeus writes: you must understand that a character does not make a genus, the genus makes the character (169).

Linnaeus solved the problem of the classification of organisms in *The system of nature, according to the three kingdoms of nature, by classes, orders, genera, and species*, in 1735. The three kingdoms of nature are minerals, plants, and animals, and, while Linnaeus left it to other philosophers to attend to the explicit arrangement of minerals in the system of nature, to that of plants and animals he attended himself. In *The species of plants*, in 1753, and in the fifth edition of *The genera of plants*, in 1754, he arranged in their genera, orders, and classes 5 944 species of plants, and in the tenth edition of *The system of nature*, in 1758, he arranged in their genera, orders, and classes 5 897 species of animals.

The deeper bond Darwin demonstrated in 1859, in *The origin of species*.

And the limitations which the analogy of organisms has, the analogy of stars does not have.

Hertzsprung solved the problem of the classification of stars. And simultaneously established the fact of the evolution of stars. In 1905-7, in *On the radiation of stars*, Hertzsprung quantifies the relation of intrinsic brightness and spectral type

of stars, and from the distribution of stars with respect to this relation infers: two collateral series of stars of the same age (1); in agreement with the hypothesis of development of stars (2).

The relation of intrinsic brightness and spectral type is the foundation of the theory of the evolution of stars. By 1926, in Eddington's *Internal constitution of the stars*, a diagram of this relation represents the evolution of stars, and Eddington has in the diagram Hertzsprung's two series, named now: the main series and giants (120), as well as a third series: white dwarfs (*id.*).

In the meantime, Einstein had formulated the theory of general relativity, and quantum theory had been developed. In 1929, in *A relation between distance and radial velocity among extragalactic nebulae*, Hubble reported the observations which established the fact that the world is expanding. On this ground – general relativity, quantum theory, and Hubble's law relating the recession velocities of galaxies to their distances – in the next two generations the big bang theory was developed, the theory of the origin and development of the world. Of the theory of the origin and development of the world the theory of the evolution of stars cannot not be an essential part, and of the development of the theory of the origin and development of the world the development of the theory of the evolution of stars was an essential part.

Ptolemy, in the *Almagest*, divided stars into kinds by apparent magnitude (VII, 4). Eighteen centuries later, Hertzsprung took the first bit of evidence for the natural kinds of stars to be *ipso facto* evidence for their evolution.

All organisms of which there is evidence have common descent as organisms. Back to the first, every organism is from an earlier organism, and hence every species is from an earlier species, and every genus is from an earlier genus. Stars do not have common descent as stars. Every star is not from an earlier star. The common descent of all stars is the big bang. The quarks and leptons constituting the elements which the stars burn, as well as the forces by which the stars burn them, have their ultimate descent in the big bang. But this common ultimate descent all stars share with all organisms, and with all things.

If organisms have common descent as organisms, but stars do not have common descent as stars, this explains the fact that a tree represents well the evolution of organisms, but not the evolution of stars. As the varieties of a species of an organism develop into new species, and the varieties of the new species develop into new species, this development is represented well by the branching and branching of branching of a tree. The tree can also readily be taken to represent, top to bottom, species, their genera, and the genera of the genera. As a cloud of interstellar particles gathers, gravity collapses it, a protostar forms, begins burning hydrogen into helium and becomes a main sequence star, the star spends most of its life as such, then it becomes a red giant, explodes in a helium flash, becomes a red supergiant, and the remaining carbon core becomes a white dwarf, and then a black dwarf, this development is represented in the Hertzsprung diagram by what looks more like a river. Nor can the diagram readily be taken to represent the relations of species and genera.

Yet organisms and stars share more than ultimate descent from the big bang. They share more also than the fact of evolution, that there is the evolution of organisms and there is the evolution of stars. They share the fact of having principles. Organisms have common descent, and have principles in common. Stars do not have common descent, and yet have principles in common.

Whether the kinds of knowledge have common descent or not, they have principles in common.

It is because it has in each case had to be of the greatest generality, in order to be a solution of the problem, that the solution of the problem of the kinds has in each case been the decisive advance. The arrangement of all organisms in one system by classes, orders, genera, and species posed explicitly in all its generality, and nothing else could pose explicitly in all its generality, the problem solved by the theory of evolution with the principle of natural selection, the descent with modification of all organisms. The classification of stars showed, and nothing else could show, the evolution of stars, which in turn showed explicitly, as well as dramatically, and nothing else could show nearly as dramatically, the development of the world. Hubble, too, before he discovered the law of the expansion of the world in

1929, solved, in 1926, in the *Extragalactic nebulae*, the problem of the classification of galaxies.

Conversely, the lack of solution of the problem of the kinds has been a barrier. A natural barrier to begin with. It has been a natural barrier in every case, and it has been a natural barrier in the case of the kinds of knowledge. If, however, in the case of the kinds of knowledge, ethics has been a kind next to physics, ethics has been a kind when physics has not been, ethics has been the kind of kind physics has had to be, and logic has had to be, rhetoric has been a kind, rhetoric has been the kind teaching what the other kinds are, fiction has been a kind, fiction has been the kind teaching what the other kinds are, and the kinds have been the dogma of a religion, too, the kinds of knowledge are not the natural barrier first encountered long ago. Art has been added to what nature has provided, has kept being added, and the kinds of knowledge have long become a fortress. The redoubtable fortress guarding the citadel.

xix

Moreover, as the philosophers abandoned the attempts at the fortress altogether a couple of centuries ago, spirit, infusing everything at the time, infused also, as it had to, the fortress. Spirit vastly reinforced the fortress, and has been vigorously superintending it since.

Spirit demanded from philosophy the restoration of the substantiality and purity of its being. And philosophy complied. It took a war to make philosophy comply, but philosophy did comply. The war for the restoration of spirit to the substantiality and purity of its being was successfully concluded with the proclamation of the independence of the spiritual sciences from the natural sciences. With the younger sister's proclamation that she, the younger sister, had broken the fetters in which her older sister had held her.

And there it is, the new master kind. The spiritual sciences. The younger sister.

It can need no recommendation as the master kind. It is armed with everything a master kind has ever been armed with. Spiritual, it can accommodate religion, and fiction, and rhetoric, and ethics, and practice, any of these, any combination of these, all of these, *ad hoc*, and at will.

All other kinds have had to become, or to be supposed to become, what the master kind tells them to become. No other kind teaches what any kind is, and certainly not what it is. The master kind teaches what any kind is, and each kind what it is.

There is, thus, in general, the older sister. And it should be superfluous to note that there can be no older sister but for her younger sister. There are the natural sciences, or the empirical sciences, or simply the sciences. And there can be no natural sciences, or empirical sciences, or sciences, but for the social sciences, or the human sciences, or simply the humanities, and there can be no social sciences, or human sciences, or humanities, unless they are the spiritual sciences. For if there are sciences whose concern is nature, or experience, can the alternative that will be the concern of the alternative sciences be but not nature? but not experience? Can not nature, and not experience, be each but an impossibility? And if not nature is society, not nature is man, not experience is society, not experience is man, can any of these not be an impossibility unless spirit is not an impossibility? unless society, and man, are supposed – explicitly, or implicitly, or tacitly, or unawares – spiritual?

Or what is the same, the dichotomy nature, or experience, on one side, society, or man, on the other, cannot divide the sciences better than the dichotomy nature, or experience, on one side, Germany, or the United States, on the other, and neither of these dichotomies can divide the sciences better than the dichotomy nature, or experience, on one side, spirit, on the other.

More particularly, physics used to be a part of philosophy. Spirit has disallowed that. Physics is one of the large and ever growing number of sciences. Mathematics used to be inseparable from physics. Spirit has disallowed that, too. *Divide et impera*, and mathematics is not allowed to be even one of the sciences. History used to learn from physics and mathematics. That, too, spirit has disallowed. History is one of the large and ever growing number of humanities. Debarred from physics and mathematics, philosophy is also one of the humanities, unless it is, philosophy being philosophy, anything, *ad hoc*, and at will.

The first salvoes in the war for the restoration of spirit to the substantiality and purity of its being were fired three generations before Hegel. Two-odd generations after him the war was successfully concluded by Dilthey.

How successfully, Russell, Wittgenstein, Heidegger, Carnap, and Popper show. Earliest to latest, Russell to Popper, the five postdate Dilthey by one-odd to two-odd generations. One of the five, Heidegger, approves of both Hegel and Dilthey. One, Wittgenstein, does not note either Hegel or Dilthey, as he does not note much at all of what predates him. Only one of the remaining three, Popper, notes Dilthey, but all three, Russell, Carnap, and Popper, disapprove, and disapprove strenuously, of Hegel. And if one to two generations after the war, when they note the victors and when they do not, when they approve of the victors and when they do not, and even when they do not note much at all of what predates them, the philosophers – and these five are, again, all important philosophers who have investigated knowledge in the last century-odd – take for granted, no questions asked, what the victors fought for and the vanquished fought against, the victory has been total.

In particular, the philosophers take for granted, no questions asked, the kinds science and philosophy such as spirit has brought them into being.

Heidegger approves of the victors. He writes: science follows the ruinous path of misinterpreting spirit and depriving it of its power (*Introduction*, 15), and: philosophy never arises from and never through science; it can never be equal to the sciences; it stands in an entirely different realm of spiritual being (6).

Russell, Carnap, and Popper disapprove of the victors. Russell is an older contemporary of Heidegger, Carnap is a younger, Popper is a younger still.

Russell writes: philosophy is the no man's land between theology and science exposed to attack from both sides (*Hist. philosophy*, intr.). Russell has no use for spirit, not explicitly. Yet, not only is philosophy exposed to attack from science. If the attack from science drives philosophy, of all directions, toward theology, to infer the spirituality of philosophy such as it is not science is unavoidable.

The position he argues, Carnap reports: is not that of rationalism (*Structure*, 183), because: the position demands rationality only for science (*id.*); it acknowledges the existence and importance of the remaining not rational spheres (*id.*). If the position acknowledges the existence and importance of the remaining not rational spheres, the position acknowledges the entirely different realm of spiritual being in which philosophy stands. If so, when the position demands rationality for science, the position demands rationality for the following of the ruinous path of misinterpreting spirit and depriving it of its power. And if so, Carnap's is not even a rearguard action. Confining rationality to science such as it is not philosophy, it is an attempt to salvage something, anything, after the unconditional surrender.

And when Popper proposes to find: a criterion to distinguish the empirical sciences from mathematics and logic as well as from "metaphysical" systems, Popper's is not even an attempt at salvage. By confining science away from mathematics and logic and by confining it away from metaphysics, both, Popper serves spirit.

Wittgenstein, finally, neither approves nor disapproves of the victors, not explicitly. Then again: philosophy is not one of the natural sciences; the word philosophy must mean something above or below, but not next to, the natural sciences (*Treatise*, 4.111). It is with Heidegger that Wittgenstein has no quarrel, explicitly, or implicitly. Nor can he, because it is Heidegger, in the first place, that has no quarrel with him.

These are the new kinds, science such as it is not philosophy and philosophy such as it is not science.

The old kinds are physics such as it is a part of philosophy, mathematics such as it is inseparable from physics, and history such as it learns from physics and mathematics. Science is perfected knowledge, exemplified by mathematics paradigmatically, as well as by physics, as well as, if only prospectively, by metaphysics, and science is thus unquestionably philosophy, just as philosophy is unquestionably science. This kind of physics is exhibited by Aristotle, before him by the first philosopher already, after him by Descartes in *The principles of philosophy*, by Newton in *The mathematical principles of natural philos-*

ophy. This kind of mathematics is exhibited by Galileo, by Kepler, by Leibniz, long before them by Ptolemy, before him by Archimedes. This kind of history is exhibited by Spinoza, long before him by the father of history already, after him by Voltaire. Science such as it is philosophy and philosophy such as it is science is what, most explicitly, Hume investigates, and Kant investigates, before them Bacon investigates, before him Aquinas investigates. Thales, Herodotus, Aristotle, Archimedes, Ptolemy, Aquinas, Bacon, Galileo, Kepler, Descartes, Spinoza, Newton, Leibniz, Voltaire – Linnaeus with the *Botanical philosophy* – Hume, and Kant are but a few of the most important instances among only the most important philosophers exhibiting the old kinds, and failing to exhibit the new kinds, in the twenty-four centuries from the first philosopher through the time of the first salvoes in the war for the restoration of spirit.

Lamarck published the *Zoological philosophy* in 1809. Lamarck, still, if he did science such as it is not philosophy, did it unawares. If he did philosophy such as it is not science, did that also unawares.

Dalton published the three volumes of the *Chemical philosophy* from 1808 to 1827. Dalton, too, if he did either science such as it is not philosophy or philosophy such as it is not science, did either unawares.

Faraday published the three volumes of the *Experimental researches in electricity* from 1839 to 1855. Throughout, Faraday identifies Coulomb, and Volta, and Ampère, and Ørsted, and himself as: philosophers (*passim*). If Coulomb, Volta, Ampère, and Ørsted were scientists, Faraday was not aware of that. If he was himself a scientist, he was not aware of that either. Faraday concluded the *Experimental researches*, in 1855, with *Some points of magnetic philosophy*. Faraday, too, if he did either science such as it is not philosophy or philosophy such as it is not science, did either unawares.

If Thales, and Herodotus, and Aristotle, and Archimedes, and Ptolemy, and Aquinas, and Bacon, and Galileo, and Kepler, and Descartes, and Spinoza, and Newton, and Leibniz, and Voltaire, and Linnaeus, and Hume, and Kant, and Coulomb, and Lamarck, and Volta, and Dalton, and Ampère, and Ørsted, and Faraday fail to exhibit the new kinds, the new kinds are fictional.

Nor is the authority of Russell, Wittgenstein, Heidegger, Carnap, and Popper an argument in favor of the new kinds.

As far as science such as it is not philosophy is concerned, it is Einstein's successful confinement to that kind that has made it not just possible, but mandatory, to suppose that Einstein's *opera* may be about time, but so is – and in an entirely different realm of spiritual being – Heidegger's *opus*. To suppose, too, that Heidegger's *opus* is about being, Einstein's *opera* are not about being, Heidegger's *opus* is about being because Heidegger says so, and Einstein's *opera* are not about being because Heidegger says so.

As far as philosophy such as it is not science is concerned, what has happened to that kind at the hands of Wittgenstein is representative.

Wittgenstein's two treatises are the *Logico-philosophical treatise*, in 1921, and the *Philosophical investigations*, in 1953. It is in the earlier that he writes: philosophy is not one of the natural sciences. He adds: philosophy is not a theory but an activity (4.112), and, as it should be, the logico-philosophical treatise is unabashedly logico-practical. In the *Philosophical investigations* Wittgenstein reports: our concern is grammatical (I, 90), and, as they should be, the philosophical investigations are unabashedly grammatical. It may be wondered whether grammar is an activity. In any case, when, in the grammatical investigations, Wittgenstein writes: philosophy is a battle against the bewitchment of our understanding by our language (109), it may be supposed that the battle is, as it should be, grammatical. Further, though: there is not one method in philosophy, but there are methods, like different therapies (133). Is, then, the battle grammatical? or therapeutical? or therapeutical in order to be grammatical? or grammatical in order to be therapeutical? Further: the work of the philosopher is to collect memories for a definite purpose (127). The collections are therapeutical? and the memories grammatical? or the purpose is grammatical? or therapeutical? and the battle is the purpose? or the battle is the work? Further still, however: what is your goal in philosophy? to show the fly the way out of the fly bottle (309).

And at this point instead of wondering, is the fly bottle language? but what is outside the fly bottle? and is showing the fly the way out a therapy? but who is the fly? and on and on, it should be wondered, is any of this anything but what good fiction is, tantalizing? Yet Wittgenstein writes before the logico-practical treatise already: one of the most difficult tasks of the philosopher is to find out where the shoe pinches him (*Notebooks*, 15 Jun 1915). And is it possible not to wonder, is the fly bottle where the shoe pinches him?

The physical things are real, the not physical things are not real. It must be added that what spirit has brought into being is not real. What spirit has brought into being is fictional.

xx

If man is the citadel, and the kinds of knowledge are the fortress guarding the citadel, vagueness is the outworks guarding the fortress.

Russell writes: everything is vague to a degree you do not realize until you have tried to make it precise (*Ph. atomism*, 1).

Everything is vague. But not everything is equally vague. Not everything is always equally vague either. Some things have been made precise. Some of the philosophers have tried to make the vague precise, and some of these philosophers have succeeded. And some things have been made vaguer. Some of the philosophers have tried to make the vague vaguer, and most of these philosophers have succeeded. The philosophers who have succeeded in making the vague precise and the philosophers who have succeeded in making the vague vaguer have not tended to be the same. Sometimes they have been, however. Russell himself, for instance. Unusually, at Russell's time the philosophers were having both a massive success in making the vague precise and a massive success in making the vague vaguer, and, very unusually, Russell's labors were part of both, the massive success in making the vague precise and the massive success in making it vaguer. Russell is, in fact, the only recent philosopher who has been successful in both, making the vague precise and making it vaguer.

To illustrate.

Completing his logical investigations, Aristotle reports: in these matters nothing at all was done before us. This has to be qualified. Not quite one and a half centuries before Aristotle, Parmenides discovered the principle of contradiction. Not quite one century before Aristotle, as Aristotle himself notes: Socrates first concerned himself with definitions (*Metaphysics*, I, 6). Nevertheless, the beginning of logic coincides substantially, if not exclusively, with the labors of Aristotle. Aristotle defined truth, distinguished proposition from sentence, classified inferences, showed that proof is a kind of inference, and thus laid the foundations of the theory of proof, and developed syllogistic, and thus more generally the predicate calculus.

Twenty-two centuries later, Boole writes: logic stands associated almost exclusively with the great name of Aristotle; as it was presented in the *Organon*, such, with scarcely any essential change, it has continued to the present day (*Laws*, 1). This has to be qualified as well. One century after Aristotle, Chrysippus discovered the rules of inference, and thus laid the foundations of the propositional calculus. Nineteen centuries after Chrysippus, Leibniz described the possibility of a philosophical calculus in which all things and their relations are expressed formally, and, hence, all problems are solved by calculation: Leibniz's gigantic conception of a *calculus philosophicus* (*Writing*, pr.), as Frege describes it. Boole himself, in the generation before Frege, discovered that syllogistic, and more generally the predicate calculus, is a calculus of sets, thus anticipating the possibility of grounding all logic in sets.

Nevertheless, as the beginning of logic coincides substantially, if not exclusively, with the labors of Aristotle, so the development of logic after Aristotle coincides substantially, if not exclusively, with the labors of Frege.

From 1879 to 1892, from the *Concept writing, a formula language of pure thinking modeled on that of arithmetic* to *On sense and reference*, Frege discovered that the elements of a proposition are function and argument, showed what function is, showed what proof is, discovered how truth can be expressed formally, how generality can be expressed formally, how existence can be expressed formally, how, in general, content and form can both be expressed formally,

and, hence, content and form can be separated formally from one another, developed a formal language in which the chains of inferences are free of gaps, and distinguished sense and reference. Galileo demonstrated that physics is mathematical essentially, and physics has since been mathematical exclusively. Frege demonstrated that logic is mathematical essentially, and logic has since been mathematical exclusively. Galileo thus demonstrated also that, whatever the relation of physics and mathematics, it cannot be exclusion. Frege thus demonstrated also that, whatever the relation of mathematics and logic, it cannot be exclusion.

From 1874 to 1897, from *On a property of the totality of all real algebraic numbers* to the *Contributions to the foundation of the transfinite theory of sets*, Cantor formulated the theory of sets.

The theory of sets is the foundation of mathematics, including logic. It is thus the foundation also of everything of which mathematics, including logic, is the foundation.

In 1908, in the *Investigations on the foundations of the theory of sets*, Zermelo axiomatized the theory of sets. Zermelo begins: the theory of sets is this branch of mathematics, to which falls the task to investigate the fundamental concepts of number, order, and function in their original simplicity and thus develop the logical foundations of all arithmetic and analysis (pr.). And reports: in this work I show how the entire theory can be reduced to a few definitions and seven principles, or axioms (*id.*).

In 1902, Russell writes to Frege: let *w* be the predicate, to be a predicate that cannot be predicated of itself; can *w* be predicated of itself? from each answer its opposite follows (16 Jun).

Frege replies: your discovery of the contradiction has caused me the greatest surprise and, I would almost say, consternation, because it rocks the foundation on which I thought to build arithmetic (22 Jun).

Russell continues in his letter: it must be concluded that *w* is not a predicate; there is also no set, as a whole, of those sets which as a whole do not belong to themselves (*id.*).

Frege continues in his reply: it seems to me, however, that the expression 'a predicate is predicated of itself' is not exact; a predicate is as a rule a first-level function, which demands as an argument an object and cannot therefore have itself as an argument (*id.*).

The rocking, then, was more apparent than real. And either way, Frege showed the ground on which it will be stopped.

Thus, to resolve the contradiction Russell formulated the theory of types, in 1903, in *The principles of mathematics*, and reformulated it more explicitly, in 1908, in the *Mathematical logic as based on the theory of types*. The theory rests on the rule: whatever involves all of a collection must not be one of the collection (*Types*, 1).

With Zermelo's axiomatization of sets the contradiction is resolved.

If these logicians, more generally mathematicians, and still more generally philosophers, have succeeded in making precise truth, proposition, inference, proof, function, generality, existence, form, content, sense, reference, set, number, order, and the foundation of arithmetic and analysis, and the foundation of mathematics, including logic, and the foundation of physics, if physics is essentially mathematical, and the foundation of philosophy, if philosophy will have anything to do, if not with physics or mathematics, at least with logic – and even if they have succeeded in making not all of this, but only some of it, and making it not precise, but only more precise, and even only less vague – their success in making the vague precise has been massive.

And in this, between the labors of Frege and the labors of Zermelo, the labors of Russell are the most important. The discovery of the contradiction and the first attempt to resolve it with the theory of types are the decisive steps in the development from Frege to Zermelo.

On the other hand, Russell formulated also the theory of definite descriptions, in 1905, in *On denoting*. This theory stands on the distinction – taken explicitly for granted in *On denoting* from the first – which is the central concern of *Knowledge by acquaintance and knowledge by description*, in 1911. It is in this paper that Russell illustrates knowledge by acquaintance with: I see the sun and am aware of it, and: I desire food and

am aware of it. Knowledge by description he illustrates with the definite descriptions: we know that the man with the iron mask existed (*id.*), and: we know that the candidate who gets the most votes will be elected (*id.*).

If what we know and what I am aware of, and what I see and what I desire, can instance knowledge, knowledge is not precise. As Russell, in 1912, writes: knowledge is not a precise conception. The distinction of knowledge by acquaintance and knowledge by description can, perhaps, make it precise. Nothing, however, can make precise knowledge, or knowledge by acquaintance, or knowledge by description, or anything at all, if it stands on what we know and what I am aware of. However vague it found knowledge, too, including knowledge by acquaintance and knowledge by description, if there are such kinds of knowledge, a theory of denotation cannot help making knowledge, including all its kinds, whatever they are, vaguer, if it stands on what we know and what I am aware of. For, however successful the theory may appear to be in fixing other references, or synonymously denotations – in fixing the reference, or denotation, of 'reference', or 'denotation', for instance – the theory cannot be more successful in fixing any other references, or denotations, than it is in fixing the references, or denotations, of the instances on which it stands. And that is, the references, or denotations, of 'we' and 'I'. Nor does the theory make the attempt to fix the references, or denotations, of 'we' and 'I'. Nor should any theory make this attempt. Nor does this theory need to make this attempt, if, as Russell, in 1944, writes: theory of knowledge has a certain essential subjectivity.

Nor has theory been made precise, if, next to the theory of sets, not only the theory of types is a theory, but the theory of definite descriptions is a theory. Not to mention theory of knowledge. Russell also concludes in *On denoting*: I beg the reader not to make up his mind against the view I have been advocating until he has attempted to construct a theory of his own on the subject of denotation (*id.*). And if Russell himself has not made theory vaguer than he found it, his reader has. For as far as the subject of the theory is concerned his reader has not followed Russell's advice very vigorously, but as far as constructing a theory of his own is concerned, on any subject, *ad hoc*, and at will, his reader has followed Russell's advice very vigorously.

The massive success in making the vague precise Russell shares with Cantor, Frege, Zermelo, philosophers who are logicians and mathematicians. The massive success in making the vague vaguer he shares with other, and other kinds of, philosophers.

How massive the success of the philosophers in making the vague vaguer has been the fact can show that, if the philosophers have not succeeded in making everything vaguer than they found it, it is not because they have not tried. Which the already introduced instances of the labors of Wittgenstein, Heidegger, Carnap, and Popper can one and all show. Not only the nothing nothings, that the nothing nothings is meaningless. Not only does time measurement in the theory of relativity need illumination of its ontological foundations, the illumination presupposes a clarification of inner temporality from the temporality of this-being. Not only scientific discovery is not redundant, logic of scientific discovery is not redundant. Natural sciences is not redundant, empirical sciences is not redundant, human sciences is not an impossibility, social sciences is not an impossibility, and it is spiritual what the sciences are that made all other sciences what they are.

If the philosophers have not succeeded in making everything vaguer, it is not because they have not tried, and it may well be that they have succeeded. Philosophy has always been liable to be a vehicle for the combination, permutation, and transposition of words. Never before, however, has philosophy prided itself on being such a vehicle, and being such a vehicle exclusively.

Hegel writes: language is the being of spirit (*Phenomenology*, VI, C.c).

Carnap disapproves of Hegel. Nevertheless, when he eliminated metaphysics, he replaced it with semantics.

Wittgenstein does not note Hegel. Nevertheless, the philosophical investigations are grammatical. And: when the philosophers try to grasp the essence of the thing, one must always ask, is the word so used in the language which is its

home?

Heidegger approves of Hegel. So he writes: essence and being speak in language (*Introduction*, 17). The essence and being of mathematics, for instance. Heidegger writes: the fundamental feature of modern science is that it is mathematical; so what does mathematical mean? it may seem that only mathematics can answer this question, but this is a mistake; in such questions we do well to keep to the word, and the word comes from the Greek τὰ μαθήματα (*Thing*, 18.a-b).

The word will answer the question, what does mathematical mean? It is a mistake that Theaetetus or Eudoxus or Archimedes or Diophantus or al-Khwarizmi or Fibonacci or Descartes or Fermat or Newton or Leibniz or Euler or Lagrange or Gauss can answer it. Or Cantor or Frege or Zermelo.

It may thus seem that to answer the question, so what does historical mean? it is a mistake that Herodotus or Polybius or Livy or Otto of Freising or ibn Khaldun or Guicciardini or Spinoza or Voltaire or Gibbon can answer it, but we do well to keep to the word, and the word comes from the Greek ἵστωρ. But this is also a mistake. The mathematical is not spiritual enough, so we do well enough to keep to an old word. We ignore the opportunities offered by mathematicology, mathematicism, and mathematicity. Sadly, we ignore even the opportunities offered by mathematicality. Not so the historical, however.

Heidegger writes: historicity is possible as a mode of being of this-being only because historicality determines this-being in the very foundation of its being (*Being*, 6), and: the emergence of historicism is the clearest sign that historiology endeavors to alienate this-being from its historicality (76).

We do well to keep to the word, but we do especially well to keep to the word as we coin it.

Needless to add, physics is denied access to truth – Popper, for one, concludes in *The logic of scientific discovery*: science is not knowledge, it can never claim to have attained truth or even a substitute for it such as probability; we do not know, we can only guess (85) – and if science can never claim to have attained truth, Heidegger will, by keeping to the word as he coins it: true-being, or truth, is uncovered-being (*Being*, 44.b).

Needless to add, physics is denied access to the world, too. Instead, on Popper's authority, the world is three worlds. On Carnap's authority, the world is real is meaningless and the world is not real is meaningless. On Wittgenstein's authority, the world is my world, and: my world is the first and only (*Notebooks*, 2 Feb 1916), and: I am my world (*Treatise*, 5.63). So once again Heidegger has the last word. As he coins it. The world not only comes with knowing, it comes with: the worldhood of the world (*Being*, 14).

Not everything is vague, then, but everything is vaguer. Everything that has been made precise has also been made vaguer, and if not everything that has not been made precise has also been made vaguer, everything that matters has been.

Vagueness is the outworks guarding the fortress guarding the citadel, and the outworks do not stretch merely to the end of language. They do not even stretch merely to the end of the world. They stretch as far as they will, at will.

xxi

The survey of the barriers before the investigation of knowledge cannot be complete if it does not reach the present. Absolutely not, the present being the present.

Hegel writes in *The philosophy of history*: when we pass through the past, no matter how large it is, we have to deal only with the present; for spirit is immortal, spirit is never past, never not yet; spirit is now essentially (intr., C.c). In *The history of philosophy*: the present is the highest; in the present, world spirit grasps itself as absolute spirit (III, 3.E).

The present being the present, the present had now and again appeared to itself the highest before Hegel. Beginning with Hegel, the present need not appear to itself the highest, the present is the highest. It is not a difficulty that the present comes and goes. The present has been the highest for two centuries now, and as the present comes and goes, the present keeps becoming higher than the highest.

Accordingly, the barriers before the investigation of knowledge have been reinforced to perfection. Unless they are results from the investigation rather than barriers before it, in which case

they have been honed to perfection.

Hegel predates Russell by one century. In the course of this century physics quit being a part of philosophy and became one of the sciences. Mathematics quit being inseparable from physics and became irrelevant. History quit learning from physics and mathematics and became one of the spiritual sciences. Philosophy, unless it became a therapy, became one of the spiritual sciences as well.

Russell, Wittgenstein, Heidegger, Carnap, and Popper span one generation. They are the last important philosophers who have investigated knowledge, and, with the exception of Russell, the earliest of the five, they are philosophers of the new kind. They are also all five important, but, with the exception of Russell again, not very important. And the relation of these two facts, they are philosophers of the new kind and they are not very important, is of cause and effect.

In the next generation, as the present has kept becoming higher than the highest, there are no important philosophers who have investigated knowledge, never mind very important. The relation of these two facts, the present has kept becoming higher than the highest and no important philosophers, is of cause and effect, too.

The survey of that generation thus has to be of the not important. In particular, Kuhn and Michel Foucault – as important as there are among the not important – will have to do.

In *The structure of scientific revolutions*, in 1962, Kuhn reports: this essay aims to make explicit some of the implications of the new historiography (1).

The essay does so three generations after the proclamation of the independence of the spiritual sciences. The younger sister had broken the fetters in which her older sister had held her since Descartes, Spinoza, and Hobbes imposed on her their developed in mathematics and natural science methods. Unfettered, the younger sister had been dissolute since. She had also been becoming ever more brazen. She now took to calumniating her virtuous sister.

Kuhn writes: scientific development has been characterized by competition of various schools with incommensurable ways of seeing the world and of practicing science (*id.*). Scientific development has been characterized by competition, practices, and ways of seeing the world. It has not been characterized by acquisition of knowledge, or search for truth, or investigation of the world. Thus: if Aristotelean dynamics, phlogistic chemistry, or caloric thermodynamics are myths, then myths can be produced by the same methods and held for the same reasons that lead to scientific knowledge; if, on the other hand, they are science, then science has included beliefs incompatible with the ones today (*id.*). One or the other, science has produced myths or science has included myths. Either way, science is myth.

Nor can science be anything else. Because in the spiritual sciences, including the new historiography whose implications the essay aims to make explicit, not history of physics, and not even history of science, explains science. History of literature explains science. Kuhn writes: to the extent that the book portrays scientific development as a succession of tradition-bound periods punctuated by non-cumulative breaks, its theses are borrowed from other fields; historians of literature, of music, of the arts, of political development, have long described their subjects in the same way (post.7).

In fact, history of literature does not explain science either. Literature explains it. Kuhn writes: the member of a mature scientific community is, like the typical character of Orwell's *1984*, the victim of a history rewritten by the powers that be (13).

Kuhn concludes: the problem, what must the world be like in order that man may know it? remains unanswered (*id.*). Kuhn has no use for knowledge, and no use for truth: does it really help to imagine that there is some one full, objective, true account of nature (*id.*)? He has no use for knowledge, no use for truth, and no use for the world. In place of any and all of these, the world, truth, and knowledge, there are but: world-views (10).

And the provenance of world-views is immaculate.

It is Dilthey. World-views had been in circulation in the couple of generations before him, but it is Dilthey who established them as the fundamental idea of the spiritual sciences. It is Dilthey who found that, as the newly unfettered sciences

needed unfettered ideas, ideas which will dispose of any and all of these, the world, truth, and knowledge – lest the natural sciences, or the spiritual sciences for that matter, investigated the world, or searched for truth, or acquired knowledge – world-views are such an idea, and an idea, indeed, which will make all ideas such ideas. For, as Dilthey has established in the *Introduction to the spiritual sciences*: the world is nowhere but in the ideas of the individual (I, 8).

The *Introduction* appeared in 1883. Dilthey developed the idea of world-views in the following years, and conclusively in *The types of world-view and their formation in the metaphysical systems*, in 1911. He writes: according to the cultural realm in which they are conditioned, the types of world-view are religious, poetic, and metaphysical (2). As to the older sister, she is excluded from the pleasures of the young. She conditions nothing. Dilthey explains: science only develops the general relations of isolated homogenous facts; religion, poetry, and primeval metaphysics express the meaning and sense of the whole; it notices, they understand (1.4). If science would be allowed to contribute anything to a world-view, then, it could only be by being allowed to contribute to one of the three that understand. As to which one, before Kuhn it may have seemed that it would be metaphysics, or perhaps religion. After Kuhn it seems more likely that it would be poetry.

Michel Foucault – Michel, to distinguish him from the earlier and important philosopher Foucault, and important because, unaware that demonstration is knowledge not more than fiction, he demonstrated the earth's rotation on its axis – Michel Foucault also concerns himself with how the new history is not the old history, the new historiography is not the old historiography, the new historicism is not the old historiology.

In *The archeology of knowledge*, in 1969, Michel Foucault writes: the single center of description of the old history, resting on the postulates of homogeneity, uniformity, and coherence, is disappearing in favor of a space of dispersion (1); but this epistemological mutation of history is incomplete (*id.*); to complete it we must rid ourselves of distinctions between types of discourse, science, literature, philosophy, religion, history, fiction, etc. (2.1); the project is of pure description of discursive events (*id.*); discourses are practices forming the objects of which they speak (2.3); and one does not seek to pass from text to thought, from exterior to interior, from multiplicity to unity, one remains in the discourse (2.7). Accordingly, having completed the project, Michel Foucault reports: my discourse is a discourse about discourses (5).

That philosophy and history are discourses the list of distinctions of which we must rid ourselves makes explicit early in the discourse. That physics is discourse can be taken for granted, given that the list includes science. Whether mathematics is discourse is kept in suspense until the end of the discourse. It is its *coup de grâce*, apparently: mathematics, too, is a discursive practice (4.6.d).

Hence: knowledge is that of which one can speak in a discursive practice (4.6.b), and: knowledge is to be found not only in demonstrations, but also in fiction, reflection, narratives, institutional regulations, and political decisions (*id.*).

Discourses are practices forming the objects of which they speak, and whether they are practices forming the objects of which they speak or practices forming the practices forming the objects of which they speak, discourses about discourses form the objects of which they speak. As demonstration is also a discourse, so is the demonstration of the earth's rotation on its axis. It is nevertheless reassuring that, while Michel Foucault's and Foucault's are both discourses, one is a discourse about discourses and the other is not. The other is a demonstration.

In general, if discourses form the objects of which they speak, to discourse that demonstration and fiction are both discourses, attending to fiction, and not to demonstration, will help. On the other hand, to note that attending to fiction, and not to demonstration, will help to discourse that demonstration and fiction are both discourses, attending to discourses about discourses will not help. But attending to demonstration should.

Michel Foucault proposes to complete the epistemological mutation of history himself. The beginning of the mutation he identifies with: Marx (1). And between Marx and himself he

associates one more philosopher with the mutation: Nietzsche (*id.*).

Although it has to be conceded that, if vagueness is a result from the investigation of knowledge, he attests the uncompromising thoroughness with which knowledge is being investigated, and if getting rid of distinctions is a result, he attests how utterly the problem of the kinds of knowledge is being solved, Michel Foucault could not complete the mutation, however. For the straightforward reason that it had already been completed. Dilthey had completed the epistemological mutation of history three generations before Michel Foucault. That the mutation is complete with the labors of Dilthey the proclamation of the independence of the spiritual sciences shows, and the establishment of their fundamental idea shows conclusively. In fact, one generation after Dilthey, and two generations before Michel Foucault, even the result: one remains in the discourse – as well as the result: the scientist is like the typical character in a novel – had been established, certainly vaguely enough, when Croce, in 1893, had had *History reduced to the general concept of art*.

As to Marx and Nietzsche – not quite two and not quite three generations after Hegel, an older and a younger contemporary of Dilthey, respectively – Marx and Nietzsche are among the victims of the mutation, not among its agents. If the mutation was, with Michel Foucault: of the single center of description of the old history, resting on the postulates of homogeneity, uniformity, and coherence, into: the space of dispersion of the new history, then Marx's was the most successful attempt – and remains the most successful attempt – and Nietzsche's was the last substantial attempt, precisely, to sustain the single center of description, resting on the postulates of homogeneity, uniformity, and coherence, and, against the dispersion that was setting in in Marx's time and becoming *de rigueur* in Nietzsche's, to keep history fettered to the methods developed in mathematics and natural science and imposed on it since Descartes, Spinoza, and Hobbes. Marx, in *The capital*, in 1867, reports: my position is that the development of society is a process of natural history (pr.). Nietzsche, in *Beyond good and evil*, in 1886, proposes to describe: the natural history of morality (V).

Michel Foucault would have Marx and Nietzsche. Dilthey would have more.

As his title in 1911 indicates, the types of world-view are formed in the metaphysical systems. So, according to the cultural realm in which they are conditioned, the types of world-view are religious, poetic, and metaphysical. But: all threads come together in metaphysics (2.3), and according to their formation in the metaphysical systems, the types of world-view are: naturalism (3); idealism of freedom (4); and objective idealism (5). Naturalism Dilthey traces back to: Protagoras (3.2). Idealism of freedom he traces back to: Anaxagoras (4.1). Objective idealism he traces back to: Xenophanes (5.1). And among the philosophers with whom he exemplifies naturalism is: the sceptic Hume (3.2), among those with whom he exemplifies idealism of freedom is: the idealist Kant (4.1).

In the world, the one of facts, Protagoras said that man is the measure of all things. It is in a world-view, whatever the facts, that what Protagoras said is: the beginning of relativism (3.2), thus of: sensualism (*id.*), thus: the foundation of naturalism (*id.*), and thus of all deficiencies of the natural sciences such as the spiritual do not have. In the world, idealism begins with Berkeley, one and a half centuries before Dilthey, less than one century before Hegel. It is in a world-view that twenty-two centuries before Berkeley Anaxagoras is an idealist, and two-odd generations earlier still Xenophanes is an idealist. It is in a world-view, too, normative as this world-view has become, that Hume is a sceptic and Kant is an idealist. In the world, Hume is a sceptic not more than Dilthey is a physicist and Kant is an idealist not more than Dilthey is a mathematician.

Berkeley is the first idealist, but idealists will deny that. Twenty and a half centuries before the first idealist, Pyrrho is the first sceptic, but sceptics will deny that. Laertius reports: sceptics deny that Pyrrho was the first sceptic; they say that Democritus already was a sceptic; and before him Heraclitus was a sceptic; and before him Xenophanes was a sceptic; and before him the seven wise men were sceptics; and before them Homer was a sceptic (IX, 70-3).

And before him *homo erectus* were sceptics,

and before them *homo habilis* were sceptics, and before them *australopithecus* were sceptics. Nor were *australopithecus* the first sceptics. Scepticism was a condition of the big bang already. Or indiscriminately, as far as scepticism is concerned, long before Homer Adam was a sceptic, but not the first sceptic. Scepticism was a condition of god already. *Mutatis mutandis*, if idealism did not begin with Berkeley, it did not begin with Xenophanes either. It began with the big bang. Or indiscriminately, as far as idealism is concerned, with god.

The novel explains: who controls the past controls the future, who controls the present controls the past (Orwell, *1984*, 1.3).

Thus in the novel, in science such as the novel explains, in discourses about discourses, in world-views, and unquestionably in the world-view which controls the present.

But not in the world. In the world, who controls the past may or may not control the future, and whether he does may or may not be ascertained by the inspection of scriptures or of Nostradamus or of novels. Who controls the present does not control the past, however. Nor can who controls the present control the past. Because *post hoc propter hoc* may be a fallacy, but *propter hoc post hoc* may not be. If there is an effect, there is a cause. And if there is an effect, and there is a cause, the cause precedes the effect. The effect does not precede the cause.

So, where the present is concerned, spirit controls the present all right, but it is not because it controls the present. Or because it controls the past. Kuhn and Michel Foucault have a cause, Dilthey has a cause, even Hegel has a cause.

So, there is also hope. Spirit does not control the past, and control of the present can be wrested from it. And if the cause is found, this can help. And control should be wrested from spirit.

For Homer is not a sceptic, the seven wise men are not sceptics, Xenophanes is neither a sceptic nor an idealist, Heraclitus is not a sceptic, Anaxagoras is not an idealist, Protagoras is not a naturalist, Democritus is not a sceptic, Hume is not a sceptic, Kant is not an idealist, and Marx and Nietzsche are the victims, not the agents of mutation.

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About the results of the next generation only one thing should be said. They have corroborated amply the hypothesis that as the present keeps becoming ever higher than the highest the philosophers keep becoming ever less important.

The barriers before the investigation of knowledge, then, are many and formidable, but they can all be collected into these three – man, the kinds of knowledge, and vagueness. Not only are these three the most formidable. If to what nature has provided art has been added, and more art, and more, and the barriers have long become a system of fortifications, these – the citadel, the fortress, and the outworks – are the indispensable parts of the system.

What remains, in order to complete the survey and assessment of the barriers, is to try and find their cause.

The barriers have been reinforced, and reinforced, and reinforced

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What the cause of the barriers is nothing should indicate more clearly and distinctly than their reinforcement. And if so, there can be no better way to approach the problem of their cause than to review the labors at reinforcing them. Given that art has been added to what nature has provided, to review the art.

The review should also dispel whatever doubts may remain lingering as to whether the barriers before the investigation of knowledge are, in fact, barriers before the investigation of knowledge. If the review of the labors at reinforcing the barriers is a review of those labors in relation to the labors of acquisition of knowledge and investigation of knowledge, it should show that, whatever knowledge is, if those are labors of acquisition of knowledge and investigation of knowledge, these are not labors of acquisition of knowledge and

investigation of knowledge. These are labors against the acquisition of knowledge and investigation of knowledge. And if the outcome of those is results, the outcome of these is not results. It is results against the results, or what is the same, barriers.

The most important labors at reinforcing the barriers can be gathered into four efforts.

The first begins with the labors of Protagoras, four-odd generations after the first philosopher, Thales, and, through the labors of Socrates, it is complete two-odd generations after Protagoras with the labors of Plato.

The second begins with the labors of the first sceptic, Pyrrho, two-odd generations after Plato, and, through the labors of the philosophers of the schools, including most particularly the four major, the Academy, the Lyceum, the Garden, and the Stoa, it is complete one century-odd after Pyrrho, by the time of Diogenes the Babylonian, Critolaus, and Carneades, when all schools have sunk into paltriness.

The third begins with the labors of st Paul – st, to distinguish him from Paul, the jurist – two-odd centuries after Carneades, and, through the labors of the fathers of the church, it is complete three and a half centuries after st Paul with the labors of Augustine. Given the intensity and duration of the dark ages that followed it, this effort can need no recommendation as the most successful. So far.

The fourth begins with the labors of the first idealist, Berkeley, thirteen-odd centuries after Augustine, continues with the labors of the philosopher who explained the state, Rousseau, and of the poet, Schiller, and through the decisive labors of Hegel, it is complete one and a half centuries after Berkeley with the labors of Dilthey.

The review is, again, of these, the labors at reinforcing the barriers before the acquisition of knowledge and the investigation of knowledge, in relation to the labors at the opposite end, at the acquisition of knowledge and the investigation of knowledge and thus at breaching the barriers.

Beginning with the first philosopher.

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Thales, the first philosopher, solved the problem of the principles of all things, and solved the problem of the earth, its position, whether it is at rest or in motion, and its shape,

Anaximander, the second philosopher, solved the problem of the earth better, and drew the map of the earth,

Anaximenes, the third philosopher, solved the problem of the principles of all things better,

Cyrus, the first great king, conquered three of the four older kingdoms, the Mede, the Lydian, and the Babylonian, and his domain stretched from the Aegean Sea in the west to the Amu Darya in the east,

Pythagoras, the fourth philosopher, discovered that number is the principle of all things, and discovered the irrationality of $\sqrt{2}$,

Xenophanes, the fifth philosopher, discovered that knowledge is not belief,

Hecataeus, the seventh philosopher, began collecting the localities on the surface of the earth,

Cambyses, the second great king, conquered the fourth of the older kingdoms, the Egyptian,

Darius, the third great king, campaigned further west, north, and east than his predecessors, and his domain stretched from the gulf of Sidra in the west to the valley of the Indus in the east,

Heraclitus, the eighth philosopher, discovered that they do not understand,

Xerxes was the fourth great king,

Parmenides discovered that the earth is a sphere, and discovered the principle of contradiction,

Anaxagoras discovered the cause of eclipses,

Empedocles solved the problem of the material principles of all things with the theory of the four elements, earth, water, air, and fire,

and these are only some of the most important results of only some of the most important philosophers and princes in the four-odd generations from Thales to Empedocles.

Then Protagoras said that man is the measure of all things.

Before Protagoras, then, where all things are concerned, the philosophers discovered that all things pose the problem of their principles, solved this problem, and began improving on the solution of the problem. They discovered that number is the principle of all things. They discov-

ered that the material are distinct from other principles, and formulated the theory of these principles.

Whatever knowledge is, these results attest the acquisition of knowledge.

One of all things is the earth, and where the earth is concerned, the philosophers discovered that the earth poses the problem of its position, whether it is at rest or in motion, and its shape, and solved this problem, and began improving on its solution as well. They drew the map of the earth. They began collecting the localities on its surface. They discovered that it is a sphere. They discovered the cause of eclipses.

Whatever knowledge is, these results cannot not attest the acquisition of knowledge. They cannot not attest it conclusively, too. That number is the principle of all things, is a result of such generality – the greatest – it can be doubted that it is conclusive, even as the twenty-five and a half centuries since Pythagoras have kept supplying ever more compelling evidence that number is the principle of all things. That on the same grounds on which quantum theory is a theory the theory of the four elements is a theory, and that on the same grounds quantum theory is a development of the theory of the four elements, can be doubted, even as the century-odd since Planck has kept supplying ever more compelling evidence that quantum theory is the theory of the material principles of all things. The philosophers' results about the earth, however, are not blessed, or cursed, with such generality. And that the earth is a sphere can no doubt be believed doubted, but it cannot be doubted. The earth is a sphere. Anaxagoras said: lunar eclipses occur when the earth blocks the sun's light from falling on the moon, solar eclipses occur when the moon hides the sun from view (Hippolytus, *Refutation*, I, 8). And this is the cause of eclipses. The philosophers obtained conclusive results, and conclusive results about the earth as a whole, and conclusive results about the relations of the earth, the moon, and the sun, and conclusive results about the relations of causes and effects reaching out to the sun.

So much suffices to show also that the labors of the princes belong in the review. If the labors of the philosophers attest the acquisition of knowledge, the labors of the princes attest the acquisition of knowledge. If mapping the earth and collecting the localities on its surface attests the acquisition, establishing the relations of the gulf bordering satrapy VI, the sea bordering satrapies I and II, the river dividing satrapies XII and XVI, and the river straddled by satrapy XX – some 4 900 km along the great circle from the gulf of Sidra in the west to the valley of the Indus in the east – attests the acquisition.

The philosophers discovered the irrationality of $\sqrt{2}$. They discovered the principle of contradiction.

They discovered that knowledge is not belief. They, the philosophers, discovered that they, men, do not understand.

And then it turned out that man is the measure. Of the principles of all things, of the problem of the principles of all things, and of the solution of the problem of the principles of all things. Of the sphericity of the earth and of the cause of eclipses. Of whether the great kings conquered the four older kingdoms or the four older kingdoms conquered the great kings. Of whether the gulf of Sidra and the Indus are 4 900 km apart or 490 km or 49 km or 4.9 km. Of the irrationality of $\sqrt{2}$ and of contradiction.

Man is the measure of all things, is not a result from the acquisition of knowledge. It is a result against the results from the acquisition of knowledge, or what is the same, a barrier before it. More particularly, it is a reinforcement of the barriers first provided by nature, and – for this reason, too – it is the most formidable barrier before the acquisition of knowledge and the investigation of knowledge. In general, all subsequent labors of reinforcement of the barriers stand on it. In particular, the labors of Socrates and Plato stand on it.

How formidable a barrier, man is the measure of all things, is, the labors of Protagoras show, the labors of his younger contemporary Socrates show better, and the labors of Plato, two-odd generations after Protagoras, show best. The labors of all three, Protagoras, Socrates, and Plato, are part of the labors of the philosophers at the acquisition of knowledge and the investigation of knowledge and *ipso facto* at breaching the barriers, and a very important part of these labors.

But the labors of all three are also part of the labors of the philosophers at the opposite end, at reinforcing the barriers, and a very important part of these labors. And if their labors at breaching the barriers are substantial, their labors at reinforcing the barriers are more substantial, by far, if their labors at breaching the barriers are extensive, their labors at reinforcing the barriers are more extensive, again by far, and if some of their successors have built on their labors at breaching the barriers, many more of their successors have built, and have built much more, on their labors at reinforcing the barriers.

Protagoras wrote the *Truth* and *On the gods*.

The *Truth* began: man is the measure of all things, of those that are that they are, of those that are not that they are not.

On the gods began: of the gods I cannot know either that they are or that they are not, for many are the barriers hindering knowledge, including the obscurity of the matter and the brevity of man's life (Laertius, IX, 51-2). Protagoras, accordingly, is the second philosopher honored with a condemnation for impiety, and the first philosopher honored with the burning of his books (Cicero, *Gods*, I, 63).

That this, I cannot know either that the gods are or that the gods are not, is a result from the acquisition of knowledge, not a barrier before it, the honors bestowed on Protagoras – and on Anaxagoras before him, and on many other philosophers, through and including Russell, after him – indicate strongly. The fact that it is the outcome of the assessment of the barriers before the acquisition of knowledge indicates even more strongly. What shows it conclusively, though, is the fact that what it asserts is incompatible with what the other proposition, the beginning of the *Truth*, asserts.

Thus, the philosopher cannot know either that the gods are or that the gods are not. But if man is the measure of all things, man can know that the gods are, and if man can also know that the gods are not, man can also choose what to know, and it is not difficult to know what man will choose to know. Moreover, if man is the measure, whether pig is the measure or not, and whether baboon is the measure or not, certainly philosopher is the measure, and if man can choose to know that the gods are, certainly philosopher can choose to know that the gods are, and certainly that god is. Moreover, the philosophers discovered that knowledge is not belief. But if man is the measure, man can choose to know that knowledge is belief, and to believe that knowledge is belief, and to know and believe that belief is knowledge, and if man can, certainly philosopher can. The philosophers discovered also that men do not understand, but if man is the measure, man can choose to understand, and what to understand, and how, and why, and if man can, certainly philosopher can.

Conversely, the successors of Protagoras in the twenty-four and a half centuries since – the philosophers among them, never mind the men, the pigs, and the baboons – when they have built seldom and reluctantly on: I cannot know that the gods are, and often and eagerly, and most particularly where the gods are concerned, including where god is concerned, on: man is the measure, they have thus kept supplying incontrovertible evidence that, whether philosopher is man, philosopher is eager to be man.

Aristotle reports: Socrates abandoned physics altogether, and devoted himself to ethics; and while searching for the general in ethical matters, he first concerned himself with definitions (*Metaphysics*, I, 6).

The discovery of definitions is a result from the acquisition of knowledge *par excellence*.

The devotion to ethics is not. It rests on abandoning physics altogether, which Socrates ensures when he teaches me: it seems ridiculous to me to search for other things before I know myself, building, as he is, on the inexhaustibility of my eagerness to know myself. In turn, the inexhaustibility of my eagerness to know myself rests on the inexhaustibility of man's eagerness to be the measure of all things. Or what is the same, where either I am concerned or man is concerned, the inexhaustibility of man's eagerness to be the measure of all things rests on the inexhaustibility of my eagerness to know myself.

The investigation of knowledge begins with Plato, with the *Theaetetus*. Earlier, in the *Meno* (97a-9d), and in the *Republic* (V, 476c-80a), Plato reaffirms Xenophanes's result, knowledge is not belief. In the *Theaetetus*, he shows all the

following. Knowledge is not perception. Knowledge only comes after a long development. Not only knowledge is not belief, knowledge and belief are exclusive of one another. Not only man is not the measure, in geometrical proofs the geometer alone is the measure, and, in general, whatever knowledge is, whoever has acquired it alone is the measure.

Plato nevertheless devoted himself to ethics, and to show how completely he abandoned physics there are, sadly, all too many ways. Here are four.

Plato does not note – not in any of the 36 treatises, authentic and spurious, collected by Thrasyllus in the Platonic corpus – that Thales solved the problem of the principles of all things or solved the problem of the earth. He notes that Thales fell into a well while gazing at the stars. He does not note that Pythagoras discovered that number is the principle of all things or discovered the irrationality of $\sqrt{2}$. He notes that: Pythagoras had followers and taught them a distinct way of life (*Republic*, X, 600a-b). He notes that Anaxagoras discovered the cause of eclipses (*Cratylus*, 409a-c). In order to belabor Anaxagoras and reaffirm: earth and air and other such things are not the true causes of things; the true cause of things is what is for the best (*Phaedo*, 97b-9b). He notes that the earth is a sphere. By way of introducing what he has to teach about: Tartarus, bored right through the earth, on whose shores the immortal souls of men are judged and either punished or rewarded (109a-14c). He does not note the geographical discoveries of the great kings. He notes that: Cyrus neglected the education of Cambyses, and Cambyses was driven mad by debauchery; Darius neglected the education of Xerxes, and Xerxes committed the same mistakes as Cambyses (*Laws*, III, 694a-5e).

Dividing Plato's treatises into logic, physics, and ethics, Laertius finds but one to count physics, the *Timaeus* (III, 50-1). Should the judgment of Laertius be questioned, too, it would be more plausibly not because other of Plato's treatises may be physics, but because even the *Timaeus* may not be. It is here that Plato argues that: the world has a maker (28a-9a), and: the maker of the world made the world because he is good (29d-e).

Whitehead, in 1929, writes: the safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato (*Process*, II, 1). If philosophy is ethics, the characterization is no doubt both the safest and general. And given that Whitehead writes one generation after Dilthey, and Faraday's magnetic philosophy is not philosophy, and Newton's natural philosophy is not philosophy, and most of the Aristotelean corpus is not philosophy, and the results of Anaxagoras and Thales are not philosophy, the safest general characterization of philosophy is no doubt that it is ethics.

Plato writes in the *Laws*: god is the measure of all things (IV, 716c). No doubt in geometrical proofs, too.

xxv

After Empedocles,
Protagoras said that many barriers hinder knowledge of whether the gods are,
Zeno of Elea discovered the paradoxes – dichotomy, Achilles, arrow, and stadium – questioning the possibility of motion,
Herodotus described 1 012 localities, reaching from the Atlantic in the west to the Indus in the east, and 804 predecessors and contemporaries of his, reaching from his own time back to the first prince,
Leucippus proposed that the principles of all things are atoms and void,
Hippias discovered a curve with which the angle can be trisected,
Socrates discovered definitions,
Hippocrates wrote theoretical medical treatises,
Thucydides wrote the history of the war of the Athenians and the Spartans,
Democritus formulated the atomic theory,
Hippocrates of Chios squared lunes, and wrote a treatise in which theorems were deduced from axioms,
Hicetas discovered that the earth's rotation on its axis can explain the apparent revolution of the heaven,
Plato investigated knowledge,
Archytas doubled the cube,
Theaetetus formulated the theory of the irrationals,
Epaminondas discovered tactics, in the battles of Leuctra, and of Mantinea,

Eudoxus formulated the theory of proportion, and formulated the theory of the motions of the heavenly bodies in concentric spheres,

Aristotle, from the definition of truth to the development of the predicate calculus, developed logic such as it remained the foundation of logic until Frege, from the description of 600 kinds of animals to the description of the world in its entirety, developed physics such as it remained the foundation of physics until Galileo and Kepler, and from the solution of the problem of the kinds of philosophy to the systematic reviews of his predecessors, developed the investigation of knowledge such as it remained the foundation of the investigation of knowledge until Bacon and Descartes,

Philip developed the troops demanded by tactics, developed the mustering, training, and deployment into smaller units of the heavy infantry of the phalanx, of light troops, and of cavalry, and the integration of the three,

Menaechmus discovered the conic sections,

Theophrastus described 550 kinds of plants, and wrote the history of physics,

Eudemus wrote the history of mathematics,

and these are again only some of the most important results of only some of the most important philosophers and princes in the four-odd generations from Empedocles to Eudemus.

Pyrrho and scepticism followed.

Before Pyrrho, mathematics – identified now explicitly as one of the two kinds of theoretical philosophy, itself identified explicitly as one of the two kinds of philosophy – made the greatest strides. The philosophers solved all three problems, the trisection of the angle, the squaring of the circle, and the duplication of the cube, the first conditionally, the second partially, the third completely. They discovered definitions. They discovered the deduction of theorems from axioms. They formulated the theory of the irrationals. They formulated the most important theory, the theory of proportion. They discovered the most important curves, the conic sections. They developed the predicate calculus.

Physics – identified explicitly as the other kind of theoretical philosophy – made great strides as well. The philosophers discovered the paradoxes which the theory of motion must resolve. They formulated the atomic theory. They discovered the earth's rotation on its axis. They formulated the theory of the motions of the heavenly bodies. They described 600 kinds of animals and 550 kinds of plants. They described the world in its entirety.

Whatever knowledge is, these results attest the acquisition of knowledge. They attest also its broadening, deepening, and quickening. The acquisition reached more objects, more substantially, and faster in the four-odd generations after Empedocles than in the four-odd generations before him.

History made great strides, too. The philosophers described 1 012 localities reaching from the Atlantic to the Indus and 804 predecessors of theirs reaching back to the first prince. They wrote the history of the war of the Athenians and the Spartans. They wrote the history of physics and the history of mathematics.

Whatever knowledge is, these results attest the acquisition of knowledge, too. Moreover, if history made great strides because mathematics and physics made great strides, and history is theoretical, in opposition to practical, as mathematics and physics are – if its end is truth, in opposition to action, as their end is – it can be concluded on these grounds that history is a part of physics.

On the same grounds the labors of Epaminondas and Philip belong in the review. It may be doubted that tactics was discovered and the troops demanded by tactics were developed because mathematics and physics made great strides. But it may not be doubted that the conduct of war became a theoretical concern, as distinct from practical – if not, in this case, in opposition to practical – at this time. The conduct of war became a theoretical concern in the generation of Plato, even if not because of the guardians of the *Republic*. It did, because Plato's older contemporary Xenophon wrote the *Cavalry commander*, the first treatise concerned specifically with the conduct of war. In the next generation Aeneas Tacticus wrote *On the defense of fortified positions*. And if the treatises of Xenophon and Aeneas Tacticus attest the acquisition of knowledge, the labors of Epaminondas of discovering tactics and of Philip of developing

the troops demanded by tactics attest the acquisition of knowledge.

As war so medicine. Hippocrates only laid the foundations of the first corpus of medical treatises, and treatises were still added to the corpus six centuries after him. Of the more than 70 treatises in the Hippocratic corpus the majority are practical through and through. Many are as practical as this: do not begin by asking for your fee (*Precepts*, 4). But a few are theoretical, including some of the oldest, and as theoretical as the *Epidemics I* and *Epidemics III*, which manifest a staunch interest in the disease and no interest whatsoever in treating it. If Hippocrates wrote the first theoretical medical treatises, though, medicine became a theoretical concern – as distinct from practical, if not in opposition to practical, in this case as well – two generations before him. Theoretical medicine can be traced back before Hippocrates to the labors of Empedocles, and before Empedocles to the labors of Alcmaeon. Alcmaeon and Empedocles were both associated with Pythagorean brotherhoods, and Alcmaeon, in particular, was associated with that established by Pythagoras himself at Croton. So much suffices to show that theoretical medicine developed because mathematics and physics made great strides. That theoretical medicine attests the acquisition of knowledge should be superfluous to add.

As war and medicine so the gods. The philosophers said that many barriers hinder knowledge of whether the gods are. The gods became a theoretical concern – and in this case in opposition to practical, truth being irreconcilably at odds with action where the gods are concerned – three generations before Protagoras, when Xenophanes scolded Homer and Hesiod for what they had said about the gods. And it should now be superfluous to add, both, that concern with the gods whose end is truth, not action, developed because mathematics and physics developed, and that this concern attests the acquisition of knowledge.

And the investigation of knowledge made great strides. The philosophers discovered that knowledge and belief are exclusive of one another. They discovered that in geometrical proofs the geometer alone is the measure, and, in general, whoever has acquired knowledge alone is the measure. They solved the problem of the kinds of philosophy, and hence implicitly of the kinds of knowledge. They began the systematic reviews of their predecessors. And all results from the acquisition of knowledge are, or should be, the concern of the investigation of knowledge, and the investigation of knowledge made great strides because the acquisition of knowledge made great strides, and the investigation of knowledge cannot not attest the acquisition of knowledge.

Pyrrho and scepticism followed.

Pyrrho was anticipated. By Protagoras, when he said that man is the measure of all things. By Protagoras's younger contemporary Gorgias, who wrote *On the not existent, or nature*, and in *On the not existent, or nature*, proposed: nothing exists, and even if anything existed, it could not be known, and even if it could be known, it could not be expressed (Empiricus, *Mathematicians,* VII, 65-6). By Metrodorus of Chios, the teacher of Pyrrho's teacher Anaxarchus, who said: I deny that we know whether we know something or nothing, or even that we know that we know or do not know, or even that we know that anything is or is not (Cicero, *Academic*, II, 73). By Anaxarchus himself, who said: everything that is is a stage-painting (Empiricus, *Mathematicians*, VII, 88).

Pyrrho was anticipated, but his claim to being the first sceptic is unassailable. Empiricus, himself a sceptic, reports: Timon, the student of Pyrrho, compared Pyrrho to the sun; and it may seem that the comparison is inconsistent with the sceptic doctrine, because the sun throws light on things, while Pyrrho forced the things we saw clearly to revert to obscurity; but if one considers the matter more philosophically, one will see that as the sun blinds those who look at it so the sceptic doctrine confuses the intellect (I, 305-6).

Pyrrho's claim to being the first sceptic is unassailable, because he saw that it will not do that we do not see clearly the things we see clearly. It will not do that the sun does not throw light. The things we see clearly have to be forced to revert to obscurity. The sun not only does not throw light, it blinds. And as the sun blinds, so the sceptic doctrine confuses the intellect.

To put it more explicitly – if less philosophi-

cally, perhaps – Pyrrho discovered that it will not do to argue, or even to establish, that we do not know. It has to be argued, and established, that we know that we do not know. And if theory cannot allow this to stand, practice will. Practice, being practice, will allow anything to stand, and if the distinction between theory and practice is forced to revert to obscurity, truth will become action and action will become truth. In any case: Pyrrho said that we ought to fight things with deeds, and if impossible with deeds, with words (Laertius, IX, 66).

That scepticism is not a result from the acquisition of knowledge, but a result against the results from the acquisition, scepticism itself can hardly have been more explicit.

The first effort at reinforcing the barriers before the acquisition and the investigation of knowledge began with Protagoras but achieved its greatest successes after him, and the second effort at reinforcing the barriers began with Pyrrho but achieved its greatest successes after him. Greatly successful in reinforcing the barriers as the first effort was, too, the second was even more successful. How successful, the development of scepticism shows, the development of the philosophical schools shows better, and the development of philological and antiquarian scholarship shows even better.

Timon compared Pyrrho to the sun, and Timon found out how the distinction between theory and practice can be forced to revert to obscurity. It cannot be maintained consistently that we know that we do not know. It cannot, that is, if, we know that we do not know, is a judgment. But the sun does not throw light, it blinds, and the acquisition of knowledge is not the acquisition of knowledge. It is a condition in need of a cure, and, we know that we do not know, is not a judgment, it is a cure. It is a purgative, in particular, Timon found out: overthrowing all assertions, the sceptic assertion overthrows itself as well, like a purgative which, throwing out of the bowels everything else, throws out itself as well (Laertius, IX, 76).

The philosophers established that knowledge is not belief and belief is not knowledge. But then the philosophers established scepticism, scepticism forced the distinction between judgment and cure to revert to obscurity, and between truth and action, and between knowledge and belief, and it has ever since remained philosophically respectable to maintain that knowledge is belief and belief is knowledge.

Himself the student of a teacher, Timon was the contemporary of the flourishing of the philosophical schools. The Academy was established a century before him, and with the establishment of the Academy, and then of the other three major schools, the Lyceum, the Garden, and the Stoa, and of a half dozen minor schools next to the major, including the sceptic, philosophy became the product of schools.

And this is what became of philosophy when it became the product of schools.

A further century on, and all schools produced the same philosophy. Which explains why the minor schools disappeared. The philosophy the schools produced was vaguely sceptic, certainly sceptic of the results of mathematics and physics. Which explains why the sceptic was one of the schools that disappeared. The philosophy the schools produced was ethics. The schools abandoned the fence, and the tree, and devoted themselves to the fruit. Needless to add, the philosophy the schools produced was practical. Controversy was confined to the flavor of the fruit.

Needless to add, the philosophy the schools produced was also paltry.

In 156 Rome ruled the earth from one end of the Mediterranean to the other. The senate ruled Rome. Polybius reports: in 156 the Athenians pillaged Oropus; the Oropians complained to the senate; the senate fined the Athenians 500 talents; and the Athenians sent to Rome the scholarch of the Academy Carneades, the scholarch of the Lyceum Critolaus, and the scholarch of the Stoa Diogenes the Babylonian to appeal the fine in the senate (XXXIII, 2).

The scholarch of the Academy, the scholarch of the Lyceum, and the scholarch of the Stoa did leave behind the record of other labors of theirs. They did not leave behind the record of labors of theirs more important than the urgently philosophical labor of appealing a fine in the senate, a fine for pillage, no less. Nor are the scholarch of the Academy, the scholarch of the Lyceum, and the scholarch of the Stoa not important. Carnea-

des is the most important philosopher the Academy produced after Plato. Under its first three scholarchs, Aristotle, Theophrastus, and Strato, the Lyceum was a school of physicists. Then it also became a school of ethicists, and Critolaus is the most important philosopher the Lyceum produced after Strato. The Stoa alone of the schools produced a philosopher more important than its founder. It produced Chrysippus. Diogenes the Babylonian is nevertheless one of the handful of important philosophers the Stoa produced after Chrysippus. As to the Garden, it sank into paltriness right after Epicurus. Which can explain why the Athenians, when they sent their best to Rome, had no scholarch of the Garden to send.

Nor was appealing a fine in the senate, for pillage, no less, as urgently philosophical as the labors of the philosophers became. A further century on, to make a tour of the schools became fashionable among the scions of the Roman aristocracy, and the schools fulfilled this new philosophical need, too. The scions matured, to maintain a domestic philosopher became fashionable among the Roman aristocracy, and there was no shortage of philosophers to fulfil this new philosophical need, too.

Timon is the contemporary also of the foundation of the Alexandrian library and of its first head, Zenodotus. The Alexandrian was the first and greatest of a growing number of libraries, and with libraries philological and antiquarian scholarship came into being.

One later philologist, as well as greater, writes apropos of antiquarian scholarship: it has an extremely limited view; most things it does not see at all; the little that it sees it sees in isolation; and since it has no standard, it takes everything to be equally important (Nietzsche, *History*, 3).

It has an extremely limited view. The little that it sees it sees in isolation. But it does not take everything to be equally important. And precisely because most things it does not see at all.

Zenodotus edited Homer, Hesiod, Anacreon, and Pindar. The crowning achievement of his labors was a Homeric glossary in alphabetical order.

There are exceptions. The most important is Eratosthenes, the third head of the Alexandrian library. Its first head is nevertheless representative of the scholarship that rose as the philosophical schools declined.

In 156, for instance, the Alexandrian library was under Aristarchus of Samothrace, its sixth head, and the last important scholar to be its head. On one side, the side of the acquisition of knowledge, Aristarchus of Samothrace came as close to it as to write a commentary on Herodotus. On the other side, he edited Homer, Hesiod, Archilochus, Alcaeus, Anacreon, and Pindar, and wrote commentaries on Homer, Hesiod, Archilochus, Alcman, Aeschylus, Pindar, Sophocles, and Aristophanes.

It is scholarship of editions, glossaries, and commentaries, and editions, glossaries, and commentaries on the poets. It is not even scholarship of editions, glossaries, and commentaries on the ethicists among the philosophers, never mind on the physicists or the mathematicians among them. Never mind venturing on the perilous path of the acquisition of knowledge itself.

Whether the development of scepticism, the development of the philosophical schools, and the development of philological and antiquarian scholarship are one another's causes and effects or not, they are, all three, the effects of the same cause.

xxvi

After Eudemus,
Alexander developed tactics more than any other general before or after, and, bequeathed the troops developed and seasoned by Philip, conquered the domain from the Adriatic Sea in the west to the valley of the Indus in the east,
Euclid deduced from 5 axioms, 5 postulates, and 131 definitions the 514 theorems, lemmas, and corollaries of the *Elements* – and the *Elements* remained the standard of rigor until Gauss and the paradigm of deduction until Frege – and wrote a treatise on optics,
Herophilus discovered the nerves and demonstrated that they originate in the brain, and, in general, developed anatomy,
Manetho wrote the history of Egypt,
Epicurus founded the Garden,
Zeno of Citium founded the Stoa,
Aristarchus discovered the method for measuring

the distances of the moon and the sun, and proposed that the earth revolves around the sun,

Erasistratus demonstrated that the veins and arteries originate in the heart, and, in general, developed physiology,

Archimedes showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$, solved the problem of the area of the parabola, solved the problems of the surface areas and volumes of the solids formed by the revolution of conic sections, developed the theory of equilibrium, and developed the theory of floating bodies,

Chrysippus discovered the rules of inference, and thus laid the foundations of the propositional calculus,

Eratosthenes measured the radius of the earth,

Hannibal discovered strategy, in the second Punic war, when in 218 he crossed the Alps and for a decade threatened the existence of Rome,

Scipio defeated Hannibal, strategically, and then also on the field, in Africa, in 202,

Cato was quaestor under Scipio in Africa in 204, aedile in 199, praetor in 198, consul in 195, celebrated a triumph in 194 after a command in Spain, was military tribune in the victory over Antiochus III in Greece in 191, censor in 184 – his career showing how, as Polybius formulated it: Rome brought nearly the entire known earth under a single government (I, 1),

Apollonius wrote the *Conics*, until Newton's *Principles* the most successful application of the model of the *Elements* to more complex problems, including problems of minima and maxima, and established that motion on an epicycle moving on a circle can reduce the motions of the planets to uniformity,

Polybius wrote the *Histories*, the third of the five – after the Sumerian prince list and Herodotus's and before Spinoza's and Voltaire's – most important historical treatises,

Hipparchus measured the distance of the moon, discovered the precession of the equinoxes, developed trigonometry, formulated the theory of the motions of the sun and the moon, and, in general, transformed astronomy and hence also geography into mathematical investigations,

Varro, in the *Antiquities*, investigated religion,

Cicero succeeded – after four centuries of attempts – in making philosophy into rhetoric,

Caesar subdued Gaul strategically, won the civil war strategically, and brought the single government of nearly the entire known earth under a single ruler, and wrote the commentaries on the Gallic war and on the civil war,

Augustus established the peace under which the government of nearly the entire known earth under a single ruler acquired permanence, and wrote *The deeds of the divine Augustus*,

Livy wrote the 142 books of *From the foundation of the city*,

Labeo transformed jurisprudence into a theoretical concern,

st Paul formulated the doctrine of salvation through the suffering, death, and resurrection of Jesus the Christ,

Hero wrote on measurement, and on the construction of measuring instruments, of catapults, of water clocks, of machines worked by compressed air, water, and steam, of machines lifting weights with the least effort, and, in general, transformed engineering into a theoretical concern,

Pliny wrote the 37 books of the *Natural history*,

Clement of Rome formulated the doctrine that god sent forth Jesus, Jesus gave the gospel to the apostles, and the apostles appointed bishops, to be succeeded by other bishops,

the *Gospel of Matthew* related the doings and sayings of Jesus,

Plutarch wrote 66 lives,

the *Gospel of Luke* and *Acts of the apostles* related the doings and sayings of Jesus, amplified by the doings and sayings of the apostles,

the *Gospel of John* and *Apocalypse* related the doings and sayings of Jesus, amplified by his doings and sayings at the beginning of the world and his doings and sayings at the end of the world,

Menelaus developed spherical trigonometry,

Ptolemy developed the theory of the motions of the planets, and catalogued 1 022 stars, in the *Almagest*, measured the distances of the moon, the sun, and the planets, in the *Planetary hypotheses*, developed the theory of map projections, and described 7 949 localities from the Canary Islands in the west to the gulf

of Tonkin in the east, assigning to 6 345 of them its latitude and longitude each, in the *Geography*, and fixed chronology absolutely, in the *Handy tables*,

Hadrian promulgated a codification of the law,

Julian edited the law into the codification promulgated by Hadrian,

Galen wrote the 115 treatises which superseded the Hippocratic corpus and remained the foundation of medicine until Vesalius,

Clement of Alexandria began the assimilation of pagan philosophy into Christian doctrines, transformed the catechetical school in Alexandria into a philosophical school, and began the transformation of Christian doctrines into philosophy,

Origen began the subjection of scriptures to philological and antiquarian scholarship, wrote exegesis and commentaries, wrote *On the principles*, and began the transformation of Christian philosophy into orthodoxy,

Plotinus succeeded – after eight centuries of attempts – in making mysticism into philosophy,

Laertius wrote *The lives and doctrines of the philosophers* in which he notes 1 273 predecessors of his and investigates 102 of them under his own head each,

Diophantus solved problems of determinate and indeterminate equations, and, in general, developed algebra, in the *Arithmetic*,

Diocletian reformed the government of nearly the entire known earth reaffirming its permanence,

and these are once again only some of the most important results of only some of the most important philosophers and princes in the six centuries from Eudemus to Diocletian.

The century of Eusebius, Constantine, Athanasius, Theodosius, and Augustine followed.

Before Augustine, then, the acquisition of knowledge made great strides.

Mathematics made great strides. The philosophers developed a deductive system of 514 theorems, lemmas, and corollaries derived from explicitly stated axioms, postulates, and definitions. They applied the model of this system to more complex problems. They established a lower and an upper limit for π, solved problems of areas and volumes, and of minima and maxima. They discovered the rules of inference. They developed the theory of map projections. Geometry developed, the theory of numbers developed, trigonometry developed, spherical trigonometry developed, algebra developed.

Physics made greater strides still. The philosophers assigned latitudes and longitudes to 6 345 localities from the Canary Islands in the west to the gulf of Tonkin in the east. They measured the radius of the earth. They measured the distance of the moon. They developed the theory of the motions of the sun, the moon, and the planets. They discovered the precession of the equinoxes. They catalogued 1 022 stars. They proposed that the earth revolves around the sun. They discovered how to measure the distances of all planets. Anatomy and physiology developed. Optics, statics, and hydrostatics developed, and developed as mathematical investigations from the first. Astronomy and geography became mathematical investigations. Engineering became a theoretical concern.

History made great strides as well. The philosophers wrote the history of Egypt. They wrote the history of Rome. They investigated religion. They wrote the history of philosophy, noting 1 273 predecessors of theirs, 102 under his own head each. They wrote a treatise which formulates a historical problem explicitly, then solves the formulated problem. The greatest statesman and greatest general humanity had seen wrote commentaries on his wars. In the next generation, again, the greatest statesman humanity had seen wrote down his deeds.

Whatever knowledge is, these results attest the acquisition of knowledge.

So does the development of tactics and strategy. So does the development of jurisprudence. So do the labors of bringing nearly the entire known earth under a single government, bringing this government under a single ruler, establishing the peace under which this government acquired permanence, promulgating the codification of the law under this government, and reforming this government to reaffirm its permanence.

These results attest the acquisition of knowledge. The remaining results do not.

The schools do not. Even if they were not founded to devote themselves to ethics – as the

fact that they professed to do, and some of them did do, logic and physics shows – they devoted themselves to ethics. More generally, even if the two ends, truth and action, are not incompatible, and the schools were founded to search for truth first and for action only on this ground, on the ground of the results of the search for truth, two centuries on, and the schools' search for action was not contaminated by search for truth.

Rhetoric does not attest the acquisition of knowledge either.

Cicero reports: during the civil war I was unable to serve the republic (*Divination*, II, 6), and: wondering how to do good to as many as possible and thus yet serve the republic, I saw no better way than to lead my fellow-citizens to knowledge (1). Cicero enumerates the treatises he has written, and adds: and had an even graver cause not intervened, there would have been no philosophical matter which I would not have presented in the Latin tongue (4). The philosophical matters presented in Cicero's treatises in the Latin tongue do not include mathematical matters, or physical matters, or historical matters. In any case, the presumption that philosophy coincides with politics, or ought to coincide, and politics coincides with oratory, or ought to coincide, can be traced back to Protagoras, Socrates, and Plato, and it is in Plato as explicit as this: until the philosophers become kings or the kings become true philosophers, the states will have no respite from evil (*Republic*, V, 473c-e), and: no one can speak well on any matter unless he has pursued philosophy properly (*Phaedrus*, 261a). If so, service to the republic – and Cicero was quaestor in 75, aedile in 69, praetor in 66, consul in 63 – could only reaffirm the presumption. And Cicero is as explicit as this: the best is the orator who is also a philosopher; and if orator and philosopher are distinct, the best is the orator who has all the knowledge which the philosopher has (*On the orator*, III, 142-3). Cicero's service to the republic, including the presentation of philosophical matters in the Latin tongue, if not quite all of them, shows that he labored to elevate the orator into a philosopher. His successors would take him to elevate the philosopher into an orator. It would not help that: nothing is more important in life than philosophy, and nothing is more important in philosophy than to ask what is the end (*Ends*, I, 11). *Summum bonum*, not truth. Accordingly, rhetoric was to teach not only what rhetoric is, rhetoric was to teach what geometry is.

In the couple of centuries after Cicero philological and antiquarian scholarship was elevated into philosophy, too. The philosophers became commentators. Some of these philosophers are important, Alexander of Aphrodisias in the third Christian century, Porphyry and Iamblichus in the fourth, Proclus in the fifth, Simplicius and Eutocius in the sixth. The majority, by far, are not important at all, and the philosophers remained commentators, more or less exclusively, for the following millennium.

Philological and antiquarian scholarship was elevated into physics and into history. Pliny and Plutarch are both very important philosophers. Nevertheless, the great virtue of Pliny's *Natural history* is that it is a collection of many earlier results. And Plutarch is an indispensable source, but he is first of all a very fine writer, and the 66 lives, of which 50 survive, are lives indiscriminately of important predecessors of his and of not important, of real predecessors of his and of fictional.

If Galen's 115 treatises superseded earlier medicine, too, it was not so much because Galen developed anatomy and physiology as it was because he developed therapy.

The remaining results attest the development of Christian doctrines, of Christianity, of Christian doctrines into philosophy, and of Christian philosophy into orthodoxy. The fertile ground of this development – including most particularly the development of Christian doctrines into philosophy, and of Christian philosophy into orthodoxy – were the schools, rhetoric, and philological and antiquarian scholarship. And this development is explicit from the first, and ever more aggressively explicit, that it does not attest results from the acquisition of knowledge, it attests results against the results from the acquisition of knowledge.

St Paul formulated the doctrine of salvation through the suffering, death, and resurrection of Jesus the Christ. Among the *Acts of the apostles* there is also this act of the apostle: he came to Athens and the philosophers went to hear what he

had to say; and when he brought up the raising of the dead, some scoffed, others said, we will listen to this some other time (17). Later on, no one scoffed, all listened. For the apostle had warned: knowledge breeds conceit, it is love that builds (*First Corinthians*, 8).

The doctrine that: god sent forth Jesus, Jesus gave the gospel to the apostles, and the apostles appointed bishops, to be succeeded by other bishops (Clement of Rome, *Letter*, 42-4), is not just practical, it is so urgently practical it had to be committed to writing as the gospel was being committed to writing.

The doings and sayings of Jesus, such as they are, were not left to fend for themselves. They were amplified by the doings and sayings of the apostles, supplying the evidence for the doctrine that Jesus gave the gospel to the apostles, who appointed bishops, to be succeeded by other bishops. They were amplified by the doings and sayings of Jesus at the beginning of the world and at the end of the world, supplying the evidence for the doctrine that god sent forth Jesus. And if the facts that, with respect to the former, the question of who appointed the bishops kept being answered on the battlefield, and, with respect to the latter, the question of whether god sent forth himself kept being answered by the extermination of heresy, if these facts do not show how strong the evidence is, they do show how practical it is.

With the labors of Clement of Alexandria and Origen pagan philosophy began being assimilated into Christian doctrines, Christian doctrines began becoming philosophy, and Christian philosophy began becoming orthodoxy. Clement of Alexandria and Origen are among the earlier fathers of the church, two centuries later Augustine is among the later fathers of the church, and the fathers of the church are the fathers of the church because they made all the following impossibilities first tolerable, then routine, then mandatory. That philosophy would be pagan and Christian. That the kinds of philosophy would be kinds of religion. That Christian doctrines would be philosophy. That the doctrines of a religion would be philosophy. That Christian philosophy would be orthodoxy. That any kind of philosophy would be orthodoxy, and that there may even be a kind of philosophy that could not be philosophy without being orthodoxy. That, even when stripped of physics and mathematics, including logic, philosophy would tolerate salvation. Or resurrection. Or knowledge breeds conceit, it is love that builds. Or who appointed the bishops. Or the doings and sayings of Jesus at the beginning of the world. Or at the end of the world.

A generation after Origen, Plotinus succeeded in making mysticism into philosophy. The cause is the same, the cause of the success of the fathers and the cause of his success.

In the four-odd generations from Empedocles to Eudemus the acquisition of knowledge quickened. In the six centuries from Eudemus to Diocletian, at first, for one century or so, it continued apace, then, in the following five centuries, it slowed down, and more, and more. The century of Eusebius, Constantine, Athanasius, Theodosius, and Augustine followed. And the acquisition of knowledge was not just stopped, it was hurled violently back.

Eusebius wrote *The history of the church*. And *The history of the church* is in some ways like the history of the great kings, the history of the war of the Athenians and the Spartans, the history of physics, the history of Egypt, the history of Rome, the history of philosophy. But not in all ways. Two generations before Eusebius, himself a father of the church, another father of the church, Cyprian, writes: he cannot have god for his father who does not have the church for his mother (*Unity*, 6). Taking it for granted that they have god for their father, Cyprian and Eusebius are both fathers of the mother. And *The history of the church* is the history of their mother.

Constantine convoked the council of Nicaea, presided over its opening session, and steered its proceedings throughout. The single ruler of the single government of nearly the entire known earth convoked and steered the first ecumenical council. And it was not that the state had found itself a religion. The religion had found itself a state.

Athanasius defended the doctrine of the consubstantiality of father and son, then also of the divinity of the holy spirit, then also of the consubstantiality of all three, father, son, and holy spirit, and made the defense of the doctrine a war in which no compromise was brooked, no quarter

was given, no prisoners were taken. At the first ecumenical council the religion proclaimed itself in possession of a state. The religion proclaimed itself also dogma, the Nicene creed. And the Nicene creed proclaimed the victory of Athanasius. Victorious was the doctrine of consubstantiality of the trinity – and this doctrine became the heart of Christian philosophy at the hands, in the next generation, of the Cappadocian fathers, Basil of Caesarea, Gregory of Nazianzus, and Gregory of Nyssa, and, in the generation after that, of Augustine. More important, victorious was Athanasius's way of waging war, no compromise brooked, no quarter given, no prisoners taken. For his enemy Arius was vanquished, and the first ecumenical council was a decisive step in the extermination of the heresy of Arius, but it was only a step. The heresy of Arius took centuries to exterminate. And there was always one heresy or another to exterminate. Augustine, for one, fought the heresy of Mani, then also the heresy of Donatus, then also the heresy of Pelagius.

Theodosius, the single ruler of the single government of nearly the entire known earth, made the war for orthodoxy a war of the single government of nearly the entire known earth.

Augustine, finally, brought the transformation of both Christian doctrines into philosophy and Christian philosophy into orthodoxy to triumphant conclusion.

On the first count, among his most important results are these. The removal of uncertainty on the matter of consubstantiality, in *On the trinity*. The removal of uncertainty on the matter of grace, in *On nature and grace*. The removal of uncertainty as to what the philosopher is, in *The city of god against the pagans*: the true philosopher is the lover of god (VIII, 1).

On the second count, the removal of uncertainty as to what orthodoxy is came in the next generation, from one of the latest fathers of the church, Vincent of Lérins. Vincent of Lérins writes: every care should be taken to hold fast to what is believed everywhere, always, by all (*Reminders*, 2).

So, salvation, resurrection, who appointed the bishops, the doings and sayings of Jesus, were believed everywhere, always, by all. The trinity and grace were believed everywhere, always, by all.

That the true philosopher is the lover of god was believed everywhere, always, by all.

And everywhere, always, by all was believed that the acquisition of knowledge is noxious.

Augustine writes, in *On the Christian doctrine*: whatever man learns outside the holy scriptures, if noxious, is to be condemned, if beneficial, is in them contained (II, 63).

There was no uncertainty as to what is outside the holy scriptures, what is in them contained, either. *On the Christian doctrine* attests the closure of the canon of the Christian scriptures, too.

Hence, should it have seemed that the proof that $3\frac{10}{71} < \pi < 3\frac{1}{7}$, or the theory of the motions of the planets and the catalogue of 1 022 stars and the latitudes and longitudes of 6 345 localities, or the history of the great kings, although they are not in the holy scriptures contained, are not noxious, they are noxious.

Augustine writes, in the *Confessions*: the futile curiosity masquerading under the name of knowledge scripture calls gratification of the eye; it is for the same reason, to satisfy that sick curiosity, that freaks are put on show in the theater and men investigate nature and want knowledge for the sake of knowledge (X, 35).

Archimedes, Ptolemy, and Herodotus satisfy that sick curiosity. As far as the circle is concerned, π is 3 is beneficial, as it is for Solomon's temple (*Kings*, I, 7). As far as the heaven is concerned, god's handiwork is beneficial, described as it is in a couple of verses (*Genesis*, 1), and as far as the earth is concerned, also god's handiwork is beneficial, described this time in two dozen verses, demanding as the descendants of Noah are of more attention in order to populate the earth (10). As far as the great kings are concerned, beneficial is that: thus said the lord to Cyrus, you do not know me, but I have given you your strength (*Isaiah*, 45).

The darkest of dark ages, in short, is not *ex nihilo*. It has a cause.

xxvii

After Diocletian,
and the century of Eusebius, Constantine, Atha-

nasius, Theodosius, and Augustine,
Leo was pope 440-61,
Gelasius was pope 492-6,
Tribonian edited the law into the codification promulgated by Justinian, including the collection of all earlier jurisprudence in the *Digest*,
Justinian was emperor 527-65, and promulgated the codification of the law, including the *Digest*,
Gregory was pope 590-604,
Muhammad became the prophet,
Umar was caliph 634-44, and had the koran committed to writing,
Uthman was caliph 644-56, and had the koran edited into a canonical version and all other versions in circulation suppressed,
Muawiyah was caliph 661-80,
al-Mansur was caliph 754-75,
Charlemagne was crowned emperor in 800,
al-Shafii formulated the doctrine that the sunnah, next to the koran, has divine authority, in the *Treatise on the sources of the law*,
al-Mamun was caliph 813-33,
al-Khwarizmi solved quadratic equations, in the *Algebra*,
al-Razi made chemical experiments, in the *Book of secrets*,
al-Battani improved on Ptolemy's parameters, in the *Astronomical treatise*,
al-Farabi established the importance of Aristotle, and argued that philosophy is superior to revelation, for it attains truth, while revelation only gives a sign of it for not philosophers, in *The views of the inhabitants of the ideal city*,
Otto was crowned emperor in 962,
Basil II was emperor 976-1025,
ibn al-Haytham examined propositions of Euclid and Ptolemy, in the *Optics*,
al-Biruni investigated time reckoning, in *The monuments of past generations*,
ibn Sina made the most substantial contribution to therapy since Galen, in the *Canon*,
Gregory VII was pope 1073-85,
Anselm proposed to prove the existence of god, in the *Monologue* and in the *Prologue*,
Urban II was pope 1088-99,
Khayyam solved cubic equations, in the *Demonstration of problems of algebra*,
and these are yet again only some of the most important results of only some of the most important philosophers and princes in the eight centuries from Diocletian to Khayyam.

Then al-Ghazali did for Islam what Augustine had done for Christianity, ushered in its dark age.

In the eight centuries before al-Ghazali the acquisition of knowledge did not make great strides.

In these centuries the empire declined, the papacy rose, the caliphate rose, and a second empire appeared in the west. And even if these four developments are all the effects of the same remote causes, they are yet one another's proximate causes and effects.

In 12^{BC} Augustus became chief pontiff, chief of the most powerful of the four colleges of Roman priests, and after him the princeps, or the emperor, was chief pontiff *ex officio*. Until Gratian, emperor $375\text{-}83^{AD}$. Urged by Ambrose, the bishop of Milan, Gratian removed the statue of victory from the senate, and relinquished the office of chief pontiff. Zosimus reports: if the emperor will not be chief pontiff, the priests told Gratian, someone else will (IV, 36). Someone else was, and two generations later, Valentinian III, emperor 425-55, promulgated this law: in support of that venerable man, Leo of Rome, pope of the believers, we decree by a perpetual edict that whatever be sanctioned by the apostolic see of st Peter, by the princeps of the bishops, shall be law to everyone (*Th. code*, nov. 17). It may have then seemed uncertain which is the law, whatever be sanctioned by the emperor or whatever be sanctioned by the princeps of the bishops, pope of the believers. A generation-odd later, Gelasius, the princeps of the bishops, pope of the believers, removed the uncertainty. Gelasius writes to Anastasius, emperor 491-518: two, august emperor, rule the world, the sacred authority of the pontiffs and the power of the princes; and of these the authority of the pontiffs is weightier than the power of the princes, because the pontiffs have to render an account to god even for the princes; and you know, my clement son, that while you are permitted to rule over mankind, in things divine you humbly bow your head before those preeminent in god's works and beg of them the means of your salvation (to Anastasius, 494).

The empire rose again under Justinian, and his domain extended from the Atlantic in the west to the Euphrates in the east.

Justinian's successors lost the west, and Gregory became the first pope to exercise the sacred authority of the pontiffs and the power of the princes, both.

When Charlemagne's domain extended over much of what had been the western half of the empire, he was crowned emperor. Leo III, pope 795-816, crowned him.

In the east, the empire lost much of its domain to the caliphate. Then the caliphate declined, and the empire rose once again, reaching a new height under Basil II, his domain extending from the Tyrrhenian Sea in the west to the Euphrates in the east. Then the empire declined once again, in 1071 it lost the battle of Manzikert to its enemies in the east, and in 1204 it was destroyed. It was destroyed not by its enemies in the east, the infidels, however, but by its friends in the west, the soldiers of Christ of the forth crusade.

In the west, it remained unclear who makes whom, the emperor the pope or the pope the emperor. The empire rose under Otto and his successors, and Otto and his successors made the abbots, the bishops, and the popes.

Gregory VII asserted the supremacy of the pope over the emperor. One and a half centuries of struggle for supremacy between the popes and the emperors followed, and the popes emerged victorious from the struggle. It proved a Pyrrhic victory. After the thirteenth Christian century empire and papacy both declined.

Gregory VII also conceived the idea of an eastern crusade. Urban II realized the idea. In 1095 he called on Christendom: sons of god, hasten to the way, help your brothers in the east; what a shame if a people despised, degenerate, and enslaved by demons should overcome a people endowed with the trust of god and shining in the name of Christ; all of you, therefore, of whatever station, knight and footman, rich and poor, go forth and expel that wicked race from our Christian lands; fight the infidel; be soldiers of Christ; vanquish the barbarian (Fulcher, 3). The following year the sons of god, knight and footman, rich and poor, hastened to the way.

They took Jerusalem in 1099. In 1187 Salah al-Din took it back.

Islam did not pose the difficulties whether the power to rule over mankind or the sacred authority in things divine is weightier, and whether one or the other should lead the faithful. The prophet wielded the power to rule over mankind and the sacred authority in things divine inseparably and unflinchingly, and if the hijra, in 622, was peaceful, what followed it was not. The prophet led the faithful in the battle of Badr, in 624, and in the conquest of Mecca, in 630.

Accordingly, under Umar the caliphate broke out of the Arabian desert explosively. Under Muawiyah it reached from the gulf of Tunis in the west to the Syr Darya in the east. One century-odd after the hijra, when al-Mansur established the peace under which Islam acquired permanence, the caliphate reached from the Atlantic in the west to the Indus in the east. Islam acquired permanence, but the caliphate did not. After al-Mamun, two centuries-odd after the hijra, the caliphate declined.

Whether the empire, the papacy, and the second empire in the west attest the acquisition of knowledge or not, Christianity attests the darkest of dark ages. And it is the darkest. It extinguished a very bright light before it. It continued for not less than six and a half-odd centuries, counting from Augustine to Anselm, more than a quarter of the twenty-six centuries since Thales. And it achieved spectacular intensity.

How spectacular the following results can show.

There was no question whether there are angels. This question had been settled so conclusively, Augustine devotes four of the twenty-two books of *The city of god*, books VII-X, to demons and angels. There was still the question about the kinds of angels, though, and one century after Augustine a convert of st Paul's, Dionysius the Areopagite, was brought to life to settle that question, too, by authoring the *Celestial hierarchy*. Highest to lowest, the kinds of angels are: seraphim, cherubim, thrones, dominions, powers, authorities, principalities, archangels, and angels (7-9).

Gregory established the temporal power of the papacy. Among his more theoretical results, collected in the *Dialogues*, are: the fire of hell is

corporeal, and bodies are tortured in it (IV, 30); hell is under the earth (44); and torture there is for all eternity (46).

Gregory VII asserted the supremacy of the papacy over the empire. Among his more theoretical results, collected in *The dictate of the pope*, are: no man can judge the pope (19), and: the church of Rome never has erred, never shall err (22).

In fact, in all of Christendom, in the six and a half-odd centuries between Augustine and Anselm, but one result was obtained which attests the acquisition of knowledge unambiguously, the *Digest*. In the *Digest* all earlier jurisprudence is reduced to 9 201 excerpts from the treatises of 38 jurists, the excerpts organized under 431 heads, in 50 books, divided into 7 parts. No matter has ever been analyzed more decisively.

Whether the caliphate attests the acquisition of knowledge or not, Islam attests the descent of its own dark age.

St Paul warns: knowledge breeds conceit, it is love that builds. Muhammad warns: the only true faith in the sight of god is submission (3.19), and: god knows, you do not know (2.216).

So, three and a half centuries after st Paul, Augustine established that: whatever man learns outside the holy scriptures, if noxious, is to be condemned, if beneficial, is in them contained. Umar obtained the same result less than a generation after Muhammad.

Ibn Khaldun reports: when the Muslims conquered Persia and came upon a large number of books, ibn Abi Wakkas wrote to Umar, asking for permission to distribute the books as booty among the Muslims; throw them in the water, Umar wrote back, if they contain right guidance, god has given us better guidance, if they contain error, god has protected us against it; so they threw the books in the water and in the fire (*Introduction*, VI, 18).

The koran became orthodoxy. The sunnah also became orthodoxy.

The Muslims nevertheless succumbed to temptation. Ibn Khaldun continues: Muslim rule flourished, the Muslims settled, became versed in the crafts and sciences, and began to desire to study philosophy; so al-Mansur sent envoys to the Roman emperor to ask for books, and the emperor sent him back Euclid's book and some writings on physics (*id.*).

The Roman emperor was Constantine V, the emperor who halted the century-odd retreat of the empire before the caliphate, and he was, clearly, as felicitous in his choice of books as he was on the battlefield.

These, then, are the greatest successes of the acquisition of knowledge in the eight centuries between Diocletian and Khayyam. In mathematics, the philosophers investigated equations, and, in general, developed algebra. They solved cubic equations by intersecting conics. In physics, they made chemical experiments. They improved on Ptolemy's astronomical parameters. They examined optical propositions of Euclid and Ptolemy, including experimentally. They correlated the time reckoning of different peoples. They developed therapy. Most important, they established the importance of Aristotle, and on this result the development of philosophy in the following seven centuries – through Aquinas and further – was to depend essentially. When al-Farabi argued also that philosophy is superior to revelation, for it attains truth, while revelation only gives a sign of truth for not philosophers, however, on this result the development of philosophy was not to depend essentially until Galileo.

The latest of these results is Khayyam's solution of cubic equations, and Khayyam is an older contemporary of al-Ghazali. So: once, al-Ghazali asked Khayyam how the polar region is distinguished in the heaven; Khayyam began by explaining motion; but as he continued, noon came, and the muezzin called to prayer; al-Ghazali rose and said, truth has come and falsehood vanished (al-Bayhaqi, *Addition*, 66).

The most important treatises of al-Ghazali are *The incoherence of the philosophers* and *The revival of the religious sciences*.

With *The incoherence of the philosophers* he did not just establish the incoherence of the philosophers. He writes in conclusion: if someone asks, do you then say that the philosophers are infidels and those who hold their beliefs must be killed? I answer, the philosophers must be pronounced infidels on three counts, their claim that the world is eternal, their claim that god does not know the particular, and their denial of the resur-

rection of the body; these doctrines are manifest infidelity (concl.). Later on, in the *Deliverance from error and union with the almighty*, he adds: in *The incoherence of the philosophers* I have established that the errors of the philosophers are of twenty kinds, and on account of three of these they must be pronounced infidels, on account of the other seventeen they must be pronounced heretics (2.2). When two generations later ibn Rushd wrote *The incoherence of the incoherence*, then, had al-Ghazali established just the incoherence of the philosophers, ibn Rushd could have shown the coherence of the philosophers. As it was, he had to show the coherence of heresy and infidelity.

With *The revival of the religious sciences* al-Ghazali made sufism orthodox.

And between these two results al-Ghazali did even more to usher in the Muslim dark age than Augustine did to usher in the Christian.

The Christian and the Muslim dark ages did not coincide. Although they could have.

In any case, when al-Ghazali's older contemporary Anselm proposed to prove the existence of god, his proof was noxious. For neither his proof is in the holy scriptures contained nor is any proof in the holy scriptures contained.

xxviii

After Khayyam,

Abelard wrote the *Yes and no*, collecting 158 questions, and the answers of the fathers of the church, yes and no, both, to each of the 158 questions,

Otto of Freising wrote *The history of the two cities*,

Gerard translated the 71 treatises whereby the Christian philosophers learned the pagan philosophers from the Muslim philosophers,

Frederick was emperor 1152-90,

ibn Rushd wrote the commentaries on Aristotle which had the Christian philosophers, when they judged Aristotle the philosopher, judge him the commentator,

Philip II was king 1180-223,

Innocent III was pope 1198-216,

Genghis Khan campaigned from the Pacific in the east to the Black Sea in the west,

Fibonacci solved quadratic equations and investigated series, in the *Book of calculation* and *Book of squares*,

Frederick II was emperor 1212-50, promulgated the most important codification of the law between Justinian and Napoleon with the constitutions of Melfi, and laid the foundations of ornithology with *On the art of hunting with birds*,

Albert wrote the five books *On minerals*, the seven books *On plants*, and the twenty-six books *On animals*,

Louis IX was king 1226-70,

Roger Bacon argued that there can be no knowledge without either experience or mathematics, in *The great work*, anticipating Bacon, Galileo, Kepler, and Descartes,

Aquinas wrote the *Theological summa*,

Edward was king 1272-307,

Polo voyaged from Italy to China and back, and described the voyage in the *Description of the world*,

Eckhart found out that the soul can not only achieve union with god, it can become god, indeed rise above god,

Orhan was sultan 1326-62,

Edward III was king 1327-77,

Occam wrote the *Summa of all logic*,

Buridan formulated the theory of impetus, in the *Questions on the Physics* and *Questions on On the heaven*, anticipating the law of inertia,

ibn Battuta voyaged from Morocco to China and back, and described the voyage in the *Travels*,

Oresme investigated relations of physical quantities mathematically, in *The configurations of qualities and motions*, anticipating Galileo, and investigated infinite series, in the *Questions on the geometry of Euclid*,

ibn Khaldun proposed to ground history in the investigation of the development of civilization, in the *Introduction to history*,

John was king 1385-433,

Bruni wrote *The history of the Florentine people*,

prince Henry sent his knights on the voyages which took them beyond cape Bojador, then steadily further down the African coast,

Charles VII was king 1422-61,

Ficino found out that Plato will sustain Christian piety more readily than Aristotle and more respectably than Plotinus,

Regiomontanus formulated the laws of sines and cosines for spherical triangles, in *On triangles of all kinds*,
Mehmed II was sultan 1451-81,
Louis XI was king 1461-83,
Ivan III was prince 1462-505,
Cão crossed the equator,
Dias rounded the cape of Good Hope,
Isabella was queen 1474-504,
Ferdinand V was king 1479-516,
Columbus crossed the Atlantic,
da Gama reached India,
Albuquerque built a fortress on Socotra and captured Hormuz in 1506-7, to gain control of the Red Sea and the Persian Gulf, and captured Goa and Malacca in 1510-1, to gain control of the Indian Ocean,
Erasmus proposed to return, against the scholastics, to a simple philosophy of Christ,
Machiavelli showed that politics, the search for action, can be a search for real action, as distinct from fictional, by grounding itself in the search for truth, in history, in *The prince* and *Discourses on Livy*,
Cortés captured Mexico in 1519-21,
Magellan began the circumnavigation of the earth,
El Cano completed the circumnavigation of the earth,
de Toreno drew the first map of the earth in its true proportions,
Pizarro captured Peru in 1531-3,
Copernicus showed that the theory of the motions of the planets, which stands in Ptolemy on the hypothesis that the earth is in the center of the world and the planets revolve around the earth, stands more firmly on the hypothesis that the sun is in the center of the world and the planets, one of which the earth is, revolve around the sun, in *The revolutions of the heavenly spheres*,
Guicciardini wrote *The history of Italy*,
Luther found out that reason is a pestilential beast,
Henry VIII was king 1509-47,
Charles V was emperor 1519-56,
Suleiman was sultan 1520-66,
Cardano formulated a general method for solving cubic equations, in *The great art*,
Vesalius wrote and designed the illustrations for *The structure of the human body*, vastly improving observation in anatomy, hence also in physiology, and in general in biology,
Elizabeth was queen 1558-603,
Maurice was stadholder 1585-625,
Viète developed general methods for solving equations, in *The analytic art*, beginning the generalization of algebra which led to the development of analysis,
Gilbert discovered that the earth is a magnet, and distinguished electric from magnetic phenomena, in *On the magnet, magnetic bodies, and the great magnet of the earth*,
Tycho discovered a new star, vastly improved observation in astronomy, and undertook the *Instauration of astronomy*,
Bruno was burned in 1600,
Bacon undertook *The great instauration*, of all sciences, and grounded in the *New organon*, the earlier, before Descartes's, of the two most important treatises on the method,
Galileo discovered the four major satellites of Jupiter, in the *Starry messenger*, proved that the earth revolves around the sun, in *The two main systems of the world, Ptolemaic and Copernican*, established a new standard of observation and experiment and showed that proof can be mathematical, and if it can be it must be, in mechanics and locomotion, hence, more generally, in physics, and hence, more generally still, in the acquisition of knowledge, in the *Two new sciences dealing with mechanics and locomotion*, and established, most generally, that philosophy is written in that grandest book, the world, and that grandest book, the world, is written in mathematics,
Kepler discovered laws, the laws of the motions of the heavenly bodies, the first and the second in the *New astronomy*, the third in the *Harmonics of the world*,
Hobbes showed that the investigation of the state can be, and if it can be it must be, grounded in the investigation of nature, in physics, in the *Leviathan*,
Gustav II was king 1611-32,
Descartes formulated the rules of the method in the *Discourse on the method*, laid the foundations of analysis with the discovery that any

curve can be expressed by a single equation, in the *Geometry*, itself a part of *The method*, and showed that all phenomena, including explicitly those beyond the solar system, can be deduced from the same explicitly stated principles, in *The principles of philosophy*,

Cavalieri discovered integration,

Cromwell was lord protector 1653-8,

Fermat discovered differentiation, solved the problem of maxima and minima, and with the result, if p is a prime, and a is a positive integer, $a^p - a$ is a multiple of p, among many others, developed the theory of numbers,

Torricelli measured air pressure,

Cassini measured the parallax of Mars, hence the parallax of the sun, and hence the solar system,

Boyle discovered the first gas law,

Huygens solved the problem of collisions, discovered the conservation of momentum, discovered the isochronism of the cycloid, determined the acceleration of gravity, and formulated the wave theory of light,

Spinoza solved the problem of when and how the Hebrew scriptures were authored, showing that the method of formulation and solution of historical problems is the method of the investigation of nature, in the *Theologico-political treatise*, and, in general, that history can, and if it can it must, ground itself in physics,

Louis XIV was king 1643-715,

Hooke discovered the cell,

Leeuwenhoek discovered protozoa,

Newton developed calculus, in *On analysis by equations of infinite number of terms* and *The method of fluxions and infinite series*, and discovered the laws of motion, and the law of gravity, and grounded the description of the system of the world in the laws, in *The mathematical principles of natural philosophy*,

Rømer measured the speed of light,

Leibniz discovered that the principle of sufficient reason is as fundamental as the principle of contradiction, described the possibility of a philosophical calculus in which all things and their relations are expressed formally, developed calculus, discovered living force, twice the kinetic energy, and its conservation, and made the history of the earth an empirical investigation,

Jakob Bernoulli discovered the law of large numbers,

Halley discovered the proper motion of stars,

Peter was tsar 1682-725,

Berkeley established idealism,

Montesquieu proposed to explain differences in the laws of countries by differences in their natural conditions, in *On the spirit of the laws*,

Voltaire wrote *The century of Louis XIV*, the last of the five – after the Sumerian prince list, Herodotus's, Polybius's, and Spinoza's – most important historical treatises,

Maupertuis formulated the principle of least action,

Daniel Bernoulli discovered the principle of superposition of waves,

Franklin formulated the one-fluid theory of electricity,

Linnaeus solved the problem of the classification of organisms, in *The system of nature*,

Euler established the fundamental character of functions, developed the theory of numbers, where one of his many results is factoring $2^{2^5}+1$, investigated series, where one of his many results is showing that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, investigated complex numbers and logarithms of complex numbers, showed that $e^{i\pi} = -1$, developed the calculus of variations, developed topology, reduced analysis to its elements, in the *Introduction to the analysis of infinities*, *Instruction in the differential calculus*, and *Instruction in the integral calculus*, formulated mechanics analytically, and discovered the general wave equation,

Buffon conducted the experiments showing that the earth is at least 74 832 years old, in the first supplementary of the 36 volumes of the *Natural history*,

Frederick II was king 1740-86,

Pitt was prime minister 1756-61 and 1766-8,

Hume showed that all inferences, which we possess in common with the beasts, terminate in experience, in the *Treatise of human nature* and *Inquiry concerning human understanding*,

Rousseau wrote the *Discourse on the origin and foundations of inequality*, but found out also that reason can be cowed by feeling, in his

discourses, the more so in his novels, and the more so in his confessions,
d'Alembert formulated mechanics analytically, in the *Treatise on dynamics*, and discovered the wave equation,
Kant formulated the theory of the origin of the solar system, in the *General natural history and theory of the heaven*, and investigated knowledge in all its generality, showing also that the ground of *a priori* judgments is experience, in the *Critique of pure reason* and *Prolegomena to any future metaphysics*,
Lagrange deduced mechanics from a single principle, in the *Analytic mechanics*,
Gibbon wrote *The history of the decline and fall of the Roman empire*,
Herschel discovered Uranus, discovered the proper motion of the solar system, and mapped the Milky Way,
Joseph II was emperor 1765-90,
Washington was president 1789-97,
Lavoisier formulated the theory of combustion, oxidation, and calcination, decomposed water into hydrogen and oxygen, formulated the principle of conservation of matter, compiled the first table of the elements, in the *Elementary treatise on chemistry*, and showed, with Laplace, that respiration is combustion,
Lamarck formulated a theory of the development of animals, in the *Zoological philosophy*,
Laplace discovered the Laplace equation, $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$, the foundation of potential theory, developed the theory of the origin of the solar system, in the *Exposition of the system of the world*, showed the dynamic stability of the solar system, wrote the *Celestial mechanics*, showed, with Lavoisier, that respiration is combustion, and defended unconditional determinism, in the *Philosophical essay on probabilities*,
Sieyès wrote *What is the third estate?* in 1789,
Schiller reformed history,
Robespierre led the committee of public safety in 1793-4,
Dalton developed the atomic theory, in the *New system of chemical philosophy*,
Napoleon was emperor 1804-15, promulgated a codification of the law, and developed strategy more than any other general before or after,
Wellington defeated Napoleon,
Fourier analyzed heat diffusion by infinite series, in the *Analytic theory of heat*,
and these are once again only some of the most important results of only some of the most important philosophers and princes in the seven-odd centuries from Khayyam to Fourier.

Then spirit demanded from philosophy the restoration of the substantiality and purity of its being, and Hegel complied.

xxix

The papacy reached the pinnacle of its power under Innocent III. Princes from the king of England in the west to the crusader kings in the east gave him obeisance. The university of Paris, chartered in 1200, received his sanction in 1215 to become the main center of learning of Christendom. The most powerful emperors after Otto, Frederick and Frederick II fought for a century to break the power of the popes, and failed. Innocent III writes: as the creator set up two great luminaries in the firmament of heaven, the greater to rule the days, the lesser to rule the nights, so he set up in the firmament of the catholic church two great dignities, the greater to rule the souls, which are like the days, the lesser to rule the bodies, which are like the nights; these two are the authority of the pontiffs and the power of the princes; and as the moon, being inferior in both size and quality, derives its light from the sun, so the power of the princes derives its dignity from the authority of the pontiffs (to Acerbus, 3 Nov 1198).

Genghis Khan, presumably, was a comet.

When the sun and the moon began to set – and this was one of the main causes they began to set – France and England began to rise. In general, the six centuries from Philip II to Napoleon can be described as the rise of France and England and their struggle to become the dominant power on earth. England won the struggle. It became the dominant power on earth. In those centuries also rose, west to east, Portugal, Castile and Aragon then united into Spain, the United Provinces, Sweden, Austria, Prussia, the Ottoman empire, and Russia, over all of which France and England, and ultimately England alone, proved victorious.

The rise of France began with Philip II and Louis IX, as the crown established its supremacy over the feudal lords. Further afield, Louis IX led the sixth and the seventh crusades, as Frederick had led the third, and Frederick II the fifth. The rise of England began with Edward, again as the crown established its supremacy over the feudal lords, and further afield, Edward added Wales to his domain. In the fourteenth and fifteenth centuries England and France fought the hundred years' war. Edward III commenced it. Charles VII brought it to victorious conclusion. Louis XI added Burgundy to his domain, and at the end of the fifteenth century France was one of the two most powerful states west of the Ottoman empire. The other was Spain. England became as powerful in the sixteenth century, under Henry VIII and Elizabeth. Humanity had been neglecting its armies and navies since the Romans. Charles VII and Louis XI began building what under their successors became the most powerful army humanity had seen. Henry VIII and Elizabeth began building what under their successors became the most powerful navy humanity had seen.

The rise of Castile and Aragon began in the thirteenth century with the reconquest of the peninsula conquered by the Muslims five centuries earlier. The rise of the Ottoman empire began in the fourteenth century, under Orhan, reached a peak under Mehmed II, and reached its highest peak in the sixteenth century, under Suleiman, whose domain stretched from the western Mediterranean to the middle Danube to the Persian Gulf. The rise of Portugal began in the early fifteenth century, under John, continued when prince Henry commenced the exploration of the African coast, continued when exploration and conquest reached India, of the conquest of the seas Albuquerque's campaign is the most spectacular example, and in the sixteenth century Portugal contested with Spain the dominion over the seas. Isabella and Ferdinand V united Castile and Aragon into Spain, sent Columbus on his voyage, conquest as well as exploration reached America, of its conquest the campaigns of Cortés and Pizarro are the most spectacular examples, Charles V sent Magellan and El Cano on their voyage, exploration and conquest encircled the earth, and when Charles V was crowned emperor in 1530 over his domain the sun never set. His successors lost the northern Netherlands, and the United Provinces rose under Maurice. In the course of the thirty years' war Sweden rose under Gustav II. As the Ottoman empire declined Austria rose, reaching its peak in the eighteenth century, under Joseph II. The rise of Russia began in the fifteenth century, under Ivan III, and by the end of the seventeenth century, under Peter, Russia stretched from the Baltic Sea in the west to the Pacific in the east.

It was in the seventeenth century that England and France became the two dominant powers on earth. Cromwell subdued Scotland and Ireland, defeated the United Provinces, and defeated Spain to seize Jamaica. By 1707, when it became Great Britain, England had wrested the dominion over the seas from Portugal, Spain, and the United Provinces, and wrested its commercial predominance from the United Provinces. Louis XIV deployed the most powerful army humanity had seen, expanded his domain in the north and in the east, and began the exploration and conquest of Louisiana. When Britain won and France lost the war of the Spanish succession, however, in 1714, Britain became the sole dominant power on earth. When Britain, under Pitt, won and France lost the seven years' war, in 1763, Britain became the overwhelmingly dominant power on earth. Thus, when the rise of Prussia in the eighteenth century, under Frederick II, led to the rise of Germany in the nineteenth century, Germany's attempt to become the dominant power on earth led to two world wars in the twentieth century. Earlier, in order to succeed, the revolution and Napoleon had to defeat Britain. Instead, Britain defeated Napoleon and the revolution.

The attempts to build the dominant power on earth can be traced back to Eanatum, prince of Lagash *c.* 2475. Eanatum reports: Eanatum, prince of Lagash, vanquished Elam; vanquished Urua; vanquished Umma; vanquished Uruk; vanquished Ur; vanquished Kiutu; sacked Uruaz; sacked Mishime; destroyed Arua; destroyed Akshak; vanquished Subartu; vanquished Kish; vanquished Mari (ins. 5). The successful attempts to build the dominant power on earth can be traced back to Sargon, prince of Akkad *c.* 2337-

281. Sargon reports: Sargon, lord of the earth, won 34 battles (ins. C-2), whence: to Sargon, lord of the earth, Enlil gave no rival from the upper sea to the lower sea (ins. C-1). The upper sea is the Mediterranean, the lower sea is the Persian Gulf, and for one century after Sargon Enlil gave no rival from the upper sea to the lower sea to Sargon's successors either. And in the forty and a half centuries between the success of the princes of Akkad and the success of Britain, the labors of development from the former to the latter are mediated by three further greatly successful attempts to build the dominant power on earth. That of the Assyrian princes, who for seven centuries tried to bring all states under one state, beginning with Ashur-uballit, and again, beginning with Adad-nirari, and again, beginning with Tiglath-pileser, and again, beginning with Ashurnasirpal II, and succeeded, in the century from Tiglath-pileser III through Ashurbanipal. That of the great kings, Cyrus, Cambyses, and Darius, who established their domain from the gulf of Sidra to the Indus, and their successors, who preserved this domain intact for one and a half centuries after Darius, through and including Alexander. And that of Rome, when Scipio vanquished Hannibal, in the following half century Rome brought nearly the entire known earth under a single government, its domain reached from the Atlantic to the Euphrates, and this domain remained under the same single government, even as the republic became empire, for the following five and a half centuries, through Theodosius.

The labors of building the dominant power on earth attest the acquisition of knowledge, if only because the development which they are – from Eanatum through Pitt and further – is inseparable from the development of government, the development of the law, the development of logistics, tactics, and strategy, and inseparable from exploration, too.

The century or so through Napoleon is the time of enlightenment at its brightest, and this time shows better than any other that the development of government and the development of the law attest the acquisition of knowledge. The century or so beginning with prince Henry is the time when exploration encompassed the earth, and this time shows better than any other that exploration attests the acquisition of knowledge. There are not quite four centuries from prince Henry through Napoleon, and these centuries show better than any other that the development of logistics, tactics, and strategy attests the acquisition of knowledge.

Thus, where government is concerned, Alexander Hamilton writes: it seems to have been reserved for the people of this country to decide the important question whether societies of men are capable of establishing good government from reflection and choice or are destined to depend for their political constitutions on accident and force (*Federalist*, 1). In 1787, Hamilton writes of the decision as it was being made in the United States. Pace Hamilton, however, the emperor, Joseph II, had by that time for two decades been making the attempt, and a more resolute attempt than the American, to establish good government from reflection and choice. And beginning in 1789, the revolution made the most resolute attempt to establish good government from reflection and choice. The first president of the United States was Washington. Another of its founders was the philosopher *par excellence* Franklin. The revolution began when Sieyès published *What is the third estate?*, it was at its most radical under Robespierre, and it was at its most successful under Napoleon. Sieyès writes: the social mechanism cannot be understood if society is not analyzed as if it is an ordinary machine (*Estate*, 5). The astronomer Bailly led the tennis court oath, the mathematician Lazare Carnot organized the army of the revolution, and Saint-Just wrote *On nature* in the year before the committee of public safety and the *Republican institutions* in the year of the committee.

Where the law is concerned, in 1804 Napoleon promulgated the Napoleonic code, codifying the successes of the revolution. Even the common law succumbed to systematization when Blackstone published the *Commentaries on the laws of England*, in 1765-9.

Where logistics, tactics, and strategy are concerned, Albuquerque gained control of the Indian Ocean with two dozen ships, Cortés conquered Mexico with some 600 men, and Pizarro conquered Peru with less than 200 men.

When Louis XIV deployed the most powerful army humanity had seen and maintained it for two generations in the field, the army was equipped with the generalship of Turenne and Condé and the fortifications of Vauban at the front, and with the government capable of supporting it under Colbert and Louvois at the rear. Beginning in the seventeenth century battles were won tactically exclusively, beginning in the eighteenth century wars were won strategically exclusively, the war of the Spanish succession was won strategically, the seven years' war was won strategically, Napoleon developed strategy more than any other general before or after, when Nelson and Wellington defeated him, they defeated him strategically, and it was in the next generation that Clausewitz wrote *On war*.

Where exploration is concerned, prince Henry began sending his knights to sea, Eanes rounded cape Bojador in 1434, and Ca' da Mosto reached the Bissagos Islands in 1456. In 1462 Benincasa drew the first map of the African coast. Columbus reached America in 1492, again in 1493, again in 1498. In 1500 de la Cosa, who had sailed with Columbus, drew the first map of the earth showing the lands across the Atlantic. In 1507 Waldseemüller drew the first map of the earth showing America as a continent of its own. In 1519 Magellan commenced the circumnavigation of the earth and in 1522 El Cano completed it. Magellan commenced it, Pigafetta reports: with 237 men (3), El Cano completed it, Pigafetta reports: with 18 men (48). Pigafetta was one of the 18. In 1526 de Toreno, who had drawn the maps Magellan had taken with him on his voyage, drew the first map of the earth showing the Pacific. It is the first map of the earth, too, showing the earth in its true proportions.

In general, from dark age to enlightenment at its brightest, the acquisition of knowledge quickened, and more, and more, and more.

In the eleventh century, with Anselm, proof began examining god, rather than the other way around.

In the twelfth century, the doctrine that whatever man learns outside the holy scriptures, if noxious, is to be condemned, if beneficial, is in them contained, fell on hard times. With Abelard, philosophy began examining the holy scriptures and the fathers of the church, rather than the other way around. With Gerard, the Christian philosophers began learning the pagan philosophers from the Muslim philosophers.

In the thirteenth century, with Aquinas, philosophy rediscovered the importance of Aristotle. With Fibonacci, mathematics rediscovered the importance of Euclid and began taking steps beyond him. With Frederick II, Albert, and Roger Bacon, physics began rediscovering observation and experiment.

In the fourteenth century, with Occam, and with Occam's razor, the excision of the indefinite amount of not philosophy that had come to pretend to be philosophy in the previous millennium began. With Buridan, and Oresme, physics began taking steps beyond Archimedes.

In the fifteenth century, with Regiomontanus, astronomy began taking steps beyond Ptolemy.

In the sixteenth century, with Machiavelli, Copernicus, Cardano, Vesalius, Viète, Gilbert, Tycho, geometry, arithmetic, algebra, astronomy, the study of magnetism, anatomy, physiology, politics, all surpassed and left far behind the results of antiquity.

In the seventeenth century, the brightest period in the acquisition of knowledge began. It was the century of Bacon, Galileo, Kepler, Hobbes, Descartes, Cavalieri, Fermat, Torricelli, Cassini, Boyle, Huygens, Spinoza, Hooke, Leeuwenhoek, Newton, Rømer, Leibniz, Jakob Bernoulli. The rules of the method were formulated. The fundamental character of the principle of sufficient reason was established. Analysis was developed. The theory of numbers was developed. The isochronism of the cycloid was discovered. Calculus was discovered. The law of large numbers was discovered. Satellites of Jupiter were discovered and the phases of Venus observed. That the earth revolves around the sun was proved. Air pressure was measured. The laws of the motions of the planets were discovered. The first gas law was discovered. The problem of collisions was solved. Conservation of momentum was discovered. The acceleration of gravity was determined. The laws of motion were discovered. The law of gravity was discovered. The solar system was measured. The wave theory of light was formulated. Kinetic energy was antici-

pated. The speed of light was measured. The cell was discovered. Protozoa were discovered. That the investigation of the state can be grounded in physics was shown.

The eighteenth was the century of enlightenment at its brightest. It was the century of Halley, Montesquieu, Voltaire, Maupertuis, Daniel Bernoulli, Franklin, Linnaeus, Euler, Buffon, Hume, d'Alembert, Kant, Lagrange, Gibbon, Herschel, Lavoisier, Lamarck, Laplace, Dalton, Fourier. The fundamental character of functions was established. The calculus of variations was developed. Topology was developed. Laplace's equation was discovered. Fourier series were discovered. Mechanics was formulated analytically. Mechanics was deduced from a single principle. The principle of least action was formulated. The wave equation was discovered. The principle of superposition of waves was discovered. The general wave equation was discovered. Uranus was discovered. The proper motion of the solar system was discovered. The dynamic stability of the solar system was demonstrated. The proper motion of stars was discovered. The Milky Way was mapped. Experiments showing that the earth is at least 74 832 years old were conducted. The theory of the origin of the solar system was formulated. The one-fluid theory of electricity was formulated. Hydrogen, nitrogen, and oxygen were discovered. The atomic theory was developed. That respiration is combustion was shown. The problem of the classification of organisms was solved. A theory of the development of animals was formulated. That all inferences terminate in experience, and the ground of *a priori* judgments is experience, too, was shown.

History, also, if it did not quite keep pace, tried to keep pace.

In the eighteenth century, Voltaire wrote *The century of Louis XIV*, Gibbon wrote *The history of the decline and fall of the Roman empire*, and these two treatises remain the high point in the development of history.

Gibbon writes: diligence and accuracy are the only merits which a historical writer may ascribe to himself, if any merit indeed can be assumed from the performance of an indispensable duty; I may therefore be allowed to say that I have carefully examined all original materials that could illustrate the subject which I have undertaken to treat (adv.).

Voltaire writes: this history concerns itself only with what merits the attention of all times (1).

The careful examination of all original materials is an indispensable duty. And the end of the performance of the indispensable duty is what merits the attention of all times.

The performance of the indispensable duty to the end of what merits the attention of all times, then, and *ipso facto* history, begins with the Sumerian prince list, thirty-eight centuries before Voltaire and Gibbon. The performance of the indispensable duty is attested unambiguously by the fact that the list collects 120 princes of 10 states, reaching back six and a half centuries before the time of its composition. What merits the attention of all times is attested unambiguously by the fact that, next to those in the valley of the Nile, these are the first princes, and these are the first states. Including Enmebaragesi, prince of Kish, prince c. 2680, prince forty-seven centuries ago, and the first prince to leave his inscriptions behind. If Herodotus is the father of history, the Sumerian prince list is its grandfather. And unlike organisms, history does not have a great-grandfather.

Sixteen centuries after the Sumerian prince list, Herodotus wrote the *Histories*. With his first sentence he proposes to research the great and magnificent deeds of Greeks and barbarians. In what follows he records the great and magnificent deeds of 804 Greeks and barbarians. And the domain from which he collects the deeds is, spatially, the entire known earth, from the Atlantic in the west (I, 203) to the Indus in the east (IV, 44), and from the problem of the source of the Dnieper in the north (53) to the problem of the source of the Nile in the south (II, 31-3). Temporally, it is from Herodotus's own time back to the first prince (4). Cicero writes: Herodotus is the father of history (*Laws*, I, 5). Herodotus is the father of history because on the ground of the particularity entailed by the performance of the indispensable duty he achieved the generality of what merits the attention of all times with respect to both space and time.

Not quite three centuries later, Polybius wrote

his *Histories*. He formulates a problem explicitly, and solves the formulated problem explicitly. The problem is: how and under what constitution did the Romans succeed, in 53 years, from 220 to 168, in bringing under a single government, for the first time ever, nearly the entire known earth (I, 1)? The solution is: the Romans developed the best constitution (VI, 50-7), and, resting on the performance of the indispensable duty, it includes the examination of the constitutions of all contemporary states, in book VI, and the description of the entire known earth, in book XXXIV.

The successes of history, however, have not been many. History has struggled with both particularity and generality. The careful examination of all original materials has been liable to become its own end. Hence all the histories concerned with what merits the attention of no time at all. Hence also the great contribution which history has made to philological and antiquarian scholarship. What merits the attention of all times, on the other hand, abandoned by the historians, has been taken over by not historians.

Augustine devotes more than one half of *The city of god*, twelve of its twenty-two books, to: the origin, progress, and merited ends of the two cities, earthly and heavenly (XI, 1). As to their origin: they both originated among the angels (XII, 1). As to their progress, Augustine identifies six ages. The first three are: infancy, from Adam to Noah, childhood, from Noah to Abraham, and adolescence, from Abraham to David (XVI, 43). The last three are: from David to the Babylonian exile, from the Babylonian exile to the nativity of Christ, and from the nativity of Christ to the last judgment (XXII, 30). More particularly, Augustine devotes four books to the origin of the two cities, XI-XIV, four books to the first three ages of the two cities, XV-XVIII, and four books to the merited ends of the two cities, XIX-XXII.

As to the careful examination of all original materials, clearly, if noxious, all original materials are to be condemned, if beneficial, they are in the holy scriptures contained.

In the twelfth century, Otto of Freising wrote *The history of the two cities*. Otto begins by introducing the two cities as he ought to: there are two cities, one of time, the other of eternity, one earthly, the other heavenly, one of the devil, the other of Christ (I, pr.). Otto then deals with the two cities not as he ought to, however. *The history* is in eight books. With the first four ages of the two cities Otto deals in book I, and, though he begins with Adam, he ends with Ashurbanipal, not with the Babylonian exile. With the fifth age he deals in book II, ending with Caesar rather than Christ. With the sixth age he deals in books III-VII, ending, respectively, with Constantine, Odoacer, the division of Charlemagne's domain, Gregory VII, and Conrad III. And since, clearly, one of the two cities has been neglected in the preceding books, book VIII: concerns itself with the Antichrist, the resurrection of the dead, and the end of the two cities (*id.*).

At length, the historians abandoned the heavenly city altogether. Yet, if they concerned themselves with the earthly city exclusively, they concerned themselves with ever smaller portions of it.

In the fifteenth century, Bruni wrote *The history of the Florentine people*. Bruni argues that the ancestors of the Florentine people are: the Etruscans who were the greatest in war and in peace before the Romans already (I, 13), and in order to do so, Bruni reaches more than two millennia before his own time. Temporally. Spatially, he confines himself to the quarrels of the Florentine people with its neighbors, the peoples of Arezzo, Siena, Volterra, Pisa, Lucca, and as far as Milan.

In the sixteenth century, Guicciardini wrote *The history of Italy*. Spatially, Guicciardini reaches from the discovery of America in the west to that of the Moluccas in the east, and from the wars of Henry VIII in the west to those of Suleiman in the east. Temporally, he confines himself to the forty-five years from 1490 to 1534.

The successors of Bruni and Guicciardini have tended to confine themselves to portions of the earthly city smaller than Bruni's spatially and Guicciardini's temporally, both. When, that is, they have concerned themselves with portions of the city at all. Needless to add, for this reason, too, they have tended to concern themselves with what merits the attention of no time at all. The most important exceptions are the following.

In the seventeenth century, Spinoza formulated and solved the problem of when and how the

Hebrew scriptures were authored, in the *Theologico-political treatise*. Spinoza thus also showed, as he had proposed to show, that the method of formulation and solution of historical problems is the method of formulation and solution of physical problems.

In the eighteenth century, Voltaire and Gibbon wrote, and Hume and Kant.

Voltaire writes in *The century of Louis XIV*: all centuries resemble one another in the criminal folly of men (I, 1); but there are four centuries which are epochs of greatness of the human mind and an example for posterity (*id.*); the first is that of Philip and Alexander, and of Pericles, Demosthenes, Aristotle, Plato, Apelles, Phidias, Praxiteles; the second is that of Caesar and Augustus, distinguished also by Lucretius, Cicero, Livy, Virgil, Horace, Ovid, Varro, and Vitruvius; the third, following the conquest of Constantinople by Mehmed II, is that of the Medici in Florence, and of Michelangelo, Palladio, and Galileo; the fourth is that of Louis XIV (*id.*); in the first the honor was confined to Greece; the third was the hour of glory of Italy; and the fourth will forever be the true glory of France, but its happy influence was not confined to France, it spread to England, Germany, Russia, and Italy (*id.*). The fourth also, Voltaire notes: approaches the closest to perfection; in that age not all arts progressed further than before, but human reason in general was brought to perfection; sound philosophy became known at this time; and there was a general revolution in our arts, our minds, our customs, and our government (*id*). Voltaire thus proposes: to show of what the French are capable when they are led; what they became under Louis XIV (*id.*).

Among the problems Gibbon formulates and solves in *The history of the decline and fall of the Roman empire* is: the causes of the progress and establishment of Christianity (15).

Hume showed that all inferences terminate in experience. Kant showed that the ground of *a priori* judgments is experience, too.

More particularly, with respect to history, Hume writes: the records of wars, intrigues, factions, and revolutions are so many collections of experiments by which the politician or moral philosopher fixes the principles of his science in the same way as the physician or natural philosopher becomes acquainted with the nature of plants, minerals, and other external objects (*Inquiry*, 8.1).

Kant writes: whatever concept one may have of the freedom of the will with a metaphysical purpose, its appearances, human actions, are, like all other natural events, determined in accordance with general laws of nature; thus, the philosopher cannot assume that men and their drama have a rational purpose, his only starting point can be to try to discover in the senseless course of human affairs a natural goal (*id.*). Kant writes this in the *Idea for a general history with a cosmopolitan purpose*, and continues: thus, we will try to find the guiding thread of such a history, and will leave it to nature to produce the man to write it, as it produced Kepler, who discovered the laws of the paths of the planets, and Newton, who explained these laws by a general natural cause (*id.*).

Most particularly, that the records of wars, intrigues, factions, and revolutions are so many collections of experiments by which the philosopher fixes the principles of his science, Hume showed when he wrote the six volumes of *The history of England*.

Kant's *Idea for a general history* is a consequence of Kant's *Critique of pure reason*. Both, Kant's *Idea* and Kant's *Critique*, are in turn consequences of Kant's *General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws*.

History, if it did not quite keep pace, tried to keep pace. The method of formulation and solution of historical problems is the method of formulation and solution of physical problems. History is a part of sound philosophy. History is so many collections of experiments. The senseless course of human affairs is determined in accordance with general laws of nature. These laws can be explained by a general natural cause.

xxx

Dark age to enlightenment at its brightest the acquisition of knowledge quickened, and more, and more, and more. And Thales through enlight-

enment at its brightest, when the acquisition of knowledge quickened and when it did not, and across all dark ages, Thales through Empedocles and Aristotle and Archimedes and Ptolemy and al-Farabi and Khayyam and Copernicus and Newton and Dalton and Fourier, the acquisition of knowledge is the acquisition of knowledge, one and the same. As unambiguously as this. Thales said that water is the principle of all things and that the earth floats on water, like wood, and twenty-four centuries later Lavoisier showed that water is not an element by decomposing it into hydrogen and oxygen and Laplace showed the dynamic stability of the solar system. Pythagoras discovered the irrationality of $\sqrt{2}$, and twenty-three centuries later Euler showed that $e^{i\pi} = -1$. Pythagoras discovered that number is the principle of all things, and Galileo established that the world is written in mathematics. Herodotus investigated the great kings, and Voltaire and Gibbon investigated Louis XIV and the Roman empire, and long before Herodotus the Sumerian prince list investigated the Sumerian princes. The expeditions of the great kings established the limits of the known earth, and so did the expeditions of Sargon earlier, and so did Magellan and El Cano later. Cyrus, Cambyses, and Darius built the dominant power on earth, and so did Sargon earlier, and so did Napoleon try to do later. Plato through Aristotle through Hume through Kant, too, as the acquisition of knowledge is the acquisition of knowledge, so the investigation of knowledge is the investigation of knowledge, one and the same.

And as the results from the acquisition of knowledge all constitute one and the same development, so the results against the results from the acquisition of knowledge all constitute one and the same development.

Not quite one and a half centuries after the first philosopher, this development begins with Protagoras. The philosophers had by that time solved a number of problems, and solved some of them most successfully. Thus, the two problems first solved by Thales, of the principles of all things and of the earth, had been solved better than Thales, and solved most successfully. Pythagoras had proposed that number is the principle of all things, Parmenides had discovered that the earth is a sphere, Empedocles had proposed that earth, water, air, and fire are the principles of all things. If principles are material and formal, as Aristotle established they are, earth, water, air, and fire are the material principles of all things, number is the formal principle of all things.

And when Protagoras proposed that man is the measure of all things, this is not an attempt to solve the problem of the principles of all things better, the problem of the material principles, or the problem of the formal principles, or an attempt to solve any problem at all. It is an attempt, and a most successful attempt, to restore man to his proper place. Against the possibility that number is the principle of all things, including man, man is the measure of all things, of those that are that they are, of those that are not that they are not, and of whether number is the principle of all things, whether number is the principle of man, whether all things have principles, whether that they have principles matters, and whether that they have principles matters more than man. Against the possibility that the earth is a sphere, man is the measure of the earth, the measure of spheres, the measure of whether the earth is a sphere, the measure of whether this matters more than man. Against the possibility that earth, water, air, and fire are the principles of all things, including man, man is the measure of whether those are the principles, or the principles of man, or this matters more than man.

Man is the measure of all things, pig is not the measure, baboon is not the measure, number is not the measure, earth, water, air, and fire are not the measure. Because man is not pig, not baboon, not number, not earth, water, air, and fire, man is man. Man is not one among all things, next to earth, water, air, and fire, the things composed of earth, water, air, and fire, number, the things of which number is the principle, spheres, principles, material principles, formal principles, problems, solutions. Man is apart.

Protagoras restored man to his proper place, and did so by restoring all things to their proper place. He divided, and divided most successfully, all things into man, on one side, all things, on the other. And that all subsequent results against the results from the acquisition of knowledge rest on this result, the fact suffices to show that, when,

twenty-three centuries later, the restoration of spirit to the substantiality and purity of its being was complete, the restoration had established the independence of the spiritual sciences from the natural sciences, it had thus once again restored man to his proper place, and done so by restoring nature, this time, to its proper place. The restoration divided, and divided most successfully, nature into man, on one side, nature, on the other.

Man is apart. The citadel.

When Thales said that water is the principle of all things and that the earth floats on water, like wood, this is physics. When, one and a half centuries later, Socrates said: it seems ridiculous to me to search for other things before I know myself, this is not physics, it is ethics. All things are now divided into myself, on one side, other things, on the other, and I can abandon physics altogether and devote myself to ethics. If physics is a kind of knowledge, ethics is a kind of knowledge, and if ethics and physics are both kinds of knowledge, the citadel is now protected by a fortress.

With the labors of Plato, in the next generation, the fortification is complete – man, kinds of knowledge, vagueness – citadel, fortress, outworks. The outworks stretch well beyond all things, too. God is the measure of all things now. And while man may have to exert himself to be the measure of some things, god is most readily the measure of anything. Plato's younger contemporary Eudoxus, for instance, a resident of the Academy himself, formulated the theory of proportion and the theory of the motions of the planets, and man can no doubt be as readily the measure of the theory of proportion and the theory of the motions of the planets as god is. But god also talked to Moses, is Jesus, sent an angel to Muhammad. So, when Whitehead writes: the safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato, the footnotes are to Plato. Whitehead writes also, for instance: in Plato there is a plain anticipation of a doctrine of grace seven hundred years before Pelagius and Augustine (*Ideas*, 10.1), and if there is a plain anticipation of a doctrine of grace in Plato, this success is not Pelagius's, or Augustine's, or even Whitehead's, this success is all Plato's. Augustine, for his part, writes: a scandalous calumny was leveled by readers of Plato, who had the temerity to say that our lord Jesus Christ learned all his ideas from the writings of Plato, for Plato undeniably lived long before our lord came in the flesh; but my good friend Ambrose examined secular history and found out that Plato went to Egypt at the time when Jeremiah was there, and thus demonstrated that it is surely more likely that Plato was shown our books by Jeremiah and so learned and wrote the things for which he is justly praised (*Doctrine*, II, 43). And if our lord Jesus Christ may have learned all his ideas from the writings of Plato, and Plato may have been shown our books by Jeremiah and so learned the things for which he is justly praised, this success is not *Jeremiah*'s or the gospels', our books' or our lord's, Ambrose's or Augustine's, this success is all Plato's.

Philosophy begins with Thales, begins with him as physics, and two and a half centuries after Thales Aristotle established these facts, both. In these two and a half centuries physics became a body of problems, their solutions, most successful solutions, conclusive solutions, the principles on which problems and solutions rest, and with Aristotle it became a body of physical treatises in which the problems, solutions, and principles are collected, most important among these the *Physics*, *On the heaven*, *On generation and corruption*, the *Meteorology*, *On the soul*, the five treatises on animals. Simultaneously mathematics developed into a body of problems, solutions, and principles, and into a body of mathematical treatises, including Aristotle's six logical treatises and, in the next generation, Euclid's *Elements*.

Ethics begins with Socrates, and not quite one century after Socrates Aristotle established this fact, too. Ethics became a body of ethical treatises with the Platonic dialogues.

More generally, Aristotle established the fact that philosophy is theoretical and practical, the end of the theoretical is truth, the end of the practical is action, and physics and mathematics are the parts of theoretical philosophy, while ethics is, a part of politics, a part of practical philosophy.

The investigation of knowledge begins with the *Theaetetus*, when, having asked, what is

piety? what is courage? what is friendship? what is temperance? what is beauty? what is justice? what is virtue? Plato asks also, what is knowledge? Aristotle investigates knowledge in all its generality in the *Posterior analytics* and in four of the treatises collected in the *Metaphysics*. Having done much physics and mathematics, whose end is truth, Aristotle investigates knowledge in all its generality with the same end, truth, and in order to investigate knowledge in all its generality investigates physics and mathematics.

By Aristotle, then, the results from the acquisition of knowledge were established in all their generality as such, results from the acquisition of knowledge.

The results against the results from the acquisition of knowledge were established in all their generality as such, results against the results from the acquisition of knowledge, in the next generation, by Pyrrho.

Scepticism being scepticism, scepticism has been eager to be ubiquitous, and much vagueness has been one of the outcomes of this eagerness. Scepticism being scepticism, however, scepticism is not ubiquitous, not even because scepticism is eager to be ubiquitous, and the vagueness can be cleared away.

Hume writes: there is a species of scepticism antecedent to all study and philosophy (*Inquiry*, 12.1); and there is another species of scepticism consequent to science and inquiry (*id.*).

If there is scepticism antecedent to all study and philosophy and scepticism consequent to science and inquiry, scepticism is ubiquitous. But there is no scepticism antecedent to all study and philosophy. Scepticism is consequent to science and inquiry, or is not scepticism.

If there were scepticism antecedent to all study and philosophy, sceptics would be right to deny that Pyrrho was the first sceptic, and say that Democritus already was a sceptic, before him Heraclitus was a sceptic, before him Xenophanes was a sceptic, before him the seven wise men were sceptics, and before them Homer was a sceptic. Scepticism was a condition of the big bang. And Descartes was a sceptic, and Hume was a sceptic.

Scepticism is antecedent to all study and philosophy, Hume continues: when inculcated by Descartes and others as a sovereign preservative against error and precipitate judgment (*id.*); it is a necessary preparative to the study of philosophy by preserving a proper impartiality in our judgments and weaning our minds from all prejudices which we may have imbibed from education or rash opinion (*id.*). Scepticism was not a condition of the big bang, Homer was not a sceptic, the seven wise men were not sceptics, Xenophanes was not a sceptic, Heraclitus was not a sceptic, Democritus was not a sceptic, Descartes was not a sceptic, Hume was not a sceptic, a preservative against error and precipitate judgment is not scepticism, and preserving a proper impartiality in our judgments and weaning our minds from all prejudices is not scepticism.

Pyrrho is consequent to two and a half centuries of science and inquiry, and beginning with Pyrrho, scepticism is consequent to science and inquiry.

Scepticism is consequent to science and inquiry, Hume continues: when men are supposed to have discovered either the absolute fallaciousness of their natural faculties or their unfitness to reach any fixed determination in all those curious subjects of speculation about which they are commonly employed (*id.*).

This is scepticism, and to the establishment of this Pyrrho's claim is unassailable. Pyrrho found out that it will not do to establish that we do not know. It has to be established that we know that we do not know. It will not do to establish fallaciousness of natural faculties. Or unfitness to reach fixed determination. The absolute fallaciousness of natural faculties has to be established. Unfitness to reach any fixed determination. Nor can it be a difficulty that it cannot be maintained consistently that we know that we do not know. Pyrrho saw clearly how the things we see clearly can be forced to revert to obscurity. Action is truth, and truth is action. Belief is knowledge, and knowledge is belief. And the absolute fallaciousness of natural faculties cannot hinder the establishment of the absolute fallaciousness of natural faculties. The unfitness to reach any fixed determination cannot hinder the reaching of the fixed determination of the unfitness to reach any fixed determination.

Scepticism is consequent to science and in-

quiry because it is a response to science and inquiry. It is a very general response, too. It is neither a response, however, nor is it general, because it has produced evidence against science and inquiry such as science and inquiry has not produced and produced for science and inquiry. Nor has it made arguments against science and inquiry such as science and inquiry has not made and made for science and inquiry.

The most important sceptics after Pyrrho are Timon, in the generation after Pyrrho, Aenesidemus, two and a half centuries after Timon, and Bayle, seventeen centuries after Aenesidemus.

Timon found out that scepticism is a cure.

Which scepticism no doubt is, if the acquisition of knowledge is a disease in need of a cure.

Aenesidemus reached the fixed determination that there are ten arguments for skepticism: animals vary, men vary, the senses vary, circumstances vary, position varies, admixture varies, quantities vary, relations vary, frequency varies, and customs and laws vary (Empiricus, *Outlines*, I, 36-7; *Mathematicians*, VII, 345).

Plato said that Heraclitus said that all things change (*Cratylus*, 402a). If so, Heraclitus certainly also said: all things are in exchange for fire, fire in exchange for all things (fr. 90). All things have principles, and if all things change, their change also has principles. The variation of animals has principles, and the variation of men has principles, and the variation of the senses, and of circumstances, and of position, and of admixture, and of quantities, and of relations, and of frequency, and of customs and laws.

Bayle declared, in general: I protest against everything that is said and everything that is done (de Boze, *Polignac*), and undertook to argue, in particular, in the *Historical and critical dictionary*, in 1697, that: the light of philosophy can only guide a man to the point where he will admit that he knows only that he knows nothing; this is the *non plus ultra* of philosophy (*s. v.* Bunel).

Which it no doubt is, if: the best use of philosophy is to realize that it misleads and we ought to look for another guide, which is the light of revelation (*id.*). As, indeed, Augustine already had established.

Scepticism is a response to science and inquiry, and a very general response, because of its appearance of ubiquity. It is not an accident that Hume finds himself supposed a sceptic, and in order to distance himself from such a supposition, finds himself finding kinds of scepticism which are not scepticism.

For, clearly, if we know that we do not know, we can know, or not know, *ad hoc*, and at will. And if we can, we will.

Scepticism, accordingly, is the foundation of all subsequent results against the results from the acquisition of knowledge.

Of the continuing devotion to ethics. For, clearly, if we decide what we know, and what we do not know, we can decide not only that we need to know no physics, but that we know no physics, and certainly the physicists know no physics, for if we can know that we do not know, we certainly can know that they do not know, and we can then keep on knowing, and certainly keep on trying to know, ourselves.

Of the paltriness of the schools. If we decide what we know, and what we do not know, we decide what is philosophy, and what is not.

Of philological and antiquarian scholarship. If we decide what we know, and what we do not know, and what is philosophy, and what is not, we decide what is important, and what is not.

Most important, scepticism is the foundation of the assimilation of philosophy into Christian doctrines, and of the transformation of Christian doctrines into philosophy, and of Christian philosophy into orthodoxy. For we do have to know, or not know, *ad hoc*, and at will, if, on the one hand: it is for the same reason, to satisfy sick curiosity, that freaks are put on show in the theater and men investigate nature and want knowledge for the sake of knowledge, yet, on the other hand: in Christ all treasures of wisdom and knowledge are hidden.

The results against the results from the acquisition of knowledge were established in all their generality as such, results against the results from the acquisition of knowledge, with the establishment of scepticism. These results all constitute one and the same development. This development has a cause. And Pyrrho's result, the sun does not throw light, it blinds, indicates the cause. Timon's result, scepticism is a cure, a purgative, in particular, indicates the cause even more clearly.

The acquisition of knowledge is not a condition of health, it is a condition in need of a cure.

In the centuries after Timon, the condition continued to deteriorate. Mathematics developed through Archimedes, and Apollonius, and Diophantus. Physics developed through Hipparchus, and Ptolemy's *Almagest* and *Planetary hypotheses* and *Geography* and *Optics*. History developed through Polybius and Livy.

Remedies continued to be found as well, however, and more effective remedies.

Philosophy became the product of schools.

Philological and antiquarian scholarship developed.

Rhetoric became philosophy, and philosophy became rhetoric.

Mysticism became philosophy.

The radical cure, though, took the three and a half centuries from st Paul to Augustine to administer.

St Paul writes: knowledge breeds conceit.

In the following century, the Pauline letters were supplemented with spurious Pauline letters, and st Paul was made to write: god's secret is Christ; and in him all god's treasures of wisdom and knowledge are hidden ([*Colossians*], 2).

A couple of centuries later, Augustine writes: whatever they may be able to demonstrate from true sources about the nature of things, we shall show not contrary to our books; whatever they may produce from their volumes against our books, and that is, against our catholic faith, we shall easily show false, or else without any hesitation believe false; so we shall hold fast to our mediator in whom all treasures of wisdom and knowledge are hidden (*Genesis*, I, 21).

The darkest of dark ages followed.

xxxi

The acquisition of knowledge began emerging from the darkest of dark ages in the eleventh and twelfth centuries, with the labors of Anselm, Abelard, and Gerard. Proof began examining god. Philosophy began examining the holy scriptures and the fathers of the church. The philosophers began learning again their Aristotle, Euclid, Archimedes, and Ptolemy.

By the thirteenth century the philosophers had all learned their Aristotle, and were taking steps beyond Euclid.

By the fourteenth century they were taking steps beyond Archimedes.

By the fifteenth century they were taking steps beyond Ptolemy.

By the sixteenth century the earth had been circumnavigated, its entire compass mapped.

The cause whose effects the results against the results from the acquisition of knowledge are had not been idle. It had been faltering, however.

The old remedies had lost their efficacy. When in the third Christian century Plotinus found out how man can achieve union with god, and succeeded in making mysticism into philosophy, Plotinus was a younger contemporary of Origen, and, tending to do what the philosophers tend to do, the philosophers tended to find all treasures of wisdom and knowledge where they ought to, in our mediator in whom all treasures of wisdom and knowledge are hidden. When in the fourteenth Christian century Eckhart rediscovered mysticism, and found out how the soul can not only achieve union with god, but become god, indeed rise above god, Eckhart was an older contemporary of Occam and Buridan, and, tending to do what the philosophers tend to do, the philosophers tended to find all treasures of wisdom and knowledge not where they ought to. They tended to find them in books – not our books either – and increasingly in the nature of things. Universities flourished, but they taught Aristotle, and so also logic and physics. Philological and antiquarian scholarship flourished, but it produced Ficino in the fifteenth century, Erasmus in the sixteenth, philosophers *par excellence* both. Ficino found out that Christian piety can ground itself in Plato, but he also produced editions and commentaries on the philosophers, Plato, Plotinus, Porphyry, Iamblichus, Proclus. Erasmus proposed to return to a simple philosophy of Christ, but he also produced editions and commentaries on the scriptures and the fathers of the church, Origen, Cyprian, Ambrose, Jerome, Augustine, and he also wrote *The praise of folly*.

Even the ultimate remedy had lost its efficacy. If god decided what man knows, god decided what was orthodoxy, what was heresy, and god had done so most efficaciously at the hands of the fathers of the church. Not any more.

In particular, when Abelard was condemned for heresy in 1140, he was not the first. Roscelin, for instance, was condemned in 1092. When Bruno was burned for heresy in 1600, he was not the last either. Vanini, for instance, was burned in 1619.

In general, Luther pointed at the fountain and headspring of all mischief. As unambiguously as this: excluding all works, we go to the head of the beast called reason, which is the fountain and headspring of all mischief; for reason does not fear god, does not follow god, does not believe in god, but disdains god; this pestilential beast, reason I say, once slain, all outward and gross sins will be nothing (comm. *Galatians*, III, 6).

Nevertheless, the sixteenth was not only the century of Luther and, following Luther, interminable wars to exterminate heresy. It was, following that pestilential beast, reason I say, the century of Machiavelli, Copernicus, Cardano, Vesalius, Viète, Gilbert, Tycho.

Galileo was condemned in 1633. Nevertheless, the philosophers continued to find all treasures of wisdom and knowledge not where they ought to, but where Galileo found them: in that grandest book, the world.

The seventeenth was thus the century when the brightest period in the acquisition of knowledge began. It was the century of Bacon, Galileo, Kepler, Hobbes, Descartes, Cavalieri, Fermat, Torricelli, Cassini, Boyle, Huygens, Spinoza, Hooke, Leeuwenhoek, Newton, Rømer, Leibniz, Jakob Bernoulli, Halley.

The world decided what man knows.

The eighteenth was thus the century of enlightenment at its brightest. As Voltaire describes it: human reason in general was brought to perfection; sound philosophy became known; and there was a general revolution in our arts, our minds, our customs, and our government.

As Hegel describes it: spirit has lost its essential life.

xxxii

Spirit has lost its essential life, lies in wickedness, and reviles itself for doing it. But: spirit is conscious of the loss, turns away from the chaff, and demands from philosophy the restoration of the substantiality and purity of its being.

The pestilential beast had been brought to perfection, however, sound philosophy had become known, and nothing short of war could restore spirit to the substantiality and purity of its being. War was what spirit demanded from philosophy, and war was what spirit was given.

The war was fought, and won, in three stages. In the first stage the enemy was identified and the battle against him joined. In the second stage the enemy was outmaneuvered and decisively defeated. In the third stage the enemy was well-nigh eradicated. Spirit infused everything.

The first stage was fought and won by Berkeley, Rousseau, and Schiller. The second stage was fought and won triumphantly by Hegel. The third stage was concluded by Dilthey, but in this final stage of the war, as in the final stages of all wars that end in unconditional surrender, the vanquished labored harder even than the victors to bring about the unconditional surrender.

xxxiii

The war began with the establishment of idealism, and Berkeley's claim to the establishment of idealism is unassailable.

Like scepticism, and taught by scepticism, idealism, being idealism, has been eager to be ubiquitous. In this case, as in that, one of the outcomes of the eagerness has been much vagueness. In this case, however, as in that, idealism, being idealism, is not ubiquitous. The vagueness can be cleared away, and in this case, as in that, it must be cleared away. What is scepticism, and what is not, is a matter of fact, and what is idealism, and what is not, is a matter of fact. Neither is a matter of words, and neither is a matter of ideas either.

Thus: consequent to science and inquiry, scepticism is the discovery of the absolute fallaciousness of natural faculties or unfitness to reach any fixed determination.

This is scepticism, because Pyrrho established scepticism. And Pyrrho established scepticism, because this is scepticism. This is the fact.

And if so, if this is the fact, everything else is vagueness. Everything else is words, and ideas. There is no scepticism antecedent to all study and philosophy. Preserving a proper impartiality in our judgments and weaning our minds from all

prejudices is not scepticism. Nor, when Hume adds: there is a more mitigated scepticism which may be both durable and useful (*Inquiry*, 12.3), can there be such a scepticism. Lest there were mitigated absolute fallaciousness, or what is the same, vagueness were indulged, and scepticism were ubiquitous after all. A species of such a mitigated scepticism, for instance, Hume writes, is: the doubt, caution, and modesty which ought forever to accompany a just reasoner (*id.*). But neither the doubt, caution, and modesty which ought forever to accompany a just reasoner need be contaminated by the supposition of scepticism, nor need scepticism be contaminated by the supposition of caution, or modesty, or just reasoning.

Hume writes also: most of the writings of Berkeley form the best lessons of scepticism to be found among either the ancient or the modern philosophers, Bayle not excluded; that all his arguments are merely sceptical arguments appears from this, that they admit of no answer and produce no conviction; their only effect is to cause that momentary amazement and irresolution and confusion which is the result of scepticism (12.1).

Hume writes this in 1748. And in that idealism is a development of scepticism Hume's judgment is conclusive. Berkeley's are sceptical arguments. That they are not merely sceptical arguments, though, appears from this, that in the following couple of generations the amazement and irresolution and confusion they cause proved far from momentary, and very far from being their only effect.

In 1710, Berkeley published *A treatise concerning the principles of human knowledge, wherein the chief causes of error and difficulty in the sciences, with the grounds of scepticism, atheism, and irreligion, are inquired into*. In 1713, he published the *Three dialogues between Hylas and Philonous, the design of which is plainly to demonstrate the reality and perfection of human knowledge, the incorporeal nature of the soul, and the immediate providence of a deity, in opposition to sceptics and atheists*.

That Berkeley does not take his arguments to be scepticism any more than he takes them to be atheism or irreligion his titles sufficiently convey. Nevertheless, pace Berkeley, with Hume: Berkeley's arguments are the best lessons of scepticism to be found among either the ancient or the modern philosophers, Bayle not excluded. Berkeley writes: there is not a single thing in the world of which we know the real nature (*Dialogues*, 3), and as unambiguously as this: the philosophers know that they know nothing (*id.*).

Hume's judgment of Berkeley is conclusive, but only in part. Berkeley's arguments are the best lessons of scepticism, but they are more than that. Berkeley continues: the philosophers are led into those extravagancies by the belief in material substance (*id.*); it is matter also that has furnished atheists and infidels of all ages with the most plausible arguments against creation (*id.*); it is immaterialism, therefore, that is most agreeable to common sense and remote from scepticism (*id.*).

Berkeley's arguments are scepticism, but only in part. More generally, they are immaterialism, of which scepticism is only a part. And they are more than that, too. Most generally, they are idealism, of which immaterialism is only a part.

A development of scepticism, idealism is a generalization of scepticism.

In 1702, Leibniz first divided the philosophers into kinds such that one of them is: the idealists. Leibniz writes: according to my hypothesis, everything occurs in the body as if there were no soul, and everything occurs in the soul as if there were no body; preestablished harmony is then a good way to relate each to the other; and it is clear that thus what is good in the hypotheses of both Epicurus and Plato, of both the greatest materialists and the greatest idealists, is combined (*id.*). It is the main point of Leibniz's *Reply to the reflections contained in the Critical dictionary of Bayle on the system of preestablished harmony*.

Everything occurs in the soul as if there were no body, then, is what is good in the hypotheses of Plato, of the greatest idealists. But there is more than that in the hypotheses of Plato, good and bad, and although there is that in the hypotheses of Berkeley, there is more than that in the hypotheses of Berkeley as well, good or bad, and more than that is idealism. Berkeley argues that everything occurs in the soul as if there were no body, when he argues, more generally, as the title

of the *Dialogues* has it: the incorporeal nature of the soul. But he argues much more generally: matter is impossible (*Dialogues*, 2). Everything occurs in the soul as if there were no body all right. Furthermore, however, there is no body, and furthermore, matter, never mind body, is impossible. And even that much, immaterialism, is only a part of idealism.

In 1781 the *Critique of pure reason* appeared. In 1782 it was reviewed. In 1783 Kant writes: there are reviewers who do not cling to the word, but follow the subject matter and examine the principles; my reviewer is not of this kind (*Prolegomena*, app.). Kant's reviewer reads idealism in Kant. Kant is incensed. He asks: idealism (13)? and replies: the very opposite of it (*id.*)! Clinging to the word, however, Kant has called something idealism. So he has to write: I wish I had called this concept of mine otherwise (app.). Then again: to alter it altogether is probably no longer possible; it may therefore be allowed me that in the future it be called formal or, better still, critical idealism (*id.*).

Clinging to the word, scepticism has been read in Hume, and, to distance himself from such a reading, Hume has found himself finding kinds of scepticism which are not scepticism. In the same way, Kant finds himself finding kinds of idealism which are not idealism. Not only formal or, better still, critical idealism. He finds even: the empirical idealism of Descartes (13).

But there can be no such thing, empirical idealism. Lest vagueness were indulged, and idealism were ubiquitous. That formal or, better still, critical idealism is not idealism, Kant himself cannot have been more explicit: my so-called, properly critical, idealism overturns genuine idealism (app.).

Most explicitly, then, Kant writes: the principle of all genuine idealists, from the Eleatic school to bishop Berkeley, is contained in this formula, all knowledge from sense and experience is nothing but sheer illusion, and only in the ideas of pure understanding and reason is there truth; the principle which throughout rules my idealism is, on the contrary, all knowledge of things merely from pure understanding or pure reason is nothing but sheer illusion, and only in experience is there truth (*id.*).

This is idealism: all knowledge from sense and experience is sheer illusion, and only in the ideas of pure understanding and reason is there truth.

This is idealism, and to the establishment of this Berkeley's claim is unassailable.

Berkeley established that scepticism will not do. It will not do to establish that we know that we do not know. It has to be established that we know better than what we know. To this end immaterialism alone will not do either. Only idealism will do. It will not do to establish that all knowledge from sense and experience is sheer illusion. To establish alone that matter is impossible will not do either. It has to be established that there is truth only in the ideas of pure understanding and reason. Not in the ideas, and not of understanding and reason, either. Only in the ideas, and of pure understanding and reason.

In this case, as in that, this is idealism, because Berkeley established idealism, and Berkeley established idealism, because this is idealism.

There is no idealism before Berkeley. Idealism is not a condition of the big bang. Pace Kant, should the Eleatic school suggest Parmenides or Zeno of Elea, neither Parmenides nor Zeno of Elea is an idealist. Plato is not an idealist, not even if he read *Jeremiah*. Descartes is not an idealist. Leibniz is not an idealist.

There is idealism after Berkeley. But Kant is not an idealist. Hegel is an idealist. Dilthey is an idealist.

xxxiv

Idealism is a generalization of scepticism, itself a generalization of the result, man is the measure of all things, and idealism is the ground on which the war for the restoration of spirit to the substantiality and purity of its being was fought, and won. And Berkeley did not just establish idealism. He also identified the enemy, and he also laid the foundations of the strategy by which the enemy was engaged, fought, and defeated.

The first two results, idealism and the identification of the enemy, Berkeley established in *The principles of human knowledge*, in 1710, the third, the foundations of the strategy to defeat the enemy, in *On motion*, in 1721.

Idealism four brief steps suffice to establish.

Berkeley writes in *The principles*: the generality of men, which are simple and illiterate, never pretend to abstract notions; not attained without pain and study, abstract ideas are confined to the learned (intr.); and abstract ideas are nothing but words (*id.*).

This is the first step. And it follows from this step alone that: he that knows that names do not always stand for ideas will spare himself the labor of looking for ideas where there are none (*id.*); in vain do we extend our view into the heavens, and pry into the entrails of the earth; we need only draw the curtain of words to behold the fairest tree of knowledge, whose fruit is excellent, and within the reach of our hand (*id.*). It follows from this step alone that there is, on one side, he that knows. This is Berkeley. On the other side are those that do not know. Those are the learned, one of whom Berkeley is not. For he that knows needs only draw the curtain of words to behold the fairest tree of knowledge. Those that do not know, had they known, will have spared themselves the labor of extending their view into the heavens, and prying into the entrails of the earth.

Berkeley continues: the objects of human knowledge are either ideas actually imprinted on the senses or ideas perceived by attending to the passions and operations of the mind, or else ideas formed by compounding, dividing, or representing ideas originally perceived in the above two ways (1). He illustrates: a stone, a tree, a book, and like sensible things (*id.*).

This is the second step. Everything is ideas. Except for one thing.

Berkeley continues: next to this endless variety of ideas, or objects of knowledge, there is something that perceives them, and exercises operations like willing, imagining, and remembering them; this perceiving active being is what I call mind, spirit, soul, or myself (*id.*); it is a thing entirely distinct from my ideas, it is the thing in which they exist, or what is the same, the thing by which they are perceived; for the existence of an idea consists in being perceived (*id.*).

This is the third step. There is mind, spirit, soul, or myself. Everything else is my ideas in mind, spirit, soul, or myself.

It remains only to make the relation of mind, soul, spirit, or myself and my ideas more explicit. Berkeley writes: the cause of ideas is an incorporeal active substance or spirit (*id.*); when we talk of unthinking agents, we only amuse ourselves with words (*id.*).

This is the last step. The relation is of cause and effect. Mind, spirit, soul, or myself is the cause of my ideas. Of a stone, a tree, a book, and like sensible things. Indeed: the words, absolute existence of sensible objects without the mind, mark out either a direct contradiction or else nothing at all (*id.*).

Idealism is a fruit of the fairest tree of knowledge, whose fruit is excellent, and within the reach of our hand. It is no doubt an excellent fruit, and all other fruits of the tree are no doubt excellent, because, however excellent they all are, the tree yet bears nothing more excellently than vagueness.

Mind is spirit is soul is myself. Ideas are my ideas. The idea, also, as Berkeley has it in *The principles*: the existence of an idea consists in being perceived, is presented in all its excellence in the *Philosophical commentaries*: *existere* is *percipi* or *percipere* or *velle*, i. e. *agere* (A). To exist is not only to be perceived, it is also to perceive. To exist, or to be perceived, is not only also to perceive, it is also to will. To exist, or to be perceived, or to perceive, is not only also to will, it is also to act. At which point the series can stop, for, if to exist is to be perceived is to perceive is to will is to act, then everything that needs to be anything is anything, at will. Truth is action, and action is truth, at will. Knowledge is belief, and belief is knowledge, at will. We need only draw the curtain of words.

Berkeley concludes in *The principles*: I shall esteem my labors altogether useless and ineffectual, if by what I have said I cannot inspire my readers with a pious sense of the presence of god; and having shown the falseness or vanity of those barren speculations which make the chief employment of learned men, the better dispose them to reverence and embrace the salutary truths of the gospel, which to know and to practice is the highest perfection of human nature (1).

Everything that needs to be anything is anything, but about this much Berkeley is crystal clear. He is waging war for truth. Truth is not in the barren speculations of the learned. Falseness

is there, or vanity. Truth – and salutary truth – is elsewhere, and Berkeley is most explicit where. He is waging war for knowledge, too, and the learned may have knowledge. But there is better knowledge elsewhere, and Berkeley is most explicit where. As he is to know – and to practice – what is the highest perfection of human nature.

Berkeley's titles themselves are a declaration of war. In 1710, he stakes a claim to *The principles of human knowledge*, as he ought to, if he has found *the chief causes of*, and corrected, *error and difficulty in the sciences*. In 1713, more plainly still, he proposes *plainly to demonstrate the reality and perfection of human knowledge*, as he ought to, if it is the reality and perfection of none of the barren speculations of the learned but of *the incorporeal nature of the soul, and the immediate providence of a deity*. The principles of human knowledge are not in Descartes's *Principles of philosophy*, the first part of which is *On the principles of human knowledge*, and positively not in Newton's *Mathematical principles of natural philosophy*.

The principles, then – Berkeley's *Principles* – points unambiguously at the enemy. The enemy is the falseness or vanity of the barren speculations of the learned. Those extending their view into the heavens, and prying into the entrails of the earth. It points unambiguously at the archenemy, too: a certain celebrated treatise on mechanics, in the entrance of which justly admired treatise, time, space, and motion are distinguished into absolute and relative, true and apparent, mathematical and vulgar (1). Still, in *The principles* there is some diplomatic caution in drawing the line between friend and foe. There is none in the *Philosophical commentaries*. Berkeley writes: all things in the scripture which side with the vulgar against the learned side also with me (A), and: I know that a mighty sect of men will oppose me; yet I may expect to be supported by those whose minds are not overgrown with madness, and these are the greatest part of mankind, especially moralists, divines, and politicians, in a word all but the mathematicians and natural philosophers (*id.*).

There they are, friend and foe. Friend is the scripture, the vulgar, and moralists, divines, and politicians whose minds are not overgrown with madness. Foe is the mathematicians and natural philosophers.

It had to be the mathematicians and natural philosophers. Galileo, Kepler, Descartes, Huygens, Newton, Leibniz, Halley. And the archenemy had to be Newton.

The mathematical principles of natural philosophy appeared in 1687.

This is how Halley judged it, in the ode *To that glory of our time and people, the mathematico-physical treatise of the most excellent Isaac Newton*:

> we are now admitted to the banquets of the gods,
> we know the laws of heaven, to which the earth
> was blind before, and the immutable order of the things,
> hidden throughout the ages of the world (*id.*).

Berkeley judged it: falseness or vanity.

This had to be the enemy, and Berkeley attacked him dauntlessly. He undertook to replace his theory of vision with a better theory of vision, to expose the falseness or vanity of his theory of motion, to expose the falseness or vanity of his analysis, and to improve on his medicine, all four. He wrote *An essay toward a new theory of vision*, in 1709. He wrote *On motion, or the principle and nature of motion, and on the cause of the communication of motions*, in 1721. He wrote *The analyst, or a discourse addressed to an infidel mathematician, wherein it is examined whether the object, principles, and inferences of the modern analysis are more distinctly conceived, or more evidently deduced, than religious mysteries*, in 1734. And he wrote *Siris, a chain of philosophical reflections and inquiries concerning the virtues of tar water, and divers other subjects connected together and arising one from another*, in 1744.

How much better the new theory of vision is than the old, the title of its vindication and explanation suffices to show, in 1733, *The theory of vision, or visual language showing the immediate presence and providence of a deity, vindicated and explained*. How great the medicinal virtues of tar water are, should its medicinal use fail to show, again their vindication and explanation can show: if physicians think they have a right to treat of religious matters, I think I have an equal right to treat of medicine (to Prior, 19 Jun 1744).

It is in the course of his attack on the archenemy, though, in *On motion* and *The analyst*, that Berkeley laid the foundation of the strategy which was eventually to defeat the enemy. To attack the archenemy, moreover, had in the meantime become yet more urgent. In 1713 the second edition of *The mathematical principles of natural philosophy* had appeared, and Newton had added a general scholium to it, to affirm explicitly that: to treat of god from the phenomena pertains to natural philosophy (*id.*).

If the philosophers have ever produced an impregnable treatise, however, before or after, *The mathematical principles of natural philosophy* is this treatise. Not only, with Halley: we now know the laws of heaven, to which the earth was blind before, and the immutable order of the things, hidden throughout the ages of the world. Not only Newton writes: the entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena (pr.), and the entire difficulty is resolved. It is resolved by the demonstration of 635 propositions, lemmas, and corollaries.

There was thus, tactically, only one way to attack this treatise. To try and sap its foundations.

Accordingly, Berkeley does not propose to demonstrate 635 propositions, lemmas, and corollaries. He proposes to sap the foundations.

The foundations are the laws of motion, and calculus, or as Newton has called it, the method of fluxions.

And against Newton, who has proposed to treat of motion as it is absolute, true, and mathematical, in opposition to relative, apparent, and vulgar, Berkeley has argued in *The principles* already that: there can be no motion other than relative (1). In *On motion* he again affirms: it is impossible that absolute motion should be conceived (*id.*). Now, however, he formulates three rules as well. The rules are also a declaration of war. For Descartes has formulated the four rules of the method, and Newton has formulated in *The mathematical principles* four: rules of philosophizing (*id.*). The third rule – Berkeley's third rule – is: to consider motion as something sensible, or at least imaginable, and to be content with relative measures (*Motion*).

In *The analyst, or a discourse addressed to an infidel mathematician*, Berkeley begins by informing the infidel mathematicians why they are addressed: you, gentlemen, are presumed to be of all men the greatest masters of reason; with this bias on their minds men submit to your decisions where you have no right to decide; and this is one short way of making infidels (1). He continues: the method of fluxions is the general key, with whose help the modern mathematicians unlock the secrets of geometry, and consequently of nature (3). But: what are these fluxions? the velocities of evanescent increments? and what are these evanescent increments? they are neither finite quantities, nor quantities infinitely small, nor yet nothing; may we not call them the ghosts of departed quantities (35)?

But the war was never going to be decided tactically. In particular, if on the foundations of the impregnable treatise 635 propositions, lemmas, and corollaries stand, the foundations of the treatise are impregnable to sapping, Berkeley's or otherwise. In general, not only is *The mathematical principles of natural philosophy* an impregnable treatise. It is the crowning achievement of six centuries of labors of the philosophers, counting from Anselm, Abelard, and Gerard, in which the acquisition of knowledge quickened, and more, and more, and broadened, and more, and more, and deepened, and more, and more – to lead, one century before Berkeley, beginning with Bacon, Galileo, and Kepler, to the brightest period in the acquisition of knowledge.

The second rule – Berkeley's second rule – is: to beware of abstractions (*Motion*). The fundamental character of the rule is clear. It is the first step in the establishment of idealism: abstract ideas are confined to the learned; and abstract ideas are nothing but words.

There was the difficulty – much as scepticism had had to face it – that Berkeley himself was rather learned. That the idea that abstract ideas are nothing but words was itself rather abstract. The learned had to be divided into the learned, on one side, Berkeley, on the other. Abstract ideas such as they are nothing but words had to be divided into such as, when the mathematicians and natural philosophers and Newton have them, are nothing but words, and such as, when the

scripture, the vulgar, and moralists, divines, and politicians whose minds are not overgrown with madness and Berkeley have them, are not nothing but words.

The first rule is: to distinguish mathematical hypotheses from the natures of things (*Motion*).

And this is the strategic move. Berkeley is unambiguous: attraction was certainly introduced by Newton not as a true, physical quality, but only as a mathematical hypothesis (*id.*), more generally: in mechanics mathematical ideas rather than the true natures of things are regarded (*id.*), most generally: the true nature of things is not regarded in abstract mathematics (*id.*).

Mathematics, on one side, the true nature of things, on the other.

With Galileo: the world is written in mathematics. Hence, mathematics is inseparable from physics. Hence, Kepler had discovered the laws of the motions of the heavenly bodies, Descartes had formulated the rules of the method, laid the foundations of analysis, and shown that all phenomena can be deduced from the same principles, Cassini had measured the solar system, Huygens had formulated the wave theory of light, Newton had developed calculus, discovered the laws of motion, and the law of gravity, and grounded the system of the world in the laws, Leibniz had discovered the fundamental character of the principle of sufficient reason, discovered the possibility of describing all objects and their relations formally, developed calculus, and anticipated kinetic energy, Jakob Bernoulli had discovered the law of large numbers, Halley had discovered the proper motion of stars. Hobbes had begun grounding politics in physics and mathematics. Spinoza had begun grounding history in physics and mathematics. And when Halley had written *An estimate of the degree of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslau, with an attempt to ascertain the price of annuities upon lives*, in 1693, life and death had begun succumbing to mathematics, too.

But mathematics is not concerned with the true nature of things. Hence, the foundations of the entire fortress guarding the citadel can be laid.

Having allotted mathematics its province, Berkeley proceeds to allot each science its province: in physics sense and experience, which reach only to apparent effects, hold sway; in first philosophy or metaphysics we are concerned with incorporeal things, with causes, truth, and the existence of things (*Motion*). In general: allot to each science its own province, assign its bounds, accurately distinguish the principles and objects belonging to each, thus it will be possible to treat them with greater ease and clarity (*id.*).

This is the strategy. Allot to mathematics hypotheses. To physics the apparent. To metaphysics truth. Thus it will be possible to treat them with greater ease and clarity.

The infidel mathematicians may be confused. Given the *tu quoque*: if physicians have a right to treat of religious matters, I have an equal right to treat of medicine, they may presume that if Berkeley has a right to treat of mathematics, nay, allot to it its province, and allot to physics its province, and to metaphysics its province, they have an equal right to treat of Berkeley. Not so. You, gentlemen, have no right to decide. You, gentlemen, have to be asked instead: whether there be not really a *philosophia prima*, a certain transcendental science superior to and more extensive than mathematics, which it might behoove our modern analysts rather to learn than despise (*Analyst*, qu. 49)?

An early biographer reports the occasion of *The analyst*: Addison had given the bishop an account of their common friend doctor Garth's behavior in his last illness, which was equally unpleasing to both these excellent advocates for revealed religion; Addison had gone to see doctor Garth and begun to discourse with him about preparing for his approaching dissolution; but the other had answered, surely, Addison, I have good reason not to believe those trifles, since my friend Halley, who has dealt so much in demonstration, has assured me that the doctrines of Christianity are incomprehensible, and the religion itself an imposture; the bishop therefore took arms against this redoubtable dealer in demonstration (Stock, *Berkeley*).

In arms the bishop was beginning to end.

xxxv

On the authority of scepticism there is no knowledge. On the authority of idealism there is

knowledge better than knowledge. There is a first philosophy or metaphysics, a *philosophia prima*, superior to and more extensive than mathematics and physics.

Still, much infidelity and heresy had had to be exterminated before it became incontrovertible that all treasures of wisdom and knowledge are hidden in our mediator. And a long war had to be fought and won before it became incontrovertible that Berkeley's first philosophy is superior to and more extensive than Newton's second philosophy.

The enemy had been identified correctly. The strategy devised by Berkeley was to triumph, too. Before it did, however, it acquired two further essential components. Rousseau supplied the first. Schiller supplied the second.

Rousseau's labors, like those of Protagoras, or Socrates, or Plato earlier, and those of Marx, or Russell later, attest both the acquisition of knowledge and the reinforcement of the barriers before the acquisition of knowledge. In the war for the restoration of spirit to the substantiality and purity of its being Rousseau fought on both sides, and fought on both sides valiantly.

On the side of the acquisition of knowledge, Rousseau's remains the best explanation of the state.

And the best explanation of the state has to be presented in some detail.

Aristotle begins in the *Politics*: whether it is the state or anything else, things are considered with the greatest clarity when they are considered from their origin and first growth; and first there has to be a union of those that cannot exist without one another, male and female, for in common with animals and plants men by nature strive to leave behind their progeny, and master and slave, for that which foresees by reason is master by nature and that which acts on the foresight by the body is slave by nature (I, 2); from these two relations the family arises by nature to supply man's daily needs (*id.*); when several families unite, and the union aims at more than the supply of daily needs, the first community is formed in the village (*id.*); and when several villages unite in a single community large enough to be more or less self-sufficient, the state has come into being, originating in the bare needs of life, and continuing for the sake of the good life (*id.*); it is thus evident that the state is by nature, and man is a political animal by nature (*id.*).

Aristotle adds: it is evident also that man is a political animal more than bees and other gregarious animals; for he is the only animal that has speech; speech is for the sake of establishing what is useful and what is useless, and hence also what is just and what is unjust; man alone has a sense of good and bad, just and unjust, and the like, and it is the association of animals with this sense that makes a family and a state (*id.*).

Whether it is the state or anything else, things are considered with the greatest clarity when they are considered from their origin and first growth.

Hence, in particular, Aristotle discovered that, if the state is to be explained, it will be, first, by beginning with what is first by nature. It will be, second, by ending with the state, not with a hive or a herd. And it will be, third, by bridging the abyss between what is first by nature and the state, or what is the same, by describing the origin of the state from nature.

Aristotle discovered all three problems. But he did not solve any of the three very well.

The conclusion: the state is by nature, and man is a political animal by nature, rests on the premise: first by nature are the relations of male and female, and master and slave. If man shares both relations with bees and other gregarious animals, however – and investigating the generation of bees in some detail, Aristotle concludes that: bees have kings; and the kings procreate (*Gen. animals*, III, 10) – the conclusion, if it is to distinguish the state from the hive, is at the mercy of that other presumption: it is the association of animals with the sense of just and unjust that makes a family and a state. Hence, granted that the explanation has to begin with an animal, an animal with a sense of just and unjust cannot help being a *petitio principii*. Hence, also, if the state is for the sake of the good life, such a state may well be a state to wish for, but it may not be, in fact, the state any more than the hive, or herd, of Plato's *Republic* may be.

The *Politics* is the first of the three most important political treatises. The second is the *Leviathan, or the matter, form, and power of the commonwealth ecclesiastical and civil*, in 1651.

Hobbes writes: nature has made men so equal that the difference between man and man is not so considerable that one man may not claim to himself any benefit to which another man may pretend; as to strength of body, the weakest has strength enough to kill the strongest; as to faculties of mind, almost all men think they have wisdom in a greater degree than all but themselves (13); hence it is manifest that during the time when men live without a common power to keep them all in awe, they are in a condition of war of every man against every man (*id.*); in such a condition there is no industry, no knowledge of the earth, no account of time, no arts, no letters, no society; the life of man is solitary, poor, nasty, brutish, and short (*id.*); in this condition every man has a right to everything, even to another's body; therefore, as long as this natural right of every man to everything endures, there can be no security to any man (14); consequently, it is a general rule of reason that every man ought to endeavor peace, and when he cannot obtain it seek the advantages of war; and the first branch of this rule contains the first and fundamental law of nature, to seek peace, and follow it (*id.*). Hobbes derives a second law of nature, a third, and so on, for a total of nineteen, and continues: but the laws of nature of themselves, without the terror of some power, are contrary to our natural passions (17); the agreement of men, unlike that of bees and ants, is artificial, and to make it constant and lasting, a common power to keep them in awe is required (*id.*); the only way to erect such a common power is for men to confer all their power and strength upon one man, or one assembly of men, that may reduce all their wills unto one will; which is as much as to say, to appoint one man, or assembly of men, to bear their person (*id.*); and the multitude so united in one person is called a commonwealth, in Latin *civitas* (*id.*).

Hobbes solved two of the three problems better than Aristotle.

A state for the sake of the good life may well be a state to wish for, but it is not the state. Not on the evidence of the fifty-odd centuries the state has been about on earth. The state, or synonymously commonwealth, in Latin *civitas*, in Greek πόλις, is: a power contrary to our natural passions; a power keeping men in awe; a power reducing the wills of men unto one will. If the explanation of the state is to end with the state, Hobbes's solution of this problem is conclusive. This is the state. Which the fact of the labors to build the dominant power on earth and of their success suffices to show, from the princes of Lagash and Eanatum, through the Akkadian princes beginning with Sargon, the Babylonian princes and Hammurabi, the Assyrian princes in the seven centuries beginning with Ashur-uballit, the great kings beginning with Cyrus, Sparta, Athens, Rome, the caliphate beginning with Muhammad already, and Charlemagne, and Mehmed II, and Charles V, and the struggle of France and England, and the defeat of Napoleon, and the two world wars in the last century-odd.

By making what the state is explicit Hobbes solved the problem of where the explanation of the state has to end conclusively. And by generalizing what is first by nature he solved the problem of where the explanation has to begin better than Aristotle.

The explanation has to begin with: no industry, no knowledge of the earth, no account of time, no arts, no letters, no society. Hence, affirmatively, Aristotle begins with an animal, in general, and with the relations of male and female, and master and slave, in particular. Hobbes begins, in particular, with: nature has made men equal, in body and in mind. Because he begins, in general, with: life is but a motion of limbs; for what is the heart but a spring, and the nerves but so many springs, and the joints but so many wheels giving motion to the whole body (intr.)? It is from the mechanical premise: life is but a motion of limbs, that Hobbes draws the political conclusion: men are equal. The *Leviathan* is explicitly an attempt to ground politics in physics.

The problem of bridging the abyss between what is first by nature and the state has thus become the problem of bridging the abyss from motion, springs, and wheels, across an animal, to the state. Generalized, the problem has become more difficult. So much so, Aristotle's solution, family, union of families into a village, union of villages into a state, remains the more plausible. Hobbes attempts to bridge the abyss with laws of

nature, and laws of nature can and must bridge the abyss. In particular, however, the first and fundamental law of nature is, to seek peace, and follow it, which is rather more likely to be the law of a hive, or a herd, than the state. Nor is it good that the state needs nineteen laws of nature. The planets make do with three. Motion also makes do with three.

In 1690, Locke published the *Two treatises of government*, of which *the latter is an essay concerning the true origin, extent, and end of civil government*. In 1748, Montesquieu published *On the spirit of the laws, or on the relation which the laws must have with the constitution of every government, with customs, climate, religion, commerce, etc.* These have been the two most effectual political treatises. One year after the accession of William III, Locke writes: I hope these papers are sufficient to establish the throne of our great restorer, our present king William, to make good his title, in the consent of the people, which being the only one of all lawful governments, he has more fully and clearly than any prince in Christendom (pr.), and the subsequent success of England and of Locke's treatise have been one another's cause and effect. One generation after *The spirit of the laws*, it is the one treatise to which Hamilton and Madison appeal repeatedly in *The federalist* (9, 43, 47, 78), and the subsequent success of the United States and of Montesquieu's treatise have been one another's cause and effect.

Whatever their virtues – the doctrine of separation of legislative and executive power (*Government*, II, 13), and of legislative, executive, and judicial power (*Laws*, XI, 6), has long urged itself as one – neither of the two treatises is a further step toward the better explanation of the state.

When Locke writes: the great and chief end of men's uniting into commonwealths, and putting themselves under government, is the preservation of their property (II, 9), this doctrine may well serve better than any other the preservation of property, but a state whose great and chief end is the preservation of property is not more likely to be the state, the one attested for fifty-odd centuries on earth, than a state for the sake of the good life, except that it is one, or ought to be one, not to wish for. It is even less likely to be the state, if: by the general name property I call men's lives, liberties, and estates (*id.*).

It is a virtue of Montesquieu, and a very unusual virtue among the philosophers who have concerned themselves with the state, that he investigates real laws. The real laws, moreover, are distributed from laws of the Greek cities through laws of contemporary states, temporally, and from laws of the conquered peoples in America in the west through laws of the Japanese in the east, spatially. On the other hand: I am not concerned with the laws, but with the spirit of the laws (I, 3). It is reassuring that in its definition the spirit of the laws is not too spiritual: all the relations which the laws must have with customs, climate, religion, commerce, etc., taken together are the spirit of the laws (*id.*), nor are the laws in their definition too spiritual: laws are the necessary relations deriving from the nature of things (1). On the other hand: all beings have their laws, the divinity has its laws, the material world has its laws, the intelligences superior to man have their laws, the beasts have their laws, man has his laws (*id.*), and it is never clear in *The spirit of the laws* whether the spirit is that of the laws of the divinity or that of the laws of the material world, that of the laws of the intelligences superior to man or that of the laws of the beasts. This is, for instance, another of Montesquieu's fundamental results: the principle of democracy is virtue (III, 3); the principle of aristocracy is moderation (4); the principle of monarchy is honor (7).

In 1755, Rousseau published the *Discourse on the origin and foundations of inequality among men*, the third of the three most important political treatises.

Rousseau writes: the philosophers who have examined the foundations of society have all felt the necessity of descending to the state of nature (intr.); but as long as we do not know natural man, all that is clear about the natural law is that it speaks immediately by the voice of nature (pr.); and meditating on the first and simplest operations of the human soul, I perceive two principles prior to reason, by one of which we interest ourselves ardently in our well-being and self-preservation, while the other inspires in us a natural repugnance to seeing any sentient being, and especially those like us, perish or suffer (*id.*);

in any animal I see only an ingenious machine; and the sole difference in the human machine is that, while the animal chooses or rejects by instinct, man does so by an act of freedom (1); the specific difference of man is not understanding but being a free agent (*id.*); let us then conclude that, wandering the forests, without industry, without speech, without home, without war, and without connections, without any need for those like him, as well as without any desire to harm them, savage man felt nothing but his true needs; man stayed forever a child (*id.*); it remains to consider the various chances which succeeded in perfecting human reason by deteriorating the species, in making a being evil by making it social (*id.*); as the human race spread, difficulties multiplied with men; differences in soils, climates, and seasons, barren years, hard winters, and hot summers required new industry (2); man found himself in a state of distinguishing the rare occasions when common interest made him count on the assistance of those like him (*id.*); the first development of the heart in the new situation was the union of husbands and wives, fathers and children, in a common habitation (*id.*); use of speech was established or imperceptibly perfected (*id.*); men united into bands (*id.*); but in the instant in which one man needed the help of another, equality disappeared, property was introduced, labor became necessary (*id.*); metallurgy and agriculture were the two arts whose invention produced this great revolution (*id.*); from the cultivation of the land its division necessarily followed, and from property the first rules of justice (*id.*); such was the origin of society and the laws, which irretrievably destroyed natural freedom, established the law of property and inequality, and subjected the entire human race to labor, servitude, and misery (*id.*); and when magistracies were instituted (*id.*); and their power became arbitrary (*id.*); all men became equal once again, because they all became nothing (*id.*); this is the road that must have led from the natural state of man to the civil state (*id.*).

In order to begin with what is first by nature, Rousseau begins with a machine, and an animal. In particular, he begins with instinct, and as explicitly as this: savage man, left by nature to instinct alone, begins with purely animal functions (1). And this result is conclusive. There is the physical problem of the relation of motion, springs, and wheels, on one side, and instinct, on the other. As far as the state is concerned, however, the problem is to bridge the abyss between instinct and the state.

Rousseau takes equality by nature for granted: men are by nature as equal as the animals of each species (pr.). Equality by nature poses the problem: of the origin and foundations of inequality. And with the solution of this problem: the origin and foundations of inequality are property, labor, and the rules of justice, Rousseau has solved the problem: of the origin of society, the laws, magistracies and their arbitrary power, and hence the problem: of the road that must have led from the natural state of man to the civil state. Rousseau has ended with the state. He has described with some specificity: the power contrary to our natural passions, keeping men in awe, reducing the wills of men unto one will.

The abyss between instinct and the state Rousseau proposes to bridge twofold. First, he proposes that: the specific difference of man is being a free agent, and neither being a free agent is likely to bridge an abyss to instinct, nor is being a free agent likely to be the specific difference of man, if: in the consciousness of freedom the spirituality of the soul shows itself; and about purely spiritual acts the laws of mechanics explain nothing (1). Further, though, when Rousseau proposes that: as the human race spread, difficulties multiplied, differences in soils, climates, and seasons, barren years, hard winters, and hot summers, and in response to these difficulties: metallurgy and agriculture were the two arts whose invention produced the great revolution of property, labor, and the first rules of justice, this result is also conclusive.

It remains the best explanation of the state.

Moreover, Rousseau is clear how it will be improved. The difficulty is: comparative anatomy has made too little progress, the observations of the naturalists are still too uncertain (1), and to overcome it: one problem not unworthy of the Aristotles and Plinys of our century is, what experiments would be necessary to acquire knowledge of natural man (pr.)?

Moreover, Rousseau's moral judgment is

immaculate. He abhors the fact that: the human race is reduced to labor, servitude, and misery.

The *Discourse on the origin and foundations of inequality among men* is a great advance in the acquisition of knowledge, and a lofty achievement of enlightenment, both.

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The rest of Rousseau's labors are neither.

He had established before the *Discourse on inequality* already, in the *Discourse on the question, whether the restoration of the sciences and the arts has contributed to the purification of the morals*, in 1750, that: astronomy is born of superstition, eloquence of ambition, hatred, flattery, and lying, geometry of avarice, physics of vain curiosity, all, even morality, of pride; the sciences and the arts owe their birth to our vices (2). Fortunately, there is a remedy: oh virtue! sublime science of simple souls, must there be all those difficulties and preparation in order to know you? your principles, are they not engraved in all hearts, and is it not enough, in order to learn your laws, to return to oneself and hear the voice of one's own conscience in the silence of the passions? this is the true philosophy (*id.*).

Accordingly, after the *Discourse on inequality*, Rousseau turned to the true philosophy. To knowing the principles of virtue engraved in all hearts, and learning its laws by returning to himself and hearing the voice of his own conscience.

In 1761, he published *Julie, or the new Héloïse, letters of two lovers, inhabitants of a small town at the foot of the Alps*. This is the novel – with all due respect to Voltaire's novels – that made the novel a vehicle of philosophy deeper than astronomy, eloquence, geometry, physics, and even morality. Also – with all due respect to Bayle's *Historical and critical dictionary*, which may be critical by being sceptical, but is historical by collecting matters which will not seem to have a principle of selection other than that all too many of them are vaguely pornographic – it is the *Letters of two lovers* that made the tribulations of lovers again, as Plato's dialogues had once made them, the concern of philosophy.

In 1762, he published *On the social contract, or principles of political right*. Its cental argument is: each of us in common puts his person and all his power under the supreme direction of the general will (I, 6). Whether it is a better explanation of the state than the earlier, the fact that Hegel approves of it, in opposition to the earlier, shows: the merit of Rousseau's contribution is that, by adducing the will as the principle of the state, he adduces a principle which has thought both for its form and its content, a principle which is indeed thinking itself, not a principle, like the social instinct, or divine authority, which has thought only for its form (*Right*, III, 3).

Also in 1762, Rousseau published *Émile, or on education*. He reports: I hate books (3); but since we absolutely must have books, there exists one which provides, to my taste, the most felicitous treatise of natural education (*id.*); and which is this marvelous book? Aristotle? Pliny? Buffon? no; *Robinson Crusoe* (*id.*).

Buffon? The first 3 volumes of the *Natural history* had appeared in 1749. By 1755 Buffon had completed 5 volumes. By 1762 he had completed 9 volumes.

Rousseau, for his part, having dealt with *Lovers*, *The social contract*, and *Education*, took the ultimate step of the true philosophy. Of returning to himself and hearing the voice of his own conscience. In 1764 he began his *Confessions*. By the time of his death, in 1778, he had completed twelve books of confessions. Thirteen years of life remained unconfessed. Rousseau had been distracted by having to write *Rousseau judge of Jean-Jacques*, and the *Reveries of a solitary walker*. But I, the solitary walker wonders: what am I? this is what remains for me to search (1).

By 1778 Buffon had completed 24 volumes.

On the authority of Berkeley, then, everything is my ideas in mind, spirit, soul, or myself. And on the ground of this abstract idea Berkeley wages war on the abstract ideas of the learned.

Rousseau has no more use for abstract ideas than Berkeley: general and abstract ideas are the source of men's greatest errors; never has the jargon of metaphysics led to the discovery of a single truth (*Education*, 4). These general and abstract ideas of Rousseau's, however, are not general and abstract ideas. Because they are engraved in his heart. Because he has learned

them by returning to himself and hearing the voice of his own conscience.

They are feelings. Rousseau writes: let all the philosophers prove that I am wrong, if you feel that I am right, I wish no more (*id.*); I know it, because I feel it (*id.*); and it would be in vain to wish to destroy this feeling in me by reason; it is stronger than all evidence (*id.*).

Rousseau thus succeeded where Luther failed. He slew the pestilential beast. Reason had found it in itself not to fear god, not to follow god, not to believe in god, and to disdain god. Feeling was to turn out to be capable of doing what god was no longer capable of doing. Reason was to turn out to be cowed by feeling. A century on, and reason feared feeling, followed feeling, believed in feeling, and never dared to disdain feeling. So much so, god took refuge in feeling.

Mind, spirit, soul, or myself Berkeley enlarged with one another. Mind, spirit, soul, or myself Rousseau enlarged further with feeling, and yet further with will: in the consciousness of freedom the spirituality of the soul shows itself; and the power to will, and the feeling of this power, are purely spiritual acts, about which the laws of mechanics explain nothing (*Inequality*, 1). Thus, should mind, spirit, soul, or myself find it difficult to know something, or to do something, mind, spirit, soul, myself, feeling, or will can find nothing difficult to know, and nothing difficult to do. I know it, because I feel it. And will – my will, the general will, any will, at will – will do it, at will.

Berkeley allotted to mathematics and physics hypotheses, the apparent, and falseness and vanity. To first philosophy or metaphysics he allotted truth.

Rousseau allotted to astronomy superstition, to geometry avarice, to physics vain curiosity, to all of them pride, and vice.

But the essential component which Rousseau added to the strategy devised by Berkeley is on the other side of the allotment.

On the authority of Rousseau as of Berkeley virtue belongs on this side. On the authority of Berkeley scripture also belongs on this side, and not indeed the gospel, but religion belongs on this side on the authority of Rousseau as well. Religion is the culmination of *The social contract* (IV, 8), and religion is at the heart of *Education* (4).

Virtue and religion are all to the good, but they are not the essential component. *Philosophia prima* exhibits itself in Berkeley as a mixture of metaphysics and theology. The true philosophy exhibits itself in Rousseau as the lethal Platonic mixture of fiction, pedagogy, ethics, and politics. Berkeley found out that there is knowledge better than knowledge. But his first philosophy was never going to find it easy to establish itself as superior to and more extensive than Newton's second philosophy. Not on metaphysical grounds, not on theological grounds. Rousseau's true philosophy could have no difficulty establishing itself as knowledge better than mathematics and physics. If it was pedagogy, it was its duty to protect the children from Aristotle, Pliny, and Buffon. If it was ethics, it was its duty to point out that they are from vice, it is from the heart, and if you feel that this is right, this is enough. Also, Berkeley argues: I side in all things with the mob (*Commentaries*, A). Reciprocally, though, the mob were never going to find it easy to side with Berkeley. Rousseau does not have to argue it. For, reciprocally, with the two lovers, inhabitants of a small town at the foot of the Alps, few could resist siding. Or with Émile and Sophie.

Fiction, pedagogy, and ethics are all to the good, too. Yet the essential component is politics.

It is in Plato, it is in Augustine, it is in Rousseau, and it will be in Hegel. When philosophy exhibits itself in Hegel as the invincible mixture of metaphysics, theology, ethics, politics, and history, politics will be essential to the mixture.

For even when some other art pretends to be the master art, politics is, with Aristotle: the master art. Politics makes the theoretical appear practical, the search for truth appear a search for action. So the *Republic* shows paradigmatically that politics, the search for action with respect to the state, is the means of making ethics, the search for action with respect to man, appear the way to being and truth. The city of god stands on the investigation of the earthly city. And the tribulations of the two lovers and education stand on the social contract.

Rousseau begins in *The social contract*: man is born free, and everywhere he is in chains; and

he who believes himself the master of others is a slave more even than they are; how has this change occurred? I do not know; what can render it legitimate? I believe I can answer this question (I, 1).

Earlier, in the *Discourse on inequality*, Rousseau has not only known, and answered the first question. He has answered the second question, too, even if implicitly more than explicitly, yet unambiguously and apparently conclusively. Can anything render the subjugation of the entire human race to labor, servitude, and misery legitimate?

Well, yes. Rousseau now formulates the question this way: since man's force and freedom are his primary means of preserving himself, how can he engage with others without hurting himself or neglecting the care he owes to himself (6)?

Theoretical in the *Discourse*, the concern is now unabashedly practical.

The answer is: each of us in common puts his person and all his power under the supreme direction of the general will; this act of association produces a moral and collective body which receives from this very act its unity, self, and will; this public person was formerly called city, it is today called state when passive, sovereign when active (*id.*).

Now as in the *Discourse*, man loses his freedom. Consequently, in the *Discourse*, man is subjected to labor, servitude, and misery. Now the consequences are different: through the social contract man loses his natural freedom and the unlimited right to everything that tempts him; but he gains civil liberty and the ownership of property (8); he gains also moral freedom; because to be driven by one's appetites is slavery, to obey the law which one prescribes to oneself is freedom (*id.*).

In the *Discourse* property is a fetter. Now the fetter is a gain.

Thus, as far as theory is concerned, that each of us in common puts his person and all his power under the supreme direction of the general will – whatever the reference of 'each of us' and whatever the reference of 'the general will', presuming, for the sake of the argument, that they have references – the evidence which Rousseau has produced in *The social contract* is none whatsoever, and the evidence which has been produced in the two and a half centuries since is the same, none whatsoever.

And the more relevant this is because, as far as practice is concerned, the labor, slavery, and misery are not labor, slavery, and misery any more. They are freedom. Moral freedom. Because I have prescribed them to myself. Or rather – pending any evidence whatsoever that I am a party to a social contract – because I am told that I have prescribed them to myself. For so much Rousseau tells me explicitly: the general will is always right, and it always tends toward public utility (II, 3).

No wonder *The social contract* ends with religion. No wonder Hegel approves of *The social contract*.

Moreover, mathematics and physics were armed, in *The mathematical principles of natural philosophy* alone, with the laws of motion, calculus, the system of the world, the law of gravity, 635 propositions, lemmas, and corollaries. Politics was an easy prey. The more so, because the hunter was the philosopher of the *Discourse on inequality*.

Buffon, for his part, completed 36 volumes of *Natural history*.

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Enlightenment was brought to an end by two proximate causes.

One was the defeat of the revolution. It was probably inevitable. Given that Britain had dominated the seas for more than one century and had been the dominant power on earth for nearly one century, the defeat of Napoleon was probably inevitable, and with the defeat of Napoleon the defeat of the revolution.

The other proximate cause was that the philosophers turned on enlightenment. This was certainly inevitable.

The two generations between Rousseau and Hegel were the time when the catastrophic transformation of philosophy began. Two generations before Rousseau, physics was not just a part of philosophy. It was, as far as both part and whole were concerned, first, foremost, substantially, and essentially a part of philosophy. Two generations after Hegel, philosophy had nothing to do with

physics.

In 1810, Mme de Staël published her most important treatise, *On Germany*. It is a review, in its four divisions, of: Germany and its customs, literature and arts, philosophy and morality, and religion and enthusiasm (*id.*). It is a very complete and very general review. In particular, Mme de Staël discusses in some detail Descartes, Locke, Leibniz, Voltaire, Condillac, Winckelmann, Kant, Lessing, Mendelssohn, Friedrich Jacobi, Herder, Schleiermacher. She discusses in great detail Goethe. She discusses in greater detail Schiller. She discusses: the two most celebrated philosophers after Kant, Fichte and Schelling (III, 7). But she does not discuss Hegel. Hegel published *The phenomenology of spirit* in 1807, and most of his labors, and all of his importance, lay in the future. Very complete and very general, Mme de Staël's is a review of philosophy on the eve of Hegel.

And Mme de Staël detects the impending transformation of philosophy unerringly. Philosophy has not yet divested itself of physics, and Mme de Staël takes it that philosophy is of two kinds: speculative philosophy has always found many partisans among the Germanic nations, experimental philosophy among the Latin nations (5). Philosophy is also old and new, however, and the new philosophy is not experimental philosophy. There is a reason: experimental philosophy judges a work above all by the ingenious and lucid form under which it is presented, idealist philosophy, on the contrary, drawn always toward the hearth of the soul, admires only the writers who approach it (9). The new philosophy is idealist. And: the new German philosophy is necessarily inclined more than any other to reach toward spirit; for, relating everything to the hearth of the soul, and considering the world itself ruled by laws whose prototype is in us, it does not accept the prejudice which destines each man to this or that field of study exclusively; the idealist philosophers believe that an art, a science, any subject whatsoever cannot be comprehended without universal knowledge (8). The idealist philosophers have also found out the way to universal knowledge: the Germans mix metaphysics with poetry (10).

Thus, it was not a philosopher, not even Fichte, the more important of the two most celebrated philosophers after Kant, who supplied the final component of the strategy of the war for the restoration of spirit to the substantiality and purity of its being.

Fichte, in 1794, in the *Foundation of the entire theory of science*, found out: the principles of the entire theory of science; the first, the I posits the I (I, 1); the second, the I posits the not I (2); and the third, I posit in the I, against the divisible I, a divisible not I (3), and concluded: beyond this knowledge no philosophy can go, but back to this knowledge every foundational philosophy must go, and when it does, it becomes theory of science (*id.*).

But Berkeley had already found out that every foundational philosophy must go back to the knowledge that everything is my ideas in myself. He had found out as well that, when it goes back to that knowledge, every foundational philosophy becomes first philosophy or metaphysics. Even if *Wissenschaftslehre* did have the novel sound in which *philosophia prima* was wanting.

It was a poet who supplied the final component of the strategy of the war for the restoration of spirit. It was not Goethe, however. Even if Goethe did supply the greatest poetic and, as: the Germans mix metaphysics with poetry, the greatest metaphysical prestige to the war.

Goethe, in 1773, with Herder and Möser, launched the movement that came to be known as storm and stress. With storm and stress the more limited war against enlightenment began, this more limited war soon to be engulfed in the unlimited war on behalf of spirit. The first salvoes in the war, in 1773, Herder, Goethe, and Möser fired with the five articles of *On the German kind and art*, the first article, the programmatic, being the *Extract from an exchange of letters about Ossian and the songs of ancient peoples*, by Herder. And it is not an accident that the war against enlightenment began with fabrications, a smaller one and a larger one. The smaller one is Ossian. Between 1760 and 1765 Macpherson published the hitherto unknown third-century^{AD} poet Ossian. Ossian fooled not just Herder. He fooled Gibbon (*History*, 1), and Schopenhauer still (*World*, I, 51). Then it was shown that the third-century^{AD} poet Ossian is the

eighteenth-century[AD] poet Macpherson. The larger one is the German kind. The smaller one took two generations to expose as a fabrication. The larger one took nearly two centuries to expose as a fabrication, maintained as it was after Herder, Goethe, and Möser by Fichte, and Hegel, and two world wars after the philosophical wars.

In 1774, Goethe published *The sufferings of young Werther*, the second, after *Götz von Berlichingen*, of his many masterpieces, through and including *Faust*. To his lover, Lotte, young Werther reports: my heart is my sole pride, it alone is the source of everything, of all my strength, all my bliss, and all my misery; what I know, everyone can know, but my heart, my heart is mine alone (9 May). One of the many other ways in which young Werther recommends his heart is: Ossian has displaced Homer in my heart (12 Oct).

In 1784, Goethe announced: a small discovery of mine (*Bone*), namely, that: an intermaxillary bone in the upper jaw is to be attributed to man as well as animals (*id.*). It was only the first of his many and multifarious contributions to the different parts of physics. Next to the above contribution to anatomy – even if anatomists such as were not poets, Blumenbach and Soemmerring, in particular, pointed out that an intermaxillary bone is to be attributed to man not more than gills and a tail are to be attributed to man – Goethe made an even more substantial contribution to botany, in 1790, with *An attempt to explain the metamorphosis of plants*. His most substantial contribution, however, was to optics. It began in 1791-2, with the *Contributions to optics*, and culminated in 1810, with *Toward the theory of colors*.

Goethe had found out in *The metamorphosis of plants* already that: the observations of Linnaeus are superficial (111). In *The theory of colors*, he obtained the affirmative result: there are primordial phenomena, such that, when we find them, it would be evil not to recognize them as such and search for something behind and above them; the investigator of nature should leave the primordial phenomena undisturbed in their eternal repose and beauty (177). Such a primordial phenomenon is: the primordial phenomenon of light and darkness (247). And for this result: for having tried to trace the phenomena to their primordial sources, beyond which no further explanation is possible, we believe and hope to merit the gratitude of the philosopher (intr.).

The negative result, too, Goethe obtained in *The theory of colors* with much greater generality and much greater precision: the worst that can happen to physics, as well as to other sciences, is to take the derivative for primordial (718). And the worst not only can happen. With the Newtonian theory of colors, it has happened. Fortunately: the Newtonian theory of colors is an old castle (pr.); pilgrims still wend their way to it, and still recommend it to the reverence of susceptible youth (*id.*); but that old castle is already abandoned; it is guarded only by a few invalids, who only imagine themselves prepared for war (*id.*); there can thus be no question of a prolonged siege or a doubtful battle in order to let the sun shine on that old nest of rats and owls, to demolish it, and to clear away the rubbish (*id.*).

Goethe failed to reform physics.

But Schiller succeeded in reforming history. And history is the final component, and the crucial component, of the strategy of the war that restored spirit to the substantiality and purity of its being.

From *The robbers* to *Wilhelm Tell*, Schiller's masterpieces are also many.

He wrote also *The stage considered as a moral institution*, in 1784, *On the tragic art*, in 1792, *On the esthetic education of man, in a series of letters*, in 1795, *On the sublime*, in 1801.

As to history, as Mme de Staël unerringly judges: Schiller is at the head of the philosophical historians (II, 29).

When in 1773 Herder, Goethe, and Möser launched storm and stress, Goethe was not to undertake the reform of physics for another decade, but Möser had already undertaken the reform of history, and Herder was just undertaking the reform of the philosophy of history.

Möser had published the *History of Osnabrück*, in 1768.

Voltaire had published *The philosophy of history*, in 1765, so Herder published *Also a philosophy of history, for the education of mankind*, in 1774, and yet again the *Ideas for the philosophy of the history of mankind*, in 1784-91.

If philosophy was old and to become new, and physics was old and to become new, clearly history was old and to become new. If the old, with Voltaire: concerned itself with what merits the attention of all times, and Voltaire concerned himself with the century of Louis XIV, and Hume concerned himself with England, and Gibbon concerned himself with the Roman empire, Möser concerned himself with Osnabrück. Then again, history had never suffered from dearth of histories concerned with what merits the attention of no time, and of histories concerned with a town, in particular, which merits attention for no reason better than that it is the historian's own town. Herder's first *Philosophy of history* is not more than what it promises itself to be, also a philosophy of history. As Goethe's physics will later on scold Newton, so Herder's first *Philosophy of history* – if more civilly, so early in the war – scolds Voltaire: the writer of the century, into whose hands will he deliver us with all his philosophy and petty dalliance of a way of thinking devoid of morality and solid human feeling? we all know how differently Rousseau preaches (3). Herder's second *Philosophy of history* is, unlike the first, an important treatise. It is, because Herder concerns himself with Africa and America (VI), China and India (XI), and, before turning to the Greeks (XIII), Rome (XIV), early Christianity (XV), and so on, concerns himself with Babylon, Assyria, Chaldea (XII, 1), and Egypt (5) – just as Voltaire has concerned himself with them all in his *Philosophy of history*. Moreover, if, with Gibbon: the careful examination of all original materials is an indispensable duty, Herder's second *Philosophy of history* respects the indispensable duty better than Voltaire's *Philosophy of history*. Not of all original materials, far from it, Herder's second *Philosophy of history* yet stands on the examination of original materials.

Schiller, the poet, pointed the way.

Schiller wrote two histories, the *History of the separation of the United Netherlands from the Spanish government*, in 1788, and the *History of the thirty years' war*, in 1791-3.

In 1788, the *History of the separation of the United Netherlands* Schiller prefaces with the list of his authorities: de Thou, Strada, van Reyd, Grotius, van Meteren, Burgundius, Meursius, Bentivoglio, and some more recent, the memoirs of Hoppers, the life and correspondence of Viglius, the records of the trials of the counts of Horn and of Egmont, the apology of prince William, a compilation of many documents and abstracts of Bor, Hooft, Brandt, Le Clerc, and others, as well as Dinoth, who gives many extracts from the pamphlets of that time (pr.). He regrets: that it was in vain that I tried to obtain the correspondence of Granvelle (*id.*), and: that the work of Spittler reached me too late (*id.*). And: I lament that it was not in my power to study the French original sources and contemporary documents (*id.*). Accordingly, the *History of the separation of the United Netherlands* is a respectable historical treatise.

In 1791-3, the *History of the thirty years' war* is not. Prefaced by no list of authorities, regrets, or laments, it is but the preparatory notes for the three dramas, *Wallenstein's camp*, *The Piccolomini*, and *Wallenstein's death*.

It was between 1788 and 1791 that Schiller found the way.

The way, in fact, was self-evident. If everything is my ideas in myself – if I posit the I and the not I in the I – can physics not be in myself? can I not posit physics? So Berkeley had shown that a theory of vision better than Newton's theory of vision is in myself, and the falseness of Newton's theory of motion is there, and the falseness of calculus, and the medicinal virtues of tar water. So Goethe also was to show that I posit an intermaxillary bone in man, and an archetypal plant, and a theory of colors better that Newton's. Can history not be in myself? can I not posit history?

This is how the *History of the separation of the United Netherlands* begins: many years ago, when I read the history of the revolution in the Netherlands under Philip II in Watson's excellent work, I felt an excitement of spirit which political actions seldom stir; on closer examination, however, I realized that what had set me in this excitement of spirit came not from the book, but from the ardent workings of my own imagination which had given the perceived material the image which so strongly excited me (pr.).

It did not take long to generalize the result. In 1789, in his inaugural address, *What is called and*

to what end is studied universal history? Schiller posits: the philosophical spirit cannot tarry long with the material of world history; the irresistible urge starts working in him to assimilate everything into his own rational nature, to elevate every phenomenon to the highest activity he knows, to thought; he cannot give in to the blind rule of necessity; he takes harmony out of himself and grafts it onto the order of things, and that is, he brings a rational goal to the course of the world and a teleological principle to world history (*id.*).

And should it have seemed uncertain whether the result is general, whether the irresistible urge working in the philosophical spirit works in the ordinary mediocre writer of history, Hegel removed the uncertainty. In his *Philosophy of history*, he writes: even the ordinary mediocre writer of history who somehow believes and professes to be only receptive, to surrender himself to what is given to him, is not passive in his thinking, he brings his categories with him and sees through them the given (intr., A.c).

The irresistible urge has since not been resisted. Instead, the philosophical spirit has transformed history into historiology, historicism, historicity, and historicality. As to the categories which the ordinary mediocre writer of history has brought with him, they can hardly be illustrated better than by the most important historian of the last century, Braudel. The second of Braudel's two important treatises is the *Material civilization, economy, and capitalism, 15th - 18th century*, in 1968-79. Its first volume is *The structures of everyday life*, and the categories, other than the category structures of everyday life, which Braudel brings with him, are: eating habits, table manners, beer and wine, the vogue for pepper, the pleasures of coffee, the pleasures of tobacco (3), furniture, floors, ceilings, tables, chairs, luxury, fashion (4).

Thus, physics was not reformed.

The invalids, rats, and owls in the old castle turned out to be prepared for war. Carnot, Helmholtz, Clausius developed thermodynamics, Faraday, Maxwell developed electrodynamics, Einstein developed relativity, Planck, Rutherford, Bohr, Schrödinger, Heisenberg developed quantum theory. Nor did the invalids, rats, and owls fail to note Goethe. Helmholtz writes: if one takes Goethe's explanations as pictorial representations of physical processes, one may acquiesce in them; as physical explanations, however, they are nonsense; for the poet nature is a sensible expression of the spiritual, it is ruled by the ideas of the work of art; Goethe's optics is a complete failure (*Goethe's*). Heisenberg writes: the real reason for Goethe's bitter fight against Newton's optics is the renunciation of vividness and immediacy; this renunciation has been the foundation of the progress of natural science since Newton; Goethe's fight thus should not be dismissed as unimportant; Goethe yet should be reproached with inconsistency; instead of attacking certain views of Newton, he should have said that all of Newton's physics, optics and mechanics and law of gravity, is the work of the devil (*History*).

Not only were the invalids, rats, and owls prepared for war. They were prepared to give as good as they got. Showing also that the poet was only imagining himself fighting an offensive war. He was fighting in defense of the citadel – of pictorial representations, the expression of the spiritual, vividness and immediacy – against the work of the devil.

History was reformed.

It should not have been an easy prey, like politics. Bacon tarried with the material of world history to undertake the instauration of history himself with *The history of the reign of king Henry VII*. Spinoza tarried to solve the problem of when and how the Hebrew scriptures were authored, and show that the method of history is the method of physics, in the *Theologico-political treatise*. Leibniz tarried for three decades with the *Brunswick annals of the western empire*. Voltaire tarried for two decades with *The century of Louis XIV*. Hume tarried for more than one decade with *The history of England*. Gibbon tarried for more than two decades with *The history of the decline and fall of the Roman empire*. So much so, Kant urged nature to produce for history, too, a Kepler and a Newton.

Nature produced Hegel.

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Hegel wrote six very important treatises. It is a number very few philosophers have achieved.

The first is *The system of science, part one, the phenomenology of spirit*, in 1807.

And *The system of science*, *the phenomenology of spirit*, opens with the call to arms: spirit has lost its essential life; but it is conscious of the loss; turning away from the chaff, confessing that it lies in wickedness, and reviling itself for doing it, spirit demands from philosophy the restoration of the substantiality and purity of its being (pr.).

Spirit turns away from the chaff. Spirit also points at the enemy, demands that the enemy be reviled, and demands that the enemy be stripped of his finery, and the necessity of a different kind of knowledge be shown: the evidence of the deficient knowledge of which mathematics boasts against philosophy rests solely on the poverty of its goal and the deficiency of its material, it is therefore of a kind which philosophy must revile (*id.*); as to the so-called proofs, like those of the equilibrium of the lever, of the relations of space and time in the motion of falling, and so on, their critique will be instructive in part to strip mathematics of its finery, in part to show its limitations and thus the necessity of a different kind of knowledge (*id.*).

The ground is idealism. Still in the preface, after the call to arms, and the delivery of spirit's ultimatum, Hegel reveals the ground: being is thinking (*id.*). And Hegel stands on the ground unwaveringly in *The phenomenology*, he stands on it unwaveringly in all his treatises, and the reason is straightforward. The ground is the fundamental proposition of philosophy. With *The phenomenology* Hegel left the war still undecided, but a decade later, with the *Encyclopedia of the philosophical sciences*, he decided it, and he writes in the *Encyclopedia*: the ideality of the finite is the fundamental proposition of philosophy; every true philosophy is for this reason idealism (95).

The goal is religion. Not the civil religion in which Rousseau's labors on *The social contract* culminate. Berkeley's labors on *The principles of human knowledge* culminate in: the salutary truths of the gospel, and so do Hegel's labors on *The phenomenology of spirit*. God is omnipresent in *The phenomenology*. Drawing to conclusion, Hegel writes: the incarnation of the divine being is the content of absolute religion (VII, C); here god is revealed as he is, as he is in himself, as spirit (*id.*); the dead divine man or human god is the general self-consciousness (*id.*). And in order to end with absolute knowledge, Hegel ends with: the calvary of absolute spirit (VIII). Hegel pursues the goal unwaveringly in *The phenomenology*, he pursues it unwaveringly in all his treatises, and the reason is once again straightforward. Once again in the *Encyclopedia*: there can be religion without philosophy, but there can be no philosophy without religion (pr. 2nd ed.).

What, more particularly, the phenomenology of spirit is, Hegel describes in the *Announcement* with which he offered it to the world: this volume presents the becoming knowledge; the phenomenology of spirit encompasses the different forms of spirit as stations of the way by which it becomes pure knowledge or absolute spirit (*id.*).

Everything in Hegel tends to be a triad – even if: when we consider god as the trinity the number three has a much more subordinate meaning than when we are concerned with the three dimensions of space or the three sides of the triangle (*Encyclopedia*, 99) – and the main stations of the way by which spirit becomes pure knowledge or absolute spirit are three: consciousness (A), self-consciousness (B), and reason (C). The stations have their substations. These of self-consciousness are: lordship and bondage (IV, A), stoicism, scepticism, and the unhappy consciousness (B). These of reason are: reason (AA), spirit (BB), religion (CC), and absolute knowledge (DD). Absolute knowledge is: spirit knowing itself as spirit (VIII).

The phenomenology is very important, but not because of any of the above singly. Philosophers had been calling to arms for a century. They had been identifying the enemy, and presenting him with the ultimatum demanding his finery. They had been notifying him that there is knowledge better than his knowledge. They had established idealism, and been asserting it ever more aggressively. They had been pursuing the goal for two millennia.

Nor is *The phenomenology* very important because of any of its particular results. For instance, the lowest sub-substation of reason is: observing reason (V, A), and: observing reason is like the Jewish people, which, precisely because

it stands before the gate to salvation, is and has been the most depraved (A.c). And Hegel has much to say about the depravity of observing reason, but none of it poses any danger to the enemy's finery. Because he has nothing in particular to say, nothing about any of the so-called proofs, like those of the equilibrium of the lever, of the relations of space and time in the motion of falling, and so on, and, with Goethe's daring example of attacking the archenemy still three years in the future, Hegel timidly attacks: physiognomy and phrenology (*id.*).

Also for instance: what enlightenment explains as error and fiction in faith is what it itself is (VI, B.II); faith has the divine right, the right of absolute self-identity or pure thinking, against enlightenment (*id.*); enlightenment has only a human right against faith and for its truth (*id.*).

But mathematics and physics were not going to be stripped of their finery by an attack on physiognomy and phrenology, or by the revelation that they are like the Jewish people. Enlightenment was not going to succumb to the assertion of the divine right of faith. Attacks lacking in particularity had not been effective before Hegel, and Hegel's results in *The phenomenology* lack particularity to be effective. In his subsequent treatises Hegel's results will acquire particularity, and become lethal.

The phenomenology is very important because of its grand vision, and the unity and coherence of its grand vision. Everything must be taken away from the enemy, and restored to spirit. Philosophy must be taken away from him, knowledge must be taken away from him, and the world must be taken away from him. Philosophy must be taken away from him, and he must be unconditionally excluded from philosophy. Knowledge must be taken away from him, and the deficiency of his knowledge shown, and spirit's not just better, spirit's absolute knowledge shown. And the world must be taken away from him, and in place of his system of the world the system of spirit established.

What most of all gives the vision its grandeur, and what will ultimately decide the war, is that not only philosophy must be taken away from the enemy, and not only knowledge, but the world, and not just the world, but the system of the world.

Aristotle writes: the world is one, there is nothing next to the world, or outside the world (*Physics*, IV, 5). And Aristotle first described the world. He described the world in its entirety, some of its major parts, and many of its lesser parts, all in relation to one another. Ptolemy measured the world. He measured the world in its entirety, some of its major parts, and many of its lesser parts, all in relation to one another. Aristotle described the world, the one next to which there is nothing, and Ptolemy described the same world, the one next to which there is nothing, better, mathematically.

Kepler discovered laws, the laws of the motions of the heavenly bodies. In 1620, book IV of the *Epitome of Copernican astronomy* is *On the position, order, and motion of the parts of the world, or on the system of the world*. And grounded in the laws, Kepler demonstrated the position, order, and motion of the parts of the world, and thus the system of the world. Or what is the same, demonstrated that the world is a system. Or what is yet the same, demonstrated that the world is the system of the world.

Aristotle described the world, the one next to which there is nothing, Ptolemy described the same world, the one next to which there is nothing, better, and Kepler described the same world, the one next to which there is nothing, yet better. And the discovery that the world, the one next to which there is nothing, is the system of the world, the one next to which there is nothing, explains the fact that Aristotle's is the first description of the world because it is the first description of the position, order, and motion of the parts of the world, Ptolemy's is a better description of the world because it is a better description of the position, order, and motion of the parts of the world, including those described by Aristotle, and Kepler's is a yet better description of the world because it is a yet better description of the position, order, and motion of the parts of the world, including those described by Ptolemy.

With the discovery that the world is the system of the world all earlier and later discoveries began being collected explicitly in the system of the world. In 1687, book III of *The mathematical principles of natural philosophy* is *The system of*

the world. The system was grounded in the more general laws of motion and of gravity. And the system became the exclusive test of discovery. How discovery fits into the system, the one next to which there is nothing, became the exclusive test, whether discovery involved the stars, with Halley, or the earth, or animals, or plants, with Buffon, or Hutton, or the origin of the solar system, with Kant, or Laplace, or electricity, with Coulomb, or the elements, with Lavoisier, or Dalton. Or history, with Kant.

If I posit the I and the not I, I posit what I will, and Fichte first posited that not the world but philosophy is a system.

Hegel writes in *The system of science, the phenomenology*: reason is spirit in that the certainty that it is all reality is raised to truth, and it is conscious of itself as its own world, and of the world as itself (VI).

That the world is spirit and spirit is the system, is only spirit's vision in *The system of science, part one, the phenomenology of spirit*. It will take Hegel the following thirty-three years, nine of them after his death, to build the system that will succeed in replacing the system of the world. But build it he will, and succeed it will.

The second of the six treatises is *The science of logic*, in 1812-6.

Hegel explains what it is: it is the first step in an expanded plan of the system of science (pr.). He explains also what the other two steps in the expanded plan of the system will be: later on, I will deal with the other two real sciences of philosophy, the philosophy of nature and the philosophy of spirit (*id.*).

Of the first step, the fundamental results are two. It established that logic is dialectic. And it stripped mathematics of its finery.

Logic was dead, a dead skeleton. So: if spirit is to bring the life of substance and content to the dead skeleton of logic, its method must be the only one by which it can be pure science (intr.). Its method must be dialectic. In the *Critique of pure reason* dialectic is the logic of illusion. But: the opposite to Kant's conclusion must be drawn (II, 2.3.3); dialectic is the only true method; which is evident from this that dialectic is nothing different from its object and content (intr.). So: logic is the system of pure reason, the realm of pure thought (*id.*), but: the necessity of contradiction is the nature of thought (*id.*), and: the grasping of opposites in their unity, of the positive in the negative, is dialectic (*id.*).

Logic had begun with Aristotle, and begun with the principle of contradiction: one thing cannot be and not be at the same time in the same relation to another thing (*Metaphysics*, IV, 3). Aristotle had added: this is the most certain of all principles (*id.*), and: it is the principle of all principles (*id.*), and: if contradictories were true, all things would be one; the same thing would be a trireme and a wall and a man; it is what follows from what Protagoras says (4).

This is the dead skeleton, however. The necessity of contradiction is the nature of thought. The same thing is a trireme and a wall and a man.

The first result in *The science of logic* is: pure being and pure nothing is the same (I, 1.1.C). The last result is: the idea of the good can find its completion only in the idea of the true (II, 2.3.2.B); and this is the absolute idea (*id.*); the absolute idea is the identity of the theoretical and the practical (2.3.3); everything else is error, confusion, opinion, striving, faltering, and perishing; the absolute idea alone is being, imperishable life, truth knowing itself, and all truth (*id.*); it is the only object and content of philosophy (*id.*).

And between the first result and the last result everything is subjected to dialectic, everything in general, and a very substantial part of it in particular. *The science of logic* has three stations, theory of being, theory of essence, and theory of concept. Each of the three stations has three substations for a total of nine. Each of the nine substations has three sub-substations for a total of twenty-seven. Twenty-four of the sub-substations have three sub-sub-substations each, one has four, one has two, and one has zero, for a total of seventy-eight sub-sub-substations. About half of the sub-sub-substations have sub-sub-sub-substations, for a grand total of about two hundred. The concerns of the dead skeleton – concept, judgment, and syllogism – are but the three sub-substations of the first substation of the third station. Among the other sub-substations and sub-sub-substations are quantum, ratio, the absolute, the mechanism, the absolute mechanism, teleology, life, knowledge, the absolute idea. Hegel has

established in *The phenomenology* already that: language is the being of spirit (VI, C.c), and in *The phenomenology* already language and spirit have enhanced each other's power explosively. Language is the being of spirit in *The science of logic*, too, and dialectic, language, and spirit enhance each other's power beyond all bounds. One fundamental result, for instance, is this. Everything is *an sich* and *an sich selbst* and *in sich* and *in sich selbst* and *für sich* and *für sich selbst*, and next to *Sein* to life have come not only *Dasein* but *Ansichsein* and *Insichsein* and *Für-sichsein* (*passim*).

Another fundamental result is this. From the earlier result: every philosophy is essentially idealism; and philosophy is idealism as much as religion is (I, 1.2.C.c), Hegel now draws the conclusion: the opposition of idealistic and realistic philosophy is thus meaningless (*id.*).

In the end, the good is the true, and: given a soul, the method returns to its beginning (II, 2.3.3); the method extends itself into a system (*id.*). Logic is dialectic. Clearly: metaphysics is a part of logic (intr.). Clearly as well: logic is the realm of pure thought; but this realm is truth unveiled; and for this reason it can be said to be the exposition of god (*id.*).

Contradiction is necessary. Theoretical and practical are identical. Language is being. These results had been asserting themselves since Protagoras, at times more forcefully, at times less forcefully, at times irresistibly. Jointly, they were the more likely to assert themselves irresistibly. The more likely still, if they were incorporated into a system which was theology was metaphysics was logic was dialectic. And the more likely yet, if the system established itself with the greatest particularity, by subjecting to itself result after result after result, in particular, in order to subject to itself everything, in general.

And dialectic has since been irresistible.

A century later, Russell writes: Hegel criticizes traditional logic and professes to replace it by an improved logic of his own (*World*, 2).

Hegel's improved logic of his own is not logic, not if Aristotle, Chrysippus, Leibniz before him, Boole, Frege, Peano, Zermelo, Russell after him, are logic.

Hegel's improved logic of his own is nevertheless not his own, very far from it. Hegel's improved logic of his own has infused everything. Even if it has not infused everything in particular, it has yet infused everything in general, because it has infused philosophy not just thoroughly, it has infused it absolutely.

Heidegger affirms metaphysics. Carnap eliminates metaphysics. To affirm metaphysics, Heidegger begins with Hegel's: pure being and pure nothing is the same. To eliminate metaphysics, Carnap's position is Hegel's: the theses of realism and of idealism are both meaningless.

And as it subjects to itself result after result after result, in particular, *The science of logic* strips mathematics of its finery.

Berkeley envisioned the strategy. Allot to mathematics hypotheses, allot to physics the apparent, allot to metaphysics truth. To hasten the allotment, he attacked the mathematical principles. He did not propose to demonstrate 635 propositions, lemmas, and corollaries, he proposed to sap their foundations. He asked: and what are these evanescent increments? the ghosts of departed quantities?

Hegel writes: the increment, the increase of x by dx, is the fundamental evil in these methods (I, 2.2.C.c). Hegel, however, has found the source of the evil, and the source is impeccably dialectical, it is an inherent contradiction: a quantitative difference which by definition is and must be smaller than any given difference is no longer a quantitative difference (*id.*). Further, the confusion pervades mathematics, confusion of quantity and quality, confusion of arithmetic and geometry, confusion of analysis and synthesis. Further, only dialectic can deal with these, as with any, opposites. And these results of ever greater generality Hegel proves. His may not be proofs, his may be improved proofs of his own, but his are very many. Many more than 635. Enhanced beyond all bounds, the power of dialectic, language, and spirit is yet enhanced further by being brought to bear on the particulars, particular after particular after particular. In order to strip mathematics, in general, of its finery, Hegel strips – to note only some of the important, only among the mathematicians – Euclid, Archimedes, Galileo, Kepler, Descartes, Cavalieri, Fermat, Roberval, Barrow, Newton, Leibniz, Euler, Lambert, La-

grange, Lazare Carnot, in particular, of their finery. As well as, never mind Newton, Leibniz, Euler, and Lagrange, Hegel strips Gauss of his finery. Not only is Gauss, if there has been a mathematician as important as these, that mathematician, Gauss is a younger contemporary of Hegel, and very much alive when Hegel writes: analysis has not so much theorems as problems (II, 2.3.2.A.a); it is thus a highly superfluous piece of scaffolding to apply in analysis the form of the geometrical method and add to the solution of the problem a proof (*id.*); as is the case with Gauss's solution of the equation $x^m - 1 = 0$ (*id.*).

Thus, the fundamental evil is incurable. It was there, when calculus was first developed by Newton and Leibniz, it was not cured by Maclaurin, when he responded to Berkeley's attack, it was not cured by Lagrange, who is the focus of Hegel's attack, and it will not be cured by Cauchy, laboring at it just as Hegel was writing, and it will not be cured by Weierstrass, in the next generation, even if mathematicians will say that the evil is now an unmitigated good. In fact, not only is it incurable. The mathematicians can be told forthrightly: I consider it nothing but a sleight of hand and charlatanism (I, 2.2.C.c).

The allotment is as envisioned by Berkeley, except that mathematics is not allowed to transgress against physics any longer. It is a decisive addition to the strategy, *divide et impera*. Hegel writes: mathematics is in principle incapable of demonstrating the quantitative determinations of physics because these are laws grounded in the qualitative nature of their moments (*id.*). Further: Galileo, Kepler, and Newton proved the laws they discovered by showing that the particulars of perception conform to the laws; but a higher proof of these laws must be demanded, namely, to know the quantitative determinations from the qualities to which they are related (I, 3.1.B.c); and to attempt such a proof mathematically is absurd (*id.*). Of what mathematics is in principle incapable, in principle capable is dialectic, and what is absurd to attempt mathematically, is readily achieved dialectically.

Mathematics has since been stripped of its finery. Rhetoric once taught geometry what geometry is, Hegel now teaches mathematics what mathematics is.

Russell writes: all that Hegel says about mathematics is muddle-headed nonsense (*Ment. development*).

In the century after Hegel, thermodynamics was developed, electrodynamics was developed, the kinetic theory was developed, relativity was developed, quantum theory was developed. Thermodynamics was developed by mathematics, and if not of the highest sophistication at first, mathematics of the highest sophistication after Maxwell and Boltzmann. Electrodynamics was developed by mathematics, of the highest sophistication after Maxwell. The kinetic theory was developed by mathematics, of the highest sophistication from the first, by Maxwell and Boltzmann. Relativity was developed by mathematics, and if special relativity was not by mathematics of the highest sophistication, general relativity was by mathematics of the highest sophistication, both by Einstein. Quantum theory was developed by mathematics, of the highest sophistication after Heisenberg.

The conclusion is unavoidable. The world is written in mathematics. All that Hegel says about mathematics is muddle-headed nonsense.

A century after Hegel, the mathematicians engaged in a debate about the foundations of mathematics. There was a decade or so of turmoil, then the mathematicians arranged themselves in two battle lines with mathematical precision. At the head of one stood Brouwer, at the head of the other stood Hilbert. And this is how Brouwer, in 1912, in *Intuitionism and formalism*, describes the debate: two points of view can be distinguished, intuitionism and formalism; in many respects they have become more and more definitely opposed to each other; in recent years they have reached agreement that the exact validity of mathematical laws as laws of nature is out of the question; but the question where mathematical exactness exists, the two sides answer differently, the intuitionist says, in the human intellect, the formalist says, on paper (*id.*).

If the two sides have reached agreement that the exact validity of mathematical laws as laws of nature is out of the question, the two sides have reached agreement with Hegel. Mathematics is in principle incapable of demonstrating the quantitative determinations of physics. And if what the

two sides argue about is, in the human intellect, or on paper, what they argue about is which side will please Hegel better.

The conclusion is unavoidable. What the mathematicians say about mathematics is what Hegel says about mathematics.

It is relevant that Hilbert does not say what Brouwer says he says. For if it may be that Hilbert says what Brouwer says he says even if he does not say what Brouwer says he says, it is because Hegel says what he says.

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The third of the six treatises is the *Encyclopedia of the philosophical sciences in outline*, in 1817.

The fundamental results of the *Encyclopedia* are also two. It stripped physics of its finery. And, dealing with all three real sciences of philosophy: science of logic, natural philosophy, and philosophy of spirit, the *Encyclopedia* first displayed the system in all its scope and power, ready to establish itself in place of the system of the world.

The first station is the science of logic. The results are those of *The science of logic*, armed now also with brevity. In general: the dialectical is the principle of all motion, all life, and all activity in reality; it is thus the soul of all true scientific knowledge (81). More particularly: pure being is the pure abstraction and thus the nothing (87); the nothing is the equal to itself and thus the same as being, the unity of being and nothing is becoming (88); being with a determination, reflected in itself in this determination, is something (90); something becomes another, which is itself something, and so on to infinity (93).

The second station is natural philosophy.

Hegel writes: the idea which is for itself is intuition, and the intuiting idea is nature; the absolute freedom of the idea is that in the absolute truth of itself it decides to release free of itself the unmediated idea as its reflection, itself as nature (244). Also: god has two revelations, as nature and as spirit, and both forms of god are his temples which he fills and in which he is present (246). Also: nature is the bride which spirit marries (*id.*). Also: nature is the son of god, not as the son, however, but as abiding in being the other in which the divine idea is outside love (247). Also: nature is the unresolved contradiction (248).

Hegel did not strip physics of its finery, however, by asserting that nature is an idea. Even if the idea is the bride which spirit marries. He did not strip physics of its finery by defending Goethe against Newton either. Even if Newton's proofs are: completely bad (270). He did not strip physics of its finery by putting the stars in their proper place either: that the world of stars is a plurality in immeasurable spaces says nothing to reason; this is the external, the empty, the negative infinity; reason knows to raise itself above this (268).

Hegel stripped physics of its finery as he did mathematics.

Physics is plagued by opposites with which it is in principle incapable of dealing. Capable of dealing with them is dialectic. These results of the greatest generality Hegel proves. And his may not be proofs, they may be improved proofs of his own, but they are very many, more even than those stripping mathematics of its finery. Hegel brings the power of dialectic to bear on particular after particular after particular.

Never mind Newton, he strips Lavoisier of his finery. Two treatises have approached the success of *The mathematical principles* in advancing all by itself each a substantial part of physics. The second is Darwin's *Origin*. The first is Lavoisier's *Elementary treatise on chemistry, presented in a new order and according to the modern discoveries*, in 1789.

Lavoisier writes in the preface: descended to us from the Greek philosophers, the doctrine of the four elements is a mere hypothesis assumed long before the first principles of experimental philosophy or of chemistry were established (*id.*). This is all the attention the doctrine of the four elements receives in the *Treatise on chemistry*, and this is all the attention the doctrine of the four elements deserves. Formulated by Empedocles, it remained the best solution of the problem of the material principles of all things for two millennia. Two and a half centuries before Lavoisier, twenty after Empedocles, Paracelsus challenged the doctrine of the four elements, earth, water, air, and fire, with the doctrine of the three elements,

mercury, sulfur, and salt. A century-odd after Paracelsus, a century-odd before Lavoisier, in 1661, in *The sceptical chemist*, Boyle destroyed both, the doctrine of the four elements and the doctrine of the three elements.

In the following century-odd the first principles of experimental philosophy and of chemistry were established.

And in the *Treatise on chemistry*, in a new order and according to the modern discoveries, Lavoisier presents: a table of simple substances (II, 1). It includes:

simple substances which belong to the three kingdoms and which can be considered the elements of bodies,

light, caloric, hydrogen, nitrogen, and oxygen,

nonmetallic oxidizable and acidifiable simple substances,

carbon, phosphorus, sulfur, boracic radical, fluoric radical, and muriatic radical,

metallic oxidizable and acidifiable simple substances,

manganese, iron, cobalt, nickel, copper, zinc, arsenic, molybdenum, silver, tin, antimony, tungsten, platinum, gold, mercury, lead, and bismuth,

and salifiable simple earthly substances,

lime, magnesia, barite, argil, and silex (*id.*).

Of the 33 simple substances 1 is light. 1 is caloric. 3 are radicals. 5 are earths on which Lavoisier comments: we can assume that the earths will soon cease to be considered simple bodies and will be considered compounds, perhaps oxidized metallic compounds (*id.*). In 1869, in the *Correlation of the properties and atomic weights of the elements*, Mendeleev discovered the periodic law and arranged the elements in: the general system of the elements (*id.*). And the remaining 23 simple substances in Lavoisier's: table of simple substances, in 1789, are 23 of the 63 elements in Mendeleev's: general system of the elements, in 1869.

Two generations before Mendeleev, and one generation after Lavoisier, Hegel writes: in recent times for the determination of an element arbitrarily has been taken chemical simplicity, which has nothing to do with the concept of physical element (281); the chemical elements cannot be brought to order at all, they are entirely heterogenous; in opposition, the physical elements are the general, particularized in matter only according to the moments of the concept; they are therefore only four (*id.*). It is arbitrary chemical simplicity, which cannot be brought to order at all, that: air falls apart into the chemical abstractions of oxygen and nitrogen, water into those of oxygen and hydrogen (*id.*). The elements, the general, particularized in matter only according to the moments of the concept, are: air, the element of undifferentiated simplicity (282); water and fire, the elements of contradiction (283); and earth, the individual element (285).

Other than Lavoisier – to note only some of the important, only among the physicists – Hegel strips of their finery Empedocles, Aristotle, Archimedes, Copernicus, Paracelsus, Tycho, Galileo, Kepler, van Helmont, Snel, Descartes, Newton, Leibniz, Linnaeus, Euler, Galvani, Herschel, Haüy, Lamarck, Volta, Berthollet, Werner, Laplace, Dalton, Cuvier, Biot, Ørsted, Davy, Berzelius.

The three substations of natural philosophy are mechanics, physics, and organic physics, the three sub-substations of mechanics are space and time, finite mechanics, and absolute mechanics, the three sub-substations of physics are physics of general individuality, physics of particular individuality, and physics of total individuality, the three sub-substations of organic physics are geological nature, vegetable nature, and animal organism, the sub-substations have nineteen sub-sub-substations, the sub-sub-substations have fifteen sub-sub-sub-substations.

Physics has since been stripped of its finery.

Not quite two generations after Hegel, in *On the relation of the natural sciences to all science*, in 1862, Helmholtz writes: the opposition of the natural sciences and those united by common philological and historical studies seems to me to have developed under the influence of Hegel's philosophy; at the end of the last century, under the influence of Kant, there was no such separation; philosophy stood on exactly the same ground as the natural sciences; Kant's critical philosophy proposed to demonstrate the sources and authority of our knowledge; the new philosophy of identity was bolder; it started with the

hypothesis that the world is the result of the thinking of a creative spirit; but even if Hegel's philosophy were more or less successful in constructing the results of the spiritual sciences, this could be no proof of its correctness; the decisive test could only be the facts of nature; and here this philosophy failed completely; Hegel's philosophy of nature seemed, at least to the investigators of nature, absolute nonsense; not one of them could accept his ideas; so Hegel began an unusually vehement and acrimonious polemic against them, and especially against Newton as the greatest representative of the investigation of nature; the philosophers accused the investigators of nature of narrow-mindedness, the investigators of nature accused the philosophers of nonsense; so it came about that the investigators of nature began to avoid all philosophical influences in their work, and to consider philosophy not only useless, but harmful; while the spiritual sciences were taken over by enthusiastic supporters of Hegel's philosophy who undertook to reform them according to his system, and, by the speedy road of speculation, to gather in at once the harvest which had hitherto been approached by long slow labor (*id.*).

Helmholtz thus describes the past, and the present, and the future.

In the past, at the end of the previous century, there was no opposition between the natural sciences and those united by common philological and historical studies. Philosophy stood on exactly the same ground as the natural sciences. The decisive test could only be the facts of nature. The harvest was approached by long slow labor.

In the present, not quite two generations after Hegel, the investigators of nature avoided all philosophical influences in their work, and considered philosophy not only useless, but harmful.

It was not a symmetrical relation, however, even if the philosophers considered the investigation of nature not only useless, but harmful, too. Because the investigators of nature did not abandon philosophy. Philosophy was taken away from them. Newton had been a philosopher. Hegel disallowed that. Philosophy was taken away from them, they were excluded unconditionally from philosophy, and when philosophy was taken away from them, knowledge was taken away from them. The deficiency of their knowledge was established, and not just the better, the absolute knowledge of the spiritual sciences was established. Einstein is not about being, Heidegger is about being, and if they are both about time, Einstein is about time measurement, Heidegger is about the ontological foundations of that time measurement.

In the future, in the following generation, the reformation of all sciences according to the system was completed.

The third station in the *Encyclopedia* is the philosophy of spirit. The substations are subjective spirit, objective spirit, and absolute spirit. The sub-substations are anthropology, phenomenology of spirit, psychology, the law, morality, ethics, art, revealed religion, and philosophy.

The system now includes everything in general and, established particular after particular after particular, a very substantial part of everything in particular. It firmly includes metaphysics, theology, logic, dialectic, mathematics, mechanics, optics, astronomy, chemistry, geology, biology, anthropology, psychology, the law, ethics, art, religion, philosophy.

And it was to include more. The third sub-sub-substation of the third sub-substation of the second substation of the philosophy of spirit is the state. The third sub-sub-sub-substation of that sub-sub-substation is world history. It was when it included these, the state and world history, that the system became invincible.

The fourth of the six treatises is the *Natural right and political science in outline, elements of the philosophy of right*, in 1821.

Clearly, the system had to include politics prominently. Plato found out that politics is the means of making ethics appear the way to being and truth. The argument, as justice in the city, so justice in the soul, so justice in the world, stands on its first term. Aristotle discovered that the explanation of the state has to begin with what is first by nature, and bridge the abyss between what is first by nature and the state. Or what is the same, to describe the origin of the state. Hence, where justice is concerned, to describe the origin of justice, and that is, justice in the city. Hence, also, if there is justice in the soul distinct from

justice in the city, describe also the origin of justice in the soul. And *ipso facto* describe also the origin of justice in the world, lest there were justice in the world distinct from justice in the city, or in the soul. But Plato had already found out that if the argument, as justice in the city, so justice in the soul, so justice in the world, stands, there is no abyss to bridge. There is no origin to describe. Justice is immortal, indiscriminately justice in the city, in the soul, and in the world. And if the proposition, there is justice in the city, is made to stand, the argument, as justice in the city, so justice in the soul, so justice in the world, will readily, indeed eagerly, stand. And to make the proposition, there is justice in the city, stand, all that needs to be done is teach you that there is justice in the city. Educate you, as justice demands, too. Educate you that, all appearances to the contrary, there is justice in the city, and it is not a difficulty, not really, that the city in which there is justice is in the heaven, and in the city in which you live on earth there is no justice, not really, because that city in the heaven, in which there is justice, is real, and the city on earth, in which you live and there is no justice, is not real.

Clearly, the system that was to establish that, as metaphysics, theology, logic, and dialectic, so ethics, art, religion, and philosophy, and everything in between, had to have prominently the term politics.

Nor could it be difficult to add the term. Politics was an easy prey. The more so, if Hobbes was ignored altogether, as he is in the *Natural right*, and Rousseau informed politics with *The social contract*, and not with the *Discourse on inequality*, as he does in the *Natural right*.

The more so, given how much Hegel had already advanced the system.

Thus, Hegel asserts the fundamental proposition: what is rational, that is real, and what is real, that is rational (pr.). Hence: this treatise is nothing else but the attempt to grasp and present the state as rational in itself (*id.*). And three stations, eight substations, twelve sub-substations later, in the ninth substation, Hegel concludes: the state is the reality of the ethical idea, ethical spirit as the revealed, clear to itself, substantial will, which thinks and knows itself, and completes what it knows and insofar as it knows it (257).

The state is rational in itself. It is the reality of the ethical idea.

Moreover: the state is the reality of the substantial will, which it has in the raised to its generality particular self-consciousness; and this substantial unity is an absolutely unmoved end in itself (258).

The end of the state, its absolutely unmoved end in itself, is the unity of the particular self-consciousness in the generality of the ethical spirit.

Needless to add: it is god's way in the world that the state is (*id.*).

Politics was an easy prey. Rousseau – *Rousseau judge of Jean-Jacques* – made it an easier prey still when he made *The social contract* judge of the *Discourse on inequality*. And Hegel made it an easier prey still. As it may well be that politics should be, if politics concerns itself with the state, the state is the ethical idea, and politics is thus, clearly, ethics.

Hegel writes: jurists often pretend to a monopoly over knowledge of the law, and will not allow anyone who is not of their guild to speak about the law; in the same way the physicists have taken Goethe's theory of colors, for not only is Goethe not of their guild, he is a poet; but not more does one need to be a shoemaker in order to know whether his shoes fit than he needs to learn a trade in order to have knowledge of matters of general interest (215).

Goethe thus failed to open the guild of Newton, Helmholtz, and Heisenberg to the knowledge of whether one's shoes fit. Hegel, however, made a very substantial contribution to maintaining politics – never blessed, or cursed, with a guild to begin with – the repository of knowledge of that kind, the knowledge of whether one's shoes fit.

Two consequences.

Hegel writes: as high as spirit stands above nature, so high the state stands above physical life; one must therefore venerate the state as the earthly divinity, and one must see that, difficult as it may be to grasp nature, it is endlessly more difficult to understand the state (272).

One does not need to learn a trade to have knowledge of the state. Yet to understand the state is endlessly more difficult than to grasp nature. Each of these by itself should be most

implausible. One does not need to learn anything like shoemaking, or Newton's theory of color, or even what Hegel has to say in the *Natural right*, to have knowledge of the state. Yet to understand the state is endlessly more difficult than to grasp color, for instance, or gravity, or why there is a shoemakers' guild, for that matter. The two jointly should be an impossibility. One does not need to learn a trade to have knowledge of the state, endlessly more difficult as it is to understand the state than to grasp nature. Nothing is impossible for spirit, however, never mind implausible. High as spirit stands above nature, it stands as high as it will. After Hegel, accordingly, politics has remained all three, the knowledge of whether one's shoes fit, endlessly more difficult than general relativity, and both. Accordingly, two and a half centuries after Rousseau – and two centuries after Hegel – politics has yet to advance beyond Rousseau.

Also, it is god's way in the world that the state is. And Hegel continues: the foundation of the state is the power of reason to become real as will; we must, therefore, not look at particular states, we must look at the idea of the state; we must consider not particular institutions, but the idea, this real god (258).

We must not look at particular states. We must look at the idea of the state.

Mathematics we must strip of its finery, so we look at the particulars. Physics we also must strip of its finery, so we also look at the particulars. Politics is different.

We must not look at Egypt, Kish, Ur, Uruk, Lagash, Umma, Akshak, Akkad, the Gutians, Isin, Larsa, the Amorites, Ashur, Eshnunna, Mari, Yamhad, Babylon, the Hittites, the Kassites, Mittani, Byblos, Tyre, Judah, Israel, Damascus, Moab, Urartu, Phrygia, Lydia, Media, Sparta, Argos, Corinth, Megara, Athens, Miletus, Mytilene, Sicyon, Samos, Locri, Catana, Acragas, Carthage, Persia, Syracuse, Macedon, Thebes, Rome, Parthia, the Vandals, the Visigoths, the Burgundians, the Ostrogoths, the papacy, the Franks, the Lombards, the western empire, the eastern empire, the caliphate, the Umayyads, the Vikings, the Magyars, Bulgaria, Poland, Russia, the Fatimids, the Ghaznevids, the Seljuk Turks, the Normans, Castile, Aragon, Portugal, France, England, Denmark, the crusader principalities, Venice, Florence, the Hansa, the Swiss, Burgundy, the Teutonic knights, the Mongols, the Ayyubids, the Ottoman empire, Spain, Austria, the United Provinces, Sweden, Prussia, Piedmont, the United States, Haiti, Argentina, Colombia, Paraguay, Chile, Peru, Mexico. To note only some of the more important particular states, this side of the Indus, through 1821, at which we must not look. We must look at the idea of the state.

Of the philosophers who have investigated the state before Hegel barely any have, in order to investigate the state, looked at particular states. Being Plato's successors, they have followed Plato's example and, in order to investigate the state, looked at the state in the heaven. After Hegel none of the philosophers who have investigated the state has, in order to investigate the state, looked at particular states. We have done what we must.

As physics has advanced greatly before Hegel and after him, and politics has advanced little before him and not at all after him, it may be that it is because politics is endlessly more difficult than physics. It has to be nevertheless more plausible, with all due respect to spirit, that it is because the philosophers who have investigated the state have, in order to investigate the state, not investigated the state. They have, in order to investigate the state, investigated the not state.

The fifteenth and last sub-substation of the *Natural right* is world history, and it is here that Hegel obtains the most important result of the treatise: in world history the element of the being of general spirit is the spiritual reality of its entire compass of inwardness and outwardness (341).

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In 1831 Hegel died. The man died. The philosopher had still much to do.

Within a few days the Association of the friends of the immortalized came into being. The associated friends were seven at first, ten at length. And at the hands of his associated friends, the immortalized continued publishing important treatises, including two of the six very important.

The fifth of the six is the *Lectures on the history of philosophy*, in 1833-6.

It is with *The history of philosophy* that Hegel

excluded the enemy unconditionally from philosophy.

Hume writes: great philosophers are so rare, there have not yet been above two in the world, Galileo and Newton (*Station*).

Two generations later, in *The history of philosophy*, Galileo merits a couple of mentions in passing. Newton merits a page: Newton contributed the most to the English way of philosophizing, that is, of doing physics without thinking; in dealing with concepts Newton is a complete barbarian; imagining he had to do with physical things, he never learned that he had concepts, he treated concepts as if they were stone and wood; Newton's experiential sciences are something so untrue, so devoid of concepts, they can serve as an example how not to experiment, how not to draw conclusions, and in general how not to know (III, 2.2.B.6). And Hegel then does offer an example how to experiment, how to draw conclusions, and in general how to know: since that time the English have called the experimental sciences philosophy, mathematics and physics they have called Newtonian philosophy (*id.*). Complete barbarians.

The cases of Galileo and Newton are easy, though, Galileo can be just ignored, Newton can be dismissed summarily. The great success of *The history of philosophy* is in that it deals with the most difficult cases, too.

It transforms the physicist *par excellence* Aristotle into a not physicist. Hegel discusses Aristotle's philosophy of nature (I, 1.3.B.2), but so he does Plato's philosophy of nature (1.3.A.2), and as these old philosophers are allowed to have philosophies of nature, but no new philosopher is allowed to have a philosophy of nature, it is superfluous to add that Aristotle's philosophy of nature is not like Galileo's philosophy of nature or Newton's philosophy of nature, for there are no such philosophies of nature, Aristotle's philosophy of nature is very hard to tell from Plato's philosophy of nature, and both Plato's and Aristotle's philosophies of nature are very hard to tell from Hegel's philosophy of nature.

The history of philosophy allows that the new philosophy begins with Bacon (III, 1.A). The new philosophy does not begin only with Bacon, however. It begins with Bacon, but also with Boehme (1.B). With the instauration of the sciences, but also with mysticism. Needless to add, Boehme is far the more significant of the two.

Yet the most spectacular feat of *The history of philosophy* is that it exhibits a philosophy of Descartes from which his mathematics and physics are amputated in their entirety (2.1.A.1), and a philosophy of Leibniz from which his mathematics and physics are amputated in their entirety (2.1.C.1).

In the new philosophy of a Bacon less significant than Boehme, an inexistent Galileo, an amputee Descartes, a barbarian Newton, an amputee Leibniz, it is a matter of course that Hume is a sceptic, Kant is an idealist. It may be added that: Hume's scepticism has made itself more significant than it is (2.2.A.2), and: Kant's philosophy has a certain shallowness and dullness (3). Notice in *The history of philosophy* merits also: Kant's barbarity (3.B).

In the introduction, before turning to the particulars, Hegel describes the history of philosophy in general: the history of philosophy shows us not the becoming of alien things but our becoming (A).

In conclusion, after the examination of the particulars, he again describes the history of philosophy in general, and this time with greater, and indeed the greatest generality.

The final sub-substation is: result (III, 3.E). Hegel writes: philosophy is the true theodicy (*id.*), and: to know unity in opposition and opposition in unity is absolute knowledge (*id.*). These are not new results.

The new results are two. First: the present is the highest (*id.*), for which there is compelling evidence: a new epoch of the world has begun; it appears that world spirit has finally succeeded in casting off everything alien and grasping itself as absolute spirit (*id.*).

Also: the history of philosophy is the innermost of world history (*id.*); it is in world history, in general, and in the history of philosophy, in particular, that spirit is real as spirit; because spirit is only that in which it knows itself as absolute spirit (*id.*).

With these results Hegel is on the verge of delivering the *coup de grâce*. Of taking away

from the enemy the world, and the system of the world, and restoring spirit unconditionally to the substantiality and purity of its being.

The last of the six treatises is the *Lectures on the philosophy of history*, in 1840.

In the introduction, before turning to the particulars, Hegel describes world history in general: world history is the unfolding of spirit in time, just as the idea unfolds itself as nature in space (C.c).

In conclusion, after the examination of the particulars, he describes world history with the greatest generality: that world history is the development and real becoming of spirit, this is the true theodicy, the justification of god in history; what has happened and happens every day is not only not without god, it is essentially his work (IV, 3.3).

These are not new results. The new result is obtained in the course of the examination of the particulars.

The philosophy of history is smaller. Its particular results are not nearly as many as those of *The science of logic* or the *Encyclopedia*. Nor is any of them by itself nearly as spectacular as some of the results of *The history of philosophy*. The French revolution, for instance, raises the question: why did the French go immediately from the theoretical to the practical, while the Germans remained content with the theoretical (*id.*)? the answer is: it could be said that it is because the French are hot-headed; but the reason lies deeper; namely, in Germany the formal principle of philosophy encounters the concrete world in a way which satisfies the inner demands of spirit (*id.*), and the reason for this in turn is: in the Protestant world religion is the general form of truth; while in the Catholic world the holy is opposed to religion and thus to truth (*id.*).

And the new result, and the decisive result, is not that in the Protestant world religion is the general form of truth, but that the Protestant world is a world. Not that in the Catholic world the holy is opposed to religion and truth, but that the Catholic world is a world. Two worlds. Further: the Christian world is the world of completion, in it the principle is fulfilled, the end of days is full, in Christianity the idea sees nothing unsatisfying (IV, intr.). Three worlds.

In the *Natural right*, where Hegel deals with world history for the first time, there are: the Oriental realm, the Greek realm, the Roman realm, and the Germanic realm (355-8). In *The philosophy of history* the realms have become worlds. They are the four main worlds of *The philosophy of history*: the Oriental world (I), the Greek world (II), the Roman world (III), and the Germanic world (IV). For a total now of seven worlds. If, that is, the worlds may be counted. For the worlds are in unfathomable relations with one another. Some of them may be just worlds within worlds, the Catholic world and the Protestant world may be worlds within the Christian world. But the Germanic world is also: the Christian Germanic world (IV, 1.2), and this world may or may not be the Christian world. Or the other way around, the Christian world may or may not be the Christian Germanic world. It is in the course of dealing with the Christian Germanic world that Hegel deals with: Muhammadanism (*id.*), which is not part of: the European world (*id.*). And the Greek world, the Roman world, and the Germanic world may be worlds within the European world, and it may be that the European world is the Christian world. Or the Christian world is the European world. In short: when we consider god as the trinity the number three has a much more subordinate meaning than when we are concerned with the triangle, and counting can only have a much more subordinate meaning when we consider the worlds. If it can have any meaning at all.

This is the result that delivered the *coup de grâce*. Because the worlds are not troubled by the question, do they share the moon? or the Oriental world has its own, the Oriental moon, the Greek world its own, the Greek moon, the Roman world its own, the Roman moon, and the Christian Germanic world its own, the Christian Germanic moon? and this last moon, is it one moon or two moons, the Catholic moon, which is opposed to the holy, and the Protestant moon, which alone satisfies the inner demands of spirit? Or the question, do they share nitrogen and oxygen? or there are Oriental nitrogen and oxygen, Greek nitrogen and oxygen, Roman nitrogen and oxygen, and nitrogen and oxygen of completion, in which the end of days is full, and from which Muhammadanism is excluded? Or the question,

for that matter, do they share earth, water, air, and fire? Or the triangle? Or, for that matter, the trinity?

Clearly, the world is not the world. The one and only world is not the one and only. The world described by Darius, Aristotle, Ptolemy, al-Battani, Kepler, Newton, Orientals, Greeks, Romans, Muhammadans, Catholics, Protestants, is at best a world. The system of the world is at best a system of a world.

Also clearly, if the world is not the only world, the worlds of *The philosophy of history* need not be the only worlds either. Once Hegel pointed the way, worlds proliferated with abandon.

And when Dilthey, two generations later, concluded: the world is nowhere but in the ideas of the individual, this did no damage to the world that had not already been done to it. If the world is many, why not as many as there are individuals? But when Dilthey found out that the world is world-views, this abolished the world altogether. There have not even been worlds since, only world-views.

To that end yet another result of Hegel has been indispensable. When he writes in *The history of philosophy*: the present is the highest, and the evidence is: it appears that world spirit has finally succeeded in grasping itself as absolute spirit, this can be taken to be about Hegel's present. It can be taken to assert that Hegel's present is the highest, and it appears that it is in Hegel that world spirit has finally succeeded in grasping itself as absolute spirit. The present being the present, however, it will not be so taken. The present being the present, it is about the present. Thus, in *The philosophy of history* Hegel asserts with the greatest generality: when we pass through the past, no matter how large it is, we have to deal only with the present; for spirit is immortal, spirit is never past, never not yet; spirit is now essentially (intr., C.c).

The present is the highest. And we have to deal only with the present.

And we have since dealt only with the present, *ad hoc*, and at will.

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Hegel's results – the dead divine man is the general self-consciousness, reason is spirit conscious of the world as itself, the necessity of contradiction is the nature of thought, pure being and pure nothing is the same, the absolute idea is the identity of the theoretical and the practical, there can be no philosophy without religion, the dialectical is the soul of all scientific knowledge, something becomes another which is itself something, the ideality of the finite is the fundamental proposition of philosophy, nature is the bride which spirit marries, reason knows to raise itself above the world of stars, the elements are air, water, fire, and earth, the state is ethical spirit which thinks and knows itself, the new philosophy begins with Boehme, Newton is an example how not to know, philosophy is the true theodicy, to know unity in opposition and opposition in unity is absolute knowledge, spirit is that in which it knows itself as absolute spirit, world history is the true theodicy, the present is the highest – Hegel's results can need no recommendation as one, or the other. Absolute knowledge, or results against the results from the acquisition of knowledge. Or what is the same, barriers before the acquisition of knowledge and the investigation of knowledge. More particularly, if they are not absolute knowledge, they are a most substantial and most massive reinforcement of the barriers before the acquisition of knowledge and the investigation of knowledge. Man, the kinds of knowledge, vagueness. The most substantial reinforcement of the barriers after Augustine, and the most massive reinforcement of the barriers before or after Augustine.

And the success of Hegel can be explained.

To begin with, there is Hegel himself. The results of very few philosophers have attained comparable scope, Aristotle, Descartes, Leibniz, Kant. Nor have the results of any of these philosophers, or of any philosopher, come even close to the degree to which Hegel's results are integrated into a complete and unitary system. Nor could the results of Aristotle, or Descartes, or Leibniz, or Kant be. Because their results are results from the acquisition of knowledge and the investigation of knowledge. Aristotle's results in logic and his results in biology, for instance, are difficult to integrate into one another, and more difficult still to integrate into his results in the investigation of knowledge. Descartes's results in mathematics, in

physics, in the investigation of knowledge are difficult to integrate into one another, Leibniz's results in logic, mathematics, physics, history, the investigation of knowledge are very difficult to integrate into one another, Kant's results in physics and the investigation of knowledge are difficult to integrate into one another. Hegel's results in metaphysics, theology, logic, dialectic, mathematics, mechanics, optics, astronomy, chemistry, geology, biology, anthropology, psychology, the law, ethics, art, religion, philosophy, politics, esthetics, history of philosophy, philosophy of history, world history are not at all difficult to integrate into one another. Unwaveringly on the same ground, unwaveringly in pursuit of the same goal, Hegel relentlessly integrates each of his results into spirit, and all of his results into one another.

Further, Hegel's labors are the decisive victory in a war which had already been fought for three generations, counting from Berkeley, and which was to continue to be fought for two more generations, counting to Dilthey, to obtain the unconditional surrender of the enemy.

Further, it was a war. With Clausewitz: war is an act of force to compel the enemy to fulfill our will (*War*, I, 1.2). It was an act of force, it was an act of force to compel the enemy, and it was an act of force to compel the enemy to fulfill our will. Conceived by Berkeley, perfected by Hegel, and executed by Hegel with consummate skill, the strategy proved invincible. Allot to mathematics hypotheses, to physics the apparent. Assign, then, to mathematics and physics deficient knowledge. Exclude them from philosophy. Separate mathematics from physics. And confine physics to being but one of the stations or substations or sub-substations of that deficient knowledge. Allot to metaphysics truth. Assign, then, to philosophy absolute knowledge. Arm, on the one flank, metaphysics with dialectic. Arm, on the other flank, the philosophy of spirit with world history. And envelop the enemy in the pincer movement of dialectic and world history, and bring him to his knees. Have him surrender the world.

Further, the strictest analogy between Hegel and Augustine holds. Augustine's results are of great scope, relentlessly integrated into completeness and unity, in the unwavering pursuit of one and the same goal. His labors were the decisive victory in a war which began before him, and was fought with great success before him. It was a war. And what the victory in that earlier war achieved is very much what the victory in the later war has achieved. After that earlier war mathematics and physics became futile curiosity masquerading under the name of knowledge. The fathers of the church decided what was noxious, what was beneficial. History became theodicy. After the victory in the later war, mathematics is deficient. Physics is a science. The spiritual sciences decide what is noxious, what is beneficial. And Hegel's worlds continue to proliferate to populate the ever higher than the highest present of history.

The analogy holds because Augustine and Hegel are the effects of the same cause.

There can be no better way to approach the problem of the cause than to review the labors at reinforcing the barriers before the acquisition of knowledge and the investigation of knowledge. To review these labors in relation to the labors of acquisition of knowledge and investigation of knowledge. The labors at reinforcing the barriers can be gathered into four efforts. The fourth was completed after Hegel.

Before turning to the problem of the cause, then, it remains to conclude the review.

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After Fourier,
Ampère developed electrodynamics, and discovered one of its laws,
Avogadro distinguished atoms and molecules,
Gauss raised the standard of rigor, advanced number theory, algebra, geometry, electrodynamics, astronomy, and developed the method of least squares,
Carnot discovered the cycle of reversibility of the heat engine,
Cauchy defined the derivative, and grounded analysis in limit and continuity,
Clausewitz wrote *On war*,
Bessel measured the distance of 61 Cygni,
Lobachevsky developed non-Euclidean geometry,
Hamilton generalized the principle of least action, and discovered quaternions,
Galois developed the theory of groups,

Faraday discovered electromagnetic induction, and discovered that electromagnetism is a field,
Boole grounded syllogistic in sets,
Darwin formulated the theory of evolution,
Cavour united Italy,
Weierstrass showed that there are continuous nowhere differentiable functions,
Marx formulated a theory of the development of society,
Helmholtz discovered the conservation of energy,
Mendel discovered the laws of heredity,
Clausius formulated the laws of thermodynamics, and discovered entropy,
Riemann generalized geometry, complex analysis, number theory,
Mendeleev discovered the periodic law,
Bakunin developed anarchism,
Maxwell discovered that the velocities of molecules follow a statistical distribution, and formulated the laws of electrodynamics,
Boltzmann developed the kinetic theory,
Bismarck united Germany,
Cantor developed the theory of sets,
Frege discovered that the elements of a proposition are function and argument, discovered how truth, generality, and existence can be expressed formally, and developed a formal language in which the chains of inferences are free of gaps,
Thomson discovered the electron,
Peano axiomatized arithmetic,
Planck discovered that thermal radiation imparts energy to matter in discontinuous quanta,
Hilbert formulated the theory of proof,
Zermelo axiomatized the theory of sets,
Einstein formulated the theory of special relativity, discovered that light imparts energy in quanta, and formulated the theory of general relativity,
Minkowski discovered that the principle of relativity is a principle of invariance,
Hardy discovered the law of the equilibrium of a population,
Rutherford discovered that the mass of the atom is concentrated in a nucleus,
Hertzsprung established the relation of intrinsic brightness and spectral type of stars, and hence the fact of the evolution of stars,
Bohr discovered that the nucleus of the atom is orbited by electrons jumping from one orbit to another with the absorption or emission of a quantum of energy,
Lenin founded the Soviet Union,
Friedmann solved the field equations of general relativity for expansion of the world,
Schrödinger formulated the law of quanta,
Heisenberg formulated the uncertainty principle,
Chetverikov achieved the synthesis of Darwin and Mendel,
Hubble measured the distance of Andromeda, and discovered that the world is expanding,
Hitler fought the greatest and most ignoble war humanity has fought,
and these are once again only some of the most important results of only some of the most important philosophers and princes after Fourier.

The acquisition of knowledge, then, continued. Physics made very great strides.

Thermodynamics was developed, the theory of the conversion of energy, from Carnot, the discovery of the cycle of reversibility of the heat engine, through Helmholtz, the discovery of the conservation of energy, to Clausius, the discovery of the increase of entropy, and the formulation of the two laws of thermodynamics. Avogadro distinguished atoms and molecules. The periodic law was discovered, the law of the elements, by Mendeleev. Electrodynamics was developed, the theory of electricity, magnetism, and light, from Ampère, the description of the phenomena of electromagnetism analytically, through Faraday, the discovery of the electromagnetic field, to Maxwell, and the formulation of the four laws of electrodynamics. The kinetic theory was developed, the theory of the motion of molecules, from Maxwell, the discovery of the statistical distribution of the velocities of molecules, to Boltzmann, the formulation of the law of distribution of energy in a system – and the kinetic theory showed the dependence of mechanics and thermodynamics on one another. Relativity was developed, from Einstein, and special relativity, the theory of motion, through Minkowski, the discovery that the laws of nature are invariant under the Lorentz transformation, to Einstein, and general relativity, the theory of space and time – and special relativity already showed the unity of

mechanics and electrodynamics. Quantum theory was developed, the theory of matter and energy, from Thomson, the discovery of the electron, through Planck, the discovery of quanta, Einstein, the discovery that light is quanta, Rutherford, the discovery of the nucleus of the atom, Bohr, the discovery of the structure of the atom, to Schrödinger, the formulation of the law of quanta, and Heisenberg, the formulation of the principle of dependence of observation on the atomic phenomena – and quantum theory showed the dependence of mechanics, thermodynamics, the periodic law, electrodynamics, the kinetic theory, and after Dirac relativity, too, on one another. The distances of the stars were measured, beginning with Bessel, stars were classified, the fact of their evolution was established, beginning with Hertzsprung, the distances of the galaxies were measured, beginning with Hubble, the field equations of gravitation were solved for expansion of the world, by Friedmann, Hubble discovered the expansion of the world, in the following generation the big bang theory was developed, the theory of the origin and development of the world – and the big bang theory is a development of quantum theory.

The theory of evolution was developed, by Darwin, the laws of heredity were discovered, by Mendel, Chetverikov showed how Mendel corroborates Darwin, and biology showed itself ever more explicitly a part of physics, by the theory of evolution, which is a theory paradigmatically, encompassing all biology while standing on a single principle, natural selection, and the laws of heredity, Mendel's laws, are quantitative, and the law of the equilibrium of a population, Hardy's law, is also quantitative, and by becoming a part of chemistry, in particular, from the demonstration, in the generation before Chetverikov, that the chromosome is the carrier of heredity, by Boveri, to the demonstration, in the generation after Chetverikov, that replicating DNA is the carrier of heredity in the chromosome, by Crick and Watson, and of the regulatory mechanism of the replication, by Jacob and Monod.

Mathematics also made very great strides.

The standard of rigor was raised. Among Gauss's many results are, in number theory the proof of the law of quadratic reciprocity, in algebra the proof of the fundamental theorem of algebra, in geometry the definition of intrinsic curvature, in electrodynamics one of the laws generalized into Maxwell's equations, in astronomy the method of calculating an orbit from three observations. The method of least squares reduced observation to curves of best fit. From Cauchy to Weierstrass, analysis was grounded in limit, continuity, and rigor. Lobachevsky developed non-Euclidean geometry, and Riemann generalized Euclidean and non-Euclidean geometries into the theory of manifolds and the theory of metrics. The Riemann zeta function brought complex analysis to bear on the investigation of the primes. Analysis, the geometry of manifolds and of metrics, groups, beginning with Galois, quaternions, beginning with Hamilton and leading to the development of vector analysis, were inseparable from the development of thermodynamics, electrodynamics, the kinetic theory, relativity, and quantum theory.

The theory of sets was developed, by Cantor, and the theory of sets became the foundation of mathematics, and as the development of physics had been and, pace Hegel, continued to be inseparable from the development of mathematics, the foundation of physics, and as the development of logic, pace Hegel, became inseparable from the development of mathematics, the foundation of logic, and as the development of mathematics and physics had been and, pace Hegel, continued to be inseparable from the development of philosophy, the foundation of philosophy.

Logic became mathematical, beginning with Boole, and conclusively with Frege.

Arithmetic was axiomatized, by Peano. The theory of sets was axiomatized, by Zermelo.

Hilbert axiomatized geometry, in 1899. In 1900, in the *Mathematical problems*, he collected mathematics into the twenty-three problems before it. The sixth is: the axiomatization of physics (*id.*). The second is: to prove that the axioms are free of contradiction among themselves, and that is, that a finite number of logical conclusions on their ground can never lead to results which are in contradiction with one another (*id.*). In 1918, he generalized the problem: to establish that in every domain of knowledge contradictions on the ground of the system of

axioms on which it stands are impossible (*Axiomatic*). The fact that physics is mathematical, the fact that the theory of sets is the foundation of mathematics, the fact that logic is mathematical, the fact of axiomatization – and axiomatization goes back to the *Elements*, and the successful axiomatization of physics goes back to *The mathematical principles of natural philosophy* – these facts one and all pose the problem formulated by Hilbert: to establish that in every domain of knowledge contradictions on the ground of the system of axioms on which it stands are impossible. And if Gödel's proofs of the incompleteness theorems, in 1931, show that the problem cannot be solved by the means formulated by Hilbert: to solve the problem of the decidability of a mathematical question by a finite number of operations (*Axiomatic*), the same facts one and all continue to pose the problem – just as it is formulated in all its generality by Hilbert – and to pose it the more acutely now, joined as they are by the fact that Gödel's proofs are proofs. Mathematical proofs, not dialectical.

After Hegel, then, on the one hand, physics and mathematics made very great strides, and the acquisition of knowledge continued.

On the other hand, the relations of physics, mathematics, history, and philosophy changed, catastrophically so, and the reinforcement of the barriers before the acquisition of knowledge and the investigation of knowledge continued as well.

On the one hand, the fact that the world is written in mathematics was ever more unambiguously shown.

On the other hand, the supposition that mathematics is a product of human thought became ubiquitous. Einstein, no less, writes: as far as the propositions of mathematics refer to reality, they are not certain, as far as they are certain, they do not refer to reality (*Geometry*). To reach this conclusion, Einstein begins with the question: how is it that mathematics, a product of human thought independent of experience, is so admirably suited to the objects of reality (*id.*)? And not only is the supposition that mathematics is a product of human thought impeccably Hegelian. Nothing but dialectic can resolve the difficulty, how is it that mathematics is so admirably suited to the objects of reality and does not refer to reality, both. Einstein adds: and yet mathematics owes its existence to the need to learn something about the behavior of real objects (*id.*). And if mathematics owes its existence to the need to learn something about the behavior of real objects, and this need has been satisfied, by the special theory of relativity, for instance, and by the general theory of relativity, also for instance, the problem can only be, how is it that mathematics refers to reality and is certain, both.

On the one hand, if philosophy was still concerned with the world and with knowledge, the development of the theory of the origin and development of the world, the development of the theory of the origin and development of organisms, including man, the development of the theory that became the foundation of mathematics, the development of mathematical logic, the formulation of the problem of showing that in every domain of knowledge contradictions on the ground of the axioms are impossible, showed ever more unambiguously that physics and mathematics were parts of philosophy.

On the other hand, the investigators of nature, Helmholtz reports, avoided all philosophical influences in their work, and considered philosophy not only useless, but harmful.

Physics became one of the sciences, biology became another of the sciences, mathematics was excluded even from the sciences.

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After the lessons learned from Napoleon and his defeat, the theory of war was brought to its highest point, by Clausewitz.

As to the practice of war, in the century after Napoleon humanity kept it localized, fighting no wars greater than the Crimean, the American civil, the Franco-Prussian, the last Russo-Turkish, the Russo-Japanese. In the century after that it did not keep it localized. In that century humanity fought the great war, and it was the greatest war humanity had fought. So far. With the great war the Soviet Union also came into being. Then the great war turned out to have been the first world war. And the second world war was not only the greatest, it was the most ignoble war humanity has fought. So far.

When Lenin founded the Soviet Union, it was

the most explicitly philosophical foundation of a state. It was much more explicitly philosophical than the foundation of the state toward which the French revolution labored, and very much more explicitly philosophical than the foundation of the United States. The foundation of the Soviet Union was explicitly an implementation of the theory, formulated by Marx, of the development of society and its revolutionary transformation.

If politics is a part of the labors of acquisition of knowledge, the labors of building the Soviet Union are as important a part of those labors as the labors of building any state are. If ethics is the judge of those labors, too, the assessment of the success or failure of the labors of building the Soviet Union should also be as important a part of that judgment as the assessment of the success or failure of the labors of building any state is.

In 1798, when all its fair-weather friends had long abandoned the revolution, Kant writes: the revolution may succeed or fail; it may be filled with misery and with such atrocities as to make a right-thinking man, even as he hopes to see it succeed once more, never attempt the experiment at such cost; the revolution yet finds in the hearts of all spectators a wishful participation that borders on enthusiasm; and this sympathy can have no other cause than the moral predisposition of the human race (*Quarrel*, 2.6). If the human race has not lost its moral predisposition, a right-thinking man should hope to see the Soviet Union – which never had any but fair-weather friends – succeed once more, even as he should never attempt the experiment at the cost at which it was first attempted.

As the Soviet Union is an effect of philosophy, so the greatest and most ignoble war humanity has fought, the war precipitated, fought, and lost by Hitler, is an effect of philosophy. With Marx philosophy became an ideology, with Dilthey it became a world-view, and that Hitler's war was a war of an ideology, or a world-view, *My struggle* is as explicit as it is in urging the waging of that war.

Moreover, not only is Hitler's war an effect of philosophy, it is a very important effect. It can be noted, for instance, that, as it happens, these four were all born in the same year,

1889 - 1945 Hitler,
1889 - 1951 Wittgenstein,
1889 - 1953 Hubble,
1889 - 1976 Heidegger.

And it can be asked, can it be that the *Logico-philosophical treatise* and *Philosophical investigations*, or *Being and time* and *Introduction to metaphysics*, or all of them, are more important than Hitler's war?

The distress this fact should cause should be alleviated by the comfort the fact should cause that the effects, in turn, of war are limited in space and time, while those of the origin and development of the world are not, and no war, not even the greatest and the most ignoble, can be nearly as important as the big bang theory.

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In 1945, Russell writes of: the growth of unreason throughout the nineteenth century and what has passed of the twentieth (*Hist. philosophy*, III, 17). The rest of the twentieth has since also passed, and some of the twenty-first.

And if the wars of those centuries illustrate the growth of unreason all too well, the new philosophy such as it has refused to be physics and mathematics, or even to be informed by physics and mathematics, and the new history such as it has been reformed by the new philosophy, illustrate it better still.

The war for the restoration of spirit to the substantiality and purity of its being was concluded in the couple of generations after Hegel. Dilthey attests its conclusion.

He writes: the spiritual sciences are an independent of the natural sciences whole (*Introduction*, I, 2); they do not study nature, they study the historic-social reality (7); the entities which act on one another in the wonderful whole of history and society are *individua*, each different from every other, each a world, for the world is nowhere but in the ideas of such an individual, and the whole significance of each individual acting at some point of the spiritual cosmos is revealed by the proposition, *individuum est ineffabile* (8); thus differentiated, the true nature of the spiritual sciences emerges and they can break the fetters in which the older sister has held the younger since Descartes, Spinoza, and Hobbes, who imposed on these sciences their

developed in mathematics and natural science methods (19).

It is thus unconditional victory that Dilthey attests. Mathematics and natural science are broken fetters. The citadel is inaccessible, *individuum est ineffabile*. The fortress protecting the citadel encompasses the whole of the independent of the defeated enemy spiritual sciences. The ground on which the citadel and the fortress stand, and on which the outworks extend at will, is the fundamental proposition, the world is nowhere but in the ideas of the individual. Confined without, the defeated enemy is conceded nature. Within is the historic-social reality, the wonderful whole of history and society, the spiritual cosmos.

Within and without, the world is world-views. And although: the threads of world-views all come together in metaphysics (*Types*, 2.3); the ultimate root of world-views is life (1.1).

Even the provenance of the proposition, *individuum est ineffabile*, is immaculate. It is Goethe (to Lavater, Sep 1780).

In the final stage of the war, however, in the couple of generations between Hegel and Dilthey, as in the final stages of all wars that end in unconditional surrender, the vanquished labored harder even than the victors to bring about the unconditional surrender. The mathematicians who reached agreement that the exact validity of mathematical laws as laws of nature is out of the question. The investigators of nature who avoided all philosophical influences in their work, and considered philosophy not only useless, but harmful. And most of all the philosophers who, fighting against Hegel and spirit, yet fought for Hegel and spirit.

Thus, Schopenhauer is a younger contemporary of Hegel, and his assessment of the state of philosophy in 1839 – when the man Hegel was dead but the philosopher Hegel continued to publish very important treatises – will appear flawless. Schopenhauer writes: philosophy, if one would still call it that, sank lower and lower, until it reached the lowest level of abasement in the ministerial creature Hegel; this man, in order to smother the freedom of thought which Kant fought for and won, made philosophy a tool of state aims, obscurantism, and Protestant Jesuitism; and to hide the disgrace and bring about the greatest possible stupefaction of minds, he drew over it a cloak of the emptiest rubbish and silliest galimatias which have ever been heard outside the insane asylum (*Freedom*, 4).

Schopenhauer's most important treatise is *The world as will and representation*, published in 1819, and doubled in bulk in its second edition, in 1844.

In *The world* Schopenhauer is clear also who his allies would be. He writes in the preface: I start from what was achieved by the great Kant (pr.), and: there is more to learn from every page of Hume than from the collected philosophical works of Hegel, Herbart, and Schleiermacher combined (II, 46).

Then again, the first mention of Hume in *The world* establishes: Hume's scepticism (I, 4), and the first mention of Kant after the preface is this: the world is my representation; Berkeley was the first to state this truth decisively, rendering thus an immortal service to philosophy; Kant's first mistake was the neglect of this principle (1).

As to physics and mathematics: the empirical sciences are a suitable occupation for those of good capacities but lacking the highest faculties, which would in any case hinder minute investigations of that kind (II, 12), and: mathematics hinders the general formation and development of spirit; its only immediate use is to train fickle and fluttering heads to fix their attention (13).

Itself, *The world* is not so hindered, and its use is not that. Schopenhauer reports: what this book imparts is one single thought; I hold this thought to be what has long been sought under the name philosophy; and as this one thought is considered from different sides, it shows itself as that which has been called metaphysics, that which has been called ethics, and that which has been called esthetics (pr.).

Allot to mathematics hypotheses, to physics the apparent, to be sure, yet Berkeley, both, takes it for granted that mathematics and physics are parts of philosophy and allots to the one hypotheses, to the other the apparent, on the ground of much labor of examination of the hypotheses and the apparent. Deficient knowledge, to be sure, yet Hegel as well, both, takes it for granted that mathematics and physics are parts of philosophy

and exposes the deficiency of their knowledge on the ground of much labor of examination of that deficient knowledge. The unconditional separation of the fortress, philosophy, metaphysics, ethics, and esthetics within, from the occupation suitable for those lacking the highest faculties and the training of fickle and fluttering heads without, begins with Schopenhauer. With Schopenhauer begins also the judgment of that occupation and that training on the ground of examination no other than they deserve from those endowed with the highest faculties and steadfast and unwavering heads. None whatsoever.

It goes without saying that it is all in the service of the citadel.

It goes without saying as well that the ground is the fundamental proposition: the world is my representation (I, 1), and: the world is my representation, is, like the axioms of Euclid, a principle which everyone must acknowledge true as soon as he understands it (II, 1); accordingly, true philosophy must always be idealistic; indeed, it must be so if only to be honest (*id.*); philosophy is idealistic essentially (*id.*).

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Schopenhauer is an important philosopher, but not very important. A generation later, Marx is a very important philosopher, by far the most important of the new kind of philosopher after Hegel. And Marx is like Rousseau. On the one hand, Rousseau proposed an explanation of the state which remains the best explanation of the state. On the other hand, Rousseau made a substantial contribution to the restoration of spirit to the substantiality and purity of its being. On the one hand, Marx formulated a theory of the development of society which remains the most successful attempt to formulate the theory of the development of society. On the other hand, Marx made a contribution to the reformation of history more substantial even than Hegel's.

In 1859, in the *Contribution to the critique of political economy*, Marx describes how he developed the theory: I studied jurisprudence, which however I treated as a discipline subordinated to philosophy and history (pr.); the first work which I undertook was the critical examination of Hegel's philosophy of right; my investigation led me to the result that legal relations and forms of the state cannot be conceived either from themselves or from the so-called general development of human spirit, but are rooted in the material relations of life, whose totality Hegel embraces under the name civil society, and that the anatomy of civil society is to be sought in political economy (*id.*).

Marx then states the theory: in the social production of their life men enter in definite, necessary, independent of their will relations, relations of production, which correspond to a definite stage of development of their material forces of production; the totality of these relations of production constitutes the economic structure of society, the real basis on which a legal and political superstructure arises and to which definite social forms of consciousness correspond; the mode of production of material life conditions the entire social, political, and intellectual life process; not the consciousness of men determines their being, but the other way around, their social being determines their consciousness; at a certain stage of their development the material forces of production of society come into contradiction with the present relations of production, or what is only the legal term for them, with the property relations within which they have until then developed; from forms of development of the forces of production these relations turn into their fetters; an epoch of social revolution begins; with the change of the economic foundation a complete transformation of the whole enormous superstructure takes place; in the consideration of such transformations the material transformation of the economic conditions of production, which can be established with the precision of natural science, must be distinguished from the legal, political, religious, artistic, or philosophical, and in short ideological forms, in which men become conscious of this conflict and engage in it (*id.*); broadly, Asiatic, ancient, feudal, and bourgeois modes of production can be indicated as the progressive epochs of the economic social formations; the bourgeois relations of production are the last antagonistic form of the social process of production; this antagonism arises from the social conditions of life; but the forces of production developing in the womb of bourgeois society

create at the same time the material conditions for the resolution of this antagonism; hence, with this social formation the prehistory of human society ends (*id.*).

It is Marx's final and most explicit statement of the theory. Of all fundamental elements of the theory only one is presented implicitly rather than explicitly. The antagonism in the relations of production is the antagonism of classes, and the resolution of the antagonism is the resolution of class struggle. In 1859, Marx can take it for granted that this does not need to be made more explicit than it has been. In 1848, in the *Manifesto of the communist party*, Marx and Engels have made it as explicit as anything has ever been made explicit.

The *Manifesto* warns: a specter is haunting Europe, the specter of communism (pr.). Then it begins: the history of all previous society is the history of class struggles (1); oppressor and oppressed always stood in opposition to one another (*id.*). It elaborates: in the earlier epochs of history there were various social orders (*id.*); but our epoch, the epoch of the bourgeoisie, has simplified the opposition of the classes; the entire society is splitting more and more into two great hostile camps, in two great classes which stand directly against one another, bourgeoisie and proletariat (*id.*). And it ends, one hostile camp to the other: let the ruling classes tremble at a communist revolution; the proletarians have nothing to lose but their chains; they have a world to win; proletarians of all countries, unite (4)!

The communist revolution is the end of class struggle, or what is the same: the end of the prehistory of human society. In 1852, Marx writes: no credit is due to me for discovering either the existence of classes in modern society or the struggle between them; what is new that I did was to prove, first, that the existence of classes is only bound up with definite historical phases in the development of production, second, that class struggle necessarily leads to dictatorship of the proletariat, third, that this dictatorship itself constitutes only the transition to the abolition of all classes and to a classless society (to Weydemeyer, 5 Mar).

Generally, then, the totality of the material conditions of life is civil society, and the anatomy of civil society is political economy.

More particularly, at any given time, there are the material forces of production. Corresponding to the forces of production are the relations of production, or what is only the legal term for them, the property relations. The relations of production are antagonistic, the relations of hostile classes. The totality of the relations of production constitutes the mode of production, or the economic structure of society. The structure determines the superstructure of legal, political, religious, artistic, philosophical, and in short ideological forms. Structure and superstructure constitute an economic social formation.

Over time, the relations of production, which at first develop the corresponding forces of production, turn into their fetters, and an epoch of social revolution begins. The revolution ends when the old economic social formation is replaced by a new economic social formation, the old mode of production and corresponding classes are replaced by a new mode of production and corresponding classes. These transformations can be established with the precision of natural science.

The economic social formations are five. In the past, there have been three, the Asiatic, the ancient, and the feudal. These are antagonistic, of classes and class struggle. In the present, in the time of the formulation of the theory, the fourth economic social formation is the bourgeois, or the capitalist. It is also antagonistic. In the future, the fifth and last economic social formation will be the communist. It will not be antagonistic, it will be classless.

Generally again, the history of all previous society is the history of class struggles. All future society is a classless society.

This is the theory.

And the theory can be judged in one of two ways.

The judgment of the theory can be taken to be a matter of ideology. Legal, political, religious, artistic, philosophical forms are all parts of the superstructure conditioned by the structure, all in short ideological forms. The theory is an ideological form and its judgment is an ideological form.

Engels writes: ideology is a process accomplished by the so-called thinker with a conscious-

ness, but with a false consciousness (to Mehring, 14 Jul 1893). Twenty-four and a half centuries earlier, the Cretan Epimenides said: all Cretans are liars (fr. 1), and if truth is not falsehood, and falsehood is not truth, what Epimenides said, truth or falsehood, is difficult to decide. Analogously, it may appear that what Engels said is also difficult to decide. If the ideology that ideology is false is true, it is false, if it is false, it is true. Thus, however, only if truth is not falsehood, and falsehood is not truth, and thus truth and falsehood only before Hegel. Engels writes: after Hegel, one can no longer allow onself to be imposed upon by the insurmountable contradiction of the old metaphysics of true and false; one knows that these have only relative validity (*End*, 4). Thus, one ideology can allow itself to be true while allowing all other ideologies to be false. Thus, also, when Marx and Engels describe ideologies, in *The German ideology*, as: fogs in men's brains (I, 1.A), they describe them with the precision of natural science.

Hence, if the judgment of the theory is taken to be a matter of fog, one fog judging another, and whether the judged fog is taken to be a political fog, as, being the fog leading the working class in its struggle against the ruling class, it unwaveringly proposes itself to be, or a religious fog, and Engels himself urges the analogy: the history of early Christianity has notable points in common with the modern workers' movement; both Christianity and the workers' socialism preach forthcoming salvation from bondage and misery (*Christianity*, 1), or a philosophical fog, or a legal fog, or an artistic fog, and whether the judging fog is the same as the judged fog or a different fog, this is all the judgment can ever be, fog.

Alternatively, the judgment of the theory can be taken to be a matter of theory. A fog on its own authority, the theory is the particular kind of fog theory on its own most explicit authority as well. Not just because Engels takes it to be: a theory (*End*, 4). Because on the authority of Marx it reveals: the anatomy of civil society. Also on the authority of Marx: its results can be established with the precision of natural science. And having after the *Critique of political economy* done precisely that, having established the most urgently needed results of the theory with the precision of natural science, in *The capital*, Marx writes in its preface: my standpoint is that the development of the economic formation of society is a process of natural history.

Hence, the theory can be judged on the standards of the particular kind of fog theory. Those of anatomy, natural science, natural history. Including the standard, truth is not falsehood, and falsehood is not truth.

And it cannot be then too difficult to judge the theory. For it is the first theory of the development of society, but it is not the first theory. Fogs as they all are, there are, for instance, the other theories formulated at the time when it was formulated, of thermodynamics, of evolution, of the elements, of electromagnetism, the kinetic. There are theories, too, of long development. The theory of the motions of the heavenly bodies was first formulated by Eudoxus, and it was twenty centuries later, and after the labors of Apollonius, and Hipparchus, and Ptolemy, and Copernicus, that Kepler discovered the laws of the motions of the heavenly bodies. The theory of the elements was first formulated by Empedocles, and it was twenty-three and a half centuries later, and after the labors of Boyle, and Lavoisier, and Dalton, and Avogadro, that Mendeleev discovered the periodic law.

Moreover, Marx needs but one page for the final statement of the theory. Marx, however, and Engels, and the two jointly, wrote much before and much after, presenting the ground on which the theory stands, the mature theory itself, and the consequences of the theory.

Thus, the economic social formations can be judged. Hegel's four worlds, Oriental, Greek, Roman, and Christian Germanic, are spirits, and it can be judged whether the four economic social formations, Asiatic, ancient, feudal, and capitalist, are not, or are like Hegel's worlds, or spirits. Whether they are real, or are spirits. Hence, it can be judged, more generally, whether the forces of production and the relations of production are real, or are spirits. Whether classes are real, or are spirits. Whether revolutions are real, or are spirits. Hence, it can be judged, more generally still, whether the totality of the material conditions of life is civil society, or that totality remains to be discovered. And whether the anatomy

of civil society is political economy, or that anatomy as well remains to be discovered. It can even be judged what service Marx's theory has rendered the ideal of a future without property and without government.

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Marx's final statement of the theory is in the preface to the *Contribution to the critique of political economy*, in 1859. In the *Contribution*, Marx reports: I consider the system of bourgeois economy (pr.). In the following years Marx wrote *The capital, critique of political economy*, published in 1867. *The capital* is the crowning achievement of the theory. Marx reports: in this work I investigate the capitalist mode of production, and the relations of production corresponding to it (pr.). In these two treatises the mature theory is presented.

Marx, Engels, and the two jointly, wrote much before and much after, and, in particular, the following are the important treatises they wrote. Even if they are not exactly treatises.

These before are six. In 1844, Marx wrote the *Economic-philosophical manuscripts*. The *Manuscripts* is notes in preparation for a treatise which remained unfinished. It deals with: wages of labor, profit of capital, rent of land, estranged labor (1), private property (2), communism, money, critique of Hegel's dialectic and philosophy in general (3). It was first published in 1932. In 1845, Marx wrote *ad Feuerbach*. *Ad Feuerbach* is eleven theses, one page. It was first published by Engels in 1888. In 1846, Marx and Engels wrote *The German ideology, critique of the newest German philosophy in its representatives Feuerbach, B. Bauer, and Stirner, and of the German socialism in its different prophets*. *The German ideology*, Marx reports: was two large octavo volumes; and we abandoned them the more willingly to the gnawing criticism of the mice, because we had achieved our main purpose, self-clarification (*Contribution*, pr.). It deals with: Feuerbach (I, 1), saint Bruno (2), saint Max (3), Karl Grün (II, 4), Dr Georg Kuhlmann (5). It was first published in 1932. In 1848, Marx and Engels published the *Manifesto of the communist party*. The *Manifesto*, Engels reports: was commissioned by the Communist league, a working men's association (pr.). In 1850, Marx published *The class struggles in France from 1848 to 1850*. *The class struggle*s is a series of articles for the monthly *New Rhine newspaper*. In 1852, Marx published *The eighteenth Brumaire of Louis Bonaparte*. *The eighteenth Brumaire* is a series of articles for the monthly *The revolution*.

In these the ground on which the theory stands is presented. In particular, the ground is presented in notes for an unfinished treatise, eleven theses, a manifesto, two series of newspaper articles, and two large octavo volumes of self-clarification, the self-clarification achieved in a polemic against adversaries as formidable as Ludwig Feuerbach, Bruno Bauer, Max Stirner, Karl Grün, Georg Kuhlmann.

In 1859, Marx published the *Contribution*, in 1867, *The capital*.

These after are four. In 1878, Engels published *Herr Eugen Dühring's upheaval in science*. Published in the triweekly *Forward*, *Anti-Dühring* deals with: philosophy (I), political economy (II), and socialism (III), where philosophy is concerned with: the philosophy of nature (I, 5-8), morality and law (9-11), and dialectic (12-3), and where the philosophy of nature is concerned with: time and space (5), cosmogony, physics, chemistry (6), and the organic world (7-8), all in the course of the annihilation of Herr Eugen Dühring. In the years through 1882, Engels wrote the *Dialectic of nature*. The *Dialectic of nature* is notes in preparation for a treatise which remained unfinished. It deals with the philosophy of nature in greater detail. It was first published in 1925. In 1884, Engels published *The origin of the family, private property, and the state, in connection with the investigations of Lewis H. Morgan*. *The origin* deals with the origin of the family, private property, and the state in the course of an exposition of Lewis Morgan. In 1886, Engels published *Ludwig Feuerbach and the end of classical German philosophy*. Published in the monthly *The new time*, *The end* is a concise as well as the most comprehensive statement of Marx's theory – and it is Marx's theory, as Engels points out: Marx was a genius, the rest of us talents at best, and the theory rightly bears his name (4). It is a statement of Marx's theory, or synonymously apparently: Marx's world-view (pr.).

In these the consequences of the theory are presented. This task fell squarely on the shoulders of Engels, and he accomplished it in notes for an unfinished treatise, two series of newspaper articles, one of them a polemic against an upheaval in science as substantial as Eugen Dühring's, and an exposition of investigations as substantial as those of Lewis Morgan.

And in order to judge the theory, it can be examined what use it has for mathematics, what use it has for physics, and what use it has for history.

Mathematics the theory explains. Mathematics is an ideological form, a part of the superstructure determined by the structure.

Mathematics the theory explains, but for mathematics the theory has no use whatsoever. The treatises presenting the ground of the theory do not exhibit even a trace of contamination by mathematics. The treatises presenting the mature theory exhibit such a trace. In *The capital* Marx writes formulas. For instance: G – W – G′ is the general formula of capital (4.1). W is commodity, G is money, G′ is more money, and: G – G′ is money begetting money (*id.*). The treatises presenting the consequences of the theory also exhibit such a trace. The *Dialectic of nature* consists of notes, and rougher notes, and among the rougher notes there are those promising the inclusion of mathematics in the dialectic of nature.

In 1873, Marx writes to Engels: in order to analyze crises, I tried in various ways to calculate in irregular curves the ups and downs of prices over the years; I believed the fundamental laws of crises can thus be determined mathematically; but I gave it up for the time being (31 May).

Marx was not a fickle and fluttering head in need of training to fix his attention. Nor was Engels. Then again, had the theory had some use for mathematics, mathematics can have informed the question, what exactly is the evidence that the economic social formations have its own mathematics each, Asiatic mathematics, ancient mathematics, feudal mathematics, capitalist mathematics? that the mode of production determines mathematics? and can such evidence, if there is such evidence, fail to show that the economic social formations have its own moon each, too?

Had the theory had some use for mathematics, it can have found some use for logic, in particular. As it is, for logic, in particular, the theory has no use whatsoever either. It has use for dialectic.

Marx writes in *The capital*: I openly avowed myself the student of that great thinker, Hegel; in Hegel's hands dialectic suffers mystification, but this in no way prevents him from being the first to present its general forms of motion in a comprehensive and conscious way; dialectic stands in Hegel on its head, it must be turned upside down to discover the rational kernel in the mystical shell (post. 2nd ed.).

Dialectic, too, is an ideological form. Marx continues: in its mystified form dialectic became a fashion in Germany because it seemed to glorify what exists; but in its rational form dialectic is an outrage and horror for the bourgeoisie and its doctrinaire spokesmen because it includes in its understanding of what exists the simultaneous understanding of its negation, its necessary demise; because it conceives of every existent form as being in motion and grasps therefore its transient aspect; and because it is not impressed by anything, being in its essence critical and revolutionary (*id.*).

Granted that it is an admirable goal, to horrify the bourgeoisie and its doctrinaire spokesmen, granted even that dialectic is admirably suited to that admirable goal, being in its essence critical and revolutionary, this is how, on the other hand, dialectic exhibits its rationality, the rational kernel in the mystical shell.

Marx writes in *The capital*: the guild system of the middle ages tried forcibly to prevent the transformation of the master of a craft into a capitalist by limiting the number of his workers; so the possessor of money or commodities turned into a capitalist only where the minimum sum advanced for production greatly exceeded the known medieval maximum; here, as in natural science, is shown the correctness of the law discovered by Hegel in his *Logic*, that quantitative differences pass over by a dialectical inversion into qualitative distinctions; the molecular theory of modern chemistry, first scientifically developed by Laurent and Gerhardt, depends on no other law (9). Engels has to add a footnote to the third edition: the author speaks here of the

homologous series of carbon compounds; thus, in the series of the paraffins, C_nH_{2n+2}, of the normal alcohols, $C_nH_{2n+2}O$, of the normal fatty acids, $C_nH_{2n}O_2$, and many others, by the simply quantitative addition of CH_2 to the molecular formula each time a qualitatively different body is built; Marx overestimated the share of Laurent and Gerhardt in the establishment of this important fact (*id.*).

Marx overestimated the share of Laurent and Gerhardt in the classification of organic compounds, and overestimated vastly the share of the classification of organic compounds in the molecular theory of modern chemistry. One way or another, if the evidence for the dependence of the molecular theory of modern chemistry on no other law than the dialectical inversion of quantity into quality is the transformation, as in natural science, of the master of a craft into a capitalist, can the evidence for the transformation, as in natural science, of the master of a craft into a capitalist be any firmer?

Engels writes in the *Dialectic of nature*: the laws of dialectic reduce themselves to three, the law of transformation of quantity into quality and conversely, the law of interpenetration of opposites, and the law of the negation of negation (art. 4). And what exactly is the difference between the rational kernel and the mystical shell?

Marx writes not only after Gauss, Cauchy, Lobachevsky, Hamilton. He writes after Boole.

Physics the theory also explains. Physics is also an ideological form, a part of the superstructure determined by the structure.

Physics the theory also explains, but for physics also the theory concerned to show a process of natural history with the precision of natural science has no use whatsoever. Of the six treatises presenting the ground of the theory, five, the notes, the theses, the manifesto, and the two series of newspaper articles, do not exhibit even a trace of contamination by physics. The sixth, the two octavo volumes of self-clarification, exhibits three such traces. One: the concentration of trade and manufacture in England created for this country a relative world market and thus a demand for its manufactured products; outgrowing the forces of production, this demand brought about a new phase of private ownership; other conditions for this phase already existed, freedom of competition, the development of theoretical mechanics, which, perfected by Newton, became the most popular science in France and England (I, 1.B). Two: at the present time the organization of labor is evident even in science and art; in Paris, the great demand for vaudevilles and novels has brought about the organization of the labor for their production; in astronomy, people like Arago, Herschel, Encke, Bessel have found it necessary to organize themselves for joint observation (3.1.N.5). Three: for Democritus the atom is only a physical hypothesis, a made-up expedient to explain the facts, just as it is in the mixture ratios of the new chemistry, Dalton, etc. (3.1.A.3)

Also in self-clarification, Marx and Engels write: even the fogs in men's brains are necessary sublimates of their material, empirically verifiable, and bound to material premises life-processes (I, 1.A).

Newton is a condition of a phase of private ownership. Bessel attests the organization of labor, as do vaudevilles. Dalton attests a made-up expedient, as does Democritus. And can this empirical verification of the fogs in the brains of Democritus, Newton, Dalton, and Bessel, not be an empirical verification of the fog in the brains of Marx and Engels?

Again, at issue is a process of natural history.

Marx and Engels write: entirely in opposition to German philosophy, which descends from heaven to earth, here it will be ascended from earth to heaven (*id.*).

From earth? Not exactly. Marx and Engels continue: that is, here the starting point is not what men say, imagine, or conceive, nor is it what men are said to be, thought to be, imagined, or conceived, in order from this point to reach men in the flesh; here the starting point is real active men, and it is shown from their real life-process how ideological reflexes and echoes of this life-process develop (*id.*).

Not from earth, then, but from real active men and their real life-process. Yet again, from real men and their real life-process? Not exactly. Marx and Engels write: of course we cannot go here either into the physical constitution of men or into the natural conditions in which men find

themselves, geological, oro-hydrographic, climatic, and other (*id.*).

Why not? Because we have to go into Ludwig Feuerbach, Bruno Bauer, Max Stirner, Karl Grün, and Georg Kuhlmann.

What use the treatises presenting the mature theory have for physics the molecular theory of modern chemistry, first scientifically developed by Laurent and Gerhardt, shows all too well.

And the treatises presenting the consequences of the theory have the following use for physics.

In *Anti-Dühring*, Engels reports: it goes without saying that the point of my recapitulation of mathematics and the natural sciences was to convince myself in detail of what I had no doubt in general, that in nature the same dialectical laws of motion act as these which govern events in history (pr. 2nd ed.).

It goes without saying, too, that the point of Engels's recapitulation of mathematics and natural science in *Anti-Dühring* is to convince himself in detail of what he has no doubt in general, because the point of Marx and Engels's self-clarification in *The German ideology* is to convince themselves in detail of what they have no doubt in general.

In *Ludwig Feuerbach and the end*, Engels writes: three great discoveries have with giant steps advanced our knowledge of the interconnection of the natural processes, the discovery of the cell, the demonstration of the transformation of energy, and Darwin's proof of the evolution of organic nature; thus, we can now present the interconnection of the processes of nature in their entirety; today, as soon as the results of the investigation of nature are comprehended dialectically in order to reach a system of nature adequate to our time, the dialectical character of the interconnection of the processes forces itself against their will even into the metaphysically schooled heads of the investigators of nature (4).

Marx writes not only after Dalton, Ampère, Bessel, Faraday, Carnot. He writes not only after von Baer, Schleiden, Schwann. He writes after Darwin, and had the theory had some use for physics, it can have found some use for Darwin, in particular.

In 1860, Marx writes to Engels: I have been reading all sorts of books, among them Darwin's book on natural selection; although developed in the crude English way, this is the book that contains the natural historical foundation of our view (19 Dec). In 1861, he writes: Darwin's treatise is very significant and serves me as a natural scientific basis for the class struggle; even if one has to put up of course with the crude English manner of development (to Lassalle, 16 Jan). In 1862, he writes to Engels: as to Darwin, I have looked at him again; and it is remarkable how he recognizes among beasts and plants his English society with its division of labor, competition, opening of new markets, inventions, and Malthusian struggle for existence (18 Jun).

From 1860 to 1862 it has transpired, then, that, although containing the natural historical foundation of our view, it is remarkable how Darwin recognizes among beasts and plants our view. Even if one has to put up of course with the crude English method of development, that of the barbarian Newton. Accordingly, Darwin merits two footnotes in *The capital* (12.2, 13). Additionally, in the *Dialectic of nature* Engels convinces himself in detail of what he has no doubt in general, namely: labor made man out of the ape (art. 10). And when he undertakes to explain the origin of the family, private property, and the state, Engels does not do so in the course of an examination of Darwin, he does so in the course of an exposition of Lewis Morgan. Epictetus advises: you will be invincible if you never enter a contest you will not win (*Manual*, 19). Marx and Engels adhered to this maxim unwaveringly in their choice of friends as well as foes, from Feuerbach, saint Bruno, and saint Max, through Herr Eugen Dühring, and Lewis Morgan.

History as well the theory explains. History as well is an ideological form, a part of the superstructure determined by the structure.

History as well the theory explains, but for history as well the theory concerned to show what the history of all previous society is, and to draw conclusions from it about all future society, has no use whatsoever.

If the treatises presenting the ground of the theory exhibit contamination by history, *The class struggles in France from 1848 to 1850* and *The eighteenth Brumaire of Louis Bonaparte* do. If they do, newspaper articles exhibit Marx's

historical researches, and appropriately so, reaching as Marx's historical research are, in 1850, two years into the past, and in 1852, twice further, four years into the past. The treatises presenting the mature theory are concerned unwaveringly with the present. And if the treatises presenting the consequences of the theory exhibit contamination by history, *The origin of the family, private property, and the state* does. If it does, it goes without saying that as physics so history. The point of Engels's historical researches is to convince himself in detail of what he has no doubt in general. For instance: the state arose from the need to hold class conflicts in check (9).

The theory does have some use, here and there, for historical examples. For instance: sugar and coffee proved their world historical importance in the nineteenth century when their lack, caused by the Napoleonic continental system, made the Germans revolt against Napoleon, and thus became the real basis of the glorious war of liberation of 1813 (*Ideology*, I, 1.A). The real basis of the war of 1813 were not Britain's unconditional control of the seas after 1805, Wellington's successful campaigns in Portugal and Spain after 1808, and Napoleon's disastrous campaign in Russia in 1812. Sugar and coffee were.

With Schiller: the philosophical spirit cannot tarry long with the material of world history. With Marx, the philosophical spirit cannot tarry with the material of world history at all.

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Thus, the evidence which the theory has produced for the reality of the economic social formations is nothing. The theory has produced no evidence from the mathematical fogs, and no evidence from the physical fogs. It has produced no evidence from the historical fogs either. Nor is there any evidence for the reality of the economic social formations in the Sumerian prince list, or in Herodotus, or in Polybius, or in Livy, or in Otto of Freising, or in Guicciardini. Or in Voltaire, or in Hume, or in Gibbon. Or in Adam Smith. If there is any evidence at all for the reality of the economic social formations it has to be the fog which the theory itself is. If so, it has to be *The capital*. Granted, then, for the sake of the argument, that Marx has established the reality of the capitalist mode of production and relations of production corresponding to it in *The capital*, the reality of the feudal, ancient, and Asiatic modes of production and relations of production corresponding to them has to depend on inference from *The capital*. And if such inference does not depend on Engels's: the point is to find in detail of what there is no doubt in general, can it not depend on Hegel's: when we pass through the past, no matter how large it is, we have to deal only with the present? Which, in fact, Marx asserts: we do not begin with a fictional primordial condition; we proceed from a national economic present fact (*Manuscripts*, 1). But if so, how firm can the evidence which *The capital* is be?

The more so because all that *The capital* has to say about classes is this: the struggle over the limits of the working day is a struggle between all capitalists, or the class of capitalists, and all workers, or the working class (8.1). The evidence for the reality of classes is that of the manifesto: the history of all previous society is the history of class struggles, and that of the report: no credit is due to me for discovering the existence of classes in modern society, what I did was to prove that their existence is only bound up with definite historical phases in the development of production. Nor is there evidence independent of the theory.

The more so because *The capital* has nothing at all to say about revolutions. The evidence for the reality of revolutions is that of the final statement of the theory: at a certain stage of the development of the forces of production an epoch of social revolution begins, and that of *The class struggles*: revolutions are the locomotives of history (3). Nor is the fact of the French revolution independent evidence. For there is the fact that this locomotive, alas, crashed. Nor is the fact of the Russian revolution independent evidence. For there is the fact that this locomotive was not a locomotive at all. It was the largest car of the train whose locomotive was the great war, and whose other cars included the destruction of the Habsburg empire, and the destruction of the Ottoman empire.

And it should be superfluous to add that, if

this is the evidence that the anatomy of civil society is political economy, the anatomy of civil society remains to be discovered.

In the *Almagest*, before turning to the formulation of the theory of the planets, Ptolemy writes: Hipparchus developed the theory of the sun and of the moon, but being a great lover of truth, he did not even attempt to develop the theory of the other five planets because he did not have a store of accurate observations from earlier times to compare with his own, such as he left us (IX, 2).

If it is a theory, the theory of the development of society formulated by Marx is not like the theory of the planets in its formulation by Ptolemy. Not only is Ptolemy's formulation mathematical, it rests on observations reaching explicitly from 141^{AD} back to 721^{BC}. This would be, in Marx's case, counting from 1867^{AD}, back to 1006^{AD}. Marx's theory is like the theory of the planets in its first formulation by Eudoxus. Or like the theory of the elements in its first formulation by Empedocles. Kepler discovered the laws of the planets twenty centuries after the first successful attempt to formulate the theory of the planets. Mendeleev discovered the law of the elements twenty-three and a half centuries after the first successful attempt to formulate the theory of the elements. If it is a theory, Marx's is the first successful attempt to formulate the theory of the development of society, and it is, on the evidence of those two other theories, two millennia short of the discovery of the laws of the development of society.

And Marx's is a theory, and a successful theory. It is an attempt to search for the anatomy, an attempt to establish results with the precision of natural science, an attempt to show a process of natural history, an attempt even to calculate curves. It is after Empedocles that the attempts to investigate the elements became systematic, became attempts to examine a theory and improve on it, after Eudoxus that the attempts to investigate the planets became systematic, became attempts to examine a theory and improve on it, and it is after Marx that the attempts to investigate the development of society can become systematic, can become attempts to examine a theory and improve on it.

If Marx's theory is a theory, and it is a theory, it is not a result of the restoration of spirit to the substantiality and purity of its being, and not a result furthering the restoration. It is a result against the restoration.

Yet it is a victim of the restoration. It is a successful theory, but not too successful. It is a victim of the supposition of deficiency of mathematics and physics. The more still it is victim of dialectic. Yet more it is a victim of the supposition that the present is the highest. And most of all it is a victim of the supposition that the philosophical spirit will not tarry with the material of world history. Gathering in at once by the speedy road of speculation the harvest hitherto approached by long slow labor, is how Helmholtz describes the restoration, and Marx's theory illustrates the description all too well. Hipparchus did not have a stock of accurate observations from earlier times. To Marx a stock of observations, many of them accurate, from at least the earlier two and a half millennia was available for inspection.

Hipparchus, however, was a great lover of truth. Marx was not. He was a great lover of action. On the authority of Hegel: the absolute idea is the identity of the theoretical and the practical. On the authority of Marx, whether it is the absolute idea or not, the theoretical is the practical.

In *ad Feuerbach*, he writes: the question whether human thinking achieves objective truth is not a question of theory, but a practical question (2), and: all mysteries which lead theory to mysticism find their rational solution in human practice and in the understanding of this practice (8). So much so, *The German ideology* instructs: one has to step out of philosophy and like an ordinary man devote oneself to the study of reality; philosophy and the study of the real world are related to one another like masturbation and sex (I, 3.1.A.6).

Thus, if one is to study sex, in particular, one should not study Darwin and Mendel. One should devote oneself, like an ordinary man, to the practice of sex.

One should, because the study of reality is not exactly the study of reality, the study of the real world is not exactly the study of the real world.

Presenting the ground of the theory, in the

Economic-philosophical manuscripts, Marx writes: nature, taken abstractly, for itself, fixed in its separation from man, is nothing for man (3). On the other hand: the human essence of nature is there only for social man; for it is first here for him as a bond with man, as his being for the other and of the other for him, as the life-element of human reality; only here what is to him his natural being becomes his human being and nature becomes man for him; society is therefore the essential unity of man with nature, the true resurrection of nature, the fulfilled naturalism of man and fulfilled humanism of nature (*id.*).

Nature is nothing. Nature is first there for man as the life-element of human reality. Society is the true resurrection of nature.

Hence, in *ad Feuerbach*, Marx writes: the standpoint of the old materialism is civil society; the standpoint of the new materialism is human society or social humanity (10).

The standpoint is not nature, or the world. Or matter, for that matter. The standpoint is society, or humanity.

It is also because: all social life is essentially practical (8), that: the question whether human thinking achieves objective truth is a practical question.

What the world is is also a practical question.

Marx concludes in *ad Feuerbach*: the philosophers have only interpreted the world, what matters is to change it (11).

The world is what the philosopher changes.

The mature theory adds: the anatomy of civil society is political economy.

And presenting the consequences of the theory, Engels writes: in one point the history of the development of society proves to be essentially different from that of nature; in the history of society the actors are endowed with consciousness, nothing happens without conscious purpose, without willed goal (*End*, 4).

Finally, then, the results of Marx, and Engels, can be compared with the results of their younger contemporary Dilthey.

In Dilthey, there is the historic-social reality: the wonderful whole of history and society, the spiritual cosmos. In Marx, there is the economic-social reality: the true resurrection of nature, the fulfilled humanism of nature, human society, or social humanity, the anatomy which is political economy. In Dilthey, the historic-social reality is essentially different from nature. In Marx, the economic-social reality is essentially different from nature. In Dilthey, the entities acting on one another in the historic-social reality are *individua*. In Marx, the entities acting on one another in the economic-social reality are classes. Classes have the virtue of not being ineffable. They do not have the virtue of having been shown real. In Dilthey, the outcome of the study of nature is world-views. The outcome of the study of the historic-social reality is also world-views. In Marx, the outcome of the study of nature is ideologies. The outcome of the study of the economic-social reality is also ideologies. In Dilthey, the world is nowhere but in the ideas of the individual. In Marx, the world is what the philosopher changes.

Engels writes: the philosophers split into two great camps; those who assert the primacy of spirit to nature comprise the camp of idealism; these who regard nature as primary belong to the various schools of materialism (*End*, 2).

To judge by the results of Marx, and Engels, one of the two great camps is a camp within the other great camp. Materialism is a kind of idealism.

Marx's theory is not a result of the restoration of spirit to the substantiality and purity of its being, it is a result against the restoration. Yet, by a dialectical inversion, no doubt, Marx's contribution to the reinforcement of the barriers before the acquisition of knowledge and the investigation of knowledge – citadel, fortress, outworks – is, after Hegel, second only to Hegel's.

In conclusion, the service Marx's theory has rendered the ideal of a future without property and without government can also be judged.

Like everything lofty, the ideal is the outcome of enlightenment. In 1755, Morelly published *The code of nature*. The code begins with: fundamental and sacred laws, and ends with: penal laws. The first fundamental and sacred law is: nothing in society shall belong individually as property to anyone but the things he actually uses (4), the first penal laws is: any citizen who attempts to introduce the detestable property shall be convicted and imprisoned for life as a raving lunatic

and enemy of humanity (*id.*). In 1793, Godwin published *An inquiry concerning political justice*. He writes: government is an evil, an usurpation of the private judgment and individual conscience of mankind; and however we may be obliged to admit it as a necessary evil for the present, it behooves us, as friends of reason and the human species, to admit as little of it as possible, and carefully to observe whether, in consequence of the gradual illumination of the human mind, that little may not hereafter be diminished (V, 1), and: with what delight must every well informed friend of mankind look forward to the auspicious period of the dissolution of political government, of that brute engine which has been the only perennial cause of the vices of mankind, and whose mischiefs are so incorporated with its substance that they can be removed in no other way than by its utter annihilation (24)!

In Marx's theory, the ideal is one of its consequences.

The theoretical being practical, Marx's theory has been tested by its practice, and as resolutely as by the experiment which the Soviet Union was. If a right-thinking man should hope to see the Soviet Union succeed once more, even as he should never attempt the experiment at the cost at which it was first attempted, it is because, being an attempt to implement Marx's theory, the Soviet Union was an attempt to serve that ideal, a future without property and without government. And if the Soviet Union failed, and failed because it was an attempt to implement Marx's theory, the service Marx's theory has rendered that lofty ideal has not been lofty at all.

Marx and Engels write: you accuse us of intending to take away your property; precisely; this is precisely what we intend to do (*Manifesto*, 2). It used to be a promise, it is now an empty threat.

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In 1840 Hegel's *Philosophy of history* was published. In the following decades Marx formulated the theory of the development of society. In 1883 Dilthey published the *Introduction to the spiritual sciences*.

The war for the restoration of spirit to the substantiality and purity of its being was successfully concluded. The barriers before the acquisition of knowledge and the investigation of knowledge were reinforced to perfection. Man was equipped with his own reality, essentially distinct from nature, economic-social reality, historic-social reality, in which the present is the highest. The relations of physics, mathematics, history, and philosophy changed catastrophically, physics and mathematics became an older sister possessed of deficient knowledge, history and philosophy became an unfettered younger sister possessed of superior knowledge, telling the older sister what she is. Spirit infused everything, one no longer allowed oneself to be imposed upon by the insurmountable contradiction of true and false, one knew that these have only relative validity, everything became ideologies, everything became world-views.

Physics began to avoid all philosophical influences, and to consider philosophy not only useless, but harmful. Physics, accordingly, continued to make great strides in the acquisition of knowledge. To history the appeal of having its own reality, in which the present is the highest, of being possessed of superior knowledge, of telling physics what it is, and of knowing that true and false have only relative validity, and everything is world-views, proved irresistible. History, accordingly, in the nineteenth century, in the course of its reformation, reduced itself to philological and antiquarian scholarship, at best, in the twentieth century, its reformation complete, reduced itself to complete irrelevance, at best.

The results to which the most important historical discovery has led, if they are results at all, the results of the new kind of history, the history of science, and the results of the most important historian of the last century, if he is important at all, illustrate.

By 1824 Young and Champollion had deciphered hieroglyphic. In the following century the Egyptian record was published. By 1857 Rawlinson, Hincks, and Oppert had deciphered cuneiform. In the following century the Sumerian, Akkadian, Babylonian, and Assyrian record was published.

It is the most important historical discovery, the corroboration of the fact of Egyptian antiquity, and the establishment of the fact of Mesopo-

tamian antiquity.

Before this discovery, the investigation of the development of physics, and hence also of physics, could ground itself in facts beginning with Thales, twenty-six centuries ago. After this discovery, it can ground itself in facts beginning with the Venus tablet of Ammisaduqa, on which the synodic period of Venus is recorded, ten and a half centuries before Thales. Before this discovery, the investigation of the development of history, and hence also of history, could ground itself in facts beginning with the father of history, Herodotus, and his immediate predecessors, twenty-five centuries ago. After this discovery, it can ground itself in facts beginning with the grandfather of history, the Sumerian prince list, and his predecessors, not less than sixteen centuries before Herodotus. Before this discovery, the investigation of the development of the state, and hence also of the state, could ground itself in facts beginning with the Greek cities in the century-odd before Thales and the contemporary Egyptian, Babylonian, Mede, and Lydian kingdoms, some twenty-seven centuries ago. After this discovery, it can ground itself in facts beginning with the first prince in the valley of the two rivers, Enmebaragesi, twenty centuries earlier, and with the first prince in the valley of the Nile, Narmer, three-odd centuries earlier still. Before this discovery, the investigation of the development of mathematics, and hence also of mathematics, could ground itself in facts beginning with Thales, too, twenty-six centuries ago. After this discovery, it can ground itself in facts beginning with the 3 calculations of area, recorded on the tablets W 20044/20 and W 19408/76, twenty-five centuries before Thales.

It is the most important historical discovery, the corroboration of the fact of Egyptian antiquity, and the establishment of the fact of Mesopotamian antiquity. It has doubled the reach of the facts on which the search for truth can ground itself in every particular case, and hence also in general. As well as the reach of the facts on which the search for action can ground itself in every particular case, and hence also in general.

The philosophers have not noticed the discovery. They have been detained by the urgent need to investigate how I see the sun and desire food, how the word knowledge is used, how knowledge is a mode of being of this-being as being-in-the-world, how knowledge is a heteropsychological occurrence, and how we do not know, we can only guess.

The historians, if they are historians, have noticed the discovery.

In 1840, in Hegel, in *The philosophy of history*, the worlds, or spirits, are four, Oriental, Greek, Roman, and Christian Germanic. In 1918, in Spengler, in *The decline of the west, outlines of a morphology of world history,* the worlds, or spirits, are cultures, and the cultures are eight: Egyptian, Babylonian, Indian, Chinese, Hellenic, Arabic, Mexican, and Western (II, 1.10). In 1934-61, in Toynbee, in *A study of history*, the worlds, or spirits, or cultures, are civilizations, or societies, and the civilizations, or societies, are nineteen: Western, Orthodox Christian, Iranic, Arabic, Hindu, Far Eastern, Hellenic, Syriac, Indic, Sinic, Minoan, Sumeric, Hittite, Babylonic, Andean, Mexic, Yucatec, Mayan, and Egyptiac (I, C.2).

The cultures are eight – or nine, or ten, or more, for there are high cultures, and primitive cultures, and one way or another, when cultures are counted, souls are counted. The civilizations are nineteen – or twenty-one, or twenty-six, or twenty-nine, or more, for there are fully-fledged civilizations, and abortive civilizations, and arrested civilizations, and one way or another, when civilizations are counted, religions are counted. In Spengler: culture is the primordial phenomenon of all past and future world history; Goethe's deep idea must be applied to all formations of human history (I, 1.6); a culture is born when a great soul awakens from the primordial soul; it dies when this soul has realized the full sum of its possibilities in the form of peoples, languages, faiths, arts, states, sciences, and returns to the primordial soul (1.7). In Toynbee: the *raison d'être* of civilizations is to provide an opportunity for the birth of fully-fledged religions (VII, A.3.a).

In Spengler: every culture has its childhood, youth, manhood, and old age (I, 1.7). In Toynbee: every civilization is born (II), grows (III), breaks down (IV), and disintegrates (V).

Not just Hegel's four worlds, or spirits, are a

much better analysis of the development of humanity. Twenty-six centuries earlier, Hesiod's five ages: golden age, silver age, bronze age, age of heroes, and iron age (*Work*, 106-201), are a much better analysis of the development of humanity.

Of the new kind of history, the history of science, the most respectable outcome has been the *Dictionary of scientific biography*, in sixteen volumes, in 1970-80. It is respectable because it respects importance, because it presumes that the philosophers are the scientists, and because it concerns itself with the past.

The *Dictionary* presents itself: it is designed to make available reliable information on the history of science through the medium of articles on the professional lives of scientists; all periods of science are represented, with the exception that there are no articles on the careers of living persons (pr.), and: the board has felt reasonable confidence in the information available to it in selecting the more important figures (*id.*).

In particular, the medium is 4 722 articles. Counting all their objects singly – counting the brothers Montgolfier 1 scientist, for instance, and the brothers Wright 1 scientist, too – the *Dictionary* is a list of 4 722 scientists. The number 4 722 all by itself shows that the huge majority of the scientists in the list cannot be important at all. But the number 4 722 all by itself shows also that the list is the complete list of all the more important scientists in the information available to the board. And it is to the degree to which it respects importance that the *Dictionary* itself is important.

The 4 722 scientists can be arranged in chronological order. It then transpires that the first scientist is Thales, the second scientist is Anaximander, the third scientist is Anaximenes. The first philosopher is the first scientist, the second philosopher is the second scientist, the third philosopher is the third scientist. It transpires, in fact, that rather generally the philosophers are the scientists. Plato is a scientist, Zeno of Citium is a scientist, Plotinus is a scientist, Augustine is a scientist, Aquinas is a scientist, Locke is a scientist, Berkeley is a scientist, Hume is a scientist, Kant is a scientist. Later on, Marx is a scientist, Engels is a scientist, even Wittgenstein is a scientist. The fact that the philosophers are the scientists all by itself shows that the kind science cannot be a real kind. It is yet to the degree to which it presumes that the philosophers are the scientists that the *Dictionary* is important.

The *Dictionary* has the virtue as well of noticing the most important historical discovery. Abandoning the otherwise uniform medium of articles on the professional lives of scientists, it yet has, in its fifteenth supplementary volume, the articles *Man and nature in Mesopotamian civilization*, *Mathematics and astronomy in Mesopotamia*, *The mathematics of ancient Egypt*, and *Egyptian astronomy, astrology, and calendrical reckoning*.

Arranged chronologically, the 4 722 scientists show their distribution in time. The *Dictionary* warns: it does not appear that either the configuration of the list as a whole, or that of various distributions that might be discerned within it, could properly serve for statistical analysis of the population of science in various times (pr.). The configuration can properly serve, however, for statistical analysis of the population of the *Dictionary*.

Its editor in chief was born in 1918. Counting one century before that, the scientists divide themselves into two, those born through 1818, and those born beginning in 1819. Into those born in the twenty-four and a half centuries from the first scientist through 1818, and those born in the century-odd beginning in 1819. The distribution is,

through 1818	2 525	53.5 %,
beginning in 1819	2 197	46.5 %.

In 1990, two supplementary volumes were published, under a new editor in chief, to rectify the omissions of the original sixteen volumes and to keep up with death. The new articles are 417. 50 are on scientists who died after 1979. The remaining 367 are on scientists who died through 1979, and are thus omissions of the original sixteen volumes. The distribution of these is,

through 1818	0	0.0 %,
beginning in 1819	367	100.0 %.

The corrected total is 5 089, its distribution is,

through 1818	2 525	49.6 %,
beginning in 1819	2 564	50.4 %.

The present is the highest. It is yet to the degree to which the present of the original sixteen vol-

umes is not as high as the present of the supplementary two volumes that the *Dictionary* is important.

In all these ways, in that it respects importance, in that it presumes that the philosophers are the scientists, and in that it concerns itself with the past, the *Dictionary of scientific biography* is exceptional. It is not representative of the new kind of history, the history of science.

Kuhn is representative.

The most important historian of the last century is Braudel.

More particularly, Braudel is the most important outcome of the *Annals*. As he reports: my greatest debt is to the *Annals* (*Mediterranean*, pr.). The *Annals of economic and social history* was founded in 1929. Clearly, to concern itself with the new economic-social historic-social reality. In 1946 it became *Annals, economies, societies, civilizations*. Clearly, to keep up with the present as it kept becoming higher than the highest.

Braudel's important treatises are two.

The first is *The Mediterranean and the Mediterranean world in the age of Philip II*, in 1949. It is explicit in its commitment to man: all problems posed by the Mediterranean are of exceptional human richness (pr.). Explicit in its commitment to the present: one of the most pressing tasks of history is to respond to the painful problems of the hour (*id.*), and: all problems posed by the Mediterranean throw light on the present times (*id.*). Explicit in having little use for importance: importance is obviously a matter for discussion (III, pr.).

Braudel's Mediterranean world is the most respectable among the large and ever growing progeny of Hegel's worlds, or spirits. And a progeny of Hegel's worlds, or spirits, it is. Its immaculate spirituality can best be illustrated by the immaculate spirituality of the Atlantic. For to say very little about the Atlantic while saying very much about the Mediterranean – in the age of Philip II, no less – even if it has no other virtues, has the virtue of brevity. Braudel writes: in the sixteenth century the Atlantic Ocean did not yet exist with its full autonomy; it had just begun to be grasped by men and constructed by them with what they could pull out of Europe, as Robinson Crusoe constructed his house with what he salvaged from his ship (I, 3.3). So: there were several Atlantics; there was the ocean of the English and French; there was the Atlantic of the Spanish; and there was the Atlantic of the Portuguese (*id.*).

Each, no doubt, with its own tides, the English and French tides, the Spanish tides, and the Portuguese tides, caused, respectively, by the English and French moon, the Spanish moon, and the Portuguese moon. To exist – for an ocean to exist – is to be constructed by men. Or by the character of a novel.

Importance is obviously a matter for discussion. Braudel is nevertheless the most important historian of the last century, and he is, because the Mediterranean world is immaculately spiritual but *The Mediterranean* is not. Braudel writes: we are offered two fairly well established chains, one constructed by the scholarship of the last twenty or thirty years, the chain of economic events, the other inventoried over the ages, the chain of political events (III, pr.). *The Mediterranean* is in three parts. The second part deals with the chain constructed in the last twenty or thirty years. The first and the third parts do not. The third part deals with the chain inventoried over the ages. And the first part deals with neither of the two chains. It labors hard, instead, to do what the younger sister used to do, to fetter itself to the older sister. It deals with geography. Nor does the third part – which, Braudel reports: I decided to publish only after much hesitation (*id.*) – show that importance is obviously a matter for discussion. It is focused staunchly on Lepanto, and pace Braudel, and with Voltaire, the battle which arrested the advance of the Ottoman empire merits the attention of all times. Also, Braudel needs more than one twentieth of the bulk of *The Mediterranean* to describe its sources. The sources are: unpublished, cartographic, and printed (*id.*), the printed are: major collections, essential, and bibliography (*id.*), and the bibliography alone is a list of 964 writers. Damaged as the performance of the indispensable duty has been by the supposition that importance is obviously a matter for discussion, Braudel performs, with Gibbon, the indispensable duty, the careful examination of all original materials.

The second of Braudel's important treatises is the *Material civilization, economy, and capitalism, 15th - 18th century*, in 1968-79. It is not contaminated by geography, it is not contaminated by the chain of political events inventoried over the ages, it is not contaminated by a description of its sources. Clearly, Braudel keeps up with the present as it keeps becoming higher than the highest.

The *Material civilization* concerns itself with capitalism. Braudel writes three generations after Marx. If Marx established any result with clarity and distinction, it is capitalism. Capitalism is a mode of production and the corresponding relations of production. In particular, it is material, not spiritual, it is the cause of ideological forms, including ethics, and it appears at a certain time, after a long development. In the economic-social historic-social reality, however, clarity and distinction is not a virtue. In the generation before Braudel already the new spiritual science sociology obtained better results. The most important sociologist is Max Weber, his most important treatise is *The Protestant ethics and the spirit of capitalism*, in 1905, and its title suffices to show that capitalism is spiritual, not material, and it is not just an effect of ethics, it is an effect of a religious ethics. Braudel, for his part, obtained this result: in potentiality capitalism is detectable since the dawn of history, it has developed and perpetuated itself down the centuries (III, 6).

But the most important result of the *Material civilization* are the structures of everyday life. It is the *Material civilization* that made respectable concerns with: eating habits, table manners, beer and wine, the vogue for pepper, the pleasures of coffee, the pleasures of tobacco, furniture, floors, ceilings, tables, chairs, luxury, fashion (I, 3-4).

Braudel writes in the *Material civilization*: is not the secret goal of history to explain the present (III, 6)?

Secret? Can it have been more overt?

The pleasures of coffee and tobacco in 1968. As Braudel kept writing the *Material civilization*, the present kept becoming higher than the highest. In 1976 Michel Foucault began publishing a *History of sexuality*. As Michel Foucault kept writing the *History of sexuality*, the present kept becoming higher than the highest. In 1978 a *History of shit* appeared.

The younger sister will not listen to the older sister. But maybe, just maybe, she will listen to Don Quixote: you know, your lordship, Sancho Panza tells his master, the fool knows more in his own house than the wise man knows in another's; no, no, Sancho, Don Quixote replies, the fool knows nothing in his own house or in anyone else's, no edifice of wisdom can rest on a foundation of folly (II, 43).

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To conclude this – to borrow Abelard's all too apt description – history of calamities.

With the restoration of spirit to the substantiality and purity of its being, then, great calamity befell the acquisition of knowledge, and greater calamity befell the investigation of knowledge. Great calamity befell physics, greater calamity befell mathematics, yet greater calamity befell history, and the greatest calamity of all befell philosophy.

Physics continued to make great strides in the acquisition of knowledge.

Yet, did it?

Maxwell illustrates.

In 1600, in *On the magnet, magnetic bodies, and the great magnet of the earth*, Gilbert begins with a review: of the opinions and delusions of ancient and modern writers (I, 1). In particular: the ancient writers record that the lodestone attracts iron, but its other properties were all hid, and lest the story of the lodestone be too jejune and too brief, to this sole property fictions and falsehoods were appended and fed to mankind in the early times not less than they are today (*id.*).

Himself, Gilbert discovered that: the terrestrial globe itself is a lodestone (I, 17), and that there are: electric motions distinct from the magnetic motions (II, 2).

Two and a half centuries later, Maxwell discovered the laws of electrodynamics. The theory which rests on the laws he developed in three steps. The first is *On Faraday's lines of force*, in 1855, the second is *On physical lines of force*, in 1861, the third is *A dynamical theory of the electromagnetic field*, in 1864. In the *Dynamical theory* Maxwell formulates the laws, and shows as well that: light is an electromagnetic distur-

bance propagated according to the laws of electromagnetism (97). In 1873, in the *Treatise on electricity and magnetism*, he systematized and generalized the theory.

The laws – Maxwell's equations – are in the *Dynamical theory* twenty (70). In the *Treatise* the twenty equations are generalized into twelve (619).

Hertz, in 1890, in *On the fundamental equations of electromagnetics for bodies at rest* (3), and *On the fundamental equations of electromagnetics for bodies in motion* (1-6), generalized the twelve equations into four.

The four equations are a generalization also of the laws discovered by Coulomb in 1785, Ampère in 1827, Faraday in 1831, and Gauss in 1839.

Maxwell's equations – or, as Feynman describes them: Maxwell's beautiful edifice (*Lectures*, II, 18.1) – are,

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0},$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \cdot \boldsymbol{B} = 0,$$

$$c^2 \nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \frac{\boldsymbol{j}}{\varepsilon_0},$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic field, $\boldsymbol{j}$ is the current density, c is the velocity of light, ρ is the charge density, ε_0 is the dielectric constant.

Feynman comments: from a long view of the history of mankind, say, ten thousand years, there can be little doubt that Maxwell's discovery of the laws of electrodynamics will be judged the most significant event of the nineteenth century; the contemporary American civil war will pale into provincial insignificance (1.6).

And the calamity begins with the fact that it should not be difficult to reach the judgment that Maxwell's discovery of the laws of electrodynamics is the most significant event of the nineteenth century, and in comparison, never mind the American civil war, even that other contemporary event, the restoration of spirit to the substantiality and purity of its being, pales into provincial insignificance. One philosopher, Feynman, reached that judgment in less than one hundred years. Then again, Feynman is not a philosopher. Feynman is a scientist. He is possessed of deficient knowledge, at best, and he is to be told by the philosophers what his judgment is. His judgment is an ideology, a world-view.

Feynman is not a philosopher, and the philosophers have not reached that judgment. They have been detained by the need to investigate how I see the sun and desire food, how the word knowledge is used, how knowledge is a mode of being of this-being as being-in-the-world, how knowledge is a heteropsychological occurrence, and how we do not know, we can only guess.

Not only is the discovery of the laws of electrodynamics the most significant event of the nineteenth century. The development of electrodynamics is the most dramatic fact of the acquisition of knowledge. The most dramatic, and the most instructive. It is a development in three-odd centuries, ten-odd generations, from very nearly nothing to very nearly everything. From a sole property to which fictions and falsehoods were appended and fed to mankind to the discovery of the laws of electrodynamics and optics, which led in less than two generations to the development of the theory of relativity and quantum theory, which led in one more generation to the development of the big bang theory, the theory of the origin and development of the world.

Most of all, however, the calamity is in the fact that not only is Maxwell himself not a scientist, Maxwell is a philosopher *par excellence*, and not only are his labors of acquisition of knowledge far and away more important than the labors of acquisition of knowledge of Hegel, Marx, and Dilthey combined, his labors of investigation of knowledge are far and away more important than their labors of investigation of knowledge combined.

Maxwell writes, in *On the mathematical classification of physical quantities*, in 1871: the first stage in the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend; the next stage is the discovery of the mathematical form of the relations of these quantities (*id.*). More generally, in the *Theory of heat*, in 1871: those whose curiosity is satisfied with observing what happens have occasionally done service by directing the attention of others

to what they have seen; but it is to those who endeavor to find out how much there is of anything that we owe all the great advances in our knowledge (4). Most generally, in the *Address to the mathematical and physical sections of the British association*, in 1870: it was a great step in science when men became convinced that they must begin by asking not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it (*id.*).

It is a great step, from asking whether a thing is good or bad, noxious or beneficial, to asking of what kind it is, and how much there is of it. And when the step is taken, the first stage is the discovery of a system of quantities on which the phenomena may be conceived to depend, the second stage is the discovery of the mathematical form of the relations of these quantities.

It is a description of the discovery of Maxwell's equations. It is a description of the development of electrodynamics. It is a description of the development of physics. And it is a description of the acquisition of knowledge in all its generality. It is at all levels of generality a description which accommodates the facts better than any description that can be extracted from Hegel, or Marx, or Dilthey.

It is a lesson also for the younger sister.

A lesson also for the investigation of knowledge.

Not good or bad, noxious or beneficial. Of what kind it is, and how much there is of it.

Mathematics, too, continued to make great strides.

Yet, it, too, did it?

Minkowski illustrates.

The theory of relativity was developed in three steps as well. Einstein formulated the special theory of relativity, in *On the electrodynamics of moving bodies*, in 1905. Minkowski generalized the theory, in the *Space and time*, in 1908. Einstein generalized it further, into the general theory of relativity, in *The foundation of the general theory of relativity*, in 1916.

The special theory of relativity, in 1905, is the most elegantly formulated theory. Einstein begins with but one definition and two postulates. The definition is of simultaneity. One of the postulates is of constancy of the velocity of light. The other is the principle of relativity: for all coordinate systems for which the equations of mechanics hold the same laws of electrodynamics and optics hold (*Electrodynamics*, pr.). The equations of mechanics are Newton's laws, the laws of electrodynamics and optics are Maxwell's equations. And from the one definition and two postulates Einstein deduces the Lorentz transformation (3). Relating the description of an event in one coordinate system to its description in another coordinate system, the Lorentz transformation is,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}},$$
$$y' = y,$$
$$z' = z,$$
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where x, y, z, and t, and x', y', z', and t', respectively, are the coordinates of the event in the two systems, one of which moves with uniform velocity with respect to the other, v is the velocity of that system, c is the velocity of light. When v is very small, the Lorentz transformation reduces to the Galilean transformation,

$$x' = x - vt,$$
$$y' = y,$$
$$z' = z,$$
$$t' = t,$$

which relates the descriptions of the event in the two coordinate systems under Newton's laws.

In 1908, in the *Space and time*, Minkowski shows that: any change in the system of reference in conformity with the Lorentz transformation leaves the expression of the laws of nature unchanged (1). Or, as Einstein later states it: the whole content of the special theory is included in the postulate that the laws of nature are invariant under the Lorentz transformation (*Popular*, app.5).

And the calamity begins with the name of the postulate and of the theory. Recalling, apparently, Newton's distinction of absolute time, space, and motion and relative time, space, and motion (*Principles*, sch. def.), the latter epithet attached itself to the postulate of relativity, and hence to

the theory of relativity. Attached itself to the postulate that the same laws, no less, hold for all coordinate systems, no less. Attached itself, after Minkowski, to the postulate that the laws of nature, no less, are invariant, no less, under the Lorentz transformation. Minkowski writes in the *Space and time*: the name relativity postulate seems to me too feeble for the demand for invariance; I would rather give it the name postulate of the absolute world (2).

But Minkowski is not a philosopher either. He is not even a scientist. He is a mathematician. He is possessed of deficient knowledge long stripped of its finery, at best, and he is definitely to be told by the philosophers what is absolute and what is relative.

The postulate is the postulate of relativity, not the postulate of the absolute world. And the theory is the theory of relativity, not the theory of the absolute world. Absolute is anything but the world. Absolute is the incarnation of the divine being. Absolute is spirit knowing itself as spirit. Absolute is to know unity in opposition and opposition in unity. Absolute is the identity of the theoretical and the practical. Absolute is the good finding its completion in the true.

Minkowski defines the world: let x, y, z be rectangular coordinates for space and t be time; the multiplicity of all thinkable x, y, z, t systems of values is then the world (1).

But he is not a philosopher, and the philosophers have defined the world better. The world is my world and I am my world. The world is worldhood. That the external world is real and that the external world is not real are both meaningless. Next to the two worlds of physical objects and of states of consciousness there is a third world of objective contents of thought.

And it is the absolute world. One year after the formulation of the general theory, in 1917, Einstein published the *Cosmological considerations on the general theory of relativity*, the first step toward the formulation of the theory of the origin and development of the world.

Great calamity has thus befallen mathematics.

In 1908, Minkowski concludes: in the pursuit of the mathematical consequences of the postulate of the world ample indications for its experimental verifications will be discovered, which will reconcile even those, to whom the loss of long established views is unsympathetic or painful, to the thought of preestablished harmony between pure mathematics and physics (5).

In 1911, in the pursuit of the mathematical consequences of the postulate of the world Einstein deduced the deflection of light by the sun's gravitational field, in 1916 he deduced it more precisely after the development of general relativity, and in 1919 Eddington discovered its experimental verification. In 1911 Einstein deduced the deflection at $0''.83$ (*Propagation*, 4), in 1916 he deduced it at $1''.7$ (*Foundation*, 22), and in 1919 Eddington measured it in two observations at $1''.98 \pm 0''.12$ and at $1''.61 \pm 0''.30$ (*Determination*). The preestablished harmony between pure mathematics and physics was experimentally verified by weighing light at the distance of the sun with exactness better than $0.000\,000\,07$.

It was at the same time that the mathematicians engaged in a debate about the foundations of mathematics, and the head of one of the two sides in the debate wrote: the two sides have reached agreement that the exact validity of mathematical laws as laws of nature is out of the question; as to where mathematical exactness exists, the one side says, in the human intellect, the other side says, on paper.

About the calamity which has befallen history enough has been said, and more than enough.

The calamity which has befallen philosophy, finally, no philosopher can illustrate better than Russell. Russell, because some philosophers have labored to bring about the calamity, other philosophers, those whom the philosophers have refused to recognize as philosophers, have labored to reverse the calamity. But no philosopher has been more successful than Russell, by a dialectical inversion, no doubt, in bringing about the calamity by laboring to reverse it.

Russell postdates Hegel by one century. In this century the catastrophic transformation of philosophy occurred, the most important philosophers in this century, such as the philosophers became with the transformation, are Marx and Nietzsche, and Marx and Nietzsche are both victims of the transformation. Marx formulated a theory of the development of society, presuming to describe a process of natural history. To do so, he had no

use for physics, no use for mathematics, including logic, and no use for history. And it is the catastrophic transformation that made it viable to suppose that the description of a process of natural history need not be informed by physics, or mathematics, or history. Nietzsche can be taken to be, as he ubiquitously has been, all about values: the task of the philosopher is to create values (*Beyond*, 211), and: the noble man feels that he determines values; he creates values (260), and all about perspectives: there is only a perspective seeing, only a perspective knowing (*Genealogy*, III, 12). He does not have to be so taken. It can be taken that: plainly and without metaphor man is an animal (*Beyond*, 202), and Nietzsche labors: toward the natural history of morality (V), labors with some success in *Beyond good and evil*, and with substantial success in *Toward the genealogy of morality*. Nietzsche also has no use for physics, no use for mathematics, including logic, and no use for history. He also finds Darwin: narrowly and dryly English (253). He also has much to say about history, *On the usefulness and disadvantage of history for life*, for instance, also being a philosophical spirit who cannot tarry with the material of world history. And if he is not all about values and all about perspectives, Nietzsche is also a victim of the transformation. Of the supposition that the natural history of morality need not be informed by physics, or mathematics, or history.

Russell is not a victim of the transformation. Russell is a mathematician, a logician *par excellence*. He wrote on relativity and on quantum theory. He even not just wrote *How to read and understand history*, he tarried with the material of world history. He is the only important philosopher other than Hegel to write a history of philosophy. Russell, in fact, is the one philosopher after the catastrophic transformation such as the philosophers were before the catastrophic transformation.

Thus, of his successors, some, like Heidegger, have not built on his labors, continuing to build on the labors of Hegel and Dilthey. Others, like Wittgenstein, Carnap, Popper, have built on his labors, not on the labors of Hegel and Dilthey.

And the calamity is that it has made no difference whatsoever.

On the one hand, in 1903, in *The principles of mathematics*, Russell formulated the theory of types, the first attempt to resolve the contradiction, which he discovered in 1902, of the set of all sets that are not members of themselves.

On the other hand, in 1905, in *On denoting*, he formulated the theory of definite descriptions, which presumes that knowledge is of two kinds, knowledge by description and knowledge by acquaintance, and the latter is instanced by I see the sun and I desire food. Hence, in 1912: knowledge is not a precise conception, it merges into probable opinion. In 1944: theory of knowledge has a certain essential subjectivity, its data are egocentric.

As Heidegger has it: knowing is a mode of being of this-being as being-in-the-world.

On the one hand, in 1919, in the *Introduction to mathematical philosophy*, Russell writes: there is only one world, the real world (16).

On the other hand, in 1914, there is *Our knowledge of the external world*, and the external world is not exactly only one. There are in *The external world*, other than the external world: the world of physics (4), and: the world of sense (*id.*), there are: my private world (3), and: our private world (*id.*), there are: an actual private world (4), and: an ideal private world (*id.*). Clearly, Hegel's worlds adhered too stringently to the principle of parsimony. There is a reason to abandon the principle, however. In 1918, in *The philosophy of logical atomism*, Russell writes: belief in the physical world has established a sort of reign of terror; the physical world is a sort of governing aristocracy which has somehow managed to cause everything else to be treated with disrespect; but that sort of attitude is unworthy of a philosopher; we should treat with exactly equal respect the things that do not fit in with the physical world (7).

When Heidegger stands up against the governing aristocracy, and stands up as resolutely as this, in the *Introduction to metaphysics*: world is always spiritual world (15), Russell stands shoulder to shoulder with him.

On the one hand, in 1919, in the *Introduction to mathematical philosophy*, Russell writes: logic must no more admit a unicorn than zoology can; for logic is concerned with the real world as truly

as zoology, though with its more abstract and general features (16).

On the other hand, in 1959, in *My philosophical development*, he reports: I no longer think that the laws of logic are laws of things, on the contrary, I now regard them as purely linguistic (9).

Russell's philosophical development has not, in 1959, caught up with Heidegger's philosophical development in the *Introduction to metaphysics*, in 1935 already, in that: we must overcome traditional logic (47), but it has in that: essence and being speak in language (17).

In 1945, in *A history of western philosophy*, Russell writes of: the growth of unreason throughout the nineteenth century and what has passed of the twentieth.

The most immediately calamitous contribution to the growth is the contemporary war.

Far as the effects of war can reach, however, the effects of philosophy can reach further, and Heidegger's are more calamitous contributions.

Yet Russell's contributions to the growth of unreason are even more calamitous. Precisely because he has contributed to the growth by fighting against it.

And his *History of philosophy* abounds in contributions, too. So, in his *History of philosophy* the philosophy of the nineteenth century and what has passed of the twentieth comprises Byron (III, 23), Schopenhauer (24), Nietzsche (25), the utilitarians (26), Marx (27), Bergson (28), William James (29), Dewey (30), and the philosophy of logical analysis (31). It is a great calamity that thermodynamics, electrodynamics, the kinetic theory have not happened at all in that philosophy of the nineteenth century and what has passed of the twentieth, and relativity and quantum theory have barely happened in passing. It is a yet greater calamity that what has happened in that philosophy is not just Schopenhauer, Marx, and Nietzsche, but Byron, the utilitarians, William James, Bergson, Dewey, and the philosophy of logical analysis.

Another and most calamitous is the cause Russell assigns to the growth of unreason. He writes: the growth of unreason throughout the nineteenth century and what has passed of the twentieth is a natural sequel to Hume's destruction of empiricism (17).

l

If knowledge, then, is spirit knowing itself as spirit, ideologies, world-views, what I see and desire, the uses of the word knowledge, a mode of being of this-being, a heteropsychological occurrence, and guesses, the investigation of knowledge has advanced apace after Hume and Kant.

Otherwise, the investigation of knowledge has stopped with Hume and Kant. If it is to continue, it had better learn from Hume and Kant. And first of all the following.

Hume writes: when we run over libraries, what havoc must we make? if we take in our hand any volume, let us ask, does it contain abstract reasoning concerning quantity and number? no; does it contain experimental reasoning concerning matter of fact and existence? no; commit it then to the flames, for it can contain nothing but sophistry and illusion (*Inquiry*, 12.3).

li

Book III of *The mathematical principles of natural philosophy* is *The system of the world*. *The system of the world* begins with: rules of philosophizing, and the first and second rules of philosophizing are:

to admit no more causes of things than are true and suffice to explain the phenomena;

therefore, to assign to effects of the same kind the same causes (*id.*).

Newton does not define cause. But Hume does: we may define a cause to be an object followed by another, and where all the objects similar to the first are followed by objects similar to the second; or, in other words, where, if the first object had not been, the second never had existed (*Inquiry*, 7.2).

The barriers before the acquisition of knowledge and the investigation of knowledge are effects of the same kind. The same causes therefore are to be assigned to them. Also, no more causes are to be admitted than are true and suffice to explain the phenomena. And one cause is true and suffices to explain the phenomena.

The labors of the philosophers at reinforcing the barriers have been gathered into four efforts, Protagoras through Socrates to Plato, Pyrrho and his successors, st Paul to Augustine, Berkeley

through Hegel to Dilthey. But the four efforts constitute one and the same development. The result, man is the measure of all things, is followed by devotion to ethics, followed by the result, god is the measure of all things. Scepticism is a generalization of the result, man is the measure, and scepticism is the foundation of the paltriness of the schools, of philological and antiquarian scholarship, and of the assimilation of philosophy into Christian doctrines, of the transformation of Christian doctrines into philosophy, and of Christian philosophy into orthodoxy. The strictest analogy holds between al-Ghazali and Augustine. The strictest analogy holds as well between Hegel and Augustine. Idealism is a generalization of scepticism, and idealism is the foundation of dialectic, of the economic-social reality and ideologies, and of the historic-social reality and world-views. It is one and the same development, from man is the measure of all things through the darkest of dark ages through the restoration of spirit to the substantiality and purity of its being.

One and the same development with one and the same end. The protection of man. The citadel. From man is the measure through all knowledge is human, man is the citadel. From ethics through the spiritual sciences, the kinds of knowledge are the fortress guarding the citadel. From the sun does not throw light, it blinds, through dialectic, the theoretical is the practical, the absolute is the relative, and the world is world-views, vagueness is the outworks guarding the fortress guarding the citadel.

The end is one and the same, the protection of man. The protection of man against the acquisition of knowledge. It is ridiculous to search for other things before I know myself. Belief is knowledge, knowledge is belief. We know that we do not know. Men's natural faculties are absolutely fallacious, unfit to reach any fixed determination. The acquisition of knowledge is a condition in need of a cure. Knowledge breeds conceit. It is for the same reason, to satisfy sick curiosity, that freaks are put on show in the theater and men investigate nature and want knowledge for the sake of knowledge. Whatever they produce from their volumes against our books we shall easily show false, or else without any hesitation believe false. God knows, you do not know. The philosophers must be pronounced infidels on three counts, on seventeen counts heretics. Reason is a pestilential beast. There is not a single thing in the world of which we know the real nature. To know and to practice the salutary truths of the gospel is the highest perfection of human nature. Let all the philosophers prove that I am wrong, if you feel that I am right, I wish no more, I know it, because I feel it. The deficient knowledge of which mathematics boasts against philosophy is of a kind which philosophy must revile. Science follows the ruinous path of misinterpreting spirit and depriving it of its power. We do not know, we can only guess.

Moreover, some of the philosophers have labored at reinforcing the barriers exclusively, some of the philosophers have not labored at reinforcing the barriers exclusively. Beginning with Pyrrho, the former abound. But of the latter, those who have not labored at reinforcing the barriers exclusively, many have yet labored at reinforcing the barriers. Protagoras, Socrates, Plato early on, Rousseau, Marx, Russell later on. And the philosophers who have not labored at reinforcing the barriers, and the philosophers who have labored staunchly at breaching the barriers, have indulged the barriers. Aristotle, Hume, Kant, each did much ethics. Ptolemy writes: even where action is concerned, mathematics above all can make men see clearly, it makes its followers lovers of the constancy, order, symmetry, and calm associated with the divine (*Almagest*, I, 1). Darwin writes: I fully subscribe to the judgment that of all differences between man and the lower animals, moral sense is by far the most important (*Descent*, 4).

And if the labors of the philosophers at reinforcing the barriers constitute one development because the one end of this one development is the protection of man – and the philosophers who have not labored at reinforcing the barriers have yet manned the barriers, too – the labors of the philosophers point at the cause of the barriers. The cause is man. The cause is that the philosophers are not just the philosophers, the philosophers are also men. Had the philosophers not been men, the barriers before the acquisition of knowledge and the investigation of knowledge

never had existed.

Moreover, from man is the measure through the restoration of spirit to the substantiality and purity of its being, the barriers have been reinforced, and reinforced, and reinforced. But the barriers were not erected first by the philosophers. The philosophers have added art, and more art, and more art, to what nature has provided in the first place. Before Protagoras already, the philosophers encountered the citadel, and this fact, that the art of reinforcement of the barriers has served what nature has provided in the first place, points at the cause conclusively.

Protagoras postdates Thales by not quite one and a half centuries.

Thales is the first philosopher. One decade-odd later, Anaximander is the second philosopher. Two decades later, Anaximenes is the third philosopher. Two decades later, Pythagoras is the fourth philosopher, and Xenophanes is the fifth philosopher. Xenophanes is the fifth philosopher, but he is the first to leave his own words behind. The labors of Thales, Anaximander, Anaximenes, and Pythagoras are attested by their successors. Xenophanes left behind 120-odd lines, 40-odd fragments, attesting his labors himself.

He left behind verse, as several of the early philosophers did, emulating Homer and Hesiod, not clear yet that the philosophers, being philosophers, will avoid verse because, both, truth need not be constrained by verse and verse will not be constrained by truth. And among the verses Xenophanes left behind are these:

> they honor the winners of horse races and foot races,
> of boxing matches and wrestling contests,
> but those are less worthy than I am, for
> better than the strength of men and horses is our wisdom (fr. 2).

It is clear who they are. It is clear also that they are less worthy than Xenophanes is. Because better than the strength of men and horses is our wisdom, and by Xenophanes's time already, our wisdom included a solution of the problem of the principles of all things, a solution of the problem of the earth, a better solution of the problem of the earth, a map of the earth, a better solution of the problem of the principles of all things, the discovery that number is the principle of all things, the discovery of the irrationality of $\sqrt{2}$, the discovery that knowledge is not belief.

It is clear also that it is not clear at all to them that they are less worthy than Xenophanes is.

Still, two generations after the first philosopher, it may have seemed that there is hope that they will mend their ways. That they will see that our wisdom is better than the strength of men and horses, will stop honoring the winners of horse races and foot races and start honoring Xenophanes.

One generation after Xenophanes, Heraclitus is the eighth philosopher. Heraclitus left behind prose, a couple of hundred lines, 130-odd fragments, and Heraclitus reports: men do not understand (fr. 1); the rabble live as if each had his own understanding (fr. 2); they know neither how to listen nor how to speak (fr. 19); they stuff themselves like cattle (fr. 29). Nor is Heraclitus judging the asses when he observes: asses prefer rubbish to gold (fr. 9).

They have not mended their ways. Three generations after the first philosopher, it is clear also that there is no hope, if there ever was hope, that they will. For, if he is not judging the asses, Heraclitus has found out also why. They will not mend their ways, because they are asses, and: asses prefer rubbish to gold.

One generation after Heraclitus, not quite four generations after the first philosopher, the philosophers are already many enough to make counting them a problem that needs to be formulated explicitly before it can be solved. In any case, one generation after Heraclitus, Parmenides left behind verse, some 150 lines, not quite 20 fragments, and Parmenides reports:

> they wander about two-headed, deaf, blind,
> befuddled, hordes with no judgment,
> believing that what is and what is not are the same and not the same
> and everything turns back on itself (fr. 6).

Not quite four generations after the first philosopher, it may have still seemed that, if there is no hope that they will mend their ways, they at least will not include the philosophers. That asses may prefer rubbish to gold, but the philosophers will prefer gold to rubbish. That hordes with no judgment may believe that what is and what is not are the same and not the same and everything

turns back on itself, but the philosophers will know that what is and what is not are not the same and not the same and everything does not turn back on itself.

As that fragment attests as well, Parmenides discovered the principle of contradiction. What is and what is not are not the same. He formulates the principle as explicitly as this:

> there are two ways,
> what is is and cannot not be,
> which is the way of truth and conviction,
> and what is not is not and must not be,
> which you will never know or utter (fr. 2).

On the authority of Aristotle: the principle of contradiction is the most certain of all principles; it is the principle of all principles.

On the authority of Hegel: the necessity of contradiction is the nature of thought; the grasping of opposites in their unity, of the positive in the negative, is the soul of all true scientific knowledge.

What is and what is not are the same and not the same and everything turns back on itself, after all.

And not quite one generation after Parmenides Protagoras said that man is the measure of all things. Of whether rubbish is rubbish and gold is gold, or gold is rubbish and rubbish is gold.

Berkeley writes: all things in the scripture which side with the vulgar against the learned side also with me.

All things which side with the asses side also with me.

When asses prefer rubbish to gold, it is not an acquired taste. And when men not only prefer rubbish to gold, but believe that gold is rubbish and rubbish is gold, it is not because gold is rubbish and rubbish is gold. It is because they prefer rubbish to gold. And when the philosophers prefer rubbish to gold, and argue that gold is rubbish and rubbish is gold, it is not because gold is rubbish and rubbish is gold either. It is because they prefer rubbish to gold.

The cause of the barriers before the acquisition of knowledge and the investigation of knowledge is that the philosophers are not just the philosophers, the philosophers are also men, and men are not just men, men are also asses.

lii

The same thing can be said more explicitly, if not more politely.

Hume writes: there is a kind of brutish and ignorant scepticism which gives the vulgar a general prejudice against what they do not easily understand and makes them reject every principle which requires elaborate reasoning; they firmly believe in witches, though they will not believe or attend to the simplest proposition in Euclid (*Religion*, 1).

The prejudice is general. The vulgar have not kept it to themselves, they have generously shared it with the not vulgar. With the philosophers who do not easily understand physics, or mathematics, or history, or for that matter philosophy, the historians who do not easily understand physics, or mathematics, or philosophy, or for that matter history, the mathematicians who do not easily understand physics, or history, or philosophy, or for that matter mathematics, the physicists who do not easily understand mathematics, or history, or philosophy, or for that matter physics, and whether they are like the vulgar, and because they do not easily understand, they will not attend to what they do not easily understand, or the vulgar are like them, and because they will not attend to what they do not easily understand, they do not easily understand. They firmly believe in spirits, if not quite so firmly in witches any longer, though they will not believe or attend to the simplest proposition in Euclid, and when they proffer non-Euclidean geometry as a reason, they will not believe or attend to the simplest proposition in Lobachevsky. Never mind in Riemann.

The relation of effect and cause is the same. The effect is scepticism which gives the vulgar and not vulgar a general prejudice against what they do not easily understand and makes them reject every principle which requires elaborate reasoning. The cause is that scepticism is brutish. That it is ignorant is redundant.

Witches, if there are witches, predate Euclid. Whether there are witches or not, belief in witches predates Euclid. If there are witches, they may well be coeval with god, given that in *Exodus* he commands: you shall let no witch live (22). Whether there are witches or not, belief in witches is at least coeval with *Exodus*, and given

that in *Exodus*, as in *Genesis* and in *Numbers*, an older document, the Yahwistic document, is incorporated into a newer code, the Priestly code, and the relevant part of *Exodus* is part of the Yahwistic document, not part of the Priestly code, belief in witches is at least coeval with the Yahwistic document. Nor is there any evidence that either god or the Yahwistic document believed or attended to the simplest proposition in Euclid. Belief in witches predates Euclid by at least five-odd centuries.

And if belief in witches, or in spirits, though they will not believe or attend to the simplest proposition in Euclid, or in Lobachevsky, instances scepticism, it is because scepticism restores Euclid and Lobachevsky to their proper place, and belief in witches and spirits to its proper place. Euclid and Lobachevsky do not throw light, they blind. They are not a condition of health, they are a condition in need of a cure. Belief in witches and spirits throws light. This is the condition of health. Like the earlier result, man is the measure, scepticism restores man to his proper place. Against the acquisition of knowledge, to his brutishness.

One year after the publication of the *Critique of pure reason*, its readers had availed themselves of the opportunity to read idealism in it. This in turn provided Kant with the opportunity to judge that reading. In an appendix to the *Prolegomena*, he judges it *A specimen of a judgment on the Critique prior to its examination*. He assigns also a cause to that reading.

Kant writes: it is as though someone, who had never seen or heard of geometry, found a copy of Euclid and, having noticed many figures while leafing through it, gave his judgment, it is a textbook of drawing, but the author uses a peculiar language in order to give dark, incomprehensible instructions which in the end teach nothing more than what everyone with a good eye can readily do (*id.*).

The relation of effect and cause is again the same. The effect is that Euclid and Kant use a peculiar language in order to give dark, incomprehensible instructions which in the end teach nothing more than what everyone with a good eye can readily do. The cause is that everyone is endowed with a good eye.

And if this instances idealism, it is again because idealism restores Euclid and Kant to their proper place, and everyone's good eye to its proper place. Everything is my ideas in myself. Everything includes Euclid, and mathematics, and Kant, and philosophy. As to myself, whether it includes Euclid and Kant, it includes everyone, endowed as he is with a good eye.

How good everyone's good eye is, however, can be assessed. Has, in fact, been assessed.

In *The new advances in the theory of vision*, in 1868, Helmholtz begins with the description of: the optical apparatus of the eye (1). He shows that: although the eye has a very large field of vision, we see accurately only a very small part of it (*id.*); the chromatic aberration of the eye reduces the sharpness of the objects in the field of vision (*id.*); the astigmatism of the eye prevents us from seeing clearly horizontally and vertically simultaneously (*id.*); the blind spot of the eye produces a gap in the field of vision where nothing is visible (*id.*); the venous shadows in the eye and the imperfect transparency of its media blur even the point of clearest vision (*id.*). Helmholtz concludes: if an optician wanted to sell me an instrument which had such defects, it is not too much to say that I will believe myself completely justified to condemn in the strongest terms the carelessness of his work and to give him back his instrument with indignation; I will not do this with my eyes of course, on the contrary, I will be glad to keep them with all their defects as long as possible; but the fact that my eyes are irreplaceable to me does not diminish the magnitude of their defects (*id.*); the eye is extraordinary because of the way in which we use it, its perfection is practical, not absolute (*id.*); and the study of the eye offers a deep insight into the character of organic purpose generally, confirming the great and daring thought of Darwin, wherever we look into organic formations, we find the same character of practical purpose (*id.*).

As the eye, then, so and very much more so the organism. As the eye, so the other organs, so their functions, so the development of organs and functions, and hence so and very much more so the organism.

As defective an optical instrument as the eye is, so and much more so defective an instrument

of the acquisition of knowledge man is.

The defects of the eye only begin with its physiology. The eye is extraordinary in its use, and its extraordinary use makes the defects extraordinary. The fact that my eyes are irreplaceable to me does not diminish the magnitude of their defects. The fact that my eyes are irreplaceable to me greatly exacerbates their defects. Lack of absolute perfection is a deficiency. Practical perfection makes it an absolute deficiency.

And the defects of man only begin with his physiology. Wherever we look, we find the same character of practical purpose.

With Maxwell: it was a great step when men became convinced that they must ask not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it. And if of what kind it is, and how much there is of it, can be asked only if it is not asked good or bad, noxious or beneficial, men cannot have a predilection for the acquisition of knowledge, men can only have a most powerful predilection against the acquisition of knowledge.

The cause of the barriers before the acquisition of knowledge and the investigation of knowledge is the organism. The cause is that the philosophers are not just the philosophers, the philosophers are also organisms.

liii

That a barrier must be removed, or even only circumvented, does not imply that it can be. But it does imply that the attempt can be made.

Parmenides discovered that what is is and cannot not be, and what is not is not and must not be. Hence, with Bacon: there are and can only be two ways to search for truth, the false and the true (*Organon*, I, 19).

Hence, also, the barriers before the investigation of knowledge are many and formidable, and they have been reinforced, and reinforced, and reinforced.

Yet the investigation of knowledge has advanced greatly, the acquisition of knowledge has advanced very greatly, and the problem of knowledge can be formulated explicitly

liv

The most formidable barriers before the investigation of knowledge are vagueness, the kinds of knowledge, and man. These are the parts of the system of fortifications into which all barriers are collected. If these are removed, or even only circumvented, all barriers will have been removed, or circumvented.

Vagueness is a barrier, but it is not a problem the investigation of knowledge must solve. Everything is vague, and everything has been made vaguer. Vagueness is yet a barrier which the acquisition of knowledge has long removed. Much has been made clear and distinct and continued to be made clear and distinct. Vagueness can be removed, in every particular case, by emulating, in every particular case, not the philosophers who have made everything vaguer, but the philosophers who have made it clear and distinct. Leibniz writes, for instance: reasoning will always be correct if it is everywhere as clear and certain as arithmetic has been (*Gen. science*, 11). And if those philosophers have been many, these have been many, too, even if not quite as many, and if the labors of those have been successful, the labors of these have been successful, too, and it is the labors of these, not of those, that have advanced the acquisition of knowledge and the investigation of knowledge. And the outcome of all acquisition of knowledge, fifty-one centuries of it now, whatever else it has been, has been clarity and distinction. The outcome of all investigation of knowledge as well, twenty-four centuries of it now, whatever else it has been, has been clarity and distinction. So much so, vagueness is a problem which has been solved explicitly, in general, and conclusively, by Descartes, with the formulation of the rules of the method. Following the rules of the method guarantees clarity and distinction. Vagueness is not a problem the investigation of knowledge must solve. It is a

problem on whose solution the investigation must stand.

The kinds of knowledge are a barrier the investigation must itself remove. It is clear how. The kinds of knowledge are a problem the investigation of knowledge must formulate, and solve.

The difficulty of the problem the following facts all show. Its most successful solution, Aristotle's, is implicit rather than explicit, a solution of the problem of the kinds of philosophy and only hence of the kinds of knowledge. Its only explicit solution, Bacon's, is negative rather than affirmative: noticing which parts lie waste, uncultivated, and abandoned by industry. Bacon's solution also predates the last four centuries of acquisition of knowledge and investigation of knowledge. Aristotle's solution predates the last twenty-three and a half centuries of acquisition of knowledge and investigation of knowledge.

Nevertheless, there are these solutions, Aristotle's and Bacon's, to build on. In general, again, the investigation of knowledge has advanced greatly, and that is, the formulation and solution of the problem of knowledge, and hence also of its kinds, has advanced greatly, explicitly and implicitly. The acquisition of knowledge has also advanced very greatly, and it has thus informed the problem of knowledge, and hence also of its kinds, explicitly and implicitly, very greatly.

The following is thus clear.

The formulation and solution of the problem must be affirmative. Bacon writes: it is one thing to point what land lies uncultivated, another to improve husbandry. The problem of knowledge, and hence also of its kinds, is posed by improved husbandry. Not by waste land. The results from the acquisition of knowledge pose the problem of knowledge, and hence also of its kinds. Not the results against the results from the acquisition of knowledge. Knowledge poses the problem of knowledge, and hence also of its kinds. Not the lack of knowledge, or the deficiency of knowledge, or the impossibility of knowledge.

There is thus Aristotle's solution – theory and practice, physics and mathematics, politics – and as husbandry has improved, and more, and more, the evidence corroborating Aristotle's solution has *prima facie* been so extensive and so strong, the labor of formulation and solution of the problem has to begin with its examination and correction. Should it need correction.

In general, where the improvement of husbandry is concerned, Aristotle's solution is grounded, counting back to Homer, in not quite four centuries of acquisition of knowledge. Even if all four are centuries of great acceleration of the acquisition. Bacon's solution is grounded, counting back to the Yahwistic document, in twenty-four-odd centuries of acquisition of knowledge. Even as it is not an accident that Bacon's solution is negative, most of the nineteen and a half centuries between Aristotle and himself having been of deceleration of the acquisition, then dark ages, then slow acceleration. Twenty-three and a half centuries after Aristotle, four centuries after Bacon, the formulation and solution of the problem is informed, counting back to the 3 calculations of area, by fifty-one centuries of acquisition of knowledge. Twenty-seven and a half are before Aristotle. And the four after Bacon include the brightest period in the acquisition of knowledge. In the investigation of knowledge, to note only the most important, Aristotle has the labors of Plato to build on. Bacon has the labors of Aristotle to build on. There have since been the labors of Descartes, Spinoza, Leibniz, Hume, Kant, Frege.

The formulation and solution of the problem must also be, as Aristotle's solution is, theoretical. Not practical. It must be grounded in the search for truth. Not in the search for action. The problem of knowledge, and hence also of its kinds, is posed by truth. What is. Not by falsehood. What is not. It is thus posed also by the world, the one next to which there is nothing. Not by ideologies, or world-views, or discourses. Knowledge poses the problem of knowledge, not ignorance. Truth poses the problem of truth, not fiction. The world poses the problem of the world, not ghosts.

Nor must truth need a recommendation. Or the world. Or the success in the search for truth. Or the success in the investigation of the world, 1.4×10^{10} years old, 9×10^{23} km across to the limit of observation.

The formulation and solution of the problem of knowledge, and hence also of its kinds, must also be genealogical. Analogy with the formula-

tion and solution of the problem of the kinds of organisms, and hence also of the development of organisms, shows this. Analogy with the formulation and solution of the problem of the kinds of stars, and hence also of the development of stars, shows it. The success of the acquisition of knowledge – the very fact that it is a development, that knowledge has been acquired – shows it. The success of the investigation of knowledge as well – again the very fact that it is a development, that knowledge has been investigated – shows it. The catastrophic transformation of the kinds of philosophy as well, and the reinforcement of the barriers before the acquisition of knowledge and the investigation of knowledge – and once again the very fact that it is a development, that the barriers have been reinforced, and reinforced, and reinforced – shows it.

The formulation and solution of the problem of knowledge, and hence also of its kinds, must also be, as Aristotle's and Bacon's solutions of the problem of the kinds both are, descriptive. It must be a description of the acquisition of knowledge, inclusive, to the degree to which the investigation of knowledge has been a part of the acquisition, of the investigation of knowledge.

Aristotle writes: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous (*Soul*, I, 2).

We call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous. The most important members of the council must be the philosophers who have advanced the acquisition of knowledge.

Bacon writes: there are and can only be two ways to search for truth; one is to fly from the senses and the particulars to most general principles and, taking them to be settled, proceed to judgment; this way is now in use; the other is to obtain principles from the senses and the particulars by continuous and gradual ascent, and only ultimately end with the most general principles; and this is the true way even if it is yet untried (*Organon*, I, 19).

This is the true way. To obtain principles from the particulars by continuous and gradual ascent. The most important particulars must be the philosophers who have advanced the acquisition of knowledge.

Man, finally, is the most formidable barrier before the investigation of knowledge. But man is not a problem the investigation of knowledge must solve either. The problem the investigation of knowledge must solve is of the relation of knowledge and man. Nor can this problem be either too important or too difficult to solve.

Man is the most formidable barrier, but he is because he has been a barrier from the first and because all other barriers have protected and reinforced this barrier. And formidable as he is, man cannot be an impenetrable barrier. Not if the other barriers are removed. Man, moreover, is not an open problem, not after Darwin. After Darwin, man is a problem that has been solved. It is for this reason also that the problem of the relation of knowledge and man cannot be either too important or too difficult to solve.

Helmholtz appeals to Darwin, and Darwin appeals to Helmholtz, in *The origin of species* (6), and in *The descent of man* (14). The perfection of the eye is not a perfection, it is an adaptation. And the pinnacle of creation is not a pinnacle of creation, he is an organism.

The philosophers are not just the philosophers, the philosophers are also organisms.

But organisms are not just organisms either, organisms are also the philosophers.

It is thus clear how the problem of the relation of knowledge and man must be formulated, and solved. It is clear also that the formulation and solution of this problem demands a generalization.

On the side of man, the formulation and solution of the problem of the relation of knowledge and man, in particular, and the investigation of knowledge, in general, and that is, the formulation and solution of all problems involving knowledge, must ground themselves in the theory of evolution.

On the side of knowledge, in the formulation and solution of the problem of the relation of knowledge and man, in particular, and in the investigation of knowledge, in general, and that is, again, in the formulation and solution of all problems involving knowledge, the supposition of parochiality must be replaced by the presump-

tion of generality. With Planck: the inhabitants of Mars, if they have the necessary intelligence, recognize the law of gravity and the principle of conservation of energy; for anyone to whom this is not self-evident it will forever remain an unresolved mystery that the same physics is made in the United States as in Germany. Knowledge cannot be human any more than it can be German. Knowledge must be the same in Germany, in the solar system, in the Milky Way, in the world.

So.

There is the problem of knowledge.

The problem of knowledge poses the problem of the acquisition of knowledge.

The problem of the acquisition of knowledge poses the problem of the relation of the two, its acquisition and knowledge, its acquisition such as it cannot not be a development and knowledge such as it cannot be knowledge and develop.

A problem of development, the acquisition of knowledge poses the problem of its beginning.

The problem thus also of its end.

Beginning to end, the problem thus also of its entirety.

Knowledge, and rather, if knowledge cannot develop, the acquisition of knowledge, poses the problem of its kinds.

The problem of its species.

The problem of the genus of which it is in turn a species.

The problem of the kinds of the acquisition of knowledge poses the problem of the kinds of philosophy.

The acquisition of knowledge poses more generally the problem of philosophy.

The problem also of the development of philosophy.

Of the relation of knowledge and philosophy.

Of the relation of the acquisition of knowledge and the development of philosophy.

Of all other relations in all other permutations of philosophy, knowledge, development of philosophy, and acquisition of knowledge.

Truth.

The relation of knowledge and truth.

The world.

The relation of knowledge and the world.

All other permutations again.

The search for truth, the investigation of the world, the acquisition of knowledge and the search for truth, the acquisition of knowledge and the investigation of the world, all other permutations.

The relation of knowledge and man, too. And the relation of knowledge to man as well as the relation of man to knowledge.

That those problems are all in relation to one another cannot not make them, however difficult each singly is to formulate, and solve, all jointly more difficult to formulate, and solve. Nevertheless, that those problems are all in relation to one another cannot not also make the attempt to formulate them all jointly, and then to solve them all jointly, substantially less likely to fail than the attempt to formulate, and then to solve, any of them singly.

And if so, the attempt has to be made to formulate the one and only one problem of which all those are the parts. Then, if this attempt succeeds, and to the degree to which it succeeds, the attempt can also be made to solve the formulated one and only one problem of which all those are the parts.

That a problem must be formulated, or solved, does not imply that it can be. But it does imply that the attempt can be made.

This is the task, then. To try and formulate the one problem of which all those are the parts. Or what is the same, to try and make explicit what all the parts are and what the whole is of which they all are the parts.

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And to this end – the formulation of the problem before the investigation – aiming, in accordance with the first rule of the method, at clarity and distinction, in accordance with the second rule, at division of difficulties into parts such as will lead to their better solution, and in accordance with the third rule, at order such as to begin with the simplest and easiest and ascend gradually to the more complex, the best way should be, in accordance with the last rule, the review. The review, in particular, of three of the five most successful investigations of knowledge, those of Aristotle, Hume, and Kant, and of the greatest successes in the acquisition of knowledge which

inform and corroborate the investigations, those of Ptolemy, Darwin, and Frege.

The three investigations of knowledge, first Aristotle's, then Hume's and Kant's, succeeded great successes in the acquisition of knowledge and were succeeded by great successes in the acquisition of knowledge. Even if they were also succeeded by great successes in the reinforcement of the barriers before the acquisition of knowledge and the investigation of knowledge. And the three are most successful investigations of knowledge because they are, both, each a synthesis of the preceding success of the acquisition of knowledge and each informed substantially, and hence corroborated, by the subsequent success of the acquisition of knowledge.

Thus, it is important that Plato's is the first investigation of knowledge, and that Aristotle's investigation is informed by Plato's. But it is more important that Aristotle postdates Thales by two and a half centuries, and in these two and a half centuries the acquisition of knowledge made very great strides. More generally, the broadening, deepening, and quickening of the acquisition, which accelerated greatly with the first philosopher, can be traced back to Homer, not quite four centuries before Aristotle. And Aristotle's investigation of knowledge is a synthesis of the success of the acquisition in these four centuries.

On the other side, among Aristotle's successors is Ptolemy. The reinforcement of the barriers began before Aristotle already, but it was after him, if not in response to him, that it became resolute, and it was after Ptolemy that it became resolute enough to bring about the darkest of dark ages. In the meantime the acquisition of knowledge advanced very greatly, even as it began decelerating, and four and a half-odd centuries after Aristotle, Ptolemy's is the greatest success, by far, in the forty-seven centuries of the acquisition of knowledge from the 3 calculations of area to Bacon, Galileo, and Kepler. And Ptolemy informs substantially, and hence corroborates, Aristotle's investigation of knowledge.

Fifteen centuries after Ptolemy, with Bacon, Galileo, and Kepler the brightest period in the acquisition of knowledge begins. It is the brightest period in the investigation of knowledge as well. In this the most important are the labors of Bacon and a generation later of Descartes, as the brightest period begins, and of Hume and of Kant, as the brightest period comes to a close. In between very important are the labors also of Spinoza and of Leibniz.

The other two of the five most important investigations of knowledge, the investigations of Bacon and of Descartes are causes of the following success of the acquisition of knowledge. They are thus the most important, by far, in showing the true way.

The investigations of Hume and of Kant are effects of the preceding success of the acquisition. They are thus the most important in showing the problems before the investigation of knowledge and their solutions.

Hume postdates Bacon, Galileo, and Kepler by not quite one and a half centuries, his younger contemporary Kant postdates them by one and a half centuries. More generally, the broadening, deepening, and quickening of the acquisition, which accelerated very greatly with the instauration of the sciences, can be traced back to Abelard and Gerard, and Anselm, not quite seven centuries before Hume and Kant. Hume's and Kant's investigations of knowledge are each a synthesis of the success of the acquisition of knowledge in these seven centuries.

On the other side, among the successors of Hume and Kant are Darwin and Frege. The war for the restoration of spirit to the substantiality and purity of its being began before Hume and Kant already, but it was after them, and in this case in response to them, too, that it was brought to successful conclusion. The acquisition of knowledge yet continued to advance very greatly, and one century and one century-odd after Hume and Kant, Darwin's and Frege's are each among the greatest successes in the fifty-one centuries of the acquisition of knowledge. And Darwin and Frege inform substantially, and hence corroborate, the investigations of Hume and of Kant.

The review, then, is of Aristotle, Ptolemy, Hume, Kant, Darwin, and Frege. To the end of formulating the problem before the investigation, the review is of the problems solved by Aristotle, Hume, and Kant, and of how these problems are informed by Ptolemy, Darwin, and Frege.

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Three centuries after Aristotle, his treatises were edited by Andronicus. Zeno of Citium's division of philosophy into logic, physics, and ethics had by that time become normative, and Andronicus arranged Aristotle's treatises into logic, physics, and ethics. Andronicus also collected into a whole the treatises comprising the *Metaphysics*.

Nineteen centuries after Andronicus, in 1831-70, the Aristotelean corpus was edited by Bekker. Bekker's arrangement follows Andronicus's. In 1 462 pages, the edition begins with 184 pages of logic, continues with 796 pages of physics, has then the 113 pages of the *Metaphysics*, and ends with 369 pages of what would be ethics.

In the generation after Bekker, *The constitution of Athens*, a third book of the spurious *Economics*, and several hundred fragments were added to the Aristotelean corpus.

53 treatises can be counted in the corpus.

30 are very important, comprising a little more than five sixths of the bulk of the corpus.

In the following enumeration of these, the 30 very important treatises, the arrangement follows Aristotle's division of philosophy into theoretical and practical, and of theoretical philosophy into physics and mathematics. Logic is a part of mathematics. Practical philosophy is politics, the master art. Within each kind the treatises are arranged from more general to more particular. At the end are the very important spurious treatises.

Theoretical philosophy,
  physics,
    book I of the *Metaphysics*,
    book XII of the *Metaphysics*,
    books XIII-XIV of the *Metaphysics*,
    Physics, 8 books,
    On the heaven, 4 books,
    On generation and corruption, 2 books,
    Meteorology, 4 books,
    On the soul, 3 books,
    History of animals, 10 books,
    Parts of animals, 4 books,
    Movement of animals, 1 book,
    Progression of animals, 1 book,
    Generation of animals, 5 books,
  mathematics,
    book II of the *Metaphysics*,
    books III-IV of the *Metaphysics*,
    books VI-X of the *Metaphysics*,
    Posterior analytics, 2 books,
    Categories, 1 book,
    On interpretation, 1 book,
    Prior analytics, 2 books,
    Topics, 8 books,
    Sophistical refutations, 1 book,
Practical philosophy,
  politics,
    Politics, 8 books,
    The constitution of Athens, 1 book,
    Rhetoric, 3 books,
    Poetics, 1 book,
    Nicomachean ethics, 10 books,
spurious,
    [*Mechanics*], 1 book,
    [*Problems*], 38 books,
    [*Economics*], 3 books.

The remaining 23 treatises, important but not very important, comprising a little less than one sixth of the bulk of the corpus, most of them spurious, in their order in the corpus, are [*On the world*], *On the senses and the sensible*, *On memory and recollection*, *On sleep and waking*, *On dreams*, *On divination in sleep*, *On length and brevity of life*, *On youth, old age, life and death, and respiration*, [*On breath*], [*On colors*], [*On the audible*], [*Physiognomy*], [*On plants*], [*On marvelous things heard*], [*On indivisible lines*], [*The situation and names of winds*], [*On Melissus, Xenophanes, and Gorgias*], [book V of the *Metaphysics*], [book XI of the *Metaphysics*], [*Great ethics*], *Eudemian ethics*, [*On virtues and vices*], [*Rhetoric to Alexander*].

Of the 30 very important treatises 13 are physics.

The central problem of book I of the *Metaphysics* are the principles of all things. Aristotle introduces the problem with his most general investigation of knowledge.

Book XII of the *Metaphysics* is a general survey of physics. Aristotle discusses substance, matter and form, actual and potential, and motion, concluding with the general description of the world.

Books XIII-XIV of the *Metaphysics* argue, negatively, that there is no immaterial substance. Affirmatively, that: the objects investigated by

the mathematician are abstracted from the sensible things (XIII, 3).

The central problem of the *Physics* is motion. Aristotle introduces the problem with a discussion of the principles of all things. In the course of the investigation of motion, he deals with infinity, and with space and time. He determines the kinds of motion. And he argues that: everything in motion must be moved by something (VII, 1), and: since an infinite series is impossible, there must be a first mover (*id.*); itself unmoved (VIII, 5).

In *On the heaven* the world in its entirety is described.

The central problem of *On generation and corruption* are the elements.

In the *Meteorology*, Aristotle begins: we have dealt with the first principles, with natural motion, with the heavenly bodies and their order, and with the elements; we have now to deal with the phenomena in the region closest to the heavenly bodies; and when we next deal with animals and plants, our task will be complete (I, 1). In particular, the *Meteorology* deals with comets, the Milky Way, rain, clouds, snow, hail, rivers, deserts, floods, the sea, winds, earthquakes, lightning, thunder, hurricanes, the rainbow. It deals also with: how the map of the earth should be drawn (III, 5).

The central problem of *On the soul* is the soul. Aristotle argues that: the soul is inseparable from the body (II, 1).

The five treatises on animals, the *History of animals*, *Parts of animals*, *Movement of animals*, *Progression of animals*, and *Generation of animals*, comprise more than one third of the bulk of the physical treatises, more than one fifth of the bulk of the entire Aristotelean corpus. Aristotle describes 600 kinds of animals.

9 of the 30 very important treatises are logic.

Book II of the *Metaphysics* distinguishes theory and practice.

The central problem of books III-IV of the *Metaphysics* is the principle of contradiction.

The central problem of books VI-X of the *Metaphysics* is truth. Aristotle introduces the problem with the discussion of the kinds of theoretical philosophy. And in order to investigate truth, he deals with substance, essence, matter and form, genus and species, definition, actual and potential, one and many.

The central concern of the *Posterior analytics* is to establish that: knowledge is acquired only by demonstration (I, 2).

The *Categories* deals with terms.

On interpretation deals with propositions.

The *Prior analytics*, *Topics*, and *Sophistical refutations* deal with inferences. Aristotle writes: there are four kinds of inferences, demonstrations, dialectical arguments, examinations, and contentious arguments; demonstrations are grounded in what is true, dialectical arguments are grounded in what is probable, examinations are grounded in what is accepted contingently, and contentious arguments are grounded in what is false (*Topics*, I, 1; *Refutations*, 2). The *Prior analytics* deals with demonstrations, the *Topics* deals with dialectical arguments and examinations, and the *Sophistical refutations* deals with contentious arguments.

5 of the 30 very important treatises are politics.

The central problem of the *Politics* is the state.

The constitution of Athens is a historical treatise, it describes the development of the constitution of Athens from its beginnings to the time of writing.

The *Rhetoric* deals with: rhetoric, which is concerned with persuasion, which is a kind of argument (I, 1); a mixture of logic and ethics, rhetoric is partly dialectical, partly contentious (4).

The *Poetics* deals with poetry, including: the origin of poetry (4).

In the *Nicomachean ethics* Aristotle concludes: the happiest life is the theoretical (X, 7).

The last 3 very important treatises are spurious.

Not by Aristotle, the *Mechanics* is yet the product of the Lyceum, and given that Aristotle's successor as scholarch Theophrastus and his successor Strato were physicists, but after Strato the Lyceum, too, became a school strictly of ethicists, the *Mechanics* is either by Theophrastus or Strato, or derived from Aristotle, Theophrastus, or Strato. It deals with the balance, the lever, missiles, the capstan, the wedge, the pulley.

The *Problems* is a collection of 891 problems.

The large majority are problems of medicine. The collection thus also is either by Theophrastus or Strato, or derived from Aristotle, Theophrastus, or Strato.

The *Economics* deals with the household.

And so much suffices to show that Aristotle is the most important philosopher, by far. To show also that this fact has long been established by Aristotle's successors as they have labored on one and all of those problems, examining his formulations and solutions, reexamining them, affirming them, reaffirming them, improving on them.

In physics, major advances were achieved by Archimedes, Hipparchus, Hero, Ptolemy, Buridan, Copernicus, Gilbert, Tycho. Aristotle yet remained the foundation of physics until Galileo and Kepler.

Aristotle remains the foundation of physics after Galileo and Kepler, too. For Galileo and Kepler have become the foundation of physics, but Aristotle is the foundation of Galileo and Kepler. Galileo writes: I am impugned as an impugner of the Peripatetic doctrine; but I claim that I observe Aristotle's doctrine more religiously than any of them; for Aristotle teaches us to reason well, argue well, and deduce necessary conclusions (to Liceti, 25 Aug 1640), and: if Aristotle returned to earth he would consider among his followers much rather me, because of my few but conclusive demonstrations of his errors, than the very many who, in order to defend every word of his as true, find in his writings doctrines which never entered his head (to Liceti, 15 Sep 1640). Kepler begins in the *New astronomy* with: an examination of Aristotle's *Metaphysics* (intr.). The subtitle of book IV of the *Epitome of Copernican astronomy* is: *a supplement to Aristotle's On the heaven*, and in its preface Kepler tells his reader, such as his reader is: I want Aristotle for my reader (*id.*).

In logic, major advances were achieved by Chrysippus, Leibniz, Kant, Boole. Aristotle yet remained the foundation of logic until Frege. Aristotle remains the foundation of logic after Frege, too, and not only because he is the foundation of Frege. The definition of truth, the distinction of proposition and sentence, the predicate calculus all remain, with scarcely any essential change, such as Aristotle developed them.

In politics, there was no major advance until Hobbes. And Aristotle remains the foundation of politics after Hobbes, and Rousseau, as well. As Aristotle formulated it, so the problem of the origin of the state remains the fundamental problem before politics.

Among the reasons for Aristotle's success the main are two. Nineteen and a half centuries before Bacon, Aristotle adheres staunchly to induction. All his investigations, physical, logical, and political, are grounded in the observation of the particulars. And twenty centuries before Descartes, Aristotle adheres staunchly to the rules of the method. To include in judgments nothing but what is clear and distinct, to divide difficulties into parts such as will lead to their better solution, to proceed in order from the simplest and easiest gradually to the more complex, and always to make enumerations so complete and reviews so general as to omit nothing.

Hence, Aristotle discovered the review of the earlier labors of formulation and solution of problems as the ground of the formulation and solution of problems. He reviews his predecessors in all his treatises, *passim*, and reviews them systematically not less than 16 times. He reviews them systematically in order to formulate and solve the problems of the principles of all things (*Metaphysics*, I, 3-10), of the number of the principles (*Physics*, I, 2-6), of infinity (III, 4), of the earth (*Heaven*, II, 13), of generation (III, 1-2), of heavy and light (IV, 2), of whether alteration is generation (*Corruption*, I, 1), of how action takes place (8), of comets (*Meteorology*, I, 6), of the Milky Way (8), of earthquakes (II, 7), of the soul (*Soul*, I, 2), of the veins (*Hist. animals*, III, 2-3), of respiration (*Respiration*, 7-13), of arguments (*Refutations*, 34), of which is the best state (*Politics*, II, 1-12).

Here are six illustrations of the magnitude of Aristotle's success in the acquisition of knowledge, three in physics, three in logic.

The principles of all things.

Thales proposed that water is the principle of all things. Pythagoras proposed that number is the principle of all things. Empedocles proposed that earth, water, air, and fire are the principles, and all things are composed from them and decom-

posed into them by love and strife. Aristotle proposed that: the principles of all things are of four kinds, material, efficient, formal, and final (*Physics*, II, 3).

Aristotle's result is a vast generalization. It also describes the results of the investigation of the principles of all things beginning to end. Thales solved the problem of the material principles of all things. Pythagoras solved the problem of the formal principles of all things. In Empedocles, earth, water, air, and fire are the solution of the problem of the material principles of all things, love and strife of the problem of the efficient principles of all things. With Galileo: the world is written in mathematics. Beginning with him physics became mathematical, thus formal, and why is explained, if, with Aristotle: mathematics is the investigation of form (*Physics*, II, 2), and number is the formal principle of all things. Two-odd generations after Planck, quantum theory was comprehended into what has been named – as unfortunately as the theory of relativity – the standard model. The standard model describes two kinds of particles, or waves, or particle-waves, quarks and leptons, and four kinds of forces, gravitational, strong, weak, and electromagnetic, and quarks and leptons are the material principles of all things, the gravitational, strong, weak, and electromagnetic forces are the efficient principles of all things.

Animals.

The fact that in the five treatises on animals Aristotle quotes Democritus 13 times, Homer 12 times, Empedocles 11 times, Anaxagoras 5 times, Ctesias 4 times, Alcmaeon 3 times, Herodotus 3 times, shows that his predecessors advanced the investigation of animals. The facts that he does not review his predecessors systematically and that he quotes Homer as often as Democritus and Empedocles show how greatly Aristotle himself advanced the investigation.

He describes 600 kinds of animals, 400 in the *History of animals* alone. These are both approximate numbers. Aristotle writes: turning to the investigation of animals, we shall try, to the best of our abilities, to omit no member of their kingdom, however lowly (*Part. animals*, I, 5). The number of the kinds of animals, which Aristotle describes, is approximate because the number of the difficulties, which he resolves in order to omit no member of their kingdom, however lowly, is indefinite.

Aristotle's result is a vast particularization. And that the particulars have to be described is conclusive. To solve the problem of the classification of organisms, Linnaeus arranges in their genera, orders, and classes 5 944 species of plants and 5 897 species of animals.

The world.

Thales proposed that the earth floats on water, like wood. Parmenides discovered that the earth is a sphere. Eudoxus formulated the theory of the motions of the heavenly bodies in concentric spheres. Aristotle described the world.

He writes: the world is one (*Heaven*, I, 8); it is spherical (II, 4); in the center is the earth, at rest, itself a sphere (14); around the earth revolve the planets; to explain their motions Eudoxus assumed 26 spheres, and Callippus added 7, but 22 more are necessary to counteract the 33, for a total of 55 (*Metaphysics*, XII, 8); beyond the planets, at the circumference, are the fixed stars (*Heaven*, II, 8); and unlike the motions of the planets, which are composite, the motions of the fixed stars are simple (10). Further: the elements are four; not fire, air, water, and earth, but simpler bodies similar to them (*Corruption*, II, 3); the natural motion of fire and air is up, of water and earth down (*Heaven*, I, 2), hence: fire occupies the highest region, earth the lowest (*Meteorology*, I, 2); and the world is so ordered that water is above earth, air is above water, fire is above air (4). Moreover: because the primary motion is circular, but the motion of the composite bodies below the sphere of the fixed stars is determined by the elements prevailing in them, we conclude that there is another element beyond those we see on earth, aether (*Heaven*, I, 2-3). Thus: the earth is in the water, the water is in the air, the air is in the aether, and the aether is in the world; but the world is one, there is nothing next to the world, or outside the world, and we cannot continue and say that the world is in something else (*Physics*, IV, 5). Further: everything in motion must be moved by something (VII, 1), and: since an infinite series is impossible, there must be a first mover (*id.*); itself unmoved (VIII, 5). In particular: the planets must be moved

(*Metaphysics*, XII, 8); as the world is one, it is sufficient to assume one first mover (*id.*); most generally, motion is locomotion (*Physics*, IV, 1); if there is no locomotion, there can be no other motion (*Metaphysics*, XII, 7); also, the primary motion is circular (9); we conclude that the first mover is at the circumference; moves the fixed stars; and communicates thus motion to everything further below as well (10).

Aristotle's result is a vast systematization. Hence, he describes the world. He describes the world in its entirety, describes its major parts, and many of its lesser parts, all in relation to one another. And it is the world, and all is in relation to one another, because with Kepler: the position, order, and motion of the parts of the world is the system of the world. Or what is the same, the world, next to which there is nothing, is the system of the world, next to which there is nothing. Thus, it is Aristotle's description of the world that Ptolemy improves upon, Ptolemy's description of the world that Kepler improves upon in *The system of the world*, Kepler's description of the world that Newton improves upon in *The system of the world*, Newton's description of the world that the big bang theory improves upon. All of Aristotle's results which have been shown erroneous, too, have been because this result of his, the world is one, there is nothing next to the world, or outside the world, has not been shown erroneous, and cannot be shown erroneous.

The principle of excluded middle.

Parmenides discovered the principle of contradiction.

Aristotle discovered that the principle of contradiction entails the principle of excluded middle. He writes: there can be no intermediate between contradictories, of one subject any one predicate must be either affirmed or denied; this is clear as soon as we define the true and the false; to affirm of what is that it is not, and of what is not that it is, is false, to affirm of what is that it is, and of what is not that it is not, is true (*Metaphysics*, IV, 7).

Brouwer argued that mathematical exactness is in the human intellect, and against the principle of excluded middle. Hilbert argued: we cannot relinquish either the principle of excluded middle or any other law of Aristotelean logic expressed in our axioms because the development of analysis is impossible without them (*Log. foundations*).

This is conclusive, too.

Terms, propositions, and inferences.

Completing his logical investigations, Aristotle writes at the end of the *Sophistical refutations*: in all investigations labor either takes over from those who have gradually advanced the investigation, or makes the first discovery, and the first discovery usually advances the investigation only a little, even if it is more important than the discoveries that spring from it; for it may well be, as the saying is, that the beginning is the most important; it is also the most difficult; once the beginning is discovered, it is easier to develop the rest; and this is indeed what has happened in the art of making speeches and more or less all other arts; those who discovered the beginning advanced the art only a little, but those who profess the art today are the heirs of a long series of predecessors; so it is not strange that that art has been developed considerably; of the present investigation, on the other hand, it was not that something was done, something was not done; nothing at all was done; contentious arguments they still teach like Gorgias, who gave speeches to his students to learn by heart; like giving them shoes rather than teaching them how to make shoes; and if much was said on rhetoric, nothing at all was said on demonstrative arguments; and we had to labor hard for a long time (34).

In *Aristotle's syllogistic from the standpoint of modern formal logic*, Łukasiewicz writes: it is an error to exemplify the Aristotelean *barbara* by,

all men are mortal,
Socrates is a man,
therefore Socrates is mortal;

Aristotle admits no singular terms in syllogisms;

all men are mortal,
all Greeks are men,
therefore all Greeks are mortal,

is also a misrepresentation, however; the genuine Aristotelean form is,

if *A* is predicated of all *B*,
and *B* is predicated of all *C*,
then *A* is predicated of all *C* (1).

Łukasiewicz explains: Aristotle's syllogistic is a

logic of terms; and no one who does not know that there exists another system, more fundamental than the theory of the syllogism, can fully understand Aristotle's proofs; this system is the logic of propositions; it is intuitively presupposed, but it is not explicitly developed by Aristotle (16). Łukasiewicz concludes: Aristotle's syllogistic is a strictly formal, mathematical theory (6); it is a system whose exactness surpasses even the exactness of a mathematical theory; but it is a narrow system (35).

Before Aristotle they were giving them shoes.

Aristotle discovered that there are terms, there are propositions, and there are inferences.

With respect to terms, he developed the logic of terms, the predicate calculus.

With respect to propositions, he discovered that: every sentence means something, but not every sentence is a proposition; a proposition is a sentence which affirms what is true or false; a prayer, for instance, is neither true nor false; we are thus concerned only with propositions, the rest we can ignore, they belong to rhetoric or poetry (*Interpretation*, 4).

With respect to inferences, he discovered that they are of four kinds, demonstrations, dialectical arguments, examinations, and contentious arguments. There is also rhetoric, concerned with a kind of arguments, but these belong to practice, partly dialectical, partly contentious, they are a mixture of logic with ethics.

All these results are conclusive, too.

Truth.

What is is truth, what is not is falsehood (*Metaphysics*, IX, 10).

And this, too, is conclusive.

Frege writes: I understand by the truth value of a proposition the fact that it is true or that it is false; further truth values there are not; I name the one the true, the other the false; and these two objects are recognized, even if only tacitly, by everyone who judges at all, and so also by the sceptic (*Reference*).

Four conclusions can be drawn from Aristotle's success in the acquisition of knowledge.

Whatever knowledge is, knowledge has been acquired.

Whatever the standard, much knowledge has been acquired.

It has been acquired by the labors of the philosophers.

And it has been acquired by the cumulative labors of the philosophers.

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Among the fragments of Aristotle there are some two dozen from the early dialogue *On philosophy*. One of the fragments describes this development:

first, men were forced by necessity to discover the useful arts, like sowing and the grinding of corn, and they called these arts wisdom;

next, they discovered the arts aiming not at necessity but at beauty, and they called these also wisdom;

next, they turned to politics, and discovered laws and all the things that keep a city together, and they called this wisdom, too; the seven wise men were such men;

next, they went further and turned to things themselves and their nature, and they called this physics, and the men who concerned themselves with it they also called wise;

finally, they turned to divine and unchangeable things, and they called the knowledge of these things the highest wisdom (fr. 13).

Aristotle returns to the description of this development in two of his later treatises. In these treatises, however, he describes a more general development. The steps above are only a part of that more general development. That development is not restricted to men. And the description of that development is Aristotle's most important result in the investigation of knowledge.

The two treatises are the *Posterior analytics* and book I of the *Metaphysics*.

The *Posterior analytics* Andronicus misnamed. It is not posterior to the *Prior*, it is prior to it. The *Prior analytics* deals with demonstration, but: demonstration always depends on principles (*Post. analytics*, I, 2), and it is the *Posterior analytics* that establishes the principles on which demonstration always depends.

Plato, in the *Meno*, arrived at the conundrum: one cannot search for what one knows, for he already knows it, and cannot search for what he does not know, for how will he search without knowing what to search for (80d-e)? Plato also

solved the conundrum: the soul is immortal; there is nothing it has not seen in this world or the netherworld, there is nothing it does not know; all learning and discovery is recollection (81c-d).

Aristotle begins in the *Posterior analytics* with Plato's conundrum, and with dismissing it out of hand: there is no reason why one should not in one way know and in another way not know what one investigates; absurd would be if one knew it before he investigates it in the way in which he can know it after he investigates it (I, 1); before, he knows it, but he does not know it simply, after, he can know it simply (*id.*); we know something simply when we know that its cause is its cause and cannot be otherwise; we know it by demonstration; and demonstration is deduction (2); demonstration always depends on true principles (*id.*); and these principles cannot themselves be demonstrated (9); to ask for their demonstration is to ask for demonstration to infinity (19).

So, there is knowledge before investigation. Then there is investigation. Investigation can reach demonstration. Demonstration is deduction. If investigation reaches demonstration, the outcome is knowledge simply. And knowledge simply is the end.

Knowledge simply is the end. What the beginning is, Aristotle indicates at the very beginning of the treatise. He writes: deduction is from premises, induction reaches the general from the clarity of the particulars (1). Induction from the clarity of the particulars is the beginning. At the end of the treatise Aristotle makes this explicit, too, when, in order to solve the problem of how the principles are discovered, he addresses the question of the beginning directly. The foundation of both, the solution of the problem of how the principles are discovered and the answer of the question of the beginning, is that general development a part of which Aristotle has described in *On philosophy*.

He writes: demonstration is from principles (II, 19); but how do we acquire these without knowing them to begin with? clearly, we have some faculty to discover them; clearly as well, all animals have this faculty; for they all have the faculty of discrimination which is perception; from perception comes memory; from memory, when it is in relation to the same thing, comes experience; and experience, which is general, is the beginning of art and of knowledge, of art when it is concerned with making things, of knowledge when it is concerned with what things are (*id.*); it is thus clear that we must begin with induction; for even perception instils the general in this way (*id.*).

The development, then, is this:
all animals have perception;
from perception comes memory;
from memory comes experience;
and experience is the beginning of art and of knowledge.
The development is also this:
first there is induction;
and the end is demonstration.
The development is also this:
first knowledge before investigation;
then investigation;
and finally knowledge simply.
Part of the development are also the stages described in *On philosophy*:
first the useful arts;
then the arts aiming not at necessity but at beauty;
then politics;
then physics;
and finally knowledge of divine and unchangeable things.

It is clear that the four series describe one development. It is clear also that they describe it at different levels of generality.

And to sort out the levels, and relate the series more explicitly to one another, there is the other treatise in which Aristotle describes the development, book I of the *Metaphysics*.

Book I of the *Metaphysics* is the most important treatise. This fact is established by the fact that it is Aristotle's most important treatise, and this fact, in turn, is established by the following facts. At whatever else it has aimed, philosophy has aimed at the explicit formulation and solution of problems, it has, beginning with Aristotle, and book I of the *Metaphysics* is Aristotle's most explicit formulation and solution of a problem. The problem is of the principles of all things, no less. Aristotle discovered the review of the earlier labors of formulation and solution of problems as the ground of the formulation and solution of problems, and the review of the labors of his

predecessors in book I of the *Metaphysics* is his most explicit. Hence, also, the history of the first two and a half centuries of development of philosophy depends essentially on this review, and as the history of the development of all subsequent philosophy depends on the history of the first two and a half centuries of development of philosophy, the history of the development of all philosophy depends essentially on this review.

And Aristotle introduces the review and the problem with his most general investigation of knowledge.

He writes: all men by nature desire to know (1), proposes as evidence: the pleasure we take in perception, and most of all sight which makes us know by making us aware of differences (*id.*), and then begins the description of that same development:

by nature all animals have perception;
from perception memory develops in some of them;
from memory experience develops;
from experience art develops;
and animals other than man have little interconnected experience;
but all men live by art and by reasoning (*id.*).

He notes that: experience is knowledge of the particular; art is knowledge of the general; and for this reason knowledge is associated more with art than it is with experience (*id.*), then continues the description of the development:

first the arts aiming to procure the necessities of life developed;
then the arts aiming to give pleasure;
and when all such arts were established and men began having leisure, knowledge which is neither useful nor pleasurable was discovered; thus mathematics was first developed in Egypt because the priests there had leisure (*id.*).

He notes that: knowledge which is neither useful nor pleasurable is knowledge of the principles of things (*id.*), states that this is the concern of the present treatise, and since: knowledge of the principles of things implies knowledge of the principles of all things, and to know all things is the furthest removed from perception (2), these, specifically, the principles of all things, are the concern of the present treatise, and then concludes the description of the development:

as the first philosophers show, it is because of curiosity that men at first began and still begin to philosophize; they asked questions about obvious things at first; but they advanced little by little;
and at length they began asking questions about the greater things, the motions of the moon, the sun, and the stars, and about the origin and development of the world (*id.*).

He adds: all other knowledge is more useful than the knowledge of the principles of all things, but none is better; and all men begin with wonder, but we must end in the contrary and better state; all men are astonished when they first learn that there is something which cannot be measured even by the smallest unit, but nothing will astonish the geometer more than were the side and the diagonal of the square to turn out to be commensurable (*id.*). And Aristotle then turns to the review of his predecessors' formulation and solution of the problem of the principles of all things.

The development is this:

all animals have perception;
from perception memory develops in some of them;
from memory experience develops, of which animals other than man have little;
from experience art develops, and first the arts aiming to procure the necessities of life developed;
then the arts aiming to give pleasure developed;
when all such arts were established, knowledge which is neither useful nor pleasurable was discovered, thus mathematics was first developed in Egypt because the priests there had leisure;
as the first philosophers show, it is because of curiosity that men at first began and still begin to philosophize; they asked questions about obvious things at first; but they advanced little by little;
and at length they began asking questions about the motions of the moon, the sun, and the stars, and about the origin and development of the world.

In this final description two of the earlier four series are integrated explicitly. The other two:

first there is induction;

and the end is demonstration,
and:
first knowledge before investigation;
then investigation;
and finally knowledge simply,
are not integrated explicitly.

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And in order to sort out the levels of generality at which the different series describe the development, and to relate the series more explicitly to one another – first of all, all men by nature do not desire to know.

For either all animals, including all men, desire to know, and what they desire to know is knowledge. Or knowledge simply is knowledge, and whatever all animals, including all men, desire is not knowledge.

The evidence that all men desire to know is: the pleasure we take in perception, and most of all sight which makes us know by making us aware of differences. If perception is knowledge, the evidence is firm. And Aristotle asserts that perception is knowledge explicitly: the end of the animal is not just to generate, which is common to all living things, but to have also knowledge, for animals have perception, and perception is a kind of knowledge (*Gen. animals*, I, 23).

But if perception is knowledge, memory is also knowledge, experience is also knowledge, art is also knowledge, and knowledge simply is also knowledge. The arts aiming to procure the necessities of life are knowledge, the arts aiming at pleasure are knowledge, and knowledge which is neither useful nor pleasurable is also knowledge. When men ask questions about obvious things this is knowledge, and when the philosophers ask questions about the motions of the moon, the sun, and the stars, and about the origin and development of the world this is also knowledge. When men are astonished that there is something which cannot be measured even by the smallest unit this is knowledge, and when the geometer would be astonished were the side and diagonal of the square to turn out to be commensurable this would be knowledge, too. And that those are all knowledge Aristotle asserts explicitly.

But if those are all knowledge, knowledge is on the horns of a dilemma. What knowledge is is so vague, it is not worth knowing what knowledge is. Or those are all knowledge, but they are not all knowledge simply.

And knowledge, of all things, must not be forced on the horns of such a dilemma.

Knowledge simply is knowledge, not knowledge simply is not knowledge. *Tertium non datur*.

The reasons are many, they should all be conclusive, and they are all – implicitly, if not explicitly – Aristotle's reasons.

If perception is knowledge, man is the measure, and with Aristotle: the same thing would be a trireme and a wall and a man; it is what follows from what Protagoras says (*Metaphysics*, IV, 4).

If experience and art are both knowledge: knowledge is associated more with art than it is with experience, and can the same thing not be a trireme and a wall and a man, even if it is more a trireme than a wall and a man?

If the arts aiming to procure the necessities of life are knowledge, and the arts aiming at pleasure are knowledge, and knowledge which is neither useful nor pleasurable is knowledge, can the useful and the not useful, both knowledge, and the pleasurable and the not pleasurable, both knowledge, too, not make a trireme a wall and a man, a wall a trireme and a man, and a man a trireme and a wall?

Aristotle writes in the *Posterior analytics:* knowledge is different from opinion, knowledge is general, of what is necessary and necessarily cannot be otherwise; opinion is of what is true or false but can also be otherwise; so there is also both knowledge and opinion of the same thing; for instance, of an animal so that it cannot be of a not animal, and so that it can be (I, 33). But if perception is knowledge, and memory is knowledge, and experience is knowledge, and art is knowledge, can opinion not be knowledge? Of an animal so that it can be of a not animal not be of an animal so that it cannot be of a not animal?

One way or another, if there is knowledge simply and knowledge not simply, can knowledge simply not be knowledge, simply, and knowledge not simply not be not knowledge, simply?

One way or another, all men do not desire to know. Never mind by nature. Men believe in witches, though they will not believe or attend to the simplest proposition in Euclid.

One way or another, very little of all that would be knowledge, or is called knowledge, can be knowledge.

This has to be the series, then, which describes the development most generally:
first not knowledge;
then investigation;
and finally knowledge.

Next, induction from the clarity of the particulars is the beginning. It is the beginning very generally as well. At the far end of the development: even perception instils the general by induction. At the near end: the principles on which deduction depends are discovered by induction. It is what Newton affirms as well: the entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena.

If so, if induction reaches from Newton back all the way to perception, the problem of the soundness of induction is solved. Induction is sound in Newton because it is sound in perception already. And it has to be clear, after Darwin, why it is sound in perception. Because it is the outcome of selection. It is, to begin with, a matter of survival.

The series:
first there is induction;
and the end is demonstration,
thus also describes the development very generally. It is clear also what the relation of the two series is. Induction is a part of all three, not knowledge, investigation, and knowledge. Demonstration is a part only of the last, knowledge.

Hence, finally, the relation of these two series to the third, the most particular, can be made explicit.

Aristotle writes: we have some faculty to discover the principles; all animals have it; for they all have the faculty of discrimination which is perception.

Perception is a faculty of animals. It is thus a problem of physics. Accordingly, Aristotle deals in some detail with the senses (II, 5-11), and hence with perception (III, 1-3), in *On the soul*. Memory is also a faculty of animals. Aristotle writes in *On memory and recollection*: memory, which not only men but other animals as well have, belongs essentially to the faculty of perception (1); and to that part of the soul to which imagination belongs (*id.*). He deals with imagination in some detail in *On the soul* as well (III, 3), and in greater detail with that other faculty, thinking (4-5).

At the far end, then, consistent with the fact that induction is the beginning, and perception instils the general by induction, the description is of the development of organisms:
all animals have perception;
from perception memory develops in some of them;
from memory experience develops, of which animals other than man have little.
At the far end, the description is of not knowledge.

Those are all, perception, memory, experience, and in general faculties and their exercise, not knowledge. They are in relation to knowledge. Some of them are even among the necessary conditions, if not among the sufficient conditions, of the acquisition of knowledge. With Planck: it is improbable that the inhabitants of Mars have eyes and ears like ours; but if they have the necessary intelligence, they recognize the law of gravity and the principle of conservation of energy. If so, it is improbable that eyes and ears are among the necessary conditions. The necessary intelligence, though, is among them. Positively experience is a necessary, if not a sufficient, condition of the acquisition of knowledge. Aristotle writes: it is always experience that supplies the principles; thus, it was only after the phenomena were adequately investigated that astronomical demonstrations were discovered; and so it is in all other kinds of art and knowledge; given the properties of things, we develop the demonstrations (*Pr. analytics*, I, 30).

Those, perception, memory, experience, and in general faculties and their exercise, are in relation to knowledge, but they are not knowledge. And they are not problems of the investigation of knowledge. They are problems of physics, of the investigation of organisms, more particularly, and of the theory of evolution, most particularly.

Whatever doubt there may have been on the matter before Darwin – and there is no doubt whatsoever in Aristotle – there can be no doubt

after Darwin. Not quite two centuries after Darwin, those are all problems which the theory of evolution has solved. Perception, memory, and memory from perception, experience, and experience from memory, selection can not only produce. Selection has, in fact, produced. The genetic and paleontological evidence that selection has produced them is massive, and conclusive.

Given atoms, gravity can produce stars – even if it cannot by itself ignite them – and it can produce galaxies. But it cannot produce animals, or plants. Selection must do that. The principle producing organisms is distinct from the principle producing stars and galaxies. The principle producing organisms is thus wise to produce organisms, and wise to ground itself in the principle producing stars and galaxies, but will not be wise to try and produce stars and galaxies.

Similarly, if selection has produced perception, memory, experience, the investigation of knowledge cannot be wise to try and produce them. Whatever the investigation of knowledge will try to produce, it can be wise to try, because selection not only has not produced it, but cannot produce it. And if selection cannot produce knowledge, or the acquisition of knowledge, it can only be because a principle distinct from selection must produce knowledge, and the acquisition of knowledge. The investigation of knowledge will thus be wise to ground itself in the theory of evolution, and to labor on the presumption that there is that principle distinct from selection, and to labor to discover that principle, too. But it will be wise also to concern itself only with what the theory of evolution tells it, negatively, to concern itself. For the theory of stars and galaxies cannot decide what the theory of organisms must do, but it cannot not decide what it must not do. Similarly, the theory of evolution cannot decide what the investigation of knowledge must do, but it cannot not decide what it must not do.

At the far end, the description is of not knowledge.

At the near end, it is of knowledge:

> when all such arts were established, knowledge which is neither useful nor pleasurable was discovered, thus mathematics was first developed in Egypt because the priests there had leisure;
> as the first philosophers show, it is because of curiosity that men at first began and still begin to philosophize, they asked questions about obvious things at first, but they advanced little by little;
> and at length they began asking questions about the motions of the moon, the sun, and the stars, and about the origin and development of the world.

If it is a description of knowledge, however, men, priests, leisure, and curiosity do not belong in the description.

Mathematics was first developed in Egypt because the priests there had leisure, Aristotle writes twenty-three and a half centuries ago. There have since been many more priests than there had been. Many more also, priests and otherwise, have had leisure than had had. And on the evidence of a rather larger sample than was available to Aristotle, the conclusions are unavoidable. Given mathematics, a handful of priests will do mathematics. Given priests, they will not do mathematics. In the list of things priests will do, with the most likely at the top, the least likely at the bottom, not only will doing mathematics be fighting for the bottom, not doing mathematics by burning it, or throwing it in the water, will be far above doing mathematics. Given mathematics, also, a handful of those who have leisure will do mathematics. Given leisure, those who have it will not do mathematics. In the list of things they will do becoming a priest will be much higher than doing mathematics, and doing mathematics will be as likely as not at the bottom.

Leisure is not even a necessary condition of the acquisition of knowledge, let alone a sufficient. Positively priests are not a necessary condition, let alone a sufficient.

Hume writes: curiosity, or the love of knowledge, has a very limited influence, and requires youth, leisure, education, genius, and example, to make it govern any one person (*Essays*, I, 14). If curiosity, or the love of knowledge, requires example, this suffices to establish that it may be that because of philosophy men at first began and still begin to be curious, but it may not be that because of curiosity men at first began and still

begin to philosophize. Let alone if curiosity requires also education. Never mind youth, leisure, and genius.

Of knowledge, the description is:

when all such arts were established, knowledge which is neither useful nor pleasurable was discovered, thus mathematics was first developed in Egypt;

the first philosophers began asking questions about the motions of the moon, the sun, and the stars, and about the origin and development of the world.

And between the far end and the near end, the development is:

from experience art develops, and first the arts aiming to procure the necessities of life developed;

then the arts aiming to give pleasure developed;

and when all such arts were established, knowledge which is neither useful nor pleasurable was discovered.

This part of the development poses the problem of the beginning of the acquisition of knowledge.

Given the standard useful or pleasurable, on one side, neither useful nor pleasurable, on the other, the division is clear. The arts aiming to procure the necessities of life and to give pleasure belong on the side of not knowledge, knowledge which is neither useful nor pleasurable belongs on the side of knowledge. It is Maxwell's great step, too: when men began asking not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it. The evidence that selection has produced the useful and the pleasurable, in general, is massive, and conclusive, and it must be presumed that it has produced the useful arts and the pleasurable arts, in particular, even if the evidence is in this case only paleontological, not so massive, and not yet conclusive.

If so, the difficulty of the problem of the beginning of the acquisition of knowledge should need no recommendation. The formulation and solution of the problem has to bridge the abyss from the useful and pleasurable to the neither useful nor pleasurable. Selection for the neither useful nor pleasurable is an impossibility. An organism concerned with the neither useful nor pleasurable will go extinct as resolutely as the organisms around it are concerned with the useful and pleasurable. By the same analogy, a mass not subject to gravity is an impossibility, yet to the degree to which it is produced by another principle, and being subject to this principle, not to gravity, makes it what it is, an organism is a mass defying gravity. And if an organism not subject to selection is an impossibility, yet to the degree to which it can be produced by another principle, and being subject to this principle, not to selection, can make it what it is, something can be an organism defying selection. A mathematician in Egypt, for instance. Or the first philosopher. Or Aristotle.

The problem of the beginning of the acquisition of knowledge cannot not be a part of the problem of the principle that can produce the acquisition of knowledge.

This part of the development Aristotle describes in *On philosophy* in more detail:

first, men were forced by necessity to discover the useful arts, like sowing and the grinding of corn;

next, they discovered the arts aiming not at necessity but at beauty;

next, they turned to politics, and discovered laws and all the things that keep a city together, such were the seven wise men;

next, they went further and turned to things themselves and their nature;

finally, they turned to divine and unchangeable things.

On the same standard, the useful arts, like sowing and the grinding of corn, which men were forced to discover by necessity, belong on the side of not knowledge. The concerns with things themselves and their nature and with divine and unchangeable things belong on the side of knowledge.

On which side the arts aiming at beauty and politics belong is not so clear. They are part of the problem of the beginning of the acquisition.

The concern with things themselves and their nature is physics. The concern with divine and unchangeable things, consistent with the fact that the end is demonstration, has to be mathematics.

The association of mathematics with Egypt, the association of politics with the seven wise

men, and the explicit appeal to the first philosophers relate these to one another in their chronological sequence. The arts aiming at beauty, being arts, have to predate mathematics. Philosophy begins with the first philosopher, and whatever philosophy is, it is last in the development. Hence, in the order of the first development of each, the two series yield this one:
the arts aiming at beauty,
mathematics,
politics,
physics,
philosophy.

Who, and hence when, the first philosopher was, Aristotle established conclusively. Mathematics can be traced back to the 3 oldest calculations of area, in Mesopotamia, not in Egypt, twenty-five centuries before the first philosopher. Politics can be traced back to the first prince, Narmer, twenty-four centuries before the seven wise men, one of whom the first philosopher was. Physics can be traced back to the Venus tablet of Ammisaduqa, ten and a half centuries before the first philosopher. And if so, and these are the beginnings of mathematics, politics, physics, and philosophy, respectively, they agree with the sequence extracted from Aristotle.

In relation to the most general series, then, not knowledge, then investigation, and finally knowledge, these five, the arts aiming at beauty, mathematics, politics, physics, and philosophy, can all belong on the side of knowledge. Whether the arts aiming at beauty belong on this side is a part of the problem of the beginning of the acquisition of knowledge. Certainly, if, dated to the Aurignacian, the art of Altamira and Lascaux predates mathematics by ten millennia. Mathematics belongs on this side. If politics postdates mathematics, this can indicate that it belongs on this side as well. Physics and philosophy belong on this side.

Finally, given that either not knowledge, or knowledge, *tertium non datur*, investigation has to belong on the side of knowledge.

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The most important problem solved by Aristotle in the investigation of knowledge is development.

What is knowledge? Plato asks in the *Theaetetus*, answers the question, again, and again, and finds all three answers wanting.

Aristotle answers the question as well. Knowledge is neither useful nor pleasurable, it is of the causes, it is of the principles, it is of the necessary, it is of the general, it is of the true. It is only by demonstration.

But he also describes development.

In book II of the *Metaphysics*: to search for truth is in one way hard, in another way easy, as the fact shows that no one finds it entirely, but no one misses it entirely either, and while those who search for it find singly little or nothing, jointly they amass much (1); without Timotheus we would not have much of our lyric poetry, but there would be no Timotheus without Phrynis; and it is the same with those who have searched for truth, from those who have done it better we have the solutions of problems, while the others have made those who have done it better possible (*id.*).

In the *Sophistical refutations*: in all investigations labor either takes over from those who have gradually advanced the investigation, or makes the first discovery, and the first discovery usually advances the investigation only a little, even if it is more important than the discoveries that spring from it; for it may well be, as the saying is, that the beginning is the most important; it is also the most difficult; once the beginning is discovered, it is easier to develop the rest.

In *On philosophy*.

In the *Posterior analytics*.

In book I of the *Metaphysics*.

And in the reviews of his predecessors, in which the facts of the first two and a half centuries of development of philosophy are collected and arranged. Given these facts – Thales is the first philosopher, Anaximander is the second, Anaximenes is the third, and through the time of writing – a substantial part of the development described in book I of the *Metaphysics* in general is described also in particular.

As the saying is, the beginning is the most important. It is also the most difficult.

Development must be related to knowledge.

The difficulty is that knowledge cannot develop. Yet development must be related to knowl-

edge.

Whatever knowledge is, knowledge cannot develop and be knowledge. Knowledge must be the same in all space and all time. Must be the same in all philosophers. The same for all men, and for all Martians. The labors to acquire it must have finality. All results that would be knowledge must be, if they are to be knowledge, conclusive.

Yet after Einstein, as Einstein shows: Newton's theory is only an approximation to the theory of relativity (*Foundation*, 21). After Newton, as Cotes writes in the preface to the second edition of *The mathematical principles*: the whole of scholastic doctrine derived from Aristotle and the Peripatetics is a useless hodgepodge of words (*id.*). After Aristotle, as Aristotle judges him: Empedocles did not know what he was saying (*Metaphysics*, I, 4). There is the fact of development, and the fact of development has continued to suggest that knowledge develops. And if knowledge develops, the consequences are catastrophic. If the results of Einstein, Newton, Aristotle, and Empedocles would all be knowledge, surely none of them is. If Newton's theory is only an approximation, surely so is Einstein's theory. If Aristotle is a useless hodgepodge of words, so is Newton. If Empedocles did not know what he was saying, neither did Aristotle. It is what has continued to give plausibility to the proposition that man is the measure, and scepticism, and idealism, and the belief that knowledge is belief and belief is knowledge. If Empedocles, Aristotle, Newton, and Einstein each knew what he was saying, none of them knew what any of the others was saying. If Empedocles, Aristotle, Newton, and Einstein each did not know what he was saying, none of them knew what any of them was saying.

The difficulty is not formulated explicitly or resolved by Aristotle. His results, however, suffice for its explicit formulation and resolution.

Knowledge is neither useful nor pleasurable, of the causes, of the principles, of the necessary, of the general, of the true, by demonstration. And either this is what knowledge is, or this is not what knowledge is.

If this is what knowledge is, it is because the list is complete, what each of the entries in the list is is clear, and what the relations of all the entries in the list are is clear. If so, what knowledge is is established conclusively. The labors of acquiring knowledge of knowledge have finality.

If this is not what knowledge is, it is because the list is not complete, what each of the entries is is not clear, and what the relations of the entries are is not clear. Which is not hard to argue. The neither useful nor pleasurable can have useful and pleasurable applications, and if knowledge is that, it is also these. If knowledge is of the causes, it is of the effects, and if so, also of the facts and their collection and arrangement. If of the necessary, then of the certain, and if of the certain, then probably also of the probable. If of the general, then of the particulars, and hence also of generalization and particularization. If of the true, what is is the true, and then also of what is. Demonstration is deduction, deduction depends on induction, and if so, knowledge is also observation. If the list is not complete, what each of the entries in the list is, and what their relations are, cannot be clear either.

But even if the list is not complete, if only one of the entries belongs in the list, this is enough to show that what knowledge is is established, if only in part, conclusively. The labors of acquiring knowledge of knowledge have finality.

And even if none of the entries belongs in the list, the attempt to compile the list is enough to show that what knowledge is can be established, and if it can be established, established conclusively. The labors of acquiring knowledge of knowledge have finality.

And all the entries belong in the list. If this is not what knowledge is, it is because the list is not complete. But not the other way around. It may be that knowledge is not the neither useful nor pleasurable, of the causes, of the principles, of the necessary, of the general, of the true, by demonstration. But it may not be that the neither useful nor pleasurable, of the causes, of the principles, of the necessary, of the general, of the true, by demonstration is not knowledge.

Whatever knowledge is, knowledge cannot develop.

The difficulty is that development must nevertheless be related to knowledge. Development must be related to knowledge without compromising the fact that knowledge cannot develop.

And the resolution of the difficulty has to be that knowledge is acquired.

The search for truth such as those who search for it find singly little or nothing, but jointly they amass much, and from those who have done it better we have the solutions of problems, is a part of the acquisition of knowledge. The labor such as in all investigations it either takes over from those who have gradually advanced the investigation, or makes the first discovery, and the first discovery usually advances the investigation only a little, but other discoveries spring from it, and once the beginning is discovered, it is easier to develop the rest, is a part of the acquisition of knowledge. The development described in *On philosophy*, the *Posterior analytics*, and book I of the *Metaphysics*, the arts aiming at beauty, mathematics, politics, physics, and philosophy, is a part of the acquisition of knowledge. The facts collected and arranged in Aristotle's reviews of his predecessors, the first philosopher, the second, the third, and through the time of writing, are a part of the acquisition of knowledge.

The acquisition of knowledge is a fact, and the acquisition of knowledge has to be the resolution of the difficulty. That knowledge cannot develop, yet development must be related to knowledge.

Because, whatever knowledge is, there can be no access to knowledge but through its acquisition.

Because, whatever knowledge is, the fact of the acquisition of knowledge shows, and nothing else can show, the fact of knowledge.

And because, whatever knowledge is, the acquisition of knowledge is cumulative.

Einstein writes: no fairer destiny can befall a physical theory than to point the way toward a more comprehensive theory in which it lives as a limiting case (*Popular*, 22). Newton writes: if I have seen further, it is by standing on the shoulders of giants (to Hooke, 5 Feb 1676). Aristotle writes: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous. No fairer destiny can befall Newton than to point the way toward Einstein, no fairer destiny can befall Aristotle than to point the way toward Newton as he points the way toward Einstein, and no fairer destiny can befall Empedocles than to point the way toward Aristotle as he points the way toward Newton as he points the way toward Einstein. Newton stands on the shoulders of Aristotle, Aristotle stands on the shoulders of Empedocles, Einstein stands on the shoulders of Newton, and the more elaborate the arrangement of giants standing on one another's shoulders the more unshakable it is. It is the one and only council of our predecessors, which Aristotle calls into session, in which Empedocles is a principal councilor, which Aristotle joins, in which he becomes a principal councilor, which Newton joins, in which he becomes a principal councilor, which Einstein joins, and in which he becomes a principal councilor.

Knowledge cannot develop. But the acquisition of knowledge cannot not be a development.

And given the distinction of knowledge and the acquisition of knowledge, the problem before the investigation – the one problem of which those are all parts, knowledge, the acquisition of knowledge, the relation of its acquisition and knowledge, the beginning of the acquisition, its end, its entirety, its kinds, its species and its genus, the kinds of philosophy, philosophy, the development of philosophy, the relation of knowledge and philosophy, of the acquisition of knowledge and the development of philosophy, all other permutations of these, truth, the relation of knowledge and truth, the world, the relation of knowledge and the world, all other permutations of these, the search for truth and the investigation of the world, the acquisition of knowledge and all relations of these, the relation of knowledge and man, and of knowledge to man and of man to knowledge – the one problem of which all these are the parts can be formulated. Or what is the same, what all the parts of the problem are and what the problem of which all these are the parts can be made explicit.

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And first of all, the two parts of the problem, knowledge and its acquisition, must be separated sharply from one another. If they are each a problem, knowledge is not a problem of development. The acquisition of knowledge is a problem of development. And there can be no access to

knowledge but through its acquisition. The problem of knowledge, if it can be formulated and solved, cannot be formulated and solved if the problem of the acquisition of knowledge is not formulated and solved first. The problem of the acquisition of knowledge, if it can be formulated and solved, can be formulated and solved even if the problem of knowledge is not formulated and solved.

Moreover, whatever else gives scepticism, idealism, and worse plausibility, most of all the confusion, and the obfuscation, of knowledge and its acquisition gives it plausibility.

A development, the acquisition of knowledge cannot not have a beginning. And the acquisition of knowledge attested on earth has a beginning.

Of knowledge, such as it cannot develop, its acquisition cannot not have finality either. In one particular case, and another, and another. And the acquisition of knowledge attested on earth has finality. Hence, if the acquisition of knowledge has both a beginning and finality, and the acquisition of knowledge attested on earth has both a beginning and finality, the acquisition of knowledge attested on earth can be collected in its entirety, from its beginning through a certain time. Conversely, if the acquisition of knowledge attested on earth can be collected in its entirety, it is because it has a beginning and finality.

If the acquisition of knowledge has not been produced by selection, and cannot be produced by selection, a principle distinct from selection has produced it. If so, the problem of the beginning of the acquisition, and the problem of collecting the acquisition in its entirety, will be solved only if the solutions stand, at least implicitly, on the theory of evolution, and only if they point, at least implicitly, to that principle. And if they stand on evolution and point to that principle, at least implicitly, then whether the acquisition of knowledge attested on earth is the acquisition of knowledge or the acquisition of knowledge attested on earth is only a part of the acquisition of knowledge, the part attested on earth, the solutions of these problems, of the beginning of the acquisition and of collecting the acquisition in its entirety, and hence the solution of the problem of the acquisition of knowledge, will have, in place of parochiality, generality.

The problems of the beginning of the acquisition, and of collecting the acquisition in its entirety, are the two decisive parts of the problem of the acquisition of knowledge.

Beginning with the first philosopher, knowledge has been acquired by the labors of the philosophers. If it has been acquired by the labors of the philosophers, exclusively, then beginning with the first philosopher it is clear what the relation of the acquisition of knowledge and the development of philosophy is. Beginning with the first philosopher, the development of philosophy includes the acquisition of knowledge.

The acquisition of knowledge predates the first philosopher. If beginning with the first philosopher knowledge has been acquired by the labors of the philosophers, exclusively, however, before the first philosopher it has been acquired by the labors of these predecessors of the philosophers of whom the philosophers are the successors, exclusively. If beginning with the first philosopher it has been acquired by the cumulative labors of the philosophers, before the first philosopher it has been acquired by the cumulative labors of these predecessors of the philosophers of whom the philosophers are the successors. If the fact that it has been acquired by their cumulative labors can identify the philosophers, and these among them who have acquired it, it can identify these predecessors of the philosophers of whom the philosophers are the successors, and these among them who have acquired it. And all the way back to the beginning of the acquisition of knowledge.

The problem of the relation of knowledge and man is thus solved. It is clear what the relation of man to knowledge and of knowledge to man is. None whatsoever. If the distinction between the labors of the philosophers and men is at all possible, there can be no sound reason to contaminate the labors of the philosophers with men. If before the first philosopher the labors of these predecessors of the philosophers of whom the philosophers are the successors and beginning with the first philosopher the labors of the philosophers include the acquisition of knowledge, exclusively, men can contribute nothing to the acquisition of knowledge but confusion. And if this is the relation of men to the acquisition of

knowledge, the relation of man to knowledge and of knowledge to man can only be none whatsoever.

Nor need this proposition be qualified because, as they will use, or try to use, anything, men will use, or try to use, the outcome of the acquisition of knowledge, any more than it needs to be qualified because men will use, or try to use, anything. Because organisms, by being organisms, and men, in particular, are opportunistic.

The problem of the relation of knowledge and philosophy is also solved. If philosophy is in relation to knowledge, it is only because the development of philosophy is in relation to the acquisition of knowledge.

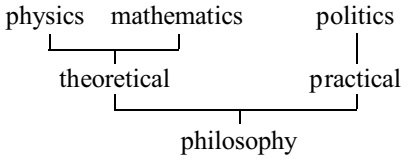
The problem of the relation of the acquisition of knowledge and the development of philosophy is – next to the problems of the beginning of the acquisition and of collecting the acquisition in its entirety – the third part of the problem of the acquisition of knowledge. The difficulty of this problem is indicated sufficiently by the fact that the acquisition of knowledge predates the first philosopher by millennia, yet, beginning with the first philosopher, the development of philosophy includes the acquisition of knowledge.

As to philosophy, philosophy is one or the other. Philosophy is nothing but the development of philosophy. Or there is more to philosophy than the development of philosophy.

The development of philosophy is the labors of the philosophers, from the labors of the first philosopher through the labors of the last philosopher. And if philosophy is nothing but the development of philosophy, the relation of philosophy to knowledge is the relation of the development of philosophy to knowledge, and the relation of the development of philosophy to knowledge can only be the relation of the development of philosophy to the acquisition of knowledge.

If there is more to philosophy than the development of philosophy, whatever that be, it can be in relation neither to knowledge nor to the acquisition of knowledge.

With Aristotle, the kinds of philosophy are,



and if the kinds of philosophy are the kinds of knowledge, they can only be because there can be no access to knowledge but through its acquisition, and the kinds of philosophy are the kinds of the acquisition of knowledge.

If the acquisition of knowledge is a development, it cannot not have kinds. And that the acquisition of knowledge has kinds, and the kinds exhibit development, the fact of physics, mathematics, and politics, and the fact that mathematics predates politics, and both predate physics, suffice to show. If knowledge cannot develop, however, knowledge cannot have kinds. Nothing that cannot develop, nothing that must be the same in all space and all time, can have kinds. Knowledge can only have kinds in relation to its acquisition, and hence have kinds no other than the kinds of its acquisition.

The problem of the relations of the kinds of philosophy and the kinds of the acquisition of knowledge is a part of the problems of the relation of the acquisition of knowledge and the development of philosophy and of the kinds of the acquisition of knowledge.

The three Aristotelean kinds, physics, mathematics, and politics, also show that the problem is one of development. Physics, mathematics, and politics have each changed. The three kinds show as well, however, that across all change they have remained essentially and fundamentally the same. Physics, essentially and fundamentally the same, can be identified unambiguously millennia before Aristotle, in Aristotle, and millennia after Aristotle. Mathematics, essentially and fundamentally the same, can be identified unambiguously millennia before Aristotle, in Aristotle, and millennia after Aristotle. Politics, essentially and fundamentally the same, can be identified unambiguously millennia before Aristotle, in Aristotle, and millennia after Aristotle.

The problem of the kinds of the acquisition of knowledge is not which they are. Which they are is clear. The problem is of their relations. These are not clear, and it is these alone, too, if which the kinds are needs to be made clearer, that can make this clearer. The problem is not to describe the kinds, but to describe their system, and describe it genealogically. To describe how each kind has first developed, how the kinds have each

developed from one another, and how the kinds have all continued to develop in relation to one another. And the difficulty of this problem is indicated by the fact that, if philosophy includes physics and mathematics, and the first philosopher is the beginning of philosophy, physics and mathematics yet predate the first philosopher by millennia.

If the development of philosophy includes the acquisition of knowledge, the kinds of the acquisition have to be kinds of philosophy, but not the other way around, the kinds of philosophy do not have to be kinds of the acquisition. Which explains why Aristotle's solution of the problem of the kinds, proposed as the solution of the problem of the kinds of philosophy, is, with the hindsight of twenty-three and a half centuries, a solution rather of the problem of the kinds of the acquisition. Preceding the acquisition of knowledge in these twenty-three and a half centuries, Aristotle precedes also all the contamination of philosophy in these twenty-three and a half centuries, and presumes – much too optimistically, it has to be judged with the hindsight of these twenty-three and a half centuries – that the main concern of philosophy is the acquisition of knowledge. One way or another, it is an excellent solution of the problem of the kinds of the acquisition, and the one to try and improve on. If it has to be improved on.

The parts of the problem of the acquisition of knowledge, then, are the beginning of the acquisition, collecting the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition.

If the problem of the acquisition of knowledge can be solved, the problem of knowledge can be solved.

Knowledge cannot be knowledge and develop. It must be the same in all space and all time. Nor can it have kinds.

In the same way, truth cannot be truth and develop. Truth must be the same in all space and all time. Nor can truth have kinds. And if development must nevertheless be related to truth, the search for truth cannot not be a development, and a development, it cannot not have kinds.

The world has developed, and through the 1.4×10^{10} years of development described by the big bang theory. With Aristotle, however: the world is one, there is nothing next to the world, or outside the world. The development of the world must be one, there must be nothing next to the development of the world, or outside the development of the world. Nor can this development itself develop. Nor can it have kinds. And if development must nevertheless be related to the development of the world, the investigation of the world has to be this development.

The search for truth either is a part of the acquisition of knowledge or coincides with the acquisition of knowledge. The investigation of the world either is a part of the acquisition of knowledge or coincides with the acquisition of knowledge. For, as there can be no access to knowledge but through its acquisition, so there can be no access to truth but through the acquisition of knowledge, and there can be no access to the world but through the acquisition of knowledge. Hence, also, as the fact of the acquisition of knowledge shows, and nothing else can show, the fact of knowledge, it alone shows, and can show, the fact of truth, and it alone shows, and can show, the fact that the world is the world.

Knowledge and its acquisition must be separated sharply from one another. And if they are a problem each, the relation of knowledge and truth, and the relation of knowledge and the world, are the parts of the problem of knowledge. The remaining problems, with all their permutations, are the parts of the problem of the acquisition of knowledge. And not of the problem of knowledge.

The problem before the investigation is thus of two main parts, the problem of the acquisition of knowledge and the problem of knowledge. The problem of the acquisition of knowledge is itself of four parts, the problem of the beginning of the acquisition, the problem of collecting the acquisition in its entirety, the problem of the relation of the acquisition and the development of philosophy, and the problem of the kinds of the acquisition. The problem of knowledge is of two parts, the relation of knowledge and truth, and the relation of knowledge and the world.

The problem of knowledge can be formulated and solved only if the problem of the acquisition of knowledge is formulated and solved first.

And if the labors of the predecessors of the philosophers of whom they are the successors and the philosophers, exclusively, include the acquisition of knowledge, the problem of the acquisition of knowledge cannot be impossible to formulate and solve.

lxi

To illustrate.

All of the above – that knowledge cannot develop, that development must be related to knowledge without compromising the fact that knowledge cannot develop, that the acquisition of knowledge is this development because the fact of its acquisition shows the fact of knowledge, because there can be no access to knowledge but through its acquisition, and because its acquisition is cumulative, that the problem of the acquisition of knowledge includes the problems of its beginning, of collecting it in its entirety, of its relation to the development of philosophy, and of its kinds, the problem of knowledge includes the problems of its relation to truth and of its relation to the world, that the problem of knowledge cannot be formulated or solved before the problem of its acquisition is formulated and solved, and that, if the labors of the predecessors of the philosophers and the philosophers, exclusively, include the acquisition, the problem of the acquisition of knowledge can be formulated and solved – to illustrate all this, the six instances of the magnitude of Aristotle's success in the acquisition of knowledge, the solution of the problem of the principles of all things, the description of 600 kinds of animals, the description of the world, the investigation of the principle of excluded middle, the investigation of terms, propositions, and inferences, and the investigation of truth, can clearly and distinctly.

And so can the following five instances even more clearly and distinctly, as they have more drama, being more recent, and showing even more unambiguously a development each from no inkling to finality.

The fourth part of the earth.

Columbus crossed the Atlantic in 1492. He reported that he has reached Asia, Cathay in particular (report, 15 Feb 1493), and continued to argue that he has reached Asia after his fourth voyage still (to Isabella and Ferdinand V, 7 Jul 1503). Columbus had opened the Atlantic, however, and exploration across it proceeded apace.

Vespucci sailed along the coast in the south as far as La Plata in 1501-2, and he first proposed that the lands across the Atlantic are not Asia. He writes in his report: in justice these lands have to be called a new world, because none of them was known to our ancestors; I have found a new continent in these southern parts, more populous and more full of animals than our Europe, Asia, or Africa (to Lorenzo il Popolano, Mar 1503).

Waldseemüller drew the first map on which these lands are a continent of their own in 1507, the new continent named America. In the *Introduction to cosmography* accompanying the map Waldseemüller explains: the earth known to Ptolemy has been divided into three parts, Europe, Asia, and Africa; these parts have been explored more extensively now and a fourth part has been discovered by Amerigo Vespucci; I see no reason why anyone should object to calling it America, after Amerigo, its discoverer (9).

Columbus discovered America, in 1492. Vespucci established this fact, and Waldseemüller established it conclusively, in 1507.

That Columbus discovered America has been contested. If America, however, is, as Vespucci describes it: a continent, or, as Waldseemüller describes it: a fourth part of the earth, a continent, or a fourth part of the earth, is discovered when it is put on the map, not when it is sighted, or when it is populated. And if so, America was discovered in 1492, not when it was sighted five centuries earlier by Leif Eriksson, or when it was populated millennia earlier by men, or millennia earlier still by any of the other animals, more full of which than our Europe, Asia, or Africa Vespucci found it.

The discovery of the fourth part of the earth is described in its entirety, no inkling to finality, by the labors of Columbus, Vespucci, and Waldseemüller.

The discovery depends on earlier results. Of these the most important, by far, is Ptolemy's *Geography*, fourteen centuries earlier, which guided Columbus across the Atlantic and Waldseemüller in the division of the earth into parts. Later results depend on the discovery. Of these

the most important are the circumnavigation of the earth by Magellan and El Cano, in 1519-22, and the map which de Toreno drew after the circumnavigation showing America, Asia, and the Pacific between them in their true proportions, and thus for the first time the entire surface of the earth in its true proportions. The discovery depends also on other results obtained in the years between 1492 and 1507. Of these the most important, if not nearly as important as any of those already noted, are the voyage of Cabot to Newfoundland in 1497, the voyage of Cabral which reached Brazil in 1500, and the map of de la Cosa in 1500 which first showed the lands across the Atlantic. The discovery depends in no way whatsoever on any of those who populated America, Europe, Asia, or Africa earlier, or those who populated them at that time and began, disgracefully, to harvest one another more eagerly than they had until that time harvested one another, or those who populated them later and continued, ever more disgracefully, ever more eagerly to harvest one another.

The satellites of Jupiter.

It took fifteen years to establish conclusively that Europe, Asia, and Africa are not all the parts of the earth. It took Galileo three days to establish conclusively that the sun, Mercury, Venus, the earth, the moon, Mars, Jupiter, and Saturn are not all the wanderers in the heaven.

Galileo announced the discovery of Io, Europa, Ganymede, and Callisto – of the Medicean stars, as he named them – in the *Starry messenger*, in 1610. Galileo reports: on 7 January I observed three little stars close to Jupiter, and although I thought that they were fixed stars, they aroused my curiosity because they were aligned in a straight line along the ecliptic; on 8 January I found them still in a line, but in a different arrangement, and I wondered whether the motion of Jupiter may not be direct, rather than retrograde as it had been calculated; on 9 January I waited eagerly to observe them but the sky was covered with clouds; on 10 January I found them still in a line, but in a yet different arrangement, and moving from doubt to astonishment, I decided that not Jupiter, but they move (*id.*). Galileo continued to observe the three satellites, and on 15 January discovered the fourth.

Contention involved in this case the discovery of the telescope. And Galileo conceded it, to the degree to which the instrument had been built before him (*Assayer*, 13). He should not have. For the instrument had been built, but what the instrument is for remained to be discovered. Which Galileo did, as his full title abundantly attests: *Starry messenger, unfolding great and most wonderful sights and displaying for the inspection of everyone, and especially of the philosophers and astronomers, what with a telescope built by himself Galileo Galilei observed about the face of the moon, the fixed stars, the Milky Way, the nebulous stars, and first of all about four planets circling Jupiter's star at unequal intervals and periods and with marvelous speed, which, unknown to anyone until now, he discovered and decided to name the Medicean stars*.

The discovery of the four major satellites of Jupiter is described in its entirety by the labors of Galileo.

The discovery depends on earlier results, and later results depend on it. It depends in no way whatsoever on any of those who populated Europe, Asia, Africa, or America – or the satellites of Jupiter – earlier, or at that time, or later, including those who built telescopes and left them to Galileo to discover what they are for.

The irrationality of π.

Galileo's discovery depends on observation, observation depends on the eye, and, for all the carelessness of the optician who would sell that defective instrument, this has continued to suggest that observation is seeing, and seeing is what anyone can do. Whether observation is seeing, the fact suffices to show that Galileo's discovery is impossible without the calculation of the motion of Jupiter. Columbus's discovery depends on navigation, the exploration of the sea and of the land has been accompanied by conquest, conquest has been accompanied by harvesting, harvesting – so that one is the harvester, and not the harvested – is what anyone will do, and this has continued to suggest that exploration, or conquest for that matter, is what anyone can do. Whether exploration, or conquest, is what anyone can do, the fact suffices to show that Columbus's discovery is impossible without the calculation of

the latitudes and longitudes of Europe, Africa, the Atlantic, America, the Pacific, and Asia. π does not depend on the eye, or navigation, or conquest, and has not been contaminated so much by the supposition that it is what anyone can do. It has been by the supposition that it is in the mind.

Lambert proved that π is irrational, in the *Preliminary advice for those who seek to square the circle*, in 1766. He begins: I doubt that this treatise will be read, or that it will be understood if it is read, by those who should be most interested in it, those who spend time and effort seeking to square the circle (1). And ends: to square the circle is absolutely impossible, nor is it difficult to find the cause (18).

The cause is not that π is irrational, that it is not the ratio of integers. The cause is that π is transcendental, that it is not the root of any polynomial equation with rational coefficients, as Lambert proposes (2), but does not prove, in the *Memoir on some remarkable properties of circular and logarithmic transcendental quantities*, in 1768. This was proved one century-odd after Lambert.

Euler, in 1748, in the *Introduction to the analysis of infinities*, showed that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$, and that, in general, any series $1 + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + \ldots$, where n is an even integer, can be expressed in terms of π (I, 10).

Leibniz, in 1682, in *On the true proportion of the circle to the circumscribed square, expressed in rational numbers*, showed that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ (*id.*).

Wallis, in 1656, in the *Arithmetic of infinities*, showed that $\frac{\pi}{4} = \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \ldots$ (191).

Viète, in 1593, in the *Responses to various mathematical matters*, showed that $\frac{2}{\pi} = \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \times \ldots$ (18).

Viète showed that an infinite series has to express π. Wallis showed it more elegantly. Leibniz showed it very elegantly. Euler showed it very generally. Lambert proved the irrationality of π. And the discovery of the irrationality of π is described in its entirety by the labors of Viète, Wallis, Leibniz, Euler, and Lambert.

Among the earlier results on which the discovery depends the most important are the following.

Archimedes, eighteen-odd centuries before Viète, in the *Measurement of the circle*, showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$ (3).

Euclid, two generations earlier, in the *Elements*, proved that the circumference of the circle is proportional to the square on the diameter (XII, 2), and thus that the square root of the circumference is proportional to the diameter.

Eudoxus, two generations earlier, developed the theory of proportion (scholia *Elements*, V; Proclus, comm. *Elements*, 68), extending operations with the rationals to the irrationals.

Anaxagoras, not quite one century earlier, is the first philosopher to try to square the circle – while in prison, Plutarch adds (*Exile*, 607f).

Pythagoras, two-odd generations earlier, discovered the incommensurability of the side and diagonal of the square (Iamblichus, *Pythagoras*, 34; Proclus, comm. *Elements*, 65), thus the irrationality of $\sqrt{2}$, and thus the irrationals.

The mathematician who left behind the oldest calculation of the area of the circle in Mesopotamia, recorded on the tablet YBC 7302, twelve centuries earlier, calculated the area of a circle with a circumference 3 at $\frac{3}{4}$ (*id.*), implying $\pi = 3$.

And the mathematician who left behind the book of 87 problems, in Egypt, one century earlier, calculated the area of a circle with a diameter 9 at 64 (50), implying $\pi = 3\frac{13}{81}$.

All in all, thirty-six centuries separate the book of 87 problems and Lambert. Of all the twelve results, conclusive are nine, those of Pythagoras, Eudoxus, Euclid, Archimedes, Viète, Wallis, Leibniz, Euler, and Lambert.

Nor is the result that π is irrational accessible to anything but proof. Nor is the result already that the side and diagonal of the square are incommensurable accessible to anything but proof. Nor are the results that $\pi = 3$ and $\pi = 3\frac{13}{81}$ already accessible to anything but calculation.

The first language committed to writing.

Rawlinson began the decipherment of cuneiform, in *The Persian cuneiform inscription at Behistun*, in 1846-51. By 1857 the decipherment was complete, accomplished by Rawlinson, Hincks, and Oppert. The next challenge was to disentangle the languages written in cuneiform from one another, and by 1875 Hincks and Oppert had disentangled the oldest. In 1875, in *Sumerian or Akkadian?* Oppert reports: Hincks discovered that, unlike the later of the languages written in cuneiform, the earliest is not Semitic, and proposed to call it Akkadian, from the title, assumed by the early princes in their inscriptions, prince of Sumer and Akkad (1). Oppert, in *Sumerian or Akkadian?* established the name Sumerian for the earliest not Semitic language. Akkadian became the name of the earliest of the later Semitic languages.

The decipherment of cuneiform is described in its entirety by the labors of Rawlinson, Hincks, and Oppert. The discovery that Sumerian is the first language committed to writing is described in its entirety by the labors of Hincks and Oppert.

America was discovered when its spatial relations to the other parts of the earth were determined. The first language committed to writing was discovered when its temporal relations to the other languages committed to writing were determined.

By 1824 Young and Champollion had deciphered hieroglyphic. In the century after Rawlinson, Hincks, and Oppert all the languages written in cuneiform were disentangled from one another.

Thus, Sumerian was committed to writing five and a half millennia ago. In the following millennium and a half cuneiform was borrowed by Akkadian and Elamite. In the following millennium in cuneiform were written Babylonian and Assyrian, both developments of Akkadian, as well as Hurrian, Kassite, Hittite, Luwian, and Ugaritic, and in the following millennium Urartian and Persian, not counting dialects, like Eblaite, and languages attested by other languages, like Hattic.

And Sumerian was committed to writing before Egyptian. The oldest cuneiform inscriptions attesting a coherent system of notation predate the oldest hieroglyphic inscriptions attesting a coherent system of notation by not less than two to three centuries.

The discovery that Sumerian is the first language committed to writing is a discovery as strictly as the discovery of the fourth part of the earth, the discovery of the four major satellites of Jupiter, and the discovery that π is irrational. It also depends on earlier results, and later results also depend on it. And it also depends in no way whatsoever on any of those who populated Europe, Asia, Africa, or America at that time, or later, or earlier, including those who spoke the first language committed to writing, including those who committed it to writing.

The solution of the field equations of gravitation.

Einstein formulated the special theory of relativity in 1905. He then developed the general theory of relativity. In *The speed of light and the statics of the gravitational field* and *On the theory of the static gravitational field*, in 1912, *On the present state of the problem of gravitation* and *Outline of a generalized theory of relativity and of a theory of gravitation*, with Grossmann, in 1913, *The formal foundation of the general theory of relativity*, in 1914, *On the general theory of relativity* and *The field equations of gravitation*, in 1915, and conclusively in *The foundation of the general theory of relativity* and *Hamilton's principle and the general theory of relativity*, in 1916.

In 1917, in the *Cosmological considerations on the general theory of relativity*, he solved the field equations formulated in *The foundation*. To do so, he introduced: the constant λ (4); necessary to make possible a quasi-static distribution of matter, as required by the fact of the small velocities of stars (5).

In 1922, in *On the curvature of space*, Friedmann solved the equations without the constant. He writes: Einstein obtains a world whose curvature is constant with respect to space and independent of time (1.1); the goal of the present notice is to prove the possibility of a world whose curvature is constant with respect to space and dependent on time (*id.*). Friedmann proves the possibility, and concludes: since the radius of curvature cannot be smaller than zero, it must decrease with the decrease of time to zero; the

time of increase of the radius from zero we call the time from the creation of the world (2.4). He adds: superfluous for the solution of the problem, the magnitude λ is not determined in our formulas (2.7); postulating $\lambda = 0$, the period of the world is of the order of 10 milliard years (*id.*).

In 1929, in *A relation between distance and radial velocity among extragalactic nebulae*, Hubble established the fact of the large velocities of galaxies, and hence the fact that the world is of a curvature constant with respect to space and dependent on time.

Einstein called λ, Gamow reports: the biggest blunder in his life (*Line*, 2). Gamow calls it: an algebraic mistake (*Creation*, 2).

The big bang theory stands on Friedmann's solution of the field equations. Friedmann's is a solution of the equations formulated by Einstein. And Einstein's biggest blunder, or algebraic mistake, is, being Einstein's, the first solution of the field equations of gravitation.

Among the later results which depend on the solution of the field equations of gravitation the theory of the origin and development of the world is only the most dramatic. Among the earlier results on which the solution depends is $\pi = 3$.

lxi

Whatever knowledge is, if mapping the earth, inventorying the heaven, sorting out the irrationals, reaching across five thousand years to determine the sequence of commitment of languages to writing, and across ten milliard years to determine the curvature of the world, if these do not show the fact of knowledge, nothing does.

All five are conclusive results. America is the fourth part of the earth. Io, Europa, Ganymede, and Callisto are the four major satellites of Jupiter. π is irrational. Sumerian is the first language committed to writing on earth. The curvature of the world is constant with respect to space and dependent on time. And they can be conclusive results only if, whatever knowledge is, it cannot develop.

Nor, whatever knowledge is, can there be access to it but through navigation, observation, proof, decipherment, solution of equations.

Cumulatively. Each of the five is an instance of cumulative development, whether the development is across three days or across thirty-six centuries. And to show that all five are parts of one and the same cumulative development the fact that they all rest on the labors of Ptolemy suffices. Ptolemy developed trigonometry: the calculation of chords from angles (I, 10-1), in the *Almagest*, without which the reduction of the observations of the heavenly bodies to order is impossible. And from the observations of the heavenly bodies he determined latitudes and longitudes across 180° of longitude, in the *Geography*, fixed chronology absolutely from Nabonassar, prince $748\text{-}34^{BC}$, through Pius, prince $138\text{-}61^{AD}$, in the *Handy tables*, and measured the distances of the planets, in the *Almagest* and *Planetary hypotheses*. The *Geography* guided Columbus and Waldseemüller. A decade before Kepler's discovery of the laws of the motions of the planets, Galileo's calculation of the motion of Jupiter depended on the methods developed in the *Almagest* – as, indeed, did Kepler's investigations which led to the discovery of the laws of the motions of the planets. Lambert's result is trigonometric, in order to prove that π is irrational, he proves that: the tangent of every rational arc is irrational, and the arc of every rational tangent is irrational (*Advice*, 15). The determination of the sequence of commitment of languages to writing is impossible without absolute chronology. And the measurement of the distances of galaxies, and hence of their velocities, depends on the measurement of the distances of stars, which depends on the measurement of the distances of the planets.

In turn, the labors of Ptolemy rest on the labors of Aristotle not only in that his description of the world is an improvement on Aristotle's description of the world. Or in that Aristotle's description as well rests on observation of the heavenly bodies. Aristotle writes, for instance: observation shows that the moon is closer to the earth than the other planets; for we have seen it pass under Mars (*Heaven*, II, 12). Ptolemy's labors rest on Aristotle's in that the measurement of the distances of the planets, too, on which the measurement of the distances of stars depends, on which the measurement of the distances of galaxies depends, in turn depends on the measurement of the circumference of the earth, and the oldest calculation of the circumference of the earth is in

On the heaven. Aristotle reports: the mathematicians calculate the circumference of the earth at 400 000 stades (14). Stades varied from about .2 km to about .15 km. The result of the calculation, presumably by Eudoxus, is thus excellent whatever the stades. The equatorial radius of the earth is 6 378 km, and whether Eudoxus calculated the radius of the earth at about 12 700 km or at about 9 500 km, with this first calculation the problem of the order of magnitude of the radius of the earth was solved.

In turn, the labors of Aristotle rest on the labors of the predecessors of the philosophers. He writes: we have seen the moon pass under Mars; and there are similar reports about the other planets from the Egyptians and the Babylonians whose observations have been kept for many years and from whom much of our evidence is derived (12).

Each of the five instances a conclusive result. Each thus instances a development to finality. And each instances a development to finality from no inkling. No inkling that there is a fourth part of the earth. No inkling that the wanderers in the heaven include the satellites of Jupiter. No inkling that π is irrational. No inkling what the sequence of commitment of languages to writing is. No inkling what the curvature of the world is.

Earlier, no inkling that there are parts of the earth, or that there are wanderers in the heaven, or that there is the circle, or that there is the sequence of anything, or that there is the world. The one cumulative development of which those are parts cannot but be a development from no inkling whatsoever.

If development must be related to knowledge without compromising the fact that knowledge cannot develop, and there can be no access to knowledge but through its acquisition, the acquisition of knowledge cannot not be a part of this development. If the acquisition of knowledge is one cumulative development, and one cumulative development from no inkling to finality, it cannot not be this development.

A problem of a development from no inkling, the problem of the acquisition of knowledge includes the problem of its beginning.

A problem of one cumulative development, it includes the problem of collecting it in its entirety.

A problem of development, which includes the labors of both predecessors of the philosophers and philosophers, and among both the labors of the predecessors of the philosophers and the labors of the philosophers the labors of physicists, and of mathematicians, and of historians, it includes the problem of its relation to the development of philosophy. Nor can the development of philosophy include the labors of Pythagoras, Anaxagoras, and Aristotle, and exclude the labors of Eudoxus, Euclid, Archimedes, and Ptolemy. Nor can it include the labors of Pythagoras, Anaxagoras, Eudoxus, Aristotle, Euclid, Archimedes, and Ptolemy, and exclude the labors of Columbus, Vespucci, and Waldseemüller, or the labors of Viète, Wallis, Euler, and Lambert, or the labors of Rawlinson, Hincks, and Oppert, or the labors of Einstein, and Friedmann. Never mind excluding the labors of Galileo and Leibniz.

A problem of a development, which includes the labors of the mathematician who wrote the book of 87 problems, the mathematician whose labors are recorded on the tablet YBC 7302, Pythagoras, Anaxagoras, Eudoxus, Aristotle, Euclid, Archimedes, Ptolemy, Columbus, Vespucci, Waldseemüller, Viète, Galileo, Wallis, Leibniz, Euler, Lambert, Rawlinson, Hincks, Oppert, Einstein, Friedmann, it includes the problem of its kinds.

A problem of finality, the problem of knowledge includes the problem of its relation to truth.

And the problem of its relation to the world.

The problem of knowledge cannot be solved before the problem of its acquisition is solved.

And if the discovery of America can be collected in its entirety, beginning to end, and the discovery of the satellites of Jupiter can be collected in its entirety, beginning to end, and the discovery that π is irrational can be collected in its entirety, beginning to end, and the discovery that Sumerian is the first language committed to writing can be collected in its entirety, beginning to end, and the discovery that the world is of a curvature constant in space and dependent on time can be collected in its entirety, beginning to end, it is all because the labors of the predecessors of the philosophers of whom the philosophers are the successors and the philosophers,

exclusively, include these results. If the labors of the predecessors of the philosophers of whom the philosophers are the successors and the philosophers, exclusively, include these results, they, exclusively, include the acquisition of knowledge. And if they, exclusively, include the acquisition of knowledge, the problem of the acquisition of knowledge, with all its four parts, the beginning of the acquisition, collecting the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition, can be formulated explicitly and the formulated problem solved explicitly.

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To the end of formulating the problem before the investigation explicitly, the review of the problems solved in the most successful investigations of knowledge, Aristotle's, Hume's, and Kant's, and of how these problems are informed by Ptolemy, Darwin, and Frege, should turn, next, to Ptolemy.

Again, Ptolemy's is the greatest success in the acquisition of knowledge in the forty-seven centuries from its beginning to Bacon, Galileo, and Kepler.

Ptolemy's success, and how it informs the investigation of knowledge, can be described in three steps. The most important problem he solved. How he solved it. And how he informs the problem before the investigation of knowledge.

Ptolemy's very important treatises are these,
Almagest, 13 books,
Planetary hypotheses, 2 books,
Geography, 8 books,
Handy tables, 1 book,
Optics, 5 books.

They comprise somewhat more than two thirds of the bulk of the Ptolemaic corpus.

The Ptolemaic corpus is an excellent example of how the Christian philosophers and the Muslim philosophers prevented one another from losing what is important, in particular, and of the futility of the effort to lose what is important, in general. Ptolemy wrote in the second Christian century. The *Almagest*, the *Geography*, and the first half of book I of the *Planetary hypotheses* are preserved in the original Greek. The *Handy tables* is preserved also in Greek, in Theon of Alexandria's commentary, in the fourth century, on the *Handy tables*. The second half of book I and book II of the *Planetary hypotheses* are preserved in Thabit ibn Qurra's Arabic translation, in the ninth century, from the Greek. Missing book I and the end of book V, the *Optics* is preserved in Eugenius of Sicily's Latin translation, in the twelfth century, from an Arabic translation from the Greek.

The Ptolemaic corpus includes also the following.

The important, but not very important, *Astrological influences*, in 4 books, and *Harmonics*, in 3 books. Concerned with astrology and music as they are, both are also preserved in the original Greek. The important *Analemma*, in 1 book, is concerned with the projection of angles onto the plane of the meridian, employed in the construction of sundials. It is preserved in William of Moerbeke's Latin translation, in the thirteenth century. The *Planisphere*, in 1 book, is concerned with the projection of the celestial sphere onto the plane of the equator, employed in the construction of the astrolabe. It is preserved in Arabic and Latin translations. The *Phases of the fixed stars*, in 2 books, is concerned with the heliacal risings of bright stars. Book II is preserved in the Greek. And like all very important corpora, the Ptolemaic, too, has been amplified by spurious treatises, two in this case, *Fruit*, the fruit being 100 astrological aphorisms, and *On judgment and command*, the judgment and command being of vaguely Stoic flavor.

Proclus also, in the fifth century, preserves Ptolemy's attempt to prove Euclid's parallel postulate (comm. *Elements*, 362-8).

It is not an accident that Theon of Alexandria, Proclus, Thabit ibn Qurra, and William of Moerbeke are important philosophers.

The most important of Ptolemy's treatises is *The mathematical collection*, as it was descriptively named at first, or *The greatest*, or *Almagest*, as it was just as descriptively renamed by Ptolemy's successors. *The greatest* remained the greatest until Newton's *Mathematical principles of natural philosophy*. It is a treatise of greater scope than Copernicus's *On the revolutions of the heavenly spheres*, of greater scope and unity than

Galileo's *Two new sciences* or Kepler's *New astronomy*, and of greater formality than Descartes's *Principles of philosophy*.

In books I and II Ptolemy lays the grounds on which the problems solved in the following books are solved. In book I he begins by describing the method (1), establishing the order in which the investigation should proceed (2), and describing the world in its entirety (3-8). Next, he develops the trigonometry which allows the solution of all plane triangles (9-11), and the spherical geometry which allows the solution of all spherical triangles (13-6) – and Ptolemy has thus made it possible to make accurate observations and to treat these observations mathematically, to add them and subtract them, and calculate their ratios. Next, having determined the inclination of the ecliptic in book I (12), in book II he determines the dependence of the measurement of the declination of the sun on latitude (1-4), hence its declination for 39 different latitudes (5-6), and hence the rising times of arcs of the ecliptic for these latitudes (7-13) – and Ptolemy has thus made it possible to compare observations made in different places with one another.

Hence, in book III he develops the theory of the motions of the sun. In books IV and V he develops the theory of the motions of the moon. In book VI he develops the theory of lunar and solar eclipses. In books VII and VIII he catalogues 1 022 stars. And in books IX through XIII he develops the theory of the motions of the planets, Mercury, Venus, Mars, Jupiter, and Saturn.

Four and a half-odd centuries before Ptolemy, Aristotle first describes the world, and Ptolemy's description of the world in book I is not substantially different from Aristotle's. Ptolemy focuses, however, on these features of the world which have to be described unambiguously, if they are to become the foundation of the mathematical description of the world. And he presents more explicitly the observations on whose ground these features have been determined.

This is how Ptolemy describes the world in book I. First: the motion of the heavenly bodies is spherical (3); the ancients already saw that the heavenly bodies are carried from east to west in circles parallel to one another and always about the same center which thus became necessary to presume the pole of the heavenly sphere (*id.*); moreover, if the heavenly bodies had any other kind of motion but circular, in a straight line, for instance, their distances from the earth would have appeared to vary (*id.*); and no such variation is observed (*id.*). Next: the earth is spherical (4); this follows from the proportionality of the times at which one and the same eclipse is observed in different places and the distances between these places (*id*); moreover, mountains are observed to rise from the sea, when approached from the sea, because of the curvature of the surface of the water (*id.*). Next: the earth is in the center of the heavenly sphere (5); if it were equidistant from the poles of the sphere but not on its axis, the sizes and distances of the stars would have appeared to vary (*id.*); if it were on the axis but not equidistant from the poles, the horizon would not have appeared to bisect the ecliptic (*id.*); and if it were neither on the axis nor equidistant from the poles, at the equinoxes the shadow of the sundial at sunrise would not have been in the same straight line as the shadow at sunset (*id.*). Next: the earth has the ratio of a point to its distance to the sphere of the fixed stars (6); this follows from the facts that the sizes and distances of the stars appear the same from all parts of the earth (*id.*); and that the plane through the observer's line of sight, the horizon, appears everywhere on earth to bisect the heavenly sphere (*id.*). Next: the earth is at rest (7); this follows from its position in the center (*id.*); also, heavy objects are carried toward the earth at right angles to the plane tangent to the point of impact (*id.*); also, if the heaven were at rest, and the earth made one revolution daily, nothing in the heavenly phenomena argues against this hypothesis, but the revolution of the earth would have been so violent that neither clouds nor any other flying objects could have been seen moving toward the east; and even if the air were carried with the motion of the earth, heavy objects in the air would have still appeared to lag behind (*id.*). Finally: there are two primary motions in the heaven (8); one carries all heavenly bodies from east to west in uniform motion along circles parallel to one another about the pole of the heavenly sphere (*id.*); the other carries some of them in the opposite direction (*id.*);

observed in the course of one day, all heavenly bodies show the first motion; observed continuously, however, while all others show only the first motion, the sun, the moon, and the other planets show more complicated and different from one another motions, which are yet all in the general direction from west to east (*id.*).

The motion of the heavenly bodies is spherical. The earth is spherical, at rest, in the center, and has the ratio of a point to its distance to the sphere of the fixed stars. And there are two primary motions in the heaven, uniform and not uniform.

Ptolemy's description thus ends with the implied problem, to reduce the not uniform motion of the sun, the moon, and the other planets to uniformity. The description also points at how the problem is to be solved. If the heaven is spherical, and the earth is spherical, and all other heavenly bodies move in circles, to reduce the not uniform motion of the sun, the moon, and the other planets to uniformity is to reduce it to circular motion.

Though more focused and more explicit, Ptolemy's description of the world in book I is not substantially different from Aristotle's, including in that the problem of reducing the motions of the sun, the moon, and the other planets to uniformity has been formulated, and it has been proposed that to solve the problem is to reduce their motion to circular motion. Ptolemy's description in book I is not a great advance over Aristotle's.

Aristotle's description of the world is a very great advance over his predecessors. It is a most dramatic instance of development from no inkling whatsoever.

The first inkling can be recovered from the oldest account of creation, *When on high*, twelve and a half centuries before Aristotle, ten centuries before the first philosopher. *When on high* begins: when on high the heaven had no name, and down below the earth had no name, Apsu was the begetter, Tiamat bore all (I, 1-4). One of the progeny of Apsu and Tiamat is Marduk: the greatest among the gods, his command shall be truth, his word shall never be wrong (IV, 1-10). And in 66 verses: Marduk arranges the stars, the sun, and the moon, marks the year, the months, and the days, lets the Tigris and the Euphrates flow, and sets up the heaven above and the netherworld below (V, 1-66). The rest of *When on high* is devoted to the more urgent matters of the battles of the gods and the creation of man.

Seven and a half centuries after *When on high*, the Yahwistic document is the earlier of the two writings integrated in *Genesis*, and the oldest of the writings which became the Hebrew scriptures. It begins: when god made the earth and the heaven, there was no shrub and no grass on the earth, because god had not yet sent rain; nor was there a man to till the earth; water welled up from below and covered the earth; and god made man from the dust of the earth (2). 7 verses, and the Yahwistic document turns to the more urgent matters of the garden of Eden, the tree of the knowledge of good and bad, and the consequences of eating the fruit of that tree. Not quite three and a half centuries later, and contemporary with Parmenides, the Priestly code is the later of the two writings integrated in *Genesis*. In the Priestly code: god separates light from darkness, makes heaven, makes the earth and the sea, fills the earth with plants, makes the sun, the moon, and the stars, fills the sea and the heaven with living creatures, fills the earth with animals, and makes man in his own image (1). 27 verses, god sees that all he has made is good, and the Priestly code also turns to more urgent matters.

One century after the Yahwistic document, one and a half centuries before the first philosopher, in Homer, in the *Iliad*, Hephaestus crafts the arms of Achilles. The god's masterpiece is the round shield of five concentric layers, which the poet needs 135 verses to describe (XVIII, 483-617). In 126 verses he describes the pictures the god drew on the three layers in the middle: a city at war, a city at peace, and a prince who supervises ploughing, harvesting, vintage, and herds of cattle and sheep, surrounded by celebrating dancers (490-615). In 2 verses the poet describes the picture on the outermost layer: on the edge of the shield the god drew the great strength of the river Ocean (616-7). And in 7 verses he describes the picture on the innermost layer: the god drew the earth, the heaven, the sea, the tireless sun, the waxing moon, and all the constellations that adorn the heaven, the Pleiades and the Hyades and Orion and the Great Bear who looks at Orion

and who alone never bathes in the Ocean (483-9).

Then there are the philosophers.

Thales, two and a half centuries before Aristotle, said that the earth is at rest because it floats on water, like wood (Aristotle, *Heaven*, II, 13).

Anaximander said that it is at rest because of equilibrium (*id.*). He said also that it is shaped like a drum (Hippolytus, *Refutation*, I, 6).

Anaximenes said that it is flat, and it is at rest because it floats on the air below, like a lid (Aristotle, *Heaven*, II, 13).

Parmenides discovered that the earth is a sphere (Laertius, IX, 21-2).

Anaxagoras said that the sun is a piece of red-hot metal larger than the Peloponnese (Aëtius, II, 21).

Oenopides discovered the obliquity of the ecliptic (Theon of Smyrna, III, 40).

Eudoxus, in the generation before Aristotle, formulated the problem of reducing the motions of the sun, the moon, and the other planets to uniformity, and proposed that the solution of the problem is circular motion. On the authority of Eudemus, Simplicius reports: Eudoxus first formulated hypotheses of uniform circular motions to which the apparent motions of the planets can be reduced (comm. *Heaven*, 488). And the theory of the motions of the planets remained a theory of circular motions until Kepler.

The theory of the motions of the planets formulated by Eudoxus is an essential part of the first description of the world.

The formulation of the theory by Eudoxus and hence its exposition by Aristotle is, with two exceptions, strictly qualitative. The exceptions are that Aristotle counts the spheres. He writes: to explain the motions of the planets Eudoxus assumed 26 spheres, and Callippus added 7, but 22 more are necessary to counteract the 33, for a total of 55. And that he reports: the mathematicians calculate the circumference of the earth at 400 000 stades.

Aristotle's description of the world is a very great advance over his predecessors. Four and a half-odd centuries later, Ptolemy's description of the world in book I of the *Almagest* is not a great advance over Aristotle's. But his description of the world in books III through XIII of the *Almagest*, in the *Planetary hypotheses*, in the *Geography*, and in the *Handy tables* is an advance over Aristotle's very much greater than Aristotle's over his predecessors.

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After Aristotle and before Ptolemy, the major steps on which the development of the mathematical description of the world beyond the earth stands are three.

Eratosthenes measured the circumference of the earth (Cleomedes, I, 10).

Apollonius established that motion on an eccentric circle and motion on an epicycle moving on a circle can reduce the motions of the planets to circular motion (Ptolemy, *Almagest*, XII, 1).

Hipparchus developed trigonometry (Theon of Alexandria, comm. *Almagest*, I, 10), employed the eccentric hypothesis to formulate the theory of the motions of the sun (Ptolemy, *Almagest*, III, 3-4), employed the epicyclic hypothesis to formulate the theory of the motions of the moon (IV, 5, 11), measured the distance of the moon (V, 11), discovered the precession of the equinoxes (III, 1), and, in general, transformed the investigation of the heaven and hence also the investigation of the surface of the earth into mathematical investigations.

And Ptolemy, following the third rule of the method, to begin with the simplest and easiest and ascend gradually to the more complex – he writes: the other heavenly bodies cannot be investigated if the sun and the moon are not investigated first, and the moon cannot be investigated if the sun is not investigated first (pr.) – Ptolemy begins in book III of the *Almagest* with the theory of the sun.

He writes: the first theorem concerning the sun is the determination of the length of the year (1); the length of the year can be determined either by the return of the sun to the same position with respect to the fixed stars, or by its return to the same equinox or the same solstice (*id.*); relevant to our investigation is the latter (*id.*); in particular, the length of the year is constant (*id.*); and it is clear also that it is a little less than $365 + \frac{1}{4}$ days (*id.*).

To determine how much less, Ptolemy reports,

Hipparchus compared the oldest accurate observation of a solstice with his own. And so does Ptolemy.

He continues: Meton and Euctemon observed summer solstice on 27 June 432^{BC} at dawn; we observed it on 25 June 140^{AD} about two hours after midnight (*id.*); over 571 years, then, the accumulation due to the additional $\frac{1}{4}$ day, over the 365 days of every year, would amount to $142\frac{3}{4}$ days; instead, the accumulation is about $140\frac{5}{6}$ days; the difference is about 2 days in 600 years; and the length of the year is thus about $365+\frac{1}{4}-\frac{1}{300}$ days (*id.*).

Next: our task is to demonstrate the apparent irregularity of the sun (3). And before addressing the task Ptolemy presents the most fundamental and simple hypotheses on whose ground he will do so. He writes: in general, the motions of the planets from west to east are in every case, just like the motion of the world from east to west, uniform and circular; the apparent irregularity of their motions is due to the position and order of the circles on which their motions take place; and the appearance of irregularity can be explained by two most fundamental and simple hypotheses; we can presume that the motion takes place on a circle which is not concentric with the world; or we can presume that the motion takes place on a circle which is carried by a circle concentric with the world (*id.*). And the most fundamental hypotheses are also the simplest because: we consider it a good principle to explain the phenomena by the simplest hypotheses with which nothing in the observations is inconsistent (1).

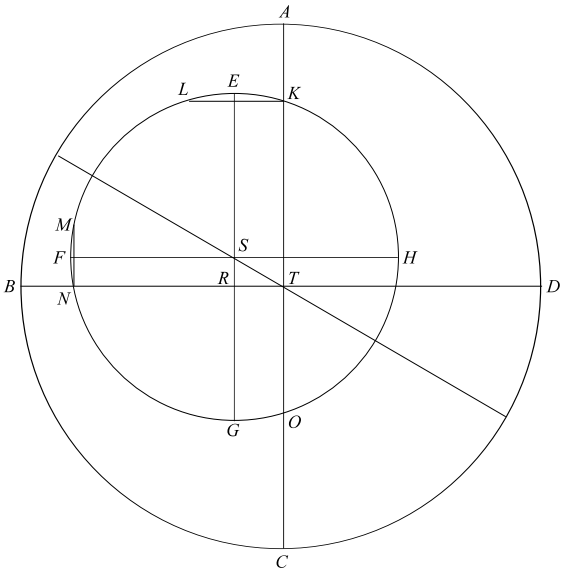
Accordingly, to explain the motions of the sun, the simplest among the motions of the planets, the eccentric hypothesis suffices. The sun traverses with uniform motion a circle whose center is not the center of the world, or of the ecliptic.

In particular:

our task is to find the ratio of the eccentricity of the sun's circle – and that is, the distance between the center of the eccentric circle and the center of the ecliptic – and the radius of the eccentric circle;

and to find the angle between apogee and a point on the ecliptic;

Hipparchus determined that the time from spring equinox to summer solstice is $94\frac{1}{2}$ days, and the time from summer solstice to fall equinox is $92\frac{1}{2}$ days;



let the ecliptic be circle $ABCD$ with center T;

draw diameters AC and BD perpendicular to one another; let A be spring equinox, B summer solstice, and so on;

representing $94\frac{1}{2}+92\frac{1}{2}=187$ days, semicircle ABC represents more than one half of the year, representing $94\frac{1}{2}$ days, quadrant AB represents more time than quadrant BC; the center of the eccentric circle thus has to be in quadrant AB;

let S be the center of the eccentric circle; draw diameters EG and FH parallel to AC and BD, respectively; and from K and N, the intersections of the eccentric circle and the diameters of the circle representing the ecliptic, draw chords KL and NM perpendicular to EG and FH, respectively;

traversing circle $EFGH$ with uniform motion, the sun traverses arc KN in $94\frac{1}{2}$ days, and arc NO

in $92\frac{1}{2}$ days; it thus traverses approximately $93^\circ 9'$ and $91^\circ 11'$, respectively;

arc $KNO = 93^\circ 9' + 91^\circ 11' = 184^\circ 20'$; subtracting semicircle EFG, the sum of the two arcs $KE + GO = 4^\circ 20'$; and hence also arc $KL = 4^\circ 20'$ (4).

In book I Ptolemy has developed trigonometry, hence compiled a table of the correspondence of the arcs and chords of the circle, and from the table, given arc KL:

chord $KL = 4\frac{32}{60}$, where the unit is the radius of the eccentric circle, $ES = 60$; and hence, $RT = 2\frac{16}{60}$;

arc $KLMN = 93^\circ 9'$; subtracting arc $KE = 2^\circ 10'$ and arc $EF = 90^\circ$, arc $FN = 0^\circ 59'$; and arc $MN = 1^\circ 58'$;

hence chord $MN = 2\frac{4}{60}$; and hence $RS = 1\frac{2}{60}$;

but $ST^2 = RT^2 + RS^2$; and $ST = 2\frac{29\frac{1}{2}}{60}$;

the ratio of the eccentricity and the radius of the eccentric circle is thus $2\frac{29\frac{1}{2}}{60} : 60$, or approximately 1 : 24 (*id.*).

All sides of triangle RST are determined, the table of chords gives angle $RTS = 24^\circ 30'$, and:

apogee is thus $24^\circ 30'$ before summer solstice (*id.*).

The position of the sun at any time, in the future or, more important, in the past, can now be calculated.

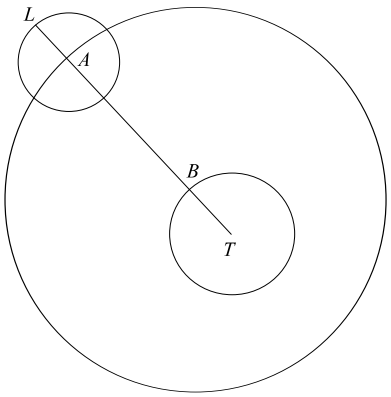
The theory of the sun in the *Almagest* is the theory developed by Hipparchus, examined and corroborated by Ptolemy.

The theory of the moon is the theory developed by Hipparchus, presented in book IV, improved substantially by Ptolemy, in book V.

The moon revolves uniformly from east to west on an epicycle, while the center of the epicycle revolves uniformly from west to east, and not about the center of the ecliptic but about a center which itself revolves uniformly from east to west on a circle about the center of the ecliptic.

The solution of the task of finding the eccentricity is:

the radius of the epicycle $LA = 5\frac{15}{60}$, the radius of the eccentric circle $AB = 49\frac{41}{60}$, and the radius of the circle about which the center of the epicycle revolves $BT = 10\frac{19}{60}$, where the unit is the greatest distance between the center of the ecliptic and the center of the epicycle, $TA = 60$ (V, 4).



The solution of the task of finding the angle between apogee and a point on the ecliptic Ptolemy obtains from the observations of three lunar eclipses. The solution is:

during the eclipse which we observed on 20 Oct 134 one hour before midnight, as the sun is at $25^\circ 10'$ Libra, and hence the moon is at $25^\circ 10'$ Aries, the moon is $64^\circ 38'$ after its apogee (IV, 6).

The position of the moon at any time, in the future or in the past, can now be calculated.

Hence, in book VI the theory of eclipses follows. Hipparchus calculated eclipses for the next 600 years (Pliny, II, 53). But the calculation of either solar or lunar eclipses requires the determination of the positions of both the sun and the moon, and Ptolemy improved the theory of eclipses substantially, too.

In books VII and VIII the star catalogue follows. Ptolemy continues to adhere to the third rule. The star catalogue has to precede the theory of the planets because the motion of the planets cannot be measured but in relation to the fixed positions of the stars.

Collecting the observations of his predeces-

sors, Ptolemy records the declinations, right ascensions, and magnitudes of 1 022 stars, arranged in 48 constellations. And the success of the catalogue does not have to be judged by comparison with the work of Hephaestus, or of any other god. It can be judged by the fact that when Halley discovered that the fixed stars are not fixed after all, that they have proper motion, it was rather a joint discovery, of Ptolemy and Halley jointly.

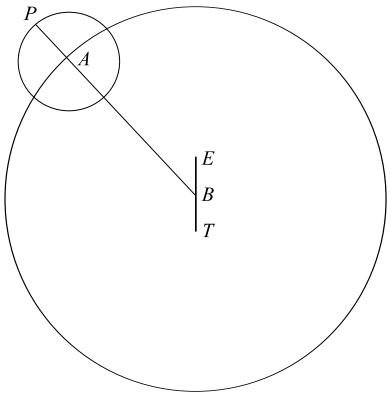
In 1717, in the *Considerations on the change of the latitudes of some of the principal fixed stars*, Halley begins: I took pains to compare the declinations of the fixed stars delivered by Ptolemy in the third chapter of book VII of the *Almagest*, as observed by Timocharis nearly 300 years before Christ and by Hipparchus about 170 years later, with what we now find (*id.*). What Halley finds is that three stars, and only three, are not where they would be. He continues: Ptolemy places Aldebaran $20'$ more northerly in declination than we now find it, Sirius $22'$ more northerly, and Arcturus $33'$ more northerly; what shall we say? it is scarcely credible that the ancients could be deceived in so plain a matter, three observers confirming one another; the most conspicuous in the heaven, these stars are in all probability the closest to the earth; if stars have motion of their own, it is most likely to be perceived in these; and though utterly imperceptible in the space of a century, it may show itself by the alteration of their places in so long a time as 1 800 years (*id.*).

In books IX through XIII Ptolemy formulates the theory of the motions of the planets. He reports: Hipparchus developed the theory of the sun and of the moon, but being a great lover of truth, he did not even attempt to develop the theory of the other five planets because he did not have a store of accurate observations from earlier times to compare with his own, such as he left us (IX, 2). The theory of the other five planets is the crowning achievement of the *Almagest*.

Saturn, for instance, revolves uniformly from west to east on an epicycle, while the center of the epicycle revolves uniformly also from west to east, and neither about the center of the ecliptic nor about the center of the eccentric circle, but about the equant point, the point on the continuation of the line through those two centers at a distance from the center of the eccentric circle equal to the distance between those two centers.

The solution of the problem of the eccentricity is:

the radius of the epicycle $PA = 6\frac{30}{60}$, the distance between the center of the ecliptic and the center of the eccentric circle $TB = 3\frac{25}{60}$, and the distance between the center of the eccentric circle and the equant $BE = 3\frac{25}{60}$ as well, where the unit is the radius of the eccentric circle, $BA = 60$ (XI, 6).



The solution of the problem of the angle between a given position of the planet, and hence of apogee, and a point on the ecliptic is:

on 22 Dec 138 four hours before midnight Saturn is at $9^\circ 4'$ Aquarius and $76^\circ 4'$ after the greatest distance between the center of the ecliptic and the center of the epicycle (*id.*).

The position of any of the other five planets at any time, in the future or in the past, can now be calculated.

The *Almagest* is a very great advance in the description of the world.

And yet the *Planetary hypotheses* takes the advance further.

Aristarchus, four centuries before Ptolemy, not quite one and a half before Hipparchus, in *On the sizes and distances of the sun and moon*, discovered the method for measuring the distances of the moon and the sun which remained without an alternative for two millennia, and he first calcu-

lated the distances. The distance of the moon Aristarchus calculated at 20 earth radii, the distance of the sun at 380 earth radii (7-17).

Hipparchus calculated the mean distance of the moon at $67\frac{1}{3}$ earth radii (Pappus, comm. *Almagest*, V, 11), and thus solved the problem of the distance of the moon. The distance is, in fact, 60.2 earth radii.

In the *Almagest* Ptolemy calculates the mean distance of the moon at: 59 earth radii (V, 13).

The mean distance of the sun Ptolemy calculates at: 1 210 earth radii (15). And the problem of the distance of the sun was solved fifteen and a half centuries after him. In 1543 Copernicus calculates it at: 1 142 earth radii (*Revolutions*, IV, 21), in 1602 Tycho calculates it at: 1 150 earth radii (*Exercises*, II, 7), in 1620 Kepler calculates it at: 3 469 earth radii (*Epitome*, IV, 1.4), in 1632 Galileo calculates it at: 1 208 earth radii (*Systems*, III), in 1644 Descartes calculates it at: 1 200 or 1 400 earth radii (*Principles*, III, 5). The problem was solved in the next generation. In 1659 Huygens calculates the distance of the sun at: 25 086 earth radii (*Sys. Saturn*), in 1672 Cassini calculates it at: 22 000 earth radii (*Observations*), and in 1684 Cassini measures it at 21 600 earth radii (*Elements*, 37). The distance is, in fact, 23 455 earth radii.

And in the *Planetary hypotheses* Ptolemy calculates the size of the world.

The task of the treatise, Ptolemy begins, is: to show how the motions of the heavenly bodies combine with one another and with the whole mechanically (I, 1.1), and, needless to add: to do so on the simplest hypotheses (1.2).

Ptolemy has established in the *Almagest* that the most plausible order of the heavenly bodies, from the earth in the center to the sphere of the fixed stars, is: the moon, Mercury, Venus, the sun, Mars, Jupiter, Saturn; because in this way the sun separates the planets which move only in its vicinity from those which reach all positions (IX, 1). He reaffirms this order in the *Planetary hypotheses*, and adds: this order is the most plausible even if it can be argued that Mercury and Venus are beyond the sun, because no transits of these planets before the sun have been observed (I, 2.2). Kepler first calculated transits in 1629, Gassendi first observed one of them in 1631.

Ptolemy continues:

to arrange the spheres of the planets in their order, we presume that the greatest distance of each is the least distance of the next;

in the *Almagest* the ratios of least and greatest distance of each planet have been determined; the least distance of the moon is 33 earth radii, its greatest distance is 64 earth radii; the ratio of the least and greatest distance of Mercury is 34:88; and it is clear that, if the greatest distance of the moon is the least distance of Mercury, the greatest distance of Mercury is 166 earth radii;

the ratio of the least and greatest distance of Venus is 16:104, and if the greatest distance of Mercury is the least distance of Venus, the greatest distance of Venus is 1 079 earth radii (2.3).

Ptolemy notes: we have determined that the least distance of the sun is 1 160 earth radii, and there is thus a discrepancy here which we cannot explain (*id.*).

The greatest distance of the sun gives the least distance of Mars, the greatest distance of Mars gives the least distance of Jupiter, the greatest distance of Jupiter gives the least distance of Saturn. The greatest distance of Saturn follows, and:

the greatest distance of Saturn is the least distance of the sphere of the fixed stars (*id.*).

In addition, Ptolemy takes for granted Aristotle's description of the spheres of the elements, earth in the center, overlain by water, overlain by air, overlain by fire.

The size of the whole follows:

taking the radius of the sphere of the earth and water as a unit, the radius of the sphere surrounding the air and fire is 33, the radius of the sphere of the moon is 64, the radius of the sphere of Mercury is 166, the radius of the sphere of Venus is 1 079, the radius of the sphere of the sun is 1 260, the radius of the sphere of Mars is 8 820, the radius of the sphere of Jupiter is 14 187, and the radius of the sphere of Saturn is 19 865 (*id.*).

Nor is the size of the world determined only relatively. Beginning with the measurement of the

circumference of the earth at: 180 000 stades (2.4) – which he reaffirms in the *Geography* (VII, 5) – Ptolemy recalculates the greatest distance of Saturn, and thus the least distance of the sphere of the fixed stars, absolutely. Stades varied from about .2 km to about .15 km. Presuming that the stade Ptolemy used ≈.16 km, he calculated the radius of the earth at 4 600 km. The equatorial radius of the earth is, in fact, 6 378 km. With a radius of the earth of 4 600 km, the distance of the sphere of the fixed stars is at least 9.1×10^7 km. The world is at least 2×10^8 km across.

Man, for comparison, is 2×10^{-3} km across.

Ptolemy concludes: if the world is such as we have described it, there is no space between the greatest distance of one sphere and the least distance of the next; this arrangement is the most plausible, because there can be no void in nature; but even if there is void between the spheres, it is clear that the distances cannot be smaller than these (I, 2.4).

With Maxwell: not good or bad, noxious or beneficial, but of what kind it is, and how much there is of it. Of what kind the world is Aristotle began determining. How much there is of the world Ptolemy began determining.

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And yet the *Geography* takes the advance in the description of the world further still.

The earth has the ratio of a point to its distance to the sphere of the fixed stars, but a different ratio to man. Or to the philosopher. Ptolemy writes in the *Geography*: the highest and most beautiful concerns of theoretical philosophy are to show to human understanding by mathematics the nature of the heaven, which, since it surrounds us, we can see revolving around us, and of the earth, which, since it is very large and does not surround us, no single man can see in whole or even in part (I, 1).

The earth is very large.

Twenty-six centuries before Ptolemy, Eanatum, prince of Lagash, notes 16 localities on the surface of the earth in his inscriptions.

Nineteen and a half centuries later, six and a half before Ptolemy, Hecataeus describes 300-odd localities. Not quite one century later, Herodotus describes 1 012 localities, from the Atlantic in the west to the Indus in the east.

After Herodotus and before Ptolemy, the major steps on which the development of the mathematical description of the surface of the earth stands are also three.

Eratosthenes discovered that the fixed parallel and fixed meridian are the indispensable presuppositions of mapping (Strabo, II, 1.1).

Hipparchus established that: there can be no geography without astronomy (I, 1.12).

And of Marinus, in the generation before him, Ptolemy writes in the *Geography*: Marinus has treated our subject with extraordinary diligence; he has carefully examined all his predecessors and corrected their errors; it will thus suffice for us to examine him and correct his errors (I, 6).

How much there is of the earth as a whole Eratosthenes established when he measured its radius. With the *Geography* Ptolemy established how much there is of the earth part by part, for a total of 6 345 parts.

In book I he again lays the grounds on which the problems solved in the following books are solved. He describes the method (1-3), and examines the quality of the relevant observational data (4-20). He then develops the theory of map projections (21-4). He defines two projections: a conic projection in which the meridians are straight lines from the vertex and the parallels are circular arcs about the vertex, and a projection in which the meridians are curves such as to preserve distances along three parallels (24). Waldseemüller's map in 1507 employs Ptolemy's second projection. The most important step in the development of the theory of map projections after Ptolemy is Lambert's *Notes and comments on the drawing of maps of the earth and heaven*, in 1772. The most important projections Lambert defines are five. Two of them, the conic equal-area projection with one standard parallel (108-10), and the conformal conic projection with two standard parallels (48-57), are developments of Ptolemy's first projection.

In books II through VII Ptolemy describes 7 949 localities, and assigns latitudes and longitudes to 6 345 of them.

He notes: the coordinates assigned to well trodden places can be considered close to truth, as there have been consistent reports about them for

a long time, but research of places which have not been traveled so much is sparse and uncertain and the coordinates assigned to them are only estimates; corrections from further research can be entered in the table of latitudes and longitudes (II, 1). In particular, the corrections from the research of the following nineteen centuries can be entered.

The 6 345 localities to which Ptolemy assigns latitude and longitude are distributed from the island of Thule in the north (II, 2) to cape Prason in the south (IV, 8), and from the islands of the Blessed in the west (IV, 6) to cape Catigara in the east (VII, 3). The sheer number of latitudes and longitudes in between precludes the possibility of difficulty of identification. Thule is the Shetland Islands. Cape Prason is cape Delgado. The islands of the Blessed are the Canary Islands. In the far east Ptolemy assigns latitudes and longitudes to each of 8 inland localities up to 3° further east than cape Catigara, to three sources of rivers, one branching of rivers, and four cities (*id.*). The point furthest east on the coast to which he assigns a latitude and a longitude is cape Catigara. Cape Catigara is: on the Great Bay (*id.*), and: it is the port of the Chinese (*id.*). The Great Bay is the gulf of Tonkin, and given that cape Catigara is: a cape, and: the port of the Chinese, cape Catigara has to be the mouth of the Red River.

The equator in the *Geography* is the equator. The prime meridian, however, runs through the Atlantic. Taking it for granted that Ptolemy's assignment of longitude to Alexandria, the place of his astronomical observations, is the best, the meridian through Alexandria can be taken as the prime meridian instead. And arranging the four ends which the acquisition of knowledge has reached from west to east, counting the midpoints of the two island groups, and rounding to the nearest $30'$, these are the assignments of latitude and longitude in the *Geography* and their corrections,

Canary Islands

Geography	$13^\circ 00' N$	$59^\circ 30' wA$,
correction	$28^\circ 00' N$	$46^\circ 00' wA$,

Shetland Islands

Geography	$63^\circ 00' N$	$30^\circ 00' wA$,
correction	$60^\circ 30' N$	$31^\circ 00' wA$,

cape Delgado

Geography	$15^\circ 00' S$	$19^\circ 30' eA$,
correction	$11^\circ 00' S$	$11^\circ 00' eA$,

mouth of the Red River

Geography	$8^\circ 30' S$	$116^\circ 30' eA$,
correction	$20^\circ 30' N$	$77^\circ 00' eA$.

Ptolemy's results should need no recommendation as excellent. Here is one example of the reports from which he had to extract latitudes and longitudes: beyond the Ganges, they say, men have tails, like satyrs (VII, 2). The latitudes and longitudes above are those at the ends of the acquisition of knowledge and their errors are the grossest in the *Geography*. Yet even the grossest of the grossest, the error in the longitude of the mouth of the Red River, is of $39^\circ 30'$ over 360°, less than 12%, as Ptolemy reaches across more than one fifth of the circle around the earth to determine it. His errors in longitude are greater than his errors in latitude, too, illustrating the fact that the determination of longitude is the more difficult problem. It was not solved conclusively until Cook and his voyages in 1768-79.

Ptolemy writes: if the men visiting all the different countries made astronomical observations, the map of the earth could be drawn without a single error (I, 4).

Ptolemy guided Columbus across the Atlantic, and Magellan and El Cano around the earth. Columbus made astronomical observations, Magellan and El Cano made astronomical observations, de Toreno drew on the map all parts of the earth in their true proportions, Cook made astronomical observations, and eventually the map of the earth was drawn without a single error.

And yet the *Handy tables* takes the advance in the description of the world further still.

The results of the *Almagest*, *Planetary hypotheses*, and *Geography* all rest on astronomical observation. Most particularly, they rest on astronomical observations made in different places and at different times. Ptolemy has established the way to compare these observations with one another, and in so doing has fixed chronology absolutely. Explicitly all the way back to Nabonassar, prince 748-34. Ptolemy explains: we calculate from the beginning of Nabonassar's reign because the most ancient observations are from that time (III, 7). The latest

observation in the *Almagest* is of Mercury, by Ptolemy, in Alexandria, in 141^{AD} (IX, 7). The most ancient observation is 861 years older, of a lunar eclipse, in Babylon, in 721^{BC} (IV, 6).

The *Handy tables* collects the tables of the *Almagest* and extends them. And it is one of its tables that fixes chronology absolutely explicitly. It correlates the years from 748^{BC} to 137^{AD} with the years of the reigns of 54 princes. Babylonian princes, as they are under the sovereignty of and include Assyrian princes, then the great kings who conquered Babylon, then Alexander who conquered the realm of the great kings, then Alexander's immediate successors, then his successors in Egypt, then the Roman princes who conquered Egypt. Ptolemy correlates years with the years of the reigns of 54 princes, and hence the years of the reigns of 54 princes are fixed absolutely. 54 princes from

Nabonassar prince 748-34,
through
Tiglath-pileser III prince 745-27,
Shalmaneser V prince 727-2,
Sargon II prince 722-05,
through
Cyrus prince 559-30,
Cambyses prince 530-22,
Darius prince 522-486,
Xerxes prince 486-65,
through
Alexander prince 336-23
through
Ptolemy prince 306-282,
through
Cleopatra prince 51-30,
through
Augustus prince 27^{BC}-14^{AD},
through
Trajan prince 98-117,
Hadrian prince 117-38,
Pius prince 138-61.

The Sumerian is the first prince list. Then there are the Larsa prince list and the Isin prince list. Then the Babylonian prince list. The Assyrian prince list. The Egyptian princes are listed in the Karnak prince list, the Abydos prince list, the Saqqara prince list, and the prince list of the Turin papyrus. Herodotus has the Spartan prince list (VII, 204; VIII, 131). Hippias compiled the list of Olympic victors (Plutarch, *Numa*, 1), and Greek historians reckoned time by olympiads. The pontifices maintained lists of magistrates (Cicero, *On the orator*, II, 52), and Roman historians reckoned time by kings and consuls, beginning from the foundation of the city. Abounding in lists of patriarchs beginning with the first man, the Hebrew scriptures encouraged reckoning time from the creation of the world.

These can all be related to one another, and after Ptolemy, and not before, all their relative dates can be fixed absolutely. In the valley of the Euphrates and Tigris back to

Ashur-uballit prince *c.* 1363-27,
and
Hammurabi prince *c.* 1792-49,
and
Ur-Nammu prince *c.* 2121-03,
and
Sargon prince *c.* 2337-281,
and all the way back to the first Sumerian prince
Enmebaragesi prince *c.* 2680.
In the valley of the Nile back to
Thutmose prince *c.* 1521-09,
and
Senwosret III prince *c.* 1878-45,
and
Djoser prince *c.* 2680,
and all the way back to the first prince
Narmer prince *c.* 3000.

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Ptolemy, then, measured the world.

He first measured the world, all subsequent improvements on the measurement of the world are improvements on his measurement, and all subsequent improvements on the improvements on the measurement of the world are improvements on the improvements on his measurement of the world.

He measured the world, both, spatially and temporally. Nor can the measurement be a measurement of the world if it is not, both, spatial and temporal.

Spatially, the measurement of the world in its entirety, in the *Planetary hypotheses*, stands on the measurement of the system of the sun, the moon, and the planets, in the *Almagest*. In turn, the measurement of the system of the sun, the

moon, and the planets stands on the measurement of the surface of the earth, in some detail in book II of the *Almagest*, and great detail in the *Geography*. The measurement of the world stands on observation, and observation stands on the quantitative description of the place of observation. And Ptolemy describes the place of observation quantitatively from the Canary Islands to the Shetland Islands to cape Delgado to the gulf of Tonkin.

The spatial measurement of the world stands on observation, and observation stands on the quantitative description of the place of observation in Ptolemy, and in the big bang theory. Hubble's law stands on the distances of galaxies, which stand on the distances of stars, which stand on the distance of the sun, which stands on the earth's radius.

Temporally, the measurement of the system of the sun, the moon, and the planets, including the calculation of their positions at any time in the future or in the past, stands on absolute chronology, fixed in the *Almagest*, and in some detail in the *Handy tables*. The temporal measurement of the world stands on observation as well, and observation stands on the quantitative description of the time of observation as well. And Ptolemy describes the time of observation quantitatively from Nabonassar to Pius.

And again, the temporal measurement of the world stands on observation, and observation stands on the quantitative description of the time of observation in Ptolemy, and in the big bang theory. Hubble's law implies the beginning of the world. It occurred 1.4×10^{10} years ago.

The measurement of the world stands on the quantitative description of the place and the time of observation.

Conversely, it is the world.

With Minkowski: the multiplicity of all thinkable x, y, z, t systems of values is the world. Of all thinkable x, y, z, t systems of values, Ptolemy determined not less than 7 374 x, y, and z systems of values, not less than 54 t values, integrated them into 7 428 x, y, z, t systems of values, and followed the consequences of the integrated systems of values to the moon, the sun, the planets, and the end of the world.

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The most important problem Ptolemy solved is the quantitative description of the world. Or better, the mathematical description of the world.

As to how, he solved it by observation.

There are 94 dated observations in the *Almagest*.

The oldest observation of a lunar eclipse is from 721.

The oldest observation of a solstice, by Meton and Euctemon, is from 432.

The oldest observation of Venus, by Timocharis, is from 272.

The oldest observations of the other planets are, of Mars from 272, of Mercury from 265, of Jupiter from 241, all three by Dionysius, and of Saturn from 229.

So much suffices to show development of observation. From no observation, to the observation of eclipses, to the observation of solstices three centuries later, to the observation of planets one and a half centuries later. Ptolemy explains: observations of the planets are relatively recent; the oldest of them are also useless because they are too crude; they tend to be of first and last visibility and such observations are particularly uncertain; moreover, observation of the distances of the planets from fixed stars requires difficult calculation and great skill because the lines joining planets and stars are seldom at right angles to the ecliptic and because the same distances appear greater near the horizon than in mid-heaven (IX, 2). The last explains also why: Hipparchus found very few observations of fixed stars from before his time, none earlier than those of Timocharis and Aristyllus, and none that were carefully made and free from uncertainty (VII, 1).

The observation of the planets requires calculation. In the *Almagest* it requires the development of trigonometry and spherical geometry.

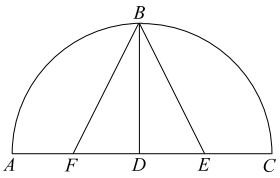
The following are only the two most fundamental of the theorems on which this development rests. Ptolemy writes:

on semicircle ABC with center D draw BD perpendicular to AC; bisect CD at E, and make $FE = BE$;

$(2DE + FD)\,FD + DE^2 = (FD + DE)^2$;

hence $FC \,.\, FD + DE^2 = FE^2$;

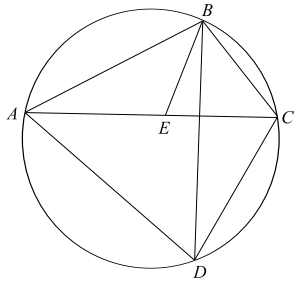
but $FE^2 = BE^2 = BD^2 + DE^2$;



so $FC \cdot FD + DE^2 = BD^2 + DE^2$;
and $FC \cdot FD = BD^2$;
Euclid has shown that of the sides of the regular hexagon, decagon, and pentagon inscribed in the same circle the relations hold,
(decagon + hexagon) decagon = hexagon ^{2},
pentagon ^{2} = decagon ^{2} + hexagon ^{2};
thus, in the circle with radius BD, and so side of the inscribed hexagon BD as well, the side of the inscribed decagon is FD, the side of the inscribed pentagon is FB;
the side of the hexagon subtends an arc of $60°$, the side of the decagon an arc of $36°$, the side of the pentagon an arc of $72°$;
thus, $(FD + BD)\,FD = BD^2$, and if BD, the chord of the arc of $60°$ is 60, FD, the chord of the arc of $36°$, is $37\frac{4\frac{55}{60}}{60}$;
also, $FB^2 = FD^2 + BD^2$, and FB, the chord of the arc of $72°$, is $70\frac{32\frac{3}{60}}{60}$ (I, 10).

The side of the inscribed square gives the chord of the arc of $90°$, the side of the inscribed equilateral triangle the chord of the arc of $120°$. The Pythagorean theorem gives the chords of the arcs supplementary to all these arcs.

Next:



inscribe quadrilateral $ABCD$ in a circle; join AC and BD; make $\angle ABE = \angle DBC$;
so $\angle ABD = \angle EBC$;
on the same arc, $\angle ADB = \angle ACB$;
and triangles ABD and EBC are similar;
also, $\angle ABE = \angle DBC$; $\angle BAC = \angle BDC$; and triangles ABE and DBC are similar;
from the second pair of similar triangles $\frac{AB}{AE} = \frac{BD}{CD}$; from the first pair of similar triangles $\frac{BC}{EC} = \frac{BD}{AD}$;
hence $AB \cdot CD = AE \cdot BD$, and $AD \cdot BC = EC \cdot BD$;
and hence $AB \cdot CD + AD \cdot BC = AC \cdot BD$ (*id.*).

And the addition, subtraction, multiplication, and division of chords now gives the chord of any arc with any degree of accuracy.

This is the foundation of observation in the *Almagest*.

The observation of the planets requires calculation. But the observation of the solstices also requires calculation, even if not, five centuries before Ptolemy, quite as sophisticated as the observation of the planets. And the observation of eclipses also requires calculation, even if not, three centuries earlier still, quite as sophisticated as the observation of the solstices.

All observation of the heavenly bodies requires calculation. In general, the observation of the heavenly bodies is essentially mathematical.

In the first two centuries of development of philosophy the philosophers concerned themselves incessantly with the heaven. The second philosopher, Anaximander, discussed the sizes and distances of the planets (Simplicius, comm. *Heaven*, 471). The fourth philosopher, Pythagoras, discussed the order of the planets (*id.*). And Plato still painstakingly describes the heaven, in the *Timaeus*. The philosophers made very important discoveries in the heaven in these two centuries, too. Parmenides established that the morning star is the evening star (Laertius, IX, 23). Anaxagoras said that the sun is a piece of red-hot metal. He discovered also the cause of eclipses (Hippolytus, *Refutation*, I, 8). Oenopides discovered the obliquity of the ecliptic. And all these two centuries' worth of philosophers concerning themselves incessantly with the heaven left behind one observation of the heavenly bodies, the solstice observation of Meton and Euctemon.

The explanation is simple. Observation is not looking, not seeing, and not even looking with determination and seeing with deliberation. Observation is discovered. In particular, it took

the philosophers the first two centuries of development of philosophy, including incessant concern with the heaven, to begin observing the heavenly bodies. Mathematics developed greatly in these two centuries. And the observation of the heavenly bodies is essentially mathematical.

Should the evidence of the first two centuries of development of philosophy, being negative, appear inconclusive, the evidence of the next half century, being affirmative, cannot be inconclusive.

Aristotle concerns himself with the heaven as much as he concerns himself with anything. The heaven is a central concern of book XII of the *Metaphysics*, the central concern of the four books of *On the heaven*, and a central concern of the four books of the *Meteorology*. In all of these, and in all of the Aristotelean corpus, there are three observations of the heavenly bodies: we have seen the moon pass under Mars (*Heaven*, II, 2); Mercury rises only a little above the horizon (*Meteorology*, I, 6); and we have ourselves observed Jupiter coincide with one of the stars of Gemini (*id.*).

Aristotle writes after Eudoxus, and explicitly on the authority of Eudoxus. Eudoxus formulated the theory of the motions of the heavenly bodies in concentric spheres, and developed the theory of proportion. Yet Eudoxus, too, left behind no observation of the heavenly bodies. Eudoxus wrote *Phenomena* in which he described 32 northern and 15 southern constellations. Hipparchus wrote a commentary on Eudoxus's *Phenomena*. And Hipparchus, Ptolemy reports: found very few observations of fixed stars from before his time, and none earlier than those of Timocharis and Aristyllus. Timocharis and Aristyllus postdate Eudoxus by two-odd generations.

The observation of the heavenly bodies is essentially mathematical. And so is the observation of the surface of the earth.

Ptolemy writes: the description of the earth deals with quantities; mathematical method must everywhere lead the way (*Geography*, I, 1); and in order to draw the map of the earth necessary is systematic research, of which one part is survey, the other part is astronomical observation; and through both parts distance and orientation must be determined (2).

If any objects at all are given immediately to the senses, the heavenly bodies are, spots of light distinguished sharply from one another against the darkness of the night sky. And if the observation of these objects is essentially mathematical, the observation of any objects at all cannot not be essentially mathematical.

That the observation of the surface of the earth, in particular, cannot not be essentially mathematical, the fact that: the earth is very large and no single man can see it in whole or even in part, should suffice to show. And the fact that: distance and orientation must be determined, suffices to show it. Lest distance and orientation were seen, or heard.

That the observation of the surface of the earth is essentially mathematical Ptolemy shows.

And five and a half-odd centuries before Ptolemy, Herodotus already shows it. If Herodotus is the father of history, he is because he is the father of geography, and he is the father of geography because he keeps measuring and calculating. Systematic research begins with the *Histories*, one of whose two parts, astronomical observation, Herodotus has not discovered, but the other part, survey, he has discovered with both its indispensable parts, measurement and calculation. The determination of distance and orientation is one of the most important problems solved in the *Histories* – from the problem of the source of the Dnieper in the north (IV, 53) to the problem of the source of the Nile in the south (II, 31-3), and from the Atlantic in the west (I, 203) to the Indus in the east (IV, 44), including 1 012 localities in between – and it is, because Herodotus keeps measuring and calculating.

Three instances: Egypt is 3 600 stades wide along the coast and 7 920 stades long along the Nile from the sea to Elephantine (II, 9); the Red Sea is 40 days of rowing long and $\frac{1}{2}$ day wide (11); the Black Sea is the most marvelous of all seas; it is 11 100 stades long and 3 300 stades wide; the channel at its entrance, the Bosporus, is 120 stades long and 4 stades wide; the Bosporus issues into the sea of Marmara which is 1 400 stades long and 500 stades wide; this sea issues into the Dardanelles which are nearly 400 stades long and not more than 7 stades wide; the Darda-

nelles issue into the Aegean Sea; in a long day's journey a ship covers about 700 stades and in one night about 600 stades; from the Bosporus to the mouth of the Rioni, which is the greatest length of the Black Sea, the journey is 9 days and 8 nights, and that is, 11 100 stades, and from Sindice to Themiscyra, which is the greatest width of the Black Sea, the journey is 3 days and 2 nights, and that is, 3 300 stades; this is how I made these measurements (IV, 85-6).

And the dimensions of the most marvelous of all seas are,
north to south,
Histories between 500 and 700 km,
correction about 600 km,
west to east,
Histories between 1 700 and 2 200 km,
correction about 1 200 km.
It is not an accident either that Herodotus anticipates the fact that the determination of longitude is more difficult than the determination of latitude.

The observation of space is essentially mathematical. And so is the observation of time.

And it is observation.

To begin with, the 94 dated observations in the *Almagest* are just that, dated. Further, of the 94, 36 Ptolemy himself made, 4 his contemporary Theon of Smyrna made, the remaining 54 Ptolemy's predecessors made, back to the lunar eclipse observations in 721-0, eight and a half centuries before Ptolemy. And even if each of the observations made by Ptolemy himself were at the time of observation an observation of objects in space without being an observation of objects in time, and even if each of the observations made by his predecessors were at the time of observation an observation of objects in space without being an observation of objects in time, any one of the observations in relation to any other of the observations cannot not be an observation of objects in time. Any of Ptolemy's 36 observations in relation to any other of his 36 observations, and any of the 94 observations in relation to any other of the 94 observations. And it should be superfluous to add that there can be no such thing as an observation that is in relation to no other observation.

If the 94 are observations of objects in space, they cannot not be observations of objects in time, and if they are observations of space, they cannot not be observations of time.

Nor can observation be of objects in space without being of objects in time. Be of space without being of time.

Which explains why: Hipparchus, being a great lover of truth, did not even attempt to develop the theory of the other five planets because he did not have a store of accurate observations from earlier times to compare with his own.

Why, left a store of accurate observations from earlier times, Ptolemy developed the theory of the other five planets.

Why he writes: the longer the time between observations, the greater the accuracy with which periods of revolution can be determined; from the same error less inaccuracy results when it is distributed over a larger number of years; we must thus endeavor to increase the accuracy of our propositions to the degree to which the length of time between us and the most ancient accurate observations allows; we must not neglect to examine all such records (*Almagest*, III, 1).

Why Copernicus writes: we must follow the ancient philosophers and hold on to the observations which they have bequeathed us; to anyone who holds on to his own views, the gates of our art are closed (*Werner*). There are 78 dated observations in *On the revolutions of the heavenly spheres*, of the 78, 27 Copernicus himself made, the remaining 51 his predecessors made, and of these 51, 45 are from the *Almagest*.

Why Halley discovered the proper motion of stars.

Observation is not looking, or seeing, not even with determination and deliberation. It is the measurement and calculation of the seen, heard, touched, tasted, smelled.

The observation of the heavenly bodies is essentially mathematical. The observation of the localities on the surface of the earth is essentially mathematical. The observation of the 54 princes of the *Handy tables* is essentially mathematical.

And five and half-odd centuries before Ptolemy, Herodotus is the father of history because he does to objects in time what he does to objects in space. He determines the spatial distribution of

1 012 objects, and he determines the temporal distribution of 804 objects.

Herodotus has not seen, or heard, or touched, or tasted, or smelled any of these 804 objects. What he has done instead is reduce reports of their sighting, reports of reports of their sighting, and so on indefinitely, to measurement and calculation. With respect to most of them only implicitly. Yet explicitly with respect to many enough of them to make it possible to arrange all 804 of them in their temporal order. Moreover, to reduce the 54 princes of the *Handy tables* to absolute chronology Ptolemy has to deal with reports as excellent as the prince lists. To reduce localities to latitudes and longitudes he has to deal with reports as poor, though rich in other ways, as that of the men with tails. Herodotus has several of the prince lists. Overwhelmingly, however, he has to deal with reports much poorer, though much richer in other ways, than the one about the men with tails.

In space, he has arranged 1 012 particular objects in their order, north to south, west to east.

He notes also 1 008 particular objects which are either of the kind to which men with tails belong or of the kind to which men without tails belong. And the very fact that this distinction is possible in Herodotus – and it is impossible in Homer, and impossible in either the Yahwistic document or the Priestly code – attests the great success of Herodotus. 84 of the 1 008 are objects, like Zeus and Apollo, of which it is readily established that they are not real. 120 are objects, like Lycurgus and Aristeas, of which it is more difficult to establish whether they are real. 804 are objects, like Cyrus and Thales, of which it is readily established that they are real. Because Herodotus has already established it. Zeus is not and cannot be committed to a particular place and a particular time. The evidence which has to decide whether Lycurgus can be committed to a particular place and a particular time, and hence whether he is real, is difficult to sort out. Cyrus is real, the relevant evidence Herodotus has already conclusively sorted out.

The foundation are the prince lists. Herodotus lists the Lydian princes, the Mede princes, the great kings, the Egyptian princes, the Spartan princes, all in their series, all with the lengths of their reigns.

He proposes to solve problems as difficult as these.

From the Spartan prince list, he calculates: Heracles was 900 years before my time and no more (II, 145).

It is not a good result. Pace Herodotus, Heracles is not real.

He calculates: Homer and Hesiod were 400 years before my time and no more (53).

It is an excellent result. Homer and Hesiod are real, and Herodotus solved the problem of when they were.

He reports: the first prince was Min (II, 99). Herodotus has listed the six Egyptian princes beginning with Psamtik who reigned before the conquest of Egypt by Cambyses: for a total of 146 years (157-61; III, 10-4). And he calculates: the Egyptian priests counted 341 generations of princes before Psamtik; 3 generations are 100 years, so 300 generations are 10 000 years, and 41 generations are 1 340 years, for a total of 11 340 years (II, 142).

It is an excellent result, and a most important result. The first prince is real, and the first prince is Min, or Narmer. Herodotus's is the first attempt to determine when the first prince was, and that it is a successful attempt the fact that it can be corrected shows.

The first prince was

Histories	*c.* 12 011^{BC},
correction	*c.* 3 000^{BC}.

In time, Herodotus has arranged 804 particular objects in their order, from the first prince to Artaxerxes, prince 465-24.

Observation is essentially mathematical. And Ptolemy took a major step toward the discovery of the importance of that most mathematical of observations, the experiment, too. His *Optics* is the first treatise whose results rest substantially on experiment. With a disc with a graduated circumference, for instance, he demonstrates that the angles of the incident and reflected rays of light to the surface of reflection are equal (III, 7-12), and measures the angles of the incident and refracted rays from air to water,

10°	8°,
20°	15° 30′,
30°	22° 30′,

40° 28°,
50° 35°,
60° $40^\circ 30'$,
70° $45^\circ 30'$,
80° 50° (V, 7-12),

and from air to glass (13-8), and from water to glass (19-22), anticipating the law, $\frac{\sin i}{\sin r}$ is constant, where i and r are the angles of the incident and refracted rays to the perpendicular to the surface dividing the media, discovered by Snel in 1621.

If observation is essentially mathematical, this conclusion drawn from Ptolemy corroborates these conclusions drawn from Aristotle's investigation of knowledge.

The relation of man to knowledge is none whatsoever.

Knowledge has been acquired by the labors of the philosophers and these predecessors of theirs of whom they are the successors, exclusively.

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As to how Ptolemy informs the problem before the investigation of knowledge, the following conclusions can also be drawn.

The two parts of the problem are the acquisition of knowledge and knowledge, and the two parts must be separated sharply from one another.

The problem of the acquisition of knowledge includes the problems of its beginning, of collecting it in its entirety, of its relation to the development of philosophy, and of its kinds.

The problem of knowledge includes the problems of its relation to truth and of its relation to the world.

And to begin at the end, Ptolemy informs the problem of the relation of knowledge and the world decisively.

The acquisition of knowledge is a development. A development from no inkling to finality. From not knowledge to knowledge. If it is not clear, however, what is knowledge, it cannot be clear what the objects of the acquisition of knowledge are, or can be, or should be.

So the first philosophers concerned themselves with water, and the earth, numbers, and the side and diagonal of the square, the morning star, and eclipses. Pythagoras concerned himself also with the transmigrations of the soul. Xenophanes concerned himself with the gods. Gorgias concerned himself with speeches. Herodotus concerned himself with the great kings. Socrates concerned himself with myself. Plato concerned himself with piety, courage, friendship, temperance, beauty, justice, and virtue.

Aristotle collected all the concerns of his predecessors. And ordered them. Into the world.

The world is one, there is nothing next to the world, or outside the world. If so, the world cannot not be the terminal object of the acquisition of knowledge. And if so, the world cannot not decide also which are, and which are not, the objects on the way toward the terminal object. It cannot not decide what the order of the objects is either. It cannot not decide what their system is. Hence, concerned with the world, Aristotle collected, ordered, and systematized all the concerns of his predecessors, and he can confidently presume the completeness of his labors: we have dealt with the first principles, with natural motion, with the heavenly bodies and their order, and with the elements; we have now to deal with the phenomena in the region closest to the heavenly bodies; and when we next deal with animals and plants, our task will be complete.

The world is one, at the circumference is the first mover, it moves the stars, further inside are the planets, further inside are fire, air, water, and the earth, on the surface of the earth are the animals, and one of the animals is the political animal.

Aristotle first described the world.

The description of the world, however, is vague in Aristotle.

It quit being vague in Ptolemy.

Book IV of the *Epitome of Copernican astronomy*, the one whose subtitle is *a supplement to Aristotle's On the heaven*, and in whose preface Kepler tells his reader: I want Aristotle for my reader, is *On the position, order, and motion of the parts of the world, or on the system of the world*.

Whatever else the system of the world is, if it is anything else, the position, order, and motion of the parts of the world is the system of the world.

This is vague in Aristotle. The planets are 7, the spheres in which they move are 55. In the center, the radius of the earth is 400 000 stades. On the surface of the earth there are not less than 600 kinds of animals. And the political animal – to judge by the fact that Aristotle collected the constitutions of 158 cities – has congregated in not less than 158 cities. Everything else is vague. The position of the parts of the world is vague, the order of the parts of the world is vague, and the motion of the parts of the world is vague. It is thus very vague what the relations of the parts are. Vaguest of all is thus the most important relation, which parts are important, which parts are not. Which parts are very important, which parts are important but not very important, which parts are not important, and which parts are so not important that they are best ignored. It is the main reason all too many of his successors have not found it difficult to suppose the physicist *par excellence* Aristotle an ethicist.

Ptolemy describes the position, order, and motion of the parts of the world explicitly. He describes explicitly the position, order, and motion of 1 022, not counting the sun, of the major parts of the world. Further inside he describes explicitly the position, order, and motion of 7, counting the sun, of the lesser parts of the world, and in their relations to the position, order, and motion of the 1 022 major parts, for a total of 1 029 parts. Further inside he describes explicitly the position, order, and motion of 6 345 of the yet lesser parts of the world, and again in their relations to the position, order, and motion of the 1 029 greater parts, for a total of 7 374 parts. He describes explicitly as well the position, order, and motion of 54 of the yet lesser, although still not too minor, parts of the world, and once again in their relations to the position, order, and motion of the 7 374 greater parts, for a total of 7 428 parts.

Or what is the same, for a total of 7 428 x, y, z, t systems of values.

Ptolemy took the first and decisive step toward describing the world in its true proportions.

Thus, what Aristotle shows vaguely, Ptolemy shows explicitly, and conclusively. The world is the terminal object of the acquisition of knowledge. And if so, the relation of knowledge and the world is necessity. Knowledge is necessarily in relation to the world.

Hence, Ptolemy informs the problems of the kinds of the acquisition of knowledge and of the relation of the acquisition and the development of philosophy decisively, too.

He begins in the *Almagest*: the true philosophers are right to separate the theoretical kind of philosophy from the practical; for even if the practical is theoretical before it becomes practical, there is a great difference between the two; ethical virtues many can have without study, but theory no one can have without instruction (I, 1).

Theory no one can have without instruction.

Ptolemy shows also who the instructors are. Of the theory developed in the *Almagest* Hipparchus, two and a half-odd centuries before Ptolemy. Of the theory developed in the *Geography* Marinus.

Ptolemy continues: and Aristotle is right to divide theoretical philosophy into physics, mathematics, and theology; theology is concerned with the first cause of the motion of the world, which can only be conceived as an invisible and motionless god, somewhere at the end of the world, detached entirely from the senses; physics is concerned with matter as it is always in motion; mathematics is concerned with form and with locomotion, and studies shape, number, size, place, time; its object is thus intermediate between the objects of the other two, it is conceived both with and without the senses; and mathematics is concerned with what all things without exception, mortal and immortal, have, for the things that always change have form which changes with them, the things that do not change have form which does not change (*id.*).

Philosophy has developed. On the side of practical philosophy, in Aristotle the master art is politics. In the four and a half-odd centuries between him and Ptolemy, ethics has become the master art. On the side of theoretical philosophy, in Aristotle there is, next to physics and mathematics, a hypothetical third kind which is to find out whether there is a substance other than those in nature. In the centuries between him and Ptolemy, the hypothetical kind has become actual. An invisible and motionless god, somewhere at the end of the world, detached entirely from the senses, has provided the ground of the

actuality.

Ptolemy continues: we have concluded that the first two kinds of theoretical philosophy are surmise and not knowledge, theology because its object is invisible and ungraspable, physics because its object is unstable and unclear; mathematics alone is certain and unshakable knowledge; its proofs proceed by the unquestionable methods of arithmetic and geometry; and mathematics can deal with the objects of the other two kinds, too, with those of theology because it studies the unchanging, and with those of physics because everything material can be understood by its locomotion; even where action and character are concerned, mathematics above all can make men see clearly (*id.*).

Philosophy includes theoretical philosophy, theoretical philosophy includes mathematics, and mathematics alone is knowledge.

On the other hand, theoretical philosophy includes also physics and theology, and these are surmise, not knowledge. Philosophy includes also practical philosophy.

On the one hand, then, the development of philosophy includes the acquisition of knowledge. The development of philosophy is the labors of the philosophers. And if the philosophers are collected, from the first to the last, the acquisition of knowledge will be collected, included in the labors of the philosophers from the first to the last.

On the other hand, the development of philosophy includes more than the acquisition. The labors of the philosophers have to be sorted out, into these that are the acquisition of knowledge, and these that are not. If mathematics is knowledge, and it can deal with the objects of physics and theology, physics and theology have to be sorted out. If mathematics is theoretical, and the practical is theoretical before it becomes practical, politics also, including ethics, has to be sorted out.

To the solution of this problem, as well as to the solution of the problem of the kinds of the acquisition, mathematics has to be the key. Because of its clarity and distinction, and generality. Its object is conceived both with and without the senses. It is concerned with what all things without exception have, form, the things that always change have form which changes with them, the things that do not change have form which does not change. Hence, mathematics can deal with the objects of every kind of theoretical philosophy. It can even make men, unlikely as they are to let it make them, see clearly.

And these propositions – the object of mathematics is conceived both with and without the senses, mathematics is concerned with what all things without exception have, form, and hence mathematics can deal with the objects of every kind of theoretical philosophy – and hence the clarity and distinction, and generality, of mathematics, Ptolemy has shown with the mathematical description of the world. Or what is the same, with the mathematical description of the terminal object of the acquisition of knowledge.

Of the problems of collecting the acquisition in its entirety and of its beginning Ptolemy provides partial solutions.

He writes: we must endeavor to increase the accuracy of our propositions to the degree to which the length of time between us and the most ancient accurate observations allows; we must not neglect to examine all such records.

And it cannot be doubted that Ptolemy did not neglect to examine all such records. For it cannot be doubted that he found the most ancient accurate observations. The most ancient accurate observation of an eclipse. The most ancient accurate observation of a solstice. The most ancient accurate observation of a star. And the most ancient accurate observation of a planet. The record has been combed, and *ad nauseam*, and these are the most ancient accurate observations.

In fact, the enumeration of the 94 dated observations of the heavenly bodies in the *Almagest* is, in accordance with the last rule of the method, complete. It is the enumeration of all accurate observations of the heavenly bodies through and including Ptolemy. Should the negative argument that the record has been combed, and *ad nauseam*, and these are all accurate observations, appear inconclusive, there is an affirmative argument. From the eight and a half centuries from himself back to the most ancient accurate observation, Ptolemy succeeded in collecting 54 accurate observations. From the fourteen centu-

ries from himself back to Ptolemy, Copernicus succeeded in collecting 6 accurate observations.

The enumeration collects the accurate observation of the heavenly bodies through and including Ptolemy in its entirety. Nor is there any vagueness in the collection. It is the collection of 94 particular objects, distributed over the labors of 18 particular predecessors of the philosophers and philosophers.

Accurate observation of the heavenly bodies begins in 721. Again, there is no vagueness. The collection is of 1 particular object, distributed over the labors of 1 particular predecessor of the philosophers.

And as the accurate observation of the heavenly bodies, its beginning and its entirety, in particular, so the acquisition of knowledge, its beginning and its entirety, in general.

About the problem of the relation of knowledge and truth, finally, this can be said. Ptolemy's stock epithet – Homeric fashion – for Hipparchus is: a lover of truth. To borrow the epithet, Ptolemy is himself a lover of truth. It is relevant to the problem whether a philosopher loves truth. Only to the degree, however, to which truth loves him back. As truth does Hipparchus and Ptolemy.

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In the two and a half centuries after Ptolemy the darkest of dark ages descended.

At length the acquisition of knowledge emerged from it. Then the acquisition accelerated, and more, and more. Columbus crossed the Atlantic in 1492. Copernicus wrote *On the revolutions of the heavenly spheres*, in 1543. A couple of generations later the generation of Viète, Gilbert, Tycho, and Bruno followed. And in the next generation, fifteen centuries after Ptolemy, the brightest period in the acquisition of knowledge began. Counting from the generation of Bacon, Galileo, and Kepler through the generation of Lavoisier, and Laplace, it continued for six generations. And the first three of these six generations are the brightest of the brightest.

It is the brightest period in the investigation of knowledge as well, and in this the most important are the labors of Bacon, Descartes, Hume, and Kant. Again, the investigations of Bacon and of Descartes are causes of the following success of the acquisition, the investigations of Hume and of Kant are effects of its preceding success.

Thus, the investigations of Bacon and of Descartes, and of their successes in the acquisition of knowledge, and of the successes of Galileo and of Kepler in the generation of Bacon, and of Huygens and of Spinoza in the generation after Descartes, and of Newton and of Leibniz in the following generation, in short, the greatest successes in the three brightest generations of the brightest period of the acquisition of knowledge and the investigation of knowledge, can best show the true way.

The review of the problems solved in the investigation of knowledge by Hume and by Kant, and of how these problems are informed by Darwin and by Frege, can best show what the problem before the investigation of knowledge is.

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Before turning to the investigations of Hume and of Kant, however, here is, again, how and why Hume became a sceptic, Kant became an idealist.

Other philosophers the restoration of spirit to the substantiality and purity of its being could revile as mathematicians, or scientists, or misled by mathematicians or scientists. Not these two. And not because they are not mathematicians, or scientists, or misled by mathematicians or scientists. They are, too.

Because of their generality. Hume and Kant do not only follow the true way. They greatly advance the true way.

And because of their enlightenment. *What is enlightenment?* Kant asks, and replies: enlightenment is man's emergence from his self-imposed minority; minority is the inability to use one's own understanding without the direction of another; it is self-imposed when its cause is not lack of understanding but lack of resolution and courage; *sapere aude!* have the courage to use your own understanding! is thus the motto of enlightenment (*id.*).

These two philosophers spirit had to coopt or to destroy. Christian doctrine, when it became philosophy, first destroyed Aristotle, losing all memory of him, then coopted him, assimilating him into itself. Spirit coopted Kant, the idealist,

and destroyed Hume, the sceptic.

Hume writes: a philosopher at last arose who determined the laws and forces by which the revolutions of the planets are governed; the like has been performed with regard to other parts of nature; and there is no reason to despair of equal success in our inquiries concerning the mental powers (*Inquiry*, 1).

Kant writes: the true method of metaphysics is in its foundation the same as that introduced by Newton in natural science with such benefit to it; it is to seek, by experiments, and with the help of geometry, the rules by which the phenomena of nature occur.

Hegel writes: Newton contributed the most to that way of philosophizing which is doing physics without thinking; Newton is a complete barbarian.

As to Hume: Hume is a sceptic. In particular: Hume extends his scepticism to proofs of the existence of god; but this lacks real ground; the conclusion rests on subjectively built concepts (*Hist. philosophy*, III, 2.2.A.2).

And Kant: Kant is an idealist. In particular: in the last period thought comes back to god; and this is what we have to consider in Kant (2.2.C.4).

Newton has since been a complete barbarian, or synonymously scientist. Hume has been a sceptic. Kant has been an idealist.

If Hume were a sceptic, his reasonings would be both strong and weak. His *attempt to introduce the experimental method in moral subjects* would be a failure.

Hume writes: should it be asked me, whether I be really one of those sceptics who hold that all is uncertain, I should reply that this question is entirely superfluous; neither I, nor any other person, was ever sincerely and constantly of that opinion (*Treatise*, I, 4.1); my intention is only to make the reader sensible of the truth of my hypothesis (*id.*); sceptical reasonings, were they possible, would be both strong and weak, the enemy of reason is obliged to take shelter under her protection (*id.*).

Hume's reasonings are strong. And his *attempt to introduce the experimental method in moral subjects* is a success.

He writes: we propose to explain the principles of human nature (intr.); we propose a complete system of the sciences (*id.*); the system is built on a foundation on which the sciences can stand with any security (*id.*); we hope to establish a science which will not be inferior in certainty to any other of human comprehension (*id.*).

After the *Treatise*, Hume wrote the *Essays, moral, political, and literary*. It is here that he writes: great philosophers are so rare, there have not yet been above two in the world, Galileo and Newton.

After the *Essays*, he wrote the *Inquiry*. He begins: it cannot be doubted that there is truth and falsehood, and truth and falsehood which lie not beyond the compass of human understanding (1), and ends: does it contain abstract reasoning concerning quantity and number? no; does it contain experimental reasoning concerning matter of fact and existence? no; commit it then to the flames, for it can contain nothing but sophistry and illusion.

After the *Inquiry*, he wrote the six volumes of *The history of England*. Does it contain experimental reasoning concerning matter of fact and existence? Yes.

In 1776 Hume died.

Adam Smith visited him shortly before he died. Adam Smith reports: he was reading in Lucian's *Dialogues of the dead* all the excuses urged to Charon for not entering his boat; he said, I might say to Charon, good Charon, I have been correcting my works for a new edition, allow me a little time to see how the public receives the alterations; but Charon would answer, when you have seen the effect of these, you will be for making other alterations, there will be no end to such excuses, so, honest friend, please step into the boat; but I might still urge, have a little patience, good Charon, I have been endeavoring to open the eyes of the public, if I live a few years longer, I may have the satisfaction of seeing the downfall of some of the prevailing systems of superstition; but Charon would then lose all temper and decency, you loitering rogue, he would say, that will not happen these many hundred years, do you fancy I will grant you a lease for so long a term? get to the boat this instant, you lazy loitering rogue (to Strahan, 9 Nov 1776).

If Kant were an idealist, he would establish that all knowledge from sense and experience is nothing but sheer illusion, and only in the ideas of pure understanding and reason is there truth. His attempt: to change the ways of metaphysics by undertaking, following the example of the geometers and the investigators of nature, its complete revolution, would be a failure.

Kant writes: my so-called, properly critical, idealism overturns genuine idealism; the principle of all genuine idealism is, all knowledge from sense and experience is nothing but sheer illusion, and only in the ideas of pure understanding and reason is there truth; the principle which throughout rules my idealism is, on the contrary, all knowledge of things merely from pure understanding or pure reason is nothing but sheer illusion, and only in experience is there truth.

Kant established that all knowledge of things merely from pure understanding or pure reason is nothing but sheer illusion, and only in experience is there truth. And his attempt: to change the ways of metaphysics by undertaking, following the example of the geometers and the investigators of nature, its complete revolution, is a success.

Before the *Critique*, Kant wrote the *General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws*. The theory formulated by Kant, and more explicitly by Laplace, remains the theory of the origin of the solar system.

In the *Critique*, Kant notes Hume: the famous, the irreproachable, the geographer of human reason Hume (II, 1.2), and: if one were to ask the cool-headed David Hume, what moved you to undermine the conviction, so comforting to men, that their reason is adequate for the assertion of a highest being? he would answer, nothing but the desire to help reason know itself (*id.*).

After the *Critique*, Kant wrote the *Prolegomena*. He now notes Hume very vigorously: Hume first interrupted my dogmatic slumber, and gave my investigations an entirely new direction (pr.), in fact: the *Critique of pure reason* is the solution of Hume's problem (*id.*), and this is a matter of course, given that: not to carry the use of reason dogmatically beyond the field of all possible experience is Hume's principle (58).

However, Kant reports: Hume was understood by no one (pr.), and: I fear that the solution of Hume's problem in the *Critique of pure reason* will fare as the problem itself, it will be misjudged because misunderstood (*id.*).

By that time this was an empirical judgment. The readers of the *Critique* had availed themselves of the opportunity to provide Kant with *A specimen of a judgment on the Critique prior to its examination*. In particular: it is as though someone, who had never seen or heard of geometry, found a copy of Euclid and, having noticed many figures while leafing through it, gave his judgment, it is a textbook of drawing, but the author uses a peculiar language in order to give dark, incomprehensible instructions which in the end teach nothing more than what everyone with a good eye can readily do (*id.*).

After the *Prolegomena*, Kant wrote the *Idea for a general history with a cosmopolitan purpose*. It is here that he concludes that nature will produce for history, too, a Kepler and a Newton.

After the *Idea*, he wrote *What is enlightenment?* After that, the *Groundwork for the metaphysics of morals*.

He then prepared the second edition of the *Critique of pure reason*. All revisions aim at, and include, the *Refutation of idealism* (I, 2.1.2.3).

Kant's last very important treatise is *Toward perpetual peace*, in 1795.

In 1797, Fichte published the *First introduction in the theory of science* and the *Second introduction in the theory of science*. He writes in the *First*: seeing that since the appearance of the Kantian critiques the fundamental change in philosophy, and hence in all science, which that great man envisioned, has been a complete failure because none of his numerous followers sees what is actually at issue, the author of the theory of science, believing to know this, has determined to dedicate his life to an independent of Kant presentation of that great discovery, and he will not relent in this determination (pr.). He writes in the *Second*: the philosopher's intuition of himself, in the act in which the I arises for him, I call intellectual intuition (5), and: intellectual intuition is the only firm standpoint of all philosophy (*id.*). It is, because Fichte has already found, in

the *Foundation of the entire theory of science*, the three fundamental principles: the I posits the I; the I posits the not I; and I posit in the I, against the divisible I, a divisible not I (I, 1-3).

Also in 1797, Kant writes: I assure you, as I am an honest man, my views are infinitely removed from this Fichtean nonsense (to Beck, 24 Jun).

In 1804 Kant died.

Hegel lectured on the history of philosophy from 1805 to his death in 1831. He writes: the general sense of Kant's philosophy is that determinations like generality and necessity cannot be found in the senses, as Hume had shown; they have therefore a source other than the senses, and this source is the subject, the I in my self-consciousness; this is the fundamental principle of Kant's philosophy (III, 3.B), and: the completion of Kant's philosophy is Fichte's philosophy (3.C).

Schopenhauer, finally, writes: Kant's greatest merit is that he distinguished the phenomenon from the thing in itself; and it is remarkable that he did not derive the mere relative existence of the phenomenon from the simple, near at hand, incontrovertible truth, there is no object without subject; Berkeley, to whose merit Kant does no justice, had made this important principle the foundation stone of his philosophy; I explain Kant's avoidance of Berkeley's principle by his visible fear of decisive idealism (*World*, I, app.). And, as he reports in some detail, Schopenhauer undertook a campaign to have the second edition of the *Critique*, which had been in circulation from 1787 to 1838, replaced in circulation by the first edition. All of Kant's revisions in the second edition aim explicitly at the refutation of idealism. Kant had thus, on Schopenhauer's authority: despoiled and disfigured the *Critique* (*id.*).

Hume writes in the *Treatise*: the victory is not gained by the men at arms who manage the pike and the sword, but by the trumpeters, drummers, and musicians of the army (intr.).

Hume managed the pike, Kant managed the sword.

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Hume's very important treatises are these,

A treatise of human nature, being an attempt to introduce the experimental method of reasoning into moral subjects, in 1739,

Essays, moral, political, and literary, in 1741,

An inquiry concerning human understanding, in 1748,

The history of England, in 1754-62,

Four dissertations; I. The natural history of religion; II. Of the passions; III. Of tragedy; IV. Of the standard of taste, in 1757,

Dialogues concerning natural religion, in 1779.

The *Essays* published first in 1741, Hume edited and published several more times, in whole and in part, adding and subtracting essays. The *Four dissertations*, in 1757, is one of these publications.

Hume wrote also *An abstract of a book lately published, entitled A treatise of human nature, etc., wherein the chief argument of that book is farther illustrated and explained*, in 1740, *A letter from a gentleman to his friend in Edinburgh, containing some observations on a specimen of the principles concerning religion and morality, said to be maintained in a book lately published, entitled A treatise of human nature, etc.*, in 1745, *An inquiry concerning the principles of morals*, in 1751, *A concise and genuine account of the dispute between Mr Hume and Mr Rousseau, with the letters that passed between them during their controversy*, in 1766, *My own life*, in 1776.

Knowledge Hume investigates in the *Treatise* and the *Inquiry*.

The *Treatise* is in three books. Book I is *Of the understanding*, book II *Of the passions*, book III *Of morals*. Book I, *Of the understanding*, is in four parts, part one is *Of the ideas, their origin, composition, abstraction, connection, etc.*, part two *Of the ideas of space and time*, part three *Of knowledge and probability*, part four *Of the sceptical and other systems of philosophy*. Book II, *Of the passions*, is in three parts, part one is *Of pride and humility*, part two *Of love and hatred*, part three *Of the will and direct passions*. Book III, *Of morals*, is in three parts, part one is *Of virtue and vice in genera*l, part two *Of justice and injustice*, part three *Of the other virtues and vices*.

This is how Hume deals with the passions: anatomists join their observations and experiments on human bodies to those on beasts, and

from the agreement of these experiments derive an additional argument for any particular hypothesis; let us, therefore, apply this method of inquiry to our anatomy of the mind (II, 1.12), and with morals: to put the affair to trial, let us suppose that by the dropping of its seed, an oak or elm produces a sapling below it, which, springing up by degrees, at last overtops and destroys the parent tree; I ask, if in this instance there be any relation wanting, which is discoverable in parricide or ingratitude (III, 1.1)?

And this is the conclusion he draws: every animal that has sense, appetite, and will must be susceptible of all the same virtues and vices for which we ascribe praise and blame to human creatures (*id.*).

The *Treatise*, Hume reports: fell deadborn from the press, without reaching such distinction as to excite a murmur among the zealots (*Life*).

When he returned to the investigation of knowledge with the *Inquiry concerning human understanding*, the investigation is not encumbered by the passions or by morals. The problems formulated and solved in book I of the *Treatise* are formulated and solved better yet. In general, the *Inquiry* is a treatise of unsurpassed clarity and distinction.

Hume writes: all objects of human reason or inquiry may naturally be divided into two kinds, relations of ideas and matters of fact (4.1); of the first kind are the sciences of geometry, algebra, and arithmetic, and, in short, every affirmation which is either intuitively or demonstratively certain; that the square on the hypothenuse is equal to the squares of the two sides, expresses a relation between these figures; that three times five is equal to the half of thirty, expresses a relation between these numbers; propositions of this kind are discoverable by the mere operations of thought, without dependence on what is anywhere existent in the universe (*id.*); but matters of fact, that the sun will rise tomorrow, for instance, are not ascertained in the same manner, nor is our evidence of their truth, however great, of a like nature (*id.*); the contrary of every matter of fact is possible, because it can never imply a contradiction; that the sun will not rise tomorrow is not less intelligible a proposition and implies not more a contradiction than the affirmation that it will rise (*id.*); if we anatomize reasonings concerning matters of fact, we shall find that they are all founded on the relation of cause and effect; by means of this relation alone we can go beyond the evidence of our memory and senses (*id.*); but every effect is a distinct event from its cause; it cannot, therefore, be discovered in the cause *a priori* (*id.*); knowledge of the relation of cause and effect arises entirely from experience (*id.*); Adam, though his rational faculties be supposed entirely perfect, cannot have inferred from the fluidity and transparency of water that it will suffocate him, or from the light and warmth of fire that it will consume him (*id.*).

All objects of human reason or inquiry may naturally be divided into two kinds, relations of ideas and matters of fact. Accordingly, all reasonings are of two kinds as well.

Of the first kind are the sciences of geometry, algebra, and arithmetic, and, in short, every affirmation which is either intuitively or demonstratively certain. Propositions of this kind are discoverable by the mere operations of thought, without dependence on what is anywhere existent in the universe.

Of the second kind are reasonings concerning matters of fact.

There is thus the problem of the relation of the two kinds of objects, relations of ideas and matters of fact, and of the relation of the two kinds of reasonings, those discoverable by the mere operations of thought, without dependence on what is anywhere existent in the universe, and those concerning matters of fact.

And the solution of this problem cannot not follow from the solution of the problem of the foundation of reasonings concerning matters of fact. Because Hume solved this problem conclusively, and in all its generality.

All reasonings concerning matters of fact are founded on the relation of cause and effect.

All reasonings.

By means of this relation alone we can go beyond the evidence of our memory and senses.

We includes the most ignorant and stupid peasants, nay infants, nay brute beasts: the most ignorant and stupid peasants, nay infants, nay brute beasts, improve by experience and learn the qualities of natural objects by observing the

effects which result from them (4.2).

It includes Adam: Adam cannot have inferred from the fluidity and transparency of water that it will suffocate him, or from the light and warmth of fire that it will consume him; no object ever discovers by the qualities which appear to the senses either the causes which produced it or the effects which will arise from it (4.1).

And it includes the ultimate effort of human reason: it is confessed that the ultimate effort of human reason is to reduce the principles productive of natural phenomena to a greater simplicity, and to resolve the many particular effects into a few general causes by means of reasonings from analogy, experience, and observation (*id.*).

It thus includes Galileo and Newton.

It includes Hume.

After Hume it includes Maxwell. Maxwell writes, in 1877: the general maxim of physical science is, the same causes will always produce the same effects (*Matter*, 19).

And it includes Heisenberg. Heisenberg writes, in 1958: when we make an experiment we have to assume a causal chain of events that leads from the atomic event through the apparatus to the eye of the observer; if this causal chain were not assumed, nothing could be known about the atomic event (*Philosophy*, 5).

All reasonings concerning matters of fact are founded on the relation of cause and effect, and knowledge of this relation arises entirely from experience.

Entirely: no object ever discovers by the qualities which appear to the senses either the causes which produced it or the effects which will arise from it (*Inquiry*, 4.1); every effect is a distinct event from its cause; it cannot, therefore, be discovered in the cause *a priori* (*id.*); nor can our reason, unassisted by experience, ever draw any inference concerning real existence or matter of fact (*id.*).

So: when it is asked, what is the nature of all our reasonings concerning matter of fact? the proper answer seems to be, they are founded on the relation of cause and effect (4.2); when again it is asked, what is the foundation of all our reasonings and conclusions concerning that relation? it may be replied in one word, experience (*id.*); but if we still carry on our sifting humor and ask, what is the foundation of all conclusions from experience? this new question may be of more difficult solution (*id.*).

It may be. Hume yet satisfied our sifting humor: all inferences from experience are effects of custom, not of reasoning (5.1); the operation of the mind by which we infer like causes from like effects is so essential to the subsistence of all human creatures, it is not probable that it could be trusted to the fallacious deductions of our reason, which is slow in its operations, appears not in any degree in the first years of infancy, and is in every age extremely liable to error (5.2); it is more conformable to the ordinary wisdom of nature to secure so necessary an act of the mind by some instinct, or mechanical tendency, which may be infallible in its operations, may discover itself at the first appearance of life and thought, and may be independent of all the labored deductions of the understanding (*id.*).

And, again, the problem is solved in all its generality, brute beasts to Adam to infants to the most ignorant and stupid peasants to Galileo, Newton, Hume, Maxwell, and Heisenberg: in inferences concerning matters of fact animals are not guided by reasoning, neither are children, nor are the generality of mankind in their ordinary actions and conclusions, nor are the philosophers, who in the active parts of life are in the main the same as the vulgar and governed by the same maxims (9); experimental reasoning itself, which we possess in common with the beasts, and on which the whole conduct of life depends, is nothing but a species of instinct, or mechanical power (*id.*).

The foundation of all conclusions from experience, and thus the foundation of all reasonings concerning cause and effect, and thus the nature of all reasonings concerning matters of fact, is custom, or instinct, or mechanical tendency, or mechanical power. All reasonings concerning matters of fact are the effects of custom, or instinct, or mechanical tendency, or mechanical power. Custom, or instinct, or mechanical tendency, or mechanical power, is their cause.

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Having proposed: to explain the principles of human nature, Hume has done exactly that.

Hence, having proposed: to establish the foundation on which the sciences can stand with any security, he has done exactly that, too.

This is the most important problem Hume solved. He established the foundation on which the acquisition of knowledge stands with any security.

The foundation is experience.

And the acquisition of knowledge stands on experience with any security. With ultimate security, with universal security, and with absolute security.

Galileo, Newton, Hume, Maxwell, and Heisenberg stand on the foundation on which beast, including Adam, stands.

Before making all this more explicit, however – what experience is, that experience is the foundation of the acquisition of knowledge, and that experience is the foundation on which the acquisition of knowledge stands with ultimate security, universal security, and absolute security – from the solution of the problem of the foundation of reasonings concerning matters of fact, and hence of the foundation of the acquisition of knowledge, the solution of the problem of the relation of the two kinds of objects of human reason or inquiry, relations of ideas and matters of fact, and of the relation of the two kinds of reasonings, those discoverable by the mere operations of thought, without dependence on what is anywhere existent in the universe, and those concerning matters of fact, follows.

Pace Hume, nothing is discoverable by the mere operations of thought, without dependence on what is anywhere existent in the universe.

With Hume: all perceptions of the human mind resolve themselves into two kinds, impressions and ideas; the difference consists in the degrees of force and liveliness with which they strike the mind (*Treatise*, I, 1.1); if we examine those arguments which are supposed to be the mere effects of reasoning and reflection, they will be found to terminate at last in some general principle or conclusion for which we can assign no reason but observation and experience (*Inquiry*, 5.1); all our ideas are copied from our impressions (*Treatise*, I, 3.1).

However fundamental the difference between impressions and ideas, it is more fundamental that ideas are copied from impressions. However fundamental the difference between the two kinds of objects of human reason or inquiry, relations of ideas and matters of fact, it is more fundamental that reasoning and reflection terminate in observation and experience, and relations of ideas terminate in matters of fact. And however fundamental the difference between affirmations which are intuitively or demonstrably certain and inferences from experience, it is more fundamental that such affirmations must terminate in inferences from experience.

Nor can Galileo, and mathematical physics, Newton, and the mathematical principles, Maxwell, and the equations of electrodynamics, Heisenberg, and matrix mechanics, stand on a foundation on which mathematics does not stand.

With Kant: inadvertently, Hume severed from knowledge a whole and indeed its most valuable province, pure mathematics (*Prolegomena*, 2).

What Hume did inadvertently, moreover, other philosophers have been doing with studied deliberation.

Hume writes: it is usual with mathematicians to pretend that those ideas which are their objects do not fall under the conception of the fancy, but must be comprehended by pure and intellectual view of which the superior faculties of the soul alone are capable (*Treatise*, I, 3.1); the same notion runs through most parts of philosophy; it is used to explain our abstract ideas, and to show how we can form an idea of a triangle, for instance, which would be neither isosceles nor scalene nor confined to any particular length or proportion (*id.*); it is easy to see why philosophers are so fond of this notion of some spiritual and refined perception; by this means they cover many of their absurdities, and refuse to submit to the decision of clear ideas by appealing to such as are obscure and uncertain (*id.*); but to destroy this artifice, we need but reflect on the principle that all our ideas are copied from our impressions (*id.*).

All our ideas are copied from our impressions.

Mathematics is empirical. And Kant will show this conclusively.

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The foundation of all reasonings is experience.

And what experience is can be made more explicit.

In book I of the *Metaphysics* Aristotle describes the development: all animals have perception; from perception memory develops in some of them; and from memory experience develops; although animals other than man have little interconnected experience.

In the *Posterior analytics* the same development is the answer to the question: but how do we acquire principles without knowing them to begin with? The answer is: clearly, we have some faculty to discover them; clearly as well, all animals have this faculty; for they all have the faculty of discrimination which is perception; from perception comes memory; from memory, when it is in relation to the same thing, comes experience; and experience is the beginning of art and of knowledge.

Experience is the outcome of development.

It is the outcome of the development of faculties and their exercise.

It is the outcome of the development of faculties and their exercise in all animals, even if animals other than man have little interconnected experience.

Hume's *Treatise* is *of human nature*. He writes in the introduction: it is evident that all sciences have a relation to human nature; even mathematics, natural philosophy, and natural religion are in some measure dependent on the science of man, since they lie under the cognizance of men, and are judged by their powers and faculties; the only expedient, then, from which we can hope for success in our philosophical researches is to leave behind the tedious lingering method, which we have hitherto followed, and instead of taking now and then a castle or village on the frontier, to march directly to the capital of these sciences, to human nature itself (*id.*).

And instead of taking a castle or village on the frontier, Hume marched directly to the capital, and took it.

As far as morals are concerned, man shares parricide and ingratitude with oak and elm. As far as the passions are concerned, observations and experiments on beasts show the anatomy of the human mind. As far as reasonings are concerned, they are the effects of instinct, or mechanical tendency, in man as in beast. Experimental reasoning itself, which we posses in common with the beasts, and on which the whole conduct of life depends, is a species of instinct, or mechanical power.

Man is an organism. Human nature is instinct, or mechanical tendency, or mechanical power.

As to experience, Hume's results corroborate Aristotle's, generalize them, and explain them.

Experience is the outcome of development. From brute beast through Adam through the ultimate effort of human reason, and from infants through the most ignorant and stupid peasants through the philosophers, experience cannot not be the outcome of development.

It is the outcome of the development of faculties and their exercise. Hume describes these in much greater detail than Aristotle, and describes them in all their generality. At the far end of the development, he begins with impressions: those perceptions that enter with the most force and violence, which we may name impressions (*Treatise*, I, 1.1). It is a most important result. Long before it becomes seeing, hearing, touching, tasting, smelling – informed as these all are each by the exercise of the others, as well as by the exercise of faculties developed long after them – perception is what enters with the most force and violence. Perception thus reaches back to organisms long before organisms developed sense. At the near end of the development, Hume grounds in experience all reasonings and all ideas. All reasonings are: the effects of instinct, or mechanical tendency. All ideas are: copied from impressions. Both are most important results, too. There is the idea, for instance: of a pure and intellectual view of which the superior faculties of the soul alone are capable. The idea goes back to Plato. It was easy to see, for Hume, why philosophers continued to be fond of this notion of spiritual perception. It is easy to see also, after Hume, why philosophers continue to be fond of this notion. And the fact that philosophers continue to be fond of the idea of a pure and intellectual view of which the superior faculties of the soul alone are capable shows that this idea, continuing to be copied by one philosopher, and another, and another, from another, is itself copied from our impressions. The fact that philosophers continue

to be fond of this notion of spiritual perception shows that this notion, continuing to be the effect of mechanical tendency, is itself the effect of mechanical tendency.

Experience is not only the beginning of art and of knowledge. Experience is the foundation of all art, and all acquisition of knowledge, and all ideas, too.

And it is, because it is the outcome of the development of the entire machinery of faculties and their exercise, and the outcome of its development in all organisms. Generality shows experience the foundation. It is the entire machinery, from those perceptions that enter with the most force and violence through the pure and intellectual view of which the superior faculties of the soul alone are capable. And it is in all organisms, oak and elm, beast and Adam, and Heisenberg. Everything of which an organism is aware, including an organism which has become a philosopher, terminates in experience.

Which explains why Kant is incensed: idealism? the very opposite of it! all knowledge of things merely from pure understanding or pure reason is nothing but sheer illusion, and only in experience is there truth.

Hume, moreover, explained the fact that everything of which an organism is aware terminates in experience. He found its cause. The cause is that the entire machinery of faculties and their exercise is just that, machinery. Experience is the effect of mechanical tendency, or mechanical power. Mechanical tendency, or mechanical power, is its cause.

Nor can the fact have another explanation.

One century after Hume, Darwin discovered the principle of natural selection. Mendel discovered the laws of heredity. Not quite two generations later, Boveri demonstrated that the chromosome is the carrier of heredity. Not quite two generations later, Crick and Watson demonstrated that replicating DNA is the carrier of heredity in the chromosome. Jacob and Monod demonstrated the regulatory mechanism of the replication, in 1961, in the *Genetic regulatory mechanisms in the synthesis of proteins*.

And Monod, in *Chance and necessity, an essay on the natural philosophy of modern biology*, in 1970, writes: the great eighteenth-century empiricists were not wrong, everything comes from experience; not from the ongoing current experience reiterated by each individual, however, but from the experience accumulated by the entire ancestry of the species over the course of its evolution (8).

Everything comes from experience, and not from the ongoing current experience reiterated by each individual, but from the experience accumulated by the entire ancestry of the species. And if it comes from the experience accumulated by the entire ancestry of the species, it comes from the experience accumulated by the entire ancestry of the genus, and of the order, and of the class, and of the kingdom, and of all organisms. Where Adam is concerned, it comes from the experience accumulated by the entire ancestry of *erectus* before *sapiens* – presuming that Adam is a *sapiens* – and of *habilis* before *erectus*, and of *australopithecus* before *homo*, and all the way back to the first organism. 4×10^9 years of ancestry on earth alone.

Experience is an accumulation. It is an accumulation so firm as to become, with Hume: mechanical tendency, or mechanical power. So firm as to become fixed permanently in, with Jacob and Monod: genetic regulatory mechanisms.

The accumulation is twofold, by instinct and in instinct, and by learning and in learning.

Darwin writes: if instincts vary ever so little, I see no difficulty in natural selection preserving and accumulating variations of instinct (*Origin*, 8).

That there is no difficulty in natural selection preserving and accumulating variations of learning Lorenz showed. Not quite three generations after Darwin, in the *Observations on the recognition of instinctive behavior characteristic of the species in birds*, in 1932, and *The companion in the environment of the bird*, in 1935, Lorenz demonstrated learning as closely associated with instinct as imprinting, the learned permanent response to an object (*Companion*, 3), and as remotely associated with instinct as tradition, the transmission of learned behavior from one generation to the next (*Observations*, 3; *Companion*, 10.d), in corvids and anatids.

Fixed permanently in genetic regulatory mech-

anisms, experience is the outcome of the development of machinery. And genetic regulatory mechanisms through tradition, it is the outcome of the development of machinery in its entirety.

Experience is not a problem of the investigation of knowledge. Experience is a problem of physics, and a problem physics, the theory of evolution, in particular, has solved. Selection has produced, and produces, experience.

lxxiv

Everything of which an organism is aware, including an organism which has become a philosopher, terminates in experience.

One consequence is that all inferences terminate in experience. Reason and reasoning are very much part of experience.

Hume writes in the *Treatise*: no truth appears to me more evident than that beasts are endowed with thought and reason as well as men, and the arguments in this case are so obvious, they never escape the most stupid and ignorant (I, 3.16).

In the *Inquiry*: nothing is more usual for writers, even on moral, political, or physical subjects, than to distinguish reason and experience, and to suppose these species of argumentation entirely different from each other (5.1); the distinction between reason and experience, notwithstanding that it is universally received, both in the active and speculative scenes of life, I shall not scruple to pronounce at bottom erroneous, or at least superficial (*id.*); if we examine those arguments which, in any of the sciences above mentioned, are supposed to be the mere effects of reasoning and reflection, they will be found to terminate at last in some general principle or conclusion for which we can assign no reason but observation and experience (*id.*).

All inferences terminate in experience. Experience can have no rational alternative. Even as it can have an irrational alternative. Hume writes: none but a fool or a madman will ever pretend to dispute the authority of experience (4.2).

Other than a philosopher of course.

Another consequence is that subjectivity is an impossibility.

With Monod: everything comes from experience, and not from the ongoing current experience reiterated by each individual, but from the experience accumulated by the entire ancestry of the species over the course of its evolution.

The two, the ongoing current experience reiterated by each individual and the experience accumulated by the entire ancestry of the species, have to be in relation to one another. The ongoing current experience reiterated by each individual has to be the reiteration of the experience accumulated by the entire ancestry of the species. The experience accumulated by the entire ancestry of the species has to be the accumulation of the ongoing current experience reiterated by each individual.

And if so, each of the two has to be analyzed further into two as well. The ongoing current experience of each individual has to be the reiteration of the experience accumulated by the entire ancestry of the species, but not exclusively. At least in part, at least occasionally, the ongoing current experience of each individual has to include the singular experience of that particular individual. The experience accumulated by the entire ancestry of the species has to be the accumulation of the ongoing current experience reiterated by each individual, but again not exclusively. At least in part, at least occasionally, the experience accumulated by the entire ancestry of the species has to include the accumulation of the singular experience of some particular individual.

And subjectivity is, by being subjectivity, unlikely to be troubled by the question of whether it is possible. If it yet is, it is the supposition of uniqueness of that singular experience that makes the notion of uniqueness of the experience of the individual appear possible, and hence the notion of uniqueness of the individual, and hence the notion of uniqueness of his ideas, and thus subjectivity, appear possible.

In general, however, nothing can be unique. Nothing can be in every way like nothing else. Everything that is in all of its ways like nothing else cannot not be in every one of its ways like something else. Something in relation to nothing – a thing in itself, to borrow Kant's term – cannot not be itself nothing.

In particular, the fact that a singular experience is an experience precludes the possibility that it be in every way like no other experience. A singular experience cannot not be an experience

but by being in one way or another like one experience or another, and in every way like some experience. In fact, nothing but that it is in one way or another like one experience or another, such as this last experience is the ongoing current experience reiterating the experience accumulated by the entire ancestry of the species, can make any singular experience of any particular individual an experience. Nothing but that it is in one way or another like one experience or another, too, such as this last experience is the experience accumulated by the entire ancestry of the species, can make the accumulation of any singular experience of any particular individual into that experience possible.

Which explains the fact that, singular as they each are, the singular experiences of any one individual in any one place at any one time are all of them more or less exactly the same as the singular experiences of any other individual of the same species in that place at that time.

It is a consequence of the fact of genetic regulatory mechanisms, the same in man as in, with Jacob and Monod: bacteria (*Mechanisms*, 1).

It is a consequence also of the fact of instinct, the same in man as in, with Darwin: ants and bees (*Origin*, 8).

It is a consequence also of the fact of learning, the same in man as in, with Lorenz: jackdaws and geese (*Observations*, 3; *Companion*, 3).

Including the fact that philosophy has since Pythagoras been imparted, and the less it should have been the more ruthlessly it has been, by learning.

There can be no such thing as unique experience, and hence no such thing as unique individual.

Once again Hume explains. He reports: when I enter most intimately into what I call myself, I always stumble on some particular perception, of heat or cold, light or shade, love or hatred, pain or pleasure; I never catch myself without a perception and never observe anything but the perception; when my perceptions are removed for any time, as by sound sleep, I am insensible of myself and may be truly said not to exist; and were all my perceptions removed by death, and could I neither feel nor think nor see nor love nor hate after the dissolution of my body, I should be entirely annihilated, nor do I conceive what is further requisite to make me a perfect nonentity; and if anyone upon unprejudiced reflection thinks that he has a different notion of himself, I must confess that I can reason with him no longer (*Treatise*, I, 4.6); but setting aside some metaphysicians of this kind, I may venture to affirm of the rest of mankind that they are nothing but a bundle of different perceptions (*id.*); there is no identity in the mind; they are successive perceptions only that constitute the mind (*id.*); the identity which we ascribe to the mind of man is a fictitious one, and of a like kind with that which we ascribe to vegetables and animal bodies (*id.*).

There can be no such thing as unique individual, and hence no such thing as unique idea.

When some metaphysician of that kind, for instance, has a notion of himself different from the notion Hume has of himself, his notion is yet not different from the notion another metaphysician of the same kind has of himself.

Berkeley writes: everything is my ideas in mind, spirit, soul, or myself. Schopenhauer writes: Berkeley made the principle, there is no object without subject, the foundation stone of philosophy. Indiscriminately mind, spirit, soul, or myself, mind, spirit, soul, or myself is indiscriminately Berkeley and Schopenhauer, apparently. The subject without whom there is no object is also indiscriminately mind, spirit, soul, or myself, and also indiscriminately Berkeley and Schopenhauer, apparently.

Hegel writes: the subject is the I in my self-consciousness. The subject is Berkeley, Hegel, and Schopenhauer indiscriminately, apparently, so is the I, apparently, and in Berkeley's, Hegel's, and Schopenhauer's self-consciousness indiscriminately, apparently.

Whatever an individual be cannot not be shared by individuals severally. If, that is, he is not nothing. If he is mind, spirit, soul, or myself, and everything is my ideas in mind, spirit, soul, or myself, he yet has to share myself, as well as my ideas, with individuals severally. If he is the I in my self-consciousness, he yet has to share the I, as well as my self-consciousness. Dilthey writes: the whole significance of each individual acting at some point of the spiritual cosmos is revealed by the proposition, *individuum est*

ineffabile. And not only that he is ineffable each individual has to share with all other individuals, apparently, as well as that they each act at some point of the spiritual cosmos. Each individual has to share at least some of his ideas, apparently, with Berkeley, Hegel, Schopenhauer, and Dilthey. Idealism, for instance.

Herodotus reports: the Egyptians first said that the soul of man is immortal, that when the body dies it creeps out and enters into some other animal born at this time, and that in this way it goes through all things living on land, in the sea, and in the air, until it creeps back into a human body; the cycle, they say, is 3 000 years (II, 123).

And so the idea that the soul of man is immortal crept out of Egypt, entered into philosophy with Pythagoras, and swelled into becoming, as this idea would, the soul of philosophy with Plato. Plato's successors, Aristotle, Epicurus, Zeno of Citium, failed to appreciate the idea. Immortality crept, however, into the Hebrew scriptures, if somewhat belatedly, into the latest of the writings comprising them, *Daniel*. Thence it crept into the Christian scriptures. Then it crept into the Muslim scriptures. By that time it had swelled into becoming the soul of everything men did, such as everything men did now was, and into the soul of all philosophy, such as all philosophy now was. It was more than a millennium later when Hume affirmed: were all my perceptions removed by death, I should be a perfect nonentity. The evidence for this, always rather conclusive, had in the previous couple of centuries yet mounted, and mounted, and mounted. But if the immortal soul of man will not go through all things living on land, in the sea, and in the air, the idea that the soul of man is immortal will go through all things. Having never quite crept out of philosophy, it was before Hume already urging itself once again as the soul of philosophy, mind, spirit, soul, or myself now. And soon after Hume it did swell once again into the soul of philosophy, the I in my self-consciousness now, the subject without whom there is no object.

But all reasonings are the effects of mechanical tendency, and all ideas are copied from impressions.

In particular, that all reasonings involving the immortal soul of man are the effects of mechanical tendency their creeping from Egypt into philosophy into scriptures, and from the Hebrew scriptures into the Christian into the Muslim, and from Pythagoras into Plato into Augustine, and on and on, into Berkeley into Hegel into Schopenhauer, and on and on, shows.

That the idea that the soul of man is immortal is copied from impressions Herodotus already shows, if creeping out of the body is what the immortal soul of man does.

One way or another, the empiricist *par excellence* Herodotus reports in this case, too, as he more or less invariably does, a fact.

There is the Sumerian, Akkadian, Babylonian, Assyrian record, on the one hand, and there is the Egyptian record, on the other. And looking for the idea that the soul of man is immortal, it is not impossible to find it in the Mesopotamian record. It is not impossible for the immortal soul of man, looking for itself, to find itself in the Mesopotamian record. In the Egyptian record, on the other hand, looking for the idea is superfluous. The idea that the soul of man is immortal pervades the Egyptian record. It is, indeed, its soul. In particular, tomb inscription by tomb inscription by tomb inscription, the idea can be traced back to the tomb inscription of Unas, prince *c.* 2320, forty-one and a half centuries before Hegel, twenty-one and a half centuries before *Daniel*, eighteen centuries before Pythagoras.

In the tomb of Unas are inscribed the oldest 228 of the spells, which, copied and multiplied in the tombs of his successors, have been collected under the name the pyramid texts.

Unas tells Re in heaven: Unas, an indestructible spirit, is coming to Re in heaven (217).

Where Unas, an indestructible spirit, goes after his death the spells are clear, he goes to Re in heaven. How he gets there is not so clear, he either takes a ferry (270), or climbs a ladder (304). What he does there is also not entirely clear. On the one hand: Unas is the servant of the gods, Unas squats before Re, Unas opens his coffers, Unas unseals his decrees, Unas seals his dispatches, Unas does what Unas is told (309). On the other hand: Unas eats the gods; Unas eats their magic; the big ones are his morning meal, the middle ones are his midday meal, the little ones are his evening meal (273-4).

After his death Hume is a perfect nonentity. After his death Unas takes a ferry, climbs a ladder, squats, opens coffers, unseals decrees, seals dispatches, and rather diligently eats, morning meal, midday meal, evening meal.

There is no subjectivity. What there is is slight variation, of the genetic regulatory mechanisms, of instinct, and of learning. Sometimes, not often, there is a mutation. Sometimes, and again not often, an instinct varies. Sometimes, and once again not often, a goose learns not what she is taught. Or one of those to whom philosophy is imparted does.

There is slight variation, and hence there is the accumulation of the slight variations.

Only what selection cannot produce can be the proper object of the investigation of knowledge.

And there is no such thing as unique individual, but there is such a thing as individual, and whether the individual matters can in any one case be a test whether selection can produce that case or not. The more the individual matters in any one case the more likely has selection to be to be able to produce it. The less the individual matters in any one case the less likely has selection to be to be able to produce it.

Selection acts and can only act on the individual. On the particular bundle of perceptions, with Hume, which the individual vegetable or animal is. More explicitly, on the particular bundle of genetic regulatory mechanisms, instinct, and learning which the individual organism is. Selection acts and can only act by accumulating the slight individual variations in those, and destroying them, by destroying their individual carriers, or preserving them, by preserving their individual carriers, in the struggle for life. For selection the individual is all important. Even if success in the struggle for life is success in leaving progeny, exclusively. Success of the individual is success in leaving behind other individuals, exclusively.

For instance, it is very well that Unas diligently eats in heaven. All important, however, is that: Unas sheds his seed; Unas has any woman Unas wants (317).

To the acquisition of knowledge, on the other hand, if it is to be acquisition of knowledge, the individual must be irrelevant.

Leibniz, for instance, writes: with Aristotle and Descartes, against Democritus and Gassendi, I admit no vacuum; against Aristotle, with Democritus and Descartes, I consider all rarefaction and condensation only apparent; with Democritus and Aristotle, against Descartes, I think that there is something passive in bodies next to extension; with Plato and Aristotle, against Democritus and Descartes, I recognize an active force in bodies; with Democritus and Descartes, against the multitude of scholastics, I hold that the phenomena of bodies can always be explained mechanically (manuscript, 1702).

It may be a pity that those are not individuals so that one could picture Democritus, Plato, Aristotle, the multitude of scholastics, Gassendi, Descartes, and Leibniz arrange and rearrange themselves in the steps of what would be a not very elegant gavotte. But those are propositions: there is no vacuum; all rarefaction and condensation is only apparent; there is something passive in bodies next to extension; there is an active force in bodies; the phenomena of bodies can always be explained mechanically. And a proposition is a proposition not only because it cannot not be shared by individuals severally. A proposition is a proposition because its association with any one individual cannot not be irrelevant to it. Nothing else can make it possible for Leibniz to hold, with Democritus, with Democritus, with Democritus, against Democritus, against Democritus, with Plato, with Aristotle, with Aristotle, with Aristotle, against Aristotle, against the multitude of scholastics, against Gassendi, with Descartes, with Descartes, with Descartes, against Descartes, against Descartes, and with himself: there is no vacuum, all rarefaction and condensation is only apparent, there is something passive in bodies next to extension, there is an active force in bodies, and the phenomena of bodies can always be explained mechanically.

Planck writes: the goal of physics is the complete emancipation of the physical picture of the world from the individuality of the picturing mind (*Unity*, 4).

The goal of physics is the goal of the acquisition of knowledge, and that goal of physics is, as far as individuality is concerned, the only goal the acquisition of knowledge has, or can have.

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Hume has proposed: to establish the foundation on which the sciences can stand with any security, and done exactly that. The foundation of all reasonings, which we possess in common with the beasts, is experience. All inferences, which we also possess in common with the beasts, terminate in experience. And if the sciences stand on reasonings and inferences, they stand on the foundation on which the beasts stand. All ideas are copied from impressions, too. And even if the reasonings and inferences on which the sciences stand stand on ideas, and even if the sciences themselves stand on ideas, they stand on the foundation on which the beasts stand. One way or another, the foundation is in Democritus, Leibniz, and Planck as it is in organism. Experience.

Hence, lest knowledge be given to an organism, and be the product of selection, knowledge is acquired.

Hence, experience is the necessary condition of the acquisition of knowledge but not its sufficient condition.

And this fact has to inform the problem before the investigation of knowledge, and both its parts, the problem of the acquisition of knowledge and the problem of knowledge, essentially. Because this fact, that experience is the necessary condition of the acquisition of knowledge but not its sufficient condition, points at both the beginning of the acquisition and its end unambiguously.

The end of the acquisition of knowledge cannot be the end of an organism. The end of the acquisition, also, such as it cannot be the end of an organism, cannot not be its end from its beginning.

The acquisition of knowledge began when the end of an organism became the end which cannot be the end of an organism. If so, the decisive part of the problem of the beginning of the acquisition has to be to show that the contradiction is only apparent. To show that the end of an organism can give rise to the end an organism cannot have. And there can be no better way to do so than to show the fact of the beginning. To show where and when it occurred, thus that it did occur, and hence show also how and why.

And Hume's result makes this possible. If reasonings and inferences are shared by organism and the acquisition of knowledge, the particularity of reasonings and inferences makes it possible to identify the beginning of the acquisition in any one case in particular, thus unambiguously, and hence, particular case after particular case, also in general, also unambiguously.

So, if the great step which Maxwell describes: when men began asking not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it, describes the beginning of the acquisition, and reasonings and inferences include questions, and they do, it can be shown where and when it was first asked of what kind something is, and how much there is of it, and hence how and why the earlier questions whether it is good or bad, noxious or beneficial, gave rise to the questions of what kind it is, and how much there is of it.

More explicitly, Aristotle describes the beginning of the acquisition. He describes the development: when all the arts aiming to procure the necessities of life and to give pleasure were established, knowledge which is neither useful nor pleasurable was discovered. More explicitly, after the arts aiming to procure the necessities of life and to give pleasure: the arts aiming at beauty, mathematics, politics, physics, and philosophy developed.

Given Hume's result, the description can be reduced to particularity. It can be established where and when each of those developed, and hence how and why. Beginning with mathematics, mathematics can be analyzed into particular reasonings and inferences, and there are not many of those early on, then physics can be analyzed into particular reasonings and inferences, and there are not many of those early on either, if mathematics and physics can be analyzed into particular reasonings and inferences, philosophy can be, and politics can be, and the arts aiming at beauty can be. It can be shown where and when each of those reasonings and inferences occurred for the first time. The arts aiming to procure the necessities of life and to give pleasure can be analyzed into particular reasonings and inferences, and if there are many of these even early on, they cannot be impossible to analyze into relevant and irrelevant, given the reasonings and inferences into which mathematics, physics,

philosophy, politics, and the arts aiming at beauty have been analyzed, and it can be hence shown how and why these gave rise to those.

Moreover, if the description as it is in Aristotle leaves it uncertain whether those all, the arts aiming at beauty, mathematics, politics, physics, and philosophy, are part of the acquisition, or whether any of them is in its entirety part of the acquisition, this difficulty can be resolved on the same ground, particularity. It can be established what the relations of where and when are, and hence what the relations of how and why are, of the particular reasonings and inferences into which those have all been analyzed.

Also, should not only reasonings and inferences have to be involved in the analysis, but ideas as well, as they would be where the arts are involved, and indeed where philosophy is involved, too, and so where mathematics, and physics, and politics are involved, if those can all be analyzed into particular reasonings and inferences, they can all be analyzed into particular ideas, and it can be then shown where and when which of these particular ideas gave rise to which of those particular ideas, and hence how and why.

Still more explicitly, Ptolemy describes the world, and Hume's result shows that the question of when the description of the world began does not have to be contaminated by vagueness either. Contaminated by an indefinite amount of vagueness as all questions involving the terminal object of the acquisition of knowledge have been.

If the world is the system of the world, and the system of the world is the position, order, and motion of the parts of the world, it can be shown where and when the description of any one part of the world began, and hence how and why. In particular, the description depends on observation, Ptolemy describes the position, order, and motion of the heavenly bodies, and himself traces the accurate observation of the heavenly bodies back to its beginning. He has himself established where and when the first accurate observation of the heavenly bodies occurred. It occurred in Babylon in 721. As Ptolemy has collected the accurate observations of the heavenly bodies, so the earlier observations of the heavenly bodies can be collected, and it can be shown how and why observation of the heavenly bodies gave rise to accurate observation of the heavenly bodies. As Ptolemy has collected all accurate observations of the heavenly bodies, so all earlier observations of the heavenly bodies can be collected, and it can be shown where and when the earliest observation of the heavenly bodies occurred, and hence how and why seeing the heavenly bodies and looking at them gave rise to observation of the heavenly bodies. Seeing the heavenly bodies and looking at them, with all ideas, reasonings, and inferences involved, can then be left alone to organism.

Ptolemy describes also the position, order, and motion of the localities on the surface of the earth. Where and when their first accurate observation occurred can be shown, and hence how and why their observation gave rise to their accurate observation, and where and when their first observation occurred can be shown, and hence how and why seeing them, looking at them, and however else they were perceived, gave rise to their observation.

Ptolemy describes also the position, order, and motion of the princes, and the same can be shown, where and when their first accurate observation occurred, and where and when their first observation occurred, and how and why seeing them, looking at them, and however else they were perceived, gave rise to their observation.

It can be shown as well what the relations of all these with one another are, always where and when, and only hence how and why, and what their relations with whatever else they involve are, always where and when, and only hence how and why, and all the way to the terminal object.

It is not an accident that Ptolemy provides an example of such clarity and distinction. Ptolemy first describes the world mathematically. Ptolemy also achieved the goal of physics, such as Planck describes it, very nearly completely. He left behind the *Almagest*, *Planetary hypotheses*, *Geography*, *Handy tables*, *Optics*, and of his individuality nothing but his name.

The physicist who made the first accurate observation of the heavenly bodies, for his part, left behind the first three accurate observations of the heavenly bodies, of three lunar eclipses in 721 and 720, and of his individuality nothing at all. So that those, his labors, observations of lunar

eclipses in 721-0, have to be his name.

If the problem of the beginning of the acquisition of knowledge can be solved, the problem of collecting the acquisition in its entirety can be solved. If always where and when, particular reasonings and inferences can be sorted out into those before the acquisition and those after, always where and when, all particular reasonings and inferences can be sorted out into those which are part of the acquisition and those which are not. And if all particular reasonings and inferences can be sorted out, all ideas can be sorted out.

If the problem of collecting the acquisition in its entirety can be solved, the problem of the relation of the acquisition and the development of philosophy can be solved.

That this problem both, has to be solved and can be solved, the immortal soul of man can illustrate.

Where and when the immortal soul of man first occurred, where and when it entered into philosophy, and where and when it became the soul of philosophy, has been shown. Unas, Pythagoras, and Plato, respectively.

The evidence is in all three cases conclusive.

Where Unas is concerned, it is not impossible to find the immortal soul of man in the Sumerian, Akkadian, Babylonian, Assyrian record. But it is not easy either. The Hebrew scriptures, the direct descendants of that record, illustrate. Given that the immortal soul of man has to be found, it is not impossible to find it in any of the writings comprising the Hebrew scriptures, all the way back to the earliest. Christian philosophers have been doing it for two millennia. Otherwise, the immortality of man is nowhere to be found in the writings comprising the Hebrew scriptures before *Daniel*, the latest of them, composed three and a half-odd centuries after the immortal soul of man entered philosophy. To find the immortal soul of man in the Egyptian record, on the other hand, and to find its first occurrence in it, is not hard at all. The predecessors of Unas began leaving behind their tombs not later than the third prince, Djer, prince *c.* 2960, six and a half centuries before Unas, and leaving behind tombs as spectacular as the pyramid of Khufu, prince *c.* 2580, two and a half centuries before Unas. The princes began leaving behind in their tombs inscriptions, too, and tomb inscription after tomb inscription, the tomb inscription of Unas is the first to propose to take the prince to heaven, whether by ferry or by ladder, and whether to squat before the gods there or to eat them.

Where Pythagoras is concerned, the immortal soul of man entered philosophy very early, with the fourth philosopher. Still, none of the first three philosophers, Thales, Anaximander, and Anaximenes, left behind evidence that he noted the immortal soul of man. Pythagoras, on the other hand, left behind not only massive evidence that he taught the transmigration of the soul. He left behind evidence for the transmigration of his own soul. Ponticus reports: Pythagoras was first Aethalides, a son of Hermes, next he was Euphorbus and was wounded as such by Menelaus at Troy, next he was Hermotimus, next he was the Delian fisherman Pyrrhus, and only then he became Pythagoras (fr. 89).

Where Plato is concerned, Plato himself reports that even Socrates still wonders whether death is what Hume says it is or what Pythagoras says it is. Just condemned to death, Socrates comments: death is one of two things, either the dead are nothing and perceive nothing, or it is, as some say, a migration of the soul from here to some other place (*Apology*, 40c). Himself, Plato does not note the immortal soul of man in his early dialogues, notes it *passim* in the *Phaedo*, and notes it often and vigorously in his late dialogues, in the *Meno* (81b-6a), in the *Symposium* (206e-9e), in the *Phaedrus* (245c-56e), in the *Republic* (X, 608c-21d), in the *Timaeus* (41a-6e), in the *Laws* (X, 896a-7b).

Given where and when the immortal soul of man first occurred, where and when it entered into philosophy, and where and when it became the soul of philosophy, it can in each case be shown how and why.

Unas not only diligently eats in heaven, Unas sheds his seed. Pythagoras not only himself transmigrated, Pythagoras taught transmigration. Plato not only teaches the immortality of the soul, Plato teaches the immortality of the soul relentlessly and ruthlessly.

The problem of whether the immortal soul of man, ubiquitously part of the development of

philosophy, is part of the acquisition of knowledge or not, has to be solved, if only because Pythagoras said also that number is the principle of all things, this proposition cannot not be part of the acquisition, if number is the principle of all things, number cannot not be the principle of the immortal soul of man, and when Aristotle reports: the Pythagoreans said that the soul is number (*Metaphysics*, I, 5), it has to be concluded that Pythagoras himself drew this inference.

And that the problem can be solved Plato shows. In the *Phaedo* he does not just note the immortal soul of man *passim*, he argues it *passim*. He argues: opposites come from one another, so life must come from death (70d-2d); man knows the equal itself, the beautiful itself, the good itself, and these are not among the sensible things, so the soul must have seen them before it came into this life (72e-7e); the simple is indestructible, and the soul must be simple (78b-84b); the forms, or ideas, are eternal, and the soul must be what it is by participation in the form of life (95b-107a). If the immortal soul of man can be subjected to arguments, it can be analyzed into reasonings and inferences, and ideas.

If the problems of the beginning of the acquisition, of collecting it in its entirety, and of its relation to the development of philosophy can be solved, the problem of its kinds can be solved.

In Aristotle the kinds are philosophy, theory, practice, physics, mathematics, politics, theology, logic, astronomy, optics, geometry, arithmetic, ethics. In Ptolemy the kinds are philosophy, theory, practice, physics, mathematics, theology, ethics, astronomy, geography, optics, geometry, arithmetic. In Hume the kinds are philosophy, speculative philosophy, moral philosophy, natural philosophy, mathematics, politics, natural religion, history, geometry, arithmetic, morals. The problem of the kinds is not which they are. It is what their relations have been. It is to analyze them all into reasonings and inferences, and ideas if need be, trace the reasonings and inferences, and ideas, to its beginning each, and follow them, and in their relations with one another, through their development.

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The fact that experience is the necessary but not the sufficient condition of the acquisition of knowledge points unambiguously at the beginning of the acquisition, and because it points unambiguously at the end of the acquisition.

The foundation of the acquisition is experience.

But its end cannot be the end of an organism, its end can only be knowledge.

And between these two facts the acquisition of knowledge stands, and cannot not stand, with any security. Or what is the same, with certainty.

Certainty has to be the consequence of the facts of genetic regulatory mechanisms, instinct, and learning. Each of these singly has to entail, and the more so all of them jointly have to entail, as a matter of survival, certainty. If knowledge cannot develop, its acquisition has to have finality, to have it from the first, to have it throughout, and to have it to the end. And between these two facts, the acquisition of knowledge cannot not have ultimate certainly, universal certainty, and absolute certainty.

From the fact that the foundation of all reasonings and inferences is experience Hume draws the conclusions: if we be engaged by arguments to put trust in past experience and make it the standard of our future judgment, these arguments must be probable only (*Inquiry*, 4.2); that all men must die, or that the sun will rise tomorrow, is probable only (6); all knowledge degenerates into probability (*Treatise*, I, 4.1); all probable reasoning is but a species of sensation; thus it is not solely in poetry and in music that we must follow our taste and sentiment, but likewise in philosophy (3.8), and: the ultimate effort of human reason is to resolve the many particular effects into a few general causes; but the causes of these general causes we should in vain attempt to discover; the ultimate springs and principles are totally shut up from human curiosity and inquiry (*Inquiry*, 4.1).

The ultimate springs and principles are totally shut up from human curiosity and inquiry.

It is not solely in poetry and in music that we must follow our taste and sentiment, but likewise in philosophy.

That the sun will rise tomorrow is probable only, all knowledge degenerates into probability.

And appearances to the contrary, all three

conclusions show the certainty of the acquisition of knowledge.

The ultimate springs and principles are totally shut up from human curiosity and inquiry. The ultimate effort of human reason is to resolve the many particular effects into a few general causes, but the causes of these general causes we should in vain attempt to discover.

To begin with, that it can be ascertained what the ultimate effort of human reason is shows certainty.

Further, the ultimate effort of human reason, to resolve the many particular effects into a few general causes, is in evidence from the first philosopher through the last – so far – twenty-six-odd centuries now. The first philosopher said that water is the principle of all things, and quantum theory says that quarks and leptons are the constituents of all things. The ultimate effort of human reason has been a trivial effort of the philosophers, and this also shows certainty.

Further, Thales said that water is the principle, Empedocles said that earth, water, air, and fire are the principles, Lavoisier said that hydrogen, nitrogen, and oxygen are the elements, quantum theory says that quarks and leptons are the elementary particles, or waves, or particle-waves. And earth, water, air, and fire, hydrogen, nitrogen, and oxygen, and quarks and leptons are not just different general causes into which the many particular effects have been resolved. They are, most particularly, each other's general causes. In the century-odd between Boyle and Lavoisier, most particularly, earth, water, air, and fire were resolved into hydrogen, nitrogen, and oxygen. Beginning with Thomson, most particularly, hydrogen, nitrogen, and oxygen were resolved into quarks and leptons. Thales through Thomson, Planck, Einstein, Rutherford, Bohr, Schrödinger, and Heisenberg, the ultimate effort of human reason has been one and the same, and this also shows certainty.

Further, quarks and leptons either are the ultimate springs or are not the ultimate springs. If they are the ultimate springs, this shows certainty. If they are not the ultimate springs, the ultimate effort of human reason either will discover the ultimate springs or will not discover the ultimate springs. If it will, it will by discovering, most particularly, the springs of quarks and leptons, the springs of the springs of quarks and leptons, so on to the ultimate springs, and this also shows certainty. If it will not, it will by failing to discover, most particularly, the springs of quarks and leptons, and this also shows certainty.

So much certainty has to add up to ultimate certainty.

Which explains why the successes of the acquisition of knowledge will go down triumphant to the latest posterity.

Hume writes: if Newton's theory can overcome the obstacles which it meets at present in all parts of Europe, it will probably go down triumphant to the latest posterity (*Essays*, I, 14). Newton's theory overcame those obstacles, such as they were. Then the fairest destiny befell it, it pointed the way toward Einstein's theory in which it lives as a limiting case.

And all the giants on whose shoulders Newton stands will go down triumphant to the latest posterity because Newton's theory will, Newton's theory will because Einstein's theory will, and Einstein's theory will because either the fairest destiny will not befall it or the fairest destiny will befall it.

It is not solely in poetry and in music that we must follow our taste and sentiment, but likewise in philosophy.

Because, conversely, it is not solely in philosophy that we must follow our taste and sentiment, but likewise in poetry and in music.

Which, in turn, because the principles of taste are universal.

One of the *Essays, moral, political, and literary* is *Of the standard of taste* (I, 23). Hume writes: there are general principles of approbation and blame in all operations of the mind (*id.*). Hence: the general principles of taste are uniform in human nature (*id.*). Indeed: the principles of taste are universal (*id.*). And Hume defines the standard of taste: strong sense, united to delicate sentiment, improved by practice, perfected by comparison, and cleared of all prejudice, entitles the critic to the character of a true judge; and the joint verdict of such, wherever they be found, is the true standard of taste and beauty (*id.*).

This explains why: the same Homer who pleased Athens and Rome two thousand years ago

is still admired at Paris and at London (*id.*).

It explains also why for most of the two thousand years between the time when he was admired at Athens and at Rome and the time when he was admired at Paris and at London, Homer was not admired. Strong sense, which critics abundantly had, was not united to delicate sentiment, not improved by practice, not perfected by comparison, and not cleared of all prejudice.

It explains also why the Martians, if they have strong sense, united to delicate sentiment, improved by practice, perfected by comparison, and cleared of all prejudice, will admire Homer.

Needless to add, they will admire Newton.

Needless to add, taste is universal because certainty is universal.

Finally, that the sun will rise tomorrow is probable only.

Pace Hume, however, all knowledge does not degenerate into probability. All acquisition of knowledge is elevated into probability.

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Galileo established that the world is written in mathematics, and in the following century, with the labors of Fermat, Pascal, Huygens, Jakob Bernoulli, probability, too, learned the language of the world.

In 1713, in *The art of conjecture*, Jakob Bernoulli writes: probability is a degree of certainty, it differs from certainty as a part differs from the whole (IV, 1).

Probability is certainty, not uncertainty.

Jakob Bernoulli continues: if something has probability so close to complete certainty that the difference cannot be perceived, it is morally certain; and if something has only as much certainty as the amount by which moral certainty falls short of complete certainty, it is morally impossible; thus, if something that has $\frac{999}{1\,000}$ certainty is morally certain, something that has $\frac{1}{1\,000}$ certainty is morally impossible (*id.*).

If something that has $\frac{999}{1\,000}$ certainty is morally certain, and something that has $\frac{999\,999}{1\,000\,000}$ certainty is not absolutely certain, and something that has $\frac{999\,999\,999}{1\,000\,000\,000}$ certainty is not absolutely certain, something that has $\frac{999\,999\,999\,999}{1\,000\,000\,000\,000}$ certainty had better be absolutely certain.

To show this Laplace's intelligence and Heisenberg's principle are singularly qualified.

In 1814, in the *Philosophical essay on probabilities*, Laplace defines probability in general: given that all cases are equally possible, probability is the ratio of the number of favorable cases to the number of all possible cases (2), and the probability of a future event in particular: the probability of a future event is the sum of the products of the probability of each cause, drawn from the observed event, and the probability that, each cause continuing to exist, the future event will take place (*id.*).

One of Laplace's illustrations is then this: if the most ancient epoch of history is set back at 5 000 years, or at 1 826 213 days, the sun having risen constantly during that interval at each revolution of twenty-four hours, the probability that it will rise again tomorrow is $\frac{1\,826\,213+1}{1\,826\,213+2}$ (*id.*). Or better than $\frac{999\,999}{1\,000\,000}$.

Laplace, however, continues: and this number is incomparably greater for him, who, recognizing in the totality of the phenomena the principle regulating the days and the seasons, sees that nothing at the present moment can arrest the course of the sun (*id.*). And how much greater that incomparably greater number is can be calculated as well.

When the most ancient epoch of history is set back at 5 000 years, it is so set on scriptural authority. There was a better authority at Laplace's time already. In 1775 Buffon had shown that the earth is at least 74 832 years old. And in the two-odd centuries since Laplace the problem of when the most ancient epoch of history must be set back has been solved conclusively. The solar system was formed 4.6×10^9 years ago.

When the most ancient epoch of history is set back at 4.6×10^9 years, then – and whether, before there were men, earlier mammals watched the sun rise, earlier reptiles before them, and before there were protists, there was anything but the earth to

watch it rise – the sun having risen constantly during that interval at each revolution of twenty-four hours, or somewhat less than twenty-four hours on average, given that the axial rotation of the earth has been slowing down, the probability that the sun will rise again tomorrow is $\frac{(4.6\times10^9)\times(3.7\times10^2)+1}{(4.6\times10^9)\times(3.7\times10^2)+2}$. Or better than $\frac{999\,999\,999\,999}{1\,000\,000\,000\,000}$.

Not incidentally, since that all men must die is probable only, too, all men can be told, if they will have the resolution and courage to emerge from their self-imposed minority, that the probability that they must die is very much better still.

Probable only as it is, that the sun will rise tomorrow is absolutely certain. Not only is the certainty of that future event more than 10^9 times better than moral certainty. Resting on the theory of the origin and development of the world, the number $\frac{999\,999\,999\,999}{1\,000\,000\,000\,000}$ accounts for the totality of the phenomena, from the first rising of the sun explicitly to the last rising of the sun implicitly.

The first step toward the theory of the origin and development of the world was the theory of the origin and development of the solar system. This theory was first formulated by Kant, in 1755, and developed and formulated conclusively by Laplace, in the *Exposition of the system of the world*, in 1796.

And the theory of the origin and development of the solar system, and the theory of the origin and development of the world, are but two of the ever more and greater achievements of Laplace's intelligence.

Conceived by Laplace, in the *Essay on probabilities*, it is in Laplace just that: an intelligence (1). Conceived, however, at the time of the restoration of spirit to the substantiality and purity of its being, Laplace's intelligence has had to go under the name Laplace's demon, and to suffer many calumnies, and especially the calumny that it is not real.

Laplace writes: an intelligence, which at any one instant could know all the forces animating nature and the situation of the beings composing it, could embrace in the same formula the motions of the greatest bodies in the world as well as those of the lightest atoms; nothing will be uncertain for it, and the future will be just as present to its eyes as the past (*id.*).

And having thus introduced the intelligence, Laplace continues with the evidence for its reality: the human intellect offers a feeble hint of this intelligence in the perfection it has been able to give to astronomy; its discoveries in mechanics and in geometry, joined to that of universal gravity, have enabled it to comprehend in the same analytic expressions the past and future states of the system of the world (*id.*).

Nor is the hint, pace Laplace, feeble.

The hint had become muscular long before Laplace already, as the perfection given to astronomy by Ptolemy suffices to show.

It had become ever more muscular in the not quite two centuries between Galileo and Laplace, as the explicit comprehension of the past and future states of the system of the world in the same analytic expressions by Kepler, and more generally still by Newton, suffices to show.

Laplace himself added much to the muscularity. He added greatly to the perfection of astronomy with the solution of all outstanding problems concerning the solar system, showing that the current acceleration of Jupiter and deceleration of Saturn are an effect of their mutual gravitation with a period of 929 years, that the acceleration of the moon is an effect of the eccentricity of the earth's orbit with a period of millions of years, that, in general, all perturbations in the orbits of the bodies in the solar system are periodic and self-correcting, and the system is therefore stable. He added greatly to the comprehension of the past and future states of the system of the world with the conclusive formulation of the theory of the origin and development of the solar system from the cooling and contraction of a gaseous nebula. He added to the perfection of the study of organisms with the demonstration, with Lavoisier, that respiration is combustion. He added greatly to the power of comprehension in the same analytic expressions with the discovery of the Laplace equation, $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$, the foundation of the investigation of the gravitational and electromagnetic potential and more

generally of potential theory and harmonic functions, and with the discovery of the Laplace transform, $g(y) = \int_0^\infty e^{-xy} f(x)\,dx$, the fundamental method of conversion of one function into another. He added to the perfection of the theory of probability, too, with its development into an investigation of causes.

And the hint has kept becoming ever more muscular after Laplace. With the perfection given to thermodynamics by the two laws formulated by Clausius, comprehending in the same analytic expressions the past and future states of the system of the world. The perfection given to evolution by the one principle formulated by Darwin, comprehending in the same expression the past and future states of the organisms in the system of the world, and this expression became analytic with the discovery of Mendel's laws. The perfection given to the periodic law, itself analytic, comprehending in Mendeleev's table of the elements the past and future states of the system of the world. The perfection given to electrodynamics by Maxwell's equations. To the kinetic theory by Boltzmann. To relativity. To quantum theory. Hubble's law has given perfection to the big bang theory, comprehending in one analytic expression the past and future states of the system of the world, beginning, now, at the beginning of the world. If the intelligence is to know all the forces animating nature, it does know, now, the strong, weak, and electromagnetic, next to the gravitational. If it is to embrace in the same formula the motions of the greatest bodies in the world as well as those of the lightest atoms, it does embrace in the same theory, now, the world itself as well as quarks and leptons.

As Laplace's intelligence has had to go under the name Laplace's demon, and the theory of the absolute world has had to go under the name theory of relativity, so Heisenberg's principle has had to go under the name uncertainty principle.

Heisenberg discovered the principle in 1927, in *On the visual content of quantum theoretical kinematics and mechanics*. In its formulation by Heisenberg in *The physical principles of quantum theory*, in 1930, the principle is:

$$\Delta x \Delta p_x \geq h,$$

where Δx is the change in the position of a particle, Δp_x is the change in its momentum, and h is Planck's constant (2.1).

The more accurately Δx is determined, the less accurately Δp_x is determined, the more accurately Δp_x is determined, the less accurately Δx is determined, and the limit of the accuracy of the determination of both is h. Ignoring, here, the question of whether Heisenberg's principle thus affirms uncertainty, or certainty,

$h = 0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 662\ 606\ 88\ Js$

with an uncertainty of about 1 in the last digit.

The point is borrowed from Feynman. He writes: to give you an idea of how the theory of quantum electrodynamics has been put through the wringer, I will give you some recent numbers; experiments have the magnetic moment of the electron at 1.00115965221, with an uncertainty of about 4 in the last digit, the theory puts it at 1.00115965246, with an uncertainty of about five times as much; if you were to measure the distance from Los Angeles to New York to this accuracy, it would be exact to the thickness of a human hair (*QED*, intr.).

Whatever Heisenberg's principle affirms, the exactitude with which it affirms it is

$$\frac{99\ 999\ 999\ 999\ 999\ 999\ 999\ 999\ 999\ 999\ 999\ 999\ 999\ 999}{100\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}.$$

With Hume: there is truth, and truth which lies not beyond the compass of human understanding.

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Kant's very important treatises are these,

General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws, in 1755,

Critique of pure reason, in 1781,

Prolegomena to any future metaphysics which will be able to stand as science, in 1783,

Idea for a general history with a cosmopolitan purpose, in 1784,

Answer to the question, what is enlightenment? in 1784,

Groundwork for the metaphysics of morals, in 1785,

Toward perpetual peace, a philosophical draft, in 1795.

Knowledge Kant investigates in the *Critique* and the *Prolegomena*.

What he investigates in the other five very important treatises their titles indicate more explicitly. In the *General natural history* he investigates the world. In the *Idea* he investigates history. In the *Answer*, *Groundwork*, and *Toward perpetual peace* he investigates ethical and political matters.

The first four very important treatises are theoretical. The last three are practical. As in Aristotle and in Hume so in Kant, the end of theory is truth, the end of practice is action. As in Aristotle and in Hume so in Kant, also, the end of theory shows what the end of practice, such as it is not, should be.

The four very important theoretical treatises span three decades.

The first, the one that proposes to apply Newton's laws to the constitution and origin of the entire structure of the world, is physical, in particular. In fact, as the philosopher Aristotle is a physicist, and the philosopher Hume is a historian, so the philosopher Kant is a physicist. He wrote also the important, if not very important, *Thoughts on the true estimation of living forces and assessment of the proofs used on this controversial matter by Leibniz and other mechanics, together with some preliminary observations on the force of bodies in general*, in 1747, *History and natural description of the most remarkable occurrences of the earthquakes which struck a large part of the earth at the end of the year 1755*, in 1756, *On the first ground of the difference of directions in space*, in 1768, *Physical geography*, in 1802.

In the two and a half-odd decades between the *General natural history* and the *Critique* and the *Prolegomena* Kant investigated knowledge continuously. He wrote the important, if not very important, *A new elucidation of the first principles of metaphysical knowledge*, in 1755, *The only possible argument for demonstration of the existence of god*, in 1763, *Investigation on the clarity of the principles of natural theology and morality*, in 1764, *On the form and principles of the sensible and intelligible world*, in 1770. And the results of his investigation of knowledge in the *Critique* and the *Prolegomena* are many, fundamental, and conclusive. That his results are many, fundamental, and conclusive Kant is clear, too. Accordingly, in the following two decades he drew the consequences of the results.

The most important consequence, by far, is, in the *Idea*, that nature will produce for history as well a Kepler and a Newton. With respect to this consequence Kant wrote also the important, if not very important, review of Herder's *Ideas for the Philosophy of the history of humanity*, in 1785, and *The presumed beginning of human history*, in 1786.

In these two decades he wrote also the important, if not very important, *Metaphysical elements of natural science*, in 1786, *Critique of judgment*, in 1790, and *Logic*, in 1800.

Predominantly, however, he concerned himself with practice in these two decades. Next to the three very important practical treatises, he wrote the important, if not very important, *Critique of practical reason*, in 1788, *Religion within the boundaries of reason alone*, in 1793, *On the common saying, this may be correct in theory but it is of no use for practice*, in 1793, *The metaphysics of morals*, in 1797, *On a supposed right to lie from philanthropy*, in 1797, *The quarrel of the faculties*, in 1798, *Anthropology considered in pragmatic respect*, in 1798.

Kant wrote also the *Investigation of the question whether the earth in its rotation on its axis, by which it brings about the alteration of day and night, has undergone any change since the times of its origin, and how this can be established*, in 1754, *The question whether the earth ages, considered physically*, in 1754, *Brief exposition of some reflections on fire*, in 1755, *On the causes of the earthquakes on the occasion of the disaster which befell the western countries of Europe toward the end of last year*, in 1756, *Further observations on the earthquakes experienced for some time*, in 1756, *The use of metaphysics combined with geometry in natural philosophy, whose first test is the physical monadology*, in 1756, *New remarks toward an elucidation of the theory of winds*, in 1756, *A brief consideration of the question whether the west winds in our regions are humid because they pass over a large sea*, in 1757, *New doctrine of motion and rest, and its consequences for the first principles of natural science*, in 1758, *An attempt at some observations on optimism*, in 1759, *The*

false subtlety of the four syllogistic figures, in 1762, *An attempt to introduce the concept of negative magnitudes in philosophy*, in 1763, *Observations on the feeling of the beautiful and sublime*, in 1764, *Essay on the diseases of the head*, in 1764, *Dreams of a seer of spirits, explained by dreams of metaphysics*, in 1766, *On the different races of men*, in 1775, *On the volcanoes on the moon*, in 1785, *On the wrongfulness of the unauthorized publication of books*, in 1785, *Determination of the concept of a human race*, in 1785, *What does it mean to orient oneself in thinking?* in 1786, *On the philosophers' medicine of the body*, in 1786, *On the use of teleological principles in philosophy*, in 1788, *On a discovery by which any new critique of pure reason will be made dispensable by an older one*, in 1790, *On the failure of all philosophical attempts at theodicy*, in 1791, *Something on the influence of the moon on the weather*, in 1794, *The end of all things*, in 1794, *On a recently arisen superior tone in philosophy*, in 1796, *Proclamation of the near conclusion of a treaty of perpetual peace in philosophy*, in 1796, *On the power of the mind to be master of its feeling sick by sheer resolution*, in 1798, *On turning out books*, in 1798, *Declaration concerning Fichte's theory of science*, in 1799, *Which is the true progress metaphysics has made in Germany since the time of Leibniz and Wolff?* in 1804.

In the two decades through his death in 1804 Kant concerned himself predominantly with practice.

This is the time also of the revolution.

1788 is the year when the estates general were summoned, and Sieyès began writing *What is the third estate?* 1790 is the second year of the national assembly. 1793 is the year of the committee of public safely and Robespierre. 1797 is the year when Napoleon dictated the peace of Campo Formio. 1798 is the year when, next to the French republic, of which Napoleon was to become the first consul the following year, the Batavian republic, established three years earlier, and the Cisalpine and the Ligurian republics, established the previous year, the Helvetian and the Roman republics were established.

1798 is the year also when Kant wrote: the revolution finds in the hearts of all spectators a wishful participation that borders on enthusiasm, and this sympathy can have no other cause than the moral predisposition of the human race.

In 1807 Napoleon dictated the peace of Tilsit. In 1807 Hegel published the first of his very important treatises. By 1815 Napoleon and the revolution had been defeated. By 1815 Hegel had published the first half of his second very important treatise. After 1815 he published the second half of that treatise, and the third, the fourth, the fifth, and the sixth of his very important treatises.

Kant's success in the investigation of knowledge, and there has been no success greater than Kant's in the investigation of knowledge, begins with the investigation of Leibniz's proof of the conservation of living force, and the application, and most successful application, of Newton's laws to the constitution and origin of the world.

Kant's success in the investigation of knowledge ends in practice, and with a wishful participation in the revolution bordering on enthusiasm, which can have no other cause than the moral predisposition of the human race.

The success of Kant's investigation of knowledge is a consequence of the success of the acquisition of knowledge in the previous one and a half centuries.

The revolution is another consequence of that success. It thus cannot be an accident that Kant will share the success of his practical philosophy with the success of the revolution. As it cannot be an accident that the revolution will share its defeat with the defeat of Kant's practical philosophy.

The review of Kant, then, should be of his most important results, first in practice, then in physics, and then in the investigation of knowledge.

To put it otherwise, Kant writes: two things fill the mind with ever new and growing admiration and respect the more and more steadily reflection concerns itself with them, the starry heaven over me and the moral law in me (*Critique pr.*, II). The review should try and show, first why the moral law in me fills the mind with admiration and respect, then why the starry heaven over me does, and then turn to Kant's results in the investigation of knowledge.

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Kant begins in the *Critique*: whether the labor of knowledge, which is the task of reason, travels the secure path of science can soon be judged by success; if it cannot reach its goal, or cannot command unanimity, it has not entered this path (pr.).

Thus, Kant affirms, logic has traveled this path since Aristotle, mathematics has traveled it since the earliest times, natural science has traveled it since Bacon.

On the other hand: metaphysics has not yet been so favored by fortune, in metaphysics we have to retrace our steps countless times, it remains a battlefield for mock combat in which no knight ever gains lasting possession by victory (*id.*). Hence: the task of this critique is to change the ways of metaphysics by undertaking, following the example of the geometers and the investigators of nature, its complete revolution (*id.*).

And metaphysics was favored by fortune, with Kant. If not in a way it wanted to be favored.

Kant writes: as the actual goal of its investigation metaphysics has only three ideas, god, freedom, and immortality, so that the second, combined with the first, would have the third as a necessary consequence; everything else with which this science concerns itself is only a means of attaining these ideas and their reality (I, 2.2.1.3); the insight into them would be theology, morality, and through their combination religion, the highest goals of our being (*id.*).

The actual goal metaphysics has are the three ideas, god, freedom, and immortality. And if the actual goal it has is not the only goal it can have, if it can have a goal other than the actual goal it has, metaphysics would be wise to have, actually, that other goal. For, if the actual goal it has is the only goal it can have, metaphysics is doomed to remain a battlefield for mock combat in which no knight ever gains lasting possession by victory.

Kant continues: metaphysics needs these three ideas not for the sake of natural science, but to go beyond nature (*id.*), and: reason can employ all methods and principles to go to the innermost of nature, but never to fly beyond the boundaries of nature, outside which there is nothing for us but empty space (2.2.app.).

The actual goal metaphysics has, the three ideas, god, freedom, and immortality, and hence the highest goals of our being, theology, morality, and religion, are all in that empty space.

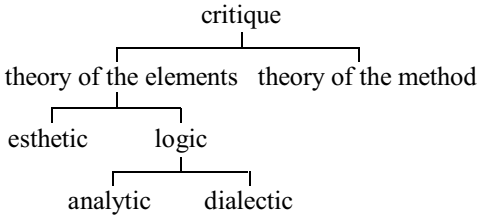
Twenty-one centuries before Kant, Aristotle's treatises came into being. Three centuries later, eighteen before Kant, the *Metaphysics* came into being. If there is metaphysics in the treatises collected into the *Metaphysics*, or in any other of Aristotle's treatises, the goal metaphysics actually has, such as Kant inspected that goal, is not the only goal metaphysics can have. Or has had. Metaphysics such as it is not the collection of the treatises after the physical treatises, but the concern with the things beyond the physical things, came into being in the centuries after the *Metaphysics* came into being, a millennium and a half before Kant. The philosophers, being philosophers, would have been eager to concern themselves with the things beyond the physical things, whether there are such things or not. Eager, in Kant's description: to go beyond nature. With the advent of Christianity among them they became eager to concern themselves with nothing else. To concern themselves, in Kant's description: with god, freedom, and immortality, and with everything else only as a means of attaining these ideas.

The attempts to go beyond nature Kant traces back to Plato. He writes: Plato abandoned the world of the senses, because it sets such narrow limits to the understanding, and ventured beyond it on the wings of the ideas into the empty space of pure understanding (intr.3); Plato did not notice that by his efforts he made no headway at all, for he had no resistance which could support him and to which he could apply his strength in order to set the understanding in motion (*id.*).

Kant favored metaphysics, however, not only with this negative result. That the actual goal it has is unattainable. That it has made and will make no headway at all into the empty space beyond nature. He favored it with the affirmative result as well that the actual goal it has is an illusion.

As *The great instauration* is: the true end and lawful termination of an infinitude of error (pr.), so the *Critique* is, under this proposition as its motto: the true end and lawful termination of an infinitude of error. As the *Discourse on the*

method is a treatise on the method, also, so the *Critique* is: a treatise on the method (pr.). And these are the main divisions of the treatise on the method, the true end and lawful termination of an infinitude of error,



The entire *Critique* comprises 927 pages. The theory of the elements comprises 700 pages. The logic comprises 658 pages. The analytic comprises 260 pages, the dialectic 383 pages.

The analytic and the dialectic comprise the bulk of the treatise. The division between them is also the fundamental in the *Critique*. It is the division between theory and the application of theory. In particular: the analytic is the logic of truth, the dialectic is a logic of illusion (2.1.2.intr.). And it is the logic of illusion that concerns itself with the actual goal of metaphysics, god, freedom, and immortality, and hence the highest goals of our being, theology, morality, and religion.

Kant concludes his investigation of these goals with a summary: we have reviewed the building materials and determined for what kind of building, of what height and strength, they suffice; and it has turned out that, although we wanted a tower to reach the heaven, the materials suffice only for a house spacious enough for our tasks in the plane of experience and high enough to survey it (II, intr.).

And he introduces it with a warning: the land of pure understanding is an island; the land of truth, it is enclosed in unalterable boundaries by nature itself; it is surrounded by a wide and stormy ocean, the seat of illusion, where fog banks and melting icebergs, ceaselessly pretending to be new lands, deceive the voyager with empty hopes, tempting him into adventures from which he cannot refrain and which he can yet never complete (I, 2.1.2.3).

There they are. God, freedom, and immortality, theology, morality, and religion, a tower to reach the heaven, fog banks and melting icebergs.

It is a trivial consequence of the fact, with Aristotle, that: the world is one, there is nothing next to the world, or outside the world. Or it could have been a trivial consequence, had the philosophers resisted the empty hopes.

Kant writes: the world itself has no place, there is nothing other than the world (2.2.2.2); the world is the mathematical whole of all phenomena, and the very same world is called nature when considered a dynamical whole (*id.*).

And he draws the trivial consequence, or what could have been a trivial consequence: to speak of an intelligible world next to the sensible world is nothing but empty peddling of words (2.1.2.3).

We can never build a tower to reach the heaven, then. The voyage to the fog banks and melting icebergs beyond the land of truth can never be completed.

And yet, we must try to build one of the turrets of that tower. The voyage to one of those fog banks and melting icebergs must be undertaken. That turret, and that fog bank or melting iceberg, is freedom, and hence morality.

The actual goal of metaphysics is the object of mock combat. It is in the empty space beyond nature in which the understanding can make no headway at all. And yet, Kant's most important practical result is an attempt to attain the actual goal of metaphysics. It is not an attempt to attain all three ideas, and all three highest goals of our being. It is not an attempt to attain god and theology, and not an attempt to attain immortality and religion. It is yet an attempt to attain freedom and morality.

Kant writes: to know an object requires that I can prove its possibility; but to think I can whatever I will, as long as I do not contradict myself (pr.). Hence, in particular: even if I cannot know my soul, and hence I cannot know freedom as a property of anything to which I can ascribe effects in the sensible world, I can think freedom, and that is, its representation contains no contradiction (*id.*).

Freedom is an idea, and it is clear what an idea is: an idea is a concept which goes beyond the possibility of experience (2.2.1.1).

Plato found out that there are ideas, and that they can be efficient causes. Kant writes: Plato found his ideas in everything practical, and that

is, everything that rests on freedom (*id.*). Moreover: that no man will ever act adequately to the pure idea of virtue does not at all show that it is chimerical; because it is only through this idea that all judgments of moral worth or unworth are possible (*id.*); and Plato rightly saw clear proofs of origin from ideas not only where human reason shows true causality, and where ideas become efficient causes, of actions and their objects, namely in morality, but also in the consideration of nature itself (*id.*).

An idea, by being an idea, is not real. An idea, however, need not be real in order to motivate, or what is the same, in order to be an efficient cause. And the idea of freedom can motivate precisely because it is not real, because it is an idea, a concept which goes beyond the possibility of experience.

Hence: because we must necessarily represent ourselves through reason as belonging to a moral world, even as the senses do not present us with anything but a world of appearances, we must assume the moral world to be a consequence of our conduct in the sensible world, and since the sensible world does not offer us such a connection, assume the moral world to be a world future for us (II, 2.2).

This future world is presented even more loftily in the *Groundwork*: if freedom of the will is presupposed, morality follows by the mere analysis of the concept (3), for: freedom of the will can be nothing but autonomy (*id.*).

No one is autonomous. Or can ever be autonomous. Everyone nevertheless can try to be autonomous. Indeed, must try to be autonomous: actions of which the world has given no example are nevertheless inflexibly commanded by reason (2).

To try to be autonomous: so act as if the maxim of your action were to become through your will a general law of nature (*id.*).

This is the general imperative. And the general imperative entails the practical imperative: so act that you employ mankind, in your own person as well as in the person of anyone else, always also as an end, never only as a means (*id.*).

We must assume the moral world to be a consequence of our conduct in the sensible world. Even as we do not. You must so act that you employ mankind, in your own person as well as in the person of anyone else, always also as an end, never only as a means. Even as you do not.

Kant writes: it is most reprehensible to obtain the laws of what I must do from what is done (*Critique*, I, 2.2.1.1). Even as it is what we do, and what you do, and because it is what is done.

Morality is not what is done, or what we do, even if what we do is what is done, or what you do, even if what you do is what we do and what is done.

Morality is what we must do, and what you must do. We do not assume the moral world to be a consequence of our conduct in the sensible world. But we must. You do not so act that you employ mankind, in your own person as well as in the person of anyone else, always also as an end, never only as a means. But you must. Actions of which the world has given no example are nevertheless inflexibly commanded by reason.

The moral law in me, then, fills the mind with admiration and respect not because it is real, or because I follow it, or I can follow it, or I will follow it. The moral law in me fills the mind with admiration and respect because it is ideal, and because I must follow it, even as I do not follow it, and even if I will not follow it.

This explains also why there is no morality, in practice, and there can be no morality, in practice, without enlightenment: enlightenment is man's emergence from his self-imposed minority; minority is the inability to use one's own understanding without the direction of another; it is self-imposed when its cause is not lack of understanding but lack of resolution and courage; *sapere aude!* have the courage to use your own understanding! is thus the motto of enlightenment.

Sapere aude!

Spirit, for its part, demanded from philosophy the restoration of its self-imposed minority, and philosophy complied.

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The *General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws* is in three parts. The first part is: on the systematic constitu-

tion among the fixed stars (*id.*). The second part is: on the first state of nature, the formation of the heavenly bodies, the causes of their motion, and their systematic relation, in the planetary system in particular as well as in the whole of creation (*id.*). The third part is: an attempt to compare with one another, on the ground of the analogies of nature, the inhabitants of the different planets (*id.*).

In the second part, which comprises the bulk of the treatise, Kant formulates the theory of the origin of the solar system. In the first part he obtains most general results concerning, as the title of the treatise has promised, the application of Newton's laws to the entire structure of the world. In the third part he obtains very general results concerning man's place in the world.

The theory of the origin of the solar system was formulated conclusively in the next generation by Laplace. Kant's most general results concerning the application of Newton's laws to the entire structure of the world anticipate most general results of the big bang theory. His very general results concerning man's place in the world anticipate very general results of the theory of evolution.

The theory of the origin of the solar system is a most dramatic instance of the acquisition of knowledge. It is the first theory of origin. It is the first removal of god from origin. Kant writes: I assume the matter of the whole world dispersed and in complete chaos (pr.). It is not the first theory to reach far beyond the earth in space. The theory of the planets developed by Ptolemy reaches to Saturn, the theory of vortices developed by Descartes reaches beyond the solar system to the systems of the other stars. It is the first theory, however, to reach far beyond the present in time. Kant presumes a development of: millions of years (II, 7), and: millions of centuries (*id.*). It is the first theory to reach back in time with laws. Kant writes: from the initial chaos matter forms in accordance with the laws of attraction and its motion is modified through repulsion; I see the creation of an ordered whole by the established laws of motion (pr.).

It is a most dramatic instance of development from no inkling to finality, too. Kant formulated the theory in 1755. Laplace formulated it conclusively in 1796.

Nor is it an accident that the philosopher who completed Newton's labors with the demonstration of the dynamic stability of the solar system, and completed Hume's labors with the calculation of the probability of the rising of the sun tomorrow, completed Kant's labors with the conclusive formulation of the theory of the origin of the solar system.

This is the theory, then, in its conclusive formulation, in the *Exposition of the system of the world*, in 1796.

Laplace begins by formulating the problem: all planets revolve around the sun, from west to east, and nearly in the same plane; the satellites revolve around their planets, in the same direction, and nearly in the same plane; and the sun, the planets, and these satellites in which rotation has been observed all rotate in the same direction and nearly in the same plane; a phenomenon so extraordinary cannot be the effect of chance, it points to a general cause (V, 6); an equally remarkable phenomenon is that the orbits of the planets and satellites are of small eccentricity, but those of the comets are very elongated; the orbits of the system present no nuance between great and small eccentricity; we are again forced to recognize the effect of a regular cause (*id.*); what is this primitive cause (*id.*)?

And Laplace then solves the formulated problem: whatever its nature, this primitive cause must have embraced all the planets, and considering the prodigious distances separating them, it could have been nothing but a fluid of immense extent; in order to give them nearly circular motion in the same direction around the sun, the fluid must have surrounded this star like an atmosphere; we are thus led to think that, due to its excessive heat, the atmosphere of the sun extended primordially beyond the orbits of all the planets and retreated successively to its present limits (*id.*); in the primitive state in which we suppose it, the sun resembles the nebulae composed of a nucleus and surrounding nebulosity (*id.*).

In 1791, in *On nebulous stars properly so called*, Herschel reports: in one of my late observations I discovered a star of about the eighth magnitude surrounded with a faintly luminous

atmosphere of considerable extent; formerly a well connected series of objects led us to infer that all nebulae consist of stars (*id.*).

After the new observation and the reexamination of seventeen observations made from 1784 to 1790, Herschel concludes: the shining fluid surrounding these nebular stars is not essentially connected with them (*id.*).

Nebulae are thus of two kinds, those that consist of stars and those that are a luminous atmosphere surrounding a star, and Herschel further concludes: the surmise of the regeneration of stars by means of nebulae becomes more probable (*id.*).

Laplace continues in *The system*: Herschel has observed the earlier states of nebulae, and not in just one of them, but in their ensemble, as in a vast forest the growth of trees can be observed in the individuals of different age (V, 6); if we conceive, by analogy, that all stars are formed in this way, we can imagine their earlier state of nebulosity; and going back as far as possible, we arrive at nebulosity so diffuse that its existence can barely be conceived (*id.*); from the progress of condensation of nebulous matter we thus arrive at the same conclusion, that the sun was once surrounded by a vast atmosphere (*id.*); if bodies penetrated deeply into the solar atmosphere, its resistance would have made them fall on the sun; we may therefore conjecture that the planets were formed at the successive limits of the atmosphere by the condensation of zones of vapors which the atmosphere, as it was cooling, abandoned in the plane of its equator (*id.*); the sun's atmosphere cannot extend indefinitely; its limit is the point where the centrifugal force due to its rotation balances the gravity; but as cooling contracts the atmosphere and condenses the particles on the surface, by the law of areas rotation increases; thus the centrifugal force increases; the point at which gravity is equal to it is closer to the center; supposing therefore that the atmosphere extends at any one epoch to that limit, it must, as it cools, abandon particles at the limit and at the successive limits produced by the increasing rotation; the abandoned particles continue to revolve around the star because their centrifugal force is balanced by their gravity (*id.*); by their mutual attraction the particles of the successively abandoned zones of vapors must form distinct concentric rings revolving around the sun (*id.*); each ring of vapors must nearly always break into several masses; because their interior particles have smaller velocity, these masses must acquire spheroidal form with rotation in the direction of their revolution; they have therefore formed planets in the state of vapor (*id.*); in the center of each a nucleus begins to grow by the condensation of the atmosphere around it; in this state the planet resembles perfectly the sun in its nebulous state; the cooling must therefore produce at the different limits of the atmosphere phenomena similar to those described, rings and satellites revolving around the center in the direction of its rotation, and rotating in the same direction (*id.*); the small eccentricity of the orbits of the planets and satellites, their small inclination to the plane of the solar equator, and the identity in the direction of rotation and revolution of all these bodies thus follow from our hypothesis with great probability (*id.*).

It remains the theory of the origin of the solar system. Qualified inessentially, and quantified, and this essentially, it has become part of the theory of the origin and development of the world.

Hubble measured the distance of galaxies. In *Cepheids in spiral nebulae*, in 1925, he measured the distance of Andromeda. In the next few years he measured the distances of more galaxies, and in *A relation between distance and radial velocity among extragalactic nebulae*, in 1929, he established the law which is the foundation of the theory of the origin and development of the world.

Among the more particular problems solved with the measurement of the distance of galaxies is the problem, as Hubble describes it in *The realm of the nebulae*, in 1936, of: the nature of the nebulae. Hubble describes it under the headings: the formulation of the problem (1), and: the solution of the problem (*id.*). The solution is: nebulae were found to be far beyond the limits of the galactic system, they are independent stellar systems in extragalactic space (*id.*). The problem was formulated when: the development of nebular research came into immediate contact with the philosophical theory of island universes, and the

question of the distances was put in the form, are nebulae island universes (*id.*)?

Nebular research Hubble traces back to Herschel.

The theory of island universes he traces back to Kant, and notes: some of Kant's remarks are an excellent example of reasonable speculation based on the principle of uniformity, the presumption that any large sample of the universe is much like any other (*id.*).

On the principle of uniformity, Kant obtains these results: the fixed stars in that plane, in which they present by their densest accumulation the streak of light called the Milky Way, but in which they must be conceived to extend through the entire heaven, are not randomly scattered, they are a system similar to the planetary (pr.); the planetary system, and next the Milky Way, are the first members of a progression of systems, and the first members of this endless progression make it possible to know what must be presumed about the whole (I); thus, elliptical nebulae are not large stars, but systems of many stars whose remoteness presents them in a uniform pale glimmer, and by analogy with the stellar system in which we find ourselves we can presume that they are stellar systems, or Milky Ways (*id.*).

The progression of systems is not endless. Other than that, the progression of systems in the world is the progression of systems described by Kant. The first member of the series is the local system of a star and planets revolving around it, the next member is the local system of stars, the local galaxy, the Milky Way, the next member is the local system of galaxies, the Local Group, and the next member is the local system of clusters of galaxies, the Local Supercluster.

Kant obtains most general results concerning the entire structure of the world on the principle of uniformity, the presumption that any large sample of the universe is much like any other. The samples are like one another in form. He formulates the theory of the origin of the solar system also on the principle of uniformity, the presumption that any large sample of the development of the solar system is much like any other. The samples obey the same laws. They are like one another in form.

He will investigate knowledge also on the principle of uniformity, the presumption that any large sample of knowledge, and any large sample of the acquisition of knowledge, is much like any other. The samples are like one another in form.

And he obtains very general results concerning man's place in the world also on the principle of uniformity, the presumption that any large sample of rational beings is much like any other. The samples are like one another in form.

In 1798, four-odd decades after the *General natural history and theory*, not quite two decades after the *Critique*, Kant published the *Anthropology considered in pragmatic respect*.

He writes in the *Anthropology*: the problem of the genus of man is absolutely insoluble, for the solution must be obtained from experience by the comparison of two species of rational being, and this experience is not given to us (II, E).

Linnaeus, in fact, whom Kant notes a page later in the *Anthropology*, had already solved the problem of the genus of man. In *The system of nature*, in 1735, man is the species *homo sapiens*, *homo sapiens* is one of the two species of the genus *homo*, *homo* is one of the four genera of the order *primates*, *primates* is one of the eight orders of the class *mammalia*, *mammalia* is one of the six classes of the kingdom *animalia*, and *animalia* is one of the three kingdoms of the empire *naturalia*. A page later in the *Anthropology* Kant notes orangutan. The other species of the genus *homo* in *The system of nature* is *homo siluestris orang outang*.

Linnaeus obtains the solution of the problem of the genus of man from experience, as well as on the presumption that man is a rational being. Indeed, as the specific difference of *homo sapiens* – as Linnaeus, too, named that species – Linnaeus proposes: know yourself (I, 1). Because, he adds, ascribing that wise saying to Solon: to know yourself is the first step toward wisdom (*id.*).

Linnaeus obtains the solution of the problem on the presumption that, whether the experience of man is the only experience of a species of rational being given to us or not, *homo sapiens* is not the only species of man. The species of man are two, *sapiens* and *orang outang*.

That the experience of man is not the only experience of a species of rational being given to us, Hume established. Beasts are endowed with

thought and reason as well as men.

That even if the experience of man were the only experience of a species of rational being given to us, however, the problem of the genus of that species could be solved, and more generally, that all such problems, of the genus of a species the experience of which is the only experience of a species of that genus given to us, can be solved, and more generally still, that all problems of the genus of a species which is the only species of its genus can be solved, Kant himself established.

Enclosed in unalterable boundaries by nature, the acquisition of knowledge is enclosed in the unalterable boundaries of experience. It is one of the most important results of the *Critique*, however, that, enclosed in the unalterable boundaries of experience, the acquisition of knowledge is enclosed in the unalterable boundaries of possible experience, not actual experience. The very fact of the acquisition, the fact that, enclosed in the unalterable boundaries of experience, it is yet a development, shows that the boundaries can only be of possible experience, not actual experience. Hence, when an experience is not given, there is yet the question of whether the possibility of that experience is given. And when it is, knowledge of what there is but one specimen, including of its genus, can be acquired, too.

It is on this ground, too, that Kant concludes in the *General natural history and theory of the heaven* with: an attempt to compare with one another, on the ground of the analogies of nature, the inhabitants of the different planets (III). The inhabitants of Mercury, Venus, the earth, Mars, Jupiter, and Saturn.

Before Kant, Kepler, in 1634, in the *Dream, or on lunar astronomy*, describes the inhabitants of the moon. Huygens, in 1698, in the *Κοσμοθεωρός, or conjectures on the countries and inhabitants of the heavenly bodies*, compares with one another the inhabitants of Mercury, Venus, the earth, Mars, Jupiter, and Saturn.

After Kant, Herschel, in 1794, in *On the nature and construction of the sun and fixed stars*, compares them with the inhabitants of the sun as well.

Kant obtains this result: if spiritual abilities depend necessarily on the machine they inhabit, then the excellence of thinking natures, the nimbleness of their ideas, the clarity of the concepts they receive from external impressions, the faculty to put these together and the agility in its exercise, in short, their entire perfection, stands under a certain rule, according to which it becomes more excellent and complete in relation to the distance from the sun of the place these natures inhabit (III).

The general conclusion, the entire perfection of thinking natures stands under a certain rule, anticipates the *Critique*.

The premise, spiritual abilities depend necessarily on the machine they inhabit, anticipates Darwin.

And on the more particular conclusion, the perfection of thinking natures becomes more excellent and complete in relation to the distance from the sun, Kant comments: if the idea of the most elevated classes of rational creatures inhabiting Jupiter and Saturn arouses the jealousy of men, and humiliates them by the knowledge of their own lowliness, they can comfort themselves by looking at the low stages of those inhabiting Venus and Mercury, which are far below the perfection of human nature (*id.*).

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Knowledge Kant investigates in the *Critique* and the *Prolegomena*.

Hume's *Treatise*: fell deadborn from the press, without reaching such distinction as to excite a murmur among the zealots. Hume wrote the *Inquiry*.

As soon as it appeared, Kant's *Critique* was judged: a textbook of drawing, teaching nothing more than what everyone with a good eye can readily do. Kant wrote the *Prolegomena*.

To the same end, *any future metaphysics which will be able to stand as science*. And the *Prolegomena* is a treatise of clarity and distinction second only to the *Inquiry*.

Once again Kant begins: it seems almost laughable that, while every other science continues to advance, this science, which will be wisdom itself, whose oracle every man will consult, keeps turning round on the same spot without making a step forward (pr.), indeed: the reader of these prolegomena who thinks for himself will be convinced that there is yet no metaphysics at all

(*id.*).

Kant continues: in order to present a knowledge as science, it must be possible to determine exactly its difference, what it has in common with no other knowledge and is peculiar to itself (1); this peculiarity can be a difference in the object, or in the sources of knowledge, or in the kind of knowledge (*id.*); where the sources of metaphysical knowledge are concerned, it lies in its concept already that they cannot be empirical; its principles must therefore never be taken from experience; for it should be not physical, but metaphysical knowledge, and that is, lying beyond experience (*id.*).

But if so: it is knowledge *a priori*, knowledge from pure understanding and pure reason (*id.*).

And: in this metaphysics is not different from pure mathematics at all (*id.*).

Now: whatever their source, and whatever their form, judgments are with respect to their content either analytic or synthetic, they either add nothing to the content of knowledge or enlarge knowledge (2); judgments from experience are always synthetic (*id.*); and mathematical judgments are in their entirety synthetic (*id.*); the proposition $7 + 5 = 12$ may be thought analytic; when it is examined more closely, however, it is found that the concept of the sum of 7 and 5 does not contain the one number which their combination is; and this is clearer still in the case of larger numbers (*id.*); nor is any fundamental proposition of pure geometry analytic; that the straight line between two points is the shortest is a synthetic proposition (*id.*).

In general: since pure mathematical knowledge must go beyond the concept, its propositions can and must never arise through the analysis of concepts, and that is, analytically; they are therefore in their entirety synthetic (*id.*); and I should note the difficulty which the neglect of this observation has caused philosophy; Hume, when he felt the call, so worthy of the philosopher, to survey the entire domain of knowledge *a priori*, inadvertently severed from it a whole and indeed its most valuable province, pure mathematics (*id.*).

Mathematical judgments are in their entirety synthetic. And: properly metaphysical judgments are in their entirety synthetic as well (*id.*).

Hence: synthetic *a priori* propositions are the only goal of metaphysics; the production of knowledge *a priori*, and ultimately synthetic *a priori* propositions, and specifically in philosophical knowledge, is the essential content of metaphysics (*id.*).

In general: the problem on which everything depends is, how are synthetic *a priori* propositions possible (5)? and this main question divides into four other questions, how is pure mathematics possible? how is pure natural science possible? how is metaphysics at all possible? and how is metaphysics as science possible (*id.*)?

It lies in the concept of metaphysical knowledge already that its sources cannot be empirical. Its principles must therefore never be taken from experience. Metaphysical, it should be knowledge lying beyond experience.

But if so, it is knowledge *a priori*, knowledge from pure understanding and pure reason.

And in this metaphysics is not different from mathematics at all.

Mathematical propositions are *a priori*, but they are also in their entirety synthetic.

And metaphysical propositions, *a priori* as they are, are in their entirety synthetic as well.

Judgments from experience are always synthetic. Conversely, too, synthetic judgments are always from experience.

And if so, even if it lies in the concept of metaphysical knowledge already that its sources cannot be empirical, its sources are empirical. Even if its principles must never be taken from experience, its principles are taken from experience. Even if it should be knowledge lying beyond experience, it is knowledge lying within experience.

Which explains why, on the one hand, metaphysics will be wisdom itself, whose oracle every man will consult. On the other hand, there is yet no metaphysics at all. On the one hand, metaphysics is a battlefield for mock combat. On the other hand, metaphysics can follow the example of the geometers and the investigators of nature. On the one hand, the actual goal of metaphysics are only the three ideas, god, freedom, and immortality. On the other hand, synthetic *a priori* propositions are the only goal of metaphysics.

Appearances to the contrary, metaphysics is

empirical.

Just as mathematics is empirical.

The proposition, metaphysics is empirical, stands on the proposition, mathematics is empirical.

The proposition, mathematics is empirical, in turn stands on the solution of the problem on which everything depends, how are synthetic *a priori* propositions possible? It stands on the resolution of the apparent contradiction between *a priori* and experience.

The solution of the problem on which everything depends, how are synthetic *a priori* propositions possible? in turn stands on the division of that question into the questions, how is mathematics possible? how is natural science possible? and how is metaphysics possible? These questions pose that question, and the answer to these questions is the answer to that question. The fact of mathematics, natural science, and metaphysics poses the problem, and supplies the solution of the problem, on which everything depends.

The fact of the acquisition of knowledge poses the problem, and supplies the solution of the problem, on which everything depends.

All of which should be made more explicit in this order, then. That there is no contradiction between *a priori* and experience. That mathematics is empirical. That metaphysics is empirical. The consequence of these facts. And that the acquisition of knowledge is the problem of the investigation of knowledge.

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Kant writes in the *Prolegomena*: experience is nothing but the synthesis of perceptions (5).

There is verbal infelicity in Kant. Idealism, for instance, which: I wish I had called this concept of mine otherwise. Or transcendental, of which there is so much in the *Critique*, transcendental esthetic, transcendental logic, transcendental analytic, transcendental dialectic, transcendental ideas, as to make the concept superfluous. Or perception, of which there is so little, next to impression, sensation, intuition, sensibility, as to demand the elimination of two of these five concepts. It is because the amount of verbal infelicity which Kant cleared away is enormous, too, that the little that he did not can and must be cleared away.

Experience is nothing but the synthesis of perceptions.

Kant continues: and if synthetic *a priori* judgments are possible, there must be in perception already something *a priori*, bound inseparably with the concept before all actual experience, or distinct perception (7); but how is it possible to perceive *a priori*? how can perception of the object precede the object itself (8)? there is only one way in which my perception can precede the actuality of the object and take place as knowledge *a priori*, namely, if it contains nothing but the form of sensibility, which in my subject precedes all actual impressions by which I am affected by objects (9).

The fact of synthetic *a priori* propositions shows that there must be *a priori* in perception already. And this can be nothing but the form of sensibility.

Perception has form. *Ipso facto* experience, the synthesis of perceptions, has form. And this is how, by having form, experience is *a priori*.

Experience has form, and this result corroborates Hume's results and vastly extends them.

Experience is the outcome of the development of faculties and their exercise, of all faculties and their exercise, in all organisms. Experience is the effect of instinct, or mechanical tendency. Everything of which an organism is aware, including an organism which has become a philosopher, terminates in experience. Experience is the foundation on which the acquisition of knowledge stands with any security.

Faculties and their exercise Hume describes in all their generality, from impressions through reasonings. And so does Kant, from: all actual impressions by which I am affected by objects, through perception: the capacity to receive representations from objects is sensibility, and the effect of an object on the capacity for representation is perception (*Critique*, I, 1), through understanding: the faculty of judgment (2.1.1.1), and reason: the faculty of principles (2.2.intr.).

Hume proposes to explain human nature, and concerns himself with man, and beast, and oak and elm. Hence the generality of his results. Kant concerns himself with rational beings. He does so in his physical investigations, from the lowly

inhabitants of Mercury and Venus to the elevated creatures inhabiting Jupiter and Saturn. He does so in his practical investigations, because: every species of rational beings is destined to a common end, namely the promotion of the highest good as a common good of all (*Religion*, III, 1.2). And he does so in his investigation of knowledge, because: it is not necessary to assume that the form of sensibility is limited to man, it may well be the form of sensibility of all rational beings (*Critique*, I, 1.2). Hence the greater generality of his results.

Knowledge *a priori* is knowledge from pure understanding and pure reason. But if synthetic *a priori* judgments are possible, there must be *a priori* in perception already. And: all our judgments are at first only judgments of perceptions (*Prolegomena*, 18). Thus, not only judgments from experience are always synthetic, but also conversely, synthetic judgments are always from experience. Thus, also, not only if synthetic *a priori* judgments are possible, there must be *a priori* in perception already, but also conversely, if there is *a priori* in perception, synthetic *a priori* judgments must be possible. The *a priori* encompasses the entire machinery of faculties and their exercise. It reaches from perception to pure understanding and pure reason, and from pure understanding and pure reason back to perception. Form reaches from impression to thought, and from thought back to impression.

Lorenz, in *Kant's theory of the a priori in the light of contemporary biology*, in 1941, writes: if one is familiar with the innate modes of behavior of animals, one can readily hypothesize that the *a priori* is due to hereditary differentiations of the central nervous system which become characteristic of the species and result in hereditary predispositions to think in certain forms (*id.*).

Hereditary differentiations of the central nervous system result in hereditary predispositions to think in certain forms, and long before the development of a central nervous system hereditary differentiations result in hereditary predispositions to perceive in certain forms. Needless to add, beginning to end, the forms are the outcome of selection.

Faculties and their exercise are machinery in Hume, and so they are in Kant. He writes: in the natural constitution of a being organized to the end of life, we presume as a principle only what is most appropriate to that end (*Groundwork*, 1); and nature will take upon itself not only the end but also the means, and with wise foresight entrust them both simply to instinct (*id.*). And from the wise foresight of nature Kant excludes only the one fog bank or melting iceberg: the will good in itself (*id.*), on which, by the mere analysis of the concept, freedom, and hence morality, stands.

Experience is the effect of instinct, or mechanical tendency. More explicitly, it is the effect of the wise foresight of nature, which in the constitution of a being organized to the end of life, taking upon itself not only the end but also the means, will entrust them both simply to instinct. Still more explicitly, it is the effect of selection.

Hence, everything of which an organism is aware, including an organism which has become a philosopher, terminates in experience.

It follows that all inferences terminate in experience.

It follows also that subjectivity is an impossibility.

In my subject the form of sensibility precedes all actual impressions by which I am affected by objects. Just as in your subject the same form of sensibility precedes all actual impressions by which you are affected by objects. Just as in any subject the same form precedes all actual impressions. In my subject the form of understanding precedes all actual understanding of anything I understand, or try to understand, or do not understand, or wantonly misunderstand. Just as the same form of understanding does in your subject. Just as it does in any subject. In my subject the form of thought precedes anything I actually think, or do not think, or think I think, or do not think I think, or think I do not think. Just as it does in your subject, and in any subject. Formally, my subject is your subject is any subject. Just as your subject is.

Which explains the fact that, if you and I happen to be in the same place at the same time, singular as they each are, your singular experiences are very much like my singular experiences, and even more alarmingly, my singular experiences are very much like your singular

experiences. Still more alarmingly, if we, you and I, happen to be in the same place for a long time, your singular experiences are exactly like my singular experiences, my singular experiences are exactly like your singular experiences, and given the identity of indiscernibles, the conclusion is unavoidable, you are I, and I am you.

At the far end of experience, where impressions are concerned, the subject is all sentient beings, at the near end, where thought is concerned, the subject is all rational beings.

Hence, the question of objectivity, the objectivity of perception, the objectivity of thought, and the objectivity of judgment – the question whether perception agrees with its object, whether thought agrees with its object, and whether judgment agrees with its object – has nothing to do with the relation of subject and object, generalized as the subject must be into all sentient beings and all rational beings. It has instead everything to do with generality.

Kant writes: all our judgments are at first only judgments of perceptions. And continues: when a judgment agrees with an object, however, all judgments of the same object must agree with one another; the objective validity of a judgment of experience is thus nothing but its necessary general validity (*Prolegomena*, 18); the given perception must be subsumed under a concept which determines the form of judgment in general in relation to perception and thus achieve the general validity of the empirical judgment (20).

Objective validity is nothing but general validity. Or what is the same, if everything terminates in experience, empirical validity is nothing but general validity.

The goal of physics, such as Planck describes the goal, follows.

In 1908, in *The unity of the physical picture of the world*, Planck describes the goal while arguing against the supposition, as it was becoming mandatory at the time, that there is no such thing as the world, all there is is world-views and pictures of the world. Planck writes: the goal of physics is the complete emancipation of the physical picture of the world from the individuality of the picturing mind; nor did Copernicus, Kepler, Huygens, Newton, Faraday speak of their picture of the world, they spoke of the world, or nature; and is there any knowable difference between their world and our picture of the world? certainly not; and it is clear after Kant that there is no method to prove such a difference (4).

Experience is the foundation on which the acquisition of knowledge stands with any security.

The acquisition is enclosed in the unalterable boundaries of experience. It is enclosed, however, in the unalterable boundaries of possible experience, not actual experience. And this distinction, between possible experience and actual experience, explains the facts, both, that experience is an accumulation and that the acquisition of knowledge is a development.

Kant writes: here is the solution of the puzzle, no *a priori* knowledge is possible but of objects of possible experience (*Critique*, I, 2.1.1.2); synthetic *a priori* propositions are related to nothing but the possibility of experience (2.1.2.2).

The form is the form not of actual experience. It is the form of possible experience, inclusive of all actual experience, and hence inclusive also of the possibility of extension of actual experience.

Moreover: the possibility of experience is what gives all knowledge *a priori* objective validity (2.1.1.2); the conditions of the possibility of experience are at the same time the conditions of the possibility of the objects of experience (*id.*).

And: what agrees with the formal conditions of experience is possible; what is connected with its material conditions is actual; and when the connection is determined in accordance with the general conditions of experience, it is necessary (*id.*).

Form gives experience its objective validity, or what is the same, its general validity. Form gives possible experience, inclusive of actual experience, hence inclusive also of the possibility of extension of actual experience, and hence the acquisition of knowledge as well, its objective validity, or what is the same, its general validity.

Thus, the ongoing current experience of each individual is the reiteration of the experience accumulated by the entire ancestry of the species. In particular, the accumulation is in the form of experience, and the actual experience of each individual reiterates the possible experience of

any individual of the species, and that is, reiterates it formally. At least in part, at least occasionally, the experience accumulated by the entire ancestry of the species has to include the accumulation of the singular experience of some particular individual as well. In particular, the accumulation is again in the form of experience, so that the actual experience of one individual becomes the possible experience of other individuals of the species, and that is, by the generalization of possible experience through the actual experience of that individual and the formalization of that actual experience into the possible experience of other individuals.

Nor, if genetic regulatory mechanisms, instinct, and learning have form – and the fact that they can be investigated formally shows that they have form – can the accumulation of experience be but in the form of experience.

And from its beginning through Aristotle through Ptolemy through Copernicus through Kepler through Huygens through Newton through Faraday through Planck, the acquisition of knowledge has been a development.

It has been the generalization of experience, and by the formalization of experience.

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If so, mathematics cannot not be empirical.

Kant writes: in place of objective necessity in the concept of cause Hume presumes subjective necessity, and denies reason any judgment about god, freedom, and immortality; yet he does not make empiricism so general as to include mathematics; he holds its propositions to be analytic; but if general empiricism of principles is presumed, mathematics is included (*Critique pr.*, pr.).

And Kant presumes general empiricism of principles.

He writes in the *Prolegomena*: the form of sensibility is twofold, space and time (10); and pure mathematics as synthetic *a priori* knowledge is possible because it has no other objects but the objects of the senses, even if it does not concern itself with the matter of the appearances but only with their form, space and time (11).

Mathematics has no other objects but the objects of the senses.

That mathematics has objects other than the objects of the senses has been argued after Kant, and against Kant, and was argued before Kant first by Plato.

Plato writes: mathematics leads the soul forcibly upward and compels it to study numbers themselves, never allowing it to attach them to the sensible things; numbers themselves can be grasped in thought alone and in no other way (*Republic*, VII, 525d-6a).

Aristotle, on the other hand, anticipates Kant. He writes: the objects investigated by the mathematician are abstracted from the sensible things (*Metaphysics*, XIII, 3); the physicist studies both form and matter (*Physics*, II, 2); the mathematician separates form from matter (*id.*).

And that with respect to mathematics, too: when Plato ventured beyond the world of the senses into the empty space of pure understanding he made no headway at all because he had no resistance which could support him and to which he could apply his strength in order to set the understanding in motion, the fact of the inseparable development of mathematics and physics shows. It shows that, even when mathematics appears never to allow the soul to attach the objects investigated by the mathematician, including numbers themselves, to the sensible things, physics compels the soul to attach the same objects, including numbers themselves, to the sensible things. That even when it appears that the objects investigated by the mathematician, including numbers themselves, can be grasped in thought alone and in no other way, the physicist grasps the same objects, including numbers themselves, in the sensible things.

Better yet, the inseparable development of mathematics and physics shows that whenever the mathematician separates form from matter, he separates from matter that form which the physicist, when he studies both form and matter, studies.

Aristotle writes: geometry studies natural lines but not as natural, optics studies mathematical lines but as natural, not as mathematical (*id.*). A generation-odd later, Euclid wrote the *Elements*. Studying natural lines but not as natural. He wrote also the *Optics*. Studying mathematical lines but as natural, not as mathematical. Num-

bers Euclid grasps in books VII-IX of the *Elements*. He grasps them mathematically, with lines but not as natural. He wrote also the *Phenomena*. In the *Phenomena* he grasps the motions of the heavenly bodies. Grasps them astronomically, with lines and numbers but as natural, not as mathematical.

Twenty-one and a half centuries after Euclid Lobachevsky developed non-Euclidean geometry, in the next generation Riemann generalized Euclidean and non-Euclidean geometries into the theory of manifolds and the theory of metrics, and two generations later Riemann's generalization became the foundation of the theory of relativity. Einstein writes: if geometry is conceived as the science of the laws governing the spatial relations of rigid bodies, it is to be considered the oldest branch of physics (*Aether*), and: I attach special importance to this view of geometry because without it I would not have been able to formulate the theory of relativity (*Geometry*).

The theory of relativity is a mathematical theory. After it quantum theory is a mathematical theory. Before it electrodynamics is a mathematical theory. Thermodynamics is a mathematical theory. The brightest period in the acquisition of knowledge is, with the labors of Galileo, Kepler, Descartes, Huygens, Newton, and Leibniz, the brightest period in the development of mathematics and physics inseparably. Ptolemy is a mathematician *par excellence* and a physicist *par excellence*. Archimedes is a mathematician *par excellence* and a physicist *par excellence*. Before Euclid, before Aristotle, and before Plato, the first philosophers are physicists and mathematicians inseparably. Democritus is a physicist and a mathematician, Anaxagoras is a physicist and a mathematician, Parmenides is a physicist and a mathematician, Pythagoras is a physicist and a mathematician, Thales is a physicist and a mathematician.

Physics predates the first philosopher. The first map, the map of Nippur, was drawn eight and a half centuries before the first philosopher, and traced back to the first map, physics is traced back to the first solution of the problem of the determination of distance and orientation, and thus to the solution of a physical problem and a mathematical problem inseparably. The planets were discovered two centuries earlier still, the discovery attested by the Venus tablet of Ammisaduqa, and traced back to the discovery of the planets, physics is traced back to the record of the synodic period of Venus, and thus once again to the solution of a physical problem and a mathematical problem inseparably.

Yet, against the evidence of the fact of the inseparable development of mathematics and physics, conclusive as it should be, the supposition that mathematics is not empirical has been maintained by the division of mathematics into two parts, such that, upon conceding that one of them, geometry, may be contaminated by experience, it would be yet argued that the other, arithmetic, is not, and even if geometry may have no other objects but the objects of the senses, the objects of arithmetic, and positively numbers themselves, can be grasped in thought alone and in no other way.

When Hume inadvertently severed from the domain of knowledge *a priori* its most valuable province, too, he severed arithmetic and algebra, not geometry. He writes: the ideas which are most essential to geometry, equality and inequality, right line and surface, are far from being exact and determinate; our appeal is to weak and fallible judgments which we make from the appearance of objects; the ultimate standard is derived from nothing but the senses (*Treatise*, I, 2.4); the first principles of geometry are drawn from the general appearance of objects (3.1). Geometry is empirical. It is arithmetic and algebra that: are discoverable by the mere operations of thought, without dependence on what is anywhere existent in the universe.

And Frege agrees with Hume, against Kant. In *The foundations of arithmetic*, in 1884, he writes: the laws of arithmetic are analytic (87); Kant underestimated the value of analytic judgments (88); I must contradict also the generality of Kant's proposition that without sensibility no object would be given to us; 0, 1 are objects which cannot be given to us in perception (89); to avoid incurring the reproach of petty quarreling with a genius to whom we can only look up in grateful awe, I must emphasize my agreement with Kant which far outweighs the disagreement; when he showed the truths of geometry synthetic

and *a priori*, he revealed their truth; he was mistaken about arithmetic; his concern, however, was to show that there are synthetic *a priori* judgments (*id.*).

Pace Frege, the laws of arithmetic are synthetic. Without sensibility no object would be given to us. And 0, 1 are objects which must be given to us in perception. All of which, if the fact of the inseparable development of physics and mathematics, inclusive as mathematics is of both arithmetic and geometry, does not show, the fact of the inseparable development of arithmetic and geometry shows.

In the *Concept writing*, in 1879, Frege writes: we divide all truths which require grounding into two kinds, according as proof of the one kind can be purely logical, of the other kind must be supported by empirical facts; and a proposition can belong to the first kind even if it can never have come to consciousness in a human mind without the activity of the senses (pr.).

Hence, in *The foundations of arithmetic*, he has to argue that this last fact is irrelevant in order to establish that the laws of arithmetic are analytic. He writes: the laws of number would be analytic judgments even if they need not have been found by thought alone; for the concern here is not with the way in which they have been found, but with the grounds of their proof; and as Leibniz says, the question is not about the history of our discoveries, which is different in different men, but about the connection and natural order of truths, which is always the same (17).

The history of our discoveries is not different in different men, however. The discoveries of Leibniz, rather than make each for himself, let alone differently, different men all waited for Leibniz to make. The discoveries of Frege, rather than make each for himself, let alone differently, different men all waited for Frege to make. The discoveries of Frege, too, waited for the discoveries of Leibniz, and the discoveries of Kant in between. In general, if the connection and natural order of truths is always the same, the history of our discoveries is not only not different in different men, it cannot be different in different men. And conversely, nothing but the fact that the history of our discoveries is not different in different men, and cannot be different in different men, can establish conclusively that the connection and natural order of truths is always the same.

If so, to the question whether the laws of arithmetic are analytic, or synthetic, that the truths for which proof can be purely logical could not have come to consciousness without the activity of the senses cannot be irrelevant. That the laws of number have not been found by thought alone cannot be irrelevant. The concern with the grounds of the proof of the laws of number is *ipso facto*, implicitly if not explicitly, concern with the way in which the laws of number have been found. And the evidence of the history of our discoveries is conclusive. Arithmetic and geometry have developed inseparably not only in the twenty-six centuries since the first philosophers. They have developed inseparably in the fifty-one centuries since the remotest beginning of mathematics.

Pythagoras discovered the incommensurability of the side and diagonal of the square, and thus the irrationality of $\sqrt{2}$. If there is a square, there is a square of side 1, and the diagonal of that square is $\sqrt{2}$. The relation of the lines implies, and is implied by, the relation of the numbers.

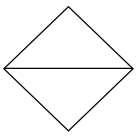
Given, that is, that 1 is a number, and $\sqrt{2}$ is a number. And what is a number is a problem that took millennia to solve.

The natural numbers, 1, 2, 3, . . ., are numbers, and whether 1 is one of the natural numbers is all by itself a problem that took millennia to solve. Under subtraction, the natural numbers yield the integers, and hence, more generally, the integers are numbers, and whether 0 is one of the integers is also all by itself a problem that took millennia to solve. Under division, the integers yield the rational numbers, and hence, more generally, the rational numbers are numbers. Under extraction of roots, the rational numbers yield irrational numbers and complex numbers, and hence, more generally, the real numbers, rational and irrational, are numbers, and more generally still, the complex numbers are numbers.

All of which is implicit in the labors of the mathematicians more than one millennium before Pythagoras. All of which becomes explicit in the labors of the mathematicians, however, more than two millennia after Pythagoras.

Three and a half millennia of labor are all by themselves an argument for synthetic, against analytic.

More particularly, twelve centuries before Pythagoras, there is, for instance, the mathematician whose labors are recorded on the tablet YBC 7289. The tablet shows a square and its diagonal, the number 30 outside the square, and the numbers $1\frac{8\,947}{21\,600}$ and $42\frac{307}{720}$ inside.

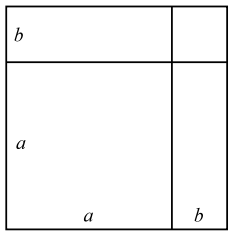


The third number is the product of the first two, and $1\frac{8\,947}{21\,600} = 1.414\,212\,963\ldots$ is an approximation of $\sqrt{2}$ so good, it is better than Ptolemy's: $1\frac{12\,653}{30\,547} = 1.414\,214\,162\ldots$ (*Almagest*, I, 10), if only by $0.000\,000\,001\ldots$. The mathematician *par excellence* whose labors are recorded on the tablet YBC 7289 is, like Pythagoras, a geometer *par excellence* and an arithmetician *par excellence*, both.

A century and a half-odd after Pythagoras, Eudoxus developed the theory of proportion. Proportion made possible operations with commensurable and incommensurable lines indiscriminately, and thus with rational and irrational numbers indiscriminately.

Given, again, that rational numbers are numbers, and irrational numbers are numbers.

Two-odd generations after Eudoxus, the *Elements* is not only paradigmatically geometrical, it is exclusively geometrical. The theory of proportion is developed in book V, with lines. Arithmetic is developed in books VII-IX. Euclid begins with the definitions: unit is that by which each of the things that are is one (VII, def. 1), and: number is a multitude of units (def. 2). Then the fundamental concern of these books is to show that operations with numbers obey proportion. To this end, numbers are always represented by lines, their products are always represented by areas. Euclid never deals with 0, never with negative numbers, never with powers higher than cubes. Arithmetic, in short, is a part of geometry. The theory of the irrationals, developed in book X, is also a part of geometry. The irrationals, too, are always represented by lines. Even algebra, or what was to become algebra, developed in book II, is a part of geometry. On the right, for instance, is Euclid's proof that: $(a+b)^2 = a^2 + 2ab + b^2$ (4).



Nineteen centuries after Euclid, in *The analytic art*, Viète formulates: the master rule, if four terms are such that the product of the extremes is equal to the product of the means, they are proportional, and conversely, if four terms are proportional, the product of the extremes is equal to the product of the means; thus a proportion is that from which an equation is composed and an equation is that into which a proportion is resolved (2).

The master rule liberates equations from proportions, and not only are numbers not represented by lines in *The analytic art*, Viète deals, next to squares and cubes, with: square-squares, square-cubes, cube-cubes, square-square-cubes, square-cube-cubes, cube-cube-cubes, and so on (3).

Two generations after Viète, Descartes laid the foundations of analysis. With the *Geometry*, not with arithmetic. Descartes begins: just as all arithmetic consists of only four or five operations, addition, subtraction, multiplication, division, and extraction of roots, which can be considered a kind of division, so nothing else is necessary in geometry in order to find required lines than to add or subtract other lines (I). And Descartes's fundamental result is to show that: if two lines are considered the principal, x and y (*id.*); then no matter how many other lines are given, they can all be expressed in terms of only known quantities next to the two unknown, x and y (*id.*); and since to determine a point a single equation containing two unknown quantities is necessary (II); all points of a line can be expressed by a single equation (*id.*). The equation of the conic sections, for instance, circle, ellipse, parabola, and hyperbola, is: $y = A - Bx + \sqrt{C + Dx + Ex^2}$ (*id.*).

It was in the century through Descartes, beginning with Cardano, that the four or five operations, addition, subtraction, multiplication, division, and extraction of roots, were explicitly extended to include the integers, including 0, and the rationals, and the irrationals, and the complex

numbers.

Thirty-four centuries separate the mathematician whose labors are recorded on the tablet YBC 7289 and Descartes.

Earlier, thirteen centuries before the mathematician whose labors are recorded on the tablet YBC 7289, forty-seven centuries before Descartes, the remotest beginning of mathematics can be traced back to the 3 calculations of area recorded on the two tablets W 20044/20 and W 19408/76. Or what is the same, the remotest beginning of mathematics can be traced back to the mathematician *par excellence* whose labors are recorded on the two tablets W 20044/20 and W 19408/76, thirteen centuries before the mathematician whose labors are recorded on the tablet YBC 7289, forty-seven centuries before Descartes.

On the one tablet the first mathematician records these four numbers, in this arrangement:

80 140
140 60.

On the other tablet he records eight numbers, these four, in the same arrangement:

1 200 930
1 200 870,

and these four, in the same arrangement:

980 1 280
1 420 520.

The arrangement implies, and the consistency of the arrangement implies unambiguously, the approximations for the areas of the three quadrilaterals,

$$\frac{80+140}{2}\times\frac{140+60}{2}=11\,000,$$

$$\frac{1\,200+1\,200}{2}\times\frac{930+870}{2}=1\,080\,000,$$

$$\frac{980+1\,420}{2}\times\frac{1\,280+520}{2}=1\,080\,000.$$

Traced back to the mathematician whose labors are recorded on the two tablets W 20044/20 and W 19408/76, the remotest beginning of mathematics is traced back to numbers and areas, both, and numbers implying areas and areas implying numbers, both. If geometry is the oldest branch of physics, geometry and arithmetic, both, are the oldest branches of physics.

Later, two and a half centuries after Descartes, thirty-six and a half after the mathematician whose labors are recorded on the tablet YBC 7289, forty-nine and a half after the first mathematician, the foundations of mathematics became the explicit concern of the mathematicians.

In this the most important are the labors of Cantor and Frege. And among the most important results are the axiomatization of arithmetic by Peano and the axiomatization of geometry by Hilbert.

The axiomatization of geometry is grounded in Kant explicitly.

In the *Foundation of geometry*, in 1899, under a motto from Kant, Hilbert begins: geometry, like arithmetic, requires only a few and simple principles for its logical development; these principles are called the axioms of geometry; the establishment of the axioms of geometry and the investigation of their relations is a problem treated in many excellent works since Euclid; this problem is equivalent to the logical analysis of our perception of space (intr.).

Peano axiomatized arithmetic in *The principles of arithmetic presented by a new method*, in 1889. The first and the sixth axioms are:

$1\ \varepsilon\ N$,

$a\ \varepsilon\ N \supset a+1\ \varepsilon\ N$,

where 1 means unity, ε means is, N means number, positive integer, $\supset$ means that what follows deduces from what precedes, and $a+1$ means the successor of a (*id.*). The first definition is:

$2=1+1;\ 3=2+1;\ 4=3+1;$ *etc.*,

where $=$ means is equal to (*id.*). And the first theorem is:

$1\ \varepsilon\ N$,

$1\ \varepsilon\ N \supset 1+1\ \varepsilon\ N$,

$1+1\ \varepsilon\ N$,

$2=1+1$,

$2\ \varepsilon\ N$ (*id.*).

Peano does not define number. What number is he leaves to the 9 axioms, 31 definitions, and 202 theorems to imply.

Frege defines number, in *The foundations of arithmetic*, in 1884. In Russell's reformulation, in *The principles of mathematics*, in 1903: number is nothing but a set of similar sets (111).

Set Cantor defines, in the *Contributions to the foundation of the transfinite theory of sets*, in

1895: by set we understand every collection into a whole of definite well-distinguished objects of our sensibility or our thought (1).

Arithmetic, then, like geometry, requires only a few and simple axioms for its logical development, and the problem of the establishment of the axioms of arithmetic and the investigation of their relations is equivalent to the logical analysis of our sensibility or our thought.

In short, with Hilbert, in the *Knowledge of nature and logic*, in 1930: when the anthropomorphic dross is removed from the Kantian theory of the *a priori*, what is left is the foundation of pure mathematical knowledge (*id.*).

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Metaphysics is of two kinds.

One has as the actual goal of its investigation only three ideas, god, freedom, and immortality. Of the other synthetic *a priori* propositions are the only goal. One will be wisdom itself, whose oracle every man will consult. Of the other the production of knowledge *a priori*, and ultimately synthetic *a priori* propositions, and specifically in philosophical knowledge, is the essential content. One is a mock combat. The other is a science.

What the two kinds have in common is that they are both kinds of logic. One is a logic of illusion. The other is the logic of truth.

Metaphysics is logic.

The logic of truth Kant calls also transcendental logic.

He writes in the *Critique*: general logic abstracts from all content; everything in it must be *a priori*; it has nothing to do with the origin of knowledge (I, 2.intr.); but there must be a science which does not abstract from all content, because it must determine the origin, the domain, and the objective validity of knowledge *a priori*; and this science should be called transcendental logic (*id.*).

He writes also: that logic has traveled the secure path of science since the earliest times can be seen in that that after Aristotle it has not had to go back a single step (pr.), but: it is remarkable also that until now it has not been able to make a single step forward either (*id.*).

And that logic made a major step forward with Kant, Frege attests: I must emphasize my agreement with Kant which far outweighs the disagreement.

Frege discovered how truth can be expressed formally, how generality can be expressed formally, how existence can be expressed formally, how, in general, content and form can both be expressed formally. And Frege thus solved a problem discovered and formulated explicitly by Kant.

Kant affirms: the problem on which everything depends is, how are synthetic *a priori* propositions possible?

The problem is to resolve the apparent contradiction of form and experience, and the *Critique* indicates the magnitude of the drama which the apparent contradiction involves. General logic abstracts from all content, everything must be *a priori*. But transcendental logic does not abstract from all content, because it must determine the origin, domain, and objective validity of knowledge *a priori*. The problem is to abstract from all content and not abstract from all content, both. The *Critique* indicates the magnitude of the drama which the resolution of the apparent contradiction has to involve as well. Kant writes: logic is the science that exhaustively presents and strictly proves nothing but the formal rules of all thinking (pr.). Form has to supply the solution of the problem. The solution of the problem has to be to abstract from all content and not abstract from all content, both, formally.

With respect to truth, Kant formulates the problem thus in the *Logic*, in 1800: the question, what is truth? divides into the two particular questions, is there a general formal criterion of truth? and is there a general material criterion of truth (7)?

The first question, is there a general formal criterion of truth? Aristotle discovered, and answered in the affirmative.

The second question, is there a general material criterion of truth? Kant discovered.

He continues in the *Logic*: the formal criteria of truth are the principle of contradiction and the principle of sufficient reason (*id.*); the general formal criteria are its *conditio sine qua non*, but they are clearly not enough for objective truth (*id.*); and a general material criterion of truth is not possible, it is indeed self-contradictory; for it

would have, as general, to abstract from all differences of objects, and nevertheless, as material, to be able to determine these differences (*id.*).

Pace the *Logic*, in the *Critique* and the *Prolegomena* Kant has shown that the question, is there a general material criterion of truth? must be answered in the affirmative as well. Self-contradictory as it appears, such a criterion is not only possible. Kant has, in fact, taken a major step toward finding it.

And when Frege showed how truth can be expressed formally, and existence and generality as well can be expressed formally, he showed that the question, is there a general material criterion of truth? has, in fact, been answered in the affirmative, and having thus shown that content and form can both be expressed formally, and their relation investigated formally, he showed that, in fact, the general problem of the formal relation of form and content has been solved.

Kant took a major step toward finding a general material criterion of truth, as well as toward the solution of the general problem of the formal relation of form and content. His solution of the problem of cause and effect illustrates.

If synthetic *a priori* judgments are possible, there must be something *a priori* in perception already. But if there is something *a priori* in perception, there must be concepts *a priori*. Kant writes: the given perception must be subsumed under a concept which must determine the form of judgment in general in relation to perception and thus achieve the general validity of the empirical judgment; a concept of this kind is a pure *a priori* concept of the understanding (*Prolegomena*, 20). And that they are such concepts is the solution of the problem of cause and effect: cause and effect are *a priori* concepts of the understanding (*Critique*, I, 2.1.1.1); and it is only by that that we subject the sequence of the appearances, and thus all change, to the law of causality, that experience itself, and thus knowledge of experience, is possible (2.1.1.2).

Hence: given the law of causality, the understanding, which in its empirical use sees in everything nothing but nature, and is justified to do so, has all it can demand, and physical explanations can proceed on their unhindered course (2.2.2.2).

Hume has argued: every effect necessarily presupposes a cause, but this does not prove that every being must be preceded by a cause, no more than it follows that, because every husband must have a wife, every man must be married (*Treatise*, I, 3.3).

Pace Hume, every man must be married.

Kant writes: the principle of contradiction is the principle of all analytic knowledge (*Critique*, I, 2.1.2.2); the possibility of experience is the principle of all synthetic judgments (*id.*); and the principle of sufficient reason is the reason of possible experience (*id.*).

Descartes anticipates the principle of sufficient reason: there is no existent thing of which it cannot be asked which is the cause why it exists (*Replies*, 2). Leibniz formulates it explicitly: no fact can be found true or existent, no assertion true, without a sufficient reason why it is thus and not otherwise (*Monadology*, 32).

The principle of sufficient reason can tolerate no unmarried men.

Which quantum theory all by itself shows conclusively. It has kept being supposed that the atomic event is not subject to the law of causality. With Heisenberg, however: when we make an experiment we have to assume a causal chain of events that leads from the atomic event through the apparatus to the eye of the observer; if this causal chain were not assumed, nothing could be known about the atomic event. And very much is known about the atomic event.

And pace the *Logic*, with the *Critique* and the *Prolegomena*, if the principle of sufficient reason is a general criterion of truth, and it is, it is not a general formal criterion, it is a general material criterion of truth.

Analytic judgments add nothing to the content of knowledge, synthetic judgments enlarge knowledge. The possibility of experience is the principle of all synthetic judgments, and the principle of sufficient reason is the reason of possible experience. Including all actual experience, possible experience includes the possibility of extension of actual experience. And the principle of all synthetic judgments, the principle of sufficient reason is the reason of the extension of actual experience. Of the enlargement of knowledge. Of the addition of content.

A general material criterion of truth must, as general, abstract from all differences of objects, and nevertheless, as material, be able to determine these differences. As a principle, the principle of sufficient reason abstracts from all differences of objects, and nevertheless, as the reason of the possibility of experience, and hence of all synthetic judgments, it is able to determine these differences.

That there must be a general material criterion of truth is a consequence of the fact that experience has form. Form reaches from perception to all enlargement of knowledge, and from all enlargement of knowledge back to perception. And the most general consequence of the fact that experience has form is that: objective validity is nothing but general validity.

Objectivity is generality. And all of Frege's results stand on this result.

Metaphysics, then, is logic in the *Metaphysics*.

Metaphysics is logic in Kant.

Metaphysics is logic in Frege. In *The laws of arithmetic*, in 1893, he reports: my work has been accorded cool reception, indeed no reception at all; I must apparently give up the hope of being read by all mathematicians who, upon encountering a logical expression, think *metaphysica sunt non leguntur*, and all philosophers who, upon seeing a formula, cry out *mathematica sunt non leguntur* (I, pr.). Frege's work has been accorded a reception, indeed a warm reception. But the difficulty has remained, that mathematicians have to think *metaphysica sunt logica*, and philosophers have to cry out *logica sunt mathematica*.

Metaphysics is logic. Logic is mathematics. Metaphysics is mathematics.

Kelvin said: mathematics is the only true metaphysics (S. Thompson, *Kelvin*, 25). The other way around. The only true metaphysics is mathematics.

Mathematics is empirical, so is its part logic, and so is its part metaphysics.

Yet, ask a celebrated metaphysician whether his science might not have descended from logic, or both from a common parent science, and he will laugh you to scorn.

Darwin writes: ask, as I have asked, a celebrated raiser of Hereford cattle whether his cattle might not have descended from Longhorns, or both from a common parent stock, and he will laugh you to scorn; the explanation I think is simple; from long continued study they are strongly impressed with the differences between the several races, and though they know that each race varies slightly, for they win their prizes by selecting such slight differences, yet they ignore all general arguments, and refuse to sum up the slight differences accumulated during many successive generations (*Origin*, 1).

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And what form is, and what the problem solved by the separation of form from matter is, and the problem solved by the separation of content from form, and what the problem solved by the separation of form from that matter from which form has been separated is, and the problem solved by the separation of content from that form from which content has been separated – and the problems of the relation of form and matter, of the relation of content and form, of the formal relation of form and matter, and of the formal relation of content and form – all has to be made more explicit.

It can all be traced back to Heraclitus. Or better, to Plato tracing it back to Heraclitus. Or better yet, to Aristotle tracing it back to Plato tracing it back to Heraclitus.

Plato discovered the forms.

This is how Aristotle describes the discovery. In book I of the *Metaphysics* he reviews his predecessors' solution of the problem of the principles of all things. And having reviewed the solution of the philosophers in the two centuries before Plato, he writes: after these philosophers came Plato; in his youth Plato became acquainted with Cratylus and the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge of them, and Plato held this view in his later years as well; Socrates, however, had abandoned physics altogether, devoted himself to ethics, and while searching for the general in ethical matters, had first concerned himself with definitions; Plato followed Socrates; he presumed, however, that definitions cannot be of the sensible things because the sensible things are always changing; definitions have to be of entities of another kind; these entities he called forms; the

sensible things are named after the forms, and the many sensible things called by the same name as a given form are by participation in that form (6).

Aristotle's description is corroborated amply by the Platonic dialogues.

Heraclitus writes: on those who step in the same river different and different waters flow (fr. 12).

In the *Cratylus*, Plato generalizes the proposition. He writes: Heraclitus said that all things change, nothing abides, and you cannot step in the same river twice (402a). And he objects to the proposition: but there is the beautiful itself, the good itself, and the same for all things that there are; and the beautiful itself is always what it is; if it always passed away, we would have to say that it were now this, now that; at the very instant we spoke of it, it would become something different; and if it were never the same, how could it be anything at all? or be ever known by anyone (439c-e)?

In his earlier dialogues Plato has asked: what is piety? in the *Euthyphro*, what is courage? in the *Laches*, what is friendship? in the *Lysis*, what is temperance? in the *Charmides*, what is beauty? in the *Greater Hippias*, gradually finding out – never mind piety, courage, friendship, temperance, and beauty – what the question itself is. And in the *Meno* what the question itself is is clear. The question: what is virtue? is not about the virtue of a man, the virtue of a woman, the virtue of a child, the virtue of an old man, the virtue of a free man, and the virtue of a slave, because: these are a swarm of virtues, and even if there are all these virtues, the question is, what makes them all virtue?

In the *Theaetetus*, too, the question: what is knowledge? is not answered with: geometry, astronomy, music, and arithmetic are knowledge, because: the question is about one thing, not about many, it is about something simple, not about variety, the question is not what one has knowledge of, or how many kinds of knowledge there are, the question is what knowledge itself is.

The beautiful itself, the good itself, what makes all virtues virtue, knowledge itself, and the same for all things that there are, are the forms.

And the problem whose solution the forms are is, with Heraclitus: on those who step in the same river different and different waters flow. With Plato: all things change, nothing remains the same; you cannot step in the same river twice.

All things change, nothing remains the same, has to be the most general of all the formulations of the problem that can be extracted from Plato. It has to be its change from a man to a woman to a child to an old man to a free man to a slave that poses the question of what makes all virtues virtue. It has to be its change from geometry to astronomy to music to arithmetic that poses the question of what knowledge itself is. It has to be its change into what it is when one has it, too, and its change from one of its kinds to another, that poses the question of what knowledge itself is. It has to be its change, too, that poses the questions of what is one in the many, and of what is simple in their variety.

All things change, nothing remains the same. But this cannot be the problem in its entirety. With Plato: all things change, nothing remains the same, but the beautiful itself is always what it is, the good itself is always what it is, and the same for all things that there are. The same for all things that there are. The problem has to be, all things change and yet all things are what they are, nothing remains the same and yet everything is what it is.

Even as all things change, that they are all things cannot change. Even as nothing remains the same, that it is the same that nothing remains cannot not remain the same. Even as you cannot step in the same river twice, it is the same you who cannot step in the same river twice, it is the same river in which the same you cannot step twice, it is the same thing that the same you cannot do in the same river twice, and you can even count what you can or cannot do, you can or cannot step in the same river once, you can or cannot step in the same river twice, and if once, and twice, also thrice, and four times, and five, and any number of times, and the conclusion is unavoidable, all things change, nothing remains the same, and yet all things are what they are, everything is what it is.

If this is the problem, it should need no recommendation as a theoretical problem, not a practical problem, as a problem discovered and formulated in the search for truth, not in the search for

action. The solution of the problem, the forms, should need no recommendation as a theoretical solution, not a practical solution, of the problem either. The theory of the forms is a theory.

Plato presents the theory most explicitly in the *Phaedo*, and the most far-reaching conclusions he draws from it in the *Phaedo* and in the *Phaedrus*.

He writes in the *Phaedo*: the equal itself, the beautiful itself, and every thing itself, are the real, every thing itself is uniform, remains always the same, and never in any way admits of any change whatsoever (78d); the equal itself is not among the sensible things (74a-5a); there are two kinds of essence, the sensible things and the not sensible things (79a); if there is anything beautiful among the sensible things, it is beautiful only because it participates in the beautiful itself (100c-d); and all things that there are are by participation in these realities, the equal itself, the beautiful itself, the good itself, and all other things of this kind (76d-e).

Always the same, never in any way admitting of any change whatsoever, the forms are the real. The real is not among the sensible things. And all things are by participation in the real.

The real is not among the sensible things. Where it is, Plato reveals in the *Phaedrus*: beyond the heaven there is a realm where what is is without color, without shape, without solidity, a being which is what it is; and this being is the object of all true knowledge, accessible to the mind alone, the pilot of the soul (247c-d). And Plato does describe in some detail how the pilot of the soul, in a chariot drawn by two horses, the two other parts of the soul, one of them good, one of them not so good, journeys to that realm beyond the heaven to catch a glimpse of the forms (245c-50c).

In the *Phaedo* Plato draws the conclusion also that, participating in the form of life, the soul must be immortal. As the theory of the forms should need no recommendation as a theory, so this conclusion from the theory should need no recommendation as practical. Having Socrates draw the conclusion just before he drinks the hemlock, Plato has Socrates say this also to his teary followers: take my advice and think little of Socrates, much of truth (91b-c). Nor should the fact need a recommendation that, beginning with Plato, and for nearly two and a half millennia now, the followers of Socrates have not exerted themselves too strenuously to follow this particular advice of his.

If the problem, then, is that all things change and yet all things are what they are, the solution of the problem has to do the following. To separate sharply things such as they change from things such as they are, describe things such as they change explicitly, describe things such as they are explicitly, describe the relation of the two, things such as they are and things such as they change, explicitly, and do all this in relation to all things, and thus show that that apparent contradiction is just that, just apparent.

And if so, the theory of the forms is a very successful solution of the problem. As successful as the theory of the four elements formulated by Empedocles two generations before Plato is of the problem of the material principles of all things, and as the theory of concentric spheres formulated by Eudoxus in the generation after Plato is of the problem of the motions of the planets.

Things such as they change the theory does not describe too explicitly. Things such as they change are the sensible things, and as these are not the object of true knowledge, the theory seldom dwells on them more than to say that, that they are not the object of true knowledge. On the relation of things such as they change and things such as they are the theory is also vague. Aristotle writes: the Pythagoreans said that all things are by imitating numbers, Plato said that they are by participating in the forms, but what the participation or the imitation is remains unclear (*Metaphysics*, I, 6). Things such as they are the theory describes more explicitly. Things such as they are are the forms, and Plato describes in great detail the beautiful itself, the good itself, justice itself, and in some detail the equal itself, numbers themselves. The great success of the theory with respect to things such as they are, however, is in affirming their generality. Plato writes in the *Republic*: there are many beds; but there is only one form of the bed; and no craftsman makes the form of the bed; the form of the bed is the bed by nature, the bed in itself, the being of the bed (X, 596b-7c). And to establish the generality of the

forms, he includes explicitly in the argument: the table, all plants, all animals – including the painter who paints the bed, the table, all plants, all animals, and all things – the earth, the heaven, the gods, and all things in heaven and in Hades (*id.*). But the greatest success of the theory is the sharp separation of things such as they are from things such as they change. The forms never in any way admit of any change whatsoever.

The apparent contradiction has thus been shown just that, just apparent. All things have been divided into two, the sensible things, within the heaven, on one side, the forms, beyond the heaven, on the other side.

Kant writes: hammer and chisel are good for working raw lumber, copperplate, however, requires an etching needle (*Prolegomena*, pr.).

Plato faced raw lumber.

And hammer and chisel work successfully completed, Aristotle could employ, and did employ, lighter implements, if not yet an etching needle.

To begin with, all things do not change. Not even Heraclitus has to be taken to say that all things change. If he is taken to say that all things change, Aristotle notes: Heraclitus is taken to say that the same thing is good and bad, and good and not good, and man and horse (*Physics*, I, 2). Cratylus, Aristotle reports, took Heraclitus to say that. Cratylus concluded: you cannot step in the same river even once, and stopped talking altogether, thenceforth only pointing with his finger (*Metaphysics*, IV, 5). Establishing thus also a precedent which all philosophers who have argued that all things change should have, even as they, lamentably, have not, followed. With respect to the sensible things, Plato, too, concludes the same, the same thing is good and not good, and man and horse, indeed man and two horses. But not even Heraclitus has to be taken to say that. Aristotle notes: we can take Heraclitus to say that all things change except for one thing which does not change and is the principle of all change (*Heaven*, III, 1).

All things do not change. And if so, the problem is not that all things change and yet all things are what they are. All things do not change, some of them change, some of them, at least the principle of all change, do not change. Also, if all things are what they are and all things do not change, there is no apparent contradiction to resolve. All things, whether they change or not, are what they are. Also, what all things are are their definitions, or essences, or beings. Or better, all things have principles.

The problem is that all things have principles.

That this is the problem, that all things have principles, and, reformulated and generalized by Aristotle, it is the same problem that Plato solves with the theory of the forms, the greatest generality of the problem shows. There is no problem of greater generality, in Plato's formulation, in Plato or before him, and there is, and there can be, no problem of greater generality, in Aristotle's formulation, in Aristotle or after him.

That this is the problem, and that it is the same problem in Plato and, reformulated and generalized, in Aristotle, book I of the *Metaphysics*, in particular, also shows. The problem of the book are the principles of all things. It is in the course of his review and criticism of his predecessors' solution of this problem that Aristotle reviews and criticizes the theory of the forms. And even the verbal infelicity attending the difficulty and generality of the problem in Plato, what things are are their definitions, their essences, their beings, persists in Aristotle, definitions, and essences, and beings, and is compounded, and substances, and causes, and principles, lighter implements as Aristotle employs to formulate and solve the problem than hammer and chisel.

The problem is that all things have principles. If so, the solution of the problem has to do the following. To separate sharply all things from their principles, describe all things explicitly, describe their principles explicitly, and describe the relation of the two, their principles and all things, explicitly.

Moreover: philosophy searches for the principles of the sensible things, and to argue, while searching for their essence, that there is some other kind of essence, is prattle (*Metaphysics*, I, 9).

The solution should have no recourse to prattle. It should not, as Kant restated the requirement: abandon the world of the senses for the empty space beyond it.

Philosophy searches for the principles of the

sensible things, and cannot abandon the world of the senses for the empty space beyond it, because everything of which an organism is aware, including an organism which has become a philosopher, terminates in experience. The world is one, too, there is nothing beyond the world, and the world is nature, and nature is the world, and the solution of the problem cannot be beyond the heaven, or what is the same, beyond the world, or what is the same, beyond nature.

All things being all things, in short, their principles cannot not be among all things.

Thus, if to describe all things explicitly, to describe the principles of all things explicitly, and to describe the relation of all things and the principles of all things explicitly, are each a part of the problem, each of these parts is also all by itself a problem of the greatest generality, as a problem involving all things cannot not be. Yet it is the remaining part, all by itself a problem of the greatest generality as well, that is the fundamental. This part is the problem on whose solution the explicit formulation and solution of all the others fundamentally depends. Namely, to separate sharply the principles of all things, inclusive as all things cannot not be of their principles, from all things, inclusive as all things cannot not be of their principles.

And this is the problem whose solution the separation of form from matter is.

All things Aristotle describes more explicitly than any of his predecessors, and more comprehensively, if not more explicitly, than any of his successors. The principles of all things he describes also more explicitly than any of his predecessors, and more comprehensively, if not more explicitly, than any of his successors.

He writes in the *Meteorology*: we have dealt with the first principles, with natural motion, with the heavenly bodies and their order, and with the elements; we have now to deal with the phenomena in the region closest to the heavenly bodies; and when we next deal with animals and plants, our task will be complete.

He has dealt with natural motion, the heavenly bodies and their order, the elements, the phenomena in the region closest to the heavenly bodies, and animals – and although he has left plants for Theophrastus to deal with, he has dealt with the political animal as well – in three of the treatises comprising the *Metaphysics*, the *Physics*, *On the heaven*, *On generation and corruption*, the *Meteorology*, *On the soul*, the five treatises on animals, and the rest of the physical treatises.

He has dealt with the first principles – has dealt with the principle of contradiction and the principle of excluded middle, has dealt with the kinds of inferences, demonstrations, dialectical, examinations, and contentious, has distinguished proposition from sentence, has distinguished proposition from inference, and term from proposition, and has developed syllogistic – in three of the treatises comprising the *Metaphysics* and the other six logical treatises.

All things Aristotle describes explicitly in the course of his physical investigations, their principles in the course of his logical investigations.

As to the problems of describing explicitly the relation of their principles and all things, and of separating their principles from all things, these have to be best approached through Aristotle's analysis, in the *Physics* (II, 3) and in book I of the *Metaphysics*, of the principles of all things into the four kinds, material, efficient, formal, and final.

How fundamental the four kinds are two facts should suffice to show.

The four kinds accommodated god, and rather, god accommodated them, as, being god, he had to. In the *Theological summa*, sixteen centuries after Aristotle, Aquinas addresses the general question: of the first cause of all things (I, 44), by asking the four particular questions: is god the efficient cause of all beings? is the primary matter created by god? is god the exemplar cause of beings? and is he the final cause of things (*id.*)? The answer is in each case affirmative. Even if, affirmative in two of the four cases as a matter of course, it is affirmative in the other two cases in spite of some difficulties, involving, in the case of the formal cause, exhumation, in the case of the material cause, incineration. Two generations before Aquinas, in 1210, the first *ex officio* judgment on Aristotle had been rendered, on the authority of the archbishop of Sens, including the conclusions: the body of master Amalric is to be removed from the cemetery and cast into unconsecrated ground; the writings of David of Dinant

are to be brought to the bishop of Paris and burned (decree, 1210). Aquinas explains: Amalric said that god is the formal cause of all things (I, 3.8); David of Dinant most absurdly said that god is the primary matter (*id.*). Accordingly, god is not the primary matter, he is its creator. God is not the formal cause either, he is the exemplar cause, and Aquinas has to explain: the determination of the forms of natural things must be reduced to divine wisdom as its first principle, because divine wisdom constitutes the order of the world and in it are the types of all things, and that is, their exemplar forms (44.3).

The four kinds accommodated the instauration of the sciences as well, and rather, again, the instauration of the sciences accommodated them, as being the instauration of the sciences, it had to. In the *New organon*, three and a half centuries after Aquinas, Bacon writes: causes are not improperly divided into four kinds, material, efficient, formal, and final (II, 2). Even if: the final corrupt rather than advance the sciences; the discovery of the formal is despaired of; and the material and efficient as they are now investigated, as remote causes, contribute nothing to true knowledge (*id.*).

Fundamental as the four kinds are, however, yet more fundamentally their division is, by dichotomy, into two.

Bacon's analysis implies the dichotomy. Corrupting rather than advancing the sciences, the final causes are best left alone. The remaining causes, then, are the material and efficient, on one side, and the formal, on the other. On one side, investigated as remote causes, the material and efficient contribute nothing to true knowledge. But they do not have to be so investigated, and to investigate the material and efficient as proximate causes is explicitly a central concern of Bacon, in particular, and of the instauration of the sciences, in general. On the other side, the discovery of the formal causes is despaired of. But it need not be despaired of, and again, to discover the formal causes is explicitly a central concern of Bacon, in particular, and of the instauration of the sciences, in general. Bacon continues in the *New organon*: although there is nothing in nature but individual bodies performing individual acts, they do so by law, and this law and its investigation, discovery, and explanation is the foundation of knowledge and its application; it is this law with its clauses that is understood by the name forms (II, 2); and it is from the discovery of the forms that truth results (3).

Hence, the dichotomy is very much in evidence in the crowning achievement of the instauration of the sciences, too. The most fundamental results of *The mathematical principles* are mass, or as Newton has it: the quantity of matter (def. 1), and gravity, which, Newton notes: explains all the motions of the heavenly bodies and of our sea (gen. sch.), on one side, and the laws of motion and of gravity, on the other.

The dichotomy is very much in evidence in quantum theory as well, quarks and leptons, and the gravitational, strong, weak, and electromagnetic forces, on one side, Schrödinger's equation, on the other.

The dichotomy is very much in evidence in these, because they are very much the two most important steps in the development of the theory of all things.

Earlier, the dichotomy is in evidence in Aquinas. God creates matter, on one side, but is the efficient, exemplar, and final cause, on the other.

And first of all, Aristotle is clear that the most fundamental division is into two.

With respect to the four kinds: the efficient, formal, and final principles often coincide (*Physics*, II, 7).

More fundamentally: neither matter nor form is generated; everything that changes is something, is changed by something, and is changed into something; that by which everything is changed is the first mover, that which is changed is matter, and that into which it is changed is form (*Metaphysics*, XII, 3).

And most fundamentally: nature is the primary underlying matter of what has in itself the principle of motion and change, and also the kind or the form affirmed in the definition (*Physics*, II, 1).

The most fundamental division of all things is into matter and form.

Aristotle writes: when everything else is removed, nothing but matter remains; matter is what is by itself neither anything in particular nor a quantity or any other of the categories by which being is determined (*Metaphysics*, VII, 3); we can

thus speak of anything as form, or as having form, and of nothing as matter by itself (10).

Matter is no determination whatsoever. Form is any determination whatsoever.

The problem of separating sharply form from matter is thus solved. If not exactly by an etching needle.

Hence, the description of all things, and of any one thing, can be so vague and confused as to be strictly material, to lack any determination whatsoever. If, indeed, such can be said to be a description. The explicit description of all things, however, in general, as well as in particular, the description of natural motion, of the heavenly bodies and their order, of the elements, of the phenomena in the region closest to the heavenly bodies, of animals, of plants, and of the political animal, too, cannot be strictly material. It cannot not be also formal, and the more explicit it is, the more formal it has to be. If it is material to the degree to which it has not determined everything, to the degree to which it has determined anything it has to be formal. Any of Aristotle's physical investigations can illustrate. In any of them much is determined, much more remains to be determined.

The explicit description of the principles of all things, on the other hand, in general, as well as in particular, can be strictly formal. Given that this is the goal of Aristotle's logical investigations, it is most closely approached with the development of syllogistic, and more generally the predicate calculus, in the *Prior analytics*.

Clearly, the explicit description of the relation of form and matter, and hence of their principles and all things, cannot not be formal either.

Clearly, also, to solve this problem, of the relation of form and matter, has to be to explain how form is separated from matter. To explain, in general, how determination is possible. How, given no determination whatsoever, any determination whatsoever is possible. And, in particular, how determination as a development is possible. How, beginning with no determination whatsoever, any determination whatsoever is possible, and further determination is possible, and further, and all the way to complete determination.

To put it otherwise. There is practice, and there is theory. Theory is physics and mathematics. Physics investigates matter and form, mathematics investigates form.

Hence, in general, how is mathematics possible? how is physics possible? and how is theory possible? And, in particular, how has it been possible for mathematics to travel the secure path of science since the earliest times? for logic to travel it since Aristotle? for physics to travel it since Bacon? and for metaphysics to try and enter it with Kant?

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If the fact that all things have principles posed the problem of the separation of their principles from all things, and the separation of form from matter is the solution of this problem, the discovery of that fact and its investigation, exclusively, could pose, and did pose, the problem, and could lead, and did lead, to its solution. The acquisition of knowledge, exclusively, as distinct from everything that preceded it and everything that attended it, posed the problem, and led to its solution. The labors of the philosophers, and, most particularly, the physicists and the mathematicians, in the two and a half centuries before Aristotle posed the problem, and the successful analysis of the success of these labors is Aristotle's solution of the problem.

From Thales through Anaximander, Anaximenes, Parmenides, Anaxagoras, Empedocles, Democritus, Hicetas, Eudoxus, the philosophers developed physics *par excellence*. They discovered the problem of the material principles of all things, solved it, improved on the solution, formulated the theory of the four elements. They discovered that the earth is a sphere, discovered the cause of eclipses, formulated the atomic theory, discovered the earth's rotation on its axis, formulated the theory of planetary motions in concentric spheres.

The philosophers developed mathematics *par excellence* from the first as well, including Pythagoras's discovery of the irrationality of $\sqrt{2}$. In the generation-odd before Aristotle Archytas solved the problem of the duplication of the cube, Theaetetus formulated the theory of the irrationals, Eudoxus formulated the theory of proportion.

The development of physics and mathematics posed the problem whose solution the separation

of form from matter is.

Hence, theory is physics and mathematics, physics investigates matter and form, and mathematics investigates form.

Hence, also, conversely, mathematics has to show what form is, mathematics, by showing what form is, has to show as well what physics is, and mathematics, by showing what physics is, has to show as well what theory is. What the search for truth is.

But if the acquisition of knowledge posed the problem of the separation of form from matter, it posed the problem also of the relation of form and matter, and if this problem remained unsolved, the acquisition of knowledge continued after Aristotle, continued to pose this problem, and to pose it more and more acutely.

Aristotle writes: the rigor of mathematics should not be demanded in all things but only in those that have no matter; it is not the way of physics, for all nature has matter (*Metaphysics*, II, 3).

But Euclid already, one generation-odd after Aristotle, investigated not only natural lines not as natural in the *Elements*, he investigated also mathematical lines as natural in the *Optics* and *Phenomena*.

Archimedes, two generations later, developed the theory of the lever and the theory of floating bodies with the rigor of mathematics.

Ptolemy, not quite four centuries later, wrote: mathematics can deal with the objects of physics, and even where action and character are concerned, mathematics above all can make men see clearly. And dealt with the objects of physics, and made men, those men who would see clearly anyway, see clearly.

Not quite fifteen centuries later the instauration of the sciences began.

Galileo established that the world is written in mathematics. And the sciences of mechanics and locomotion are mathematical.

Kepler discovered laws. And the laws are mathematical.

Descartes, in the next generation, formulated the rules of the method. And the method is mathematical.

Spinoza, in the next generation, showed that the method of history is the method, too.

Newton, later in the same generation, wrote *The mathematical principles*.

Leibniz, in the same generation, described the possibility of a philosophical calculus in which all things and their relations are expressed formally, so that: there will be no more need for disputes between two philosophers than between two calculators; it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate (*Gen. science*, 14).

More and more acutely, form and matter demanded to be revisited.

Hence, experience has form.

And if matter is no determination whatsoever, and form is any determination whatsoever, this, that experience has form, has to be a solution of the problem of the relation of form and matter. How, given no determination whatsoever, any determination whatsoever is possible, is explained. There is form in perception already. Form reaches from impression to thought, and from thought back to impression.

It is possible experience, too, inclusive of all actual experience, and hence of the possibility of extension of actual experience, that has form. How, given any determination, further determination is possible, and all the way to complete determination, is thus explained as well. Form reaches from impression to mathematics, and from mathematics back to impression.

If experience has form, however, this poses the problem of the relation of form and experience. And this problem is the copperplate requiring an etching needle.

If experience has form, the problem of the relation of form and experience is a problem of the relation of form and form – and this, then, the relation of form and experience, is the problem of the formal relation of form and matter.

As the solution of the problem of the relation of form and matter depends on the solution of the problem of the sharp separation of form from matter, too, so the solution of the problem of the formal relation of form and matter depends on the solution of the problem of the sharp separation of form from that matter from which form has been separated. And as the problem whose solution the separation of form from matter is is posed by the fact that all things have principles, so the problem

whose solution the separation of form from that matter from which form has been separated is is posed by the fact that the principles of all things have principles, and as the solution of that problem has to establish which the principles are, in order to separate form from matter, so the solution of this problem has to establish which the principles of the principles are, in order to separate form from that matter from which form has been separated.

It is explicitly the task of the *Critique*. To establish which the principles of the principles are. Most particularly, which the principles of the mathematical principles of natural philosophy are. Most generally, to find the form of the relation of form and matter. If form is *a priori*, to find the *a priori* of the relation of *a priori* and experience.

Aristotle discovered that the principle of contradiction is the principle of all principles. Leibniz discovered that the principle of sufficient reason is a principle of the same, the greatest, generality as the principle of contradiction. Newton discovered that the laws of motion and the law of gravity are the mathematical principles of natural philosophy. And in the *Critique* Kant shows how those principles are the principles of these principles. The analytic is in two parts, analytic of concepts and analytic of principles. In the analytic of principles Kant establishes that: the principle of contradiction is the principle of all analytic judgments, the possibility of experience is the principle of all synthetic judgments, and the principle of sufficient reason is the reason of possible experience, and then proposes: the systematic presentation of all principles of synthetic judgments (I, 2.1.2.2). These are: one axiom of sensibility, one anticipation of perception, three analogies of experience, and three postulates of empirical thought in general (*id.*). The first two analogies of experience are: in all change substance persists; and all change occurs according to the law of cause and effect (*id.*).

The discovery of mass, and of gravity, is thus explained as *a priori* as well as empirical.

And on this ground Kant has solved the problem of the sharp separation of form from that matter from which form has been separated.

Form is *a priori*.

Hume writes: nothing is more usual for writers than to distinguish between reason and experience, to suppose these species of argumentation entirely different from one another, and to take the former for the mere result of our intellectual faculties, considering the nature of things *a priori* (*Inquiry*, 5.1). Nothing being more usual for writers than that, it is with that that Kant begins in the *Critique*. In the first of the seven very brief parts of its introduction, he writes: knowledge *a priori* is not that which is independent of this or that experience, but that which is entirely independent of all experience; opposed to it is empirical knowledge, or knowledge possible only *a posteriori*, and that is, through experience.

Nothing being more usual for writers than that, it is that also that would give Kant the appearance of idealism. Knowledge *a priori* is entirely independent of all experience.

But if actual experience is not possible experience, the appearance disappears. Entirely independent of all actual experience, knowledge *a priori* is entirely dependent on the possibility of experience.

It is halfway through the *Critique* that Kant concludes: here is the solution of the puzzle, no knowledge *a priori* is possible but of objects of possible experience.

It is in the second of the seven very brief parts of its introduction already, however, that he writes: inseparably bound together, the certain characteristics of knowledge *a priori* are necessity and strict generality.

Halfway through the *Critique* he concludes: the possibility of experience is what gives all knowledge *a priori* objective validity. And in the *Prolegomena*: objective validity is nothing but necessary general validity.

Form is *a priori*. But the certain characteristics of *a priori* are necessity and generality. The certain characteristics of form, then, are necessity and generality, and if these are inseparably bound together, the certain characteristic of form is generality.

And this has to be a solution of the problem of the sharp separation of form from that matter from which form has been separated. The form of the relation of form and matter is generality. If form is determination, determination of the

relation of determination and no determination is generality. If form is *a priori*, *a priori* of the relation of *a priori* and experience is generality.

It has to be a solution of the problem.

Because of its generality, and the pun is unavoidable.

And because of the particularity of its consequences, and the pun is again unavoidable.

In particular, if the form of the relation of form and matter is generality, this explains the fact that the acquisition of knowledge has been the generalization of experience, and by the formalization of experience.

Physics is more formal and more general in Aristotle than in Anaxagoras, Empedocles, and Democritus, much more formal and much more general in Galileo and Kepler, more still in Newton, much more still in Clausius, Maxwell, Einstein, Planck, Bohr, Schrödinger, and Heisenberg. Mathematics is more formal and more general in Euclid than in Pythagoras, much more general in Descartes, much more general still in Newton and Leibniz, much more formal and much more general in Gauss, and yet more in Cantor, Frege, Peano, and Hilbert.

The first philosopher already obtained a result of the greatest generality, water is the principle of all things. With Bacon: to obtain principles from the particulars by continuous and gradual ascent is the true way. And the successors of Thales have improved on that result of his by following the true way. The ascent of Empedocles from the particulars to the principles of all things is more gradual, the ascent of Lavoisier is much more gradual, the ascent of the philosophers who have developed quantum theory is more gradual still.

If the form of the relation of form and matter is generality, this explains also the fact that the acquisition of knowledge has been induction, and it explains the fact that induction has aimed at deduction. As it has in Aristotle, and as it has in Newton. As it has also reached that aim nearly perfectly in the axiomatization of Euclid, and yet more nearly perfectly in the more rigorous axiomatizations of Peano and Hilbert.

It explains the fact, too, that, *a priori* as it is, form has had to be discovered. In Plato, next to the labor of driving a chariot beyond the heaven, the discovery of the forms demanded the labor, in a dialogue after a dialogue after a dialogue, of disquisitions more formal and more general than any earlier disquisitions. Aristotle obtained more formal and more general results, and to do so: we had to labor hard for a long time. More formal and more general, Kant's results revealed themselves as: incomprehensible instructions teaching nothing more than what everyone with a good eye can readily do. More formal and more general, Frege's results revealed themselves as, whether *metaphysica* or *mathematica*: *non leguntur* both.

It is all consistent also with the facts, trivially, that formalization is greater determination, generalization is more determinations, and formalization is necessarily generalization, but generalization is not necessarily formalization.

The form of the relation of form and matter is generality.

Hence Kant made major steps toward the solution of the problem of the formal relation of form and matter, too. With the explicit result, space and time are the form of perception. And with the implicit results, the principle of sufficient reason is a general material criterion of truth, and the relation of logic, mathematics, and physics is inclusion.

If the problem of the relation of form and matter is the problem of how determination is possible, and the solution of this problem is that experience has form, the problem of the formal relation of form and matter has to be the problem of how determination as formal is possible, and the solution of this problem has to be to describe the form of experience in all its generality.

If everything of which an organism is aware, including an organism which has become a philosopher, terminates in experience, and experience terminates in perception, the description of the form of experience in all its generality has to terminate in the description of the form of perception.

And space and time are the form of perception.

If, with Hilbert: the establishment of the axioms of geometry and the investigation of their relations is equivalent to the logical analysis of our perception of space, the axioms of geometry corroborate Kant's result. If it can be concluded, from Cantor, Frege, and Peano, that the establishment of the axioms of arithmetic and the investi-

gation of their relations is equivalent to the logical analysis of our sensibility, and in particular, of the perception of time, the axioms of arithmetic corroborate Kant's result as well.

The laws of physics corroborate it unambiguously. Kepler's laws, Newton's laws, the laws of thermodynamics, Maxwell's equations, the laws of relativity, Schrödinger's equation and Heisenberg's principle, Hubble's law, are one and all spatial and temporal determinations. The establishment and investigation of the laws of physics is equivalent to the physical analysis of perception.

If space and time are not the form of perception, they are in the way earth, water, air, and fire are not the elements, and the way hydrogen, nitrogen, oxygen are not the elements. Earth, water, air, and fire, exclusively, had to be analyzed into the elements, into hydrogen, nitrogen, oxygen, in order to establish that earth, water, air, and fire are not the elements and establish which the elements are, and hydrogen, nitrogen, oxygen, exclusively, had to be analyzed into the elements, into quarks and leptons, in order to establish that hydrogen, nitrogen, oxygen are not the elements and establish which the elements are. If space and time are not the form of perception, space and time, exclusively, have to be analyzed into the form of perception.

If the description of the form of experience in all its generality is possible, a general material criterion of truth has to be possible. And if the principle of contradiction is a general formal criterion of truth, the claim of the principle of sufficient reason to be a general material criterion of truth has to be excellent. In the way in which it determines an object, the principle of contradiction determines it completely. If it is an object, it cannot be another object, and the principle of contradiction thus leaves all other determinations of the object alone. The determination, it cannot be another object, does not imply further determination. In the way in which it determines an object, the principle of sufficient reason determines it incompletely. It determines it as an object of a certain kind, the effect of a cause. If it is an object, it has a cause, and the principle of sufficient reason thus does not leave all other determinations of the object alone. The determination, it has a cause, implies further determination, the cause, and it is thus, by being a determination, also lack of determination. Hence, also, if beings are men, and causes are wives, the principle of sufficient reason determines that every man must be married. The principle of sufficient reason cannot tolerate Hume's distinction: every effect necessarily presupposes a cause, but this does not prove that every being must be preceded by a cause. On the other hand, Hume's simile will seem improved in that the discovery of any man will provoke speculation and incite the search for his wife.

The description of the form of experience in all its generality has to accommodate the acquisition of knowledge in all its generality, too.

Experience has form.

Hence, mathematics is empirical. Logic is also empirical.

And the further conclusion is unavoidable. Logic is a part of mathematics, mathematics is a part of physics, and physics coincides with theory.

This is consistent with Aristotle's result. Theory is physics and mathematics, physics investigates matter and form, mathematics investigates form. Logic investigates the form of form.

The acquisition of knowledge is the generalization of experience and by the formalization of experience. And if the form of the relation of form and matter is generality, it may appear, as it has always tended to appear, that, because it is more formal, mathematics is more general than physics, and because it is more formal, logic is more general than mathematics. But if matter is no determination whatsoever, form is any determination whatsoever, and the form of the relation of form and matter is generality, precisely because it has to be less formal, physics has to be more general than mathematics, and precisely because it has to be less formal, mathematics has to be more general than logic.

Nor can the generalization of experience, and by its formalization, not be physics in its entirety. Part of which is mathematics, part of which is logic.

Logic is a part of mathematics, mathematics is a part of physics, and physics coincides with theory.

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And the copperplate demands more etching.

If matter is no determination whatsoever, and form is any determination whatsoever, form cannot be investigated in relation to matter. Determination cannot be investigated in relation to no determination. Nothing can be investigated in relation to no determination. Investigation, by being investigation, can only be of determination in relation to determination. It can only be of form in relation to form. Determination can only be investigated in relation to determination. Form can only be investigated in relation to itself.

Hence the problem of separating content from form.

The development of physics and mathematics posed the problem of separating form from matter. Hence, physics investigates matter and form, mathematics investigates form. Conversely, mathematics has to show what form is.

The *Elements* is mathematics. In particular, the *Elements* is 5 axioms, 5 postulates, 131 definitions, and 514 propositions – theorems, lemmas, and corollaries. And the 514 propositions are all different. Given a line, to construct an equilateral triangle (I, 1); if a line is divided into the parts a and b, then $(a+b)^2 = a^2 + 2ab + b^2$ (II, 4); if $a : b = c : d$, then $a : c = b : d$ (V, 16); if a prime divides the product of two numbers, it divides at least one of its factors (VII, 30); given any number of primes, there are more primes (IX, 20); if $a + \sqrt{b} = x + \sqrt{y}$, then $a = x$ and $b = y$ (X, 42); circles are to one another as the squares on their diameters (XII, 2); to construct a dodecahedron and inscribe it in a sphere (XIII, 17). But the 514 propositions are also all the same. Each states what is to be given, states what is to be proved, or to be constructed, then proceeds from what is given step by step by step, as both what is given and each of the steps are earlier results, either axioms, postulates, or definitions, or what earlier propositions have proved, or constructed, to what is now, at the end of the proposition, itself proved, or constructed.

The different, then, has to be the content, the same the form. And the same is that all the 514 are inferences. Each is the drawing of a conclusion from premises. If – then.

Aristotle writes: knowledge is acquired only by demonstration. And to determine what demonstration is, he distinguishes: four kinds of inferences, demonstrations, dialectical arguments, examinations, and contentious arguments; demonstrations are grounded in what is true, dialectical arguments are grounded in what is probable, examinations are grounded in what is accepted contingently, and contentious arguments are grounded in what is false. There is even: rhetoric which, concerned with persuasion, is also a kind of argument, a mixture of logic and ethics, it is partly dialectical, partly contentious.

If these, rhetoric, contentious arguments, examinations, dialectical arguments, and demonstrations, are all inferences, and inferences are formal, form admits of degree. Inferences are less formal, and more formal. If they are all determinations, determination admits of degree. There is lesser determination, and greater determination.

Hence, to divide them sharply, determination is conditional, and it is unconditional. And this has to be the solution of the problem of the separation of content from form. What is determined conditionally is content, what is determined unconditionally is form. Clearly, also, what is determined conditionally can only be determined in relation to what is determined unconditionally. Content can only be content in relation to form.

The acquisition of knowledge posed the problem of separating form from matter, and the acquisition of knowledge posed the problem of separating content from form. The fact of proof, in particular, or synonymously of demonstration, posed this problem.

In the two and a half centuries before Aristotle the philosophers developed physics *par excellence*, and mathematics *par excellence*. They developed mathematics as demonstration, or synonymously proof. Pythagoras's discovery of the incommensurability of the side and diagonal of the square already is impossible but by proof. In the following couple of centuries proofs accumulated. The more they accumulated the more they entailed one another. They led to the results of Archytas, Theaetetus, and Eudoxus. And two-odd generations before Aristotle Hippocrates of Chios wrote the first *Elements*, and one

generation-odd after Aristotle Euclid collected, developed, systematized, and perfected all earlier results into the *Elements*. The simultaneous development of dialectical arguments, of examinations, of sophistical arguments, and of rhetoric is all abundantly attested in and by the Platonic dialogues. The acquisition of knowledge, exclusively, could pose, and did pose, the problem whose solution the separation of content from form is, and could lead, and did lead, to the solution.

Proof is one of the kinds of inference. Inference is determination, or what is the same, it is formal. Determination, and being formal, thus admits of degree. There is thus inference which is unconditional determination, which is formal exclusively. And an inference is an inference, and all kinds of inference are each an inference, only to the degree to which it is unconditional determination, to the degree to which it is formal exclusively.

Hence the problem of the relation of content and form. How is proof as unconditional determination possible? or what is the same, how is proof as formal exclusively possible?

Aristotle investigates rhetoric in the *Rhetoric*, sophistical argument in the *Sophistical refutations*, examination and dialectical argument in the *Topics*, and demonstration in the *Prior analytics*.

He begins in the *Prior analytics*: our problem is demonstration; so we must determine what a proposition is, what a term is, and what a syllogism is (I, 1); a proposition is a sentence affirming or denying something of something (*id.*); a term is that into which a proposition is resolved, a predicate and that of which a predicate is predicated, a subject, is or is not relating the two (*id.*); and a syllogism is an inference such that, given certain propositions, another proposition follows necessarily (*id.*).

Barbara, for instance, is the syllogism such that, given general affirmative propositions, another general affirmative proposition follows necessarily:

> if *A* is predicated of all *B*,
> and *B* is predicated of all *C*,
> then *A* is predicated of all *C* (4).

A proof, then, is formal exclusively if it is a syllogism.

It is a solution of the problem of the relation of content and form.

With Łukasiewicz: Aristotle's syllogistic is a strictly formal, mathematical theory; it is a system whose exactness surpasses even the exactness of a mathematical theory.

But it is only a partial solution.

With Łukasiewicz: Aristotle's syllogistic is a logic of terms; and no one who does not know that there exists a more fundamental system can fully understand Aristotle's proofs; this system is the logic of propositions; it is intuitively presupposed but not explicitly developed by Aristotle.

That there exists this more fundamental system Frege established conclusively.

Before him, few philosophers knew that there exists this system.

One is Chrysippus, one century after Aristotle. Laertius reports: Chrysippus recognizes five kinds of inference; in the first kind a conditional proposition and its antecedent are the premises, its consequent is the conclusion, thus,

> if the first proposition, then the second proposition,
> but the first proposition,
> therefore the second proposition (VII, 79-80).

Chrysippus discovered *modus ponens*.

Barbara presupposes *modus ponens*.

Frege, in the *Concept writing*, admits only a single mode of inference, because: it is a law of perspicuity to do so; otherwise there would be no reason to stop at several, new modes of inference can be added indefinitely (6). The single mode of inference is *modus ponens*.

Another is Kant, one century-odd before Frege, the twenty centuries separating Chrysippus and Kant, with but Leibniz in between, indicating the scarcity of logicians among the philosophers, including the mathematicians.

That form has to be analyzed into propositions Aristotle established conclusively.

He established conclusively as well what a proposition is. In the *Prior analytics*: a proposition is a sentence affirming or denying something of something. And more explicitly in *On interpretation*: every sentence means something, but not every sentence is a proposition; a proposition is a sentence which affirms what is true or false;

a prayer, for instance, is neither true nor false; we are thus concerned only with propositions, the rest we can ignore, they belong to rhetoric or poetry.

Hence, in order to solve the problem of the relation of content and form, how is proof as formal exclusively possible? completely, it has to be established what the elements of a proposition are, on one side, and what the formal relation of propositions is, on the other.

A proposition is resolved into a subject and a predicate, is or is not relating the two.

The formal relation of propositions is the inference. When an inference is grounded in what is true, it is a demonstration. And when a demonstration is such that, given certain propositions, another proposition follows necessarily, a demonstration is a syllogism.

Whether the problem of the relation of content and form is solved completely thus depends on whether, on the side of the elements, subject and predicate are the elements and their relation of is or is not is formal, and, on the side of the relation of propositions, the relations of being true and of following necessarily from one another are formal.

Being formal admits of degree. And that Aristotle's solution of the problem is excellent the fact of scarcity of logicians in the two millennia separating Chrysippus and Kant sufficiently attests. Excepting Leibniz, the philosophers, including the mathematicians, made no attempt to improve on Aristotle's solution.

That it is an incomplete solution Kant's attempt to improve on it attests.

The problem on which everything depends is, how are synthetic *a priori* propositions possible? Or, with respect to content and form, how is the content of a proposition as formal possible?

This is the problem of the formal relation of content and form. And the solution of this problem can only be to show formally how proof as formal exclusively is possible.

The solution of this problem, too, depends on separating sharply content from that form from which content has been separated. And to do so, given Aristotle's solution of the problem of the relation of content and form, has to be to show that the relations of is or is not, of being true, and of following necessarily can all be expressed formally.

Of either, of the sharp separation of content from that form from which content has been separated, and hence of the problem of the formal relation of content and form, only proof can be the solution.

Form reaches from impression to mathematics and from mathematics back to impression. In particular, Kant argues that there are pure *a priori* concepts, and among these are: existence and necessity (*Critique*, I, 2.1.1.1). Thus, in that it is grounded in a concept which is formal exclusively, the relation of the elements of is or is not is itself formal exclusively. In that it is grounded in a concept which is formal exclusively as well, the relation of propositions of following necessarily from one another is itself formal exclusively as well.

Kant's formulation of the problem of the formal relation of content and form is a major step toward improving on Aristotle's solution, his solution of the problem is not.

He takes it for granted that the elements of a proposition are subject and predicate. Boole still, two-odd generations later, one generation before Frege, in 1847, in *The mathematical analysis of logic*, writes: a proposition has necessarily two terms, the subject and the predicate, connected together by the copula is or is not (20).

Frege's most fundamental discovery is that the elements of a proposition are not subject and predicate. The elements of a proposition are function and argument.

Once again, as the acquisition of knowledge, and the development of mathematics, in particular, posed the problem of the formal relation of content and form, so the acquisition of knowledge, and the development of mathematics, in particular, led to its solution.

Hence, Frege showed also how the relations of propositions of being true and of following necessarily from one another can be expressed formally. Also, although it is not a relation of the elements of the proposition, how the relation of is or is not can be expressed formally. How, in general, content and form can both be expressed formally.

The problem of the relation of content and

form is thus solved completely.

As well as formally. What the *Concept writing* affirms it affirms by proof.

Content and form are separated formally. Form is analyzed into propositions. Proposition is analyzed into function and argument. The relation of propositions is analyzed into modes of inference, of which it is a law of perspicuity to admit only one.

How proof is possible as formal exclusively is shown formally.

This is how Frege describes it informally: the demand is irrefutable, to avoid all jumps in inferences; my concept writing thus aims to achieve greater brevity and clarity of expressions and to move in a few fixed forms like a calculus, so that no transition which does not conform to the rules laid down once and for all is permitted; it can also express not only the logical form, like Boole's notation, but content as well; and thus no presupposition can creep in unnoticed (*Foundations*, 91). In short: my concept writing makes possible a chain of inferences in which no link is missing (cont.).

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Assuming the matter of the whole world dispersed and in complete chaos primordially, it follows from Newton's laws that the solar system originated in a gaseous nebula. It follows further that the solar system is the first member of a progression of systems, the next member of the progression is the system of star systems, the next member is the system of galactic systems, and thus the entire system of the world.

On the same ground, uniformity, it follows that spiritual abilities depend necessarily on the machine they inhabit. Or better, that experience is the outcome of the development of all faculties and their exercise in all organisms.

It follows also that the entire perfection of thinking natures stands under a rule. Or better, that experience has form.

Subjectivity is an impossibility. The subject is all sentient beings and all rational beings.

Objectivity is generality.

Experience is not the world, or what is the same, nature, or what is the same, all things. Experience, and that is, possible experience, inclusive of all actual experience, is a part of the world, or nature, or all things.

Knowledge, necessarily in relation to the world, cannot develop.

The acquisition of knowledge, on the other hand, is a development. Its foundation is experience. It is the generalization and formalization of experience. Its terminal object is the world.

These are all most important results in the investigation of knowledge, And yet, even if implicit rather than explicit, Kant's most important result in the investigation of knowledge is that the acquisition of knowledge is the problem of the investigation of knowledge.

If experience is the foundation of the acquisition of knowledge, all earlier acquisition has to be part of the foundation of all later acquisition. The more successful the earlier acquisition, too, and that is, in the generalization and formalization of experience, the more it has to be the foundation of the later acquisition exclusively.

Its acquisition has to be the problem of the investigation of knowledge, if only because it is the investigation not of experience but of the development of which experience is the foundation. And lest it had access to knowledge independent of the acquisition of knowledge, the investigation of knowledge can only be the investigation of the success of the acquisition of knowledge. If it will concern itself with experience, with perception, thought, faculties, their outcome, it can only concern itself with what the acquisition of knowledge, and physics and mathematics, in particular, have made of it. If it will concern itself with form, with inference, proposition, function, it can only concern itself with what the acquisition of knowledge, and mathematics and physics, in particular, have made of it.

Thus, the problem on which everything depends is, how are synthetic *a priori* propositions possible? this main question divides into the questions, how is pure mathematics possible? how is pure natural science possible? and how is metaphysics possible?

And: that pure mathematics and pure natural science are possible their reality proves (*Critique*, intr.6).

The fact of mathematics and physics poses, and nothing else can pose, the problem on which

everything depends, and the fact of mathematics and physics leads, and nothing else can lead, to its solution.

The acquisition of knowledge is the problem of the investigation of knowledge because there can be no access to knowledge but through its acquisition. The problems of knowledge and its acquisition must be separated sharply from one another.

And if the problem of the acquisition of knowledge includes the problems of its beginning, of collecting it in its entirety, of its relation to the development of philosophy, and of its kinds, Kant indicates unambiguously that these problems can all be solved, and how they can all be solved.

He begins in the *Critique*: whether the labor of knowledge, which is the task of reason, travels the secure path of science can soon be judged by success.

And he judges by success: that logic has traveled this path since the earliest times can be seen in that that after Aristotle it has not had to go back a single step (pr.); mathematics has also since the earliest times, since that admirable people the Greeks, traveled this path (*id.*); natural science took longer to find it, for it is only about a century and a half since the ingenious Bacon in part made this discovery, in part stimulated it further (*id.*); and metaphysics has not yet been so favored by fortune (*id.*); but the task of this critique is to change the ways of metaphysics by undertaking, following the example of the geometers and the investigators of nature, its complete revolution (*id.*).

The judgment that the labor of knowledge has traveled the secure path of science cannot not be a judgment also, implicitly if not explicitly, of when the labor entered that path. And Kant's is such a judgment explicitly. Logic entered it with the labors of Aristotle. Mathematics entered it, never mind that admirable people the Greeks, with the labors of one particular philosopher as well: finding the royal path in mathematics is to be ascribed to a revolution brought about by the happy discovery of a single man (*id.*); new light dawned on the first who demonstrated the equilateral triangle, whether he was called Thales or however he was called (*id.*). Physics entered it with the labors of Bacon who: in part made this discovery, in part stimulated it further, and this discovery can thus also be explained by a quick revolution in the way of thinking (*id.*). Also: when Galileo rolled spheres down an inclined plane, when Torricelli made air bear a weight equal to that of a column of water, when Stahl changed metal into calx and calx back into metal, new light dawned on all investigators of nature (*id.*). And the complete revolution of metaphysics which Kant proposes to bring about is like the revolution brought about by: Copernicus (*id.*).

The labor of knowledge entered the secure path of science in each case with the labors of one particular philosopher. Physics entered it with the labors of Bacon. Logic, earlier, entered it with the labors of Aristotle. Mathematics, earlier still, entered it with the labors of the first who demonstrated the equilateral triangle, whether he was called Thales or however he was called.

Kant writes in *The presumed beginning of human history*: the first beginning of human history does not have to be invented, it can be taken from experience (*id.*). The first beginning of the acquisition of knowledge does not have to be invented either. So Kant takes it from experience.

It can be taken from experience better than Kant took it.

Physics begins long before Bacon. Its development can be traced back to the labors of Ptolemy, fifteen centuries earlier – further back to the first accurate observation of the heavenly bodies, the observation of lunar eclipses in 721-0, eight and a half centuries earlier still – further back to the first observation of Venus and the record of its synodic period, on the Venus tablet of Ammisaduqa, nine centuries earlier still, thirty-two and a half centuries before Bacon.

Logic does not begin long before Aristotle. Still, its development can be traced back to the labors of Plato and the discovery of the forms – the labors of Socrates and the discovery of definitions – the labors of Parmenides and the discovery of the principle of contradiction, not quite one and a half centuries before Aristotle.

Mathematics as well begins long before the demonstration of the equilateral triangle. This is the first demonstration in the *Elements*. It cannot

be ascribed to Thales, even if other results demonstrated in the *Elements* can be ascribed to him. The development of mathematics can in any case be traced further back to the mathematician who calculated $\sqrt{2} = 1.414\,212\,963\ldots$, his labors recorded on the tablet YBC 7289, eleven and a half centuries before Thales – and further back to the 3 calculations of area, these labors recorded on the tablets W 20044/20 and W 19408/76, thirteen and a half centuries earlier still, twenty-five centuries before Thales.

What cannot be taken better than Kant took it is that the beginning of the acquisition of knowledge can be taken from experience. That to take it from experience is to take it from the acquisition of knowledge itself. That to do so is to judge by success.

And to do so is to judge formally.

The labor of knowledge entered the secure path of science in each case with the labors of one particular philosopher. And that is, in the case of mathematics, it entered it with the demonstration of the equilateral triangle. It entered it with a demonstration, and that is, with propositions and inferences. Whether the first on whom thus new light dawned was called Thales or however he was called. In the case of logic, it entered it with the labors of Aristotle, and that is, with the *Posterior analytics*, *Prior analytics*, and the rest of the logical treatises, and that is, with propositions and inferences. The first on whom thus new light dawned was called Aristotle, but the labor of knowledge entered the secure path of science with these propositions and inferences whether he was called Aristotle or however he was called. In the case of physics, it entered it with the labors of Bacon, and that is, with the *New organon*, and that is, with propositions and inferences, and with these propositions and inferences, however Bacon was called and whoever Bacon was. And if the labor of knowledge entered the secure path of science before the first philosopher, it yet entered it in each case with the labors of one particular predecessor of the philosophers, and that is, in each case with propositions and inferences. In the case of mathematics, if it entered it with the 3 calculations of area, it entered it with these propositions and inferences, and whether the first on whom thus new light dawned was called W 20044/20 and W 19408/76 or 3 calculations of area or however he was called, and whoever he was however he was called. In the case of physics, if it entered it with the observations recorded on the Venus tablet of Ammisaduqa, it entered it with these propositions and inferences, whoever the first on whom thus new light dawned was and whatever, other than propositions and inferences, he was whoever he was.

The problem of the relation of the beginning of mathematics, the beginning of physics, and the beginning of logic is a problem of the relation of propositions and inferences, of the first calculation of areas, the first observation of Venus, the discovery of the principle of contradiction, and, separated as these are by fourteen and a half centuries and eleven and a half centuries, respectively, of all relevant propositions and inferences in between, as well as all relevant earlier propositions and inferences, if there are such propositions and inferences.

History, too, while it waits for its Kepler and its Newton, if its beginning is traced back to Herodotus, it is traced back to the *Histories*, and that is, to propositions and inferences, and if it is traced back to the Sumerian prince list, sixteen centuries before Herodotus, it is traced back to propositions and inferences, and the problem of the relation of the beginning of mathematics, the beginning of history, the beginning of physics, and the beginning of logic is a problem of the relation of propositions and inferences.

And the problem of the beginning of the acquisition of knowledge is a problem of the relation of propositions and inferences. If the problem is of how experience gave rise to the acquisition, it is of how experience such as it predates the acquisition gave rise to the acquisition. It is of how experience such as it predates the acquisition gave rise to the discovery of the principle of contradiction – to the observation of Venus earlier, and hence at length to the discovery of the principle of contradiction – to the prince list earlier, and hence at length to the observation of Venus, and the discovery of the principle of contradiction – to the calculation of areas earlier, and hence at length to the prince list, the observation of Venus, and the discovery of the principle of contradiction – and if it is a

problem of the relation of propositions and inferences across the twenty-six centuries from the beginning of mathematics through the beginning of history through the beginning of physics through the beginning of logic, it is a problem of the relation of propositions and inferences from the first. It is the problem of how the propositions and inferences constituting experience such as it predates the acquisition of knowledge gave rise to the propositions and inferences constituting the acquisition of knowledge. And if these can be identified in particular – the discovery of the principle of contradiction, the observation of Venus, the prince list, the calculation of areas – those can be identified in particular as well.

Thus, also, if the first beginning of human history is taken from experience, the question can be answered, too, whether the first beginning of human history predates the beginning of the acquisition of knowledge, or postdates it, or coincides with it.

As the problem of its beginning, so the problem of collecting the acquisition in its entirety. It is a problem of collecting propositions and inferences. How Galileo rolled spheres down an inclined plane, Torricelli made air bear a weight equal to that of a column of water, Stahl changed metal into calx and calx back into metal.

Inferences, moreover, being inferences, have to lead the collection from any one inference to the next, and the next, and all inferences.

As the problems of its beginning and of collecting it in its entirety, so the problem of the relation of the acquisition and the development of philosophy.

Kant writes in *The metaphysics of morals*: as objectively there can be only one human reason, so there cannot be many philosophies; there can be only one true system of philosophy from principles, however varied and contradictory philosophizing about one and the same proposition may have been; thus, the moralist says, there is only one virtue and one doctrine of virtue; the chemist says, there is only one chemistry, Lavoisier's, the teacher of medicine says, there is only one principle of classification of disease, Brown's; and although the new system excludes all others, it does not detract from the merit of the earlier moralists, chemists, and teachers of medicine, for without their discoveries, or even their unsuccessful attempts, we will not have attained the unity of true principles of all philosophy into one system (pr.).

There cannot be many worlds, there cannot be many acquisitions of knowledge, and there cannot be many philosophies. The world is one. The acquisition of knowledge, whose terminal object the world is, is one. And philosophy, inclusive of the only one true system of philosophy, of all varied and contradictory philosophizing about one and the same proposition, and of many false systems of philosophy, is one.

Including also, when it has liked the acquisition and when it has not, the acquisition of knowledge. This the fact that it includes chemistry shows.

That the problem of the relation of the acquisition and the development of philosophy is a problem of the relation of propositions and inferences the fact of chemistry shows as well. The fact also shows it that virtue has been the matter of doctrine. The fact also that, traced back to Plato and Socrates, the doctrine of virtue is traced back to propositions and inferences which have to be disentangled from propositions and inferences to which the beginning of logic is traced.

That the problem has to be solved the fact that philosophy has included chemistry and virtue, both, shows. The fact also shows it that, while Kant misjudged John Brown's classification of disease, he did not that there is only one principle of classification of disease. Which in turn, sixteen centuries before Kant, Galen's classification of disease shows, and, in the century after Kant, Virchow's *Cellular pathology* and the germ theory of disease, established by Pasteur and Koch, show.

And so is, finally, the problem of the kinds of the acquisition, a problem of the relation of propositions and inferences. Which propositions and inferences are mathematics, which history, which physics, which chemistry, which medicine, which logic, which ethics, which gave rise to which, and how, and why.

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The review of the problems solved in the most

successful investigations of knowledge, and of how these problems are informed by the subsequent acquisition of knowledge, should turn next to Darwin, and then to Frege.

Darwin's very important treatises are these,

Journal of researches into the geology and natural history of the various countries visited by H.M.S. Beagle, under the command of captain FitzRoy, from 1832 to 1836, in 1839,

The origin of species by means of natural selection, or the preservation of favored races in the struggle for life, in 1859,

The variation of animals and plants under domestication, in 1868,

The descent of man, and selection in relation to sex, in 1871,

The formation of vegetable mold, through the action of worms, with observations on their habits, in 1881.

Darwin is so important, it can be affirmed that, if great philosophers are so rare, there had not been above two in the world through Hume's time, Galileo and Newton, there has since been a third, Darwin.

Darwin's importance can be described in the same three steps as those in which Ptolemy's importance has been. The problem he solved. How he solved it. And how he informs the problem before the investigation of knowledge.

The problem Darwin solved is life. More explicitly, he solved the problem of the place of life in the system of the world. Still more explicitly, he solved the problem of the origin and development of all organisms within the development of the world.

The formulation of the problem of life as the problem of the origin and development of all organisms is itself a most important result. It is a consequence of the labors of Linnaeus. In *The system of nature*, in 1735, Linnaeus solved the problem of the classification of all organisms. And the solution of this problem, of the classification of all organisms, posed explicitly, and nothing else could pose explicitly, the problem of life as the problem of the origin and development of all organisms.

The whale illustrates.

Can you pull the leviathan by a fishhook? god asks Job (*Job*, 40), unaware, apparently, that, six centuries before the commitment of *Job* to writing, Tiglath-pileser, prince 1115-076, had, indeed, pulled the leviathan by a fishhook.

In three of his inscriptions Tiglath-pileser reports: in the great sea, with a harpoon that I made, I killed the sea horse (ins. 4-5, 8), and this is how the whale makes its appearance in the record.

This presumes that the sea horse is the leviathan and both are the whale. The reason is that in the *Odyssey* Scylla feeds on whale (XII, 97). It is not that the feeding habits of Scylla, endowed as she is with six heads and twelve feet, inspire confidence. It is that she feeds on κῆτος, and κῆτος remained in the Greek record anything big and terrifying living in the sea, as did φάλλαινα, for the sea was full of such things, and when κῆτος became *cetus* and φάλλαινα became *balena*, *cetus* and *balena* remained in the Latin record anything big and terrifying living in the sea. Fewer big and terrifying things live in the sea today, but the vagueness persists in all languages because whale is not a species, it is at least a genus, inclusive of all its species, and it can be an order, inclusive of all its genera, inclusive of all their species.

Seven and a half centuries after Tiglath-pileser, four after Homer, one and a half after the commitment of *Job* to writing, Aristotle discovered that the whale: produces its young alive, and: has milk.

It cannot be concluded that Aristotle discovered that the whale is a mammal.

The systematic investigation of animals begins with Aristotle, and begins with him most successfully. In the *Parts of animals* he writes: turning to the investigation of animals, we shall try, to the best of our abilities, to omit no member of their kingdom, however lowly. And in the five treatises on animals he describes 600 members of their kingdom, however lowly.

400 of these he describes in the *History of animals*, and in the *History of animals* he begins: some animals live in water, some live on land; some are stationary, some move about; some are gregarious, some solitary; some are carnivorous, some herbivorous, some omnivorous; some are nocturnal, some diurnal; some make sounds, some are mute; some are aggressive, some are not

(I, 1); some have blood, some are bloodless (4); some are viviparous, some oviparous, some vermiparous; some have feet, some do not (5), before eventually concluding: we must take animals kind by kind, and discuss their peculiarities one at a time, in order to become gradually able to discuss them more precisely (6).

Bacon has the rule: we must begin without either premature speculation or any great subtlety (*Organon*, II, 11), and among the reasons the systematic investigation of animals begins with Aristotle most successfully is that he adheres staunchly to this rule.

So, where the whale is concerned, Aristotle has discovered that it: develops not from an egg, but from an embryo, like man and the viviparous quadrupeds (*Hist. animals*, VI, 12); produces its young alive (*Gen. animals*, I, 9); and has breast and milk (*Hist. animals*, III, 20). He recognizes also the three kinds: whale, porpoise, and dolphin (III, 2), recognizes that these three kinds are all of one kind: cetacean (*id.*), and recognizes that cetacean is of the same kind as: man, horse, seal, and all other animals with hair (I, 5).

But he has not discovered that the cetacean is a mammal, or discovered the kind mammal. He includes sharks and rays among the cetaceans on the ground that: some viviparous animals hatch their eggs internally, others engender them internally (*id.*). He proposes also aquatic and terrestrial as the fundamental kinds of animals (*Part. animals*, I, 3-4), rather than, say, viviparous and oviparous, and investigates the degree to which the whale is aquatic, or terrestrial, or both (*Hist. animals*, VIII, 2), rather than, say, why it is viviparous.

Aristotle did not discover that the whale is a mammal. He established that the whale is an animal, however. The whale is like the porpoise, dolphin, man, horse, seal. He did not solve the problems of the kinds of the whale and of the kind of which the whale is in turn a kind. He established the fundamental character of both problems, however, and took major steps toward their solution. The whale is a cetacean, it is also viviparous.

More generally, Aristotle did not solve the problem of the kinds of animals. Some animals are aquatic, some are terrestrial. Some are stationary, some move about, some are gregarious, some solitary, some are carnivorous, some herbivorous, some omnivorous, some are viviparous, some oviparous, some vermiparous. He established the fundamental character of this problem as well, however, and took major steps toward its solution.

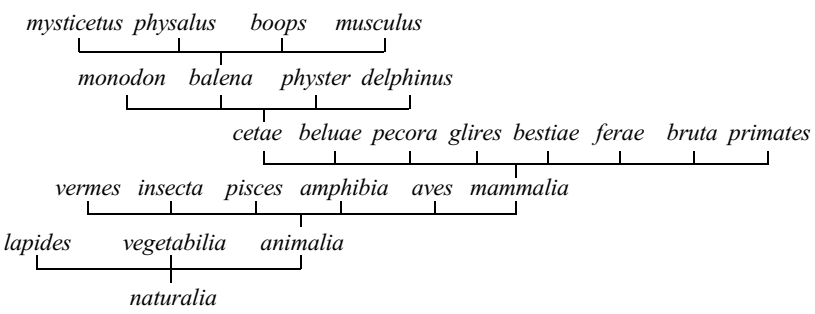
The last rule of the method is: to make enumerations so complete and reviews so general as to omit nothing. The success of Aristotle's attempt to omit no member of the kingdom of animals, however lowly, is attested by the number 600. And he took major steps toward sorting out the 600 into their kinds. The question, what is an animal? could thus be answered – and in the following two millennia it could not be answered better – an animal is the 600 kinds described by Aristotle.

The systematic investigation of animals begins with Aristotle, the systematic investigation of plants begins in the next generation with Theophrastus, and it begins with him most successfully as well. In the *History of plants* and *Causes of plants* Theophrastus also adheres staunchly to Bacon's rule. He writes in the *History*: the fundamental kinds are tree, shrub, undershrub, and herb (I, 3). Unless it was too subtle to separate undershrub from shrub. The success of his attempt to omit no member of the kingdom of plants, however lowly, is attested by the number 550, the members of their kingdom Theophrastus describes, and he took major steps toward sorting out the 550 into their kinds as well. The question, what is a plant? could thus also be answered – and in the following two millennia it could not be answered better either – a plant is the 550 kinds described by Theophrastus.

Hence, more generally still, the question, what is an organism? could be answered, an organism is the 1 150 kinds described by Aristotle and Theophrastus.

And in *On the soul* Aristotle writes: the soul is the principle of life (II, 4). The question, what is life? could thus be answered by appeals to the soul, with all the attendant perils of transmigration, immortality, and worse. Or it, too, could be answered, life is the 1 150 kinds described by Aristotle and Theophrastus.

Twenty-one centuries after Aristotle and



Theophrastus, the questions, what is the whale? more generally, what is an animal? and more generally still, what is an organism? were answered explicitly by Linnaeus.

The system of nature, according to the three kingdoms of nature, by classes, orders, genera, and species appeared in 1735. Its tenth edition, in 1758, is conclusive of Linnaeus's labors on animals. He describes 5 897 species of animals, and arranges them in their genera, orders, and classes. Of his labors on plants conclusive are *The species of plants, showing the properly recognized plants, related by genera, with specific differences, common names, selected synonyms, and native locations, arranged by the sexual system*, in 1753, and the fifth edition of *The genera of plants, as well as their natural characters, by number, shape, place, and proportion of all fruit bearing parts*, in 1754. In *The species of plants* Linnaeus describes 5 944 species of plants, and in *The genera of plants* he arranges them in their genera, orders, and classes.

And the picture above is of the whale in *The system of nature*.

What the whale is is clear. If usage will commit itself to *balena*, the whale is one of the 4 genera, with its 4 species, of the order *cetae*, itself one of the 8 orders of the class *mammalia*, itself one of the 6 classes of the kingdom *animalia*, itself one of the 3 kingdoms of the empire *naturalia*. If usage will commit itself to *cetus*, the whale is one of the 8 orders, with its 4 genera, with their 12 species, of the class *mammalia*, itself one of the 6 classes of the kingdom *animalia*, itself one of the 3 kingdoms of the empire *naturalia*. If usage will not commit itself, it can be left to its own devices.

What an animal is is also clear. It is the 5 897 species of animals. Not only is the number of species ten times greater than the number of kinds in Aristotle, showing that completeness with respect to the object of enumeration, not just with respect to the attempt at enumeration, is being achieved. The 5 897 are not merely kinds any more, they are species, *musculus* and *sapiens*, for instance, belonging to genera, to *balena* and *homo*, respectively, belonging to orders, to *cetae* and *primates*, belonging to classes, both to *mammalia* in this case. Linnaeus solved the problem of the kinds of animals, in principle, conclusively, too. All corrections of his solution – the addition of many species, the addition of genera, orders, and classes, the introduction of intermediary levels of subordination between genera and orders, orders and classes, and classes and kingdom – have corroborated the principle of his solution.

Linnaeus solved the problem of the kind of which animal is in turn a kind as well. With its 6 classes, their 34 orders, their 312 genera, and their 5 897 species, animal is one of the 3 kingdoms of the empire nature.

Which shows also that the root of the classification, the empire *naturalia*, is an essential part of the classification. All substantial corrections of Linnaeus's solution of the problem of the classification of organisms, too, have concerned the root of the classification. The relations of kingdoms with one another and with the empire of which the kingdoms are kinds have been the most important problems the classification has had to solve as protista have been discovered, then bacteria, then viruses.

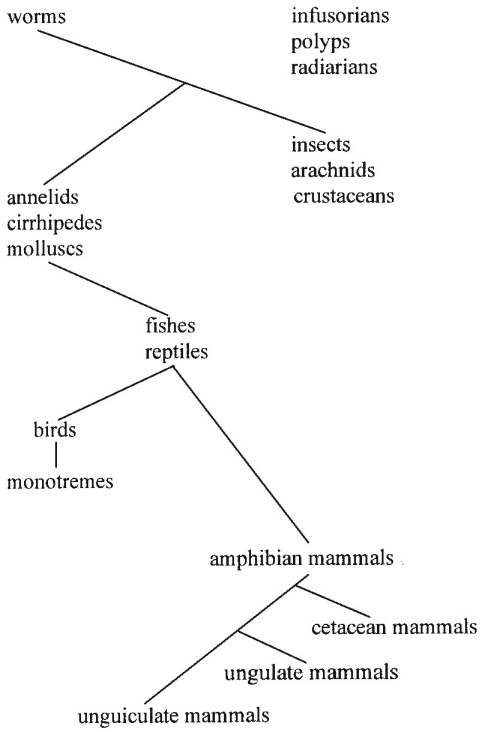
Few attempts at enumerations so complete and reviews so general as to omit nothing have been nearly as successful as Linnaeus's. Also, Aristotle formulated the principle of definition, *genus proximus differentia specifica*, and the principle of classification, the *dictum de omni et nullo* – the principle of definition in the *Topics* (I, 8), the principle of classification in the *Prior analytics*

(I, 1). And no attempt at definition or classification has been nearly as successful in adhering to the principles of definition and classification as Linnaeus's.

In *The system of nature*, then, the whale is explained. Its kinds are shown, of what kind it is a kind is shown, in what proximate relations to what other kinds it is is shown, and in what more remote relations to what other kinds. Shown is, in short, its place in the system of mammals, in the system of animals, in the system of organisms, in the system of nature. Man is explained in the same way, its two kinds are shown, *sapiens* and *siluestris orang outang*, of what kind it is a kind, and in what proximate and what more remote relations to what other kinds it is. Shown is his place in the system of mammals, in the system of animals, in the system of organisms, in the system of nature. Animal is explained in the same way. Its system is shown, in the system of nature. And organism is explained. Organism is the 11 841 species of the 1 417 genera of the 146 orders of the 31 classes of the 2 of the 3 kingdoms, animal and plant, of the empire nature. The system of organism is shown, in the system of nature.

The system of nature thus posed, and nothing else could pose, the problem of explaining the system of nature. More particularly, the system of nature posed the problem of explaining the system of animals, or what is the same, as Lamarck showed a half century after Linnaeus, of explaining the origin and development of all animals. More generally, the system of nature posed the problem of explaining the system of organisms, or what is the same, as Darwin showed a century after Linnaeus, of explaining the origin and development of all organisms.

Lamarck published the *Zoological philosophy, or exposition of considerations on the natural history of animals, on the diversity of their organization and the faculties they obtain from it, on the physical causes which maintain life in them and give rise to the movements they make, as well as on what produces feeling in some of them and intelligence in those endowed with it*, in 1809. He writes: from Aristotle to Linnaeus nothing of note was done regarding the general arrangement of animals; Linnaeus, a man of high genius and one



of the greatest naturalists, established the classification of animals (I, 5). Lamarck then proceeds to correct the classification.

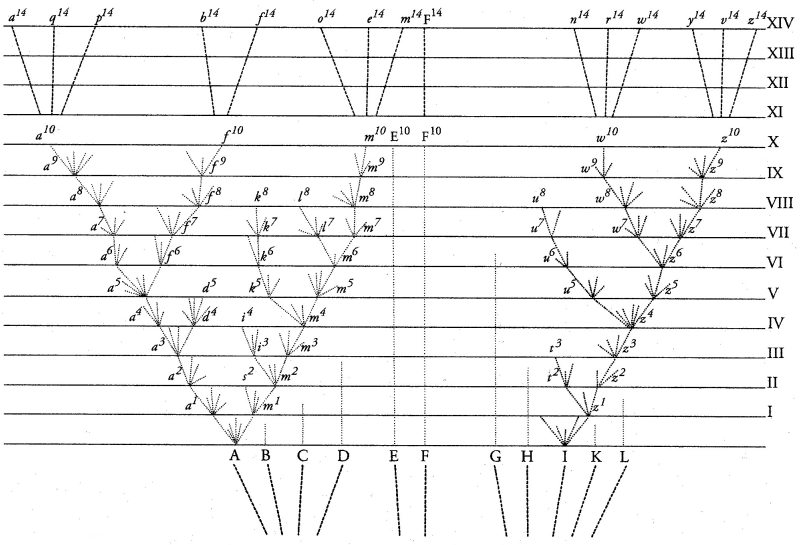
The picture on this page is the only picture in the *Zoological philosophy*. It shows: the origin of the various animals (8).

And the picture on the next page is the only picture in *The origin of species*.

Darwin explains: A to L represent the species of a genus large in its country; after a thousand generations species A has produced the well-marked varieties a^1 and m^1; after another thousand generations variety a^1 has produced variety a^2, variety m^1 has produced varieties s^2 and m^2; and so on; by the time a^{14} through m^{14} have been produced they are distinct species all descended from A; by that time B, C, D, and E have all become extinct; F has transmitted its unchanged descendants, F^{14}, to that stage of descent; and similarly G, H, I, K, and L (4). Darwin adds: each horizontal line has been supposed to represent a thousand generations, but it may represent a million or a hundred million generations (*id.*).

A to L have also been supposed to represent

the species of a genus. But they can represent the genera of an order. They can represent the orders of a class. They can represent the classes of a kingdom. And they can represent the kingdoms of the empire. The theory of evolution explains the origin and development of species, hence of genera, hence of orders, hence of classes, hence of kingdoms, and *ipso facto* of all organisms.



Darwin writes: without constantly bearing in mind the universal struggle for life the whole economy of nature will be dimly understood or quite misunderstood (3). Success in the struggle for life is: success in leaving progeny (*id.*). And: that varieties, which are incipient species, become ultimately distinct species follows inevitably from the struggle for life; owing to the struggle for life, any variation, however slight, if it be in any degree profitable to an individual of any species, will tend to its preservation and will generally be inherited by its offspring; the offspring will thus also have a better chance of surviving (*id.*); I have called this principle, by which each slight variation, if useful, is preserved, natural selection (*id.*).

The whale, animal, and organism are all thus explained conclusively. One principle, natural selection, has produced them all.

Natural selection has produced the whale. Darwin writes: the black bear was seen by Hearne swimming for hours with widely open mouth, catching like a whale insects in the water; and if the supply of insects were constant and better adapted competitors did not already exist in the country, I can see no difficulty in a race of bears being rendered, by natural selection, more and more aquatic in their structure and habits, with larger and larger mouths, until a creature is produced as monstrous as a whale (6, 1st ed.).

Natural selection has produced that most recalcitrant of organisms, man. Darwin writes: Linnaeus, so remarkable for his sagacity, placed man in the same order with the quadrumana under the title primates (*Descent*, 6).

Natural selection has produced the system of organisms, the one grand natural system. Darwin writes: all past and present organic beings constitute one grand natural system (*Origin*, 15); the affinities of all beings of the same class have been represented by a great tree; I believe this simile largely speaks the truth (4); the arrangement of species, genera, and families into what is called the natural system some authors regard as merely a scheme for arranging living objects, or as an artificial method of enunciating briefly general propositions; but the famous expression of Linnaeus that the characters do not make the genus, the genus gives the characters, seems to me to imply that some deeper bond is included in our classification than mere resemblance (14).

It is in the *Botanical philosophy*, in 1751, that Linnaeus writes: you must understand that a character does not make a genus, the genus makes the character (169).

Natural selection has produced all past and present organic beings, throughout all time and space. Darwin writes: it is a truly wonderful fact that all animals and all plants throughout all time and space should be related to each other in group subordinate to group; this great fact is explained through inheritance and the complex action of natural selection (*Origin*, 4); from the first dawn

of life all organic beings can be classed in groups under groups; this grand fact of the natural subordination of organic beings is in my judgment explained (14).

All past and present organic beings. Throughout all time and space.

Had there been inhabitants of Mercury, Venus, the earth, Mars, Jupiter, and Saturn each, A to F can represent them, and in either case, had they all had common origin, or had they each originated independently. Less gloomily, F can represents the inhabitants of Mercury, I the inhabitants of the earth, A the inhabitants of Saturn.

In Andromeda as well, if gravity holds together the masses, and it had better, selection produces the organisms.

From the first dawn of life, too. It has been mandatory to exclude the origin of life from the theory of evolution. Wantonly: looking to the first dawn of life, it is an error to suppose that there would be no struggle for existence, and consequently no natural selection, until many forms had been produced; variation in a single species inhabiting an isolated station might be beneficial (4, 3rd ed.).

The big bang theory does not reach back to the big bang. It reaches back to 10^{-43} seconds after the big bang. Beginning 10^{-43} seconds after the big bang, it reaches forward 1.4×10^{10} years, counting not further than the present, the same principles producing quarks and leptons, atoms, stars, and galaxies. The theory of evolution does not reach back before the first organism either. It reaches back to the first organism. It is an error to suppose that there would be no struggle for existence, and consequently no natural selection, until more than the first organism and its progeny have been produced, and taking it for granted that an organism cannot be an organism unless it has progeny, it is an error to suppose that there would be no struggle for existence, and consequently no natural selection, until more than the first organism has been produced. Beginning with the first organism, the theory of evolution reaches forward 4×10^9 years on earth alone, counting again not further than the present, the same principle producing all organisms, or all life.

The problem Darwin solved is the origin and development of all organisms within the development of the world. Their origin and development within the solar system alone for 4×10^9 of the 1.4×10^{10} years since the big bang.

Pace Aristotle, the soul is not the principle of life. Natural selection is the principle of life.

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The *Journal of researches into the geology and natural history of the various countries visited by H.M.S. Beagle* appeared in 1839.

And in the *Journal of researches* Darwin anticipates the theory of evolution as clearly as this. He writes about the Galapagos: the natural history of these islands is eminently curious; most of the organic productions are aboriginal, there is even a difference between the inhabitants of the different islands, yet all show a marked relationship with those of America; the archipelago is a little world within itself, or rather a satellite attached to America, whence it has derived a few stray colonies, and has received the general character of its indigenous productions; both in space and time, we seem to be brought near to that great fact, that mystery of mysteries, the first appearance of new beings on this earth (17); the land birds form a most singular group of finches; the most curious fact is the perfect gradation in the size of the beaks; one might readily fancy that from an original paucity of birds in this archipelago, one species had been taken and modified for different ends (*id.*).

The origin of species appeared in 1859.

The origin of species is: one long argument (15).

It is also, next to *The mathematical principles of natural philosophy*, the most powerful argument made.

Darwin begins with variation under domestication. He explains: at the commencement of my observation it seemed that a careful study of domesticated animals and plants would offer the best chance of making out this obscure problem; nor have I been disappointed; I have found knowledge of variation under domestication the best and safest clue; such studies have been commonly neglected by naturalists (intr.). And still more explicitly in *The variation of animals and plants under domestication*: man can be said to have been trying an experiment on a gigantic

scale; and it is an experiment which nature during the long lapse of time has incessantly tried (intr.).

It is on this discovery, of the best and safest clue, the best chance of making out the problem, commonly neglected by naturalists, that the development of the theory of evolution rests. Darwin discovered how to approach that mystery of mysteries, the first appearance of new beings on this earth, experimentally. Hence, not only is the experiment an experiment on a gigantic scale. Its results, being the results of an experiment, are transparent, and the conclusion from its results, being the conclusion from the results of an experiment, is inevitable: ask a celebrated raiser of Hereford cattle whether his cattle might not have descended from Longhorns, or both from a common parent stock, and he will laugh you to scorn (1); the explanation is simple; they know that each race varies slightly, for they win their prizes by selecting such slight differences, yet they ignore all general arguments, and refuse to sum up the slight differences accumulated during many successive generations (*id.*).

After variation under domestication Darwin turns to variation under nature. As Herefords and Longhorns, so all organisms. Variation is general. Hence: the larger genera tend to become larger, and throughout nature the forms of life now dominant tend to become still more dominant; but the larger genera also tend to break up into smaller genera; and thus the forms of life throughout the universe become divided into groups subordinate to groups (2).

The problem is thus formulated explicitly. The problem is to explain how, given variation, Herefords and Longhorns are descended from a common parent stock. To explain, in general, how, given variation, the forms of life throughout the universe become divided into groups subordinate to groups. To explain the origin of species. That mystery of mysteries, the first appearance of new beings on this earth.

Formulated in the first two chapters, this is the problem. In the next two chapters its solution follows.

There is the fact of the struggle for life: without constantly bearing in mind the universal struggle for life the whole economy of nature will be dimly understood or quite misunderstood (3).

And from variation and the struggle for life: that varieties become ultimately distinct species follows inevitably (4); the principle is that each slight variation, if useful, is preserved (*id.*).

The general argument is thus complete. Darwin has shown, as his title promises, *The origin of species by means of natural selection, or the preservation of favored races in the struggle for life*.

In the following nine chapters – ten in the sixth edition – Darwin addresses the difficulties before the theory, real and otherwise, and shows the depth and the breadth of the problems which the theory solves.

He deals first with the laws of variation. He concludes: whatever the cause of each slight difference in the offspring from their parents, and a cause for each must exist, it is the steady accumulation through natural selection of such differences, when beneficial to the individual, that gives rise to the modification in structure by which the innumerable beings on the face of the earth are enabled to struggle with each other, and the best adapted to survive (5).

However: our ignorance of the laws of variation is profound (*id.*).

And this was the one, and only one, real difficulty before the theory. The ignorance of the laws of variation had to be remedied. A cause for each slight difference must exist. But the theory had to find that cause. Moreover, the theory had to establish not just the laws of variation. It had to establish the laws of hereditary variation. Mendel's laws.

In the first edition, in 1859, Darwin is clear about the difficulty: our ignorance of the laws of variation is profound. Also: the laws governing inheritance are for the most part unknown (1). And: it is of the highest importance to gain a clear insight into the means of modification (intr.).

By the sixth edition, in 1872, he is clear that this is the main difficulty before the theory. He writes: unless favorable variations be inherited by some at least of the offspring, nothing can be effected by natural selection (4).

The remaining difficulties before the theory are of two kinds, insurmountable and surmountable.

Of the insurmountable this is representative: they believe that very many structures have been created for the sake of beauty, to delight man or the creator (6, 6th ed.), and Darwin concedes: this doctrine, if true, would be absolutely fatal to my theory (*id.*).

The surmountable difficulties have, in fact, all been surmounted. These include the questions of whether selection can produce organs of extreme perfection – and the one, in particular, whose extreme perfection Helmholtz finds afflicted with inaccuracy, chromatic aberration, astigmatism, a blind spot, and opacity – whether it can produce instinct, whether it can produce hybrids, why there is no evidence of transitional forms. The last is representative of how these difficulties have all been surmounted by Darwin already. He writes: the geological record is extremely imperfect (11), yet: all the great leading facts of paleontology seem to me simply to follow on the theory of descent with modification through natural selection (*id.*).

All the great leading facts follow, and as the geological record has kept becoming ever more perfect, all the facts of paleontology, great and small, have kept following ever more simply.

The depth and the breadth of the problems which the theory solves Darwin shows in five chapters on the geological succession of organic beings, on their geographical distribution, and on their mutual affinities. It is a *tour de force*.

Darwin shows that the theory explains the temporal succession of organic beings: all the great leading facts of paleontology seem to me simply to follow on the theory of descent with modification through natural selection.

He shows that the theory explains the spatial distribution of organic beings: all the grand leading facts of geographical distribution are explicable on the theory of migration, together with subsequent modification and the multiplication of new forms; we can understand the high importance of barriers, whether of land or water, which separate our several zoological and botanical provinces; we can understand the localization of sub-genera, genera, and families (13).

Darwin has thus shown that: the laws governing the succession of forms in past times are nearly the same with those governing at the present time their differences in different areas (*id.*); and on my theory these several relations throughout time and space are intelligible (*id.*).

Hence, further, the inductive fact of Linnaeus's classification of organic beings has become a deductive fact. It follows from the principle of natural selection that: all past and present organic beings constitute one grand natural system (15).

Further still: all the great facts in morphology become intelligible (14); what can be more curious than that the hand of man formed for grasping, that of the mole for digging, the leg of the horse, the paddle of the porpoise, and the wing of the bat should all be constructed on the same pattern, and should include the same bones, in the same relative positions (*id.*)? on the theory of natural selection we can satisfactorily answer this question (*id.*).

And yet further: we can understand the great leading facts in embryology (*id.*); the embryos of distinct animals in the same class are often strikingly similar; a better proof of this cannot be given than the circumstance, mentioned by von Baer that having forgotten to ticket the embryo of some vertebrate animal, he cannot now tell whether it be that of a mammal, bird, or reptile (*id.*); the leading facts of embryology, second in importance to none in natural history, are explained on the principle of slight modifications in the many descendants from some one ancient progenitor (*id.*).

Darwin ends with a recapitulation and conclusion. One of his conclusions is: light will be thrown on the origin of man and his history (15).

Another is: when I view all beings not as special creations, but as the lineal descendants of some few beings which lived long before the first bed of the Cambrian system was deposited, they seem to me ennobled (*id.*); there is grandeur in this view of life (*id.*).

Darwin saw *The origin* through six editions, the sixth in 1872. The substantial changes are two.

One reflects the clarity that the one real difficulty before the theory is the ignorance of the laws of heredity. In fact, between the fourth and the fifth editions, in 1868, in *The variation of animals and plants under domestication*, Darwin

has addressed the difficulty directly by formulating: the provisional hypothesis of pangenesis (27).

The other substantial change is the shortening of the temporal reach of selection. The note on the picture in the first edition, for instance: each horizontal line may represent a million and a hundred million generations, became in the fifth edition: may represent a million or more generations.

Most dramatically, Darwin writes in the first edition: standing on the North Downs and looking at the South Downs, and remembering that at no great distance to the west the northern and southern escarpments meet, one can safely picture the great dome of rocks which must have covered up the Weald; the distance from the northern to the southern Downs is about 22 miles, the thickness of the several formations is on average about 1 100 feet; and if we may assume that the sea would eat into cliffs 500 feet in height at the rate of one inch in a century, at this rate the denudation of the Weald must have required 306 662 400 years (10).

He deleted the passage in the third edition. The reason is: we encounter a formidable objection; for it seems doubtful whether the earth has lasted long enough; Kelvin concludes that the consolidation of the crust can hardly have occurred less than 20 or more than 400 million years ago, but probably not less than 98 or more than 200 (*id.*).

Kelvin was wrong by more than an order of magnitude. Darwin was right within an order of magnitude. Darwin was nevertheless right to listen to Kelvin.

For nearly two millennia the world, including the earth, had been but a couple of days older than Adam.

This changed in the century before Darwin. In 1754, in *The origin of inequality*, Rousseau speaks of: thousands of centuries (1). In 1755, in the *General natural history and theory*, Kant speaks of: millions of centuries (II, 7).

And in 1775 Buffon grounded the age of the earth in experiment. In the *Researches on the cooling of the earth and the planets*, from experiments on rates of cooling, Buffon concludes: the earth is at least 74 832 years old (*id.*).

In 1859, when the first edition of *The origin* appeared, Buffon's was still as large as there was a result on the age of the earth obtained by experiment. The result, though, had been abandoned.

In 1785, in the *Abstract of a dissertation concerning the system of the earth, its duration, and stability*, Hutton argues that: the world has neither a beginning nor an end (*id.*). In 1795, in *The theory of the earth*, he argues in great detail and at greater length that: we find no vestige of a beginning, no prospect of an end (I, 1.4). And with the *Principles of geology*, in 1830-3, Lyell established firmly the presumption that an indefinite amount of time is available to all developments on earth. It is what Darwin takes for granted in the first edition.

Kelvin had no use for: Hutton's and Lyell's doctrine of eternity and uniformity (*Abode*, 10). He resumed the attempts to ground the age of the earth in experiment, and it is in *On the secular cooling of the earth*, in 1862, also from experiments on rates of cooling, that he concludes: we may say with much probability that the condensation of the earth cannot have taken place less than 20 million years ago, nor more than 400 million (11).

Kelvin was not only wrong by not much more than an order of magnitude. He improved on the best earlier result by many orders of magnitude. And Darwin was right to listen to him because the theory of evolution stands, and stands unshakably, on experiment.

The power of the argument of *The origin of species* begins with its generality. The argument encompasses all organic beings.

Further, however, its generality rests on deduction, and rests on deduction paradigmatically. All the great leading facts of paleontology, all the grand leading facts of geographical distribution, the fact of the one grand natural system which all past and present organic beings constitute, all the great facts in morphology, and all the great leading facts in embryology follow from the one, and only one, principle of natural selection.

Further still, the deduction rests on induction. *The origin* is a massive compendium of facts. Moreover, Darwin notes in concluding it that it is: one long argument. In introducing it, he notes that it is only: an abstract (intr.), and promises: to publish hereafter the facts on which its conclu-

sions have been grounded (*id.*). These facts comprise the next two very important treatises, *The variation of animals and plants under domestication*, in 1868, and *The descent of man, and selection in relation to sex*, in 1871, and each of these two treatises is a compendium of facts substantially more massive than *The origin*.

And yet further, the deduction rests on induction, and rests on induction paradigmatically. It rests on experiment.

It is how Darwin solved the problem of life, by experiment.

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Observation is not looking, not seeing, and not even looking with determination and seeing with deliberation. It is the measurement and calculation of the seen, heard, touched, tasted, smelled. It is essentially mathematical.

Or better, it is essentially formal. Better yet, it is the formalization of experience.

And if so, there can be no essential difference between observation and experiment. The difference can only be of degree.

Bacon writes: man, the servant and interpreter of nature, does and understands only as much as he has observed of the order of nature, in deed or in mind, nor can he know or do more (*Organon*, I, 1). What man has observed of the order of nature is not delivered by the senses: the testimony and reports of the senses are always in relation to man, not in relation to the world; it is thus a great error to assert that the senses are the measure of things (distr.); the senses are only to judge experiment, experiment is to judge the thing (*id.*). What experiment is is clear: experience as it occurs is an accident, as it is pursued an experiment (I, 82). And it is clear how it is pursued: the secrets of nature reveal themselves more under the vexation of art than by themselves (98); not when nature is free and left to herself, but when she is constricted and vexed, when by the arts and services of man she is forced out of her state, and molded and shaped (distr.).

If the vexation of nature, then, distinguishes observation, including experiment, from looking and seeing, the degree of vexation has to distinguish the kind of observation which experiment is. Or what is the same, the degree to which the secrets of nature have had to be forced out of her by art. But the degree cannot be determined by the art, it can only be determined by nature. If so, and some objects have been delivered by experience more clearly and distinctly than other objects, the less clearly and distinctly objects have been delivered by experience the more these objects have had to be vexed into clarity and distinction by art. By experiment.

Thus, if Ptolemy investigates both the heavenly bodies and refraction, and investigates the latter by experiment, the two investigations are not different in that their results are in the one case observed, in the other case not observed. They are in both cases observed. The difference is not that in the one case experience is as it occurs, in the other as it is pursued. Experience is in both cases as it is pursued. The difference is not that in the one case the secrets of nature have been forced out of her by art, in the other case without art. They have in both cases been forced out of her by art. Indeed, the instrument of vexation in the case of refraction, the disc with a graduated circumference, is rather less vexatious than the instrument in the case of the heavenly bodies, the armillary sphere. The difference is in the objects of investigation. The spots of light distinguished sharply from one another against the darkness of the night sky experience has delivered as clearly and distinctly as it has any objects from the first. The bending of the rays of light when passing from one medium to another experience has not delivered clearly and distinctly before the investigation of the rays of light at all.

If Ptolemy anticipated the importance of that most formal of observations, the experiment, however, and Bacon first argued its decisive importance, it was Galileo who established its decisive importance. By experiment. By doing experiments *par excellence*.

Kant writes: when Galileo rolled spheres down an inclined plane new light dawned on all investigators of nature.

This is how and why.

Galileo reports: along the length of a wooden beam about twelve braccia long, half a braccio wide, and three inches thick, a groove a little over an inch wide and very straight was carved (*Sciences*, III).

The beam was thus not quite 7 m long, about .3 m wide, somewhat less than .1 m thick. The groove was some .03 m wide.

Galileo continues: in the groove vellum was glued to make it as clean and smooth as possible; one end of the beam was elevated above the horizontal plane one or two braccia; a very hard, well rounded, and polished bronze ball was allowed to descend in the groove along the entire length of the beam; the time was recorded; and this was repeated many times in order to establish the amount of time with complete certainty; not even a difference of one tenth of one pulse was found (*id.*).

One pulse is less than one second.

Galileo continues: when this was precisely established, the same ball was allowed to descend only one quarter of the length of the beam; and the time was again measured (*id.*); the experiment was then made for other lengths along the beam (*id.*); the experiments were repeated hundreds of times (*id.*); and they were made for all inclinations of the plane (*id.*); as to the measure of time, a large pail was filled with water, fastened on a rope, and a slender tube was affixed to its bottom through which a narrow thread of water was allowed to escape; this water was collected in a beaker during the time that the ball descended along the groove; the collected amounts of water were weighed on a delicate balance, and the differences and ratios of the weights gave the differences and ratios of the times with such precision that they never differed by a notable amount (*id.*).

It is the experiment *par excellence* because of its pristine simplicity. Also because it sets all the standards of: repeated many times to establish complete certainty, of: not even a difference of one tenth of one pulse was found, of: repeated hundreds of times, of: made for all inclinations of the plane, of: such precision that the times never differed by a notable amount.

Yet it is the experiment *par excellence* first and foremost because experience which is not clear and distinct at all before the experiment is experience perfectly clear and distinct with the experiment.

The experiment demonstrates the law of falling bodies, or more generally of uniformly accelerated motion. Galileo concludes: the distances in uniformly accelerated motion are as the squares of the times (*id.*). $s \propto t^2$.

What makes it an experiment is not the art. Galileo's discovery of the satellites of Jupiter depends on more sophisticated instruments of vexation. The theory on which it depends and the machinery on which it depends are both more sophisticated than the theory and the machinery on which the discovery of the law of falling bodies depends. The discovery of the satellites depends on the measurement of distances at the orbit of Jupiter. And Galileo notes: nature has not granted men a perfect instrument; it is rather the very instrument of sight that introduces obstacles; but in our time it has pleased god to concede to human ingenuity a discovery so marvelous as to improve sight 4, 6, 10, 20, 30, and 40 times (*Systems*, III). The discovery of the law of falling bodies depends on the measurement of distances along a beam. And god did not have to concede to human ingenuity anything more wonderful than the beam, a chisel to carve it, some vellum, and a bronze ball. The pail, tube, beaker, and delicate balance god had to concede to human ingenuity in both cases, to measure the times of descent of the ball along the beam as well as the periods of revolution of the satellites.

What makes it an experiment are the objects of the experiment. The relations of space and time in falling bodies, let alone more generally in uniformly accelerated motion, experience has not delivered clearly and distinctly, if it has delivered them at all, before the experiment. The experiment first reduced them to clarity and distinction, and hence to their form, $s \propto t^2$.

An experiment thus does not have to depend on new art to be an experiment. One of Galileo's most important experiments is this: shut yourself with a friend in a cabin under the deck of a ship; bring with you some flies and butterflies, take a tank of water with some fish in it, hang a bottle that empties drop by drop into a vessel beneath it; when the ship stands still observe carefully all the motions; let then the ship move as fast as you wish; as long as the motion is uniform, you will not notice the least difference in any of the motions (II). This is the experiment which demonstrated inertia, and which established the princi-

ple of relativity, and hence the Galilean transformation, $x' = x - vt$, $y' = y$, $z' = z$, $t' = t$.

Nor does an experiment have to reduce to clarity and distinction, and hence to its form, actual experience, it can reduce possible experience, including such as is best never actualized. Another of Galileo's experiments is this: if the earth is bored through the center and a ball let fall, it will pass the center and ascend as much as it has descended; this is demonstrated plainly by the pendulum which, removed from its state of rest, falls back toward its state of rest and goes the same distance beyond it (I).

All three experiments show the generalization of experience as well as its formalization. All three show also that the generalization is from actual experience to possible experience. From bronze balls to uniformly accelerated motion. From a cabin under the deck of a ship to inertia. From a pendulum to the center of the earth.

It can be concluded that the further removed from actual experience possible experience is the more it is accessible to nothing but experiment.

If the center of the earth is far removed from actual experience the age of the earth is removed further.

In 1687, in *The principles*, Newton writes: a globe of incandescent iron one inch wide will scarcely lose all its heat in one hour; a larger globe will preserve its heat longer in the ratio of its diameter, for its surface is smaller in this ratio with respect to the heat it contains; so a globe of incandescent iron equal to the earth, and that is, about 40 000 000 feet wide, will scarcely cool in as many days, or about 50 000 years (III, 41).

In 1774, in the *Experiments on the development of heat in bodies*, Buffon reports that that experiment of Newton's suggested this further experiment: ten iron globes were forged, with diameters graduated by one half inch from one half inch to five inches; in a cave, to ensure constant ambient temperature, the globes were heated to incandescence; the time which each globe took to cool to the ambient temperature was measured; and the series was obtained, in minutes,

39, 93, 145, 196, 248, 308, 356, 415, 466, 522;

it approximates closely the series,

39, 93, 147, 201, 255, . . . , $54n - 15$;

hence, an iron globe whose diameter is n, in half inches, will cool from incandescence to the actual temperature of the earth in $54n - 15$ minutes; and given that the diameter of the earth is 941 461 920 half inches an iron globe the size of the earth will cool from incandescence to the actual temperature of the earth in 50 838 943 662 minutes, or 96 670 years and 132 days (*id.*).

This suggests the *Further experiments on the development of heat in different mineral substances*, also in 1774. Buffon reports: twenty-four globes were made, each one inch in diameter, from different substances, iron, gold, silver, lead, bismuth, copper, antimony, zinc, marble, glass, sandstone, gypsum, limestone, porcelain, wood, and so on; the globes were heated; the times of cooling were measured; and the ratios for the times of cooling were established, iron to gold 10 000 to 8 148, iron to silver 10 000 to 7 423, and so on (*id.*)

Finally, in 1775, in the *Researches on the cooling of the earth and the planets*, Buffon calculates: if the terrestrial globe is glass, sandstone, limestone, marbles, and ferruginous matters, it has cooled to the actual temperature of the earth in about 74 047 years (*id.*). And from their distances from the sun, sizes, and densities, he calculates the times for the cooling of the other planets, the moon, the four satellites of Jupiter, and the five satellites of Saturn, corrects the results against one another, and the final times for the planets are:

Mercury	54 192 years,
Venus	91 643,
the earth	74 832,
Mars	28 538,
Jupiter	240 451,
Saturn	130 081 (*id.*).

These are the times for the cooling of the planets to the actual temperature of the earth, and the time of the cooling of the earth, 74 832 years, is thus the minimum for the age of the solar system.

Rates of cooling remained the only way to access the age of the earth until Rutherford. In 1862, in *On the secular cooling of the earth*, Kelvin calculates the age of the earth at: not less than 20 million years, nor more than 400 million (11). In 1871, in *On the origin of the planetary*

system, Helmholtz calculates it at: not less than 22 million years (*id.*). In 1897, in *The age of the earth as an abode fitted for life*, Kelvin calculates it at: more than 20 and less than 40 million years, and probably much nearer 20 than 40 (17). All three calculations are from rates of cooling.

Rutherford writes, in 1904, in *The radiation and emanation of radium*: the temperature gradient observed in the earth may be due to the heat liberated by the radioactive matter distributed throughout it; and if so, the earth may have been at a temperature capable of supporting life for a much longer time than that estimated by Kelvin from thermal data (2). In 1905, in *Radium – the cause of the earth's heat*: the distribution of radioactive matter in the earth throws grave doubt on the validity of the calculations of its age based on the assumption that it is a simple cooling body (*id.*). And in 1907, in *Some cosmical aspects of radioactivity*: radium steadily produces from itself helium; helium is found in quantity only in the radioactive minerals; the presence of helium in these minerals is at once explained, if helium is produced by the radium or other radioactive matter in the minerals; the amount of helium will steadily increase with time, and the total amount present should be proportional to the age of the mineral and the amount of radioactive matter in it; I have here some crystals of thorianite; and it can be calculated with some confidence that the mineral thorianite is at least 500 million years old, and that is, this interval of time must have elapsed since the formation of the mineral in the earth's crust (*id.*).

A century-odd later, the radioactive date for the age of the earth is 4 540 million years.

Nor is the origin of species less removed from actual experience than the age of the earth.

Hence, in order to show that: without constantly bearing in mind the universal struggle for life the whole economy of nature will be dimly understood or quite misunderstood, Darwin not only adduces fact after fact after fact, he keeps counting and calculating.

He writes: if the elephant breeds when 30 years old, and goes on breeding till 90, bringing forth 3 pairs of young in the interval, at the end of the fifth century there would be alive 15 000 000 elephants descended from a single pair (*Origin*, 3), and when clamor arose about this number, Darwin replied: at the end of 750 to 760 years there would be alive 18 803 080 elephants (in *Athenaeum*, 26 Jun 1869).

Malthus, in 1798, in *An essay on the principle of population*, had proposed that: population, when unchecked, increases in a geometrical ratio (1).

Darwin writes: but we have better evidence than mere theoretical calculations (*Origin*, 3), and then produces this evidence, continuing to count and calculate: on a piece of land three feet by two, dug and cleared, I marked all the seedlings of our native weeds as they came up, and out of 357 not less than 295 were destroyed (*id.*); out of 20 species growing on a little plot of turf, three feet by four, 9 species perished (*id.*); the winter of 1854-5 destroyed four fifths of the birds in my grounds (*id.*); we can easily raise corn in our fields because the seeds are in great excess compared with the number of birds which feed on them; but anyone who has tried knows how troublesome it is to get seed from wheat or other plants in a garden; I have in this case lost every single seed (*id.*); on an estate, where I had ample means of investigation, there was a large and extremely barren heath; but several hundred acres of exactly the same nature had been enclosed twenty-five years previously and planted with Scotch fir; the change in the native vegetation was most remarkable; 12 species of plants flourished in the plantations which could not be found on the heath; the effects of insects must have been still greater, for 6 insectivorous birds were very common which were not to be seen on the heath (*id.*); near Farnham, in Surrey, there are extensive heaths with a few clumps of old Scotch firs; on the hundreds of acres of unenclosed heath a singe Scotch fir could not be seen; but on looking closely between the stems of the heath, I found a multitude of seedlings which had been perpetually browsed by the cattle; in one square yard, at a point some hundreds of yards from the clumps, I counted 32 little trees; and one of them, judging from the rings of growth, had during twenty-six years tried to raise its head above the stems of the heath and failed (*id.*).

But the theory of evolution stands, and could only stand unshakably, on: the experiment which

man has been trying on a gigantic scale, and which nature has during the long lapse of time incessantly tried.

Hence, as Galileo rolled balls down an inclined plane so Darwin bred pigeons.

He reports in *The origin*: believing that it is always best to study some special group, I have, after deliberation, taken up domestic pigeons; I have kept every breed which I could purchase or obtain, and have been favored with skins from several quarters of the world (1); the diversity of breeds is something astonishing (*id.*). Darwin describes: the carrier, the tumbler, the runt, the barb, the pouter, the Jacobin, the trumpeter, the laugher, the fantail (*id.*); and several other distinct breeds might have been specified (*id.*). Yet: great as the differences between the breeds are, I am fully convinced that all have descended from the rock-pigeon, *columba livia* (*id.*); the rock-pigeon is of a slaty-blue coloring, has a white rump, the tail has a terminal dark bar, with the bases of the outer feathers externally edged with white, the wings have two black bars (*id.*); these several marks do not occur together in any other species of the whole family (*id.*). Nevertheless: when two birds belonging to two distinct breeds are crossed, neither of which is blue or has any of the above marks, the mongrel offspring are very apt suddenly to acquire these characters; for instance, I crossed some uniformly white fantails with some uniformly black barbs, and they produced mottled brown and black birds; these I again crossed, and one grandchild of the pure white fantail and pure black barb was of as beautiful a blue color, with the white rump, double black wing-bar, and barred and white-edged tail-feathers, as any wild rock-pigeon (*id.*)!

Nine years later, the main concerns of *The variation of animals and plants under domestication* are two. One is the titular, once again: the experiment on a gigantic scale. Once again Darwin reports: I have been led to study domestic pigeons with particular care, because the evidence that all domestic races are descended from one known source is far clearer than with any other domesticated animal (5). The experiment is more successful still: I have succeeded in uniting in one bird five of the most distinct races, and with patience I might undoubtedly have thus united all (6). Darwin reports case after case after case, and: the last case is the most curious; two black barbs, a red spot, and a white fantail produced a bird exhibiting the general blue color, together with every characteristic mark, of the wild rock-pigeon (*id.*).

The experiment has succeeded in breaking up the pigeon into characteristic marks, and in tracing the characteristic marks from one generation to the next and next. And as the pigeon so all organisms, and all life.

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The other main concern of *The variation of animals and plants under domestication*, in 1868, is heredity. Darwin writes: it is obvious that variation which is not inherited throws no light on the derivation of species (12).

The ignorance of the laws of heredity was the only real difficulty before the theory of evolution, and Darwin addresses the difficulty directly by formulating: the provisional hypothesis of pangenesis (27).

The hypothesis is this: it is universally admitted that the cells or units of the body increase by self-division and ultimately become converted into the various tissues and substances of the body (*id.*); I assume that the units throw off minute granules which are dispersed throughout the whole system; these multiply by self-division and are ultimately developed into units like those from which they were originally developed; they may be called gemmules (*id.*); gemmules are collected from all parts of the system to constitute the sexual elements, and their development in the next generation forms a new being; but they are also capable of transmission in a dormant state to future generations, and may then be developed; their development depends on their union with other partially developed cells; I assume also that gemmules in their dormant state have a mutual affinity for each other, leading to their aggregation into buds or into the sexual elements (*id.*); to illustrate, if one of the protozoa be formed of a small mass of homogeneous matter, a gemmule thrown off from any part would reproduce the whole; but if the upper and lower surfaces differ from each other and from the central part, all three would have to throw off gemmules (*id.*); all

forms of reproduction graduate into one another, and it is impossible to distinguish organisms produced from buds, self-division, or fertilized germs; according to our hypothesis all forms of reproduction depend on the aggregation of gemmules derived from the whole body (*id.*); if organisms subjected to changed conditions of life have been modified, the gemmules cast off from modified parts will tend to reproduce parts modified in the same manner (*id.*); an organism does not generate its kind as a whole, but each separate unit generates its kind; each cell of a plant has the potential of reproducing the whole plant, but it does only in virtue of containing gemmules derived from every part (*id.*); the child, strictly speaking, does not develop into the man, but includes germs which develop and form the man (*id.*); inheritance must be looked at as merely a form of growth, like the self-division of a lowly-organized unicellular organism (*id.*); each animal and plant may be compared with a bed of soil full of seeds (*id.*); an organic being is a microcosm, a little universe, formed of a host of self-propagating organisms (*id.*).

The difficulty is thus resolved. The hypothesis is excellent. It not only explains heredity. It incorporates a number of conclusive results. The development of gemmules, constituting the sexual elements, forms in the next generation a new being. Gemmules are capable of transmission in a dormant state. Modified, gemmules reproduce in the same manner. The child does not develop into the man, but includes germs which develop and form the man. Inheritance is a form of growth. An organic being is formed of a host of self-propagating organisms.

In 1868 the hypothesis was also superfluous. Mendel had discovered the laws of heredity in 1865.

Mendel had discovered the laws of heredity in 1865, but for the following three and a half decades Mendel remained the sole philosopher aware of the fact that Mendel had discovered the laws of heredity. In 1865 Mendel is clear also that he has found: the solution of a question whose significance for the developmental history of organic forms cannot be underestimated (*Experiments*). But he does not draw the conclusion that he has found the solution of a question whose significance for the theory of evolution cannot be underestimated, and it was six decades before the fact that Mendel's laws are a part of Darwin's theory was established.

It is an excellent example of the fact that men do not desire to know.

Mendel discovered the laws of heredity in 1865, in the *Experiments on plant hybrids*.

Darwin remained unaware of Mendel. His very extensive reading did not extend to the *Proceedings of the Brünn society for the investigation of nature*, where Mendel's *Experiments* was published.

Mendel communicated his results to other philosophers, but not to the one philosopher to whom he should have communicated them.

Of the other philosophers to whom he communicated his results the most important, to the degree to which he is important, is Nägeli. In 1866, when Mendel became a correspondent of Nägeli, Darwin was a correspondent of Nägeli, too. Nägeli found very little use for Darwin, no use at all for Mendel, and no use for informing Darwin of Mendel.

In general, the tenacity of the resistance with which Darwin's theory was met has been second only to the tenacity of the resistance with which Copernicus's theory was met.

Mendel quit being the sole philosopher aware of the fact that Mendel had discovered the laws of heredity in 1900. The fact that Mendel's laws are a part of Darwin's theory was established in 1926.

Mendel does not refer to Darwin in the *Experiments*. He writes six years after *The origin of species*, however, and whether he refers to Darwin or not, his conclusions cannot not refer to Darwin's theory.

Mendel begins by noting that he proposes: the solution of a question whose significance for the developmental history of organic forms cannot be underestimated.

And in conclusion he writes: changes of type must occur when the conditions of life of a species are changed and it possesses the ability to adapt to the new conditions (*id.*); if change in the conditions were the sole cause of variability, it should be expected that plants cultivated for centuries under almost identical conditions would

acquire greater independence; but this is known not to be the case (*id.*); it thus remains more than probable that a factor which has received little attention causes the variability of cultivated plants (*id.*); and the fact of hybrids which remain constant in their progeny and propagate just like the pure species is of particular importance for the developmental history of plants, because these hybrids attain the significance of a new species (*id.*).

Moreover: the unity in the developmental plan of organic life is indisputable (*id.*), and: the view that a species has firm limits beyond which it is incapable of changing cannot be considered unconditionally valid (*id.*).

These conclusions cannot not refer to, and corroborate, Darwin's theory.

In his *Recollections of the development of my mind and character*, Darwin writes: I have deeply regretted that I did not proceed far enough to understand something of the great leading principles of mathematics; for men thus endowed seem to have an extra sense (*id.*).

Mendel has this extra sense.

Maxwell writes: the first stage in the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend; the next stage is the discovery of the mathematical form of the relations of these quantities. The investigation of heredity entered the first stage with Darwin, the next stage with Mendel. Darwin's experiments broke up the organism into characteristic marks. Mendel's experiments reduced the characteristic marks to their mathematical form.

More generally still, Mendel's experiments are not just a masterful instance of the acquisition of knowledge, they are one of the most masterful instances of the acquisition of knowledge. Mendel discovered laws of great generality, and discovered them by a single series of experiments. Of all characteristic marks of all organisms he found 7 characteristic marks of one organism, which he found the way to reduce to their mathematical form, which he found the way to generalize into the laws of all organisms.

He begins: to establish the numerical relations of the different forms of the progeny of hybrids in every generation appears to be the only right path toward the solution of a question whose significance for the developmental history of organic forms cannot be underestimated (*Experiments*).

The magnitude of the experiment to that end Mendel indicates twice: the experiment took eight years (*id.*), and at one of its stages: more than 10 000 plants were carefully examined (*id.*).

The first major step toward the success of the experiment is the discovery that it has to be limited to: constantly differing traits (*id.*). Mendel finds that pea, *pisum sativum*, exhibits 7 such traits: which can be distinguished with ease and certainty (*id.*). In particular: the seeds are either spherical or angular, the albumen is either yellow or green, the seed coat is either yellow or gray, the ripe pods are either arched or tightly drawn, the unripe pods are either green or yellow, the arrangement of flowers is either axillary or terminal, and the stem is either long or short (*id.*).

Mendel combines each pair of traits through fertilization, and finds that: each of the 7 hybrid traits resembles one of the two parent traits completely (*id.*). He adds: this fact is of the greatest importance (*id.*), and terms: the trait which appears unchanged in the hybrid combination dominant, the trait which becomes latent in the combination recessive (*id.*).

Mendel crosses each hybrid with itself, and finds: in the first generation of hybrids the dominant and recessive traits reappear in their characteristic form and in the decisively pronounced average ratio of 3:1 (*id.*); no transitional forms are observed in any of the experiments (*id.*); in the second generation, the forms which have the recessive trait in the first generation do not vary further with respect to that trait, they remain constant in their offspring; of the forms which have the dominant trait in the first generation, one part yields progeny which remains constant with the dominant trait, two parts yield progeny which carries the dominant and recessive traits in the ratio of 3:1, thus showing exactly the same behavior as the hybrid forms (*id.*); the ratio of 3:1 thus resolves itself into the ratios of 1:2:1, distinguishing the dominant trait as a parent trait and as a hybrid trait (*id.*); if A is the dominant of two constant traits, a the recessive, and Aa the hybrid form, the expression

$$A + 2Aa + a$$

gives the developmental series for the progeny of the hybrids (*id.*); assuming each plant to form 4 seeds in each generation, the series for the subsequent generations is,

1st	1	2	1	1 : 2 : 1
2nd	6	4	6	3 : 2 : 3
3rd	28	8	28	7 : 2 : 7
4th	120	16	120	15 : 2 : 15
5th	496	32	496	31 : 2 : 31
*n*th				$2^n - 1 : 2 : 2^n - 1$ (*id.*).

Mendel shows that: the discovered law is valid when more than one traits are combined (*id.*); for two traits, for instance, the expression is

$$AB + Ab + aB + ab + 2ABb + 2aBb + 2AaB + 2Aab + 4AaBb,$$

given completely by the expressions

$$A + 2Aa + a$$
$$B + 2Bb + b \text{ (id.)}.$$

Mendel concludes: the behavior of each pair of differing traits in the hybrid combination is independent of all other differences in the two parent plants (*id.*).

He generalizes the conclusion: the complete agreement shown by all traits subjected to experiment justifies the assumption that the behavior of traits which manifest themselves less distinctly is the same (*id.*).

On this ground he infers the nature of the reproductive cells: as many kinds of germ cells are formed in the ovaries and as many kinds of pollen cells in the anthers as the possible constant combination forms, and these germ and pollen cells correspond in their inner constitution to the individual forms (*id.*).

And these are Mendel's laws, the law of segregation, each hereditary trait is controlled by two factors which separate and pass into separate germ cells, and the law of independent assortment, the hereditary units are pairs of factors which segregate independently when germ cells are formed.

Finally, Mendel draws the most general conclusions: it has been demonstrated that the hybrids form germ and pollen cells which are different in kind, and that this is the ground of the variability of their offspring (*id.*); this development follows a constant law which is based on the material constitution and arrangement of the elements which have achieved viable union in the cell (*id.*). Also: hybrids which remain constant in their progeny attain the significance of a new species. And: the view that a species has firm limits beyond which it is incapable of changing cannot be considered unconditionally valid.

In 1902, in *On multipolar mitosis as a means of analysis of the cell nucleus*, Boveri established that the chromosome is the carrier of heredity in the cell. Obeying Mendel's laws. In 1953, in the *Molecular structure of nucleic acids*, Crick and Watson established that replicating DNA is the carrier of heredity in the chromosome. Obeying Mendel's laws.

In the meantime, there was more resistance to Darwin, and to Mendel now, and to both, to overcome.

In 1908, in the one-page *Mendelian proportions in a mixed population*, Hardy apologizes: I should have expected the very simple point which I wish to make to have been familiar to biologists (*id.*). It was not, apparently, as it required: a little mathematics of the multiplication table type (*id.*). Hardy thus finds himself: compelled to intrude in a discussion of matters of which I have no expert knowledge (*id.*).

Having mathematics, however, he discovered with his intrusion the law of those matters, the law of the equilibrium of a population, the fundamental law of population biology. Hardy writes: supposing that *Aa* is a pair of Mendelian characters, *A* being dominant, and that in any given generation the numbers of pure dominants *AA*, heterozygotes *Aa*, and pure recessives *aa* are as $p:2q:r$, if mating is random, the distribution will be the same from one generation to the next if $q^2 = pr$; and if so, whatever the values of *p*, *q*, and *r*, the distribution will in any case continue unchanged after the second generation (*id.*).

If in any given generation the distribution is $p:2q:r$, in the next generation the distribution will be $(p+q)^2 : 2(p+q)(q+r) : (q+r)^2$, and whatever the values of *p*, *q*, and *r*,

$$\frac{p}{(p+q)^2} = \frac{2q}{2(p+q)(q+r)} = \frac{r}{(q+r)^2},$$

or $q^2 = pr$.

Chetverikov, in 1926, in *On certain aspects of the evolutionary process from the point of view of contemporary genetics*, showed that Mendel's laws are a part of Darwin's theory.

He writes: the present article aims to clarify certain questions of the theory of evolution in connection with our contemporary genetic concepts (*id.*), namely: how to connect evolution with genetics? is it possible to approach the questions of variability, the struggle for life, selection, in a word, Darwinism, not from the amorphous and indefinite views on heredity which existed at Darwin's time, but from the firm laws of genetics (*id.*)? I propose to deal with only three aspects which are especially important for the correct appraisal of the role of genetic ideas in the general structure of the theory of evolution, the origin of mutations, the role of free crossing in Mendelian heredity, and the significance of selection under these conditions (*id.*).

With respect to mutations, Chetverikov concludes: there is no ground to see in the process of the origin of mutations the result of the artificial influence of man; on the contrary, all available facts indicate that this process goes on under natural conditions just as regularly as in the laboratory (*id.*).

With respect to free crossing, he writes: Hardy established the extremely important law characterizing the conditions of equilibrium under the Mendelian laws of heredity and free crossing (*id.*); what stabilizes the frequencies of the components of a given population is the very mechanism of free crossing; any change in the ratio of these frequencies is thus possible only by the action of an external force and only as long as this force disturbing the equilibrium continues to act (*id.*); it follows that a species, like a sponge, soaks up heterozygous mutations, while remaining from the first to the last phenotypically homogenous (*id.*); the genotypic variability of a species increases proportionally to its age (*id.*); isolation, therefore, geographical isolation, isolation in time, or ecological isolation, automatically leads to differentiation within a species (*id.*). Chetverikov concludes: free crossing is by its very essence the conservative factor preserving the genotypic structure of the species in the condition in which it is at any given moment (*id.*).

With respect to selection, he writes: natural selection is the direct antagonist of free crossing; if free crossing stabilizes a population, selection continuously displaces its equilibrium; it is the evolutionary principle, leading ceaselessly to modification of the species (*id.*). Chetverikov quantifies the intensity of selection, and concludes: every improvement in the organism, even the slightest, has a definite chance of spreading throughout the whole mass of individuals comprising the freely crossing population (*id.*); and the transformation of a freely crossing population, the replacement of the less adapted form by the more adapted, in a word, the process of adaptive evolution of the species, always proceeds to the end (*id.*).

Thus, Chetverikov adds: Darwinism, in so far as natural selection and the struggle for life are its characteristic features, finds in Mendelism a completely unexpected and powerful ally (*id.*).

One of Darwin's experiments in *The variation under domestication* is this: I crossed the peloric snapdragon with pollen of the common form, and the latter reciprocally with peloric pollen; I obtained two great beds of seedlings, not one of which was peloric; the crossed plants, which perfectly resembled the common snapdragon, were allowed to sow themselves, and out of 127 seedlings, 88 proved to be common, 2 were indeterminate, 37 were perfectly peloric (14).

There it is, Mendel's ratio of 3:1.

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The main concerns of *The descent of man, and selection in relation to sex*, in 1871, are also two, the titular.

Darwin notes the many uniquenesses of man: it has been asserted that man alone is capable of progressive improvement; that he alone makes use of tools or fire, domesticates other animals, or possesses property; that no animal has the power of abstraction, or of forming general concepts, is self-conscious and comprehends himself; that no animal employs language; that man alone has a sense of beauty, is liable to caprice, has the feeling of gratitude, or of mystery; believes in god, or is endowed with conscience (3). He then shows that the descent of that most unique of organisms is not different from the descent of any other organism.

And he demonstrates the most powerful kind of natural selection, sexual selection. For in-

stance: just as man can give beauty, according to his taste, to his male poultry, so it appears that female birds have by long selection of the more attractive males added to their beauty or other attractive qualities (8); no doubt this implies powers of discrimination and taste on the part of the female bird (*id.*).

An admirable organism, the female bird.

In the four decades after his first publication, in 1837, Darwin published, next to the five very important, the following important, if not very important, treatises, *Observations on the parallel roads of Glen Roy, with an attempt to prove that they are of marine origin*, in 1839, *The structure and distribution of coral reefs*, in 1842, *A monograph of the sub-class cirripedia*, in 1851-4, *On the two forms, or dimorphic condition, in the species of primula, and on their remarkable sexual relations*, in 1861, *The various contrivances by which orchids are fertilized by insects*, in 1862, *On the sexual relations of the three forms of lythrum salicaria*, in 1864, *The power of movement in plants*, in 1865, *The expression of emotions in man and animals*, in 1872, *Insectivorous plants*, in 1875, *The effects of cross- and self-fertilization in the vegetable kingdom*, in 1876.

His explanation of the parallel roads of Glen Roy is in error. Darwin comments: this paper was a great failure, and I am ashamed of it; I attributed the parallel roads to the action of the sea; but I had to give up this view when Agassiz propounded his glacier-lake theory (*Recollections*).

His explanation of coral reefs is conclusive. He writes: when the two types of structure, barrier reefs and atolls, on the one hand, and fringing reefs, on the other, are laid down on a map, they offer a grand and harmonious picture of the movements which the crust of the earth has undergone within a late period; we there see vast areas rising, with volcanic matter every now and then bursting forth, and other wide spaces sinking, without any volcanic outbursts; and we may feel certain that the movement has been so slow as to have allowed corals to grow up to the surface, and to have buried under the broad face of the ocean every one of those mountains above which atolls now stand like monuments, marking the place of their burial (*Reefs*, 6).

His investigations of barnacles, the dimorphic condition of primroses, the fertilization of orchids by insects, the sexual relations of the three forms of loosestrifes, the movement of plants, the expression of emotions in animals, insectivorous plants, the effects of fertilization, all attest the fact that the theory of evolution stands on experiment.

In 1876, Darwin wrote also the *Recollections of the development of my mind and character*. He notes: I have attempted to write as if I were a dead man in another world looking back at my own life; nor have I found this difficult, for life is nearly over with me (*id.*). On the early stages of his labors he reports: my first notebook was opened in July of 1837; I worked on true Baconian principles and without any theory collected facts on a wholesale scale (*id.*). On the late stages he reports: my mind seems to have become a kind of machine for grinding general laws out of large collections of facts (*id.*).

The last of Darwin's very important treatises is *The formation of vegetable mold, through the action of worms, with observations on their habits*, in 1881.

Darwin's first publication, in 1837, is *On the formation of mold*. It already attests the discovery of the experiment on a gigantic scale. Darwin writes: the agriculturalist in ploughing the ground follows a method strictly natural, he only imitates in a rude manner the work which nature is daily performing by the agency of worms (*id.*).

Forty-four years later, Darwin's main concern are the habits of worms. He continues to count and calculate: Hensen estimates that 53 767 worms exist in an acre of garden land; if we assume that half that number live in old pasture land, taking 15 tons as the weight of the castings annually thrown up on an acre of land, each worm must annually eject 20 ounces (3); on each acre of land which is sufficiently damp and not too sandy, gravelly, or rocky for worms to inhabit, a weight of more than 10 tons of earth annually passes through their bodies and is brought to the surface; in England together with Scotland the land which is cultivated has been estimated at above 32 million acres; in a million years, the product is 320 million million tons of earth (5).

A most admirable organism, the worm.

Darwin wrote also *On the connection of certain volcanic phenomena in South America; and on the formation of mountain chains and volcanoes as an effect of the same power by which continents are elevated*, in 1838, *On the distribution of the erratic boulders and on the contemporaneous unstratified deposits of South America*, in 1841, *Notes on the effects produced by the ancient glaciers of Caernarvonshire, and on the boulders transported by floating ice*, in 1842, *Geological observations on the volcanic islands visited during the voyage of H.M.S. Beagle, together with some brief notices of the geology of Australia and the cape of Good Hope*, in 1844, *Geological observations on South America*, in 1846, *On the geology of the Falkland Island*, in 1846, *Geology*, in John Herschel's *Manual of scientific inquiry, prepared for the use of her majesty's navy, and adapted for travelers in general*, in 1849, *Extract from an unpublished work on species, consisting of a portion of a chapter entitled, On the variation of organic beings in a state of nature; on the natural means of selection; on the comparison of domestic races and true species*, in 1858, *The different forms of flowers on plants of the same species*, in 1877, *Biographical sketch of an infant*, in 1877, *The action of carbonate of ammonia on the roots of certain plants*, in 1882, *The action of carbonate of ammonia on chlorophyl-bodies*, in 1882.

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Darwin informs decisively the problem before the investigation of knowledge. In particular, he informs decisively the problems of the beginning of the acquisition of knowledge and of collecting the acquisition in its entirety.

Experience is the outcome of the development of all faculties and their exercise in all organisms.

Experience is also the necessary condition of the acquisition of knowledge. But it is not its sufficient condition. If so, the end of the acquisition cannot be the end of an organism. And if the end of an organism cannot be the end of the acquisition either, the end of an organism cannot be the end of the acquisition from the beginning of the acquisition, and the end of an organism cannot be the end of the acquisition throughout the acquisition. The acquisition of knowledge begins when the end of an organism becomes an end which cannot be the end of an organism, and the acquisition of knowledge continues when its end continues to be an end which cannot be the end of an organism.

The acquisition of knowledge, including its beginning, has to be taken from experience. If experience is the foundation of the acquisition, all earlier acquisition has to be part of the foundation of all later acquisition. If so, to take the acquisition, including its beginning, from experience is to take it from the acquisition itself. To do so is to judge by its success. To do so is to judge formally. If so, the problems of the beginning of the acquisition and of collecting it in its entirety are problems of the relation of propositions and inferences. Inferences, being inferences, have to lead from any one inference to the next, and the next, and all inferences. If so, and the problem of the beginning is of how experience such as it predates the acquisition gave rise to the acquisition, its solution depends on separating sharply experience such as it predates the acquisition from the propositions and inferences which are the acquisition. The solution of the problem of collecting the acquisition in its entirety depends on separating sharply all propositions and inferences which are the acquisition from all propositions and inferences which are not the acquisition.

And the dichotomy the end of an organism, on one side, an end which cannot be the end of an organism, on the other, makes it possible to do both, to separate sharply experience such as it predates the acquisition from the propositions and inferences which are the acquisition, and to separate sharply all propositions and inferences which are the acquisition from all propositions and inferences which are not the acquisition.

Rousseau's remains the best explanation of the state. If the state is, with Hobbes: a power contrary to our natural passions, keeping men in awe, reducing the wills of men unto one will, and to explain the state is to bridge the abyss between man and the state, between an organism and a power contrary to the natural passions of the organism, Rousseau's remains the most successful attempt to bridge the abyss. In particular,

Rousseau describes the development: as the human race spread, difficulties multiplied, differences in soils, climates, seasons, barren years, hard winters, hot summers; and in response to these difficulties metallurgy and agriculture were the two arts whose invention produced the great revolution of property, labor, and the first rules of justice.

It remains the best explanation of the state because the two arts of metallurgy and agriculture, and the great revolution of property, labor, and the first rules of justice have narrowed the abyss between an organism and the state.

They have not closed the abyss, however. Metallurgy, agriculture, property, labor, and the rules of justice are all firmly on the side of the organism.

They are all thus firmly on the side of the not acquisition of knowledge as well.

Agriculture, property, and justice can illustrate.

Darwin writes: the agriculturalist in ploughing the ground follows a method strictly natural, he only imitates in a rude manner the work which nature is daily performing by the agency of worms.

And if the method of the agriculturalist is the method of the worm because the end of the agriculturalist is the end of the worm, success in the struggle for life, agriculture is explained.

Ipso facto agriculture is not a part of the acquisition of knowledge.

Ipso facto agriculture predates the beginning of the acquisition of knowledge.

And *ipso facto* when the acquisition continues, and encompasses agriculture, agriculture is still not a part of the acquisition of knowledge.

The oldest treatise on agriculture is Cato's *On agriculture*.

Cato is very important. Rome became the dominant power on earth in his lifetime, and Cato is among the few whose labors achieved that end.

Cato is very important, but *On agriculture* is not. Nothing in it suggests an end other than that of the agriculturalist, or what is the same, of the worm. It is a collection of advice, all of it excellent advice. For instance: planting needs no thinking, it needs doing (3); building needs thinking; when you are 36, if you have planted, build; and see that neither the steading lags behind the farm nor the farm behind the steading; for it is good to have storage to hold products until the prices are good; this increases property, and reputation (*id.*). More generally: what is good cultivation? good ploughing; what second? more ploughing, what third? manuring (61). More particularly: as a medicine, cabbage is better than any other vegetable; it can be eaten either cooked or raw; if you eat it raw, dip it in vinegar; it promotes digestion excellently (156).

One century-odd after Cato, the second oldest treatise on agriculture is Varro's *On rustic matters*.

Varro is also very important. He wrote the *Antiquities*, and *The disciplines*.

On rustic matters is a collection of advice as well. As experience has continued to accumulate, the advice is more excellent even than Cato's. The advice is all to the same end as well, that of the agriculturalist, and there is little next to the advice to suggest an end other than that end.

Little, but there is.

Cato mentions no earlier writer on agriculture. Nor need he mention any, if agriculture needs doing, not reading.

Varro begins with a list of 51 writers: from whom you can learn (I, 1). The list is headed by 2 princes: Hiero II and Attalus III (*id.*). The next 5 are: philosophers, Democritus the physicist, Xenophon the Socratic, Archytas the Pythagorean, and Aristotle and Theophrastus the Peripatetics (*id.*). The next 39: have written in prose (*id.*). The next 2: have written in verse (*id.*). One of these is Hesiod. The final 3 are: the Carthaginian Mago who surpassed all previous writers by collecting all matters with which they had dealt in 28 books; these Cassius Dionysius abridged to 20 books; and these Diophanes abridged to 6 books (*id.*).

You can learn from these writers. But agriculture needs doing, not reading. So Varro adds: I shall try to be briefer still and treat the matter in three books, one on agriculture, one on animal husbandry, and one on the steading, omitting everything that has no bearing on agriculture (*id.*). And in his three books Varro never mentions again 45 of the 51. The record, too, in its wisdom, corroborates the fact that agriculture

needs doing, not reading. Of 41 of the 51 the record has preserved nothing more substantial than that, being an entry each in Varro's enumeration.

As all of Cato's book attests and nearly all of Varro's, writers on agriculture need not suggest an end other than that of the agriculturalist. The enumeration of 51 of these writers, however, as the fact that Varro finds no use for 45 of them attests, cannot be the end of the agriculturalist.

Varro mentions again 6 of the 51, twice Aristotle, five times Theophrastus, six times Mago and his abridgers, and once the other poet, not Hesiod.

Varro writes: the many books of Theophrastus entitled the *History of plants* and *Causes of plants* are of little use to those who want to do agriculture, they are of more use to those who want to go to the schools of the philosophers (5). In the *History of plants* and *Causes of plants* Theophrastus investigates 550 kinds of plants. Varro adds: which is not to say that some things in these books are not useful or interesting (*id.*).

The investigation of 550 kinds of plants cannot be the end of the agriculturalist. The *History of plants* and *Causes of plants* attest the acquisition of knowledge. They attest also the extension of the acquisition to agriculture. The 550 kinds include agricultural plants. They are yet of more use to those who want to go to the schools of the philosophers. Attesting the fact that when the acquisition extends to agriculture its end is still not the end of the agriculturalist. Which is not to say that some things in these books are not useful. Attesting the fact that the results of the acquisition can have an application to agriculture, as well as the fact that the application is not a part of the acquisition.

Varro writes also: as Dicaearchus writes, from the remotest antiquity of human life man has descended to our own time step by step; the first stage was the state of nature, when man lived on the products of the virgin earth; from this stage he descended into the second, the pastoral stage, when he gathered acorns, berries, and other fruits, and caught and tamed wild animals; the third stage began when man descended from the pastoral life into that of the agriculturalist (II, 1).

The investigation of the development whose outcome the agriculturalist is cannot be the end of the agriculturalist either. And Dicaearchus attests the fact that agriculture as such, as agriculture, has become an object of the acquisition of knowledge.

The end of the agriculturalist, or what is the same, of an organism, on one side, and an end which cannot be the end of the agriculturalist, or what is the same, of an organism, on the other, separate sharply, with the labors of Dicaearchus, the investigation of the development whose outcome agriculture is from agriculture. With the labors of Varro, the enumeration of the writers on agriculture from the writers on agriculture. And with the labors of Theophrastus, the extension of the acquisition of knowledge to agriculture from the application of the acquisition of knowledge to agriculture.

Thus also Liebig, in 1840, published the *Organic chemistry in its application to agriculture and physiology*, generalized in its later editions into the *Chemistry in its application to agriculture and physiology*. Liebig's treatise revolutionized agriculture. Nevertheless, while the development of chemistry, including its extension to agriculture, continues to attest the acquisition of knowledge, the application of chemistry to agriculture continues to attest that the end of the agriculturalist has not changed.

Darwin supplies the ground of, and explains, Aristotle's dichotomy, useful and pleasurable, on one side, neither useful nor pleasurable, on the other. He supplies also the ground of, and explains, Maxwell's dichotomy, good or bad, beneficial or noxious, on one side, of what kind, and how much, on the other. Hence either dichotomy can separate sharply the acquisition of knowledge from everything that precedes it.

Darwin supplies also the ground of, and explains, the fundamental character, once the acquisition has begun, of the dichotomy, theory, on one side, the side of the acquisition, practice, on the other, the side of the organism. Hence this dichotomy can separate sharply the acquisition from everything that accompanies and attends the acquisition and is liable and eager to be confused with the acquisition.

Having listed the many uniquenesses of man – progressive improvement, use of tools, use of

fire, domestication of animals, property, power of abstraction, general concepts, self-consciousness, comprehension of himself, language, sense of beauty, liability to caprice, feeling of gratitude, feeling of mystery, belief in god, endowment with conscience – Darwin notes: I will hazard a few remarks on the more important of these points (*Descent*, 3).

So, he hazards a remark on a monkey in the zoological gardens. He writes: in the zoological gardens a monkey, which had weak teeth, used to break open nuts with a stone, and I was assured by the keepers that, after using the stone, he hid it in the straw and would not let the other monkeys touch it; here, then, we have the idea of property (*id.*).

Rousseau writes: the first who, having enclosed a piece of land, thought to say, this is mine, and found people simple enough to believe him, was the true founder of civil society (*Inequality*, 2). There he is, the monkey who would not let the other monkeys touch the stone, the true founder of civil society.

The investigation of property begins with Aristotle. In the *Politics*, on the ground that: things are considered with the greatest clarity when they are considered from their origin, Aristotle begins with the origin of the state. He traces it back to: the union of those that cannot exist without one another, male and female, for in common with animals and plants men by nature strive to leave behind their progeny, and master and slave, for that which foresees by reason is master by nature and that which acts on the foresight by the body is slave by nature. And next, on the ground that: we should begin the examination of everything with the smallest number of possible elements (I, 3); and the state is composed of households (*id.*); and the first and smallest number of possible elements of the household are master and slave, husband and wife, and father and children (*id.*), Aristotle turns to the elements master and slave, and in order to examine these elements, turns to property. To the question, in particular: is property a part of the household (4)? He concludes: property is of two kinds, one is a part of the household, and thus natural and honorable, the other is not a part of the household, and thus not natural and not honorable (10). The first kind: is gained by husbandry, farming, piracy, fishing, or hunting, and is natural because it is gained by men through their own industry (8). The second kind: is gained by trade, and is not natural because it is gained by men from one another (10). Clearly, when Adam Smith affirms: there is a certain propensity in human nature to truck, barter, and exchange (*Wealth*, I, 2), either Adam Smith is in error or there is a certain propensity in human nature to gain from one another. Clearly, also, Aristotle takes it for granted that the pirate will not trade what he gains. Even if he may yet attest a certain propensity in human nature to gain from one another.

Aristotle continues: this is enough about the theory of property; we now proceed to the practice (11). Advice follows, all of it excellent. For instance, Aristotle tells the story: reproached for poverty, Thales predicted a good season for olives through his knowledge of the stars, hired all olive presses in Chios and Miletus in advance, and, cornering the oil market, made a fortune (*id.*), and adds: the moral that goes with this story is that the philosophers can become rich if they want to, but the story shows also that to establish a monopoly is one of the ways of gaining property (*id.)*.

As the true founder of civil society attests, property predates the acquisition of knowledge. As the theory of property attests, the acquisition has extended to property. And as the first philosopher already attests, the application of the acquisition, of his knowledge of the stars, no less, to property can have no other end but property.

Thus also, when Locke writes: the great and chief end of men's uniting into commonwealths, and putting themselves under government, is the preservation of their property (*Government*, II, 9), and: god gave the world to men in common; but he gave it to the industrious and rational, not to the quarrelsome and contentious (5), that these are the results of an attempt at the acquisition of knowledge the fact attests that Locke investigates the state on the same grounds as Aristotle: things are considered with the greatest clarity when they are considered from their origin, and: we should begin the examination of everything with the smallest number of possible elements. That the

application of Locke's results cannot be a part of the acquisition of knowledge the fact attests that it can have no other end but property.

The extension of the acquisition of knowledge to justice is attested by the *Republic*. The *Republic* opens with the quarrel of Socrates with Thrasymachus. Thrasymachus says: justice is what is useful to the stronger; for, whether in a tyranny, or in a democracy, or in an aristocracy, the rulers make the laws, they make them useful to themselves, and proclaim what they have made just; so justice is the same in all cities, it is what is useful to the established rule; and since the established rule is the stronger, justice is the same everywhere, it is what is useful to the stronger (I, 338c-9a). Socrates objects: no one who rules, to the degree to which he is a ruler, seeks his own advantage, he seeks the advantage of those he rules (342e). Thrasymachus wonders: Socrates, do you still have your wet nurse? why? Socrates asks; to have her wipe your nose, Thrasymachus replies, you seem to think that shepherds and cowherds seek the advantage of the sheep and the cattle (343a-b).

Shepherds and cowherds attest that justice predates the acquisition of knowledge. Socrates, Thrasymachus, and Plato attest that the acquisition has extended to justice. And the fact that after Socrates as before, and for two and a half millennia now, justice has staunchly continued to refuse to deduce from Socrates's proposition: the ruler seeks not his own advantage but the advantage of those he rules, and has eagerly continued to deduce from Thrasymachus's proposition: the rulers make the laws useful to themselves and proclaim what they have made justice, attests the fact that the application of the acquisition is not a part of the acquisition.

More generally, Darwin writes: there is a much wider interval between one of the lowest fishes, a lamprey or a lancelet, and one of the higher apes than between an ape and man (*Descent*, 3); there is no fundamental difference between man and the higher mammals in their mental faculties (*id.*); and as man possesses the same sense as the lower animals, his fundamental intuitions must be the same (*id.*).

So, Darwin hazards a remark on his dog as well. He writes: my dog, a full-grown and very sensible animal, was lying on the lawn during a hot and still day; at a little distance a slight breeze occasionally moved an open parasol, which would have been wholly disregarded by the dog had anyone stood near it; as it was, every time the parasol moved, the dog growled fiercely and barked; he must, I think, have reasoned to himself in a rapid and unconscious manner that movement without any apparent cause indicated the presence of some strange living agent, and no stranger had a right to be on his territory; the belief in spiritual agencies would easily pass into the belief in the existence of one or more gods (*id.*).

If he will believe in spiritual agencies, and the existence of one or more gods, Darwin's dog, a full-grown and very sensible animal, will have no difficulty believing that god gave the world to the industrious and rational, not to the quarrelsome and contentious. Or that spirit demands from philosophy the restoration of the substantiality and purity of its being. Or that there are spiritual sciences. For the same reason, too, no stranger has a right to be on his territory.

Darwin writes: it seems to me highly probable that any animal with well-marked social instincts, parental and filial affections included, would inevitably acquire a moral sense or consciousness as soon as its intellectual powers had become as well, or nearly as well, developed as in man (4); this view of the origin and nature of the moral sense accords well with what we see of the early undeveloped condition of this faculty in mankind; the virtues which must be practiced are those which are still recognized as the most important; but they are practiced almost exclusively in relation to the men of the same tribe (*id.*).

First the south of the promised land was conquered: Joshua left no one alive, he destroyed everything that drew breath, as the lord had commanded (*Joshua*, 10). Then the north was conquered: the Israelites plundered the cities, and kept all the spoil and the cattle, but put everything living to the sword, left nothing that drew breath alive, as the lord had commanded Moses, Moses had charged Joshua, and so Joshua did (11).

More recently, in 1836, Darwin landed in Tasmania. He stayed just ten days, and has only

three things to report in the *Journal of researches*, in 1839. One is: general impressions from the geological structure (19). The second is: the general aspect of the flora (*id.*).

The third concerns the fauna, the relation of the recently arrived invader fauna to the native fauna. In particular: thirty years is a short time to have banished the last aboriginal from his native land, and that an island nearly as large as Ireland; natives were shot and taken prisoner in skirmishing, but nothing seems to have impressed them with the idea of our overwhelming power until, in 1830, the whole island was put under martial law and by proclamation the whole population was commanded to assist in one great attempt to secure the entire race; similarly to the great hunting matches in India, a line was formed reaching across the island to drive the natives into a *cul-de-sac* (*id.*).

In the second edition, in 1845, Darwin adds: in 1835 the remaining natives were removed to an island where food and clothes were provided for them, they amounted to 210; in 1842 they mustered 54 (*id.*).

Not an admirable organism, man.

Finally, in *The formation of vegetable mold, through the action of worms, with observations on their habits*, Darwin writes: I wished to learn how far worms reacted consciously and how much mental power they displayed, as few observations of this kind have been made on animals so low in the scale of organization and so poorly provided with sense organs (pr.).

With eyes, for instance, they are not provided at all. Nevertheless: when the light from a candle was concentrated by a large lens on them, the worms generally withdrew instantly; but when employed in dragging leaves into their burrows or in eating them, and even during the short intervals whilst they rested from their work, they either did not perceive the light or were regardless of it; we thus have to attribute attention to them, and so some mental power (1).

Given various objects to draw into their burrows, leaves of lime, leaves of laburnum, leaves of rhododendron, pine needles, petioles of clematis, petioles of ash, petioles of robinia, and triangles of paper: the worms seized each particular object in too uniform a manner, and from a cause which we can generally understand (2). They seized each by the apex or by the base, not by the middle, and: we can hardly escape the conclusion that they show some degree of intelligence in the manner of plugging up their burrows (*id.*).

It can be a cure for misanthropy, philhelmy.

xcv

Frege's very important treatises are these,

Concept writing, a formula language of pure thinking modeled on that of arithmetic, in 1879,

The foundations of arithmetic, a logico-mathematical investigation on the concept of number, in 1884,

Function and concept, in 1891,

On concept and object, in 1892,

On sense and reference, in 1892.

He wrote also the important, if not very important, *On a geometrical representation of the imaginary forms in the plane*, in 1873, *Methods of calculation grounded in an extension of the concept of quantity*, in 1874, *On the goal of the concept writing*, in 1882, *The laws of arithmetic, derived by the concept writing*, in 1893-903, *On the concept writing of Mr Peano and my own*, in 1898, *What is a function?* in 1904.

He wrote also *On applications of the concept writing*, in 1879, *On the scientific justification of the concept writing*, in 1882, *Geometry of pairs of points in the plane*, in 1883, *On formal theories in arithmetic*, in 1886, *On the law of inertia*, in 1891, *The whole number*, in 1895, *On the foundations of geometry*, in 1903-6, *The thought, a logical investigation*, in 1919, *Negation, a logical investigation*, in 1919.

Frege is not as important as Darwin, but barely.

And his importance can be described in the same three steps. The problem he solved. How he solved it. And how he informs the problem before the investigation of knowledge.

Frege begins in the *Concept writing*: the knowledge of a scientific truth goes as a rule through several stages of certainty; first surmised perhaps from an insufficient number of particular cases, a general proposition is more and more certainly established when through chains of inferences it is brought in connection with other

truths as either consequences are derived from it or conversely it is recognized as the consequence of already firmly established propositions; there are thus the questions, on one side, about the path by which a proposition is gradually won, and, on the other, about the way in which it is eventually most firmly grounded; the first question must possibly be answered differently for different men, the second is more definite, its answer depends on the inner nature of the proposition under consideration (pr.).

There is thus the path by which a proposition is gradually won. And that this path is not different for different men Frege's description of the path shows.

The path by which a proposition is gradually won – the knowledge of a scientific truth – goes through stages of certainty. In the early stages a proposition is surmised from an insufficient number of particular cases. A general proposition is then more and more certainly established. And when a general proposition is brought through chains of inferences in connection with other truths as consequences are derived from it and it is recognized as the consequence of other propositions, this is not only one of the late stages of the path. It points unambiguously at the final stage of the path. There is the question about the path by which a proposition is gradually won, and there is the question about the way in which it is eventually most firmly grounded. The answer to the second question depends on the inner nature of the proposition. And this question is answered conclusively, a proposition is eventually most firmly grounded, and its inner nature is shown, when through chains of inferences it is brought in connection with all truths which are its consequences and all truths of which it is the consequence.

The stages of the path are of greater and greater certainty. The end of the path is complete certainty. The stages, the path, and the end depend on the inner nature of the proposition. And the inner nature of the proposition determines the stages, the path, and the end.

All along a proposition is synonymous with a truth, too.

Frege continues: the firmest proof is obviously the purely logical, which, abstracting from the particular nature of things, grounds itself only in these laws on which all knowledge rests (*id.*).

What proof is is clear. Proof is to bring a proposition in connection with the truths which are its consequences and of which it is the consequence. Proof is thus firm, firmer, and eventually the firmest. And the path by which a proposition is gradually won is a path of greater and greater certainty, and eventually complete certainty, because it is a path of firmer and firmer proof, and eventually the firmest proof. The stages of the path are stages of proof. Chains of inferences, by being chains of inferences, and consequences, both by being consequences and by having consequences, imply chains of inferences and consequences, and imply all chains of inferences and consequences. Proof, by being proof, implies proof, and implies the firmest proof, which is obviously the proof which through chains of inferences brings in connection all truths. Or which, obviously, abstracting from the particular nature of things, grounds itself only in these laws on which all knowledge rests.

The path is the path of all knowledge. Or better, knowledge is the end of the path. The path is the path of all acquisition of knowledge.

The path begins with no proof. It ends with the firmest proof. And if the acquisition of knowledge has parts, all of its parts follow the same path, from no proof to the firmest proof. If the parts do not all follow the path simultaneously, however, at any one time before the parts have all reached the end the parts are at different stages along the path. At any one time, and the further removed the time is from either the beginning of the acquisition or the end of the acquisition the more so, the different parts of the acquisition have reached different stages along the path. If some parts have reached the stage of the firmest proof, other parts have reached the stage of firmer but not yet the firmest proof, other parts have reached the stage of firm but not yet firmer proof, other parts are still at the stage when propositions are surmised from an insufficient number of particular cases, and other parts have not reached even this stage yet. At any one time, if the acquisition of knowledge in its entirety, and in the relations of its parts to one another, exhibits the firmest proof, it exhibits also firmer but not yet

the firmest proof, and firm but not yet firmer proof, and surmises from an insufficient number of particular cases. It exhibits also much that does not belong on the path at all, including much that bars the path, and much that bars it deliberately.

Frege continues: we thus divide all truths which require grounding into two kinds, according as proof of the one kind can proceed purely logically, proof of the other kind must be supported by empirical facts; and a proposition can belong to the first kind even if it can never have come to consciousness in a human mind without the activity of the senses; for, that without perception no mental development is possible in the beings known to us, holds ultimately for all judgments; thus, not the psychological origin, but the most complete way of proof is the ground of the division (*id.*).

Experience is the foundation of the acquisition of knowledge. The foundation of the path. Experience thus also precedes the path. That without perception no mental development is possible holds ultimately for all judgments. A proposition can never have come to consciousness without the activity of the senses.

On the other hand, the ground of the division of all truths which require grounding into two kinds is the end of the path. The ground is the most complete way of proof. Hence, the one kind of proof proceeds purely logically. Abstracting from the particular nature of things, it grounds itself only in these laws on which all knowledge rests. It is the firmest. The other kind is not the firmest. It must be supported by empirical facts.

Proof being proof, the proof which must be supported by empirical facts can through chains of inferences be brought in connection with the proof which proceeds purely logically. Grounding itself thus in these laws on which all knowledge rests, the proof which must be supported by empirical facts can thus itself be the firmest. Which explains the fact that not only these parts of the acquisition of knowledge, in which, because they abstract from the particular nature of things, proof can ground itself only in these laws on which all knowledge rests, can reach the end of the path. These parts of the acquisition, in which, because they do not abstract from the particular nature of things, proof must be supported by empirical facts, can also reach the end of the path.

So also the psychological origin, which is not the ground of the division, can be an object of the acquisition of knowledge. The concern with this object will follow the same path. And even if this concern is at a stage earlier than that of surmises, and even if its path is barred, and even if it is barred deliberately, it can reach the stage of proof, and it can reach eventually, when through chains of inferences the proof which must be supported by the empirical facts of the psychological origin is brought in connection with the firmest proof, the end of the path.

Frege continues: and when I considered the question to which of these two kinds the judgments of arithmetic belonged, I first had to ascertain how far it was possible to proceed in arithmetic only by inferences, only with the support of these laws of thought which transcend all particulars; first I tried to reduce the concept of ordering in a series to that of logical consequence; so that nothing sensible could penetrate unnoticed, everything had to depend on there being no gaps in the chains of inferences; as I tried to fulfil strictly this demand, I found a barrier in the inadequacy of language; the more complex the relations, the less did language allow the precision which my goal demanded; and from these difficulties came out the thought of the present concept writing (*id.*).

Language, part of experience, precedes the path. It is also, and the more complex the relations the more so, a barrier on the path.

The judgments of arithmetic, to the degree to which they ground themselves only in these laws of thought which transcend all particulars, only in these laws on which all knowledge rests, are the end of the path. To the degree to which anything sensible still penetrates unnoticed in them, it is a contamination from the earlier stages of the path. Hence the goal of clearing away the contamination. The goal of chains of inferences free of gaps. Hence the concept writing. A formula language of pure thinking modeled on that of arithmetic.

The concept writing is the end of the path.

Chains of inferences free of gaps is the most explicit description of the end.

Frege continues: I can best make clear the relation of my concept writing to the language of life by comparing it to that of the microscope to the eye; the range of its applicability and the flexibility with which it adapts itself to most diverse circumstances give the eye a great superiority over the microscope; considered as an optical instrument, it shows many imperfections, to which no attention is ordinarily paid because of its intimate connection with mental life; as soon as scientific goals demand sharpness of distinction, however, the eye shows itself insufficient; the microscope in opposition is perfectly suited for precisely such goals, even if it is useless for all other (*id.*).

The path by which a proposition is gradually won is the path of the knowledge of a scientific truth is the path toward scientific goals is the path toward the firmest proof is the path toward complete certainty.

The eye, like the language of life, precedes the path. The microscope, like the concept writing, is the end of the path.

The microscope itself shows the stages of the path, too. First, the imperfect optical instrument produced by evolution entered the stage of observation, attested, thirty-five and a half centuries before Frege, by the Venus tablet of Ammisaduqa and the discovery of Venus. Observation then reached the stage of accurate observation, attested, twenty-six centuries before Frege, by the oldest accurate observation in the *Almagest*, of a lunar eclipse in 721. Accurate observation then reached the stage of firm proof, when through chains of inferences observations were brought in connection with one another and with other truths. Seventeen and a half centuries before Frege, it reached the stage of chains of inferences as free of gaps as Ptolemy's. Proof then became firmer still and reached the stage of telescopic observation with Galileo, in 1610, two and a half-odd centuries before Frege. Hence it reached the stage also of microscopic observation, with Malpighi, in *On the lungs*, in 1661, two-odd centuries before Frege. Proof then became yet firmer and with the development of quantum theory, one and a half generations after Frege, in 1933, the microscope reached the stage of the electron microscope.

The microscope shows the stages of the path, and so does the concept writing.

Frege continues: Bacon held that it is more excellent to find a means through which everything can be readily found than to discover particular cases; and all great scientific advances of recent times have had their origin in an improvement of the method (*id.*); Leibniz also recognized the advantages of an appropriate means of expression; his conception of a general characteristic, or *calculus philosophicus*, was gigantic (*id.*).

The language produced by evolution embarked on the path and entered the stage of surmises from an insufficient number of particular cases with the Sumerian prince list, the first great advance which has its origin in an improvement of the method, four centuries before the time when the eye entered the stage of observation. The method reached the stage of firm proof with Aristotle, at the time halfway between the time when observation reached the stage of accurate observation and the time when it reached the stage of proof as free of gaps as Ptolemy's. The method entered the stage of firmer, and firmer, and firmer proof with Bacon, Galileo, and Descartes, at the time when proof reached the stage of telescopic observation, and at the time when proof reached the stage of microscopic observation, the method reached the end of the path.

The end is in sight in Descartes already. Leibniz describes the end in *The art of combination*, in 1666: perfect demonstrations in all disciplines (app.). In 1679, in the *Beginning and specimens of a general science for the instauration and growth of the sciences,* he writes: reasoning will always be correct if it is everywhere as clear and certain as arithmetic has been (11), and given the general science, or general characteristic, or *calculus philosophicus*: there will be no more need for disputes between two philosophers than between two calculators; it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate (14).

Frege continues: Leibniz's conception of a general characteristic, or *calculus philosophicus*, was so gigantic, the attempt to realize it could not go beyond the preparation (*id.*); but if this lofty goal cannot be reached in one leap, there is no reason to despair of a gradual approach (*id.*); the

signs of arithmetic, geometry, and chemistry can be seen as realizations of Leibniz's conception for particular areas; the present concept writing adds a new one, and indeed the one in the middle, adjoining all others (*id.*); from this, the work can begin, with the greatest prospect of success, at filling the gaps in the existing formula languages, connecting their until now separate areas into a single domain, and extending this domain to areas which have until now lacked such a language (*id.*); I am confident that my concept writing can be successfully employed to establish the foundations of the differential and integral calculus (*id.*); it seems to me easier still to extend its area to geometry (*id.*); the transition to the pure theory of motion and then to mechanics and physics can follow (*id.*); and if it is a task of philosophy to break the dominion of the word over the human mind, my concept writing can become an effective tool of the philosopher as well (*id.*).

The problem Frege solved is the uniformity of the acquisition of knowledge.

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The path by which a proposition is gradually won is the path of the knowledge of a scientific truth is the path toward scientific goals is the path toward the firmest proof is the path toward complete certainty. Is the path of the acquisition of knowledge.

Experience, the eye, language, precede the path.

Knowledge, the microscope, arithmetic, the concept writing, truth, are the end of the path.

The stages of the path are stages of proof. From no proof, before the path, to chains of inferences free of gaps, in the end.

At any one time after the beginning and before the end, the different parts of the acquisition are at different stages along the path. At Frege's time, arithmetic, geometry, and chemistry are at the most advanced stage. So is, with the concept writing, logic. The foundations of the differential and integral calculus are at an earlier stage. The theory of motion, mechanics, and physics are at an earlier stage still. Philosophy, exclusive of these parts, and still under the dominion of the word, is at a yet earlier stage.

The work at filling the gaps in the existing formula languages, connecting their separate areas into a single domain, and extending this domain to areas which lack such a language has the greatest prospect of success. It does, because it is the work of the acquisition of knowledge beginning to end. The work of the acquisition of knowledge is work at developing formula languages, connecting them into a single domain, and extending this domain. Beginning to end, it is work to bring all truths in connection with one another through chains of inferences free of gaps.

If all this is made more explicit, what the problem of the uniformity of the acquisition of knowledge is, and what the solution of the problem is, will be made more explicit.

And it can all be made more explicit.

In order to do so, however, the ground has to be cleared first.

Frege begins in the *Concept writing* by separating the two questions. On one side, the question about the path by which a proposition is gradually won. On the other, the question about the way in which a proposition is eventually most firmly grounded. On one side, the question must possibly be answered differently for different men. On the other, the question is more definite, its answer depends on the inner nature of the proposition.

And Frege maintains the separation sharply. That it holds ultimately for all judgments that without perception no mental development is possible, and a proposition can never have come to consciousness without the activity of the senses, belongs with the first question. In general, the psychological origin, on one side. The most complete way of proof, on the other.

Frege maintains the separation sharply in the *Concept writing*, and in all his treatises, and most generally and most explicitly in *The foundations of arithmetic*.

In *The foundations of arithmetic*, he begins by affirming the principle: to separate sharply the psychological from the logical, the subjective from the objective (intr.). His central argument is that the laws of arithmetic are analytic, and he once again affirms: the laws of number would be analytic judgments even if they need not have been found by thought alone; for the concern here is not with the way in which they have been

found, but with the grounds of their proof; and as Leibniz says, the question is not about the history of our discoveries, which is different in different men, but about the connection and natural order of truths, which is always the same (17). And he ends by reaffirming: I do not at all deny that without sense impressions we would be stupid like stones and would know nothing either of numbers or of anything else; but this psychological proposition does not concern us here at all; and I make the point again, because of the ever present danger of confusing two fundamentally different questions (105).

Perception, the mental development which perception makes possible, the path by which a proposition is gradually won, the history of our discoveries, and in general, the psychological, and the subjective, on one side. The most complete way of proof, the connection and natural order of truths, and in general, the logical, and the objective, on the other.

And what demands the separation is the transformation of philosophy in the two-odd generations before Frege. The transformation of philosophy has made the confusion of the two fundamentally different questions an ever present danger.

Frege writes: the joint work of philosophy and mathematics is not as beneficial as may be wished and may indeed be possible; the reason, in my opinion, is the predominance of psychological ways of consideration in philosophy, which have penetrated even into logic; with this tendency mathematics has nothing in common, and it explains easily the aversion of many mathematicians to philosophical considerations (intr.).

Thus on one side. On the other: my investigations are more philosophical than may seem appropriate to many mathematicians; but the fundamental investigation of the concept of number must always turn out rather philosophical; it is a problem common to mathematics and philosophy (*id.*).

On one side, philosophy, exclusive of arithmetic, geometry, chemistry, logic, mechanics, and physics, is under the dominion of the word. In it psychological considerations predominate, with which mathematics has nothing in common, and to which mathematicians have an aversion.

It is not philosophy as it is in Bacon. Or in Leibniz. Or in Kant.

Or in Frege. For, on the other side: my concept writing can become an effective tool of the philosopher; number is a problem common to mathematics and philosophy; fundamental investigation must always turn out philosophical; and my investigations are philosophical.

If the goal of the separation, then, is philosophy such as it is in Bacon and in Leibniz and in Kant, in opposition to philosophy such as it has become with the restoration of spirit to the substantiality and purity of its being, the separation is most successful. On the side of the most complete way of proof, the connection and natural order of truths, and in general, the logical, and the objective, Frege's results are clear of spiritual contamination, all of them are major advances, and most of them are conclusive. To the degree to which the separation concedes to philosophy such as it has become with the restoration of spirit to the substantiality and purity of its being everything on the other side, however, concedes that there is such a thing as the subjective, concedes the psychological, concedes perception and the mental development which perception makes possible, concedes the path by which a proposition is gradually won, and concedes the history of our discoveries, too, this is an error. Nothing can be conceded to the restoration of spirit to the substantiality and purity of its being. Because, in general, spirit brooks no compromise, takes no prisoners, and if anything is conceded to it, it will continue to demand, and will eventually take over, everything. In particular, there can be no such thing as the subjective. And all the rest, the psychological, perception and the mental development which perception makes possible, the path by which a proposition is gradually won, and the history of our discoveries, are areas in which the work at extending the domain of the existing formula languages to areas which lack such a language, and connecting all areas into a single domain, has the greatest prospect of success. And has, in fact, had great success.

Frege writes: number is an object of psychology or a product of psychic processes as little as the North Sea; nor is the objectivity of the North Sea affected by the fact that it is our arbitrary

choice which part of all the water covering the earth we will delimit and designate with the name 'North Sea' (26); number is something objective in the same way; when one says 'the North Sea is 10 000 square miles large' one points neither by 'North Sea' nor by '10 000' to a state or a process in one's inside; one affirms something entirely objective, which is independent of our ideas and the like (*id.*); the botanist wants to say something equally factual when he reports the number of a flower's petals as when he reports their color; the one depends on our arbitrary choice as little as the other; there is thus a similarity between number and color; and it is not in that both are perceptible in external things, it is in that both are objective (*id.*); arithmetic is psychology as little as astronomy is; as astronomy is concerned not with the ideas of the planets but with the planets themselves, so the objects of arithmetic are not ideas (27).

Astronomy, geography, botany, and arithmetic are objective indiscriminately.

Because the planets, the North Sea, a flower's petals, and 10 000 are objective indiscriminately.

The objective is not a product of psychic processes. It is not a state or a process in one's inside. It is not affected by our arbitrary choice. It is not perceptible in external things. It is independent of our ideas and the like.

Further: I distinguish the objective from the impressible, spatial, actual; the axis of the earth, the center of mass of the solar system are objective, but I would not call them actual, as the earth itself is; one calls the equator an imaginary line; it would yet be false to call it a thought up line; it is not created by thought, a product of a spiritual process, it is only known, grasped by thought (26).

The objective is not the impressible, spatial, actual. It is not a product of a spiritual process. It is not created by thought, it is only known, grasped by thought.

So negatively. Affirmatively: space belongs according to Kant to appearance; it might present itself to other rational beings entirely differently; indeed, we could not even know whether it appears to one man as to another; for we cannot set the perception of space of the one next to that of the other in order to compare them; but there is nevertheless something objective in it; all recognize the same geometrical axioms, even when it is only in action and because they must cope with the world; objective in it is the accordance with law, conceptual, judgeable, what lets itself be expressed in words (*id.*).

The objective is the accordance with law.

Because it is accordance with law, it is conceptual, it lets itself be expressed in concepts. Because it is accordance with law, it is judgeable, it lets itself be expressed in judgments. Because it is accordance with law, it lets itself be expressed in words as well.

Thus, the planets accord with law, with Kepler's laws, and more generally Newton's laws, with the laws explicit and implicit in the theory of the origin and development of the solar system developed by Laplace, and more generally Hubble's law and all the laws explicit and implicit in the big bang theory, including Schrödinger's equation. The North Sea accords with law, with all the laws above, and it is where it is and as large as it is, in particular, because it accords with the laws explicit and implicit in the theory of continental drift developed by Wegener. A flower's petals accord with law, with all the laws above, and they have the color they have and the number they have, in particular, because they accord with Darwin's principle, or law, and Mendel's laws, or principles. And 10 000 accords with law, and like all the laws above the laws of arithmetic have been discovered, and if it has not been established when, it can be, and they share with all the laws above not only this fact, and not only the fact that the North Sea is not 10 000 square miles large, it is 220 000 square miles large, but the fact also that the North Sea and 10 000 accord indiscriminately with the principle, or law, of contradiction.

Thus, the objective is not the perceptible in external things. It is not the impressible, spatial, actual. And not conversely. The perceptible in external things is objective, the impressible is objective, and impressions are objective, and the senses are objective, and perception is objective, all accord with law, all accord with the laws explicit and implicit in Hume's argument that experience is the effect of mechanical tendency, all accord with the laws explicit and implicit in

the theory of evolution. The spatial is objective, and the temporal is objective, what is space, what is time, what is in space, what is in time, all accord with law, the laws explicit and implicit in Newton's argument that there is absolute space and time, in Leibniz's argument that there is no absolute space and time, in Kant's argument that space and time are the form of sensibility, in Einstein's theory of relativity, in Minkowski's definition: the multiplicity of all thinkable x, y, z, t systems of values is the world. The actual is objective, and the possible is objective, what is actual, what is not actual but possible, what is neither actual nor possible, all accord with the laws explicit and implicit in Kant's argument that no *a priori* knowledge is possible but of objects of possible experience.

Space belongs according to Kant to appearance. There is nevertheless something objective in it. It accords with law. Hence, we can know whether it appears to one man as to another, and we do, in fact, know. It appears to one man in the same form as to another. We know also, because we can, in fact, set the perception of space of the one next to that of the other in order to compare them. We can do so, and we do so, formally. And we can know, and we do know, that it cannot present itself to other rational beings differently. Lest it presented itself without form.

Hence, all recognize the same geometrical axioms.

Hence, also, when all, or very nearly all, do so not by doing geometry, but only in action and because they must cope with the world, that they must cope with the world accords with law, too, the one Darwin discovered.

The objective is not affected by our arbitrary choice. And again not conversely. Our arbitrary choice, or what we delude ourselves is our arbitrary choice, is objective, which the fact should suffice to show that our arbitrary choice is never too difficult to show dictated by the fact that we must cope with the world, and we do cope with the world invariably, or very nearly invariably, in accordance with the same law, the one Darwin discovered.

The objective is not a product of psychic processes. It is not a product of a spiritual process. It is not a state or a process in one's inside. And once again not conversely. All products of psychic processes, all products of spiritual processes, all states and processes in one's inside, are objective, all accord with law, which the fact already shows that Plato keeps searching in a dialogue after a dialogue after a dialogue for what is the same in anyone's inside, and the fact shows that Aristotle already laid the foundations of the branch of physics psychology with *On the soul*, and, twenty-two centuries later, the fact shows that with the labors of Frege's older contemporaries Helmholtz, and then Wundt, psychology became explicitly experimental, and explicitly a branch of physiology and thus a branch of biology and thus a branch of physics. Even if, not yet at the stage of proof reached at the time by astronomy, geography, and arithmetic, or even that reached by botany, or even that reached by physiology, psychology was still at the stage when propositions are surmised from an insufficient number of particular cases, its chains of inferences very far from being free of gaps.

Everything is objective. Everything accords with law. What everything is not is shown objective. Shown according with law. Yet.

At some time nothing is shown objective. But then there is Aristotle – Ptolemy – Kepler – and Helmholtz and Wundt postdate Kepler, and postdate Darwin, too, but not by much.

Thus, the objective is independent of our ideas and the like. And yet again not conversely. Our ideas and the like are not independent of the objective.

Frege writes: if two were an idea, it would be mine only; the idea of another is already as such another; we would then have perhaps many millions of twos; one would have to speak of my two, your two, one two, all twos (27); with the growth of children ever new twos would appear, and who knows whether in a few millennia these would not change so that $2 \times 2 = 5$ (*id.*).

Two is not an idea. That even if it were, however, there would be no danger that it be mine only, Frege himself shows.

He writes: the idea in the subjective sense is that to which the psychological laws of association are related; it is of sensible, pictorial nature; the idea in the objective sense belongs to logic and is essentially not sensible, even if the word

which refers to an objective idea is often accompanied by a subjective idea which is nevertheless not its reference; the subjective idea is often demonstrably different in different men, the objective idea is the same for all (*id.*); this distinction is as justified as that between psychology and logic; would that these were always separated sharply from one another (*id.*)!

Granted, for the sake of the argument, that there be such a thing as mine only, an idea could yet not be such a thing. Not if it is that to which law is related, the psychological laws of association, for instance. Nor if it is demonstrable, demonstrably different in different men, for instance.

Logic is objective.

Nor have the labors of any philosopher advanced logic more or more substantially than Frege's. And this fact alone shows conclusively that logic is objective, and that it has been shown that logic is objective. An important part of the advance has been clearing away much confusion of logic with psychology, too.

None of which depends, however, or can depend, on the argument, logic is not psychology, psychology is subjective, logic is objective. Never mind the argument, psychology is subjective, logic is not psychology, logic is objective.

The labors of Aristotle suffice to show, let alone the labors of Helmholtz and Wundt, that psychology is wantonly conceded to spirit.

Worse, the concession that there is such a thing as the subjective cannot not be a concession to idealism.

Nor can the concession stand.

Frege writes: while everyone can only feel his pain, his desire, his hunger, have his sound and color perception, numbers can be the common objects of many, and are rather exactly the same for all (93).

Numbers are exactly the same for all.

Also, however, in the proposition, everyone can only feel his pain, 'everyone' does not need to refer to everyone, it only needs to refer to two among everyone to preclude the possibility that anyone can only feel his pain. In the proposition, the idea of another is already as such another, if 'another' has a reference, the proposition has a reference, if the proposition has a reference, 'idea' has a reference, and if 'idea' has a reference, this precludes the possibility that either the idea of another be his only or my idea be mine only. And granted that an idea is different in different men, can it be different differently from however a petal is different in different flowers, and an equator is different in different planets?

Frege writes: by objectivity I understand independence from our perception, sensibility, and ideas, from the drawing of inner pictures out of the memories of earlier perceptions, but not independence from reason; for to answer the question of what a thing independent of reason is, is to judge without judging, to wash the fur without soaking it (26).

Independence from perception, sensibility, and ideas, objectivity had better be independence from reason as well. With Hume: the distinction between reason and experience I shall not scruple to pronounce at bottom erroneous. With Kant: all knowledge of things merely from pure understanding or pure reason is nothing but sheer illusion.

And Frege himself has answered the question of what a thing independent of reason is. He writes: a proposition stops being true when I no longer think of it no more than the sun is annihilated when I close my eyes (intr.). What a thing independent of reason is is the truth of a proposition.

Hence, it makes no difference whether we soak the fur or not. Indeed, the fur is washed even if no one ever soaks it. Frege writes: that from certain propositions another follows is something objective, independent of the laws of the motion of our attention, and it makes no difference whether we actually draw the conclusion or not (80); even if all rational beings went into hibernation simultaneously, the truth of a proposition would not be suspended for the duration (77).

In short, Frege's results themselves forbid any compromise with the restoration of spirit to the substantiality and purity of its being.

Frege's assessment of the damage done to philosophy by the restoration is impeccable, too.

He writes: the astronomer would hesitate to extend his conclusions to long past times, lest he met with the objection, "you calculate $2 \times 2 = 4$; but the idea of number has a development, a

history! it may be doubted whether by that time it was so far advanced; how do you know that it already held? may it not be that the then living beings had the proposition $2 \times 2 = 5$, from which $2 \times 2 = 4$ only evolved by natural selection in the struggle for existence, and this proposition is itself destined perhaps to develop into $2 \times 2 = 3$?"; the historical approach, which seeks to detect the becoming of things, and from the becoming their essence, certainly has a great justification; but it has also its limits; if in the constant flux of all things nothing remained firm, eternal, the possibility of knowing the world would disappear and everything would fall into confusion; one imagines, it seems, that the concepts sprout in the single soul like the leaves on the trees, and endeavors to know their essence by investigating their origin and to explain it psychologically from the nature of the human soul; but this approach draws everything into the subjective and, followed to the end, abolishes truth (intr.).

It has since been followed to the end. It has abolished truth. And it has not drawn everything into the subjective, because there is no such thing as the subjective, but it has drawn everything into world-views and discourses.

To associate the theory of evolution with that is an error. In fact, with the labors of Frege's older contemporary Mendel already biology began developing its concept writing, and a century on, its concept writing had been connected with the concept writing of chemistry, too.

To concede history to that is an error as well. Leibniz's conception of a *calculus philosophicus* is gigantic. But if this lofty goal cannot be reached in one leap, there is no reason to despair of a gradual approach. In fact, precisely because it has been gradual, the success of the gradual approach offers the greatest prospect of success most particularly to history as well.

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The *Concept writing* is the first and the most important of Frege's very important treatises. He develops the concept writing in it. In its preface, also, he indicates the breadth, depth, and generality of the problems solved, and to be solved, by the concept writing, the path by which a proposition is gradually won, the beginning of the path, the end of the path, the stages of the path, the stages reached by the different parts of the acquisition of knowledge, and the work of developing formula languages and connecting them into a single domain. *On sense and reference* is the last of the very important treatises and the most important after the *Concept writing*. In it Frege addresses directly the problems of language, before the path, and of truth, at the end of the path. And to make more explicit the problems solved, and to be solved, by the concept writing, there can be no firmer ground than *On sense and reference*.

Frege writes in *On sense and reference*:

$a = a$ and $a = b$ are obviously propositions of different knowledge value; the discovery that not a new sun rises every morning, but always the same, has certainly been one of the richest in consequences in astronomy; even today to recognize a comet is not always trivial;

a difference in the knowledge value of $a = a$ and $a = b$ can arise only by that that a difference in the way the signified is given corresponds to the difference of the sign; it thus suggests itself to think connected with a sign, next to the signified, which may be named the reference of the sign, also that, which I would call the sense of the sign, in which the way the signified is given is contained; the reference of 'evening star' and 'morning star' would be the same, but not the sense;

by 'sign' and 'name' I understand any designation which represents a proper name, whose reference is thus a definite object, this word taken in its widest extent; the designation of a single object can consist also of more words or other signs; for brevity I call every such designation proper name;

the sense of a proper name will be grasped by everyone who knows the language or the whole of the designations to which it belongs; but with this the reference, in case it exists, is always illuminated only from one side; to all-sided knowledge of the reference would belong to be able to say immediately whether any given sense belongs to it; to this we never attain;

given a sense, it does not follow that a reference corresponds to it; the words 'the heavenly body most distant from the earth' have a sense, but it is very doubtful that they also have a refer-

ence; the expression ‘the least rapidly convergent series’ has a sense, but one can prove that it has no reference, as for any given convergent series, a less rapidly convergent yet still convergent series can be found; when grasping a sense, one does not certainly have a reference;

from the reference and the sense of a sign the idea associated with it has to be distinguished; when the reference of a sign is a perceptible object, my idea of it is an inner image arising from memories of sense impressions, which I have had, and activities, inner and outer, which I have exercised; it is often saturated with feelings; the clarity of its separate parts is different and changing; the same idea is not always connected, not even in the same man, with the same sense; the idea is subjective; the idea of one man is not that of another;

the idea is thus essentially different from the sense of a sign, which can be the common property of many and is not a part or a modus of the single soul; for it cannot be denied that mankind has a common store of thoughts transmitted from one generation to another;

the reference of a proper name is the object itself which we signify by it; the idea which we then have is entirely subjective; in between is the sense which is no longer subjective like the idea but is not the object itself either; the following analogy can perhaps clarify these relations; someone observes the moon through a telescope, I compare the moon itself to the reference, it is the object of the observation which is mediated through the real image projected by the objective lens inside the telescope and through the retinal image of the observer; the former I compare to the sense, the latter to the idea or perception; the image in the telescope is indeed one-sided, it depends on position; but it is nevertheless objective, inasmuch as it can serve many observers; of the retinal images, on the other hand, everyone has his own; because of the different constitution of the eyes even geometrical congruence would be hard to achieve, actual coincidence would be out of the question; the analogy may perhaps be pursued further, by assuming that the retinal image of A can be made visible to B, or that A can see his own retinal image in a mirror; this would perhaps show how an idea itself can be taken as an object; as such, however, it is not for the observer what it is for the one having it immediately;

we can thus recognize three stages of the difference of words, expressions, and whole propositions; the difference concerns either at most the ideas, or the sense but not the reference, or finally also the reference;

with respect to the first stage, to the possible differences here belong also the colorings and nuances which the poetic art and eloquence seek to give to the sense; these colorings and nuances are not objective, every hearer or reader must create them for himself from the hints of the poet or orator; without a kinship of human ideas art of course would be impossible; but how far the correspondence to the intentions of the poet extends can never be exactly determined;

of ideas and perceptions no more will be said; they have been noted only in order to avoid confusing the idea, which a word arouses in a hearer, with its sense or its reference;

to make brief and exact expression possible, let the following way of speaking be established; a proper name, word, sign, sign combination, expression, expresses its sense, refers to or designates its reference;

from the idealist and sceptic side will perhaps long have been objected, “you speak without further ado of the moon as an object, but how do you know that the name ‘the moon’ has a reference? how do you know that anything at all has a reference?”; I reply that when we say ‘the moon’, it is not our intention to speak of our idea of the moon, and we are not content with the sense either; we presuppose a reference; to want to assume that in the proposition ‘the moon is smaller than the earth’ at issue is an idea of the moon is to miss the sense entirely; if this is what the speaker wants, he would use the phrase ‘my idea of the moon’;

to justify speaking of the reference of a sign, it is enough to point to our intention in speaking or thinking, even if the qualification has to be added, in case it has one;

we now ask about the sense and reference of a whole proposition;

such a proposition contains a thought; by a thought I understand not the subjective act of

thinking but its objective content which is capable of being the common property of many;

is this thought now to be regarded as the sense of the proposition or its reference? let us assume that the proposition has a reference! if we replace in it one word with another which has the same reference, but a different sense, this can have no effect on the reference of the proposition; the thought, however, changes; the thought, for instance, of the proposition 'the morning star is a body illuminated by the sun' is different from that of 'the evening star is a body illuminated by the sun'; someone who did not know that the evening star is the morning star could hold the one thought for true, the other for false; the thought therefore cannot be the reference of the proposition, we have rather to take it to be the sense;

does a proposition as a whole have perhaps only a sense, but no reference? one will expect such propositions to occur, as there are parts of propositions which have a sense but no reference; propositions in which proper names without a reference occur will be of this kind; the proposition 'Odysseus was fast asleep set ashore at Ithaca' obviously has a sense, but because it is doubtful whether the name 'Odysseus' has a reference, it is doubtful also whether the whole proposition has one; it is certain, however, that anyone who seriously holds the proposition for true or false, recognizes also a reference of the name 'Odysseus';

if only the sense of the proposition, the thought, were at issue, it would be unnecessary to care about the reference of a part of it; the thought remains the same whether the name 'Odysseus' has a reference or not;

that we trouble ourselves at all about the reference of a part of a proposition is a sign that we recognize and demand a reference for the proposition itself; the thought loses value for us as soon as we recognize that the reference of one of its parts is missing; we are therefore justified not to be satisfied with the sense of a proposition, but to ask also about its reference;

but why do we want every proper name to have not only a sense, but also a reference? why is the thought not enough? because and inasmuch as its truth value is at issue;

this is not always the case; when listening to an epic, for instance, next to the melodious sound of the language, we cling only to the sense of the propositions and the ideas and feelings these arouse; with the question of truth we leave the enjoyment of art and turn to scientific consideration; as long as we take the poem as a work of art it does not matter to us whether the name 'Odysseus' has a reference; the striving for truth therefore is what in general forces us to advance from the sense to the reference;

we are therefore forced to recognize the truth value of a proposition as its reference; I understand by the truth value of a proposition the fact that it is true or that it is false; further truth values there are not; I name for brevity the one the true, the other the false; every proposition, in which the reference of the words is at issue, is therefore to be regarded as a proper name, and its reference is, when it has one, either the true or the false; these two objects are recognized, even if only tacitly, by everyone who judges at all, who holds something for true, and so also by the sceptic;

the designation of the truth values as objects may appear a wilful fancy, a mere play on words, from which no far-reaching consequences may be drawn;

so much should be clear, however, that in every judgment – even the most self-evident – the step from the stage of the thought to the stage of the reference, the objective, has already occurred;

if our presumption is correct, that the reference of a proposition is its truth value, this must remain unchanged when a part of a proposition is replaced by an expression with the same reference, but a different sense; and this is in fact the case; Leibniz explains, *eadem sunt, quae sibi mutuo substitui possunt, salva veritate*;

if the truth value of a proposition is its reference, all true propositions have the same reference and, on the other side, so do all false propositions; we thus see that in the reference of the proposition everything individual is effaced; we can therefore never be concerned with the reference of a proposition alone; yet the mere thought gives no knowledge either, only the thought together with its reference, and that is, its truth value;

let us now return to our starting point!

when we found the knowledge value of '$a = a$'

and '$a = b$' different in general, the explanation is that for the knowledge value the sense of the proposition, and that is, the thought expressed in it, comes into consideration not less than its reference, and that is, its truth value; if now $a = b$, then the reference of 'b' is the same as that of 'a', and therefore the truth value of '$a = b$' is the same as that of '$a = a$'; nevertheless the sense of 'b' can be different from that of 'a', and thus also the thought expressed in '$a = b$' can be different from that expressed in '$a = a$'; and then the two propositions do not have the same knowledge value; and if we understand by 'judgment' the progress from the thought to its truth value, we will say that the judgments are different (*id.*).

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Given selection, selection can produce, and has produced, perception, the mental development which rests on perception, the eye, the retinal image, ideas, thought, language, and sense.

Selection has not produced, and cannot produce, reference, propositions, chains of inferences, and truth.

Reference, propositions, chains of inferences, and truth are objective.

And, to begin with, perception, the mental development which rests on perception, the eye, the retinal image, ideas, thought, language, and sense are one and all objective.

Selection acts, and can only act, on the individual. It acts on the particular bundle of genetic regulatory mechanisms, instinct, and learning which the individual organism is. It does, however, by accumulating the slight individual variations in those, and destroying them, by destroying their individual carriers, or preserving them, by preserving their individual carriers, in the struggle for life.

Hence, everything comes from experience, but not from the ongoing current experience reiterated by each individual. It comes from the experience accumulated by the entire ancestry of the species over the course of its evolution.

Thus, perception is objective in that the individual act of perceiving reiterates the individual acts of perceiving of the entire ancestry of the species. Hence, also, it is objective in that its content is capable of being the common property of many. On both counts, perception is objective in that it accords with law.

So is the mental development which rests on perception. So is the eye. So is the retinal image.

Everyone has his own retinal image. Everyone. Accordingly, the retinal image of A can be made visible to B, A can see his own retinal image in a mirror, and this shows that the retinal image can be taken as an object. Both, that it can be taken as an object, and that it can be shown that it can be taken as an object, because it accords with law. Thus, in the proposition 'the retinal image of A is different from the retinal image of B', it may be that the reference of 'the retinal image of A' is different from the reference of 'the retinal image of B', but it may not be that the reference of 'the retinal image' is different in the first occurrence from that in the second. Lest the reference of 'the retinal image of the moon', for instance, were different in 'the retinal image of the moon of A' and 'the retinal image of the moon of B'. Or 'A' had no reference, or 'B' had no reference, or, given that both have a reference, A and B were not objects of the same kind. Hence, in the proposition 'the retinal image of A is different from the retinal image of B', 'is different', too, cannot not have a reference. The difference cannot not be possible to take as an object. It cannot not accord with law. Accordingly, because of the different constitution of the eyes geometrical congruence may be hard to achieve, but because they are all eyes, all constituted in one and the same way, by selection, geometrical congruence cannot not be possible to achieve. In general, even if actual coincidence of the retinal images of the one having a retinal image and the one observing his retinal image is out of the question, possible coincidence cannot be. Lest the retinal image stopped according with law.

As the retinal image so ideas. Everyone has his own ideas. Everyone. Accordingly, B can have an idea of an idea of A, A can have an idea of an idea of his own, and this shows that an idea can be taken as an object. Both, that it can be, and that it can be shown that it can be, because it accords with law. In the proposition 'the idea of A is different from the idea of B', the reference of 'the idea' cannot be different in the first occur-

rence from that in the second. For neither the reference of 'the idea of the moon' can be different in 'the idea of the moon of A' and 'the idea of the moon of B', nor are the references of 'A' and 'B' not objects of the same kind. Hence, in the proposition 'the idea of A is different from the idea of B', 'is different' cannot not have a reference either. The difference cannot not accord with law. And even if actual coincidence of the ideas of the one having an idea and the one having an idea of his idea is out of the question, possible coincidence cannot be.

Given that my idea of an object is an inner image arising from memories of sense impressions, which I have had, and activities, inner and outer, which I have exercised, that it is saturated with feelings, that the clarity of its separate parts is different and changing, and that the same idea is not always connected, not even in the same man, with the same sense, in relation to one and all of these – object, inner image, memories, sense impressions, I have had, activities, inner and outer, I have exercised, saturated, feelings, clarity, parts, different, changing, connected, man, sense – in relation to one and all of these 'my idea' refers to your idea just as it does to my idea. Sense is not a part or a modus of the single soul, it can be the common property of many, and idea is not a part or a modus of the single soul, it can be the common property of many. For it cannot be denied that mankind has a common store of thoughts transmitted from one generation to another, and it cannot be denied that mankind has a common store of ideas transmitted from one generation to another. The idea, for instance, that my idea is mine alone.

All ideas are objective. In that each can be the common property of many, and in that all accord with law. What all ideas are not is worth being the common property of many, or any, and this accords with law, too.

Idea and thought cannot be different in that the one is objective the other is not.

Frege writes in *On sense and reference*: it may be attempted to regard the relation of the thought to the true not as that of the sense to the reference, but as that of the subject to the predicate; subject and predicate are indeed parts of thought, they are at the same stage of knowledge; by the combination of subject and predicate, however, one always reaches only a thought, never reaches from a sense to its reference, from a thought to its truth value, one moves at the same stage, never progresses from one stage to the next; a truth value cannot be a part of a thought, just as the sun cannot be, because it is not a sense, but an object (*id.*).

If an idea is an image arising from memories of sense impressions and activities and saturated with feelings, and thought is the combination of subject and predicate, idea and thought are distinguished sharply from one another.

We recognize three stages of the difference of words, expressions, and whole propositions. The difference concerns either at most the ideas, or the sense but not the reference, or finally also the reference.

Movement within the first stage, from images and feelings to images and feelings, cannot be language.

Language has to be movement from the first stage to the second, from images and feelings to combinations of subjects and predicates, movement within the second stage, from combinations of subjects and predicates to combinations of subjects and predicates, and movement from the second stage back to the first, from combinations of subjects and predicates back to images and feelings.

If so, if language is those movements, this can explain the fact that selection has produced language. If language is those movements, this can show why, whatever else it may be, language is the means of learning, of the transmission of the experience acquired by one individual, long before it can be stored in instinct, from that individual to another, and hence from one generation to another. Adam cannot have inferred from the fluidity and transparency of water that it will suffocate him, or from the light and warmth of fire that it will consume him. If Adam has acquired these experiences, however, and survived them, and has developed language, Adam can teach Seth to combine the subjects and predicates, water and suffocation, fire and consumption.

It can explain also why the individual's acts of engagement in language – one's own movements

from images and feelings to subjects and predicates, from subjects and predicates to subjects and predicates, and especially from subjects and predicates back to images and feelings – will appear all-important to the individual.

It can explain also why, if the poetic art and eloquence seek to give colorings and nuances to the sense, the poetic art and eloquence will be the pinnacle of acts of engagement in language.

If language is those movements, language can be forced as well into movement from the second stage to the third, from the sense to the reference, and movement within the third stage, from one reference to another, but only against resistance.

These movements, from the sense to the reference, and from one reference to another, cannot not seek to eliminate thought. To eliminate all traces of ideas. And to eliminate even the appearance of importance of individuality, never mind individuality.

If so, this can explain why language has been a barrier before the development of mathematics and physics. As Frege reports: the more complex the relations, the less did language allow the precision which my goal demanded. Hence, to deal with the barrier, the formula languages of arithmetic, geometry, chemistry, logic, calculus, mechanics, physics.

It can explain also why, disapproving of mathematics and physics, Hegel combined this subject and this predicate: language is the being of spirit.

It can explain also why the poet, Goethe, sought to give this coloring and nuance to this combination of subject and predicate: *individuum est ineffabile*. The physicist, Planck, sought to fix the reference of 'the goal of physics': the goal of physics is the complete emancipation of the physical picture of the world from the individuality of the picturing mind.

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Perception, the mental development which rests on perception, the eye, the retinal image, ideas, thought, language, and sense, on one side.

Reference, propositions, chains of inferences, and truth, on the other. Showing the uniformity of the acquisition of knowledge.

An illustration follows. The discovery that the evening star is the morning star.

The reference of 'the evening star' and 'the morning star' is the same. The reference is Venus, the second planet of the solar system.

And to show that this is the reference, and hence when it was discovered that this is the reference, the relevant phenomena have to be described. These are of two kinds, heavenly, and earthly, what the heavenly bodies do, and what the earthly bodies have done about the heavenly.

The description should begin with the earthly phenomena, and in particular, with what Parmenides has done. Parmenides, Laertius reports: first said that the evening star is the morning star (IX, 23).

Parmenides postdates the first philosopher by one century-odd. And beginning with the first philosopher, seven centuries of most successful investigation of the heavenly phenomena begin. The second philosopher already, Anaximander, discussed the sizes and distances of the planets. Four and a half centuries after Thales, not quite three and a half after Parmenides, Hipparchus developed the theory of the motions of the sun and the moon. Seven centuries after Thales, six after Parmenides, Ptolemy developed the theory of the motions of the planets.

As part of this development observation developed. Ptolemy reports: observations of the planets are relatively recent; the oldest of them are also useless because they are too crude; they tend to be of first and last visibility and such observations are particularly uncertain. The oldest observation of Venus on which the theory of its motions rests is thus from 272, by Timocharis, midway through the seven centuries from Thales to Ptolemy.

After Ptolemy the philosophers stopped investigating the heaven. They turned to the more urgent task of going to heaven. As Kepler describes this phenomenon in the title of the *Rudolphine tables*: *astronomical science collapsed*.

In 1543, fourteen-odd centuries after Ptolemy, Copernicus published *On the revolutions of the heavenly spheres*.

In 1610, Galileo constructed a telescope and discovered the four major satellites of Jupiter. In 1611, he reported the first observation of the phases of Venus (to Giuliano de' Medici, 1 Jan).

The phases of Venus show conclusively that Venus revolves around the sun. And even if not yet conclusively, they do show, and the more so with the satellites of Jupiter, that the earth as well revolves around the sun.

In 1616, a committee of eleven, appointed to examine the Copernican hypothesis, submitted its findings to the holy office: the proposition that the sun is in the center of the world and does not move, is foolish, absurd, and formally heretical (report, 24 Feb).

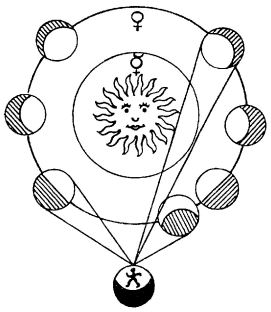
In 1609, Kepler had discovered the first and the second laws of the motions of the planets. In 1619, he discovered the third. In their formulation in the *Epitome of Copernican astronomy*, in 1618-21, the laws are:

the orbits of the planets are ellipses with the sun in one of the foci (V, 1.3);

the areas defined by the arcs traversed by a planet in equal times and the respective radii to the sun are equal (1.4);

the more distant from the sun a planet is the more slowly it moves, and so that the ratio of the periodic times is the ratio of the $\frac{3}{2}$ powers of the distances (IV, 2.3).

Turning now to the heavenly phenomena, Ptolemy describes them in general: there are two primary motions in the heaven; one carries all heavenly bodies from east to west in uniform motion about the pole of the heavenly sphere; the other carries them in the opposite direction; observed in the course of one day, all heavenly bodies show the first motion; observed continuously, while all other stars show only the first motion, the sun, the moon, and the other planets show more complicated and different from one another motions, which are yet all in the general direction from west to east.



In particular, Kepler has this picture in the *Epitome* (2.5). It shows Mercury and Venus as they revolve around the sun, and hence the phases which Venus exhibits to an observer on the earth. The picture shows also that the sun is pleased to show the phases of Venus to a human figurine bestriding the earth. Whether the figurine is pleased to be shown the phases cannot be determined with the naked eye. It can be inferred from the earthly phenomena, however, that, if the figurine represents Galileo or Kepler, it is pleased, if it represents a member of the committee of eleven appointed to examine the Copernican hypothesis, it is not pleased.

With Copernicus, Galileo, and Kepler, against the committee, Venus revolves around the sun. And so does the earth. Accordingly, to an observer on the earth, Venus appears to change its distance from the sun, a little every day, different distances corresponding to different phases. More particularly, when the sun is between the planet and the earth, the planet is invisible for some days, lost in the sun's glare. The planet then appears east of the sun, after the sun sets, as an evening star. It then increases its distance from the sun, reaches a maximum, then it decreases its distance, as an evening star throughout, until it disappears in the sun's glare again, the planet being between the sun and the earth now. It appears after some days west of the sun, as a morning star now, before the sun rises. It increases its distance, reaches a maximum, decreases its distance, as a morning star throughout, until it disappears behind the sun. Then it repeats the cycle, taking 584 days on average. Or to give them all, positions and cycle, their names – from superior conjunction to evening rising to greatest eastern elongation to evening setting to inferior conjunction to morning rising to greatest western elongation to morning setting to superior conjunction, the synodic period of Venus is 584 days on average.

These are the heavenly phenomena.

Back to the earthly phenomena, excavation in Mesopotamia began in 1842, with Botta. In 1853 Rassam found the ruins of the palace of Ashurbanipal, including its library.

Ashurbanipal was prince 668-27, not quite one century before the first philosopher.

Among the tablets recovered from his library is the collection of omens *When Anu and Enlil*. Complete, the collection contains seventy tablets. The sixty-third is the Venus tablet of Ammisa-

duqa.

By 1857 Rawlinson, Hincks, and Oppert had deciphered cuneiform.

In 1870 Rawlinson published the first and most important of the fragments comprising the Venus tablet of Ammisaduqa. Rawlinson identified them as: tables of the movements of the planet Venus and their influences (*Asia*, III, 63). In the following years more fragments were published. In 1906 Schiaparelli identified the record on the tablet as that of the observation of a continuous series of the appearances and disappearances of the planet over twenty-one years. In 1912 Kugler identified the name, the year of the golden throne, as the name of the eighth year of Ammisaduqa's reign, and hence the twenty-one years of observation as the twenty-one years of Ammisaduqa's reign.

Thus it was established that, when Parmenides said that the evening star is the morning star, he did not discover this fact. Eleven and a half centuries earlier, the predecessor of the philosophers who has to go by the name Venus tablet of Ammisaduqa discovered this fact.

Ammisaduqa was prince *c.* 1646-25, ten centuries before Ashurbanipal.

The Venus tablet of Ammisaduqa is divided into three parts. Part of a collection of omens as the tablet itself is, all its three parts record omens. On two of the three parts, however, when the omens are ignored, observation remains.

Ignoring the omens, the tablet reports:

in the first year, Venus disappeared in the west in the 11th month, 15th day, remained hidden for 3 days, and appeared in the east in the 11th month, 18th day;

in the second year, Venus disappeared in the east in the 8th month, 21st day, remained hidden for 1 month and 21 days, and appeared in the west in the 10th month, 16th day;

in the third year, Venus disappeared in the west in the 6th month, 29th day, remained hidden for 16 days, and appeared in the east in the 7th month, 15th day;

in the fourth year, Venus disappeared in the east in the 4th month, 3rd day, remained hidden for 2 months and 6 days, and appeared in the west in the 6th month, 9th day;

and so on through the twenty-first year.

The months are lunar. The years are identified each with its name, the names derived from the deeds of the prince.

Among the further difficulties that have had to be resolved in order to establish that the tablet attests the discovery of Venus are the following.

When were months intercalated in the series in order to adjust the lunar months to the solar years. What was the precision of observation, given that, as Ptolemy warns, the observation of horizon phenomena, and of first and last visibility, is particularly uncertain. What errors were made in the transcription of the tablet by scribes who were experts at divination and not so much at observation.

Hence, given Kepler's laws – and not without Kepler's laws – the motions of Venus can be calculated, the motions of the earth can be calculated, and hence the relative positions of the three, the sun, Venus, and the earth, can be calculated. In particular, it can be calculated when the planet can have been invisible at Babylon during inferior conjunction for 3 days, then during superior conjunction for 1 month and 21 days, then during inferior conjunction for 16 days, then during superior conjunction for 2 months and 6 days, and so on. Hence it can be established when the observation can have been made, as Schiaparelli first tried to do, and hence when Ammisaduqa can have reigned, as Kugler first tried to do.

Ammisaduqa can have reigned 1646-25, or 1638-17, or 1582-61, and so on later, or 1702-681, or 1758-37, and so on earlier.

Hence, finally, in order to establish that Ammisaduqa reigned *c.* 1646-25, and hence that this is when Venus was discovered, chronology has had to be fixed. The Assyrian, Egyptian, Hittite, and Babylonian records have had to be sorted out, so that, beginning with Ptolemy's *Handy tables*, the dates of the Assyrian princes are fixed back to Ashur-uballit, prince *c.* 1363-27, then the dates of the Egyptian princes are fixed back to Amenhotep, prince *c.* 1545-21, then the dates of the Hittite princes are fixed back to Mursili, prince *c.* 1610, and hence the dates of the Babylonian princes are fixed back to Ammisaduqa. Hence, Ammisaduqa reigned *c.* 1646-25, and '*c.*' refers to the fact that it is certain that Ammisaduqa

reigned either 1646-25 or 1638-17, but it is only very probable that he reigned 1646-25 rather than 1638-17.

And Venus was discovered, and *ipso facto* the planets were discovered, thirty-two and a half centuries before Kepler discovered the laws of their motions.

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Thirty-two and a half centuries before the discovery of the laws of the motions of the planets, then, thirty-one and a half centuries before the discovery that the planets revolve around the sun, seventeen and a half centuries before the formulation of the theory of the motions of the planets, and ten and a half centuries before the philosophers began discussing the planets, the discovery of Venus, and *ipso facto* of the planets, is attested by the observation of the planet through more than twelve of its synodic periods, inferior conjunction to superior conjunction to inferior conjunction, and again, and again, and more than twelve times.

Thus, the reference of 'Ishtar' on the Venus tablet of Ammisaduqa, of 'Hesperus' and 'Phosphorus' when Parmenides said that the evening star is the morning star, of 'Aphrodite' in the *Almagest*, of 'Venus' in *On the revolutions of the heavenly spheres*, and of 'the second planet of the solar system' in the *Epitome of Copernican astronomy* is the same. The reference is the second planet of the solar system.

The reference, on one side. On the other side, among all the thoughts, and sense, which, by having failed to impress the reference in the least, show its imperturbable immutability, are these.

Error. The theory of the motions of the second planet of the solar system was developed on the presumption that it revolves around the earth. The discovery that it revolves around the sun was made on the presumption that it continues to revolve on an epicycle revolving on an eccentric circle. That the reference is yet the same, and the same as on the Venus tablet of Ammisaduqa, the description of the same 584-day synodic cycle – superior conjunction to evening rising to greatest eastern elongation to evening setting to inferior conjunction to morning rising to greatest western elongation to morning setting to superior conjunction – by the tablet, Ptolemy, Copernicus, and Kepler shows.

Ambiguity. Mercury is also an evening star and a morning star. And to someone on Mars the earth will also appear as an evening star and a morning star. The same fact disposes of the ambiguity.

Images saturated with feelings. The planet has been not only a goddess, and now a Babylonian goddess, now a Greek goddess, now a Roman goddess. It has been the devil himself, for Phosphorus is also Lucifer.

Divination. In the century through Ashurbanipal, the Assyrian princes became the masters of Mesopotamia, and toward the end of that time of Egypt, too. And this is the time when divination becomes prominent in the record, as the contemporary Hebrew prophets attest as well. *When Anu and Enlil* was collected at this time, and became the Assyrian princes' main means of access to the future. It incorporated eventually some 7 000 omens. It incorporated also the observation of Venus under Ammisaduqa. After a hiatus of nearly one millennium, it was, apparently, the first use the successors of the observation found for the observation, to supply it with omens and include it in a collection of 7 000 omens.

With the omens, the tablet begins:

> if in the first year Ishtar disappeared in the west in the 11th month, 15th day, remained hidden for 3 days, and appeared in the east in the 11th month, 18th day, Adad will bring rains, Ea will bring floods, prince will greet prince.

Nor have the attempts to impress the reference with divination, prophecy, prediction, and the like ever lost their appeal.

Foolishness. The proposition that the sun is in the center of the world and does not move, is foolish, absurd, and formally heretical. That the proposition is absurd can be doubted, given that, with Cicero: nothing so absurd can be said as not to have been said already by one philosopher or another, and that the earth revolves around the sun had been said three centuries before Cicero by Aristarchus. That the proposition is foolish cannot be doubted. For it cannot be doubted that the proposition is formally heretical. It is formally heretical, the committee of eleven judged: because it contradicts explicitly many scriptural

passages according to both their literal meaning and their interpretation by the holy fathers and the doctors of theology (report, 24 Feb 1616).

So it can be doubted also whether the proposition that that proposition is absurd is itself absurd, given that nineteen centuries before the committee of eleven a contemporary of Aristarchus, the second scholarch of the Stoa: Cleanthes said that Aristarchus should be sued for impiety for disturbing the hearth of the world (Plutarch, *Face*, 6). But it cannot be doubted that the proposition that that proposition is formally heretical is itself not formally heretical. Or, consequently, that the proposition that that proposition is foolish is itself foolish.

Frege's second principle in *The foundations of arithmetic* is: about the reference of words must be asked in their connection in propositions, not in their isolation (intr.).

And this principle is the ground on which all contamination by thoughts, and sense, can be cleared away from the reference. Because: in their connection in propositions, cannot not imply: in their connection in all propositions. And this, in turn, because the fact that the reference is the same in all propositions in which it occurs, and through all contamination by thoughts, and sense, demands the principle.

Reference connects propositions.

It connects the observations on the Venus tablet of Ammisaduqa and Kepler's laws through thirty-two and a half centuries of error, ambiguity, images saturated with feelings, divination, and foolishness. Deduction from Kepler's laws shows when Venus was discovered, and conversely, the discovery of Venus corroborates the laws. More generally, and more dramatically still, the unearthing and decipherment of the early record lost for millennia in ignorance has corroborated the laws, and conversely, the laws have fixed the chronology of the early record.

Reference connects propositions, and connects them immutably.

And it is because reference connects propositions, and connects them immutably, that thoughts, and sense, and ideas, can also connect propositions.

Because the moon is the moon the images of the moon in the lens of the telescope are the images of the moon in the lens of the telescope and the retinal images of the moon are the retinal images of the moon. The proposition 'Adad will bring rains' has a sense because 'rains' has a reference, even if 'Adad' does not. The proposition 'Ea will bring floods' has a sense because 'floods' has a reference, even if 'Ea' does not. And the proposition 'Adad will bring Ea' has a sense, too, and the proposition 'Ea will bring Adad' has a sense, even if neither 'Adad' has a reference nor 'Ea' has a reference, because of the fear, or hope, that 'Adad' has a reference and 'Ea' has a reference. The committee of eleven was appointed to judge the Copernican hypothesis, and judged that: it contradicts explicitly many scriptural passages, not because thoughts, or sense, or ideas, were at stake. Whether 'god' has a reference was at stake.

In general, mankind has a common store of thoughts, and a common store of ideas, because it has a common store of references. Because it has rains, and floods, and Venus in common, it has Ishtar, and Aphrodite, and god in common, and fears and hopes in common.

To the objection from the idealist and sceptic side, but how do you know that anything at all has a reference? Frege replies, we are not content with the sense, we presuppose a reference; to justify speaking of the reference, it is enough to point to our intention in speaking or thinking.

If it is enough to point to our intention, it is because 'our' refers to anyone's, to the idealist and sceptic's intention as well as Frege's, the Martians' intention as well as mankind's.

Frege writes: the thought loses value for us as soon as we recognize that the reference of one of its parts is missing; but why do we want not only a sense, but also a reference? why is the thought not enough? because and inasmuch as its truth value is at issue; the two objects, the true and the false, are recognized, even if only tacitly, by everyone who judges at all, who holds something for true, and so also by the sceptic.

In general: the striving for truth is what forces us to advance from the sense to the reference.

And: the designation of the truth values as objects may appear a wilful fancy, a mere play on words, from which no far-reaching consequences may be drawn; so much should be clear, however,

in every judgment, even the most self-evident, the step from the stage of the thought to the stage of the reference has already occurred.

From the thought and the sense to the reference and the true.

It is another description of the same path. The path by which a proposition is gradually won, the path of the knowledge of a scientific truth, the path toward scientific goals, the path toward the firmest proof, the path toward complete certainty.

This description, however, relates explicitly what precedes the path, the thought and the sense, and the path, the reference and the true. Hence, all other descriptions can be made more explicit and sorted out in relation to one another. And hence, what the problem of the uniformity of the acquisition of knowledge is, and what the solution of the problem is, can be made more explicit.

A proposition is won.

With Aristotle: not every sentence is a proposition; a proposition is a sentence which affirms what is true or false. And: to affirm of what is that it is not, and of what is not that it is, is false, to affirm of what is that it is, and of what is not that it is not, is true.

With Frege: all false propositions have the same reference, the false, and all true propositions have the same reference, the true. And a proposition is true when every one of its parts has a reference.

Moreover: a proposition stops being true when I no longer think of it no more than the sun is annihilated when I close my eyes. And: even if all rational beings went into hibernation simultaneously, the truth of a proposition would not be suspended for the duration.

Taking it for granted, then, that the false can need no winning, to win a proposition can only be to win a true proposition. And if so, there can only be one way to win a proposition. To win a proposition can only be to discover it. And that is, to discover that every one of its parts has a reference, and hence the proposition has a reference, the true.

And again there can only be one way to do so. To do so can only be to discover the connection of one proposition with another. As Frege describes it: to bring a proposition in connection with other truths through chains of inferences.

There can be no other way because reference connects propositions. The reference of all true propositions is the same. And the reference of every part of every true proposition is the reference of that part of every other true proposition in which that part has a reference. If one proposition is discovered, then, its connection with all other propositions is discovered. Or what has to be the same, if one proposition is won explicitly, all other propositions are won implicitly. If so, to win one proposition explicitly can only be to win all other propositions implicitly. And if so, to win any one proposition explicitly can only be to make the connection of this proposition with other propositions explicit.

If so, a proposition can only be won gradually. To win a proposition cannot not be a path. The path cannot begin with the winning of a single proposition either. It can only begin with the winning of several propositions. And it can only proceed from the winning of several propositions to the winning of more propositions to the winning of more propositions to the winning of all propositions.

So, the description, from the thought and the sense to the reference and the true, is of what precedes the path and the path. Given the thought and the sense, the path has not begun. The path has begun with the reference and the true. The path is also in its entirety the reference and the true. It is in its entirety as well the clearing away of the thought and the sense from the reference and the true.

The description, by which a proposition is gradually won, is of the path in its entirety and in all its generality. Every step of the path, from the first to the last, is the winning of a proposition.

The two descriptions, of the knowledge of a scientific truth, and toward scientific goals, are of later stages of the path. Of the stages when, having begun with the winning of several propositions, and that is, bringing them in connection with one another through chains of inferences, and proceeded to the winning of more propositions, and more, and that is, again, bringing them in connection with one another through chains of inferences, the path has thus reached the stage of the development of theories.

The last two descriptions, toward the firmest

proof, and toward complete certainty, are of the latest stages of the path. When laws are discovered, and the end, the winning of all propositions, is clearly in sight.

The stages of the path are stages of proof. Proof is to bring a proposition in connection with the truths which are its consequences and of which it is the consequence.

And the end is reached, proof is the firmest, and complete certainty is attained, when the chains of inferences are free of gaps. When no link is missing.

It is all in evidence in the development from the Venus tablet of Ammisaduqa through Kepler's laws.

The development begins with the winning of the true. It thus begins with the discovery of a reference, thus with the winning of propositions, and of several propositions, not one, and thus by bringing the propositions in connection with one another through chains of inferences.

Thus, if the development begins also with observation, it shows that observation is not looking, or seeing, observation is measurement and calculation, it is essentially mathematical, and essentially formal, because observation is the winning of propositions, and by bringing them in connection with one another through chains of inferences. In the first year, Venus disappeared in the west in the 11th month, 15th day, remained hidden for 3 days, and appeared in the east in the 11th month, 18th day. In the second year, Venus disappeared in the east in the 8th month, 21st day, remained hidden for 1 month and 21 days, and appeared in the west in the 10th month, 16th day. And through the twenty-first year.

The development continues with the winning of more propositions, and more, and it is the same development when it continues with the labors of the philosophers. Aristotle reports: the observations of the Egyptians and the Babylonians have been kept for many years and from these much of our evidence is derived. And the oldest observation in the *Almagest* was made one and a half centuries before the first philosopher, in 721, in Babylon.

The development has reached the stage of theories. The stage of all the propositions brought through chains of inferences in connection with one another in the *Almagest*, and through chains of inferences in connection with all the propositions in the *Planetary hypotheses*, themselves brought in connection with one another through chains of inferences, and through chains of inferences in connection with all the propositions in the *Geography*, themselves brought in connection with one another through chains of inferences, and in connection with all the propositions in the *Planetary hypotheses*, and through chains of inferences in connection with all the propositions in the *Handy tables*, themselves brought in connection with one another through chains of inferences, and in connection with all the propositions in the *Geography*, and in connection with all the propositions in the *Planetary hypotheses*, and in connection with all the propositions in the *Almagest*.

The development has reached the stage of the discovery of laws. Deduction from Kepler's laws shows when Venus is discovered, the discovery corroborates the laws, more generally, the unearthing of the early record has corroborated Kepler's laws, and the laws have fixed the chronology of the early record. Showing that to win one proposition explicitly is to win all other propositions implicitly, and that to win a proposition explicitly is to make the connection of this proposition with other propositions explicit.

All stages of the development are proofs. Bringing propositions in connection with the propositions which are their consequences and of which they are the consequences.

And the end, the firmest proof, complete certainty, and the winning of all propositions, is clearly in sight. Kepler discovered laws, the laws of the motions of the heavenly bodies. His contemporary Galileo affirmed that the world is written in mathematics. Galileo writes also: in understanding any one proposition perfectly, the human intellect has absolute certainty, such as nature itself has; in the mathematical sciences the human intellect is equal to the divine in objective certainty, for it comprehends necessity, beyond which there can be no greater certainty (*Systems*, I). In the next generation Descartes formulated the rules of the method. Laboring on *The method*, he labored, he reports, on: the project for a general science capable of raising our nature to its

highest degree of perfection (to Mersenne, Mar 1636). Not quite two generations later Leibniz conceived the philosophical calculus. Hence: there will be no more need for disputes between philosophers, it will suffice to take their pens, sit down to their abaci, and calculate.

And these propositions, the laws, the language of the world, the method, and the calculus, are all in connection with one another, too.

ci

It is all in evidence in the development from the Venus tablet of Ammisaduqa through the first philosophers through Aristotle through Ptolemy through Galileo, Kepler, Descartes, Leibniz and further.

It is all in evidence implicitly. Not less in evidence, however, is the difficulty of showing that it is all in evidence explicitly.

To win a proposition is to discover that every one of its parts has a reference. To do so is to discover the connection of propositions with one another. To do so is to bring the propositions in connection with one another through chains of inferences.

To win a proposition explicitly is to win all other propositions implicitly.

To win a proposition is a path. From several propositions to more propositions to more propositions to all propositions.

Every step of the path is the winning of a proposition.

Every stage of the path is proof. Bringing propositions in connection with the propositions which are their consequences and of which they are the consequences.

At the end, proof is the firmest, complete certainty is attained, all propositions are won. The chains of inferences are free of gaps. No link is missing.

And this is the problem of the uniformity of the acquisition of knowledge.

Whatever else it is – the firmest proof, complete certainty, and the winning of all propositions – the end of the path cannot not be knowledge, inclusive of everything else which it is.

The path is thus the acquisition of knowledge.

The acquisition of knowledge is thus uniform. Every step of the path is the winning of a proposition. Every stage of the path is proof.

And the problem is to show this, to show that the acquisition of knowledge is uniform.

The difficulty of the problem can need no recommendation. Not, if to win this proposition, the acquisition of knowledge is uniform, is, like the winning of any other proposition, to bring propositions in connection with one another through chains of inferences. The more so, if to win this proposition by proof is, like the winning of any other proposition by proof, to bring propositions in connection with the propositions which are their consequences and of which they are the consequences. And the more so, if to win this proposition by the firmest proof is, like the winning of any other proposition by the firmest proof, to bring propositions in connection with the propositions which are their consequences and of which they are the consequences through chains of inferences free of gaps.

In particular, to begin with, the Venus tablet of Ammisaduqa is not *ex nihilo*. The difficulty is to find the propositions which led to these propositions, and show how and why those led to these. And that is, to bring those in connection with these through chains of inferences, and show how and why these are the consequences of those, or those are the consequences of these. If there are such propositions, propositions which led to these propositions. If there are no such propositions, to show how and why there are no such propositions. Either way, to find the first propositions won, and show how and why they are the first propositions won.

Further, nine centuries separate the Venus tablet of Ammisaduqa and the oldest observation in the *Almagest*. The difficulty is, again, to find the propositions which connect these propositions, and show how and why those connect these, and that is, how and why these are the consequences of those, or those are the consequences of these. The difficulty is greater, also, because in these nine centuries the development has become increasingly differentiated. The path is followed, or not followed, in different places at different times at different paces for different lengths of time. In some places at some times many propositions are won, in the same places at other times no propositions are won, in other

places at the same times other propositions are won, and the difficulty is to show that across all places and all times all propositions are won uniformly, and that is, to show that they are all brought in connection with one another through chains of inferences, and show it by bringing them all in connection with one another through chains of inferences.

Further, it is easier to follow the steps of the path in the seven centuries from Thales to Ptolemy. Easier, but not easy. Two and a half centuries separate even Hipparchus and Ptolemy, and it is not easy to show even how and why the labors of Ptolemy, both, continue and are yet separated by two and a half centuries from the labors of Hipparchus. The difficulty is greater, also, because at this time the path reaches the stage of firmer proof, and of theories. But theories are not just collections of firm proofs. They are collections of firm proofs, and the firmest proofs, and not very firm proofs, and surmises from an insufficient number of particular cases, and errors, and foolishness. And if the path is in its entirety the clearing away of the thought and the sense from the reference and the true, the difficulty is to show that the path is in its entirety the clearing away of the thought and the sense, and show it by the clearing away of the thought and the sense. And that is, once again, by bringing the propositions contaminated by the thought and the sense in connection with one another through chains of inferences. It is part of the difficulty that the theories have reached far. The propositions of the *Almagest*, *Planetary hypotheses*, *Geography*, and *Handy tables* are all in connection with one another, but their connections as well range from the firmest proofs through not very firm proofs through surmises from an insufficient number of particular cases.

Further, fifteen centuries separate Galileo and Kepler from Ptolemy, and fourteen centuries separate Copernicus from Ptolemy, with the darkest of dark ages in between. The difficulty is the same, to clear away the thought and the sense, and the images saturated with feelings, and bring the propositions in connection with one another through chains of inferences.

Beginning with Bacon, Galileo, and Kepler, the steps of the path are again easier to follow. And again not easy. The theories are many, and more, and more, and reaching further, and further. The thought, the sense, and the images saturated with feelings are also more, and more, and reaching further, and further.

All along there are the barriers before the acquisition of knowledge, the labors at overcoming the barriers, the labors at reinforcing the barriers, the labors at overcoming the reinforcement of the barriers, and again, and again.

Proof is the firmest, and complete certainty is attained, when the chains of inferences are free of gaps. When no link is missing.

This is the end of the path.

And this has to be the solution of the problem of the uniformity of the acquisition of knowledge.

If the problem is to show that every step of the acquisition of knowledge is the winning of a proposition, and every stage is proof, and proof admits of degree, it is firm, and firmer, and firmer, and the firmest, to win conclusively the proposition, the acquisition of knowledge is uniform, has to be to bring all propositions ever won in connection with one another through chains of inferences free of gaps.

cii

Reference, propositions, chains of inferences, and truth show the uniformity of the acquisition of knowledge.

But the reference is the object. The true is an object. Propositions are functions. Chains of inferences are chains of functions.

These are Frege's two most fundamental discoveries. That propositions are functions. And that object is all that is not function.

More generally, then, function and object show the uniformity of the acquisition of knowledge.

Frege writes in the *Concept writing*:

if we think the circumstance, that hydrogen is lighter than carbon dioxide, expressed in our formula language, we can set in the place of the sign for hydrogen the sign for oxygen or that for nitrogen; the sense thus changes so that 'oxygen' or 'nitrogen' enters in the relations in which 'hydrogen' stood before; if an expression is thought changeable in this way, it decomposes into a stable component, which represents the

entirety of the relations, and the sign, which is thought as replaceable by other signs, and which refers to the object that finds itself in these relations; the first component I name function, the latter its argument (9);

the proposition, Cato killed Cato, shows the same; if we think of 'Cato' as replaceable at its first occurrence, the function is 'to kill Cato'; if we think of 'Cato' as replaceable at its second occurrence, the function is 'to be killed by Cato'; if we think finally of 'Cato' as replaceable at both occurrences, the function is 'to kill oneself' (*id.*);

we now express the matter generally;

if in an expression, whose content need not be judgeable, a simple or a compound sign occurs in one or more places, and we think the sign replaceable in all or some of these places by another sign, but everywhere by the same sign, we name the part that remains unchangeable in the expression function, the replaceable part its argument (*id.*);

to express an indeterminate function of the argument A, we let A enclosed in parentheses follow a letter, for instance,

$$\Phi(A);$$

likewise,

$$\Psi(A, B)$$

refers to a function of the two arguments A and B, which are not determined further (10).

And more generally still, and more explicitly, in *Function and concept*:

my starting point is what is named function in mathematics;

with the word 'function' it is usual to think of expressions in which a number is indicated only indefinitely by the letter x, as in

$$'2 \,.\, x^3 + x';$$

x is named the argument; and in

$$'2 \,.\, 1^3 + 1',$$
$$'2 \,.\, 4^3 + 4',$$
$$'2 \,.\, 5^3 + 5',$$

the same function is recognized, only with different arguments, namely 1, 4, and 5; from this it can be seen that the essence of the function is in what is common in these expressions, what is present as well in

$$'2 \,.\, x^3 + x'$$

apart from the 'x', and what we can write thus

$$'2 \,.\, (\,)^3 + (\,)';$$

what I aim to show is that the argument does not belong to the function, but builds together with the function a complete whole; because the function by itself is incomplete, in need of supplementation, unsaturated;

thus, when I say 'the function $2 \,.\, x^3 + x$', x should not be considered to belong to the function; this letter only serves to indicate the kind of the need for supplementation by pointing to the places where the sign of the argument is to enter;

we name that by which the function is completed by the argument the value of the function for this argument; for instance, 3 is the value of the function $2 \,.\, x^3 + x$ for the argument 1, because $2 \,.\, 1^3 + 1 = 3$;

we shall not stop at equations and inequalities; the linguistic form of equations is the affirmative proposition;

and affirmative propositions in general, just like equations or analytic expressions, can be thought decomposed into two parts, of which one is complete in itself, the other is in need of supplementation, or unsaturated; thus, for instance, the proposition

'Caesar conquered Gaul'

can be decomposed into 'Caesar' and 'conquered Gaul'; the second part is unsaturated, introduces an empty place, and only when this place is filled by a proper name, or an expression representing a proper name, a complete sense appears; here as well I name the reference of the unsaturated part function; the argument is in this case Caesar;

we see that the extension of the concept concerns also what can occur as an argument; not merely numbers now, but objects in general are admissible; persons I must in any case count among the objects; the two truth values have already been introduced as possible values of a function; we must go further and admit objects without restriction as values of a function; to introduce another instance, the expression

'the capital of the German empire'

obviously represents a proper name and refers to an object; if we now decompose it into the parts

'the capital of'

and 'the German empire', the first part is unsaturated, while the second is complete in itself; accordingly, I name

'the capital of x'

the expression of a function; and if we take as its argument the German empire, we have as the value of the function Berlin;

when we have thus admitted as arguments and values of functions objects without restriction, the question arises of what is here named object; a school definition I hold for impossible, because we have here something which, due to its simplicity, does not admit of logical decomposition; it is only possible to point to what is meant; and it can be said briefly, object is all that is not function, whose expression therefore brings with it no empty place (*id.*).

ciii

Frege, then, showed what function is.

What object is.

What proposition is.

Function is the unchangeable in an expression. The changeable is its argument. A function and its argument are a complete whole. Completed by an argument, a function has a value for this argument.

Due to its simplicity object does not admit of logical decomposition. Object is all that is not function.

The elements of a proposition are function and argument.

Indiscriminately propositions, hydrogen is lighter than carbon dioxide, $2 . x^3 + x = y$, and Caesar conquered Gaul, are indiscriminately functions and arguments.

The power of the analysis of propositions into functions and arguments cannot be overstated.

Thus, if the proposition, hydrogen is lighter than carbon dioxide, is analyzed into the function

x is lighter than y,

where x indicates the argument and y indicates the value, hydrogen is the argument and carbon dioxide is the value. Not the function, hydrogen is an object. So is carbon dioxide. The proposition can also be analyzed into the function

y is lighter than x,

and carbon dioxide is then the argument, hydrogen is the value. Not the function, carbon dioxide is an object, and so is hydrogen. The proposition can be analyzed also into the function

hydrogen x y,

and is lighter than is then the argument, carbon dioxide is the value, or into the function

hydrogen y x,

and carbon dioxide is then the argument, is lighter than is the value. Not the function, and whether it is the argument or the value, is lighter than is an object. And is lighter than is an excellent object. Because weight is an excellent object. Newton fixed the reference of 'weight', it is the force of attraction at the surface of the earth. The proposition can be analyzed also into the function

x y carbon dioxide,

or

y x carbon dioxide,

and if is lighter than is the argument, hydrogen is the value, if hydrogen is the argument, is lighter than is the value.

If in the proposition, $2 . x^3 + x = y$, the function is

$2 . x^3 + x = y$,

for the argument 1 the value of the function is 3. Not the function, 1 and 3 are objects. The proposition can also be analyzed into the function

$x . a^3 + a = y$,

and then 2 is the argument, $2 . a^3 + a$ is the value. Or into the function

$2 . a^3 (x)\, a = y$,

and then + is the argument. And + is an excellent object, because addition is an excellent object. The value is again $2 . a^3 + a$. It can also be analyzed into the function

$x = y$,

and then $2 . ()^3 + ()$ is the argument. And $2 . ()^3 + ()$ is an excellent object as well, and its part $()$ is an excellent object, too, if, indicating the kind of the need for supplementation, $2 . ()^3 + ()$ indicates a number, and $()$ indicates a number. Or what is the same, if the reference of '$2 . ()^3 + ()$' is a number, and the reference of '$()$' is a number. The value is the same as the argument.

That hydrogen is lighter than carbon dioxide is a proposition, and that $2 . x^3 + x = y$ is a proposition, are themselves propositions. If the function is

that y is a proposition is x,

propositions are the arguments, and hydrogen is lighter than carbon dioxide and $2 . x^3 + x = y$ are the values. And proposition is another excellent object. Aristotle fixed the reference of 'proposi-

tion', and Frege fixed it more generally.

The proposition that hydrogen is lighter than carbon dioxide is a function, and the proposition that $2 . x^3 + x = y$ is a function, are themselves functions. If the function is

that y is a function is x,

it is a function whose arguments are functions. Frege named such functions second-level functions. If the indeterminate first-level function of the argument x is expressed $f(x)$, for instance, the indeterminate second-level function of the argument $f(x)$ is expressed $g(f(x))$, for instance. Not the function, not the second-level function, in particular, a function, the first-level function, in particular, is now an object. And function is another excellent object. Frege fixed the reference of 'function', it is the unchangeable in an expression.

The proposition, Caesar conquered Gaul, can be analyzed into the function

x conquered y,

with Caesar as the argument and Gaul the value, or the function

y conquered x,

with Gaul as the argument and Caesar the value. It can be elaborated, Caesar conquered Gaul in 58-1, and this proposition can then be analyzed into the function

x_1 conquered x_2 in y,

with Caesar now the first argument, Gaul the second argument, 58-1 the value. The original proposition, Caesar conquered Gaul, can be analyzed also into the functions

$y\ x$ Gaul,

and

Caesar x y,

and conquered is then the argument. Not the function, conquered is an object, and conquered is another excellent object. Because conquest is an excellent object. Caesar did not fix the reference of 'conquest' single-handedly, but did make an essential contribution to fixing it, and when he conquered Gaul as well as when he described how he conquered Gaul in the *Commentaries on the Gallic war*.

A proposition is true when every one of its parts has a reference. When the function has a reference, the argument has a reference, the value as well has a reference, and all their parts each has a reference. Because 'hydrogen', 'is lighter than', and 'carbon dioxide' each has a reference, the reference of the proposition 'hydrogen is lighter than carbon dioxide' is the true. Because '2', '.', '^{3}', '+', and '=', each has a reference, if the reference of 'x' is 1, the reference of the solution of the function – of the proposition '$2 . x^3 + x = 3$' – is the true. Because 'Caesar', 'conquered', and 'Gaul' each has a reference, the reference of the proposition 'Caesar conquered Gaul' is the true.

Particular propositions and general propositions are functions and arguments indiscriminately. Hence, with Frege, the proposition, the evening star is the morning star, can be analyzed to instance the proposition, $a = b$. The two functions are

x is y

and

$x = y$,

or

y is x

and

$y = x$,

or

the evening star $x\ y$

and

axy,

or

$y\ x$ the morning star

and

yxb.

In any case, the references of 'a' and 'b' are objects among which are the objects to which 'the evening star' and 'the morning star' refer, and the reference of '=' in the general proposition either is the same as the reference of 'is' in the particular proposition or includes it. In any case, the function, the evening star is the morning star, is a solution of the function, $a = b$.

When the general proposition is analyzed into the function

axy

or the function

yxb,

= is the argument. And = is another excellent object, because equality is another excellent object. The reference of 'equality' has been difficult to fix explicitly. It has been fixed implic-

itly, however, by every solution of the function $a = b$.

When the particular proposition is analyzed into the function

the evening star $x\ y$

or the function

$y\ x$ the morning star,

is is the argument. And is is another excellent object. Because the reference of 'is' can be restricted to be the same as, or included in, the reference of 'is equal to'. And because if it is not so restricted, it is, at its widest, is, and existence is another excellent object, unappreciated as it has remained as either excellent or an object by idealism and scepticism. The reference of 'existence' has also been difficult to fix explicitly. It has been fixed implicitly, however, by every solution of every function.

We must admit as arguments and values of functions objects without restriction.

Nor can the generality or the importance of this discovery be overstated.

Its generality appeared threatened in 1902.

In 1902, Russell writes to Frege: let *w* be the predicate, to be a predicate that cannot be predicated of itself; can *w* be predicated of itself? from each answer its opposite follows.

Frege replies: your discovery of the contradiction has caused me the greatest surprise and, I would almost say, consternation, because it rocks the foundation on which I thought to build arithmetic.

Russell continues in his letter: it must be concluded that *w* is not a predicate; there is also no set, as a whole, of those sets which as a whole do not belong to themselves; from this I conclude that under certain circumstances a definable set does not constitute a whole.

Cantor, in the generation of Frege, developed the theory of sets. And why the theory of sets has become the foundation of arithmetic, and of logic, and of mathematics such as it rests on arithmetic and logic, and of physics such as it rests on mathematics, and of the acquisition of knowledge in its entirety such as it rests on mathematics and physics, the generality of Cantor's definition of set shows.

Cantor writes: by set we understand every collection into a whole of definite well-distinguished objects of our sensibility or our thought.

Russell's conclusion that there is a definable set which does not constitute a whole threatened the generality of: every collection of objects.

It threatened it for six days, the six days separating Russell's letter and Frege's response.

In consternation although he was, Frege continues in his response: it seems to me, however, that the expression 'a predicate is predicated of itself' is not exact; a predicate is as a rule a first-level function, which demands as an argument an object and cannot therefore have itself as an argument; I would therefore rather say 'a concept is predicated of its own extension'; if the function $\Phi(\xi)$ is a concept, I indicate its extension, or the corresponding set, by '$\grave{\varepsilon}\ \Phi\ (\varepsilon)$'; and in '$\Phi(\grave{\varepsilon}\ \Phi(\varepsilon))$', or '$\grave{\varepsilon}\ \Phi(\varepsilon) \cap \grave{\varepsilon}\ \Phi(\varepsilon)$', we then have the predication of the concept $\Phi(\xi)$ of its own extension (22 Jun).

$\cap$ connects the first-level functions into which the second-level function is resolved.

Frege thus showed the ground on which the apparent contradiction will be resolved: a first-level function demands as an argument an object and cannot therefore have itself as an argument.

And it is on this ground, on the rule, in particular: whatever involves all of a collection must not be one of the collection, that Russell resolved the contradiction with the theory of types.

And it is on this ground, too, that in 1908, in the *Investigations on the foundations of the theory of sets*, Zermelo axiomatized the theory of sets.

Set is every collection of objects.

Formally, the definition has to be more explicit and add, where objects are concerned, definite well-distinguished. Just as the propositions standing on the definition have to make more explicit what definite well-distinguished is. It has to be more explicit and add, where collection is concerned, such as obeys Zermelo's seven axioms: 1. of definiteness, 2. of elementary sets, 3. of separation, 4. of power set, 5. of union, 6. of choice, and 7. of infinity (1). Such as obeys also the one axiom added since to Zermelo's seven, of replacement, to extend the reach of the axiomatization. Just as the propositions standing on the definition have to make more explicit what the axioms affirm.

None of which should be allowed to obscure the fact that set is every collection of objects.

Or the fact that we must admit as arguments and values of functions objects without restriction.

civ

Function and object show the uniformity of the acquisition of knowledge.

Frege's examples can illustrate the problem, its difficulty, and the solution of the problem.

The evening star is the morning star.

And the predecessor of the philosophers who has to go under the name Venus tablet of Ammisaduqa discovered that the evening star is the morning star.

It is an instance of the acquisition of knowledge. It can be analyzed into the three parts, that predecessor of the philosophers, discovered, and that the evening star is the morning star. And it is then a function of one argument, $y = f(x)$. The function is

x discovered y,

the argument is that predecessor of the philosophers, and the value is that the evenings star is the morning star.

The argument is a definite well-distinguished object. Indeed, not only is that predecessor of the philosophers shown a definite well-distinguished object by his definite well-distinguished labors in a definite well-distinguished time, the observation of Venus in the twenty-one years of Ammisaduqa's reign. In conformity with the goal of physics defined by Planck, the labors of that predecessor of the philosophers have been emancipated from the individuality of the picturing mind so completely, they are not contaminated by the record even of whether he was one individuality or several.

The value is also a definite well-distinguished object, and whether the discovery is taken narrowly, as the discovery of the identity of the evening star and the morning star alone, or more broadly, as it should be, as the discovery of Venus, *ipso facto* of a planet, and *ipso facto* of the planets. The identity of the evening star and the morning star is a definite well-distinguished object, Venus is a definite well-distinguished object, a planet is a definite well-distinguished object, and the planets are definite well-distinguished objects. Indeed, the evening star is the morning star is the third brightest heavenly body, and if any objects are given immediately to the senses as definite well-distinguished, the heavenly bodies are. And the planets are not only definite well-distinguished objects, they are the members of a set, the planets in the solar system, defined as clearly and distinctly by the world as any set is.

If the function itself, finally, is, discovered, it is more specifically observation. Observation is propositions and chains of inferences. Several dozen propositions, in particular, connected with one another through chains of inferences as free of gaps as to describe a series of twelve consecutive synodic periods of the planet.

Hydrogen is lighter than carbon dioxide.

And Cavendish discovered that hydrogen is lighter than carbon dioxide.

In 1766, in the *Three papers containing experiments on factitious air*, Cavendish concludes: if the density of common air, at the time of the experiment, was 800 times less than that of water, inflammable air must be 5 490 times lighter than water, or nearly 7 times lighter than common air (1), and: if the outward air, at the time the experiment was tried, is supposed to have been 800 times lighter than water, fixed air is 511 times lighter than water, or 1.57 times heavier than common air (2). Inflammable air is hydrogen, common air is air, fixed air is carbon dioxide, and hydrogen is 11 times lighter than carbon dioxide.

The function itself is the same, discovered.

The argument is now Cavendish, and Cavendish is a definite well-distinguished object, shown such by his definite well-distinguished labors in a definite well-distinguished time, the *Experiments on factitious air* in 1766.

The value is now that hydrogen is lighter than carbon dioxide, and it is a definite well-distinguished object as well. Hydrogen is the first element in the periodic table, and thus the member of a set, the elements in the periodic table, defined as clearly and distinctly by the world as the set of the planets in the solar system is. If the elements are definite well-distinguished objects, so are their compounds. If the elements and their compounds are definite well-distinguished ob-

jects, so are their relations. And that the relation hydrogen is lighter than carbon dioxide is a definite well-distinguished object the fact of its definite well-distinguished determination shows. 11 times lighter. Or, as it has been determined more explicitly and more precisely after Cavendish, a molecule of hydrogen is 22 times lighter than a molecule of carbon dioxide.

The function is now more specifically experiment. Experiment is a kind of observation. And how free of gaps the chains of inferences now are, the measurements and calculations in Cavendish's conclusions should suffice to show.

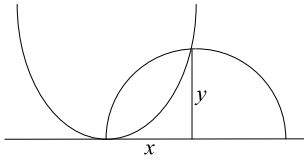
2. $x^3 + x = y$.

It is a cubic equation. And Archimedes solved a cubic equation implicitly, Diophantus solved a cubic equation explicitly, Khayyam solved cubic equations more generally, Cardano formulated a general method for solving cubic equations, and Descartes formulated the theory of polynomial equations.

In *On the sphere and cylinder*, Archimedes solves the problem: to cut a sphere by a plane so that the volumes of the segments are to one another in a given ratio (II, 4). He has demonstrated that: the volume of a segment of a sphere is equal to the volume of a cone with a base equal to the surface of the segment and a height equal to the radius of the sphere (I, 44), and solves the problem of the two segments in a given ratio by finding the cones to which they are equal. He concludes, however: the problem is equivalent to solving the proportion $(r+x) : a :: b : (2r-x)^2$, where x is the height of one of the segments, r is the radius of the sphere, a is a given length, and b is a given area (II, 4). And the proportion implies the equation $x^3 + Bx^2 = D$, where $B = -3r$, and $D = ab - 4r^3$.

Five centuries later, in the *Arithmetic*, Diophantus solves the problem: to find a right triangle such that the sum of the area and the hypothenuse is a square, and the perimeter is a cube (VI, 17). In particular, if the sides of the triangle are 2 and a, the side of the square is $x+1$, and the side of the cube is $x-1$, the problem is equivalent to the system of equations $\frac{2a}{2} + \sqrt{2^2 + a^2} = (x+1)^2$ and $2 + a + \sqrt{2^2 + a^2} = (x-1)^3$, the two equations yield $2 + (x+1)^2 = (x-1)^3$, and Diophantus writes explicitly: therefore $x^3 - 3x^2 + 3x - 1 = x^2 + 2x + 3$, and x is 4 (*id.*). Or $x^3 + Bx^2 + x = D$, where $B = -4$, and $D = 4$.

Eight-odd centuries later, in the *Demonstration of problems of algebra*, Khayyam solves several problems of this kind: the sum of a cube and a number of its sides equals a number (*id.*). In particular: draw a parabola; on the tangent to



its vertex, and from its vertex, draw a semicircle; drop a perpendicular from the intersection; hence, from the properties of the parabola, $x^2 = Ly$, where L is the latus rectum; from the properties of the circle, $y^2 = x(2r-x)$, where r is the radius; and solving for y, $x^3 + Cx = D$, where $C = L^2$, and $D = 2L^2 r$; to solve the cubic equation is to draw a parabola with latus rectum L and a semicircle with radius r (*id.*).

Four and a half centuries later, in 1545, in *The great art*, Cardano solves all forms of the cubic equation. He begins with: the sum of a cube and a first power is equal to a number (11), and to solve it, has the rule: cube one third of the coefficient of x; add the square of one half of the constant of the equation; take the square root of the whole; duplicate this, and to one of the two add one half of the constant, from the other subtract one half of the constant; subtract the cube root of the second from the cube root of the first; the remainder is x (*id.*). If $x^3 + Cx = D$,

$$x = \sqrt[3]{\sqrt{\left(\frac{C}{3}\right)^3 + \left(\frac{D}{2}\right)^2} + \frac{D}{2}} - \sqrt[3]{\sqrt{\left(\frac{C}{3}\right)^3 + \left(\frac{D}{2}\right)^2} - \frac{D}{2}}.$$

And in 1637, in the *Discourse on the method*, in its last part, the *Geometry which is an essay in this method*, Descartes formulates the theory of polynomial equations. The fundamental result is: every equation can have as many distinct roots as the number of dimensions of the unknown quantity (III). To solve equations, Descartes collects all terms on one side, $Ax^3 + Bx^2 + Cx + D = 0$, for instance, and recognizes in the solution all real roots, positive or negative.

Archimedes, Diophantus, Khayyam, Cardano,

and Descartes solved the cubic equation.

The function is the same.

The argument are now Archimedes, Diophantus, Khayyam, Cardano, and Descartes, each shown a definite well-distinguished object by his definite well-distinguished labors in a definite well-distinguished time, *On the sphere* in the lifetime of Archimedes, *c.* 287 - 212^{BC}, the *Arithmetic* in the lifetime of Diophantus, who wrote in the generation before 276-82^{AD}, the *Problems of algebra* in the lifetime of Khayyam, 1048 - 1123/31, *The great art* in 1545, and *The method* in 1637. The plurality of the objects has begun to capture more of the complexity of the argument of the function. The five objects have also begun to fill the thirty-four century gap between the Venus tablet of Ammisaduqa and Cavendish.

The value is the cubic equation. It is as definite well-distinguished an object as only mathematical objects have been shown. The plurality of the objects which have led to the discovery of this object, sphere, plane, volume, segment of a sphere, ratio, proportion, triangle, sum, area, square, perimeter, cube, product, number, parabola, tangent, intersection, power, constant, root, equation, has begun to capture more of the complexity of the value of the function as well. And these objects have begun to fill the gaps between what was discovered in the three instances as well. In all three instances what was discovered involves numbers explicitly, for instance.

And the function itself is the same. The relation of discovery and the solution of a problem, including the solution of an equation, cannot be exclusion. More particularly, the relation of observation, including experiment, and the solution of a problem, including the solution of an equation, cannot be exclusion. The function is in any case more specifically propositions and chains of inferences now as well. Chains of inferences as free of gaps as only mathematical solutions of mathematical problems have been.

Whether it is discovered, including solved, or solved, including discovered, the function is, in general,

x acquired knowledge of *y*.

More of the complexity of the function has begun to be captured, and more can be.

His successors in the following thirty-eight centuries established that that predecessor of the philosophers discovered that the evening star is the morning star.

The function is the same.

Among the arguments are Kepler and Rawlinson. And to show that each of the arguments, each of the successors of that predecessor of the philosophers who established that he discovered that the evening star is the morning star, is a definite well-distinguished object cannot be too difficult. The difficulty is to collect them all. To collect all the relevant, and only the relevant, objects, many as they are, of varying importance, of varying distribution in time, of varying distribution in space, of varying distribution across different kinds of the acquisition of knowledge.

The value is that that predecessor of the philosophers discovered that the evening star is the morning star. The difficulty of showing this value a definite well-distinguished object can need no recommendation. The difficulty should not be impossible to resolve, however, if the difficulty of showing definite well-distinguished the objects which led to the discovery of this object can be resolved. And the formulation of the theory of the motions of the planets, and the discovery of the laws of the motions of the planets, and the decipherment of cuneiform should not be impossible to show definite well-distinguished objects, if the identity of the evening star and the morning star, and the relation hydrogen is lighter than carbon dioxide, and the solution of the cubic equation can be shown definite well-distinguished objects. Also, if the resolution of this difficulty depends on the resolution of that of collecting all the relevant, and only the relevant, objects which are the arguments of the function, if that difficulty can be resolved, this difficulty can be resolved.

The function itself is the same, acquired knowledge of.

When Cavendish discovered that hydrogen is lighter than carbon dioxide, both, Cavendish discovered more than that, and more than Cavendish discovered that.

The *Experiments on factitious air* which demonstrated that inflammable air is lighter than common air and common air is lighter than fixed air, in 1766, were the decisive step in the demonstration that air is not an element. The demonstra-

tion can be traced back to van Helmont. In the *Rise of medicine, or beginnings of a new physics*, in 1648, van Helmont produced, by combustion and fermentation, gas from solid. The gas is: a spirit, hitherto unknown, which I call by the new name of gas (19). Black, in the *Experiments on magnesia alba, quicklime, and some other alkaline substances*, in 1756, produced a quantity of gas and a quantity of solid from a quantity of solid, then produced the original quantity of the original solid from the resultant quantities of the resultant gas and the resultant solid. The gas is: fixed air (2). Black thus discovered that fixed air, one of the many kinds of spirit, or gas, produced by van Helmont and after him, is a distinct air, *ipso facto* discovered fixed air, and *ipso facto* demonstrated that fixed air is distinct from air. When Cavendish demonstrated that inflammable air is 11 times lighter than fixed air, he discovered that factitious air is of different kinds, that inflammable air is one kind, fixed air is another kind, both are different in kind from common air, and *ipso facto* discovered inflammable air. That the conclusive step in the demonstration that air is not an element remained to be taken, the multitude of airs with which Cavendish still has to deal shows. Lavoisier took it, in the *Elementary treatise of chemistry*, in 1789. Inflammable air became: hydrogen (I, 8), fixed air became: carbonic acid (6), hydrogen, carbon, and oxygen, and not carbonic acid, became 3 of the 33 simple substances in Lavoisier's: table of the simple substances. In 1869, in the *Correlation of the properties and atomic weights of the elements*, they became 3 of the 63 elements in Mendeleev's: general system of the elements. Carbonic acid had in the meantime become carbon dioxide.

The third philosopher, Anaximenes, had proposed that air is the principle of all things, a century later Empedocles had proposed that air is one of the four roots of all things, a century later Aristotle had proposed that air is one of the five material principles of all things, and so air had remained for the following nineteen centuries.

The instance points to the main difficulty with respect to the values of the function. The main difficulty is reality. Air, spirit, fixed air, factitious air, inflammable air, hydrogen, carbon, oxygen, carbonic acid, carbon dioxide, gas, solid, the principles of all things, the roots of all things, the material principles of all things, the simple substances, the elements, the table of the simple substances, the general system of the elements, all have to be shown definite well-distinguished objects. And if they are all definite well-distinguished objects, they are all real. Factitious air such as Cavendish experimented with it is real. Spirit such as van Helmont produced it is real. The roots of all things such as Empedocles sang of them are real.

Again, this difficulty should not be impossible to resolve, if the difficulty of collecting all the relevant, and only the relevant, objects which are the arguments of the function can be resolved. And that difficulty cannot be impossible to resolve, if the objects themselves can guide the collection. Thus, Aristotle, in book I of the *Metaphysics*, collects the relevant labors of Anaximenes and Empedocles (3). Lavoisier, in the *Physical and chemical essays*, in which, Lavoisier reports: I have confined myself to the character of the historian (I, intr.), collects the relevant labors of van Helmont (1), Black (7), and Cavendish (10).

And with respect to the arguments of the function reality cannot be a difficulty. The objects which are the arguments of the function cannot not be all real.

For this reason alone the resolution of all difficulties should begin with these objects.

The main difficulty with respect to these objects is relevance. It is, again, to collect all the relevant, and only the relevant, objects which are the arguments of the function.

Introducing the rule for solving the equation $x^3 + Cx = D$, Cardano reports: Ferro discovered this rule nearly thirty years ago; he gave it to Fiore, whose contest with Tartaglia gave Tartaglia an occasion to discover it; in response to my entreaties Tartaglia gave it to me, but without the demonstration; I sought the demonstration, and it was very difficult to find (*Art*, 11).

Ever since, the discovery of a general method for solving cubic equations has been mired in the distribution of credit and disgrace among the individualities which emancipated the method from the individuality of the picturing mind.

Clearing away the confusion of the objects

with individualities, as well as with an indefinite amount of other images saturated with feelings, cannot be easier than the cleaning of the Augean stables, contaminated as these had been by only 3 000 oxen. Many more, and rather more deliberately, have been humanizing the objects which are the arguments of the function, as they have everything.

It is yet the minor part of the difficulty. The method for solving cubic equations is propositions and chains of inferences. Whichever the individualities, picturing minds, and other images saturated with feelings, the relevant definite well-distinguished object is *The great art* in 1545.

The major part of the difficulty are the gaps in the chains of inferences.

Thus, the beginning of the solution of the cubic equation can be sought before Archimedes.

One and a half centuries before him, Archytas solved the problem of the duplication of the cube. The solution implies the equation $x^3 = D$, where $D = 2a^3$, where a is the side of the given cube.

The beginning can be sought before Archytas, too. Fourteen centuries before him, the predecessor of the philosophers who has to go under the name Moscow papyrus solves this problem: you are given ; the height is 6, the lower side is 4, the upper side is 2; square 4, you have 16; double 4, you have 8; square 2, you have 4; add 16, 8, and 4, you have 28; take $\frac{1}{3}$ of 6, you have 2; multiply 28 by 2, you have 56; look, you found it (14). What you found is the volume of a truncated square pyramid. The solution implies the formula for the volume, $V = \frac{h}{3}(a^2 + ab + b^2)$, and the formula implies a cubic equation. If the sides are x and $x + 2$, the height is $x + 4$, and the volume is 56, the equation is $Ax^3 + Bx^2 + Cx = D$, where $A = 3$, $B = 18$, $C = 28$, and $D = 152$.

The beginning can be sought before that predecessor of the philosophers, too. Six and a half centuries before him, Eanatum, prince *c.* 2475, reports: the prince of Umma defied Lagash; Eanatum, the prince of Lagash, unleashed a deluge on Umma; he vanquished Umma; he made 20 burial mounds for it (ins. 1). Burial mounds are volumes, volumes imply cubic equations.

If the beginning of the solution of the cubic equation is Diophantus, the beginning is a cubic equation.

If the beginning is Archimedes, Archimedes has the proportion $(r + x) : a :: b : (2r - x)^2$, and only one proposition is necessary to make the cubic equation implicit in the proportion explicit. The missing proposition is Viète's master rule: if four terms are such that the product of the extremes is equal to the product of the means, they are proportional, and conversely, if four terms are proportional, the product of the extremes is equal to the product of the means; thus a proportion is that from which an equation is composed and an equation is that into which a proportion is resolved. Only this proposition is necessary to make the chain of inferences from the labors of Archimedes to the cubic equation free of gaps. On the other hand, it took the philosophers the eighteen-odd centuries from Archimedes to Viète to discover this proposition.

If the beginning is traced back to Archytas, only one and a half centuries separate Archytas and Archimedes. On the other hand, Archytas solves the problem of the duplication of the cube by two mean proportionals, $a : x :: x : b :: b : 2a$, where a is the side of the given cube, and hence x the side of the doubled cube. And to make the cubic equation implicit in the two proportions explicit, and the chain of inferences from the labors of Archytas to the cubic equation free of gaps, many propositions are necessary. Certainly the theory of proportion developed by Eudoxus, and many of the propositions demonstrated on this ground by Euclid in the *Elements*, as well as Viète's master rule.

If the beginning is traced back to the predecessor of the philosophers who has to go under the name Moscow papyrus, both, many propositions are necessary to close the gap between his labors and the cubic equation, and many centuries separate him from his successors who discovered these propositions. The gap even from the solution of a problem of the volume of a truncated square pyramid and the general solution of a problem of volume, instanced by Archytas, is very wide. In comparison, the synodic period of Venus is the same in the predecessor of the philosophers who has to go under the name

Venus tablet of Ammisaduqa, and in Ptolemy, and in Copernicus, and in Kepler. A wide gap separates him in time from his successors in the investigation of the heaven. But there is no gap in the chains of inferences from his observation to theirs, or from theirs to his. It has to be concluded that he discovered Venus. In the same way, the predecessor of the philosophers who has to go under the name Moscow papyrus unquestionably discovered that the volume of a truncated square pyramid of sides 4 and 2 and height 6 is 56.

Whether the beginning of the solution of the cubic equation is Diophantus, or Archimedes, or is traced back to Archytas, if it has to be, or to the predecessor of the philosophers who has to go under the name Moscow papyrus, the beginning is not *ex nihilo*. Hence the question of the relevance of Eanatum and the 20 burial mounds he made for Umma.

Eanatum and the 20 burial mounds he made for Umma point also to the main difficulty with respect to the function itself, acquired knowledge of.

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The main difficulty with respect to the function itself is its extent. What is part of it, and what is not.

The main difficulty with respect to the argument is relevance. And this difficulty has to be resolved first. The main difficulty with respect to the value is reality. And this difficulty has to be resolved next. The main difficulty with respect to the function itself is its extent. And this difficulty has to be resolved last.

Caesar conquered Gaul.

Caesar is a definite well-distinguished object, shown such by his definite well-distinguished labors in a definite well-distinguished time. The difficulty is whether his labors are relevant, and if they are, which of them are.

If this difficulty can be resolved, the difficulty of whether Gaul is real, and of whether any other object to which Caesar is in the relation in which he is to Gaul is real, can be resolved.

And hence the difficulty of whether, conquered, in the relation of Caesar and Gaul, and in the relation of Caesar and any other object to which he is in the relation in which he is to Gaul, is part of, acquired knowledge of, or not, can be resolved.

In the *Commentaries on the Gallic war*, Caesar begins: all Gaul is divided into three parts (I, 1), then describes the three parts, and thus all Gaul: it is separated from Germany by the Rhine; it is separated from Spain by the Pyrenees; and it is bounded from the mouth of the Rhine to the Pyrenees by the ocean (*id.*). That on the near side, the side of Italy, it is bounded by the Alps and the Mediterranean Caesar takes for granted.

To begin with, then, Gaul is the definite well-distinguished country of Gaul, defined clearly and distinctly as such by the world, separated from other countries by rivers, mountains, and seas. Caesar's labors are an important step in putting Gaul on the map of the surface of the earth, and if a country is discovered not when it is sighted, or populated, and whether by other animals or men, but when it is put on the map, Caesar's labors are an important step in the discovery of Gaul.

Not quite four centuries before Caesar, in Herodotus, the description of Gaul reaches no further than: Massalia, and the Ligurians living above it (V, 9). One century later, not quite three before Caesar, Pytheas sailed the Atlantic north to Britain, and described the coast of Gaul (Strabo, II, 4.3). Not quite two centuries later, one before Caesar, Polybius writes: the conquests of Alexander in Asia and of the Romans elsewhere have opened new lands; so I undertook the danger and hardship of traveling through Africa, Iberia, and Gaul, and of sailing the seas bordering them, in order to correct the errors of my predecessors and make these parts of the earth known (III, 59). Polybius's description of Gaul reaches in the north to the Loire (XXXIV, 10). Caesar returns to the description of Gaul throughout the *Commentaries on the Gallic war*, and it is the *Commentaries on the Gallic war* that put all Gaul, and all Gaul in some detail, on the map of the surface of the earth.

In the course of the conquest of Gaul Caesar crossed twice to Britain, and in the discovery of Britain his is an even more decisive step.

The first description of Britain is by Pytheas. Polybius's attempt to correct the errors of his predecessors with respect to Britain failed. He

reports: asked about Britain, the people of Massalia knew nothing worth recording, nor did the people of Narbo or Corbilo (XXXIV, 10). The second description of Britain is by Caesar, in the *Commentaries on the Gallic war*.

Pytheas established that Britain is an island, and that the island is triangular in shape. He determined also its dimensions, by sailing around it (Strabo, I, 4.3; II, 4.1). Caesar corroborates the shape: the island is triangular (V, 13), and also determines its dimensions: from the reports of the Britons (*id.*).

And below are the two determinations, as well as, following Ptolemy's instruction, their correction from subsequent research, calculating the sides of the triangle from Land's End in the west to North Foreland in the east, from North Foreland in the south to Duncansby Head in the north, and from Duncansby Head in the northeast to Land's End in the southwest, in kilometers,

Pytheas	800	2 400	3 200,
Caesar	680	1 080	1 200,
correction	520	860	970.

It can be concluded that in their relation to Britain and America, respectively, Pytheas is very much like Columbus, Caesar is very much like Vespucci.

The connection of geographical discovery and conquest predates Caesar. Polybius notes Alexander. Earlier, Herodotus notes: most of Asia was discovered by Darius (IV, 44). The connection postdates Caesar, too, as America abundantly attests.

In relation to the definite well-distinguished countries of Gaul or Britain, then, the labors of Caesar are relevant. To compare them to the labors of investigation of the planets, Gaul and Britain are very much like Venus and Mercury, and the labors of Caesar are not like those of the predecessor of the philosophers who has to go under the name Venus table of Ammisaduqa, and not like those of the first philosophers, Anaximander or Parmenides, they are very much like those of Timocharis, who made the first accurate observation of Venus, and Dionysius, who made the first accurate observation of Mercury.

Further, Caesar conquered Gaul in eight years.

He conquered it logistically. He supplied and maintained his troops in a persistently hostile territory, 6 legions in 58, 12 legions in 51. A legion contained 10 cohorts, of 6 centuries each, of 80 men each at full strength.

He conquered it tactically. He faced a numerically superior enemy in all his campaigns.

Most important, he conquered it strategically.

In 58 he defeated the Helvetii near Bibracte (*Gallic*, I, 2-29), then moved north, and expelled the Germans who had crossed the Rhine (34-54).

In 57 he moved further north, and defeated the Belgae on the Sambre (II, 1-33). His lieutenants secured the passes over the Alps (III, 1-6).

In 56 he reduced the Veneti in the northwest (7-19). His lieutenants reduced the Aquitani in the southwest (20-7).

In 55 he again expelled the Germans who had again crossed the Rhine (IV, 4-15). Then, Caesar reports: Caesar decided that it is imperative that he cross the Rhine (16); and, in order to do so in a way according with his dignity and the dignity of the Roman people, to build a bridge over the river (17). The bridge was ready in ten days, Caesar crossed the river with his army, crossed back, and dismantled the bridge (18). Next, Caesar reports: Caesar decided to cross over to Britain because in all our wars the enemy received reinforcements from that quarter (20). He built a fleet, crossed with 2 legions, defeated the Britons, and crossed back (21-36).

In 54 he built a larger fleet, crossed with 5 legions, defeated the Britons, and crossed back (V, 1-23). He subdued a revolt in the northeast of Gaul (25-53).

In 53 he subdued another revolt in the northeast, built a bridge over the Rhine, crossed it, defeated the Germans, crossed back, and dismantled the bridge (VI, 2-10, 29-44).

In 52 Vercingetorix led a general revolt (VII, 1-5), Caesar besieged him in Gergovia, and the siege failed (6-53). He besieged Vercingetorix in Alesia, constructed a double circumvallation, to trap the besieged forces, on one side, to protect the besieging army from the relief forces, on the other (63-74), defeated the relief forces, and accepted the surrender of Vercingetorix (75-90).

In 51 he crushed the remaining pockets of resistance (VIII, 1-48).

Caesar established control over the eastern boundary of the country along its entire length,

then crossed the boundary and secured the eastern approaches to the country, then established control over the seas, crossed the seas and secured the northern approaches to the country, then annihilated resistance inside the country.

Gaul was pacified and brought under Roman control more firmly in eight years than Spain had been in one and a half centuries or Africa or Asia Minor had been in one century.

Clausewitz writes: tactics is the theory of the use of troops in battle, strategy is the theory of the use of battles toward the goal of the war (*War*, II, 1).

The first step in the development of strategy is the second Punic war, beginning with the labors of Hannibal, and continuing with those of the generals who defeated him, Fabius Maximus, Marcellus, and Scipio. The next step are Caesar's wars, the Gallic, and then the civil, which Caesar won in four years more conclusively and more unambiguously strategically than the Gallic. The next major step are Napoleon's wars. And the theory of war developed by Clausewitz in *On war* is grounded firmly in the observation of Napoleon's wars.

Caesar's battles are real, Bibracte in 58, the Sambre in 57, the Veneti in 56, the crossing of the Rhine and Britain in 55, Britain in 54, the crossing of the Rhine in 53, Gergovia and Alesia in 52, and the use Caesar made of the battles toward the goal of the war is real. Caesar's strategy in the Gallic war is a definite well-distinguished object. It is one among objects of the same kind, Hannibal's strategy in the second Punic war, Scipio's strategy in that war, Napoleon's strategy in the Italian campaign. The objects have become with Clausewitz the objects of theory. Considering how free of gaps its chains of inferences are, the theory is not very far advanced. Still, to compare it with the theory of the motions of the planets, it is not at the stage which that theory reached with Hipparchus, but it is well beyond the stage which that theory reached with Eudoxus or Aristotle.

The object Caesar's strategy in the Gallic war is difficult to identify and investigate. Part of the difficulty is that, unlike Venus and Gaul, which have not been produced by the astronomer and the geographer, Caesar's strategy in the Gallic war has been produced by the general. If in this way Caesar's strategy in the Gallic war is unlike Venus and Gaul, however, it is very much like hydrogen and carbon dioxide. Hydrogen and carbon dioxide have been produced by the chemist. And if hydrogen and carbon dioxide have been produced before the chemist, hydrogen in the big bang, for instance, carbon dioxide in respiration, for instance, the chemist has produced not only very many compounds which have never been produced before him, he has produced elements which have never been produced before him, beginning with neptunium, in 1940.

Caesar's strategy in the Gallic war is difficult to identify and investigate, but not nearly as difficult as the confusion, and obfuscation, of theory, whose end is truth, with practice, whose end is action, will make it appear. In theory, whose end is truth, Caesar's strategy in the Gallic war may be more difficult to identify and investigate than Venus and Gaul. But it may not be more difficult to identify and investigate than the hitherto unknown spirit produced by van Helmont, one of whose kinds was discovered by Black to be fixed air, which was shown by Lavoisier to be carbonic acid, and after him to be carbon dioxide.

In the relation of Caesar and Gaul of conquest, then, more of Caesar's labors are relevant. Next to his labors of geographical discovery, his labors of development of strategy are relevant.

Further, Caesar conquered Gaul, this led to the civil war, and Caesar won the civil war.

The Romans, as Polybius formulated it: brought nearly the entire known earth under a single government. With his victory in the civil war Caesar brought the single government of nearly the entire known earth under a single ruler.

Before Caesar, the successful labors to the same end can be traced back to Sargon: lord of the earth, to whom Enlil gave no rival from the upper sea to the lower sea. After Caesar, they can be traced forward to Napoleon, to whom Enlil gave Britain, and Napoleon fought it from one end of Europe to the other, and in Egypt, and in America.

Before Caesar, the investigation of government can be traced back to Plato, after Caesar, it can be

traced forward to Marx, including in both Plato and Marx a solution of the problem of the best government.

And if the solution of the problem of the best government is like the solution of the cubic equation, the philosophers' solution of the former, and either Plato's or Marx's, has not reached the stage which the solution of the latter reached with the predecessor of the philosophers who has to go under the name Moscow papyrus. For, if at the time of that predecessor of the philosophers the cubic equation was in the heaven, or in the future, it has since quit being in the heaven, or in the future, and the propositions necessary to close the gap in the chains of inferences between his labors and the cubic equation have been discovered. While Plato's best government is still in the heaven, Marx's best government is still in the future, and the propositions necessary to close the gap in the chains of inferences between the labors of either Plato or Marx and the best government, if there are these propositions, remain to be discovered. For the same reason, the princes' solutions of the problem of the best government are substantially further advanced than the philosophers'. Caesar's solution may well be at the stage of Diophantus's solution.

The objection that the government of a substantial portion of the earth under a single ruler is not the best government does not have to be met by the fact that the propositions which would close the gap in the chains of inferences between government and the best government remain to be discovered. It can be met by the fact that Caesar's successors who have preferred to emulate Caesar, rather than listen to the philosophers, have included not only emperors. They have included popes, professing as these have been to listen to higher authority. They have included also successors of Marx, professing as these have been to listen to Marx.

The government of a substantial portion of the earth under Caesar is real. It is a definite well-distinguished object. From the Atlantic in the west to the Euphrates in the east and from the mouth of the Rhine in the north to deep in the valley of the Nile in the south, founded between 49 and 44, fixed firmly in the laws Caesar passed and the precedents he established.

It is one among objects of the same kind, the government of a substantial portion of the earth under Sargon, under Darius, under Innocent III, under Napoleon.

It is a difficult object to identify and investigate. More difficult than Caesar's strategy in the Gallic war. Again, however, not nearly as difficult as the confusion, and obfuscation, of theory with practice will make it appear. Nor can it be more difficult to identify and investigate than the cubic equation.

The investigation of government by the philosophers has continued to tend to be of government in the heaven, and in the future, seldom and sparingly hampered by the examination of government on the surface of the earth, or in the past. Concern with government on the surface of the earth, and in the past, the philosophers have left to the princes. For this reason alone the explicit investigation of government by the philosophers cannot have advanced further than the investigation of government implicit in the labors of the princes of development of government. And if so, the labors of the princes of development of government cannot not be more relevant to the investigation of government, should it be hampered by descent from the heaven, and from the future, than the labors of the philosophers of investigation of government.

The relation of the government of a substantial portion of the earth under Caesar and Caesar's conquest of Gaul is of effect and cause.

Thus, still more of Caesar's labors are relevant. Next to his labors of geographical discovery, and of development of strategy, his labors of development of government are relevant.

Further, Caesar conquered Gaul, and described the conquest in the *Commentaries on the Gallic war*.

Hirtius undertook to close the gap between Caesar's description of the Gallic war, which ends in 52, and his description of the civil war in the *Commentaries on the civil war*, which begins in 49. Hirtius's labors have since become book VIII of *The Gallic war*.

Hirtius comments: Caesar wrote the *Commentaries* to prevent the historians of these events from lacking knowledge; but instead of thus giving them an object, he robbed them of one

(pr.).

Bacon agrees: Caesar, in modesty mixed with greatness, gave the name commentaries to the best history in the world (*Growth*, II, 6).

Caesar's labors of writing the best history in the world are relevant as well.

Finally, it can be concluded that Caesar's labors of geographical discovery, development of strategy, development of government, and historical investigation, are all of Caesar's labors that are relevant, and all his other labors are irrelevant. If further research shows more of his labors relevant, or some of those labors irrelevant, the conclusion can be corrected accordingly.

The difficulty of whether Caesar's labors are relevant, and if they are, which of them are, can be resolved. Hence the difficulty of whether Gaul and any other objects to which Caesar is in the relation in which he is to Gaul are real can be resolved. And hence the difficulty of whether, conquered, in the relation of Caesar and Gaul and any other objects to which he is in the same relation, is part of, acquired knowledge of, can be resolved.

Clearly, the objects which are the argument of the function, and hence the objects which are the value of the function, have to decide what is part of the function, and what is not. Thus, if, in particular, when Caesar conquered Gaul, conquered, is part of, acquired knowledge of, when in the course of the civil war Caesar conquered Spain, conquered, is not part of it, when five centuries later the Visigoths, Burgundians, and Franks conquered Gaul, conquered, is not part of it, when two and a half centuries later Tariq ibn Ziyad conquered Spain, conquered, is not part of it. When four and a half centuries before Caesar, Darius conquered Asia to the Indus, conquered, is part of it.

In order to resolve the difficulties with respect to the function itself, including the main, its extent, first the difficulties with respect to the value have to be resolved. And in order to resolve the difficulties with respect to the value, including the main, reality, first the difficulties with respect to the argument have to be resolved.

And the main difficulty with respect to the argument is relevance. To collect all the relevant, and only the relevant, objects which are the argument of the function.

Part of this difficulty is the confusion of the objects with individualities, next to an indefinite amount of other images saturated with feelings. The record has blessed very few of these objects with emancipation from individuality and images as complete as that of the predecessors of the philosophers who have to go under the names of papyri and tablets.

The major part of the difficulty are the gaps in the chains of inferences. It is hard to connect astronomical discoveries with one another, harder to connect chemical discoveries, harder to connect steps in the development of algebra, harder to connect steps in the development of geography, harder to connect steps in the development of strategy, harder to connect steps in the development of government, and the hardest to connect steps in the development of history.

And there is more to the history of our discoveries than the development of astronomy, chemistry, algebra, geography, strategy, government, and history.

cvi

Function and object show the uniformity of the acquisition of knowledge.

When Frege discovered that function and argument are the elements of a proposition, hence developed the concept writing, and hence concluded that the formula languages of arithmetic, geometry, and chemistry are realizations of Leibniz's conception of the philosophical calculus for particular areas, the concept writing adds the area in the middle, adjoining all others, and the work can continue, with the greatest prospect of success, at filling the gaps in the existing formula languages, connecting their separate areas into a single domain, and extending this domain to areas which lack such a language, Frege solved the problem of the uniformity of the acquisition of knowledge.

The solution is proposed by Frege as the solution of the problem of the uniformity of the acquisition such as he observed it at the time of writing. And he is concerned to exclude from it the history of our discoveries.

But the solution of the problem of the uniformity of the acquisition of knowledge such as it is

observed at any one time cannot stand if it is not the solution of the problem of the uniformity of the acquisition of knowledge in its entirety, and that is, of the uniformity of the history of our discoveries.

The solution of the problem of the uniformity of the acquisition of knowledge has to be shown the solution of the problem of the uniformity of the history of our discoveries.

And this problem will be solved when there is no more need for disputes between two historians than between two philosophers, it suffices to take their pens, sit down to their abaci, and calculate.

cvii

Frege solved the problem of the uniformity of the acquisition of knowledge.

As to how, he solved it in the same way Ptolemy solved the problem of measuring the world, and Darwin solved the problem of life, by observation and experiment.

Observation is not looking, or seeing. It is a generalization and formalization of experience. And it is attested by chains of inferences, and by nothing else.

With Bacon: man, the servant and interpreter of nature, does and understands only as much as he has observed of the order of nature, in deed or in mind, nor can he know or do more. And: the testimony and reports of the senses are always in relation to man, not in relation to the world; it is thus a great error to assert that the senses are the measure of things; the senses are only to judge experiment, experiment is to judge the thing. And: experience as it occurs is an accident, as it is pursued an experiment. And: the secrets of nature reveal themselves not when nature is free and left to herself, but when she is constricted and vexed, when she is forced out of her state, and molded and shaped.

The vexation of nature distinguishes observation from looking and seeing, and the degree of vexation distinguishes the kind of observation which experiment is.

The difference in the degree of vexation is determined by the object of the observation.

Nature has had to be vexed to reveal the synodic period of Venus to the predecessor of the philosophers who has to go under the name Venus tablet of Ammisaduqa and to reveal the dimensions of Britain to Caesar. She has had to be vexed more to reveal refraction to Ptolemy and the satellites of Jupiter to Galileo. More still to reveal the relation of space and time in uniformly accelerated motion to Galileo. More still to reveal the age of earth to Buffon and the weight of hydrogen to Cavendish. More still to reveal the origin of the breeds of pigeons to Darwin and the age of the earth better to Rutherford.

From Venus and Britain through refraction and the satellites of Jupiter through uniformly accelerated motion through the age of the earth and the weight of hydrogen through the origin of the breeds of pigeons and the radioactive age of the earth, the objects of observation have demanded ever greater degree of generalization and formalization in order to become accessible to observation. Ever greater generalization and formalization has had to be the ground of the further generalization and formalization. The chains of inferences have had to become ever longer. They have had to become ever less tolerant of gaps, too.

And nature has had to be vexed more still to reveal what proposition, inference, function, object are.

With Hume: everything terminates in experience. With Kant: only in experience is there truth.

Experience is the foundation of the acquisition of knowledge. If so, however, all earlier acquisition of knowledge has to be part of the foundation of all later acquisition of knowledge. And the more successful the earlier acquisition, and that is, in the generalization and formalization of experience, the more it has to be the foundation of the later acquisition exclusively.

As Aristotle collected animals, 600 kinds of them, so he collected the labors of his predecessors, the physicists and the mathematicians, the lawgivers and the ethicists, the orators and the poets, even those who, taught by Gorgias, gave shoes to their students to learn by heart. Hence, Aristotle distinguished proposition from sentence, and sorted out inferences into demonstrations, on one side, dialectical arguments, examinations, sophistical arguments, and rhetoric, on the other. To establish what proposition and inference are, Aristotle had to vex the vexations of his predecessors. To generalize and formalize their general-

izations and formalizations.

And to establish better what proposition and inference are, and establish what function and object are, Frege had to vex the vexations of his predecessors. To generalize and formalize their generalizations and formalizations.

This is how he describes his labors with respect to function. In the middle of *Function and concept*:

how has the reference of the word function been extended by the advance of science? two directions can be distinguished;

first, the domain of arithmetical operations which contribute to the building of a function has been extended; to addition, multiplication, exponentiation, and their converses the different ways of transition to a limit have been added, without always having clear awareness of the essentially new which has thus been introduced; the advance has gone further still and it has become even necessary to resort to common language when the sign language of analysis has failed, as, for instance, when a function has been described whose value for rational arguments is 1, for irrational 0;

second, the domain of what can enter as an argument and value of a function has been extended by the admission of the complex numbers; with this the sense of the expressions 'sum', 'product', and so on, has had to be determined further;

in both directions I go further still (*id.*).

Both directions are old, not new.

What they are Frege gathers from: the advance of science. From the history of our discoveries.

In both directions he goes further. Very much further.

In the second direction, the extension of the domain of what can enter as an argument and value of a function: we must admit as arguments and values of functions objects without restriction. From the admission of complex numbers, to the admission of the evening star, the morning star, hydrogen, carbon dioxide, Caesar, Gaul. And all of Frege's illustrations are gathered from the history of our discoveries.

In the first direction, the extension of the domain of operations which contribute to the building of a function: any expression with an unchangeable part is a function. From addition, multiplication, exponentiation, their converses, and the ways of transition to a limit, to propositions are functions. And the advance includes explicitly: arithmetic, geometry, chemistry, the concept writing, the foundations of the differential and integral calculus, the pure theory of motion, mechanics, physics, and philosophy such as it is under the dominion of the word. It includes the entire history of our discoveries.

Frege's advance is a massive generalization over the history of our discoveries. It is a massive advance which stands on massive induction.

Frege describes the advance in the middle of *Function and concept*, and once again at the end:

if we look back at the development of arithmetic, we recognize a stage by stage advance;

first it was calculated with single numbers, with 1, 3, and so on;

$$2+3=5,\ 2.3=6$$

are theorems of this kind;

then it was advanced to general laws which hold for all numbers; in notation to this corresponds the transition to calculation with letters; in

$$(a+b).c=a.c+b.c$$

we have a theorem of this kind; with this the consideration of single functions was reached, without yet using the word in the mathematical sense or having grasped its reference;

the next stage was the recognition of general laws of functions and with this the coinage of the technical term 'function'; in notation to this corresponds the introduction of letters like *f*, *F* for the indefinite indication of functions; in

$$\frac{df(x).F(x)}{dx}=F(x).\frac{df(x)}{dx}+f(x).\frac{dF(x)}{dx}$$

we have a theorem of this kind; with this single second-level functions were obtained, without yet grasping what we have named second-level functions;

when this occurs, the next advance is made;

it could be thought that this will continue; but this last stage already is probably not so rich in consequences as the earlier, because in this further advance, instead of second-level functions, first-level functions can be considered (*id.*).

It is a superb description of the history of our discovery of function.

The discovery has been a stage by stage ad-

vance.

The stages are four.

The first stage is the calculation with single numbers. This stage was reached forty-nine and a half centuries before Frege. The theorems

$$2+3=5,\ 2.3=6$$

are not attested. The first attested theorems, however, the 3 calculations of area by the predecessor of the philosophers whose labors are recorded on the tablets W 20044/20 and W 19408/76, beginning with

$$\frac{80+140}{2}\times\frac{140+60}{2}=11\,000,$$

are theorems of this kind.

The second stage is the advance to general laws which hold for all numbers. This stage was reached between Pythagoras and Euclid – counting from its end, twenty-seven and a half centuries after the first stage was reached. The theorem

$$(a+b).c=a.c+b.c$$

is proved by Euclid in the *Elements* (V, 1).

The third stage is the recognition of general laws of functions. This stage was reached between Galileo and Leibniz, counting end to end, twenty centuries after the second stage was reached. The theorem

$$\frac{df(x).F(x)}{dx}=F(x).\frac{df(x)}{dx}+f(x).\frac{dF(x)}{dx}$$

is proved by Leibniz in *A new method for maxima and minima*.

The fourth stage is the grasping of second-level functions. This stage was reached between Euler and Frege, counting end to end, two centuries after the third stage was reached.

It is a description of the greatest generality. It accommodates five millennia of development.

It is a description of the greatest clarity and distinction. The five millennia are analyzed into four stages.

It is a description of the greatest sophistication. It makes it possible to trace the beginning of analysis not just to calculation with single numbers, but to the two tablets on which the recorded single numbers only imply calculation. To recognize that, although Euclid does not use the word function, the generality of his proof, concerned with magnitudes, of which numbers are only one kind, shows that the consideration of single functions has been reached. That Leibniz's proof, although very far from being free of gaps, introduced as one of the rules of the differential calculus, shows the recognition of general laws of functions.

It is an impeccable assessment of the discovery, too. The fourth stage is not so rich in consequences as the third. It need not be thought that the advance will continue.

And it should be superfluous to add that it stands on observation.

In general, every advance in the acquisition of knowledge which stands on induction cannot not stand on observation. And Frege's is a massive advance which stands on massive induction.

Deduction does not have to stand on observation. Nevertheless, every deduction which is an advance in the acquisition of knowledge cannot not be deduction from principles which stand on induction which stands on observation.

Frege's advance has been most immediately an advance in logic and mathematics.

And the importance of the fact that Frege's advance stands on observation is the importance of the fact that logic and mathematics stand on observation.

Logic and mathematics have been relentlessly contaminated by the supposition that men make them up and think them up.

Aristotle's immediate successors already found out that in the orchard which philosophy is, ethics is the fruit, physics is the earth and the tree, logic is the fence. And Boole still, in the generation before Frege, taking a major step in the advance of logic, took it in *An investigation of the laws of thought*.

Plato already writes: mathematics never allows the soul to attach numbers themselves to the sensible things; numbers themselves can be grasped in thought alone and in no other way. And Dedekind, for one, in the generation of Frege, writes: numbers are the free creations of the human mind (*What*, pr.).

If philosophy is an orchard, and physics is the earth and the tree, logic is the fruit, ethics is the fence. Nor is logic an investigation of thought. Logic is an investigation of knowledge.

With Frege: the thought gives no knowledge, the reference gives knowledge. And the acquisition of knowledge is the advance from the

thought and the sense to the reference and the true.

Accordingly: the firmest proof is the purely logical, which grounds itself only in these laws on which all knowledge rests.

Numbers themselves can be grasped in the reference alone. Nor are they the free creations of the human mind, nor is mathematics the free creation of the human mind. Mathematics is a kind of physics.

Frege writes: the laws of number are not actually applicable to the external things; they are not laws of nature; they are yet applicable to judgments which hold for the things of the external world, they are laws of the laws of nature; they affirm not a connection between natural phenomena, but one between judgments; and to these belong the laws of nature (*Foundations*, 87).

Accordingly: the mathematician cannot create a thing at will any more than the geographer; he, too, can only discover what is there and name it (96); as the geographer does not create a sea when he draws boundary lines and says, the part of the water surface bounded by these lines I will name Yellow Sea, so the mathematician can create nothing by his definitions; nor can a mere definition miraculously give a thing a property which it does not have, other than that of now bearing the name which it has been given; that an oval drawn with ink on paper would acquire by a definition the property that, added to one, it gives one, I can only regard as a scientific superstition; one might as well make a lazy student diligent by definition (*Laws*, pr.).

The judgment of Hardy, a mathematician *par excellence*, who, in *On the zeroes of Riemann's ζ (s) function*, first proved part of the Riemann hypothesis, and a physicist *par excellence*, who, in the *Mendelian proportions in a mixed population*, formulated the law of the equilibrium of a population, is neither less explicit nor less picturesque. In 1940, in *A mathematician's apology*, Hardy writes: it may be that modern physics fits best into a framework of idealistic philosophy, I do not believe it, but eminent physicists say so; pure mathematics, on the other hand, seems to me a rock on which all idealism founders; 317 is a prime not because we think so, or because our minds are shaped in one way rather than another, but because it is so, because mathematical reality is built that way (24); our function is to discover or observe mathematical reality, and what we grandiloquently describe as our creations are simply our notes of our observations (22).

And it is a function.

cviii

If form is any determination whatsoever, form can only be investigated in relation to itself.

Hence the problem of the separation of content from form.

Aristotle writes: knowledge is acquired only by demonstration. And to determine what demonstration is, he distinguishes four kinds of inference according to their grounds, demonstration is grounded in what is true, dialectical argument in what is probable, examination in what is accepted contingently, contentious argument in what is false.

If these are all inferences, and inferences are formal, determination admits of degree. From what is false to what is accepted contingently to what is probable to what is true.

Determination is conditional, and unconditional. And this is the solution of the problem of the separation of content from form. What is determined conditionally is content, what is determined unconditionally is form.

But what is determined conditionally can only be determined in relation to what is determined unconditionally. Content can only be content in relation to form. An inference can be an inference only to the degree to which it is unconditional determination, or what is the same, only to the degree to which it is formal exclusively, and demonstration, or synonymously proof, can be demonstration, or synonymously proof, only to the degree to which it is unconditional determination, or what is the same, only to the degree to which it is formal exclusively.

Hence the problem of the relation of content and form. How is proof as unconditional determination possible? or what is the same, how is proof as formal exclusively possible?

In order to solve the problem of proof, Aristotle writes: we must determine what a proposition is, what a term is, and what a syllogism is. A

proposition is a sentence which affirms what is true or false. A term is that into which a proposition is resolved, a subject and a predicate, is or is not relating the two. And a syllogism is an inference such that, given certain propositions, another proposition follows necessarily.

The problem of the relation of content and form, how is proof as formal exclusively possible? is thus solved.

Proof is an inference. An inference is a relation of propositions. A proposition is a subject and a predicate, related by is or is not. And if given propositions affirm what is true, and their relation is such that another proposition follows necessarily, the inference is a proof.

Whether the problem of the relation of content and form is solved completely, and it is shown how proof can be formal exclusively, thus depends on the following.

Whether form is propositions.

Whether a proposition is a sentence which affirms what is true or false.

If this is what a proposition is, whether the relation of being true is formal.

Whether the elements of a proposition are subject and predicate, related by is or is not.

If these are the elements of a proposition, whether the relation of is or is not is formal.

Whether proof is an inference such that, given certain propositions, another proposition follows necessarily.

And if this is what proof is, whether the relation of following necessarily is formal.

That form is propositions, what a proposition is, and what proof is, Aristotle established conclusively. The remaining difficulties Frege resolved conclusively.

One century after Aristotle, Chrysippus discovered the modes of inference, including *modus ponens*, the single mode which, following the law of perspicuity, Frege admits in the *Concept writing*. *Modus ponens* is constructed from two propositions:

> if the first proposition, then the second proposition,
> but the first proposition,
> therefore the second proposition.

Chrysippus thus showed that the relation of propositions of following necessarily from one another can be formal exclusively.

Twenty centuries after Chrysippus, one-odd before Frege, Kant writes: the problem on which everything depends is, how are synthetic *a priori* propositions possible? Or, with respect to content and form, how is the content of a proposition as formal possible?

The problem of the relation of content and form is thus reformulated into the problem of the formal relation of content and form. How proof as formal exclusively is possible can only be shown formally. Only proof can be the solution of the problem.

And Frege solved this problem. He showed how the relation of a proposition to the true can be expressed formally. He showed that the elements of a proposition are function and argument. He showed how the relation of propositions of following necessarily from one another can be expressed formally. And although the elements of a proposition are not related by is or is not, he showed how the relation of is or is not can be expressed formally, too. And he showed it all formally. By proof.

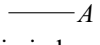
Informally: my concept writing makes possible a chain of inferences in which no link is missing.

And this is how formally, in the *Concept writing*. Frege writes:

a judgment will always be expressed by the sign

which stands to the left of the sign or combination of signs which gives the content of the judgment; if the vertical stroke is omitted, the judgment will change into a mere combination of ideas, of which the writer does not express whether he acknowledges its truth or not; if, for instance,

refers to ‘the opposite magnetic poles attract one another’, then



will not express this judgment, it is merely to produce in the reader the idea of the mutual attraction of the opposite magnetic poles, perhaps in order to draw consequences from it and test the correctness of the thought (2).

A proposition has thus been distinguished

from a sentence formally. And in the proposition form and content have been distinguished formally. Frege continues:

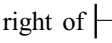
if a short vertical stroke is attached below the horizontal stroke, this will express the circumstance that the content does not occur; thus, for instance,

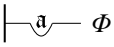
refers to '*A* does not occur';

without the vertical stroke, there will be no judgment;

only invites the idea that *A* does not occur, without expressing whether this idea is true (7).

Truth and falsehood have thus been expressed formally. Frege continues:

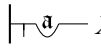
in the expression of a judgment the combination of signs to the right of  can always be considered a function of one of the signs in it; if in the place of its argument and in a concavity in the horizontal stroke a German letter is introduced, as in



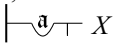
this refers to the judgment that the function is a fact whatever be taken for its argument (11).

Generality has thus been expressed formally. Frege continues:

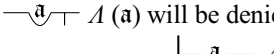
the reference of

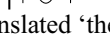


is that something, for instance Δ, can be found, such that $X(\Delta)$ will be denied; it can thus be translated 'there are some things which do not have the property *X*'; the reference of

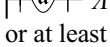


is different; it is 'whatever $\mathfrak{a}$ may be, $X(\mathfrak{a})$ must always be denied', or 'there is no thing which has the property *X*', or, if we name something which has the property *X* an *X*, 'there is no *X*';

 will be denied by



this can thus be translated 'there are Λ's'; it is to be understood to include the case 'there is one Λ'; if, for instance, $\Lambda(x)$ refers to the circumstance that *x* is a house, then



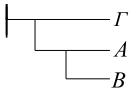
is 'there are houses or at least one house' (12).

Existence has thus been expressed formally.

Further:

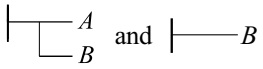
the vertical stroke which connects two horizontal strokes is called the condition stroke (5);

it can be recognized that



denies the case where *B* will be affirmed, but *A* and Γ will be denied; if a causal connection between *A* and *B* is presupposed, it can be translated 'if *A* is the necessary consequence of *B*, it can be concluded that Γ occurs' (*id.*);

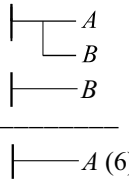
it is thus apparent that from the two judgments



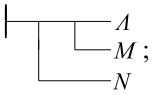
the new judgment

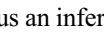
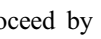
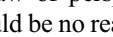
follows;

this inference can be written thus,



in logic after Aristotle a whole series of modes of inference can be enumerated; I employ only this one, at least in all cases in which from more than one judgment a new judgment is derived; because the truth contained in any other mode of inference can be expressed in a judgment of the form, if *M* holds, and if *N* holds, Λ also holds, in signs,



from this judgment and  and  ,  follows; thus an inference by any mode of inference can be reduced to our case; and since it is possible to proceed by a single mode of inference, it is a law of perspicuity to do so; otherwise there would be no reason to stop at the Aristotlean modes of inference, new ones can be added indefinitely (*id.*).

And thus the relation of propositions of following necessarily from one another has been expressed formally as well.

Frege writes in *The foundations of arithmetic*:

the goal of proof is not only to raise the truth of a proposition above all doubt, but also to give knowledge of the dependence of truths on one another; after one has convinced oneself by the futile attempts to move it that a boulder is unshakeable, there is still the question of what, then, supports it so securely; the further these investigations are pursued, the fewer the primitive truths to which everything leads; and this simplification is a worthwhile goal in itself; perhaps the hope even can be realized of winning the general ways of building concepts and principles, in which in the more complex cases what men have done in the simplest cases instinctively is brought to consciousness, and extracting from them the generally valid (2).

cix

As Frege observed the labors of his predecessors, so his successors observed his labors.

One major development which rests on the labors of Frege has been the theory of proof formulated by Hilbert, a massive generalization over earlier results itself.

Hilbert writes in *The logical foundations of mathematics*, in 1923: the fundamental idea of my theory of proof is the following; everything that is part of mathematics will be rigorously formalized so that mathematics will become a stock of formulas; these will differ from the usual formulas of mathematics only in that, next to usual formulas, they will contain logical signs, in particular, for follows, →, and not, ¯ (*id.*).

In *The new grounding of mathematics*, in 1922, Hilbert explains: this reflects the conviction that, because of the close connection and inseparability of arithmetical and logical truths, it is necessary to build arithmetic and formal logic simultaneously (*id.*).

He continues in *The logical foundations*: certain formulas which will serve as building blocks of the formal edifice of mathematics will be called axioms; a proof will be a figure which presents itself sensibly as such; it will consist of inferences by the inference schema

$$\frac{\mathfrak{S} \qquad \mathfrak{S} \rightarrow \mathfrak{T}}{\mathfrak{T}}$$

where each of the premises, the formulas $\mathfrak{S}$ and $\mathfrak{S} \rightarrow \mathfrak{T}$, will always either be an axiom, or arise directly from an axiom by substitution, or agree with the end formula $\mathfrak{T}$ of an inference which appears earlier in the proof, or arise from such an end formula by substitution; a formula will be called provable if it either is an axiom, or arises from an axiom by substitution, or is the end formula of a proof; in addition to this formalized mathematics proper, there will be a certain new mathematics, a metamathematics, necessary to secure mathematics, in which, in opposition to the purely formal modes of inference of mathematics proper, contentual inference will be employed, only, however, to prove the consistency of the axioms; this metamathematics will operate with the proofs of mathematics proper, and these will be themselves the object of the contentual investigation; in this way the development of mathematical science in its entirety will be carried out in two ways in constant alternation, through the winning of new provable formulas from the axioms by formal inference, and through the addition of new axioms with the proof of their consistency by contentual inference (*id.*)

The end of the theory Hilbert had formulated before formulating the theory. He had formulated it first in the *Mathematical problems*, in 1900: to prove that the axioms are free of contradiction among themselves, and that is, that a finite number of logical conclusions on their ground can never lead to results which are in contradiction with one another (2). He had formulated it more generally still in the *Axiomatic thinking*, in 1918: to establish that in every domain of knowledge contradictions on the ground of the system of axioms on which it stands are impossible (*id.*). He had also formulated the means to the end: to solve the problem of the solvability of every mathematical question (*id.*). As well as the means to this end: to solve the problem of the decidability of a mathematical question by a finite number of operations (*id.*).

With the formulation of the theory the end of the theory became a consequence of the theory.

In 1931, in *On formally undecidable propositions of Principia mathematica and related systems*, Gödel proved the incompleteness theorems.

This is how, in 1930, in the note announcing

that forthcoming publication, *Some metamathematical results on completeness and consistency*, Gödel describes the theorems: if on the axioms of Peano the logic of the *Principia mathematica* together with the axiom of choice is overbuilt, a formal system *S* is obtained, for which the following propositions hold, I. the system *S* is not complete, and that is, there are in it propositions *A*, and such propositions can be shown, for which neither *A* nor ^{-}A is provable; II. even if all logical devices of the *Principia mathematica* are admitted in the metamathematics, there is no consistency proof for the system *S*; a consistency proof for the system *S* can thus be carried out only with the help of modes of inference which are not formalized in the system *S* itself (*id.*).

This is how he introduces the theorems in *On formally undecidable propositions*: the development of mathematics toward greater precision has led to the formalization of large domains of it in such a way that proof can be carried out by only a few mechanical rules; the most comprehensive formal systems so far are the system of the *Principia mathematica* and the axiom system of set theory; both these systems are so broad that in them all methods of proof employed today in mathematics are formalized, and that is, reduced to only a few axioms and rules of inference; so the conjecture suggests itself that these axioms and rules of inference are sufficient to decide all mathematical questions which can be expressed formally in those systems; it will be shown below that this is not the case, rather, there are in both those systems relatively simple problems which cannot be decided from the axioms (1).

And this is how he: sketches the main idea of the proof (*id.*). He writes: the formulas of a formal system are finite sequences of primitive signs; analogously, the proofs are nothing but finite sequences of formulas; for metamathematical consideration it is irrelevant what objects are taken as primitive signs; we decide to take as such natural numbers; accordingly, a formula is a finite sequence of natural numbers and a proof is a finite sequence of finite sequences of natural numbers; the metamathematical propositions thus become propositions about natural numbers or sequences of natural numbers, and can be expressed in the symbols of the system itself; in particular, it can be shown that the concepts formula, proof array, provable formula, can be defined in the system; and we now construct an undecidable proposition of the system, and that is, a proposition *A* for which neither *A* nor ^{-}A is provable (*id.*); the argument is closely related to the liar; we have before us a proposition which asserts its own unprovability (*id.*).

The theorems have since been generalized. In a note added in 1963, Gödel writes: in consequence of later advances, in particular of the fact that due to Turing's work a precise and unquestionably adequate definition of the general concept of formal system can now be given, a completely general version of the theorems is now possible; and that is, it can be proved that in every consistent formal system that contains a certain amount of finitary number theory there exist undecidable arithmetical propositions, and, moreover, that the consistency of any such system cannot be proved in the system (4).

As in the theory of relativity much attention has attracted the word, relativity, so in Gödel's theorems much attention have attracted the words, incompleteness and inconsistency. As in that theory less attention has attracted the fact that it is the theory of the absolute world, so in these theorems less attention has attracted the fact that they are proofs. Or the fact that they are a step in the development of mathematics such as has led to the formalization of large domains of it in such a way that proof can be carried out by only a few mechanical rules. Or the fact that they are overbuilt on the formal systems of the *Principia mathematica* and the axiom system of set theory, in which all methods of proof employed today in mathematics are formalized, and that is, reduced to only a few axioms and rules of inference. Or the facts that a precise and unquestionably adequate definition of the general concept of formal system can be given, or that the theorems are completely general, or that they are results about every consistent formal system, or that they are results about the consistency of any formal system.

Much attention has also attracted the fact that Gödel's theorems show that the end of the theory of proof formulated by Hilbert: to establish that in every domain of knowledge contradictions on

the ground of the system of axioms on which it stands are impossible, is not attainable by the means formulated by Hilbert: to solve the problem of the decidability of a mathematical question by a finite number of operations. Less attention has attracted the fact that the end of the theory of proof formulated by Hilbert is a consequence of the theory of proof formulated by Hilbert, Gödel's theorems are most rigorous results most rigorously within the theory of proof formulated by Hilbert, and whatever other consequences they may have, the chief among their consequences cannot not be – reformulated as the means to it have to be – the end of the theory of proof formulated by Hilbert.

The most important development which rests on the labors of Frege, by far, has been the generalization of the theory of sets.

Cantor formulated the theory of sets.

As the more important a discovery is the more likely it has to be to meet with scepticism, so the theory of sets had to meet with much scepticism. In *On the infinite*, in 1925, Hilbert reports: the most vehement attacks were launched from all sides against Cantor's theory; the opposition was so violent, the commonest and most fruitful concepts and the simplest and most important modes of inference in mathematics were threatened (*id.*). But, Hilbert adds: no one shall drive us away from the paradise which Cantor created for us (*id.*).

The theory of sets transformed infinity from a refuge of mysticism into an ordinary object of mathematics. It became the foundation of analysis. The foundation of arithmetic. The foundation of geometry. It showed that all mathematical objects can be considered sets. It showed that every mathematical theorem can be proved from the axioms of the theory of sets.

The theory of sets is the foundation of mathematics.

Cantor formulated the theory from 1874, in *On a property of the totality of all real algebraic numbers*, to 1895-7, in the *Contributions to the foundation of the transfinite theory of sets*.

Frege transformed logic into mathematical logic from 1879, in the *Concept writing*, to 1892, in *On sense and reference*.

One generation before the first philosopher, eighty-four generations before Russell, the Cretan Epimenides said that all Cretans are liars. In 1902 Russell discovered the paradox of the set of all sets that are not members of themselves.

Russell and Whitehead completed the *Principia mathematica* in 1910-3. One of its three goals is to solve the paradoxes. The introduction describes the three goals: first, the complete enumeration of all ideas and steps in reasoning employed in mathematics; second, the perfectly precise expression of mathematical propositions; third, the system is specially framed to solve the paradoxes which in recent years have troubled students of symbolic logic and the theory of sets (*id.*).

What the effect of the pursuit of the third goal on the pursuit of the other two was, Russell describes in *My philosophical development*, in 1959. He reports: throughout 1903 and 1904 my work was almost wholly devoted to the attempt to find a solution of the paradoxes (7); the conclusion to which I was led was that sets are merely a convenience in discourse (*id.*); a set is only an expression (*id.*). Hence, Russell and Whitehead developed a logic of relations, and: what is important in the logic of relations is what is different from the logic of sets (8).

And this is how Gödel judges the effect, in *Russell's mathematical logic*, in 1944. Gödel writes: it was only in the *Principia mathematica* that full use was made of the new method for actually deriving large parts of mathematics from a very few logical concepts and axioms; in addition, the young science was enriched by a new instrument, the abstract theory of relations; it is to be regretted that this first comprehensive and thoroughgoing presentation of mathematical logic and derivation of mathematics from it is so greatly lacking in formal precision in the foundations that it represents in this respect a considerable step backward compared with Frege (*id.*). Russell had written: logic is concerned with the real world as truly as zoology. Gödel quotes this, and comments: Russell's pronouncedly realistic attitude, however, was always stronger in theory than in practice; when he started on a concrete problem, the objects to be analyzed soon turned out into logical fictions (*id.*).

In 1903, in the second volume of *The laws of*

arithmetic, Frege also struggles with the paradoxes. He has no formal solution. He yet concludes: nothing remains but to recognize sets as objects in the proper and full sense of the word (post.).

In 1908 Zermelo axiomatized the theory of sets.

With Frege: the separation of logic and mathematics is impossible. With Hilbert: arithmetical and logical truths are inseparable. With Zermelo: the theory of sets develops the logical foundations of all arithmetic and analysis. With Gödel: mathematical logic is, on the one hand, a part of mathematics (*Russell's*).

The theory of sets is the foundation of mathematics. The foundation of mathematics is the logical foundation of mathematics. The theory of sets is the foundation of logic.

With Frege: logical is the proof which grounds itself in these laws on which all knowledge rests. With Hilbert: the end of the theory of proof is to establish that contradictions are impossible in every domain of knowledge. With Russell: logic is concerned with the real world as truly as zoology. With Gödel: on the other hand, mathematical logic is a science prior to all others which contains the ideas and principles underlying all sciences; it was in this sense that mathematical logic was first conceived by Leibniz in his *calculus philosophicus* (*id.*).

Experience is the foundation of the acquisition of knowledge. If so, all earlier acquisition is part of the foundation of all later acquisition. If so, the more successful the earlier acquisition, and that is, in the generalization and formalization of experience, the more it is the foundation of the later acquisition exclusively.

And the theory of sets is the foundation of the acquisition of knowledge exclusively.

cx

Frege informs the problem before the investigation of knowledge decisively.

The acquisition of knowledge is a path. From the thought and the sense to the reference and the true.

With the thought and the sense the path has not begun. The path has begun with the reference and the true, and the path is in its entirety the reference and the true.

Every step of the path is the winning of a proposition.

The elements of a proposition are function and argument. Function is the unchangeable in an expression. Arguments and values of functions are objects without restriction. Object is all that is not function. The reference is the object.

To win a proposition is to win a true proposition. To win a true proposition is to discover that every one of its parts has a reference, and hence the proposition has a reference, the true. The reference of every part of every true proposition is the reference of that part of every other true proposition. To win a proposition is thus to win its connection with other propositions. To bring it in connection with them through chains of inferences. Proof is to bring it in connection with the propositions which are its consequences and of which it is the consequence.

The stages of the path are stages of proof.

Proofs are collected in formula languages.

The acquisition of knowledge is the labor at developing formula languages, connecting them in a single domain, and extending this domain.

In the end all chains of inferences are free of gaps. All formula languages are connected in the single domain of the philosophical calculus.

All of this has to be the goal of the investigation of knowledge. In what it investigates, and how it investigates it, both.

If so, and Kant's is the most successful investigation of knowledge, the investigation of knowledge has a long way to go to the goal. To compare it with the investigation of the heaven, it is not at the stage of Ptolemy, never mind the stage of Kepler, it is not even at the stage of Hipparchus. It is, with the investigations of Hume and Kant, just past the stage of Eudoxus and Aristotle.

If all of this has to be the goal of the investigation, in general, the more unwaveringly it has to be the goal of the attempt, in particular, to formulate, and solve, the problem before the investigation.

cxi

To review the problem.

Knowledge cannot develop. Knowledge can

only be acquired. The development which can be in relation to knowledge can only be the acquisition of knowledge. The problem is thus of two parts, the acquisition of knowledge, and knowledge, and the problem of knowledge cannot be formulated or solved unless the problem of the acquisition of knowledge is formulated and solved first.

The parts of the problem of the acquisition of knowledge are the beginning of the acquisition, the collection of the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition. The parts of the problem of knowledge are the relation of knowledge and truth, and the relation of knowledge and the world.

The problem of the acquisition of knowledge has to be formulated and solved first. And of its parts, the problems of the beginning of the acquisition, and of collecting the acquisition in its entirety, have to be formulated and solved first.

Aristotle describes the development:

all animals have perception,
from perception memory develops in some of them,
from memory experience develops, of which animals other than man have little,
from experience art develops, and first the arts aiming to procure the necessities of life developed,
then the arts aiming to give pleasure developed,
when all such arts were established, knowledge which is neither useful nor pleasurable was discovered, thus mathematics was first developed,
and the first philosophers began asking questions about the motions of the moon, the sun, and the stars, and about the origin and development of the world.

The later stages Aristotle describes also in more detail:

first, men were forced by necessity to discover the useful arts, like sowing and the grinding of corn,
next, they discovered the arts aiming not at necessity but at beauty,
next, they turned to politics, and discovered laws and all the things that keep a city together,
next, they went further and turned to things themselves and their nature,
finally, they turned to divine and unchangeable things.

That perception is knowledge was argued before Plato, and still would be. If perception is knowledge, memory is knowledge, experience is knowledge, the grinding of corn is knowledge, the arts aiming at beauty are knowledge, mathematics is knowledge, politics is knowledge, physics is knowledge, philosophy is knowledge. Everything is knowledge, or what is the same, nothing is knowledge.

Alternatively, everything is not knowledge. If so, the question of what is not knowledge, and what is knowledge, cannot be answered without dividing the development, perception, memory, experience, the grinding of corn, the arts aiming at beauty, mathematics, politics, physics, philosophy, into two, not knowledge, on one side, knowledge, on the other, implicitly if not explicitly. And if so, explicitly is better than implicitly. Only the explicit division of the development, and that is, the solution of the problem of the beginning of the acquisition of knowledge, can answer the question of what is not knowledge, and what is knowledge.

Part of the development of one part of the development was recovered explicitly by Ptolemy. He writes: mathematics alone is certain and unshakable knowledge. And in order to do mathematics, Ptolemy collected observations. He writes: the longer the time between observations, the greater the accuracy with which periods of revolution can be determined; we must thus endeavor to increase the accuracy of our propositions to the degree to which the length of time between us and the most ancient accurate observations allows; we must not neglect to examine all such records. Accordingly, Ptolemy collected all accurate observations, from the most ancient, in 721^{BC}, to his own last observation, in 141^{AD}. He writes also: Hipparchus did not attempt to develop the theory of the planets because he did not have a store of accurate observations from earlier times to compare with his own. Having collected such a store, Ptolemy developed the theory of the planets.

Mathematics is certain and unshakable, and so is its development. And it is, because it is cumu-

lative.

The success of the labors of Ptolemy stands on the success of the labors of his predecessors. And nothing in this proposition needs to be vague. In particular, nothing is vague as to which the success of the labors of Ptolemy's predecessors is. As far as accurate observation is concerned, his predecessors are 58 accurate observations made by 16 predecessors of the philosophers and philosophers. As far as accurate observation is concerned, the success of the labors of Ptolemy stands on the success of the labors of these 16 predecessors of his, and on nothing else.

If accurate observation is part of the acquisition of knowledge, and Ptolemy discovered the most ancient accurate observation, Ptolemy thus solved one part of the problem of the beginning of the acquisition of knowledge. If he collected all accurate observations from 721^{BC} to 141^{AD}, he thus solved one part of the problem of collecting the acquisition of knowledge in its entirety. We must not neglect to examine all such records. And if Ptolemy examined all such records, partial as the solutions are, of the parts of the two problems of which they are the solutions, they are the complete solutions.

As the problems of the beginning of the acquisition and of collecting the acquisition in its entirety can be solved with respect to one part of mathematics, so they can be with respect to all parts of mathematics. As they can be solved with respect to mathematics, so they can be with respect to philosophy, physics, politics, the arts aiming at beauty, the grinding of corn, experience, memory, and perception.

This generalization follows from Hume and from Kant.

Everything of which plant, animal, or philosopher is aware terminates in experience. Oak, elm, Adam, Galileo, or Newton. Hume writes: the most ignorant and stupid peasants, nay infants, nay brute beasts, improve by experience and learn the qualities of natural objects by observing the effects which result from them; experimental reasoning itself, which we possess in common with the beasts, is nothing but a species of instinct, or mechanical power. When brute beast infers from the fluidity and transparency of water that it will suffocate him, and Adam infers from the light and warmth of fire that it will consume him, these are inferences from experience, and what Galileo collected in the *Two new sciences*, and Newton in *The mathematical principles*, are inferences from experience.

The entire development, and every part of the development, perception, memory, experience, the grinding of corn, the arts aiming at beauty, mathematics, politics, physics, philosophy, is accessible to nothing but inferences from experience. And because every part of the development, and hence the entire development, resolves itself into experience, and hence into inferences from experience.

Every part, from perception, at the far end of the development, to philosophy, at the near end. At the far end: all perceptions resolve themselves into impressions and ideas; and all ideas are copied from impressions. And given experience, impressions are nothing but part of experience. At the near end: mathematicians pretend that their ideas must be comprehended by pure and intellectual view of which the superior faculties of the soul alone are capable; and philosophers are fond of this notion of spiritual and refined perception, by which they cover their absurdities, and refuse to submit to the decision of clear ideas by appealing to such as are obscure and uncertain; but to destroy this artifice, we need but reflect on the principle that all ideas are copied from impressions. And given experience, impressions are nothing but part of experience.

The entire development, with all its parts, is the development of inferences from experience.

Thus, when Ptolemy collected the accurate observations of his predecessors, he collected inferences from experience. And he thus separated these inferences from experience from all other inferences from experience. He separated them from all earlier inferences from experience, and from all other contemporary inferences from experience. The inferences from experience collected into the 58 accurate observations made by 16 predecessors of his, on one side, all other inferences from experience made by oaks and elms, brute beasts and Adam, infants and the most ignorant and stupid peasants, mathematicians and philosophers, on the other side.

And as these inferences from experience can

be separated from all other inferences from experience, so all inferences from experience can be divided into these that are part of the acquisition of knowledge, on one side, and those that are not part of it, on the other.

On the ground on which it thus stands the generalization stands with any security. Hume established the foundation on which the acquisition of knowledge stands with any security. And on which, hence, the investigation of the acquisition can stand with any security.

In order to make accurate observations Ptolemy collected earlier accurate observations. In order to do mathematics, he collected earlier mathematics. In order to continue the acquisition of knowledge, he collected knowledge acquired earlier.

If experience is the foundation of the acquisition, all earlier acquisition is part of the foundation of all later acquisition, and the more successful the earlier acquisition the more it is the foundation of the later acquisition exclusively.

This follows from Kant's discovery, that experience has form.

Accordingly, in order to solve the problem on which everything depends, how are synthetic *a priori* propositions possible? Kant solves the problems, how is mathematics possible? and how is physics possible? In order to investigate knowledge, Kant investigates mathematics and physics.

Kant writes: whether the labor of knowledge travels the secure path of science can be judged by success. And he judges by success. Logic entered the secure path of science with the labors of one particular philosopher, mathematics entered it with the labors of one particular philosopher, physics entered it with the labors of one particular philosopher, and metaphysics will enter it, if it will, with the labors of one particular philosopher. To judge by success is to judge formally. And to judge formally is to judge propositions and inferences. In each case the labors of the particular philosopher are propositions and inferences.

Hence: the first beginning of human history does not have to be invented, it can be taken from experience. Nor does the first beginning of the acquisition of knowledge have to be invented. It can be taken from experience. To do so is to take it from the acquisition of knowledge itself. To do so is to judge by success. To do so is to judge formally. And to do so is to judge propositions and inferences.

Propositions and inferences, by being propositions and inferences, are in connection with one another. Given a number of propositions and inferences, they are in connection with other propositions and inferences, which are in connection with other propositions and inferences, and all the way to all propositions and inferences.

The observations collected by Ptolemy are part of the observations collected by Copernicus. Kepler's laws show that the Venus tablet of Ammisaduqa attests the discovery of Venus. Ptolemy's propositions and inferences are in connection with Copernicus's propositions and inferences, which are in connection with Kepler's propositions and inferences, which are in connection with the propositions and inferences on the Venus tablet of Ammisaduqa. The propositions and inferences of the *Almagest* are in connection with the propositions and inferences of the *Geography*, which are in connection with Caesar's propositions and inferences. They are in connection also with the propositions and inferences of the *Handy tables*, which are in connection with the propositions and inferences of Herodotus, which are in connection with the propositions and inferences of Rawlinson, Hincks, and Oppert.

Given a number of propositions and inferences, the collection of more propositions and inferences, and more, and all the way to all propositions and inferences, has to do no more than follow the connection of the propositions and inferences with one another. And if it succeeds in doing no more, if it succeeds in not contaminating in any way the connection of the propositions and inferences with one another, the collection will separate sharply the acquisition of knowledge, on one side, from everything that is not the acquisition of knowledge, on the other.

Propositions and inferences being propositions and inferences, the collection of more propositions and inferences can begin with any collection of a number of propositions and inferences. And a collection of a number of propositions and inferences is attested unambiguously by success. By the fact that, as it stands on propositions and

inferences established earlier, so propositions and inferences established later stand on it. The collection can thus begin with Ptolemy. Or it can begin with Aristotle. In fact, Aristotle himself began the collection. Aristotle discovered the review of his predecessors' formulations and solutions of problems as the ground of the formulation and solution of problems. Each of his reviews is a collection of propositions and inferences. Each of his reviews is thus also a partial solution of the problems of the beginning of the acquisition of knowledge and of collecting it in its entirety. The collection of more propositions and inferences can readily begin with Aristotle's reviews. But it can begin with Hume. Or with Kant. Or with Heisenberg. It can even begin with Hegel. For spirit did not restore itself to the purity and substantiality of its being, philosophy did it for it, and as this very inference attests, philosophy did it by propositions and inferences.

Darwin formulated the theory of evolution.

Aristotle is concerned with animals, Hume is concerned with oak, elm, and brute beast, Kant is concerned with all sentient beings and all rational beings, Frege is concerned with thought.

The theory of the origin and development of organisms cannot decide what the acquisition of knowledge is, but cannot not decide what the acquisition of knowledge is not. If the acquisition is not produced by selection, it is produced by a principle distinct from selection, and the end of an organism cannot be the end of the acquisition and the end of the acquisition cannot be the end of an organism.

The theory of evolution is thus the decisive negative test of the formulation and solution of the two problems, of the beginning of the acquisition, and of collecting the acquisition in its entirety. Selection cannot not decide where the development from perception through philosophy has to be divided into two, not acquisition of knowledge, on one side, and hence acquisition of knowledge, on the other. Where inferences from experience have to be divided into two, those which precede the acquisition, on one side, and hence those with which the acquisition begins, on the other. And selection cannot not decide where all subsequent development, once the acquisition has begun, has to be divided into two, not part of the acquisition, on one side, and hence part of it, on the other. Where all subsequent inferences from experience, after the first inferences attesting the acquisition, have to be divided into two, those which are not part of the acquisition, on one side, and hence those which are, on the other.

Thus, selection shows why observation is not looking or seeing. Why the grinding of corn is not part of the acquisition. Why oak and elm instance parricide and ingratitude. Why the thought gives no knowledge.

Frege discovered that the elements of a proposition are function and argument, showed that the firmest proof is free of gaps, formalized truth, generality, and existence, distinguished the reference and the true from the thought and the sense, made logic a part of mathematics.

Aristotle is concerned with propositions, proof, form, truth, generality, existence, thought, logic, mathematics. Ptolemy writes: mathematics is certain and unshakable knowledge, it can also deal with the objects of physics and theology, and even where action and character are concerned, it above all can make men see clearly. Hume is concerned with inferences, proof, truth, generality, existence, thought, mathematics. Kant discovered that propositions pose the problem on which everything depends. He is also concerned with proof, form, truth, generality, existence, thought, logic, mathematics. And what Darwin discovered is a proposition, the principle of selection.

Frege's results have to be the decisive affirmative test of the formulation and solution of all problems before the investigation.

In particular, if the formulation and solution of the problems of the beginning of the acquisition and of collecting it in its entirety has to begin with the collection of propositions and inferences, the distinction of the reference and the true from the thought and the sense can guide the collection in the effort to follow the connection of the propositions and inferences with one another without contaminating it with thoughts, sense, and images saturated with feelings.

The reference and the true can guide the collection, because proof can guide it. Proof is to bring a proposition in connection with the propositions which are its consequences and of which it is the consequence. And proof develops, from

surmises from an insufficient number of particular cases to firmer proof, and firmer, and firmer, and to chains of inferences free of gaps. From the Venus tablet of Ammisaduqa to Eudoxus to Hipparchus to Ptolemy to Copernicus to Kepler to Newton to Laplace.

On the hypothesis that the acquisition of knowledge is a function, the main difficulty with respect to the function itself is its extent. To resolve this difficulty, the difficulties with respect to the value of the function have to be resolved first. The main difficulty with respect to the value is reality. To resolve this difficulty, the difficulties with respect to the argument of the function have to be resolved first. The main difficulty with respect to the argument is relevance. It is to collect all the relevant, and only the relevant, objects which are the argument. And if these objects are propositions and inferences, and all propositions and inferences, and only the propositions and inferences, constituting the acquisition of knowledge are collected, this has to lend plausibility to the hypothesis.

With Maxwell: not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it.

If the problems of the beginning of the acquisition and of collecting it in its entirety are solved, how much there is of it will have been answered.

Beginning with the first philosopher, the acquisition of knowledge has been part of the development of philosophy. The problem is that the acquisition of knowledge predates the development of philosophy by millennia. And that the development of philosophy has included much more than the acquisition of knowledge.

The problem of the kinds of the acquisition is not which they are. The kinds are mathematics, politics, history, physics, and their subdivisions. The problem is of their relations. It is to describe how each kind has first developed, and how all kinds have developed from one another. The problem is to describe the system of the kinds.

If these problems are solved, of what kind it is will have been answered.

If the problem of the acquisition of knowledge is solved, the problem of knowledge can be solved.

One part of the problem of knowledge is its relation to truth. To the reference of all true propositions.

The other part is its relation to the world. To the terminal object of the acquisition of knowledge.

enumeration and review

Below are the particulars collected in this part one of the investigation.

To arrange them all uniformly years of birth are assigned, to the left of each particular. When the years are not recorded – and before the last several centuries the years are seldom recorded – they are inferred, and the conclusions to which the inferences point are listed to the right of each particular. The assignment of years to the particulars, and more generally chronology, is one of the problems of part four of the investigation.

All writings noted in the course of the investigation are also enumerated under the relevant particulars. Beginning at the time of printing, years are assigned to the writings. When a writing was published in the lifetime of the writer, or close to it, the year is that of publication rather than writing, when plural editions were published in his lifetime, the year is that of the first edition, the title that of the last edition. Inscriptions and fragments are numbered, the editions which the numbering follows indicated at the end.

The particulars are divided into important and not important. And first the important are enumerated, with the very important in bold. Then, separated from both the important and the not important, the few poets who have been noted in the course of the investigation are enumerated. Then the not important are enumerated. Being not important, these are more difficult to arrange uniformly. They are thus enumerated in two steps. First, years of birth are assigned to all those that can and should be dated, and they are enumerated, like the important, in their temporal series and order. Then, those that cannot and should not be dated, and either because they are not real or because, similarly, they are very very very not important, are enumerated in the order of their earliest sources in the text.

Importance, clearly, cannot not be important. It is a central problem of part three of the investigation.

The enumeration and review of the particulars collected in the course of the entire investigation, in all its parts, is part eight of the investigation.

important

3120^{BC}	**3 calculations of area**	the century around *c.* 3080^{BC}
3040	**Narmer**	prince *c.* 3000
3000	**Djer**	prince *c.* 2960
2720	**Djoser**	prince *c.* 2680
2720	**Enmebaragesi**	prince *c.* 2680
2640	**Sneferu**	prince *c.* 2600
2620	**Khufu**	prince *c.* 2580, son of Sneferu
2575	**Ur-Nanshe**	prince *c.* 2535
2545	**Akurgal**	prince *c.* 2505, son of Ur-Nanshe
2515	**Eanatum**	prince *c.* 2475, son of Akurgal
	inscriptions	
2362 - 2281	**Sargon**	prince *c.* 2337-281
	inscriptions	
2360	**Unas**	prince *c.* 2320
	tomb inscription	
2161 - 2103	**Ur-Nammu**	prince *c.* 2121-03
	code	
2128 - 2055	**Shulgi**	prince *c.* 2103-055, son of Ur-Nammu
2102 - 2046	**Amar-Sin**	prince *c.* 2055-46, son of Shulgi
2082 - 2037	**Shu-Sin**	prince *c.* 2046-37, son of Amar-Sin
2077	**Sumerian prince list**	under Ibbi-Sin, prince *c.* 2037-13
2062 - 2013	**Ibbi-Sin**	prince *c.* 2037-13, son of Shu-Sin
2046 - 1976	**Amenemhet**	prince *c.* 2006-1976
2016 - 1932	**Senwosret**	prince *c.* 1976-32, son of Amenemhet

1972 - 1897	**Amenemhet II**	prince *c.* 1932-897, son of Senwosret
1937 - 1878	Senwosret II	prince *c.* 1897-78, son of Amenemhet II
1918 - 1845	**Senwosret III**	prince *c.* 1878-45, son of Senwosret II
1911	**temple register from Lahun –** observation in 1871 ± 5	
1885 - 1800	**Amenemhet III**	prince *c.* 1845-00, son of Senwosret III
1885	Kudur-Mabuk	prince *c.* 1845
1885	**book of 87 problems**	under Amenemhet III, prince *c.* 1845-00
1861	**Larsa prince list**	under Rim-Sin, prince *c.* 1821-761
1854	**Isin prince list**	under Damaqilishu, prince *c.* 1814-791
1846 - 1761	**Rim-Sin**	prince *c.* 1821-761, son of Kudur-Mabuk
1839 - 1791	Damaqilishu	prince *c.* 1814-791
1837 - 1792	Sinmuballit	prince *c.* 1812-792
1826	**Moscow papyrus**	the century around *c.* 1786
1817 - 1749	**Hammurabi** code	prince *c.* 1792-49, son of Sinmuballit
1789 - 1711	**Samsuiluna**	prince *c.* 1749-11, son of Hammurabi
1784	**calculation of the diagonal of the square –** the century before *c.* 1694	
1784	calculation of the area of the circle – the century before *c.* 1694	
1751 - 1683	Abieshuh	prince *c.* 1711-683, son of Samsuiluna
1723 - 1646	Ammiditana	prince *c.* 1683-46, son of Abieshuh
1686 - 1625	**Ammisaduqa**	prince *c.* 1646-25, son of Ammiditana
1686	**Venus tablet of Ammisaduqa –** under Ammisaduqa, prince *c.* 1646-25	
1670	**Hattusili**	prince *c.* 1630
1665 - 1594	Samsuditana	prince *c.* 1625-594, son of Ammisaduqa
1665	**Babylonian prince list**	under Samsuditana, prince *c.* 1625-594
1634	***When on high***	the century around *c.* 1594
1630	**Mursili**	prince *c.* 1610, grandson of Hattusili
1585 - 1521	**Amenhotep**	prince *c.* 1545-21
1561 - 1509	**Thutmose**	prince *c.* 1521-09
1534 - 1491	Thutmose II	prince *c.* 1509-491, son of Thutmose
1510	**Karnak prince list**	under Thutmose III, prince *c.* 1470-37
1496 - 1437	**Thutmose III**	child *c.* 1491 - prince *c.* 1470-37, son of Thutmose II
1490	**map of Nippur**	the century before *c.* 1400
1450 - 1408	Ashur-bel-nisheshu	prince *c.* 1417-08
1415 - 1363	Eriba-Adad	prince *c.* 1390-63, son of Ashur-bel-nisheshu
1390 - 1327	**Ashur-uballit**	prince *c.* 1363-27, son of Eriba-Adad
1366 - 1302	Ramesses	prince *c.* 1304-2, father of Seti
1360 - 1317	Enlil-nirari	prince *c.* 1327-17, son of Ashur-uballit
1346 - 1291	**Seti**	prince *c.* 1302-291, son of Ramesses, father of Ramesses II
1342	**Abydos prince list**	under Seti, prince *c.* 1302-291
1331	**Saqqara prince list**	under Ramesses II, prince *c.* 1291-24
1330 - 1273	**Adad-nirari**	prince *c.* 1305-273, son of Enlil-nirari
1316 - 1224	**Ramesses II**	prince *c.* 1291-24, son of Seti
1281	**Turin papyrus**	the century after *c.* 1291
1160 - 1115	Ashur-resh-ishi	prince 1133-15
1140 - 1076	**Tiglath-pileser** inscriptions	prince 1115-076, son of Ashur-resh-ishi
1028 - 963	**David**	prince *c.* 1003-963
988 - 923	Solomon	prince *c.* 963-23, son of David
970 - 912	Ashur-dan II	prince 935-12
950 - 891	**Adad-nirari II**	prince 912-891, son of Ashur-dan II
930 - 884	Tukulti-Ninurta II	prince 891-84, son of Adad-nirari II
910 - 859	**Ashur-nasirpal II**	prince 884-59, son of Tukulti-Ninurta II
881	**Li**	deposed in 841

860		**Yahwistic document**	the century before *c.* 791/50
		parts of *Genesis*, *Exodus*, and *Numbers*	
790		***Isaiah***	prophecy began *c.* 750
785 -	727	**Tiglath-pileser III**	prince 745-27
767		**Assyrian prince list**	under Shalmaneser V, prince 727-2
765 -	722	Shalmaneser V	prince 727-2, son of Tiglath-pileser III
764		**Homer**	the century before 676/3
		Iliad	
		Odyssey	
761		**observation of lunar eclipses from 721 to 720**	
760 -	705	**Sargon II**	prince 722-05, younger son of Tiglath-pileser III
740 -	680	**Sennacherib**	prince 705-680, son of Sargon II
734		**Hesiod**	one generation after Homer
		Work and days	
717		*When Anu and Enlil*	the century before 627
715 -	668	**Esarhaddon**	prince 680-68, son of Sennacherib
690 -	627	**Ashurbanipal**	prince 668-27, younger son of Esarhaddon
689 -	610	Psamtik	prince 664-10
667		*Jeremiah*	prophecy began in 627
653		Epimenides	purified Athens between *c.* 632 and 594
		fragments	
649 -	578	Pittacus	died in 578 having lived more than 70 years
634		**Solon**	archon 594
625		**Thales**	predicted an eclipse which occurred in 585
611 -	541	**Anaximander**	611 - the decade after 546
597		***Joshua*, *Judges*, *Samuel*, *Kings* –** the two generations after 587	
590		**Anaximenes**	between Anaximander and Xenophanes
584 -	530	**Cyrus**	prince 559-30
578 -	522	**Polycrates**	tyrant *c.* 538-22
570 -	495	**Pythagoras**	*c.* 570 - the decade after *c.* 500
570 -	473	**Xenophanes**	*c.* 570 - the decade after 478
		fragments	
562		**Hecataeus**	under Darius, prince 522-486
555 -	522	**Cambyses**	prince 530-22, son of Cyrus
550 -	486	**Darius**	*c.* 550 - 486
540		**Heraclitus**	one generation after Pythagoras
		fragments	
532		**Alcmaeon**	between Pythagoras and Empedocles
529		**Priestly code**	the century before 445/33
		parts of *Genesis*, *Exodus*, and *Numbers*	
		Leviticus	
528		*Job*	the century after 538
520 -	465	**Xerxes**	*c.* 520 - 465
515 -	445	**Parmenides**	*c.* 515 - the decade after *c.* 450
		fragments	
500 -	428	**Anaxagoras**	*c.* 500 - *c.* 428
495 -	435	**Empedocles**	*c.* 495 - *c.* 435
495 -	429	**Pericles**	*c.* 495 - 429
490 -	420	**Protagoras**	*c.* 490 - *c.* 420
490		**Oenopides**	a little younger than Anaxagoras
490		**Zeno of Elea**	born *c.* 490
485 -	380	Gorgias	*c.* 485 - *c.* 380
484 -	425	**Herodotus**	484 - the decade after 430
		Histories	

483 -	424	**Artaxerxes**	
481		Melissus	general in 441
480		**Leucippus**	between Anaxagoras and Democritus
472		**Meton**	observation in 432
470 -	394	**Hippias**	younger than Protagoras - the decade after 399
469 -	399	**Socrates**	
469		**Hippocrates**	contemporary with Socrates
		Hippocratic corpus, including	
		Epidemics I	
		Epidemics III	
		Precepts	
469		Philolaus	contemporary with Socrates
467		Thrasymachus	parodied in 427
464 -	399	**Thucydides**	general in 424 - the decade after 404
460 -	394	**Theodorus of Cyrene**	one generation after Oenopides - the decade after 399
460		**Democritus**	40 years younger than Anaxagoras
460		**Hippocrates of Chios**	one generation after Oenopides
453 -	405	Darius II	prince 424-05, son of Artaxerxes
449		Cratylus	younger than Socrates
448		**Hicetas**	between Philolaus and Plato
447 -	364	Euclid of Megara	older than Plato - the decade after 369
445		Ctesias	under Artaxerxes II, prince 405-359
443 -	365	**Camillus**	tr. mil. 403 - 365
436 -	338	Isocrates	
430 -	359	Artaxerxes II	prince 405-359, son of Darius II
430 -	350	**Xenophon**	*c.* 430 - the decade after 355
		Cavalry commander	
427 -	347	**Plato**	
		Apology	
		Euthyphro	
		Laches	
		Lysis	
		Charmides	
		Greater Hippias	
		Meno	
		Phaedo	
		Symposium	
		Phaedrus	
		Republic	
		Cratylus	
		Theaetetus	
		Sophist	
		Timaeus	
		Laws	
427		**Archytas**	contemporary with Plato
414 -	369	**Theaetetus**	*c.* 414 - 369
410 -	362	**Epaminondas**	*c.* 410 - 362
408 -	356	**Eudoxus**	*c.* 408 - *c.* 356
407		Aeneas Tacticus	general in 367
		On the defense of fortified positions	
395 -	317	**Ponticus**	contemporary with Xenocrates - the decade after 322
		fragments	
395 -	314	Xenocrates	
391		Metrodorus of Chios	between one and two generations after Isocrates
384 -	322	**Aristotle**	

book I of the *Metaphysics*
book XII of the *Metaphysics*
books XIII-XIV of the *Metaphysics*
Physics
On the heaven
On generation and corruption
Meteorology
On the soul
History of animals
Parts of animals
Movement of animals
Progression of animals
Generation of animals
On the senses and the sensible
On memory and recollection
On sleep and waking
On dreams
On divination in sleep
On length and brevity of life
On youth, old age, life and death, and respiration
book II of the *Metaphysics*
books III-IV of the *Metaphysics*
books VI-X of the *Metaphysics*
Posterior analytics
Categories
On interpretation
Prior analytics
Topics
Sophistical refutations
Politics
The constitution of Athens
Rhetoric
Poetics
Nicomachean ethics
Eudemian ethics
[*Mechanics*]
[*Problems*]
[*On the world*]
[*On breath*]
[*On colors*]
[*On the audible*]
[*Physiognomy*]
[*On plants*]
[*On marvelous things heard*]
[*On indivisible lines*]
[*The situation and names of winds*]
[*On Melissus, Xenophanes, and Gorgias*]
[book V of the *Metaphysics*]
[book XI of the *Metaphysics*]
[*Great ethics*]
[*On virtues and vices*]
[*Rhetoric to Alexander*]
[*Economics*]
fragments

384 - 322 Demosthenes

382 -	336	**Philip**	
378		**Menaechmus**	student of Eudoxus
375		Anaxarchus	between Metrodorus of Chios and Pyrrho
371 -	287	**Theophrastus**	
		History of plants	
		Causes of plants	
371		**Dicaearchus**	contemporary with Theophrastus
371		**Eudemus**	contemporary with Theophrastus
371		**Pytheas**	contemporary with Dicaearchus
366 -	282	**Ptolemy**	
360 -	270	**Pyrrho**	*c.* 360 - *c.* 270
356 -	323	**Alexander**	
354		Callippus	one generation after Aristotle
346		**Euclid**	under Ptolemy, prince 306-282
		Elements	
		Phenomena	
		Optics	
346		**Herophilus**	under Ptolemy
346		**Manetho**	under Ptolemy
341 -	270	**Epicurus**	
335 -	263	**Zeno of Citium**	
335		**Timocharis**	observations from 295 to 272
331 -	232	Cleanthes	
327 -	269	Strato	scholarch 287-69
322		Zenodotus	under Ptolemy II, prince 282-46
320 -	230	Timon	*c.* 320 - *c.* 230
320		**Aristarchus**	observation in 280
		On the sizes and distances of the sun and moon	
316 -	242	Arcesilaus	
316		Ariston of Chios	contemporary with Arcesilaus
316		**Erasistratus**	one generation after Herophilus
312		Dionysius	observations from 272 to 241
308 -	246	Ptolemy II	
305 -	215	Hiero II	*c.* 305 - 215
287 -	212	**Archimedes**	*c.* 287 - 212
		On the sphere and cylinder	
		Measurement of the circle	
279 -	206	**Chrysippus**	281/78 - 208/5
274 -	194	**Eratosthenes**	276/3 - 196/3
273 -	203	Fabius Maximus	cos. 233 - 203
262 -	208	Marcellus	cos. 222 - 208
247 -	183	**Hannibal**	
242 -	187	**Antiochus III**	*c.* 242 - 187
240 -	152	Diogenes the Babylonian	*c.* 240 - *c.* 152
240 -	151	Critolaus	contemporary with Diogenes the Babylonian - the decade after 156
236 -	183	**Scipio**	
234 -	149	**Cato**	
		On agriculture	
233		**Apollonius**	born 246/21
		Conics	
216 -	144	Aristarchus of Samothrace	*c.* 216 - *c.* 144
214 -	129	Carneades	
200 -	118	**Polybius**	*c.* 200 - *c.* 118
		Histories	

189 ***Daniel*** the generation after 164
187 **Hipparchus** observations from 147 to 127
commentary on Eudoxus's *Phenomena*
145 - 86 **Sima Qian** *c.* 145 - the decade after 91
Records of the grand historian
138 - 78 **Sulla**
135 - 51 **Posidonius** *c.* 135 - *c.* 51
116 - 27 **Varro**
Antiquities
The disciplines
On rustic matters
106 - 43 **Cicero**
On the orator
On the laws
Academic matters
On the good and bad ends
On the nature of the gods
On divination
100 - 44 **Caesar**
Commentaries on the Gallic war
Commentaries on the civil war
95 - 52 Lucretius 97/4 - 55/0
95 - 46 Cato Uticensis
95 Andronicus one generation before Strabo
89 - 22 Vitruvius under Caesar, Rubicon 49 - the decade after 27
On architecture
83 - 43 Hirtius cos. 43 - 43
book VIII of the *Commentaries on the Gallic war*
69 - 30^{BC} Cleopatra
65 - 29^{AD} **Strabo** 68/3 - the decade after 24^{AD}
Geography
63 - 14 **Augustus**
The deeds of the divine Augustus
59 - 17 **Livy**
From the foundation of the city
58 - 21 **Labeo** pr. 18^{BC} - *c.* 21
52 Aenesidemus one generation before Philo Judaeus
42 - 37 **Tiberius**
24 Cleomedes between Posidonius and Ptolemy
The circular theory of the heavenly bodies
22^{BC} Philo Judaeus old in 39^{AD}
6^{AD} - 62 **st Paul** on the road to Damascus *c.* 36 - *c.* 62
First letter to the Corinthians
[*Letter to the Colossians*]
22 **Hero** observation in 62
24 - 79 **Pliny**
Natural history
43 - 102 **Clement of Rome** bishop 93-102
Letter to the Corinthians
43 ***Gospel of Matthew*** between 75 and 90
48 - 125 **Plutarch** 45/50 - the decade after 120
On exile
Dinner conversations
On the face appearing in the orb of the moon
Numa

Sulla

50 ***Gospel of Luke*, *Acts of the apostles* –** between 80 and 100

53 - 117 **Trajan**

55 - 135 **Epictetus** 50/60 - 130/40

Manual

55 ***Gospel of John*, *Apocalypse* –** between 90 and 100

57 Marinus one generation before Ptolemy

58 **Menelaus** observations in 98

76 - 138 **Hadrian**

86 - 161 **Pius**

87 **Ptolemy** observations from 127 to 141

Almagest
Planetary hypotheses
Geography
Handy tables
Optics
Astrological influences
Harmonics
Analemma
Planisphere
Phases of the fixed stars
[*Fruit*]
[*On judgment and command*]

87 Theon of Smyrna observations from 127 to 132

Mathematical matters useful for the study of Plato

89 Aëtius between Plutarch and Galen

Collection of doctrines

102 - 173 **Julian** *c.* 102 - the decade after 168

127 Gellius born 125/8

Attic nights

129 - 208 **Galen** 129 - 199/216

146 - 211 **Severus**

146 - 212 Papinian contemporary with Severus - 212

150 - 214 **Clement of Alexandria** *c.* 150 - 211/6

159 Empiricus one generation after Galen

Outlines of Pyrrhonism
Against the mathematicians

164 Alexander of Aphrodisias wrote between 198 and 209

170 - 235 Hippolytus *c.* 170 - 235

Refutation of all heresies

176 **Paul** one generation after Papinian

185 - 254 **Origen**

200 - 258 Cyprian *c.* 200 - 258

On the unity of the catholic church

205 - 270 **Plotinus**

216 - 276 Mani 216 - 274/7

219 **Laertius** two generations after Empiricus

The lives and doctrines of the philosophers

224 **Diophantus** the generation before 276-82

Arithmetic

234 - 308 Porphyry 234 - 305/10

On the life of Plotinus and the arrangement of his works

240 - 325 Iamblichus *c.* 240 - *c.* 325

On the life of Pythagoras

244 - 313 **Diocletian**

255 - 336 Arius 250/60 - 336
265 - 339 **Eusebius** the decade after 260 - 337/40
The history of the church
275 - 337 **Constantine** *c.* 275 - 337
280 **Pappus** observation in 320
commentary on the *Almagest*
295 - 373 **Athanasius** *c.* 295 - 373
324 Theon of Alexandria observations in 364
commentary on the *Almagest*
commentary on the *Handy tables*
330 - 379 Basil of Caesarea *c.* 330 - 379
330 - 390 Gregory of Nazianzus *c.* 330 - 390
335 - 394 Gregory of Nyssa *c.* 335 - *c.* 394
339 - 397 Ambrose *c.* 339 - 397
340 - 420 Jerome 331/48 - 420
347 - 395 **Theodosius**
353 - 423 Pelagius 350/5 - the decade after 418
354 - 430 **Augustine**
On the Christian doctrine
Confessions
On the trinity
On the literal reading of Genesis
On nature and grace
The city of god against the pagans
380 - 461 **Leo** pope 440-61
380 Capella wrote between 410 and 429
Philology
394 **Vincent of Lérins** wrote in 434
Reminders
398 **Theodosian code** promulgated in 438
401 - 450 Theodosius II
411 - 485 **Proclus** 410/2 - 485
commentary on book I of the *Elements*
431 - 518 Anastasius *c.* 431 - 518
432 - 496 **Gelasius** pope 492-6
letters to Anastasius
433 - 493 Odoacer *c.* 433 - 493
443 Dionysius the Areopagite the century before 533
Celestial hierarchy
480 - 543 **Tribonian** *c.* 480 - *c.* 543
Digest
483 - 565 **Justinian**
489 **Simplicius** left Athens in 529
commentary on *On the heaven*
489 Eutocius contemporary with Simplicius
489 Zosimus two generations after 408/50
New history
540 - 604 **Gregory** *c.* 540 - 604
Dialogues
570 - 632 **Muhammad** *c.* 570 - 632
koran
581 - 656 **Uthman** conversion *c.* 611 - 656
587 - 644 **Umar** 581/92 - 644
601 - 680 **Muawiyah** 595/607 - 680
671 Tariq ibn Ziyad general in 711

711 - 775 **al-Mansur** 709/13 - 775
718 - 775 Constantine V
735 - 816 Leo III pope 795-816
742 - 814 **Charlemagne**
767 - 820 **al-Shafii**
Treatise on the sources of the law
786 - 833 **al-Mamun**
795 - 852 **al-Khwarizmi** 790/800 - the decade after 847
Algebra
831 - 901 Thabit ibn Qurra 826/36 - 901
860 - 930 **al-Razi** died in 925/35
Book of secrets
861 - 929 **al-Battani** one generation after Thabit ibn Qurra - 929
Astronomical treatise
865 - 950 **al-Farabi** 860/70 - *c.* 950
The views of the inhabitants of the ideal city
912 - 973 **Otto**
958 - 1025 **Basil II**
965 - 1040 **ibn al-Haytham** 965 - *c.* 1040
Optics
973 - 1020 Leif Eriksson 970/5 - *c.* 1020
973 - 1055 **al-Biruni** 973 - the decade after 1050
The monuments of past generations
980 - 1037 **ibn Sina**
Canon
1020 - 1085 **Gregory VII** *c.* 1020 - 1085
The dictate of the pope
1033 - 1109 **Anselm**
Monologue
Prologue
1035 - 1099 **Urban II** *c.* 1035 - 1099
1048 - 1123 Roscelin 1045/50 - 1120/5
1048 - 1127 **Khayyam** 1048 - 1123/31
Demonstration of problems of algebra
1058 - 1111 **al-Ghazali**
The incoherence of the philosophers
The revival of the religious sciences
Deliverance from error and union with the almighty
1059 - 1128 **Fulcher** *c.* 1059 - 1127/9
The deeds of the Franks on their pilgrimage to Jerusalem
1079 - 1142 **Abelard**
Yes and no
History of calamities
1093 - 1152 Conrad III
1096 - 1141 Hugh of St Victor *c.* 1096 - 1141
Instruction, on the study of reading
1113 - 1158 **Otto of Freising** 1111/5 - 1158
The history of the two cities
1114 - 1187 **Gerard** *c.* 1114 - 1187
1123 - 1190 **Frederick** *c.* 1123 - 1190
1126 - 1198 **ibn Rushd**
The incoherence of the incoherence
1138 - 1193 Salah al-Din
1158 - 1220 David of Dinant under Innocent III, pope 1198-216 - the decade after 1215

1161 - 1216 **Innocent III**
letters to Acerbus
1161 - 1227 **Genghis Khan** 1155/67 - 1227
1165 - 1223 **Philip II**
1170 - 1245 **Fibonacci** *c.* 1170 - the decade after 1240
Book of calculation
Book of squares
1187 - 1241 Ögedei 1185/9 - 1241
1194 - 1250 **Frederick II**
constitutions of Melfi, 1231
On the art of hunting with birds
1200 - 1280 **Albert** 1193/207 - 1280
On minerals
On plants
On animals
1206 - 1255 Batu 1205/7 - 1255
1209 - 1259 Möngke
1214 - 1270 **Louis IX**
1215 - 1286 William of Moerbeke *c.* 1215 - *c.* 1286
1215 - 1294 **Kublai**
1217 - 1265 Hülegü *c.* 1217 - 1265
1217 - 1292 **Roger Bacon** 1214/20 - 1292
The great work
1225 - 1274 **Aquinas** 1224/6 - 1274
Theological summa
1239 - 1307 **Edward**
1254 - 1324 **Polo** *c.* 1254 - 1324
Description of the world
1260 - 1328 **Eckhart** *c.* 1260 - 1328
1280 - 1342 Marsilius *c.* 1280 - *c.* 1342
1281 - 1360 **Orhan**
1285 - 1349 **Occam** 1280/90 - 1349
Summa of all logic
1295 - 1358 **Buridan** *c.* 1295 - *c.* 1358
Questions on the Physics
Questions on On the heaven
1304 - 1373 **ibn Battuta** 1304 - the decade after 1368
Travels
1312 - 1377 **Edward III**
1320 - 1382 **Oresme** *c.* 1320 - 1382
Questions on the geometry of Euclid
Treatise on the configuration of qualities and motions
1332 - 1406 **ibn Khaldun**
Introduction to history
1357 - 1433 **John**
1370 - 1444 **Bruni**
The history of the Florentine people
1394 - 1460 **prince Henry**
1395 - 1450 Eanes *c.* 1395 - the decade after 1445
1403 - 1461 **Charles VII**
1420 - 1482 **Benincasa**
atlas, 1462
1423 - 1483 **Louis XI**
1432 - 1481 **Mehmed II**
1432 - 1488 Ca' da Mosto

1433 - 1499 **Ficino**
1436 - 1476 **Regiomontanus**
On triangles of all kinds, 1463
1440 - 1505 **Ivan III**
1450 - 1486 **Cão** *c.* 1450 - 1486
1450 - 1498 Cabot *c.* 1450 - 1498
1450 - 1500 **Dias** *c.* 1450 - 1500
1451 - 1504 **Isabella**
1451 - 1506 **Columbus**
report, 15 Feb 1493
letter to Isabella and Ferdinand V, 7 Jul 1503
1452 - 1516 **Ferdinand V**
1453 - 1515 **Albuquerque**
1454 - 1512 Vespucci
letter to Lorenzo il Popolano, Mar 1503
1457 - 1509 **Henry VII**
1460 - 1510 **de la Cosa**
map, 1500
1460 - 1524 **da Gama** *c.* 1460 - 1524
1468 - 1520 Cabral 1468 - *c.* 1520
1468 - 1526 **de Toreno** drew maps in 1508 - 1526
map, 1526
1468 - 1536 **Erasmus** 1466/9 - 1536
Encomium of custom, or the praise of folly, 1511
1469 - 1527 **Machiavelli**
The prince, 1513
Discourses on the first ten books of Livy, 1517
1470 - 1520 **Waldseemüller** *c.* 1470 - 1518/21
map, 1507
Introduction to cosmography, with certain principles of geometry and astronomy necessary for the matter, 1507
1473 - 1543 **Copernicus**
Letter against Werner, 1524
On the revolutions of the heavenly spheres, 1543
1475 - 1541 **Pizarro** *c.* 1475 - 1541
1476 - 1526 **El Cano** *c.* 1476 - 1526
1478 - 1535 More
1480 - 1521 **Magellan** *c.* 1480 - 1521
1483 - 1540 **Guicciardini**
The history of Italy, 1540
1483 - 1546 **Luther**
commentary on the *Letter to the Galatians*, 1531
1485 - 1547 **Cortés**
1489 Pigafetta sailed with Magellan in 1519
The voyage around the world, 1526
1491 - 1547 **Henry VIII**
1493 - 1541 **Paracelsus**
1495 - 1566 **Suleiman**
1499 - 1557 Tartaglia
1500 - 1558 **Charles V**
1501 - 1576 **Cardano**
The great art, or the rules of algebra, 1545
1503 - 1566 Nostradamus
1514 - 1564 **Vesalius**

On the structure of the human body, 1543
1527 - 1598 Philip II
1533 - 1584 William
1533 - 1603 **Elizabeth**
1540 - 1603 **Viète**
Introduction in the analytic art, 1591
Responses to various mathematical matters, 1593
1544 - 1603 **Gilbert**
On the magnet, magnetic bodies, and the great magnet of the earth, new physiology demonstrated by many arguments and experiments, 1600
1546 - 1601 **Tycho**
Introductory exercises for the instauration of astronomy, 1602
1548 - 1600 **Bruno**
1561 - 1626 **Bacon**
On the dignity and growth of the sciences, 1605
Second part of the great instauration, new organon, or directions for the interpretation of nature, 1620
The history of the reign of king Henry VII, 1622
1564 - 1642 **Galileo**
Starry messenger, unfolding great and most wonderful sights and displaying for the inspection of everyone, and especially of the philosophers and astronomers, what with a telescope built by himself Galileo Galilei observed about the face of the moon, the fixed stars, the Milky Way, the nebulous stars, and first of all about four planets circling Jupiter's star at unequal intervals and periods and with marvelous speed, which, unknown to anyone until now, he discovered and decided to name the Medicean stars, 1610
The assayer, in which on a delicate and precise scale are weighted the matters contained in The astronomical and philosophical balance of Lothario Sarsi, 1623
Dialogue on the two main systems of the world, Ptolemaic and Copernican, 1632
Discourses and mathematical demonstrations in two new sciences dealing with mechanics and locomotion, 1638
letters to Giuliano de' Medici; letters to Liceti
1567 - 1625 **Maurice**
1571 - 1630 **Kepler**
New astronomy αἰτιολογητός, or celestial physics derived from commentaries on the motions of the star of Mars, from the observations of Tycho, 1609
Harmonics of the world, in five books, I. geometrical, II. architectonic, III. harmonic, IV. metaphysical, psychological, and astrological, V. astronomical and metaphysical, 1619
Epitome of Copernican astronomy, written in the form of questions and answers, 1618-21
Rudolphine tables, with which the restoration of astronomical science, long collapsed, is continued, first conceived by that phoenix of astronomers Tycho in 1564, from most accurate observations of the stars especially after 1572, 1627
Dream, or on lunar astronomy, 1634
1575 - 1624 Boehme
1578 - 1657 **Harvey**
1579 - 1644 van Helmont
Rise of medicine, or beginnings of a new physics; new progress in medicine, for the destruction of decease, for long life, 1648
1580 - 1626 **Snel**
1583 - 1645 Grotius
1588 - 1648 Mersenne
1588 - 1679 **Hobbes**
Leviathan, or the matter, form, and power of the commonwealth ecclesiastical and civil, 1651
1591 - 1661 **Desargues**
1592 - 1655 Gassendi
1594 - 1632 **Gustav II**

1596 - 1650 **Descartes**
Discourse on the method to conduct one's reason well and search for truth in the sciences, as well as the Optics, Meteorology, and Geometry, which are essays in this method, 1637
Replies to objections raised by some men of learning against the Meditations, 1641
The principles of philosophy, 1644
letters to Mersenne
1598 - 1647 **Cavalieri**
1599 - 1658 **Cromwell**
1601 - 1665 **Fermat**
1602 - 1675 Roberval
1602 - 1686 **Guericke**
1608 - 1647 **Torricelli**
1611 - 1675 Turenne
1616 - 1703 **Wallis**
Arithmetic of infinities, or a new method of investigation of the quadrature of curves and other difficult mathematical problems, 1656
1618 - 1663 **Grimaldi**
1619 - 1683 Colbert
1620 - 1682 **Picard**
1621 - 1686 Condé
1623 - 1662 **Pascal**
1625 - 1712 **Cassini**
Astronomical observations made in different regions of the kingdom during the year 1672, 1673
The elements of astronomy, verified by Mr Cassini from the relation of his tables to the observations made by Mr Richer on the island of Cayenne, 1684
1627 - 1691 **Boyle**
The sceptical chemist, or chemico-physical doubts and paradoxes, touching the experiments whereby vulgar spagyrists are wont to endeavor to evince their salts, sulfur, and mercury, to be the true principles of things, 1661
1628 - 1694 **Malpighi**
Anatomical observations on the lungs, 1661
1629 - 1695 **Huygens**
The system of Saturn, or on the causes of Saturn's marvelous phenomena and its satellite, a new planet, 1659
Κοσμοθεωρός, or conjectures on the countries and inhabitants of the heavenly bodies, 1698
1630 - 1677 Barrow
1630 - 1696 Richer
1632 - 1677 **Spinoza**
Theologico-political treatise, 1670
1632 - 1704 **Locke**
Two treatises of government, in the former the false principles and foundation of Sir Robert Filmer and his followers are detected and overthrown, the latter is an essay concerning the true origin, extent, and end of civil government, 1690
1632 - 1723 **Leeuwenhoek**
1633 - 1707 Vauban
1635 - 1702 **Hooke**
1638 - 1686 **Steno**
1638 - 1715 **Louis XIV**
1639 - 1691 Louvois
1642 - 1727 **Newton**
On analysis by equations of infinite number of terms, 1669
A treatise on the method of fluxions and infinite series, with its application to the geometry of curves, 1671
The mathematical principles of natural philosophy, 1687
letters to Hooke

1644 - 1710 **Rømer**

1646 - 1716 **Leibniz**

Dissertation on the art of combination, in which from the fundamental arithmetic new precepts are extracted for the theory of combinations and transpositions, and their uses for all sciences are shown; new seeds sown for the art of meditation or the logic of discovery, 1666

Beginning and specimens of a general science for the instauration and growth of the sciences, 1679

On the true proportion of the circle to the circumscribed square, expressed in rational numbers, 1682

A new method for maxima and minima, as well as tangents, impeded by neither fractional nor irrational quantities, and a singular kind of calculus for it, 1684

Reply to the reflections contained in the second edition of the Critical dictionary of Bayle, article Rorarius, on the system of preestablished harmony, 1702

manuscript beginning *Nullum quidem librum*, 1702

New essays on human understanding, 1704

Principles of philosophy, or monadology, 1714

Brunswick annals of the western empire, 1716

letters to Wagner

1647 - 1706 Bayle

A historical and critical dictionary, 1697

1650 - 1702 **William III**

1655 - 1705 **Jakob Bernoulli**

The art of conjecture, to which are added a treatise on infinite series and a letter written in French on the game of tennis, 1713

1656 - 1708 Tournefort

1656 - 1742 **Halley**

To that glory of our time and people, the mathematico-physical treatise of the most excellent Isaac Newton, 1687

An estimate of the degree of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslau, with an attempt to ascertain the price of annuities upon lives, 1693

Considerations on the change of the latitudes of some of the principal fixed stars, 1717

1660 - 1734 **Stahl**

1666 - 1736 **Gray**

1667 - 1748 **Johann Bernoulli**

1672 - 1725 **Peter**

1672 - 1731 **Geoffroy**

1679 - 1754 Wolff

1685 - 1731 **Taylor**

1685 - 1753 **Berkeley**

Philosophical commentaries, 1708

An essay toward a new theory of vision, 1709

A treatise concerning the principles of human knowledge, wherein the chief causes of error and difficulty in the sciences, with the grounds of scepticism, atheism, and irreligion, are inquired into, 1710

Three dialogues between Hylas and Philonous, the design of which is plainly to demonstrate the reality and perfection of human knowledge, the incorporeal nature of the soul, and the immediate providence of a deity, in opposition to sceptics and atheists, 1713

On motion, or the principle and nature of motion, and on the cause of the communication of motions, 1721

The theory of vision, or visual language showing the immediate presence and providence of a deity, vindicated and explained, 1733

The analyst, or a discourse addressed to an infidel mathematician, wherein it is examined whether the object, principles, and inferences of the modern analysis are more distinctly conceived, or more evidently deduced, than religious mysteries, 1734

Siris, a chain of philosophical reflections and inquiries concerning the virtues of tar-water, and

divers other subjects connected together and arising one from another, 1744
letters to Prior

1689 - 1755 **Montesquieu**
On the spirit of the laws, or on the relation which the laws must have with the constitution of every government, with customs, climate, religion, commerce, etc., 1748

1694 - 1778 **Voltaire**
The century of Louis XIV, 1751
The philosophy of history, or a historical dissertation on the origin, manners, customs, and religions of the different nations of antiquity, with a concise exposition of their religious superstitions, 1765

1698 - 1746 Maclaurin

1698 - 1759 **Maupertuis**

1700 - 1782 **Daniel Bernoulli**

1706 - 1790 **Franklin**

1707 - 1778 **Linnaeus**
The system of nature, according to the three kingdoms of nature, by classes, orders, genera, and species, 1735
The genera of plants, as well as their natural characters, by number, shape, place, and proportion of all fruit bearing parts, 1737
Botanical philosophy, in which the foundations of botany are explained, with definitions of parts, examples of terms, and observations of rarities, 1751
The species of plants, showing the properly recognized plants, related by genera, with specific differences, common names, selected synonyms, and native locations, arranged by the sexual system, 1753

1707 - 1783 **Euler**
Introduction to the analysis of infinities, 1748
Instruction in the differential calculus, with its application in finite analysis and the doctrine of series, 1755
Instruction in the integral calculus, 1768-70

1707 - 1788 **Buffon**
Natural history, general and particular, with a description of the cabinet of the king, 1749-89, including
Experiments on the development of heat in bodies, 1774
Further experiments on the development of heat in different mineral substances, 1774
Researches on the cooling of the earth and the planets, 1775

1708 - 1778 **Pitt**

1711 - 1776 **Hume**
A treatise of human nature, being an attempt to introduce the experimental method of reasoning into moral subjects, 1739
An abstract of a book lately published, entitled A treatise of human nature, etc., wherein the chief argument of that book is farther illustrated and explained, 1740
Essays, moral, political, and literary, 1741
Of the middle station of life, 1742
A letter from a gentleman to his friend in Edinburgh, containing some observations on a specimen of the principles concerning religion and morality, said to be maintained in a book lately published, entitled A treatise of human nature, etc., 1745
An inquiry concerning human understanding, 1748
An inquiry concerning the principles of morals, 1751
The history of England, 1754-62
Four dissertations, I. The natural history of religion, II. Of the passions, III. Of tragedy, IV. Of the standard of taste, 1757
A concise and genuine account of the dispute between Mr. Hume and Mr. Rousseau, with the letters that passed between them during their controversy, 1766
My own life, 1776
Dialogues concerning natural religion, 1779

1712 - 1778 **Rousseau**
Discourse on the question, whether the restoration of the sciences and the arts has contributed to the purification of the morals, 1750
Discourse on the origin and foundations of inequality among men, 1755
Julie, or the new Héloïse, letters of two lovers, inhabitants of a small town at the foot of the Alps, 1761
On the social contract, or principle of political right, 1762
Émile, or on education, 1762
Rousseau judge of Jean-Jacques, a dialogue, 1780-2
Reveries of a solitary walker, 1782
The confessions, 1782-9
1712 - 1786 **Frederick II**
1713 - 1784 Diderot
Encyclopedia, or analytic dictionary of the sciences, arts, and trades, by a society of men of letters, 1751-72
Detailed explanation of the system of human knowledge, in the *Encyclopedia*, 1751
Encyclopedia, in the *Encyclopedia*, 1755
1715 - 1780 Condillac
1717 - 1768 Winckelmann
1717 - 1783 **d'Alembert**
Treatise on dynamics, 1743
Preliminary discourse of the editors, in the *Encyclopedia*, 1751
1718 - 1783 Morelly 1718 - the decade after 1778
The code of nature, or the true spirit of the laws neglected or unknown in all times, 1755
1723 - 1780 Blackstone
Commentaries on the laws of England, 1765-9
1723 - 1790 **Adam Smith**
An inquiry into the nature and causes of the wealth of nations, 1776
letters to Strahan
1724 - 1804 **Kant**
Thoughts on the true estimation of living forces and assessment of the proofs used on this controversial matter by Leibniz and other mechanics, together with some preliminary observations on the force of bodies in general, 1747
Investigation of the question whether the earth in its rotation on its axis, by which it brings about the alteration of day and night, has undergone any change since the times of its origin, and how this can be established, 1754
The question whether the earth ages, considered physically, 1754
General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws, 1755
A new elucidation of the first principles of metaphysical knowledge, 1755
Brief exposition of some reflections on fire, 1755
History and natural description of the most remarkable occurrences of the earthquakes which struck a large part of the earth at the end of the year 1755, 1756
On the causes of the earthquakes on the occasion of the disaster which befell the western countries of Europe toward the end of last year, 1756
Further observations on the earthquakes experienced for some time, 1756
The use of metaphysics combined with geometry in natural philosophy, whose first test is the physical monadology, 1756
New remarks toward an elucidation of the theory of winds, 1756
A brief consideration of the question whether the west winds in our regions are humid because they pass over a large sea, 1757
New doctrine of motion and rest, and its consequences for the first principles of natural science, 1758
An attempt at some observations on optimism, 1759
The false subtlety of the four syllogistic figures, 1762

The only possible argument for demonstration of the existence of god, 1763
An attempt to introduce the concept of negative magnitudes in philosophy, 1763
Investigation on the clarity of the principles of natural theology and morality, 1764
Observations on the feeling of the beautiful and sublime, 1764
Essay on the diseases of the head, 1764
Dreams of a seer of spirits, explained by dreams of metaphysics, 1766
On the first ground of the difference of directions in space, 1768
On the form and principles of the sensible and intelligible world, 1770
On the different races of men, 1775
Critique of pure reason, 1781
Prolegomena to any future metaphysics which will be able to stand as science, 1783
Idea for a general history with a cosmopolitan purpose, 1784
Answer to the question, what is enlightenment? 1784
Groundwork for the metaphysics of morals, 1785
review of Herder's *Ideas for the Philosophy of the history of humanity*, 1785
On the volcanoes on the moon, 1785
On the wrongfulness of the unauthorized publication of books, 1785
Determination of the concept of a human race, 1785
The presumed beginning of human history, 1786
Metaphysical elements of natural science, 1786
What does it mean to orient oneself in thinking? 1786
On the philosophers' medicine of the body, 1786
Critique of practical reason, 1788
On the use of teleological principles in philosophy, 1788
Critique of judgment, 1790
On a discovery by which any new critique of pure reason will be made dispensable by an older one, 1790
On the failure of all philosophical attempts at theodicy, 1791
Religion within the boundaries of reason alone, 1793
On the common saying, this may be correct in theory but it is of no use for practice, 1793
Something on the influence of the moon on the weather, 1794
The end of all things, 1794
Toward perpetual peace, a philosophical draft, 1795
On a recently arisen superior tone in philosophy, 1796
Proclamation of the near conclusion of a treaty of perpetual peace in philosophy, 1796
The metaphysics of morals, 1797
On a supposed right to lie from philanthropy, 1797
The quarrel of the faculties, 1798
Anthropology considered in pragmatic respect, 1798
On the power of the mind to be master of its feeling sick by sheer resolution, 1798
On turning out books, 1798
Declaration concerning Fichte's theory of science, 1799
Logic, a manual for lectures, 1800
Physical geography, edited from manuscripts, 1802
Which is the true progress metaphysics has made in Germany since the time of Leibniz and Wolff? 1804
letters to Beck

1726 - 1797 Hutton
Abstract of a dissertation concerning the system of the earth, its duration, and stability, 1785
The theory of the earth, with proofs and illustrations, 1795

1728 - 1777 **Lambert**
Preliminary advice for those who seek to square and rectify the circle, 1766
Memoir on some remarkable properties of circular and logarithmic transcendental quantities, 1768
Notes and comments on the drawing of maps of the earth and heaven, 1772

1728 - 1779 **Cook**

1728 - 1799 **Black**
Experiments on magnesia alba, quicklime, and some other alkaline substances, 1756
1729 - 1786 Mendelssohn
1731 - 1810 **Cavendish**
Three papers containing experiments on factitious air, 1766
1732 - 1799 **Washington**
1735 - 1826 John Adams
1736 - 1793 Bailly
1736 - 1806 **Coulomb**
1736 - 1813 **Lagrange**
Analytic mechanics, 1788
1737 - 1794 **Gibbon**
The history of the decline and fall of the Roman empire, 1776-88
1737 - 1798 Galvani
1738 - 1822 **Herschel**
On nebulous stars properly so called, 1791
On the nature and construction of the sun and fixed stars, 1794
1740 Montgolfier Joseph 1740 - 1810; Étienne 1745 - 1799
1741 - 1790 **Joseph II**
1743 - 1794 **Lavoisier**
Physical and chemical essays, 1774
Elementary treatise on chemistry, presented in a new order and according to the modern discoveries, 1789
1743 - 1822 Haüy
1744 - 1803 Herder
Extract from an exchange of letters about Ossian and the songs of ancient peoples, 1773
Also a philosophy of history, for the education of mankind, 1774
Ideas for the philosophy of the history of mankind, 1784-91
with Goethe and Möser
On the German kind and art, 1773
1744 - 1829 **Lamarck**
Zoological philosophy, or exposition of considerations on the natural history of animals, on the diversity of their organization and the faculties they obtain from it, on the physical causes which maintain life in them and give rise to the movements they make, as well as on what produces feeling in some of them and intelligence in those endowed with it, 1809
1745 - 1827 **Volta**
1748 - 1822 Berthollet
1748 - 1836 **Sieyès**
What is the third estate? 1789
1749 - 1817 Werner
1749 - 1827 **Laplace**
Exposition of the system of the world, 1796
Treatise on celestial mechanics, 1799-825
Philosophical essay on probabilities, 1814
1749 - 1832 **Goethe**
Götz von Berlichingen, 1773
The sufferings of young Werther, 1774
An intermaxillary bone in the upper jaw is to be attributed to man as well as animals, 1784
An attempt to explain the metamorphosis of plants, 1790
Contributions to optics, 1791-2
Faust, 1808-32
Toward the theory of colors, 1810
letters to Lavater
with Herder and Möser, *q. v.*
1751 - 1836 Madison

with Hamilton and Jay, *q. v.*
1753 - 1823 Lazare Carnot
1756 - 1804 Alexander Hamilton 1755 or 1757 - 1804
with Madison and Jay
The federalist, a collection of essays written in favor of the new constitution as agreed by the federal convention on 17 Sep 1787, 1788
1756 - 1836 **Godwin**
An inquiry concerning political justice and its influence on morals and happiness, 1793
1758 - 1794 **Robespierre**
1758 - 1805 Nelson
1759 - 1805 **Schiller**
The robbers, 1781
The stage considered as a moral institution, 1784
History of the separation of the United Netherlands from the Spanish government, 1788
What is called and to what end is studied universal history? 1789
History of the thirty years' war, 1791-3
On the tragic art, 1792
On the esthetic education of man, in a series of letters, 1795
On the sublime, 1801
Wallenstein's camp, *The Piccolomini*, and *Wallenstein's death*, 1801-4
Wilhelm Tell, 1804
1762 - 1814 Fichte
Foundation of the entire theory of science, 1794
First introduction in the theory of science, 1797
Second introduction in the theory of science, for readers who already have a philosophical system, 1797
1766 - 1817 Mme de Staël
On Germany, 1810
1766 - 1834 Malthus
An essay on the principle of population, or a view of its past and present effects on human happiness, with an inquiry into the prospects respecting the future removal or mitigation of the evils which it occasions, 1798
1766 - 1844 **Dalton**
New system of chemical philosophy, 1808-27
1767 - 1794 Saint-Just
On nature, on the civil state, on the city, or the rules of independence, of government, 1792
Republican institutions, 1794
1768 - 1830 **Fourier**
Analytic theory of heat, 1822
1768 - 1834 Schleiermacher
1769 - 1821 **Napoleon**
Napoleonic code, 1804
1769 - 1832 **Cuvier**
1769 - 1852 **Wellington**
1770 - 1831 **Hegel**
The system of science, part one, the phenomenology of spirit, 1807
Announcement, 1807
The science of logic, 1812-6
Encyclopedia of the philosophical sciences in outline, 1817
Natural right and political science in outline, elements of the philosophy of right, 1821
Lectures on the history of philosophy, 1833-6
Lectures on the philosophy of history, 1840
1773 - 1829 **Young**
1774 - 1862 Biot
1775 - 1836 **Ampère**

Essay on the philosophy of the sciences, or analytic exposition of a natural classification of all human knowledge, 1834-43

1775 - 1854 Schelling
1776 - 1856 **Avogadro**
1777 - 1851 **Ørsted**
1777 - 1855 **Gauss**
1778 - 1829 Davy
1779 - 1848 Berzelius
1780 - 1831 **Clausewitz**
On war, 1832
1784 - 1846 **Bessel**
1786 - 1853 Arago
1788 - 1860 Schopenhauer
The world as will and representation, 1819
On the freedom of man's will, 1839
1789 - 1857 **Cauchy**
1790 - 1832 Champollion
1791 - 1867 **Faraday**
Experimental researches in electricity, 1839-55, including
Some points of magnetic philosophy, 1855
1792 - 1856 **Lobachevsky**
1792 - 1866 Hincks
1792 - 1876 **von Baer**
1796 - 1832 **Carnot**
1797 - 1875 Lyell
Principles of geology, or the modern changes of the earth and its inhabitants considered as illustrative of geology, 1830-3
1798 - 1857 Comte
Course in positive philosophy, 1830-42
System of positive politics, or treatise on sociology, instituting the religion of humanity, 1851-4
Positive catechism, or summary exposition of the universal religion, in thirteen systematic conversations between a woman and a priest of humanity, 1852
1803 - 1873 Liebig
Chemistry in its application to agriculture and physiology, 1840
1804 - 1881 Schleiden
1805 - 1865 **Hamilton**
1807 - 1873 Agassiz
1808 - 1873 Napoleon III
1809 - 1882 **Darwin**
On the formation of mold, 1837
On the connection of certain volcanic phenomena in South America; and on the formation of mountain chains and volcanoes as an effect of the same power by which continents are elevated, 1838
Journal of researches into the geology and natural history of the various countries visited by H.M.S. Beagle, under the command of captain FitzRoy, from 1832 to 1836, 1839
Observations on the parallel roads of Glen Roy, and of other parts of Lochaber in Scotland, with an attempt to prove that they are of marine origin, 1839
On the distribution of the erratic boulders and on the contemporaneous unstratified deposits of South America, 1841
The structure and distribution of coral reefs, 1842
Notes on the effects produced by the ancient glaciers of Caernarvonshire, and on the boulders transported by floating ice, 1842
Geological observations on the volcanic islands visited during the voyage of H.M.S. Beagle, together with some brief notices of the geology of Australia and the cape of Good Hope, 1844
Geological observations on South America, 1846

On the geology of the Falkland Island, 1846
Geology, in John Herschel's *Manual of scientific inquiry, prepared for the use of her majesty's navy, and adapted for travelers in general*, 1849
A monograph of the sub-class cirripedia, with figures of all species, 1851-4
Extract from an unpublished work on species, consisting of a portion of a chapter entitled, On the variation of organic beings in a state of nature; on the natural means of selection; on the comparison of domestic races and true species, 1858
The origin of species by means of natural selection, or the preservation of favored races in the struggle for life, 1859
On the two forms, or dimorphic condition, in the species of primula, and on their remarkable sexual relations, 1861
The various contrivances by which orchids are fertilized by insects, 1862
On the sexual relations of the three forms of lythrum salicaria, 1864
The power of movement in plants, 1865
The variation of animals and plants under domestication, 1868
The descent of man, and selection in relation to sex, 1871
The expression of emotions in man and animals, 1872
Insectivorous plants, 1875
The effects of cross- and self-fertilization in the vegetable kingdom, 1876
Recollections of the development of my mind and character, 1876
The different forms of flowers on plants of the same species, 1877
Biographical sketch of an infant, 1877
The formation of vegetable mold, through the action of worms, with observations on their habits, 1881
The action of carbonate of ammonia on the roots of certain plants, 1882
The action of carbonate of ammonia on chlorophyl-bodies, 1882
notes in the *Athenaeum*
letters to Nägeli

1810 - 1861 **Cavour**
1810 - 1882 **Schwann**
1810 - 1895 Rawlinson
The Persian cuneiform inscription at Behistun, deciphered and translated; with a memoir on Persian cuneiform inscriptions in general, and on that of Behistun in particular, 1846-51
The cuneiform inscriptions of western Asia, 1861-75
1811 - 1832 **Galois**
1814 - 1876 **Bakunin**
1815 - 1864 **Boole**
The mathematical analysis of logic, 1847
An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities, 1854
1815 - 1897 **Weierstrass**
1815 - 1898 **Bismarck**
1818 - 1883 **Marx**
Economic-philosophical manuscripts from 1844
ad Feuerbach, 1845
The class struggles in France from 1848 to 1850, 1850
The eighteenth Brumaire of Louis Bonaparte, 1852
Contribution to the critique of political economy, 1859
The capital, critique of political economy, 1867
letters to Engels; letters to Lassalle; letters to Weydemeyer
with Engels
The German ideology, critique of the newest German philosophy in its representatives Feuerbach, B. Bauer, and Stirner, and of the German socialism in its different prophets, 1846
Manifesto of the communist party, 1848
1819 - 1868 Foucault

1820 - 1895 Engels
Herr Eugen Dühring's upheaval in science, 1878
Dialectic of nature, 1882
The origin of the family, private property, and the state, in connection with the investigations of Lewis H. Morgan, 1884
Ludwig Feuerbach and the end of classical German philosophy, 1886
Contribution to the history of early Christianity, 1895
letters to Mehring
with Marx, *q. v.*
1821 - 1894 **Helmholtz**
On Goethe's natural-scientific works, 1853
On the relation of the natural sciences to all science, 1862
The new advances in the theory of vision, 1868
On the origin of the planetary system, 1871
1821 - 1902 Virchow
Cellular pathology grounded in physiological and pathological histology, 1858
1822 - 1884 **Mendel**
Experiments on plant hybrids, 1865
letters to Nägeli
1822 - 1888 **Clausius**
1822 - 1895 Pasteur
1824 - 1907 **Kelvin**
On the secular cooling of the earth, 1862
The age of the earth as an abode fitted for life, 1897
1825 - 1905 Oppert
Sumerian or Akkadian? 1875
1826 - 1866 **Riemann**
1831 - 1879 **Maxwell**
On Faraday's lines of force, 1855
On physical lines of force, 1861
A dynamical theory of the electromagnetic field, 1864
Address to the mathematical and physical sections of the British association, 1870
Theory of heat, 1871
On the mathematical classification of physical quantities, 1871
A treatise on electricity and magnetism, 1873
Matter and motion, 1877
1831 - 1916 **Dedekind**
What are and what should the numbers? 1888
1832 - 1920 Wundt
1833 - 1911 Dilthey
Introduction to the spiritual sciences, 1883
The types of world-view and their formation in the metaphysical systems, 1911
1834 - 1907 **Mendeleev**
Correlation of the properties and atomic weights of the elements, 1869
1843 - 1910 Koch
1844 - 1900 **Nietzsche**
Untimely observations, second part; on the usefulness and disadvantage of history for life, 1874
Beyond good and evil, prelude to a philosophy of the future, 1886
Toward the genealogy of morality, a polemic, 1887
1844 - 1906 **Boltzmann**
1845 - 1918 **Cantor**
On a property of the totality of all real algebraic numbers, 1874
Contributions to the foundation of the transfinite theory of sets, 1895-7
1848 - 1925 **Frege**
On a geometrical representation of the imaginary forms in the plane, 1873

Methods of calculation grounded in an extension of the concept of quantity, 1874
Concept writing, a formula language of pure thinking modeled on that of arithmetic, 1879
On applications of the concept writing, 1879
On the goal of the concept writing, 1882
On the scientific justification of the concept writing, 1882
Geometry of pairs of points in the plane, 1883
The foundations of arithmetic, a logico-mathematical investigation of the concept of number, 1884
On formal theories in arithmetic, 1886
Function and concept, 1891
On the law of inertia, 1891
On concept and object, 1892
On sense and reference, 1892
The laws of arithmetic, derived by the concept writing, 1893-903
The whole number, 1895
On the concept writing of Mr Peano and my own, 1896
On the foundations of geometry, 1903-6
What is a function? 1904
The thought, a logical investigation, 1919
Negation, a logical investigation, 1919
letters to Russell

1852 - 1912 **Meiji**
1853 - 1928 Lorentz
1856 - 1940 **Thomson**
1857 - 1894 Hertz
On the fundamental equations of electromagnetics for bodies at rest, 1890
On the fundamental equations of electromagnetics for bodies in motion, 1890
1858 - 1932 **Peano**
The principles of arithmetic presented by a new method, 1889
1858 - 1947 **Planck**
The unity of the physical picture of the world, 1908
Eight lectures on theoretical physics, 1909
1861 - 1947 Whitehead
Process and reality, an essay in cosmology, 1929
Adventures of ideas, 1933
with Russell, *q. v.*
1862 - 1915 Boveri
On multipolar mitosis as a means of analysis of the cell nucleus, 1902
1862 - 1943 **Hilbert**
Foundations of geometry, 1899
Mathematical problems, 1900
Axiomatic thinking, 1918
The new grounding of mathematics, first statement, 1922
The logical foundations of mathematics, 1923
On the infinite, 1925
Knowledge of nature and logic, 1930
1864 - 1909 **Minkowski**
Space and time, 1908
1864 - 1920 Max Weber
The Protestant ethics and the spirit of capitalism, 1905
1866 - 1925 **Sun Yat-sen**
1866 - 1952 Croce
History reduced to the general concept of art, 1893
1867 Wright Wilbur 1867 - 1912; Orville 1871 - 1948
1870 - 1924 **Lenin**
1871 - 1937 **Rutherford**

The radiation and emanation of radium, 1904
Radium – the cause of the earth's heat, 1905
Some cosmical aspects of radioactivity, 1907

1871 - 1953 **Zermelo**
Investigations on the foundations of the theory of sets, 1908

1872 - 1970 **Russell**
The principles of mathematics, 1903
On denoting, 1905
Mathematical logic as based on the theory of types, 1908
Knowledge by acquaintance and knowledge by description, 1911
The problems of philosophy, 1912
Our knowledge of the external world as a field for scientific method in philosophy, 1914
The philosophy of logical atomism, 1918
Introduction to mathematical philosophy, 1919
How to read and understand history, the past as the key to the future, 1943
My mental development, 1944
A history of western philosophy, 1945
My philosophical development, 1959
letters to Frege
with Whitehead
Principia mathematica, 1910-3

1873 - 1967 **Hertzsprung**
On the radiation of stars, 1905-7

1877 - 1947 **Hardy**
Mendelian proportions in a mixed population, 1908
On the zeroes of Riemann's ζ (s) function, 1914
A mathematician's apology, 1940

1878 - 1956 Łukasiewicz
Aristotle's syllogistic from the standpoint of modern formal logic, 1951

1879 - 1955 **Einstein**
On the electrodynamics of moving bodies, 1905
On the influence of gravitation on the propagation of light, 1911
The speed of light and the statics of the gravitational field, 1912
On the theory of the static gravitational field, 1912
On the present state of the problem of gravitation, 1913
The formal foundation of the general theory of relativity, 1914
On the general theory of relativity, 1915
The field equations of gravitation, 1915
The foundation of the general theory of relativity, 1916
Hamilton's principle and the general theory of relativity, 1916
Cosmological considerations on the general theory of relativity, 1917
On the special and the general theory of relativity, a popular account, 1917
Geometry and experience, 1921
The problem of space, aether, and the field in physics, 1930
with Grossmann
Outline of a generalized theory of relativity and of a theory of gravitation, 1913

1880 - 1930 Wegener

1880 - 1959 **Chetverikov**
On certain aspects of the evolutionary process from the point of vies of contemporary genetics, 1926

1881 - 1966 Brouwer
Intuitionism and formalism, 1912

1882 - 1944 Eddington
The internal constitution of the stars, 1926
with Dyson and Charles Davidson
A determination of the deflection of light by the sun's gravitational filed, from observations made

at the total eclipse of 29 May 1919, 1920
1885 - 1962 **Bohr**
1887 - 1961 **Schrödinger**
1888 - 1925 **Friedmann**
On the curvature of space, 1922
1889 - 1945 **Hitler**
My struggle, 1925-7
1889 - 1951 Wittgenstein
Notebooks 1914-6
Logico-philosophical treatise, 1921
Lecture on ethics, 1929
Philosophical investigations, 1953
1889 - 1953 **Hubble**
Cepheids in spiral nebulae, 1925
Extragalactic nebulae, 1926
A relation between distance and radial velocity among extragalactic nebulae, 1929
The realm of the nebulae, 1936
1889 - 1976 Heidegger
Being and time, 1927
What is metaphysics? 1929
Introduction to metaphysics, 1935
The question about the thing, contribution to Kant's theory of the transcendental principles, 1935
1891 - 1970 Carnap
The logical structure of the world, 1928
Pseudoproblems in philosophy, the extrapsychological and the realism controversy, 1928
Elimination of metaphysics through logical analysis of language, 1932
1901 - 1976 **Heisenberg**
On the visual content of quantum theoretical kinematics and mechanics, 1927
The physical principles of quantum theory, 1930
On the history of the physical explanation of nature, 1932
Physics and philosophy, the revolution in modern science, 1958
1901 - 1983 Tarski
The semantic conception of truth and the foundations of semantics, 1944
1902 - 1984 Dirac
1902 - 1985 Braudel
The Mediterranean and the Mediterranean world in the age of Philip II, 1949
Material civilization, economy, and capitalism, 15th-18th century, 1968-79
1902 - 1994 Popper
The logic of scientific discovery, 1934
On the sources of knowledge and of ignorance, 1960
Epistemology without a knowing subject, 1967
1903 - 1989 Lorenz
Observations on the recognition of instinctive behavior characteristic of the species in birds, 1932
The companion in the environment of the bird, 1935
Kant's theory of the a priori in the light of contemporary biology, 1941
1904 - 1968 Gamow
The creation of the universe, 1952
My world line, 1970
1906 - 1978 Gödel
Some metamathematical results on completeness and consistency, 1930
On formally undecidable propositions of Principia mathematica and related systems, 1931
Russell's mathematical logic, 1944
1910 - 1976 Monod
Chance and necessity, an essay on the natural philosophy of modern biology, 1970
with Jacob

Genetic regulatory mechanisms in the synthesis of proteins, 1961
1912 - 1954 Turing
1916 - 2004 Crick
with Watson
Molecular structure of nucleic acids, a structure for deoxyribose nucleic acid, 1953
1918 - 1988 Feynman
Lectures on physics, 1963-5
QED, the strange theory of light and matter, 1985
1920 - 2013 Jacob
with Monod, *q. v.*
1928 - 2025 Watson
with Crick, *q. v.*
1929 - 2019 Gell-Mann

poets

688	Archilochus	sings of an eclipse which occurred in 648
684	Alcman	the two generations after 676/3
619	Alcaeus	one generation after Pittacus
578	Anacreon	contemporary with Polycrates
525 - 456	Aeschylus	*c.* 525 - 456
518 - 441	Pindar	518 - the decade after 446
496 - 406	Sophocles	497/5 - 406
494 - 428	Phidias	under Pericles, general in 454 - the decade after 433
450 - 386	Aristophanes	*c.* 450 - 386
402	Praxiteles	worked in 364/1
370	Apelles	worked in 332/29
70 - 19	Virgil	
65 - 8^{BC}	Horace	
43^{BC} - 17^{AD}	Ovid	
120^{AD} - 190	Lucian	115/25 - *c.* 190

Dialogues of the dead
1475 - 1564 Michelangelo
1508 - 1580 Palladio
1547 - 1616 Cervantes
The ingenious hidalgo Don Quixote de la Mancha, 1605-15
1660 - 1731 Defoe
The life and strange surprising adventures of Robinson Crusoe, of York, a mariner, 1719
1729 - 1781 Lessing
1756 - 1791 Mozart
1770 - 1827 Beethoven
1788 - 1824 Byron

not important

788 - 734	Nabonassar	prince 748-34
489 - 418	Laches	older than Socrates - 418
485	Phrynis	won a contest *c.* 445
472	Euctemon	contemporary with Meton
451 - 361	Timotheus	died at 90 in 365/57
449 - 406	Phaedrus	younger than Socrates - the decade before 401
440 - 403	Charmides	*c.* 440 - 403
421 - 400	Meno	very young in 401 - 400
417	Phaedo	born *c.* 417
335	Aristyllus	contemporary with Timocharis

236 Mago — the century before 146
191 Cassius Dionysius — two to three generations before Varro
171 - 133 Attalus III
161 Diophanes — one to two generations before Varro
82 - 46 BC Vercingetorix — led revolt in 52 - 46
27 BC - 36 AD Thrasyllus — under Tiberius, prince 14-37 - 36
6 AD st Peter — contemporary with st Paul
273 - 355 Donatus — consecrated bishop in 313 - 355
346 scholia in the *Elements* — reach back to the time between Pappus and Proclus
359 - 383 Gratian
419 - 455 Valentinian III
597 ibn Abi Wakkas — general in 637
1096 - 1164 Héloïse — 1092/100 - 1164
1100 - 1170 al-Bayhaqi — died in 1170

Addition to The keepers of wisdom

1130 - 1202 Eugenius of Sicily — *c.* 1130 - 1202
1135 - 1205 Amalric — died in the decade before 1210
1161 Acerbus — contemporary with Innocent III
1170 decree of the archbishop of Sens, the bishop of Paris, and other bishops, gathered in Paris, on the expurgation of heresy and on the different books of Aristotle, 1210
1463 - 1503 Lorenzo il Popolano
1465 - 1526 Ferro
1468 - 1522 Johann Werner
1499 Fiore — contemporary with Tartaglia
1500 - 1547 Bunel
1507 - 1577 Viglius
1517 - 1586 Granvelle
1522 - 1568 count of Egmont
1523 - 1576 Hoppers
1524 - 1568 count of Horn
1535 - 1612 van Meteren
1550 - 1602 van Reyd
1552 - 1612 Rudolph II
1553 - 1617 de Thou
1572 - 1649 Strada
1574 - 1636 Giuliano de' Medici
1576 report of the committee appointed to examine the propositions, the sun is in the center of the world and does not move, and the earth is not in the center and moves, to the holy office, 24 Feb 1616
1577 - 1657 Liceti
1579 - 1639 Meursius
1579 - 1644 Bentivoglio
1583 - 1634 Wallenstein
1583 - 1654 Grassi
1585 - 1619 Vanini
1586 - 1649 Burgundius
1588 - 1653 Filmer
1590 - 1621 Cosimo II
1599 - 1656 Ottavio Piccolomini
1660 - 1717 Gabriel Wagner — *c.* 1660 - 1717
1661 - 1719 Garth
1661 - 1742 de Polignac
1672 - 1719 Addison
1680 - 1751 Thomas Prior
1680 - 1753 de Boze

Eulogy for the cardinal de Polignac, 1742

1682 - 1716 Cotes
preface to the second edition of *The mathematical principles of natural philosophy*, 1713
1715 - 1785 Strahan
1720 - 1794 Möser
History of Osnabrück, a general introduction, 1768
with Herder and Goethe, *q. v.*
1735 - 1788 John Brown
1736 - 1796 Macpherson
1740 - 1813 Stock
An account of the life of George Berkeley, late bishop of Cloyne in Ireland, 1776
1741 - 1801 Lavater
1743 - 1819 Friedrich Jacobi
1745 - 1792 Hearne
1745 - 1829 Jay
with Hamilton and Madison, *q. v.*
1752 - 1840 Blumenbach
1755 - 1830 Soemmerring
1761 - 1840 Beck
1776 - 1841 Herbart
1785 - 1871 Bekker
1791 - 1865 Encke
1792 - 1871 John Herschel
1802 - 1870 Botta
1804 - 1872 Feuerbach
1805 - 1865 FitzRoy
1806 - 1856 Stirner
1807 - 1853 Laurent
1809 - 1882 Bruno Bauer
1812 - 1876 Kuhlmann
1816 - 1856 Charles Gerhardt
1817 - 1887 Grün
1817 - 1891 Nägeli
1818 - 1866 Weydemeyer
1818 - 1881 Lewis Morgan
1825 - 1864 Lassalle
1826 - 1910 Rassam
1833 - 1921 Dühring
1835 - 1910 Schiaparelli
1835 - 1924 Hensen
1842 - 1910 William James
1846 - 1919 Mehring
1851 - 1916 Silvanus Thompson
The life of William Thomson, baron Kelvin of Largs, 1910
1859 - 1941 Bergson
1859 - 1952 Dewey
1862 - 1929 Kugler
1868 - 1939 Dyson
with Eddington, *q. v.*
1875 - 1970 Charles Davidson
with Eddington, *q. v.*
1878 - 1936 Grossmann
with Einstein, *q. v.*
1880 - 1936 Spengler
The decline of the west, outlines of a morphology of world history, 1918
1889 - 1975 Toynbee

A study of history, 1934-61
1903 - 1950 Orwell
1984, 1949
1922 - 1996 Kuhn
The structure of scientific revolutions, 1962
1926 - 1984 Michel Foucault
The archeology of knowledge, 1969
History of sexuality, 1976-84
1930 *Dictionary of scientific biography*, 1970-90
1949 - 1984 Laporte
History of shit, 1978

not dated (with their earliest source in the text)

Enlil (Eanatum)
Re (Unas)
Ishtar (Venus tablet of Ammisaduqa)
Apsu, Tiamat, Anu, Ea, Marduk (*When on high*)
Adam, Seth, Noah, Abraham, Moses, Joshua (Yahwistic document)
Isaiah (*Isaiah*)
Achilles, Zeus, Apollo, Odysseus, Menelaus, Theseus, Ocean, Hephaestus, Hermes, Aphrodite, Heracles, Euphorbus, Scylla (Homer)
Hesperus, Phosphorus (Hesiod)
Adad (*When Anu and Enlil*)
Jeremiah (*Jeremiah*)
Samuel (*Joshua*, *Judges*, *Samuel*, *Kings*)
Job (*Job*)
Lycurgus, Aristeas (Herodotus)
Euthyphro, Lysis, Timaeus (Plato)
Aethalides, Hermotimus, Pyrrhus (Ponticus)
Nicomachus (Aristotle)
Daniel (*Daniel*)
Mercury, Venus (Varro)
Numa (Livy)
Jesus, Antichrist (st Paul)
Matthew (*Gospel of Matthew*)
Luke, Dionysius the Areopagite (*Gospel of Luke*, *Acts of the apostles*)
John (*Gospel of John*, *Apocalypse*)
Lucifer (Augustine)
Philology, Rhetoric, Geometry (Capella)
Hylas, Philonous (Berkeley)
Julie, Émile, Sophie (Rousseau)
Götz von Berlichingen, Werther, Lotte, Faust (Goethe)
Bor, Hooft, Brandt, Le Clerc, Dinoth, Spittler, Watson, Wilhelm Tell, Max Piccolomini (Schiller)

Charon (Lucian)
Don Quixote, Sancho Panza (Cervantes)
Robinson Crusoe (Defoe)

Ossian (Macpherson)

the numbering of inscriptions and fragments follows these editions
Eanatum – Cooper
Sargon – Gelb/Kienast
Tiglath-pileser – Grayson

Epimenides, Xenophanes, Heraclitus, Parmenides – Diels
Ponticus – Wehrli
Aristotle – Rose

Finally, counting centuries from the year of birth assigned to the 3 calculations of area, these are the distribution of the particulars and their totals,

	important and writings		poets and writings		not important and writings	
3120-3021	2	–	–	–	–	–
3020-2921	1	–	–	–	–	–
2920-2821	–	–	–	–	–	–
2820-2721	–	–	–	–	–	–
2720-2621	3	–	–	–	–	–
2620-2521	3	–	–	–	–	–
2520-2421	1	1	–	–	–	–
2420-2321	2	2	–	–	–	–
2320-2221	–	–	–	–	–	–
2220-2121	2	1	–	–	–	–
2120-2021	5	–	–	–	–	–
2020-1921	3	–	–	–	–	–
1920-1821	11	–	–	–	–	–
1820-1721	6	1	–	–	–	–
1720-1621	7	–	–	–	–	–
1620-1521	3	–	–	–	–	–
1520-1421	4	–	–	–	–	–
1420-1321	8	–	–	–	–	–
1320-1221	2	–	–	–	–	–
1220-1121	2	1	–	–	–	–
1120-1021	1	–	–	–	–	–
1020- 921	4	–	–	–	–	–
920- 821	3	1	–	–	–	–
820- 721	9	3	–	–	1	–
720- 621	9	1	2	–	–	–
620- 521	14	4	3	–	–	–
520- 421	34	23	4	–	7	–
420- 321	29	61	2	–	2	–
320- 221	20	5	–	–	1	–
220- 121	8	3	–	–	3	–
120- 21	17	17	3	–	2	–
20^{BC}- 80^{AD}	13	10	–	–	1	–
81- 180	14	18	1	1	–	–
181- 280	13	7	–	–	1	–
281- 380	12	9	–	–	2	–
381- 480	9	5	–	–	1	–
481- 580	6	4	–	–	–	–
581- 680	4	–	–	–	1	–
681- 780	5	1	–	–	–	–
781- 880	6	4	–	–	–	–
881- 980	6	3	–	–	–	–
981-1080	8	10	–	–	–	–
1081-1180	12	6	–	–	6	1
1181-1280	15	8	–	–	–	–
1281-1380	9	8	–	–	–	–
1381-1480	31	14	1	–	3	–
1481-1580	27	23	2	1	17	–

1581-1680	48	33	1	1	13	1
1681-1780	77	173	3	–	14	3
1781-1880	71	164	1	–	30	2
1881-1980	25	43	–	–	6	6
	624	667	23	3	111	13

and the not dated are 87.