

An investigation of knowledge

part two of eight

Radoslav Datchev

2025

As its title page announces it, this is part two of the investigation. Part two of eight.

Clearly, if brevity is a virtue, it is not a virtue of this investigation. If brevity is a virtue. For, if it is, it is rather more likely to be a virtue of some things, and not of others, and not of these others, because it is a virtue of those others. Thus, in particular, if there are and can only be two ways to search for truth, the false and the true, it is not the true that is and can only be the short one.

On the one hand, then, this part of the investigation, being part two of eight, continues the labors of part one, as part three continues its labors, and so on, and through part eight. The

eight parts are one another's premises and consequences. On the other hand, nevertheless, each of the eight parts, and this part two, in particular, should stand, to one degree or another, on its own. To the degree to which the premises and the consequences are important, the labor of establishing the premises and of drawing the consequences should stand on its own.

What the investigation in its entirety, in all its eight parts, proposes, the introduction to part one states, briefly. Also, the titles of the eight parts, listed in the following contents – all eight borrowed, from Bacon, Descartes, Aristotle, Spinoza, Newton, Leibniz, Planck, and again Descartes, respectively – are one way to summarize the argument.

There are and can only be two ways to search for truth, the false and the true	11
It is much better never to search for truth than to search without the method, the method must be followed as staunchly in the search for truth as the thread of Theseus in the labyrinth	331
In order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous	595
The method of history does not differ from the method of physics at all, but agrees with it entirely.	1341
The entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena	
There will be no more need for disputes between philosophers, it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate	
That the inhabitants of Mars, if they have the necessary intelligence, recognize the law of gravity and the principle of conservation of energy, should be self-evident, to anyone to whom this is not self-evident, on the other hand, it will forever remain an unresolved mystery that the same physics is made in the United States as in Germany	
enumeration and review	

**It is much better never to search for truth than to search without the method,
the method must be followed as staunchly in the search for truth
as the thread of Theseus in the labyrinth**

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Bacon pointed the way.
In 1620, in the *New organon*: there are and can only be two ways to search for truth; one is to fly from the senses and the particulars to most general principles and, taking them to be settled, proceed to judgment; this way is now in use; the other is to obtain principles from the senses and the particulars by continuous and gradual ascent, and only ultimately end with the most general principles; and this is the true way even if it is yet untried (I, 19).

And the true way is the method, and Descartes formulated the rules of the method.

In 1637, in the *Discourse on the method*:
first, to include in judgments nothing but what is so clear and distinct that it cannot be doubted;
second, to divide difficulties into parts such as will lead to their better solution;
third, to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex;
and last, always to make enumerations so complete and reviews so general as to omit nothing (2).

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It is thus clear what the true way is, and how to follow it.

Above, in part one, the problem before the investigation has been formulated. And if it is clear what the problem before the investigation is,

and what the true way is, and how to follow it, the investigation could turn to the attempt to solve the problem without further ado.

This would be an error, however. The investigation can improve substantially its odds of success by the observation of how the true way has been followed. This is what the investigation should do next, in this part two. And there can be no better way to do it than to observe the labors of Bacon, Galileo, Kepler, Descartes, Huygens, Spinoza, Newton, and Leibniz.

Descartes writes: the method is not new, for nothing is older than truth (*Meditations*, ded.). And the labors at following the true way, and the rules of the method, predate Bacon and Descartes. Thirty-six and a half centuries before Bacon and Descartes, the success of the first greatly successful investigation, the Sumerian prince list, is a success of the method. Nineteen and a half centuries before Bacon and Descartes, all successes of Aristotle are successes of the method. Fifteen centuries before Bacon and Descartes, all successes of Ptolemy are successes of the method.

It is nevertheless Bacon who pointed the way. And it is Descartes who discovered the rules of following the true way.

The earlier labors at following the true way were partial and halting, even if decisively more resolute from one step to the next. Nor were the labors at following the true way frequent. So it was not that the true way was altogether untried before Bacon. Beginning with him, however, it

was tried staunchly, and the brightest period in the acquisition of knowledge followed, the 6 generations from that of Bacon, Galileo, and Kepler through that of Lavoisier and Laplace. And the first 3 of these 6 generations are the brightest of the brightest.

The false way, on the other hand, has never been in need of being pointed. It has always been the highway of choice in the search for truth. It was the way of choice before Aristotle. It was the way of choice in the four and a half centuries between Aristotle and Ptolemy. It was the only way for most of the fifteen centuries between Ptolemy and Bacon. In the 6 generations after Bacon the true way was well trodden, and it may have appeared that, having fallen into abeyance, the false way may be abandoned altogether. Then philosophy returned to it with a vengeance, and it once again became the highway of choice of philosophy such as it had quit being physics and mathematics, and of history such as it had quit learning from physics and mathematics.

Descartes writes: it is much better never to search for truth than to search without the method; disjointed studies and obscure meditations confound the intellect; and whoever walks in the dark long enough to become used to it can no longer bear the light of day (*Rules*, 4).

It is much better never to search for truth than to search without the method. As the labors at reinforcing the barriers before the acquisition of knowledge and the investigation of knowledge, and again, and again, and again, attest.

They attest also that to follow the method is hard. To walk in the light of day is hard.

Descartes's advice is this: the method must be followed as staunchly in the search for truth as the thread of Theseus in the labyrinth (5).

Ariadne gave Theseus the thread. Thus, the rules of the method are the thread, Descartes's

successors are Theseus, Descartes is Ariadne. And the apter the simile is in that a Minotaur is lurking in the labyrinth. In that Theseus abandons Ariadne, too.

Counting 157 generations from the generation of the predecessor of the philosophers who left behind the 3 oldest calculations of area through the generation before that of Bacon, Galileo, and Kepler – 3 generations from that of Bacon, Galileo, and Kepler through that of Newton and Leibniz – and 11 generations since – the total is 171 generations of search for truth on earth. And of these 171 generations it is in the 3 from that of Bacon, Galileo, and Kepler through that of Newton and Leibniz that the search for truth broke into the light of day. The search for truth advanced in these 3 generations more and more substantially than in the other 168 combined. So much so, the previous 157 can be described as the preparation of the ground for the search in these 3, the following 11 can be described as the collection of the consequences of the search in these 3.

The method is the cause. The light of day is the effect.

These 3 generations are the time of Bacon, Galileo, Kepler, Harvey, Snel, Hobbes, Desargues, Descartes, Cavalieri, Fermat, Guericke, Torricelli, Wallis, Grimaldi, Picard, Pascal, Cassini, Boyle, Malpighi, Huygens, Spinoza, Locke, Leeuwenhoek, Hooke, Steno, Newton, Römer, Leibniz, Jakob Bernoulli, Halley.

And in order to observe how the method has been followed so as to follow it as staunchly as the thread of Theseus must be followed in the labyrinth – so as to walk firmly in the light of day – there can be no better way than to observe the labors of the most important among the most important philosophers in these 3 generations, Bacon, Galileo, Kepler, Descartes, Huygens, Spinoza, Newton, and Leibniz.

Bacon, Galileo, and Kepler

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Hume writes: Bacon pointed out the road to true philosophy (*History*, 49, app.). Kant's treatise

on the method is, like Bacon's treatise on the method, and under this motto from that treatise: the true end and lawful termination of an infinitude of error. Darwin reports: I worked on true

Baconian principles. And Frege traces the labors on which the concept writing stands back to Bacon: Bacon held it more excellent to find a means through which everything can be readily found than to discover particular cases, and all great scientific advances of recent times have had their origin in an improvement of the method.

Bacon's is a great advance, and by being an advance of the method.

Before showing more explicitly the advance, the magnitude of the advance can be shown.

And how great the advance is can be shown in three ways. By showing how great an advance it is over the earlier major steps in the development of the method, which are also the greatest earlier successes in the acquisition of knowledge, and are the greatest successes in the acquisition of knowledge because they are the major steps in the development of the method, the Sumerian prince list, Aristotle, and Ptolemy. By showing how great an advance it is over the greatest successes of Bacon's immediate predecessors in the generation before him, Viète, Gilbert, Tycho, and Bruno. And by showing how great an advance Bacon's one strictly physical result is.

The Sumerian prince list is a list of 120 princes of 10 cities, the 120 princes arranged in their successions in the 10 cities.

And there it is, the first major step on the true way. The true way begins with the collection of the particulars. The collection of the particulars is attested unambiguously by the collection of the 120 princes, and it is attested even more unambiguously by the fact that the 120 princes are collected from 10 cities.

The 10 cities are 10 states as well as 10 localities on the surface of the earth.

West to east, the localities are,

Mari, Kish, Akkad, Akshak, Adab, Uruk,

Ur, the Gutian horde, Hamazi, Awan.

From Mari in the north to Ur in the south, the Sumerian prince list reaches across not quite 4° of latitude along the meridian. From Mari in the west to Awan in the east it reaches across 7° of longitude along the parallel. From Mari in the northwest to Awan in the southeast it reaches across 730 km along the great circle, $\frac{1}{55}$ of the circumference of the earth.

The Sumerian prince list was emulated. By the Larsa prince list – the Isin prince list – the Babylonian prince list – the Assyrian prince list – the lists of princes and patriarchs in the Hebrew scriptures – the lists of princes, other predecessors of his, and localities in Herodotus. The investigation of the past, when it is aware of the fact and when it is not, stands on the Sumerian prince list, and *ipso facto* on the method.

Compiled under Ibbi-Sin, prince of Ur c. 2037-13, the Sumerian prince list predates Bacon by thirty-six and a half centuries.

To begin with the collection of the particulars is the first element of the true way.

How resolutely Aristotle collects the particulars the 600 kinds of animals he describes suffice to show. He is clear also that the collection of the particulars is the first element of the true way: we begin with induction, and: induction reaches the general from the clarity of the particulars.

The most explicit anticipation of the true way, such as Bacon describes it with all its essential elements, is at the end of the *Nicomachean ethics*. Aristotle writes: we now turn to the general study of constitutions; so first we will review whether anything has been well said on the matter by our predecessors; then we will study in the constitutions we have collected what kinds of influences preserve and destroy particular constitutions; and then we will be better able to judge which constitution is the best (X, 9).

We review our predecessors, then we study the particular constitutions we have collected, then we judge which constitution is the best.

And dramatically as the 600 kinds of animals he describes attest Aristotle's success in the collection of the particulars, the constitutions he collected attest it even more dramatically.

Cicero reports: the constitutions of all states, Greek as well as barbarian, can be studied in Aristotle (*Ends*, V, 11). And in the long list of Aristotle's writings Laertius has the item: the constitutions of 158 states (V, 27).

Doubt in these reports may at one time have appeared warranted, if doubt in the reports of Cicero and Laertius may ever be warranted. It may have been urged that, had they been collected, 158 constitutions would not have been all altogether lost. Editing the Aristotelean corpus in

1831-70, for instance, Bekker had no constitutions to include. By 1890, however, *The constitution of Athens* had been unearthed in Egypt. In the previous couple of decades Aristotle's fragments had been collected as well, and these attest 66 more constitutions (fr. 472-603).

67 of the 158 can thus be enumerated. And they are 67 states as well as 67 localities on the surface of the earth in this case as well.

West to east, the localities are,

Massalia, Acragas, Himera, Neapolis, Gela, Syracuse, Rhegium, Locri, Sybaris, Croton, Tarentum, Corcyra, Epirus, Leucas, Ithaca, Ambracia, Acarnania, Elis, Aetolia, Achaea, Cyrene, Arcadia, Thessaly, Bottieia, Sparta, Tegea, Delphi, Pellene, Methone, Sicyon, Argos, Corinth, Cythera, Orchomenus, Opus, Epidaurus, Thebes, Megara, Troezen, Aegina, Chalcis, Athens, Ceos, Cythnos, Melos, Crete, Tenos, Delos, Naxos, Samothrace, Chios, Tenedos, Phocaea, Samos, Antandrus, Adramyttium, Cyme, Parium, Colophon, Miletus, Iasus, Rhodes, Lycia, Chalcedon, Cyprus, Soli, Sinope.

From Massalia in the north to Cyrene in the south Aristotle's collection reaches across 10° of latitude along the meridian. From Massalia in the west to Sinope in the east it reaches across 30° of longitude along the parallel. From Massalia in the west to Sinope in the east it reaches across 2 400 km along the great circle, $\frac{1}{17}$ of the circumference of the earth.

Herodotus, one century earlier, describes 1 012 localities, from the Atlantic in the west to the Indus in the east, and from the lower course of the Dnieper in the north to the middle course of the Nile in the south. Of these 1 012 countries, cities, rivers, mountains, islands, seas, not more than one quarter can have even the remotest claim to being states. Further west, beyond the Atlantic, further north, beyond the lower course of the Dnieper, and further south, beyond the middle course of the Nile, there were no states. Further east, beyond the Indus, there were. Pace Cicero, then, Aristotle did not collect the constitutions of all states. Rather with Cicero, however, Aristotle did collect the constitutions of not less than one

half of all contemporary states.

The true way begins with the collection of the particulars. And ends with the most general principles.

And Aristotle ends with the most general principles. In practical philosophy, in politics, and in theoretical philosophy, in physics and in mathematics. So much so, he describes the world in its entirety.

The true way ends with the most general principles.

It is by sharing this with the true way that the false way pretends to be the true.

Bacon writes: both ways begin from the senses and the particulars and end in the greatest generality; yet the difference between them is immense; one glances at experiments and the particulars in passing, the other duly and orderly verses itself in them, one establishes at once abstract and useless principles, the other rises gradually to what is general by nature (*Organon*, I, 22).

The true way duly and orderly verses itself in the particulars and rises gradually to what is general by nature. In Bacon's other description: it obtains principles from the particulars by continuous and gradual ascent and only ultimately ends with the most general principles.

Not to end with the most general principles, only ultimately to end with the most general principles is the final element of the true way.

Because to ascend continuously and gradually is the middle element.

And Aristotle does not ascend continuously and gradually.

Ptolemy does.

Ptolemy's success in the collection of the particulars is attested by the 7 949 localities which he describes and the 6 345 of these to which he assigns latitudes and longitudes. From the Shetland Islands in the north to cape Delgado in the south across 72° of latitude along the meridian. From the Canary Islands in the west to the mouth of the Red River in the east across 123° of longitude along the parallel. From the Canary Islands in the west to the mouth of the Red River in the east across 13 700 km along the great circle, $\frac{1}{3}$ of the circumference of the earth.

It is attested by the 54 princes, from Pius back to

Nabonassar, whose reigns Ptolemy synchronized with astronomical observation and with one another, and thus fixed chronology absolutely, and hence back to the princes of the Sumerian prince list, too. It is attested by the 94 accurate observations, from his own in 127-41^{AD} back to the observation of lunar eclipses in 720-1^{BC}, which are all accurate observations from the beginning of accurate observation through Ptolemy's own.

Ptolemy ends with the most general principles. So much so, he measures the world.

He anticipates also the sharp separation of the true way from the false, as well as their relation of exclusion: physics and theology are surmise and not knowledge, mathematics alone is certain and unshakable knowledge; it can deal also with the objects of the other two; and even where action is concerned, it above all can make men see clearly.

And his ascent from the particulars is continuous and gradual.

That Aristotle's is not the fact can show that the mathematician Ptolemy builds on his labors, but so does the mystic Plotinus. Galileo writes: I claim that I observe Aristotle's doctrine more religiously than any of them. But any of them had for centuries claimed to observe Aristotle's doctrine religiously, and have for centuries after Galileo continued to claim that they, not Galileo, observe Aristotle's doctrine religiously. Aristotle's ascent in his logical investigations is more continuous and gradual. It is less continuous and gradual in his physical investigations, for all his success in the collection of the particulars. It is not continuous and gradual at all in his political investigations, again for all his success in the collection of the particulars. And this can explain why it has never been difficult for his successors to take him to be another of the philosophers who fly from the particulars to most general principles and, taking them to be settled, proceed to judgment.

That Ptolemy's ascent is continuous and gradual the fact that it is mathematical shows. Mathematical theory leads chains of inferences substantially free of gaps in time, synchronized reign by synchronized reign, back to Nabonassar, and in space, latitude and longitude by latitude

and longitude, around much of the earth's surface, and, calculated orbit by calculated orbit, to the moon, to Mercury, to Venus, to the sun, to Mars, to Jupiter, to Saturn, to the fixed stars, and to the world in its entirety.

These are the greatest successes of the method before Bacon.

Bacon showed that there are and can only be two ways to search for truth, the false and the true.

He made the most successful attempt to close the false way for good.

He showed the elements of the true way. To begin with the collection of the particulars. To ascend continuously and gradually. And only ultimately to end with the most general principles.

All of this he did by following the true way.

He thus explained the acquisition of knowledge, too. The fact of the acquisition of knowledge. Its failure, when it has failed. And its success, when it has succeeded.

The most important philosophers of the generation before Bacon are Viète, Gilbert, Tycho, and Bruno.

Viète greatly advanced mathematics. Mathematics was fundamentally geometry. Problems were solved by proportions. In the century from Cardano through Viète through Descartes, mathematics became fundamentally analysis. Problems began being solved by equations. In *The analytic art*, Viète formulated: the master rule (2), which reduced proportions to equations. More important still, the analytic art: uses not numbers, but symbols or signs for things, say, the letters of the alphabet (4). Representing terms in equations by letters made operations involving complex series of terms transparent. It made also the generality of problems possible to express in equations. One of the more particular problems Viète solved is:

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \times \dots$$

Another is: the powers of the binomial; given the binomial $a + b$, its square is

$$a^2 + 2ab + b^2,$$

its cube

$$a^3 + 3a^2b + 3ab^2 + b^3,$$

its fourth power

$$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

its fifth power

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5,$$

its sixth power

$$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6,$$

and the construction of higher powers is not different (*Logistic*, 11). Another is the construction of the regular heptagon (*Supplement*, 24). The analytic art, moreover, Viète writes: claims for itself the greatest problem of all, to solve all problems (*Art*, 8).

Gilbert greatly advanced physics. The investigation of magnetism and electricity begins with him. He discovered that the earth is a magnet (*Magnet*, I, 17). He distinguished electric from magnetic phenomena (II, 2). He also argued that: the force of the moon extends to the earth, and the magnetic force of the earth reaches the moon; the two correspond to one another and agree in their action; the earth attracts and repels the moon, and the moon attracts and repels the earth (*World*, II, 19).

Tycho greatly advanced physics as well. He showed that the firmament is mutable. He reports: in November 1572, while observing the stars, as is my habit, I noticed a new and unusual star, surpassing all others in brightness, in a place in the heaven where there had never been a star before; it was a miracle as great as the darkening of the sun at the time of the crucifixion (*Star*). He determines that the star shows neither parallax nor contrary to the diurnal motion, and concludes: this star is not a comet, nor a fiery meteor, nor is it below the moon, nor is it among the planets; it is a star shining in the firmament itself, never seen before our time since the beginning of the world (*id.*). By February 1573 the new star faded to first magnitude, by April to second, by August to third, by October to fourth, by January 1574 to fifth, by February to sixth, then it faded altogether. In November 1577 Tycho observed a comet with a long tail. He measures the direction of its motion and its parallax, hence calculates its distance: from 173 earth radii away in November 1577 to 1 733 earth radii away in January 1578 (*World*, 9), and since: the moon is 52 earth radii away (8), concludes: this comet existed not among the elements, not below the moon, but deep in the aether among the planets (concl.).

Both discoveries rest on vastly improved observation. Ptolemy measured the inclination of the orbit of the moon to the ecliptic at 5° (*Almagest*, V, 12). Tycho showed that the inclination varies from $4^\circ 58' 30''$ to $5^\circ 17' 30''$ (*Exercises*, I, 1).

Bruno greatly advanced enlightenment. He argued that Copernicus is right, the earth revolves around the sun (*Supper*, 1, 3). He argued that the world is not created, it is infinite in space and in time (*Infinity*, 1-2). He said that Christ was a rogue, and did his roguish deeds to seduce the people (Mocenigo, deposition, 23 Mar 1592). He said that Christian doctrine is asinine (*id.*). And he told his executioners: perhaps your fear in passing judgment on me is greater than mine in receiving it (Schopp, to Rittershausen, 17 Feb 1600).

Bacon did not advance mathematics, and he did not greatly advance physics. He obtained no mathematical results, never mind as important as those of Viète, and with one exception, he obtained no physical results as important as those of Gilbert and Tycho. And nothing shows better how great his advance is than the fact that his advance is greater than theirs, even as it is not an advance in mathematics, in particular, or a great advance in physics, in particular. It is an advance in philosophy, in general, such as it includes both physics, in particular, and mathematics, in particular. Such as it includes practical philosophy, in particular, too. Bacon obtained results of the greatest generality, and, without precedent, obtained them by following the true way with all its elements. He ended with the most general principle that the true way is to end with the most general principles by continuous and gradual ascent from the particulars by continuous and gradual ascent from the particulars.

The one exception, Bacon's one very important physical result, is the form of heat.

The true way was untried. So Bacon proposes to try it, in book II of the *New organon*, and in particular: to investigate the form of heat (11).

He writes: first of all we have to prepare a natural and experimental history, sufficient and good; this is the foundation of the thing (10); and we have to begin with the collection of all known instances which agree in the nature under investigation, however different they may be in matter

(11). The collection proceeds:

1. the rays of the sun,
2. the rays of the sun reflected and condensed,
3. fiery meteors,
4. burning thunderbolts,
5. eruptions of flame from mountains,
6. all flame,
7. ignited solids,
8. hot springs,
9. boiling liquids,
10. hot vapors and fumes,
11. fine seasons regardless of the time of year,
12. air in caverns,
13. all villous substances, like wool and skins,
14. all bodies near a fire,
15. sparks from flint and steel,
16. all bodies rubbed strongly,
17. vegetables kept together,
18. quicklime sprinkled with water,
19. iron and tin dissolved in water,
20. animals,
21. horse dung and like excrements,
22. strong oil of sulfur and of vitriol,
23. oil of marjoram and similar oils,
24. strong spirit of wine,
25. aromatic and hot herbs,
26. all acids,
27. even keen cold induces sensation of burning,
28. other (*id.*).

Bacon adds: we call this the table of essence and presence (*id.*), and continues: second, we have to make a presentation of instances in which the given nature is lacking (12); we call this the table of deviation or of absence in proximity (*id.*). This table has 32 entries, each corresponding to an entry in the first table: one deviation corresponding to the first affirmative instance, six to the second, one to the third, and so on (*id.*).

Bacon continues: third, we have to make a presentation of the instances in which the nature under investigation is found in greater or lesser degree (13); for the form of a thing is the thing itself, the thing differs from the form only as the apparent differs from the real, or as the thing in relation to man differs from the thing in relation to the world, and it follows that no nature can be the true form unless it decreases when the nature decreases and increases when the nature increases (*id.*); this table we call the table of degrees or of

comparison (*id.*). The table of degrees or of comparison has 41 entries (*id.*). Next: presentation done, induction itself has to be set to work (15). Induction itself is set to work also in a table. This table is: of exclusion or of rejection of natures (18), and has 14 entries (*id.*).

And after the four tables with their 115 entries, Bacon concludes: the common notion of fire is worthless (20); the genus of heat is motion; heat is motion and nothing else (*id.*); the first specific difference is that heat is an expansive motion of the body (*id.*); the second specific difference is a modification of the first, and it is that heat is a motion toward the circumference and upward (*id.*); the third specific difference is that heat is a motion not of the whole body, but of its smaller parts (*id.*); the fourth specific difference is a modification of the last, and it is that heat is a rapid motion of minute particles, and not of the smallest, but of those a degree larger (*id.*).

The notion of fire is worthless. Heat is a rapid motion of minute particles, and not of the smallest, but of those a degree larger.

Thus Bacon in 1620.

The notion of fire had been worthful for twenty-one centuries before him. Heraclitus had first said that fire is the principle of all things, two generations later Empedocles had formulated the theory that fire is one of the four material elements, and fire had remained a material element since.

And fire, or phlogiston, or caloric – in any case material and elemental – remained worthful for more than two centuries after Bacon.

In 1661, in *The sceptical chemist*, Boyle destroyed the theory of the four elements. Nevertheless, in 1669, in the *Subterranean physics*, Becher proposed that air, water, and three kinds of earth, vitreous, inflammable, and fluid, are the elements, and in 1697, in Stahl's *General theory of fermentation*, inflammable earth became phlogiston.

In 1783, in the *Reflections on phlogiston*, Lavoisier destroyed the theory of phlogiston. Nevertheless, in 1789, in the *Elementary treatise on chemistry*, he writes: the cause of heat, or the exquisite elastic fluid which produces it, we distinguish by the name caloric; nor do we have to suppose it real matter, it is sufficient to con-

sider it a certain repulsive cause which separates the particles of matter (I, 1). And it is yet the exquisite elastic fluid, rather than the certain repulsive cause, that has to be one of: the simple substances which belong to the three kingdoms and which can be considered the elements of bodies, light, caloric, oxygen, nitrogen, and hydrogen (II, 1).

In 1824, in the *Reflections on the motive power of fire*, Carnot discovered the cycle of operations of the reversible heat engine. In 1847, in *On the conservation of force*, Helmholtz discovered the principle of conservation of energy. In 1850, in *On the moving force of heat*, Clausius formulated the laws of thermodynamics. The development of thermodynamics rests essentially on Carnot's results, and Carnot's results in the *Reflections on the motive power of fire* all rest on the presumption that caloric is material, from: the production of motive power in heat engines is due not to the actual consumption of caloric, but to its transfer from a warm body to a cold body (*id.*), to: the motive power of heat is independent of the agents employed to realize it, its quantity depends only on the temperature of the bodies between which the transfer of caloric is effected (*id.*).

How worthwhile the notion of fire was before Bacon, its longevity shows. How worthwhile it remained after Bacon, its worth for the development of chemistry, including by Stahl and Lavoisier, and for the development of thermodynamics, including by Carnot, shows. How much more worthwhile the true way is the notion of a rapid motion of minute particles, and not of the smallest, but of those a degree larger, shows.

The kinetic theory, developed by Maxwell and Boltzmann, grounded thermodynamics in the molecular theory, and this theory showed conclusively that the notion of fire is worthless.

In 1871, in the *Theory of heat*, Maxwell writes: heat is a form of energy (22); energy is known to us in two forms, kinetic, the energy of motion (*id.*); and potential, the energy of a body which depends not on its own state but on its position with respect to other bodies (*id.*); the energy of a hot body must be an energy arising from the motion of its parts (*id.*); it must be a motion of parts too small to observe separately

(*id.*); we thus arrive at the conception of a body as consisting of a great many small parts, each in motion; we call any of these parts a molecule of the substance (*id.*); the doctrine that visible bodies consist of a determinate number of molecules is called the molecular theory of matter; the opposite doctrine is that, however small the parts into which we divide a body, each part retains all the properties of the substance; this is the theory of the infinite divisibility of bodies; we do not assert that there is an absolute limit to the divisibility of matter; what we assert is that after we have divided a body into a certain finite number of constituent parts called molecules, any further division of these molecules will deprive them of the properties which give rise to the phenomena observed in the substance (*id.*).

Heat is a motion of molecules.

cxvi

Bacon's very important treatises are four, *On the dignity and growth of the sciences*, in 1605, *Second part of the great instauration, new organon, or directions for the interpretation of nature*, in 1620, *The history of the reign of king Henry VII*, in 1622, *New Atlantis*, in 1627.

He wrote also the important, if not very important, *Essays or councils, civil and moral*, in 1597, *On the wisdom of the ancients*, in 1609, *Natural and experimental history to ground philosophy, or phenomena of the universe, which is the third part of the instauration of the sciences, part one, history of winds*, in 1622, *History of life and death*, in 1623, *Forest of forests, or a natural history*, in 1627, *Maxims of the law*, in 1630.

He wrote also *An advertisement touching controversies of the church of England*, in 1589, *Religious meditations*, in 1597, *On the colors of good and evil*, in 1597, *A declaration of the practices and treasons attempted and committed by Robert, late earl of Essex, and his accomplices*, in 1601, *A brief discourse touching the happy union of the kingdoms of England and Scotland*, in 1603, *Certain considerations touching the better pacification and edification of the church of England*, in 1604, *Apology in certain*

imputations concerning the late earl of Essex, in 1604, *The charge touching duels*, in 1614, *Apothegms new and old*, in 1625.

cxvii

Bacon published the *Second part of the great instauration, new organon, or directions for the interpretation of nature* in 1620. It contains:

a proem,
a dedicatory letter,
a preface to the great instauration,
the distribution of the works,
a preface to the new organon,
the new organon, in two books, book I comprising 130 aphorisms, book II comprising 52 aphorisms,
a preparative toward natural and experimental history,
and a catalogue of particular histories.

The distribution of the works – following the preface to the great instauration and preceding the preface to the new organon – enumerates the six parts of the great instauration:
first, division of the sciences;
second, new organon, or directions for the interpretation of nature;
third, phenomena of the world, or natural and experimental history for the foundation of philosophy;
fourth, ladder of the intellect;
fifth, forerunners, or anticipations of the restored philosophy;
sixth, restored philosophy, or active science (*id.*).

Bacon completed the first and the second parts, and began the work on the third part.

He completed the first part when he expanded *The advancement of learning*, first published in 1605, into *On the dignity and growth of the sciences*, in 1623.

He completed the second part with the *New organon*.

And he began the work on the third part. The work includes the two brief appendices to the *New organon*, the preparative toward natural and experimental history and the catalogue of particular histories, in 1620, the two treatises, the *Natural and experimental history to ground philosophy, or phenomena of the world, which is the third part of the instauration of the sciences*, part

one, history of winds, in 1622, and the *History of life and death*, in 1623, as well as the incomplete *Forest of forests, or a natural history*, in 1627.

In 1622 Bacon published also *The history of the reign of king Henry VII*. It is a major step in the instauration of history.

In 1627 the *New Atlantis* was published. The Royal Society was founded in 1660 and chartered in 1662. It claimed Baconian pedigree not only in that it proposed itself a society for the instauration of the sciences. It proposed itself the realization of the society described in the *New Atlantis*. One century-odd before Bacon, More had described the island of Utopia. Bacon describes the island of New Atlantis, where: preeminent among all the excellent deeds of the king is the institution of a society, the noblest foundation on earth, dedicated to the study of the works and creatures of god (*id.*).

The first part of the great instauration is the division of the sciences. And four centuries after Bacon the sciences remain to be divided better than Bacon divided them. One century-odd after him, in order to present in the *Preliminary discourse* to the *Encyclopedia*: the world map of knowledge, d'Alambert borrowed the world map of knowledge: from the immortal chancellor of England Francis Bacon, whom one is tempted to regard the greatest, most general, and most eloquent of philosophers. And in the two and a half-odd centuries since no attempt has been made to improve on Bacon's division of the sciences by obtaining it from the particulars by continuous and gradual ascent either.

All of Bacon's conclusions are thus inductive, not deductive. They are generalizations over a massive collection of the particulars. Over, as Bacon describes it in *On the dignity*: the survey of the circuit of knowledge (II, 1), and in the distribution of the works: the exhibition of the knowledge which the human race possesses at present (*id.*).

Thus: we are zealous to recede as little from antiquity as is compatible with truth and the advancement of knowledge (*Growth*, III, 4). More generally: it is good to review what is received; in this way both the old may be more easily perfected and the new more easily approached; for the goal is the improvement of the

old as much as the acquisition of more (*Organon*, distr.).

The survey of the circuit of knowledge, however, shows: deserted tracts, waste, uncultivated, and abandoned by the industry of man (*Growth*, II, 1). What the human race possesses at present is: the ubiquitous corruption of philosophy by superstition and theology (*Organon*, I, 65), and: all the variety of books is everywhere repetition of the same thing, the whole stock, numerous as it appears, proves on examination very scanty (pr. *Instauration*).

And Bacon draws the most general conclusion: there are and can only be two ways to search for truth; one is to fly from the senses and the particulars to most general principles and, taking them to be settled, proceed to judgment; and this way is now in use.

It is a most important result, the sharp separation of the false way from the true. It is a most important result, also, because it is obtained by following the true way. Because it is a generalization over the description of the knowledge which the human race possesses.

An even more important result, however, and for the same reason, is the attempt to close the false way, and for good.

In the *New organon, or directions for the interpretation of nature* the directions concern both ways, the false and the true. Where the false way is concerned, first of all, Bacon lists: the causes of errors and of why errors have so long continued (78). The causes are:

the times favorable to the advancement of knowledge were short (*id.*);
the least diligence was given to natural philosophy (79);
natural philosophy scarcely ever possessed a man wholly, being always a means to something else (80);
the end was not right, which is discoveries (81);
the means was not right either, which is experiment (82);
the true way was not just deserted, it was barred and blocked; experience was not just abandoned or poorly managed, it was scorned (83);
instead, antiquity was venerated dogmatically and childishly (84-7);
pusillanimity followed (88);

and superstition and zeal for religion (89);
and in the schools everything was adverse to the advancement of knowledge (90).

And if the errors have so long continued, and against Bacon's impeccable analysis of their causes have long continued after Bacon, too, it is because they are in possession of the human understanding and have taken deep root in it. Hence, further, Bacon points at the root.

He writes: the idols and false notions which are now in possession of the human understanding and have taken deep root in it not only so beset men's minds that truth can hardly find entrance; even after entrance is obtained, in the very instauration of the sciences they will recur and will molest us, unless men, forewarned by the danger, fortify themselves fiercely against them (I, 38); these idols are of four kinds; we call them idols of the tribe, idols of the cave, idols of the market, and idols of the theater (39);

the idols of the tribe are grounded in human nature itself, in the tribe or race of man; for it is false that sense is the measure of things; on the contrary, all perceptions are according to the measure of man, not according to the measure of the world (41);

the idols of the cave are those of the individual man; for everyone has a cave of his own, which distorts and corrupts the light of nature, due to education and conversation, to the reading of books, and to the authority of those esteemed and admired (42);

the idols of the market are those from the intercourse and association of men with one another; for men associate by discourse, and words are impressed according to the understanding of the vulgar; and the ill and inept choice of words marvelously obstructs understanding (43);

lastly, the idols that have immigrated into the minds of men from the dogmas of the philosophers we name the idols of the theater; for all received philosophies are but so many fables and stage plays, presenting fantastic and theatrical worlds; nor do we speak only of those now in fashion or the ancient; many more fables can yet be composed (44).

It remains the most valiant and most successful attempt to close the false way for good.

Even if many more fables have since been composed. The idols and false notions which are in possession of the human understanding and have taken deep root in it not only so beset men's minds that truth can hardly find entrance. Even after entrance is obtained, in the very instauration of the sciences they will recur and will molest us.

And yet the most important result is the affirmative. The description of the true way with all its essential elements.

Bacon begins in *The great instauration*: there was never the slightest hope that the errors that have flourished and will flourish forever would put themselves right by the native power of the mind or with the help of dialectic; so nothing remained but to try the whole thing anew and undertake the complete instauration of the sciences, arts, and all human learning on proper foundation (proem). And in the *New organon*: there remains but one remedy, to undertake the entire work of the understanding anew, and from the very beginning not leave the mind to itself, but guide it continuously, so that the whole thing is done by machinery (pr.).

The whole thing is done by machinery.

Thus, the whole thing begins with the collection of the particulars.

With the collection and ordering of the particulars. Bacon writes: let no one be alarmed at the multitude of the particulars (I, 112), because: the particulars are like an army, little is to be hoped for unless all particulars pertaining to the matter are marshaled (102); we have to lead men to the particulars and their series and order (36).

The whole thing begins with the marshaling of the particulars in their series and order.

The means is experiment. Bacon writes: experience as it occurs is an accident, as it is pursued an experiment (82); the secrets of nature reveal themselves more under the vexation of art than by themselves (98); not when nature is free and left to herself, but when she is constricted and vexed, when by the arts and services of man she is forced out of her state, and molded and shaped (distr.).

And Bacon has shown the marshaling of the particulars in their series and order from experience not as it occurs but as it is pursued. In the division of the sciences. In the investigation of

the form of heat. And in the catalogue of particular histories appended to the *New organon*. In the catalogue, the particular history of winds is # 6. The particular history of life and death is # 58. All the particular histories are 130.

Next, the whole thing proceeds by continuous and gradual ascent.

And this is more difficult still. Not only: there was never the slightest hope that the errors that have flourished and will flourish forever would put themselves right by the native power of the mind. There is the irresistible propensity: to fly to most general principles and, taking them to be settled, proceed to judgment.

The remedy is: human understanding does not need wings, it needs to be hung with weights (I, 104).

The remedy is conclusive. What exactly the weights are, Galileo, Kepler, and Descartes will show. But Bacon already is clear that the weights are a machine. The whole thing is done by machinery. Also: when they sailed by the stars, men could only follow the coasts and cross small seas; before the ocean could be traversed and the new lands found, the mariner's needle had to be discovered as a more faithful and certain guide; similarly, practice, meditation, observation, and argument could lead to the discoveries so far made; but before the remoter and more obscure parts of nature can be found, a guide for the better use and operation of the human mind and understanding has to be discovered (pr. *Instauration*). Also: to draw a straight line or a circle by hand depends on its steadiness and skill, to draw them with a rule or a compass depends on steadiness and skill little or not at all; and so it is with reasoning (I, 61). Also: in mathematics, with a machine, a demonstration is easy and transparent, without a machine, all seems involved and subtler than it is (distr.).

Galileo, Kepler, and Descartes will show that the weights are mathematics.

Finally, the whole thing ends with the most general principles.

And this is yet more difficult. Because the propensity to fly, irresistible as it is, is yet fortified and impelled by the propensity to prefer fabulous worlds to the world.

Nevertheless, Bacon's remedy is once again

conclusive.

That those who seek to discover and know, if not those who seek to guess and divine, will discover and know the most general principles – if they will discover and know them – in the world, Aristotle established. Bacon concurs: those who seek to guess and divine devise fabulous worlds of their own, those who seek to discover and know examine and dissect the nature of the world itself (distr.); the instauration of the sciences seeks knowledge not, arrogantly, in the tiny cells of the human mind, but, reverently, in the great world itself (pr. *Instauration*).

That the principles are formal Aristotle established as well.

Form, however, was in Aristotle also one of the four kinds of principles. Aristotle never distinguished principles, ἀρχαί, and causes, αἰτίαι, sharply either. And in the following two millennia the ever growing confusion of forms and principles and causes became an ever more powerful idol of the theater.

Bacon first distinguished principles, *axiomata*, and causes, *causae*, sharply. Principles should be distinguished by their generality, exclusively. Causes should be in relation to effects, exclusively. And the principles should be principles rather than axioms, because Descartes established principles as the more general term.

Bacon's analysis of principles and causes begins with Aristotle. He writes: it is right that true knowledge is knowledge by causes; and causes are not improperly divided into four kinds, material, efficient, formal, and final (II, 2).

Of the four kinds, however: the final corrupt rather than advance the sciences (*id.*); they are in relation to the nature of man, not in relation to the world (I, 48).

Of the remaining three, on one side: the material and the efficient, as they are now investigated, as remote causes, and without reference to the form, are shallow and superficial, and contribute nothing to true knowledge (II, 2).

On the other side: of a given nature to discover the form is the work and end of knowledge (1); it is knowledge of the forms that embraces the unity of nature in various matters (3); it is from the discovery of the forms that truth results (*id.*).

Bacon generalizes the result. In *On the dignity*:

speculative natural philosophy, or the search for causes, is of two kinds, physics and metaphysics, and, because the real difference between them must be drawn from the nature of causes, physics inquires into the efficient and the material causes, metaphysics inquires into the form and the end (III, 4). More generally, and more explicitly, in the *New organon*: from the two kinds of principles arises a true division of philosophy and the sciences; the investigation of forms constitutes metaphysics, the investigation of efficient causes, and matter, and processes, and configurations, constitutes physics (II, 9).

It is, more generally still, Aristotle's division of theoretical philosophy into physics and mathematics.

Truth results from the discovery of the forms. And the forms are the laws. Or better, the law with its clauses.

Bacon writes: although there is nothing in nature but individual bodies performing individual acts, they do so by law, and this law and its investigation, discovery, and explanation is the foundation of knowledge and its application; and it is this law with its clauses that is understood by the name forms (2); when we speak of forms we understand nothing but the laws and determinations which govern and constitute a simple nature; the form of heat and the form of light is the same as the law of heat and the law of light (17).

It will take the labors of Galileo, Kepler, Descartes, Huygens, Spinoza, Newton, and Leibniz to clear away the remaining ambiguity in the relation of laws and principles. With the labors of Bacon already, however, it is clear where the whole thing ends, and hence also how.

It ends with the law with its clauses.

And hence this is how: hard to follow, the method is easy to state; it is to establish stages of certainty (pr. *Organon*); our road is not level, but ascends and descends; it first ascends to principles, then descends to works (I, 103); the directions for the interpretation of nature have two general parts, first, how to educe principles from experience, second, how to deduce new experiments from the principles (II, 10); our method is not to extract works from works or experiments from experiments, but from works and experiments to extract causes and principles and from

these causes and principles new works and experiments (I, 117).

Establishing stages of certainty, our road ascends from experiments to principles, descends from these principles to new experiments, ascends from these experiments to higher principles, descends from these again, ascends again, descends again, ascends again, and until it reaches the law with its clauses.

So, in Bacon's generation Kepler reached the three clauses of the motions of the heavenly bodies. Three generations later, Newton reached the three very general clauses of motion and one very general clause of gravity.

In short: we do not propose to look, like augurs, in order to take auspices; we propose to march in, like commanders, in order to take possession (distr.).

And so we did.

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Bacon, Galileo, and Kepler are contemporaries. Their years of birth and death are,

1561 - 1626 Bacon,

1564 - 1642 Galileo,

1571 - 1630 Kepler.

Their very important treatises appeared, Bacon's from 1605, *The growth of the sciences*, to 1627, the *New Atlantis*, Galileo's from 1610, the *Starry messenger*, to 1638, the *Two new sciences*, Kepler's from 1596, *The cosmographic mystery*, to 1627, the *Rudolphine tables*.

Nor is it an accident that they are contemporaries.

Bacon begins the enumeration of the causes of errors and of why they have so long continued with the most general and most fundamental: the times favorable to the advancement of knowledge were short. He explains: out of the twenty-five centuries over which the memory and learning of men extends, there were hardly six fertile in the sciences and favorable to their development; only three periods of revolution in learning can properly be counted; one among the Greeks, one among the Romans, one among us, the nations of western Europe; and to each of these hardly two centuries can justly be assigned (*Organon*, I, 78).

The times favorable to the advancement of knowledge were short. And with the hindsight of

the four centuries of development since Bacon, which these times were can be judged more precisely.

The 3 oldest calculations of area, recorded on the tablets W 20044/20 and W 19408/76, are from the century around *c.* 3080^{BC}. For the sake of uniformity a year of birth can be assigned to that predecessor of the philosophers, 3120^{BC} 3 calculations of area.

From that predecessor of the philosophers to Bacon, Galileo, and Kepler the memory and learning of men extends over forty-seven centuries.

And out of these forty-seven centuries, three have been the most fertile in the sciences and most favorable to their development.

The first is, thirteen and a half centuries after that predecessor of the philosophers, the century from the mathematician who calculated the value of $\sqrt{2}$ at 1.414 212 963 ..., his labors recorded on the tablet YBC 7289, from the century before *c.* 1694, to the physicist who discovered the planets, his labors recorded on the Venus tablet of Ammisaduqa, in the reign of Ammisaduqa, prince *c.* 1646-25. Assigning years of birth, it is the century from

1784 calculation of the diagonal of the square

to

1686 Venus tablet of Ammisaduqa.

The second is, thirteen centuries later, the century from

384 - 322 Aristotle

to

287 - 212^{BC} Archimedes.

The third is, seventeen and a half centuries later, the century from

1473^{AD} - 1543 Copernicus

to Bacon, Galileo, and Kepler.

Each of the three followed centuries, and was substantially the result of these centuries, which were increasingly fertile in the sciences and favorable to their development.

The first, the century from the calculation of the diagonal of the square to the Venus tablet of Ammisaduqa, followed an acceleration in the acquisition of knowledge which can be traced back to the historian who compiled the Sumerian prince list. Counting back to

2077 Sumerian prince list,
the century from the calculation of the diagonal
of the square to the Venus tablet of Ammisaduqa
followed three centuries, 10 generations, which
were increasingly fertile in the sciences and
favorable to their development.

The second, the century from Aristotle to
Archimedes, followed an acceleration in the
acquisition of knowledge which can be traced
back to the physicist who made the oldest accu-
rate observation, recorded by Ptolemy in the
Almagest. Counting back to

761 observation of lunar eclipses from
721 to 720,

the century from Aristotle to Archimedes fol-
lowed four centuries, 13-odd generations, which
were increasingly fertile in the sciences and
favorable to their development. If this century,
the second of the three, was also more fertile and
favorable than the first, it was because it was the
second, and thus building on the successes of the
first, and because it followed more centuries
which were increasingly fertile and favorable.

The third, the century from Copernicus to
Bacon, Galileo, and Kepler, was also more fertile
and favorable than the second, and it was because
it was the third, and thus building on the suc-
cesses of the second, and the first, and because it
followed more centuries which were increasingly
fertile in the sciences and favorable to their
development. It followed an acceleration in the
acquisition of knowledge which can be traced
back to the labors of the philosophers who
brought the darkest of dark ages to an end. The
philosophers brought the darkest of dark ages to
an end beginning with the labors of

1033 - 1109 Anselm,
and decisively with those of
1079 - 1142 Abelard
and
1114 - 1187 Gerard.

Counting back to Anselm, the century from
Copernicus to Bacon, Galileo, and Kepler fol-
lowed nearly four and a half centuries, 15 genera-
tions, which were increasingly fertile in the
sciences and favorable to their development.

Bacon, Galileo, and Kepler thus followed,
counting back to Anselm, nearly five and a half
centuries, more than 17 generations, which were

increasingly fertile and favorable. Two and a half
centuries before them, with Buridan and Oresme,
physics was taking steps beyond Archimedes. Not
quite one and a half centuries before them, with
Regiomontanus, physics was taking steps beyond
Ptolemy. In the next generation Columbus
crossed the Atlantic. In the next generation
Magellan and El Cano circumnavigated the earth,
and this is the generation also of Copernicus.
Cardano and Vesalius followed in the next genera-
tion. Viète, Gilbert, Tycho, and Bruno in the
next. And Bacon, Galileo, and Kepler in the next.

Hence, all other errors had been corrected as
well. In the century before Bacon, Galileo, and
Kepler, their predecessors already excelled in all
the following, and in all the following Bacon,
Galileo, and Kepler surpassed their predecessors.
Diligence was given to natural philosophy. Natu-
ral philosophy possessed a man wholly, becoming
an end in itself. The end was right, which is
discoveries. The means was right, which is exper-
iment. The true way was not just unbarred and
unblocked, it was frequented, experience was not
just rediscovered and well managed, it was em-
braced. Antiquity was examined, the sound in it
adopted, the erroneous avoided. Grandeur fol-
lowed. And enlightenment and zeal for knowl-
edge.

The relative importance of Bacon, Galileo, and
Kepler can also be judged, and has to be, pre-
cisely because the importance of all three is very
great.

Hume writes: if we consider Bacon merely as
an author and philosopher, the light in which we
view him at present, though very estimable, he
was yet inferior to his contemporary Galileo,
perhaps even to Kepler; Bacon pointed out at a
distance the road to true philosophy; Galileo both
pointed it out to others, and made considerable
advances in it (*History*, 49, app.).

If how important a philosopher is is measured
cumulatively, by all the advances of his succes-
sors which rest on his advances, Bacon is the
most important of the three. This fact, with the
hindsight of the two and half-odd centuries of
development since Hume, the judgment of Kant,
Darwin, and Frege suffices to attest.

If, on the other hand, how great a philosopher
is is measured by the advances he singlehandedly

made, Galileo and Kepler are each greater than Bacon. More generally, when Hume writes: great philosophers are so rare, there have not yet been above two in the world, Galileo and Newton, Hume's judgment can be qualified only in that, before Hume, Ptolemy belongs in this list and, after Hume, Darwin.

If, also, how great a philosopher is is measured by how much he advanced enlightenment, the greatest philosophers are two, the greatest by very far, Galileo and Spinoza.

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Galileo's very important treatises are six, *Starry messenger, unfolding great and most wonderful sights and displaying for the inspection of everyone, and especially of the philosophers and astronomers, what with a telescope built by himself Galileo Galilei observed about the face of the moon, the fixed stars, the Milky Way, the nebulous stars, and first of all about four planets circling Jupiter's star at unequal intervals and periods and with marvelous speed, which, unknown to anyone until now, he discovered and decided to name the Medicean stars*, in 1610, *History and demonstrations concerning sunspots and their appearances, contained in three letters to Mark Welser*, in 1613, *Letter to Mme Christina of Lorraine, grand duchess of Tuscany*, in 1615, *The assayer, in which on a delicate and precise scale are weighted the matters contained in The astronomical and philosophical balance of Lothario Sarsi*, in 1623, *Dialogue on the two main systems of the world, Ptolemaic and Copernican*, in 1632, *Discourses and mathematical demonstrations in two new sciences dealing with mechanics and locomotion*, in 1638.

He wrote also the important, if not very important, *On motion*, in 1592, *Defense against the calumnies and impostures of Baldesar Capra in his Astronomical consideration on the new star of 1604, as well as, and much more, in publishing as his invention the construction and uses of the geometric and military compass under the title Usus et fabrica circini proportionis*, in 1607, *Discourse on bodies floating on water or moving*

in it, in 1612, *Discourse on the ebbs and flows of the sea*, in 1616, *Letter to Francesco Ingoli in response to the Disputations on the location and rest of the earth*, in 1624, *Report on flood control*, in 1630, *On Euclid's definitions of proportion*, in 1642.

He wrote also *The little balance*, in 1586, *Treatise on fortifications*, in 1593, *On mechanics*, in 1600, *Dialogue on the new star*, in 1605, *Treatise on the sphere, or cosmography*, in 1606, *The operations of the geometric and military compass*, in 1606, *Discourse on the comets*, in 1618, *Astronomical operations*, in 1637, *Letter to prince Leopold of Tuscany, concerning chapter fifty of the Litheosphorus of Fortunio Liceti*, in 1641.

He left behind also many letters. Favaro, in 1890-909, edited Galileo's corpus, in 20 volumes. The letters comprise 9 of the 20, volumes X-XVIII.

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In 1623, in *The assayer*, Galileo writes: he seems to think that philosophy is a book of man's fantasy, like the *Iliad* or *The furious Orlando*, books in which the least important thing is whether what is written in them is true; but it is not like that; philosophy is written in that grandest book continually open before our eyes, the universe, which cannot be understood without learning to understand the language and to know the letters in which it is written; it is written in mathematical language and the letters are triangles, circles, and other geometrical figures, without which it is impossible to understand a single word, without which one wanders in vain in a dark labyrinth (6).

Philosophy is not a book of man's fantasy, in which the least important thing is whether it is true. Philosophy is written in the world, and the world is written in mathematics.

Galileo showed all this. What philosophy is not. What philosophy is. That what philosophy is not, it is not, and what philosophy is, it is, because the important thing is whether it is true. And the world decides what is true.

To show all this, he had to wage a war, too, and win it.

In 1610, Galileo published the *Starry messen-*

ger, unfolding great and most wonderful sights and displaying for the inspection of everyone, and especially of the philosophers and astronomers, what with a telescope built by himself Galileo Galilei observed about the face of the moon, the fixed stars, the Milky Way, the nebulous stars, and first of all about four planets circling Jupiter's star at unequal intervals and periods and with marvelous speed, which, unknown to anyone until now, he discovered and decided to name the Medicean stars.

And the *Starry messenger* was the messenger of war. It proposed to display great and most wonderful sights for the inspection of everyone. That there are mountains on the moon. That the fixed stars are many more than appear to the eye. That the Milky Way is a swarm of stars. That nebulae are groups of stars. And first of all that there are four satellites of Jupiter circling it at unequal intervals and periods and with marvelous speed.

The *Starry messenger* proposed to display these great and most wonderful sights for the inspection of everyone. Everyone, however, did not propose to have these great and most wonderful sights displayed to him, never mind for his inspection. Mountains on the moon and satellites of Jupiter argued strongly that the earth revolves around the sun. And the earth was content to have the sun revolve around it. Galileo also promises: to treat these matters in a book on the system of the world (*id.*), in which: we will show with many arguments and experiments that the earth is not excluded from the dance of the stars; it moves; it surpasses the moon in brightness; it is not the dump heap of the filth and dregs of the world (*id.*). And the earth was content to be excluded from the dance of the stars, and be the dump heap of the filth and dregs of the world.

The clouds of war had been gathering for some time.

A century earlier, in 1514, Copernicus wrote the *Little treatise on the hypotheses concerning the heavenly motions*. He writes: arguments for the immobility of the earth are based on the appearances, and all such arguments collapse, because my arguments against its immobility are based on the appearances, too (*id.*).

From the first, then: all arguments for the

immobility of the earth collapse. And much else was to collapse as well.

The *Little treatise* acquired wide circulation. How wide Luther attests. In 1539, even his table talk piously recorded now, Luther found an opportunity, in his table talk, to become a judge of Copernicus: that new astrologer who makes the earth move and not the heaven, sun, and moon; whoever wants to be clever agrees with nothing that others respect; for myself, I trust scripture, and Joshua ordered the sun to stop, not the earth (table talk, 4 Jun 1539).

Table talk elsewhere, among others who trusted scripture, followed similar lines. That new heretic in Germany, whoever wants to be clever agrees with nothing that others respect. Where that new astrologer was concerned, however, that new heretic in Germany saw clearly what was at stake: for myself, I trust scripture.

By the time of Copernicus and Luther, the church had been exterminating heretics, and exterminating them most successfully, for fourteen centuries. So it could look with confidence at the prospective extermination of that new heretic in Germany as well. Four generations on, however, when the wars to that end ended with the peace of Westphalia, in 1648, that new heretic, equipped with his own church now, remained unexterminated.

By the time of Copernicus and Luther, Christian philosophy had proved itself capable of assimilating anything. It had for several centuries adhered staunchly to Augustine's admonition: whatever man learns outside the holy scriptures, if noxious, is to be condemned, if beneficial, is in them contained. Then, in spite of some resistance, it had learned the pagan philosophers from the Muslim philosophers. In spite of great resistance, it had then assimilated Aristotle, and assimilated him eventually in just the way it rested on the holy scriptures, chapter and verse. Having assimilated Aristotle, it had assimilated Ptolemy as well. It had found it possible to read *On the heaven* in *Genesis* and in *Joshua*. It had found it possible to read the *Almagest* in *Genesis* and in *Joshua*. It had found it possible, too, to make it mandatory to read *On the heaven* and the *Almagest* in *Genesis* and in *Joshua*. So Christian philosophy could look with confidence at the

prospective assimilation of Copernicus into itself as well. If Ptolemy could be read in *Genesis*, surely Copernicus could be. If Joshua could order the sun to stop, surely he could order the earth to revolve around the sun.

Christian philosophy could assimilate anything. This time, though, the philosophers would not let it.

In 1543 *On the revolutions of the heavenly spheres* appeared. Copernicus writes: there will be some who, when they hear that I ascribe motion to the earth, will clamor that I must be silenced immediately; but the philosopher is not subject to the judgment of the vulgar, for, to the extent permitted by god, he searches in all things for truth (pr.). And: there will be babblers who, although completely ignorant of astronomy, will claim to be its judges and, distorting some scriptural passage to their purpose, will find faults with my work and censure it (ded.).

In 1514 arguments had opposed arguments. On one side, arguments which show that the arguments on the other side collapse. On the other side, arguments which collapse. Three decades later, in 1543, arguments continued to oppose arguments. But the arguments involved much more now. On one side, the philosopher who searches for truth. On the other side, the babblers who distort scriptural passages.

Copernicus, philosophy, and truth, on one side. Babblers and scriptural passages, on the other. Copernicus, clearly, was looking for war.

That the other side was not, the fact suffices to show that it waited for two and a half generations, until 1616, and until Galileo's provocations, before it committed itself to the censure of Copernicus.

When Bruno was burned in 1600, too, he had been disparaging Christ, the church, Christian doctrine, and the holy scriptures, in word and in print, for three decades. His trial had gone on for eight years. And it was Bruno, with his defiance: your fear is greater than mine, who had forced his judges to become his executioners.

In 1610 Galileo reported the observation of mountains on the moon and of satellites of Jupiter. In 1611 he reported the observation of phases of Venus and of sunspots. With these discoveries, if any arguments against Copernicus had not yet

collapsed, collapsed.

In 1613, in the *History and demonstrations concerning sunspots*, Galileo writes: propitious winds now universally blow on the great Copernican system and guide us so clearly, there is little fear of crosswinds (*id.*).

And the great Copernican system established, it was time to deal with the babblers.

Galileo writes: as to those who philosophize without any desire for truth and for the causes of things, who deny the new discoveries and ridicule them, and say that they are illusions, it is time we ridicule them right back and say that they have themselves become invisible and inaudible (*id.*).

The invisible and inaudible responded by deploying their most powerful weapon. In February 1615 a formal complaint was filed with the holy office against Galileo. In March the complaint was supplemented by a deposition charging him with the suspicion of heresy.

Galileo responded by deploying his most powerful weapon. He wrote the *Letter to Mme Christina of Lorraine, grand duchess of Tuscany*.

In the *Letter*, Galileo ignores altogether the practical matter of defending himself. The theoretical matter of showing that the earth revolves around the sun he does not ignore. He addresses it not by defending the fact, however. He addresses it by attacking its opponents. By showing them what is at issue in all its generality, theoretically and hence also practically, and by resolving the issue, theoretically and hence also practically, and resolving it in all its generality and hence also in particular, where the earth's revolution around the sun is concerned.

He writes in the *Letter*:

a few years ago I discovered in the heaven many particulars which had remained invisible until our time; because of their novelty, and because they contradict propositions accepted by the philosophers in the schools, they roused no small number of these professors against me, as if I had placed those things in the heaven with my own hands to confuse nature and the sciences;

confounded perhaps by the truth of the propositions which I affirm, they lack the confidence to defend themselves on the battlefield of philosophy; they have thus resolved to shield the fallacy of their arguments with the authority of the holy

scriptures;

it is most pious to say and most prudent to consider that the holy scriptures never lie, provided their true meaning is penetrated; but no one can deny that their true meaning is often recondite and very different from the literal; the holy scriptures attribute to god feet, hands, eyes, anger, contrition, hatred;

it is reasonable to conclude that the propositions dictated by the holy spirit the sacred writers rendered so as to accommodate the capacities of the vulgar and the undisciplined; and whenever the holy scriptures have seen fit to draw conclusions about nature, they have followed the same course;

therefore, in disputes about problems of nature we must begin not with the authority of scriptural passages, but with the evidence of experience and with necessary demonstrations; the holy scriptures and nature derive equally from the divine word, the one is the dictation of the holy spirit, the other is the most obedient executrix of god's orders; and while the holy scriptures, in order to accommodate the common understanding, say many things which are different from the absolute truth, nature is inexorable and immutable, never transgresses against the laws imposed on her, and does not care whether her reasons and modes of operation are understood by men or not;

there is so little astronomy in the holy scriptures one cannot find in them even the names of the planets, except for the sun and the moon, and once or twice Venus under the name Lucifer;

no one will say that geometry, astronomy, music, and medicine are treated better in the holy scriptures than in Archimedes, Ptolemy, Boethius, and Galen;

conclusions about nature which are truly demonstrated should not have lower authority than the holy scriptures;

conclusions about nature, once they are certain, because they are most true and in concord with demonstrated truth, must be the ground of the true exposition of scriptural passages and of the investigation of their meaning;

in order to remove from the world the new theory it will be necessary to prohibit not only Copernicus and all the writings of those who follow the same theory, but to prohibit the sci-

ence of astronomy altogether, as well as to forbid men to look toward the heaven;

and no one can doubt that the supreme pontiff has the absolute power to permit and condemn propositions; but no creature has the power to make them true or false (*id.*).

Theoretically, as far as truth is concerned, this was the end of the authority of the holy scriptures.

Practically, as far as action is concerned, to prohibit the science of astronomy altogether, as well as to forbid men to look toward the heaven, had been tried earlier with the greatest success. Alas, the right course of action as it now was as well, the resolution of those who had done it earlier was now lacking. To prohibit Copernicus and all the writings of those who follow the same theory was all that was tried.

In February 1616 a committee of eleven appointed to examine Copernicus submitted its findings to the holy office: the proposition that the sun is in the center of the world and devoid of local motion, is foolish and absurd in philosophy, and it is formally heretical because it contradicts explicitly many scriptural passages according to both their literal meaning and their interpretation by the holy fathers and the doctors of theology; the proposition that the earth is not the center of the world and not motionless but moves as a whole and with diurnal motion, receives the same judgment in philosophy, and with respect to theological truth it is at least erroneous (report, 24 Feb 1616).

The supreme pontiff, committees of eleven, the holy fathers, and the doctors of theology may have the power to permit and condemn propositions. Alas, no creature has the power to make propositions true or false.

Alas, Galileo did not relent either.

In 1623, in *The assayer*, he reports: long experience has taught me that the less men know and understand, the more resolutely they judge (21). Also: he believes good philosophers can be enclosed within walls; I believe they fly, and not like starlings, but like eagles, alone (9). Also: reasoning is like running, not like carrying burdens; one Arab steed will outrun a hundred pack horses (45). And he made them ponder not only who is the eagle and who are the pack horses. He

made them ponder which books are not like *The furious Orlando*, and which books are like *The furious Orlando*. The books of the eagle. Or the books of the pack horses. Nay, the books of the holy fathers and the doctors of theology. Nay, the holy scriptures.

In 1632, in *The two main systems*, Galileo continues to advise them: anyone who considers it more reasonable for the whole world to revolve in order to let the earth stay still, should climb to the top of the cupola and, in order to get a view of the city, demand that the city revolve around him so that he would not have to trouble to turn his head (II). And he continues to describe them: I often encounter heads so thick, what I say does not penetrate them even after I have repeated it a thousand times (I); the vain presumption to understand everything can have no other ground than having never understood anything (*id.*); I cannot even censure them without doing them more honor than they deserve (III); and I have heard things which I should refrain from repeating, not to avoid discrediting their authors, but to avoid tarnishing so greatly the honor of the human race (*id.*).

In June 1633 sentence was passed on Galileo. The ten cardinals passing it write: we sentence you, the above mentioned Galileo, that you have rendered yourself vehemently suspected of heresy (sentence, 22 Jun 1633); so that you be more cautious in the future and that others abstain from similar crimes, we order that the book *The two main systems* be prohibited by public edict (*id.*); we condemn you to formal imprisonment; and as a salutary penance we impose on you to recite the seven penitential psalms once a week for the next three years (*id.*).

In the book to be prohibited by public edict the above mentioned Galileo also writes: such is the power of truth, when you attack it, your attacks strengthen it and confirm it (II).

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Philosophy is written in the world, and the world is written in mathematics.

And to show how Galileo showed this, it can be shown how he advanced philosophy, how he advanced the investigation of the world, and how he advanced mathematics.

Philosophy is the labors of the philosophers. The cumulative labors of the philosophers. And there is and can be only one way in which the labors of the philosophers can be and are cumulative. Accordingly, the accumulation of the labors of the philosophers begins with the second philosopher. The first philosopher said that water is the principle of all things, and that the earth is at rest because it floats on water, like wood. The second philosopher said that the earth is at rest because of equilibrium. The third philosopher said that air is the principle of all things, and that the earth is at rest because it floats on the air below, like a lid. Two-odd centuries later, Aristotle describes it in general: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

That they can save themselves the labor of adopting the sound in what our predecessors have said and avoiding the erroneous, some of the followers of the fourth philosopher found out. Of some of the Pythagoreans Iamblichus reports: they follow precepts as divine revelations, say nothing of their own, think it wrong to do so, and consider whoever knows the greatest number of precepts the wisest (*Knowledge*, 25; *Pythagoras*, 18).

Philosophy is, or what is the same, the labors of the philosophers are, theoretical and practical, search for truth and search for action. Theoretical are physics and mathematics, practical are politics and ethics.

And with the labors of the first philosopher philosophy is physics, it is physics with the labors of the second philosopher, and of the third philosopher, and philosophy remained first and foremost physics for the one and a half centuries after the first philosopher. Philosophy is first and foremost physics with the labors of Aristotle as well. On the evidence of the Aristotelean corpus, more than one half physics.

One and a half centuries after the first philosopher Socrates abandoned physics altogether and devoted himself to ethics. Because it is ridiculous to search for other things before I know myself.

Being physics, philosophy is the search for truth. If philosophy will be both physics and

ethics, truth being truth, physics has to guide ethics, the search for truth has to guide the search for action. But if philosophy abandons physics and devotes itself to ethics, action being action, the search for action need not be troubled by the search for truth. Philosophy can abandon truth altogether, or, action being action, take it that truth is action and action is truth, at will.

Physics, truth, and adopting the sound in what our predecessors have said and avoiding the erroneous are inseparable from one another. If only because, with Aristotle: those who search for truth find singly little or nothing, jointly they amass much. They find singly little or nothing, because they all have nothing at all – and most particularly no physics at all – to begin with. Jointly they amass much because they adopt the sound in what our predecessors have said and avoid the erroneous.

Inseparable from one another as well, if only for the same reason as well, are the abandonment of physics, the abandonment of truth, and following precepts as divine revelations and considering whoever knows the greatest number of precepts the wisest.

And this is how Galileo advanced philosophy. He showed that philosophy is physics. That physics, truth, and adopting the sound in what our predecessors have said and avoiding the erroneous are inseparable from one another. And that the abandonment of physics, the abandonment of truth, and following precepts as divine revelations and considering whoever knows the greatest number of precepts the wisest are inseparable from one another.

Two and a half centuries separate the first philosopher and Aristotle. Nineteen and a half centuries separate Aristotle and Galileo.

And when the first two and a half centuries of the development of philosophy are analyzed into physics, its abandonment, truth, its abandonment, and adopting the sound in what our predecessors have said and avoiding the erroneous, and its abandonment, they are so analyzed first of all by Aristotle. He established that the first philosophers were physicists. He established that Socrates abandoned physics and devoted himself to ethics. He established that those who search for truth find singly little or nothing, jointly much.

He established that in order to formulate the problems we are to solve we have to call the council of our predecessors. He is also clear what philosophy is. Clear what physics is. Clear what ethics is. Clear what truth is. Clear what the council does. When the first two and a half centuries of the development of philosophy are so analyzed, then, these results of Aristotle are results of the labor of adopting the sound in what our predecessors have said and avoiding the erroneous, too. They are theoretical results. With these results, too, Aristotle amassed truth.

When the nineteen and a half centuries of the development of philosophy from Aristotle to Galileo are analyzed into physics, its abandonment, truth, its abandonment, and adopting the sound in what our predecessors have said and avoiding the erroneous, and its abandonment, they are so analyzed on the authority of Aristotle, on the authority of Bacon and of Galileo, and, with a very few and partial exceptions, Ptolemy, in particular, on the authority of no philosopher in between. Or what is the same, when the nineteen and a half centuries of the development of philosophy from Aristotle to Galileo are so analyzed, the sound in what Aristotle, Bacon, and Galileo have said is adopted, the erroneous in what the philosophers in between have said is avoided.

The century beginning with Aristotle is one of the centuries most fertile in the sciences and most favorable to their development. Other than Aristotle, it is the century of Theophrastus and the description of 550 kinds of plants, Euclid and the *Elements*, Herophilus and the discovery that nerves originate in the brain, Aristarchus and the proposition that the earth revolves around the sun and the first measurement of the distances of the moon and the sun, Erasistratus and the discovery that veins and arteries originate in the heart, Archimedes and the solutions of problems of areas and volumes defined by conics and the discovery of the law of equilibrium and of the law of floating bodies, Chrysippus and the discovery of the rules of inference, Eratosthenes and the measurement of the radius of the earth. Physics advanced greatly. The philosophers amassed much truth.

It is the century also of the establishment of

the schools, the Academy, Lyceum, Garden, and Stoa. A further century on, and the schools produced philosophy. With Bacon: in the schools everything was adverse to the advancement of knowledge. The schools abandoned physics and devoted themselves to ethics. Veneration of the founder and following precepts as divine revelations began among the Academics and Epicureans with their founders, among the Peripatetics and Stoics one century after their founders.

The three and a half-odd centuries from Eratosthenes to Ptolemy were thus not so fertile in the sciences and favorable to their development. They were also increasingly less fertile and favorable. The first of these centuries is still the century of Apollonius and the *Conics* and Hipparchus and the theory of the motions of the sun and the moon. In the last of these centuries, in order to adopt the sound in what our predecessors have said and avoid the erroneous and formulate the theory of the motions of the planets, Ptolemy had to reach back to Hipparchus, more than two and a half centuries earlier.

In the two and a half-odd centuries from Ptolemy to Augustine philosophy became Christian philosophy and Christian philosophy became orthodoxy. And it did not by just abandoning adopting the sound in what our predecessors have said and avoiding the erroneous. It did by the extermination of heresy. Two and a half centuries after st Paul, three generations before Augustine, Eusebius wrote *The history of the church*. *The history of the church* is the history of bishops, martyrs, books, and heresies. These are the heresiarchs Eusebius notes: Simon (II, 13), Menander (III, 26), Cerinthus (28), Nicolaus (29), Saturninus, Basilides, Carpocrates (IV, 7), Valentinus, Cerdo (10), Marcus, Marcion (11), Thebuthis, Simon, Cleobius, Dositheus, Gorthaeus (22), Montanus (27), Tatian, Severus (29), Bardaisan (30), Apelles, Potitus, Basilicus, Syneros (V, 13), Priscilla, Maximilla (14), Florinus, Blastus (15), Miltiades (16), Themiso, Alexander (18), Alcibiades, Theodotus (23), Artemon, Theodotus, Asclepiodotus, Theodotus, Natalius (28), Symmachus (VI, 17), Ambrose (18), Proclus (20), Beryllus (32), Novatian (43), Nepos, Coracion (VII, 24), Sabellius (26), Paul of Samosata (27), Mani (31). 48 heresiarchs in two and a half centuries

yield an average of 5 to 6 heresiarchs per generation. More even than by the number of heresiarchs, however, the determination of the extermination is indicated by the fact that not more than 2 of the 48, Novatian and Mani, can have a claim to importance.

The seven centuries beginning with Augustine were thus as barren in the sciences and hostile to their development as there have been centuries. Truth was not amassed, falsehood was.

The five centuries after that, the five centuries before the heresiarch Galileo, were less barren and hostile at first, then gradually more fertile and favorable, and more. This is what happened to Aristotle.

Seven and a half centuries after Augustine, four and a half before Galileo, Gerard translated 71 treatises from Arabic into Latin. All the 71 are physics and mathematics, among them the *Physics*, *On the heaven*, *On generation and corruption*, *Meteorology*, and *Posterior analytics*.

Philosophy was thus exposed to the danger of adopting the sound in what Aristotle has said and avoiding the erroneous.

In 1210, a council under the archbishop of Sens decreed: neither the books of Aristotle on natural philosophy nor their commentaries shall be read in Paris in public or in secret, this we forbid on pain of excommunication (decree, 1210). In 1215, on the authority of the supreme pontiff, Innocent III, rules for the university of Paris were laid down, among them: they shall not read the books of Aristotle on metaphysics and on natural philosophy nor commentaries on them (decree, Aug 1215). In 1231, the supreme pontiff, Gregory IX, issued the bull *Mother of the sciences*, among its motherly provisions: the books on natural philosophy prohibited in provincial council for due cause we order examined and purged from all suspicion of error (*id.*). William of Auxerre was charged with the task. Gregory IX instructed him: a comely woman found among the captives is not permitted in the house before her hair is shorn and her sharp nails are trimmed; so we order you to examine the books on natural philosophy and excise what you find erroneous or likely to give offence (to William of Auxerre, 23 Apr 1231).

In the next generation, Roger Bacon, Bona-

venture, and Aquinas – the marvelous doctor, the seraphic doctor, and the angelic doctor – are contemporaries. The marvelous doctor liked the comely woman as she was, with her hair unshorn and her nails untrimmed. The seraphic doctor did not like her at all, not even with her hair shorn and her nails trimmed. The angelic doctor sheared her hair and trimmed her nails.

In *The great work*, Roger Bacon writes: they raise objections in Paris against the natural philosophy and metaphysics of Aristotle as set forth by ibn Sina and ibn Rushd; and for a long time, because of the utmost ignorance of their books, they have excommunicated those who read them (I, 9); but on the authority of all great philosophers Aristotle is the greatest of them all; nothing is philosophy except what Aristotle says it is; in philosophy Aristotle is the philosopher just as in sacred science Paul is the apostle (II, 13).

In the *Discourses on the six days*, Bonaventure writes: as god separated light and darkness among the angels, so he separated them among the philosophers, and the leader of those last is Aristotle (VI, 2); he affirms nothing about hell; never affirms that there is a devil, never that there is beatitude and life after death (3); other blindnesses follow, that the world is eternal, that the soul perishes (4); and this pit of darkness has not been locked up yet (5).

In the *Theological summa*, blindnesses and all, Aquinas assimilated that pit of darkness into Christian philosophy. Or, indiscriminately, into theology.

On the one hand, Aquinas is a philosopher *par excellence*. The first question in the *Theological summa* is not, are any doctrines besides the doctrines revealed by god necessary? It is: are any doctrines besides the disciplines of philosophy necessary (I, 1.1)? The answer is: yes, the doctrines revealed by god are also necessary (*id.*). The labor of adopting the sound in what our predecessors have said and avoiding the erroneous is also very much in evidence in the *Summa*. Aquinas's highest authority is the philosopher, that pit of darkness, his authority higher than that of any doctor of theology, nay, higher than that of any father of the church, nay, higher than that of the apostle. After the first introductory general question, for instance, in the following 25 general

questions (2-26), Aquinas treats of: the divine essence (2), and in these 25 questions he appeals to the authority of the philosopher 147 times, to that of Augustine 119 times, of the apostle 68 times, of Dionysius the Areopagite 51 times, of *Psalms* 35 times, of the gospels, *Matthew*, *Luke*, and *John*, 25 times, of Boethius 20 times, and so on. Clearly, what the philosopher had said on the divine essence was sounder than what Augustine had said, and very much sounder than what the apostle had said.

On the other hand, the *Summa* is the *Theological summa*. Aquinas answers in it 512 general questions, divided into 2 669 particular questions. It is in three parts: we shall treat of god, of the rational creature's advance toward god, and of Christ who, as man, is our way to god (I, 2). The introductory 1 general question is divided into 10 particular questions. Aquinas then treats of god in 118 general questions, divided into 574 particular, of the rational creature's advance toward god in 303 general questions, divided into 1 536 particular, and of Christ who, as man, is our way to god in 90 general questions, divided into 549 particular. Analyzed into physics and ethics, the *Theological summa* is ethics. If there is physics in it, it is in its first part, the one that treats of god. This part is itself divided into three parts: the divine essence (2-26), the distinctions of persons (27-43), and the procession of creatures from him (44-119). Among the creatures, the treatment of angels demands 24 general questions (50-64, 106-14). For the treatment of the work of the six days, matter, light, animals, plants, motion, heavenly bodies, 15 general questions suffice (65-74, 115-9).

In 1323 Aquinas was canonized.

Aristotle became unquestionably a doctor of theology. The schools could safely follow his precepts as divine revelations and consider whoever knew the greatest number of them the wisest.

Not quite three centuries separate the canonization of Aquinas and Galileo. In these centuries physics advanced, haltingly at first, then greatly. The first of the three is the century of Buridan and Oresme. The second is the century of Regiomontanus, and of prince Henry, the voyages steadily further down the African coast, and Columbus. The third is the century of Magellan

and El Cano, and of Copernicus, Gilbert, and Tycho.

Aquinas writes in the *Summa*: besides the disciplines of philosophy the doctrines revealed by god are necessary (I, 1.1); for the things that can be learned in philosophy, inasmuch as they can be known by natural reason, can also be taught by another science, inasmuch as they fall within revelation (*id.*); this science is sacred doctrine; and it is a science; just as the musician accepts on authority the principles taught by the mathematician, so sacred doctrine rests on the principles revealed by god (1.2); and sacred doctrine stands above all other sciences; it is nobler than all of them; for it surpasses all of them, both, by its greater certainty and by the higher worth of its subject matter (1.5).

Counting from Clement of Alexandria and Origen to Augustine, it took the fathers of the church two centuries to make philosophy Christian philosophy.

After Augustine, counting to Anselm and Abelard, for seven centuries philosophy was Christian philosophy on the ground of the proposition: whatever man learns outside the holy scriptures, if noxious, is to be condemned, if beneficial, is in them contained. To the degree to which it could be philosophy on the ground of this proposition.

Counting from Abelard and Gerard to Aquinas, exposed once again to the pagan philosophers, Christian philosophy revived and was for one and a half centuries in some turmoil.

Aquinas then established these propositions. The things that can be learned in philosophy can also be taught by sacred doctrine. Sacred doctrine is a science. It stands above all other sciences. It surpasses all of them by its greater certainty. It surpasses all of them also by the higher worth of its subject matter.

And counting from Aquinas to Galileo, for three and a half more centuries philosophy remained Christian philosophy on the ground of these propositions.

All in all, philosophy was Christian philosophy for fourteen centuries.

Analyzed into physics and ethics, it was ethics.

Both Buridan and Oresme discussed the possibility that the earth moves, anticipating

Copernicus. Both advanced the investigation of motion, Buridan with the theory of impetus, Oresme with the mathematical investigation of accelerated motion, anticipating Galileo. And both not only took all those propositions for granted, they argued them explicitly. With the labors of Buridan and Oresme the things that could be learned in physics could also be taught by sacred doctrine. Sacred doctrine stood above physics. It surpassed physics by its greater certainty. It surpassed it also by the higher worth of its subject matter.

If the labors of Copernicus, Gilbert, and Tycho challenged all those propositions, they did so only implicitly. The presumption that theirs are labors beset by the lack of certainty and lack of worth remained unchallenged.

Galileo showed all those propositions false.

The things that can be learned in philosophy cannot be taught by sacred doctrine. Because: philosophy is not a book of man's fantasy, philosophy is written in the world, and without learning the language in which it is written it is impossible to understand a single word.

Sacred doctrine is not a science. Precisely because it rests on the principles revealed by god: the holy scriptures, in order to accommodate the common understanding, say many things which are different from the absolute truth.

Sacred doctrine does not stand above all other sciences. On the contrary: conclusions about nature are in concord with demonstrated truth, and they must be the ground of the true exposition of the holy scriptures.

Sacred doctrine does not surpass all other sciences by its greater certainty. On the contrary. Galileo writes: extensively, with respect to the multitude of things that can be understood, the human intellect is nothing even if it comprehends one thousand propositions, because one thousand in relation to infinity is zero; but intensively, with respect to understanding any one proposition perfectly, I say that the human intellect understands some propositions perfectly, and it then has absolute certainty, such as nature itself has; such are the mathematical sciences, geometry and arithmetic; in them the divine intellect knows infinitely more propositions, since it knows them all; but with respect to the few which the human

intellect comprehends, its knowledge is equal to the divine in objective certainty, because here it arrives at the comprehension of necessity, beyond which there can be no greater certainty (*Systems*, I); the truth of knowledge by mathematical demonstrations is the same as that known by divine wisdom (*id.*).

Nor does sacred doctrine surpass all other sciences by the higher worth of its subject matter. On the contrary: the propositions dictated by the holy spirit the sacred writers rendered so as to accommodate the capacities of the vulgar and the undisciplined.

In short, philosophy is not Christian philosophy. Philosophy is philosophy.

Nor is philosophy ethics. Philosophy is physics. Philosophy is written in the world. Or synonymously, nature. Galileo writes: the great book of nature is the proper object of philosophy (*ded.*). Written in the world, or synonymously nature, philosophy is the reading of the world, or synonymously nature.

Physics and truth are thus inseparable. For: nature is inexorable and immutable, never transgresses against the laws imposed on her, and does not care whether her reasons and modes of operation are understood by men. And: no creature can make propositions true or false. What creatures can do is learn the language of the world, or synonymously nature, and read it.

Physics and truth and adopting the sound in what our predecessors have said and avoiding the erroneous are thus inseparable.

Galileo writes: they seem to think that nature first made the brain of man, then arranged things to conform to the capacity of his mind; but I should think that nature first made things in her own way, then made the mind of man good enough to understand some of her secrets, and only through hard labor (II).

Bacon and Galileo are the first philosophers after Aristotle to undertake the hard labor of calling the council of our predecessors into a plenary session.

To adopt the sound in what our predecessors have said and avoid the erroneous Bacon surveys the entire circuit of knowledge: history, philosophy, poetry, sacred history, civil history, natural history, science of god, science of man, science

of nature, logic, ethics, physics, mathematics.

To adopt the sound in what our predecessors have said and avoid the erroneous Galileo examines what all our predecessors have said, physicists or theologians, old or new. Brief as it is, the *Letter to Mme Christina* quotes Augustine 15 times, and notes – to note only the important – Tertullian, Lactantius, Jerome, and Dionysius the Areopagite among the fathers of the church, Lombard and Aquinas among the doctors of theology. All of Galileo's labors rest on the labors of: the superhuman Archimedes, whom I cannot mention without feeling awe (*Motion*). Galileo's is the first sound judgment of Gilbert: I have the greatest praise and admiration for Gilbert who framed such a stupendous theory about an object which an infinite number of men of splendid intellect had handled without paying any attention to it; I only wish he were more of a mathematician (*Systems*, III). Galileo's is the first sound judgment of the relation of Copernicus to Ptolemy: Copernicus restored astronomy to its Ptolemaic presuppositions (*id.*). And Galileo did not concede Aristotle to the doctors of theology. *The two main systems* is an examination of Aristotle at greater length and in more detail than it is of Ptolemy and Copernicus. Galileo writes: if Aristotle saw the new discoveries in the heaven, he would change his opinions and correct his books (II), and: those who defend Aristotle's every word do not see how greatly they detract from his reputation and credit (*id.*). Nor could Galileo concede Aristotle to the doctors of theology, if physics, truth, and adopting the sound in what our predecessors have said and avoiding the erroneous are inseparable. He concludes: I am impugned as an impugner of the Peripatetic doctrine; but I claim that I observe Aristotle's doctrine more religiously than any of them; for Aristotle teaches us to reason well, argue well, and deduce necessary conclusions; and if Aristotle returned to earth he would consider among his followers much rather me, because of my few but conclusive demonstrations of his errors, than the very many who, in order to defend every word of his as true, find in his writings doctrines which never entered his head.

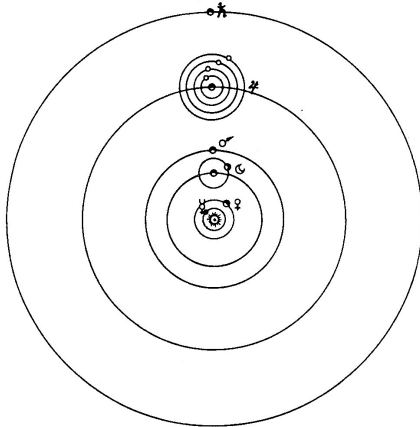
For the following one and a half centuries philosophy remained what Bacon and Galileo

showed philosophy is. Descartes advanced physics greatly. Cassini, Boyle, Huygens advanced physics greatly. Newton, Leibniz, Jakob Bernoulli, Halley advanced physics greatly. Daniel Bernoulli, Linnaeus, Euler, Buffon, d'Alembert advanced physics greatly. Herschel, Lavoisier, Laplace advanced physics greatly. Hobbes grounded politics in physics. Spinoza grounded history in physics. Hume and Kant grounded the investigation of knowledge in physics.

The main reason are Bacon and Galileo. Bacon pointed out the false way, described the false way, pointed out the true way, described the true way, and showed that there are and can only be two ways to search for truth, the false and the true. All with great clarity and distinction. Galileo showed what the results of following the false way have been, and what the results of following the true way can be. Both with the greatest clarity and distinction. With proof. The former also by fighting them no quarter given. The latter also by producing them in great abundance.

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The picture below, without the Galilean satellites, appears first in Copernicus, in 1543, in *On the revolutions of the heavenly spheres* (I, 10), then, with the Galilean satellites, in Galileo, in 1632, in *On the two main systems of the world, Ptolemaic and Copernican* (III), then in Descartes, in 1644, in *The principles of philosophy* (III, 31). The version below is from Descartes. The engraver is better.



The sun is orbited by Mercury, Venus, the earth,

itself orbited by the moon, Mars, Jupiter, itself orbited by the Galilean satellites, and Saturn.

In 1610 Galileo observed mountains on the moon and satellites of Jupiter. These were strong arguments that the earth revolves around the sun. Strong, but indirect, by analogy. As the moon, so the earth, as Jupiter, so the sun. By 1612 Galileo had measured the periods of the satellites: the first, 1 day, $18\frac{1}{2}$ hours, the second, 3 days, $13\frac{1}{3}$ hours, the third, 7 days, 4 hours, the fourth, 16 days, 18 hours (*Water*). In 1611 he observed phases of Venus and sunspots. And it was when he measured the differences in the apparent sizes of Venus and Mars, and explained sunspots, that the arguments became conclusive.

In 1623, he writes: neither Tycho nor any other astronomer, not even Copernicus, could clearly refute Ptolemy; a powerful argument stood in their way; they could not establish that Venus shows itself 40 times and Mars 60 times larger in one position than in another, as they are required by the Copernican system; I demonstrated this to be true and manifest to the senses (*Assayer*, 6).

And in 1632, in *The two main systems*, he concludes: we have seen that there are strong arguments in favor of the Copernican system, and three of them have been shown conclusive; first, the motion of the planets as they appear to stop and to go backward and to approach the earth and to recede from it; second, the revolution of the sun observed in the sunspots; third, the ebbing and flowing of the ocean (IV).

The first Copernicus showed.

The second and the third Galileo showed.

He writes: if the sun rotates around its axis, and its axis is inclined to the plane of the ecliptic, then, first, the annual motion of the earth around the sun will make the sunspots appear to traverse the sun twice a year along straight lines, and at all other times along perceptibly curved arcs; second, the curvature of the arcs will appear during one half of the year convex toward the upper part of the sun, during the other half of the year convex toward its lower part; third, as the spots appear to rise at the left side of the sun and to set at its right side, for six months the points of rising will be lower than the points of setting, for the other six

months they will appear to be higher, and on two days they will appear to be at the same level; fourth, the day of maximum obliquity of the motion of the earth will be the day on which the spots appear to traverse the sun in a straight line, and the day of equilibrium will be the day on which the spots appear to traverse it in the most curved arcs (III).

And: if there are two motions of the earth, annual along the ecliptic, and daily around its axis, its axis being inclined to the ecliptic, even if each of these motions is uniform in itself, from their composition an uneven motion in the parts of the earth will result (IV); and as the acceleration and retardation of the motion of a vessel will make the water in it run back and forth along its length and rise and fall at its extremities, who can deny that such an effect must take place in the ocean (*id.*)?

The discovery that the earth revolves around the sun, the demonstration of this fact, the demonstration that the sunspots show this fact, and the demonstration that the tides show this fact, are each a major advance in the investigation of the world.

Aristotle first described the world. Described it in its entirety, some of its major parts, many of its lesser parts, all in relation to one another. Ptolemy measured the world. In its entirety, some of its major parts, many of its lesser parts, all in relation to one another. Aristotle began determining of what kind the world is. Ptolemy began determining how much there is of it. And Copernicus's discovery and Galileo's demonstrations are major advances in the determination of how much there is of it, in the determination of what kind it is, and for the first time in the determination of how the two, the quantity and the quality, depend on one another.

Ptolemy writes: from the facts that the sizes and distances of the stars appear the same from all parts of the earth, and that the plane through the horizon appears everywhere on earth to bisect the heavenly sphere, it follows that the earth has the ratio of a point to its distance to the sphere of the fixed stars.

From the same facts, if the earth revolves around the sun, it follows that the orbit of the earth around the sun has the ratio of a point to its

distance to the sphere of the fixed stars. Copernicus writes: the size of the world is so great the distance between the earth and the sun is imperceptible in relation to the sphere of the fixed stars (*Revolutions*, I, 10).

It caused alarm. Galileo has to write: you calculate that, for the orbit of the earth around the sun to appear imperceptible in relation to the sphere of the fixed stars, the fixed stars have to be 16 506 000 earth radii away; and what impossibility or inconvenience do you find in that, Mr Ingoli (*Ingoli*)?

In *The two main systems*, Galileo calculates the distance of the fixed stars independently of the fact of the earth's revolution: if the sun is 1 208 earth radii away, and its apparent diameter is $\frac{1}{2}^\circ = 30' = 1\,800'' = 108\,000'''$, and the apparent diameter of a star of the sixth magnitude is $50'''$, the apparent diameter of the sun is 2 160 times larger, and if that star is equal to the sun and not larger, it is at a distance 2 160 times the distance of the sun (III).

Ptolemy calculates the distance of the stars at least 19 865 earth radii. Galileo calculates the distance of a star of the sixth magnitude at 2 609 280 earth radii. The world is thus at least,

Ptolemy	2×10^8 km across,
Galileo	3×10^{10} km across.

The better determination of its size is a great advance in the investigation of the world.

With Kepler: the position, order, and motion of the parts of the world is the system of the world.

And as the picture shows, the better determination of the position and order of the parts of the world is a greater advance. Before Copernicus, the problem of the position and order of the parts is solved conclusively with respect to the system of the earth and moon. By Copernicus, Galileo, and Kepler the problem of the position and order of the parts is solved conclusively with respect to the solar system, the system of the sun, planets, and satellites, through Saturn explicitly, and beyond Saturn implicitly, too.

Yet a greater advance still is the better determination of the uniformity of the parts of the world, and hence of the world. The parts, sun, planets, satellites, all revolve around one another.

As the earth also rotates on its axis so the sun rotates on its axis. There are mountains on the earth, there are mountains on the moon, and there are spots on the sun. The parts and the parts of the parts as well, the earth, the ocean on the earth, the sun, the spots on the sun, all move in relation to one another.

Hence, the greatest advance is the much better determination of the motion of the parts of the world, and hence of motion in the world.

Aristotle established the importance of motion. It is the central problem of the *Physics*. Six of its eight books, III-VIII, are concerned with motion. Motion is locomotion, if there is no locomotion, there can be no other motion. The natural motion of fire and air is up, of water and earth down. The primary motion is circular. Everything in motion must be moved by something. So there must be a first mover, itself unmoved. The first mover is at the circumference, moves the fixed stars, and so communicates motion to everything further below.

It is all vague. None of these very general results is obtained by continuous and gradual ascent from the particulars.

The description of the world quit being vague with Ptolemy. He begins with the collection of the particulars, with the observations, ascends continuously and gradually, by chains of inferences substantially free of gaps, and ends with general principles, with the theory of the motion of the heavenly bodies.

An abyss, however, remained between Ptolemy's general results and Aristotle's very general results. And not only were these very general results unexamined by the particulars. They were supplemented by even more general results.

Aquinas writes: the first mover everyone understands to be god (*Summa*, I, 2.3).

Buridan (*Heaven*, II, 22) and Oresme (*Heaven*, II, 25) both discuss the possibility that the earth moves. Both discuss also what moves the heavenly bodies. Buridan writes: the greater the amount of matter in a body, the more impetus it receives and the more intensely (*Physics*, VIII, 12.5), and concludes: thus, as the holy scriptures do not say that intelligences move the heavenly bodies, it appears unnecessary to posit such intelligences; it can be presumed instead that god,

when he created the world, moved each heavenly body as he pleased and impressed in it an impetus which continues to move it without he himself having to do so (*id.*). Oresme writes: the number of intelligences moving the heavenly bodies must accord with the number of their motions; one intelligence must therefore move each epicycle (*Heaven*, II, 2); presumably, as soon as he created the heaven, god deputed angels to move them (*id.*). Oresme adds: to examine the matter further, I will explain how angels move (*id.*), does so, and obtains this result also: an angel can be in heaven, on earth, here at Rouen, in Paris, in Rome, and in 100 000 other places simultaneously (*id.*).

Copernicus restored astronomy to its Ptolemaic presuppositions. He begins with the observations, with the same observations, ascends continuously and gradually, and ends with the great improvement of the theory of the motion of the heavenly bodies. Including in that the theory is not supplemented by those very general results.

Yet the abyss between the theory of the motion of the heavenly bodies and motion in general remained.

Galileo closed it. His treatises are a vast collection of the particulars. *The two main systems* and *Two new sciences* alone record observations and experiments on: the earth, pendula, a tunnel through the earth, cannonballs, sunspots, the moon, its inhabitants, mirrors (*Systems*, I), Jupiter, its satellites, the Pacific, air, arrows, birds, force, ships, their masts, gravity, parallax (II), stars, their sizes, their distances, clocks, refraction, Mercury, Venus, nebulae, the equinoxes, the solstices, the lodestone, the compass (III), the Mediterranean, the tides (IV), beams, ropes, columns, their strength, their resistance, pumps, balls, glass, gold, gunpowder, floating bodies, light, air, its rarefaction, its compression, its weight, friction, atoms, void, musical instruments, sound, strings (*Sciences*, I), animals, their strength, their proportions, their bones, the balance, the lever, pressure (II), centers of gravity, collisions, impact, trajectories, machines (III), projectiles (IV) – to note only some of the objects of the observations and experiments.

And *The two main systems* and *Two new sciences* follow the true way paradigmatically: ascend from the observations and experiments to

principles, descend from these principles to new observations and experiments, ascend from these observations and experiments to higher principles, descend again, ascend again, and until they reach the law with its clauses.

Here are a few of the steps of the advance from the heavenly bodies to motion in general.

In *The two main systems*:

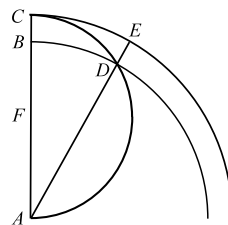
suppose that among the decrees of the divine architect was the thought of creating in the world those globes which we behold continually revolving in the heaven and of establishing an immovable center of their revolution in that place where the sun is; suppose that he created them in the same place, and in that place gave them tendencies to descend toward the center until they have acquired the speeds which seemed good to the divine mind; at what distance from the sun were they created? we take the sizes of the orbits and the periods of revolution; from these we deduce that Jupiter moves faster than Saturn, and it follows that, departing from the same height, Jupiter descended further than Saturn, as it is indeed the case, its orbit being inferior to that of Saturn; next, from the proportion of their speeds and the distance between their orbits, and from the ratio of acceleration of natural motion, we can determine at what distance from the center of their revolutions the place from which they originally departed must have been; hence, we can examine whether Mars, descending from the same place to its orbit, is observed to agree in the size of its orbit and the speed of its motion with what is found by calculation; then the same can be done for the earth, Venus, and Mercury (I);

suppose that in repeated experiments an iron ball of 100 pounds is found to fall from a height of 100 braccia in 5 seconds; since the distances traversed by a falling body are proportional to the squares of the times, the same body will traverse 14 400 braccia in 1 minute, and 51 840 000 braccia, or 17 280 miles, in 1 hour; taking, then, the distance from the orbit of the moon to the center of the earth to be 56 earth radii, or 196 000 miles, the same body will traverse that distance in 3 hours, 22 minutes, and 4 seconds (II);

a ship travels from Venice to Alexandretta; the top of its mast, being further from the center of the earth, makes a longer voyage than the foot of

its mast; and when a man walks, his head travels further than his feet (*id.*);

if the motion of a stone falling straight down along a tower from its top to its base, and thus toward the center of the earth, were uniform, and the circular motion of the earth around its axis is also uniform, the two motions could be compounded into a spiral; but since the motion of the stone is continually accelerated, the line compounded of the two motions must be described by the ever increasing successive distances from the circumference of that circle which would have been described by the center of weight of the stone had it remained on the top of the tower; the line of the compounded motion must thus deviate from the top of the tower at an ever increasing rate; and if AB is the radius of the earth, BC is the height of the tower, the tower is carried by the earth along its circumference BD and describes with its top the arc CE , and if F is the



midpoint between A and C , the center of the earth and the top of the tower, and with center F the semicircle CDA is described, I think it is very probable that a stone dropped from the top of a tower travels along the arc of the semicircle CDA (*id.*).

In book III of the *Two new sciences* Galileo begins:

concerning the oldest object we here set forth the entirely new science; for nothing in nature is older than motion, and about it volumes neither few nor small have been written by the philosophers; yet I find that its essential properties have not even been observed, let alone demonstrated; it has been noted, for instance, that in their natural motion falling bodies continually accelerate, but the proportion of the acceleration has not been found, no one has demonstrated that the distances traveled in equal times by a body descending from rest obey the same ratio as the odd numbers; it has been observed that projectiles describe a curved line, but no one has discovered that this line is a parabola; these and other things neither few nor less worthy of being known I will demonstrate, and thus, what is more important

still, the gate and the road to the largest and most excellent science will be opened, in which these labors of mine will be the elements (*id.*).

Nothing in nature is older than motion. The science of motion is the largest and most excellent. And although it concerns the oldest object, it is with the labors of Galileo entirely new. He opened the gate and the road to it. His labors are its elements. The two discoveries, the ratio which a body descending from rest obeys, and that the line which projectiles describe is a parabola, are but two of the elements. And, the elements of the science of motion, they are more generally the elements of physics, and more generally the elements of the acquisition of knowledge.

Galileo continues: we consider, first, uniform motion, second, naturally accelerated motion, third, violent motion, or the motion of projectiles (*id.*).

The order of the investigation, first uniform motion, then uniformly accelerated motion, then motion compounded of the two, uniform and uniformly accelerated, is another of the elements.

First, with respect to uniform motion, Galileo writes: concerning uniform motion we need a single definition, uniform is motion in which the distances traversed in equal times are equal (*id.*). It is Aristotle's definition (*Physics*, VI, 7). Galileo, however, continues: from the definition four axioms follow, in uniform motion the distance traversed in more time is greater than the distance traversed in less time, the time in which greater distance is traversed is more than the time in which lesser distance is traversed, the distance traversed with greater speed is greater than the distance traversed in the same time with lesser speed, and the speed with which greater distance is traversed is greater than the speed with which in the same time lesser distance is traversed (*id.*).

Distance, time, and speed are functions.

It is another of the elements, the most fundamental, and the richest in consequences.

Galileo proves six theorems concerning uniform motion. The last is: if two bodies are in uniform motion, the ratio of their speeds is the ratio compounded of the distances directly and the times inversely (*id.*).

$$v \propto \frac{s}{t}. \text{ Or, more generally, } v = f(s, t).$$

With Frege, the advance from the second to the third stage in the discovery of functions is the advance from general laws of numbers to the recognition of general laws of functions. The discovery of functions entered the third stage with Galileo. With Frege as well, the third stage is the richest in consequences.

Next, with respect to uniformly accelerated motion, Galileo writes: first of all we need to find the definition which agrees best with nature; and because nature employs acceleration in the descent of heavy bodies, we decided to search in their properties for the definition which agrees with the essence of naturally accelerated motion; we successfully demonstrated the essential properties of these bodies and the demonstrations agreed with experiments; it was as if we were led by the hand to investigate naturally accelerated motion in nature itself which always employs the first, simplest, and easiest means; just as no one of judgment would think that there can be a simpler or easier way of swimming or flying than that employed by fish and birds by natural instinct (*id.*).

Definitions from nature itself: from demonstrations, experiments, and their agreement, all three, is another of the elements.

Galileo continues: thus, this motion is uniformly accelerated which, abandoning rest, adds to itself equal increments of speed in equal times (*id.*).

The definition of uniformly accelerated motion is another of the elements.

Further: I picture to myself a falling body; it departs from rest, and that is, from the deprivation of any speed whatsoever, and begins its motion accelerating from the first instant; time being infinitely divisible, then, there will be no degree of swiftness, however small, or what is the same, no degree of slowness, however great, which the body will not have after its departure from infinite slowness, and that is, from rest (*id.*).

The discovery that, as nature always employs the first, simplest, and easiest means, these have to be investigated in the smallest increments which nature adds to itself is another of the fundamental elements. Hence the development of calculus in the following two-odd generations.

Further: place a heavy body on some yielding

material and let it press on it as much as it can; if you then raise it one or two braccia and let it fall on the same material, it will make a new impression, greater than the impression it made by its weight alone; the effect is caused by the body in conjunction with its speed; the speed of a falling body can thus be measured without error from the quantity and quality of impact (*id.*).

Measurement without error is another of the elements.

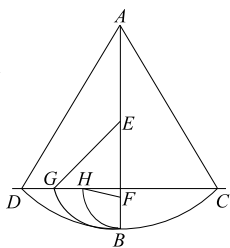
Further: the distinction between natural motion and violent motion is superfluous, and rather null; when you support a rock in your hand, what else are you doing but impressing on it upward impelling force equal to the force of its weight pulling it downward (*id.*)?

Natural motion is redundant. Motion cannot be unnatural, including supernatural. It is the element fatal to angels.

Galileo continues: given the definition of uniformly accelerated motion, we need a single additional principle, that the speeds acquired by the same body over different inclinations of planes are equal whenever the heights of the planes are equal (*id.*); and I want to increase the probability of the principle by experiment so much that it falls little short of equality with necessary demonstration (*id.*).

That experiment must leave no room for doubt in the truth of a principle is another of the elements.

In particular: suppose that this page is a vertical wall; from a nail driven at *A* a lead ball of one or two ounces is suspended by the fine thread *AB*, two or three braccia long; draw on the wall the line *CD* perpendicular to *AB*; then moving the thread *AB* with the ball to *AC*, set it free; the ball will rise very nearly to the line *CD*, prevented from arriving exactly there by the impediment of the air and the thread; we conclude that the impetus acquired by the ball at point *B* in its descent through the arc *CB* is sufficient to impel it to the same height through the similar arc *BD*; having repeated this experiment several times, drive a nail in the wall on the



line *AB* at *E*, or at *F*, so that the thread *AC*, moving through the arc *CB*, when it reaches *B*, is constrained by the nail *E*, or *F*, to travel along the circumference *BG* described about the center *E*, or *BH* described about the center *F*; we see that the same impetus drives the same body to *G*, or to *H*, always rising to the same line *CD*; this experiment leaves no room for doubt in the truth of the principle (*id.*).

Galileo proves the theorem: the time in which a body traverses a distance in uniformly accelerated motion beginning from rest is equal to the time in which the same body traverses the same distance in uniform motion with one half of the final speed in the uniformly accelerated motion (*id.*). Oresme has proved this theorem (*Configuration*, III, 7). From the definition of uniformly accelerated motion and this theorem, however, Galileo proves next this one: if a body descends from rest, the distances it traverses are proportional to the squares of the times in which it traverses them (*id.*).

$s \propto t^2$. The law of falling bodies, or more generally of uniformly accelerated motion, is another of the elements.

The corollary follows: the successive distances traversed in uniformly accelerated motion are to one another as the odd numbers, 1, 3, 5, 7, ..., which is the ratio of the differences by which successive squares exceed one another (*id.*).

The inclined plane experiments follow. As Kant describes them: when Galileo rolled spheres down an inclined plane new light dawned on all investigators of nature. The element is the standard for what an experiment is: repeated hundreds of times, for all inclinations of the plane, with such precision that the times never differ by a notable amount, until complete certainty is established.

Galileo generalizes: the speed which a body has, when all external causes of acceleration or retardation are removed, is by nature impressed indelibly on the body; it follows also that motion in a straight line is eternal, for if it is uniform, it cannot be weakened, much less removed (*id.*).

The element is the law of inertia. Galileo's is a decisive step toward its discovery. The law is formulated in all its generality in Descartes's *Principles of philosophy*, and is the first law of

motion in Newton's *Principles of philosophy*.

Finally, with respect to motion compounded of uniform motion and uniformly accelerated motion, Galileo writes: I generate this kind of motion in the following way; I conceive of a body projected on a horizontal plane, with all impediments removed; clearly, if the plane is infinite, uniform motion on it will be perpetual; but if the plane ends, and the body continues to move beyond the end, it will add to its previous uniform and indelible motion the motion caused by the downward tendency which it has from its own weight; and thus motion compounded of the uniform horizontal motion and the uniformly accelerated downward motion arises (IV).

Further: Apollonius has proved the principal properties of the parabola (*id.*); one of which is that the square on the ordinate is proportional to the abscissa (*id.*). It is the property Khayyam employs to solve cubic equations.

The theorem follows: a projectile in motion compounded of uniform horizontal motion and uniformly accelerated downward motion describes a semiparabolic line (*id.*);

suppose a body is in uniform motion along the horizontal line AB ; at B it lacks support, and from its own weight it adds to its uniform motion a natural motion downward; the straight line AB is produced beyond B , BE being now the flow or the measure of time; the equal parts BC , CD , DE are marked; from B the perpendicular BF is dropped; from points C , D , E lines parallel to BF are drawn; in the first of these a length CG is marked, in the next the length DH four times greater than CG , in the next the length EI nine times greater than CG , and so on, proportional to the squares on BC , BD , BE , and so on; if the body after time BC is at G , after time BD it is at H , after time BE it is at I , and so on; the points G , H , I , and so on, therefore, lie in one and the same parabolic line (*id.*).

The parabola is part of the law of falling bodies. And yet another element is that nature is fond of all conics, not just circles.

Galileo again generalizes: weight, speed, shape are variable in infinitely many ways; to treat such

objects scientifically abstraction is necessary; in order to demonstrate we must abstract from the impediments (*id.*); two balls of equal size, one of which weighs ten or twelve times more than the other, one of lead, the other of oak, for instance, descending from a height of 150 or 200 braccia, arrive at the ground with very little difference in speed; we can conclude without notable error that propositions demonstrated after abstracting from the effect of the medium are absolutely true (*id.*).

The elements are abstraction and, hence, absolute truth.

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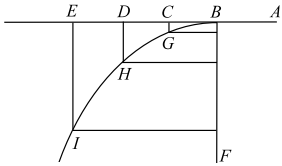
Mathematics Galileo advanced by showing that it is physics.

The labors of Galileo are the most successful synthesis of the successes of all his predecessors.

They are a synthesis of the successes of Aristotle, on one side, and Archimedes and Apollonius, on the other. He brought the power of the mathematical discoveries of Archimedes and Apollonius to bear on the physical problems discovered by Aristotle. And the other way around, too. Concluding his investigation of uniformly accelerated motion, he writes: I marvel not a little that all these matters were left untouched by Archimedes, Apollonius, Euclid, and so many other illustrious mathematicians and philosophers (III). His labors provide the answer. Their labors did not, and his labors did bring the power of the physical problems discovered by Aristotle to bear on the mathematical discoveries of Archimedes, Apollonius, Euclid, and so many other illustrious mathematicians.

His labors are a synthesis of the successes of Aquinas, on one side, and Buridan and Oresme, on the other. Aquinas advanced theology. Buridan and Oresme advanced physics, including the mathematical investigation of physical problems. Galileo brought the power of the mathematical investigation of physical problems to bear on theology. And not, in this case, the other way around. We must begin with the evidence of experience and with necessary demonstrations not only in disputes about problems of nature. Conclusions about nature must be the ground of the true exposition of the holy scriptures, too.

Most important, his labors are a synthesis of



the successes of all his immediate predecessors, Viète, Gilbert, Tycho, and Bruno.

Like Bruno, he greatly advanced enlightenment. Unlike Bruno, he did so on the ground of his great advancement of physics and mathematics.

Like Tycho, he greatly advanced physics by advancing observation. Much more than Tycho, he brought the power of observation to bear on the investigation of the system of the world.

He writes: I have the greatest praise and admiration for Gilbert who framed such a stupendous theory about an object which an infinite number of men of splendid intellect had handled without paying any attention to it; I only wish he were more of a mathematician. Galileo framed a stupendous theory about an object which an infinite number of men of splendid intellect had handled without paying any attention to it. And he was a mathematician *par excellence*.

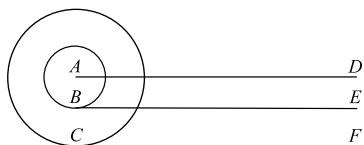
Most important, like Viète, he greatly advanced mathematics. And he did so by greatly advancing physics.

Here are a few of his more purely mathematical results.

Early in the *Two new sciences*, the problem of the constitution of bodies arises. Galileo writes: the smallest particles of metal are joined firmly together, and yet fire goes through them and finally separates them; penetrating through the smallest voids in the metal, through which the smallest particles of air and other fluids cannot pass, the particles of fire free the particles of the metal from the force with which the voids attract the particles and prevent their separation (I).

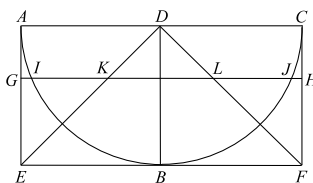
This raises the problem of the number of voids. Galileo writes: it can be demonstrated that it is not impossible that there be infinitely many voids in a finite continuous extension; at the same time an admirable problem posed by Aristotle in the *Mechanics* can be solved (*id.*); draw around the center *A*

two concentric circles; from *A* draw the line *AD*, and form the points *B* and *C* on the circles the tangents *BE* and *CF*, all parallel to one another; suppose the large



circle revolves on the line *CF* equal to its circumference; when the large circle has completed one revolution, what has the small circle done, and what the center? the center has touched the entire line *AD*, and the small circle has measured the entire line *BE*; the small circle has measured a line much longer than its circumference; the line traced by it is equal to the line traced by the large circle (*id.*); if a line is resolved into a finite number of parts, these parts cannot be arranged into an extension greater than that which they have when continuous and joint without the introduction of the same number of void spaces; but if a line is resolved into infinitely many indivisibles, the line can be conceived to expand immensely without the introduction of void spaces, although not without being resolved into infinitely many indivisible voids as well (*id.*).

Moreover: what holds for lines holds for surfaces and solids; draw semicircle *ABC* with center *D*; circumscribe rectangle *AEFC*; draw lines *DE* and *DF*; let the entire figure rotate about the fixed radius *DB*



perpendicular to *AC*; rectangle *AEFC* will describe a cylinder, semicircle *ABC* will describe a hemisphere, triangle *EDF* will describe a cone; suppose the hemisphere removed, leaving the cone and the remainder of the cylinder resembling a bowl; the volumes of the cone and the bowl are equal; suppose a plane parallel to the base of the bowl moves upward, through the line *GH*, for instance; it cuts the bowl at points *G*, *I*, *J*, and *H*, and the cone at points *K* and *L*; it always cuts a part of the cone equal to the part of the bowl whose cross section is represented by the triangles *AGI* and *CJH*; also, the base of the remaining cone is equal to the circular surface which remains from the base of the bowl resembling a ribbon; and from this the marvel follows; as the cutting plane gradually moves toward the line *AC*, the parts of the solids it cuts are always equal, and so are the surfaces which are their bases; as the plane moves further and further, the two solids finally vanish; their equality is maintained to the end; and in the instant of their

ultimate diminution, the ribbon reaches its limit in the circumference of the circle, and the base of the cone is a single point; why should the circumference of the circle and the single point not be called equal (*id.*)?

This raises the problem of infinity. Galileo writes: all numbers, including squares and not squares, are more than the squares alone (*id.*); yet the squares must be as many as all numbers, for the squares are as many as their roots, and all numbers are roots (*id.*); indeed, the number of squares diminishes at an ever increasing ratio, up to 100, $\frac{1}{10}$ of the numbers are squares, up to 10 000, $\frac{1}{100}$ are squares, up to 1 000 000, $\frac{1}{1 000}$ are squares; yet in the infinite number there must be as many squares as are all numbers together (*id.*).

This raises the problem of the speed of light. Galileo writes: does light travel instantaneously? or does it require time? an experiment occurs to me; two men each hold inside a lantern a light which each can conceal and reveal by interposing his hand; facing each other at a distance of a few braccia, they practice revealing and concealing the light from one another so that when either of them sees a light from the other, he at once uncovers his own; when they adjust this signaling so that there is no sensible difference, either of them will reply immediately to the signal from the other; they can then place themselves at a distance of 2 or 3 miles, at night, and resume the experiment, observing carefully whether their responses are the same as when they were close to one another; if they are, they can conclude that light travels instantaneously, for, if it requires time, at a distance of 3 miles, which amounts to 6 miles for the one light to go and the other to come, the interval should be quite noticeable; I have not yet tried the experiment beyond a small distance, less than 1 mile (*id.*).

It is as if we are led by the hand to investigate nature itself.

The physical problems lead to the mathematical problems which lead to the physical problems which lead to the mathematical problems. And where the mathematical problems are concerned, Galileo

anticipates not only calculus, and Newton and Leibniz, and functions, and Euler, he anticipates sets, and Cantor.

Galileo writes: to try to treat questions about nature without geometry is to try to do the impossible (*Systems*, II). And: in the mathematical sciences, geometry and arithmetic, the human intellect has absolute certainty, such as nature itself has; in them the human intellect is equal to the divine in objective certainty; the truth of knowledge by mathematical demonstration is the same as that known by divine wisdom.

Physics without mathematics is impossible. Physics with mathematics has absolute certainty, such as nature itself has, the same as the truth known by divine wisdom. And if physics has attained absolute certainty, such as nature itself has, the same as the truth known by divine wisdom, mathematics cannot not be physics.

It is what Galileo showed – in particular, proposition after proposition after proposition, and hence in general, with absolute certainty, such as nature itself has, the same as the truth known by divine wisdom – that physics without mathematics is impossible, physics with mathematics has absolute certainty, such as nature itself has, the same as the truth known by divine wisdom, and mathematics cannot not be physics.

Hence the subsequent advance of physics such as it is impossible without mathematics and of mathematics such as it is physics.

Hence, also, disapproval of the fact that physics has attained absolute certainty, such as nature itself has, the same as the truth known by divine wisdom, has had to ground itself in disapproval of the fact that mathematics is physics.

Berkeley writes: the true nature of things is not regarded in mathematics. Hegel writes: mathematics is in principle incapable of demonstrating the determinations of physics. And that spirit has been successfully restored to the substantiality and purity of its being nothing attests better than that a mathematician will write: the exact validity of mathematical laws as laws of nature is out of the question.

On the other hand, Kant's result: mathematical propositions are synthetic, is a generalization, most particularly, over the development of physics and mathematics in the one and a half centu-

ries from Galileo to himself. Minkowski writes: the experimental verification of the postulate of the world will reconcile even those, to whom the loss of long established views is unsympathetic or painful, to the thought of preestablished harmony between pure mathematics and physics. And while Minkowski underestimates the determination of those, to whom the loss of long established views is unsympathetic or painful, as Galileo opened the gate and the road to the sciences of mechanics and locomotion, showing the preestablished harmony between mathematics and physics, so the development of these two sciences after him continued to show it, and the development of thermodynamics continued to show it, and of electrodynamics, and of the molecular theory of matter, and of the theory of relativity, and of quantum theory, and of the big bang theory.

Preestablished harmony between mathematics and physics is possible in one way, and one way only. Mathematics is physics.

In general, philosophy is written in the world, and the world is written in mathematics.

cxxiv

Kepler's very important treatises are ten, *Harbinger of cosmographic treatises, containing the cosmographic mystery, on the marvelous proportion of the heavenly spheres, and on the true and proper causes of the number, magnitude, and periods of the heavens, demonstrated by the five regular geometric bodies*, in 1596,

Additions to Witelo, in which the optical part of astronomy is treated, and first of all artificial observation and estimation of the diameters and eclipses of the sun and the moon, with examples of important eclipses, in 1604,

New astronomy αἰτιολογητός, or celestial physics derived from commentaries on the motions of the star of Mars, from the observations of Tycho, in 1609,

Dioptrics, or demonstration of the consequences for seeing and for what is seen of the newly discovered telescope, in 1611,

A new year's gift, or on the hexagonal snowflake, in 1611,

New stereometry of wine casks, especially those

most in shape like the Austrian, and the best use in them of a cubical gauging rod, with a supplement to Archimedean stereometry, in 1615,

Harmonics of the world, in five books, I. geometrical, II. architectonic, III. harmonic, IV. metaphysical, psychological, and astrological, V. astronomical and metaphysical, in 1619,

Epitome of Copernican astronomy, written in the form of questions and answers, in 1618-21,

Rudolphine tables, with which the restoration of astronomical science, long collapsed, is continued, first conceived by that phoenix of astronomers Tycho in 1564, from most accurate observations of the stars especially after 1572, in 1627,

On the rare and marvelous phenomena of the year 1631, incursions of Venus and Mercury in the sun, an admonition to astronomers and students of the things in the heaven, in 1629.

He wrote also the important, if not very important, *On the new star in the foot of Serpenter, book full of disquisitions astronomical, physical, metaphysical, meteorological, and astrological*, ἐνδόξιος and παραδόξιος, in 1606, *A singular phenomenon, or Mercury in the sun*, in 1609, *Conversation with the Starry messenger sent recently to mankind by Galileo*, in 1610, *Report on observations of the four wandering satellites of Jupiter, which Galileo with the right of discoverer named the Medicean stars*, in 1611, *Repeated more exhaustive report that our lord and savior Jesus Christ was born not just one year before the beginning of our present time-reckoning as Röslin, Bünting, and others hold, nor just two years as Scaliger and Calvisius together with many old church writers hold, but full five years; the time of the birth of Christ determined from correct harmony and comparison of heathen and Jewish histories, as well as from agreement with the course of the heaven*, in 1613, *One thousand logarithms of as many round numbers; legitimate premise of the demonstration of their origin and use; a new arithmetic or compendium, of which nothing is more admirable or useful for the solution of nearly all problems of calculation, especially those in the study of triangles, without multiplication*, in 1624, *Supplement to the One thousand logarithms, contain-*

ing precepts on their use, in 1625, *Dream, or on lunar astronomy*, in 1634.

He wrote also *On the more certain foundations of astrology, new brief treatise toward a theory of the world, together with a physical forecast for the year from the birth of Christ 1602*, in 1602, *Forecast for the year from the gracious birth of our lord and savior Jesus Christ 1605, together with a thorough report on the appearance of an unusual new star*, in 1604, *On the star of third magnitude in Swan which was unknown until the year 1605 and has not yet been extinguished*, in 1606, *On the true year of the birth of our savior Jesus Christ, examination of the recent proposal of Suslyga that the customary epoch is four years short*, in 1606, *Exhaustive report on the new bearded star, or comet, which appeared in September and October of 1607, and what it means; together with an entirely new and peculiar yet well grounded discourse on what comets actually are, whence they come, by what their motion is governed, and with which of their shapes they have something to indicate for mankind*, in 1608, *Answer to Dr Röslin's Discourse on the constitution of the present, and how it develops into the future*, in 1609, *Tertius interveniens, or warning to some theologians, physicians, and philosophers, and especially Dr Feselius, that by cheap condemnation of the stargazer's superstition they may throw out the child with the bath-water and thus act inadvertently against their profession*, in 1610, *On the true year in which the eternal son of god assumed human nature in the womb of the blessed virgin Mary*, in 1614, *On comets, in three books, I. astronomical, II. physical, III. astrological*, in 1619, *New ephemerides of the heavenly motions beginning with the year of the common era 1617, from observations mainly of Tycho, physical hypotheses, and the Rudolphine tables*, in 1619-30, *Kanones pueriles, or chronology from Adam to this current year of Christ 1620, divided in six parts and confirmed from the holy scripture of the old and new testaments, with support from heathen antiquities and astronomical calculation*, in 1620, *Astronomical report on two great and peculiar lunar eclipses seen in the past year 1620, and on the great solar eclipse which will occur in the current year 1621 on 11/21 May, with a brief recapitulation of all*

great solar eclipses seen in Europe in eighty years and what in each case the state of spiritual and secular affairs before and after has been, in 1621, *Defense of my Harmonics of the world against the Analytical demonstrations of Dr Fludd*, in 1622, *Discourse on the great conjunction, or meeting, of Saturn and Jupiter in the fiery sign of Lion which occurs in July of the year 1623, as also on the many and multifarious prognostications about this year*, in 1623, *Shield bearer for the Dane Tycho against Chiaramonti's Antitycho*, in 1625, *On the letter of Dr Bartsch about the computation and edition of ephemerides*, in 1629.

CXXV

The true way ascends, then descends, then ascends again, then descends again, then ascends again. The ascent is induction, the descent is deduction. And the successes of Bacon, Galileo, and Kepler, all three, are fundamentally successes of induction. As they had to be. The true way was not altogether untried before them. It was tried partially, haltingly, infrequently, however. It was they who opened it decisively to become after them the well trodden way of the acquisition of knowledge.

The way ascends, descends, ascends, descends, ascends. And the labors of Kepler ascended the highest. They ascended so high, it was not until the labors of Newton and Leibniz, two-odd generations later, that it became clear how high.

The way ascends, ascends, ascends, and until it reaches the law with its clauses. Kepler discovered laws, the laws of the motions of the heavenly bodies. More important still, it is with his labors that it is for the first time clear what the law with its clauses is. Hence, and yet more important, it is with his labors that it is for the first time clear that the world is the system of the world, and hence also, conversely, the system of the world is the world.

To show this, first his labors of discovering the laws can be shown. Then the labors of discovering the law with its clauses before and after him, such as his labors stand on the earlier labors and the later labors stand on his labors. And finally that the world is the system of the world and the system of the world is the world.

The main steps of Kepler's advance are five. He is a master of the title, too, and the titles of the five treatises in which he took the five steps describe the advance perfectly.

The first step is the *Harbinger of cosmographic treatises, containing the cosmographic mystery, on the marvelous proportion of the heavenly spheres, and on the true and proper causes of the number, magnitude, and periods of the heavens, demonstrated by the five regular geometric bodies*, in 1596.

The second step is the *New astronomy αἰτιολογητός, or celestial physics derived from commentaries on the motions of the star of Mars, from the observations of Tycho*, in 1609.

The third step is the *Harmonics of the world, in five books, I. geometrical, II. architectonic, III. harmonic, IV. metaphysical, psychological, and astrological, V. astronomical and metaphysical*, in 1619.

The fourth step is the *Epitome of Copernican astronomy*. It is in seven books. Books I-III appeared in 1618, book IV in 1620, books V-VII in 1621. Book IV is *On the position, order, and motion of the parts of the world, or on the system of the world*, and in it, Kepler notes in a subtitle: *all magnitudes, motions, and proportions in the heaven are explained by natural or archetypal causes*.

The fifth step is the *Rudolphine tables, with which the restoration of astronomical science, long collapsed, is continued, first conceived by that phoenix of astronomers Tycho in 1564, from most accurate observations of the stars especially after 1572*, in 1627.

In 1596 already Kepler searches for the true and proper causes, and has found them. Among the true and proper causes are the five regular geometric bodies. By 1609 he has found the two true and proper causes, too, which are the first and second laws of the motions of the heavenly bodies. Astronomy, accordingly, is new, and the new astronomy is a science of causes. By 1619 he has found the third law of the motions of the heavenly bodies. In 1620 the true and proper causes are natural or archetypal causes. Among the natural or archetypal causes are the laws.

In 1596 already Kepler has solved the cosmographic mystery. By 1609 he has solved it better

yet, and a science of causes now, astronomy is celestial physics. Hence, in 1619 he describes the harmony of the world. In 1620 he describes the system of the world.

In 1596, that the mystery is solved, the marvelous proportion of the number, magnitude, and periods of the heavens shows. In 1620, that the mystery is solved completely and conclusively, the explanation of all magnitudes, motions, and proportions in the heaven by natural or archetypal causes shows.

Through 1621 Kepler's labors are fundamentally inductive, resting essentially on the observations of that phoenix of astronomers Tycho. In 1627 his labors are for the first time fundamentally deductive. And with the tables deduced from the natural or archetypal causes the restoration of astronomical science, long collapsed, is continued.

The most important of the five treatises, and steps in the advance, are the first two, *The cosmographic mystery* and the *New astronomy*.

Both follow the true way. *The cosmographic mystery* follows it hastily, eager to reach the most general principles. The *New astronomy* follows it with stately deliberation. It verses itself duly and orderly in the particulars, ascends, descends, again, and again, rising gradually to what is general by nature, and all the way to the laws.

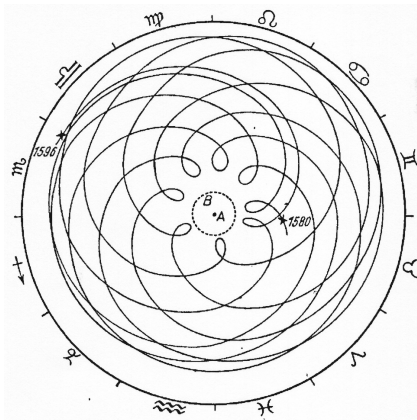
Twenty-five years after its publication Kepler reissued *The cosmographic mystery*, and in the dedication introducing it judges it: no apprentice piece was ever more admirable, more felicitous, or dealt with worthier matter; nor was this due merely to my ingenuity, it was as if an oracle from heaven dictated it to my pen; every chapter was recognized at once by those who understood it as important and true (*id.*); they have in the years since, too, more than once carried a torch before me (*id.*).

The apprentice piece is the most admirable, the most felicitous, and deals with the worthiest matter, because the oracles who dictated it to Kepler's pen were Copernicus, god, Aristotle, and Euclid jointly. Or, to put it in terms of torches, Kepler followed the torchbearers who would only all four jointly lead him to the solution of the cosmographic mystery. Not Copernicus. Not Copernicus and god. Not even Coperni-

cus, god, and Aristotle. Copernicus, god, Aristotle, and Euclid.

The mystery, in 1596, begins with an argument that the earth revolves around the sun. Kepler concludes: no religion will prevent me from listening to Copernicus, if what he says is consistent; my trust in him was first established by the beautiful agreement of what he says with everything observed in the heaven; even more important, though, the things, at which others teach us to marvel, Copernicus alone beautifully explains, removing the cause of the marvel, which is the ignorance of the causes (1).

In 1609, the *New astronomy* begins with a description of the phenomena. Kepler has this picture:



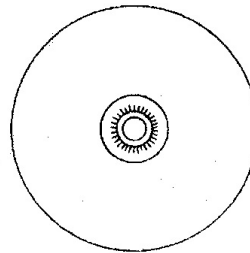
He explains: this would be an accurate depiction of the motion of Mars from 1580 to 1596, if it were true that the earth is at rest, as Ptolemy and Tycho would have it; continued further, the motion would become utterly perplexing, because the path is infinite and never the same; notice also that, as the orbit of Mars would require a vast space, the spheres of the sun, Venus, Mercury, the moon, fire, air, water, and earth would all have to be enclosed in the tiny space *B* around the earth *A*; and similar paths would have to be ascribed to the other planets (1).

Powerful as it is, it is not a conclusive argument. Galileo, in the following decade, will supply the conclusive arguments. Kepler, however, has already begun drawing the consequences of the fact that the earth revolves around the sun. The fact that the earth revolves around the sun is in Kepler from the first to the last the

ground of induction.

As Copernicus is the ground of induction so is god. Kepler writes in *The mystery*: of three things I kept seeking the causes why they are thus and not otherwise, the number, the size, and the motion of the orbits; and I dared to do so because of the quintessential harmony of the sun, the fixed stars, and the space between them with god the father, the son, and the holy spirit (pr.). More particularly: the image of god the triune is in the sphere, of the father in the center, of the son in the surface, of the holy spirit in the equality of the relation between the point in the center and the circumference (2).

And as Copernicus is the ground of induction from the first to the last so is god. In 1620, in the *Epitome*, Kepler asks: which are the principal parts of the world (IV, 1.1)? and replies: in the sphere, which is the image of god the creator and archetype of the world, there are three regions, and they are the sign of the trinity; the sun in the center is the sign of the father, the fixed stars on the surface are the sign of the son, the planetary system between the sun and the fixed stars is the sign of the spirit (*id.*), and he illustrates, too:



Descartes, in the next generation, will conclude that: nature is nothing other than god himself (*Meditations*, 6). Spinoza, in the generation after that, will conclude that, conversely: god is nature (*Ethics*, I, 29). Galileo and Kepler have each taken a major step toward this conclusion. In Kepler, god is the archetype of the world. He is the quintessential harmony. He is the principal parts of the world. Copernicus, moreover, shows where he is. Copernicus shows where in the world the father is, and hence where the son is, and where the holy spirit.

The relation of Copernicus and god is mediated by Aristotle. Kepler writes in *The mystery*: Copernicus ascribes motion to the earth by math-

emational arguments, I do so by physical, or rather metaphysical (pr.). The physical, or rather metaphysical, arguments rest on Aristotle.

Kepler writes: nature loves simplicity, she loves unity (1); and everything that Copernicus demonstrates *a posteriori* and from observation can be demonstrated *a priori* and in accordance with Aristotle (*id.*). In particular: that the whole world is enclosed in a sphere Aristotle has shown abundantly (2); the circular motion of the stars argues for the circularity as well of the inner orbits (*id.*); there are three kinds of quantity in the world, shape, number, and extension (*id.*); we have found circularity only in the shape (*id.*); the other causes must therefore be sought in the straight, lest we had to suppose that god made anything in the world at random, of which no one will ever persuade me (*id.*).

The earth revolves around the sun. There is quintessential harmony. The whole world is enclosed in a sphere and the inner orbits are circular as well.

We seek the causes. And Euclid supplies them.

Kepler writes: behold, reader, my discovery (pr.); Euclid demonstrates that the regular bodies are five (*id.*), hence: the sphere of the earth is the measure of all; circumscribe around it a dodecahedron; the sphere comprehending the dodecahedron will be that of Mars; circumscribe around this sphere a tetrahedron; the sphere comprehending the tetrahedron will be that of Jupiter; circumscribe around it a cube; the sphere comprehending the cube will be that of Saturn; in the sphere of the earth inscribe an icosahedron; the sphere which the icosahedron comprehends will be that of Venus; inscribe in it an octahedron; the sphere which the octahedron comprehends will be that of Mercury; and you have the explanation of the number of the planets (*id.*).

Euclid demonstrates that: any solid angle is contained by plane angles whose sum is less than four right angles (*Elements*, XI, 21). It follows that: 3 equilateral and equiangular triangles can contain a solid angle, 4 such triangles can, and 5 such triangles can, but 6 or more cannot, 3 squares can, but 4 or more cannot, 3 equilateral and equiangular pentagons can, but 4 or more cannot, and no other equilateral and equiangular polygons, 6-sided or more, can (XIII, 18). The

tetrahedron is the body whose solid angles are contained by 3 regular triangles, and is enclosed by 4 such triangles. The octahedron is the body whose solid angles are contained by 4 regular triangles, and is enclosed by 8 such triangles. The icosahedron is the body whose solid angles are contained by 5 regular triangles, and is enclosed by 20 such triangles. The cube is the body whose solid angles are contained by squares, and is enclosed by 6 squares. The dodecahedron is the body whose solid angles are contained by regular pentagons, and is enclosed by 12 such pentagons.

Kepler writes: if we reject all irregular bodies, and retain only those whose sides are equilateral and equiangular, only five remain, the cube, the tetrahedron, the dodecahedron, the icosahedron, and the octahedron (2); it is impossible that there be more than these five (*id.*); Euclid demonstrates also how these bodies are suited by their nature to be inscribed in a sphere and circumscribed about a sphere (*id.*); in Copernicus there are six planetary spheres (*id.*); the paths of the planets are eccentric; and the physicists agree that each of their spheres is of such thickness as the demonstration of the motions of the planet requires (14); the spheres do not touch one another, but there are great spaces between them; insert, then, in the spaces between the spheres of the planets the five regular bodies in their true proportions, and let astronomy be the judge, while Copernicus pleads for us (*id.*).

Calculation, in fact, pleads even more felicitously. Kepler reports: I calculated day and night (pr.).

The calculations are three. All three compare results of Copernicus from observation in *On the revolutions* with results from the inscription and circumscription of the five regular bodies in and about the spheres, beginning with the sphere of the earth.

First, Kepler calculates the ratios of the radius of the inner sphere of each planet to the radius of the outer sphere of the next. On the left are the ratios from book V of *On the revolutions*, on the right those from the five regular bodies:

Saturn to Jupiter	.635	.577,
Jupiter to Mars	.333	.333,
Mars to earth	.757	.795,
earth to Venus	.794	.795,

Venus to Mercury .723 .577 or .707 (14). The last number is .577 if the ratio is to the radius of the sphere inscribed in the octahedron, .707 if it is to the radius of the circle inscribed in the square of the octahedron. Kepler concludes: the numbers are close, in two cases the same (*id.*); and you can see how greatly unequal they would have been, if the attempt were contrary to nature, and that is, if god himself in creation did not look to these proportions (*id.*).

Next, Kepler corrects an error in Copernicus. He writes: Copernicus established that the sun is the center of the world, yet calculated the distances of the planets from the center of the earth's orbit, as if it were the center of the world, and even as it is at a distance from the sun depending on the eccentricity of the orbit (15).

He then calculates the least and greatest distances of the planets, with the mean distance of the earth from the center of its orbit as the unit, from Copernicus, corrected with respect to the center of the world, and from the five regular bodies:

Saturn	9.987	10.599,
	8.342	8.852,
Jupiter	5.492	5.111,
	4.999	4.652,
Mars	1.649	1.551,
	1.393	1.311,
earth	1.042	1.042,
	.958	.958,
Venus	.741	.761,
	.696	.715,
Mercury	.489	.506,
	.233	.233 (<i>id.</i>).

The agreement is better still. As to the remaining disagreement, it is not only clear how it will be eliminated: the observations are rough, inaccurate often by as much as half a degree (18); and the eccentricities are suspect, borrowed as they are from Ptolemy (*id.*). It is clear who will eliminate it: I do not despair of victory (*id.*).

The third calculation is of the periods of the planets. Kepler writes: nothing agrees with reason better, as Aristotle also says in *On the heaven*, than that the motions of the planets should depend on their distances (20). And the most powerful argument in *The mystery* follows. Kepler writes: either the moving souls of the planets are

weaker the further away from the sun they are, or the single moving soul in the center, the sun, impels each body the more strongly the closer it is (*id.*); suppose, then, as is highly probable, that motion is dispensed by the sun like light; as it spreads out from a center light weakens proportionally (*id.*); the measure of the weakening, then, must be sought in the ratio of the widening circles; in proportion as an orbit is wider so the motion of the narrower orbit is stronger; but as an orbit is wider, it also requires more time to traverse; one increase in the distance of a planet from the sun, then, acts twice in the increase of its period; one half of the decrease from a greater to a lesser period, then, should show the true ratio of the distances (*id.*); for instance, the period of Mercury is about 88 days, that of Venus is about $224 \frac{2}{3}$ days; the difference is $136 \frac{2}{3}$; one half of this is $68 \frac{1}{3}$; adding this to 88 gives $156 \frac{1}{3}$; and as 88 is to $156 \frac{1}{3}$, so the mean distance of Mercury should be to the mean distance of Venus (*id.*).

If the periods are T_1 and T_2 , and the mean distances are r_1 and r_2 ,

$$\frac{T_2 - T_1}{T_1} = 2 \frac{r_2 - r_1}{r_1},$$

or equivalently,

$$\frac{2 T_1}{T_1 + T_2} = \frac{r_1}{r_2}.$$

The ratios of the mean distances from Copernicus and from the five regular bodies are:

Saturn to Jupiter	.572	.574,
Jupiter to Mars	.290	.274,
Mars to earth	.658	.694,
earth to Venus	.719	.762,
Venus to Mercury	.500	.563 (<i>id.</i>),

and Kepler concludes: you see that we have come closer still to truth (*id.*).

So you have, in *The cosmographic mystery*, the solution of the cosmographic mystery.

Not only you have, for the first time, an explanation of the number, distances, and periods of the planets. Not only is the agreement of observation and calculation within one tenth of the order of magnitude. Not only is it clear how the remain-

ing disagreement will be eliminated.

The explanation is by true and proper causes.

Not only is the sun the center of the world. It is the physical center of the world. The single moving soul in the center, the sun, dispenses motion to the planets. Nor is the sun only the physical center of the world. It is the vastly improved mathematical center of the world. The sun is the single center of the circumscription of the five regular bodies about one another. Nor is the world just physical and mathematical. The world is physical by being mathematical. The sun dispenses motion in proportion to distance and to time, and the measure of the proportion must be sought, and is found, in the ratio of the orbits.

There is, in general, preestablished harmony of physics and mathematics. Kepler writes: body is what god created in the beginning; and it is clear to whoever has its definition why god created body and not something else; what god proposed, I say, is quantity; he needed everything that pertains to the essence of body, and quantity is the form of body which makes it body, and the origin of its definition (2).

Quantity is the form of body. God, therefore, created body.

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The advance in *The cosmographic mystery* is very great.

Nevertheless, thirteen years later, in 1609, in the *New astronomy*, three new oracles have joined the earlier four to dictate to Kepler's pen. Tycho, Gilbert, and Apollonius. The observations are no longer rough. What the single moving soul in the center dispenses is magnetic force. And the orbits are elliptical. The advance is accordingly greater yet, and all the way to the laws.

Kepler writes in the introduction: my goal in this treatise is to improve astronomy, so that the calculations from tables correspond to the heavenly phenomena; in August 1608 Mars was 4° beyond the position calculated from the old tables; my new calculation removes this error entirely (*id.*); to achieve this goal, all things are so interconnected with all things, that no other way can succeed but grounding the motions of the planets in their physical causes; and this is what I establish with this treatise (*id.*).

He continues: the first step is the demonstration that the eccentricities of all six planets intersect in a single point, the center of the sun (*id.*).

The sun is at rest, the earth moves, and Kepler offers at some length: remedies for the objections agitating souls and extinguishing the light (*id.*). He concludes with: advice for astronomers (*id.*), and: advice for idiots (*id.*). For astronomers: I implore my reader not only to extol the firmness and stability of the earth as a gift of god, but to acknowledge his wisdom in its motion as well (*id.*). For idiots: whoever is too dull to understand astronomy, or too infirm in his piety to trust Copernicus, I urge him, once he has dismissed the study of astronomy and damned whatever philosophical opinions he pleases, to keep to himself and, leaving peregrination in the world alone, go home to scratch his own little patch of dirt (*id.*).

He continues: the second step is the demonstration that the equant is essential to the theory of the earth (*id.*); the third step is the demonstration that the eccentricity of the equant of Mars has to be bisected precisely (*id.*); the fourth is the step toward the physical hypotheses; I have discovered that the path of a planet is not a circle, but a perfect ellipse; then the crown is put on the edifice; with the highest probability the proper movers of the planets are nothing but the properties of the planetary bodies themselves; all causes of the heavenly motions are purely bodily, in particular, magnetic; the only exception is the rotation of the body of the sun as it remains in the same place, which is apparently the effect of a living force (*id.*).

The goal is to improve astronomy so that calculation corresponds to the phenomena. Calculation from the old tables results in an error of 4° . Having reduced the error to $8'$, one thirtieth of the earlier error, Kepler notes: even if I thought that these $8'$ could be ignored, I would have made enough of a correction (19); but these $8'$ could not be ignored (*id.*). And his calculation removes the error entirely.

In *The mystery* the observations are rough. In the *New astronomy* they are not. In the *New astronomy* the theory of the motions of the heavenly bodies rests on vastly improved precision of

observation, and for the first time on a continuous series of observation, those of Tycho from 1580 to 1600. As the accurate depiction of what the motion of Mars from 1580 to 1596 would have been, if it were true that the earth is at rest, illustrates.

All things are interconnected with all things, and no other way can succeed but grounding the motions of the planets in their physical causes.

The first step is the demonstration that the eccentricities of the orbits of all planets intersect in the center of the sun. The demonstration is on the assumption that the orbits are circular, as they are in both Ptolemy and Copernicus, and either on the assumption that there is an equant point on the line of apsides or on the assumption that there is no equant point. The line of apsides connects in Ptolemy apogee and perigee. It connects in Copernicus aphelion and perihelion. Equant is in Ptolemy the point, other than either the center of the ecliptic or the center of the circular motion of the planet, with respect to which the motion of the planet is uniform. Copernicus rejects the equant. In the course of the demonstration Kepler calculates the eccentricity of Mars on the assumption that there is an equant at .09282 (16), on the assumption that the orbit is a perfect circle at .11332 (19), and concludes: either the assumption that the orbit is a perfect circle or the assumption that there is an equant is false, or both assumptions are false (*id.*). On the assumption that there is an equant, however, the disagreement of calculation and observation is never more than $8'$. Kepler concludes: it is clear why Ptolemy was satisfied with a fixed equant; the greatest error is $8'$, and this in Mars, whose eccentricity is the greatest of all planets (*id.*); nevertheless, since divine benevolence has given us Tycho, whose observations show this error of $8'$ in the Ptolemaic calculation, it behooves us to acknowledge and honor this favor of god (*id.*); and because they could not be ignored, these $8'$ alone have led to the restoration of all astronomy (*id.*).

Kepler notes: filled with audacity, we return to the battlefield (25).

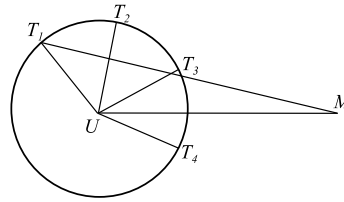
The second step is the demonstration that the equant is essential to the theory of the earth. The orbit of the earth is a perfect circle in Copernicus. This cannot be plausible on general grounds, if

the earth is a planet and the orbits of the other planets are eccentric. Kepler proves it:

I will take observations of Mars in the same position with respect to its line of apsides, and will find by trigonometry the distance of the earth from the center of its uniform motion;

Mars was observed in the same position on 5 Mar 1590 at 7:10 in the evening, on 21 Jan 1592 at 6:41, on 8 Dec 1593 at 6:12, and on 26 Oct 1595 at 5:44;

let Mars be at M ;



let U be the point about which the motion of the earth is uniform;

let at the times when Mars is at M , the earth be at T_1 in 1590, at T_2 in 1592, at T_3 in 1593, and at T_4 in 1595;

as the periodic times of Mars from one observation to the next are equal, the times of revolution of the earth about the center of its uniform motion from one position to the next are equal, and angles T_1UT_2 , T_2UT_3 , and T_3UT_4 are equal (*id.*).

Given its line of apsides and eccentricity, the observed position of Mars gives its position with respect to the center of its uniform motion. Given the line of apsides of the sun and its eccentricity, the observed position of the sun gives the position of the earth with respect to the center of its uniform motion. As Ptolemy has shown, and in both cases the center of uniform motion is given whether the sun revolves around the earth or the earth around the sun. And if the center of the two uniform motions is the same, the positions of Mars and the earth with respect to the center give the angle between the radii connecting them to the center. Kepler continues:

from Tycho's observations, angle T_1UM is $127^\circ 5' 1''$;

in its observation in 1590 Mars is at $25^\circ 6'$ Aries, and this is the direction of T_1M ; the direction of UM is toward $15^\circ 53' 45''$ Taurus; angle UMT_1 is thus $20^\circ 47' 45''$; and angle UT_1M is

$$32^{\circ} 7' 14'';$$

$$\text{but } \frac{\sin UT_1 M}{UM} = \frac{\sin UMT_1}{UT_1};$$

and if $UM = 1$, $UT_1 = .66774$ (*id.*).

Kepler writes: if UT_2 , UT_3 , and UT_4 are equal to UT_1 , my suspicion that the orbit of the earth is eccentric would be false, if they are different, my triumph will be complete (*id.*). He calculates UT_2 , UT_3 , and UT_4 in the same way, writes: I present the four lines for inspection,

$$\begin{array}{ll} UT_1 & .66774, \\ UT_2 & .67467, \\ UT_3 & .67794, \\ UT_4 & .67478 \text{ (} id. \text{),} \end{array}$$

and concludes: the circle, therefore, which Copernicus draws about the center U of uniform motion of the earth is not the path of the earth; there is instead another circle which is the path of the earth; and since the line UT_3 is the longest, the center of this circle is towards T_3 , and that is, in the direction of the sun (*id.*).

The eccentricity of the orbit of the earth is
in Kepler .018 (30),
correction .017.

The third step is the demonstration that the eccentricity of the equant of Mars has to be bisected precisely. The eccentricity of the orbit of Mars is

in Kepler .093 (42),
correction no correction.

The fourth is the step toward the physical hypotheses. The path of a planet is not a circle, but a perfect ellipse. The proper movers of the planets are nothing but the properties of the planetary bodies themselves. All causes of the heavenly motions are purely bodily, in particular, magnetic. The only exception is the rotation of the sun on its axis, which is apparently the effect of a living force.

In particular, the causes are both, that the orbits are ellipses and that the force is magnetic, and both causes are physical, purely bodily, nothing but properties of the bodies themselves.

Quantity is the form of body. God, therefore, created body. In particular, he created body magnetic and revolving in ellipses.

Kepler has shown that the eccentricities of all planets intersect in the center of the sun. It follows: the force which moves the planets resides

in the sun (33). Moreover: the body of the sun is magnetic, and it rotates in space (34); Gilbert has demonstrated that the earth is a magnet, and rotates once a day (*id.*); I presume the same thing about the sun (*id.*).

He has shown that the orbits cannot be circles. The equant is essential to the theory of all planets. He has measured the eccentricities. Apollonius has become one of his oracles (59). It follows: the orbits are ellipses (47).

And the second half of the treatise is the labor of showing that the orbits are ellipses, that the force is magnetic, and that, if the sun and the planets are magnetic bodies, each with poles attracting and repulsing one another, then depending on their distance and orientation with respect to one another as each rotates on its axis, the gradually increasing and decreasing attraction and repulsion can result in no other orbits but ellipses.

Descartes, in the next generation, will improve the hypothesis of magnetic force into the hypothesis of vortices. Newton, not quite two generations after that, will improve the hypothesis of vortices into the hypothesis of gravity.

The rest is conclusive.

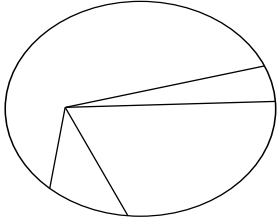
No other way can succeed but grounding the motions in their physical causes. The movers are nothing but the properties of the bodies themselves. All causes are purely bodily. Nor can the rotation of the sun on its axis be an exception. Kepler writes: a conclusion is finally made in favor of nature, repudiating mind (57).

The orbit is the result of the gradual increase and decrease of the force depending on distance.

And Kepler concludes by formulating the two causes: a planet approaches the sun and recedes from it in an elliptical orbit; and it spends time at each point in proportion to the distance of the point from the sun (60).

With respect to the first, he has shown that the sun is in one of the foci of the ellipse (59). He will thus formulate it in the *Epitome*: the orbits of the planets are ellipses with the sun in one of the foci (V, 1.3). With respect to the second, he has shown that the measure of the time is the area enclosed between an arc traversed by a planet and the radii connecting it to the center of uniform motion (40), and that: the perfect measure of the

distances of unequal arcs from the sun is the area of the ellipse (59). He will thus formulate it in the *Epitome*: the areas defined by the arcs traversed by a planet in equal times and the respective radii to the sun are equal (1.4).



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With *The mystery* and the *New astronomy*, then, Kepler established that what god created in the beginning is body. Because he thus proposed quantity. He needed everything that pertains to the essence of body, and quantity is the form of body.

These are the most fundamental results on which the subsequent acquisition of knowledge rests. That what god created is at issue. That he created body. And because quantity is its form.

God had long been preoccupied with punishments, rewards, promises, deliverance, more promises, more punishments, more rewards, salvation, more punishments, more rewards. Indulging in philosophy, he had been an unabashed ethicist. Three centuries before the Christian centuries Ariston of Chios had said: physics is above us, logic is nothing to us, ethics alone is about us. And along with the vast majority of philosophers since, god had been this kind of philosopher. He had engaged in some perfunctory physics in creation, then had focused unwaveringly on good and bad, noxious and beneficial.

Kepler proposed that he is a physicist.

Galileo proposed to engage him, the architect, in experiments: suppose that the divine architect created an immovable center of revolution in the sun, and created the planets in the same place and in that place gave them tendencies to descend toward the center until they have acquired the speeds which seemed good to him; at what distance from the sun were they created?

But god did not want to be a physicist. Along with all the philosophers to whom physics is

above us, logic is nothing to us, ethics alone is about us, he did not even want to be a Copernican, never mind a Galilean and a Keplerian.

Descartes, in the next generation, proposed what may have appeared a compromise. He writes in *The method*: in order to be free to say what I judged, without being obliged either to follow or to refute the opinions received among the learned, I resolved to leave this world entirely to their disputes, and to speak solely of what will happen in a new one, if, in some imaginary space, god created enough matter to compose it, agitated variously and without order the different parts of this matter to compose a chaos more confused than the poets imagine, and after that did no other thing but give his ordinary concurrence to nature and let it act by the laws he established (5).

It may have appeared a compromise. But it is not a compromise. Descartes continues: there could be nothing in the substance, position, motions, and all the different qualities in the heaven, stars, and earth of this world which must not be like those of the world I described (*id.*). The world Descartes described must be like this world, and this world must be like the world Descartes described. Never mind the opinions received among the learned. And god did no other thing but create matter and let it act by laws.

Nor could there be a compromise. Nor could god be left alone. What god created could not not be at issue. Not if the description of the world was at issue.

Whether he wanted to be a physicist or not, then, god had to be a physicist. He had to create body. For the movers are nothing but the properties of the bodies themselves. All causes are purely bodily.

He had to be a mathematician, too. For quantity is the form of body. There is preestablished harmony of physics and mathematics. The world is physical by being mathematical. It is written in mathematics.

Conversely, physics and mathematics cannot not explain all of god's creation.

The philosophers – if not those to whom ethics alone is about us – have continued to labor at drawing all the consequences of this fact.

And no treatise has been more valiant in the attempt to draw them all than the *Harmonics of*

the world. In the *Harmonics*, in 1619, Kepler relates, grounds, corrects, expands, generalizes, and particularizes the results of *The mystery* and the *New astronomy*.

The *Harmonics* is in five books, and its full title describes the object of each book: *the first is geometrical, on the origin and construction of the regular figures; the second is architectonic, on the congruence of the regular figures in the plane and in the solid; the third is properly harmonic, on the origin of the harmonic proportions in the figures; the fourth is metaphysical, psychological, and astrological, on the mental essence of the harmonies and on their kinds in the world, and especially on the harmony of the rays descending from the heavenly bodies to the earth and their effects on nature, or on the sublunary and human soul; the fifth is astronomical and metaphysical, on the most perfect harmonies of the heavenly motions, and the origin of the eccentricities in the harmonic proportions.*

In the *Harmonics*, like Viète earlier and Descartes later, Kepler treats geometrical problems arithmetically and arithmetical problems geometrically. In the first two books he establishes the fundamental character of the five regular solids. Along the way he formulates and solves packing problems: to fill the plane perfectly with regular not overlapping polygons (II, 18-21). In book III, the properly harmonic, with lesser esthetic subtlety than Zarlino, but with greater mathematical subtlety, he builds on Ptolemy's *Harmonics*, and deals with the consonances (1-3), the intervals (4-5), melody (5), the octave (7-8), the scale (9), the tetrachords (10), the modes (14), the relation of melody and harmony (16). In an appendix he applies the results. He writes, for instance: in a splendid passage, Bodin compares the democratic constitution to arithmetical proportion, the aristocratic constitution to geometrical proportion, the monarchic constitution to harmonic proportion (*id.*), and again provides the mathematical subtlety. Another instance is: the law of banquets (*id.*). He writes: there would be an absurd arithmetical equality if everybody is seated indiscriminately; mere geometrical arrangement is insipid, for if the learned are seated next to the learned, what good will they be to the unenlightened? if women are seated next to women, what pleasure

will there be? the seating should be harmonic (*id.*). In book IV, dealing with the harmony of the rays descending from the heavenly bodies, he deals with things more salutary than most of the things that had tended to descend from heaven, and in a much more salutary way.

In book V he collects the fundamental results and relates them to one another. Hence, one further result is: Saturn and Jupiter are the bass, Mars is the tenor, the earth and Venus are the alto, Mercury is the soprano (8). Clearly, Neptune and Uranus are the basso profundo, the asteroid belt completes the tenor. The most beautiful voice cannot be double.

Another is the third law. Discovered, Kepler reports: by continuous toil for a very long time (3).

The very long time is, all in all, twenty-two centuries. The toil becomes continuous, however, with Kepler.

Anaximander, the second philosopher, first discussed the distances of the planets (Simplicius, comm. *Heaven*, 471).

Not quite two and a half centuries later, Aristotle writes: the motions of the planets should depend on their distances (*Heaven*, II, 10).

Nineteen and a half centuries later, Kepler showed that they do. The discovery is in four steps.

In 1596, in *The mystery*, he obtains the result,

$$\frac{2T_1}{T_1 + T_2} = \frac{r_1}{r_2},$$

where T_1 and T_2 are the periods of the planets, r_1 and r_2 their mean distances. The agreement of observation and calculation is within one tenth of the order of magnitude.

Thirteen years later, in 1609, in the *New astronomy* (39), he obtains the result,

$$\frac{T_1}{T_2} = \frac{r_1^2}{r_2^2}.$$

Ten years later, in 1619, in the *Harmonics* (V, 3), the result is the third law,

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}.$$

Two generations later, in 1687, in *The principles*, in book I, Newton deduces the third law from the more general laws of motion: the peri-

ods are proportional to the $\frac{3}{2}$ powers of the major axes of the orbits (15). Not of the mean distances.

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3},$$

where a_1 and a_2 are the major axes.

As to the agreement of observation and calculation, in book III, *The system of the world*, Newton begins with four rules and six phenomena. Phenomenon 4 is the third law. Newton notes: this proportion, found by Kepler, is accepted by everyone (*id.*), and lists: the mean distances of the planets, as found by Kepler from observation and as calculated from the periods of the planets,

Saturn	9.51000	9.54000,
Jupiter	5.19650	5.20096,
Mars	1.52350	1.52369,
earth	1.00000	1.00000,
Venus	.72400	.72333,
Mercury	.38806	.38710 (<i>id.</i>).

If *The mystery*, the *New astronomy*, and the *Harmonics* suffice to show the superabundance of Kepler's results, there are yet also his many other results in the many other important and very important treatises he wrote in the meantime. Among them are the following.

Additions to Witelo, in which the optical part of astronomy is treated, and first of all artificial observation and estimation of the diameters and eclipses of the sun and the moon, with examples of important eclipses, in 1604. In this is the first calculation of the dependence of the angle of refraction on the medium (4.6). The first argument also that all conics, circle, ellipse, parabola, hyperbola, as well as the straight line, are continuously transformed into one another (4.4). On hearing of Galileo's telescopic discoveries, he wrote the *Dioptrics, or demonstration of the consequences for seeing and for what is seen of the newly discovered telescope*, in 1611. This is the first demonstration of the formation of the optical image in the telescope.

A new year's gift, or on the hexagonal snowflake, in 1611. In this he investigates why snowflakes are hexagonal, why honeycombs are hexagonal, why most flowers have five petals, why the shells of snails are spiral, why ores are crys-

talline. He solves packing problems. It is the first step in the development of crystallography.

New stereometry of wine casks, especially those most in shape like the Austrian, and the best use in them of a cubical gauging rod, with a supplement to Archimedean stereometry, in 1615. In this he treats curvilinear areas and volumes as sums of infinite number of infinitesimal elements (*passim*). It is a major step toward the development of calculus.

He labored assiduously at fixing the date of the incarnation. He wrote *On the true year of the birth of our savior Jesus Christ, examination of the recent proposal of Suslyga that the customary epoch is four years short*, in 1606. Then the *Repeated more exhaustive report that our lord and savior Jesus Christ was born not just one year before the beginning of our present time-reckoning as Röslin, Bünting, and others hold, nor just two years as Scaliger and Calvisius together with many old church writers hold, but full five years; the time of the birth of Christ determined from correct harmony and comparison of heathen and Jewish histories, as well as from agreement with the course of the heaven*, in 1613. Then *On the true year in which the eternal son of god assumed human nature in the womb of the blessed virgin Mary*, in 1614.

He wrote the *Dream, or on lunar astronomy*, published posthumously in 1634. This is the record of a voyage to the moon, told by a demon. The demon begins: we admit no one lethargic, fat, or tender; we prefer those good at horsemanship or those who have often sailed to the Indies and have become inured to subsisting on garlic, dried fish, and poor victuals; we especially like shriveled old women; we take no Germans; but we do not mind the firm bodies of the Spaniards (*id.*). The demon describes the motions of the heavenly bodies as seen from the moon, the surface of the moon, and then the inhabitants of the moon. They are: of monstrous size, rapid growth, and short life, because they have to develop bodies of such an enormous size (*id.*). Another of their peculiarities is: to some of them happens the opposite of what happens to flies among us, they die during the day because of the heat and come back to life at night (*id.*).

The *Epitome of Copernican astronomy*, in

1618-21, is in seven books. Books I-III are introductory, book I *On the principles of astronomy in general and the doctrine of the sphere in particular*, book II *On the sphere and its circles*, book III *On the doctrine of the first motion*. Book VII is *On the doctrine of the sphere and the theory jointly*. The more important books IV-VI Kepler describes thus: IV is the theory, V is the organon, VI is the practice; IV is physical, V is geometrical, VI is properly astronomical (V, 1). Book VI is *On the apparent motions of the planets*. Book V has no title, it continues the labors of book IV.

The most important, book IV – the theory, the physical – is *On the position, order, and motion of the parts of the world, or on the system of the world*. And in it, Kepler reports: *all magnitudes, motions, and proportions in the heaven are explained by natural or archetypal causes*.

The great importance of book IV of the *Epitome* is twofold. Kepler has been searching for the true and proper causes, and finding them, since *The mystery*. Three-odd decades after *The mystery*, in the *Rudolphine tables*, he assesses his success: I transferred all of astronomy from fictitious circles to natural causes (pr.). He has obtained a superabundance of results. Among them many causes. Book IV of the *Epitome*, in which all magnitudes, motions, and proportions in the heaven are explained by natural or archetypal causes, reaches the greatest clarity as to which among the many causes are the principal. It reaches the greatest clarity also as to what is the system of the world, and hence the world.

Kepler begins with the principal parts of the world. They are three, the sun in the center, the fixed stars on the surface, and the planetary system in between, the signs of the father, the son, and the holy spirit (1.1). He argues that the sun is in the center (1.2), then describes the order of the planets, and fits the five regular solids in between their orbits (1.3-4). He calculates the size of the world: the radius of the sphere of the fixed stars is 60 000 000 earth radii (1.4). The world is 8×10^{11} km across.

He describes the motions of the planets (2.1). The chapter *On the causes of the motion of the planets* follows. Kepler concludes: no mind is necessary to move the planets, nor a soul to direct their revolutions; there is the one and only solar

body in the center of the world and to it the motion of the planets can be ascribed (2.2).

The *Epitome* is in the form of questions and answers, Kepler next asks: what causes do you adduce for making the sun the moving cause of the planets (2.3)? and answers by adducing four causes:

it is apparent that the more distant from the sun a planet is the more slowly it moves, and so that the ratio of the periodic times is the ratio of the $\frac{3}{2}$ powers of the distances;

the closer a planet approaches to the sun the greater its speed is, and precisely in the ratio of the squares;

the sun is the most beautiful, absolutely round, the greatest, and the source of the light and heat from which all life flows into the plants, so it can be concluded that its heat and light are the instruments by which it causes the motion of the planets;

and the rotation of the sun on its immobile axis is in the same direction as the direction of the revolution of all the planets and in a period shorter than that of Mercury which is closest to it (*id.*).

The chapter *On the causes of the ratio of the periodic times* follows. Kepler asks: what are these causes (2.4)? and answers: four causes combine to establish the periodic times, the length of the orbit of the planet, the amount of matter in the planet, the strength of the motor force of the sun, and the volume of the planet (*id.*);

the length of the orbit is proportional to the distance;

the amount of matter is proportional to the square root of the distance (*id.*);

the strength of the force of the sun is inversely proportional to the distance (2.3);

the volume is proportional to the distance (2.4);

and compounding, the periodic time is proportional to the $\frac{3}{2}$ power of the distance (*id.*). These

causes Kepler argues on the ground that the force of the sun is magnetic and the planets are magnets (2.3). Along the way he anticipates inertia: matter has a natural inertia or rest, so that a body needs some power stronger than itself to move it

out of its position (2.2).

And in the chapter *The causes of the true irregularities* he writes: that the motions of the planets are regular, and that is, ordered according to a fixed and immutable law, is beyond controversy; otherwise there would have been no astronomy; and astronomy bears witness that the orbits are not circles but ellipses (3.1).

In short, having discovered the laws, Kepler has taken the decisive step also toward establishing that the principal among the true and proper causes of the number, magnitude, and periods of the heavens, which he has demonstrated in 1596, the physical causes, in which he has grounded the motions of the planets in the new science of causes in 1609, the natural or archetypal causes, by which he has explained all magnitudes, motions, and proportions in the heaven in 1620 – among the natural causes, to which, from fictitious circles, he has transferred all of astronomy – are the laws.

It will take the following two generations to establish it conclusively.

Leibniz writes: Kepler first noted that all phenomena can be deduced if the earth and all primary planets are assumed to move on ellipses in whose focus is the sun and it is a law of motion that the areas swept by a planet with respect to the sun are proportional to the times (manuscript, 1689).

Finally, in 1627, Kepler published the *Rudolphine tables, with which the restoration of astronomical science, long collapsed, is continued*.

The *Rudolphine tables* consist of precepts for the preparation of tables, tables proper, and a star catalogue. The catalogue is of the positions of 1 000 fixed stars. The tables proper are of calculated positions of the sun, the planets, and the moon. The precepts for the preparation of tables contain all data necessary for the recalculation of the positions of the stars, sun, planets, and moon for any latitude and longitude on the surface of the earth for any date in the past or in the future. They include precepts as well: for the reduction of the time reckoning of various nations to the time reckoning of the Julian and the new Gregorian calendars (I, 12). Also, Napier had discovered logarithms in 1614, and among the precepts are instructions for their use, and a table of loga-

rithms and antilogarithms.

The *Rudolphine tables* are not just a vast improvement on earlier tables. They are perpetual. Much had been said in the past about what will happen in the future, much was to be said in the future about what will happen in the more distant future, and much will no doubt be said in the more distant future about what will happen in the yet more distant future. What the *Rudolphine tables* said will happen in the future for the first time did, in fact, to the nearest 10' happen in the future.

More generally, what Kepler said will happen in the future for the first time did, in fact, happen in the future as he had said it will happen in the future.

While laboring at the *Tables*, and preparing ephemerides for the following years, he discovered that transits of Mercury and Venus are due in 1631, and in 1629, two years before the transits, published *On the rare and marvelous phenomena of the year 1631, incursions of Venus and Mercury in the sun, an admonition to astronomers and students of the things in the heaven*.

The philosophers, including the astronomers and students of the things in the heaven, had concerned themselves with prophecy for millenia by that time, humanity had so concerned itself longer, and both humanity and the philosophers have so continued to concern themselves. Kepler's prophecy in the *Admonition* is thus not the first, far from it. It is the first, however, to commit itself to what will happen in the future as explicitly as this:

7 Nov 1631 sun Scorpio 14° 45' 0'',
 Mercury Scorpio 14° 55' 5'' (*id.*).

By the time the prophecy could be examined Kepler was dead and could not heed his own admonition. But Gassendi, an astronomer and student of the things in the heaven, was alive and heeded the admonition. His title, upon heeding it, is itself masterful, *Mercury seen in the sun, Venus not seen*. Gassendi's is not the first report that a prophecy has been fulfilled either, far from it. It is also the first, however, of a prophecy that needed no revelation to show fulfilled.

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The true way begins with the collection of the

particulars, proceeds by continuous and gradual ascent, and ends with the most general principles.

To discover the form is the work and end of knowledge. Truth results from the discovery of the forms. The forms are the law with its clauses.

The end is the most general principles, and the end is the law with its clauses.

It will take all the labors of Bacon, Galileo, Kepler, Descartes, Huygens, Spinoza, Newton, and Leibniz to establish what the law with its clauses is, which the most general principles are, and clear away all the ambiguity in the relation of laws and principles.

What the law with its clauses is is for the first time clear with the labors of Kepler.

The difficulty of the problem thus solved can need no recommendation. Explicitly and implicitly, Kepler labored at it for three-odd decades. At the same time Bacon labored at it more explicitly, if not quite as long, and Galileo labored at it longer, if not quite as explicitly. The philosophers had labored at it since Aristotle. And they had to continue to labor at it after Bacon, Galileo, and Kepler.

To show this, the most important labors at the problem before and after Kepler's, such as his labors stand on the earlier and the later stand on his, can be shown.

And the most important earlier labors are indicated unambiguously by the pedigree of the laws he discovered. The pedigree of Kepler's laws is Aristotelean, Aquinian, and Euclidean.

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In the *New astronomy* Kepler begins with: an examination of Aristotle's *Metaphysics*, the subtitle of book IV of the *Epitome* is a supplement to Aristotle's *On the heaven*, and in its preface Kepler notifies his reader: I want Aristotle for my reader.

Beginning to end he searches for causes, and beginning to end he finds causes, the true and proper causes, the physical causes, the natural or archetypal causes, the causes of the motion of the planets, the causes of the ratio of the periodic times, the causes of the true irregularities. The new astronomy is a science of causes. And it is to natural causes that, from fictitious circles, he transferred all of astronomy. Kepler's laws can

hardly have been more ostentatious about their Aristotelean pedigree.

Aristotle investigates principles and causes in all his treatises. And he never distinguishes principles and causes sharply. His two most important investigations of principles and causes are book I of the *Metaphysics* and book II of the *Physics*, and in both he investigates principles, ἀρχαί, and causes, αἰτίαι, indiscriminately. It does not help that in both, and throughout Aristotle's treatises, principles and causes are also first principles and first causes.

But principles and causes should be distinguished sharply, and can be, and the grounds on which they should be, and can be, are Aristotelean.

Aristotle analyzes principles and causes into four kinds, material, efficient, formal, and final.

In book II of the *Physics* he defines the kinds. Material is: that out of which a thing comes to be and which persists (3), efficient is: the beginning of the change or rest of a thing (*id.*), formal is: the form and paradigm of a thing, and that is, the definition of what it is and its genus (*id.*), final is: the end of a thing or what it is for (*id.*). He exemplifies, too. Examples of material are: bronze of the statue, silver of the bowl (*id.*), of efficient: the counselor, or the father of the child (*id.*), of formal: the ratio 2:1 and in general number of the octave (*id.*), of final: health of walking, for why does he walk? to be healthy; as well as purging, drugs, surgical instruments (*id.*).

The statue, the bowl, the counselor, the child, the octave, walking, purging, drugs, and surgical instruments have thus been analyzed. Hence, more complicated examples can be analyzed, the statue of the counselor, the bowl of the child, singing the octave while walking, the child holding the bowl full of drugs, the father singing the octave to the counselor, the counselor giving surgical instruments to the child, the statue of the counselor counseling purging to either the father walking and singing the octave or the child walking with the bowl full of either drugs or surgical instruments, and so on.

If the examples are not very fortunate, are indeed very unfortunate, it is not because they do not provide guidance for the search for the principles and causes of things better than the statue,

the octave, the child, and walking. They do. It is because they are liable to reinforce the supposition, firmly grounded in the propensity to fly to most general principles, that nothing is easier than to find principles and causes. Descartes has to write: you say that nothing is easier than to fit some cause to an effect; but even if it were true that there are effects to which it is easy to fit various causes, one cause to each effect, it is not always so easy to fit the same cause to many different effects unless it is the true cause which produces them (to Morin, 13 Jul 1638).

There are other places in Aristotle liable to reinforce that supposition. In the *Rhetoric*, for instance, he begins his discussion of forensic speeches by proposing: every action is due to one of seven causes, chance, nature, compulsion, habit, reasoning, anger, or appetite (I, 10). And if not only compulsion, habit, reasoning, anger, and appetite are causes – nay, next to nature – but chance is a cause, too, how hard can it be to find a cause?

The examples are unfortunate, but the division of principles and causes into that out of which a thing comes to be and which persists in it, the beginning of its change or rest, the definition of what it is and its genus – never mind its end – is not. Material and formal are matter and form, and on Aristotle's analysis of all things into matter and form all acquisition of knowledge stands. As to efficient – never mind final – there is more about it in the *Physics* than the examples in book II, and none of it is about counselors or fathers.

The *Physics*, being physics, is the investigation of nature. And Aristotle begins with a general description of nature: nature is the principle and cause of motion and rest, and substantially and not accidentally (II, 1). After the discussion of the four kinds, he concludes: the necessary in nature is matter and the changes in it (9), and hence reaffirms the general description: nature is the principle of motion and change (III, 1). Moreover: there is no motion apart from the things that move, no change apart from the things that change (*id.*).

The investigation of nature is the investigation of motion.

Aristotle concludes: there are three kinds of motion, motion with respect to size, motion with

respect to quality, and motion with respect to position (VIII, 7); and because the others are impossible without it, the last, locomotion, must be first (*id.*); if there must always be motion, there must always be locomotion (*id.*).

Nature is the principle and cause of motion and rest. The necessary in nature is matter and the changes in it. There is no motion apart from the things that move. And if there must always be motion, there must always be locomotion.

This is what should provide the examples of principles and causes.

But principles and causes are also first principles and first causes. And what is first Aristotle is explicit, too, if not very clear.

Locomotion must be first. And Aristotle continues: locomotion is first in three ways; it is first in that, if it did not exist, other things would not exist, but if other things did not exist, it can exist; it is first in time; and it is first in being (*id.*).

Locomotion is first in these three ways.

In general, there are five ways in which the first is first. Aristotle writes in the *Categories*: one thing is first in relation to another in one of four ways; first, and most properly, by being first in time; second, by not implying the existence of another, as, for instance, one of two things is first because, if there are the two, there is one of them, but if there is one of them, there are not necessarily the two; third, by being first in knowledge; fourth, by being better, in which way it is least properly first; and there is, in fact, one more way in which a thing can be first, by being the cause of another; there are thus five ways in which one thing is first in relation to another (12).

There are five ways in which the first is first.

Better as it no doubt is, however, and rather precisely because it is no doubt better, the least proper way of being first is better dismissed out of hand. This leaves four of the five ways. But one more of them is better dismissed out of hand as well. By being first in knowledge. For this way of being first either will follow the other three ways or it will not. If it will, it is superfluous. And if it will not, it is no doubt because it will follow the better way. Because that way is better for the child, for instance. Or for the father. Or, as likely as not, for the councilor.

This leaves three of the five ways in which the first is first in the *Categories*, and these are the three ways in which locomotion is first in the *Physics*: by being first in time; by not implying the existence of another; and by being the cause of another. Taking it for granted that the third way in which locomotion is first in the *Physics*: first in being, is the last way in which the first is first in the *Categories*: by being the cause of another. Aristotle writes: locomotion is first in being, because there must be motion continuously (*Physics*, VIII, 7), and: everything in motion must be moved by something (VII, 1). If so, and there must be motion continuously, there must be causes continuously. And if so, first in being has to be first by being the cause of another.

Principles and causes are first principles and first causes.

The first is first by being first in time, by not implying the existence of another, and by being the cause of another.

Principles and causes are thus principles and causes because they are first in time, do not imply the existence of other things, and are the causes of other things.

This can explain why god is the principle and cause. He is first in time, he does not imply the existence of other things, and he is the cause of other things. The more so, if he is also first in knowledge, as he would be, and first in the least proper way of being first, too.

It can explain why beginning to end Kepler searches for causes, and finds causes. The true and proper causes. The physical causes. The natural or archetypal causes. The causes of the motion. The causes of the ratio. The causes of the irregularities. Why astronomy is a science of causes. Why Kepler transferred it to natural causes. And why the laws are causes.

In general, it can explain the millennia's worth of confusion, counting from Aristotle through and including Kepler and through and including all too many of his successors, as well as obfuscation, of principles and causes.

Even when the better way of being first is dismissed, and being first in knowledge is dismissed, too, the remaining three ways of being first are all three all too vague. One is by being the cause of another. Principles and causes are

principles and causes by being causes. Or, to take the formulation of the *Physics*, principles and causes are principles and causes by being first in being. And even if this way is dismissed as well, can first in time not be first by not implying the existence of another? and first by not implying the existence of another not be first in time?

But principles and causes are not just first.

Principles and causes are material, efficient, formal, and final.

Nature is the principle and cause of motion and rest, the necessary in nature is matter and the changes in it, there is no motion apart from the things that move, if there must always be motion, there must always be locomotion – and this should provide the examples of principles and causes.

Final is the end of a thing or what it is for. Examples are health of walking, as well as purging, drugs, surgical instruments.

But there are better examples.

Plato has Socrates report: I discovered that earth and air and other such things are not the true causes of things; the true cause of things is what is for the best (*Phaedo*, 99b).

Pace Aristotle, final principles and causes are neither principles nor causes.

With Bacon: final causes corrupt rather than advance the sciences. With Descartes: final principles are of no use whatsoever in physics (*Meditations*, 4). Bacon explains: final causes are in relation to man, not to the world.

Material, efficient, and formal are principles and causes. And for this very reason they should not be indiscriminately principles and causes.

Material is that out of which a thing comes to be and which persists. Examples are bronze of the statue, silver of the bowl.

But there are better examples. In Aristotle, too.

Earth, water, air, and fire (*Metaphysics*, I, 3). If earth, water, air, and fire, then also, and rather, hydrogen, helium, lithium, beryllium. If hydrogen, helium, lithium, beryllium, then also, and rather, protons, neutrons, electrons. If protons, neutrons, electrons, then also, and rather, quarks and leptons.

Efficient is the beginning of the change or rest of a thing. Examples are the counselor, or the father of the child.

Again, there are better examples.

Love and strife (4). If love and strife, so also, and rather, the first mover (XII, 10). If the first mover, then also god. If the first mover and god, then also, and rather, gravity. If gravity, then also, and rather, the gravitational, strong, weak, and electromagnetic forces.

Formal is the form and paradigm of a thing, and that is, the definition of what it is and its genus. Examples are the ratio 2:1 and in general number of the octave.

Again, there are better examples.

The principle of contradiction (IV, 3). If the principle of contradiction, then also the principle of sufficient reason.

And so much suffices to establish the ground on which to distinguish principles and causes sharply.

Material should be neither principles nor causes. Material should be the elements, exclusively. And the elements should be material, exclusively. Material should be in relation to matter, exclusively.

Efficient should be causes, exclusively. And causes should be efficient, exclusively. Causes should be in relation to effects, exclusively. Efficient causes should be redundant.

Formal should be principles, exclusively. And principles should be formal, exclusively. Principles should be distinguished by their generality, exclusively. Formal principles should be redundant.

This is consistent with Hume's definition of cause: a cause is an object followed by another, and where all the objects similar to the first are followed by objects similar to the second; or, in other words, where, if the first object had not been, the second never had existed.

It is consistent with Kant's result: the principle of contradiction is the principle of all analytic judgments, the principle of sufficient reason is the principle of all synthetic judgments.

It is consistent with Aristotle's analysis of all things most fundamentally into matter and form.

It is not consistent with Aristotle's division of the four, material, efficient, formal, and final, into two – material, on one side, efficient, formal, and final, on the other.

But it is consistent with Bacon's division of

the three, material, efficient, and formal, into two – material and efficient, on one side, formal, on the other.

Kepler's laws are not causes. They are principles. They are forms. The forms are the law with its clauses. And Kepler's laws are clauses of the law.

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They are laws because of their Aquinian pedigree.

As, in general, the law with its clauses is law because of its Aquinian pedigree.

By Augustine's time, the early fifth Christian century, the transformation of philosophy into Christian philosophy was complete.

The world had become a creature. Augustine writes: of all visible things the world is the greatest, of all invisible things the greatest is god; we see that the world exists, and we believe that god exists; and that god made the world, we cannot believe more securely than from god himself; and where do we hear god? nowhere more clearly than in the holy scriptures, where his prophet says, in the beginning god created heaven and earth (*City*, XI, 4); those who say that the world is eternal and without beginning, and thus not made by god, stray greatly from truth and rave in deadly madness of ungodliness; for even if the testimony of the prophets is ignored, the world itself, by the perfect order of its changes and motions, and by the great beauty of all visible things, proclaims by silent testimony both that it has been created and that it can only have been created by god ineffable and invisible in greatness and ineffable and invisible in beauty (*id.*).

Being creator of the world did not prevent god from being eternal and immutable. Augustine continues: but why did it please the eternal god to make heaven and earth when he made them and not before (*id.*)? did he do it on a whim, did a new act of will occur to him, who is in no way mutable (*id.*)? as the soul was created in time but will never perish in time, like number which has a beginning but no end, so let them also believe that the world was made in time, yet god did not on this account change his eternal purpose and will in making it (*id.*).

Nor did being either creator of the world or

eternal and immutable prevent god from being also omnipotent. Augustine writes: the power of the creator and his omnipotent and all-swaying strength is for every creature the cause of its continued being, and were his strength at any time to cease being, and to cease directing the things which have been created, at the same time their species would cease being and all nature would perish (*Genesis*, IV, 12).

So let them believe.

Nor was it, apparently, or is it, difficult to believe that god is creator of the world, eternal and immutable, and omnipotent. Difficult was, and is, for god to be creator of the world, eternal and immutable, and omnipotent. Difficult was, and is, for god to be any of these, never mind all of these, never mind any of these consistently, never mind all of these consistently.

The darkest of dark ages eventually came to a close, and seven and a half centuries after Augustine, Abelard writes: I think it has to be asked whether god can do more or better things than he does, and even whether he can cease doing things that he does; and either way, whether we affirm or deny, troublesome difficulties follow; either if he can do more or fewer things than he does or if he can cease doing things that he does, this will greatly detract from his supreme goodness (*Theology*, III, 27).

His goodness was an insurmountable obstacle to his omnipotence.

Thus: we can do things he cannot do, eat, walk, even sin (47). Worse: if he cannot be without providence and it is impossible for things not to be the consequences of this providence, it yet cannot be impossible for the foreseen things not to be; and if he cannot not will to create the world, it yet cannot be impossible for the world not to be (53).

Abelard concedes: few or no one will agree with our conclusion, which seems to disagree also with what many saints have said as well as with reason (46). Then draws the conclusion anyway: god can do and do not only what he does and does not, and only in the way and at the time in which and at which he does it and does it not, and no other (56).

Having done for nineteen and a half centuries, counting from the Yahwistic document, as he

pleases, god had to begin doing, with the labors of Abelard, only what the principle of contradiction allowed him to do.

Unlike their adherents, the difficulties were not prejudiced against any one religion. Two generations after Abelard, Maimonides writes: if we were to accept the eternity of the world as taught by Aristotle, that everything in the world happens by fixed laws, that nature is immutable, and there is nothing beyond nature, we would be necessarily in opposition to the foundation of our religion, we would disbelieve all miracles, and we would reject all hopes and fears derived from the scriptures (*Guide*, 25).

Bonaventure, not quite three generations later, agreed. Aristotle is: that pit of darkness.

Miracles, and hopes and fears, were insurmountable obstacles to his eternity and immutability.

Aquinas, in the generation of Bonaventure, assimilated that pit of darkness into Christian philosophy.

Aquinas also faced the difficulties before his omnipotence and his eternity and immutability squarely. His resolution of these difficulties did not agree with what many saints had said any more than Abelard's. His resolution of these difficulties, however, established the essential and fundamental character of the law. Such as, three and a half centuries later, Bacon will write: although there is nothing in nature but individual bodies performing individual acts, they do so by law, and this law and its investigation and discovery is the foundation, work, and end of knowledge. Galileo will write: the most obedient executrix of god's orders, nature is inexorable and immutable, and never transgresses against the laws imposed on her. Kepler will write: that the motions of the planets are ordered according to a fixed and immutable law is beyond controversy. And in the next generation Descartes will write: god created matter, agitated it, and did no other thing but let nature act by the laws he established.

God is omnipotent. Aquinas writes: all confess that god is omnipotent; yet it is difficult to explain in what precisely his omnipotence consists, for there may be doubt as to what, when we say that god can do all things, all means; but if we consider the matter carefully, power is with

respect to possible things, and the proposition, god can do all things, is correctly understood to mean that god can do all things which are possible (*Summa*, I, 25.3); all things which do not imply a contradiction, therefore, are among all possible things these with respect to which god is called omnipotent; while whatever implies a contradiction does not come within the scope of his omnipotence because it cannot have the character of possibility; it is thus better to say that such things cannot be done, not that god cannot do them (*id.*).

God is omnipotent, but the principle of contradiction is more omnipotent.

God is also eternal and immutable, and his eternity follows from his immutability. Aquinas writes: god is altogether immutable; for we have shown that there is a first being, which we call god; this first being must be pure actuality, without the admission of any potentiality, because potentiality is posterior to actuality; but everything which is mutable in any way is in some way only in potentiality; hence it is clear that god cannot be in any way mutable (9.1). Further: what is altogether immutable can have no succession, nor can it have a beginning or an end (10.1); for eternity is known in two ways, in that the eternal is interminable, it has neither a beginning nor an end, and in that eternity has no succession, it is simultaneously altogether (*id.*).

God is eternal and immutable, and as Aquinas respects the principle of contradiction not less than god should, miracles are nothing but the effects of ignorance.

Aquinas writes: from the multiplicity of causes multiplicity of orders follows, subjected to one another as cause is subjected to cause; a higher cause is not subjected to a lower cause, but conversely; for instance, the order of the household depends on the father, this order depends on the order in the city, this order depends on the ruler of the city, and this order depends on the prince who orders the entire principedom (105.6); therefore, with respect to the order of things dependent on the first cause, god can do nothing against this order (*id.*); with respect to the order of things dependent on secondary causes, god can do things outside this order (*id.*). Either way: those things that god does outside the causes

which we know are called miracles (105.7).

Clearly, if it is a creature, the world is a thing dependent on secondary causes. Like a principedom brought into being by its first prince.

The *Theological summa* is in three parts, it investigates: god, the rational creature's advance toward god, and Christ who, as man, is our way to god. On the authority of the philosopher, if not on that of the saints, it can be analyzed into theoretical philosophy and practical philosophy. The first part, then, the investigation of god, is theoretical. Some of it is as unambiguously physical and logical as Aristotle's physical and logical investigations. The second and the third parts, the investigation of the rational creature's advance toward god and of Christ who, as man, is our way to god, are practical. All of this is ethical and political. The *Summa* is essentially practical, such as, in general, Christian philosophy had unwaveringly been. Such as Christian philosophy had seldom been, however, and had only begun being in the couple of centuries before Aquinas, the practical results of the *Summa* are essentially informed by the theoretical.

The results above, those concerning god's omnipotence, and eternity and immutability, Aquinas obtains in the first part, the theoretical.

The second part, the investigation of the rational creature's advance toward god, comprises the bulk of the treatise. It is itself in three parts, investigating: the end of man (II-I, 1-5), acts in general (6-114), and acts in particular (II-II, 1-189). The second of these parts is in two parts, investigating: acts themselves (II-I, 6-48), and the principles of acts (49-114). The second of these parts is again in two parts, investigating: the intrinsic principles of acts (49-89) and the extrinsic principles of acts (90-114). And the extrinsic principles of acts are themselves twofold: law (90-108) and grace (109-114).

Aquinas's results in the investigation of the law are his most important. They are a vast systematization and generalization over earlier results. They are results, moreover, which systematize and generalize over results not only of the holy scriptures, and of the jurists, and of the philosophers, but also of the princes, and all the way back to the first prince.

The first authority to whom Aquinas appeals

in his investigation of the law is: the apostle (90.1). The second is: the jurist (*id.*). The third is: the philosopher (*id.*).

The apostle, because the Christian scriptures assert, and reassert, and reassert the law. The apostle in the *Letter to the Galatians* (2-4), the *Second letter to the Corinthians* (3), the *Letter to the Romans* (2-3, 7, 13), the gospels in *Mark* (12), *Matthew* (5, 22), *Luke* (10, 16), *John* (7, 13-5). Which the Christian scriptures do because the Hebrew scriptures do. The foundation of the Hebrew scriptures is the torah. The law. The torah developed in three steps, the Yahwistic document, *Deuteronomy*, and the Priestly code. All three assert, and reassert, and reassert the law. All three also collect the law in codes. That of the Priestly code is *Leviticus*, earlier, the most important is that of *Deuteronomy* (12-26), earlier, that of the Yahwistic document is in *Exodus* (20-3). The Yahwistic document predates the apostle by eight and a half centuries. The Hebrew scriptures assert, and reassert, and reassert the law, and collect the law in codes, because the princes do. To note but the most important, Hammurabi, prince of Babylon, promulgated a code of laws nine and a half centuries before the Yahwistic document. Ur-Nammu, prince of Ur, promulgated a code of laws three and a half centuries earlier. Two-odd centuries earlier, Urukagina, prince of Lagash, left behind the oldest inscriptions concerned with laws (ins. 1-3). Urukagina predates the apostle by twenty-four centuries. As to the earlier princes, the jurist writes: what pleases the prince has the force of law (*Digest*, I, 4.1), and if so what pleases the prince later, because so what pleases the prince earlier, and because first of all so what pleases the first prince. The first prince, Narmer, predates the apostle by thirty-one-odd centuries, Aquinas by forty-three and a half centuries.

The jurist is Ulpian, and Ulpian is the jurist for the same reason st Paul is the apostle and Aristotle is the philosopher. Under Justinian, in the sixth Christian century, the entire earlier development of jurisprudence was collected in the *Digest*. In particular, 9 201 excerpts from the treatises of 38 jurists are collected in the *Digest*. 2 468 of the 9 201 are excerpts from the treatises of 1 jurist, Ulpian.

The rise of jurisprudence begins under Augustus, with Labeo. From Labeo through Ulpian jurisprudence develops explosively for two-odd centuries. Labeo predates st Paul by two generations. Ulpian is a younger contemporary of Hippolytus, one of the first fathers of the church read in the philosophers. The following two centuries, through Augustine, are the time of the transformation of philosophy into Christian philosophy. The explosive development of jurisprudence thus in part precedes, in part, the larger part, is contemporary with the rise of Christianity. The end of the explosive development of jurisprudence is contemporary with the beginning of the transformation of philosophy into Christian philosophy.

The development of jurisprudence and the transformation of philosophy into Christian philosophy – jurisprudence concerned with human law, Christian philosophy concerned with human law not less than with divine law – cannot not be the effects of the same remote cause. And on chronological grounds alone it can be concluded not only that they are the effects of the same proximate cause. It can be concluded that the first, the development of jurisprudence, is among the immediate causes of the second, the transformation of philosophy into Christian philosophy, the second, the transformation of philosophy into Christian philosophy, is among the immediate effects of the first, the development of jurisprudence. The more so, if the development of jurisprudence remained uninformed by, and largely unaware of, the impending concern with divine law, but the transformation of philosophy into Christian philosophy was informed by, and very much aware of, the labors of the jurists.

The jurists remained also uninformed by, and largely unaware of, the theoretical concerns of the philosophers. The fathers of the church were acutely aware of, if not so acutely informed by, the theoretical concerns of the philosophers. Early on, in order to show, with Hippolytus, in the *Refutation of all heresies*, that: all heresies are rooted in the doctrines of the philosophers (I, pr.). Later on, in order to conclude, with Augustine, that: whatever man learns outside the holy scriptures, if noxious, is to be condemned, if benefi-

cial, is in them contained.

Not quite nine centuries after Augustine, Aquinas concerns himself with both human law and divine law, and on the ground of theoretical results resting firmly on theoretical results of his predecessors. The result is a vast systematization and generalization.

Aquinas analyzes the law into five kinds: eternal, natural, human, divine, and of sin (*Summa*, II-I, 91).

The law of sin is more precisely: law of the incitement to sin (91.6). Aquinas writes: when man turned his back on god, he fell under the influence of sensuality; this happens to everyone individually, the more a man deviates from the path of reason, the more he becomes like the beasts who are led by the impulse of sensuality (*id.*); and the impulse of sensuality has the nature of law (*id.*).

Anticipating Darwin, no less.

Divine law gives rise to the objection: there is eternal law, the eternal law is divine, so it seems that there was no need for divine law (91.4). Aquinas answers: there was; divine law was necessary for the direction of the conduct of man, and for four reasons (*id.*); so that man is directed to his end, which is eternal happiness and is inappropriate to his natural faculties (*id.*), so that man knows what he ought to do and ought not to do without any doubt (*id.*); so that internal acts are curbed, which human law cannot do sufficiently (*id.*); and so that no evil remains unforbidden and unpunished (*id.*).

Divine law is twofold, old and new, contained in the holy scriptures. And Aquinas's analysis of divine law, and thus of the holy scriptures, is exquisite. It seems that there was no need for the holy scriptures. There was. The holy scriptures were necessary to direct man to an end inappropriate to his natural faculties. So that he knows what he ought to do without any doubt. So that even internal acts are curbed. So that no evil remains unforbidden. In short, *par excellence*: for the direction of the conduct of man.

Human law raises the question: is every human law derived from natural law (95.2)? Aquinas answers: every human law has the nature of law precisely to the degree to which it is derived from natural law; if at any point it deviates from natu-

ral law, at every such point it is not a law but a perversion of law (*id.*).

Political doctrine cannot be more salutary.

Human law is derived from natural law. In particular: from the principles of natural law, as general and indemonstrable principles, human reason needs to proceed to more particular determinations; and these particular determinations, when they conform to the other essential conditions of law, are called human laws (91.3).

In turn: natural law is nothing but the rational creature's participation in eternal law (91.2).

Eternal law, finally, is the very idea of government of things in god the ruler of the world.

Beginning his analysis of the law, Aquinas writes: law is the rule and measure of acts by which man is induced to act or is restrained from acting (90.1). Further, however: every law is for the common good (90.2). It follows: law is nothing but the commandment of practical reason emanating from the ruler who governs a perfect community (91.1). And it follows further: clearly, given that the world is ruled by divine providence, the whole community of the world is ruled by divine providence; therefore, the very idea of government of things in god the ruler of the world has the nature of law; and since the idea of things in the divine mind is not subject to time, but eternal, this law must be called eternal (*id.*).

Law is the rule and measure. And lest the whole community of the world were but a community of men, it is the rule and measure of the world, as general and indemonstrable principles, from which the rule and measure of acts by which man is induced to act or is restrained from acting is derived. It is nothing but the commandment of theoretical reason, from which the commandment of practical reason is derived.

The appeal to providence is superfluous. Omnipotence under the principle of contradiction suffices to imply the rule and measure of the whole community of the world. God can do and not do only what he does do and not do, respectively, in the way and at the time when he does do and not do it, respectively, and in no other way and at no other time. It suffices to imply also the immutability and eternity of the rule and measure of the whole community of the world. With respect to the order of things dependent on the

first cause, god can do nothing against this order.

The world can be a creature of god not more than god can be a creature of the order against which he can do nothing.

The rule, measure, government, and commandment of the world are immutable. But the rule, measure, government, and commandment of the acts of men, derived from the rule, measure, government, and commandment of the world as general and indemonstrable principles, are mutable. There is thus hope, politically, even if at every point at which human law deviates from natural law it is not a law but a perversion of law, and the propensity of human law to deviate from natural law has been irresistibly stronger than the opposite.

Eternal law is the very idea of government of things in god the ruler of the world. But the very idea of government is the rule and measure of the order. And things are things. But in god the ruler of the world has to be doubly redundant.

Eternal law is the rule and measure of the order of things in the world.

And the five kinds of law are fundamentally two. Natural, and derived from it, human.

Divine law is human law. It was necessary for the direction of the conduct of man. And it is mutable. It mutated from the old to the new. The old itself mutated from the torah through the neviim to the kethuvim. The new itself mutated from the apostle and the gospels into the fathers of the church. Old and new, it then mutated into the koran. Which itself mutated into its amplification by the sunnah. To note but the main steps in the mutation.

The law of sin is natural law. Hence, also, since: the extrinsic principle inclining to evil is the devil (90), the devil is a rule and measure of nature.

The distinction between eternal law and the rational creature's participation in eternal law is superfluous. If only because: the irrational animals participate in eternal reason just as the rational creature does (91.2). Eternal law is natural law, and this kind of law is, in general, natural rather than eternal, because nature cannot not be synonymous with the world. The important distinction is between law and discovery of the law.

Human is the law of the city, the object of politics. Natural is the law of the world, the object of physics.

Natural law is the rule and measure of the order of things in the world.

Thus, having well-nigh annihilated philosophy when he first conquered it, and reduced the little that he did not annihilate to whim, god at length brought to philosophy order and generality greater than it had had before.

The essential and fundamental character of the law is a central presupposition of Occam, two generations later, of Buridan, in the same generation, of Oresme, in the next generation. Hence, after a hiatus longer than a millennium, logic began advancing again and physics began advancing again.

Two-odd centuries further on, the investigation and discovery of the law became the foundation, work, and end of knowledge explicitly. And that so the law is, the foundation, work, and end of knowledge, was shown conclusively when it was discovered that the motions of the heavenly bodies are ordered according to the fixed and immutable laws of Kepler.

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The pedigree of the laws is Aristotelean, Aquinian, and Euclidean.

Aristotle first described the world.

The world is one, there is nothing next to the world, or outside the world. Nature is motion. The world is spherical. The primary motion is circular. At the circumference is the first mover, it moves the sphere of the fixed stars, and thus communicates motion to everything inside. Further inside are the spheres of the planets, of Saturn, inside it of Jupiter, inside it of Mars, inside it the spheres of the sun, Venus, and Mercury, inside these the sphere of the moon. Further inside are the spheres of the elements, of fire, inside it of air, inside it of water, and the innermost is of the earth. On the surface of the earth are the animals, one of them the political animal.

The description is vague.

Aristotle established that knowledge is acquired only by demonstration, and demonstration is deduction. And the description of the world is not even in part a demonstration, not even in part

a deduction.

Aristotle distinguished terms, propositions, and inferences. He showed what a proposition is, and what an inference is. He established that when an inference is grounded in what is true, it is a demonstration. Demonstration is deduction, and deduction always depends on principles.

Demonstration demands that the principles be true. Deduction demands that the principles be general. And if the principles are to reach the description of the world, the description of the world demands that the principles be very general.

This explains why Aristotle searches for the principles in all his treatises. Whether he has found the principles, however, and whether any of his predecessors or successors has found the principles, if knowledge is acquired only by demonstration, and demonstration is deduction, only deduction can decide.

Deduction is a chain of inferences, a chain of inferences must, with Frege, be free of gaps, and only a chain of inferences free of gaps can show that a chain of inferences begins with the principles. That the principles are true. And that the principles are general enough to have reached the description of the world.

Aristotle thus developed syllogistic. When a demonstration is such that, given certain propositions, another proposition follows necessarily, a demonstration is a syllogism.

Yet but a few of Aristotle's results are reduced to syllogisms, but a few of his results in logic, and fewer of his results in physics. None of the description of the world is reduced to syllogisms.

Which the fact that syllogistic is a narrow system explains. It is a calculus of terms. Given certain propositions, syllogistic will order these propositions in a chain of inferences completely free of gaps. But it cannot produce or even judge propositions. It cannot produce or even judge inferences. And it cannot decide whether the given propositions are the principles. Or whether they are true. Or whether they are general enough to have reached the description of the world.

To this end not only the problem of the relation of terms in a proposition has to be solved better than Aristotle solved it. The problem of the relation of propositions in an inference has to be

solved. The modes of inference have to be discovered. All formally. And the problems of what is truth, and generality, and existence, have to be solved formally. So that the chains of inferences are free of gaps.

Explicitly these problems will not all be solved until Frege. Implicitly, however, the mathematicians among the philosophers had been solving them for two centuries by Aristotle's time.

Hence, Euclid, in the generation after Aristotle, wrote the *Elements*. In the *Elements* from 5 axioms, 5 postulates, and 131 definitions 514 theorems, lemmas, and corollaries are deduced.

The *Elements* has remained the paradigm of deduction. The chains of inferences are so substantially free of gaps as to recommend themselves, twenty-two centuries later, to Frege. He begins in *The foundations of arithmetic*: having for some time distanced itself from Euclidean rigor, mathematics is now returning to it and even striving beyond it (1).

With the *Elements* deduction decides which are the principles. The axioms, postulates, and definitions are the principles. Deduction always depends on principles, and the 514 propositions of the *Elements* depend on the axioms, postulates, and definitions. The 141 principles, moreover, are not equally general. The definitions are introduced in groups, 23 with the axioms and postulates at the beginning of the treatise, 2 at the beginning of book II, 11 at the beginning of book III, 7 at the beginning of book IV, and so on, as they are principles of the following, and not of the preceding, propositions. The axioms and postulates are principles of propositions in all thirteen books. And of all propositions in all thirteen books the axioms are principles:

1. things equal to the same thing are equal;
2. if equal things are added to equal things, the wholes are equal;
3. if equal things are subtracted from equal things, the remainders are equal;
4. things which coincide with one another are equal to one another;
5. the whole is greater than the part (*id.*).

These are the most general principles.

It is clear that they are principles. It is clear that they are true. If there is preestablished harmony between mathematics and physics. It is

clear that they are very general. No demonstration before or after has stood, explicitly or implicitly, but on the principle, things equal to the same thing are equal.

It is clear also that an abyss remains between these principles and the description of the world. Yet it is clear also that this is the way to close the abyss, the Euclidean.

Euclid's contemporary Autolycus and Euclid himself took the first step. Autolycus wrote *On the moving sphere* and *On the risings and settings*, Euclid wrote the *Phenomena*. In *On the moving sphere* Autolycus proves 12 propositions concerned with circles on the surface of the sphere and their intersections. It is the first explicit distinction of the circles of the equator, the ecliptic, the horizon, and the meridian. Autolycus begins with the postulates: points move uniformly when they traverse equal or similar arcs; and when a point moving uniformly traverses two arcs, the ratio of the times is the ratio of the arcs (*id.*). In *On the risings and settings* he proves 30 propositions concerned with the morning and evening risings and settings of stars, and when they are visible. He begins with the definitions of risings and settings (*id.*). In the *Phenomena* Euclid proves 18 propositions. The first is: the earth is in the center of the world; let the sphere

$ABCD$ be the world, the earth be at E , and AEC be a horizon; when Cancer is observed rising at A , Capricorn is observed setting at C ; six of the twelve constellations of the ecliptic are above AEC ; AEC is therefore a diameter of the sphere of the fixed stars and the ecliptic; later, when Leo is observed rising at B , Aquarius is observed setting at D ; BED is therefore also a diameter of the sphere of the fixed stars and the ecliptic; E is therefore in the center (*id.*). The remaining propositions are concerned with the circles on the surface of the sphere, and with the risings and settings of stars. In a preface Euclid begins by proposing: the stars are observed to move in circles (*pr.*), and with definitions of the circles of the equator, ecliptic, horizon, and meridian.

Deduction has begun bridging the abyss be-

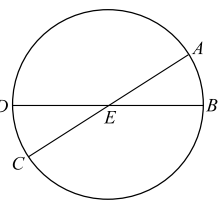
tween things equal to the same thing are equal, if equal things are added to equal things, the wholes are equal, and the description of the world. Even if the chains of inferences are not too many, 60 propositions in all three treatises, and still far from free of gaps. Deduction shows the principles, the stars are observed to move in circles, the main circles are the equator, the ecliptic, the horizon, and the meridian, points move uniformly when they traverse equal or similar arcs, and when a point moving uniformly traverses two arcs, the ratio of the times is the ratio of the arcs. The principles are true, and general. Not general enough, however. They reach only parts of the description of the world, and chains of inferences are yet to connect the parts.

Two generations later Archimedes greatly advanced mathematics. He also first showed that there is, in fact, preestablished harmony between mathematics and physics, in *On the equilibrium of planes*, in *On floating bodies*, and in *The sand-reckoner*.

In *On the equilibrium of planes* he proves 25 propositions. Including: magnitudes are in equilibrium at distances reciprocally proportional to the weights (I, 6-7). This result is the law of the lever. Galileo established why: on this single proposition of Archimedes depends reasoning not only about the lever, but about most mechanical instruments (*Sciences*, II).

This result is among the propositions in *On the equilibrium*. The principles are 7 postulates, among them: 1. equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline toward the weight which is at the greater distance; 2. when weights at certain distances are in equilibrium, if something is added to one of them, they are not in equilibrium but incline toward that weight to which something is added; 4. when equal and similar plane figures coincide, their centers of weight coincide (*id.*). The postulates show the preestablished harmony. They show that there is no gap in the chains of inferences from the *Elements* to *On the equilibrium*. Hence, also, in the *Elements* lines, figures, solids, and numbers are magnitudes, *On the equilibrium* shows that weight is a magnitude, too.

The principles are true, and general, but still



maximum. Next: the diameter of a poppy seed is greater than $\frac{1}{40}$ of 1 finger (*id.*). This result

Archimedes has also obtained, like the angle subtended by the sun, by experiment: 25 poppy seeds placed side by side measure just over 1 finger (*id.*). Finally: 10 000 grains of sand will pack, and more than pack, a sphere with the diameter of a poppy seed (*id.*).

Hence:

the sun is at the circumference of the world; its

diameter subtends $\frac{1}{720}$ of the circle; its diam-

eter is thus greater than the side of the regular chiliagon inscribed in the greatest circle in the world; and the perimeter of any regular polygon with more sides than the hexagon is greater than 3 times the diameter of the circle in which it is inscribed; so,

3 diameters of the world < 1 000 diameters of the sun;

diameter of the sun < 30 diameters of the moon;

diameter of the moon < diameter of the earth;

diameter of the sun < 30 diameters of the earth;

3 diameters of the world < 30 000 diameters of the earth;

diameter of the world < 10 000 diameters of the earth;

circumference of the earth < 3 000 000 stades;

3 diameters of the earth < circumference of the earth;

diameter of the earth < 1 000 000 stades;

diameter of the world < 10 000 000 000 stades;

1 stade = 9 600 fingers;

diameter of the world < 96 000 000 000 000 fingers;

1 finger < 40 diameters of a poppy seed;

diameter of the world < 3 840 000 000 000 000 diameters of a poppy seed;

a sphere with the diameter of a poppy seed will be packed with 10^5 grains of sand;

the world will be packed with 10^{51} grains of sand (*id.*).

Archimedes writes: Aristarchus has proposed the hypothesis that the sun and the fixed stars are at rest and the earth revolves around the sun (*id.*); and if the diameter of the sphere of the earth's revolution around the sun on Aristarchus's hypothesis is the same as the diameter of the world

on the hypothesis that the earth is in the center of the world, and as the diameter of the earth to the diameter of the world on the hypothesis that the earth is in the center of the world so the diameter of the sphere of the earth's revolution around the sun to the diameter of the world on Aristarchus's hypothesis, on Aristarchus's hypothesis the world will be packed with 10^{63} grains of sand (*id.*). On the big bang hypothesis at first the world will be packed with 10^0 grains of sand, today it will be packed with 10^{96} grains of sand.

Along the way, to prevent some from thinking that there may be no numbers greater than those, Archimedes has shown the series 10 , 10^8 , $10^{8 \times 10^8}$, $10^{8 \times 10^{8 \times 10^8}}$, and so on infinitely.

A pity the world is not packed with sand. Or with poppy seeds. Had it been, deduction would have reached the description of the world in its entirety with *The sand-reckoner*, eighteen and a half centuries before Kepler. As it is, deduction has yet shown that the principles have to reach the position and order of the parts of the world, and the position and order of the major parts, too, the earth, the moon, the sun, the fixed stars.

Archimedes's younger contemporary Eratosthenes solved the problem of measuring the circumference of the earth conclusively. A century later Hipparchus solved the problem of measuring the distance of the moon conclusively.

And with Ptolemy, two and a half-odd centuries later, fifteen before Kepler, the description of the world quit being vague. Ptolemy packed the world, and better than with sand. The region between the sphere of the elements, at 33 earth radii, and the sphere of the fixed stars, at 19 865 earth radii, he packed with the calculated orbits of the planets, in their order, moon, Mercury, Venus, sun, Mars, Jupiter, Saturn, at their calculated distances. At the circumference, the sphere of the fixed stars he packed with the declinations and right ascensions of 1 022 stars. Inside, the surface of the sphere of the earth he packed with the latitudes and longitudes of 6 345 localities. Time he packed with the correlation of the sequences of princes with the motions of the planets.

Deduction shows that the principles have to describe the position and order as well as the motion of the parts of the world.

It is still not clear, however, which are the

principles. The world is described formally, and the chains of inferences are substantially free of gaps. But Ptolemy does not present a set of principles formally. The principles are introduced informally, in the course of the description of the world in book I of the *Almagest*, such as it is not substantially different from Aristotle's. The principles are many, and it is not clear which are less general, which are more general, which are the most general.

It is for the first time in *The cosmographic mystery* that the principles are a few, and it is clear which are the most general. The principles are the sun is in the center, and the five regular solids inscribed in one another and circumscribed about one another determine the position and order of the planets. They do not determine the motion of the planets.

The laws do. The laws determine the position, order, and motion. The orbits of the planets are ellipses with the sun in one of the foci, the areas defined by the arcs traversed by a planet in equal times and the respective radii to the sun are equal, and the ratio of the periodic times is the ratio of the $\frac{3}{2}$ powers of the distances.

From the sun through the end of the solar system, including along the way the earth, and the moon orbiting the earth, the world is packed with the position, order, and motion of the heavenly bodies.

Deduction shows that the laws are the principles.

That principles are less general and more general, that there are principles, and there are principles of principles, has been clear since Aristotle. With the discovery of Kepler's laws it is for the first time clear which principles are less general, which principles are more general, which principles are principles, which principles are principles of principles. The laws are the principles, and it is clear which laws are less general, which laws are more general, which laws are laws, which laws are laws of laws. It is clear what the law with its clauses is.

Two-odd generations after Kepler Newton deduced Kepler's laws from the laws of motion. Three generations after Newton Lagrange deduced Newton's laws from the principle of least

action.

Maupertuis formulated the principle of least action one generation-odd before Lagrange.

One century before Maupertuis, one generation after Kepler, Fermat formulated the principle of least time. Fermat's principle holds for optical phenomena. Maupertuis's principle holds for mechanical phenomena. Lagrange's principle of least action is a generalization of Maupertuis's principle, and holds for mechanical phenomena as well. Two-odd generations after Lagrange Hamilton generalized the principle of least action further. Hamilton's principle holds for both optical and mechanical phenomena.

Two-odd generations after Hamilton Hilbert deduced the equations of the space-time continuum of the world from Hamilton's principle.

Later in the same generation Schrödinger deduced the equation of the position, order, and motion of all particles in the world from Hamilton's principle.

To note but a few of the clauses of the law.

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The position, order, and motion of the parts of the world is the system of the world. The system of the world is the world.

And conversely, the world is the system of the world.

How and why, the two further clauses of the law, the laws of thermodynamics, can show.

Clausius discovered the laws of thermodynamics, in 1850.

In 1847, in *On the conservation of force*, Helmholtz writes: the task of the physical sciences is to seek these laws, by which the particular processes of nature can be traced back to general rules, and from which they can be further determined (intr.). And Helmholtz accomplished the task, he found this law: the sum of the living forces and the forces of tension is constant; and in this most general form we can describe our law as the principle of conservation of force (2).

It took the two generations after their discovery to formulate Kepler's laws conclusively. It took the couple of decades after its discovery to formulate Helmholtz's principle conclusively. The living forces are the kinetic energy, the forces of tension are the potential energy, and the

principle of conservation of force is the principle of conservation of energy. In 1871, in the *Theory of heat*, Maxwell writes: the name kinetic energy is appropriate (4); the name potential energy is very felicitous (*id.*); and the principle that in any system of bodies the sum of the kinetic and potential energy remains always the same is the principle of conservation of energy (*id.*).

In 1850, in *On the moving force of heat and the laws which can be deduced from it for the theory of heat*, Clausius also accomplished the task. He found the two laws: we lay down the fundamental law, in all cases in which work is produced by heat, a quantity of heat is expended proportional to the work produced, and conversely, by the expenditure of a quantity of work an equal quantity of heat is produced (1); also, Carnot assumed that the equivalent of the work produced by heat is in the mere transfer of heat from a hotter body to a colder body, while the quantity of heat remains undiminished, and if we are to retain the fundamental law, the second part of Carnot's assumption, that the quantity of heat remains undiminished, must be rejected; but its first part then obtains in all cases (2).

It took Clausius the following decade and a half to formulate the two laws conclusively. The fundamental law is the first law of thermodynamics, and the first law of thermodynamics is nothing but the principle of conservation of energy applied to the phenomena of heat. Carnot's assumption corrected in order to retain the first law is the second law of thermodynamics. The correction, the quantity of heat does not remain undiminished, but decreases, is the discovery of the increase of entropy.

In 1865, in *On different convenient for application forms of the main equations of the mechanical theory of heat*, Clausius begins: the entire mechanical theory of heat rests on two fundamental laws, the law of the equivalence of heat and work, and the law of the equivalence of transformations (1); analytically,

$$dQ = dU + dW,$$

and

$$\int \frac{dQ}{T} \leq 0,$$

where Q is the heat in a system of bodies, U is what Kelvin has called the energy, W is the work,

and T is the temperature measured from the absolute zero (*id.*).

Hence he defines:

$$\int \frac{dQ}{T} = S - S_0,$$

and explains: S is the transformation content of the body, and I propose to name the quantity S – from the Greek word ἡ τροπή, the transformation – the entropy of the body; I have deliberately build the word entropy to be as similar as possible to the word energy, because the two quantities which should be named by these words are so closely related in their physical references (14).

And he concludes: if what I have named entropy with respect to particular bodies is thought consistently with respect to all circumstances in the world and in relation to the other simpler concept of energy, the two fundamental laws of the world corresponding to the two main laws of the mechanical theory of heat can be expressed in the following simpler form,

1. the energy of the world is constant;
2. the entropy of the world strives for a maximum (17).

In 1897, in the *Lectures on thermodynamics*, Planck begins: the method of this treatise is to start from a few very general facts of experience, mainly the so-called two fundamental laws of the theory of heat, and from these to obtain in a purely logical way a large number of other laws of physics and chemistry (*pr.*).

With Aristotle, Hume, and Kant, everything comes from experience. With Planck, the laws are facts of experience. Knowledge is acquired only by deduction. Like Aristotle, Euclid, Archimedes, Ptolemy, Kepler, Helmholtz, and Clausius, Planck deduces. Three centuries after Bacon, many clauses of the law have been discovered, and the task, part of the task of seeking the laws, is to deduce from the small number of very general the large number of less general. From Clausius's laws Planck deduces, among many others, Boyle's law (80-4), Dalton's law (18-32), Avogadro's law (36-43), Gay-Lussac's law (80-4), Kirchhoff's law (221).

In particular, Planck begins with the fact: if a body A is in thermal equilibrium with two other bodies B and C , then B and C are in thermal equilibrium with one another (2).

This fact has acquired the name zeroth law of thermodynamics. The zeroth law of thermodynamics is nothing but Euclid's first axiom applied to the phenomena of heat. Hence, heat, too, is a magnitude.

Planck writes: by the first law, the sum of the work W done on a system by an external reservoir of constant temperature T and the heat Q introduced in the system by the reservoir is

$$W + Q = 0,$$

by the second law, the change of the entropy of the reservoir, assuming only reversible changes of volume to occur in it, is

$$-\frac{Q}{T} \geq 0,$$

or

$$Q \leq 0,$$

therefore,

$$W \geq 0,$$

and that is, work has been used and heat has been added to the reservoir; this inequality expresses analytically the impossibility of a *perpetuum mobile* (139).

More generally, the second law implies the irreversibility of nature, or what is the same, of the world. Three years later, in 1900, in *On the theory of the law of the distribution of energy in the normal spectrum*, Planck discovered that thermal radiation imparts energy to matter in discontinuous quanta, hence in the next generation quantum theory was developed, and hence in the next generation the big bang theory was developed.

Back in 1897, in the *Lectures*, Planck writes: Clausius summed up the first law in that, that the energy of the world remains constant, the second law in that, that the entropy of the world strives for a maximum; and it has legitimately been objected that it makes no sense to speak without further ado of the energy or the entropy of the world, because such quantities cannot be defined precisely; but it is not difficult to formulate Clausius's propositions so that they make sense and express clearly what is essential in them (136); it can be said that the further extended in space the system under consideration is the more nearly constant its energy remains; the error is small when the energy of our solar system is considered, smaller when the system of all fixed stars known to us is considered, and in this way the

proposition that the energy of an infinitely large system, or the energy of the world, remains constant, has, in fact, a reference; analytically,

$$\lim_{R \rightarrow \infty} \frac{1}{E} \frac{dE}{dt} = 0,$$

where E is the total energy in the very large space R ; the proposition that the entropy of the world steadily increases can be understood in the same way (*id.*).

The error is small when the energy of the system of our 1 fixed star is considered. The error is infinitesimal when the energy of the system of the 2×10^{11} fixed stars in our galaxy is considered. There is no error when the energy of the system of the 2×10^{22} fixed stars in the 10^{11} galaxies in the world is considered.

Planck continues: finally, the principal question of the possible limits to the validity of the second law can be addressed; many scientists and philosophers maintain that there are such limits; if there are such limits, then, their existence can only be grounded either in an error in our starting point, the impossibility of a *perpetuum mobile*, or in a deficiency of our method of proof; the first objection is legitimate and no argument can remove it; the second objection, however, amounts to this, the practical impossibility of a *perpetuum mobile* is granted, but not the absolute impossibility, because with our limited experiments we cannot justify adequately the ideal processes assumed in our proof of the real construction of a *perpetuum mobile*, and this objection is unsound; it is altogether inconsistent to suppose that the validity of the law depends on the greater or lesser art of observation and experiment of physicists and chemists; the content of the law has nothing to do with experiment; it says, there exists in nature a quantity which in all changes in nature changes always in the same direction, and this proposition is either true or false, but it is what it is, true or false, whether thinking and measuring beings exist on earth or not, and whether, if they exist, they can control the particulars of physical and chemical processes once, or twice, or a hundred times more precisely than we can; the limits of the law, if it has any limits at all, can lie only in that domain in which its content lies, in observed nature and not in observing man (136, 7th ed.).

The first objection is legitimate and no argument can remove it. It is legitimate to produce a *perpetuum mobile* and thus remove the second law of thermodynamics, and all laws, and the acquisition of knowledge in its entirety. It has been done.

The second objection, on the other hand, is unsound. Even if many scientists and philosophers have maintained it since Protagoras, and maintained it at Planck's time already on the unshakable ground of the restored to the substantiality and purity of its being spirit.

The system of the world is the world, because energy and entropy have packed the space-time continuum.

Conversely, the world is the system of the world, because the second objection is unsound. Sound is the method. The laws hold absolutely.

The connection of the laws of the motions of the heavenly bodies and of thermodynamics is not only in that they are clauses of the law, the former less general, the latter more general. It is also

in that the latter have been discovered because the former have been. And in this the connection is as direct as this. In his review of *The mathematical principles* Halley comments on its first book: all is found to agree with the phenomena of the celestial motions as discovered by the great sagacity and diligence of Kepler (*id.*), and on its third book: here the veracity of the hypothesis of Kepler is demonstrated (*id.*). Maxwell writes in the *Theory of heat*: in a scholium to his third law Newton has stated the relation between work and kinetic energy in a manner so perfect that it cannot be improved (4). Newton writes in that passage: if action is reckoned by force and velocity jointly, and reaction is reckoned by the velocities of the individual parts and the forces of resistance arising from friction, cohesion, weight, and acceleration, action and reaction will be equal in all examples of devices and machines (*id.*). Newton has anticipated the principle of conservation of energy in a manner so perfect that it cannot be improved.

Descartes

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One generation after Bacon, Galileo, and Kepler, the labors of Descartes are the synthesis of the labors of Bacon, Galileo, and Kepler.

Descartes is the second most important philosopher, after Aristotle.

Aristotle's importance his successors took a long time to establish. In the half millennium after him the schools dominated philosophy, and in the schools everything was adverse to the advancement of knowledge. Then philosophy was transformed into Christian philosophy. Then the Muslims settled, ibn Khaldun reports, and began to desire to study philosophy. And twelve and a half centuries after Aristotle, al-Farabi established the importance of Aristotle. Importance is established by the labors of the philosophers, by the labors of the philosophers cumulatively, and the importance of Aristotle has been established by al-Farabi, ibn Sina, and their successors, Gerard, ibn Rushd, Albert, Aquinas, Scotus, Occam, and

their successors, Bacon, Galileo, Kepler, and their successors.

Descartes's importance his successors established immediately. In the generation after him alone, Wallis, Picard, Pascal, Cassini, Redi, Boyle, Ray, Malpighi, Huygens, Richer, Spinoza, Pufendorf, Leeuwenhoek, Hooke, Burnet, Swammerdam, Steno, all build on and continue his labors, essentially and substantially, in philosophy and all its parts, physics and mathematics and history, and in practical philosophy, too.

The most important of Descartes's treatises is *The principles of philosophy*, in 1644. Its title is, among the many precedents Descartes established, one of those that have found especial favor with his successors.

Here are thirty-two of the emulations, emulations of the emulations, and emulations of the emulations of the emulations.

In 1663, Spinoza published the *Parts I and II of Descartes's Principles of philosophy demonstrated in the geometrical manner*.

In 1680, Boyle, who had published *The sceptical chemist* in 1661, supplemented the title of its second edition with *to be the true principles of things*.

In 1687, Newton published *The mathematical principles of natural philosophy*.

In 1687, Leibniz published *On a general principle useful for the explanation of the laws of nature through the divine wisdom*. In 1714, he published *The principles of nature and grace founded on reason*, and *The principles of philosophy, or monadology*.

In 1697, Stahl published the *General theory of fermentation, in which its causes and effects are presented from their mechanico-physical principles*.

In 1710, Berkeley published *A treatise concerning the principles of human knowledge*.

In 1725, Vico published *The principles of a new science dealing with the common nature of nations*.

In 1747, Frederick II published *The general principles of war, applied to the tactics and discipline of the Prussian troops*.

In 1751, Hume published *An inquiry concerning the principles of morals*.

In 1755, Kant published *A new elucidation of the first principles of metaphysical knowledge*. Fond of *Critiques* later on, he was of *Principles* early on. In 1764, he published the *Investigation on the clarity of the principles of natural theology and morality*. In 1770, he published *On the form and principles of the sensible and intelligible world*.

In 1765, Euler published the *Theory of the motion of solid or rigid bodies, established from the first principles of our knowledge*.

In 1798, Fichte published *The system of the theory of morals according to the principles of the theory of science*.

In 1798, Malthus published *An essay on the principle of population*.

In 1821-2, Schleiermacher published *The Christian faith, presented in connection with the principles of the evangelical church*.

In 1829, Gauss published the *General principles of the theory of the shape of fluids in the state of equilibrium*.

In 1829, Lobachevsky published *On the prin-*

ciples of geometry. It was ignored, and in 1836-8, he published the *New principles of geometry, with a complete theory of parallels*.

In 1830-3, Lyell published the *Principles of geology*. The inspired full title of the first edition, *Principles of geology, being an attempt to explain the former changes of the earth's surface by reference to causes now in operation*, he changed in later editions, heeding criticism which needed no heeding, to *Principles of geology, or the modern changes of the earth and its inhabitants considered as illustrative of geology*.

In 1889, Peano published *The principles of arithmetic presented by a new method*.

In 1894, Hertz published *The principles of mechanics, in new interdependence*.

In 1903, Moore published the *Principia ethica*.

In 1903, Russell published *The principles of mathematics*.

In 1907, Einstein published *On the inertia of energy required by the principle of relativity*, and *On the principle of relativity and the conclusions drawn from it*, in 1910, *The principle of relativity and its consequences in modern physics*.

In 1910-3, Russell and Whitehead published the *Principia mathematica*. Borrowing either from Newton borrowing from Descartes or from Moore borrowing from Newton borrowing from Descartes.

In 1930, Heisenberg published *The physical principles of quantum theory*.

In 1930, Dirac published *The principles of quantum mechanics*.

Another fact indicating Descartes's importance is this. In the first and second stages of the war for the restoration of spirit to the substantiality and purity of its being the archenemy Berkeley, Goethe, and Hegel fought was Newton. In the concluding stage of the war Dilthey traced the evil back to its root, Descartes. And Heidegger writes: Descartes's feigned new beginning unveils itself as a baleful prejudice (*Being*, 6).

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Descartes begins in *The principles*: philosophy signifies the study of wisdom, and by wisdom is understood not only prudence in our affairs, but the perfect knowledge of all the things that man can know, for the conduct of his life, the preser-

vation of his health, and the discovery of all the arts; to be such knowledge must be deduced from first causes; and to try to acquire it, which is what is properly called to philosophize, it is necessary to begin with the search for these first causes, and that is, for the principles (pr.).

Philosophy is the perfect knowledge of all the things that can be known. And to be such it must be deduced from the principles.

Descartes continues: the principles must meet two conditions, first, they must be so clear and evident that, upon applying itself attentively to their consideration, the human mind cannot doubt their truth, and second, knowledge of other things must depend on them, so that they can be known without knowing those things, but not conversely, those things cannot be known without knowing them (*id.*).

The principles are so clear that, upon attentive consideration, their truth cannot be doubted. And other things cannot be known without knowing them.

Thus, there are the principles which Descartes discovered. Hence, there are the things that have become known with knowing the principles. And hence, philosophy has become, in fact, the perfect knowledge of all the things that can be known.

Again, in order to show this Descartes's advance can be shown.

Descartes did not write as much as other philosophers have. It is one reason for the quality of what he wrote. He did not publish all that he wrote either. It is one reason for the quality of what he published.

His very important treatises are six, and they show the steps in the advance, as they will, being Descartes's, with clarity and distinction.

He wrote the *Treatise on clear and useful rules for the direction of the mind in the search for truth*, in 1628. Published posthumously, it would have been the most important treatise on the method, had Descartes not published the treatises he published.

He wrote *The world, or a treatise on light and the other principal objects of sense*, in 1633. Also published posthumously, it would have been his most important physical treatise, had Descartes not published the treatises he published.

He published the *Discourse on the method to*

conduct one's reason well and search for truth in the sciences, as well as the Optics, Meteorology, and Geometry, which are essays in this method, in 1637.

He published the *Meditations on the first philosophy, in which the existence of god and the distinction between man's soul and body are demonstrated*, in 1641.

He published *The principles of philosophy*, in 1644.

And he published *The passions of the soul*, in 1649.

He wrote also the important, if not very important, *Replies to objections raised by some men of learning against the Meditations*, in 1641, *The description of the human body and all its functions, those that do not depend on the soul and those that depend on it, as well as the principal cause of the formation of its members*, in 1648.

He wrote also *A compendium of music*, in 1618, *Comments on a broadsheet published in Belgium at the end of 1647, entitled An account of the human mind, or rational soul, which explains what it is and what it can be*, in 1648, *The search for truth by the natural light*, in 1650.

He also left behind many letters. Adam and Tannery, in 1887-913, edited Descartes's corpus. The letters comprise 5 of the 11 volumes, I-V. Among his correspondents are Mersenne, Hobbes, Desargues, Fermat, Roberval, Arnauld, Pascal, Huygens.

The principles of philosophy is in four parts. Parts I and II contain the principles. Parts III and IV contain the things that have become known with knowing the principles.

Part I is *On the principles of human knowledge*. The principles of human knowledge are the principles of the acquisition of knowledge, and the principles of the acquisition of knowledge Descartes established in the *Rules for the search for truth* already with such clarity and distinction that, upon attentive consideration, their truth cannot be doubted. Nevertheless, he formulated them in *The method* with much greater clarity and distinction. Nevertheless, never mind the human mind, the philosophers found it difficult to apply themselves attentively to their consideration. Descartes presented them with yet greater clarity and distinction in the *Meditations*. The difficul-

ties persisted. Descartes presented them with yet greater clarity and distinction in *The principles*. The difficulties have persisted.

Part II is *On the principles of the material things*. Descartes's labors at these principles began with *The world*, continued with *The method*, and were completed with *The principles*.

Part III is *On the visible world*, part IV is *On the earth*. The things that had become known with knowing the principles Descartes began gathering in *The world*, continued gathering in *The method*, and *The principles* abounds in these things. These things cannot be known without knowing the principles. As these things become known with knowing the principles, however, so more of these things can become known with knowing the principles, and more of these things with knowing these things. These things were a program of discovery and investigation of the greatest depth and breadth. Descartes began work on this program in his treatises after *The principles*, and this program supplied the problems to which all of his successors turned.

The investigation of the principles of the acquisition of knowledge advanced greatly with Bacon. It advanced greatly further with Descartes. In turn, Spinoza and Leibniz built on him in this.

The investigation of the principles of the material things advanced greatly with Galileo and with Kepler each. Descartes is a synthesis of Galileo and Kepler in this, and a great further advance. In turn, Huygens, and then Newton and Leibniz built on him in this.

And in the advance of the investigation of the principles of the acquisition of knowledge in their dependence on the principles of the material things and of the investigation of the principles of the material things in their dependence on the principles of the acquisition of knowledge is Descartes's synthesis of Bacon, Galileo, and Kepler. It is his greatest advance. In this his only precursor is Aristotle. And in this his remains the greatest advance.

First, then, the principles can be shown, and first the principles of the acquisition of knowledge, then the principles of the material things. Then the things that have become known with knowing the principles. Then the perfection of philosophy.

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Beginning to end Descartes searches for truth, and finds it. The *Rules* are *Rules for the search for truth*. The *method* is *The method to search for truth*. And *The principles*: lay the foundations of truth (ded.).

Knowledge is truth. The principles of the acquisition of knowledge are the principles of the acquisition of truth.

And in the *Rules* already Descartes has found the principles of the acquisition of knowledge.

He begins: it is wrong to study each science separately (1); knowledge of one truth does not hinder the discovery of another, it helps it (*id.*); indeed, all sciences are so closely interconnected, it is easier to learn them all together than to separate one from the rest (*id.*).

It is a fundamental presumption of Aristotle, that all sciences are interconnected. It is a fundamental presumption of Bacon as well.

It follows, it is easier to learn them all together.

It is nevertheless much easier, however, not to learn them at all. And easiest of all is, as some of the followers of Pythagoras already found out: to follow precepts as divine revelations, say nothing of their own, think it wrong to do so, and consider whoever knows the greatest number of precepts the wisest. Two and a half centuries after the beginning of philosophy the consequences of this fact have not asserted themselves with all their irresistible force, and Aristotle does not have to deal with them. Twenty-two centuries after the beginning of philosophy, nineteen and a half after Aristotle, the consequences of this fact have asserted themselves, and reasserted themselves, and reasserted themselves, with all their irresistible force, and Bacon and Descartes have to deal with them.

Bacon begins with: the idols and false notions which are in possession of the human understanding and so beset it that truth can hardly find entrance. And concludes: nothing remains but to try the whole thing anew and undertake the complete instauration of the sciences, arts, and all human learning on proper foundation.

Descartes reports in *The method*: my entire goal was to cast aside the loose earth and the sand

in order to find the rock and the clay (3).

He cast aside the loose earth and the sand. And he found the rock and the clay. The clay is geometry, the rock is arithmetic. He continues in the *Rules*: arithmetic and geometry alone of all sciences are free from falsehood and uncertainty (2).

Nothing remains but to try the whole thing anew. The goal is to build on the clay and the rock.

Bacon writes: we are zealous to recede as little from antiquity as is compatible with truth; it is good to review what is received. Descartes writes: we should read the ancients because it is a great advantage to make use of the labors of so many men before us (3). Even if: there is the danger, if we study them too much, that their errors will infect us (*id.*).

Zealous to recede as little from antiquity as is compatible with truth, Bacon builds on Aristotle. Having found the clay and the rock, Descartes builds on Euclid, Archimedes, Apollonius, Ptolemy.

The true way begins with the senses and the collection of the particulars. It obtains principles from the particulars by continuous and gradual ascent and ends with the most general principles. It establishes stages of certainty, ascends from experiments to principles, descends from these principles to new experiments, ascends again, descends, ascends, and until it reaches the law with its clauses.

And in the *Rules* already Descartes describes the true way more explicitly.

He writes: according to its different functions, the same faculty is called pure intellect, imagination, memory, perception (12). Aristotle associates different faculties with different stages in the acquisition of knowledge. Descartes does not. It is a most important result. Distinctions between faculties are in relation to animals, including men. In relation to the search for truth, and the acquisition of knowledge, the same faculty is called pure intellect, imagination, memory, perception. Faculties can distinguish kinds of animals. They cannot distinguish kinds of truth, or knowledge. In the investigation of animals, including men, the distinction of faculties may be essential. In the investigation of the search for

truth, and the acquisition of knowledge, the distinction of faculties cannot be essential. Just as it cannot be essential whether an animal, and whether a man or another, or another machine searches for truth, and acquires knowledge. Essential are truth, and knowledge.

Descartes writes: intuition is what a pure and attentive mind, proceeding by the light of reason alone, conceives without doubt; because it is simpler, it is more certain than deduction; thus, everyone can intuit that he exists, that he thinks, that a triangle is bounded by three lines, that a sphere is bounded by a single surface, and the like (3).

Intuition had long been the intuition of souls and spirits, not of triangles and spheres. Accordingly, as soon as Descartes published, that intuition, of souls and spirits, not of triangles and spheres, read itself in Descartes. Having read itself in Descartes as soon as Descartes published, the same intuition, of souls and spirits, not of triangles and spheres, has continued to read itself in Descartes.

Intuition is what a pure and attentive mind, proceeding by the light of reason alone, conceives without doubt. But the same faculty is called pure intellect and perception. Intuition is what a pure and attentive mind, or a pure and attentive perception, proceeding by the light of reason alone, or by the light of perception alone, conceives, or perceives, without doubt. Without doubt, exclusively, is what makes it what it is.

In the *Rules* already, in fact, Descartes is explicit: we can make a true and certain judgment only by enumeration (7); enumeration, or induction, is the examination of everything relevant to the problem under investigation with such diligence and accuracy as to make it certain and manifest that nothing has been omitted (*id.*), and: the two operations of intuition and enumeration complement and perfect one another, and so thoroughly as to coalesce into a single operation (11).

Thus, everyone can intuit that he exists, that he thinks, that a triangle is bounded by three lines, that a sphere is bounded by a single surface, and the like. Everyone can, but not everyone will. Intuition and enumeration coalesce into a single operation, and enumeration, or induction, is the

examination of everything relevant to the problem under investigation with such diligence and accuracy as to make it certain and manifest that nothing has been omitted. And if anyone who can intuit will intuit, there is, alas, no helping the examination of everything relevant to the problem under investigation with such diligence and accuracy as to make it certain and manifest that nothing has been omitted. And everything relevant is in Euclid and in Euler, not in Augustine and in Hegel.

We can make a true and certain judgment only by enumeration.

Enumeration, or induction, is the examination of everything relevant to the problem under investigation with such diligence and accuracy as to make it certain and manifest that nothing has been omitted.

What a problem is is clear: a problem is everything in which there is truth or falsehood (13).

Intuition and enumeration coalesce into a single operation.

To what end is also clear: the first principles are known only by intuition (3).

On the other hand: remote conclusions are known only by deduction (*id.*); and: if performed by an intellect in the least degree rational, deduction can never be wrong (2); errors are never due to faulty inference (*id.*).

And: knowledge is complete when all particulars relevant to the problem are exhibited by a continuous and uninterrupted survey in a sufficient and well-ordered enumeration (7).

It is a description of the true way.

The true way consists of induction and deduction. Induction is enumeration. It reaches the first principles. What deduction is arithmetic and geometry show. It begins with the first principles. Both induction and deduction are the solution of problems. The true way is complete when all particulars relevant to the problem are exhibited by a continuous and uninterrupted survey in a sufficient and well-ordered enumeration.

With this the description is not more explicit than Bacon's. With the following, however, it is.

The following is the discovery of the rules of the method.

Descartes writes: method is necessary to search for truth in things (4); by method I under-

stand the certain and easy rules, which, if followed exactly, will never allow the false to be taken for true (*id.*); the method cannot teach intuition and deduction; if the intellect is incapable of these, it will not comprehend the rules of the method however easy they may be (*id.*); the great minds of the past were aware of the method; arithmetic and geometry are spontaneous fruits of the innate principles of the method; in our time algebra is another attempt to revive the method (*id.*); and when I considered the matter carefully, I saw that mathematics is concerned with nothing but problems of order and measure, whether these involve numbers, or figures, or stars, or sounds, or anything else; and I saw that there must be general knowledge of order and measure (*id.*).

Mathematics, whether it is concerned with numbers, figures, stars, sounds, or anything else, is the spontaneous fruit of the principles of the method. Conversely, mathematics led Descartes to the discovery of the rules of the method.

The method is the certain and easy rules, which, if followed exactly, will never allow the false to be taken for true. There is general knowledge of order and measure. The rules are the rules of order and measure.

In particular:

the whole method consists in the ordering and arrangement of things; in this one rule all human industry is contained (5).

This one rule: must be followed as staunchly in the search for knowledge as the thread of Theseus in the labyrinth (*id.*); and this goes also for the philosophers who ignore experience and think that truth will spring from their heads like Minerva from the head of Jupiter (*id.*). And:

the secret of the method is to attend to the simplest (6); this rule instructs us that things can be arranged in their series and order (*id.*); for we propose not to search for things singly, but to know them in their connection with one another (*id.*).

The whole method consists in the ordering and arrangement of things, and the secret is to attend to the simplest which indicates the series and order of things.

If one is incapable, or intolerant, of intuition and deduction, the revelation of the thread of Theseus and the secret will not help. Otherwise,

with the revelation of the thread and the secret the method is as easy to follow as this: the method is like the mechanical arts which need no method other than themselves to instruct them, and no instruments other than their own to make their instruments; to become a blacksmith, for instance, one can use a stone for an anvil and a rock for a hammer at first; also, he would make a hammer and an anvil before trying to make swords and helmets (8).

But a stone and a rock.

With this the description of the true way is not only more explicit. It is as explicit as it has to be.

The method needs no method other than itself to instruct itself.

It consists wholly in the ordering and arrangement of things. It can thus begin with the ordering and arrangement of any things. The secret is to attend to the simplest which indicates the series and order of things. The ordering and arrangement of things is thus enumeration, or induction. Enumeration, or induction, is the examination of everything relevant to the problem under investigation with such diligence and accuracy as to make it certain and manifest that nothing has been omitted. And the enumeration, or induction, instructs itself. Attending to the simplest instructs which the problem under investigation is, instructs which everything relevant to the problem is, and instructs which the diligence and accuracy are such as to make it certain and manifest that nothing has been omitted. The secret guides the enumeration, or induction, all along. At the end all particulars relevant to the problem are exhibited by a continuous and uninterrupted survey in a sufficient and well-ordered enumeration.

The two rules instruct themselves beginning to end.

They never allow the false to be taken for true.

The method needs no method other than itself. There are and can only be two ways to search for truth, the false and the true. There is and can only be one method, the method. The true way is the method, and the method is the true way.

The elements of the true way are the two rules. Ordering and arrangement, and the simplest indicating the series and order of things.

And the two rules are the rules of induction.

Because deduction, if performed by an intel-

lect in the least degree rational, can never be wrong. Errors are never due to faulty inference.

Because errors are always due to faulty induction. And the two rules are the end of error. If followed exactly, they will never allow the false to be taken for true.

Because we can make a true and certain judgment only by enumeration, or induction.

And because the first principles, true and certain judgments themselves, are known only by intuition, and intuition and enumeration, or induction, coalesce into a single operation.

Descartes's description of the true way is more explicit, and as explicit as it has to be, because he has found the clay and the rock. Geometry and arithmetic. Algebra also, which is an attempt to revive the method. Mathematics shows that there must be general knowledge of order and measure.

In the *Rules* already Descartes anticipates many of his other most important results, too. Among them these.

The most fundamental result in the *Geometry*, which is an essay in the method: the key is to treat the unknown terms as if they are known (17); the assumption is that we know that the unknown terms in the problem are so dependent on the known that they are entirely determined by them (*id.*).

The most fundamental result in *The principles of philosophy*: figure, extension, and motion are purely material (12).

The most misunderstood result in *The method*, in which the *Geometry* is an essay: if Socrates says that he doubts everything, it follows necessarily that he understands that he doubts, and therefore that he knows that something is either true or false (*id.*).

The principles of mathematics, then, are the principles of the acquisition of knowledge.

More explicitly, the two rules of the method are the principles of the acquisition of knowledge.

Still more explicitly, the ordering and arrangement of things, and attending to the simplest which indicates the series and order of things, are the principles of the acquisition of knowledge.

And Descartes will formulate the principles of the acquisition of knowledge still more explicitly in *The method*.

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Descartes published the *Discourse on the method to conduct one's reason well and search for truth in the sciences, as well as the Optics, Meteorology, and Geometry, which are essays in this method* in 1637.

And ever since 1637 *The method* has been examined, like everything else, in both ways, the false and the true.

The way of choice has not been the true. No doubt because: good sense is the best distributed thing in the world; everyone thinks he is so well endowed with it, even those who are otherwise the hardest to please do not wish to have more of it than they have (1). The way of choice has been to examine the introductory *Discourse on the method*, and ignoring the *Optics, Meteorology, and Geometry, which are essays in this method*, proceed to judgment. Alas, we can make a true and certain judgment only by enumeration, and enumeration is the examination of everything relevant to the problem under investigation with such diligence and accuracy as to make it certain and manifest that nothing has been omitted. Then again, the judgment need not aspire to truth and certainty. All it needs to do is confirm that one is so well endowed with good sense one cannot wish to have more of it.

Alternatively, *The method* can be examined by the continuous and gradual ascent from the *Optics, Meteorology, and Geometry* to the *Discourse*. Or what is the same, by conducting one's reason well and searching for truth in the *Optics, Meteorology, and Geometry* not less than in the *Discourse*. Or what is yet the same, with such diligence and accuracy as to make it certain and manifest that nothing has been omitted.

While still laboring on what was to become *The method*, Descartes writes to Mersenne: so that you know what I want, there will be four treatises under the title *The project for a general science capable of raising our nature to its highest degree of perfection, with the Optics, Meteorology, and Geometry, in which, in order to present the science, the author explains the most singular matters he could find, and so that even those who have never studied them can understand them* (Mar 1636).

A pity he abandoned this title. Not because of

its unwarranted respect for our nature, or for the understanding of those who have never studied, or will ever study, the matters presenting the science. Because the author does explain the most singular matters.

After publication, Descartes writes to Mersenne: since you want my opinion, in the *Optics* and *Meteorology* I have only tried to show that my method is better than the ordinary, but in the *Geometry*, I say, I have shown it; right at the beginning I solve a problem which on the authority of Pappus none of the ancients was able to solve, and none of the moderns has been able to solve it either; moreover, what I propose in book II on the nature and properties of curves is, it seems to me, as far from ordinary geometry as the rhetoric of Cicero is from the ABC of children (Dec 1637).

These last are of all the most singular matters which the author explains the most singular. And if what he proposes on the nature and properties of curves is as far from ordinary geometry as the rhetoric of Cicero is from the ABC of children, it is also as far from ordinary geometry as ordinary geometry is from the rhetoric of Cicero.

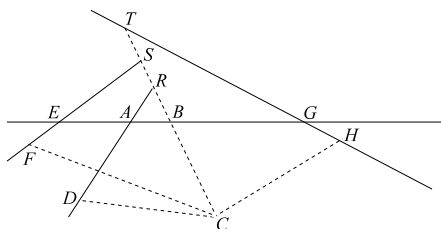
The problem which none of the ancients was able to solve, and none of the moderns has been able to solve either, is this: given any number of straight lines in position, as many angles, and a ratio, to find the locus of points in the plane such that the straight lines drawn from any of the points make with the given straight lines the given angles, and the product of some of these lines is to the product of the rest of them in the given ratio (*Geometry*, I).

For instance, given the one line AB , to find the locus of points such that, if one of the points is C , angle CBA be equal to the given angle and CB be equal to the given ratio, given the two lines AB and AD , to find the locus of points such that angles CBA and CDA be equal to the given angles and $CB:CD$ be equal to the given ratio, given the three lines AB , AD , and EF , to find the locus of points such that angles CBA , CDA , and CFE be equal to the given angles and $CB \times CD:CF$ be equal to the given ratio, given the four lines, AB , AD , EF , and GH , to find the locus of points such that angles CBA , CDA , CFE , and CHG be equal to the given angles and

$CB \times CD : CF \times CH$ be equal to the given ratio, and so on.

For one line the locus is two parallel lines, for two lines the locus is two intersecting or parallel lines. Pappus writes: for three or four lines the locus is one of the conic sections, parabola, hyperbola, or ellipse; but what it is for more than four lines is not known (*Collection*, VII, 36-7).

Thirteen centuries later, Descartes writes:



let $AB, AD, EF, GH, \dots$, be the lines given in position;

I assume the problem solved; and because so many lines are confusing, I consider two of them the principal, and relate all other lines to these two; so let segment AB be x , segment BC be y , and produce all other lines to meet these two, themselves produced if necessary; now, all angles of triangle ABR are given, so the ratio $AB:BR$ is known; let $AB:BR = a:b$, and since $AB = x$, $BR = \frac{bx}{a}$;

hence $CR = y + \frac{bx}{a}$;

or $CR = y - \frac{bx}{a}$, or $CR = -y + \frac{bx}{a}$, depending on

which of the three points, B , or R , or C , is between the other two; and similarly in all similar cases below;

triangle CDR is also given, so $CR:CD$ is known;

and if $CR:CD = a:c$, then $CD = \frac{acy + bcx}{a^2}$;

lines AB, AD , and EF are given in position, so the distance AE is known; let $AE = k$, and then $BE = k + x$;

triangle BES is given, so if $BE:BS = a:d$, $BS = \frac{dk + dx}{a}$; and hence $CS = \frac{ay + dk + dx}{a}$;

triangle CFS is given, so if $CS:CF = a:e$, $CF = \frac{aey + dek + dex}{a^2}$;

distance AG is given, and if $AG = l$, $BG = l - x$;

triangle BGT is given, so if $BG:BT = a:f$, $BT = \frac{fl - fx}{a}$; and hence $CT = \frac{ay + fl - fx}{a}$;

triangle CHT is given, so if $CT:CH = a:g$, $CH = \frac{agy + fgl - fgx}{a^2}$;

and so you see that no matter how many lines are given, the length of any line through C can be expressed by three terms, one containing the unknown quantity x , another containing the unknown quantity y , and the third containing only known quantities; except when some of the lines are parallel and one or another of the terms vanishes, but this case is too simple to need further explanation;

you see also that in the product of any number of these lines the power of any term containing x or y will not be greater than the number of lines whose product it is; thus, no term will be of a power higher than the second if two lines are multiplied, no term will be of a power higher than the third if three lines are multiplied, and so on to infinity (I);

considering again the given lines AB, AD, EF , and GH , in order to find the locus, we have to find $CB \times CD : CF \times CH$;

this is now equivalent to

$$y^2 = \frac{(acfgl - a^2dek)y - (a^2de + acfg - abcg)xy + (bcfgl)x - (bcfg)x^2}{(a^3e - a^2cg)};$$

all terms in parentheses are known, to shorten the work we can write $y^2 = Fy - Gxy + Hx - Ix^2$;

and the square root is $y = A - Bx + \sqrt{C + Dx + Ex^2}$;

to determine a point a single equation containing two unknown quantities is necessary;

and all points of a line, whether a straight line or a curve, can be expressed by a single equation (II);

thus, for three or four lines the locus is a parabola, hyperbola, or ellipse, but can also be a circle or a straight line; for five, six, seven, or eight lines it is a curve of the second kind, but can also be a conic section, circle, or straight line; for nine, ten, eleven, or twelve lines it is a curve of the third kind, but can also be one of the lower kinds; and so on (I);

the classification of geometrical curves follows from their nature; if the equation of a curve contains no term of a power higher than the second in one or both of the unknown quanti-

ties, the curve is of the first and simplest kind, comprising the circle, parabola, hyperbola, and ellipse; if the equation contains terms of the third or fourth power in one or both of the unknown quantities, the curve is of the second kind; if the equation contains terms of the fifth or sixth power in one or both of the unknown quantities, the curve is of the third kind; and so on to infinity (II).

To help them appreciate Cicero, at least, if they will not appreciate Descartes, the children can be told that: I solved the problem in five or six weeks (to Mersenne, 5 Apr 1632).

The most singular matters which the author explains are also the foundation of the theory of polynomial equations. The fundamental result is: every equation can have as many distinct roots as is the power of the unknown quantity (III).

They are also a major step in the development of functions.

And Descartes generalizes further yet: to comprehend all curves that there are in nature together and order them into kinds, I say that all points of any geometrical curve, and that is, any curve which admits of exact measurement, necessarily have a definite relation to all points of a straight line, and this relation can be expressed by a single equation (II).

On this result all subsequent mathematics and physics rests.

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In the *Rules* already, casting aside the loose earth and the sand, Descartes has found the rock and the clay. Arithmetic and geometry alone of all sciences are free from falsehood and uncertainty.

To search for truth method is necessary. Method is the certain and easy rules which, if followed exactly, will never allow the false to be taken for true.

Arithmetic and geometry are spontaneous fruits of the principles of the method. So is algebra. Descartes has considered the matter carefully, and seen that mathematics is concerned with nothing but problems of order and measure.

Hence the rules of the method. The principles. The whole method consists in the ordering and arrangement of things, and the secret is to attend

to the simplest which indicates the series and order of things.

Nine years later: in the *Geometry*, I say, I have shown that my method is better than the ordinary. More generally: in the *Geometry* I endeavor to give a general way for solving all problems that have never been solved (to Mersenne, Mar 1636). And he has given a general way for solving all problems that have never been solved.

Descartes's success can be explained. If arithmetic and geometry are free from falsehood and uncertainty, have been free from falsehood and uncertainty for twenty-one and a half centuries by Descartes's time, counting since Pythagoras, for forty-seven centuries, counting since the predecessor of the philosophers who left behind the 3 oldest calculations of area, and mathematics has advanced in the century before Descartes through Cardano, and Viète, and Galileo, and Kepler, and the principles of mathematics are the principles of the method, Descartes's success is explained. Mathematics is the spontaneous fruit of the principles of the method, and Descartes brought the spontaneous fruit under cultivation.

The fact of the fruit can also be explained. The fact that there is freedom from falsehood and uncertainty. That there is truth and certainty.

Descartes explained this fact. In the introductory *Discourse on the method*.

He writes: I had to seek a method which has the advantages of logic, geometry, and algebra and is free from their defects (2); and I found that, given the firm and constant resolution not to fail to follow them even once, these four rules are sufficient;

first, to include in judgments nothing but what is

so clear and distinct that it cannot be doubted;

second, to divide difficulties into parts such as will lead to their better solution;

third, to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex;

and last, always to make enumerations so complete and reviews so general as to omit nothing (*id.*).

These are the rules.

And following the last rule, the four rules are an enumeration so complete and a review so general as to omit nothing. Given the firm and

constant resolution not to fail to follow them even once, they are sufficient.

Following the third rule, they proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex. The third rule is thus also the foundation of the last. The firm and constant resolution to follow the last depends on the firm and constant resolution to follow the third. The gradual ascent from the simplest to the more complex is the foundation of the completeness of enumerations and generality of reviews. The last and the third rules, also, affirm more explicitly what the earlier rules, those of the *Rules*, affirm. The second and the first rules affirm more.

Following the second rule, to divide difficulties into parts such as will lead to their better solution, the second rule is the foundation of the third, and the first rule, to include in judgments nothing but what is so clear and distinct that it cannot be doubted, is the foundation of the second. The division of difficulties into parts such as will lead to their better solution is the foundation of the gradual ascent from the simplest to the more complex. And the clarity and distinction that cannot be doubted is the foundation of the division of difficulties into parts such as will lead to their better solution.

Thus, following the first rule, to include in judgments nothing but what is so clear and distinct that it cannot be doubted, the first rule is to include in judgments nothing but what is so clear and distinct that it cannot be doubted.

To explain mathematics is to explain truth and certainty. To explain truth and certainty is to explain the method. To explain the method is to explain the rules of the method. To explain the rules of the method is to explain the first rule of the method. And to explain the first rule of the method is to explain the clarity and distinction that cannot be doubted.

That knowledge, all knowledge, truth, any one truth, certainty, ultimate certainty, mathematics, and the method follow from one another Descartes affirms everywhere. In the *Discourse*, he formulates the rules, and affirms it once again. He writes: all things which fall under the knowledge of men follow from one another; there can thus be nothing either so remote that it cannot be

reached or so hidden that it cannot be discovered (*id.*); there is but one truth of each thing, and whoever finds it knows as much of it as can be known of it; a child instructed in arithmetic, having done a sum following the rules, can be certain of having found, concerning the sum it has examined, everything that the human mind will ever find (*id.*); and the method contains everything that gives certainty to the rules of arithmetic (*id.*).

He continues: in order to devote myself solely to the search for truth, I thought it necessary to do the opposite, and reject as absolutely false everything that I could imagine in the least doubtful, so as to see whether anything remained that was entirely indubitable; I decided to suppose that nothing was such as our senses made us imagine (4); to reject as false all arguments which I had previously taken for demonstrations (*id.*); to pretend that all things that had ever entered into my mind were not true any more than the illusions of my dreams; and I saw immediately that, just as I thus tried to think everything false, it was necessary that I, who thought it, was something; I noticed that this truth, I think therefore I am, is so firm and certain that all the most extravagant suppositions of the sceptics cannot sway it, and I judged that I can accept it without reservation as the first principle of the philosophy I sought (*id.*).

I cannot doubt everything. The principle of contradiction will not let me. The best that I can do is doubt everything except that I doubt everything. I cannot think everything false. The principle of contradiction will not let me. The best that I can do is think false everything except that I think true that I think false everything. Therefore I am.

Descartes has taken the first step toward this result in the *Rules*: if Socrates says that he doubts everything, it follows necessarily that he understands that he doubts, and therefore that he knows that something is either true or false. And after much misunderstanding of the *Discourse*, he will make the result more explicit in the *Meditations*, and *The principles*.

He continues in the *Discourse*: next I examined attentively what I was (*id.*); from that alone that I thought to doubt the truth of other things, it followed most evidently and most certainly that

I existed (*id.*); and from this I knew that I was a substance whose entire essence or nature is nothing but to think (*id.*); and I judged that I can take it as a general rule that the things which we conceive very clearly and very distinctly are all true (*id.*).

The first rule.

The first rule stands on the substance whose entire essence or nature is nothing but to think. The substance whose entire essence or nature is nothing but to think stands on the principle of contradiction.

Descartes continues: next, seeing clearly that to know is a greater perfection than to doubt, I resolved to search whence it was possible to think of something more perfect than I was (*id.*); this idea could only be put in me by a nature truly more perfect than I was, and which had in it all perfections which I could not have, and that is, god (*id.*); infinite, eternal, immutable, omniscient, omnipotent (*id.*).

The *Discourse* caused, and causes, much offence.

All I am is but thinking, nay, clearly and distinctly. There were not, and are not, many I's who would be but thinking, nay, clearly and distinctly.

It was well, and is well, that there is god, and he is infinite, eternal, immutable, omniscient, omnipotent. It was not so well, and is not so well, that he is the consequence of clarity and distinction. Or that what he has to do with me is be the consequence of my thinking. Galileo has allowed that next to the most obedient executrix of god's orders, there are the holy scriptures, even if, in order to accommodate the common understanding, they say many things different from the absolute truth. Kepler has allowed that the sun, the fixed stars, and the planetary system are the sign of the trinity. Descartes has no use for the trinity, and no use for the holy scriptures.

Four years later, in 1641, the *Meditations on the first philosophy, in which the existence of god and the distinction between man's soul and body are demonstrated* is dedicated: to the dean and doctors of the sacred faculty of theology at Paris (*id.*). And should the mere fact of the dedication fail to offend the dean and doctors with sufficient clarity and distinction, Descartes advises them: I

have always thought that these two matters, god and the soul, are the principal which must be demonstrated by the arguments of philosophy rather than theology (*id.*).

Nor does Descartes discriminate in favor of the dean and doctors against anyone else: I have developed a method for resolving all kinds of difficulties in the sciences; and I thought it was my duty to apply it to the matters at hand (*id.*); I will add that I think my demonstrations are such that I do not think the human mind can discover better ones (*id.*); the demonstrations of Archimedes, Apollonius, Pappus are accepted as certain and evident by everyone, yet few understand them; similarly, although my demonstrations are equal in certainty and evidence to those of geometry, or even surpass them, I fear not many will understand them (*id.*); the judgment of many is so feeble, they will rather be persuaded by the first opinion they encounter, however false and remote from reason it may be, than by a solid and true refutation of their opinions (*pr.*).

Such are the odds, then, against which Descartes tries again. It is his duty.

He writes: I establish it as a general rule that true is everything that I perceive clearly and distinctly (3).

Because: even if some malignant spirit of the utmost power and cunning employs all his industry to deceive me (1); as soon as I turn to things which I perceive clearly, let whoever will try to deceive me, he will never bring it about that I am not while I think that I am; or make it ever true that I never was if it is now true that I am; or make it true that 2 and 3 add up to more or less than 5; or make true anything in which I recognize a manifest contradiction (3).

Truth stands on clarity and distinction, clarity and distinction stands on the principle of contradiction.

Further: when I think a triangle, even if no such figure exists or has ever existed without my thought, there is still its determinate nature, or essence, or form, immutable and eternal, which is not an effect of me and not dependent on my mind (5); my thought cannot impose necessity on things (*id.*); it must be concluded that god necessarily exists (3); every clear and distinct perception must necessarily have god for its author,

because god, the sum of perfection, cannot on pain of contradiction be false, and the perception must therefore be true (4).

God, too, stands on the principle of contradiction. More particularly, truth and certainty stands on clarity and distinction, clarity and distinction stands on the substance whose entire nature is to think, the substance whose entire nature is to think stands on god, god stands on the principle of contradiction.

Spinoza, in the next generation, will take the final step: god is nature. Descartes, however, has already taken the step: nature is nothing but god himself (6).

Moreover, he has affirmed in the *Rules*: the same faculty is called pure intellect, imagination, memory, perception. In the *Meditations*, he affirms early on: whatever I have accepted as true I have acquired either from the senses or through the senses (1). In order to take the decisive step in the argument he affirms: what am I? a thing that thinks; and what is that? a thing that doubts, understands, affirms, denies, wills, wills not, imagines, perceives (2). And in conclusion he affirms: the use of the senses is first, the use of reason follows; nothing is in the intellect that has not been in the senses (6).

The preestablished harmony of mathematics and physics follows: it is possible to have full and certain knowledge of all of corporeal nature which is the object of pure mathematics (5).

That the *Meditations* failed to change the odds, the *Replies to objections raised by some men of learning against the Meditations*, also in 1641, attests. Among the men of learning are Mersenne, Hobbes, Gassendi, Arnauld. Descartes has to write: one voyager returning from America who tells us that he has seen the antipodes deserves trust more than a thousand who deny their existence for no other reason than that they do not know them (6).

Two and a half centuries later Frege will write: I must apparently give up the hope of being read by all mathematicians who, upon encountering a logical expression, think *metaphysica sunt non leguntur*, and all philosophers who, upon seeing a formula, cry out *mathematica sunt non leguntur*. Descartes writes: my work is obscure even to the most learned because, if they are versed in

metaphysics, they abhor geometry, if they have studied geometry, they will not read first philosophy (*Principles*, ded.).

In 1644, in *The principles of philosophy*, in its first part, *On the principles of human knowledge*, Descartes reaffirms the earlier results with conclusive clarity and distinction: it is contradictory to believe that what thinks, at the same time at which it thinks, does not exist (7); it is contradictory that god is false (29); sense, imagination, and pure thought are only different modes of perception (32).

But he also generalizes further still, and further, and reaches the greatest generality.

He writes in the *Meditations*: the method is not new, for nothing is older than truth (ded.). In the *Replies*: there is nothing in their philosophy which is not new, there is nothing in my philosophy which is not ancient; because I accept as principles only these which have been known to all philosophers and accepted by them generally, and which are therefore the most ancient of all (to Dinet).

And in *The principles*: although all truths which I include in my principles have always been known to everyone, no one until this time has recognized them as the principles of philosophy, and that is, that from them the knowledge of all other things in the world can be deduced (pr.).

All truths included in the principles have always been known to everyone.

Mathematics stands on truth and certainty, truth and certainty stands on clarity and distinction.

Clarity and distinction has always been known to everyone.

Clarity and distinction stands on the substance whose entire nature is to think, the substance whose entire nature is to think stands on god.

Nature is nothing but god himself.

Clarity and distinction has always been known to everyone by nature.

Descartes anticipates Hume. All reasonings are the effects of instinct, or mechanical tendency. And hence Darwin.

All truths included in the principles have always been known to everyone, but no one until Descartes has recognized them as the principles of philosophy, and that is, that from them the

knowledge of all other things in the world can be deduced.

That from the principles of philosophy the knowledge of all other things in the world can be deduced was recognized by Aristotle. He established that knowledge is acquired only by demonstration, demonstration is deduction, and deduction always depends on principles. He established also that the principle of contradiction is the principle of all principles.

To deduce knowledge of things in the world from principles the mathematicians among the philosophers had begun before Aristotle already. With Kepler they deduced knowledge of things in the world from principles as general as the laws of the motions of the heavenly bodies.

Descartes's advance is the recognition that these follow from one another. If from the principles of philosophy the knowledge of all other things in the world can be deduced, from these principles the principles from which the mathematicians have deduced knowledge of things in the world can be deduced. And conversely, if the mathematicians have deduced knowledge of things in the world from principles, from these principles the principles of philosophy from which the knowledge of all other things in the world can be deduced can be induced.

Hence Descartes's further advance, the first explicit step from mathematics toward the foundations of mathematics, or what is the same, toward logic, or what is the same, toward metaphysics. And conversely, the first explicit step from metaphysics, or logic, or the foundations of mathematics toward mathematics. It is Descartes's project for the general science capable of raising our nature to its highest degree of perfection that Leibniz develops into the conception of the general characteristic, or *calculus philosophicus*, so that: there will be no more need for disputes between two philosophers, it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate, that Frege develops into the formula language of pure thinking modeled on that of arithmetic, so that: all areas of knowledge are connected into a single domain through chains of inferences free of gaps.

Hence Descartes's further advance, the first explicit formulation of the principles of philoso-

phy. Of the principles of all principles.

And yet Descartes advanced further still. And to the greatest generality.

Descartes is the first philosopher to separate sharply the principles, on one side, from all other things in the world, on the other, and to separate sharply among the principles the principles of the acquisition of knowledge, on one side, from the principles of all other things, on the other.

No philosopher before him is clear about these distinctions. But a very few philosophers after him are clear about them.

He writes: from the principles of philosophy the knowledge of all other things in the world can be deduced.

Also: *The principles of philosophy* are divided into four parts; the first contains the principles of knowledge, which can also be called first philosophy, or metaphysics; the three other parts contain everything most general in physics, the first laws or principles of nature, how the heaven, the fixed stars, the planets, the comets, and the entire world are composed, and also the nature of this earth, and of air, water, fire, and the lodestone, which are the bodies most commonly found everywhere on earth, and also all the qualities observed in these bodies, light, heat, weight, and the like (pr.).

And the four parts are under the titles, part I *On the principles of human knowledge*, part II *On the principles of the material things*, parts III and IV *On the visible world* and *On the earth*.

All the following are as fundamental as there are results.

All things in the world are divided into the principles, on one side, all other things in the world, on the other. The principles are among all things in the world. But the principles cannot be like all other things in the world. All other things in the world are deduced from the principles. The principles are not deduced from the principles.

The relation of all other things in the world and the principles is deduction. Deduction of other things from the principles shows that the principles are principles.

The deduction of all other things in the world from the principles shows that the principles are the principles.

Generality shows the principles principles. The

greatest generality shows the principles the principles.

The principles cannot be indiscriminately principles. All other things in the world are deduced from the principles. The principles are thus of two kinds, the principles of deduction, and the principles of all other things.

More generally, the principles of knowledge, or first philosophy, or metaphysics, on one side. Everything most general in physics, the principles of the material things, the laws or principles of nature, on the other.

The principles of the acquisition of knowledge, and the laws of nature.

If they are of the greatest generality both, the principles of the acquisition of knowledge and the laws of nature cannot be deduced from one another.

Even if they are of the greatest generality both, however, that the principles of the acquisition of knowledge and the laws of nature are the principles of the same all other things in the world is so clear and distinct it cannot be doubted.

From the laws of nature all other things in the world are deduced. And from the principles of the acquisition of knowledge the same all other things in the world are deduced.

From all other things in the world the laws of nature are induced. And from the same all other things in the world the principles of the acquisition of knowledge are induced. Both by clarity and distinction, division of difficulties into parts, gradual ascent, and enumerations so complete and reviews so general as to omit nothing. Omit nothing of the same all other things in the world.

Moreover, also for the first time, Descartes formulates the principles of the acquisition of knowledge and the laws of nature, both, explicitly.

The principles of the acquisition of knowledge are:

first, to include in judgments nothing but what is so clear and distinct that it cannot be doubted; second, to divide difficulties into parts such as will lead to their better solution;

third, to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex;

and last, always to make enumerations so com-

plete and reviews so general as to omit nothing.

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The laws of nature are:

the first, any thing by itself always perseveres in the same state, and hence, once moved, always continues to move (*Principles*, II, 37);

the second, all motion is by itself in a straight line, and hence, a thing which moves in a circle always tends to recede from the center of the circle (39);

and the third, a body, upon encountering a stronger body, loses nothing of its motion, upon encountering a weaker body, loses as much of its motion as it transfers to the weaker body (40).

Nineteen centuries earlier, the investigation of motion begins with Aristotle. He obtained these general results. Motion is locomotion, if there must always be motion, there must always be locomotion. The primary motion is circular. Everything in motion must be moved by something. The first mover is at the circumference, moves the fixed stars, and so communicates motion to everything below.

Euclid writes: the stars are observed to move in circles. Ptolemy writes: the motion of the heavenly bodies is spherical; the ancients already saw that the heavenly bodies are carried from east to west in circles parallel to one another and always about the same center which thus became necessary to presume the pole of the heavenly sphere; moreover, if the heavenly bodies had any other kind of motion but circular, in a straight line, for instance, their distances from the earth would have appeared to vary; and no such variation is observed.

The ground of Aristotle's general results is, as Autolycus and Euclid show, observation. The heavenly bodies are observed to move. Their motion is observed to be locomotion. Their locomotion is observed to be circular. The multitude of the heavenly bodies is also observed, and given the multitude of the heavenly bodies, the generalizations follow, all motion is primarily locomotion, and all locomotion is primarily circular. Not all motion is observed to be circular locomotion. But if motion is communicated from

the heavenly bodies to everything below, the other results follow. Communicated below, motion degenerates from natural to violent, from locomotion to other kinds, and from circular to not circular. If motion is communicated, everything in motion is moved by something already in motion. And if there can be no infinite series, the first mover is above, at the circumference.

The theory of the motions of the moon and the sun was formulated by Hipparchus, the theory of the motions of the planets by Ptolemy.

And, on the one hand, the theory of the motions of the heavenly bodies corroborated the general results. The great success of the theory, its mathematical and hence explanatory power, is firmly grounded in the presumption that the primary motion is circular. The further development of the theory by Copernicus corroborated the general results in the same way. The great success of Copernicus is firmly grounded in the same presumption, that the primary motion is circular.

On the other hand, the greater the success of the theory the more widely it opened the abyss between its success and all other general results. The theory developed by Ptolemy, and further by Copernicus, is a theory of locomotion, but of the locomotion of the heavenly bodies alone. It has nothing to say about the locomotion of any other of the things in the world. It has nothing to say about the communication of motion, its communication in general, or its communication from the first mover to the heavenly bodies, or from one heavenly body to another, or from the heavenly bodies to anything further below. It has nothing to say about the degeneration of motion, from natural to violent, from locomotion to other kinds of motion, from circular to not circular. It has nothing to say about the relation of mover and moved in general, or of any particular mover and any particular moved.

Advances were made. Toward the motion of weights and fluids, by Archimedes, the motion of machines, by Hero, the motion of light, by Ptolemy. Three centuries before Galileo, Buridan, his contemporaries Bradwardine and Heytesbury, and in the next generation Oresme, made steps toward the formulation of the problems of velocity, acceleration, and collisions. In the generation

before Galileo, Gilbert investigated magnetic motion.

Not many advances. When they were first obtained, the general results were not obtained by the continuous and gradual ascent from the particulars, and the abyss between them and the particulars remained.

Galileo closed it: concerning the oldest object we set forth the entirely new science; for nothing in nature is older than motion, and about it volumes neither few nor small have been written by the philosophers; yet I find that its essential properties have not even been observed, let alone demonstrated.

Galileo observed its essential properties and demonstrated them.

And Descartes discovered the laws of nature.

The general result that motion is primarily locomotion was corroborated conclusively.

The other general results were variously, some of them greatly simplified, the rest shown: superfluous, and rather null.

The primary motion is not circular, it is in a straight line. The composition of motions, each in a straight line, explains all motions in lines other than straight. The distinction between natural and violent motion is thus null. All motion is natural.

Everything in motion is not moved by something already in motion, it is moved by itself. The distinction between mover and moved is thus also null. The presumption of a first mover is thus also null. Everything in motion is naturally in motion.

There is thus no degeneration of motion either. And the communication of motion is a problem of the relation of bodies in motion.

The general results were reduced to one general result. Inertia.

These are the arguments which lead Descartes to the discovery of the laws of nature.

In *The world*:

let us suppose that god creates anew so much matter that no place is empty (6); let us conceive of this matter as real perfectly solid body, which fills equally the entire length, breadth, and depth of space (*id.*); let this matter be divisible into any parts and according to any figures, and each of its parts be capable of receiving any motion (*id.*); let all distinction in it consist in the diversity of motions which god has given it; and if at first he

makes one thing move in one way, another in another, one slower, another faster, or even if he does not make anything move at all, he makes all things follow the ordinary laws of nature; for god has so marvelously established these laws that even if we suppose that he creates nothing else, and makes a chaos more confused than the poets can imagine, the laws are sufficient to make the parts of the chaos disentangle themselves and arrange themselves in such order as to have the form of a most perfect world, in which all things, general as well as particular, are just as they appear in this true world (*id.*);

the quantity of matter does not differ from its substance more than number differs from numbered things; its extension, or its property of occupying space, is not an accident, but its true form and essence (*id.*);

by nature I understand not some goddess or other imaginary power, but matter itself (7); and from the single condition that god continues to conserve it as he created it follows necessarily that the changes in it cannot be attributed to the action of god; because the action of god does not change at all, they have to be attributed to nature; and the rules according to which these changes occur I call the laws of nature (*id.*);

the first rule is that each part of matter always continues to be in the same state unless the encounter with another part compels it to change its state (*id.*); the second rule is that when one body encounters another, it cannot give it any motion except by losing at the same time as much of its own, nor take away any motion from it except by increasing its own by as much (*id.*);

the two rules follow from this alone that god is immutable, and, acting always in the same way, produces always the same effect; for if in the first instant he created it he put a certain quantity of motion in all matter, it must be either avowed that he always conserves the same quantity or believed that he does not always act in the same way (*id.*);

the third rule is that when a body moves, even if its motion follows a curved line or any other line, each of its parts always tends to continue its motion in a straight line (*id.*);

I assume no other laws, except those that follow infallibly from the eternal truths on which

the most certain and evident demonstrations of the mathematicians rest, the truths by which god himself has taught us that he has disposed all things according to number, weight, and measure (*id.*);

thus, when one examines sufficiently the consequences of these truths and our rules, he can know effects from their causes, and have *a priori* demonstrations of everything that can be produced in this new world (*id.*);

god will never make a miracle in this new world, and the intelligences, or rational souls, which we may suppose will be there, will never disturb the ordinary course of nature (*id.*).

In *The method*:

I resolved to leave this world entirely to their disputes, and speak solely of what will happen in a new one, if, in some imaginary space, god created enough matter to compose it, agitated variously and without order the different parts of this matter to compose a chaos more confused than the poets imagine, and after that did no other thing but give his ordinary concurrence to nature and let it act by the laws he established.

In the *Meditations*:

nature, considered generally, is nothing other than god himself, or the coordination of the created things instituted by god (6).

In the *Replies*:

there is no existent thing of which it cannot be asked which is the cause why it exists; and this can be asked also of god, not because he needs another cause to exist, but because the immensity of his nature is the cause or reason he needs no other cause to exist (2).

In *The principles*:

there cannot be many worlds (II, 22);

matter in the entire universe is of one and the same kind (23);

all variation of matter, or all diversity of its forms, depends on motion (*id.*);

motion is the transfer of matter (25);

motion and rest are different modes of a body in motion (27);

god is the primary cause of motion (36);

he always conserves the quantity of motion in the universe (*id.*);

and from the immutability of god we know the rules or laws of nature, which are the secondary

and particular causes of the diverse motions observed in particular bodies (37).

Then he formulates the laws.

By nature I understand not some goddess or other imaginary power, but matter itself.

All distinction in matter consists in the diversity of motions.

God always conserves the quantity of matter in the world, and always conserves the quantity of motion in the world.

From the single condition that god continues to conserve matter as he created it follows necessarily that the changes in it cannot be attributed to the action of god. They have to be attributed to nature.

The rules according to which the changes occur are the laws of nature.

And the first and the second laws of nature are, any thing by itself always perseveres in the same state, and hence, once moved, always continues to move, and all motion is by itself in a straight line, and hence, a thing which moves in a circle always tends to recede from the center of the circle.

Descartes established that nature is matter.

He established that matter is motion.

He established that the changes in nature cannot be attributed to god.

He discovered the laws of nature.

He discovered that the laws of nature are laws of conservation.

And he discovered that inertia is the fundamental law of nature.

The steps in the discovery of inertia are six. It is anticipated by Buridan. It is anticipated by Kepler. All its essential elements are recognized by Galileo. It is recognized as the fundamental law of nature, and the law is formulated in all its generality, by Descartes. The law is formulated more explicitly by Huygens. And the law is formulated most explicitly, in all its generality, and conclusively, by Newton.

Three centuries before Galileo, in the *Questions on the Physics*, Buridan writes:

in moving a body, the mover imprints in it a certain impetus in the direction in which it moves it, and by this impetus a stone is moved after the thrower stops moving it (VIII, 12.4).

In 1620, in the *Epitome of Copernican astron-*

omy, Kepler writes:

matter has a natural inertia or rest, so that a body needs some power stronger than itself to move it out of its position.

In 1638, in the *Two new sciences dealing with mechanics and locomotion*, Galileo writes:

the speed which a body has, when all external causes of acceleration or retardation are removed, is by nature impressed indelibly on the body; it follows also that motion in a straight line is eternal, for if it is uniform, it cannot be weakened, much less removed.

In 1644, in *The principles of philosophy*, Descartes formulates: three laws of nature. The first and the second are:

any thing by itself always perseveres in the same state, and hence, once moved, always continues to move; and all motion is by itself in a straight line, and hence, a thing which moves in a circle always tends to recede from the center of the circle.

In 1673, in *The pendulum clock*, Huygens formulates: three hypotheses (2). The first is:

if there were no gravity, and air did not impede the motion of bodies, every body will continue to move with uniform velocity in a straight line (*id.*).

And in 1687, in *The mathematical principles of natural philosophy*, Newton formulates: three axioms or laws of motion (*id.*). The first is:

every body perseveres in its state of rest or uniform motion in a straight line, unless it is compelled to change its state by forces impressed on it (*id.*).

Galileo discovered inertia. Descartes discovered that inertia is the fundamental law of nature.

Three generations after Newton Lagrange deduced Newton's laws from the principle of virtual velocities, itself a generalization of the principle of least action. Two-odd generations later Hamilton generalized the principle of least action further. Two generations later, in 1894, in *The principles of mechanics*, Hertz writes: we lay down the following, and only the following, fundamental law, every free system persists in its state of rest or of uniform motion in a straightest path (308-9).

Descartes also discovered that the fundamental relations of bodies in motion have to complete

the laws of nature.

The third law in *The principles of philosophy* is:

a body, upon encountering a stronger body, loses nothing of its motion, upon encountering a weaker body, loses as much of its motion as it transfers to the weaker body.

The second and the third hypotheses in *The pendulum clock* are:

by the action of gravity, whatever its origin, bodies are set in motion composed of uniform motion in one direction and motion downward due to gravity; and these two motions can be considered separately, neither impeding the other (2).

And the second and the third axioms or laws of motion in *The mathematical principles of natural philosophy* are:

change in motion is proportional to the motive force impressed and occurs along the straight line in which this force is impressed; and to action there is always contrary and equal reaction, or, the actions of two bodies on one another are always equal and in contrary directions (*id.*).

The principles of philosophy showed that from the principles of philosophy the knowledge of all other things in the world can be deduced. And in *The mathematical principles of natural philosophy* from the principles of philosophy the knowledge of all other things in the world is deduced.

cxxxix

Descartes is the synthesis of Bacon, Galileo, and Kepler, and on this synthesis all subsequent acquisition of knowledge stands.

Bacon distinguished the true way sharply from the false. He showed that the true way begins with the collection of the particulars, proceeds by continuous and gradual ascent, and until it reaches the most general principles. The law with its clauses.

Descartes distinguished the true way even more sharply. It is much better never to search for truth than to search without the method.

The true way is the method. Descartes made more explicit how it begins, and how it proceeds, and this is the synthesis of Bacon and Galileo. The method has the advantages of mathematics.

It ascends from experiments to principles, descends from these principles to new experiments, ascends again, descends again, and ascends again, by clarity and distinction, division of difficulties into parts, from the simplest to the more complex, and always by enumerations so complete and reviews so general as to omit nothing.

Philosophy is written in the world, and the world is written in mathematics. Descartes writes: Galileo examines physical matters by mathematical arguments, and in this I agree with him entirely and hold that there is no other way to find truth (to Mersenne, 11 Oct 1638).

Further, however: I admit no other principles in physics than in geometry or abstract mathematics (*Principles*, II, 64). Galileo showed the preestablished harmony of physics and mathematics by showing physics deductive. Descartes showed the preestablished harmony by showing physics axiomatic. He showed that the principles of physics are principles of mathematics, and showed which the principles of physics are.

This is the synthesis and generalization of Galileo and Kepler. Kepler discovered the laws of the motions of the heavenly bodies. Descartes discovered the laws of nature.

The laws determine the position, order, and motion of the parts of the world. The position, order, and motion of the parts of the world is the system of the world. The system of the world is the world.

Bacon, Galileo, and Kepler presume that the laws of nature determine the system of the world. Descartes shows that the laws of nature determine the system of the world.

To describe the system of the world Galileo and Kepler describe the solar system bounded by the fixed stars. To describe the system of the world Descartes describes the system of the systems of the fixed stars, one of which the solar system is.

The true way ascends until it reaches the most general principles. Descartes showed that the most general principles are the principles of philosophy, that the principles of philosophy are the principles of the acquisition of knowledge and the laws of nature, and which the principles of the acquisition of knowledge are and which the laws of nature are.

All these results, the true way, that philosophy is written in the world, the principles of the acquisition of knowledge, that the world is written in mathematics, the preestablished harmony of physics and mathematics, that philosophy is physics and mathematics, the laws of nature, that the laws determine the system of the world, that the system of the world is the world, stand on one another.

Enlightenment, too, stands on these results.

Descartes writes in *The method*:

they have an interest in my abstaining from publishing the principles of philosophy which I follow, for the principles are so simple and evident that by publishing them I will open the windows and admit the daylight in that cave to which they have descended to have their fights; but even the best minds have no reason to want to know my principles, for, if they want to know how to talk about things and acquire reputation for being learned, they will achieve this much more easily by contenting themselves with plausibility, which can be found without great pain in all kinds of matters, than by searching for truth, which discovers itself only little by little and in certain things, obliging one, when asked about other things, to confess his ignorance (6).

cxl

Things which cannot be known without knowing the principles became known with knowing the principles. Most dramatically, vortices and man became known.

Vortices.

In part III of *The principles, On the visible world*, Descartes begins: we have discovered certain principles of the material things (1); now it has to be examined whether all phenomena of nature can be explained from them alone, beginning with these phenomena which are the most general and on which the rest depend, and that is, with the general construction of the whole visible world (*id.*).

The order of investigation is paradigmatic. First the principles. Then the description of the most general phenomena on which the rest depend. Then, grounded in the principles and the description of these phenomena, the formulation of a theory. Then the examination whether all

phenomena can be explained from the theory.

The description of the most general phenomena, and that is, the general construction of the whole visible world, is quantitative: the moon is about 60 earth radii away, the sun 1 200 or 1 400 earth radii (5); the distance of Mercury from the sun is more than 400 earth radii, the distance of Venus more than 800, of Mars 1 800 or 2 000, of Jupiter more than 6 000, of Saturn 10 000 or 12 000 (6).

The formulation of the theory of vortices follows. Its principal elements are: the fixed stars are so far away that in comparison with them Saturn is very close (7); the sun and the fixed stars shine with their own light (9); the sun is one of the fixed stars (13); the heavens are fluid (24); the heavens carry with them all bodies which they contain (25); the heaven around the sun carries the planets, in the manner of a vortex with the sun in its center (30); all heavenly bodies have its own vortex each (46); equal, spherical, very quickly moving particles constitute the vortices (48-51).

The examination whether all phenomena can be explained from the theory follows. In particular: how the sun and the fixed stars were formed (54); what light is (55); comets (126); how the planets were formed (146); weight (IV, 26); air (45); the tides (49); the nature of the earth's interior (57); metals (63); salts (68); earthquakes (77); fire (80); the magnet (133), among others.

Spinoza, in 1663, in the *Parts I and II of Descartes's Principles of philosophy demonstrated in the geometrical manner*, after demonstrating in the geometrical manner parts I and II of *The principles*, demonstrates in the geometrical manner the theory of vortices as well. He writes: a good hypothesis has to meet the requirements, 1. to imply no contradiction, 2. to be the simplest possible, 3. to be very easy to know, 4. to make possible the deduction of everything observed in all of nature (3); postulate, let it be conceded, all matter of which this visible world is composed was initially divided by god into particles as nearly equal as possible (*id.*); the particles had the same quantity of motion as there is in the world now and moved uniformly; they moved around its own center each, composing the fluid body which we presume the heaven is; some

moved together around certain points, others around other points (*id.*); and so composed as many vortices as there are now stars in the world (*id.*); this hypothesis implies no contradiction because it attributes to matter nothing but divisibility and motion (*id.*); it is the simplest because it assumes no inequality or dissimilarity either in the particles or in their motion; from this it follows that it is also very easy to know (*id.*).

That it meets the requirements a good hypothesis has to meet is a great virtue of the theory of vortices.

A greater virtue is that it explains how the earth, the planets, the sun, and the fixed stars were formed. It is the first attempt to explain how the world developed. It is hence a virtue all by itself that it made the earlier explanation of how the world was made superfluous, and rather null.

Descartes writes in *The principles*: there are no secret forces, no miracles of sympathy or antipathy, nothing whatsoever in all of nature which cannot be attributed to purely corporeal causes entirely devoid of mind and thought (IV, 187). That it is a demonstration of this proposition is a yet greater virtue of the theory.

And yet a greater virtue still is that it has led to the formulation of the theory of gravity. The analogy holds. As Ptolemy's description of the world to Aristotle's description of the world so the theory of gravity to the theory of vortices.

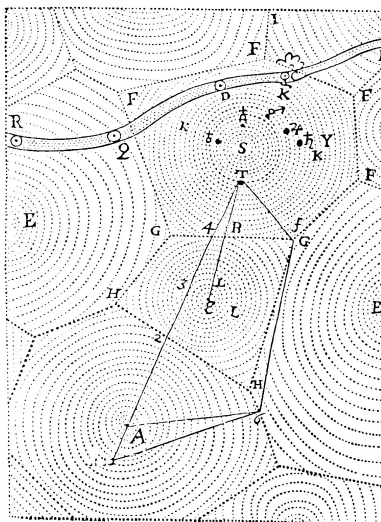
Newton writes: I have investigated the properties of vortices in order to test whether the celestial phenomena can be explained by them (*Principles*, II, 52), and: the planets revolve in ellipses with a focus in the sun, and describe by radii drawn to the sun areas proportional to the times; and the parts of a vortex cannot revolve in this way (53). In general: the hypothesis of vortices is beset with many difficulties (*gen. sch.*).

The hypothesis of gravity is beset with no difficulties. It succeeds, by adopting the sound and avoiding the erroneous in the hypothesis of vortices, in what the hypothesis of vortices first attempts.

The energy of the world remains constant. Planck writes: it is not difficult to formulate Clausius's proposition so that it expresses clearly what is essential; the further extended in space the system under consideration is the more nearly

constant its energy remains; the error is small when the energy of our solar system is considered, smaller when the system of all fixed stars known to us is considered, and in this way the proposition that the energy of an infinitely large system, or the energy of the world, remains constant, has, in fact, a reference.

The three steps in Planck's generalization, the solar system, the system of all known stars, the world, correspond to three stages in the acquisition of knowledge. The labors to demonstrate the system of the world, and thus fix the reference of 'the world', reached the first stage with Ptolemy. They reached the third stage with the big bang theory. The greatest virtue of the theory of vortices is that it is with it that they reached the second stage.



The picture is from the first publication of *The world*, in 1664: around the sun, *S*, its vortex revolves; the vortex carries the planets, Mercury, ☿, Venus, ♀, the earth, *T*, Mars, ♂, Jupiter, ♃, and Saturn, ♄; a comet, ☄, traverses the vortex; from the earth a nearby star, *ε*, and a more distant star, *A*, can be observed; and as its vortex revolves around the star *S*, so its vortex revolves around every other star, *ε*, *A*, *E* (8).

Man.

In concluding *The principles* Descartes writes: I will not add, as I previously intended, two more parts to these principles, a fifth on living things, on animals and plants, and a sixth on man; I have

not yet examined perfectly all the things I would want to discuss in these parts (IV, 188).

He adds: but I will say a few things about the objects of the senses (*id.*); even though the human soul informs the whole body, it has its principal seat in the brain; it is here that it not only understands and imagines, but also senses (189); the motions excited in the brain by the nerves affect the soul, or mind, which is closely conjoined to the brain, in different ways corresponding to the different motions; and these different affections, or thoughts, consequent to the motions, are called perceptions (*id.*).

In *The world* already – in its last chapter, which has come to go under the name *Treatise on man* – he has established: the body is nothing but a machine made of earth (18); and all functions of this machine follow entirely naturally from the mere disposition of its organs, neither more nor less than the motions of a clock or another automaton follow from those of its counterweights and wheels; it is thus not necessary to suppose that there is in it a soul, vegetative or sensitive, or any other principle of motion and life than its blood and its spirits agitated by the heat of the fire burning continuously in its heart, or to suppose that this fire is not of the same nature as all the fires in inanimate bodies (*id.*).

In *The description of the human body and all its functions*, written in 1648, published in 1664, he begins with: the general description of the entire machine (1); the heat in the heart is the great spring and the principle of all motions in the machine; the veins are pipes conducting the blood from all parts of the body to the heart, where it serves to nourish the heat; the stomach and intestines are a larger pipe perforated by little holes through which the juices from food run into the veins, and are carried by them to the heart; the arteries are yet more pipes through which the blood, heated and rarefied in the heart, passes to all other parts of the body, bringing them heat and matter to nourish them; the most agitated and most alive parts of the blood go by arteries from the heart to the brain in the straightest line; these parts, composed like air, or a very subtle wind, are called animal spirits; these spirits, dilating the brain, make it capable of receiving impressions from external objects, as well as from the soul,

and thus be the organ, or the seat, of the common sense, imagination, and memory; the same air, or spirit, runs from the brain through the nerves to the muscles, disposing the nerves to serve as the organs of external sense, inflating the muscles to give motion to all members of the body (*id.*).

Finally, the last treatise Descartes published is *The passions of the soul*, in 1649. He writes: what is a passion in the soul is an action in the body (1, 2); everything we experience in us must be attributed to our body (3); it is an error to believe that the soul gives motion and heat to the body (5); the soul is united to all parts of the body (30); but there is a little gland in the brain where the soul exercises its functions more particularly than in the other parts of the body (31); this gland is the principal seat of the soul; and I am persuaded that the soul can have no other place in the entire body, where it exercises its functions immediately, but this gland, because I have observed that all other parts of our brain are double, just as we have two eyes, two hands, two ears, and generally all our organs of external sense are double; but since we can have only one and simple thought of one and the same thing at the same time, it is necessary that there is a place where the two images from any two impressions can be assembled into one (32).

Galen first described the pineal gland (*Parts*, VIII, 14), fourteen and a half centuries before Descartes, at the time when the transformation of philosophy into Christian philosophy was beginning.

In the generation after Descartes, Redi showed that there is no spontaneous generation, Malpighi demonstrated the capillaries, Leeuwenhoek discovered protozoa, Hooke discovered the cell, Swammerdam discovered the stages in the development of insects.

cxli

Philosophy is the perfect knowledge of all the things that can be known. And to be such it must be deduced from the principles.

Philosophy is theoretical and practical.

And Descartes did not deduce only theoretical philosophy from the principles, and both mathematics and physics.

The deduction of practical philosophy from the

principles is in *The method*.

Descartes writes:

I formulated for myself provisional morals, consisting of only three of four maxims;

the first was to obey the laws and customs of my country, retaining constantly the religion in which god had given me the grace to be instructed since my childhood, and governing myself in all other things by following the most moderate and furthest removed from excess opinions of the most sensible among those with whom I had to live;

the second was to be as firm and as decisive in my actions as I could, and to follow even the most doubtful opinions, once I had determined to follow them, as constantly as if they were very certain;

the third was to try always to vanquish myself

rather than fortune, to change my desires rather than the order of the world;

finally, to conclude these morals, I decided to review the various occupations which men have in this life, in order to try to choose the best (3).

Following these morals, Descartes wrote the *Rules*, *The world*, *The method*, the *Meditations*, the *Replies*, *The principles*, *The human body*, *The passions*.

Descartes published the morals in 1637. In 1637 those following less provisional morals had been exterminating heresy for fourteen and a half centuries, counting from Galen's contemporary Irenaeus, had four years earlier found Galileo guilty of the crime of vehement suspicion of heresy, and were in the nineteenth year of the thirty years' war to determine yet again who is the keeper of his brother's morals.

Huygens and Spinoza

cxlii

Huygens's very important treatises are five.

In 1656, in *On the motion of colliding bodies*, he solved the problem of collisions, and discovered the principle of conservation of momentum.

Also in 1656, in the *Treatise on calculation in games of chance*, he showed that, if in cases p the gain or loss is a , in cases q the gain or loss is b , and $p + q$ are all cases, all with equal chance of occurring, the value of one chance is $\frac{ap + bq}{p + q}$,

and thus discovered the possibility of calculation of the probability of random events.

In 1659, in *The system of Saturn, or on the causes of Saturn's marvelous phenomena and its satellite, a new planet*, he explained the marvelous appearance of Saturn by discovering its ring, and solved the problem of the distance of the sun, and thus determined the true proportions of the solar system. He had discovered Saturn's satellite, a new planet, in 1656.

In 1673, he published *The pendulum clock, or geometrical demonstrations concerning the motion of pendula applied to clocks*. It is in five parts. Part I is *A description of the pendulum*

clock, part II is *The fall of heavy bodies and their motion in a cycloid*, part III is *The evolution and dimension of curves*, part IV is *The center of oscillation*, part V is *Another construction of the clock based on the circular motion of pendula, and theorems on centrifugal force*. In part II he formulated the three hypotheses from which the fall of heavy bodies is deduced, and proved the isochronism of the cycloid. In part III he developed the theory of evolutes. In part IV he solved the problem of centers of oscillation, derived the law of the pendulum, and determined the acceleration of gravity. In part V he investigated centrifugal force. He had discovered the pendulum clock, and constructed it, in 1657.

In 1678, in the *Treatise on light, in which are explained the causes of what occurs in reflection, in refraction, and particularly in the strange refraction of Iceland crystal*, he formulated the wave theory of light.

He wrote also the following important, if not very important, treatises. In 1654, in *On the magnitude of the circle, with an appendix on the constructions of some famous problems*, he showed that $3.1415926533 < \pi < 3.1415926538$ (20). In 1656, in the *New observation of a moon*

of Saturn, he reported the discovery of Saturn's satellite. In 1658, in *The clock*, he reported the discovery and construction of the pendulum clock. In 1690, in the *Discourse on the cause of gravity*, he analyzed Newton's theory of gravity. In 1698, in the *Κοσμοθεωρός, or conjectures on the countries and inhabitants of the heavenly bodies*, he described the inhabitants of Mercury, Venus, Mars, Jupiter, and Saturn.

He wrote also the *Theorems on the quadrature of the hyperbola, ellipse, and circle when the center of gravity of segments is given, to which is appended 'Εξέτασις of the measurement of the circle of Gregory of St Vincent*, in 1651, *Short instruction on the use of the clock to find longitude*, in 1665, *Rules for the motion of bodies in collision*, in 1669, *Compendium on star observation freed from difficulties by optical tubes*, in 1684, *Memoirs on the grinding of lenses for telescopes*, in 1685.

Huygens also left behind many letters. His corpus was edited in 1888-950, in 22 volumes. The letters comprise 10 of the 22, volumes I-X. Among his correspondents are Mersenne, Hobbes, Descartes, Hevelius, Wallis, Pascal, Cassini, Leeuwenhoek, Rømer, Leibniz.

In general, Huygens is the decisive step from Descartes to Newton. From the demonstration that all other things in the world can be deduced from the principles to the deduction of all other things in the world from the principles.

And to show his importance more particularly, the following can be shown more particularly. The solution of the problem of the distance of the sun and determination of the true proportions of the solar system. The discovery of the ring of Saturn. The solution of the problem of collisions and discovery of the principle of conservation of momentum. The discovery of the isochronism of the cycloid, the derivation of the law of the pendulum, and the determination of the acceleration of gravity. And the discovery of the importance of waves and of the speed of light.

cxliiii

The big bang theory stands on Hubble's law.

Hubble's law is $v = H_0 r$, where v is the recession velocity of a galaxy, r is its distance, and H_0 is the Hubble constant, the current rate of expan-

sion of the world.

Hubble's law thus stands on the distances of galaxies.

In 1936, in *The realm of the nebulae*, Hubble writes: the distance of the moon was known to the Greeks, the distance of the sun was established in the latter part of the seventeenth century, distances of stars were determined a century ago, and distances of galaxies in our own generation; the distances were the essential data; until they were found no progress was possible (1).

Until the distances were found no progress was possible.

The big bang theory stands on the distances of galaxies, the distances of galaxies stand on the distances of stars, the distances of stars stand on the distance of the sun.

Hubble determined the distances of galaxies, in 1925.

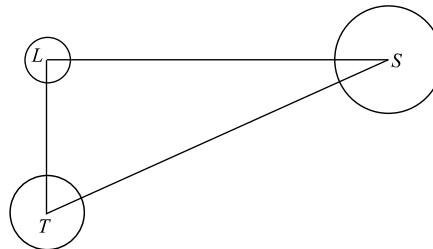
Bessel determined the distances of stars, in 1838.

And Huygens solved the problem of the distance of the sun, in 1659.

In *The system of Saturn*, he writes: the astronomers differ greatly in their evaluations of the distance from the earth to the sun, and this is not surprising, considering that no tolerable method to measure it has been found; they try to measure it from eclipses and from dichotomies of the moon, and it is easy to show that these efforts must be in vain (*id.*).

Aristarchus discovered both methods, from dichotomies of the moon and from eclipses, in *On the sizes and distances of the sun and moon*, nineteen and a half centuries before Huygens.

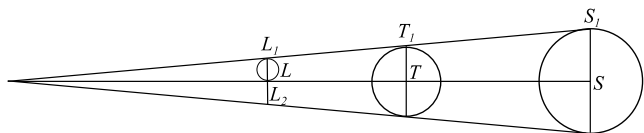
From a dichotomy of the moon.



The circle around T is the earth, the circle around L is the moon, the circle around S is the sun. When the moon is visible as a half moon, angle TLS , the angle between the line of sight of the

moon and the line connecting the moon and the sun, is right. Given angle LTS , the angle between the lines of sight of the moon and the sun, triangle LTS is solved, and the ratio of TL to TS , of the distance of the moon to the distance of the sun, can be calculated. And given distance TL , the distance of the moon, distance TS , the distance of the sun, follows.

From a lunar eclipse.



The circles are the same. During a lunar eclipse the moon is eclipsed by the earth's shadow. SS_1 , the sun's radius, TT_1 , the earth's radius, and L_1L_2 , the radius of the earth's shadow at the distance of the moon, are parallel to one another and perpendicular to STL_2 , the line through the centers of the sun, the earth, and the earth's shadow. Given distance TT_1 , the earth's radius, distance TL_2 , equal to the distance of the moon, and distance L_1L_2 , the radius of the earth's shadow at the distance of the moon, quadrilateral $T_1TL_2L_1$ is solved, and given the ratio of TL_2 to TS , of the distance of the moon to the distance of the sun, distance TS , the distance of the sun, follows.

And it can all be inferred from observation. In *On the sizes and distances*, Aristarchus concludes that the angle between the lines of sight of the moon and the sun at half moon is 87° (*id.*), and hence calculates from the dichotomy the ratio of the distance of the moon to the distance of the sun at about 1:19 (*id.*). He concludes that the radius of the earth's shadow at the distance of the moon is equal to the diameter of the moon (*id.*), and hence, given the ratio 1:19, calculates from the eclipse the ratio of the radius of the moon to the radius of the earth. He concludes that the apparent diameter of the moon is 2° (*id.*), the moon thus subtends $\frac{1}{180}$ of the circle of its motion around the earth, and the ratio of the radius of the moon to the radius of the earth gives the radius of the circle, or the distance of the moon. The moon is about 20 earth radii away. And the ratio 1:19 gives the distance of the sun. The sun is about 380 earth radii away.

Both methods are formally impeccable, and it can all be inferred from observation. The methods were perfected, too, and observation was greatly improved. Also, one generation-odd after Aristarchus, Eratosthenes determined the earth's radius. One century later, Hipparchus determined the distance of the moon.

Both methods are nevertheless, with Huygens: in vain. Very small angles have to be observed, as well as the exact moment of dichotomy of the moon, as well as the exact moments of the beginning and end of an eclipse, as well as the ratio of the radius of the moon to the radius of the earth's shadow at the distance of the moon.

Hipparchus calculated the distance of the sun at 490 earth radii (Pappus, comm. *Almagest*, V, 11). Ptolemy calculated it at 1 210 earth radii (*Almagest*, V, 15). Copernicus calculated it at 1 142 earth radii (*Revolutions*, IV, 21). Tycho calculated it at 1 150 earth radii (*Exercises*, II, 7). All four are calculations from eclipses.

In 1620 Kepler calculated the distance at 3 469 earth radii. He writes: as the apparent diameter of the sun is $30'$, the point of vision has to be a little more than 229 radii of the sun away; and because the earth is the home of the measuring creature, and the earth is the measure of the bodies in the world, and its radius is the measure of the distances, and the distance between the earth and the sun is first, nothing can be more fitting or better ordered than to postulate the equality of these two ratios, as the body of the earth to the body of the sun so the radius of the earth to the distance between the earth and the sun; and the distance is $3\,469 \frac{1}{3}$ earth radii (*Epitome*, IV, 1.4).

As the apparent diameter of the sun is $30'$, and $\tan 15'$ is a little less than $\frac{1}{229}$, if the radius of the sun is r_s , and the distance of the sun is d , $\frac{r_s}{d}$ is a little less than $\frac{1}{229}$. If the radius of the earth is r_T , the body of the earth is r_T^3 , the body of the sun is r_s^3 , and the earth is the home of the measuring creature, $\frac{r_T^3}{r_s^3} = \frac{r_T}{d}$. As r_T is the unit, $r_s^3 = d$.

But $229 r_s$ is a little more than d . The radius of the sun is thus a little more than 15 earth radii, and the distance of the sun is a little more than 3 469 earth radii.

In 1632 Galileo calculated the distance at 1 208 earth radii (*Systems*, III). In 1644 Descartes calculated it at 1 200 or 1 400 earth radii (*Principles*, III, 5).

In 1644 Wendelin calculated the distance at 14 656 earth radii (*Eclipses*). In 1651 Riccioli calculated it at 7 300 earth radii (*Almagest*, I, 3.7). Both are calculations from dichotomies of the moon.

Huygens writes in *The system*: no tolerable method to measure the distance has been found; it thus seems that only one method remains; to observe with the telescope the apparent diameters of the planets, then to compare their sizes with that of the sun, and then to adopt for the earth the relative size that seems to accord best with the order and disposition of the entire system (*id.*).

Thus: the diameter of Jupiter when it is closest to us seems to be $64''$; and as this least distance of Jupiter from the earth is to the mean distance of the sun as 26:5, by the proportion $\frac{26}{5} = \frac{5'35''}{64''}$

we find $5'35''$ for the angle of the diameter of Jupiter which would be seen if the planet is imagined placed as far from us as the sun is at mean distance; from here, the sun is observed with a diameter of $30'30''$; that of Jupiter is therefore to that of the sun as $5'35''$ to $30'30''$, and that is, a little more than 1 is to $5\frac{1}{2}$ (*id.*).

Excepting Mercury: which is too close to the horizon to measure well (*id.*), the apparent diameters of the planets, and hence their sizes compared to that of the sun are:

Venus	$85''$	$\frac{1}{84}$,
Mars	$30''$	$\frac{1}{166}$
Jupiter	$64''$	$\frac{1}{5\frac{1}{2}}$,
Saturn	$30''$	$\frac{1}{5\frac{2}{5}}$ (<i>id.</i>).

The planets are now all four as far as the sun. Huygens continues: it is manifest that the

planets do not obey the rule, the more distant from the sun the more voluminous, Jupiter is larger than Saturn, Venus is larger than Mars; nevertheless, to conserve the harmony of the entire system as much as possible, it seems most reasonable to admit that as the earth is intermediate between Venus and Mars in distance, it is intermediate between them in size as well; and taking the mean of their diameters for the diameter of the earth, we find that it is $\frac{1}{111}$ of that of the sun (*id.*).

The earth is now also as far as the sun.

Huygens concludes: the distance which separates the earth from the sun is thus greater than the results of all previous evaluations; in fact, if the diameter of the earth is $\frac{1}{111}$ of that of the

sun, and the diameter of the sun is $\frac{1}{113}$ of its

mean distance from us, which follows from its apparent diameter of $30'30''$, the diameter of the earth is $\frac{1}{12\,543}$ of its distance from the sun (*id.*).

The distance is thus 25 086 earth radii.

Huygens notes: I admit that the foundation of the calculation is not very firm (*id.*); but even if the result is two times more or less than what the distance really is, or even three times more or less, the fact that we have determined it with such exactitude should not be judged insignificant, considering that no other method exists where an error ten times greater cannot be feared (*id.*).

Pace Huygens, the foundation is firm. Aristarchus's result is sixty-two times less than what the distance really is. Aristarchus's result is nevertheless excellent, considering that there is no result at all before it, and it is excellent because its foundation is rock, considering the sand which the measuring creature, when it has not measured its distance, has had to say about the sun. The foundation of Aristarchus's result is firm, the foundation of Ptolemy's result is firmer, the foundation of Kepler's result is yet firmer, and the foundation of Huygens's result is yet firmer.

In 1672 Cassini calculated the distance at 22 000 earth radii (*Observations*). In 1673 Flamsteed calculated it at 21 000 earth radii (to Cassini, 7 Jul 1673).

In 1684 Cassini found a yet firmer foundation. In *The elements of astronomy, verified by Mr Cassini from the relation of his tables to the observations made by Mr Richer on the island of Cayenne*, from simultaneous measurements by himself in Paris and Richer in Cayenne, with the distance across the Atlantic as a base, Cassini calculated the parallax of Mars at $25\frac{1}{2}''$ (36),

hence the parallax of the sun at $9\frac{1}{10}''$ (*id.*), and

hence its distance at 21 600 earth radii (37).

In 1716 Halley found a yet firmer foundation. In *A singular method to determine the parallax of the sun, or its distance from the earth, by observing Venus in the sun*, he calculated transits of Venus due in 1761 and 1769 (*id.*). Following the example of Kepler, Halley concluded with an admonition: therefore again and again we recommend it to the curious observers of the stars to whom, when our lives are over, we entrust these observations, that, mindful of our advice, they strenuously apply themselves to these observations (*id.*).

They did not in 1761. But they did in 1769. And Lalande, in 1772, in the *Memoir on the parallax of the sun, deduced from observations in the South Sea, in the kingdom of Astrakhan, and in China*, calculated the parallax at $8''.6$ (*id.*), and hence the distance at 23 984 earth radii (*id.*).

In 1862 Foucault found a yet firmer foundation. In the *Experimental determination of the speed of light, and the parallax of the sun*, he writes: the new experiment with a rotating mirror gives the speed of light at 298 million meters per second (*id.*); if one combines this value with the constant of aberration $20''.45$ in order to deduce the parallax of the sun, which is obviously a function of the two, one finds, instead of $8''.57$, the notably greater value $8''.86$; the mean distance from the earth to the sun is thus diminished by about $\frac{1}{30}$ (*id.*). To 23 280 earth radii.

It has remained the method of measuring the distance. From the speed of light combined with the constant of aberration, of the displacement of light by the motion of the earth, the parallax of the sun is deduced, and hence its distance. The number has thus been fixed at 23 455 earth radii.

Tabulating.

The problem of the distance of the sun was solved in four steps,

Aristarchus	380 earth radii,
Ptolemy	1 210,
Kepler	3 469,
Huygens	25 086.

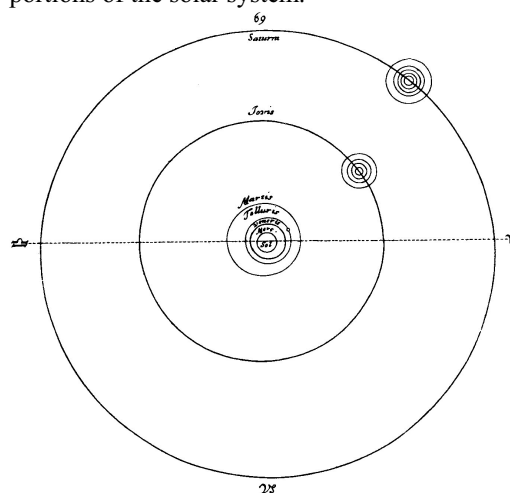
The solution was corrected in three steps,

Cassini	21 600,
Lalande	23 984,
Foucault	23 280.

And the number was refined to

23 455 earth radii.

Huygens solved the problem of the distance of the sun, and hence the problem of the true proportions of the solar system.



The picture is from the *Κοσμοθεωρός*, in 1698. Huygens writes: the orbits of the planets are drawn as nearly as possible in their true proportions; the orbits of the satellites of Jupiter and Saturn and the orbit of the moon, however, are drawn larger than their true proportions for otherwise they could not be seen (1).

The orbit of the moon can yet barely be seen. The satellites of Saturn are five, the one discovered by Huygens in 1656, and the four discovered by Cassini from 1671 to 1684.

cxliv

The ground of Huygens's solution of the problem of the distance of the sun is the best accordance with the order and disposition of the entire system. The ground of his discovery of

Saturn's ring is the same.

He writes in *The system of Saturn*:

when Galileo first used the telescope to observe the heavenly bodies, and revealed to mortals the remarkable phenomena which he thus discovered, the most amazing among them seem to me to have been those concerning Saturn; for the others, although justly celebrated as well, did not pose the question of their causes; only the changing shapes of Saturn presented a new and recondite artifice of nature, which neither Galileo nor any other astronomer since has been able to explain; Galileo first saw that Saturn shines not in a simple orb, but in the triple shape of a star with two appendages;

when I established that its moon revolves around Saturn in a period of sixteen days, I concluded that undoubtedly Saturn itself rotates on its axis in an even shorter period;

hence it seemed plausible that all the celestial matter between Saturn and its moon is subject to the same motion, and the nearer it is to the planet the more it approaches its speed; from this it followed that the appendages are either joined to the central body and revolve with it or separated from it by a certain distance and revolve not much more slowly;

and because the period of rotation of the planet is so short, this could be explained in only one way, by admitting that Saturn's globe is surrounded equally on all sides by another body, and that is, that a ring encircles it about the middle;

I established that the appendages are inclined to the ecliptic at an angle of more than 20° ; from this it followed necessarily that the ring will appear to us in different aspects, at one time as a broad ellipse, at another time as a narrow ellipse, and sometimes even as a straight line (*id.*).

Huygens reports his observations of a continuous cycle of the different aspects, from a broad ellipse to a narrow ellipse to a straight line to a narrow ellipse to a broad ellipse, and concludes:

to respond to those to whom it seems novel and unreasonable to attribute to one of the heavenly bodies a shape never before found among them, they should consider that this hypothesis is not an arbitrary invention, like the epicycles of the astronomers; I see this ring with the eyes, I perceive its shape just as we perceive the shapes

of all other things;

moreover, from the great similarity of Saturn and our earth it can be concluded with great probability that Saturn, like the earth, is in the middle of its vortex, and everything in it that has weight tends by its nature toward its center, and it follows necessarily that the ring, tending with all its parts equally toward the center, by this alone is maintained at the same distance from the center (*id.*).

The discovery of Saturn's ring is a great success of the theory of vortices. It is also a decisive step toward the theory of gravity. Saturn's vortex implies the ring. But it is not necessary to maintain the ring in its position.

The solution of the problem of collisions is an even more decisive step.

The science of motion is the largest and most excellent, and although it concerns the oldest object, it is with the labors of Galileo entirely new.

He discovered inertia.

He discovered the relativity of motion: shut yourself with a friend in a cabin under the deck of a ship; bring with you some flies and butterflies, take a tank of water with some fish in it, hang a bottle that empties drop by drop into a vessel beneath it; when the ship stands still observe carefully all the motions; let then the ship move as fast as you wish; as long as the motion is uniform, you will not notice the least difference in any of the motions.

He discovered the law of falling bodies. The distance is proportional to the square of the time.

He discovered that the motion of a projectile can be analyzed into uniform horizontal motion and uniformly accelerated downward motion. Hence also that it describes a parabola.

And he took the first step toward the formulation and solution of the problem of collisions.

He writes in the *Two new sciences*: to determine the force of its impact it is not sufficient to consider the speed of a projectile; it is necessary to consider also the state and condition of the thing that receives the impact; for the thing that is impacted suffers violence from the speed of the thing that impacts it to the degree to which it opposes it and stops its motion; if a blow lands on a thing that yields without any resistance, the

effect will be null; if one runs to strike an enemy with his lance, and as he overtakes him the enemy runs away with the same speed, there will be no blow; the impact will be the greatest when the thing that is impacted does not yield at all, but opposes and stops all the motion of the thing that impacts it (IV); the difficulty concerns the origin and principle of the immense force of impact, seen in the blow of a hammer of 8 or 10 pounds overcoming a resistance which will not yield to a weight of many hundreds of pounds exerted only by pressing; I should like to find a way of measuring the force of this impact, which I do not believe to be infinite; I think it has a limit of equality with other forces, pressure, weight, levers, screws, and other mechanical devices (*id.*).

To determine the force of impact it is not sufficient to consider the speed. To determine what else has to be considered the force has to be measured. It has a limit of equality with other forces. To find the limit the relation of collision has to be considered, the relation of the thing that is impacted and the thing that impacts it. The thing that is impacted suffers violence from the speed of the thing that impacts it to the degree to which it opposes it and stops its motion. To determine what else has to be considered, and find the limit of equality, the relation of the speed and the degree to which the thing that is impacted opposes and stops the motion of the thing that impacts it has to be considered. And to this end collision has to be considered elastic, impact has to be considered at its greatest, when the thing that is impacted does not yield at all, but opposes and stops all the motion of the thing that impacts it.

Descartes took the next step.

He discovered that god always conserves the quantity of matter in the world and always conserves the quantity of motion in the world.

He discovered that the laws of nature are laws, fundamentally, of conservation.

He formulated the three laws of nature.

The first and the second are the law of inertia.

The third is: a body, upon encountering a stronger body, loses nothing of its motion, upon encountering a weaker body, loses as much of its motion as it transfers to the weaker body.

And, from conservation, and the third law, he formulated the problem of collisions conclusively, and solved it explicitly, if not conclusively.

The problem is to make explicit what body is, and that is, what quantity of matter is. To make explicit what stronger body is, and what weaker body is, and that is, what quantity of motion is. And to show that the quantity of motion is conserved, that the quantity of motion before collision and after collision is the same.

Descartes writes in *The world*: the quantity of matter does not differ from its substance more than number differs from numbered things; the extension of matter, or its property of occupying space, is its true form and essence. In *The principles*: place, or space, does not signify a thing different from the body in a place, it designates its magnitude, figure, and situation among other bodies (II, 13).

Quantity of matter is volume.

Huygens showed that quantity of matter is not volume. Newton showed that quantity of matter is mass. Mass is the product of volume and density. It is definition 1 in *The mathematical principles*.

Descartes writes in *The principles*: in accordance with the first law, the force of a body to act on another or to resist the action of another consists in its tendency to remain in the same state; and this force is measured not only by the magnitude of the body, but also by its speed (43).

Quantity of motion is the product of volume and speed.

Huygens showed that quantity of motion is not the product of volume and speed. He showed also that not speed, but velocity, speed with its direction, has to be considered. Newton showed that quantity of motion is the product of mass and velocity. It is definition 2 in *The mathematical principles*.

After the laws, Descartes formulates seven rules. The first two are: if two bodies, equal in size and moving with equal speeds, the first toward the left, the second toward the right, collide, they rebound and continue to move, the first toward the right, the second toward the left, without either losing any of its speed (46); if two bodies, the first larger than the second, moving

with equal speed, toward the left, and toward the right, collide, only the smaller rebounds, and both continue to move each with its speed toward the left (47).

Motion is conserved. The sum of the products of the volumes and speeds before a collision is equal to the sum of the products of the volumes and speeds after the collision.

Huygens considers collisions elastic explicitly. He takes quantity of matter as weight. He considers velocity. And he shows conservation. He discovered the conservation of kinetic energy in elastic collisions. And he discovered the conservation of momentum. Weight is proportional to mass. And the sum of the products of the masses and velocities before a collision is equal to the sum of the products of the masses and velocities after the collision.

In *On the motion of colliding bodies*, in 1656, Huygens begins with three hypotheses.

Hypothesis 1: any body already in motion, if nothing obstructs it, continues to move perpetually with the same speed in a straight line (*id.*).

Inertia.

Hypothesis 2: whatever the cause of the rebound of hard bodies when they collide, we assume that when two bodies equal to one another and with equal speed collide directly, each rebounds with the same speed with which it has arrived (*id.*).

Collisions are elastic.

Huygens adds: two bodies collide directly when both their motion and their contact occur in the straight line which passes through their centers of gravity (*id.*).

Hypothesis 3: the motion of bodies and their equal or unequal speeds are to be understood in relation to other bodies considered at rest, even if both those and these are involved in some other motion; thus, when two bodies collide, even if each of them is simultaneously involved in some other equal motion, they do not act on one another with respect to that common motion at all, but in all ways act as if that motion is absent (*id.*).

Motion is relative. Huygens explains: if a voyager in a boat, which moves with uniform speed, makes two equal balls collide with equal speed as determined in relation to himself and the boat, each ball rebounds with equal speed in

relation to himself and the boat, just as it does if he makes the same balls collide with equal speed in a boat at rest or on the ground (*id.*).

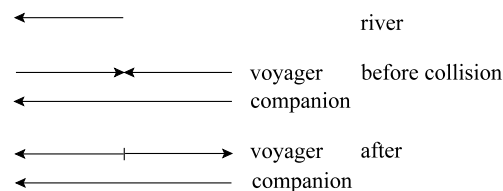
Proposition 1 follows:

if a body collides with a body at rest, after the collision the first body is at rest and the second body acquires the same speed which the first body has before the collision;

suppose a boat floats down a river and so close to the bank that a voyager in the boat can extend his hands to a companion on the bank; the voyager holds in his hands two balls suspended on strings; he moves his hands together with equal motion as determined in relation to himself and the boat and makes the balls collide with equal speed; the balls rebound with equal speed in relation to himself and the boat;

the boat has at the same time moved toward the left with the same speed with which the voyager's left hand has moved toward the right; it is clear that in relation to the bank and his companion there the voyager's left hand has remained at rest and his right hand has moved with twice the speed with which it has moved in relation to the voyager and the boat;

suppose, then, that the companion on the bank takes hold of the voyager's hands and of the strings on which the balls are suspended; it is clear that while the voyager makes the balls collide with equal speed as determined in relation to himself and the boat, at the same time his companion makes one of the balls collide with the other, which is at rest, with twice the speed as determined in relation to himself and the bank; it follows that in relation to the bank and the companion there one of the balls comes to rest after the collision, and the other, at rest before the collision, moves in the same direction with twice the speed with which it moves in relation to the voyager and the boat, which is the same speed with which the first ball moves toward the second before the collision (*id.*).



Proposition 4 is: when two bodies collide, their relative speed after the collision is the same as their relative speed before the collision (*id.*).

Speed is relative in both quantity and direction. Thus, if the velocities of the two bodies before the collision are v_1 and v_2 , and their velocities after the collision are v_1' and v_2' , then $v_1 + v_2 = v_1' + v_2'$.

Given: Galileo's demonstration that when two bodies fall with uniformly accelerated motion the distances are proportional to the squares of the final speeds (*id.*), and: the most certain axiom of mechanics that when bodies move because of their gravity, their common center of gravity cannot rise (*id.*), proposition 8 is: if two bodies, whose speeds are inversely proportional to their magnitudes, collide, each rebounds with the same speed which it has before the collision (*id.*).

If magnitude is mass, the masses of the two bodies are m_1 and m_2 , and $m_1 : m_2 = v_2 : v_1$, then $v_1 = v_1'$ and $v_2 = v_2'$.

And proposition 9 generalizes this result by showing that whatever the magnitudes and speeds of two bodies, given their motions before a collision, their motions after the collision can be calculated.

The problem of collisions is thus solved.

The solution implies the principle of conservation of momentum.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

In 1669 Huygens states the principle explicitly: the quantity of motion of two bodies can increase or decrease when they collide; but it remains always the same quantity of motion in the same direction sustaining the quantity of motion in the opposite direction (to *Journal*, 18 Mar).

Proposition 11 in *On the motion* is: if two bodies collide, the sum of the products of their magnitudes and the squares of their velocities is the same before and after the collision (*id.*).

The product of the mass and the square of the velocity is twice the kinetic energy. And kinetic energy in elastic collisions is conserved.

cxlv

The pendulum clock, or geometrical demonstrations concerning the motion of pendula applied to clocks, in 1673, is a yet more decisive

step. It builds on Galileo's *Two new sciences*, and has a greater unity and reaches further than the *Two new sciences*. It builds also on Descartes's *Principles*, and if it does not have the scope of Descartes's *Principles*, it has the rigor of Newton's *Principles*.

Huygens writes in the preface:

it is sixteen years since we published the treatise in which we described the construction of the clock which we had then discovered; we have since made many discoveries so closely related to the perfection of this clock that they can be considered its most important part and its foundation;

by its very nature the simple pendulum is not a certain and equal measure of time; its wider motions are slower than the narrower; led by geometry, however, we have discovered a different and until now unknown way of suspending the pendulum; we have found a line whose curvature is marvelously suited to give the required equality of time; this line is the path traced by a nail fixed to the circumference of a revolving wheel; the geometers of our time have given it the name cycloid; and the main part of the present treatise is devoted to the proof of the power of this line to measure time;

but it is first necessary to confirm and expand with new demonstrations the theory of falling bodies, developed by the great Galileo, whose richest fruit and highest peak is this very property of the cycloid which we have discovered;

next, in order to apply this line to the pendulum, a new consideration of curves is necessary, namely, of curves which by their evolution generate other curves; and from this arises an elegant and new theory of the comparison of the lengths of curves and straight lines;

and in order to explain composite pendula, it is necessary to consider also centers of oscillation, a subject treated by many but with little success until now (*id.*).

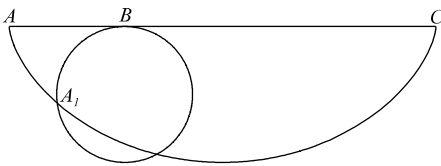
The clock which Huygens discovered in 1657, and described in *The clock*, in 1658, is the clock with a freely suspended pendulum whose motion is transmitted to the clockwork by a handle and fork. It improved the measurement of time dramatically. It also posed for the first time explicitly the problem: of the equal measure of time.

And Huygens solved the problem: by a geometrical method, only by following in the footsteps of geometry.

The simple pendulum is not isochronous, its period depends on the amplitude. Huygens discovered that if the line which the pendulum bob traces is a cycloid, however, its period is independent of the amplitude, and isochronous. Hence, the string on which the pendulum bob swings has to be forced to lead the bob in a cycloidal path. To find the shape of the plates, around which the string has to wrap itself, as the pendulum swings, in order to lead the bob in such a path, Huygens developed the theory of evolutes, of curves which by their evolution generate other curves. He discovered that the evolute of a cycloid is a cycloid. And the shape of the plates has to be cycloidal as well.

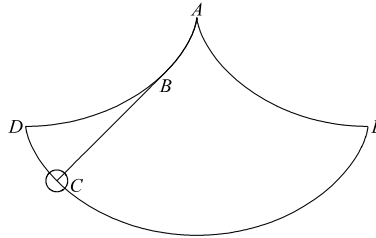
In part I, *A description of the pendulum clock*, he writes: to see that the oscillations of the simple pendulum are not equal, but occur in less time through smaller arcs, make the following experiment; suspend equal weights on two strings of the same length, pull one back a long distance, the other a short distance, and release them simultaneously; you will see that one is faster than the other; the time of descent through a quadrant is to the time of descent through the smallest arc as 34 to 29; and this difference is not due to the resistance of the air, as some have thought; it is due to the nature of motion and the properties of the circle; and this the fact that the motion of the isochronous pendulum is not circular also shows (*id.*).

Huygens then introduces the cycloid, the curve traced by a nail fixed to the circumference of a revolving wheel.



As a circle begins to roll along the straight line ABC , it touches the line at point A , as the circle rolls from A to B , the point traces the curve AA_1 , and as the circle rolls from B to C , completing one revolution, the point traces the curve A_1C , completing the cycloid AA_1C .

He introduces also the evolute of the cycloid, a cycloid itself. He writes: if a string is stretched around a curve in one direction and its end in this direction remains fixed while its other end remains taut, it is clear that this end describes another curve; this curve is called the curve described by evolution (III, def. 3); the curve around which the string is stretched is called the evolute (def. 4).



The string ABC is stretched around the evolute ABD from A to B , while its other end BC , remaining taut and thus tangent to the evolute, describes the curve DCE . If the evolute, ABD , is a cycloid, the curve described by evolution, DCE , is also a cycloid.

Huygens writes: I do not know whether any other line has this remarkable property, to describe itself by its own evolution (I).

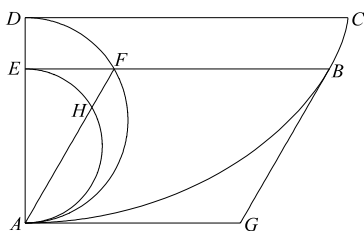
In part II, *The fall of heavy bodies and their motion in a cycloid*, he begins with 3 hypotheses and deduces from them 26 propositions. The hypotheses are: 1. if there were no gravity, and air did not impede the motion of bodies, every body will continue to move with uniform velocity in a straight line; 2. by the action of gravity, whatever its origin, bodies are set in motion composed of uniform motion in one direction and motion downward due to gravity; 3. and these two motions can be considered separately, neither impeding the other (*id.*).

Proposition 3 is: if a body falls, the distances traversed are proportional to the squares of the times. Proposition 4: if a body ascends after a fall, it traverses in equal times the same distances upward as it does downward, and it rises to the same height from which it descends. Proposition 7: the times of descent on differently inclined planes whose height is the same are proportional to the lengths of the planes. Proposition 9: a body rises to the same height from which it falls on however many planes it rises and whatever their

inclinations. Proposition 11: if a body descends on some surface and then ascends on the same surface or on a similar and similarly situated surface, it descends and ascends the same distance in equal times.

Proposition 25 shows the isochronism of the cycloid:

on a cycloid whose axis is perpendicular and whose vertex is at the bottom, the times of descent to the vertex are equal, and are to the time of a fall through the axis as the semicircumference of a circle is to its diameter;



let ABC be a cycloid whose vertex A is at the bottom and axis AD is vertical;

let a body descend by its natural impetus through the arc BA ;

draw the straight line BE parallel to the base CD ; on the axis AD draw semicircle AFD , which cuts BE at F ;

draw AF , and parallel to it BG , which is tangent to the cycloid at B , and cuts the horizontal line through A at G ;

on AE draw semicircle AHE ;

the time of descent through the arc BA is to the time of uniform motion through the line BG with one half of the final velocity acquired by a fall as the semicircular arc EHA is to the diameter EA (*id.*).

Huygens has proved this in proposition 24. He continues:

but the time of uniform motion through BG with one half of the final velocity acquired by a fall is equal to the time of a naturally accelerated fall through BG (*id.*).

Huygens has proved this in proposition 5. He continues:

BG is equal to FA , and the time of a fall through FA is equal to the time of a fall through DA (*id.*).

Galileo has proved that: the times of descent through any planes inclined from the circumfer-

ence of a vertical circle to its lowest point are equal (*Sciences*, III). Huygens concludes:

hence, the time of descent through the arc BA is to time of a fall through the axis DA as the semicircular circumference EHA is to the diameter EA ;

and given the whole cavity of the cycloid, it is clear that after a body has descended through the arc BA , it will ascend through an arc equal to BA in a time the same in the ascent as in the descent; it will then return again through A to B ; and the times of each of these oscillations, whether they are completed in large arcs or in small arcs, is to the time of a fall through the axis DA as the circumference of a circle is to its diameter (*id.*).

The time of descent on a cycloid is to the time of a fall through its axis as $\pi r : 2r$, or what is the same as $\pi : 2$, or what is yet the same, as a constant, and the times of oscillation are thus independent of the amplitude, the same through large arcs as through small.

In part III, *The evolution and dimension of curves*, Huygens begins with 4 definitions and deduces from them 11 propositions. Among these: the length of a cycloid is 4 times its axis, or 4 times the diameter of its generating circle (7); to find a straight line equal to a parabolic curve (9); to find straight lines equal to curves whose evolutions describe ellipses and hyperbolas (10); to show that for any geometrical curve there is another geometrical curve for which an equal straight line can be found (11).

In proposition 6 he concludes: it is clear that a pendulum suspended between two plates curved in the shape of cycloids describes by its motion a cycloid, and its oscillations, whatever their magnitude, are completed in equal times (*id.*).

In part IV, *The center of oscillation*, he begins with 13 definitions and 2 hypotheses and deduces from them 26 propositions. In propositions 25 and 26 he determines the length of a simple pendulum beating seconds, derives the law of the pendulum, and determines the distance through which a body falls in one second, and thus the acceleration of gravity.

In part I Huygens has shown how to regulate a clock by the fixed stars. By the end of part IV it is clear how to construct a clock keeping equal

time. And in proposition 25 he begins:
 regulate by observations of the fixed stars over
 several days a clock so that it makes one
 oscillation in one second;
 suspend next to it a simple pendulum with a
 spherical bob, and increase or decrease the
 length of the string until its oscillations over a
 quarter of an hour or a half hour are in unison
 with those of the regulated clock;
 the simple pendulum should be set in motion
 through a small arc, 5° or 6° ;
 find the center of oscillation;
 then measure the length of the pendulum; the
 length of a simple pendulum beating seconds
 is, in Paris feet, $3 \frac{8\frac{1}{2}}{144}$ (*id.*).

1 Paris foot = .3248 m. The length of a simple
 pendulum beating seconds is, in meters, .9936.
 Huygens continues:

we have shown that each oscillation of a pendu-
 lum suspended between cycloids is to a fall
 from a height one half the length of the pendu-
 lum as the circumference of a circle is to its
 diameter (*id.*).

Huygens has shown this in proposition 5 of part
 III. He continues:

it follows that the times of oscillation of two
 pendula are as the times of falls from heights
 one half the lengths of the pendula;

these half lengths, as well as the whole lengths,
 are as the squares of the times in which they
 are traversed in a fall;

the lengths are thus as the squares of the times
 which measure the oscillations;

and the very small oscillations of a simple pendu-
 lum do not differ from the very small oscilla-
 tions of a pendulum of the same length sus-
 pended between cycloids;

therefore, the lengths of simple pendula are as the
 squares of the times in which very small oscil-
 lations are completed (*id.*).

This is the law of the pendulum. If T is the
 time in which an oscillation is completed, or the
 period, and l is the length, $T^2 \propto l$.

This law, too, Galileo discovered, in the *Two
 new sciences* (I).

Huygens, however, implicitly if not explicitly,
 has determined the constant of proportionality. If t_d

is the time of descent of the pendulum, and t_f is
 the time of a fall through its axis, $\frac{t_d}{t_f} = \frac{\pi}{2}$; if t_f is

1 second, $t_d = \frac{\pi}{2}$; the squares of the times in fall

are as the distances, and if $\frac{l}{2}$ is one half of the
 length of a pendulum, and d_s is the distance in a
 fall of 1 second, $\frac{t_d^2}{\frac{\pi^2}{4}} = \frac{\frac{l}{2}}{d_s}$; the period is a descent

to the vertex, ascent to the same height, descent
 to the vertex, and ascent to the point of beginning
 of the first descent, or $T = 4t_d$; hence, $\frac{T^2}{4\pi^2} = \frac{l}{2d_s}$,

or $T = 2\pi\sqrt{\frac{l}{2d_s}}$; and twice the distance traversed
 in a fall of 1 second is equal to the acceleration of
 gravity in 1 second, $2d_s = g$, and $T = 2\pi\sqrt{\frac{l}{g}}$.

And Huygens determines g , the acceleration of
 gravity. In proposition 26:

the time of one small oscillation is to the time of
 a fall from one half the height of the pendulum
 as the circumference of a circle is to its diame-
 ter, or as 355 to 113;

hence, if $1'' = 60'''$, $\frac{355}{113} = \frac{60'''}{19\frac{1}{10}''}$, and $19\frac{1}{10}''$ is

the time of a fall though one half the height of
 a pendulum beating seconds;

the length of a pendulum beating seconds is

$3 \frac{8\frac{1}{2}}{144}$ Paris feet; and $19\frac{1}{10}''$ is then the time

of a fall through $1 \frac{76\frac{1}{4}}{144}$ Paris feet;

but in a fall the squares of the times are as the
 distances;

$$\frac{\left(19\frac{1}{10}\right)^2}{60^2} = \frac{36\,481}{360\,000}; \quad \frac{36\,481}{360\,000} \approx \frac{1\frac{76\frac{1}{4}}{144}}{15\frac{1}{12}};$$

and, very nearly, $15\frac{1}{12}$ Paris feet is the height of
 a fall in 1 second (*id.*).

15 $\frac{1}{12}$ Paris feet are 4.90 m, and twice the dis-

tance traversed in the first second of a fall gives the acceleration of gravity. The acceleration of gravity is,

Huygens 9.80 m/s^2 ,

Newton 9.81,

no further correction.

Part V of *The pendulum clock* is *Another construction of the clock based on the circular motion of pendula, and theorems on centrifugal force*. The theorems are 13.

The proof of the power of the cycloid to measure time, Huygens writes, is the richest fruit and highest peak of the theory of falling bodies developed by the great Galileo. More generally, *The pendulum clock* is the richest fruit and highest peak of the acquisition of knowledge before Newton's *Principles*, itself in turn the richest fruit and highest peak of the acquisition of knowledge. Huygens begins by confirming and expanding with new demonstrations the theory of falling bodies. Newton confirms and expands with new demonstrations all theories developed by Huygens in *The pendulum clock*. It is *The pendulum clock* that showed that the generality of Descartes's *Principles* can be achieved with the rigor of Galileo's *Two new sciences*.

The very important problems solved in *The pendulum clock* are many. The clock. Hence the certain and equal measure of time. Hence the elegant and new theory of the comparison of curves and straight lines. Hence the theory of centers of oscillation. Hence the derivation of the law of the pendulum. Hence the measurement of the height from which a body falls in 1 second, and thus of the acceleration of gravity. Hence the investigation of centrifugal force.

It is with *The pendulum clock* that time succumbed to analysis. *The pendulum clock* is an important step in the development of calculus. It is the most important step toward the development of the theory of gravity. And even more important is the unity of the treatise, achieved only by following in the footsteps of geometry.

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In 1678, in the *Treatise on light*, in which are explained the causes of what occurs in reflection,

in refraction, and particularly in the strange refraction of Iceland crystal, Huygens formulated the wave theory of light.

Nature, it has turned out, is fond of waves. In 1924 de Broglie proposed, and in 1926 Schrödinger showed that particles are waves, and thus, indeed, everything is waves.

The investigation of the musical intervals, and thus of strings, begins with Pythagoras (Porphyry, comm. *Harmonics*, 3). In 1638 Galileo investigated the relations of tension, length, thickness, and matter in strings (*Sciences*, I). In 1644 Mersenne measured the speed of sound (*Reflections*, Harm., 1.4.5). In 1660 Boyle showed that air is the medium of sound (*Experiments*, 27), and in 1666 concluded: when a thing makes a percussion of the air, it puts that fluid body into an undulating motion; and if a man's ear chances to be in the way of these motions of the air forward and backward, it gives him the perception of them which he calls sound (*Origin*, I, 2).

And in 1678 Huygens treated waves with both generality and rigor.

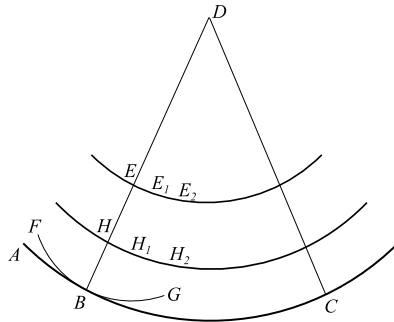
The central result of the *Treatise on light* is Huygens's principle, the principle that every point of a wave front is itself the source of a wave.

The *Treatise* is in six chapters, and in chapter one Huygens begins: most of those who have written on the various parts of optics have contented themselves with presupposing the fundamental truths drawn from experience; I find that no one has yet given a probable explanation of the principal and most notable phenomena of light, that it propagates only in straight lines, and that visible rays, coming from an infinity of different places, cross without hindering one another at all; in this book I will try, in accordance with the principles of the philosophy of the present day, to give clearer and more probable reasons for the properties of light (*id.*).

And later in the chapter Huygens formulates the principle:

it has to be considered that in the emanation of a wave each particle of the matter, in which the wave propagates, has to communicate its motion not only to the particle next to it in the straight line from the luminous point; it necessarily has to impart motion also to all others particles touching

that particle and opposing its motion; it thus follows that around each particle arises a wave of which the particle is the center;



if ABC is a wave coming from the luminous point D , the particle E , one of those within the sphere ABC , will make its own particular wave FBG touching the wave ABC at B at the same instant that the principal wave from D has reached ABC ; it is clear that the only point of the wave FBG which will touch the wave ABC is the point B in the straight line through DE ; in the same way the other particles contained in the sphere ABC , such as E_1 , H , H_1 , and so on, will each make its own wave; but each of these waves can only be infinitely feeble compared with the wave ABC , to the composition of which all the others contribute by that part of their surfaces which is the most distant from the center D ;

it is clear also that the wave ABC is determined by the limit of the motion beginning from D and attained in a certain time, as there is no motion beyond this wave, even if there is motion in the space which it encloses, namely, in the parts of the particular waves which do not touch the sphere ABC ;

nor need all this seem burdened with minuteness or subtlety, because on its ground all properties of light, and everything pertaining to its reflection and refraction can be explained (*id.*).

Principally and most notably, it explains the principal and most notable phenomena of light, that it propagates only in straight lines, and that visible rays, coming from an infinity of different places, cross without hindering one another at all.

From Huygens's principle Huygens deduces reflection (2), deduces refraction (3), demonstrates Fermat's principle of least time (*id.*), explains atmospheric refraction (4), deduces the

double refraction in Iceland crystal (5), and discovers the polarization of light: a marvelous phenomenon of which I have not yet been able to find the cause (*id.*).

In the *Treatise on light* Huygens also records the first measurement of the speed of light. He concludes that the speed of light is constant. And the fact that the speed of light is finite is a fundamental premise of the wave theory.

He writes:

it cannot be doubted that light consists in the motion of certain matter; this has to be assumed, at least in the true philosophy, in which the cause of all natural effects is sought by mechanical reasoning; otherwise, in my opinion, all hope of ever comprehending anything in physics has to be abandoned;

when we see a luminous object, this cannot be by the transport of matter from the object to us, in the way an arrow traverses the air, because this will grossly violate the two principal properties of light; light spreads in another way; and to understand how, the knowledge we have of the propagation of sound in the air leads;

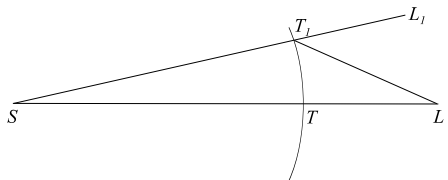
as it cannot be the transport of a body from the luminous object to our eyes, there can be no doubt that light also comes from one to the other by a motion impressed on the matter between the two;

and if light takes time for its passage, it will follow that this motion, impressed on the intervening matter, is successive; it will further follow that light spreads, like sound, in spherical surfaces and waves; and I call them waves from their similarity to those formed in water when a stone is thrown in it;

to see, then, whether the passage of light takes time, let us consider whether there are any experiments which can convince us of the contrary;

those that can be made on earth, by striking lights at great distances, can prove that light takes no sensible time to pass across these distances; but these distances are small, and from such experiments it can only be concluded that the passage of light may be extremely rapid;

Mr Descartes, who was of the opinion that the passage of light is instantaneous, grounded his opinion on a better experiment, one drawn from eclipses of the moon;



let S be the sun, TT , a part of the earth's orbit, and STL a straight line from the sun to the moon; if light requires time, 1 hour, for instance, to traverse the distance between the earth and the moon, it follows that, when the earth is at T , the shadow it casts on the moon has not yet arrived at L ; the shadow will only arrive there 1 hour later, and this occultation of the moon will only arrive back at the earth 1 hour later still;

let us assume that in these 2 hours the earth has arrived at T_1 ; when the earth is at T_1 , then, it will see the eclipsed moon at L , and at the same time the sun at S , because it is at rest, as I presume with Copernicus, and as light moves always in straight lines, the sun must always appear where it is; yet the eclipsed moon has always been observed at the point of the ecliptic opposite to the sun; while on the present argument it would appear behind that point by an amount equal to the angle LT_1L , the supplement of the angle ST_1L , and this is contrary to experience, for that angle would be very sensible, about 33° ;

it has been assumed in this argument that light takes 1 hour for the passage from here to the moon; but if it takes 1 minute, the angle LT_1L will be only $33'$; if it takes 10 seconds, the angle will be less than $6'$, and it will then not be easy to perceive at all;

it is true that this assumes a strange speed, one 100 000 times greater than that of sound; for sound, according to my observations, travels about 180 toises in 1 second (1).

1 toise = 1.949 m. The speed of sound is,
 Mersenne 448 m/s,
 Huygens 351,
 correction 343.

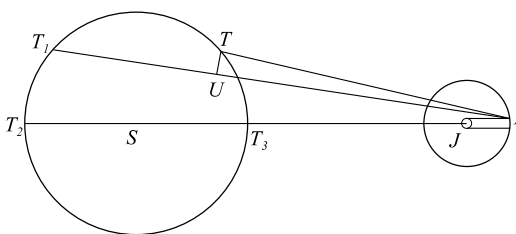
Huygens continues:

but this assumption need not seem an impossibility; because the question is not about the transport of a body at so great a speed, but about the successive motion transmitted from some bodies to other bodies;

and, in fact, what I have proposed here as a

hypothesis has recently received the probability of an established truth by the ingenious proof of Mr Rømer; it is founded, like the preceding argument, on the observation of the heavenly bodies, and it not only proves that light takes time for its passage, it demonstrates how much time it takes, and that its speed is 6 times greater even than that which I have just mentioned;

Mr Rømer makes use of the eclipses of the little planets revolving around Jupiter;



let S be the sun, $TT_1T_2T_3$ the annual orbit of the earth, J Jupiter, and the orbit around Jupiter that of the satellite nearest to it, which is best suited for this investigation because of the swiftness of its revolution; let I be this satellite as it emerges from Jupiter's shadow; and let the satellite be seen to emerge from the shadow when the earth is at T ;

it must be, then, that, if the earth remained at the same place, at T , the satellite would be seen to emerge again from the shadow after $42\frac{1}{2}$ hours,

for this is its period; and if the earth remained at T for 30 revolutions of the satellite, it would be seen to emerge from the shadow after 30 times $42\frac{1}{2}$ hours;

if the earth is carried along during this time to T_1 , however, increasing its distance from Jupiter, and light takes time for its passage, the emergence will be perceived later at T_1 than it would have been at T , and to the time of 30 times $42\frac{1}{2}$

hours must be added the time which light has taken to traverse the distance UT_1 , the difference between the distances IT and IT_1 ;

and in observations of these eclipses over 10 consecutive years the difference has been found to be considerable, 10 minutes and more; and it has been calculated that in order to traverse the whole diameter of the annual orbit T_3T_2 , which is

twice the distance from here to the sun, light takes 22 minutes;

the motion of Jupiter is included in this calculation;

the diameter T_2T_3 is according to me some 24 000 diameters of the earth; supposing it, then, to be not more than 22 000, given that light traverses it in 22 minutes, this makes the speed of light $16\frac{2}{3}$ diameters of the earth in 1 second,

which makes it 110 000 000 toises in 1 second, as the diameter of the earth is 2 865 leagues, counting 25 leagues to the degree, and each league is 2 282 toises, according to the exact measurement by Mr Picard in 1669 (*id.*).

1.10×10^8 toises is 2.14×10^8 m.

In 1728 Bradley discovered the aberration of starlight, and deduced from it the ratio of the speed of light to the speed of the earth around the sun at 10 210:1 (*Motion*). Given Picard's measurement of the earth's radius at 6 371 km, and Cassini's measurement of the distance to the sun at 21 600 earth radii, the ratio 10 210:1 gives 2.80×10^8 m/s for the speed of light.

In 1862 Foucault measured the speed of light with a rotating mirror.

The speed of light is,

Rømer	2.14×10^8 m/s,
Bradley	2.80×10^8 ,
Foucault	2.98×10^8 ,
correction	3.00×10^8 .

As the investigation of light rests on the investigation of the heavenly bodies so it does on the investigation of collisions.

Huygens writes: and if one is to search for the way in which the motion of light is communicated, one will find none that agrees with uniform progression better than the property of elasticity; if the motion of light were to become slower in proportion to the distance from its source, it could not conserve its great speed over great distances; but if the aethereal matter, which propagates light, is supposed elastic, its particles will have the property of equally rapid rebound whether they are struck strongly or feebly, and light will always propagate with equal speed (*id.*).

And nature has turned out to be fond of the constancy of the speed of light, too. Hence, in 1905, Einstein developed the theory of relativity.

With Huygens, then, waves are the consequence of the finite speed of light.

The finite speed of light is the consequence of the assumption that light consists in the motion of matter.

And this assumption is the consequence of the true philosophy, in which the cause of all natural effects is sought by mechanical reasoning. Or all hope of ever comprehending anything in physics has to be abandoned.

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In the preface added to the *Treatise on light* in 1690, Huygens writes: there will be seen in this treatise demonstrations which do not produce certainty as great as those of geometry; the geometers prove their propositions from certain and incontestable principles; here the principles are verified by the conclusions drawn from them; the nature of things will not have it otherwise; it is nevertheless always possible to attain a degree of probability which falls short of complete evidence in nothing; for if the things demonstrated from the assumed principles correspond perfectly to the phenomena which experience has manifested, the more so if the number of these phenomena is great, and the more so if new phenomena have been foreseen from the hypotheses, and effects have been found to correspond to the expectations, if there are all these proofs of probability in the matters of which I treat, it seems to me that this must be a grand confirmation of the success of my investigation (*id.*).

It is a grand confirmation of the success of the *Treatise on light*. More generally, it is a grand confirmation of the success of Huygens's earlier investigations as well. More generally still, it is a grand confirmation of the success of the investigations of the philosophers in the preceding couple of generations as well. Most generally, it is a grand confirmation of the true way.

Bacon describes the true way. The whole thing is done by machinery. It begins with the collection of the particulars. It establishes stages of certainty. It ascends from experiments to principles, descends from these principles to new experiments, ascends from these experiments to higher principles, descends from these again, ascends again, and until it reaches the law with its

clauses.

Two generations later, Huygens describes the true way with the particularity which its grand confirmation has shown.

Principles have been discovered.

The principles have been verified by the conclusions drawn from them. The things demonstrated from the principles have corresponded perfectly to the phenomena manifested by experience, the number of these phenomena has been great, and new phenomena have been foreseen from the principles.

Hence, more general principles have been discovered. The things demonstrated from these principles have again corresponded perfectly to the phenomena manifested by experience, the number of these phenomena has been greater, and new phenomena have again been foreseen from the principles.

A degree of probability has been attained which falls short of complete evidence in nothing.

Moreover, the inseparable development of physics and mathematics, with the labors of Galileo, of Kepler, of Descartes, and of Huygens, shows that, pace Huygens, as the principles of the physicists so the principles of the mathematicians. The principles are verified by the conclusions drawn from them. Even if the mathematicians have discovered some of their principles a couple of millennia earlier, while the physicists have begun discovering their principles, upon becoming mathematicians, in the couple of generations through Huygens.

The principles are verified by the conclusions drawn from them. Nothing else can show the principles certain and incontestable.

And the whole thing begins with the collection of the particulars. The whole thing is done by machinery. Induction, then deduction, then again induction, then again deduction.

The nature of things will not have it otherwise.

And nothing shows this better than the fact that, jointly, the labors of the five, Bacon, Galileo, Kepler, Descartes, and Huygens show five clear and distinct steps in the grand confirmation of the true way. Bacon pointed the way. Galileo made considerable advances in it. Kepler reached the principles. Descartes reached the most general principles. And Huygens verified the principles

by the conclusions drawn from them.

The whole thing is done by machinery.

Bacon pointed the way. In the previous generation Viète greatly advanced mathematics, Gilbert and Tycho greatly advanced physics, Bruno greatly advanced enlightenment. Their advances, however, do not inform one another explicitly at all. With the labors of Viète, Gilbert, Tycho, and Bruno the advance in analysis, the advance in the investigation of magnetism, the advance in the investigation of the heaven, and the advance in man's emergence from his self-imposed minority are each an advance of its own. It is with the labors of Bacon, the more so with the labors of Galileo and Kepler, the more so with the labors of Descartes in the next generation, and the more so with the labors of Huygens in the next generation, that these advances have all informed one another explicitly, and that the great further advances in mathematics, physics, and enlightenment have all become explicitly inseparable from one another, and all achieved explicitly in the pursuit of the true way.

Gilbert's treatises and Tycho's treatises are each a vast collection of the particulars. Galileo's treatises are not just a vaster collection. They are a complete collection. Bacon writes: the particulars are like an army, little is to be hoped for unless all particulars pertaining to the matter are marshaled. The fourth rule of the method is: always to make enumerations so complete and reviews so general as to omit nothing. *The two main systems* marshals all facts gathered in the investigation of the heaven from Aristotle through itself. Hence, Galileo proved that the earth revolves around the sun. *The two main systems* and the *Two new sciences* marshal between them all facts gathered in the investigation of motion from Aristotle through themselves. Hence, concerning the oldest object Galileo set forth the entirely new science. Galileo showed that induction can be complete. Bacon writes: experience as it occurs is an accident, as it is pursued an experiment. Galileo showed paradigmatically how to pursue it. Hence, experiment is with his labors the ground of the completeness of induction, as well as the ground of the step from induction to deduction. The *Two new sciences* showed that the generality of Aristotle can, and if

it can, it must, be achieved with the rigor of Archimedes. The true way ascends, descends, ascends, and the *Two new sciences* is the first treatise to ascend explicitly from the phenomena to definitions and axioms, and to descend explicitly from the definitions and axioms to the phenomena. Galileo showed that the chains of inferences of the physicists, as of the mathematicians, can be free of gaps. Hence, as the mathematicians prove their propositions from principles so the physicists can prove their propositions from principles. And, hence, Galileo took a major step toward the discovery of the principles.

As to enlightenment, even if the judgment which the pack horses and doctors of theology rendered on Galileo did not show that the advance in physics and mathematics endangers man's self-imposed minority, the judgment which Galileo rendered on the pack horses and doctors of theology could hardly show it more clearly and distinctly.

Kepler's treatises are a vast collection of the particulars as well. From the *New astronomy* to the *Harmonics* to the *Epitome* the collection is better ordered. There have yet been better ordered collections. There has not been a collection to achieve a greater ascent. What Kepler's treatises lack in rigor, they more than make up in the generality of the principles they have reached. The lack of rigor can explain why Kepler's laws took two generations to establish themselves conclusively. Kepler's laws, on the other hand, show paradigmatically the verification of the principles by the conclusions drawn from them. Beginning with the deduction of the perpetual tables. And of the prophecy that needed no revelation to show fulfilled.

Descartes is the synthesis of Bacon, Galileo, and Kepler. More generally, he is the synthesis of all earlier acquisition of knowledge.

Aristotle is the synthesis of all earlier acquisition of knowledge. And Aristotle's synthesis is the consequence and the ground, both, of the greatest breadth and completeness which induction has achieved. Breadth, in collecting facts in everything that has been investigated. Completeness, in collecting all facts in much that has been investigated. Descartes's synthesis is the consequence and the ground, both, of the greatest

breadth and completeness which induction has achieved and the greatest height and rigor which deduction has achieved. Height, in beginning with the most general principles. Rigor, in proceeding by chains of inferences as free of gaps as there have been. In Descartes's synthesis, moreover, the height and rigor of deduction rest on the breadth and completeness of induction, and the breadth and completeness of induction rest on the height and rigor of deduction.

Bacon pointed the true way. Descartes pointed it clearly and distinctly. Galileo showed that the world is written in mathematics. Descartes showed it mathematically. Kepler showed that the world is the system of the world. Descartes showed it systematically. And again, these are in Descartes one another's consequences and grounds, both. He pointed the true way, and pointed it mathematically, and pointed it systematically. He showed systematically as well as mathematically that the world is written in mathematics, and so written systematically, and it is the system of the world because it is written in mathematics and it is written in mathematics because it is the system of the world, and showed as well systematically as well as mathematically that the true way is written in mathematics, and systematically, and because the world is so written.

All subsequent acquisition of knowledge stands on Descartes's synthesis.

The method. Hume's and Kant's investigations are investigations and applications of the method.

Analysis. All subsequent mathematics and physics stands on the results of the *Geometry*.

The method is mathematical. Hence Frege.

The theory of vortices and the description of the world beyond the solar system. Hence the theory of gravity. Then the theory of relativity. Then the big bang theory.

The quantity of matter and the quantity of motion. It is with these that the investigation of the world began succumbing to equations.

The laws of nature. The acquisition of knowledge has been the acquisition of the laws of nature. Kepler's laws. Newton's laws. Clausius's laws. Maxwell's equations. Schrödinger's equation. Hubble's law.

Man is a machine. Hence Darwin. And Mendel.

All subsequent labors against the labors of acquisition of knowledge, too, have been labors most particularly against Descartes's synthesis. They have an interest in my abstaining from publishing the principles of philosophy. Still do. One century after Hegel still, three after Descartes: Descartes's feigned new beginning unveils itself as a baleful prejudice.

In the generation after Descartes already, his successors advanced philosophy greatly, in all its parts, by building on and continuing his labors, essentially and substantially, and all the following philosophers by building on his labors explicitly.

Wallis took a major step in the development of calculus.

Picard measured the length of 1° of the meridian, and hence the diameter of the earth. With Picard's measurement the problem of the earth's radius is solved conclusively. Picard measured the radius of the earth at 6 372 km. The equatorial radius of the earth is 6 378 km.

Pascal discovered the hydraulic press, discovered Pascal's law, in a confined fluid external pressure is transmitted uniformly in all directions, laid the foundations of the calculus of probability.

Cassini measured the parallax of the sun. With Cassini's measurement the proportions of the solar system are demonstrated.

Redi showed that there is no spontaneous generation. Miracles were excluded from generation.

Boyle showed that air transmits sound, discovered Boyle's law, at constant temperature the volume of a gas is inversely proportional to its pressure. On Boyle's law all subsequent investigation of fluids and of heat stands.

Ray took a major step in the classification of organisms, and established species as its fundamental unit.

Malpighi demonstrated the capillaries. It is with Malpighi's labors that anatomy and physiology became experimental.

Richer discovered that the beat of a pendulum is slower at the equator than further north. It was the most important experimental anticipation of the theory of gravity.

Pufendorf made the second, after Spinoza's, explicit attempt to ground politics and ethics in the method.

Leeuwenhoek discovered protozoa.

Hooke discovered the cell, discovered the law of elasticity, strain is proportional to stress.

Burnet described the development of the earth, reducing *Genesis* explicitly to physics.

Swammerdam discovered the stages in the development of insects. It is with Swammerdam's labors that the fact began being established that insects are more important than men, positively that they have been more important, and positively that they are a much more important object of investigation.

Steno began the investigation of the stratification of the earth.

Huygens and Spinoza are part of this generation.

Huygens showed that the generality of Descartes's *Principles* can be achieved with the rigor of Galileo's *Two new sciences*. Descartes's *Principles* is the treatise of the greatest generality, ranging over everything that the acquisition of knowledge has reached. No other treatise has achieved such generality. Newton's *Principles* is not so general. Newton's *Principles* is nevertheless the most successful treatise, by far – by very far – in achieving both generality and rigor. It has not sacrificed too much generality either in order to achieve rigor throughout. And the decisive step in showing that no generality at all has to be sacrificed is Huygens's. In his treatises so many results are deduced, so many different results are deduced, all in chains of inferences so free of gaps, and all connected through chains of inferences so free of gaps, as to make the conclusion, inductively, unavoidable, nothing must be sacrificed, and if nothing must be sacrificed, everything must not be sacrificed.

Leibniz more than any philosopher did just that, undertook to deduce everything, everything in general and nearly enough everything in particular.

Huygens is the decisive step toward Newton and Leibniz, both.

The principles are verified by the conclusions drawn from them. By the perfect correspondence of the things demonstrated from the principles to the phenomena manifested by experience, the great number of these phenomena, and the new phenomena foreseen from the principles.

Huygens discovered Titan, discovered the ring of Saturn, determined the true proportions of the solar system, discovered the constancy of the speed of light, discovered that light is waves, discovered a certain and equal measure of time, discovered the isochronism of the cycloid, discovered evolutes, discovered how to calculate the probability of random events, determined the constant of proportionality of the law of the pendulum, determined the height from which a body falls in 1 second, solved the problem of collisions, discovered the conservation of momentum. To note only some of his more important results.

At the same time Spinoza completed the grand confirmation of the true way. With Spinoza's labors history, too, succumbed to the method.

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One and a half centuries later, spirit had lost its essential life.

But it was conscious of the loss. Turning away from the chaff, confessing that it lies in wickedness, and reviling itself for doing it, spirit demanded from philosophy the restoration of the substantiality and purity of its being.

And philosophy complied. It restored spirit to the substantiality and purity of its being. Philosophy quit being what it was and became what it was not. What it became, what Hegel made it become in the *History of philosophy* shows.

The new philosophy begins with Boehme as well as Bacon.

Galileo does not exist.

Kepler does not exist.

Descartes has done no mathematics or physics.

Huygens does not exist.

Newton is a complete barbarian.

Leibniz has done no mathematics or physics.

Hume is a sceptic.

Kant is an idealist.

And yet what Spinoza has become has had the gravest consequences.

Hegel writes: Spinoza aroused the great hatred of the rabbis, and the even greater hatred of the Christian and especially the Protestant theologians first by his *Theologico-political treatise* (III, 2.1.A.2); in that book is his doctrine on inspiration, a critical treatment of the Mosaic

books, and the like, from the point of view that their laws are restricted to the Jews; what later Christian theologians have written critically on this, where they would show that these books were edited later and were in part younger than the Babylonian captivity, on which especially the new Protestant theologians so greatly plume themselves, all this is found in that book of Spinoza already (*id.*).

Having thus used it to strip the Christian and especially the Protestant theologians of their finery, Hegel has no more to say about the *Theologico-political treatise*.

He continues: Spinoza aroused that hatred, however, most of all with his philosophy, which we now have to consider closer (*id.*); his chief work is the *Ethics* (*id.*); Spinoza's system is very simple (*id.*); the simple thought of Spinozan idealism is this, what is true is only the one substance, whose attributes are thought and extension, or nature, and only this absolute unity is real, is the reality, only it is god (*id.*); the Spinozan idea is the same as the *ὄν* of the Eleatics; it is with Spinoza that this eastern view first spoke in the west; in general, it has to be noted that thought must put itself on the standpoint of Spinozism; this is the essential beginning of all philosophizing; to begin philosophizing, one must be first a Spinozist; the soul must bathe in that aether of the one substance, in which everything held for true is extinct; it is this negation of everything particular, to which every philosopher must have arrived; it is the freedom of spirit and its absolute foundation; the only difference from the Eleatic philosophy is that through Christianity in the modern world the concrete individuality is present everywhere in spirit (*id.*).

Spinoza's system is very simple. It is the same as that of the Eleatics. The only difference is Christianity, Parmenides is before it, Spinoza is after it.

Divide et impera. Detach Bacon and Descartes from Galileo, Kepler, and Huygens. Detach Descartes and Leibniz from mathematics and physics. Detach *The method* from the *Geometry*. Detach the *Ethics* from the *Theologico-political treatise*.

In general, detach philosophy from the chaff. Mathematics is chaff. Physics is chaff. History is

chaff. Everything held for true is chaff. Everything particular is chaff.

So that philosophizing can begin. So that the soul can bathe in that aether of the one substance, in which everything held for true is extinct, this negation of everything particular, to which every philosopher must have arrived.

The hatred of the rabbis and the Christian theologians did no damage to Spinoza, to history, and to philosophy. The love of Hegel did.

It is with Spinoza's labors that faith, religion, and theology were explicitly excluded from philosophy. It is Spinoza who most explicitly exposed the wickedness of spirit.

More important, Voltaire wrote *The philosophy of history*, Herder wrote *Also a philosophy of history*, then also *The philosophy of the history of mankind*, then Hegel also wrote *The philosophy of history*, and to write a philosophy of history became another of the pastimes of the philosophers. The only philosophy of history which is, in fact, philosophy and, in fact, of history is Spinoza's *Theologico-political treatise*.

Most important, it is with Spinoza's labors that history succumbed to the method.

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Spinoza's first treatise is the *Treatise on the correction of the understanding, and on the way in which it is best directed toward true knowledge of things*, in 1662.

Like Bacon, Galileo, Kepler, Descartes, and Huygens, Spinoza labors at the correction of the understanding from the first, and like Bacon, Galileo, Kepler, Descartes, and Huygens, to the same end, true knowledge of things.

Descartes writes: philosophy is the perfect knowledge of all the things that can be known; and to be such philosophy must be deduced from the principles.

From the first to the last Spinoza labors at the perfection of philosophy. At the deduction of philosophy from the principles. The deduction of all the things that can be known from the principles. And it is with his labors that the deduction of all the things that can be known has come closest to being, in fact, of all the things that can be known, and, in fact, their deduction. It is with his labors that the deduction has come closest to

reaching explicitly all philosophy, explicitly all its parts, explicitly all philosophy in relation to all its parts, and explicitly all its parts in relation to all philosophy.

Philosophy is theoretical and practical. Theoretical philosophy is physics and mathematics. It is also history, if history will be a part of philosophy. Practical philosophy is politics, including ethics.

And Spinoza labors at the perfection of all philosophy. At the deduction of theoretical philosophy, the deduction of practical philosophy, and the deduction of practical philosophy from theoretical philosophy. He labors at the deduction of physics, the deduction of history, the deduction of politics, the deduction of ethics, the deduction of history from physics, the deduction of politics from physics, and the deduction of ethics from physics. It is with the labors of Spinoza that philosophy has come closest to perfect unity and uniformity. It is with his labors that philosophy has come closest to being deduced in its entirety from one principle.

The principle is nature.

Spinoza's very important treatises are five, *Parts I and II of Descartes's Principles of philosophy demonstrated in the geometrical manner*, in 1663,

Theologico-political treatise, in 1670,

Ethics demonstrated in geometrical order, in 1677,

Political treatise, in which it is demonstrated how the society in which monarchy rules, as well as that in which aristocracy rules, needs to be ordered so as not to lapse into tyranny and to preserve inviolate the peace and freedom of the citizens, in 1677,

Compendium of the grammar of the Hebrew language, in 1677.

His first treatise, the *Treatise on the correction of the understanding*, in 1662, is important, if not very important.

He wrote also the *Metaphysical thoughts, in which difficulties arising in the general as well as special parts of metaphysics are explained*, in 1663.

In the *Parts I and II of Descartes's Principles*, Spinoza lays the foundation. Part I of Descartes's *Principles* is *On the principles of human knowl-*

edge, part II is *On the principles of the material things*. In Descartes these principles are the ground of deduction. From the principles of the acquisition of knowledge and the principles of the material things Descartes deduces knowledge and the material things. In Spinoza, implicitly if not yet explicitly, these principles are the ground of induction. In order to demonstrate them in the geometrical manner Spinoza orders the principles explicitly into definitions, postulates, axioms, and propositions. He thus orders them into less general and more general. And the demonstration thus shows, implicitly if not yet explicitly, that the attempt can be made to deduce the principles of the acquisition of knowledge and the principles of the material things, both, from more general principles, and from one most general principle. In the *Parts I and II of Descartes's Principles* already Spinoza has taken a step toward the formulation of the one principle from which all other principles can be deduced.

The principle is god. The demonstration of the principles in the geometrical manner has not yet shown this explicitly. It has shown most explicitly, however, that god needs further investigation.

Hence, in the *Theologico-political treatise*, Spinoza investigates what god has said about himself in scripture. In order to do so, he deduces the method of the investigation of scripture from the method of the investigation of nature.

Descartes writes: method are the certain and easy rules which, if followed exactly, will never allow the false to be taken for true.

The method is thus the method, the true way. It is the method of physics, and the method of history. The general rules are the same, and from the same general rules the particular rules have to be deduced. The particular rules of physics. And in the *Theologico-political treatise* Spinoza deduces the particular rules of history.

The rules of history deduced, Spinoza formulates and solves the problem of when and how scripture was authored. He formulates and solves the most difficult and most important problem history has solved, before or since. Both, the difficulty and the importance of the problem, are attested abundantly by the fact that, in order to formulate and solve it, Spinoza has to cut across

more than two millennia of the most vigorous falsification and most vigorous protection of the most vigorous falsification.

The further investigation of god thus shows, where scripture is concerned, that it is the object, and hence also the source, of: folly, anile devotion, arrogance, and malice.

Where god is concerned, that: god is nature.

Theology deduced from physics, Spinoza then deduces politics.

Hence, ethics can also be deduced.

The *Ethics*, accordingly, is *demonstrated in geometrical order*. The demonstration is in five parts. Part I is *On god*. It contains Spinoza's conclusive attempt to establish the one principle from which all other principles can be deduced. The principle is god. God is nature. The principle is nature. Part II is *On the nature and origin of the mind*, part III is *On the origin and nature of the emotions*. These contain Spinoza's advance of physics. Mind is body. The emotions are also body. Part IV is *On human bondage, or on the strength of the emotions*, part V is *On the power of the understanding, or on human freedom*. These contain the deduction of ethics from physics.

The *Political treatise* is unfinished. Spinoza died in 1677. He *demonstrated how the society in which monarchy rules, and that in which aristocracy rules, needs to be ordered so as not to lapse into tyranny and to preserve inviolate the peace and freedom of the citizens*. He was to demonstrate also how the society in which democracy rules needs to be ordered to the same end. The *Political treatise* contains Spinoza's conclusive deduction of politics from physics.

The importance of Spinoza cannot be overstated. Nor is it in that the soul can bathe.

To show the importance of Spinoza the following can be shown. The uniformity of philosophy. What god is. The formulation and solution of the problem of when and how scripture was authored. And that the method of history is the method.

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In part I of the *Parts I and II of Descartes's Principles* Spinoza begins with a preface, then from 10 definitions and 11 axioms deduces 21

propositions, and 10 lemmas and corollaries.

Axiom 1 is: at knowledge and certainty of an unknown thing we arrive only through knowledge and certainty of another thing prior to it in certainty and knowledge (*id.*).

At knowledge and certainty we arrive only through knowledge and certainty. Thus, given knowledge and certainty, we arrive at further knowledge and certainty. Conversely, given that we arrive at knowledge and certainty, there is prior knowledge and certainty. Given the knowledge and certainty at which the philosophers before him have arrived, Huygens arrives at knowledge and certainty at which no one before him has arrived. Conversely, given the knowledge and certainty at which Huygens arrives, there is the knowledge and certainty at which the philosophers before him have arrived.

In the same way, the principles of the acquisition of knowledge are the rules of the method, and Descartes discovered the rules of the method. The rules are knowledge and certainty, and Descartes arrives at the rules from prior knowledge and certainty. From mathematics. He writes: arithmetic and geometry are spontaneous fruits of the innate principles of the method; I considered the matter carefully, and I saw that mathematics is concerned with nothing but problems of order and measure; and I saw that there must be general knowledge of order and measure.

Descartes discovered the principles of the acquisition of knowledge by induction.

And he discovered the principles of the material things, or the laws of nature, also by induction.

Nor can the principles be discovered but by induction. That they are discovered, on the other hand, deduction shows. The grand confirmation. The principles are verified by the conclusions drawn from them. By the perfect correspondence of the things demonstrated from them to the phenomena manifested by experience, the great number of these phenomena, and the new phenomena foreseen from them.

The true way ascends, descends, ascends, descends, ascends. From spontaneous fruits to principles to experiments to more general principles to new experiments to more general principles.

The principles cannot be deduced from the principles. The principles are among all things in the world. But they cannot be like all other things in the world. All other things in the world are deduced from them. They are not deduced from themselves.

The principles thus cannot be indiscriminately principles. They have to be of two kinds, the principles of deduction, and the principles of all other things. More generally, the principles of the acquisition of knowledge, and the laws of nature.

If they are of the greatest generality both, the principles of the acquisition of knowledge and the laws of nature cannot be deduced from one another. That they are the principles and the laws of the same all other things in the world, however, both induction and deduction can show. From the same all other things in the world both the principles of the acquisition of knowledge and the laws of nature can be induced, and from both the principles of the acquisition of knowledge and the laws of nature the same all other things in the world can be deduced.

And both, in fact, implicitly if not explicitly, have been shown. The development of mathematics and physics – their preestablished harmony – has shown it. Euclid wrote the *Elements*, as well as the *Phenomena* and *Optics*. Archimedes solved problems of areas and volumes, as well as of weights and fluids. Ptolemy developed the theory of the planets mathematically. Galileo set forth the entirely new science of motion mathematically. Kepler discovered the laws of the heavenly bodies mathematically. These are all further knowledge and certainty from prior knowledge and certainty, all the results of induction and of deduction, both, and induction from, and deduction to, very much the same all other things in the world. If the development of mathematics has led to the discovery of the principles of the acquisition of knowledge, it has on the ground of physical results not less than mathematical. And that the development of mathematics has led to the discovery of the principles of the acquisition of knowledge Descartes is explicit. Before him Aristotle affirms that knowledge is acquired only by demonstration, and demonstration is deduction. Ptolemy affirms that mathematics alone is certain and unshakable knowledge, and it can

deal with the objects of theology and physics as well. Galileo affirms that the world is written in mathematics. If the development of physics has led to the discovery of the laws of nature, it has on the ground of mathematical results not less than physical. And that the development of physics has led to the discovery of the laws of nature the fact suffices to show that Euclid, Archimedes, Ptolemy, Galileo, Kepler, Huygens, all labor toward the formulation of principles, of definitions, postulates, axioms, laws.

But if the principles of the acquisition of knowledge and the laws of nature can both be induced from the same all other things in the world, and the same all other things in the world can be deduced from both the principles of the acquisition of knowledge and the laws of nature, this implies that the principles of the acquisition of knowledge and the laws of nature may not be of the greatest generality. It implies that from the principles of the acquisition of knowledge and the laws of nature a more general principle may be induced, the same one most general principle, and from this one most general principle the principles of the acquisition of knowledge and the laws of nature may both be deduced.

And, in general, the labors of Spinoza are just that, the attempt to formulate and solve the problem of the one most general principle which can be induced from the principles of the acquisition of knowledge and the laws of nature, and from which the principles of the acquisition of knowledge and the laws of nature can be deduced.

Again, axiom 1 is: at knowledge and certainty of an unknown thing we arrive only through knowledge and certainty of another thing prior to it in certainty and knowledge.

And prior is not just prior in acquisition, as it has been taken above. It is prior in certainty and knowledge, hence also prior in acquisition. And prior in certainty and knowledge implies a principle of all principles.

The problem of the principle of all principles was not new.

The first philosopher already said that water is the principle of all things.

Parmenides discovered the principle of contradiction, and Aristotle said that the principle of

contradiction is the principle of all principles.

Plato already said that god is the measure of all things. Then god conquered philosophy, and became unquestionably, should the question of principles arise, the principle of all things, and the principle of all principles.

What was new was twofold. God had not only for a long time been part of philosophy, essentially, substantially, and unquestionably the principle of all things, and the principle of all principles. For just as long the true philosopher had been, with Augustine, the lover of god. What was new was that the true philosophers discovered that in order to be true philosophers they do not have to, pace Augustine, love god. They have to investigate him. Positively if he would be the principle of all things, and the principle of all principles. And whether they loved him or not, Galileo, Kepler, Descartes, Spinoza, investigated him.

Also, Descartes's success in establishing the principles of the acquisition of knowledge and the laws of nature implied not only that there may be a principle of all principles. It implied that the attempt can be made to formulate and solve the problem of this principle explicitly.

The principles of the acquisition of knowledge are the rules of the method. The rules of the method stand on the first rule. On clarity and distinction that cannot be doubted. And: every clear and distinct perception must necessarily have god for its author, because god, the sum of perfection, cannot on pain of contradiction be false.

The principles of the acquisition of knowledge depend on clarity and distinction that cannot be doubted, clarity and distinction that cannot be doubted depends on the truthfulness of god, the truthfulness of god depends on the principle of contradiction.

Also: the world is one; nature is matter itself; matter is of one and the same kind; all variation of matter is motion; nature is nothing but god himself; god is the primary cause of motion; he always conserves the quantity of motion in the world; and from the immutability of god we know the laws of nature.

The laws of nature depend on the immutability of god, the immutability of god depends on

conservation, conservation depends on the unity and uniformity of nature. The world is one, nature is matter, matter is of one kind, all variation in it is motion, nature is god.

In order to formulate and solve the problem of the principle of all principles explicitly, then, with respect to the principles of the acquisition of knowledge, the relations of clarity and distinction that cannot be doubted, the truthfulness of god, and the principle of contradiction have to be sorted out. With respect to the laws of nature, the relations of the immutability of god, conservation, and the unity and uniformity of nature have to be sorted out. With respect to both, the principles of the acquisition of knowledge and the laws of nature, the relations of all these have to be sorted out.

If Spinoza's labors are an attempt to formulate and solve the problem of the one most general principle of all principles, the analogy with Kepler's labors holds. If Kepler's labors are an attempt to discover the laws of the motions of the heavenly bodies, his treatises show two distinct steps to that end. *The cosmographic mystery*: the most admirable and most felicitous apprentice piece, is the first step. His subsequent treatises are all one step, one continuous, and conclusively successful, effort at the discovery of the laws. *The Parts I and II of Descartes's Principles* is Spinoza's apprentice piece. His subsequent treatises are all one continuous, and as successful as there has been, effort at the formulation and solution of the problem of the one most general principle of all principles.

cli

In the preface to part I of the *Parts I and II* Spinoza begins by explaining why Descartes has to doubt all things. He has to doubt all things in order to: discover the foundation of all sciences (*id.*), and: free them from all doubt (*id.*).

Spinoza then writes:

here I should satisfy also those who argue thus, because it is not known to us by itself that god exists, we can never be certain of anything;

Descartes replies, from that that we do not know whether the author of our origin may have created us such as to be deceived even in the things which appear most evident to us, it does

not follow that we can doubt the things which we understand clearly and distinctly while we attend to them; therefore, even if it is not known to us by itself that god exists, but only through something else, we can nevertheless arrive at certain knowledge of god's existence, provided we carefully attend to everything from which we conclude it;

but some are not satisfied by this reply, and I will give another;

we cannot be certain of anything, not as long as we do not know the existence of god, but as long as we do not have a clear and distinct idea of him; if anyone wants to argue against me, then, his argument has to be this, we cannot be certain of anything as long as we do not have a clear and distinct idea of god; but we cannot have a clear and distinct idea of god as long as we do not know whether the author of our nature is deceiving us; therefore, we cannot be certain of anything;

to this I reply by conceding the major premise and denying the minor; we have a clear and distinct idea of a triangle even if we do not know whether the author of our nature is deceiving us; in the same way we have such an idea of god, as I have just shown; we can thus doubt neither his existence nor any truth of mathematics (*id.*).

It is as impossible, attending to the idea of god, to think that he is a deceiver as, attending to the idea of a triangle, to think that its three angles are not equal to two right angles. And as we can form such an idea of a triangle even if we do not know whether the author of our nature is deceiving us, so we can achieve a clear idea of god even if we doubt whether the author of our nature may not be deceiving us in all things.

To those who argue that it is not known to us by itself that god exists, Descartes replies that we cannot doubt the things which we understand clearly and distinctly while we attend to them. The existence of god follows.

The relations of clarity and distinction that cannot be doubted, the truthfulness of god, and the principle of contradiction are thus sorted out. Clarity and distinction that cannot be doubted is first. The existence of god follows. Clarity and distinction that cannot be doubted cannot on pain of contradiction not be the principle of contradic-

tion. The existence of god follows, and hence his truthfulness.

From the *Rules* to *The method* to the *Meditations* to the *Replies* Descartes has become ever more explicit. He has had to. Those who love god will not have god follow. Descartes has had to reply. It is his duty.

It is in the *Meditations* that he writes: god cannot be false on pain of contradiction.

And yet in the *Replies* he has to write: I think it is just that I absolutely ignore the judgments of those who, clinging to their preconceived opinions, do not want to meditate with me, and that I despise them as of no importance whatsoever; but I know that even those who earnestly search for truth will find my *Meditations* difficult (2); so I will append here a few things by way of a synthesis (*id.*). And he reduces the argument of the *Meditations* to 4 propositions, deduced from 10 definitions, 7 postulates, and 10 axioms.

Axiom 1 is: there is no existent thing of which it cannot be asked which is the cause why it exists.

Leibniz will discover that the principle of sufficient reason is the one, and only one, principle as general as the principle of contradiction. He writes: our reasonings are founded on two grand principles, that of contradiction, by which we judge that what involves a contradiction is false, and what is contradictory to the false is true; and that of sufficient reason, by which we consider that no fact can be found true or existent, no assertion true, without a sufficient reason why it is thus and not otherwise (*Monadology*, 31-2). Descartes's axiom 1 anticipates the principle of sufficient reason.

Descartes adds: and this can be asked even of god; not because he needs a cause in order to exist, but because the very immensity of his nature is the cause or the reason why he needs no cause in order to exist.

God is not first, god follows. From clarity and distinction that cannot be doubted, and from the answer to the question of the cause why he exists. From the principle of contradiction, and from the principle of sufficient reason.

To those who have to argue: we cannot have a clear and distinct idea of god as long as we do not know whether the author of our nature is deceiv-

ing us, Spinoza replies: as we have a clear and distinct idea of a triangle even if we do not know whether the author of our nature is deceiving us, so we have a clear and distinct idea of god even if we do not know whether the author of our nature is deceiving us, and we can thus doubt neither his existence nor any truth of mathematics.

As Descartes so Spinoza. The existence of god follows, as the truths of mathematics do, from clarity and distinction.

And the main concern of part I of the *Parts I and II* is to establish this. Granted the major premise, we cannot be certain of anything as long as we do not have a clear and distinct idea of god, to establish the minor, we have a clear and distinct idea of god. Therefore, knowledge and certainty.

Among the 10 definitions, 1 is of thought. 2 is of idea. 5 is of substance. 8 is of god: substance which we understand by itself to be the sum of perfection, and in which we conceive nothing involving any defect or any limitation of perfection, is called god (*id.*).

3 of the 11 axioms follow, beginning with: at knowledge and certainty we arrive only through knowledge and certainty prior in certainty and knowledge.

4 propositions follow, showing that, if I think, I am.

The remaining 8 axioms follow. The last, axiom 11, is Descartes's axiom 1: nothing exists of which it cannot be asked which is the cause or the reason why it exists (*id.*).

The remaining 17 propositions follow. 13 of them show the clear and distinct idea of god, the other 4 draw conclusions from the clear and distinct idea of god.

Proposition 5: the existence of god is known only from the consideration of his nature, 6: the existence of god is demonstrated *a posteriori* only from that that the idea of him is in us, 7: the existence of god is also demonstrated from that that we exist having the idea of him (*id.*).

The existence of god established, proposition 8 concludes: mind and body are distinct in reality (*id.*).

Proposition 9: god is the sum of understanding, 10: whatever perfection is found in god is

from god, 11: there cannot be many gods, 12: all things that exist are conserved only by the power of god, 13: god is the sum of truthfulness, and least of all a deceiver (*id.*).

The truthfulness of god established, the conclusions follow, 14: whatever we perceive clearly and distinctly is true, 15: error is not anything positive (*id.*).

Proposition 16: god is incorporeal, 17: god is a simplest being, 18: god is immutable, 19: god is eternal, 20: god has preordained all things from eternity (*id.*).

The immutability of god established, and hence also his eternity, part I concludes with the proposition, 21: substance extended in length, breadth, and depth exists in reality, and we are united to one part of it (*id.*).

From clarity and distinction the existence of god follows. From the existence of god that mind and body are distinct follows. The proof is: whatever we perceive clearly god can make just as we perceive it; but we perceive clearly mind, and that is, thinking substance, without body, and that is, extended substance; and conversely, we perceive clearly body without mind; therefore, mind can be without body, and body without mind (*id.*).

From the existence of god the conservation of all things also follows.

From the existence of god the truthfulness of god also follows. From the truthfulness of god truth follows.

From the truthfulness of god the immutability of god also follows. From the immutability of god the generalization of body, extension, follows, and that we are united to one part of it.

In part I, then, Spinoza has deduced from clarity and distinction not only the existence of god, and hence his truthfulness. He has deduced also the conservation of all things, he has deduced also the immutability of god, he has deduced also extension, and he has thus prepared the ground for the deduction of the laws of nature from conservation, the immutability of god, and extension. More generally, he has prepared the ground for the deduction of the principles of the material things from the principles of the acquisition of knowledge. Most generally, he has prepared the ground for the deduction of all princi-

ples from the one principle of clarity and distinction.

In part II from 1 postulate, 8 definitions, and 21 axioms he deduces 37 propositions, and 13 lemmas and corollaries.

The postulate is: demanded here is only that everyone attend to his perceptions so accurately as to distinguish the clear from the obscure (*id.*).

Among the definitions, 1 is of extension, 3 of atom, 6 of space, 8 of locomotion. 2 is of substance. Spinoza defines substance in part I: every thing which we perceive immediately in the subject is called substance (*id.*). The existence of god established, he redefines it in part II: by substance we understand that which, in order to exist, needs only the concurrence of god (*id.*).

Propositions 1-11 demonstrate the unity and uniformity of nature. Propositions 1-6 demonstrate Descartes's result: matter is of one kind. All matter is extension. Propositions 7-11 demonstrate Descartes's result: all variation in matter is motion. All variation is locomotion.

The conservation of the quantity of motion follows. Proposition 12: god is the principal cause of motion, 13: god conserves by his concurrence the same quantity of motion and rest which he originally impressed on matter.

The laws of nature follow. They are propositions 14-7.

In propositions 18-37 the solution of the problem of collisions follows.

And in the couple of pages of part III Spinoza begins the deduction of the theory of vortices.

From clarity and distinction the truthfulness of god follows, from the truthfulness of god the immutability of god follows, from the immutability of god the unity and uniformity of nature follows, and from the unity and uniformity of nature the laws of nature follow.

The attempt to deduce all principles from one principle has thus succeeded spectacularly.

Too spectacularly.

The laws of nature follow from the principles of the acquisition of knowledge.

More fundamentally, body follows from mind. Extended substance from thinking substance. If I think, I am. The existence of god follows. That what we perceive clearly god can make follows. And we perceive clearly body.

More fundamentally still, perception is thought. Definition 1 in part I already affirms: all operations of the will, intellect, imagination, and the senses are thoughts (*id.*).

In all of this Spinoza is making more explicit what is explicit or implicit in Descartes. I am a thinking substance. The existence of god follows. Body follows. And it does because perception is thought. In Spinoza as in Descartes this is the ground of the transition from the principles of the acquisition of knowledge to the principles of the material things. In *The principles of philosophy*, concluding *The principles of human knowledge*, Descartes writes: divine authority is to be preferred to perception; excepting that, it does not become a philosopher to assent to anything but what has been perceived (I, 76), and beginning *The principles of the material things*, he writes: we know with certainty that material things exist because it is not in our power to effect in ourselves one sensation rather than another; on the contrary, this plainly depends on the thing which affects our senses (II, 1). Accordingly, among the principles of the material things Spinoza has the single postulate: demanded here is only that everyone attend to his perceptions so accurately as to distinguish the clear from the obscure.

And not quite two generations later, Berkeley attended to his perceptions so accurately as to discover idealism.

Perception is not thought. The other way around, thought is perception. Or better, thought had better be grounded in perception. Perception is first, thought is, some hundreds of millions of years of evolution later, second.

Body cannot follow from mind. Again the other way around, mind had better follow from body. Indeed, even in *Genesis*, on the authority of the Yahwistic document: the lord god made earth and water (2); then he made man from the dust of the earth (*id.*). On the authority of the Priestly code, on the fifth day: god made the great sea monsters, and all the living creatures that creep, and that swarm in the waters, and all the winged birds of every kind (1), then, on the sixth day: god said, let us make man in our image (*id.*).

And the laws of nature cannot follow from the principles of the acquisition of knowledge. If the laws of nature and the principles of the acqui-

sition of knowledge have to follow from one another, once again the other way around, the principles of the acquisition of knowledge had better follow from the laws of nature.

That the principles are discovered deduction shows. The principles are verified by the perfect correspondence of the things demonstrated from them to the phenomena manifested by experience.

In particular, in 1644, Descartes deduces in the 7 rules of motion the solution of the problem of collisions.

In 1664, so does Spinoza in propositions 18-37 of part II of the *Parts I and II*.

In 1656, so does Huygens in the 13 propositions of *On the motion of colliding bodies*.

Spinoza writes: Huygens thinks that Descartes's rules of motion are false (to Oldenburg, Nov 1665); I myself do not impeach any of them except the sixth, about which I think Huygens is also in error (*id.*).

The perfect correspondence of the solution of the problem of collisions by Huygens, and not by Descartes, or Spinoza, to the phenomena manifested by experience has verified the principles.

It has verified the laws of nature. And if it will verify the principles of the acquisition of knowledge, it will only because it has verified the laws of nature. Not because clarity and distinction has been verified, inertia has been verified. Because inertia has been verified by the solution of the problem of collisions, clarity and distinction has been verified.

If deduction will verify the principles of the acquisition of knowledge, it will not by the deduction of the laws of nature from the principles of the acquisition of knowledge. It will only, given the deduction of all other things in the world from the laws of nature, by the deduction of the same all other things in the world from the principles of the acquisition of knowledge.

And as deduction so induction. The principles cannot be discovered but by induction. Both the laws of nature and the principles of the acquisition of knowledge cannot be discovered but by induction. When the laws of nature are discovered by induction, however, they are then verified by deduction. And if the principles of the acquisition of knowledge will be discovered, they will only, given both the particulars from which the

laws of nature have been induced and the particulars which have been deduced from the laws of nature, by induction from all these particulars, both these from which the laws of nature have been induced and these which have been deduced from the laws of nature.

Moreover, the true way ascends to principles, descends to new experiments, ascends to more general principles, and again, and again. The principles must admit of generalization. Lest there be no new results, the principles cannot be beholden to any particular result. And once again it is when, and because, the particulars demand the generalization of the laws of nature that they demand the generalization of the principles of the acquisition of knowledge, too.

Spinoza writes: it is just as impossible, attending to the idea of god, to think that he is a deceiver as, attending to the idea of a triangle, to think that its three angles are not equal to two right angles.

One and a half centuries later, Lobachevsky will show that, attending to the idea of a triangle, it is possible to think that its three angles are not equal to two right angles.

Spinoza himself will show that, attending to the idea of god, it is possible to think that he is a deceiver. Has, in fact, been a deceiver for not less than twenty-one and a half centuries, counting from the Priestly code.

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Kepler's *Mystery* follows the true way hastily, eager to reach the most general principles. So does Spinoza's *Parts I and II of Descartes's Principles*. The *New astronomy* follows the true way with stately deliberation. It ascends, descends, again, and again, all along versing itself duly and orderly in the particulars. So does the *Theologico-political treatise*.

And where do we hear god? Augustine asks, and replies: nowhere more clearly than in the holy scriptures.

It is one of Spinoza's most important results. That god is not *sui generis*. That god not only can be investigated as any other object can be, god has been investigated as any other object has been. He has been investigated by following the way now in use, and always in use, by flying to

most general principles and, taking them to be settled, proceeding to judgment. And as he has been investigated by following that way, so he can be investigated by following the other way, even if this way is yet untried, by the continuous and gradual ascent from the particulars.

And where are the particulars? Nowhere more than in the holy scriptures.

The *Theologico-political treatise* is in 20 chapters. 1-15 are the theological, 16-20 the political. The political conclusions follow from the theological.

The theological chapters divide into three parts. In 1-6 Spinoza examines what is revealed in the holy scriptures, in 7-11 he examines how it is revealed, in 12-5 he draws general conclusions from the preceding examination.

The collection and examination of the particulars is in chapters 1-11. Chapter 1 is representative. In its ten pages or so, Spinoza examines, counting only those identified chapter and verse, 92 passages, from the following books and chapters:

<i>Genesis</i>	1, 2, 6, 20, 22, 37, 41,
<i>Exodus</i>	7, 23, 24, 25, 31, 32, 33,
<i>Leviticus</i>	19,
<i>Numbers</i>	11, 12, 14, 22, 27,
<i>Deuteronomy</i>	4, 5, 24, 34,
<i>Joshua</i>	2, 5,
<i>Judges</i>	6, 8, 13, 14, 15,
<i>1 Samuel</i>	3, 11, 16, 30,
<i>1 Kings</i>	22,
<i>Isaiah</i>	11, 29, 30, 33, 40, 43, 48, 49, 63,
<i>Ezekiel</i>	1, 2, 27, 37, 42,
<i>Amos</i>	4,
<i>Micah</i>	2,
<i>Haggai</i>	2,
<i>Zechariah</i>	4,
<i>Psalms</i>	33, 51, 135, 139, 143,
<i>Proverbs</i>	1, 16, 25,
<i>Job</i>	27, 32, 33, 34, 37,
<i>Ecclesiastes</i>	3, 12,
<i>Daniel</i>	4, 7,
<i>Nehemiah</i>	1, 9,
<i>1 Chronicles</i>	22,
<i>Matthew</i>	3,
<i>John</i>	1,
<i>Acts</i>	2, 9, 22.

In chapters 1-6 Spinoza examines what is

revealed in the holy scriptures. 1 is *On prophecy*, 2 *On prophets*, 3 *On the vocation of the Hebrews, and whether the gift of prophecy was peculiar to the Hebrews*, 4 *On divine law*, 5 *On the reason why ceremonies were instituted, and on faith in historical accounts and the reason and necessity for them*, 6 *On miracles*.

He begins in chapter 1: prophecy, or revelation, is certain knowledge of things revealed by god to men; prophet is thus the one who interprets the things revealed by god to those who cannot have certain knowledge of those things, and can attain to them only by faith (*id.*).

The fundamental problem which the holy scriptures pose is thus knowledge. Not faith. All can attain to the things revealed by god by faith. All but the prophets, however, can attain to those things, and whether they will attain to them or not, only if the prophets will interpret those things to them. The problem is thus, first and foremost, whether the prophets, who will no doubt interpret those things, themselves have certain knowledge of those things.

Spinoza continues: it follows from its definition that prophecy includes natural knowledge, the knowledge common to all men, resting on foundations common to all men; it is because the vulgar are fond of the rare and the marvelous, and spurn the gifts of nature, that it is supposed that prophecy does not include natural knowledge (*id.*); but as my principal goal is to treat of scripture, I will turn to the other causes and means by which god reveals things to men, whether these things are beyond the bounds of natural knowledge or within it (*id.*).

And the fundamental problem which the holy scriptures pose is thus formulated explicitly. The problem is, what are the causes and means by which god reveals things to men, whether these things are beyond the bounds of natural knowledge or within it.

Spinoza continues: what must be said can be drawn solely from scripture; for what can we say of things which may exceed the limits of our intellect, except what we can conclude from the sayings and writings of the prophets? and since, as far as I know, we have no prophets now, we can do nothing but turn to the sacred volumes of the departed prophets (*id.*).

Spinoza examines the sacred volumes of the departed Moses, Joshua, Samuel, Isaiah, Ezekiel, Amos, Micah, Haggai, Zechariah, David, Solomon, Job, Daniel, Nehemiah, Matthew, John, Luke.

And concludes: we can without any scruples affirm that the prophets perceived the things revealed by god only by the imagination, by words and images; we find no other means in scripture, and must therefore invent no other means (*id.*); as the prophets perceived the things revealed by god by the imagination, they could no doubt perceive much that is beyond the limits of the intellect; for many more ideas can be composed from words and images than from the principles and axioms on which natural knowledge rests (*id.*); we can thus understand why the prophets perceived everything in parables and enigmas, and expressed everything spiritual in bodily form; such is the nature of the imagination; we can understand why Micah saw god as sitting, Daniel saw him as an old man dressed in white, Ezekiel saw him as fire, why the holy spirit descended to those with Christ as a dove, to the apostles as fiery tongues, to Paul as a light (*id.*).

Thus the Hebrew prophets and the Christian prophets.

Thus also the Muslim prophet. The prophet perceived the things revealed by god only by the imagination, by words and images. We find no other means in the koran, and must therefore invent no other means. And the prophet no doubt perceived much that is beyond the limits of the intellect, for many more ideas can be composed from words and images than from the principles and axioms on which natural knowledge rests.

Earlier, Herodotus reports: Homer and Hesiod created for the Greeks their theogony, they distinguished the gods according to their descent from each other and described their honors, arts, and appearances (II, 53). Whether Homer and Hesiod were prophets or not – whether, as the muses revealed things to them, god revealed those things to the muses – Homer and Hesiod created for the Greeks their theogony by words and images, parables and enigmas, not from the principles and axioms on which knowledge rests.

Earlier, in *When on high*, Apsu is the begetter, Tiamat bears all. Not from the principles and

axioms on which knowledge rests.

The things revealed by god to men the prophets perceived by the imagination.

Spinoza continues: as to the laws of nature by which this occurred, I confess my ignorance; I might say, as others do, that it occurred by the power of god; but this would be no better than to want to explain some singular thing by attaching to it a transcendental form; everything occurs by the power of god; the power of nature is itself nothing but the power of god; we do not understand god to the degree to which we do not know the causes of nature; it is ultimate folly to ascribe to the power of god something of which we do not know the natural cause, which is itself the power of god (*id.*); but we are not inquiring yet into the causes of prophetic knowledge, we are still only investigating the scriptural documents, and drawing conclusions from them as from natural facts; we are not concerned yet with the causes of these documents (*id.*).

The power of nature is the power of god.

Spinoza does not yet know the laws of nature by which revelation occurred. He is still only investigating the scriptural documents. Still only investigating the natural facts which the scriptural documents are. If, however, from the investigation of these natural facts, he finds the causes of prophetic knowledge, and the causes of the scriptural documents, and hence the laws of nature by which revelation occurred, he will *ipso facto* find the laws of the power of god by which revelation occurred.

And this will be believed doubted, but cannot be doubted. It cannot be doubted, because the power of nature is the power of god. It will be believed doubted, because everything can be believed doubted, and because ultimate folly is itself fond of the imagination, not of the principles on which knowledge rests.

The things revealed by god to men the prophets perceived by the imagination.

Hence: we must inquire how could the prophets be certain of things which they perceived only by the imagination and not from the certain principles of reason (*id.*).

It is the problem of chapter 2, *On prophets*.

Spinoza begins: from the above chapter it follows that the prophets were endowed not with

perfect intellects, but with vivid imaginations; and this scripture abundantly confirms (*id.*).

Spinoza shows that scripture abundantly confirms it, and continues:

the imagination by its nature does not involve certainty, as all clear and distinct ideas do; to be certain of the imagined things, the imagination must be supported by something else, namely, reasoning;

it follows that prophecy by itself cannot involve certainty; and we see that the prophets were certain of god's revelation not by the revelation itself, but by an attendant sign; Abraham, when he heard god's promise, demanded a sign; even more clearly, Gideon told god, show me a sign that it is you who speaks to me; god said to Moses, let this be a sign that I send you; and Hezekiah demanded a sign from Isaiah;

although these instances show that prophecy and revelation are things open to doubt, there is some certainty; it is in that god never deceives the pious and the elect; we see this in the story of Abigail; in the case of Micah; and even if he deceived Ahab through prophets, he did so through false prophets;

the certainty given to the prophets by signs was not mathematical, such as follows necessarily from the perception of the perceived thing, it was only moral;

and as the signs were given in order to persuade the prophet, it follows that the signs were given according to the opinions and capacities of the prophet; and a sign which could render one prophet certain of his prophecy could not convince another imbued with different opinions; revelation varied according to the bodily disposition, the imagination, and the reasoned opinions which the prophet already had; if the prophet was cheerful, victories, peace, and things that make men glad were revealed; if the prophet was miserable, sad things, wars, massacres, and disasters were revealed; if the prophet was elegant, he perceived god's mind elegantly, if he was confused, confusedly; in order to prophesy, Elisha asked for a harp, and could not perceive god's mind before he was entertained by its music; god revealed himself to Moses angry; to Cain furious; miserable and weary of life, Jeremiah prophesied nothing but calamities; Micah

never prophesied anything good to the house of Ahab, even if other true prophets had; the prophecies of Isaiah and Nahum are elegant in style, those of Ezekiel and Amos rude;

I conclude that prophecy never made a prophet learned, it left him with his preconceived opinions; therefore, we are not bound to trust scripture in speculative matters; are we to believe that the soldier Joshua was an astronomer? Solomon was ignorant of the ratio of the circumference and diameter of the circle, and thought, as the run of craftsmen do, that it was 3:1; Noah thought that the earth outside Palestine was uninhabited;

but Adam was ignorant that god was omnipresent or omniscient, too; Abraham was ignorant that god had foreknowledge of all things;

we have shown more than sufficiently, then, that god accommodated revelation to the understanding and opinions of the prophets, and that the prophets could be, and were, in fact, ignorant, not only in speculative matters, but in matters of morality and practice;

it follows that we must not run to the prophets for knowledge of either natural or spiritual things;

we can only come to the same conclusion regarding the reasonings of Christ;

and we must say absolutely the same regarding the signs and reasonings of the apostles (*id.*).

The question, how could the prophets be certain of things which they perceived only by the imagination, has thus been answered. They could not be. We must not run to the prophets for knowledge.

Spinoza has also shown more than sufficiently that god accommodated revelation to the understanding and opinions of the prophets. He has shown more than sufficiently the ignorance of the prophets in both speculative matters and matters of morality and practice. He has thus shown more than sufficiently that god accommodated revelation to ignorance. And this is the ground, drawn solely from scriptures, on which he will solve the fundamental problem which scriptures pose, what are the causes and means by which god reveals things to men.

Chapter 3 is *On the vocation of the Hebrews, and whether the gift of prophecy was peculiar to the Hebrews.*

On the one hand: by the guidance of god I

understand nothing but the fixed and immutable order of nature; the universal laws of nature are nothing but the eternal decrees of god, which always involve eternal truth and necessity; whether we say of all things that they follow natural laws or that they are from the decree and guiding ordinance of god, we say the same thing; in all things the power of nature is nothing but the very power of god (*id.*).

On the other hand: god accommodated revelation to the understanding and opinions of the prophets. Or what is the same, nature accommodated revelation to the ignorance of the prophets.

It follows, and Spinoza shows that scripture abundantly confirms this conclusion as well, that the gift of prophecy was not peculiar to the Hebrews.

It follows further that: the election of the Hebrews concerned nothing but temporal bodily happiness and freedom, and that is, government and the ways and means to obtain it, consequently also the laws necessary for the establishment of this government, as well as the ways in which this was revealed (*id.*).

Aquinas analyzed the law into five kinds: eternal, natural, human, divine, and of sin.

But of sin is natural. Eternal and natural cannot not be the same. Divine is human. The law is of two kinds, natural, and human.

Four-odd centuries after Aquinas, two generations after the discovery of Kepler's laws, Spinoza is the first philosopher who analyzes the law into the two kinds: law which depends on the necessity of nature and law which depends on the pleasure of men. The law which depends on the pleasure of men he analyzes further into the two kinds: law whose end is everything relating to the state and law whose end is only the highest good.

Spinoza is the first philosopher also, and one of a very few, who is clear about the relations of the kinds of the law.

Chapter 4 is *On divine law.*

Spinoza writes: absolutely, the name law signifies that by which an individual thing, or all things, or all things of one species, acts or act for the same certain and determinate reason, and this reason depends either on the necessity of nature or on the pleasure of men; law which depends on the necessity of nature is that which follows from

the nature, or the definition, of the thing or things; law which depends on the pleasure of men, and which is more properly called right, is that which men, in order to live more safely, or more commodiously, or for other causes, have prescribed to themselves and others (*id.*).

Everything occurs by the power of god. The power of god is the power of nature. Everything occurs by the power of nature.

If there is law which depends on the necessity of nature, then, there cannot be law which does not depend on the necessity of nature. If there is law which depends on the pleasure of men, this law cannot not be law which depends on the necessity of nature. The relation of the law which depends on the necessity of nature and the law which depends on the pleasure of men can only be the relation of genus and a species of that genus.

Spinoza writes: man, being part of nature, constitutes part of the power of nature (*id.*).

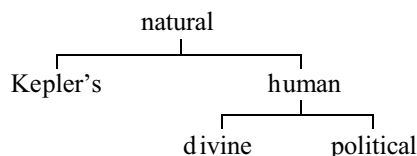
The pleasure of men, being the pleasure of men, will not appear to men part of the necessity of nature. However it appears, the problem the pleasure of men poses is to show that it is, and how it is, part of the necessity of nature. And Spinoza's is a major advance toward the formulation and solution of this problem, too.

Spinoza notes: the name law has been applied to natural things by borrowing (*id.*).

As the fact suffices to show that the distinction between law and right, *lex* and *iur*, which has to make the distinction between law which depends on the necessity of nature and law which depends on the pleasure of men clear, is borrowed from the jurists, Mucius, Servius, Alfenus, Labeo, and their successors.

Hence, Spinoza defines law which depends on the pleasure of men more particularly. He writes: to define it more particularly, law is reason to live to some end which man prescribes to himself or others (*id.*). And: law which man prescribes to himself or others can be distinguished into human and divine; human law is reason to live which serves the safety of life and everything relating to the state; divine law is reason to live which aims only at the highest good, and that is, at the true knowledge and love of god (*id.*).

To picture the relations of the kinds,



where, to avoid ambiguity, law which depends on the pleasure of men is human, but law whose end is everything relating to the state is political.

Law is that by which an individual thing, or all things, or all things of one species, acts or act for the same certain and determinate reason. The heavenly bodies act for the same certain and determinate reason. And men act for the same certain and determinate reason. In terms of pleasure, Kepler's laws depend on the pleasure of the heavenly bodies. Human laws depend on the pleasure of men.

The same certain and determinate reason of the pleasure of men is also twofold. The reason follows, as it will where pleasure is concerned, from the end of pleasure. The end of the pleasure is either the highest good or the state.

The highest good cannot not be one. Hence: divine law is universal, or common to all men; it is deduced from universal human nature; and it cannot be extracted by faith from historical accounts (*id.*).

Political law is not common to all men, it is not deduced from human nature, and it can be extracted by faith from historical accounts.

Divine law and political law differ in generality. Divine law is common to all men. Political law is common to some men.

They differ also in how they are obtained. Divine law is deduced from nature. Political law is not deduced from nature. If divine law will be obtained by following the true way, the deduction will rest on induction. That political law is obtained by induction the fact indicates that men, like all organisms, will strive to maximize their pleasure, and the fact attests that political law will strive to appeal to the authority of historical accounts. The difference is thus not so much in that divine law is deduced, political law is not. It is rather that divine law can be, and if it can be, it had better be, obtained by following the true way. Political law has been obtained by following the other way, and although it does not have to be obtained by following that way, it is all too likely

to continue to be.

These are theoretical differences. The most important practical difference, and that is, political, follows from the fact that human law men prescribe. To themselves and others. Divine law is common to all, and whether they prescribe it to themselves or others can make no difference. Political law is common to some. If they prescribe it in order to live more safely, or more commodiously, or for other causes, not only are some liable to live more safely than others, some much more commodiously than others, and some to be the causes of what others are the effects. They are liable to prescribe it to no other end but that, that some live more safely than others, much more commodiously than others, and are the causes of what others are the effects.

Divine law is common to all men, deduced from nature, and cannot be extracted by faith from historical accounts.

It follows that divine law cannot be extracted from scriptures.

What scriptures are is political law.

Spinoza has already drawn this conclusion: the election of the Hebrews concerned nothing but government, the ways and means to obtain it, the laws necessary for its establishment, and the ways in which this was revealed.

He shows, again, in chapter 5, *On the reason why ceremonies were instituted, and on faith in historical accounts and the reason and necessity for them*, that scripture abundantly confirms this conclusion. In chapter 6, *On miracles*, he shows that scripture abundantly confirms also the trivial, at this point, conclusion that there are no miracles, what there is is ignorance.

In chapters 7-11 he shows that, authored in ways not altogether holy, the holy scriptures are historical accounts. The solution of the fundamental problem which scriptures pose, what are the causes and means by which god reveals things to men, follows. The causes and means by which god reveals things to men are political. Some men prescribed to themselves and others, and much more to others than themselves, the election of the Hebrews, and prescribed to god to reveal it to others. To reveal it to themselves they could not prescribe to god, even if they wanted to, having prescribed it to god themselves.

In chapters 12-5 he draws general theological conclusions. These concern faith, religion, and theology. Pace Hegel, Spinoza did not arouse the great hatred of the rabbis, and the even greater hatred of the Christian theologians, most of all with his *Ethics*. He aroused it most of all with his *Theologico-political treatise*. He aroused it with the results of his examination of what is revealed in scriptures. Revealed is ignorance. He aroused it even more with the results of his examination of how it is revealed. It is revealed by fabrication. And he aroused it most of all with his general conclusions, those concerning faith, religion, and theology. He would have aroused the hatred of Hegel, too, had consistency been a consideration for Hegel.

In chapters 16-20 he draws political conclusions. He deduces the divine law. Then he makes the valiant attempt to rectify political law by deducing it from the divine.

cliii

Descartes has taken the step: nature is god.

And in the *Theologico-political treatise* already Spinoza has taken the final step: god is nature.

It is the final step. If nature is god, there may yet be more to god than nature. There may yet be supernature. If god is nature, there is no more to god than nature. There is no supernature.

In all things the power of nature is nothing but the very power of god. The laws of nature are nothing but the decrees of god. The guidance of god is nothing but the fixed and immutable order of nature.

God is nature.

And on this ground Spinoza solved all problems involving god, explicitly those of scriptures, of faith, of religion, and of theology, and hence implicitly all those, too, involving scriptures, faith, religion, or theology. He thus also cleared away, if only it would be cleared away, the indefinite amount of obfuscation, and hence even more indefinite of confusion, involving scriptures, faith, religion, and theology.

The scriptural documents are natural facts.

These natural facts were authored in a most natural way, by fabrication.

They were also authored to a most natural end,

the state.

Spinoza writes: the laws revealed by god to Moses were laws only of the government of the Hebrews; they were binding on no one but the Hebrews, and not on them either after the downfall of their government (pr.).

With Hobbes: the state is a power contrary to our natural passions, keeping men in awe, reducing the wills of men unto one will.

The law was fixed in ceremonies. And: the end of ceremonies was that men do nothing by their own decisions, act in all things by the mandate of others, and continually acknowledge with all their actions and thoughts that they are not under their own right, but under the right of others (5).

After Spinoza, this result is corroborated by Rousseau, then by Godwin, then by Marx.

Before Spinoza, this result is anticipated first by Critias, twenty-one centuries before Spinoza, two generations after the Priestly code.

Critias writes: at first men lived like beasts; next laws were established so that justice may rule mankind, and the laws held men back from doing evil in the open, but not from doing it in secret; so a wise man discovered the fear of gods, to frighten evildoers even in their thoughts (fr. 25).

The end was that men continually acknowledge with all their actions and thoughts that they are under the right of others.

What is faith follows.

Spinoza writes: faith is to sense the presence of god, which can be achieved, if one is ignorant, by obedience to god, and follows necessarily from this obedience (14). Clearly: faith requires not true dogmas, but pious dogmas, such as will move the soul to obedience (*id.*).

Clearly also: to catholic or universal faith pertains only what is absolutely necessary for obedience to god and whose ignorance would render this obedience absolutely impossible (*id.*).

By the time of Spinoza the Hebrew religion was more than two millennia old. The Christian religion was more than one and a half millennia old. The Muslim religion was more than one millennium old.

When Spinoza observes faith and religion, he observes faith and religion as they have been brought to perfection.

This is how he observes religion: religion has become a collection of absurdities and arcana, and it is those who outright despise reason, who reject the intellect and are the open enemies of its corrupt nature, who are believed to be in possession of the divine light (pr.).

This is how he observes faith: faith has become nothing but a collection of credulity and prejudices; and what prejudices? such as make men from rational beings into beasts, such as make them completely incapable of using their free judgment to distinguish true from false, such as appear to be, indeed, carefully designed to extinguish the light of reason altogether (*id.*).

In the final theological chapter Spinoza shows that: neither theology is the slave of reason, nor reason is the slave of theology (15). Theology is the slave of faith and religion. Reason is the slave of nothing.

And with this the whole goal of the treatise is accomplished.

Spinoza writes: the whole goal of the treatise is to separate faith from philosophy (14); philosophy has no end but truth, faith has no end but obedience and piety; the ground of philosophy are axioms obtained only from nature itself, the ground of faith are histories and words obtained only from scriptures and revelation (*id.*); between faith and philosophy there is no connection whatsoever, no relation whatsoever (*id.*).

The goal is accomplished. Faith, and with faith also religion, and with faith and religion also theology are separated from philosophy.

Theology has been the mistress of reason for more than a millennium. Religion is the possession of those who outright despise reason and are its open enemies. Faith is nothing but prejudices carefully designed to extinguish the light of reason altogether.

Philosophy is grounded in nature. And: whatever is contrary to nature is contrary to reason; whatever is contrary to reason is absurd, and therefore to be rejected (6).

Reason made the first step toward extricating itself from slavery five and a half centuries before Spinoza, with the labors of Abelard, who discovered that, omnipotent though god is, the principle of contradiction is more omnipotent. But the steps remained small, cautious, hesitant, infre-

quent, because the slavery was maintained by the most formidable of edifices. It was maintained by theology, itself maintained by religion, itself maintained by faith, itself maintained by scriptures.

Augustine writes: whatever they may produce from their volumes against our books, and that is, against our catholic faith, we shall easily show false, or else without any hesitation believe false.

It was five centuries after Abelard when the philosophers first dared to probe into the foundation of the edifice.

In 1605, ten years before Galileo's *Letter to Mme Christina*, Bacon writes: our savior laid before us two books to study, the scriptures revealing the will of god, and the creatures expressing his power; and the latter is the key to the former; it opens our understanding to the true sense of the scriptures (*Growth*, I).

Bacon, Galileo, Kepler, Descartes, Huygens studied the latter. Spinoza then used the key to open the former.

We can continue without any hesitation believing whatever they may produce from their volumes false. Continuing to supply evidence for the fact that: the end of faith is not truth, but obedience.

As the foundation so the entire edifice, as scriptures so faith, religion, and theology. Between any of that and philosophy there is no connection whatsoever, no relation whatsoever.

It is with the labors of Spinoza that philosophy has come closest to perfect unity and uniformity.

Whatever is contrary to reason is to be rejected. It is with the labors of Spinoza that philosophy has come closest to rejecting everything contrary to reason, and affirming nothing but reason. It is in his treatises that reason has come closest to pristine purity.

Hence, it is with his labors as well that philosophy has come closest to being deduced in its entirety from one principle. Nature. The ground of philosophy are axioms obtained only from nature itself. Spinoza deduces theoretical philosophy and practical philosophy. And his deduction of history from physics is unsurpassed, as his analysis of scriptures attests, in both its results and the degree to which the chains of inferences from the principle to the results are free of gaps.

His deductions of politics from physics, and hence of ethics from physics, are not as successful. The results of other philosophers have advanced politics more. Yet no results in politics have been obtained through chains of inferences as free of gaps as Spinoza's. Other philosophers have taught ethics more successfully. Again, ethics has never been taught through chains of inferences as free of gaps as those of the *Ethics demonstrated in geometrical order*.

In the remainder of the *Theologico-political treatise*, then, Spinoza turns to the deduction of politics. In the *Ethics* to the deduction of ethics. And in the *Political treatise* to the deduction of politics once again.

The final result from the deduction of politics in the *Theologico-political treatise* is: the true end of the state is freedom (20). Hence, the deduction of politics in the *Political treatise* shows *how society needs to be ordered so as to preserve inviolate the peace and freedom of the citizens*.

The true end of the state is freedom.

If so, however, as Spinoza has himself shown, the end of the state has not been the true end of the state.

And if so, first of all how and why the end of the state has been what it has been has to be deduced. Hence, how and why it has not been the true end. Hence, how and why it can become the true end. If it can become the true end.

All of this is part of Spinoza's deduction.

What is not is the collection and examination of the particulars.

Hence, as Spinoza notes in the *Political treatise*: statesmen have written far more happily about politics than the philosophers (1).

Aristotle collected the constitutions of 158 states. None of this research is in evidence in his *Politics*. And all these philosophers who concerned themselves with politics before and after him, Plato, Zeno of Citium, Cicero, Augustine, al-Farabi, Aquinas, Marsilius, Occam, ibn Khaldun, Machiavelli, More, Bodin, Grotius, Hobbes, Spinoza, Locke, Montesquieu, Rousseau, Hamilton and Madison, Sieyès, Burke, Godwin, Hegel, Marx, collected between them the constitutions of 0 states. Politics has remained obtained by flying to most general principles and, taking them to be

settled, proceeding to judgment.

On the other hand, all advances in politics have been, as they could only be, on the ground on which Spinoza deduces it.

In the *Theologico-political treatise*: by right and ordinance of nature I understand nothing but the rules of nature by which we conceive the individual determined to exist and act in a certain way (16). In the *Political treatise*: I consider the passions of men, love, hatred, anger, envy, ambition, pity, and all the other commotions of the soul, not as vices of human nature, but as its properties, just as heat, cold, storm, thunder, and the like, are properties of the air, which, however inconvenient, are necessary, and have certain causes, by which we seek to understand their nature (1); by natural right I understand the laws or rules of nature themselves from which all things follow, and that is, the power of nature itself (2); man, whether wise or ignorant, is part of nature and everything, by which he is determined to act, must be referred to the power of nature (*id.*).

The final result from the deduction of ethics is: the most useful thing in life is to perfect as much as possible the intellect, or reason; in this alone the sum of man's happiness and blessedness consists (*Ethics*, IV, app.4).

cliv

Spinoza's final investigation of god is part I of the *Ethics*, *On god*.

Definition 6 is of god: by god I understand being absolutely infinite, and that is, substance consisting in infinite attributes, each expressing eternal and infinite essence (*id.*). Definition 3 is of substance: by substance I understand that which is in itself and conceived by itself, and that is, whose conception does not need another conception in order to form (*id.*).

Substance is that whose conception does not need another conception in order to form.

It follows: but one substance exists in the nature of things (10).

But one substance exists in nature.

God is substance.

It follows: no substance but god can be conceived (14).

God is nature.

It follows: whatever is in god (15); all things are in god, and all things that occur, occur only by the laws of the infinite nature of god (*id.*); god acts only by the laws of his nature (17).

Nature acts only by its laws.

It follows: nothing in the nature of things is contingent; all things are determined by the necessity of god's nature in certain modes of existence and operation (29). Nature is thus nature naturing and nature natured, *natura naturans* and *natura naturata*: by nature naturing I understand that which is in itself and conceived by itself, or these attributes of substance which express eternal and infinite essence, and that is, god considered as a free cause; by nature natured I understand all that follows from the necessity of god's nature or from god's attributes, and that is, all attributes of the modes of god, considered as things which are in god and without which god cannot be or be conceived (*id.*).

Nature is nature naturing. All things in nature are nature natured.

Part II is *On the nature and origin of the mind*.

Definition 1 is of body: by body I understand the mode which expresses the essence of god, considered an extended thing, in a certain and determinate mode (*id.*).

God is an extended thing. Nature is body.

There is but one substance in nature.

It follows: substance thinking and substance extended are one and the same substance (7).

Substance thinking is substance extended.

It follows: the object of the idea constituting the human mind is body (13).

Mind is body.

Spinoza's advance of physics lacks in the clarity and distinction of Galileo, Kepler, Descartes, and Huygens. It not only reaches further, however, it reaches as far as the advance can reach.

clv

Nature is god, god is nature.

Occam's razor, then, must be produced, and god excised. Nature is nature.

Philosophy has no end but truth. It is grounded in nature. Whatever is contrary to nature is contrary to reason. Whatever is contrary to reason is absurd, and therefore to be rejected.

Hume said, I might say to Charon, good Charon, have a little patience, I have been endeavoring to open the eyes of the public, if I live a few years longer, I may have the satisfaction of seeing the downfall of some of the prevailing systems of superstition; but Charon would then lose all temper and decency, you loitering rogue, he would say, that will not happen these many hundred years, do you fancy I will grant you a lease for so long a term? get to the boat this instant, you lazy loitering rogue.

On the one hand, then, god must be excised. On the other hand, these many hundred years after Spinoza the excision continues.

One century-odd after Spinoza, in 1796, in the *Exposition of the system of the world*, Laplace formulated the theory of the origin and development of the solar system. He concludes: if the conjectures I have proposed stand, the stability of the planetary system follows from the laws of motion; and I cannot refrain from noting how Newton has erred on this point, straying from the method he otherwise so happily applies; he writes that the admirable arrangement of the sun, planets, and comets can only be the work of an intelligent and omnipotent being; but can the arrangement not be an effect of the laws of motion (V, 6)?

Or, as Laplace so happily put it on another occasion: *je n'ai pas eu besoin de cette hypothèse*.

The anecdote is recorded on the authority of the emperor. The emperor said: I asked him why the name of god, which came incessantly out of the pen of Lagrange, would not come even once out of his; it is because, he said, I have had no need for that hypothesis; I was caught by the anecdote, and quoted to him Lalande and some others; but he persisted in his opinion (Antomarchi, *Memoirs*, I).

This is what the emperor said.

But god will not have Laplace have the last word. The anecdote was revised accordingly. Told of the anecdote, Lagrange replied, *ah, mais c'est une belle hypothèse, ça explique beaucoup de choses*.

One and a half centuries after Laplace, in 1952, in *Is there a god?* Russell writes: if I were to suggest that between the earth and Mars there

is a china teapot revolving about the sun in an elliptic orbit, nobody would be able to disprove my assertion, provided I were careful to add that the teapot is too small to be revealed even by our most powerful telescopes (*id.*).

Lemaître formulated the hypothesis of a big bang in 1927, in *A homogenous universe of constant mass and increasing radius accounting for the radial velocity of extragalactic nebulae*. In 1958, in *The primeval atom hypothesis and the problem of the clusters of galaxies*, Lemaître comments on the hypothesis: it removes any attempt at familiarity with god; it is consonant with the words of Isaiah when he speaks of the hidden god, hidden even in the beginning of creation (*id.*).

Isaiah is in 66 chapters. In chapters 1-39 Isaiah prophesies. In chapters 40-55, two centuries after Isaiah, Isaiah is made to prophesy some more. In chapters 56-66, a couple of generations after Isaiah was made to prophesy some more, he is made to prophesy still more. The words of Isaiah when he speaks of the hidden god, in chapter 45, are the words of a postprophetic prophet pretending to be the prophet.

Thus, where the china teapot is hidden remains a mystery. It is not a mystery, however, after Spinoza, how it was hidden.

clvi

Descartes formulated the rules of the method in *The discourse on the method*, and then *the Optics, Meteorology, and Geometry are essays in the method*. Spinoza formulated the rules of the method of history in chapter 7 of the *Theologico-political treatise*, and then the solution of the problem of when and how the Hebrew scriptures were authored, in chapters 8-10, is an essay in the method.

Chapter 7 is *On the interpretation of scripture*.

Chapter 8 *In which it is shown that the pentateuch and the books of Joshua, Judges, Samuel, Ruth, and Kings are not autographs, and then it is inquired whether they are from several writers, or all from one writer, and who he is*, chapter 9 *In which it is inquired whether these books can be ascribed ultimately to the hand of Ezra, and whether the marginal notes found in the Hebrew codices are different versions*, chapter 10 *The*

remaining books of the old testament are examined in the same way as the previous.

Spinoza writes in chapter 7:

we must now present the true method of interpretation of scripture;

the method of interpretation of scripture does not differ from the method of interpretation of nature at all, but agrees with it entirely; for as the interpretation of nature consists first and foremost in this, to arrange clearly the history of nature, and from it, as from certain facts, to draw definitions of the natural things, so for the interpretation of scripture it is necessary first and foremost to order its genuine history, and from it, as from certain facts, to draw legitimate conclusions about the principal goals of the authors of scripture;

the general rule for the interpretation of scripture, therefore, is, to accept as facts of scripture nothing which is not obtained after the most careful examination of its history;

and I will now explain what this history must be, and what it first and foremost must include;

1. it must contain the nature and properties of the language in which the books of scripture were written and which their authors spoke;

2. it must collect the sentences of all the books, and arrange them under general heads;

3. finally, it must arrange the circumstances of all the books, and that is, the life, practices, and learning of the author of each book, who was he, what was the occasion, what the time, for whom did he write, and finally in what language; and also the fate of each book, how was it first received, into whose hands did it fall, how many versions were there, by whose advice was it accepted among the sacred books, and finally how did all the books, which are all the sacred books, coalesce into one corpus;

when we have this history of scripture, and have firmly decided to affirm as a doctrine of the prophets nothing which does not follow most clearly from this history, it is time to brace ourselves for the investigation of the goals of the prophets and the holy spirit;

the method and order are once again like those of the interpretation of nature from its history;

as in the scrutiny of the natural things we endeavor first of all to investigate the most gen-

eral, for instance, motion and rest, and their laws and rules, which nature always observes and by which she continually acts, and from these to proceed gradually toward the less general, so from the history of scripture we try first of all to find out what is the most general, what is the basis and foundation of all scripture, what is the doctrine which all the prophets have commended as eternal and most useful for all men, for instance, that one and omnipotent god exists, who alone is to be adored and who cares for all, and especially for those who adore him and love their neighbor like themselves, etc.;

having thus shown how to interpret scripture, we have simultaneously demonstrated that this is the only certain way of investigating its true sense;

and as our method, grounded in this, to seek knowledge of scripture in itself alone, is the only true method, plainly, whatever knowledge of scripture it cannot acquire, of that we have to despair;

we thus have to point out its difficulties, what is wanting in it to acquire complete and certain knowledge of the sacred books;

the greatest difficulty before the method arises from this that it demands complete knowledge of Hebrew; and where is that to seek? the ancients have left us no dictionary, no grammar, no rhetoric; and the Hebrew nation has lost all its ornaments, and no wonder after all the persecutions, and has nothing but fragments of the language and but a few books;

another difficulty before the method arises from this that it demands the history of all that has happened to the books of scripture; and of this we are greatly ignorant; who the authors of many of the books were we are either completely ignorant or at least in doubt; we do not know on what occasions the books were written, or at what times; we do not know in what hands they fell, or in how many versions they were written, or even whether they were written in more versions than we now have;

the interpretation of scripture must demand nothing beyond the natural light common to all, neither supernatural light, nor external authority; it must not be so difficult that only the acutest philosophers can accomplish it, it must accom-

moderate the natural and common ingenuity and capacities of men; and such we have shown our method to be; the difficulties before it have their source in the doings of men, not in the nature of the method (*id.*).

This is the method.

Spinoza then turns to the problem of when and how the Hebrew scriptures were authored.

And first of all, the difficulties before the method having their source in the doings of men, not in the nature of the method, and demanding neither supernatural light nor external authority, Spinoza resolved the difficulties.

He not only acquired knowledge of Hebrew. He wrote its grammar, the *Compendium of the grammar of the Hebrew language*, published in 1677.

And he compiled the genuine history of scripture. We are no longer ignorant who the authors of the books were, on what occasions the books were written, at what times, in what hands they fell, by whose advice they were accepted among the sacred books, and finally how all the books, which are all the sacred books, coalesced into one corpus.

Hence, whatever knowledge of scripture the method cannot acquire, of that we have to despair, and we have to despair of nothing. Nothing is wanting in the method to acquire complete and certain knowledge of the sacred books, and this is what has been acquired, complete and certain knowledge of the sacred books.

The study and investigation of the Hebrew language became routine, like that of the Greek language.

The study and investigation of the genuine history of the Hebrew scriptures became routine, like that of the history of the Greek record.

The study and investigation of the history of the Hebrew language became routine, like that of the history of the Greek language. The history of the language informed the history of the record. All the following was established. – Which are the dialogues in the Platonic corpus that Plato wrote, which are the dialogues that pretend that Plato wrote them. In the dialogues that Plato wrote, which are the passages that Plato wrote, which are the passages that pretend that Plato wrote them. Of the dialogues that Plato wrote,

which are early, which are late. – Of the sacred books, which are early, which are late. Which pretend to be early, how early they pretend to be, how late they pretend to be early. In each of the books, which are the passages that are early, which are the passages that are late, which are the late passages that pretend to be early, and in each case how early, how late.

The study and investigation of the genuine history of the Hebrew state became routine. The genuine history of the Hebrew scriptures was correlated with, and grounded in, the genuine history of the Hebrew state.

Two centuries after Spinoza, in 1878, in the *Prolegomena to the history of Israel*, Wellhausen resolved the last outstanding difficulties of the problem of when and how the Hebrew scriptures were authored.

The Egyptian and the Mesopotamian records were recovered. Knowledge of the languages was acquired. The history of the records was compiled. The history of the languages was established. The history of the states was established. The history of the records was correlated with, and grounded in, the history of the states. The solution of the problem of when and how the Hebrew scriptures were authored was corroborated by, as it was grounded in, the history of the Mesopotamian record and the Mesopotamian states.

To show, then, that complete and certain knowledge of the sacred books has been acquired, first Spinoza's solution of the problem of when and how the Hebrew scriptures were authored can be shown, then how it has been corrected. This will show, too, that Spinoza formulated the problem conclusively, that he solved it conclusively in all its essential parts, and that all corrections of his solution have corroborated his solution in all its essential parts.

The true way begins with the collection of the particulars. As the interpretation of nature consists first and foremost in this, to arrange clearly the history of nature, so for the interpretation of scripture it is necessary to order its genuine history.

From the genuine history of scripture the formulation of the problem follows: who were the authors of the books, on what occasions were the

books written, at what times, in what hands did they fall, by whose advice were they accepted among the sacred books, and how did all the books coalesce into one corpus.

The true way then ascends. As in the scrutiny of the natural things we endeavor first of all to investigate the most general, motion and rest, and their laws and rules, so from the history of scripture we try first of all to find out what is the most general.

The general rule is, to accept as facts nothing which is not obtained after the most careful examination of the history.

And the most general which Spinoza obtains after the most careful examination of the history is: the end was that men continually acknowledge with all their actions and thoughts that they are under the right of others.

With Critias: a wise man discovered the fear of gods, to frighten evildoers even in their thoughts. With Hobbes: the state is a power contrary to our natural passions, keeping men in awe, reducing the wills of men unto one will.

The end of the foundation of all scripture, the doctrine which all the prophets commended as eternal and most useful for all men, that one and omnipotent god exists, who alone is to be adored and who cares for all, and especially for those who adore him and love their neighbor like themselves, was political.

The true way then descends. We proceed gradually toward the less general.

More particularly: the laws revealed by god to Moses were laws only of the government of the Hebrews.

The sacred books were authored accordingly.

The massive corruption of the sacred books follows.

Spinoza writes: as to those who do not concede that any corruption has crept into the sacred books, but maintain that god with singular providence protected them incorrupt, and say that the various readings are signs of the profoundest mysteries, I do not know whether they say this from folly and anile devotion, or from arrogance and malice, so as to have god's arcana all for themselves; but I know this, that I find in their writings no arcana, only puerilities (9).

More particularly still:

that Moses is the author of the pentateuch has no foundation, and is completely against reason; *Joshua* is not an autograph either;

that *Judges* was written by the judges I believe no one of sound mind can persuade himself;

Samuel was written many generations after Samuel;

Kings is extracted from books of the deeds of Solomon, and from the chronicles of the kings of Judah and the chronicles of the kings of Israel;

we conclude that all these books, which we have received as autographs, rather ascribe their contents to antiquity;

if we now attend to the connections and the argument of all these books, we see clearly that they were all written by one and the same historian, who wanted to write down the antiquities of the Jews from their beginnings to the first destruction of the city;

the connections and order of the books indicate a historian who set himself a predetermined goal; in order to write down the origin and development of the Hebrew nation, he set forth the occasions and times of the establishment of the laws of Moses, then deduced everything that followed from them;

who this historian was, it is hard to show; I suspect, however, that he was Ezra, and the reasons for this conjecture are not light;

the historian continues his history down to the release of Jehoiachin; it follows that he cannot be long before Ezra;

scripture testifies of no one but Ezra, in *Ezra*, 7, he applied himself to the study of the law of god and to teaching it, and he was a scribe expert in the law of Moses; we see from this testimony that Ezra applied himself not only to the study of the law of god, but also to teaching it;

in *Nehemiah*, 8, we read, they read to them the book of the law of god, explaining it, teaching its sense, and making them understand it;

and as *Deuteronomy* contains not only the book of the law of Moses, or the greater part of it, but also many insertions explaining it, I conjecture that *Deuteronomy* is the book of the law of god, written, embellished, and explained by Ezra;

a comparison of the decalogue of *Deuteronomy* with the decalogue of *Exodus*, where its history is told at length, shows a discrepancy in

all things, in form, in length, and in rationale; I think that here as elsewhere the differences are due to Ezra, who explained the law of god to the people of his time, and then wrote this book of law; I conjecture that he wrote this book first, because it contains the laws of the fatherland, which the people needed first of all; this accomplished, I think Ezra next undertook to write down the complete history of the Hebrew nation from the beginning of the world to the destruction of the city, and inserted *Deuteronomy* in its place; the first five books he called by the name of Moses because his life is their principal object, for the same reason he called the sixth book *Joshua*, the seventh *Judges*, the eighth *Ruth*, the ninth and tenth *Samuel*, and the eleventh and twelfth *Kings* (8);

but Ezra's is not the ultimate hand of everything contained in these books; he collected his history from various writers, and he sometimes left the final examination and arrangement to his heirs (9);

the prophecies contained in the prophetic books, *Isaiah*, *Jeremiah*, *Ezekiel*, and the twelve minor prophets, were collected from other books and not always in the order in which they were spoken or written by the prophets;

Psalms was collected at the time of the second temple;

Proverbs was collected at the same time, or in any case not earlier than the reign of Josiah;

Chronicles was written after Ezra;

Ezra, *Nehemiah*, *Esther*, and *Daniel* are from one hand; we do not know who the writer was; but *Nehemiah*, 12, testifies that the descendants of the high priest went to meet Alexander; it follows that these books postdate Alexander; and I have no doubt that they postdate Judah Maccabee's restoration of the cult in the temple, because it was at that time that false books of *Daniel*, *Ezra*, and *Esther* were written by evildoers, who were without doubt sadducees, because, as far as I know, the pharisees never accepted these books;

finally, we can conclude that there was no canon of sacred books before the time of the Maccabees, and that these which we now have were selected from a multitude of others by the pharisees at the time of the second temple;

and I assert that the pharisees alone selected the books of the old testament and made them into a sacred canon, because in *Daniel*, 12, the resurrection of the dead is predicted, which the sadducees denied; and because in the talmud, in the treatise *Sabbath*, 2, the pharisees themselves clearly indicate this (10).

The dates noted in these passages are,
 Solomon prince c. 963-23,
 Josiah prince 640-09,
 destruction of Jerusalem 587,
 release of Jehoiachin 561,
 second temple 515,
 assembly under Ezra 445/33,
 Alexander prince 336-23,
 Judah Maccabee revolt in 168.

And this is how Spinoza's solution of the problem of when and how the Hebrew scriptures were authored has been corrected.

The end was that men continually acknowledge with all their actions and thoughts that they are under the right of others. The end of the doctrine which all prophets commended as eternal and most useful for all men, that one and omnipotent god exists, who alone is to be adored and who cares for all, and especially for those who adore him and love their neighbor like themselves, was political.

No correction.

The laws revealed by god to Moses were laws only of the government of the Hebrews.

No correction.

The sacred books were authored accordingly.

No correction. Moreover, it has become clearer whether those who say that the various readings are signs of the profoundest mysteries, say this from folly and anile devotion, or from arrogance and malice.

The pentateuch, *Joshua*, *Judges*, *Ruth*, *Samuel*, and *Kings*, which we have received as autographs, rather ascribe their contents to antiquity.

No correction.

These books were all written by one and the same historian. He set himself a predetermined goal. In order to write down the origin and development of the Hebrew nation, he set forth the occasions and times of the establishment of the laws of Moses, then deduced everything that followed from them. I suspect that he was Ezra.

I conjecture that he wrote *Deuteronomy* first. He next undertook to write down the complete history of the Hebrew nation from the beginning of the world to the destruction of the city, and inserted *Deuteronomy* in its place.

This part of the solution can be divided into its four elements. The goal of the writer. The sequence of his writings. Who he is. When he wrote.

The goal was predetermined. He set forth the laws of Moses, then deduced everything that followed from them.

No correction.

The sequence is, first *Deuteronomy*, then the complete history of the Hebrew nation from the beginning of the world to the destruction of the city.

No correction.

The law was promulgated in 622, and this is the date of *Deuteronomy*.

Joshua, Judges, Samuel, and Kings are from one hand, and were written in the century after 622.

The other four books of the pentateuch, *Genesis, Exodus, Leviticus, and Numbers*, constitute one fundamental document, as Wellhausen named it: the Priestly code (*Prolegomena*, intr.2). The Priestly code was composed in the century before 445/33.

Spinoza writes: Ezra collected his history from various writers. The Priestly code incorporated into itself the oldest Hebrew document, the Yahwistic document. The Yahwistic document was composed in the century before c. 791/50.

These books, Spinoza writes, were all written by one and the same historian. Counting the Yahwistic document one historian, *Deuteronomy* another historian, *Joshua, Judges, Samuel, and Kings* another, and the Priestly code another, the historians are four. Counting the incorporation of the Yahwistic document into itself by the Priestly code, and the insertion of *Deuteronomy* in its proper place by the Priestly code, and hence the pentateuch one historian, the historians are two, the Priestly code and *Joshua, Judges, Samuel, and Kings*. And the two historians share the one and the same predetermined goal, to set forth the laws of Moses, and deduce everything that followed from them.

Spinoza conjectures that the one historian is Ezra. The Priestly code was composed in the century before the assembly under Ezra. And it was Ezra, as Wellhausen writes: who made the law of Moses the constitution of Judaism (X, 2.2).

The booklet *Ruth* is contemporary with *Joshua, Judges, Samuel, and Kings*, written in the century before 538.

The prophecies contained in the prophetic books were collected from other books and not always in the order in which they were spoken or written by the prophets.

Isaiah is representative. Isaiah's prophecy began under Uzziah, prince c. 791-50, and in chapters 1-39 Isaiah prophesies accordingly. In chapters 40-55 he prophesies after the destruction of the city in 587. In chapters 56-66 he prophesies after the second temple, consecrated in 515. In each case ignoring the later passages in which all 66 chapters abound.

Psalms was collected at the time of the second temple.

No correction. Dating from the time of the Yahwistic document to that of the Priestly code, the 150 psalms were collected at the time of the second temple.

Proverbs was collected at the same time, or in any case not earlier than the reign of Josiah.

No correction. *Proverbs* was collected between the time of *Deuteronomy* and that of the Priestly code.

Chronicles was written after Ezra. *Ezra, Nehemiah, Esther, and Daniel* are from one hand, and postdate Judah Maccabee's restoration of the cult in the temple.

Ezra, Nehemiah, and Chronicles are from one hand, written in the century after 445/33. The booklet *Esther* is a little earlier, its *terminus post quem* is 465, not 445/33. *Daniel*, the one book in the Hebrew scriptures to predict the resurrection of the dead, is the latest book in the Hebrew scriptures, written in the generation of Judah Maccabee, whose revolt began in 168.

Finally, there was no canon of sacred books before the time of the Maccabees.

The canon of the pentateuch – the torah – was fixed by the Priestly code. Other than the torah, there was no canon before the time of the Macca-

bees.

The pharisees alone selected the books of the old testament and made them into a sacred canon.

No correction.

clvii

Through Bacon, Galileo, and Kepler, through Descartes, through Huygens and Spinoza, the advance is the same. The acquisition of knowledge.

Bacon writes: first of all we have to prepare a natural and experimental history, sufficient and good; this is the foundation of the thing. And: the particulars are like an army, little is to be hoped for unless all particulars pertaining to the matter are marshaled. And: we do not propose to look, like augurs, in order to take auspices, we propose to march in, like commanders, in order to take possession.

Bacon showed how to marshal the army and take possession in the *New organon*. Galileo showed it better in *The two main systems* and *Two new sciences*. Kepler showed it better in *The cosmographic mystery* and *New astronomy*. Descartes showed it better in *The method* and *Principles*. Huygens showed it better in *The pendulum clock* and *Treatise on light*. And Spinoza showed it better yet in the *Theologico-political treatise*. He prepared the natural and experimental history, sufficient and good, of the most recalcitrant particulars. He arranged the army in its divisions, *On prophecy*, *On prophets*, *On the vocation of the Hebrews*, *On divine law*, *On the reason why ceremonies were instituted*, and *On miracles*. Then he took possession.

Galileo writes: concerning the oldest object I here set forth the entirely new science; for nothing in nature is older than motion, and about it volumes neither few nor small have been written by the philosophers; yet I find that its essential properties have not even been observed, let alone demonstrated; these and other things neither few nor less worthy of being known I will demonstrate, and thus, what is more important still, the gate and the road to the largest and most excellent science will be opened, in which these labors of mine will be the elements.

The object concerning which Spinoza set forth the entirely new science is not the oldest, but it

had long pretended to be the oldest. The science to which he opened the gate and the road is not the largest and most excellent either, but it had long pretended to be the largest and most excellent, too. One way or another, concerning that object Spinoza set forth the entirely new science. About it volumes very many more and larger had been written by the philosophers. Yet its essential properties had not only not even been observed, let alone demonstrated, they had been hidden in the profoundest mysteries. These and other things neither few nor less worthy of being known Spinoza demonstrated. And thus, what is more important still, the gate and the road to a new science was opened, in which these labors of his are the elements.

Kepler writes: I transferred all of astronomy from fictitious circles to natural causes.

Spinoza transferred scriptures, hence god, hence all that had stood on scriptures and god, and hence all of philosophy, from the profoundest mysteries to natural causes.

Descartes writes: it is much better never to search for truth than to search without the method.

Bacon shows this, Galileo shows it, Kepler shows it, Descartes shows it, Huygens shows it, and yet Spinoza shows it more clearly and distinctly still. Volumes neither few nor small had been written by the philosophers. Bacon, Galileo, and Kepler cut through the folly. Galileo and Descartes began cutting through the malice as well. Spinoza cut through all the folly, anile devotion, arrogance, and malice.

There are and can only be two ways to search for truth, the false and the true. The true way is the method.

When he solved the locus problem, Descartes solved a problem which, counting back to Apollonius, eighteen centuries of philosophers had not been able to solve. When he solved the problem of when and how the Hebrew scriptures were authored, Spinoza established facts reaching back to the beginning of the world, not one day less, on the authority of the Priestly code. Counting back to *Deuteronomy*, he established facts reaching twenty-three centuries into the past.

Of all that has damaged philosophy, nothing has damaged it more effectively than the destruc-

tion of the essential and substantial unity and uniformity of the labors of Bacon, Galileo, Kepler, Descartes, and Huygens and the labors of Spinoza.

The restoration of spirit to the substantiality and purity of its being stands on the separation of spirit from the chaff. The investigation of nature, on one side, Spinoza, on the other. The investigation of god in the *Theologico-political treatise*, on one side, the investigation of god in the *Ethics*, on the other. The chaff is thus all on one side. On the other side, the soul can bathe.

And nothing has damaged history more than its exclusion from this essential and substantial unity and uniformity.

Dilthey writes: the spiritual sciences are an independent of the natural sciences whole; they do not study nature, they study the historic-social reality; the entities which act on one another in the wonderful whole of history and society are *individua*, each different from every other, each a world; thus differentiated, the true nature of the spiritual sciences emerges and they can break the fetters in which the older sister has held the younger since Descartes, Spinoza, and Hobbes, who imposed on these sciences their developed in mathematics and natural science methods.

Indeed.

Huygens and Spinoza are not just contemporaries. They share the fetters. The discovery of Titan, the discovery of the ring of Saturn, the determination of the true proportions of the solar system, the discovery of the constancy of the speed of light, the discovery that light is waves, the discovery of a certain and equal measure of time, the discovery of the isochronism of the cycloid, the discovery of evolutes, the discovery of how to calculate the probability of random events, the determination of the constant of proportionality of the law of the pendulum, the determination of the height from which a body falls in 1 second, the solution of the problem of collisions, the discovery of the conservation of momentum, the discovery that *Deuteronomy* is the book in which the laws of Moses were first set down, the discovery of the historian who set forth the laws of Moses and deduced everything that followed from them, the determination of the sequence of composition of the books of the

pentateuch, the determination of the sequence of the books in the Hebrew scriptures, the discovery of the time and place of the closure of their canon, the determination of the fundamental dependence of the development of the Hebrew scriptures on the development of the Hebrew state, the determination of the end of the laws of Moses, the determination of the end of scriptures, these discoveries one and all share the fetters. Mathematics and natural science.

The method of history is the method.

Kant writes: human actions are, like all other natural events, determined in accordance with general laws of nature; we thus leave it to nature to produce the man to write such a history, as it produced Kepler, who discovered the laws of the paths of the planets, and Newton, who explained these laws by a general natural cause.

Nature has not yet produced for history a Kepler, and a Newton. But it has a Galileo in Spinoza.

clviii

The method of interpretation of scripture does not differ from the method of interpretation of nature at all, but agrees with it entirely.

And the method of interpretation of scripture is the method of history.

Spinoza writes: as the interpretation of nature consists first and foremost in this, to arrange clearly the history of nature, and from it, as from certain facts, to draw definitions of the natural things, so for the interpretation of scripture it is necessary first and foremost to order its genuine history, and from it, as from certain facts, to draw legitimate conclusions about the principal goals of the authors of scripture.

First, to arrange clearly the history of nature and to order the genuine history of scripture. Then, from it, as from certain facts, to draw definitions of the natural things and to draw conclusions about the principal goals of the authors of scripture.

First, then, it can be shown that ordering the genuine history of scripture does not differ from arranging clearly the history of nature at all, but agrees with it entirely. Then, that, given these certain facts, drawing conclusions about the principal goals of the authors of scripture does

not differ from drawing definitions of the natural things at all, but agrees with it entirely. Then, that it follows that this is the method of interpretation of scripture, exclusively. Then, that the generalization follows, as scripture, and the things in it, so the entire record, and the things in it.

The history of nature is the history of nature to the degree to which it is the history of observation and experiment. Aristotle's history of animals in the *History of animals*, Theophrastus's history of plants in the *History of plants*, Ptolemy's collection of all accurate observations of the heavenly bodies in the *Almagest*, Bacon's history of fire in the *New organon*, Galileo's history of motion in *The two main systems* and *Two new sciences*, Kepler's history of the planets in the *New astronomy*, Huygens's history of light in the *Treatise on light*. What makes observation and experiment observation and experiment is constriction and vexation. Bacon writes: experience as it occurs is an accident, as it is pursued an experiment; the secrets of nature reveal themselves not when nature is free and left to herself, but when she is constricted and vexed.

The reading of scripture as it occurs is an accident, as it is pursued an experiment. And the secrets of scripture did not reveal themselves when scripture was free and left to itself, but when Spinoza constricted and vexed it into its genuine history.

To the degree to which the history of nature is the history of observation and experiment, the history of scripture does not differ from the history of nature at all. And the history of nature cannot be the history of nature if it is not the history of observation and experiment.

But the other way around, too. To the degree to which the history of scripture is the examination of what our predecessors have said, the history of nature does not differ from the history of scripture at all either. And the history of nature cannot be the history of nature if it is not the examination of what our predecessors have said either. Aristotle writes: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous. Thus, Aristotle adopts the sound and avoids the erroneous in what our predecessors have said. So

does Theophrastus. So does Ptolemy. So does Bacon. So does Galileo. So does Kepler. So does Huygens. Hence, the entire earlier history of animals is in Aristotle's treatises on animals. The entire earlier history of plants is in Theophrastus's treatises on plants. All earlier accurate observations of the heavenly bodies are in the *Almagest*. Bacon's history of fire is the history of the sound and erroneous in what our predecessors have said about fire. Galileo's history of motion is the history of what Aristotle, Archimedes, Ptolemy, Copernicus have said about motion. Kepler's history of the planets is the history of what Tycho has said about the planets. Huygens's history of light begins with Descartes and Rømer.

The history of nature cannot be the history of nature if it is not the examination of what our predecessors have said, because observation and experiment is observation and experiment to the degree to which it incorporates all earlier observation and experiment. It is what makes the history of nature the history of observation and experiment. With Copernicus: we must hold on to the observations which the ancient philosophers have bequeathed us; to anyone who holds on to his own views, the gates of our art are closed. And as Spinoza examines what the authors of scripture have said, so Aristotle, Theophrastus, Archimedes, Ptolemy, Copernicus, Tycho, Bacon, Galileo, Kepler, Descartes, Huygens, Rømer examine what our predecessors have said.

Spinoza writes: I will now explain what this history must be, and what it first and foremost must include; 1. it must contain the nature and properties of the language in which the books of scripture were written and which their authors spoke; 2. it must collect the sentences of all the books, and arrange them under general heads; 3. finally, it must arrange the circumstances of all the books.

These are the means and the ends of the constriction and vexation.

1. The language of the books.

Spinoza explains: we endeavor to ascertain the goal of a prophet about things which are imperceptible and can only be imagined, not about things which the intellect ascertains and of which it easily forms clear concepts; things which are by their nature easily perceived cannot be expressed

so obscurely as to be unintelligible; Euclid, who wrote about nothing but the simplest and most intelligible things, can easily be understood in any language; we can ascertain his goal perfectly and be certain of his true meaning without a thorough knowledge of the language in which he wrote; a child's knowledge is in fact sufficient; we need not research his life, or learning, or habits; nor need we inquire into what language he wrote, or when, or into the fate of his book, or its different versions, or how, or by whose advice it has been received (*Th. treatise*, 7).

The language in which the books were written and which their authors spoke is the means of access to things expressed so obscurely as to be unintelligible. The end is to reduce these things to clarity and distinction.

Hence, Spinoza, who reduced the things expressed so obscurely as to be unintelligible to clarity and distinction, can easily be understood in any language. We can ascertain his goal perfectly and be certain of his true meaning without a thorough knowledge of either Latin or Hebrew. A child's knowledge is in fact sufficient. We need not research his life, or learning, or habits. Nor need we inquire into the fate of his book, or its different versions, or how, or by whose advice it has been received.

2. The sentences of the books.

The collection of the sentences of all the books of scripture follows the last rule of the method: always to make enumerations so complete and reviews so general as to omit nothing.

Aristotle writes: every sentence means something, but not every sentence is a proposition; a proposition is a sentence which affirms what is true or false; a prayer, for instance, is neither true nor false.

Aristotle adds: we are thus concerned only with propositions, the rest we can ignore, they belong to rhetoric or poetry. We do not have to ignore them, however. A prayer is not a proposition. But there are propositions about prayers. Things expressed so obscurely as to be unintelligible are not propositions either. But there are propositions about things expressed so obscurely as to be unintelligible, too.

Hence, there are prayers and things expressed so obscurely as to be unintelligible in scripture.

There are neither prayers nor things expressed so obscurely as to be unintelligible in Spinoza. Only propositions.

3. The circumstances of the books.

And that is: the life, practices, and learning of the author of each book, who was he, what was the occasion, what the time, for whom did he write, and finally in what language; and also the fate of each book, how was it first received, into whose hands did it fall, how many versions were there, by whose advice was it accepted among the sacred books, and finally how did all the books, which are all the sacred books, coalesce into one corpus.

The enumeration and review of the circumstances of all the books of scripture follows the last rule of the method as well.

It also shows that the circumstances of the books are objects of investigation very much like the circumstances of animals, plants, and the heavenly bodies. It is what Aristotle, Theophrastus, and Ptolemy enumerate and review. Their life, and practices, and the learning of animals, too, who are they, the occasions, the times, and if they do not write, yet the language of animals, and where domesticated animals and plants are concerned, into whose hands they fall. The versions, too. The advice, sacredness, and canon are specific to the objects books. The substantial differences are two. Animals, plants, and the heavenly bodies are less recalcitrant. They are also nevertheless more difficult to investigate. Fire, motion, and light are still more difficult.

The history of scripture has reduced scripture to a collection of particulars, ordinary objects of investigation all. It is one of Spinoza's most important results. That *Deuteronomy*, the historian who set forth the laws of Moses and deduced everything that followed from them, *Joshua*, *Judges*, *Samuel*, and *Kings*, Ezra, are all ordinary objects of investigation.

In all three rules the history of scripture does not differ from the history of nature at all. And once again, the other way around as well.

1. The language of the books. Galileo writes: he seems to think that philosophy is a book of man's fantasy, like the *Iliad* or *The furious Orlando*; but it is not like that; philosophy is written in that grandest book, the universe, which cannot

be understood without learning to understand the language in which it is written; without this it is impossible to understand a single word.

2. The sentences of the books. Lagrange and Lemaître suffice to attest that not every sentence in the history of nature is a proposition either.

3. The circumstances of the books. About motion volumes neither few nor small had been written by the philosophers. The rule – which follows from Descartes's rules – is Hegel's, to separate the wheat from the chaff. With Hegel, too, the chaff is always much more than the wheat. A very few volumes, small or large, have to be separated from all the volumes neither few nor small.

These are the particular rules.

Hence, clearly, the general rule for the interpretation of scripture and the general rule for the interpretation of nature cannot differ from one another at all either, but can only agree with one another entirely.

Spinoza writes: the general rule for the interpretation of scripture, therefore, is, to accept as facts of scripture nothing which is not obtained after the most careful examination of its history.

Next: when we have this history of scripture, and have firmly decided to affirm as a doctrine of the prophets nothing which does not follow most clearly from this history, it is time to brace ourselves for the investigation of the goals of the prophets and the holy spirit.

Given the certain facts of the history, the rule is to draw definitions of the natural things, and to draw conclusions about the principal goals of the authors of scripture.

It will be objected, the goals of the authors of scripture are not the goals of the prophets, never mind the goals of the holy spirit. The rule: to affirm as a doctrine of the prophets nothing which does not follow most clearly from the history, disposes of the objection. For: what can we say of things which may exceed the limits of our intellect, except what we can conclude from the sayings and writings of the prophets? and since, as far as I know, we have no prophets now, we can do nothing but turn to the sacred volumes of the departed prophets. Pending the reappearance of a prophet, or of the holy spirit itself, the goals of the authors of scripture are the goals of the

prophets, and the goals of the holy spirit itself.

And conclusions about the principal goals of the authors of scripture do not differ from definitions of the natural things at all.

The principal goal of the authors of scripture, Spinoza established, was political.

It will be said that the goal of Spinoza, when he said that the goal of the authors of scripture was political, was itself political. The goal of saying that will itself be unabashedly political. For, if the goal of Spinoza was political, can Spinoza have established that the goal of the authors of scripture was political? It will be said also that the goals of all writers are political. The goal of saying that will appear more philosophical, being as old as Protagoras. It will yet be political. For, if man is the measure of all things, Spinoza may be the measure of scripture, and of the holy spirit for that matter, but Augustine, for instance, is also the measure of scripture, and of the holy spirit, Maimonides, also for instance, is also the measure of scripture, and of the holy spirit, and can any man not be the measure of who is the better measure, Spinoza? or Augustine? and Maimonides?

It will be said that the goal of Spinoza was political, and it will be said that the goals of all writers are political.

But it is not like that. The goal of the author of chapters 1-15 of the *Theologico-political treatise* is not practical. It is theoretical.

There is truth, and there is action. There is the search for truth, and there is the search for action. The search for truth is theoretical philosophy, the search for action is practical philosophy.

And truth, the search for truth, the search for action, and action, can readily, and will eagerly, each be confused with any and all of the others. They can also all four be fused into one invincible whole. Plato first did that in the *Republic*.

An invincible whole, as far as action is concerned. As far as truth is concerned, with Aristotle, the search for truth depends unconditionally on its separation from the search for action, let alone action. Hence the division of philosophy into theoretical and practical.

With Aristotle. With Bacon. With Descartes.

With Spinoza.

Spinoza's investigation of the state in chapters

16-20 of the *Theologico-political treatise* is practical philosophy. It is a search for action. And it is not action. This the fact that the result of his deduction: the true end of the state is freedom, is yet to lead to any action, suffices to show.

His investigation of scripture in chapters 1-15 is theoretical philosophy. It is a search for truth.

Also, we need not inquire into the circumstances of the book. But we can.

The life, practices, and learning of the author. The author was an excommunicated grinder of lenses and writer of books. He studied Bacon, Galileo, Descartes, the Hebrew scriptures, the Christian scriptures, the mishna, the talmud, Maimonides. – What was the occasion, what the time. The time was two generations after Bacon, Galileo, and Kepler, one after Descartes, the generation of Huygens. Hence the occasion. Spinoza writes: the whole goal of the treatise is to separate faith from philosophy; philosophy has no end but truth, faith has no end but obedience and piety; between faith and philosophy there is no connection whatsoever, no relation whatsoever. – For whom did he write. He writes: I know how obstinate prejudices are; therefore, I do not invite the vulgar and all those who share the passions of the vulgar to read this book; indeed, I want them to neglect it altogether rather than molest it with their perverse interpretation; they will profit nothing, but they will prevent others, who accept that reason is the slave of theology, to whom especially I believe this book should be useful, from finding it useful (pr.). – In what language. The Galilean and Cartesian, clarity and distinction, in the geometrical manner. – How was the book first received. It was burned. – Into whose hands did it fall. Into the hands of Bayle: the *Theologico-political treatise* is a pernicious and detestable book (*Dictionary*, s. v.); those who knew Spinoza say that he was sociable, affable, honest, obliging, and of well-ordered morality; this is strange, but we need not be surprised at it more than at seeing people who lead very bad lives even though they are completely convinced of the gospels (*id.*); Spinoza's system is the most monstrous hypothesis that can be imagined, the most absurd, the most diametrically opposed to what is most evident to us (*id.*); this hypothesis has been overthrown completely even by the

feeblest of its adversaries (*id.*). Then it fell into the hands of Hegel.

From the history we draw definitions of the natural things, and conclusions about the principal goals of the authors of scripture. Moreover: as in the scrutiny of the natural things we endeavor first of all to investigate the most general, motion and rest, so from the history of scripture we try first of all to find out what is the most general, what is the basis and foundation of all scripture.

In the scrutiny of the natural things Galileo endeavors to investigate the most general, and draws the definitions of motion at its most general, the definition of uniform motion and the definition of not uniform motion, of uniformly accelerated motion. From the history of scripture Spinoza tries to find out what is the most general, and draws conclusions about the principal goals of the authors of scripture at their most general, are the goals theoretical or practical, search for truth or search for action. The conclusion is, action. Namely: the end was that men continually acknowledge with all their actions and thoughts that they are under the right of others.

Finally: from the most general we proceed gradually toward the less general.

The method of interpretation of nature and the method of interpretation of scripture do not differ from one another at all in that they begin with history, in that the history is the history of observation and experiment, in that it is the history of what our predecessors have said, in that they share the three particular rules, concerning language, sentences, and circumstances, in that they share the general rule, to accept as facts nothing which is not obtained after the most careful examination of the history, in that they endeavor first of all to investigate the most general, and in that they then proceed gradually toward the less general.

It follows that this is the method of interpretation of scripture, exclusively. It is a trivial conclusion, in fact, if the scriptural documents are natural facts. The interpretation of scripture cannot differ from the interpretation of nature in any other way but in the way a particular differs from the general one of whose particulars it is.

It follows further, as scripture, and the things in it, so the entire record, and the things in it.

As the Hebrew scriptures so the Christian scriptures. The problem of when and how the Christian scriptures were authored was solved by Reimarus, in 1767, in the *Apology, or treatise in defense of the rational worshiper of god*. The method is Spinoza's. The remaining difficulties were resolved in the following century-odd. The method was Spinoza's. The results are also very much the same. St Paul authored 7 of the 14 letters in the Pauline corpus, the other 7 post-Pauline prophets authored. None of the 4 gospels is an autograph, nor is the *Acts of the apostles*. None of the 7 catholic letters is an autograph either. Nor is the *Apocalypse of John*. All in all, 20 of these 27 scriptural documents were authored in that most natural of ways, fabrication. As to the principal goal: every soul must submit to the authority above it (st Paul, *Romans*, 13).

So the Muslim scriptures. The koran is the word of the prophet. But it was the not prophet Umar, a decade after the departure of the prophet, who had the word of the departed prophet committed to writing. In the following decade, the not prophet Uthman had the word of the prophet edited into a canonical version of the word of the prophet, and all other versions of the word of the prophet in circulation suppressed. Next to the koran, divine law is also the sunnah, what the prophet did and said other than in the koran. The sunnah became divine law with the labors of al-Shafii, two centuries after the departure of the prophet. The hadith, into which the sunnah is revealed, were collected one generation after al-Shafii by al-Bukhari, 4 200 of them (ibn Khaldun, *Introduction*, VI, 11). The method of solution of the problem is the same, and the results are once again very much the same. If the labors of Umar do not attest the most natural of ways of authoring scriptural documents, the labors of Uthman do. As to the labors of al-Shafii, al-Shafii writes: a tradition is proved when it is related from one to another back to the prophet or to one next to him; the proof is established when the one who relates the tradition merits confidence in his religion, is known to be reliable in what he relates, understands what he relates, is aware of anything that may change the meaning, relates the tradition word for word as he heard it and not just its meaning, has learned it by heart, does not

attribute anything to someone he has not met or add anything he has not heard; and the same holds for all those who precede him until the one is reached who relates the tradition back to the prophet or to one who was next to the prophet (*Law*, 10). As to the labors of al-Bukhari, counting seven generations between the departure of the prophet and himself, al-Bukhari established 29 400 cases of: merits confidence in his religion, is known to be reliable in what he relates, understands what he relates, is aware of anything that may change the meaning, relates the tradition word for word as he heard it and not just its meaning, has learned it by heart, does not attribute anything to someone he has not met or add anything he has not heard. Counting al-Bukhari himself brings the cases to 33 600. As to the principal goal: the only true faith in the sight of god is submission (koran, 3.19).

If the method has the power to penetrate scriptures, in which things are expressed so obscurely as to be unintelligible, and investigate these things, it has the power to penetrate Euclid, who wrote about nothing but the simplest and most intelligible things, and investigate these things. If it has the power to penetrate scriptures and Euclid, and investigate the things in them, it has the power to penetrate everything in between, and investigate the things in it, Plato, who is more like scriptures than like Euclid, and Aristotle, who is more like Euclid than like scriptures.

If the method can investigate Plato, and that is, the Platonic corpus, it can investigate Socrates, who is one of the things in the Platonic corpus. One of the things as well in the Aristotlean corpus. In the corpus also of Xenophon. And in that of Aristophanes.

The method can investigate Aristotle, and Ptolemy, Hume, Kant, Darwin, Frege.

It can investigate Protagoras, Pyrrho, Augustine, al-Ghazali, Berkeley, Rousseau, Schiller, Hegel, Marx, Dilthey.

Bacon, Galileo, Kepler, Descartes, Huygens, Spinoza himself, Newton, Leibniz.

The method can penetrate the books, and it can investigate the things in the books, from the simplest and most intelligible to those expressed so obscurely as to be unintelligible.

The Hebrew scriptural documents were pro-

duced, counting from the Yahwistic document to *Daniel*, over six and a half-odd centuries. The method can sort out the books, and the things in the books, in their relation to one another across six and a half-odd centuries. It can sort out the Hebrew scriptures, the Christian scriptures, and the Muslim scriptures, and the things in them, in their relation to one another across seventeen centuries, counting from the Yahwistic document to the collection of hadith.

If it can sort out the relations of scriptures, and the things in them, it can sort out the relations of Euclid and Spinoza, and the things in them.

It can sort out also the relations of scriptures and Euclid and Spinoza.

Of Aristotle and his predecessors and successors.

Hegel and his predecessors and successors.

It can investigate philosophy over the entire course of its development, and with all its parts.

Including history. It can investigate Herodotus, and the things in him, Thucydides, and the things in him, Polybius, and the things in him, Livy, and the things in him.

Cyrus, Cambyses, Darius, and Xerxes are the principal things in Herodotus.

Cyrus, Cambyses, Darius, and Xerxes also left inscriptions behind. And the recovery of Mesopotamian antiquity begins with the Behistun inscription, left behind by Darius, recovered, twenty-three and a half centuries after Darius, by Rawlinson.

Arithmetic and geometry are spontaneous fruits of the innate principles of the method.

And nothing shows more clearly and distinctly that the method of interpretation of scripture is the method of history, and the method of history is the method, than its spontaneous fruit, the recovery of Egyptian and Mesopotamian antiquity.

1. The language of the books. By 1824 Young and Champollion had deciphered hieroglyphic, by 1857 Rawlinson, Hincks, and Oppert had deciphered cuneiform.

2. The sentences of the books. In the following century all papyri and tomb inscriptions, building inscriptions and cuneiform tablets, were collected, and arranged under general heads.

3. The circumstances of the books. The lives,

practices, and learning of the writers of papyri, inscriptions, and tablets were ascertained, who were they, what were the occasions, what the times, for whom did they write, and finally in what language, also the fate of each papyrus, inscription, and tablet, how was it first received, into whose hands did it fall, how many versions were there, by whose advice were some of the papyri, inscriptions, and tablets accepted as sacred, and finally how did these papyri, inscriptions, and tablets coalesce into sacred corpora.

Because the fruit has been spontaneous, investigation is yet to proceed further. It is yet, from the history, as from certain facts, to draw conclusions about the principal goals of the writers, to try and find out what is the most general, and then from the most general to proceed gradually toward the less general.

clix

Voltaire wrote the first *Philosophy of history*, in 1765.

It is a small volume. Voltaire begins with an introduction of a couple of pages in which: we examine whether the globe which we inhabit was formerly the same as it is at present (*id.*). He then has the following heads, each meriting a couple of pages itself: on the different races of men; on the antiquity of nations; on the knowledge of the soul; on the religion of the first men; on the customs and opinions of the ancient nations; on savages; on America; on theocracy; on the Chaldeans; on the Babylonians become Persians; on Syria; on the Phoenicians; on the Scythians and Gomerians; on Arabia; on Bram, Abram, or Abraham; on India; on China; on Egypt; on the Egyptian language and symbols; on the Egyptian monuments; on the Greeks; on the Jews; on the Jews in Egypt; on Moses as the chief of a nation; on the Jews from Moses to Saul; on the Romans, their empire, religion, and toleration (*id.*). What the principal goal is the full title states, *The philosophy of history, or a historical dissertation on the origin, manners, customs, and religions of the different nations of antiquity, with a concise exposition of their religious superstitions.*

Both ways begin from the particulars and end in the greatest generality, one glances at the particulars in passing, the other duly and orderly

verses itself in them, one establishes at once abstract and useless principles, the other rises gradually to what is general by nature. Voltaire wrote *The century of Louis XIV*, *The century of Louis XIV* verses itself duly and orderly in the particulars, and it is one of the five most important historical treatises. With *The exposition of the religious superstitions of the different nations of antiquity* Voltaire showed that the philosophy of history need not be hampered by versing itself, never mind duly and orderly, in the particulars.

Herder wrote *Also a philosophy of history, for the education of mankind*, in 1774. Then also the *Ideas for the philosophy of the history of mankind*, in 1784-91. The *Ideas* is large. It is in four volumes, divided into 20 books. It is also a philosophy of history. It treats of all the things of which Voltaire treats. It begins with the earth (I), it treats of America (VI, 6), the Chaldeans and Babylonians (XII, 1), the Persians (2), the Phoenicians (4), India (XI, 4), China (1), Egypt (XII, 5), the Greeks (XIII), the Jews (XII, 3), the Romans (XIV). The new is twofold. Herder treats not only of the nations of antiquity, but of all nations, ending with the nations of the present (XX). Also, he corrects Voltaire's error with respect to the principal goal. Religion is not superstition: man is built for humanity and religion (IV, 6); man is built for the hope of immortality (7).

Hegel's *Lectures on the philosophy of history* appeared in 1840. It is intermediate in size. Not in importance. Hegel, including most particularly with *The philosophy of history*, transformed philosophy, transformed history, and made the philosophy of history what it has since been.

There are four worlds, the Oriental world, the Greek world, the Roman world, and the Christian Germanic world. World history is the unfolding of spirit in time. When we pass through the past, no matter how large it is, we have to deal only with the present, for spirit is immortal, spirit is never past, never not yet, spirit is now essentially. The present is the highest.

It followed. The spiritual sciences do not study nature, they study the historic-social reality. The entities which act on one another in the wonderful whole of history and society are *individua*, each a world, for the world is nowhere but in the ideas of such an individual. The world is nothing but world-views. And the root of world-views is life.

If Galileo and Kepler are not philosophers, Newton is a barbarian, the spiritual sciences do not study nature, and the world is nothing but world-views, the *Ethics* may have its uses, if the soul will bathe in it, but the *Theologico-political treatise* is useless.

Otherwise, the only philosophy of history which is, in fact, philosophy and, in fact, of history is the *Theologico-political treatise*.

Newton and Leibniz

clx

A decade-odd after Huygens and Spinoza, Newton and Leibniz are contemporaries. Their years are,

1642 - 1727 Newton,

1646 - 1716 Leibniz.

The three generations from that of Bacon, Galileo, and Kepler through that of Newton and Leibniz are the brightest period in the acquisition of knowledge. The acquisition of knowledge advanced greatly with Bacon, Galileo, and Kepler, advanced more greatly yet with Descartes, advanced more greatly yet with Huygens and

Spinoza, and advanced more greatly yet with Newton and Leibniz. Newton and Leibniz are the culmination of the brightest period in the acquisition of knowledge.

If how important a philosopher is is measured cumulatively, by all the advances of his successors which rest on his advances, the most important philosophers are Plato, Aristotle, Euclid, Augustine, Bacon, Galileo, Descartes, Newton, Leibniz, Kant, and Hegel. The most important is Aristotle, the second most important is Descartes.

If how great a philosopher is is measured by how much he advanced enlightenment, the greatest philosophers are Galileo and Spinoza. The

greatest is Spinoza.

If how great a philosopher is is measured by how far he advanced the acquisition of knowledge, the greatest philosophers are Ptolemy, Galileo, Newton, and Darwin. The greatest is Newton.

If how great a philosopher is is measured by how broadly he advanced the acquisition of knowledge, the greatest philosophers are Aristotle and Leibniz. The greatest is Leibniz.

The attempt to order the genuine history of the acquisition of knowledge, which is first and foremost necessary, in order to accept as facts nothing which is not obtained after the most careful examination of the history, the attempt to collect the sentences of all the books, and arrange them under general heads, part of which the observation of the true way, of how it has been followed in the brightest period of the acquisition of knowledge, is, having labored to make those propositions stand, can conclude, with the observation of Newton and Leibniz, with the attempt to make them stand firmer.

clxi

Newton's very important treatises are nine, *Enumeration of lines of the third order*, written in 1668,

On analysis by equations of infinite number of terms, written in 1669,

A treatise on the method of fluxions and infinite series, with its application to the geometry of curves, written in 1671,

New theory of light and colors, published in 1672,

An account of a new catadioptrical telescope, published in 1672,

A treatise on the quadrature of curves, written in 1676,

The mathematical principles of natural philosophy, published in 1687,

A scale of the degrees of heat; description and signs of heat, published in 1701,

Optics, or a treatise on the reflection, refraction, inflection, and colors of light, published in 1704.

The following are the important, if not very important. *A treatise on the system of the world*, written in 1685. *A historical account of two*

notable corruptions of scripture, published in 1691. *On the nature of acids*, published in 1692. *Universal arithmetic, or a treatise on arithmetical composition and resolution*, published in 1707. *The chronology of ancient kingdoms amended, to which is prefixed a short chronicle from the first memory of things in Europe to the conquest of Persia by Alexander*, published in 1728, *Observations on the prophecies of Daniel and the Apocalypse of st John*, published in 1733.

He wrote also *A series of queries propounded by Mr Newton, to be determined by experiments, positively and directly concluding his new theory of light and colors, and here recommended to the industry of the lovers of experimental philosophy*, in 1672, *The origin of monarchies*, in 1693.

Newton's great many discoveries can be gathered under the two most general. He discovered calculus. And he discovered the principles. The two are inseparable. The two are also, both singly and jointly, the greatest advances in the acquisition of knowledge.

Calculus Newton developed from the *Enumeration of lines* through the *Analysis by equations* through *The method of fluxions* through *The quadrature of curves* to *The principles*.

The principles are in *The principles*. *The principles* was blessed with having Halley for its first judge. Halley prefaced an ode to it:

we are now admitted to the banquets of the gods,
we know the laws of heaven, to which the earth
was blind before, and the immutable order of the
things,

hidden throughout the ages of the world.

Halley wrote also its first review: so many and so valuable philosophical truths as are in it discovered and put past dispute were never yet owed to the capacity and industry of any one man (*id.*).

To show the importance of Newton, then, first the development of calculus can be shown. Then the principles can be shown, and more particularly, first, what is in *The principles*, next, that all of Newton's labors outside *The principles*, on light and colors, the new telescope, the corruption of scripture, acids, heat, reflection, refraction, and inflection, chronology, and the prophecies of Daniel and st John, belong among what is in *The principles*, next, how *The principles* describes the system of the world, and finally, which are the

principles.

clxii

Queen Sophie Charlotte asked Leibniz about Newton. Leibniz replied: taking mathematics from the beginning of the world to the time of Newton, what Newton has done is much the better half (Conduitt, Keynes ms. 130.07).

And in order to show the development of calculus, this can be shown, that taking mathematics from the beginning of the world to the time of Newton, what Newton has done is much the better half.

Later on it can also be shown that what Leibniz has done is the other half.

How mathematics can be taken from the beginning of the world to the time of Newton can be made explicit. The predecessor of the philosophers who left behind the 3 oldest calculations of area predates Newton by forty-seven and a half centuries. Ignoring the possibility that mathematics may have been done outside the solar system, then, no mathematics has been done from the big bang to the time of that predecessor of the philosophers. All mathematics from the beginning of the world to the time of Newton has been done in the forty-seven and a half centuries from the time of that predecessor of the philosophers to the time of Newton.

Hence, what has been done can be made explicit as well. To this end, it can be divided into five parts. What has been done to the time of that predecessor of the philosophers. What has been done to the time of the book of 87 problems. What has been done to the time of Euclid. What has been done to the time of Ptolemy. And what has been done to the time of Newton.

More explicitly, assigning years of birth, the divisions are,

3120	3 calculations of area,
1885	book of 87 problems,
346 ^{BC}	Euclid,
87 ^{AD}	Ptolemy,
1642 - 1727	Newton.

That what has been done has to be traced back to the predecessor of the philosophers, and of the mathematicians, in particular, who left behind the 3 oldest calculations of area can be taken on the authority of Frege.

Frege writes: how has the reference of the word function been extended by the advance of science? two directions can be distinguished; first, the domain of arithmetical operations which contribute to the building of a function has been extended; second, the domain of what can enter as an argument and value of a function has been extended; and if we look back at the development of arithmetic, we recognize a stage by stage advance; first it was calculated with single numbers, with 1, 3, and so on;

$$2 + 3 = 5, 2 \cdot 3 = 6$$

are theorems of this kind.

The oldest theorems of this kind are the 3 calculations of area, beginning with

$$\frac{80 + 140}{2} \times \frac{140 + 60}{2} = 11\,000.$$

On the authority of Frege, the development of arithmetic is part of the advance of science, and the advance of science has extended the reference of 'function'. There are four stages in the development of arithmetic, the advance of science, and the extension of the reference of 'function'. The first is the calculation with single numbers.

And the oldest theorems attest calculation. Calculation with the positive integers. And the calculation of areas.

Taking mathematics from the beginning of the world to that predecessor of the philosophers, what that predecessor of the philosophers has done is all mathematics.

All mathematics is calculation.

All mathematics is arithmetic, arithmetic is calculation, and, in particular, the calculation of areas. Functions are calculation with the positive integers. And all science is mathematics, which is arithmetic, which is calculation, and, in particular, the calculation of areas.

Twelve and a half centuries later, what has been done is attested by the book of 87 problems.

The book begins: the rules of calculation, the gateway to the knowledge of all things and the uncovering of all secrets; this book was copied in the thirty-third year of Apepi, the prince of all Egypt, from an old book written in the time of Amenemhet III, the prince of all Egypt; the scribe Ahmose copied it (*id.*).

Amenemhet III was the prince of all Egypt c. 1845-00. Apepi was the prince of all Egypt c.

1590.

And very much has been done.

The book divides into three parts, problems 1-23, problems 24-34, and problems 35-87.

The domain of calculation has been extended vastly, from the positive integers to the positive rational numbers. The first part, problems 1-23, introduces the rules of calculation. It introduces the rules of the addition, subtraction, multiplication, and division of fractions.

The domain of what can enter into calculation has been extended vastly as well. In the third part, problems 48-55 calculate areas. Problem 50 is: a round field of diameter 9; what is the area (*id.*)? The solution is: 64 (*id.*), implying

$$\pi = \frac{256}{81} = 3.16049\dots$$

Problems 41-6 calculate volumes. Problem 44 is: a granary of length 10, width 10, height 10; how much grain goes in it (*id.*)? Presuming that 75 heqat of grain go in 1 unit of volume, the solution is: 7 500 heqat (*id.*). 1 heqat $\approx$ 4.8 liters. Other than areas, round fields, volumes, and grain, the domain of what can enter into calculation includes: loaves (39), barley (64), fat (66), cattle (67), beer (71), small loaves exchanged for large loaves (72), beer exchanged for loaves (77), feed for geese (82), feed for oxen (84). Problem 40 is: 100 loaves are divided among 5 men; $\frac{1}{7}$ of the three largest

shares equals the two smallest shares; what is the difference between the shares (*id.*)? The solution presumes that the shares are in arithmetical progression. In particular: make the difference between the shares $5\frac{1}{2}$; the shares then are 1,

$6\frac{1}{2}$, 12, $17\frac{1}{2}$, and 23; the total is 60; 100 is $1\frac{2}{3}$ times 60; multiply the shares by $1\frac{2}{3}$; the shares are $1\frac{2}{3}$, $10\frac{5}{6}$, 20, $29\frac{1}{6}$, and $38\frac{1}{3}$; the difference is $9\frac{1}{6}$ (*id.*). Geometrical progression

has been discovered as well. Problem 79 is: 7 barns, 49 cats, 343 mice, 2 401 malt, 16 807 heqat; the total is 19 607 (*id.*).

The second stage, Frege writes: was the advance to general laws which hold for all numbers;

in notation to this corresponds the transition to calculation with letters; in

$$(a+b).c = a.c + b.c$$

we have a theorem of this kind; with this the consideration of single functions was reached, without yet using the word in the mathematical sense or having grasped its reference.

And the mathematician who left behind the book of 87 problems has taken a major step toward the second stage. Toward the general laws which hold for all numbers, the calculation with letters, and the consideration of single functions. By extending the domain of what can enter into calculation to round fields, volumes, grain, loaves, barley, fat, cattle, beer, barns, cats, mice, malt. By dealing with all these and the positive rational numbers indiscriminately. And by dealing with quantity explicitly. The second part of the book, problems 24-34, deals with quantity explicitly. Problem 31 is: the sum of a quantity, $\frac{2}{3}$

of it, $\frac{1}{2}$ of it, and $\frac{1}{7}$ of it is 33; what is the quantity (*id.*)? Or what is the same, even if it will take the thirty-two-odd centuries through Viète to write this explicitly, $x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 33$.

Taking mathematics from the beginning of the world to the mathematician who left behind the book of 87 problems, what this mathematician has done is all mathematics, or very nearly all.

What he has done is all mathematics on the following counts.

First, given what he has done, what he has done makes it possible to identify all his predecessors, all those who have done earlier, in one way or another, in one part or another, what he has done. If what he has done is mathematics, it makes it possible to identify all his predecessors in mathematics, and all his predecessors in mathematics in both the Egyptian record and the Mesopotamian record. Hence, in the Egyptian record he has but one important predecessor. One century-odd before him, under Senwosret, prince c. 1976-32, an account of work on a building was compiled, including dimensions of chambers and wages of workers (*id.*), implying calculation. In the Mesopotamian record he has five important predecessors. One left behind the 3 oldest calcu-

lations of area, from the century around c. 3080, twelve and a half centuries before him. Five centuries later, from the century around c. 2580, seven and a half centuries before him, there are two more. One calculates: a granary; 1 man receives 7 sila of grain; how many men receive grain? 164 571 men; the remainder is 3; 1 152 000 sila of grain in the granary (*id.*). 1 sila $\approx$ 1 liter. The other tabulates the squares with sides 10×60 , 9×60 , 8×60 , and so on, down to $\frac{1}{3} \times 60$, $\frac{1}{6} \times 60$, and $\frac{1}{12} \times 60$ (*id.*). And not quite four centuries later, from the century before c. 2156, three-odd centuries before him, there are two more. One draws a quadrilateral with the numbers 2, $1\frac{1}{6}$, 2, and $2\frac{5}{6}$ over the sides (*id.*), implying the calculation of the area. The other attests the highest point in the development of mathematics before the book of 87 problems, the oldest explicitly formulated problem: the length is $67\frac{1}{2}$, the area is 100; what is the width (*id.*)?

The width is: $1\frac{13}{27}$ (*id.*), implying $\frac{135}{2}x = 100$.

Next, it can be concluded that what he has done not only includes all that his predecessors have done, what he has done has made all that they have done trivial.

Next, what he has done is very much more than what his predecessors have done.

Next, in general, what he has done includes mathematics in all its scope.

Next, it includes mathematics in all its power.

And finally, what he has done shows explicitly the unity and uniformity of all mathematics. Shows it as explicitly as it will not be shown again until Euclid. In the unity and uniformity of its organization, too – from the rules of calculation, to their generalization in the treatment of quantity, to their applications from areas and volumes to cats and mice – the book of 87 problems anticipates the *Elements*.

On these counts what the mathematician who left behind the book has done is all mathematics. And if this has to be qualified, it has to be on the following grounds. Much building has been done, including the pyramids of Khufu, prince c. 2580, and of Khafre, prince c. 2540, no less, and build-

ing implies calculation. Also, twelve and a half centuries old by this time, the record abounds in numbers. For instance, in the Egyptian record, the annals carved on the Palermo stone, completed under Neferirkare, prince c. 2440, record each year the level of the high Nile: 3 cubits, 1 palm, 2 fingers (*id.*); 4 cubits, 1 span (*id.*); 8 cubits, 3 fingers (*id.*). In the Mesopotamian record, Eanatum, prince c. 2475, reports: Eanatum, the prince of Lagash, unleashed a deluge on Umma; he vanquished Umma; he made 20 burial mounds for it (ins. 1). Sargon, prince c. 2337-281, reports: Sargon, lord of the earth, won 34 battles (ins. C-2). These numbers imply calculation.

All such calculation is well within the scope and power of what the mathematician who left behind the book has done. On this ground, what he has done is still all mathematics. On the ground that not all such calculation is explicitly part of what he has done, what he has done is very nearly all mathematics.

Either way, after twelve and a half centuries of development, all mathematics is calculation.

All mathematics is arithmetic. The treatment of areas and volumes anticipates the development of geometry. If so, however, geometry is part of arithmetic, it is the calculation of areas and volumes. The treatment of grain and barley, cats and mice, anticipates, remotely, the development of physics. If so, however, physics is part of arithmetic, too, it is the calculation of grain, barley, cats, and mice. All mathematics is arithmetic, and arithmetic is calculation.

Functions are also calculation. Twofold, the generalization is in both ways the generalization of calculation. Functions are calculation with the positive rational numbers. Functions are also the calculation of quantities.

And all science is mathematics. If not retrospectively – if more of what the record attests, not just the book of 87 problems, will be science – positively prospectively. Anticipating Pythagoras, Aristotle, Ptolemy, Viète, Galileo, Descartes, Leibniz, Kant, Frege, Hilbert, what the mathematician who left behind the book of 87 problems has done is: the gateway to the knowledge of all things and the uncovering of all secrets.

In the following centuries mathematics advanced greatly. The Egyptian record attests, from

the century around *c.* 1786, the mathematician who left behind 25 problems, one of which determines the volume of a truncated square pyramid. The Mesopotamian record attests the first of the three brightest periods in the development of mathematics. It attests the following, for instance. From the century before *c.* 1694, one mathematician left behind 286 problems. As the quantity so the quality. Problem 1 is: a rectangle; the area is 600; multiplied by a number the length is 150; multiplied by another number the breadth is 80; the sum of the two numbers is 9; what are the length and the breadth (*id.*)? implying the system of equations

$$xy = 600,$$

$$ax = 150,$$

$$by = 80,$$

$$a + b = 9,$$

and, in order to solve the system, the equation

$$2x^2 - 135x + 2250 = 0.$$

Another left behind a table of 15 Pythagorean triples, from 45, 60, and 75 through 12 709, 13 500, and 18 541 (*id.*). Another calculated the value of $\sqrt{2}$ at $1\frac{8\,947}{21\,600} = 1.414\,212\,963 \dots$ (*id.*).

Under Ammisaduqa, prince *c.* 1646-25, another discovered the planets. In the century before *c.* 1400, another drew the map of Nippur, solving the problem of the determination of distance and orientation across 2 km.

The scope of mathematics increased greatly. Its power increased greatly as well. With this, however, its unity and uniformity became only implicit.

Then for a long time mathematics did not advance. Then it again advanced greatly.

clxiii

Fifteen and a half centuries after the mathematician who left behind the book of 87 problems, what has been done is attested by Euclid.

And taking mathematics from the beginning of the world to the time of Euclid, what Euclid has done is not less than three quarters of it.

First, what he has done makes it possible to identify all his predecessors.

He left behind the *Elements*, *Data*, *Phenomena*, and *Optics*.

To begin with the *Elements*, it indicates the

following earlier developments.

Demonstration was discovered. Two-odd centuries earlier, Pythagoras discovered the incommensurability of the side and diagonal of the square, and thus demonstration. Incommensurability is inaccessible to anything but demonstration. Aristotle writes: that the side and diagonal of the square are incommensurable is demonstrated by showing that, if they are supposed commensurable, even numbers turn out to be odd (*Pr. analytics*, I, 23). For instance. If the side and diagonal are commensurable, their ratio is rational; let it be $\frac{p}{q}$ in least terms; by the Pythagorean

theorem $2p^2 = q^2$; q^2 is thus even, q is thus even, and as $\frac{p}{q}$ is in least terms, p is odd; q is even, so

let $q = 2r$; then, $2p^2 = 4r^2$, and $p^2 = 2r^2$; p^2 is thus even, and p is thus even; but it is also odd.

The *Elements* is the paradigm of demonstration, and it remained the paradigm of demonstration exclusively at least until Newton's *Principles*.

Demonstration entails that the premises on which conclusions depend can be stated explicitly, and a century after Pythagoras, four generations before Euclid, Hippocrates of Chios wrote the first *Elements*, the first treatise in which theorems were deduced from axioms.

The *Elements* is the paradigm of demonstration because it is paradigmatically axiomatic.

The discovery of the incommensurability of the side and diagonal of the square is *ipso facto*, given a square of side 1, the discovery of the irrationality of $\sqrt{2}$. In the generation of Hippocrates of Chios, Theodorus of Cyrene proved that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational. The irrationals pose the problem of extending the operations of addition, subtraction, multiplication, and division to them. Two generations after Hippocrates and Theodorus, two before Euclid, Eudoxus formulated the theory of proportion. The theory of proportion extended the operations of arithmetic to the irrationals.

The theory of proportion is developed in book V of the *Elements*. Proportion is the foundation of the *Elements*, the foundation of all of Euclid's treatises, and the foundation of all mathematics until Viète. Or, as Eratosthenes, two generations

after Euclid, put it: proportion is the bond of mathematics (Proclus, comm. *Elements*, 43).

In the generation of Eudoxus, Theaetetus developed the theory of the irrationals.

The theory of the irrationals is developed further in book X of the *Elements*.

In the generation of Hippocrates of Chios, Hippias solved the problem of trisecting the angle, conditionally, with the discovery of the quadratrix. Hippocrates of Chios solved the problem of squaring the circle, partially, by squaring lunes. In the next generation, three before Euclid, Archytas solved the problem of doubling the cube completely.

In book XII of the *Elements* the problem of squaring the circle is solved completely by the method of exhaustion, of approaching the area of the circle with inscribed and circumscribed regular polygons of increasing number of sides. The method of exhaustion, too, was discovered by Eudoxus. A method of approach to a limit, it is the first unambiguous anticipation of the differential and integral calculus as well.

Hippasus, not quite two generations after Pythagoras, is associated with the construction of the dodecahedron.

The *Elements* ends, in book XIII, with the construction of the five regular solids.

The *Data* has 94 propositions showing, when something is given, what else is given. Proposition 84 is: if two straight lines contain a given area in a given angle, and one is greater than the other by a given quantity, each of them is also given (*id.*). Or what is the same, given

$$xy = a,$$

$$x - y = b,$$

x and y are given. Or what is yet the same, given

$$x^2 - bx - a = 0,$$

its two roots are given.

In this Euclid's most important predecessor is the one who left behind the 286 problems, fourteen and a half centuries before him.

The *Phenomena* has 18 propositions. Eudoxus formulated the theory of the motions of the heavenly bodies in concentric spheres, too, and the first step in the further development of this theory are the two treatises of Euclid's contemporary Autolycus and the *Phenomena*. In this Euclid's most important predecessor, other than

Eudoxus, in the generation between Eudoxus and himself, is Aristotle, with *On the heaven*.

The *Optics* has 58 propositions. Euclid's predecessors said little about light. God separated light from darkness. Aristotle reports: Empedocles said that the light of the sun moves through the space between the sun and the earth before it reaches the earth (*Senses*, 6). Aristotle also writes: Empedocles is wrong to speak of light as moving (*Soul*, II, 7).

The investigation of light begins with the *Optics*.

Pappus reports that Euclid wrote also *Conics*, in four books, and *Surface loci*, in two books (*Collection*, VII, 3, 30).

One generation before Euclid, Menaechmus discovered the conics. In the *Conics*, Pappus reports, Euclid formulated the locus problem (33-4), such as nineteen and a half centuries later Descartes solved it in five or six weeks.

Proclus reports that Euclid wrote also *Elements of music* and *Fallacies* (comm. *Elements*, 69-70).

The study of the intervals, scales, concords, melody, harmony, begins with Pythagoras. Aristoxenus, one generation before Euclid, left behind the oldest *Elements of harmonics*.

In the *Fallacies*, Proclus reports: Euclid teaches the beginner how to discover paralogisms; he enumerates the different ways of refutation, and provides for each exercises by theorems, showing the true next to the false, and showing it so that the refutations of the errors expose the seductions which we encounter (70).

In this Euclid's most important predecessor is again Aristotle, with the logical treatises, and the *Sophistical refutations*, in particular.

Next, it can be concluded that what his predecessors have done Euclid has done. What he has done has made most of what they have done trivial, too.

Next, he has done more than what they have done.

And how much, taking mathematics from the beginning of the world to his time, he has done can be judged in this way.

Philosophy is theoretical and practical. Ignoring practical philosophy, theoretical philosophy is physics and mathematics. Mathematics in-

cludes logic. Mathematics investigates form as it can be separated from matter, physics investigates form and matter as they are not separated from one another. Logic investigates the form of form.

Mathematics, such as it investigates form as separated from matter, is distinct from physics. Mathematics, such as mathematics and physics both investigate form, is part of physics. Logic, such as it investigates the form of form, is distinct from the rest of mathematics.

Mathematics such as it is distinct from both logic and physics, then, Euclid has done in its entirety, or very nearly in its entirety.

More particularly, logic investigates terms, propositions, and inferences. Inferences are of four kinds, demonstration, dialectical arguments, examinations, and contentious arguments. Aristotle has done terms and propositions. Of demonstration, Aristotle has done the predicate calculus, Euclid has done the paradigm of demonstration. Of logic such as it is distinct from the rest of mathematics, Euclid has done not less than one half, and much the better half.

More particularly, physics investigates motion, the heavenly bodies, the elements, the phenomena in the region below the moon, animals, and plants. Empedocles has done the elements, Aristotle has done motion, the phenomena in the region below the moon, and the animals, Theophrastus has done the plants. Euclid has done the heavenly bodies and their motions, and done them by demonstration. He has done also light, also by demonstration. Of physics such as it is distinct from mathematics, Euclid has done not less than one quarter, and much the best quarter.

If logic is a small part of mathematics, and mathematics is a large part of physics, taking mathematics from the beginning of the world to his time, what Euclid has done is not less than three quarters.

Next, in general, it is with respect to the scope of mathematics that what Euclid has done is not less than three quarters.

Next, with respect to power, the power of the predicate calculus is not great. What Euclid has done is very nearly mathematics in all its power.

Finally, what he has done shows explicitly the unity and uniformity of all mathematics. Shows it paradigmatically, too.

If any of this has to be qualified, it has to be on the same grounds here as above. Twenty-eight centuries old by this time, the record abounds in calculation. For instance, there is much calculation in Herodotus. The record also begins to abound in answers to the question, what is mathematics? Beginning with Plato, in particular.

Neither of this can be the ground of substantial qualification. If for no other reason, because the calculations of Herodotus, very important as they are, are accounted for in the part of physics which investigates the phenomena in the region below the moon. Plato's questions and answers are accounted for in either the part of logic dealing with dialectical arguments and examinations or the part dealing with contentious arguments.

Mathematics is part of philosophy. And in philosophy, inclusive of all its parts, Aristotle's treatises show what Aristotle has done.

If scope is the number of things and the number of the kinds of things brought under investigation, what Aristotle has done is of very great scope.

If power is the number of discoveries to which the investigation has led, what Aristotle has done is not of very great power. In physics, in the parts which he has done and Euclid has not, no substantial advance was made until Buridan. In logic, excepting Chrysippus, no substantial advance was made until Leibniz.

If unity is shown by uniformity, and uniformity is shown by the small number of fundamental propositions and modes of inference employed, what Aristotle has done is of great unity and uniformity.

What Euclid has done is not of greater scope, but it is of much greater power, and of very much greater unity and uniformity.

Taking philosophy from the beginning of the world to the present – some two centuries after Hegel's present – what Aristotle has done has been done after him. It has been done, again, very substantially, by Descartes. Then, again, substantially, by Leibniz. And, once again, differently, by Hegel.

Taking mathematics from the beginning of the world to the present, what Euclid has done has been done before and after him. First, it has been done by the mathematician who left behind the

book of 87 problems. Then, after great advances, again, by Euclid. Then, after further great advances, once again, very substantially, by Ptolemy. Then, after further great advances, yet again, very substantially, by Descartes. And then, after further great advances, yet again, very substantially, by Newton.

Taking the acquisition of knowledge from the beginning of the world to the present, this has to be as fundamental a fact for the investigation of knowledge as there are facts.

clxiv

Mathematics begins with calculation. Calculation with the positive integers. And the calculation of areas.

After twelve and a half centuries of development, all mathematics is calculation. Calculation has been generalized, it is calculation with the positive rational numbers, and the calculation of quantities, including areas and volumes, grain and loaves, cats and mice. Mathematics is arithmetic, and arithmetic is in the first stage of the development of functions, the calculation with numbers, and making steps toward the second stage, toward the general laws which hold for all numbers, the calculation with letters, and the consideration of single functions. And all science, positively prospectively, is mathematics.

After twenty-eight centuries of development, mathematics is demonstration. It is also geometry.

This poses the problem of the relation of calculation and demonstration, and of arithmetic and geometry.

The general fact that mathematics has become demonstration, and geometry, in the last two-odd centuries of its twenty-eight centuries of development has to make it probable that demonstration is a development of calculation, and hence, calculation is the foundation of demonstration, and that geometry is a development of arithmetic, and hence, geometry and arithmetic are not independent of one another parts of mathematics, but one is part of the other.

And the following more particular facts have to make this certain.

Demonstration is axiomatic, and the first three of the five axioms on which demonstration rests

are:

1. things equal to the same thing are equal;
2. if equal things are added to equal things, the wholes are equal;
3. if equal things are subtracted from equal things, the remainders are equal.

Demonstration rests on the operations of arithmetic, on addition and subtraction.

Demonstration rests also on the theory of proportion. The theory of proportion rests on the definition of proportion, and the definition of proportion, definition 5 in book V, is:

given magnitudes a , b , c , and d , and any positive integers m and n , if,

whenever $ma > nb$, also $mc > nd$,

whenever $ma = nb$, also $mc = nd$,

and whenever $ma < nb$, also $mc < nd$,

then $a:b = c:d$ (*id.*).

Proportion rests on the operations of arithmetic, too, on multiplication. Eratosthenes writes: proportion is the bond of mathematics. Viète writes: if four terms are such that the product of the extremes is equal to the product of the means, they are proportional, and conversely, if four terms are proportional, the product of the extremes is equal to the product of the means; thus a proportion is that from which an equation is composed and an equation is that into which a proportion is resolved. Klein writes: the theory of proportion is the theory of all possible algebraic transformations of equations of the type $\frac{a}{b} = \frac{c}{d}$

(*Elementary*, II, 3.2.3).

The discovery of incommensurability, and *ipso facto* of the irrationals, has led to the formulation of the theory of proportion, and the theory of proportion, in turn, has extended the operations of arithmetic to the irrationals.

Mathematics is firmly in the second stage of the development of functions. On all three counts. It has advanced to general laws which hold for all numbers. It has made the transition to calculation with lines, if not yet with letters. And it has reached the consideration of single functions. The theorem

$$(a+b) \cdot c = a \cdot c + b \cdot c$$

is proposition 1 in book V. Euclid proves that $a^2 + b^2 = c^2$, where a , b , and c are the sides of a right triangle (I, 47), or equivalently, a Pythagorean

rean triple. That $(a+b)^2 = a^2 + 2ab + b^2$, where $a+b$ is the side of a square (II, 4), or equivalently, a and b are any real numbers. With lines, too, he proves that, if a prime divides the product of two numbers, it divides at least one of its factors (VII, 30), that, given any number of primes, there are more primes (IX, 20), that, if $a + \sqrt{b} = x + \sqrt{y}$, then $a = x$ and $b = y$ (X, 42). Numbers are lines, areas and volumes are lines, the motions of the heavenly bodies are lines, rays of light are lines.

The reference of 'function' has been generalized vastly. The domain of arithmetical operations which contribute to the building of a function has been extended, to addition, multiplication, and their converses, exponentiation and its converse have been added, and the transition to a limit has been anticipated as well. The domain of what can enter as an argument and value of a function has been extended by the admission of the irrationals, and with this the sense of the expressions 'sum', 'product', and so on, has had to be determined further.

And the generalization is the generalization of calculation. Functions are calculation with the positive real numbers. Functions are also the calculation of lines.

Demonstration is a development of calculation, then, and calculation is the foundation of demonstration.

Geometry is a development of arithmetic. In particular, geometry is a generalization of arithmetic. Arithmetic has thus become part of geometry. It is the fundamental part of geometry, however. If geometry is the investigation of lines and their relations, and arithmetic is the investigation of number and its relations, the investigation of lines and their relations is the generalization of the investigation of number and its relations. Numbers are lines, and not the other way around. But it is because the axioms and definitions have established that lines do what numbers do, obey the operations of arithmetic, that geometry, of which arithmetic is part, is mathematics. Pythagoras discovered incommensurability, the irrationals, and demonstration. Pythagoras also said: number is the principle of all things. And number is the principle of all things.

Logic is also a development of arithmetic. In particular, it is also a generalization of arithmetic.

Syllogistic is a calculus, the predicate calculus. And it is a calculus. In order to develop syllogistic, Aristotle takes for granted, implicitly, the propositional calculus. And it is a calculus, too. The predicate calculus has established that terms in propositions can do, and if they can, they had better, what numbers do, obey a few mechanical rules borrowed from the rules of arithmetic. The propositional calculus, when Chrysippus will develop it, will establish that propositions in inferences can do, and if they can, they had better, what numbers do, obey a few mechanical rules borrowed from the rules of arithmetic.

Physics, all its parts that have succumbed to demonstration, is a generalization of geometry. Einstein writes: if geometry is conceived as the science of the laws governing the spatial relations of rigid bodies, it is to be considered the oldest branch of physics. The parts of physics that have succumbed to demonstration have established that spatial relations, those of the heavenly bodies and of rays of light, to begin with, obey the rules which lines obey, which obey the rules which numbers obey.

And all science is mathematics. Positively prospectively, if, with Aristotle, knowledge is acquired only by demonstration, and demonstration is mathematics.

In the century and a half after Euclid mathematics advanced greatly. Through Aristarchus, Archimedes, Chrysippus, Eratosthenes, Diocles, Apollonius, Hipparchus. Then the second of the three brightest periods in the development of mathematics ended. Then for two and a half-odd centuries mathematics advanced little. Then there was Ptolemy.

Four and a half centuries after Euclid, what has been done is attested by Ptolemy.

And taking mathematics from the beginning of the world to the time of Ptolemy, what Ptolemy has done is not less than one half, and much the better half.

Ptolemy writes: Aristotle is right to divide theoretical philosophy into physics, mathematics, and theology; and we have concluded that mathematics alone is certain and unshakable knowledge; its proofs proceed by the unquestionable methods of arithmetic and geometry; and mathematics can deal with the objects of the other two

kinds, too, with those of theology because it studies the unchanging, and with those of physics because everything material can be understood by its locomotion.

This is what mathematics is. Certain and unshakable knowledge. Which can deal with the objects of the other two kinds of theoretical philosophy, too.

Unlike all the philosophers who have answered, and will answer, the question, what is mathematics? instead of doing mathematics, Ptolemy has answered it after doing mathematics.

And what he has done is the world. He has described the position, order, and motion of not less than 7 428 parts of the world. He has described the system of the world. He has described – explicitly – the world in its entirety.

That mathematics is knowledge follows. It describes the world. What its relation to the other two kinds of theoretical philosophy is follows as well. It can deal with the objects of physics because everything material can be understood by its locomotion. Theology, Ptolemy writes: is concerned with the first cause of the motion of the world. And mathematics can deal with the object of theology as well because it studies the unchanging. Hence, also, mathematics is arithmetic and geometry, its proofs proceed by the unquestionable methods of arithmetic and geometry, and if these, arithmetic and geometry, pose the problem of their relation, the solution of this problem can only be part of the solution of the general problem, of the relation of mathematics and the other two kinds of theoretical philosophy, namely, mathematics can deal with their objects, too.

With respect to the scope of mathematics, then, if scope is the number of things and the number of the kinds of things brought under investigation, what Ptolemy has done is extend it as far as it can be extended. The scope of mathematics is the world. And it includes explicitly the objects of all of theoretical philosophy, of physics and theology as well as mathematics.

Ptolemy has done the world, and how he has done it is function.

The third stage in the development of functions, Frege writes, is: the recognition of general laws of functions and with this the coinage of the

technical term ‘function’; in notation to this corresponds the introduction of letters like f , F for the indefinite indication of functions; in

$$\frac{df(x) \cdot F(x)}{dx} = F(x) \cdot \frac{df(x)}{dx} + f(x) \cdot \frac{dF(x)}{dx}$$

we have a theorem of this kind.

The development of calculus, in the three generations between Galileo and Leibniz, will coin the technical term, introduce letters like f , F for the indefinite indication of functions, and prove that theorem. The recognition of general laws of functions, however, implicitly, if not explicitly, begins with Archimedes, including in that he deals geometrically, reducing them to the relations of lines, with weights and fluids, and the recognition of general laws of functions, implicitly, if not explicitly, is the foundation of all that Ptolemy has done.

In particular, trigonometry is the foundation, and more particularly still, the sine function is. The first thing, after its general introduction, that Ptolemy does in the *Almagest* is the calculation of the chords subtending arcs of the circle. Or what is the same, the calculation of the sine function. Hence, everything that he does next in the *Almagest*, eccentricities, lines of apsides, sizes of epicycles, equant points, positions of fixed stars, he does by sine functions. Everything that he does in the *Planetary hypotheses*, the distances of the planets, and the size of the world, rests on sine functions. Everything that he does in the *Geography*, the theory of projections, and the latitudes and longitudes of localities, rests on sine functions. Everything that he does in the *Optics*, angles of incidence, of reflection, of refraction, rests on sine functions. It is the sine function, too, that makes observation a quantity, makes it obey the rules of arithmetic.

Ptolemy has taken a major step toward the third stage in the development of functions, and it is this stage, Frege notes, that is the richest in consequences.

With respect to the power of mathematics, if power is the number of discoveries to which the investigation has led, what Ptolemy has done is taken a major step toward its extension as far as it can be extended. Because, still with Frege, function is the unchangeable in an expression, arguments and values of functions are objects

without restriction, and object is all that is not function.

The recognition of general laws of functions is the foundation of all that Ptolemy has done implicitly. Explicitly, all that he has done, observation, sine function, eccentricities, lines of apsides, epicycles, equant points, fixed stars, distances of planets, size of the world, projections, latitudes, longitudes, angles of incidence, reflection, refraction, and the absolute determination of chronology in the *Handy tables*, too, he has done by calculation.

Ptolemy writes: we have arrived at some of our assumptions not from readily available principles, but after a long period of trial and error; even if assumptions are not proved, however, if they are in agreement with the phenomena, they cannot have been found without a method (*Almagest*, IX, 2).

The method is calculation.

Mathematics is thus not just deduction. It is induction, and hence deduction. It is not just demonstration either. It is calculation, and hence demonstration.

Some of the great advances in mathematics after Euclid have been on the deductive model of the *Elements*. It is how Archimedes has solved problems of curves, areas, volumes, developed the theory of equilibrium, the theory of floating bodies. How Apollonius has developed the theory of the conics, solved problems of tangents, areas, maxima and minima. But the advances in the investigation of the heavenly bodies and the surface of the earth, with Aristarchus, Eratosthenes, Apollonius, Hipparchus, the investigation of the modes of inference, with Chrysippus, the investigation of parabolic mirrors, with Diocles, the investigation of machines, with Hero, have all been made not from readily available principles, but after a long period of trial and error. And if the paradigm of deduction is Euclid's *Elements*, the paradigm of induction, and hence deduction, is – until Newton's *Principles* – Ptolemy's *Almagest*. The *Mathematical collection*.

The difference is not that the principles of deduction in the *Elements* are proved, they are not, or that they have not been found by trial and error, they have. The difference is that generalization has forced the *Almagest* to concern itself

explicitly with the questions of which are its principles, and how, and why. They are not readily available. They have been found after a long period of trial and error. They cannot have been found without a method, however, if they are in agreement with the phenomena. And calculation shows that they are in agreement with the phenomena.

As the scope and power of mathematics increased greatly after Euclid, its unity and uniformity became only implicit. If unity is shown by uniformity, and uniformity is shown by the small number of fundamental propositions and modes of inference employed, what Ptolemy has done has not increased the unity and uniformity of mathematics quantitatively. The number of fundamental propositions and modes of inference employed has not decreased. To the degree to which generalization has posed the problem of its foundations, however, hence the problem of which the fundamental propositions and modes of inference are, and hence the problem of the unity and uniformity of mathematics, and has posed this problem on the ground of mathematics such as it has been done, as distinct from mathematics such as it ought to be done, what Ptolemy has done has greatly increased the unity and uniformity of mathematics qualitatively. The more so, if all that he has done he has by calculation.

It can be concluded that, with respect to logic, such as it is distinct from the rest of mathematics, what Ptolemy has done is anticipate the fact that mathematics has to deal with the problems of logic, such as it is distinct from the rest of mathematics. With respect to mathematics, such as it is distinct from both logic and physics, what he has done is not less than one half, and much the better half. With respect to physics such as it is distinct from mathematics, what he has done is much more than one half, and very much the best part. All in all, not less than one half, and much the better half.

After thirty-two and a half centuries of development, mathematics has described the world, taken a major step toward the richest in consequences stage in the development of functions, and discovered the problem of its principles.

Fifteen centuries later, the development of calculus is the greatest advance in the develop-

ment of mathematics, by far. The cause has to be proportionate to the effect. And if the development of calculus is an effect, whose cause are the world, functions, and the principles, the cause is proportionate to the effect.

The description of the world, the generalization of calculation, and the formulation of the principles, all three, are most explicitly the problems which the philosophers who developed calculus, beginning with Galileo and Kepler, and through Newton and Leibniz, proposed to solve, and solved.

What Ptolemy has done, then, is the remote cause of the development of calculus. What Galileo and Kepler have done is the proximate cause.

clxv

For a long time after Ptolemy mathematics advanced infrequently. Diophantus is one and a half centuries after Ptolemy, Pappus two generations later, al-Khwarizmi five centuries later, ibn al-Haytham one and a half centuries later, Khayyam three generations later, Fibonacci four generations later, Buridan four generations later, Oresme one generation later, Regiomontanus four generations later. Thirteen and a half centuries from Ptolemy to Regiomontanus. Nor did mathematics advance too much, when it did.

Then prince Henry began sending his knights to sea, and in 1434 Eanes rounded cape Bojador. In 1492 Columbus crossed the Atlantic. In 1519-22 Magellan and El Cano circumnavigated the earth.

Then, fourteen centuries after Ptolemy, Copernicus published *On the revolutions of the heavenly spheres*, in 1543, Cardano published *The great art*, in 1545, and mathematics began advancing greatly once again, and the advance accelerated, and accelerated.

Copernicus, with the discovery that the earth revolves around the sun, advanced the description of the world dramatically, qualitatively. Yet, quantitatively, the description remained the same. Observation remained the same. Calculation in general remained the same. Copernicus calculated what Ptolemy calculated, in the way Ptolemy calculated it. Which explains why the hypothesis that the earth revolves around the sun remained

for two generations a hypothesis in need of a proof.

Cardano advanced calculation greatly. The domain of arithmetical operations had been extended to the addition, multiplication, and their converses of the positive real numbers, and exponentiation and its converse to the third power. But the presumption that numbers are lines had hindered the extension of the operations to the negative numbers and 0, and exponentiation and its converse beyond the third power. Cardano made the decisive step toward all that, the extension of the operations to all real numbers, including the negative, including 0, and of exponentiation and its converse beyond the third power. He discovered the complex numbers, too.

He writes in *The great art*, for instance: if it is said, divide 10 into two parts whose product is 40, it is clear that this is impossible (37). And continues: we will nevertheless do it; divide 10 into two equal parts of 5 each; square these, obtaining 25; subtract 40 from 25; the remainder is -15, and the square root of this, added to 5 and subtracted from it, gives the parts whose product is 40, namely, $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$ (*id.*). He adds: thus advances mathematical subtlety, its end as elevated as it is useless (*id.*). In *The great art of arithmetic*, in 1550, he writes: $\sqrt{9}$ is either 3 or -3, for plus times plus and minus times minus are both plus; $\sqrt{-9}$ is therefore a third kind of thing, of stealthy nature (38).

Also, when he solved all forms of the quadratic equation, and all forms of the cubic equation, and formulated rules for the solutions, Cardano showed that equations and their solutions can be generalized. If the different forms of the quadratic can be solved, the quadratic in general can be solved, if the different forms of the cubic can be solved, the cubic in general can be solved.

The advances of Copernicus and Cardano had not informed one another.

One generation-odd later, Viète, Tycho, and Stevin are contemporaries.

Viète continued, and accelerated, the advance of calculation. In *The analytic art*, in 1591, he formulated the master rule (2), and proportion was generalized conclusively into equation. Moreover: the analytic art uses not numbers, but

symbols or signs for things, say, the letters of the alphabet (4). Viète writes: in order to be solved, equations need preparation, especially when they involve the subtraction of terms of different powers, more negative powers than positive, and fractions or irrationals (*Examination*, I, 1). Representing terms in equations by letters made the preparation of equations transparent. Viète writes: first and foremost, the analyst has to understand the origin of equations and their fundamental structure (*id.*). Representing terms by letters made the origin of equations, how they arise from one another, and their fundamental structure, how they constitute one another, transparent, too. Letters thus made it possible to deal with equations of very much greater complexity. Viète writes: zetetics no longer limits its reasoning to numbers, which was the shortcoming of the old analysis; it employs the newly discovered logistic by species, which is far more fruitful and powerful than the old numerical logistic (*Art*, 1). Letters made it possible to deal with equations of very much greater generality, and to distinguish it explicitly as generality. The following instances illustrate both, the complexity and the generality. They show also the further advance of calculation toward the third stage in the development of functions.

Viète solved the problem of the powers of the binomial. He writes the second power, the third, the fourth, the fifth, the sixth:

$$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6,$$

and adds: the construction of higher powers is not different (*Logistic*, 11).

Ptolemy discovered trigonometric identities, one century before Viète Regiomontanus discovered more, Viète discovered still more, for instance:

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

and

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

(*Canon*, app.).

Viète showed that π is an infinite series. He applies the method of exhaustion to a circle of radius $\frac{1}{2}$, calculates the chords supplementary to the sides of the inscribed square, regular octagon, regular 16-sided polygon, and so on, all the way to: the polygon of an infinite number of sides or,

if you please, circle (*Responses*, 18), at: $\sqrt{\frac{1}{2}}$,

$$\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}, \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}}, \text{ and so on, all}$$

$$\text{the way to } \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}} + \dots}} \text{ (id.)},$$

hence calculates the area of the polygon of an infinite number of sides or, if you please, circle at:

$$\frac{1}{2 \times \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \dots \text{ (id.)}, \text{ and as the area of that polygon, or circle, is } \frac{\pi}{4},$$

$$\pi = \frac{2}{\sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}} \times \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \dots$$

Two-odd centuries before Viète Oresme calculated sums of infinite series, for instance:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots = 2$$

(*Euclid*, 2). Euclid calculated the sum of a finite geometrical series:

$$\frac{S_{n-1}}{a_n - a_1} = \frac{a_1}{a_2 - a_1}$$

(*Elements*, IX, 35), where $a_1, a_2, \dots, a_n$ are the members of the series, S_n is their sum. Viète calculated the sum of an infinite geometrical series. Euclid's result yields $\frac{S_n - a_n}{S_n - a_1} = \frac{a_1}{a_2}$, and

Viète shows that: if $\frac{a_1}{a_2} > 1$, as n approaches

infinity, a_n approaches 0, and

$$S_\infty = \frac{a_1^2}{a_1 - a_2}$$

(*Responses*, 17).

In general, Viète made the calculation of infinities routine. No wonder: the analytic art claims for itself the greatest problem of all, to solve all problems (*Art*, 8).

With Viète as well begins the problem – still

to be solved – of which are the kinds of mathematics, and what are their relations. In Viète the kinds are not less than seven, geometry, arithmetic, zetetics, poristics, exegetics, logistic, analysis.

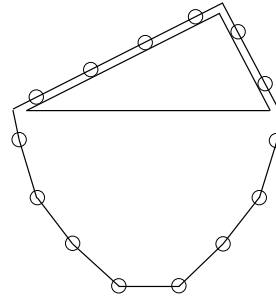
Tycho advanced observation. Beginning with his observations of the heavenly bodies from 1580 to 1600, observation became a continuous series, and hence more accurate by orders of magnitude.

With Tycho and Viète the advance in calculation, including observation, began informing the advance in the description of the world made by Copernicus. Gilbert and Bruno argued, with Copernicus, that the earth revolves around the sun. Tycho and Viète argued, against Copernicus, that the sun revolves around the earth. They also argued, however, with Copernicus, and by calculation, and much improved calculation, that Mercury, Venus, Mars, Jupiter, and Saturn revolve around the sun, not around the earth. And thus the hypothesis that the earth revolves around the sun began being proved. Tycho undertook the calculation in *On the recent phenomena in the aethereal world*, in 1588, Viète in the *Celestial harmony*, in 1600.

Stevin published *The principles of the art of weights*, in 1586. He begins: number, magnitude, and weight are essentially inseparable; while the first two, however, have reached the stage of art, in the art of calculation and the art of measurement, weight has not, because its fundamental properties have remained hidden from our predecessors (*ded.*). Hence: *The principles of the art of weights* separates weight in thought from natural matter, and in 14 definitions and 28 propositions treats of vertical weights and oblique weights (*cont.*). Stevin discovered that, maintained in equilibrium by weights less than those by which vertical weights are, oblique weights will reveal the fundamental properties of weight.

And: the principle of the properties of oblique weights is contained in the following proposition;

let a triangle be in a plane perpendicular to the horizon with its base parallel to it, and equal weights be placed on its two other sides; then, as the left side of the triangle is to the right, so the force of the weight on the right side is to the force of the weight on the left; for, let the left side be twice the right; let 14 spheres, equal in weight



and size, be strung about the triangle at equal distances, and so that 4 are on the left side, 2 are on the right; if the force of the 4 is not equal to the force of the 2, the spheres will be set in motion, and the motion will have no end, which is absurd; the spheres are therefore in equilibrium; hence, if the 8 spheres below the triangle are removed, the 6 spheres on the triangle will remain in equilibrium; the force of the 4 spheres on the longer side is therefore equal to the force of the 2 on the shorter (II, 19); the force of an oblique weight is therefore inversely proportional to the length of the oblique plane (*id.*). On the title page Stevin comments: a wonder is not a wonder (*id.*). The wonder, the law of the inclined plane, anticipates also the greater wonder, the parallelogram of forces, which Newton will deduce from the yet greater wonder, the laws of motion.

Stevin also introduced the decimal, in *On the decimal*, in 1585. He also argued, by calculation, that the earth revolves around the sun, in *The course of the heaven*, in 1608.

With Stevin the philosophers began, both, to describe the position, order, and motion of the part of the world force, and to search for the principles explicitly, and find them.

Galileo and Kepler, in the next generation, achieved the synthesis of the description of the world, generalization of calculation, and formulation of the principles, the three inseparable from one another, advanced all three very much further, and, hence, posed all the problems which the development of calculus, in the next two-odd generations, solved.

Galileo proved that the earth revolves around the sun. He determined better the size of the world, the position and order of its parts, and their uniformity. He determined better the motion

of the parts, too, and closed the abyss between the description of the motion of the heavenly bodies and motion in general. As an iron ball of 100 pounds falls from a height of 100 brachia in 5 seconds, so it falls from the orbit of the moon in 3 hours, 22 minutes, and 4 seconds. As an iron ball falls, so from the ratio of acceleration of natural motion it can be calculated, if the planets were created in the same place, at what distance from the center of their revolutions that place was. A stone falling straight down from the top of a tower is carried by the circular motion of the earth, and its motion is compounded of these two motions.

Hence the new science of locomotion. In it we consider uniform motion, naturally accelerated motion, and violent motion, motion compounded of uniform and uniformly accelerated motion.

The new science of locomotion posed the problems of speed and acceleration, of change of distance over time and change of speed over time.

A falling body departs from rest, and that is, from the deprivation of any speed whatsoever, and begins its motion accelerating from the first instant. Time being infinitely divisible, there is no degree of swiftness, however small, or what is the same, no degree of slowness, however great, which the body does not have after its departure from infinite slowness, and that is, from rest.

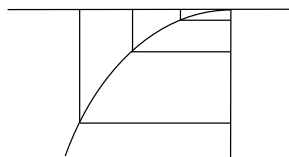
Speed and acceleration posed the problems of degrees of swiftness, however small, or what is the same, degrees of slowness, however great, and all the way to infinite slowness, and that is, rest.

In uniform motion, $v \propto \frac{s}{t}$. In uniformly accelerated motion, $s \propto t^2$.

Distance, time, and speed are functions. So is acceleration. Distance, being infinitely divisible, time, being infinitely divisible, and speed, being of degrees, however small, or great, and all the way to infinite, posed the problems of building functions of great complexity and generality. The more so acceleration.

In violent motion, motion compounded of uniform horizontal and uniformly accelerated vertical motion, a projectile describes a parabola.

The horizontal line is time, the vertical line is distance. The problems of distance, time, speed,



and acceleration, problems of infinite divisibility, and problems of the building of functions, posed the problems of the relations of straight lines and curves, of rectification of curves, of tangents to curves, and of areas under curves.

These problems posed the problem of continuity.

Nor is it impossible that there be infinitely many voids in a finite continuous extension. When a large circle revolves on a line, a small circle on the same center and the center describe a line of the same length as the large circle. What holds for lines holds for surfaces and solids. Why should the circumference of a circle and a single point not be called equal? Moreover, all numbers are more than the squares alone, yet the squares must be as many as all numbers. In the infinite number there must be as many squares as are all numbers together.

The description of the world posed the problems of motion. The problems of motion posed the problems of its generation and transformation. The problems of the generation and transformation of motion posed the problems of the relations of lines, numbers, and infinities. With Stevin physics was still fundamentally statics. Because the problems of motion posed the problems of its generation and transformation, with Galileo physics became fundamentally dynamics. With Viète mathematics was still fundamentally the building of equations. Because the problems of the generation and transformation of motion posed the problems of the relations of lines, numbers, and infinities, with Galileo mathematics became fundamentally the building of functions.

For the same reasons observation became fundamentally experiment. The calculation which observation is became calculation of fundamentally greater complexity and generality. With Bacon, the secrets of nature reveal themselves not when nature is free and left to herself, but when she is constricted and vexed. To reveal the secrets of generation and transformation, relations of

lines, numbers, and infinities, nature demanded more constriction, more vexation.

Kepler discovered the laws of the motions of the heavenly bodies.

The motions of the planets should depend on their distances. Circumscribe around the sphere of the earth a dodecahedron, the sphere comprehending it is that of Mars, circumscribe around it a tetrahedron, the sphere comprehending it is that of Jupiter, circumscribe around it a cube, the sphere comprehending it is that of Saturn, in the sphere of the earth inscribe an icosahedron, the sphere which it comprehends is that of Venus, inscribe in it an octahedron, the sphere which it comprehends is that of Mercury. In the center is the sun, and the dependence of the motions of the planets on their distances is $\frac{1}{2} \frac{T_1 + T_2}{T_1} = \frac{r_2}{r_1}$.

The heavenly bodies posed the same problems. They posed the problem of the dependence of motion on distance, and the problem of the dependence of motion on distance posed the problems of distance and period, and that is, of distance, time, and speed.

The heavenly bodies posed the problem also of packing the world. Packing it with the five regular solids, in particular. In general, Kepler kept formulating and solving packing problems. And packing problems posed the problems of maxima and minima. Viète solved problems of maxima and minima. In the first proposition of part II of the *New stereometry of wine casks* Kepler shows that: of all parallelograms inscribed in a circle the largest is the square (*id.*), and the entire part II is devoted to problems of maxima and minima. Problems of maxima and minima posed problems of generation and transformation, of areas in that case, and vice versa, problems of generation and transformation posed problems of maxima and minima.

The dependence of the motion of the planets on their distances was also shown a function as explicitly as a relation had been shown.

Thus in the apprentice piece. Thirteen years later Kepler discovered the first two laws. In their first formulation: a planet approaches the sun and recedes from it in an elliptical orbit; and it spends time at each point in proportion to the distance of the point from the sun.

Galileo discovered that nature is fond of parabolas. Kepler discovered that she is fond of ellipses, too, and ellipses posed the same problems, of the relations of straight lines and curves, of rectification, tangents, and areas under curves. These problems posed the problem of continuity.

A planet moving uniformly in a circle, as it does in Ptolemy, and Copernicus, does not pose the problem of its differential speed. A planet moving in an ellipse does. It spends time at each point in proportion to the distance of the point from the sun. And the measure of the distance, Kepler discovered, is the area. He writes in the *New astronomy*: the sum of the distances is contained in the area (60).

This is the most fundamental problem which calculus will solve. How sums of distances are contained in areas. Also in the *New stereometry*, Kepler took the first step toward its solution. In the second proposition of part I he writes: the circumference of the circle has an infinite number of points, and each of these can be regarded as the base of an isosceles triangle with sides equal to the radius; there is thus an infinite number of triangles in the area of the circle (*id.*). And in all of the *New stereometry* he analyzes curvilinear areas and volumes into an infinite number of figures. The problem of how sums of distances are contained in areas posed problems of the relations of lines, numbers, and infinities, and again vice versa, problems of the relations of lines, numbers, and infinities posed the problem of how sums of distances are contained in areas.

Ten years after the discovery of the first two laws, Kepler discovered the third. The function became $T \propto r^{\frac{3}{2}}$.

Kepler also searched for the causes from the first, and beginning with the *New astronomy* found them in the forces. In the magnetic forces, in particular. Physics became dynamics. Galileo discovered the fundamental character of speed and acceleration. Kepler discovered the fundamental character of force.

Of force in relation to tangents and areas, numbers and infinities. Mathematics became the building of functions. Kepler developed trigonometry, developed logarithms, developed greatly the theory of conics. Conics became relations of ordinates, abscissas, secants, and tangents.

Most of all, Kepler discovered the laws of the motions of the heavenly bodies by calculation. I calculated day and night, he reports in 1596. Counting through the *Rudolphine tables* and the calculation of the transits of Mercury and Venus due in 1631, he calculated for not less than thirty-five years. Thus quantitatively. Qualitatively, he brought all the power of calculation, more than Ptolemy, more than Galileo, to bear on the description of the world.

Written in mathematics, the world is read by calculation.

clxvi

The next is the generation of Descartes, Cavalieri, and Fermat.

Descartes published the *Geometry, which is an essay in the method*, in 1637.

He reports: I begin where Viète ends (to Mersenne, Dec 1637). More particularly: in the *Geometry* I endeavor to give a general way for solving all problems that have never been solved (to Mersenne, Mar 1636).

And these are the main steps of the general way, under Descartes's heads:

how the calculus of arithmetic relates to the operations of geometry;
just as all arithmetic consists of only four or five operations, addition, subtraction, multiplication, division, and extraction of roots, so nothing else is necessary in geometry in order to find required lines than to add or subtract other lines;

how symbols can be used in geometry;
often there is no need to draw the lines, it suffices to designate them by letters; thus, to add two lines, I write $a + b$, to extract the cube root of $a^3 - b^3 + ab^2$, I write $\sqrt[3]{a^3 - b^3 + ab^2}$;

how to arrive at equations with which to solve problems;

to solve a problem, first it has to be considered solved, and names have to be given to all lines which seem necessary for the construction, known as well as unknown; then the difficulty has to be resolved in the order which shows most naturally the dependence of the lines on one another, until it is found how to express the same quantity in two ways; this is what is called an equation; and as many equations have to be found as there are unknown lines;

an example from Pappus (I).

The example is the locus problem. And: you see that no matter how many lines are given, the length of any line through a point can be expressed by three terms, one containing the unknown quantity x , another containing the unknown quantity y , and the third containing only known quantities (*id.*).

Further:

which are the curves that can be admitted in geometry;

to comprehend all curves that there are in nature together and order them into kinds, I say that all points of any geometrical curve, and that is, any curve which admits of exact measurement, necessarily have a definite relation to all points of a straight line, and this relation can be expressed by a single equation;

the way to distinguish all curves into kinds and know the relation of all their points to those of straight lines;

if the equation of a curve contains no term of a power higher than the second in one or both of the unknown quantities, the curve is of the first and simplest kind, comprising the circle, parabola, hyperbola, and ellipse; if the equation contains terms of the third or fourth power in one or both of the unknown quantities, the curve is of the second kind; if the equation contains terms of the fifth or sixth power in one or both of the unknown quantities, the curve is of the third kind; and so on to infinity;

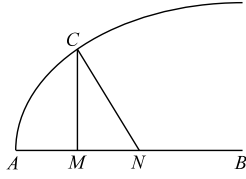
to find the properties of curves it suffices to know the relation of all their points to those of straight lines and the way to draw lines perpendicular to them;

in this way all that can be determined about the areas which curves comprehend can be found; all their other properties as well depend on the angles which they make with other lines, and the angle between two curves can be measured by the angle between two straight lines, if a straight line can be drawn perpendicular to one of the curves at the point of intersection; this is why I believe I should give a general way to draw a straight line perpendicular to a curve;

general way to find straight lines perpendicular to curves;

let AC be the curve to which it is required to

draw a perpendicular at C ; consider the problem solved, and the required line to be CN ; produce CN to meet the straight line AB ; let $CM=x$ and $AM=y$; an equation has



to be found expressing the relation of x and y ; if CMN is a right triangle, and its sides are x , $a-y$, and b , then $b^2 = x^2 + a^2 - 2ay + y^2$; hence, $x = \sqrt{b^2 - a^2 + 2ay - y^2}$ and $y = a + \sqrt{b^2 - x^2}$; and given these two equations, one of the unknowns can be eliminated, and the result will be one equation with only one unknown; for instance, if the curve is an ellipse, then from Apollonius, I, 13, $x^2 = ly - \frac{l}{t}y^2$, where l is the latus rectum, t is the transverse diameter, and y is the abscissa to the ordinate x , x can be eliminated, and the equation becomes $y^2 + \frac{ly - 2aty + a^2t - b^2t}{t-l} = 0$ (II).

Descartes comments: and I dare say that this is not only the most useful and most general problem I know, but the most useful and most general I ever desired to know in geometry (*id.*).

Further:

on the nature of equations;

an equation consists of several terms, some known some unknown, and some of which taken together are equal to the rest; or all of them can be taken to equal 0, for often this is the best form to consider;

how many roots can every equation have; every equation can have as many distinct roots, and that is, values of the unknown quantity, as is the power of the unknown quantity; if, for instance, $x-2=0$ and $x-3=0$, when these equations are multiplied $x^2-5x+6=0$ is obtained, and this is an equation in which the quantity x has the values 2 and 3;

which are the false roots;

often some of the roots are false, or less than 0; for instance, $x^4 - 4x^3 - 19x^2 + 106x - 120 = 0$ has four roots, three true, 2, 3, and 4, one false, 5;

how, when one of the roots of an equation is known, its power can be lowered;

it is evident from the above that an equation is always divisible by a binomial consisting of the unknown quantity minus one of its true roots or

plus one of its false roots; and thus the power of an equation can be lowered;

how many true roots can every equation have; it is clear also how many true and false roots every equation can have; it can have as many true roots as there are changes of sign from + to - or - to +, and as many false roots as there are + or - signs in succession; in the last equation, for instance, $+x^4$ is followed by $-4x^3$, $-19x^2$ by $+106x$, and $+106x$ by -120 , and the equation has three true roots, $-4x^3$ is followed by $-19x^2$, and it has one false root;

the true and the false roots can both be either real or imaginary;

neither the true nor the false roots are always real; for instance, the equation $x^3 - 6x^2 + 13x - 10 = 0$ has only one real root, 2, the other two, however we increase, diminish, or multiply them, always remain imaginary (III).

Descartes concludes with: a general rule to reduce equations of a power higher than the fourth (*id.*); a general way to construct all solid problems which are reduced to an equation of the third or fourth power (*id.*), and: a general way to construct all problems which are reduced to an equation of a power not higher than the sixth (*id.*).

In the *Geometry*, Descartes has collected all the power of mathematics, greatly increased in the previous century, has reduced it to unity and uniformity, and has greatly increased further both the power and scope of mathematics.

Accordingly, the general way for solving all problems that have never been solved has since been the general way for solving all problems that have never been solved.

To solve a problem has been to reduce it to an equation.

The theory of equations has been developed, beginning with the *Geometry*.

Straight lines have been reduced to equations, and curves have been reduced to equations. Descartes has reduced the conics, his successors will reduce other geometrical curves, the conchoid, the cissoid, as well as not geometrical curves, the spiral, the quadratrix, the cycloid, and ever more.

The problem of the relation of numbers and lines has been solved.

Functions have taken a further step toward the third and richest in consequences stage in their development, the recognition of general laws of functions. The last step, the coinage of the technical term 'function', and introduction of letters for the indefinite indication of functions will follow with Newton and Leibniz.

To find the properties of curves it suffices to know the relation of all their points to those of straight lines and the way to find lines perpendicular to them. The general way to find tangents is thus not only the most useful and most general problem to know, it is the most useful and most general to desire to know in geometry.

The problem of the relation of curves and straight lines has been solved. The problem of rectification has been solved, the problem of tangents has been solved, and a major step has been taken toward the solution of the problem of areas under curves.

Hence, a major step has been taken as well toward the solution of the problems of distance, time, velocity, acceleration, force, and motion. Equations, beginning with the equations of curves, will solve these problems, too. And Descartes has taken a major step toward the solution with the attempt to define the quantity of matter and the quantity of motion and discover the equations of their conservation.

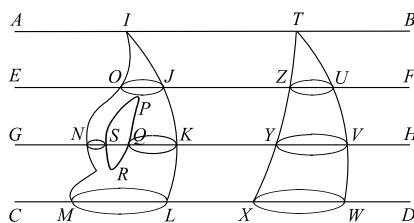
Geometry has been reduced to the calculus of arithmetic. Vastly generalized with the theory of equations, the calculus of arithmetic has become algebra and analysis.

Descartes has also formulated the rules of the method. He has formulated the theory of vortices. To describe the system of the world Galileo and Kepler still have described the solar system bounded by the fixed stars, Descartes has described the system of the systems of the fixed stars, one of which the solar system is. He has formulated the laws of nature.

How all these, the calculus of arithmetic, the rules of the method, the system of the world, and the laws of nature, depend on one another has become explicit, too. It will become still more explicit with Newton.

Cavalieri published the *Geometry of indivisibles of continua developed by a new method*, in 1635.

In book VII Cavalieri proves the theorem: if between the same parallels two plane figures are constructed, and every straight line in one of the figures at a given distance from the parallels is equal to the straight line in the other figure at the same distance from the parallels, the two plane figures are equal; and if between the same parallel planes two solids are constructed, and every plane figure in one of the solids at a given distance from the parallel planes is equal to the plane figure in the other solid at the same distance from the parallel planes, the two solids are equal;

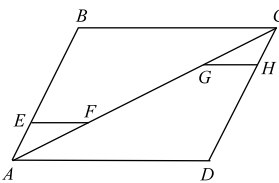


let between the parallels AB and CD the plane figures $IJKLMNO$, which has the hole $PQRS$, and $TUVWXYZ$ be constructed; let EF and GH be lines parallel to AB and CD ; if OJ is equal to ZU , the sum of NS and QK is equal to YV , and the same for all lines equidistant from AB , the figures $IJKLMNO$ and $TUVWXYZ$ are equal; and if the two are solids between parallel planes through AB and CD , the plane figures bounded at the distance at which the plane through EF is equal, the plane figures bounded at the distance at which GH is equal, and the same for all plane figures equidistant from the plane through AB , the two solids are equal (1).

Cavalieri has solved the problem discovered by Kepler, how sums of distances are contained in areas. He has solved also the problem, how sums of areas are contained in volumes.

In general, Cavalieri has discovered integration.

In book II he shows: in the parallelogram $ABCD$, divided by its diagonal into the equal triangles ABC and ADC , if EF and GH are lines equidistant from the bases BC and AD , they are equal; the same for all lines



equidistant from the bases; the parallelogram, therefore, has twice the area of either of the triangles (19).

Cavalieri has shown, barring the notation, that $\int_0^a x dx = \frac{a^2}{2}$. He comments: what in Euclid has to be proved separately for triangles, polygons, and circles, we have proved generally in one proposition; in the same way what has to be proved there separately for pyramids, cones, and spheres, we have comprehended in one proposition (16).

In the *Geometry* already, Cavalieri has shown also that $\int_0^a x^2 dx = \frac{a^3}{3}$ (24). In the *Six exercises in geometry*, in 1647, he shows also that $\int_0^a x^3 dx = \frac{a^4}{4}$, $\int_0^a x^4 dx = \frac{a^5}{5}$, and so on, tabulating the results all the way to $\int_0^a x^{100} dx = \frac{a^{101}}{101}$ (IV, 32).

Fermat published the *Method for the investigation of maxima and minima*, in 1638.

He writes: the entire theory of the investigation of maxima and minima presupposes two unknowns and the following rule;

let a be any unknown in the problem; the maximum is expressed in terms of a ; then $a + e$ is substituted for the original a and the maximum is expressed in terms of a and e ; the two expressions are, to speak like Diophantus, adequated; all terms containing e are suppressed; and the solution of the last equation gives the value of a which leads to the maximum or minimum;

here is an instance; $A \xrightarrow{\quad C \quad} B$
the straight line AB is
to be divided at C so that $AC \times CB$ is a maximum; let $AB = b$; let one of the segments be a ; the other is then $b - a$, and the product whose maximum is to be found is $ab - a^2$; let now the first segment be $a + e$; the other is $b - a - e$, and the product is $ab - a^2 + be - 2ae - e^2$; the two expressions are now adequated

$$ab - a^2 + be - 2ae - e^2 \sim ab - a^2,$$

the common terms are canceled

$$be \sim 2ae + e^2,$$

all terms are divided by e

$$b \sim 2a + e,$$

and when e is suppressed

$$b = 2a;$$

to solve the problem, therefore, one half of b must be taken (intr.).

Fermat has solved the problem of maxima and minima.

In general, Fermat has discovered differentiation.

He comments: it is impossible to give a more general method (*id.*).

And continues: we apply the method to the discovery of tangents to any curve;

let, for instance, the parabola with vertex A and diameter BA be given; let the point C be given, at which the tangent CE is to be drawn, intersecting the diameter at E ; on CE at point D the ordinate DF is taken; the ordinate CB is also taken; as point D is outside the parabola, $\frac{BA}{FA} > \frac{CB^2}{DF^2}$; by similar

triangles, $\frac{CB^2}{DF^2} = \frac{BE^2}{FE^2}$; hence, $\frac{BA}{FA} > \frac{BE^2}{FE^2}$;

let now $BA = d$, $BE = a$, and $BF = e$; hence,

$$\frac{d}{d-e} > \frac{a^2}{a^2 - 2ae + e^2},$$

or

$$a^2d - 2ade + de^2 > a^2d - a^2e,$$

canceling the common terms, and adequating

$$de^2 + a^2e \sim 2ade,$$

dividing by e

$$de + a^2 \sim 2ad,$$

and suppressing the term still containing e

$$a^2 = 2ad,$$

or

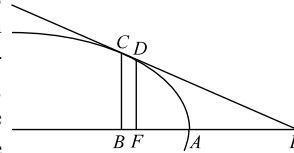
$$a = 2d;$$

we have thus proved that BE is twice BA , as is confirmed by truth (*id.*).

Fermat adds: the method never fails, and it can be extended to a number of very beautiful problems; we have thus found the centers of gravity of plane figures and solids bounded by either straight lines or curves (*id.*).

Kepler showed that: near a maximum decrements become imperceptible (*Stereometry*, II, 5).

Fermat has discovered that a function becomes stationary at a maximum or minimum, and divi-



sion of the difference of its values, as they approach 0, by its arguments gives the maximum or minimum for its values.

Or what is the same, the maxima and minima of a function are given by its derivatives. Cauchy, not quite two centuries later, will define the derivative: when the function $y=f(x)$ lies continuously between two given limits of the variable x , an infinitely small increment of the variable produces an infinitely small increment of the function itself; consequently, the two terms of the ratio of the differences

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

are infinitely small quantities; as they indefinitely and simultaneously approach the limit 0, the ratio itself can converge toward another limit; this limit, when it exists, has a determinate value for each particular value of x ; but it varies with x ; the form of this new function which serves as the limit of the ratio $\frac{f(x + \Delta x) - f(x)}{\Delta x}$ depends on the

form of the function $y=f(x)$; to indicate this dependence, the new function is given the name derived function, and is designated by y' , or $f'(x)$ (*Calculus*, 3).

Fermat, in *On the transformation and simplification of equations of location for the comparison of all forms of areas under curves, and on the use of geometrical progression for the quadrature of parabolas and hyperbolas to infinity*, in 1659, has shown also, barring the notation, as the title indicates, too, that, in general,

$$\int_0^a x^n dx = \frac{a^{n+1}}{n+1} \text{ (id.)}. \text{ With the solution of prob-}$$

lems of centers of gravity of plane figures and solids bounded by either straight lines or curves by the method of maxima and minima Fermat has taken the first step as well, as the problems and the method of their solution indicate as well, toward the demonstration that integration and differentiation are one another's converses.

Fermat also greatly advanced the theory of numbers.

The next is the generation of Wallis and Huygens.

Wallis published the *Arithmetic of infinities, or a new method of investigation of the quadrature of curves and other difficult mathematical*

problems, in 1656.

In it Wallis obtains the result:

$$\frac{\pi}{4} = \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \dots \text{ (191).}$$

And below are some of the other results which he obtains in order to obtain this result.

He writes: you will find this work entirely new (ded.); our method begins where Cavalieri's method of indivisibles ends; and this the work itself as well as its title show, for as his was the *Geometry of indivisibles* so I have called our method the *Arithmetic of infinities* (id.).

And begins: the simplest way of investigation is this, to show the thing, and observe the ratios and compare them, so that a general proposition may become known by induction; for instance,

$$\begin{array}{ll} \frac{0+1}{1+1} = \frac{1}{2}, & \frac{0+1+2}{2+2+2} = \frac{1}{2}, \\ \frac{0+1+2+3}{3+3+3+3} = \frac{1}{2}, & \frac{0+1+2+3+4}{4+4+4+4+4} = \frac{1}{2}, \end{array}$$

and the same however far we proceed (1); hence, the series of quantities in continually increasing arithmetical progression beginning with 0, and whether the series is finite or infinite, is to the series of the same number of terms equal to the greatest as $\frac{1}{2}$ (2).

As the quantities so their squares:

$$\begin{array}{ll} \frac{0+1}{1+1} = \frac{1}{3} + \frac{1}{6}, & \frac{0+1+4}{4+4+4} = \frac{1}{3} + \frac{1}{12}, \\ \frac{0+1+4+9}{9+9+9+9} = \frac{1}{3} + \frac{1}{18}, & \frac{0+1+4+9+16}{16+16+16+16+16} = \frac{1}{3} + \frac{1}{24}, \end{array}$$

and so on; as the number of terms increases the excess over $\frac{1}{3}$ continually decreases (19); if the

series proceeds to infinity, the excess must vanish completely (20); hence, the infinite series of the continually increasing squares beginning with 0 is to the series of the same number of terms equal to the greatest as $\frac{1}{3}$ (21);

As the squares so the cubes: the investigation continues by the method of induction;

$$\begin{array}{ll} \frac{0+1}{1+1} = \frac{1}{4} + \frac{1}{4}, & \frac{0+1+8}{8+8+8} = \frac{1}{4} + \frac{1}{8}, \\ \frac{0+1+8+27}{27+27+27+27} = \frac{1}{4} + \frac{1}{12}, & \end{array}$$

$$\frac{0+1+8+27+64}{64+64+64+64+64} = \frac{1}{4} + \frac{1}{16},$$

and so on; the excess over $\frac{1}{4}$ continually decreases (39); it must at length become less than any given quantity, and if the series proceeds to infinity, must vanish completely (40); hence, the infinite series of the continually increasing cubes beginning with 0 is to the series of the same number of terms equal to the greatest as $\frac{1}{4}$ (41).

Wallis has shown, barring the notation, that $\int_0^1 x \, dx = \frac{1}{2}$, $\int_0^1 x^2 \, dx = \frac{1}{3}$, $\int_0^1 x^3 \, dx = \frac{1}{4}$.

The generalization follows: by the same method can be found the ratio of the infinite series of the continually increasing fourth powers, fifth powers, sixth powers, and so on, to the series of the same number of terms equal to the greatest; the ratio is $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, and so on (43).

Barring the notation, $\int_0^1 x^n \, dx = \frac{1}{n+1}$.

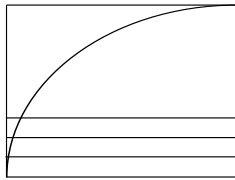
These results have been obtained earlier. Wallis has obtained them, the arithmetic of infinities being arithmetic, with numbers, not lines, areas, and volumes. The following results Wallis has obtained first. By the same: simplest way of investigation, showing the thing, and observing the ratios and comparing them, so that a general proposition may become known by induction.

He generalizes further: the ratio also can be found of the infinite series of the continually increasing square roots, cube roots, fourth roots, and so on, to the series of the same number of terms equal to the greatest; the ratio is $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$,

and so on (54);

for instance, the parabola is to the circumscribed parallelogram as $\frac{2}{3}$; the

parabola is an infinite series of square roots, the parallelogram is an infinite series of the same number of terms equal to the greatest; and they are as $\frac{2}{3}$, or $\frac{1}{\frac{1}{2}+1}$ (55).



Wallis writes: of the curves which Descartes calls geometrical formulae have been discovered (194). The formula of the parabola is $y = \sqrt{x}$. And

Wallis has shown that $\int_0^1 \sqrt{x} \, dx = \frac{1}{\frac{1}{2}+1}$.

Hence, $\sqrt{x} = x^{\frac{1}{2}}$, $\sqrt[3]{x} = x^{\frac{1}{3}}$, $\sqrt[4]{x} = x^{\frac{1}{4}}$, and so on.

Wallis has shown that n in $\int_0^1 x^n \, dx = \frac{1}{n+1}$ can be a fraction.

It can also be irrational (64).

It can also be negative (87).

Wallis generalizes further: $\int_0^1 (1-x) \, dx = \frac{1}{2}$, for

$$1 - \int_0^1 x \, dx = 1 - \frac{1}{2}, \quad \text{and} \quad \int_0^1 (1+x) \, dx = \frac{3}{2}, \quad \text{for}$$

$$1 + \int_0^1 x \, dx = 1 + \frac{1}{2} \quad (108).$$

Further: $\int_0^1 (1 \pm x^n) \, dx = 1 \pm \frac{1}{n+1}$ (111).

Further: $\int_0^1 (1-x)^n \, dx = \frac{1}{n+1} = \frac{n!}{(n+1)!}$ (117).

And further: $\int_0^1 (1-x^2)^n \, dx = \frac{(2n)!}{(2n+1)!}$ (118).

Wallis has obtained all these results by the simplest way. He has shown the thing, and observed the ratios and compared them. So also to obtain the last result, he has shown the binomial coefficients, and added and subtracted them:

$$1 - \frac{1}{3} = \frac{2}{3}, \quad 1 - \frac{2}{3} + \frac{1}{5} = \frac{2 \times 4}{3 \times 5},$$

$$1 - \frac{3}{3} + \frac{3}{5} - \frac{1}{7} = \frac{2 \times 4 \times 6}{3 \times 5 \times 7},$$

and so on (*id.*).

Finally: the quadrant of the circle is to the circumscribed square as the circle is to the square on the diameter (121). As $\frac{\pi}{4}$. Also: the quadrant

of the circle is an infinite series of square roots of differences, the circumscribed square is an infinite series of the same number of terms equal to the greatest (*id.*). The formula is $y = (1-x^2)^{\frac{1}{2}}$.

Wallis affirms: $\int_0^1 (1-x^2)^{\frac{1}{2}} \, dx = \frac{\pi}{4}$ (*id.*). And the

final third of the *Arithmetic of infinities* is devoted to applying the simplest way to the recalculation of this integral.

In 1685 Wallis published the *Treatise of*

algebra, both historical and practical, showing its origin, progress, and advancement, from time to time, and by what steps it has attained to the height at which it now is. The main steps in the origin, progress, and advancement of what had been geometry, then arithmetic, was now algebra, and was to become calculus, to the height at which it was, Wallis traces from: the method of exhaustion (73), through: Cavalieri's method of indivisibles (74), and: the arithmetic of infinities (75), to: the doctrine of infinite series further prosecuted by Mr Newton (91).

Huygens published *The pendulum clock*, in 1673.

With the labors of Galileo and Kepler the description of the world, the generalization of calculation, and the formulation of the principles posed the problems which the development of calculus solved.

The description of the world, the generalization of calculation, and the formulation of the principles posed the problems of motion. Motion posed the problems of distance, time, speed, and acceleration. These posed the problems of force. These all posed the problems of infinite divisibility. They posed the problems also of maxima and minima. They all posed the problems of the relations of lines, numbers, and infinities. They all posed the problems also of the relations of straight lines and curves, of rectification, tangents, and areas. They all posed also the problems of the building of functions. Functions also of great complexity and generality.

That the problems were essentially and substantially dependent on one another, the fact that their solutions were essentially and substantially dependent on one another suffices to show. Kepler discovered that sums of distances are contained in areas. And sums of areas are contained in volumes. Infinities are sums. Tangents are maxima and minima, obtained by infinite division. Accelerations and forces are tangents, areas, and infinite sums. Motion is forces.

Physics became dynamics because mathematics became the building of functions, and mathematics became the building of functions because physics became dynamics.

The description of the world, the generalization of calculation, and the formulation of the

principles are dependent on one another essentially and substantially, all three, explicitly in Galileo, more explicitly in Kepler, and still more explicitly in Descartes. They are not in Cavalieri, in Fermat, and in Wallis. They are once again essentially and substantially dependent on one another, all three, and still more explicitly in Huygens.

He determined the true proportions of the solar system, solved the problem of collisions, discovered the conservation of momentum, and formulated the wave theory of light.

And he wrote *The pendulum clock, or geometrical demonstrations concerning the motion of pendula applied to clocks*.

This is how he describes it in its preface: it expands the theory of falling bodies, developed by the great Galileo, whose richest fruit and highest peak is the isochronism of the cycloid which we have discovered; from this a new consideration of curves follows, namely, of curves which by their evolution generate other curves; from this arises a new theory of the comparison of straight lines and curves; this leads to the consideration of centers of oscillation, a subject treated by many but with little success until now.

To this brief description the following has to be added. In part I Huygens shows how to regulate a clock by the fixed stars. In part II he begins by formulating the three hypotheses which are the decisive step in the formulation of the principles from the three laws of nature which Descartes has formulated to the three laws of motion which Newton will formulate. The theory of evolutes in part III is a vast generalization of the theory of curves, rectification, and tangents. Part IV generalizes the static problem of centers of gravity into the dynamic problem of centers of oscillation. Huygens determines the constant of proportionality in the law of the pendulum. He determines the acceleration of gravity. His investigation of the cycloid has made explicit the dependence on one another of a curve, bodies in motion, distance, and time, all four. And his investigation of centrifugal force in part V is the decisive step toward the theory of gravity. Newton comments on the early propositions of *The principles*: it is by propositions of this kind that Huygens in his

excellent treatise on the pendulum clock compared the force of gravity to the centrifugal forces of revolving bodies (I, 4).

The increase of the power of mathematics cannot not be a cause of the increase of its scope. Conversely, the increase of the scope of mathematics cannot not be a measure of the increase of its power. In the three generations from Galileo and Kepler through Newton and Leibniz the power of mathematics, and hence its scope, increased more, very much more, than in any other three generations. And in the generation from Descartes through Huygens they increased more than in the previous or the next. Before these three generations as well as after, the increase of the power of mathematics, and the more so the increase of its scope, have tended to be achieved at the expense of its unity and uniformity, and vice versa, its unity and uniformity to be achieved at the expense of its power and scope. It is thus all by itself a measure of the magnitude of the advance of mathematics in these three generations that with the increase of its power and the increase of its scope, very great as these both were, mathematics succeeded in maintaining its unity and uniformity, too. Substantially with Descartes's *Geometry*, yet again, and more substantially, with *The pendulum clock*, and yet again, and yet more substantially, with Newton's *Principles*.

clxvii

The next is the generation of Newton and Leibniz.

Before turning to Newton, though, two matters that have dominated the history of the development of calculus can and should, respectively, be addressed.

To a degree proportionate to the importance of the development of calculus, its history has been mired in the distribution of credit and disgrace among the individualities which emancipated calculus from the individuality of the picturing mind.

Here, then, taking it for granted that the development begins with Galileo and Kepler and is complete with Newton and Leibniz, is the complete enumeration of the important philosophers who emancipated calculus from the individuality

of the picturing mind.

1564 - 1642 Galileo,
1571 - 1630 Kepler,
1596 - 1650 Descartes,
1598 - 1647 Cavalieri,
1601 - 1665 Fermat,
1602 - 1675 Roberval,
1608 - 1647 Torricelli,
1616 - 1703 Wallis,
1623 - 1662 Pascal,
1629 - 1695 Huygens,
1630 - 1677 Barrow,
1638 - 1675 James Gregory,
1642 - 1727 Newton,
1646 - 1716 Leibniz.

That the enumeration is complete the following enumeration of the more important among the not important can show.

1584 - 1667 Saint-Vincent,
1615 - 1660 van Schooten,
1620 - 1687 Nicolas Mercator,
1622 - 1685 de Sluse,
1628 - 1704 Hudde,
1632 - 1723 Wren.

The other matter is important.

The history of the development of calculus has been mired also in lamentation for the loss of rigor it brought upon mathematics.

Kepler committed that misdeed. Galileo committed the misdeed of claiming the credit due to others. Cavalieri committed both that misdeed and the misdeed of claiming the credit due to others. Descartes committed both those misdeeds and all kinds of others besides. And so on.

On the matter of rigor, then, the following can be said.

Rigor is chains of inferences free of gaps.

Rigor thus admits of degree.

In 1884 Frege writes: having for some time distanced itself from Euclidean rigor, mathematics is now returning to it and even striving beyond it.

Thus, if Euclid is rigorous, Aristotle is not so rigorous. But Aristotle is much more rigorous than Plato. And Plato is very much more rigorous than Homer.

By 1884 mathematics was striving beyond Euclidean rigor, and calculus had been rigorously grounded, too. The striving of mathematics

beyond Euclidean rigor begins with Gauss, two-odd generations before Frege. In the rigorous grounding of calculus the most important are the labors of Cauchy, two generations before Frege, and Weierstrass, in the generation before Frege.

If Gauss, Cauchy, Weierstrass, and Frege are rigorous, the contemporary development of thermodynamics and electrodynamics is not so rigorous. But Clausius and Maxwell are very much more rigorous than their older contemporary Marx. Who is much more rigorous than his younger contemporary Dilthey.

And at the time of the development of calculus, if at one end of the continuum of rigor was still Euclid, at the other end were the holy scriptures. That it was unquestionably one and the same continuum, the eagerness with which the holy scriptures examined the new development sufficiently attests. And in this continuum the least rigorous labors of Kepler, Cavalieri, and Wallis are infinitely closer to Euclid than the holy scriptures are. Moreover, not only are Fermat and Huygens closer still, and Newton and Leibniz closer still. The least rigorous labors of Kepler, Cavalieri, and Wallis are infinitely more rigorous than the labors of Berkeley. Or Hegel. Never mind Dilthey. And that the continuum is still one and the same, the eagerness with which Berkeley and Hegel examine the development of calculus sufficiently attests.

Euclidean rigor was preceded, counting from Pythagoras, by two-odd centuries of great advance of mathematics. Then rigor set in. The advance slowed down, then stopped. The great advances of Archimedes and Apollonius were within Euclidean rigor. But the greatest advance, Ptolemy's, was at the expense of Euclidean rigor.

The analogy with the recent development of mathematics holds. The century before Newton and Leibniz was a century of great advance of mathematics. The century after them was a century of great advance as well, it was the century of Jakob Bernoulli, Saccheri, Johann Bernoulli, Taylor, Daniel Bernoulli, Euler, d'Alembert, Lambert, Lagrange, Laplace. Then in the century after Gauss rigor set in. It was a century of great advance of mathematics, the century of Bessel, Cauchy, Lobachevsky, Hamilton, Galois, Weierstrass, Riemann, Cantor, Frege. But the century

after that century was not a century of great advance of mathematics.

Wallis explains: my design was not so much to show a method for demonstrating things already known as to show a way of investigation or finding out of things yet unknown (*Algebra*, 79); and I look upon induction as a very good method of investigation which very often leads to the discovery of a general rule (*id.*).

Rigor is very important. As the increase of the power of mathematics and its scope have tended to be achieved at the expense of its unity and uniformity, however, and its unity and uniformity at the expense of its power and scope, so the increase of its rigor has tended to be achieved at the expense of all three, its power, its scope, and its unity and uniformity. And all three, power, scope, and unity and uniformity, are more important than rigor. Because the increase of its power and the increase of its unity and uniformity cannot not be preconditions of the increase of its rigor. As the antecedents of both Euclidean rigor and Gaussian rigor show. And because the increase of its scope cannot not be the unconditional measure of its advance. As the same antecedents show.

The analogy between the earlier development of mathematics and its recent development extends further.

Plutarch reports: Plato disapproved of how Archytas doubled the cube; solving problems by mechanical means, Plato said, will corrupt geometry and distract it from rising to the immaterial and immortal (*Conversations*, VIII, 2).

Hobbes writes of Wallis: having examined your pannier of mathematics, I find in it no knowledge, neither of quantity, nor of measure, nor of proportion, nor of time, nor of motion, nor of anything, only of certain characters, as if a hen had been scraping there (*Lessons*, 5). In general: symbols ought no more to appear in public than the most deformed business which you do in your chambers (3).

And if Plato and Hobbes were concerned to protect one kind of mathematics against another, Berkeley and Hegel were not.

Berkeley writes: the method of fluxions is the general key, with whose help the modern mathematicians unlock the secrets of nature. And: what

are these fluxions? the ghosts of departed quantities? In opposition: I shall esteem my labors altogether useless, if, having shown the falseness or vanity of those barren speculations, I cannot dispose my readers to embrace the salutary truths of the gospel. It follows: the true nature of things is not regarded in mathematics. And: you, gentlemen, have no right to decide. And: there is a certain transcendental science superior to mathematics, which it behooves our modern analysts to learn rather than despise.

Calculus is the general key. The key is deficient because it lacks rigor. In opposition, there are the salutary truths of the gospel. With respect to these the question of rigor does not arise. The generalization follows. The true nature of things is at issue. And you, gentlemen, have no right to decide. A certain transcendental science, which it behooves you to learn, decides.

Hegel writes: the increase of x by dx is the fundamental evil. And: I consider it nothing but a sleight of hand and charlatanism. In opposition: reason is spirit in that the certainty that it is all reality is raised to truth, and it is conscious of itself as its own world, and of the world as itself. It follows: the knowledge of which mathematics boasts rests solely on the poverty of its goal and the deficiency of its material, it is therefore of a kind which philosophy must revile. And: stripping mathematics of its finery shows the necessity of a different kind of knowledge. And: the absolute idea alone is being, imperishable life, truth knowing itself, and all truth; everything else is error, confusion, opinion, striving, faltering, and perishing.

Lack of rigor exposes calculus as deficient. Which exposes mathematics as excluded from the secrets of nature, and positively from truth.

Berkeley and Hegel are not mathematicians. Mathematicians, however, have not only agreed. Mathematicians, being mathematicians apparently, have taken that chain of inferences, if, indeed, it is a chain of inferences, to the limit.

Not quite one and a half centuries after Hegel, Bourbaki published the *Elements of the history of mathematics*, in 1960.

Bourbaki writes: it is of the gradual and inevitable unfolding of a symphony, where the “Zeitgeist”, simultaneously composer and conductor,

holds the baton, that the development of the infinitesimal calculus in the seventeenth century makes one think; each executes his part with his proper timbre, but no one masters the themes he makes heard, themes which learned counterpoint has almost inextricably intertwined; it is therefore under the form of a thematic analysis that its history must be written (17).

The “Zeitgeist”, the composer and conductor holding the baton, can hardly have been more impeccably Hegelian.

The principal themes the thematic analysis unfolds are: a) the theme of mathematical rigor contrasting with that of the infinitely small, indivisibles, or differentials; b) kinematic; c) algebraic geometry; d) classification of problems; e) interpolation and calculus of differences; f) algebraization; g) function (*id.*).

The true nature of things is not among the themes. Nor can it be under the baton of a “Geist”. To the degree to which it is not a symphony, calculus can only be an instrument.

Bourbaki writes: nearly three centuries of constant use have not yet completely blunted this incomparable instrument (*id.*).

Incomparable. Three centuries of constant use. The constant use has brought about – never mind mathematics – thermodynamics, electrodynamics, the kinetic theory, relativity, quantum theory, the big bang theory. It is yet but an instrument.

Nor can it be but an instrument.

Bourbaki writes: the mathematicians have always been persuaded that they demonstrate “truths” or “true propositions”; such a conviction can obviously be only of a sentimental or metaphysical order, and it cannot be justified on the ground of mathematics, not even given a sense that would not be a tautology (1).

Berkeley and Hegel had to wage a war to wrest truth from mathematics. But the war is over. And it may not be very coherent that it is obvious that the persuasion of the mathematicians that they always demonstrate “truths” can be only of a sentimental or metaphysical order. But it is obvious. It is no doubt obvious as well that there is no such thing as truth, all there may be is “truths”. Then again, Bourbaki is not allotting “truths” to metaphysics alone. He is allotting them to metaphysics and sentimentality. May it

be, the mathematicians being sentimental, and Bourbaki being a mathematician, that he is trying to reclaim some "truths", if not for mathematics, for "mathematics"?

clxviii

Newton begins where Descartes, Fermat, and Wallis end.

He wrote the *Enumeration of lines of the third order*, in 1668, *On analysis by equations of infinite number of terms*, in 1669, *A treatise on the method of fluxions and infinite series, with its application to the geometry of curves*, in 1671, *A treatise on the quadrature of curves*, in 1676, and published *The principles*, in 1687.

In the *Enumeration of lines of the third order*, in 1668, he writes: geometrical lines are best divided into orders according to the power of the equation expressing the relation of abscissa and ordinate, or what the same, according to the number of points in which the line can be cut by a straight line; thus, a line of the first order is a straight line; lines of the second order are the conic sections and the circle; lines of the third order are the cubic parabola, the cissoid, and others; a curve of the first genus, then, as straight lines are not to be reckoned curves, is the same as a line of the second order, a curve of the second genus is the same as a line of the third order; and a line of the infinite order is one which a straight line can cut in an infinite number of points, such as the spiral, cycloid, quadratrix, and every line generated by the infinitely continued rotation of a radius (1).

Newton then shows that, if one of the axes is parallel to an asymptote, the general cubic equation can be reduced to 4 forms, and these, according to the kinds of their roots, positive, negative, real, and imaginary, yield: 72 species of lines of the third order (4).

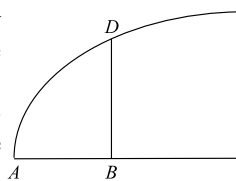
With his first treatise Newton has generalized the theory of curves.

And with his second, *On analysis by equations of infinite number of terms*, in 1669, he has discovered calculus.

He begins: the general method which I have devised for measuring the quantity of curves by means of series of infinite number of terms is rather shortly explained than accurately demon-

strated in what follows (1);

let the base AB of any curve AD have the ordinate BD ; let $AB=x$ and $BD=y$, a, b, c , etc., be given quantities, and m and n be whole numbers; then



rule 1. if $ax^{\frac{m}{n}}=y$, then $\frac{an}{m+n}x^{\frac{m+n}{n}} = \text{area } ABD$ (2);

rule 2. if the value of y is made up of several such terms, the area is made up of the sum of the areas of the terms (4);

rule 3. if the value of y , or any of its terms, is more compounded, it must be reduced in the way arithmeticians divide, extract square roots, or resolve affected equations; afterward you will find the area by the preceding rules (11).

Newton writes: the thing will be evident by example (3), and the first example of rule 1 is:

if $x^2=y$, then $\frac{1}{3}x^3 = \text{area } ABD$ (*id.*).

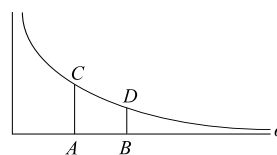
The first example of rule 2 is:

if $x^2+x^{\frac{3}{2}}=y$, then $\frac{1}{3}x^3+\frac{2}{5}x^{\frac{5}{2}} = \text{area } ABD$ (5).

And the first examples of rule 3 are:

let $\frac{a^2}{b+x}=y$, and

that is, let the curve be a hyperbola; to free the equation from the denomina-



tor I divide, a new equation arises,

$$y = \frac{a^2}{b} - \frac{a^2x}{b^2} + \frac{a^2x^2}{b^3} - \frac{a^2x^3}{b^4} + \dots,$$

and by rule 2,

$$\text{area } ABDC = \frac{a^2x}{b} - \frac{a^2x^2}{2b^2} + \frac{a^2x^3}{3b^3} - \frac{a^2x^4}{4b^4} + \dots,$$

an infinite series also, yet such that a few of the initial terms are exact enough for any use (12);

in the same way, if $\frac{1}{1+x^2}=y$, by division

arises

$$y = 1 - x^2 + x^4 - x^6 + x^8 - \dots,$$

and by rule 2,

$$\text{area } ABDC = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots;$$

or if $\frac{1}{x^2+1}=y$, there arises

$$y = x^{-2} - x^{-4} + x^{-6} - x^{-8} + \dots,$$

and by rule 2,

$$\text{area } BD\alpha = -x^{-1} + \frac{1}{3}x^{-3} - \frac{1}{5}x^{-5} + \frac{1}{7}x^{-7} - \dots;$$

you must proceed in the first way when x is small enough, but in the second way when it is supposed great enough (13).

After the examples, Newton shows: application of what has been said to other problems of that kind (37). Among these, he finds the length of an arc of the circle:

$$\text{arc} = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \dots \quad (39).$$

He notes: from these things one can guess how one ought to proceed in investigating the areas and volumes of solids, and likewise the centers of gravity (41).

And continues: to find the converse of these things (42). He writes: if, on the contrary, from the area, or length, etc., of any given curve the length of the base AB is required, then you must extract the root x out of the equations found by the preceding rules (*id.*);

thus, if from the area $ABDC$ of the hyperbola $\frac{1}{1+x} = y$ I want to investigate the base AB , I let $ABDC = z$, and extract the root out of this equation

$$z = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots \quad (43).$$

The series, Euler will show in *The analysis of infinities*, for $\ln(1+x)$ (I, 7).

Newton continues:

or if from the arc of the circle the sine is required, out of its equation

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (45);$$

if the cosine is required, out of the same equation

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (46).$$

The two series with which in *The analysis of infinities* Euler introduces trigonometric functions (I, 8).

Newton concludes: neither do I know anything of this kind to which this method does not extend, and in various ways; even tangents to mechanical curves can be drawn by it; and whatever the common analysis performs by means of equations of a finite number of terms it performs by means

of infinite equations; so I have not made any question of giving it the name of analysis; for the reasonings in it are not less certain, nor the equations less exact, even if we mortals, whose reasoning powers are confined within narrow limits, can neither express nor so conceive all the terms of these equations as to know exactly from them the quantities we want, just as the surd roots of finite equations can neither be so expressed by numbers, or any analytic contrivance, that the quantity of any of them can be so distinguished from all the rest as to be understood exactly (52).

And finally: there are two things especially which an attentive reader will see need to be demonstrated;

1. the quadrature of simple curves belonging to rule 1 (53);
2. the resolution of affected equations (56).

On the second demonstration the soundness of the solution of equations by infinite series rests. On the first the soundness of the method in general, of calculus, rests. Newton has a more rigorous demonstration in *The method*, more rigorous in *The principles*, and his successors will have more rigorous. In the *Analysis* already, however, the demonstration is, with all due respect to the reasoning powers of us mortals, as rigorous as this:

let $AD\delta$ be any curve with base $AB = x$, ordinate $BD = y$, and area $ABD = z$, as at the beginning;

let $B\beta = o$, $BK = v$, and rectangle $B\beta HK$, equal to ov , be equal to space $B\beta\delta D$;

hence, $A\beta = x + o$, and area $A\delta\beta = z + ov$;

assume any relation between x and z that you please, and seek for y in this way;

take at pleasure $\frac{2}{3}x^{\frac{3}{2}} = z$, or $\frac{4}{9}x^3 = z^2$;

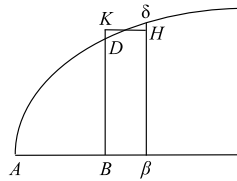
substituting $x + o$ for x , and $z + ov$ for z , there arises

$$\frac{4}{9}(x^3 + 3x^2o + 3xo^2 + o^3) = z^2 + 2zov + o^2v^2;$$

taking away equal terms, and dividing by o , there remains

$$\frac{4}{9}(3x^2 + 3xo + o^2) = 2zv + ov^2;$$

if we now suppose $B\beta$ to be diminished infinitely



and vanish, or o to be nothing, v and y will be equal, and the terms multiplied by o will vanish;

so there will remain

$$\frac{2}{3}x^2 = zv,$$

or

$$\frac{2}{3}x^2 = \frac{2}{3}x^{\frac{3}{2}}y,$$

or

$$x^{\frac{1}{2}} = y;$$

wherefore, conversely, if $x^{\frac{1}{2}} = y$, then $\frac{2}{3}x^{\frac{3}{2}} = z$

(54);

and, generally, if $\frac{n}{m+n}ax^{\frac{m+n}{n}} = z$, or, letting

$$\frac{na}{m+n} = c, \text{ and } m+n=p, \text{ } cx^{\frac{p}{n}} = z, \text{ or } c^n x^p = z^n,$$

substituting $x+o$ for x , and $z+ov$ for z , there arises

$$c^n(x^p + p ox^{p-1} + \dots) = z^n + n ov z^{n-1} + \dots,$$

neglecting terms which will at length vanish;

taking away equal terms, and dividing by o , there remains

$$c^n p x^{p-1} = n y z^{n-1},$$

or

$$p c x^{\frac{p-n}{n}} = n y,$$

or, restoring m for $p-n$ and na for pc ,

$$ax^{\frac{m}{n}} = y;$$

wherefore, conversely, if $ax^{\frac{m}{n}} = y$, then

$$\frac{n}{m+n} ax^{\frac{m+n}{n}} = z; \text{ Q.E.D. (55).}$$

Newton has discovered calculus. He has discovered the whole of which his predecessors have discovered parts.

He has shown differentiation, he has shown integration, and he has shown that the two are one another's converses.

He has shown its generality. Neither do I know anything of this kind to which this method does not extend, and in various ways. Neither will his successors know anything of this and ever more kinds to which it does not extend, and in ever more ways.

Whatever the common analysis performs, it performs. It performs much more, too. It per-

forms lines, areas, and volumes. It performs the operations of arithmetic. It performs equations. It performs rectification, tangents, and areas under curves. It performs maxima and minima. It performs centers of gravity. It performs infinite series. It will perform the solution of the problems of space and time, and force, and motion. And it will perform ever more.

The reasonings are certain, the equations are exact. And I have not made any question of giving it the name of analysis.

Most generally, as Newton himself will show, it will perform all three. The generalization of calculation. The formulation of the principles. And the description of the world. The first in *The method*, the second and the third in *The principles*.

In *The method of fluxions and infinite series, with its application to the geometry of curves*, in 1671, Newton begins: in the following treatise I have endeavored to enlarge the boundaries of analytics, and to make improvements in the doctrine of curves (*intr.*).

In particular, he begins with: an introduction, or the solution of equations by infinite series (*id.*), and begins the solution by affirming: the great conformity between the several operations of the same kind in species and in common numbers is obvious to everyone; indeed, there seems to be no difference between them except in the marks or characters they use (*id.*).

In concluding the solution, he writes: this much for the method of computation I use; it remains, in order to illustrate the analytic art, that I shall give some specimens of problems (*id.*).

Twelve specimens follow. Problems 3-12 are: to determine the maxima and minima of quantities (3); to draw tangents to curves (4); to find the quantity of curvature in any curve (5); to find the quality of curvature in any curve (6); to find any number of curves that can be squared (7); to find any number of curves whose areas can be compared with the conic sections (8); to find the quadrature of any curve (9); to find any number of curves that can be rectified (10); to find any number of curves whose lines can be compared with any curve (11); to determine the length of curves (12).

Problems 1 and 2 Newton formulates thus in

the contents:

1. from given fluents to find their fluxions;
2. from given fluxions to find their fluents.

Thus in their heads:

1. given the relation of fluents to one another, to determine the relation of their velocities;
2. given an equation including fluxions of quantities, to find the relation of the quantities to one another.

And this is how he introduces them: I shall observe that all difficulties can be reduced to these two problems only, concerning a space described by motion and how it is accelerated or retarded;

1. given the length of space described continuously, and that is, at all times, to find the velocity of motion at any time;
 2. given the velocity of motion continuously, to find the length of space described at any time;
- thus, in the equation $x^2 = y$, if y represents the length of space at any given time, and another space x , by increasing with the uniform velocity $\dot{x}$, measures the time, then $2\dot{x}x$ will represent the velocity by which the space y is measured at the same moment of time; and conversely;

hence it is that I consider things as generated by continuous increase, in the manner of a space which a thing or point in motion describes;

but since I do not consider time here any further than as measured by uniform motion, and only things of the same kind can be compared, as well as their velocities of increase and decrease, I have no regard to time formally but to suppose some of the quantities, being of the same kind, increased by uniform fluxion, which may therefore not improperly, by analogy, receive the name of time;

these quantities, then, which I consider gradually and indefinitely increasing, I call fluents, or flowing quantities, and represent by the final letters of the alphabet, v, x, y, z , to distinguish them from other quantities, which can be considered known and determinate, and are represented by the initial letters, a, b, c , etc., and the velocities by which every fluent is increased by generating motion I call fluxions, or simply velocities, and represent by the same letters pointed, $\dot{v}, \dot{x}, \dot{y}, \dot{z}$ (intr.).

Newton then solves the problems, in *The*

method as in the *Analysis*, by presenting examples of increasing difficulty, then demonstrating the solution. The demonstration of the solution of problem 1 is:

the moments of fluents, and that is, their indefinitely small parts by whose accession in indefinitely small portions of time they are continuously increased, are as the velocities of their increase;

hence, if the moment of any one, x , is represented by the product of its velocity $\dot{x}$ and an indefinitely small quantity o , the moments of the other will be represented by $\dot{v}o, \dot{y}o, \dot{z}o$, for $\dot{v}o, \dot{x}o, \dot{y}o$, and $\dot{z}o$ are to one another as $\dot{v}, \dot{x}, \dot{y}$, and $\dot{z}$;

it follows that after any indefinitely small interval of time the quantities x and y become $x + \dot{x}o$ and $y + \dot{y}o$, and therefore the equation which at all times expresses the relation of the fluents will express as well the relation of $x + \dot{x}o$ and $y + \dot{y}o$, and $x + \dot{x}o$ and $y + \dot{y}o$ can be substituted in the same equation for x and y (1).

Thus, if $y = x^n$, then $y + \dot{y}o = (x + \dot{x}o)^n$, and expanding, subtracting $y = x^n$, dividing by o , and neglecting terms which at length vanish, yields the relation of the moments

$$\dot{y} = nx^{n-1} \dot{x},$$

and thus the ratio of $\dot{y}$ and $\dot{x}$. In the notation which Leibniz will introduce

$$\frac{dy}{dt} = nx^{n-1} \frac{dx}{dt},$$

and thus the ratio of $\frac{dy}{dt}$ and $\frac{dx}{dt}$, and thus $\frac{dy}{dx}$.

Newton concludes the demonstration:

it can be observed that terms which are not multiplied by o will always vanish, as will terms multiplied by more than one power of o , and, divided by o , the rest of the terms will always acquire the form that they ought to have by the rule; Q.E.D. (1).

Finally, in preparation for the demonstration of problem 2 he writes:

that the fluents may be more easily distinguished from one another, the fluxion in the numerator may be called the related quantity, the one in the denominator the correlated; and for better understanding you may conceive that the correlated quantity is time, or any other quantity that flows uniformly by which time is measured,

that the related quantity is space, which the moving thing or point, however accelerated or retarded, describes in that time, and that the problem is from the given velocity of the motion at every instant of time to determine the space described in the whole time;

and with respect to this problem equations may be distinguished into three orders,

1. those in which two fluxions and only one of their fluents are involved,
2. those in which the two fluents are involved together with their fluxions,
3. those in which the fluxions of more than two quantities are involved (2).

Newton then presents examples of each of the three orders, then demonstrates the solution.

Having proposed to enlarge the boundaries of analysis, then, Newton has enlarged them beyond all bounds.

He has shown differentiation, integration, and that the two are one another's converses.

He has shown that all difficulties can be reduced to these two problems only, from given fluents to find their fluxions, and conversely.

He has shown that differentiation is a general method of finding a rate of change.

He has shown that integration is a general method of finding a sum by reversing the process of finding a rate of change. The fundamental theorem of calculus.

He has shown that the great conformity between operations in species and in numbers, such that, indeed, there is no difference between them except in the marks or characters they use, extends to differentiation and integration.

He has discovered differential equations.

Functions are firmly in the third stage of their development, the recognition of general laws of functions, the richest in consequences stage.

I consider things as generated by continuous increase.

As things are. Things are not static. Or immutable. Things are in motion. Generated by continuous increase. Which explains not only why physics has become dynamics. It explains why analysis has reached all things generated by continuous increase. And that is, all things.

Things are generated by continuous increase. Quantities thus cannot be just quantities.

Quantities considered gradually and indefinitely increasing are fluents, or flowing quantities. Or variables. To distinguish them from quantities which are considered known and determinate. Or constants. And the velocities by which every fluent is increased by generating motion are fluxions, or simply velocities. Or derivatives.

And what are these fluxions? Berkeley will ask, the fundamental evil, Hegel will answer.

Fluxions are the velocities by which every quantity considered gradually and indefinitely increasing is increased. They are the velocities by which every fluent is increased by generating motion. They are the indefinitely small parts of fluents by whose accession in indefinitely small portions of time the fluents are continuously increased.

The offense has begun with Cavalieri's indivisibles. As Newton writes in *The principles*: the hypothesis of indivisibles is harsh (I, 1.11).

By grounding the indivisibles, the indefinitely small parts, in motion, in gradual and indefinite increase, in increase by generating motion, in accession in indefinitely small portions of time, Newton has taken a major step toward removing the harshness.

Five years later, in 1676, in *A treatise on the quadrature of curves*, he has taken a further step. He writes: I consider mathematical quantities not as consisting of very small parts, but as described by continuous motion; lines are described and thereby generated not by the apposition of points, but by the continuous motion of points, surfaces by the motion of lines, solids by the motion of surfaces, angles by the rotation of the sides, portions of time by a continuous flux, and so on other quantities; this genesis takes place in the nature of things, and is seen daily in the motion of bodies; and in this way the ancients, too, by drawing movable straight lines along immovable straight lines, taught the genesis of rectangles (1).

And eleven years later, in 1687, in *The principles*, he has removed the harshness. He has discovered what fluxions are.

He writes: because the hypothesis of indivisibles is harsh and the method of indivisibles considered less geometrical, the proofs in what follows depend on the ultimate sums and ratios of

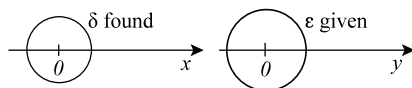
vanishing quantities and the first sums and ratios of nascent quantities, and that is, on the limits of such sums and ratios (I, 1.11); it may be objected that there is no such thing as an ultimate proportion of vanishing quantities, for before vanishing the proportion is not ultimate, and after vanishing it is not at all; by the same argument it can be contended that there is no ultimate velocity of a body with which its motion ceases (*id.*); but the answer is easy, the ultimate velocity is the very velocity with which the body arrives at its ultimate place and with which the motion ceases (*id.*); there is a limit which the velocity attains at the end of the motion and which it cannot exceed; this is the ultimate velocity (*id.*); this limit is certain and definite, and to determine it is a properly geometrical problem (*id.*).

Fluxions are limits. And limits are certain and definite. Even if it will take two more centuries to define limit as rigorously as Weierstrass will define it.

D'Alembert, in 1765, writes: one quantity is the limit of another when the second can approach the first more closely than a quantity as small as can be imagined, and such that the approaching quantity can never exceed the quantity which it approaches, so that the difference between the quantity and its limit is absolutely unassignable; for instance, the circumference of the circle is the limit of the increase of the inscribed and of the decrease of the circumscribed polygon of increasing number of sides (*Limit*).

Cauchy, in 1821, writes: when the values assigned successively to the same variable approach indefinitely a fixed value, in such a way as to end up differing from it as little as one wishes, this last is called the limit of all the others; for instance, an irrational number is the limit of the various fractions which provide values approaching it more and more (*Analysis*, pr.).

Weierstrass, in 1878, writes: a variable quantity x becomes infinitely small simultaneously with another y , means the following,



after the assumption of an arbitrarily small quantity ε , a limit δ can be established for x , such

that for every value of x , for which $|x| < \delta$, the corresponding value becomes $|y| < \varepsilon$ (*Introduction*, II, 5.3).

In 1765 d'Alembert concludes: the theory of limits is the ground of the true metaphysics of the differential calculus (*id.*). By 1821, and the more so by 1878, the mathematicians have been taught to refrain from conclusions involving either truth or metaphysics.

This is what fluxions are. They are

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

They are also the ground of the true metaphysics of the differential calculus. The ground also of the true metaphysics of the integral calculus.

The ground also of all mathematics. And all physics. And all investigation of any thing generated by continuous increase.

In the *Analysis* already Newton has shown that whatever the common analysis performs calculus performs, nor is there anything of this kind to which it does not extend, and in various ways. Viète, Galileo, Kepler, Descartes, Cavalieri, Fermat, Roberval, Torricelli, Wallis, Pascal, Huygens already have shown that whatever geometry, and arithmetic, and algebra perform the common analysis performs, nor is there anything of this kind to which it does not extend, and in various ways. And in *The method* Newton has enlarged its boundaries.

Including explicitly within them, by grounding fluxions, or simply velocities, in motion, however accelerated or retarded, the problems of space, time, velocity, acceleration, and motion. And in *The principles* he will show that as it performs all mathematics, so calculus performs all physics.

Taking mathematics from the beginning of the world to the time of Newton, then, and taking mathematics such as it is distinct from physics, Desargues, in the generation of Descartes and Fermat, has laid the foundations of projective geometry. Fermat has developed the theory of numbers. Huygens has developed the theory of probability. Logic has remained such as Aristotle and Chrysippus have developed it. How what Newton has done will inform projective geometry, the theory of numbers, the theory of probability, and logic, is only implicit in Newton. The rest of mathematics Newton has done. Considering

the power of what he has done, the scope of what he has done, and the unity and uniformity of what he has done, as the unity and uniformity of what he has done is attested by *The method* alone, what he has done is not far short of all of it.

Taking physics such as it is distinct from mathematics, Newton has not done animals and plants. What he has done will eventually inform animals and plants, too, but this is also only implicit in Newton. The rest of physics Newton has done. And considering the power, scope, and unity and uniformity of what he has done, as the unity and uniformity of what he has done is attested by *The principles* alone, including in that *The principles* attests the unity and uniformity of mathematics and physics as no other treatise before or after, what he has done is not far short of all of it either.

All in all, taking mathematics from the beginning of the world to the time of Newton, what Newton has done is much more than half, and much the better more than half.

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Newton's discoveries can be gathered under the two most general. He discovered calculus. And he discovered the principles.

And to show the importance of the latter the following can be shown. What is in *The principles*. That all of Newton's labors outside *The principles* belong among what is in *The principles*. How *The principles* describes the system of the world. And which are the principles.

Newton published *The mathematical principles of natural philosophy* in 1687, a second edition in 1713, and a third in 1726.

And this is what is in it in its third and final edition.

It opens with a preface. Newton begins by explaining why the mathematical principles of philosophy:

because the ancients found mechanics, as Pappus reports, the best in the investigation of nature, and their recent successors, rejecting substantial forms and occult qualities, have undertaken to reduce the phenomena of nature to mathematical laws, this treatise proposes to perfect mathematics as it relates to philosophy (*id.*).

And more generally still:

we set forth the mathematical principles of philosophy; because the entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena (*id.*).

The treatise proper then begins with 8 definitions, followed by a scholium. After each definition Newton provides instances. He begins:

definition 1. quantity of matter is its measure, from density and volume conjointly (*id.*).

The problem of the quantity of matter, the attempts at whose solution begin with Descartes, is thus solved. Quantity of matter is mass, the product of density and volume. Next:

definition 2. quantity of motion is its measure, from quantity of matter and velocity conjointly (*id.*).

The problem of the quantity of motion, the attempts at whose solution also begin with Descartes, is thus also solved. Quantity of motion is momentum, the product of mass and velocity.

The remaining definitions, 3-8, are of force. 3 is of: inherent force of matter (*id.*), 4 of: impressed force (*id.*). Next:

definition 5. centripetal force is that by which bodies are drawn, or are impelled, or in any way tend, toward a point as to a center;

of this kind are gravity by which bodies tend toward the center of the earth, magnetic force by which iron seeks a magnet, and the force, whatever it is, by which the planets are perpetually drawn back from rectilinear motion and compelled to revolve in curved lines (*id.*).

Huygens has defined centrifugal force, and centripetal force is its complement. Centripetal force is also force in general. As the instances show – the first and the third anticipating Newton's demonstration that gravity and the force, whatever it is, by which the planets are perpetually compelled to revolve in curved lines, are the same force, the second anticipating the labors of Newton's successors until Faraday to demonstrate that magnetic force is the same force, too. Next:

definition 6. absolute quantity of centripetal force is its measure, proportional to the efficacy of the cause (*id.*);

definition 7. accelerative quantity of centripetal

force is its measure, proportional to the velocity generated in a given time (*id.*).

$a \propto \frac{v}{t}$. And finally:

definition 8. motive quantity of centripetal force is its measure, proportional to the motion generated in a given time (*id.*).

$F \propto \frac{mv}{t}$, or $F \propto ma$. The problem of force, the attempts at whose solution begin with Kepler, is thus also solved.

The scholium to the definitions, next, is the first of many. Newton explains: so that the principles may not appear sterile, I have illustrated them with philosophical scholia, commenting on matters which are general and in which philosophy most of all appears to be grounded (III, pr.). And the matters on which he comments in this first philosophical scholium, so that the principles may not appear sterile, are general and in them all philosophy is grounded: absolute and relative, true and apparent, mathematical and vulgar (*id.*); time, space, place, and motion (*id.*). Newton writes:

time, space, place, and motion are familiar to all; it must be noted, however, that these quantities are vulgarly conceived only in relation to objects of sense; and this is the source of prejudices; to eliminate them, those have to be distinguished into absolute and relative, true and apparent, mathematical and vulgar;

absolute, true, and mathematical time, in itself and by its nature in relation to nothing external, flows uniformly and is otherwise named duration;

absolute space, by its nature in relation to nothing external, remains always the same and immovable;

absolute motion is the translation of a body from one absolute place to another absolute place; in astronomy absolute time is distinguished from relative by the equation of vulgar time; it is possible that there is no uniform motion which measures time accurately; it is possible that all motions are accelerated or retarded; yet the flow of absolute time cannot change; for the duration, or the perseverance of the existence of things, is the same, whether their motion is fast, or slow, or none; and its measure is thus

rightly distinguished from sensible things, and ascertained by the astronomical equation; and as the order of the parts of time is immutable so the order of the parts of space is;

for times and spaces are the places of themselves and of all things; they are places universally, in time in order of succession, in space in order of position; it is their essence to be places; and for primary places to move is absurd; they are therefore absolute places; and only translation between these places is absolute motion;

in philosophy abstraction from the senses is necessary;

immobile, then, are places which maintain their given positions from infinity to infinity, and therefore remain always immobile, and constitute the space which I call immobile;

the causes which distinguish true motion from relative are the forces impressed on bodies to generate motion;

the effects which distinguish absolute motion from relative are the forces of recession from the axis of circular motion;

and to know the true motions of singular bodies is difficult; but not hopeless; it is possible to find arguments, in part from the apparent motions, which are the differences of the true motions, and in part from the forces, which are the causes and effects of the true motions (*id.*).

Mathematical time, space, and motion are true time, space, and motion are absolute time, space, and motion.

The mathematical principles of philosophy are the true principles of philosophy are the absolute principles of philosophy.

The 3 axioms or laws of motion follow, with 6 corollaries, and their own scholium. The axioms or laws of motion are:

law 1. every body perseveres in its state of rest or uniform motion in a straight line, unless it is compelled to change its state by forces impressed on it;

law 2. change in motion is proportional to the motive force impressed, and is in the straight line in which that force is impressed;

law 3. to action there is always opposite and equal reaction; or, the actions of two bodies on one another are always equal and opposite in

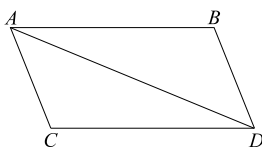
direction (*id.*).

Thus, because the ancients found mechanics the best in the investigation of nature, and their recent successors, rejecting substantial forms and occult qualities, have undertaken to reduce the phenomena of nature to mathematical laws, the recent successors of the ancients have succeeded in reducing the phenomena of nature to mathematical laws. Beginning with Galileo and Kepler, the discovery of the first law is complete with Descartes. Beginning with Descartes and Huygens, the discovery of the second law is complete with Newton. The discovery of the third law begins and is complete with Newton. With the discovery of the laws, as the formulation of the third law suffices to show, too, the phenomena of nature have been reduced to equations.

The entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena. And having discovered the forces of nature, Newton begins the demonstration of the other phenomena.

The first deduction from the laws is the parallelogram of forces:

corollary 1. a body under forces acting conjointly describes the diagonal of a parallelogram in the same time in which it describes its sides separately;



corollary 2. hence follows the composition of the direct force AD from any oblique forces AB and BD , and conversely, the resolution of any direct force AD into the oblique forces AB and BD (*id.*).

With the parallelogram of forces vector calculus begins.

In corollary 3, the second deduction is of the conservation of momentum (*id.*). And the first two deductions are representative. Results of the greatest generality and obtained with the greatest mathematical sophistication before *The principles* have been reduced to particularity and triviality in *The principles*.

The deduction of the principle of relativity follows:

corollary 4. the common center of gravity of two

or more bodies does not change its state of motion or rest from the actions of the bodies among themselves, and remains, excluding external actions and impediments, either at rest or in uniform motion in a straight line;

corollary 5. bodies enclosed in a given space have the same motions among themselves whether the space is at rest or moves uniformly in a straight line;

corollary 6. if bodies move in any way among themselves, and an equal accelerative force urges them along parallel lines, they all move among themselves in the same way as if that force did not incite them (*id.*).

Galileo discovered the principle of relativity. Newton has generalized it and formulated it quantitatively.

Then, so that the principles may not appear sterile, Newton again comments on matters which are general and in which philosophy is grounded. He comments on: the wide range and certainty of the third law (*id.*). And rather, on the wide range and certainty of all three laws. He begins:

these principles are accepted by the mathematicians and confirmed by multiple experiments; from the first two laws and the first two corollaries Galileo discovered that the descent of heavy bodies is in the square of the time and the motion of projectiles is in a parabola; from the same laws and corollaries Wren, Wallis, and Huygens, easily the principal geometers of the previous generation, discovered the rules of collision (*id.*),

he adduces multiple experiments, and concludes: if action is reckoned by force and velocity jointly, and reaction is reckoned by the velocities of the individual parts and the forces of resistance arising from friction, cohesion, weight, and acceleration, action and reaction will be equal in all examples of devices and machines (*id.*), the passage on which Maxwell, in turn, comments: Newton has stated the relation between work and kinetic energy in a manner so perfect that it cannot be improved.

The bulk of the treatise follows.

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The bulk of the treatise is divided into three books. More important, it is divided into two

parts. Books I and II are *On the motion of bodies*, comprising the first part. Book III is *On the system of the world*, comprising the second part.

Book III opens with its own preface, and Newton explains: in the preceding books I have delivered the principles of philosophy, which are, however, not just philosophical, but mathematical, and that is, from them it can be disputed in philosophical matters; it remains that from the same principles we teach the constitution of the system of the world (*id.*).

In book III Newton teaches the constitution of the system of the world.

In books I and II he delivers the principles of philosophy, which are, however, not just philosophical, but mathematical. Hence, he teaches the constitution of the system of the world from them. Hence, also, in philosophical matters it can not only be disputed from them. It had better be.

Books I and II are divided into sections.

Book I into fourteen sections.

Section 1 is *On the method of first and ultimate ratios, which is used to demonstrate what follows*. It contains 11 lemmas, 19 corollaries, and 2 scholia.

The foundation of the method is:

lemma 1. quantities, and ratios of quantities, which in finite time constantly tend to equality, and before the end of the time approach one another so that their difference is less than any given, become ultimately equal;

if this is denied, let them be ultimately unequal, and their ultimate difference be D ; therefore they cannot approach one another so that their difference is less than D , contrary to the hypothesis (*id.*).

In the scholium after lemma 11, the general matter in which philosophy is grounded is: first and ultimate sums and ratios are limits (*id.*); and limit is certain and definite (*id.*).

The method is further developed in book II, lemma 2, with 3 corollaries, a scholium, and an appeal to *The method of fluxions*.

Section 2 is *On the discovery of centripetal forces*. It contains 10 propositions, 1 lemma, 31 corollaries, and 5 scholia.

And the first proposition in *The principles* is a vast generalization of Kepler's second law: proposition 1. the areas, which bodies moved in

orbits describe by the radii drawn to an immobile center of force, are in an immobile plane, and are proportional to the times (*id.*).

Under a centripetal force the area is proportional to the time. In the second proposition Newton proves the converse:

proposition 2. every body, which moves in any curve in a plane, and by the radii drawn to a point, whether immobile or moving uniformly in a straight line, describes about that point areas proportional to the times, is urged by a centripetal force tending toward that point (*id.*).

Under a centripetal force the area is proportional to the time, and the area is proportional to the time under a centripetal force. Newton concludes: as the description of equal areas points at the center, toward which the force by which the body is affected the most is directed, and by which it is drawn away from rectilinear motion and maintained in its orbit, why will we not use in what follows the description of equal areas to point at the center, about which all circular motion propagates in free space (sch. 3)?

In the first section Newton has developed the method used to demonstrate what follows. And in the first propositions of the second section, using the method, he has discovered centripetal force. He has discovered the quantity of centripetal force, and *ipso facto* of force.

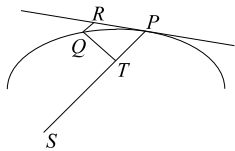
Hence, in proposition 4 he shows what the force is, and what the relations of the force and the period, the velocity, and the position of the body are, if the curve described by the body is a circle. He concludes:

from this proposition and its corollaries the ratio of a centripetal force to any force can be found; for instance, to gravity; it gives also the descent of heavy bodies; and by propositions of this kind Huygens in his excellent treatise on the pendulum clock compared the force of gravity to centrifugal forces (sch. 4).

In but four propositions Newton has reduced the results of Galileo, Kepler, and Huygens to particularity and triviality.

In proposition 6, corollary 1, the generalization is made explicit: if a body P revolves in any orbit about a center

S , PQ is the just nascent arc of its orbit described in minimal time, PR is tangent at P , QR is parallel to the distance SP , QT is perpendicular to the distance, and F is the force, $F \propto \frac{QR}{SP^2 \times QT}$ (*id.*).



For any curve described by a body, $F \propto \frac{QR}{SP^2 \times QT}$.

Section 3 is *On the motion of bodies in eccentric conic sections*. It contains 7 propositions, 2 lemmas, 20 corollaries, and 1 scholium.

In proposition 11 Newton shows that, if the curve described by a body is an ellipse, $F \propto \frac{1}{SP^2}$.

The force is inversely proportional to the square of the distance. In proposition 12 he shows that so is the force if the curve is a hyperbola. In proposition 13 that so the force is if the curve is a parabola. In proposition 15 he deduces Kepler's third law.

In the preface to book III he writes: in books I and II there are many propositions which can be found demanding even by readers proficient in mathematics; but it should be sufficient to read diligently the definitions, the laws of motion, and the first three sections of book I, then turn to the book on the system of the world, consulting earlier propositions at will (*id.*).

With the definitions, the laws, and the first three sections of book I Newton has unified and vastly generalized the results of his predecessors. In the remainder of books I and II he generalizes further, and further, and further, and further.

Sections 4 and 5 are *On the discovery of elliptical, parabolic, and hyperbolic orbits from a given focus* and *On the discovery of orbits when neither focus is given*, in 12 propositions, 13 lemmas, 16 corollaries, and 5 scholia. Newton concludes: so much about the discovery of orbits; it remains to determine the motions of bodies in the discovered orbits (sch. 29).

Section 6-8 are *On the discovery of motions in given orbits*, *On bodies in rectilinear ascent and descent*, and *On the discovery of orbits in which bodies revolve under the action of any centripetal forces*, in 13 propositions, 1 lemma, and 16

corollaries. Newton notes: in these propositions we have assumed that the centripetal force varies according to any law that can be imagined (42), and concludes: so far we have considered the motion of bodies in immobile orbits; it remains to treat the motion of bodies in orbits revolving about a center of force (*id.*).

Section 9 is *On the motion of bodies in moving orbits, and on the motion of apsides*, in 3 propositions and 8 corollaries. Newton concludes: so much about the motion of bodies in a plane which passes through the center of force; it remains to determine the motion in a plane which does not pass through the center of force (45). Section 10 is *On the motion of bodies on given surfaces, and on the oscillating motion of simple pendula*, in 11 propositions, 9 corollaries, and 1 scholium.

In section 11 Newton begins: so far I have shown the motions of bodies attracted toward an immobile center, such as hardly exist in the nature of things; attractions are directed only toward bodies and, by the third law, the actions of attracting and attracted bodies are always mutual and equal; if there are two bodies, neither the attracting nor the attracted body can be at rest but, by the fourth corollary to the laws, attracting one another, they must both revolve about a common center of gravity; if there are more bodies, attracted by another body, or attracting another body, or all attracting one another, they must move with respect to one another so that the common center of gravity either is at rest or moves uniformly in a straight line; for this reason I now proceed to show the motion of bodies attracting one another, considering centripetal forces attractions, even if, to speak physically, they are truly impulses (*id.*).

Section 11 is *On the motion of bodies drawn to one another by centripetal forces*, in 13 propositions, 32 corollaries, and 1 scholium. It contains the solution of the problems of the motions of bodies such as exist in the nature of things. The solution of the two-body problem, in propositions 57-63, and of the three-body problem, in proposition 66 and its twenty-two corollaries.

Newton concludes:

by these propositions we are directed to the analogy between centripetal forces and the central bodies toward which the forces tend; it is reason-

able to presume that forces directed toward bodies depend on their nature and quantity, as in magnets; and if this is the case, the attractions of bodies must be ascertained by assigning the forces properly to their particles and taking the sums of these forces;

I call attraction generally any endeavor of bodies to approach one another, whether this endeavor is from the actions of the bodies, or from mutual tendency, or from spirits emitted, or from the action of aether, or air, or any medium whatsoever, corporeal or incorporeal; in the same way I call impulse generally not the species and physical qualities of forces, but their quantities and mathematical proportions; in mathematics are investigated the quantities and ratios of forces which follow from any conditions whatsoever; next, descending to physics, these ratios are related to the phenomena in order to establish which general conditions apply to particular bodies; thus disputes about the physical species, causes, and reasons of the forces can be grounded;

let us see, then, what the forces of spherical bodies consisting of particles are, and what motions follow (sch. 69).

Thus midway through *The principles*. By the end the disputes will all be grounded, and settled.

Section 12 is *On the attractive forces of spherical bodies*, in 15 propositions, 1 lemma, 24 corollaries, and 3 scholia. Newton shows:

proposition 73. a particle inside a sphere is attracted by a force proportional to the distance of the particle from the center of the sphere (*id.*);

proposition 74. a particle outside a sphere is attracted by a force inversely proportional to the square of the distance of the particle from the center of the sphere (*id.*).

He writes: I have now shown the two major cases of attraction, when the centripetal force decreases as the square of the distance and when it increases as the distance; the effects in both cases, that bodies revolve in conic sections, and that centripetal forces compose spherical bodies, are effects of the same law; which is worthy of note; to continue through the other cases which lead to less elegant conclusions would be tedious; instead, I will comprehend them all in a general

method (sch. 78).

What is worthy of note, the elegant conclusion, is that Newton has discovered that the same force, by the same law, constitutes as well as moves all bodies.

Newton concludes the section, here as everywhere, with an appeal to generality and philosophy: now that the attractions of spherical bodies have been shown, the attractions of other bodies consisting of particles can be shown; but I will not treat particular cases; it will be enough to adduce some general propositions of use in philosophical matters (sch. 84).

Section 13 is *On the attractive forces of not spherical bodies*, in 9 propositions, 13 corollaries, and 1 scholium.

Section 14 is *On the motion of the smallest bodies under the action of centripetal forces tending toward each of the parts of a large body*, in 5 propositions, 2 corollaries, and 2 scholia. The philosophical matter is light. The generalization extends to: the analogy between the propagation of rays of light and the motion of bodies (sch. 96). Newton shows that the analogy holds for: the reflections and refractions of light, demonstrated by Snel and Descartes (*id.*); the fact that light propagates successively and comes from the sun to the earth in about 7 or 8 minutes (*id.*); the inflection of the rays of light, which Grimaldi recently discovered, in passing near the edges of bodies (*id.*). Also: the inflection is less at greater distances from the edges, and at still greater distances the rays are inflected in the opposite direction and form three bands of colors (*id.*). And finally: spherical shapes are the best for optical use; if the objective lenses of telescopes are made of two spherical lenses, and water is enclosed between them, the errors of refraction in the surfaces of the lenses are corrected accurately enough by the refractions of the water; such objective lenses are to be preferred to elliptical or hyperbolic (*id.*).

98 propositions, 29 lemmas, 190 corollaries, and 22 scholia in book I.

In book II the generalization continues. One difference is that, while in book I Newton reports in detail very many experiments, in book II he reports in detail very many more experiments. He continues to deliver the principles which are not

just philosophical, but mathematical, and the more general the principles which are not just philosophical, but mathematical, become, the more they rest on experiments.

Book II is divided into nine sections.

Section 1 is *On the motion of bodies resisted in proportion to the velocity*, in 4 propositions, 1 lemma, 13 corollaries, and 1 scholium. Newton concludes: the hypothesis of resistance to bodies in proportion to the velocity, however, is more mathematical than natural; in media lacking rigor resistances to bodies are as the squares of the velocities; let us see, therefore, what motions follow from this law of resistance (sch. 4).

Section 2 is *On the motion of bodies resisted as the square of the velocity*, in 6 propositions, 1 lemma, 25 corollaries, and 2 scholia. It is in this section that Newton has to develop calculus further. Section 3 is *On the motion of bodies resisted in part in proportion to the velocity, in part as the square of the velocity*, in 4 propositions, 7 corollaries, and 2 scholia. Newton concludes: the resistance to spherical bodies in fluids arises in part from the tenacity, in part from the friction, in part from the density of the medium; we have shown that the part from the density is as the square of the velocity, the part from the tenacity is uniform, or as the time (sch. 14); I have thus opened the way into the investigation of these matters; and I hasten to others (*id.*).

Section 4 is *On the circular motion of bodies in resisting media*, in 4 propositions, 1 lemma, 12 corollaries, and 1 scholium.

Section 5 is *On the density and compression of fluids, and on hydrostatics*. It opens with the ninth definition in *The principles*: definition of fluid. fluid is every body whose parts yield to any force and are moved easily among themselves (*id.*).

It contains 5 propositions, 12 corollaries, and 2 scholia. In the previous generation Boyle has discovered that at constant temperature the volume of a gas is inversely proportional to its pressure. In proposition 23 Newton deduces Boyle's law, reducing this law, too, to particularity and triviality. He concludes the section: whether elastic fluids consist of particles which repel one another is a question of physics; we have demonstrated mathematically the properties

of a fluid consisting of such particles so that the philosophers can address that question (sch. 23).

Section 6 is *On the motion and resistance of simple pendula*, in 8 propositions, 19 corollaries, and 1 scholium.

The scholium is the first of the two general scholia in *The principles*. The general philosophical matter is aether. Newton writes: from these propositions the resistance of any medium can be found by pendula oscillating in them (*id.*). He finds the resistance of air, of water, and of mercury. And continues: there is also the opinion that there is a certain aethereal medium, subtlest by far, which freely permeates the pores of all bodies, and that a resistance has to arise from it as it flows through the pores of bodies; and to test whether the resistance experienced by moving bodies is entirely on their external surfaces, or the internal parts encounter notable resistance as well, I devised the following experiment;

I suspended a round box by a cord 11 feet long;

I drew this pendulum to a distance of about 6 feet and marked the exact place; letting the pendulum fall, I marked also the places to which it returned after the first, second, and third oscillations; I repeated this many times to find the places as exactly as possible;

then I filled the box with lead and other heavy metals which were at hand;

but first I weighed the empty box and half the cord, as a stretched cord urges a pendulum with half its weight; I added the weight of the air inside the box; and the whole weight of the empty box was about $\frac{1}{78}$ of that of the full;

then, since the full box increases the length of the pendulum by stretching the cord, I shortened it so that the length be the same; and letting the pendulum fall from the first marked place, I counted about 77 oscillations until it returned to the second marked place, as many until it returned to the third, and as many until it returned to the fourth;

I conclude that the resistance of the full box is to the resistance of the empty box as not more than 78 to 77; for the inherent force of the full box is 78 times that of the empty box, and if the resistances were equal, the full box would conserve its motion and always return to each

of the marked places after 78 oscillations; but it returned after 77;

let, therefore, the resistance of the box on its external surface be A , the resistance of the empty box on its internal parts B ; and if the resistance of equally fast boxes on their internal parts is as the matter, or as the number of particles resisted, the resistance of the full box on its internal parts will be $78B$;

hence, the whole resistance of the empty box to the whole resistance of the full box will be

$$\frac{A+B}{A+78B} = \frac{77}{78};$$

and hence, $\frac{B}{A} = \frac{1}{5928}$; the resistance on the

internal parts of the empty box is more than 5000 times less than the resistance on the external surface;

this argument depends on the hypothesis that the greater resistance encountered by the full box is not from some other hidden cause but only from the action of a subtle fluid on the enclosed metal (*id.*).

Newton has thus not shown that the subtlest medium is not. But he has shown that, if it is, it is very very subtle.

Section 7 is *On the motion of fluids and the resistance of projectiles*, in 9 propositions, 4 lemmas, 32 corollaries, and 6 scholia. Newton compares the resistances to a sphere and to a cylinder in the direction of its axis (34). And concludes: by the same method the resistances to other figures can be compared and those most suitable for continuing motion in resisting media discovered (sch. 34); and this proposition will be of use for the construction of ships (*id.*).

Section 8 is *On the motion propagated through fluids*, in 10 propositions, 10 corollaries, and 1 scholium. Newton investigates waves. The last proposition is:

to discover the distances between pulses;
find the number of vibrations in a given time;
divide the space pulses travel in that time by that number; and this is the length of one pulse; Q.E.I. (50).

Newton has discovered this, too, that $\lambda = \frac{v}{f}$, where

λ is the wavelength, v is the velocity, f is the frequency. He concludes: these propositions

apply to the motion of light and sound; for as light propagates in straight lines, it cannot consist in action alone, and as sound arises from vibrating bodies, it is nothing but propagated pulses of air (sch. 50). He then calculates the speed of sound.

Section 9 is *On the circular motion of fluids*. It is the concluding section of the first part of *The principles*, in which Newton delivers the principles of philosophy. In *The principles of philosophy* Descartes has developed the theory of vortices. And Newton concludes the first part of *The mathematical principles of natural philosophy* with the investigation of vortices.

He begins with the first of the three hypotheses in *The principles*:

the resistance arising from the friction of the parts of a fluid is proportional to the velocity of separation of the parts of the fluid (*id.*).

3 propositions, 19 corollaries, and 2 scholia follow. Newton shows that vortices are incompatible with Kepler's third law (sch. 52). More generally, they are incompatible with the description of areas proportional to the times (sch. 53). Newton concludes: it is clear that the planets are not carried by corporeal vortices (*id.*).

If there is aether, it is very very subtle. But the planets are not carried by corporeal vortices either, however subtle these may be. Motion, therefore, propagates in free space.

And: how motion propagates in free space without vortices can be understood from book I, and will be now shown fully in the system of the world (*id.*).

1 definition, 1 hypothesis, 53 propositions, 7 lemmas, 149 corollaries, and 18 scholia in book II.

Book III is *On the system of the world*.

Before turning to the system of the world, however, enough has been said to make it possible to assess what is in *The principles*, qualitatively and quantitatively.

Qualitatively, the generalization is beyond all bounds.

The phenomena of nature have been reduced to mathematical laws. The entire difficulty in philosophy has been solved, from the phenomena of motion the forces of nature have been discovered, then from these forces the other phenomena

have been demonstrated.

The problem of the quantity of matter has been solved. The problem of the quantity of motion has been solved. The problem of force has been solved. Mathematical, true, and absolute time, space, and motion have been distinguished from vulgar, apparent, and relative. From the laws the parallelogram of forces, the conservation of momentum, and the principle of relativity have been deduced. The relation of work and kinetic energy has been anticipated.

The description of the motion of bodies has proceeded from the equation of force, to the orbit of a body in any curve, as force varies by any law that can be imagined, to motion in orbits revolving about a center of force, to motion in a plane which does not pass through the center of force, to the motions of two bodies in relation to one another, to the motions of three bodies in relation to one another, to ascertaining the attractions of bodies by the forces of their particles, through the demonstration that the same force, by the same law, constitutes as well as moves all bodies, through the analogy between the propagation of rays of light and the motion of bodies, including reflection, refraction, and diffraction, and the construction of telescopes, too, to motion in media and their resistance, in proportion to the velocity, as the square of the velocity, in part in proportion to the velocity, in part as the square of the velocity, due to tenacity, due to friction, due to density, the deduction of Boyle's law, the properties of fluids, the properties of pendula, the experimental examination of the aether, the relation of resistance and shape, minimizing the resistance, and the construction of ships, too, waves, the relation of wavelength and frequency, the motion of light and sound, what light is, what sound is, and the demonstration that the planets are not carried by vortices. Before turning to the system of the world.

All grounded in calculus. Itself grounded in limit.

Quantitatively, what is in *The principles* can be counted.

Book III, after its preface, begins with 4 rules of philosophizing, and 6 phenomena. Not divided into sections, it then contains 42 propositions, 11 lemmas, 50 corollaries, and 6 scholia. It has also

2 hypotheses, preceding propositions 11 and 39, respectively.

In the first edition *The principles* ended with book III, its last proposition, and comets. In the second edition Newton added the second general scholium, abolishing altogether the possibility that the principles may appear sterile. *The principles* now ends with god. Newton writes: to treat of god from the phenomena pertains to natural philosophy (*id.*). And: he rules all things; and because of his dominion he is called lord god παντοκράτωρ; he is a being eternal, infinite, and absolutely perfect; alive and intelligent; omnipotent and omniscient; omnipresent (*id.*).

All in all, 2 prefaces, 9 definitions, 3 laws, 3 hypotheses, 4 rules, 6 phenomena, 193 propositions, 47 lemmas, 395 corollaries, and 49 scholia, 2 of them general.

It can all be sorted out into three kinds.

The 25 definitions, laws, hypotheses, rules, and phenomena are the explicit premises of the deductions.

The 635 propositions, lemmas, and corollaries are the deductions.

The 51 prefaces and scholia are commentaries on matters which are general and in which philosophy most of all appears to be grounded.

The first kind, comprising as it is definitions, laws, hypotheses, rules, and phenomena, raises the question of which are the principles. The more so in relation to the second kind, given the generality of many of the propositions, lemmas, and corollaries it comprises. The more so still in relation to the third kind, given that the matters which the prefaces and scholia comprise are general and in them philosophy is grounded.

The first and second kinds, however, the explicit premises of the deductions and the deductions, also point unambiguously at the quantitative success of *The principles*.

Knowledge, with Aristotle, is acquired only by demonstration. To be perfect, with Descartes, knowledge must be deduced. To this end, with Frege, the chains of inferences must be free of gaps.

And to this end, first and foremost, deduction must be explicit. It must be clear which are the premises, and which are the deductions from the premises.

Deduction is explicit in Euclid. In the *Elements* from 141 definitions, postulates, and axioms 514 propositions, lemmas, and corollaries are deduced.

In the *Elements* explicit deduction reaches the elements – numbers, and hence ratios and proportions, and lines, and hence plane and solid figures.

Explicit deduction reaches beyond the elements in Archimedes. In *On the equilibrium of planes* and *On floating bodies* from 9 definitions and postulates 47 propositions and lemmas are deduced.

Deduction is not explicit in Ptolemy.

Explicit deduction reaches further in Galileo. In the *Two new sciences* from 7 definitions, postulates, and axioms 71 propositions, lemmas, and corollaries are deduced.

Deduction is not explicit in Kepler.

It is not in Descartes.

It is in Huygens. In *The pendulum clock* from 22 hypotheses and definitions 63 propositions are deduced.

In *The mathematical principles* from 25 definitions, laws, hypotheses, rules, and phenomena 635 propositions, lemmas, and corollaries are deduced.

In short, Halley writes:

you who relish celestial nectar,
join me in celebrating Newton;
closer to the gods a mortal cannot come (*Glory*).

Than joining Halley in celebrating Newton.

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Before turning to the system of the world, it can also be shown that all of Newton's labors outside *The principles* belong among what is in *The principles*.

This should be clear on general grounds. If the entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena, Newton's labors before *The principles* are labors from the phenomena of motion to discover the forces of nature, his labors after *The principles* are labors from these forces to demonstrate the other phenomena.

Newton's labors outside *The principles* can be divided into mathematical, optical, theological,

chemical, and historical.

The mathematical are first. Newton wrote the *Enumeration of lines of the third order*, in 1668, *On analysis by equations of infinite number of terms*, in 1669, and *A treatise on the method of fluxions and infinite series*, in 1671. These labors continue with *A treatise on the quadrature of curves*, in 1676. They include the summary of his lectures, written in 1683, and published in 1707 as the *Universal arithmetic, or a treatise on arithmetical composition and resolution*. They conclude in *The principles*, in 1687.

Next are the optical. Newton published the *New theory of light and colors*, and *An account of a new catadioptrical telescope*, in 1672. These labors as well continue in *The principles*, in 1687. They conclude with the *Optics, or a treatise on the reflection, refraction, inflection, and colors of light*, published in 1704, its fourth and final edition in 1730.

In 1685 Newton wrote *A treatise on the system of the world*. He comments on it in the preface to book III of *The principles*: I wrote an earlier treatise on the system of the world in popular form, so that it might be more widely read; but those who have not sufficiently grasped the principles will certainly neither appreciate the force of the conclusions nor lay aside the prejudices to which they have become accustomed over many years; to avoid lengthy disputations, therefore, I rendered the substance of that treatise in mathematical form, so that it might be read only by those who have mastered the principles (*id.*).

Next is *The principles*, in 1687.

Next are the theological. Newton wrote *A historical account of two notable corruptions of scripture*, in 1691, and the *Observations on the prophecies of Daniel and the Apocalypse of st John*, published in 1733.

Next are the chemical. *On the nature of acids* was published in 1692, and *A scale of the degrees of heat, description and signs of heat* in 1701.

Last are the historical. *The chronology of ancient kingdoms amended, to which is prefixed a short chronicle from the first memory of things in Europe to the conquest of Persia by Alexander* was published in 1728.

Philosophy is theoretical and practical. The

search for truth is theoretical philosophy. Theoretical philosophy is mathematics and physics.

That *The principles* is philosophy Newton is explicit. That *The principles* not only searches for truth, but for truth such as it is absolute, he is also explicit. That *The principles* is mathematics and physics he is also explicit.

Given the success of *The principles*, the weight of Newton's authority on any matter, whether addressed in *The principles* or not, should be overwhelming. The more so if it rests on Aristotle's authority. The more so if the matter is addressed in *The principles*. The more so the more general the matter. The more so the matter of what is philosophy, what is mathematics, what is physics, and what the relations of the three, philosophy, mathematics, and physics, are.

It is the catastrophic transformation of philosophy in the last couple of centuries that has restored to substantiality and purity the illusion that there is weightier authority.

Newton's labors outside *The principles* in relation to those inside show what the relations of physics and its parts optics, theology, chemistry, and history are. And what the relations of mathematics and physics are, and what the relations of philosophy and its parts mathematics and physics are.

Excepting *The principles*, the most important of Newton's labors by far, the chronological order of his other labors corresponds to the order of their importance. In reverse order, the historical are important, the chemical are more important, the theological are more important, the optical are more important, the mathematical are the most important.

Newton's historical investigations stand on research as thorough as that on which any of his investigations stands. To illustrate, in *The chronology of ancient kingdoms amended* he begins with the examination: of the chronology of the first ages of the Greeks (1), and in the first three paragraphs alone examines the following authorities: Hesiod, Epimenides, Solon, Thales, the Orphic poems, Pherecydes, Xenophanes, Acusilaus of Argos, Cadmus of Miletus, Parmenides, Empedocles, Herodotus, Hellanicus, Hippias, Pherecydes of Athens, Isocrates, Plato, Eudoxus, Ephorus, Anaximenes of Lampsacus, Aristotle,

Callisthenes, Timaeus, Timocharis, Aristyllus, Aristarchus, Eratosthenes, Polybius, Hipparchus, Apollodorus of Athens, Siculus, Dionysius of Halicarnassus, Pliny, Josephus, Plutarch, Laertius, the *Suda* (*id.*).

The chronology of ancient kingdoms amended is in six chapters. Newton examines: the chronology of the first ages of the Greeks (1); the empire of Egypt (2); the Assyrian empire (3); the contemporary empires of the Babylonians and Medes (4); the description of the temple of Solomon (5); the empire of the Persians (6).

The examination yields: a short chronicle from the first memory of things in Europe to the conquest of Persia by Alexander (*id.*). And Newton reports: I have drawn this chronological table to make chronology agree with the course of nature, with astronomy, with sacred history, with Herodotus the father of history, and with itself (*intr.*).

Chronology is not just a part of history. It is its *sine qua non*. There is and can be no history without chronology. Herodotus is the father of history because in the *Histories* chronology is for the first time fixed across a substantial portion of time and space. There is history in the Hebrew scriptures because from their lists of patriarchs and princes chronology can be extracted. And the beginning of history can be traced back sixteen centuries before Herodotus, twelve centuries before the earliest of the writings in the Hebrew scriptures, to the Sumerian prince list because from it chronology can be extracted.

Ptolemy made chronology agree with astronomy. Kepler made the birth of our savior Jesus Christ agree with the course of the heaven and with the heathen and Jewish histories.

Newton's solution of the problem of chronology is not flawless. It is hampered by the lack of the essential data, those supplied a couple of centuries later by the recovery of the Egyptian and the Mesopotamian records.

His formulation of the problem of chronology, on the other hand, is flawless. The problem is, precisely, to make chronology agree with the course of nature, with astronomy, with sacred history, with Herodotus the father of history, and with itself. More particularly, moreover, the problem is this. To make chronology agree with the course of nature, and hence astronomy. Hence

make it agree with the empire of Egypt and the Assyrian empire – and if there are earlier empires agree with them, too. Hence make it agree with the successors of these empires, with the first ages of the Greeks, with the contemporary empires of the Babylonians and Medes, and with the empire of the Persians – and that is, agree with Herodotus the father of history. Hence make it agree with sacred history. And thus make it agree with itself.

The problem has also been solved precisely as Newton formulated it. Chronology has been made to agree with the course of nature, and hence astronomy. Hence it has been made to agree with the Egyptian and the Mesopotamian records. Hence it has been made to agree with the later records, and most particularly with Herodotus, who is the father of history also because the *Histories* is the indispensable connection of the later records with the earlier. Hence it has been made to agree with sacred history. And thus it has been made to agree with itself.

Having from the phenomena of motion discovered the forces of nature, Newton demonstrates from these forces the phenomena of history as well. From the course of nature – chronology.

Physics is the foundation of its part history.

And the importance of this fact the consequences of the solution of the problem of chronology show.

One consequence is that the solution of the problem of chronology has supplied the proof of Spinoza's solution of the problem of when and how the Hebrew scriptures were authored, and either a more important or a more difficult historical problem remains to be formulated or solved. Another consequence is that sacred history has been made to agree with the course of nature, rather than the other way around.

Physics is the foundation of its part history, and so it is of its part chemistry.

Newton's chemical investigations are as thoroughly experimental as any of his investigations.

His two published chemical treatises are both brief, a couple of pages each.

A scale of the degrees of heat, description and signs of heat is but one table. These are some of the entries in the table:

- 0 heat of air in winter when water begins to freeze, found by placing a thermometer in compressed snow as it begins to thaw;
- 6 heat of the air at noon in July;
- 12 greatest heat which a thermometer receives on contact with a human body or a bird hatching her eggs;
- $14\frac{3}{11}$ – heat of blood newly drawn;
- $20\frac{2}{11}$ – heat of a bath by which melted wax hardens and loses its transparency;
- 33 heat by which water begins to boil;
- $40\frac{4}{11}$ – least heat by which a mixture of one part lead, four parts tin, and five parts bismuth is melted and kept in continuous fusion;
- 72 heat by which tin by itself is melted;
- 81 heat by which bismuth is melted;
- 114 heat by which ignited bodies in cooling cease to shine and in heating begin to shine in the dark;
- 136 heat by which ignited bodies glow at night but not in the twilight;
- 161 heat by which ignited bodies glow in the twilight immediately preceding the rising and following the setting of the sun;
- 192 heat of live coals in a small kitchen fire made of bituminous coals; the heat of such a fire made of faggots is greater, 200 or 210; that of a large fire is still greater, especially if blown with bellows (*id.*).

Newton explains:

the degrees of heat are in arithmetical progression, beginning with that which water has when it begins to freeze, presuming this to be the lowest degree of heat, or the boundary between heat and cold, and supposing the external heat of a human body to be 12 parts;

the table was constructed by a thermometer and a red hot iron; by a thermometer were found the degrees of heat down to that which melts tin; by a hot iron the rest; for the heat which a hot iron communicates to the cold bodies around it in a given time, and that is, the heat which it loses, is proportional to the whole heat of the iron, and therefore in geometrical proportion in equal times of cooling;

thus, first it was found that, if in a thermometer with linseed oil, placed in snow as it begins to thaw, the oil takes the space of 10 000 parts, the

same oil, rarefied with the heat of a human body, takes the space of 10 256 parts; rarefied with the heat of melted tin, it takes the space of 11 516 parts; hence, presuming the degrees of heat of the oil proportional to its rarefaction, and taking 12 parts for the heat of a human body, the parts for that of melting tin are 72;

next a thick piece of iron was heated red hot, then laid in a cold place where the wind blew continuously, and putting particles of several metals and other fusible bodies on it, the time of cooling was noted until all the particles hardened and the heat of the iron was equal to that of the human body; hence, supposing that, when the times are in arithmetical progression, the excess of the heat of the iron and of the particles above the heat of the atmosphere, found by a thermometer, are in geometrical progression, the several degrees of heat were discovered;

the iron was laid in a wind blowing continuously so that the air heated by the iron be always carried off and succeeded by cold air; thus equal parts of air were heated in equal times and received a degree of heat proportional to the heat of the iron; the several degrees of heat thus found had the same ratios among themselves as those found by the thermometer; and therefore the rarefaction of the oil was properly assumed proportional to the degrees of heat (*id.*).

Newton has discovered how to measure the degrees of heat, too.

In *The principles* he has shown that the revolution of bodies in conic sections and the composition of bodies by centripetal forces are effects of the same law. That all bodies mutually attract one another and all particles in all bodies mutually attract one another.

On this ground he investigates fluids, including the aether.

On this ground he shows that heat is proportional to rarefaction, and how to measure the degrees of heat.

And on this ground he concludes in *On the nature of acids*: all bodies have particles which mutually attract one another; the sums of the least of these may be called particles of the first composition, the sums of these sums particles of the second composition, and so on (*id.*).

Having from the phenomena of motion discov-

ered the forces of nature, Newton demonstrates from these forces the phenomena of chemistry as well.

And, on the one hand, more observation will be necessary before the phenomena of chemistry are demonstrated conclusively from the forces of nature.

A century later, in the *Researches on the laws of affinity*, in 1801, Berthollet writes: the forces which produce chemical phenomena are all derived from the mutual attractions of the particles of bodies and have been given the name affinity to distinguish them from astronomical attraction (15); astronomical attraction is exercised by masses at a distance, where the form of the particles, their intervals, and their particular affinities have no influence (*id.*); the effects of chemical attraction, or affinity, are on the contrary so altered by the particular conditions that they cannot be deduced from the general principle; only observation can establish them (*id.*).

On the other hand, Berthollet continues: as it is very probable that affinity does not differ in origin from general attraction, it should equally be subject to the laws which mechanics has determined for the phenomena due to the action of mass, and it is natural to think that the more general the principles to which chemical theories apply the more they have to be analogous to those of mechanics (*id.*).

And a century-odd later still, quantum theory discovered the forces of nature other than gravity, in the meantime the most general principle to which chemical theories apply, the periodic law, had been discovered, and quantum theory demonstrated the periodic law from the forces of nature.

Having discovered the forces of nature, Newton demonstrates the phenomena of history, demonstrates the phenomena of chemistry, and demonstrates the phenomena of theology as well.

In the concluding general scholium of *The principles* he writes:

the six primary planets revolve in circles concentric with the sun, in the same direction, and very nearly in the same plane; ten moons revolve about the earth, Jupiter, and Saturn in concentric circles, in the same direction, and very nearly in the same plane; and all these regular motions do not have their origin in mechanical

causes, because comets move freely in very eccentric orbits and in all parts of the heaven; and at aphelia, where their motions are the slowest, comets are at the greatest distances from one another so as to attract one another the least; and this most elegant arrangement of the sun, planets, and comets cannot have its origin but in the judgment and dominion of an intelligent and powerful being;

and if the fixed stars are the centers of similar systems, they are all similarly constructed by the judgment and dominion of one; the more so if the light of the fixed stars is of the same nature as the light of the sun, and each system sends light into all the others; and to prevent these systems from falling on one another by their gravity, he has placed them at immense distances from one another;

he rules all things not as a soul of the world but as a universal lord, and because of his dominion he is called lord god παντοκράτωρ, and that is, universal emperor; for god is a relative word and refers to servants, and divinity is the dominion of god, and not over his body, as those to whom god is a soul of the world suppose, but over servants; the supreme god is a being eternal, infinite, and absolutely perfect; and however perfect, a being without dominion is not lord god;

from true lordship follows that the true god is alive, intelligent, and powerful; from the other perfections follows that he is supreme, or supremely perfect; that he is eternal and infinite; omnipotent and omniscient; rules all things; and knows all things that are and that will be;

and he is omnipresent not only by power, but by substance, for power cannot subsist without substance; in him all things are and move, but without suffering from one another; he is not affected by the motion of bodies, bodies feel no resistance from his omnipresence;

we know him from his properties and attributes and from the wisest and best constitution and final causes of things; we admire him because of his perfections; and we venerate and worship him because of his dominion; we worship him as servants; god without dominion, providence, and final causes is nothing but fate and nature;

and so much about god, to treat of whom from the phenomena pertains to natural philosophy

(*id.*).

The most elegant arrangement of the solar system – the regular motions of the planets and satellites, on the one hand, and the very eccentric orbits of the comets and their distribution at aphelia, on the other hand – shows that the solar system has its origin in the judgment and dominion of an intelligent and powerful being. The similarity of the solar system and the systems of the fixed stars, the same nature of the light of the sun and the fixed stars, the distribution of light throughout the system of the fixed stars, and the distribution of the fixed stars, show that that being is one. Hence, he rules all things as a universal lord. Divinity is dominion. From dominion follows that he is alive, intelligent, supreme, omniscient, and omnipresent. Although bodies feel no resistance from his omnipresence. And we worship him because of his dominion. God without dominion, providence, and final causes is nothing but fate and nature.

To treat of god from the phenomena pertains to natural philosophy. Accordingly, Galileo treats of god from the phenomena. So does Kepler. Descartes concludes that nature is god. Spinoza concludes that god is nature. Newton concludes that god is not just nature. He rules all things.

Nor can the treatment of god, if the holy scriptures reveal god, exclude the holy scriptures. Accordingly, Galileo shows that conclusions about nature must be the ground of the true exposition of the holy scriptures. Spinoza shows when and how the holy scriptures were authored. And Newton examines the holy scriptures in *A historical account of two notable corruptions of scripture* and the *Observations on the prophecies of Daniel and the Apocalypse of st John*.

The main result of the *Historical account* is that the trinity is a corruption of scripture. This corruption Newton traces back to the corruption of two passages, of verse 7 in chapter 5 of the *First letter of John*, and of verse 16 in chapter 3 of st Paul's *First letter to Timothy*. Of the first, Newton writes: the history of the corruption is in short this; first some of the Latins interpreted spirit, water, and blood to prove that the father, son, and holy spirit are one, then Jerome inserted the trinity in express words in his version of scripture, then the Africans began alleging it

against the Vandals, then the Latins began inserting it in the margins of their bibles, and then it began to creep into the text itself in transcription (I, 2). Of the second, he writes: what the Latins did to that text, the Greeks did to this of st Paul; by changing $\omicron$ into Θ C, the abbreviation for $\theta\epsilon\acute{o}\varsigma$, they now read, great is the mystery of divinity, god manifested in the flesh (II, 1).

The main result of the *Observations* is that prophecy shows providence, not the future. Newton writes: the folly of interpreters has been to foretell times and things, as if god designed to make them prophets; but god's design was otherwise; he gave prophecies not to gratify men's curiosities by enabling them to foreknow things, but so that, after the prophecies are fulfilled, they may be interpreted by the events, and thus manifest his providence to the world (II, 1).

More important than the results of the *Historical account* and *Observations* is their method. Newton's theological investigations are historical. Spinoza writes: the method of interpretation of scripture does not differ from the method of interpretation of nature; it is necessary first and foremost to order the genuine history of scripture; and this history must include, 1. the language of the books, 2. their sentences, 3. their circumstances. And Newton orders the genuine history of scripture, including 1. the language of the books, 2. their sentences, 3. their circumstances.

Newton's theological investigations, too, stand on research as thorough as that on which any of his investigations stands. To illustrate, in order to establish the history of the corruption of the first of those two passages, Newton examines – next to scripture itself, its recension, Greek, Latin, Coptic, Syriac, Arabic, Armenian, Slavonic, and vulgar, numerous printed editions, and the documents of church councils – the following authorities: Clement of Alexandria, Tertullian, Cyprian, Arius, Alexander of Alexandria, Eusebius, Victorinus, Athanasius, Didymus the Blind, Hilary, Epiphanius, Phoebadius, Faustinus, Basil of Caesarea, Gregory of Nazianzus, Gregory of Nyssa, Gregory of Elvira, Ambrose, Jerome, Philastrius, Idacius, Chrysostom, Augustine, Hesychius of Jerusalem, Cyril of Alexandria, Leo, Theodoret, Eucherius, Arnobius Junior, Eugenius of Carthage, Cerealis, Vigilius of

Thapsus, Victor Vitensis, Fulgentius, Cassiodorus, Facundus, Oecumenius, Gregory, Bede, Paschasius, Cyril, Nicetas, Clairvaux, Joachim of Fiore, Aquinas, Valla, Jiménez de Cisneros, de Nebrija, Manutius, Lefèvre d'Étaples, Erasmus, de Zúñiga, de Colines, Edward Lee, Luther, de Sepúlveda, de Carranza, Bullinger, Beza, Hessels, Montano, Canter, Franciscus Lucas, Caryophilus, Grotius, Chiffletius, Brian Walton, Mabillon, Richard Simon, Gilbert Burnet (I, 1-36).

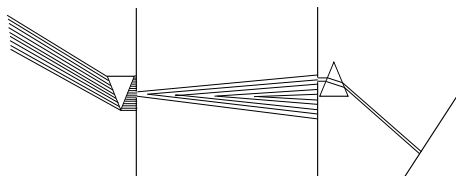
By the same method, too, it has since been shown that corruption began earlier. Neither is the *First letter of John* by John, nor is st Paul's *First letter to Timothy* by st Paul, nor is *Daniel* by Daniel, nor is the *Apocalypse* of st John by st John.

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The results of Newton's optical investigations, like those of the mathematical, are of very great importance, even if not quite as great as those of the mathematical.

His optical investigations also, like the mathematical, begin before *The principles*.

In the course of his investigation of light – Newton reports in the *New theory of light and colors* – he was led to: a crucial experiment (*id.*). He writes: I took two boards, and placed one of



them close behind a prism at the window, so that the light pass through the prism and a small hole in the board and fall on the other board, having made a small hole in it as well; then I placed another prism behind this second board so that the light pass through it also and be refracted again before arriving at the wall; this done, I turned the first prism to and fro about its axis to make the several parts of the image cast on the second board successively pass through the hole in it, and to observe to what places on the wall the second prism will refract them (*id.*).

The image produced by the first prism and cast on the second board was a spectrum of light of

different colors. Passed successively through the second prism, the several parts of the image, of a different color each, were each refracted differently, but each preserved its color.

Newton concludes: light is a heterogenous mixture of differently refrangible rays (*id.*); colors are not qualifications of light derived from refractions or reflections of bodies, as is generally believed, but original and connate properties, different in different rays (*id.*); the least refrangible rays all exhibit a red color; the most refrangible rays all exhibit a deep violet color (*id.*); colors are of two kinds, one original and simple, the other compounded; original or primary are red, yellow, green, blue, and violet purple, together with orange, indigo, and an indefinite variety of intermediate gradations (*id.*); a mixture of yellow and blue makes green; of red and yellow orange; and so on (*id.*); but the most surprising and wonderful composition is that of white; no ray alone exhibits it; it is always compounded, and to its composition all primary colors are required, mixed in due proportion (*id.*); hence it comes to pass that white is the usual color of light; for light is an aggregate of rays of all sorts of colors (*id.*).

Newton has discovered what color is, too.

He concludes also: these things being so, it can no longer be disputed whether light is a body (*id.*).

Newton has shown also that light is a body.

He concludes also: I saw that the perfection of telescopes has hitherto been limited not because of the want of glasses configured according to the perfections of optics, as all men imagine, but because light is a heterogenous mixture of differently refrangible rays, and when a glass is configured to collect rays in one point, it cannot collect them all in the same point, for they suffer different refrangibility (*id.*).

He adds: this made me take reflection into consideration (*id.*). And in *An account of a new catadioptrical telescope* he describes the design of a reflecting telescope. Having since Galileo been refractors, for the following century and a half telescopes became reflectors.

The discovery of what color is, the demonstration that light is a body, and the reflecting telescope are each by itself a major advance.

Further, however, if the entire difficulty in philosophy is from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena, the optical are the investigations, next to the mathematical, that led to the discovery of the forces of nature. These are the investigations that led to *The principles*.

How the mathematical investigations led from the phenomena of motion to the discovery of the forces of nature is clear.

With the labors of Galileo and Kepler the description of the world, the generalization of calculation, and the formulation of the principles posed the problem of motion.

Motion posed the problems of distance, time, speed, and acceleration. These posed the problem of force.

Distance, time, speed, acceleration, and force posed the problems of infinite divisibility, maxima and minima, the relations of lines and numbers, the relations of straight lines and curves, rectification, tangents, and area, and the building of functions of great complexity and generality.

The problems of infinite divisibility, maxima and minima, the relations of lines and numbers, the relations of straight lines and curves, rectification, tangents, and area, and the building of functions of great complexity and generality were solved. As the development of calculus attests.

Hence the problems of distance, time, speed, acceleration, and force were solved. As *The principles* attests.

Hence the problem of motion was solved. As, again, *The principles* attests.

And how the optical investigations, next to the mathematical, led from the phenomena of motion to the discovery of the forces of nature, the investigation of motion shows.

To investigate motion, Galileo investigates bodies in motion.

So does Kepler. He writes: body is what god created in the beginning; and it is clear why; what god proposed is quantity; he needed everything that pertains to the essence of body, and quantity is the form of body which makes it body.

As the investigation of motion became mathematical, it became the investigation of quantity.

Descartes discovered that: god always con-

serves the quantity of motion in the world, and he always conserves the quantity of matter in the world. Hence, Descartes also first defined quantity of motion and quantity of matter. Of quantity of matter he writes: the quantity of matter does not differ from its substance more than number differs from numbered things; place, or space, does not signify a thing different from the body in a place, it designates its magnitude, figure, and situation among other bodies. And: all matter of which this visible world is composed was initially divided by god into particles as nearly equal as possible; and the particles had the same quantity of motion as there is in the world now.

Galileo investigates particles in order to investigate the constitution of bodies. Descartes investigates not only how particles constitute bodies, he investigates how particles constitute the vortices which move bodies.

As the investigation of motion became the investigation of quantity, it became the investigation of matter in motion, and hence of particles.

Newton writes in *The principles*: the attractions of bodies must be ascertained by assigning the forces properly to their particles. He shows that all particles in all bodies mutually attract one another. And writes in *On the nature of acids*: all bodies have particles which mutually attract one another.

And as the investigation of motion became the investigation of matter in motion, and of particles, it posed the problems of light.

Light was separated sharply from sight. It began being investigated in relation to matter. More important still, it began being investigated in relation to motion.

Aristotle has little to say about light. He discusses the senses in *On the soul*: sight (II, 7), hearing (8), smell (9), touch (10), taste (11), and what he has to say about light, if it is about light, is part of what he says about sight.

The investigation of light begins with Euclid. In the *Optics*, from 8 definitions he deduces 58 propositions. Definition 1 is: lines drawn from the eye pass through a space of great extent (*id.*); definition 2: the space within sight is a cone with its vertex at the eye and its base at the limit of sight (*id.*). Proposition 1 is: nothing seen is seen at once in its entirety (*id.*); proposition 2: things

at a smaller distance are seen more clearly than equal things at a greater distance (*id.*). And all definitions and propositions are about what and how is seen. Euclid's *Optics* is a treatise on perspective. It is the beginning of the investigation of light because it is a mathematical investigation of perspective. Because it is mathematical, the investigation of perspective will make it possible to distinguish light from perspective, and hence more generally from sight.

Ptolemy's *Optics* is a major advance. Ptolemy investigates reflection and refraction, and investigates refraction experimentally. It is the first treatise grounded substantially in experiment. Nevertheless, it is a treatise on perspective. Ptolemy does not distinguish light from sight, and all his results, including those on reflection and refraction, are with explicit reference to the eye and sight.

In the following millennium and a half the most important optical investigations are ibn al-Haytham's *Optics*, Grosseteste's *On the motion of bodies and light*, book V of Roger Bacon's *Great work*, and Witelo's *Perspective*. They are not an advance on Ptolemy. They are also treatises on perspective.

The difficulty of distinguishing light from sight – of establishing the fact of light independent of sight – suffices to show the importance of the problem of light.

And it is once again with Galileo and Kepler that the formulation and solution of the problems of light, separated sharply from sight, begins.

Galileo writes: if condensation and rarefaction are opposite motions, where there is great rarefaction, condensation no less enormous cannot be denied; we see the boundless rarefaction of a small quantity of gunpowder when it is resolved into a vast volume of fire; and what about the attendant unlimited expansion of light? if this fire and light were to be reunited, what condensation that would be (*Sciences*, I)!

Light is in relation to motion, to the opposite motions of rarefaction and condensation.

Galileo first asks also: does light travel instantaneously? or does it require time? and first proposes to answer the question experimentally. Kepler argues that light travels instantaneously. In the *Additions to Witelo*, in which the optical

part of astronomy is treated, he writes: light has no matter; it is therefore not resisted by the medium; its speed is therefore infinite (I, 1.5). Descartes concludes that light travels instantaneously from the measurement of lunar eclipses. And in 1676, four years after Newton's first optical publications, Rømer measured the speed of light.

Light requires time.

Kepler argues: the causes of the heavenly motions are purely bodily, in particular magnetic. To establish this, he argues: it is highly probable that motion is dispensed by the sun like light.

Light is dispensed, and like motion.

In the *Additions to Witelo*, Kepler writes: as light has two aspects, of the essence by which it is light, and of the quantities which it acquires, so it has two operations, the prior of locomotion, and the posterior of illumination (1.17); it is by motion that light spreads through media, and is thrown against surfaces, and is in turn either absorbed by the medium or forced to rebound from it (*id.*).

The essence of light is locomotion.

With Descartes *The world is The world, or a treatise on light*. One of the three essays in the method is the *Optics*. And in *The principles of philosophy* he writes: light consists entirely in the force by which particles accumulate around the centers of the vortices of stars (III, 55), and: considered in itself, the force of light is nothing but a certain pressure along the straight lines from a star to the earth (IV, 28).

Light is in relation to particles, and force.

In short, from the phenomena of motion – to the forces of nature.

Moreover, Bacon writes: the secrets of nature reveal themselves not when nature is free and left to herself, but when she is constricted and vexed, when she is forced out of her state, and molded and shaped.

So she was to reveal energy and entropy, Uranus and Neptune, magnetism and electricity, molecules and atoms, electrons and galaxies. And so of her deeply hidden secrets she first revealed light. When nature was free and left to herself light was not distinguished from sight. When nature was constricted and vexed what is light was discovered.

What, in turn, constriction and vexation is is illustrated perfectly by the crucial experiment and the measurement of the speed of light. To separate light from sight the crucial experiment passes light through a prism through a small hole through another small hole through another prism as the first prism rotates to and fro. To separate light from sight the measurement of its speed traces light from the earth as it revolves around the sun to Io as it revolves around Jupiter as it revolves around the sun back to the earth as it continues to revolve around the sun. Both have sight – not see – measure and calculate. Both involve also – prisms, Io, an earth revolving around the sun, measurement in the very small and in the very large, calculation – much, and rather most, of all earlier investigation of nature.

It is with the investigation of light that the investigation of nature became an investigation of the deeply hidden secrets of nature.

Newton showed that light is a body. And in *The principles* he pursues: the analogy between the propagation of rays of light and the motion of bodies. He shows that the analogy holds for reflection, refraction, and diffraction. It explains the speed of light. It explains color. It explains telescopes.

Newton showed that light is a body in 1672. Huygens showed that light is a wave in 1678. Not quite two and a half centuries later, quantum theory showed that light is both, a body and a wave.

The success of the investigation of nature from Aristotle through Newton can be described as success first and foremost of the investigation of motion. Aristotle established the fundamental character of the problems of motion, and in the three generations from Galileo and Kepler through Newton and the formulation of the laws of motion and the law of gravity the problems of motion were solved. The success of the investigation of nature after Newton can be described as success first and foremost of the investigation of light. In the three generations from Galileo and Kepler through Huygens and Newton the fundamental character of the problems of light was established, and in the following two and a half centuries these problems were solved. Maxwell discovered the laws of electricity and magnetism,

and showed that light is an electromagnetic wave. Relativity is the theory of space and time, standing on the postulate of constancy of the velocity of light. Quantum theory is the theory of matter and light. Or, as Feynman describes it, *QED, the strange theory of light and matter*.

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The last of Newton's very important treatises is the *Optics, or a treatise on the reflection, refraction, inflection, and colors of light*.

There are no major new results in the *Optics*. In the *Optics*, on the other hand, Newton generalizes over the acquisition of knowledge in its entirety. Over its entire breadth, as well as over its development, he also foretells the future. Rather better than Daniel or st John.

He begins: my design in this book is not to explain the properties of light by hypotheses, but to propose and prove them by reason and experiments (I, pr.).

The bulk of the treatise follows. From 9 definitions and 8 axioms Newton deduces 39 propositions, 33 experiments, and 48 observations.

He then writes: when I made the foregoing observations, I designed to repeat most of them with more care and exactness and make new ones to determine how the rays of light are bent in their passage by bodies; but I was interrupted and cannot now think of taking these things into further consideration; and since I have not finished this part of my design, I shall conclude by proposing some queries so that a further search be made by others (*id.*).

He proposes 31 queries. They can be divided into two parts, queries 1-30, and query 31. Queries 1-30 are a general description of the fundamental results obtained by his predecessors and Newton himself, and hence of the further search to be made by their successors. In the concluding query 31, as long as the preceding thirty, Newton is at his most general. He concerns himself with the world, the principles, the method, and the relations of mathematics, physics, and philosophy, including moral philosophy.

Among queries 1-30 are these:

1. do not bodies act on light at a distance, and bend its rays by their action; and is this action,

ceteris paribus, strongest at the least distance?

5. do not bodies and light act mutually on one another, bodies on light in emitting, reflecting, refracting, and inflecting it, light on bodies in heating them, and putting their parts in the vibrating motion in which heat consists?
6. do not black bodies conceive heat more easily from light than those of other colors, because the light falling on them is not reflected, but enters them, and is reflected and refracted in them until it is stifled and lost?
8. do not all bodies, when heated beyond a certain degree, emit light and shine; and is not this emission performed by the vibrating motions of their parts?
9. is not fire a body heated so hot as to emit light copiously? for what else is a red hot iron than fire? and what else is a burning coal than red hot wood?
11. do not great bodies conserve their heat the longest, their parts heating one another; and may not great bodies, when heated beyond a certain degree, emit light so copiously as by its emission and reaction and the reflections and refractions of its rays in their pores to grow still hotter, until they come to a period of heat like that of the sun? and are not the sun and fixed stars great earths vehemently hot, whose heat is conserved by their greatness, the mutual action and reaction in them, and the vast weight and density of their atmospheres?
18. is not the aethereal medium exceedingly more rare and subtle than air, and exceedingly more elastic and active? and does it not readily pervade all bodies? and is it not by its elastic force expanded through all the heavens?
19. does not the refraction of light proceed from the different density of this aethereal medium in different places, light receding always from the denser parts?
23. is not vision performed chiefly by the vibrations of this medium, excited in the bottom of the eye by the rays of light?
24. is not animal motion performed by the vibrations of this medium, excited in the brain by the power of the will, and propagated from it through the solid, pellucid, and uniform capillaries of the nerves into the muscles for contracting and dilating them?

28. are not hypotheses erroneous, in which light is supposed to consist in pressure or motion propagated through a fluid medium?
29. are not the rays of light very small bodies emitted from shining substances?
30. are not bodies and light convertible into one another, and may not bodies receive much of their activity from the particles of light which enter into their composition (*id.*)?

Queries 1-30 show that when the description of the acquisition of knowledge is Newton's it makes little difference whether the description is retrospective or prospective. Nor is this difficult to explain. With Bacon, the whole thing is done by machinery. And no philosopher has been better than Newton at operating the machinery.

Accordingly, the queries describe the acquisition of knowledge in the following centuries with some precision.

Do not bodies and light act mutually on one another? do not all bodies, when heated, emit light? is not this emission performed by the vibrating motions of their parts? is not fire a body heated so hot as to emit light copiously? are not the sun and fixed stars great earths vehemently hot, whose heat is conserved by their greatness, action and reaction in them, and the weight and density of their atmospheres? are the fundamental questions thermodynamics, electrodynamics, the kinetic theory, quantum theory, the big bang theory have answered.

There is no aether. Nevertheless, Maxwell developed electrodynamics and discovered its laws on the presumption of aether, the presumption that there is an aethereal medium exceedingly more rare and subtle than air, and exceedingly more elastic and active. Because Maxwell discovered that electricity and magnetism entail a field. Einstein showed, with the special theory of relativity, that the presumption of aether is superfluous and, with the general theory of relativity, that mass suffices to entail a field.

Vision and animal motion are not performed by the vibrations of the aether. Nevertheless, is not vision performed by vibrations excited in the bottom of the eye by the rays of light? and is excitement in the brain propagated from it through the nerves into the muscles for contracting and dilating them? are among the fundamen-

tal questions physiology has answered. As they have had to be, if, with Helmholtz: in their investigation of the processes of life physiologists must expect unconditional conformity to the laws of the forces of nature (*Goal*).

The fundamental result of quantum theory is that the hypothesis that light consists in pressure or motion is not erroneous and that the rays of light are very small bodies emitted from shining substances, both.

But the most dramatic are queries 6 and 30. Quantum theory begins with Planck's investigation of black bodies, and how the light falling on them is not reflected, but enters them, and is reflected and refracted in them. And that bodies and light are convertible into one another, and bodies receive much of their activity from the particles of light which enter into their composition, anticipates Einstein's discovery that mass and energy are convertible into one another, with the velocity of light the constant of proportionality of the conversion. $E = mc^2$.

In query 31 Newton generalizes further.

First he describes the tenor and course of nature:

have not the small particles of bodies certain powers, virtues, or forces, by which they act at a distance, not only on the rays of light for reflecting, refracting, and inflecting them, but also on one another for producing a great part of the phenomena of nature? for bodies act on one another by gravity, magnetism, and electricity, and these show the tenor and course of nature, and make it not improbable that there may be more attractive powers than these; all bodies seem to be composed of hard particles; the smallest particles may cohere by the strongest attractions and compose bigger particles of weaker virtue; these may cohere and compose bigger particles whose virtue is still weaker, and so on, until the progression ends in the biggest particles on which the operations of chemistry and the colors of natural bodies depend, and which compose bodies of sensible magnitude;

nature will be conformable to herself and very simple in performing all the great motions of the heavenly bodies by the attraction of gravity and almost all the small ones of their particles

by some other attractive and repelling powers (*id.*).

Gravity, magnetism, and electricity show the tenor and course of nature. And there are more attractive powers. The powers are the four, gravitational, strong, weak, and electromagnetic. All bodies are composed of particles. Quarks and leptons cohere by the strongest attractions and compose atoms of weaker virtue, atoms cohere and compose molecules whose virtue is still weaker. And in performing all motions by four attractive and repelling powers, nature is conformable to herself and very simple.

Newton writes: god gave prophecies not to gratify men's curiosities by enabling them to foreknow things, but so that, after the prophecies are fulfilled, they may be interpreted by the events, and thus manifest his providence to the world.

More than any other philosopher Newton manifested to the world the tenor and course of nature. Hence his prophetic power. And his prophecies, interpreted by the events, manifest to the world the tenor and course of nature as well as the fact that the tenor and course of nature has been manifested to the world.

In query 31 Newton next describes the principles:

vis inertiae is a passive principle by which bodies persist in their motion or rest; by this principle alone there could never have been any motion in the world; some other principle was necessary to put bodies in motion; and now that they are in motion, some other principle is necessary to conserve the motion;

seeing that the variety of motion in the world is always decreasing, there is a necessity of conserving and recruiting it by active principles, such as are the cause of gravity, by which the planets and comets keep their motions and bodies acquire motion in falling, and the cause of fermentation, by which the heart and blood of animals are kept in perpetual motion and heat;

considering all these things, it seems probable to me that god in the beginning formed matter in solid, massy, hard, impenetrable, movable particles, of such sizes and figures, with such other properties, and in such proportion to

space, as most conduced to the end for which he formed them;

it seems to me further that these particles had not only *vis inertiae*, with such passive laws of motion as naturally result from it, but that they also are moved by certain active principles, such as that of gravity, and that which causes fermentation, and the cohesion of bodies; and these principles I consider not occult qualities, but general laws of nature (*id.*).

The principles are general laws of nature. They are passive and active. They reach from the first formation of matter to the keeping of animals in perpetual motion and heat. The passive his predecessors and Newton have discovered. Inertia is how bodies persist in their motion or rest. Gravity is how the planets and comets keep their motions and bodies acquire motion in falling. The active their successors have discovered. Entropy is why the variety of motion in the world is always decreasing. The big bang is why, and for how long, motion is conserved and recruited. Darwin's principle keeps animals in perpetual motion and heat. The big bang is also how the first formation of matter begins.

The principles show the tenor and course of nature. Or what is the same, the general laws of nature show the conformity to herself and simplicity of nature. Or what is yet the same, their conformity to themselves and simplicity shows her conformity to herself and simplicity. And conversely, the tenor and course of nature shows that the principles are the principles. Her conformity to herself and simplicity shows that her general laws have been discovered.

They have been by following the true way.

Newton thus next describes the method, and the relations of mathematics, physics, and philosophy:

as in mathematics so in natural philosophy the investigation of difficult things by the method of analysis ought always to precede the method of composition; analysis consists in making experiments and observations, and drawing general conclusions from them by induction, and admitting no objections against the conclusions except such as are taken from experiments or other certain truths;

and although arguing from experiments and

observations by induction is not demonstration of general conclusions, it is the best way which the nature of things admits, and it may be looked upon as the stronger the more general the induction is;

by this way of analysis we may proceed from effects to their causes, and from particular causes to more general, until the argument ends in the most general; this is the method of analysis; synthesis consists in assuming the causes discovered and established as principles, and explaining the phenomena from them, and proving the explanations (*id.*).

There are two differences between Bacon's description of the true way and Newton's.

One is that in the three generations from Bacon through Newton the true way has been for the first time resolutely followed, and Newton describes this fact.

The true way has ascended, descended, ascended, descended, and ascended.

First has been induction, then deduction.

Induction has been observations and experiments.

Induction has proceeded from effects to their causes, and from particular causes to more general, until the argument has ended in the most general.

Deduction has assumed the causes discovered and established as principles, and has explained the phenomena from them, and proved the explanations.

It has not only been the best way which the nature of things admits. It has been the only way which the nature of things admits. There are and can only be two ways. *Tertium non datur*. And the only way which the nature of things admits has admitted no objections, because it can admit no objections, except such as are taken from experiments or other certain truths.

The other difference is that Newton's description is also like Descartes's. It is more general, and it is explicit that the true way is the way of mathematics.

Because, with Descartes, mathematics has long been the spontaneous fruit of the innate principles of the method. With Newton, in physics as in mathematics.

Analysis ought always to precede synthesis. In

physics as in mathematics.

Accordingly, analysis precedes synthesis in Newton's development of mathematics.

The *Analysis by equations* and *Method of fluxions* are analysis. *The principles* is synthesis. The *Analysis* and *Method* are analysis as much as any earlier mathematical treatise is. *The principles* is synthesis as much as Euclid's *Elements* is. Given the dichotomies of calculation and demonstration, induction and deduction, arithmetic and geometry, and hence analysis and synthesis, the *Analysis* and *Method* are calculation, induction, arithmetic, and hence analysis, *The principles* is demonstration, deduction, geometry, and hence synthesis.

Accordingly, more generally, analysis precedes synthesis in the development of calculus.

From Galileo and Kepler through Descartes, Cavalieri, Fermat, Wallis, and Huygens through Newton and Leibniz the development is calculation rather than demonstration, induction rather than deduction, arithmetic rather than geometry, and hence analysis rather than synthesis, more than any development in mathematics after Euclid is. After Gauss, after Cauchy, and decisively after Weierstrass, the development becomes demonstration rather than calculation, deduction rather than induction, and hence paradigmatically synthesis. Even if it is no longer geometry. And even if it continues to call itself analysis – remembering its beginnings, presumably, and conforming to the division of judgments into analytic and synthetic rather than to the older dichotomy of analysis and synthesis, verbally inconsistent as the two are.

Accordingly, most generally, analysis precedes synthesis in the development of mathematics.

In the first twenty-five and a half centuries of its development, from the 3 oldest calculations of area to Pythagoras, mathematics is calculation, and not demonstration. It is also fundamentally arithmetic, geometry part of it.

Pythagoras then discovered demonstration. Two centuries later Aristotle distinguished sharply deduction and induction. One generation-odd later Euclid wrote the *Elements*. The *Elements* is the paradigm of demonstration, and of deduction. It is the paradigm also of geometry. Mathematics became demonstration, and deduc-

tion. It became fundamentally also geometry, arithmetic now part of it. The major advances in mathematics in the century after Euclid, of Archimedes and of Apollonius, are advances of demonstration, deduction, and geometry.

Six-odd centuries after Euclid, Pappus defined analysis and synthesis. Pappus writes: in analysis we assume what is sought as if it is done, and look for the thing from which it follows, and for what comes before that, and until we come to a thing already known, or to a first principle; in synthesis, conversely, we assume what is obtained last in analysis done, and what were there antecedents we arrange here as consequents, in their natural order, and in their connection with one another, in order to end with the construction of what is sought (*Collection*, VII, 1).

And twelve-odd centuries after Pappus, when in the century from Cardano through Viète to Descartes and Fermat mathematics began advancing greatly once again, synthesis was associated firmly, and more particularly, with demonstration, deduction, and geometry. Analysis was associated less firmly, and more generally, with calculation, induction, and arithmetic. It was associated also with algebra. It was associated also, very firmly, with discovery. In synthesis we arrange the consequents in their natural order. We discover the antecedents which will become consequents as well as their natural order in analysis.

Analysis ought always to precede synthesis because analysis always precedes synthesis.

It does in that mathematics was calculation for twenty-five and a half centuries before it discovered demonstration.

In that in the two and a half centuries from Pythagoras to Euclid the antecedents were discovered, by Theodorus of Cyrene, Hippocrates of Chios, Archytas, Theaetetus, Eudoxus, which then became consequents in the *Elements*.

In that the axioms never are, and by being axioms never can be, the result of synthesis, and are not the result of synthesis in Euclid, or in Archimedes, or in Apollonius, in the century of the greatest success of synthesis. In that, too, that the same is the century also of Aristarchus, Chrysippus, Eratosthenes, Diocles, all of whose labors are inductive more than deductive. The

success of Euclid's *Phenomena* and *Optics*, too, is also inductive more than deductive, and Archimedes's *Sand-reckoner* is explicitly inductive rather than deductive.

In that in the following five centuries through Pappus the greatest advances in mathematics are Ptolemy's and Diophantus's, and Ptolemy's is explicitly inductive, Diophantus's is the major step in the separation of arithmetic from geometry.

In that in the following twelve centuries the labors of al-Khwarizmi, ibn al-Haytham, Khayyam, Fibonacci, Buridan, Oresme, Regiomontanus are all analytic, not synthetic.

In that in the following one and a half centuries calculus was developed. Not synthetically.

In that in the following century-odd, with Jakob Bernoulli, Saccheri, Johann Bernoulli, Taylor, Daniel Bernoulli, Euler, d'Alembert, Lambert, Lagrange, Laplace, Fourier, mathematics continued to advance by calculation and induction rather than demonstration and deduction.

In that in the following century, finally, a century once again of demonstration and deduction – and rigor – the most important advances in mathematics are those of Lobachevsky, Riemann, Cantor, and Frege, and once again analysis precedes synthesis. Lobachevsky developed non-Euclidean geometry from *On the principles of geometry*, in 1829, through the *Geometrical investigations on the theory of parallels*, in 1840, analytically. The most important results anticipating the development of non-Euclidean geometry in the previous century, those of Saccheri and Lambert, are analytic as well. Riemann's generalization of this development, next, in *On the hypotheses which lie at the foundation of geometry*, in 1854, is synthetic. Cantor developed the theory of sets from *On a property of the totality of all real algebraic numbers*, in 1874, analytically, to the *Contributions to the foundation of the transfinite theory of sets*, in 1895-7, synthetically. And in the preface to the *Concept writing*, in 1879, Frege describes in some detail the trials and errors which led to the development of the concept writing, tracing them before his own back to Leibniz.

The method is the method. There are and can

only be two ways to search for truth, and the true way is and can only be one and the same.

And if so, in physics as in mathematics, because in mathematics as in physics.

First calculation, then demonstration, because first induction, then deduction. But induction is observation, including experiment. Observation is calculation, including measurement. And if first calculation, then demonstration, because first induction, then deduction, calculation is observation.

Mathematics is a part of physics. Nor can mathematics not be a part of physics, if mathematics is a part of philosophy, and physics is synonymously natural philosophy.

Nor is philosophy natural and human, a search for nature and a search for man. Philosophy is theoretical and practical, a search for truth and a search for action.

And Newton ends in query 31, more generally in the *Optics*, and most generally in philosophy, by drawing from the search for truth conclusions for the search for action:

and if natural philosophy in all its parts, by pursuing this method, shall at length be perfected, the bounds of moral philosophy will be enlarged; for so far as we can know by natural philosophy what is the first cause, what power he has over us, and what benefits we receive from him, so far our duty toward him, as well as toward one another, will appear to us by the light of nature;

and no doubt, if the worship of false gods had not blinded the heathen, their moral philosophy would have gone further than the four cardinal virtues; and instead of teaching the transmigration of souls, and to worship the sun and moon, and dead heroes, they would have taught us to worship our true author and benefactor, as their ancestors did under the government of Noah and his sons before they corrupted themselves (*id.*).

If god is nature, the worship of false gods has blinded and continued to blind not only the heathen.

On the other hand, if natural philosophy be perfected, the bounds of moral philosophy will be enlarged.

On the authority of Ussher, in 1650, in *The*

annals of the old testament deduced from the first origin of the world, the ultimate step in the one and a half millennia of research into the genealogies and lists of the holy scriptures: Noah was born in 2948^{BC} and died in 1998^{BC} (II). Noah was thus born not quite two centuries after the predecessor of the philosophers who left behind the 3 oldest calculations of area.

And in the five-odd millennia since, counting from the predecessor of the philosophers who left behind the 3 oldest calculations of area, not from Noah, there have been times when natural philosophy has been perfected and times when it has not been. The latter have included times when it has been abandoned altogether. There have also been times when the bounds of moral philosophy have been enlarged and times when they have not been. The latter have included times when they have been diminished severely. The times when natural philosophy has been perfected have not all been times when the bounds of moral philosophy have been enlarged. The last couple of centuries have been such a time, for instance. Conversely, however, the times when the bounds of moral philosophy have been diminished have all been times when natural philosophy has not been perfected. And the times when the bounds of moral philosophy have been enlarged have not only all been times when natural philosophy has been perfected. The enlargement of the bounds of moral philosophy has been the explicit consequence of the perfection of natural philosophy. In the time of enlightenment, for instance, from the generation of Galileo through that of Spinoza through that of Hume and of Kant.

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The principles concludes with *The system of the world*.

Newton writes: in the preceding books I have delivered the principles of philosophy; it remains that from the same principles we teach the constitution of the system of the world.

He begins by teaching 4 rules of philosophizing. He then introduces 6 phenomena. And he then deduces 42 propositions, 11 lemmas, and 50 corollaries. Along the way he introduces also 2 hypotheses, one before proposition 11, one before proposition 39. Also, so that the principles may

not appear sterile, he illustrates them with 7 scholia, including the concluding general scholium.

The rules are:

- rule 1. no more causes of natural things must be admitted than are true and suffice to explain their phenomena;
- rule 2. therefore, to natural effects of the same kind the same causes must be assigned, as much as possible;
- rule 3. qualities of bodies which cannot be increased or decreased, and which belong to all bodies on which experiments can be made, must be taken for qualities of bodies universally;
- rule 4. in experimental philosophy propositions gathered from the phenomena by induction must be taken, contrary hypotheses notwithstanding, for true, or accurate, or very nearly so, until other phenomena occur to make them either more accurate or liable to exceptions (*id.*).

On rule 1: no more causes must be admitted than are true and suffice to explain the phenomena, Newton comments:

this is why the philosophers say, nature does nothing in vain; and what can be done with less is done in vain with more; nature is simple, and things do not luxuriate in superfluous causes (*id.*).

On rule 2: to effects of the same kind the same causes must be assigned, he comments:

to the respiration of man and of beast, to the falling of stones in Europe and in America, to the light of a cooking fire and of the sun, to the reflection of light on earth and on the planets (*id.*).

On rule 3: qualities which belong to all bodies on which experiments can be made must be taken for qualities universally, he comments:

certainly idle fantasies are not to be feigned against the evidence of experiment;

the extension, hardness, impenetrability, mobility, and force of inertia of a whole are from the extension, hardness, impenetrability, mobility, and force of inertia of its parts, from which we conclude that all the smallest particles of all bodies are extended, hard, impenetrable, mobile, and endowed with force of inertia; and

this is the foundation of all philosophy (*id.*). And on rule 4: propositions gathered from the phenomena by induction must be taken for true until other phenomena make them either more accurate or liable to exceptions, he comments: this must be done so that arguments from induction not be negated by hypotheses (*id.*).

The 6 phenomena follow:

phenomenon 1. the satellites of Jupiter describe around the center of Jupiter areas proportional to the times, and their periods are as the $\frac{3}{2}$

powers of their distances from this center;

phenomenon 2. the satellites of Saturn describe around the center of Saturn areas proportional to the times, and their periods are as the $\frac{3}{2}$

powers of their distances from this center;

phenomenon 3. the five principal planets, Mercury, Venus, Mars, Jupiter, and Saturn, revolve around the sun;

phenomenon 4. the periods of the five principal planets are as the $\frac{3}{2}$ powers of their distances

from the sun;

phenomenon 5. the five principal planets describe around the sun areas proportional to the times;

phenomenon 6. the moon describes around the earth areas proportional to the times (*id.*).

On phenomenon 4 Newton comments: this proportion was found by Kepler (*id.*). This proportion is Kepler's third law. The phenomena affirm that the satellites of Jupiter, the satellites of Saturn, and the five principal planets obey Kepler's third law. They affirm also that the satellites of Jupiter, the satellites of Saturn, the five principal planets, and the satellite of the earth obey Kepler's second law, and more generally, obey the generalization of Kepler's second law demonstrated by Newton in proposition 1 of book I: bodies in orbits around a center of force describe areas proportional to the times.

In book I this result follows from the laws of motion. Here: it is established by astronomical observations (ph. 1). Newton reports observations, other than his own, of: Kepler, Boulliau, Borelli, Cassini, Huygens, Towneley, and Pound. And how firmly the result is established by astronomical observations phenomenon 1 can

illustrate. The distances of the satellites of Jupiter from its center, from the observations of Cassini, on the left, and from the periodic times, on the right, in radii of Jupiter, are:

Io	5.667	5.667,
Europa	9.000	9.017,
Ganymede	14.383	14.384,
Callisto	25.300	25.299 (<i>id.</i>).

After the rules and the phenomena the propositions follow.

And the rules point to what the propositions will teach, the phenomena point to how they will teach it, in order to teach the constitution of the system of the world.

Bacon writes: true knowledge is knowledge by causes. Galileo writes: to recognize the cause removes the marvel (*Sciences*, I). Kepler writes: no other way can succeed but grounding the motions in their causes. Descartes writes: to be perfect knowledge must be deduced from first causes. Huygens writes: in the true philosophy the cause of all effects is sought. Spinoza writes: we understand god to the degree to which we know the causes of nature.

Rule 2 is: to effects of the same kind the same causes must be assigned, as much as possible. And the propositions will succeed in assigning to effects of the same kind the same causes as much as it is possible to succeed. They will assign to all effects one and only one cause.

Gravity. Newton will assign to all major effects in the world explicitly, and hence also to all other effects in the world implicitly, this one and only one cause.

This is what the propositions will teach.

As to how, Descartes's rule 3 is: to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex, and the phenomena point to the order from the simplest and easiest to the more complex. Beginning with the satellites of Jupiter and Saturn, the propositions will ascend to the planets and the sun, then to the earth and the moon.

The simplest and easiest are the heavenly bodies. Points of light against the night sky, no objects are given more clearly and distinctly to the eye, and hence to measurement and calculation, than the heavenly bodies. Observation thus begins with the observation of the heavenly

bodies. The observation of Sirius in Egypt, and with the Venus tablet of Ammisaduqa in Mesopotamia. The first investigation brought to perfection is the investigation of the heavenly bodies. By Ptolemy. The first laws discovered are the laws of the heavenly bodies. Kepler's. And the 6 phenomena are phenomena of the heavenly bodies, established by astronomical observations.

The heavenly bodies are of two kinds, the fixed stars and the wanderers. A myriad of them, all apparently moving in circles, no objects are given more clearly and distinctly to measurement and calculation as uniform than the fixed stars. And no objects pose more clearly and distinctly the problem of reducing not uniformity to uniformity than the wanderers. They move apparently not uniformly. But they are a handful moving not uniformly among a myriad moving uniformly. They move, moreover, not uniformly but not too much so. They do not move haphazardly, they rather deviate from the uniform motion of the fixed stars, and not too much either. They move not uniformly enough to pose the problem of the not uniformity of their motion, but uniformly enough to pose it as a problem of reducing the not uniformity of their motion to the uniformity of the motion of the fixed stars. This is the problem as Ptolemy solved it. Copernicus showed that the wanderers revolve around the sun. Kepler separated the problem of the uniformity of the motion of the bodies of the solar system from the presumption that it is the uniformity of the motion of the fixed stars, and solved the so reformulated problem. Kepler thus also showed that the bodies of the solar system are the simplest and easiest. And the 6 phenomena are phenomena of the bodies of the solar system.

The bodies of the solar system are the sun, the planets, the satellites, and comets. And the simplest and easiest are the satellites of Jupiter and Saturn. When Galileo discovered the satellites of Jupiter, then Huygens and Cassini the satellites of Saturn, they discovered objects not only given to measurement and calculation as clearly and distinctly as any objects are, but given close to their centers of revolution, and with very short periods – moving with marvelous speed, as Galileo describes them. The satellites of Jupiter thus provided the first measurement of the speed

of light. And the satellites of Jupiter and Saturn are phenomena 1 and 2, their observation showing Kepler's laws more directly and more exactly than the observation of the planets.

Next are the planets and the sun, phenomena 3 to 5. Last are the earth and the moon, phenomenon 6.

The earth is more complex and difficult because it is, as Kepler describes it, the home of the measuring creature. The creature has seldom been eager to be the measuring creature. And when it has been, the difficulty with respect to its home is that its home cannot not be the ground of measurement, and if it is to be itself measured, it has to be measured on the same ground. It is the point of observation which, if it is to be itself observed, has to be observed from the same point. It is the greatest difficulty which the propositions will resolve – the ascent from the heavenly bodies to the earth, and from the earth back to the heavenly bodies.

The moon is more complex and difficult because it is large. It is the fifth largest satellite in the solar system, after Ganymede, Titan, Callisto, and Io. While Ganymede, Callisto, and Io revolve around Jupiter, however, which is more than 300 times larger than the earth, and Titan revolves around Saturn, which is nearly 100 times larger than the earth, the moon revolves around the earth. It is a large satellite of a small planet. The moon and the earth perturb one another's motions more than any other pair of bodies in the solar system. And reducing the not uniformity of the motion of the moon to uniformity will demand more of the propositions than any other problem.

The propositions will end with comets. Tycho showed that comets are not atmospheric phenomena, as they had been taken to be since Aristotle, that they move among the planets. Newton will show that comets are bodies of the solar system, like the planets and satellites, and will reduce their motions, too, to uniformity.

Aristotle first described the system of the world. With Ptolemy the description became quantitative. Copernicus showed that the system is solar. With Kepler the description was grounded in laws. With Descartes it reached beyond the solar system. And with Newton it will become systematic constitutionally. It will be

grounded in one cause. Or what is the same, in one force and its law.

Newton will assign one cause to all major effects of the bodies of the solar system. He will assign the same cause to all major effects of the earth as one of the bodies of the solar system. He will assign the same cause to major effects of bodies on the earth. He will assign the same cause to effects of bodies both on the earth and in the heaven. All this in the propositions of book III explicitly.

Hence, through the numerous explicit connections of these propositions with all results of his predecessors, and all his results in books I and II, and all his results outside *The principles*, Newton will assign the same cause implicitly to all effects.

The qualities of a whole are from the qualities of its parts, from which we conclude that they are the qualities of all parts of all bodies. And this is the foundation of all philosophy.

And all investigation in the following century-odd is labors to assign the same cause explicitly to all the effects to which Newton has assigned it only implicitly. To the smallest particles. To electric and magnetic motions. To stars and nebulae. It was in the century after that, after Faraday, that it became clear that another cause has to be assigned to electricity and magnetism.

clxxv

Analysis ought always to precede synthesis.

And propositions 1 through 7 are the analysis, propositions 8 through 42 are the synthesis.

Analysis proceeds from effects to their causes, and from particular causes to more general, until the argument ends in the most general.

And the analysis proceeds from the phenomena to their causes. It proceeds from particular causes, from the forces which maintain the satellites of Jupiter in orbits, in proposition 1, to more general causes, in propositions 2 through 6. Until it ends in the most general, in proposition 7. Until it ends in gravity. Proposition 7 affirms: there is gravity in bodies universally (*id.*).

Along the way Newton has also formulated the law of the most general cause. He has shown that the force of gravity is inversely proportional to the square of the distance between bodies, in

propositions 1-3, and proportional to the quantity of matter, or mass, of bodies, in propositions 4-7. If the force is F , the masses are m_1 and m_2 , and the distance between them is r ,

$$F \propto \frac{m_1 m_2}{r^2}.$$

Synthesis consists in assuming the causes discovered and established as principles, and explaining the phenomena from them, and proving the explanations. Or, as Newton writes here: above we have considered the motions from the phenomena; now that the principles are known, we deduce the celestial motions from the principles *a priori* (13).

And the synthesis is Newton's ultimate *tour de force*. It comprises 85 propositions, lemmas, and corollaries. 4 of these demonstrate trivial results. Trivial, that is, being results in *The principles*. The remaining 81 all break new ground.

Trivial are propositions 9: gravity decreases from the surfaces of the planets inward in the ratio of the distances from their centers, very nearly (*id.*), 15: to find the principal diameters of the orbits (*id.*), 16: to find the eccentricities and aphelia of the orbits (*id.*), and 21: the equinoctial points regress, and the axis of the earth in its annual revolution twice inclines toward the ecliptic and twice returns to its former position (*id.*). Each of these Newton demonstrates in a couple of lines by an appeal to propositions demonstrated in books I and II.

In the remaining 81 propositions, lemmas, and corollaries Newton formulates and solves many more than 81 problems. Still, these propositions, lemmas, and corollaries can be described under the following general heads, the moon, comets, the precession of the equinoxes, the heavenly bodies, the earth, the world.

The moon is the central problem of propositions 17 and 22 through 38 and their corollaries. And, again, here as everywhere, the solution of the central problem depends on the solution of many other problems and leads to solution of many other problems.

Newton affirms:

proposition 22. all motions of the moon and all inequalities in its motions follow from the principles (*id.*).

He then enumerates the inequalities:

the eccentricity of the moon is greatest when its apogee is in syzygy and least when it is in quadrature, so that the moon is swifter in perigee and slower in apogee; the apogee also advances and the nodes regress, and not uniformly; the apogee also advances more swiftly in syzygy and regresses more slowly in quadrature; the nodes also are at rest in syzygy and regress in quadrature; the greatest latitude also is greater in quadrature than in syzygy; the mean motion also is slower when the earth is in perihelion than when it is in aphelion;

these are the more significant inequalities noted by the astronomers; but there are other inequalities, which have not been observed before, and by which the motions of the moon are so perturbed that they have not until now been possible to reduce by a law to a definite rule; the velocity of the annual motions of the apogee and of the nodes, as well as the difference between the greatest eccentricity in syzygy and the least in quadrature, are as the cube of the apparent diameter of the sun, and this variation itself is very nearly as the square of the time between quadratures (*id.*).

The satellites, and in particular the moon, revolve around the planets, and the moon around the earth, as the planets revolve around the sun. The orbits are ellipses, and eccentricity is the degree of divergence of an ellipse from a circle. The greatest and least distances from the sun are the aphelion and perihelion, the greatest and least distances from the earth are the apogee and perigee. When the sun, the earth, and another heavenly body are in a straight line this is the line of syzygy, and the sun and the other body are either in opposition, when the earth is between them, or in conjunction, when the sun and the other body are on the same side of the earth. A heavenly body is in quadrature when it appears at a right angle from the sun. A heavenly body advances or regresses when in a given time it completes more or less than one revolution. The plane of the revolution of the earth around the sun is the ecliptic, the divergence from it is a divergence in latitude. And the intersection of the plane of revolution of another heavenly body and the ecliptic is the line of nodes.

In proposition 22 Newton enumerates the

inequalities, and in propositions 23 through 35 does what has not been done before: reduces the motions of the moon by a law to a definite rule. Therefore: all motions of the moon and all their inequalities follow from the principles. And he concludes in the scholium after proposition 35: with these calculations I wanted to show that the motions of the moon can be calculated from their causes by the theory of gravity (*id.*).

The difficulty of the problem of the moon is indicated by the enumeration of its inequalities. It is indicated also by the fact that its solution has to deduce from the theory the perturbations in the revolution of the moon around the earth by the sun, as the earth revolves around the sun and its revolution is perturbed by the moon, as the moon revolves around the earth and its revolution is perturbed by the sun.

To improve on Newton's solution of the problem of the moon remained one of the outstanding problems concerning the solar system for another century. Two generations after Newton, in the *Theory of the motion of the moon, accounting for all inequalities*, in 1753, Euler writes: certainly, if the motion of the apogee of the moon, such as it is found by observation to be 40° per year, could be deduced from Newton's immortal theory, there could be no stronger argument to show its truth (*intr.*). It was a generation-odd later that all outstanding problems concerning the solar system were solved, including the problem of the moon. Laplace, in seven treatises published from 1785 to 1790, showed that all perturbations in the orbits of the bodies of the solar system are periodic and self-correcting, and the system is therefore stable.

In proposition 17 Newton shows that: the libration of the moon follows from its daily motion (*id.*). Libration is partial axial rotation, and the moon exhibits libration both in longitude and in latitude.

In propositions 24, 36, and 37 Newton solves the problem of the tides:

proposition 24. the tides arise from the actions of the sun and the moon;

proposition 36. to find the force of the sun to move the sea;

proposition 37. to find the force of the moon to move the sea (*id.*).

Newton calculates first the height of the tides due to the action of the sun from the force with which the sun attracts the earth. Hence, from the observed height of the tides at conjunction and opposition of the sun and the moon, and due thus to their joint action, he calculates the height due to the action of the moon.

Hence, he calculates also the force with which the moon attracts the earth (*cor.* 2). And hence, having in proposition 8 calculated the masses and densities of the sun and of the earth, he calculates here the mass and density of the moon (*cor.* 3-4).

In proposition 38 he deduces the shape of the moon. As in attracting the earth the moon produces tides, so in attracting the moon the earth has produced its shape: a spheroid whose greatest diameter passes, if extended, through the center of the earth, and exceeds the diameter perpendicular to it by 186 feet (*id.*).

Comets are the central problem of propositions 40 through 42 with their corollaries and the preparatory lemmas 4 through 11 with their corollaries. Newton shows that comets revolve around the sun:

proposition 40. comets move in conics with foci in the center of the sun, and describe areas proportional to the times (*id.*).

He shows also how to calculate their orbits:

corollary 1. the orbits are ellipses;

corollary 2. but they are so close to parabolas that parabolas can be substituted for them without sensible error;

proposition 41. to determine the trajectory of a comet moving in a parabola from three given observations;

proposition 42. to correct the calculated trajectory (*id.*).

In *A synopsis of the astronomy of comets*, in 1705, Halley concludes: we are thus able to determine the period and axis of revolution of a comet, and hence to foretell its return; indeed, I believe that the comet which Apian observed in 1531, the one which Kepler and Longomontanus observed in 1607, and the one which I observed in 1682, are one and the same (2); and I dare venture to foretell that it will return in 1758 (*id.*).

And so in 1758 Halley's comet did.

The precession of the equinoxes is the central problem of proposition 39 and the preparatory

lemmas 1 through 3. Other than the lemmas, the preparation for the solution of this problem demands the second of the two hypotheses in book III, third of the three in *The principles*. Newton has shown in proposition 18 that the earth is not exactly a sphere, but an elliptic spheroid flattened at the poles. The earth can thus be considered a sphere surrounded by an equatorial ring of matter. And:

hypothesis 2. the motion of the equinoctial points will be the same whether this ring is fluid or consists of rigid and solid matter (*id.*).

Newton then deduces the precession of the equinoxes from the shape of the earth, from the action of the sun and the moon on the earth such as it is not a sphere, but an elliptic spheroid flattened at the poles. In particular: the annual precession due to the force of the sun is $9''7'''20''''$; that due to the force of the moon is $40''52'''52''''$; the total is thus $50''00'''12''''$; and this agrees with the phenomena; the precession is found from observation to be more or less $50''$ annually (*id.*).

Spectacular as all these results are, the following are more so still.

The analysis concludes with proposition 7: there is gravity in bodies universally, and the synthesis begin with proposition 8 and its 4 corollaries. The central problem are the heavenly bodies. And Newton demonstrates that the heavenly bodies are just that, bodies. The earth, too, is just that, a body. Newton shows how the mass and density of any heavenly body observed with another heavenly body revolving around it can be calculated, and hence the weight of a body on its surface, or what is the same, the force of attraction on its surface, can be calculated, too.

In particular, the sun, Jupiter, Saturn, and the earth are so observed, with the planets and their satellites, respectively, revolving around each.

These are Newton's results:

	diameter	density	mass	force on surface
sun	10 000	100	1	10 000,
Jupiter	997	$94\frac{1}{2}$	$\frac{1}{1\ 067}$	943,
Saturn	791	67	$\frac{1}{3\ 021}$	529,
earth	109	400	$\frac{1}{169\ 282}$	435

(*id.*).

And this is how he obtains those about Jupiter.

He writes:

the weights of equal bodies revolving in circles around planets are, by book I, proposition 4, corollary 2, as the diameters of the circles directly and the squares of the periods inversely, and the weights at the surfaces of the planets or at any other distances from the centers are as the squares of the distances inversely (cor. 1).

The weights are as the forces, the diameters of the circles are as the distances, and if the force is F , the distance is r , and the period is T ,

$$F \propto \frac{r}{T^2}, \text{ and } F \propto \frac{1}{r^2}.$$

Both follow from Kepler's third law,

$$\frac{T^2}{r^3} \text{ is constant.}$$

Newton continues:

I compared the period of Venus around the sun, which is 224 days and $16\frac{3}{4}$ hours, and that of

the outermost satellite of Jupiter, which is 16 days and $16\frac{8}{15}$ hours, with the mean distance

of Venus from the sun and the greatest heliocentric elongation of the satellite from the center of Jupiter, which is $8'16''$; and in this way I found by calculation that the weights of equal bodies equally distant from the centers of the sun and of Jupiter are as 1 to $\frac{1}{1\ 067}$

(cor. 1).

From phenomenon 4, the mean distance of Venus from the sun, with the mean distance of the earth from the sun as 1, is 0.72333 (*id.*). Also from phenomenon 4, the mean distance of Jupiter from the sun is 5.20096 (*id.*), and the sine of the greatest heliocentric elongation of Callisto multiplied by the mean distance of Jupiter from the sun gives the greatest heliocentric distance of Callisto from Jupiter at 0.01251.

The period of Venus is 224.69792, the period of Callisto is 16.68889.

$F \propto \frac{r}{T^2}$. Hence, $\frac{0.72333}{(224.69792)^2} = 0.000014326$, and $\frac{0.01251}{(16.68889)^2} = 0.000044916$, and the forces of the

sun attracting Venus and of Jupiter attracting Callisto are as 14 326 to 44 916.

$$F \propto \frac{1}{r^2}, \text{ or } \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}, \text{ or } F_1 = \frac{r_2^2 \times F_2}{r_1^2}. \text{ Hence,}$$

$$\frac{(0.01251)^2 \times 44\,916}{(0.72333)^2} = 13.43571 \text{ is the force with}$$

which Jupiter will attract Callisto at a distance such as Venus is from the sun.

$$14\,326 \text{ is to } 13.43571 \text{ as } 1 \text{ to } \frac{1}{1\,066}. \text{ And if}$$

Newton obtained 1 to $\frac{1}{1\,067}$, it is because he calculated beyond the fifth digit after the decimal point.

This is the ratio of the forces, and *ipso facto* also of the masses.

Newton continues:

the diameters of the sun and Jupiter are found to be as 10 000 to 997; and the weights of equal bodies toward the sun and Jupiter at distances of 10 000 and 997 from their centers, respectively, are as 10 000 to 943 (cor. 3).

$$\text{Again, } F_1 = \frac{r_2^2 \times F_2}{r_1^2}. \text{ And if at a distance of}$$

$$10\,000 \text{ the force is } 1, \text{ at a distance of } 997 \text{ it is } \frac{(10\,000)^2 \times 1}{1\,067} = 0.0943.$$

Finally:

$$\text{the densities are as } 100 \text{ to } 94\frac{1}{2} \text{ (id.).}$$

Given the masses and the radii, this is trivial.

If the diameter, density, mass, and force of attraction on the surface of the sun are, respectively, 10 000, 100, 1, and 10 000, the diameter of Jupiter is

$$\begin{array}{ll} \text{Newton} & 997, \\ \text{correction} & 1\,028, \end{array}$$

its density is

$$\begin{array}{ll} \text{Newton} & 94\frac{1}{2}, \\ \text{correction} & 94.4, \end{array}$$

its mass is

$$\begin{array}{ll} \text{Newton} & \frac{1}{1\,067}, \\ \text{correction} & \frac{1}{1\,048}, \end{array}$$

and the force of attraction on its surface is

$$\begin{array}{ll} \text{Newton} & 943, \\ \text{correction} & 904. \end{array}$$

This is how Newton describes Jupiter.

The central problem of propositions 18 through 20 is: this here earth.

The deduction begins with the planets generally:

proposition 18. the axes of the planets are smaller than the diameters perpendicular to them; because of the rotation of the planets on their axes their parts, by receding from the axes, endeavor to ascend at the equator; and, therefore, if the matter is fluid, it will increase the diameter at the equator by ascending and decrease it at the poles by descending (*id.*).

The planets are thus not exactly spheres, but elliptical spheroids flattened at the poles. Hence, the same body weighs less at the equator than further north or south. Or what is the same, the force of attraction on the surface of a planet is less at the equator than further north or south.

Newton finds how much less:

proposition 19. to find the ratio of the axis of a planet to the diameter perpendicular to it;

proposition 20. to find and compare with one another the weights of bodies in different regions of this here earth (*id.*).

And this is how he describes this here earth.

In proposition 19 he shows that:

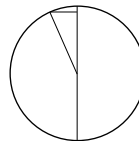
the diameter of the earth at the equator is to its diameter through the poles as 230 to 229 (*id.*).

And in proposition 20 he writes:

from this the following theorem is deduced;

the increase of weight from the equator to the poles is very nearly as the square of the sine of the latitude; and the arc of the degree of latitude on a meridian increases in about the same ratio (*id.*).

In lemmas 10 and 11 of book I he has shown that the square of the sine is proportional to the versed sine – or the difference between the radius and the cosine – and the versed sine is proportional to the force.



Newton continues:

now, the latitude of Paris is $48^\circ 50'$, latitude at the equator is $00^\circ 00'$, latitude at the pole is $90^\circ 00'$; and taking the radius as 10 000, the

squares of the sines of those arcs of latitude are as 11 334 to 00 000 to 20 000;
 gravity at the pole is to gravity at the equator as 230 to 229; the excess of gravity at the pole is thus to gravity at the equator as 1 to 229;
 hence, the excess of gravity at the latitude of Paris will be to gravity at the equator as $\frac{11\,334}{20\,000}$ to 229, or 5 667 to 2 290 000; and therefore the total gravities in these places will be as 2 295 667 to 2 290 000;
 the lengths of pendula beating equal periods are as the gravities; and since the length of a pendulum beating seconds at the latitude of Paris is 3 Paris feet and $8\frac{1}{2}$ lines, or rather, because of the weight of the air, $8\frac{5}{9}$ lines, the length of a pendulum at the equator will be shorter than the length of a pendulum with the same period at Paris by $1\frac{87}{1\,000}$ lines (*id.*).

1 line = 2.256 mm.

Newton continues:

in 1672, on the island of Cayenne, Mr Richer first observed that pendulum clocks go slower near the equator than in our regions; while observing the transit of the fixed stars across the meridian, he found that his clock goes slower than it should with respect to the mean motion of the sun by 2 minutes and 28 seconds per day; he constructed a pendulum beating seconds and recorded its length every week for ten months; when he returned to France, he compared the length of this pendulum with the length of a pendulum beating seconds at Paris and found that it was shorter by $1\frac{1}{4}$ lines (*id.*).

The latitude of Cayenne is $04^{\circ}56'N$.

Newton continues:

in England, as far as I can tell, an iron rod 3 feet long is $\frac{1}{6}$ of a line shorter in the winter than in the summer; let this quantity, because of the heat at the equator, be subtracted from the difference found by Richer; and there remains a difference of $1\frac{1}{12}$ lines, in excellent agreement with the result found by theory (*id.*)

In lines, then, theory and observation agree within 0.004 of a line. In millimeters, within 0.008 of a millimeter. In proposition 19 Newton concludes: since, from the measurement of Picard, the mean radius of the earth is 19 615 800 Paris feet, the height of the earth at the equator is about 19 658 600 Paris feet, its height at the poles about 19 573 800 Paris feet (*id.*). 12 lines are 1 Paris inch, 12 Paris inches are 1 Paris foot, and in earth radii, theory and observation agree within 0.000 000 000 002 of an earth radius.

Bessel improved on Newton's solution of the problem of the eccentricity of the earth, in 1837, in the *Determination of the axes of the elliptic spheroid of revolution*: the semiaxes of the spheroid through the equator and the poles are, in toises, 3 271 954 and 3 261 073 (5). 1 toise = 1.949 m.

The eccentricity of the earth is

Newton	0.004,
Bessel	0.003,
no further correction.	

The central problem of propositions 10 through 14 and their corollaries is the world. The problem of all propositions is the world. Of these, however, the problem is the world in its entirety.

Newton writes:

proposition 10. the motions of the planets in the heaven can be conserved for the longest time; in the scholium to proposition 40, book II, it was shown that a globe of frozen water, moving in the air, loses, because of the resistance of the air, $\frac{1}{4\,586}$ of its motion in traversing the

length of its radius;

the resistance of media decreases in the ratio of their weight and density, so that air, being 860 times lighter than water, resists 860 times less; in the scholium to proposition 22, book II, it was shown that at a height of 200 miles the air is rarer than on the surface of the earth in the ratio of about 75 000 000 000 000 to 1;

hence, Jupiter, if it moved in a medium with the density of the upper air, would not lose, because of its resistance, a millionth of its motion in a million years;

and therefore in the heaven, which is void of air and exhalations, the planets and the comets can move without any sensible resistance for

the longest time (*id.*).

Newton has discovered the problem of the duration of the solar system. Laplace will deduce the origin of the solar system from the theory of gravity. And the big bang theory will show how long the longest time is. The sun with its planetary system is 4.6×10^9 years old, and about halfway through its life cycle.

Newton has taken also a major step toward the solution of the problem of the dynamic stability of the solar system.

He writes:

proposition 13. the planets move in ellipses with foci in the center of the sun, and by radii drawn to this center describe areas proportional to the times (*id.*).

These are Kepler's first and second laws, which Newton has generalized and established firmly in the first propositions of book I. Here the problem is how much the planets perturb one another's motions. Newton concludes:

the perturbations in the motions of the planets in ellipses from their actions on one another are so minor that they can be ignored;
except the perturbations from the action of Jupiter on Saturn which cannot be altogether ignored; for at equal distances gravity toward Jupiter and gravity toward the sun are as 1 to 1 067; the distance of Saturn from Jupiter and the distance of Saturn from the sun are as 4 to 9 (*id.*),

$$\text{again } \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2},$$

and in the conjunction of Jupiter and Saturn the gravity of Saturn toward Jupiter and the gravity of Saturn toward the sun are as 81×1 to 16×1 067, or about 1 to 211; and hence in this conjunction a perturbation in the orbit of Saturn arises so sensible that the astronomers have been at a loss to explain it (*id.*).

Newton thus explained this perturbation. And it was most of all the remaining problems of the perturbations from the action of the earth and the moon and of Jupiter and Saturn on one another that Laplace, in the seven treatises published from 1785 to 1790, solved in order to show that all perturbations are periodic and self-correcting, and the solar system is therefore stable. In the *Exposi-*

tion of the system of the world, in 1796, Laplace writes: the perturbations in the motions of the planets, thought to be exceptions to the law of gravity, have become one of the striking examples of its truth; such has been the lot of Newton's brilliant discovery that every difficulty which it has encountered has provided it with an opportunity for a new triumph (IV, 2).

Descartes first describes the world as a system of systems of stars of which the solar is one. In propositions 10 through 14 Newton investigates the solar system in its entirety, and hence its place in the system of the world. And hence also the system of the world such as the solar is only one of its systems.

In proposition 10 he shows that the motions of the planets around the sun can be conserved for the longest time.

Before proposition 11 he introduces the first of the two hypotheses in book III:

hypothesis 1. the center of the system of the world is at rest;

this is conceded by all, even if some contend that the earth is at rest in the center of the system, others that the sun is; let us see what follows from this hypothesis (*id.*).

What follows is this:

proposition 11. the common center of gravity of the earth, the sun, and all the planets is at rest;
proposition 12. the sun is in perpetual motion, but never recedes far from the common center of gravity of all the planets;

corollary. hence, the common center of gravity of the earth, the sun, and all the planets has to be taken as the center of the world; for, if in the center has to be the body toward which all bodies gravitate the most, as the common opinion is, this privilege has to be conceded to the sun (*id.*).

In proposition 13 Newton shows that the planets move in ellipses with foci in the center of the sun.

Moreover:

proposition 14. the aphelia and the nodes of the orbits are at rest;

the inequalities are so minor that they can be ignored here;

corollary 1. the fixed stars are also at rest, they maintain their positions with respect to the

aphelia and the nodes;
 corollary 2. and because they have no sensible parallax arising from the annual motion of the earth, at the immense distances at which they are, the forces of these bodies can produce no sensible effect on our system; moreover, they are dispersed so evenly in all parts of the heaven that they mutually cancel their forces, by proposition 70, book I (*id.*).

If the center of the system of the world is at rest, and in this center has to be the body toward which all bodies gravitate the most, this privilege has to be conceded to the sun.

Halley, in 1717, showed that the fixed stars are not fixed, Aldebaran, Sirius, and Arcturus move. Herschel, in 1783, showed that the sun is not at rest, it moves in the direction of λ Herculis. Herschel also, in 1785, showed that the stars are not dispersed evenly in all parts of the heaven, the Milky Way is a confined cluster of stars. Bessel, in 1838, showed that the stars have a sensible parallax arising from the annual motion of the earth, he measured the parallax of 61 Cygni.

Nor is the sun the center of the Milky Way. It is 2.5×10^{17} km from the center. This privilege has had to be conceded to the point toward which all bodies in the galaxy gravitate the most. Nor is the Milky Way the center of the world. This privilege has to be conceded to the point from which all things in the world have since the big bang been receding.

The center of the world is not at rest either in that the theory of relativity has shown that motion and rest are always with respect to a frame of reference. Unless the world is one, the frame of reference is the world, the big bang theory has shown that there is nothing vague about this frame of reference, and the center of the world cannot not be, with respect to this frame of reference, absolutely at rest.

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And to conclude the observation of Newton which are the principles can be shown.

What is in *The principles* can be divided into three kinds. The 25 definitions, laws, hypotheses, rules, and phenomena. The 635 propositions, lemmas, and corollaries. And the 51 prefaces and scholia. The first are the explicit premises of the

deductions. The second are the deductions. The third are commentaries, so that the principles may not appear sterile, on matters which are general and in which philosophy most of all appears to be grounded.

And if the principles are general, and deduction indicates generality, the principles are among the 25 definitions, laws, hypotheses, rules, and phenomena. Among them, and not all of them. That they are definitions, laws, hypotheses, rules, and phenomena suffices to show that some of them are more general than others.

Which are the principles is about the most general.

Moreover, the principles are *The mathematical principles of natural philosophy*. In the opening preface Newton writes: we set forth the mathematical principles of philosophy. In the preface to book III he writes: in the preceding books I have delivered the principles of philosophy, which are, however, not just philosophical, but mathematical, and that is, from them it can be disputed in philosophical matters. And: it remains that from the same principles we teach the constitution of the system of the world.

Which are the principles cannot not be, explicitly or implicitly, about the relations of mathematics, physics, and philosophy. About the world, too. And not only here as everywhere explicitly is better than implicitly. Here more than anywhere.

To begin with, then, the 3 hypotheses are not among the principles.

Newton concludes the first part of *The principles* by showing that vortices are incompatible with the description of areas proportional to the times. To do so, he needs the first of the three hypotheses: the resistance arising from the friction of the parts of a fluid is proportional to the velocity of separation of the parts of the fluid.

The theory of gravity has long replaced the theory of vortices as the theory of the motions of the bodies in the world. And this hypothesis of Newton's has since become a definition. Two kinds of fluid are defined in hydrodynamics, Newtonian and not Newtonian, when stress is proportional to strain and when it is not.

The third hypothesis is: the motion of the equinoctial points will be the same whether the equatorial ring of matter surrounding the earth is

fluid or consists of rigid and solid matter. Newton needs it to deduce the precession of the equinoxes from the shape of the earth. And this hypothesis has since become a theorem. Laplace reports: this hypothesis of the great geometer I have replaced with a theorem (*System*, IV, 14).

The second hypothesis, finally, is: the center of the system of the world is at rest. Newton needs it to decide to which point the privilege of being the center has to be conceded. And this hypothesis has since not become a definition or a theorem.

The hypotheses are prospective definitions or theorems, and are hypotheses while it remains to be established which.

The 22 definitions, laws, rules, and phenomena divide into the 9 definitions and 3 laws, which are explicit premises of the deductions in both parts of *The principles*, books I and II, in which Newton delivers the principles of philosophy, and book III, in which from the same principles he teaches the system of the world, and the 4 rules and 6 phenomena, which are explicit premises of the deductions in the second part, in which he teaches the system of the world.

And of the 9 definitions and 3 laws the 3 laws are the principles.

Among the definitions there are propositions of great generality, the definitions of mass, of momentum, of force, of acceleration.

The greater generality of the laws is shown by all the following,

By the fact that Newton has elevated them into: axioms or laws. Axioms have been the most general propositions from which deduction proceeds since Euclid. The true way ends in the most general, and laws have been the end of the true way since Bacon, Galileo, and Kepler.

By their immediate consequences which Newton deduces in the corollaries to the laws, the parallelogram of forces, the conservation of momentum, the principle of relativity.

By the labors of Newton's predecessors. The labors of Galileo, Kepler, Descartes, Huygens, all aim at the discovery of the laws.

By the labors of Newton's successors. Mechanics has been greatly generalized analytically three times, by Lagrange, in the *Analytic mechanics*, in 1788, by Hamilton, in *On a general*

method in dynamics, in 1834, and by Hertz, in *The principles of mechanics*, in 1894. All three are generalizations of Newton's laws. Lagrange's principle of virtual velocities is a generalization of the second law. Hamilton's principle of least action is a further generalization of the second law. Hertz's fundamental law is a generalization of the first law. Mechanics has also been generalized to accommodate the phenomena discovered after Newton, by Einstein. And Einstein's is also a generalization of Newton's laws. Einstein has made Newton's laws agree with Maxwell's equations, the laws of mechanics with the laws of electrodynamics.

The 3 axioms or laws of motion are:

every body perseveres in its state of rest or uniform motion in a straight line, unless it is compelled to change its state by forces impressed on it;

change in motion is proportional to the motive force impressed, and is in the straight line in which that force is impressed;

to action there is always opposite and equal reaction; or, the actions of two bodies on one another are always equal and opposite in direction.

Three and a half centuries after Newton, that the 3 axioms or laws of motion are the principles is unlikely to be contested. That they are the mathematical principles will be contested. That they are the principles of philosophy will also be contested.

Newton cannot have been more explicit. He writes: this treatise proposes to perfect mathematics as it relates to philosophy. And: we set forth the mathematical principles of philosophy. And: I have delivered the principles of philosophy, which are not just philosophical, but mathematical, and that is, from them it can be disputed in philosophical matters.

The question is not whether the laws are the mathematical principles, and the principles of philosophy. The question is why. Why are they in Newton the mathematical principles, and the principles of philosophy. And why it will be contested today that they are the mathematical principles, and the principles of philosophy.

And the answer is the catastrophic transformation of philosophy in the last couple of centuries.

In the three generations from Bacon, Galileo, and Kepler through Descartes through Huygens and Spinoza through Newton and Leibniz the acquisition of knowledge advanced more than at any other time.

In these three generations also the philosophers affirmed more resolutely than at any other time that philosophy searches for truth, hence it searches for the principles, it searches for the principles because they are the principles of the world, or synonymously nature, and this is why, because it searches for truth, it searches for the principles, this is why philosophy is fundamentally, substantially, and essentially natural philosophy, or synonymously physics, and this is why mathematics is the language of the world, the principles are mathematical, mathematics is a part of physics, and the principles are the principles of philosophy. Nor did the philosophers only affirm it, they showed it, they discovered the principles.

The two, the greatest advance in the acquisition of knowledge and the presumption that philosophy is physics and mathematics is a part of physics, are in the relation of effect and cause.

So also Newton affirms that absolute is true is mathematical.

So also he begins in *The principles*: because the ancients found mechanics the best in the investigation of nature, and their recent successors have undertaken to reduce the phenomena of nature to mathematical laws, this treatise proposes to perfect mathematics as it relates to philosophy.

So also he affirms that because the principles are not just philosophical, but mathematical, from them it can be disputed in philosophical matters.

So also he describes the laws always as the mathematical principles as well as, and indiscriminately, either the principles of natural philosophy or the principles of philosophy.

So also he develops calculus in *The principles* as part of the investigation of *The motions of bodies*.

So also he writes: in mathematics are investigated the quantities and ratios of forces which follow from any conditions whatsoever; next, descending to physics, these ratios are related to the phenomena in order to establish which general conditions apply to particular bodies; thus

disputes about the physical species, causes, and reasons of the forces can be grounded.

The supposition that any conditions whatsoever are conditions without the world has maintained the fiction that mathematics is more general than physics. It should be superfluous to note that on the same supposition any fiction is more general than physics. And fictions being fictions, any other fiction will assert itself as more general than the mathematical.

Fortunately, the fact that mathematical propositions not only can be related to the phenomena in order to establish which general conditions apply in particular, but have been, should suffice to reduce the fiction to reality. The generality of what follows from any conditions whatsoever may be greater than the generality of the general conditions which apply in particular, but may not be greater than the generality of the relation of conditions to their consequences. And that this relation is physical, and only hence mathematical, the fact that thus disputes about the physical species, causes, and reasons not only can be grounded, but have been, should suffice to show.

It sufficed for Galileo, Kepler, Descartes, Huygens, Spinoza, Newton, Leibniz.

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Of the 9 definitions and 3 laws the 3 laws are the principles.

And of the 4 rules and 6 phenomena the 4 rules are the principles. The rules are: rules of philosophizing. And the 4 rules of philosophizing are not only more general than the 6 phenomena, they are as general as the 3 laws.

Gravity shows that they cannot not be.

Kepler had some difficulty with the sun. He writes: the sun captures the planets, it makes them revolve as it revolves, and by a bodily force, not by an animal force or a mental force; but how can the sun do this without hands? instead of hands there is the force of the sun's body which is emitted in straight lines throughout the world; can you make this clear by example? yes, the lodestone; but is it not absurd to take this material force from the body of the sun and put it in the celestial aether? no, as the example of the lodestone shows (*Epitome*, IV, 2.3).

The principles appeared in 1687.

In 1690, in the *Discourse on the cause of gravity*, Huygens expresses his admiration at some length and in some detail, except on this one point: there is only this difficulty, that Mr Newton, rejecting the vortices of Descartes, wants the celestial spaces to contain only very rare matter; but it seems that when such rarity is posed, neither the action of the planets nor that of light can be explained (*id.*).

Leibniz agrees: I do not understand how Newton conceives of gravity; it seems that according to him gravity is an incorporeal and inexplicable force (to Huygens, 1690).

Nay, Newton agrees: that gravity should be innate, inherent, and essential to matter so that one body may act on another at a distance through vacuum without the mediation of anything else, is to me so great an absurdity that I believe no man with a competent faculty of thinking in philosophical matters can ever fall into it (to Bentley, 25 Feb 1693).

Gravity, then, can explain neither the action of the planets nor that of light. It is an incorporeal and inexplicable force. It is an absurdity so great that no man with a competent faculty of thinking in philosophical matters can ever fall into it. In fact, on the authority of Leibniz, having lost some of his patience, having had to repeat the same thing five times, gravity is: invisible, intangible, not mechanical; inexplicable, unintelligible, precarious, without a ground, without an example; a perpetual miracle, and if not a miracle then false, a chimerical thing, a scholastic occult quality (to Clarke, 18 Aug 1716).

So how does gravity do it?

In this case, too, instead of hands there is the force of every body emitted in straight lines throughout the world. But can you make this clear by example? the lodestone perhaps? No. Newton has shown in book III that: the force of gravity is different in kind from the magnetic force, for magnetic attraction is not as the attracted matter (6, cor. 5). Can you at least put it in the celestial aether? No. Newton has shown in book II that, if there is aether, it is very very subtle, and has concluded in book III: if all particles of all bodies are of the same density, and cannot be rarefied without pores, there must be vacuum; and density is the same, I say, if attrac-

tion is proportional to size (cor. 4).

So how does gravity do it?

Huygens writes: it seems to me impossible without aether (*Gravity*). Leibniz writes: nor am I persuaded that the action of the surrounding aether is superfluous (to Huygens, 1690).

Nor is Newton persuaded.

He writes in the *Optics*: is not the aethereal medium exceedingly more rare and subtle than air, and exceedingly more elastic and active? and does it not readily pervade all bodies? and is it not by its elastic force expanded through all the heavens?

And in the concluding general scholium to *The principles* the matters which are general and in which philosophy most of all appears to be grounded are the difficulties besetting the hypothesis of vortices, the lord god παντοκράτωρ, gravity, philosophy, and a certain subtlest spirit. So, having dealt with vortices and the lord god, Newton concludes:

I have explained the phenomena of the heaven and of our sea by the force of gravity, but I have not yet assigned a cause to gravity;

I have not been able to deduce the reason for the properties of gravity from the phenomena, and I do not feign hypotheses; for what is not deduced from the phenomena has to be called hypothesis, and hypotheses have no place in experimental philosophy; in this philosophy propositions are deduced from the phenomena and rendered general by induction; this is how the impenetrability, mobility, and impetus of bodies and the laws of motion and gravity have been discovered; it is enough that gravity in fact exists, acts according to the laws we have expounded, and suffices to explain the motions of the heavenly bodies and of our sea; finally, a few things can be added about a certain subtlest spirit, which pervades gross bodies and is hidden in them; by the force and actions of this spirit the particles of bodies attract one another at the smallest distances and cohere when contiguous, electric bodies act at greater distances, light is emitted, reflected, refracted, inflected, and heats bodies, and all sensation is excited and the parts of animals are moved by the will, as the vibrations of the spirit propagate through the nerves from the organs of

sense to the brain and from the brain to the muscles; but this cannot be explained briefly; there are not enough experiments to determine and demonstrate accurately the laws of the actions of this spirit (*id.*).

It is enough that gravity in fact exists, acts according to the laws Newton has expounded, and suffices to explain the motions of the heavenly bodies and of our sea and very many other things.

Still. How does it do it? For it does not do it by aether, not even if it is the subtlest spirit, for there is no aether. And it does not do it by the lord god παντοκράτωρ.

Moreover, it will not stop doing it. Never mind special relativity, after general relativity: Newton's theory holds to a first approximation (Einstein, *Foundation*, 21), quite dramatically, considering that: Euclidean geometry does not hold even to a first approximation (22). Never mind quantum theory, the big bang theory becomes the theory of the world at Planck time, 10^{-43} seconds after the beginning, when from the unified until then force, of which there is no theory, first of all gravity emerges.

Gravity does it by the 4 rules of philosophizing.

The law of gravity,

$$F \propto \frac{m_1 m_2}{r^2},$$

is a result of such generality, it has come to be taken, no questions asked, as the premise of deductions. In Newton, for his part, the law of gravity is not one of the premises of the deductions, it is one of the deductions.

In this philosophy propositions are deduced from the phenomena and rendered general by induction. This is how the laws of motion and gravity have been discovered.

Analysis ought always to precede synthesis, and the analysis whose conclusion the law of gravity is shows paradigmatically how in this philosophy propositions are deduced from the phenomena and rendered general by induction.

This is the analysis:

proposition 1. the forces by which the satellites of Jupiter are perpetually drawn away from rectilinear motion and maintained in their orbits are directed to the center of Jupiter, and

are inversely as the squares of the distances; this is clear from phenomenon 1 and proposition 2 or proposition 3 and corollary 6 to proposition 4 of book I;

the same is to be understood for the satellites of Saturn;

proposition 2. the forces by which the principal planets are perpetually drawn away from rectilinear motion and maintained in their orbits are directed to the sun, and are inversely as the squares of the distances;

this is clear from phenomena 4 and 5 and propositions 2 and 4 of book I;

proposition 3. the force by which the moon is maintained in its orbit is directed toward the earth, and is inversely as the square of the distance;

this is clear from phenomenon 6 and proposition 2 or proposition 3 of book I, and from the very slow motion of the apogee of the moon (*id.*).

These propositions are deduced from the phenomena. And they are next rendered general by the most dramatic of inductions. Newton continues:

proposition 4. the moon gravitates toward the earth, and is always drawn back from rectilinear motion, and maintained in its orbit, by the force of gravity;

the mean distance of the moon in syzygy is according to Ptolemy 59 earth radii, according to Wendelin and Huygens 60, according to Copernicus $60\frac{1}{3}$, according to Streete $60\frac{2}{5}$,

according to Tycho $56\frac{1}{2}$; but Tycho makes the

refraction of the sun and the moon $4'$ to $5'$ greater than that of the fixed stars, and when this error is corrected, his number comes to about $60\frac{1}{2}$;

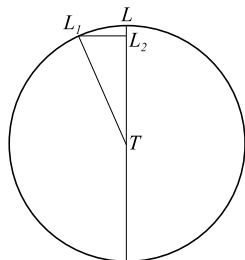
let us assume, then, that the mean distance of the moon in syzygy is 60 earth radii; that its period with respect to the fixed stars is 27 days, 7 hours, and 43 minutes, as the astronomers have established it; and that the circumference of the earth is 123 249 600 Paris feet, as the French have measured it;

and if, now, the moon were deprived of all motion and let fall to the earth with all the force

with which it is otherwise maintained in its orbit, it would fall in 1 minute $15\frac{1}{12}$ Paris

feet; this can be calculated from proposition 36 or corollary 9 to proposition 4 in book I (*id.*).

For instance. The circumference of the earth is 123 249 600 Paris feet. Hence, the radius of the earth is 19 615 783 Paris feet. Hence, the distance of the moon, LT , which is 60 earth radii, is 1 176 946 980 Paris feet.



Hence, the orbit of the moon is 7 394 975 972 Paris feet. The period is 39 343 minutes. Hence, the distance which the moon travels in 1 minute, arc LL_1 , is 187 962 Paris feet. The angle which this arc subtends at the earth, LTL_1 , is thus $0.009\,150\,31^\circ$. The cosine of this angle is .999 999 987, and hence the distance L_2T is 1 176 946 965 Paris feet. And $LT - L_2T = LL_2$, or $1\,176\,946\,980 - 1\,176\,946\,965 = 15$ Paris feet.

Thus, calculating to ten significant digits, to the nearest Paris foot. Newton, clearly, calculated far beyond ten significant digits. He continues:

the moon would fall in 1 minute $15\frac{1}{12}$ Paris feet,

or more exactly 15 feet, 1 inch, and $1\frac{4}{9}$ lines;

in approaching the earth the force increases as the inverse square of the distance; it will thus be at the surface of the earth 60^2 greater than at the distance at which the moon is (*id.*),

again $F \propto \frac{r}{T^2}$,

and it follows that a body falling with that force in our regions will describe in 1 minute $60^2 \times 15\frac{1}{12}$ Paris feet, and in 1 second $15\frac{1}{12}$

Paris feet, or more exactly 15 feet, 1 inch, and $1\frac{4}{9}$ lines;

and with this force, in fact, heavy bodies descend to the earth; for a pendulum beating seconds in the latitude of Paris is 3 feet and $8\frac{1}{2}$ lines long, as Huygens has observed; the distance

through which a heavy body falls in 1 second is to one half of the length of this pendulum as the square of the ratio of the circumference of the circle and its diameter, as Huygens has also shown; and the distance is thus 15 feet, 1 inch, and $1\frac{7}{9}$ lines (*id.*).

At a distance of 169 480 365 120 lines, the moon let fall and a pendulum in Paris agree within $\frac{1}{3}$ of

a line. Newton concludes:

therefore, the force which maintains the moon in its orbit, in descending to the surface of the earth, is equal to the force of gravity among us;

and it is thus, by rules 1 and 2, the very same force which we call gravity (*id.*).

In the following scholium the drama continues:

the demonstration of this proposition can be expounded more completely thus;

if plural moons revolved around the earth, as in the systems of Saturn or Jupiter, their periodic times would, by induction, observe the laws which Kepler discovered for the planets, and therefore their centripetal forces would be inversely as the squares of their distances from the center of the earth, by proposition 1 above; and if the lowest of them were small and nearly touched the tops of the highest mountains, the centripetal force maintaining it in its orbit would, by the preceding calculation, be nearly equal to the gravity of the bodies on the tops of these mountains;

and if this little moon were deprived of all motion and descended to the earth, it would do so with the same velocity with which heavy bodies fall on the tops of these mountains; for the forces of descent are equal; if the force by which the little moon would fall were different from gravity, and the little moon also gravitated toward the earth as the bodies on the tops of the mountains do, the little moon would descend by both forces with twice the velocity; since, then, the forces of heavy bodies and of the moons are directed to the center of the earth, and are similar, and equal, they have, by rules 1 and 2, the same cause; and, therefore, the force which maintains the moon in its orbit is

the very same force which we call gravity (*id.*).
This is the climax of the drama. Further:

proposition 5. the circumjovial satellites gravitate toward Jupiter, the circumsaturnian satellites toward Saturn, and the circumsolar planets toward the sun, and are always drawn back from rectilinear motion, and maintained in their orbits, by the force of gravity;

these are phenomena of the same kind as the revolution of the moon around the earth; therefore, by rule 2, they depend on a cause of the same kind;

corollary 1. therefore, there is gravity toward all planets universally;

scholium. it is now established that the force which maintains the heavenly bodies in their orbits is gravity, and therefore we shall henceforth call it gravity; for the cause which maintains the moon in its orbit has to be, by rules 1, 2, and 4, extended to all the planets (*id.*).

It is now revealed that it is Oedipus who killed his father and slept with his mother. The conclusion of the analysis is accordingly anticlimactic:

proposition 6. all bodies gravitate toward each planet, and at any distance from the center of any planet the weight of any body toward the planet is proportional to the quantity of matter in each;

proposition 7. there is gravity in bodies universally, and it is proportional to the quantity of matter in each (*id.*).

By rules 1, 2, 3, and 4, therefore, there is gravity in bodies universally, and it is proportional to the quantity of matter in each.

The rules are:

no more causes of natural things must be admitted than are true and suffice to explain their phenomena;

therefore, to natural effects of the same kind the same causes must be assigned, as much as possible;

qualities of bodies which cannot be increased or decreased, and which belong to all bodies on which experiments can be made, must be taken for qualities of bodies universally;

in experimental philosophy propositions gathered from the phenomena by induction must be taken, contrary hypotheses notwithstanding, for true, or accurate, or very nearly so, until

other phenomena occur to make them either more accurate or liable to exceptions.

The 3 axioms or laws of motion are the laws of nature. The 4 rules of philosophizing are the principles of the acquisition of knowledge.

This explains why causes, and induction, are the central problem of Hume, and of Kant.

It explains why Maxwell writes: the general maxim of physical science is, the same causes will always produce the same effects (*Matter*, 19). And writes so in 1877, after *A dynamical theory of the electromagnetic field*, in 1864, in which he has formulated the laws of electrodynamics.

The development of quantum theory has appeared to show that causality does not apply to the atomic event, and that there is an unavoidable indeterminacy in observation. Dirac, for one, writes in *The principles of quantum mechanics*, in 1930: causality applies only to a system which is left undisturbed (1), and: there is an unavoidable indeterminacy in the calculation of observational results (*id.*).

But the 4 rules of philosophizing are the principles of the acquisition of knowledge. And this explains why Heisenberg writes: when we make an experiment we have to assume a causal chain of events that leads from the atomic event through the apparatus to the eye of the observer; if this causal chain were not assumed, nothing could be known about the atomic event (*Philosophy*, 5). And writes so in 1958, after *On the visual content of quantum theoretical kinematics and mechanics*, in 1927, in which he has formulated the principle which goes not only under the name principle of uncertainty but under the name also principle of indeterminacy.

It is by rules 1, 2, 3, and 4 that anything could ever become known about the atomic event. It is by rules 1, 2, 3, and 4 that ever more has become known about the atomic event. It is by rules 1, 2, 3, and 4 that all that has become known about the atomic event has become known. Newton comments on the rules: idle fantasies are not to be feigned against the evidence of experiment; lest arguments from induction were negated by hypotheses. It may be that it has not been explained how causality applies to the atomic event, and how Heisenberg's principle shows the determi-

nacy of observation. It may even be that how causality applies to the atomic event, and how Heisenberg's principle shows the determinacy of observation, is invisible, intangible, not mechanical, inexplicable, unintelligible, precarious, without a ground, without an example, a perpetual miracle, and if not a miracle then false, a chimerical thing, a scholastic occult quality. But it may not be that causality does not apply to the atomic event, or that Heisenberg's principle shows the indeterminacy of observation.

It is by rules 1, 2, 3, and 4, explicitly or implicitly, tacitly or unawares, that all that has become known has become known.

clxxviii

Beginning with Bacon, Galileo, and Kepler the true way was for the first time followed resolutely. Great success in the acquisition of knowledge followed. And three generations after Bacon, Galileo, and Kepler the true way culminated in Newton. Newton is a triumph of the true way.

All results of the philosophers show this. And most clearly and distinctly the laws of nature, the principles of the acquisition of knowledge, and the two in relation to one another, the laws of nature and the principles of the acquisition of knowledge, show it.

Bacon affirmed that the end of the true way is the law with its clauses. Kepler discovered the laws of the motions of the planets. Galileo discovered the law of falling bodies and the law of the pendulum. Galileo, Kepler, and Descartes discovered the first law of motion. Descartes formulated the laws of nature. Descartes and Huygens discovered the second law of motion. Pascal discovered the law of the pressure of a fluid. Boyle discovered the law of the relation of temperature, volume, and pressure. Hooke discovered the law of elasticity. In 2 generations the philosophers discovered more laws than in the previous 157 generations combined. And rather, after 157 generations of acquisition of knowledge, in these 2 generations the discovery of the laws began.

Newton then discovered the laws of motion and the law of gravity. Some of the laws discovered earlier were generalized in these laws. The other laws discovered earlier were deduced from

these laws. And the system of the world was grounded in these laws. Newton showed, both, that the end is the law with its clauses and the end, the law with its clauses.

In the same way, beginning with Bacon it is for the first time clear that truth decides how to search for truth. Knowledge decides how knowledge is acquired. There are and can only be two ways to search for truth, the false and the true. The description of the false way is thus also an attempt to close the false way. The description of the true way is thus also an attempt to search for truth. And acquire knowledge. The true way begins with the collection of the particulars, marshals them in their series and order, proceeds by continuous and gradual ascent, and ends with the most general principles. Establishing stages of certainty, it ascends from experiments to principles, descends from these to new experiments, ascends from these to higher principles, descends from these again, ascends again, and until it reaches the law with its clauses. And among the clauses of the law have to be the laws of the search for truth. Among the most general principles have to be the principles of the acquisition of knowledge.

Galileo is also clear. Philosophy is not a book of man's fantasy, philosophy is written in the world. He is just as clear that there are and can only be two ways to search for truth, and acquire knowledge, and he describes them both in some detail. Galileo and Kepler, however, each show that the false way is the false and the true way is the true more clearly and distinctly than Bacon by following the true way, following it more resolutely, and following it further than it has ever been. Each shows how the particulars are marshaled in their series and order. How experiments are marshaled. How observations are marshaled. How the way ascends from experiments and observations to principles. How it descends from these to new experiments and observations. How it ascends again, and descends, and ascends. How it reaches the law with its clauses. And describes the system of the world. Truth decides how to search for truth. Knowledge decides how knowledge is acquired. Philosophy is written in the world.

And in the next generation, with Descartes, the

principles of philosophy are for the first time formulated explicitly. For the first time as well the principles are separated sharply into the laws of nature and the principles of the acquisition of knowledge.

There are all things in the world. All things in the world have principles. The relation of all other things in the world and the principles is deduction. The deduction of all other things in the world from the principles shows the principles the principles. The principles are thus of two kinds, the principles of deduction and the principles of all other things. More generally, the principles of the acquisition of knowledge and the laws of nature.

With Descartes the laws of nature are for the first time formulated explicitly.

With Descartes as well the principles of the acquisition of knowledge are for the first time formulated explicitly.

Beginning with Bacon, Galileo, and Kepler the acquisition of knowledge becomes the discovery of the laws. And beginning with Bacon and Descartes knowledge becomes the special object of investigation. In the generation after Descartes Spinoza and Locke investigate knowledge, in the next generation Leibniz, in the next generation Berkeley, in the next generation Hume and Kant.

With Descartes, method is necessary to search for truth. Method is the certain and easy rules, which, if followed exactly, will never allow the false to be taken for true. The true way is the method. The rules of the method are the principles of knowledge. And they are:

- to include in judgments nothing but what is so clear and distinct that it cannot be doubted;
- to divide difficulties into parts such as will lead to their better solution;
- to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex;
- and always to make enumerations so complete and reviews so general as to omit nothing.

The laws of nature and the principles of the acquisition of knowledge have both been obtained by induction. Obtained by induction from the same all other things in the world, too, they can be presumed in agreement with one another.

It is deduction, however, the deduction of all

other things in the world from the principles, that shows the principles the principles.

If, then, there are principles more general than the laws of nature and the principles of the acquisition of knowledge both, the deduction of both the laws of nature and the principles of the acquisition of knowledge from those more general principles will show them both principles, in agreement with one another, the principles of the same all other things in the world both.

Spinoza explored this possibility. He tried to formulate and solve the problem of the one most general principle which can be induced from the laws of nature and the principles of the acquisition of knowledge, and from which the laws of nature and the principles of the acquisition of knowledge can both be deduced. The principle is god, or synonymously nature. It remains the most successful attempt to formulate and solve this problem. The most successful, but not too successful. And not because the principle is not nature. If there are principles, or there is one principle, more general than the laws of nature and the principles of the acquisition of knowledge both, those principles are, or that principle is, nature. To show this, however, those principles have to be, or that principle has to be, formulated more explicitly than Spinoza formulated it, and the chains of inferences from those principles, or that principle, to the laws of nature and the principles of the acquisition of knowledge have to be substantially more free of gaps.

Nor has the discovery of more laws of nature, Clausius's, Darwin's, Maxwell's, Schrödinger's, Hubble's, or the generalization of Newton's laws by Lagrange, and by Hamilton, and of Newton's and Maxwell's laws by Einstein, even as it has continued to point ever more clearly and distinctly at the unity of the laws of nature, made it easier to formulate or solve that problem.

While it remains to be shown that there are principles more general than the laws of nature and the principles of the acquisition of knowledge both, and both can be deduced from those more general principles, and, in fact, whether there are those more general principles or not, one of these can be more general than the other – either the principles of the acquisition of knowledge can be more general than the laws of nature, or the laws

of nature can be more general than the principles of the acquisition of knowledge – and if the principles of the acquisition of knowledge are more general, the laws of nature can be deduced from them, if the laws of nature are more general, the principles of the acquisition of knowledge can be deduced from them.

The first of these cannot be a real possibility, however. It can only be a fictional possibility. The principles of the acquisition of knowledge cannot be both principles of the acquisition of knowledge and more general than the laws of nature. If there are laws of nature, the laws of nature cannot be some of the principles of the acquisition of knowledge, next to some other principles of the acquisition of knowledge which are not laws of nature. And the other way around, if there are laws of nature, the principles of the acquisition of knowledge not only have to be among the laws of nature, they can be the principles of the acquisition of knowledge only by being laws of nature. Unless, with Berkeley, everything is my ideas in mind, spirit, soul, or myself.

On pain of idealism, or worse, the laws of nature had better be more general than the principles of the acquisition of knowledge, the principles of the acquisition of knowledge had better follow from the laws of nature, and to show this, and deduce the principles of the acquisition of knowledge from the laws of nature, is, in fact, the fundamental problem before the investigation of knowledge. The investigations of Hume and of Kant are steps toward the formulation and solution of this problem. Hume and Kant both labor to show how the success of the investigation of nature, and most particularly Newton, is success of the acquisition of knowledge. And the investigations of Darwin and of Frege are major steps toward the resolution of the difficulties before the formulation and solution of this problem. Darwin and Frege have narrowed substantially the gap between the laws of nature and the principles of the acquisition of knowledge. After Darwin the chains of inferences have to reach from laws not more general than those of organisms down to the principles of the acquisition of knowledge. Also, the world is written in mathematics, the laws of nature are mathematical, the principles of the

acquisition of knowledge are induced from mathematics, there is preestablished harmony between physics and mathematics, and after Frege chains of inferences substantially free of gaps have begun to connect these propositions.

While this problem, too, remains to be formulated and solved, deduction can yet show that the laws of nature and the principles of the acquisition of knowledge are in agreement with one another.

Has, in fact, shown it.

Huygens, looking back at the greatest success of the acquisition of knowledge, having himself greatly contributed to it, affirms that, if the things demonstrated from the principles correspond perfectly to the phenomena which experience has manifested, the more so if the number of these phenomena is great, and the more so if new phenomena have been foreseen from the principles, this must be a grand confirmation of the principles.

And *The principles* is the grandest confirmation of the principles. Of the laws of nature. Of the principles of the acquisition of knowledge. And of their agreement with one another.

The things demonstrated from the laws of nature correspond perfectly to the phenomena which experience has manifested, the number of these phenomena is great, and new phenomena have been foreseen from the laws. And it is clear and distinct which the laws of nature are. They are the 3 axioms or laws of motion and the 1 law of gravity.

Also, the things demonstrated from the principles of the acquisition of knowledge correspond perfectly to the phenomena which experience has manifested, the number of these phenomena is great, and new phenomena have been foreseen from the principles. And it is clear and distinct which the principles of the acquisition of knowledge are. They are the 4 rules of philosophizing.

Also, the deduction from the principles of the acquisition of knowledge, from the 4 rules of philosophizing, begins with the induction which leads to the more general laws of nature, to the 1 law of gravity, in order to demonstrate the phenomena from the two jointly, the laws of nature and the principles of the acquisition of knowledge.

Descartes's *Principles of philosophy* is fundamentally inductive. It ascends from *The principles of human knowledge* to *The principles of the material things* and the 3 laws of nature in order to descend to *The visible world* and *The earth*. Newton's *Mathematical principles of natural philosophy* is fundamentally deductive. It ascends to the 3 axioms or laws of motion in order to descend to *The motion of bodies*, then ascends again to the 4 rules of philosophizing and hence to the 1 law of gravity in order to descend to *The system of the world*.

Newton's *Principles* is the grandest confirmation of the principles.

And the importance of this fact cannot be overstated. Because philosophy is written in the world, yet the supposition that philosophy is a book of man's fantasy has once again reasserted itself. The rules of philosophizing are the principles of knowledge, yet idle fantasies are feigned against the evidence of experiment, and arguments from induction are negated by hypotheses.

To emphasize their generality and importance the 4 rules of philosophizing can, and with the hindsight of the catastrophic transformation of philosophy since Newton should, be stated more clearly and distinctly.

In rule 1 Newton speaks of natural things. In rule 2 he speaks of natural effects. In rule 3 he speaks of the qualities of bodies. In rule 4 he speaks of experimental philosophy. But lest there were not natural things, supernatural perhaps, natural things is redundant. All things are natural. Natural effects is also redundant. All effects are natural. Lest not bodies included spirits, as the qualities of bodies so the qualities of not bodies, waves, or propositions, or discoveries. And lest not experimental philosophy were human, all philosophy is experimental. Whether already retrospectively or still only prospectively.

The rules are,
to admit no more causes of things than are true
and suffice to explain the phenomena;
therefore, to assign to effects of the same kind the
same causes;
to take qualities which belong to all things on
which experiments can be made for qualities
universally;
and to take propositions gathered from the phe-

nomena by induction for true, or accurate, or very nearly so, until other phenomena occur to make them either more accurate or liable to exceptions.

That all discovered laws of nature are one law with its clauses can be presumed on the ground that all laws have been obtained by induction, by induction, one way or another, from the same all other things in the world, and obtained by the cumulative labors of the philosophers, exclusively. Even if to show that all discovered laws of nature are one law with its clauses is a problem that remains to be formulated and solved.

That all discovered principles of the acquisition of knowledge are one principle with its clauses can be presumed on the same ground, all principles have been obtained by induction, induction from the same all other things in the world, and by the cumulative labors of the philosophers, exclusively. Even if to show that all discovered principles of the acquisition of knowledge are one principle with its clauses is also a problem that remains to be formulated and solved.

The following can nevertheless be affirmed.

Newton's rules 1 and 2 depend unconditionally on his rules 3 and 4. To admit no more causes than are true and sufficient, and assign to effects of the same kind the same causes, depends unconditionally on taking qualities on which experiments can be made universally, and taking propositions gathered by induction for true.

Causes depend on induction.

Newton's rules 3 and 4 depend unconditionally on Descartes's rules 3 and 4. To take qualities on which experiments can be made universally, and take propositions gathered by induction for true, depends unconditionally on beginning with the simplest and easiest and ascending gradually to the more complex, and always making enumerations so complete and reviews so general as to omit nothing.

Induction depends on order, completeness, and generality.

Descartes's rules 3 and 4 depend unconditionally on his rules 1 and 2. To begin with the simplest and easiest and ascend gradually to the more complex, and always make enumerations so complete and reviews so general as to omit

nothing, depends unconditionally on including in judgments nothing but what is so clear and distinct that it cannot be doubted, and dividing difficulties into parts such as will lead to their better solution.

Order, completeness, and generality depend on clarity and distinction and the better division of difficulties into parts.

Aristotle established the fundamental character of the principle of contradiction. Leibniz established the fundamental character of the principle of sufficient reason.

And Descartes's rule 1 depends unconditionally on the principle of contradiction, his rule 2 depends unconditionally on the principle of sufficient reason.

Clarity and distinction depends on not contradiction, the better division of difficulties into parts depends on there being a sufficient reason.

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Newton is a triumph of the true way.

And so is Leibniz.

And with Leibniz the observation of the true way – of how the true way has been followed in the 3 generations from Bacon, Galileo, and Kepler through Descartes through Huygens and Spinoza through Newton and Leibniz – can conclude.

To show the importance of Leibniz is, on the one hand, easy. Leibniz has made so many discoveries, a random sample can suffice to show his importance. On the other hand, Leibniz is the most difficult philosopher to observe, or describe, or analyze. He is the one philosopher who has, in fact, done everything. He is a logician *par excellence*, a mathematician *par excellence*, a physicist *par excellence*, a historian *par excellence*, a metaphysician *par excellence*. He has done geology. He has done engineering. He has done politics. He has done theology. He has done jurisprudence. He has done ethics. In general, whatever philosophy is, or has been, or will be, Leibniz is a philosopher *par excellence*. And while some of his labors or other will do excellently to exemplify any and all of those kinds, not the other way around. Everything Leibniz has done he has in connection with everything. It is of the essence of his labors that they transgress

against those kinds. Indeed, among his most important results are that logic is mathematics, mathematics is physics, physics is metaphysics, metaphysics is logic.

To describe the labors of the philosophers importance should be the most important guide in any case. Not those kinds. Because the catastrophic transformation of philosophy in the last couple of centuries has been a catastrophic transformation of those kinds. Because, before the transformation as well as after, the more important the philosophers the more their labors have transgressed against those kinds. And because, whatever the kinds, the importance of the labors of the philosophers cannot not be established by the labors of their successors, and over time conclusively. The more so the more important the philosophers. The more so the most important philosophers, including Leibniz.

And if the labors of the philosophers have to be described in kinds, and they have to, as there can be no description but in kinds, explicitly or implicitly, the kinds should be those kinds. Those have been the kinds of philosophy for two and a half millennia. They have also been, however, the kinds against which the labors of the philosophers have transgressed, the more so the more important the philosophers. The labors of the philosophers should thus be described in those kinds such as the philosophers have, both, received them from their predecessors, such as their predecessors have both received them from their predecessors and transgressed against them, and transgressed against them. The more so the more important the philosophers. The more so the most important philosophers, including Leibniz.

To show the importance of Leibniz, then, first his important treatises can be enumerated and arranged in their series and order. And then some of his most important results can be shown, under the following heads. The philosophical calculus. Calculus. Dynamics. The history of the earth. History. China. The principles.

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Leibniz's very important treatises are nineteen, *Dissertation on the art of combination, in which from the fundamental arithmetic new precepts are extracted for the theory of combinations*

and transpositions, and their uses for all sciences are shown; new seeds sown for the art of meditation or the logic of discovery, in 1666,
Beginning and specimens of a general science for the instauration and growth of the sciences, in 1679,
Calculus of consequences, in 1679,
 the untitled manuscript beginning *Cum animadverterem plerosque*, in 1679,
On the analysis of situation, in 1679,
A new method for maxima and minima, as well as tangents, impeded by neither fractional nor irrational quantities, and a singular kind of calculus for it, in 1684,
 the untitled manuscript beginning *Primae veritates sunt*, in 1685,
On a recondite geometry and the analysis of indivisibles and infinites, in 1686,
A brief demonstration of a memorable error of Descartes and others concerning a natural law, according to which god would always conserve the same quantity of motion, misused also in mechanics, in 1686,
Discourse on metaphysics, in 1686,
Essay on dynamics concerning the laws of motion, in which it is demonstrated that conserved is not the same quantity of motion but the same absolute force, or the same quantity of moving action, in 1691,
Protoearth, or treatise on the first state of the earth and the vestiges of its ancient history in the monuments of nature, in 1693,
 the untitled manuscript beginning *Defin. 1. Eadem seu*, in 1695,
A specimen of dynamics for the discovery of the admirable laws of nature of the forces of bodies and their actions on one another, and for their reduction to their causes, in 1695,
New system of the nature and of the communication of substances, as well as of the union which there is of the soul and the body, in 1695,
On the radical origination of things, in 1697,
New essays on human understanding, in 1704,
Principles of nature and grace founded on reason, in 1714,
Principles of philosophy, or monadology, in 1714.

Most are short treatises. They are yet 19.

The years are those of writing, and in eight cases also of publication. In the other eleven cases publication was posthumous.

The *Principles of nature and grace* was published in 1718. The *Principles of philosophy, or monadology* was published in 1720. The *Protoearth, or treatise on the first state of the earth* was published in 1749. The *New essays on human understanding* was published in 1765. The *Discourse on metaphysics* was published in 1840.

And six of the treatises were published two-odd centuries after they were written. Beginning with *The art of combination*, Leibniz labored on the philosophical calculus, or general characteristic, or general science, for the following three decades. It was his main concern in the years around 1679, he worked at it again in the years around 1685, and again in the years around 1695, his labors on it recorded in some four dozen manuscripts. These were collected and published by Carl Gerhardt in 1890 and Couturat in 1903. And among these manuscripts are six of Leibniz's very important treatises, the *Beginning and specimens of a general science for the instauration and growth of the sciences*, the *Calculus of consequences*, the untitled manuscript beginning *Cum animadverterem plerosque*, and *On the analysis of situation*, all four written around 1679, the untitled manuscript beginning *Primae veritates sunt*, written around 1685, and the untitled manuscript beginning *Defin. 1. Eadem seu*, written around 1695. The first of the six requires another editorial comment, too. Gerhardt collected 20 of the manuscripts under the title *General science; characteristic*. The first 18 of these 20 are from the years around 1679, the last 2 are from the years around 1695. The title *Beginning and specimens of a general science for the instauration and growth of the sciences* is firmly attested by Leibniz himself, so is the unity of the first 18 of these 20 manuscripts, and this is what the *Beginning and specimens of a general science for the instauration and growth of the sciences* is, these 18 manuscripts.

If those are Leibniz's very important treatises, many more of his treatises are important, if not very important. And below is an attempt at a complete enumeration of his important treatises,

including the very important.

The very important are representative in that, like other philosophers, Descartes or Newton, or Hegel or Marx, Leibniz left behind much more than he published. The enumeration proceeds in the order of writing.

Also like other philosophers, and more than other philosophers, Leibniz has left behind many letters, more than 15 000 of them, to more than 1 100 correspondents. Among these are Hobbes, Guericke, Arnauld, Wallis, Redi, Bossuet, Huygens, Spinoza, Pufendorf, Mabillon, Burnet, Malebranche, Newton, Jakob Bernoulli, Johann Bernoulli, Wolff. The enumeration ignores the letters altogether.

The enumeration is an attempt as well to arrange the treatises in their series and order, to assign steps in their development and to relate them to kinds.

All in all, the enumeration counts 81 important treatises. If there are errors, in this enumeration as in all enumerations of the treatises of the philosophers above or below, these should be errors of commission, not of omission.

Again, most are short. They are yet 81.

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1666.

From this year is the *Dissertation on the art of combination, in which from the fundamental arithmetic new precepts are extracted for the theory of combinations and transpositions, and their uses for all sciences are shown; new seeds sown for the art of meditation or the logic of discovery*. The art of combination is the first step in the development of the philosophical calculus, or general characteristic, or general science. And the title suffices to show that the art of combination, and the philosophical calculus, is to rest on the fundamental arithmetic, and to concern itself with all sciences.

Descartes's labors on *The project for a general science capable of raising our nature to its highest degree of perfection* were completed in *The method to conduct one's reason well and search for truth in the sciences*. The *Dissertation on the art of combination*, and the philosophical calculus, are a continuation of these labors.

And Leibniz will concern himself with the

greatest generality, and this twofold, with the grounding of all sciences in the fundamental arithmetic and with the encompassing of all sciences into a unity, beginning to end.

The *Dissertation* ends with an *Appendix containing corollaries for arithmetical disputation*. The corollaries are under four heads: I. logic; II. metaphysics; III. physics; IV. practice. Under logic, Leibniz begins: 1. the primary propositions are two; one, the principle of all theorems or necessary propositions, what is thus is or is not thus and conversely; two, the principle of all observations or contingent propositions, something exists; 2. there can be perfect demonstrations in all disciplines; 3. considered in themselves, all disciplines are theoretical; considered in their uses, they are all practical (*id.*).

Propositions are of two kinds, necessary and contingent, theorems and observations. The principle, something exists, anticipates the principle of sufficient reason. The disciplines are all disciplines as explicitly as they are in the Aristotelian dichotomy of theory and practice, and in the division of theory into logic, metaphysics, and physics. There can be perfect demonstrations in all disciplines. And these are all corollaries for arithmetical disputation. Not disputation.

The two main, and complimentary, directions of all of Leibniz's labors are thus indicated clearly from the first. One is the search for the fundamental arithmetic on which all knowledge rests, and yet more generally, on which everything rests. The other is the search for the unitary and uniform description of all knowledge, and again yet more generally, the unitary and uniform description of everything. The development of the philosophical calculus and of calculus will be among the major results of the first. The development of history, of geology, of dynamics, of the system of nature, and of the principles of philosophy will be among the major results of the second.

1667 to 1677.

In these years Leibniz wrote jurisprudence, politics, physics, and mathematics.

He wrote the *New method of teaching and learning jurisprudence, from the principles of the art of teaching and the light of experience*, in 1667. Involved in an attempt to reform the law of

the empire, he wrote the *Elements of natural law*, in 1671. And he wrote the *Jurisdiction of the emperor, on the supreme and delegated rights of the German princes*, in 1677. These do not break new ground.

He wrote the *Project for the conquest of Egypt, presented to Louis XIV*, in 1671. This does not break new ground either.

He wrote *A new physical hypothesis, by which most phenomena of nature are derived from one universal motion supposed about our globe, without disdaining either the Tychonians or the Copernicans*, in 1671. And this does not break new ground either.

New ground break the mathematical writings.

Given that the art of combination is to concern itself with all sciences, all of Leibniz's subsequent writings are steps in the development of the art. Some of these steps, however, follow more immediately from the earlier steps than other. Thus, the corollaries for arithmetical disputation under: IV. practice, begin with: 1. justice (*id.*), and justice is the main concern of the writings on jurisprudence. But why jurisprudence, of all sciences, and justice, of all things, is not clear.

In these writings, on the other hand, Leibniz judges his predecessors, and his judgment is impeccable. In the *Jurisdiction of the emperor*, for instance, he writes: Hobbes says that by nature men have a right to all things (11); the commonwealth of Hobbes thus exists neither among civilized people nor among barbarians, and I consider it neither possible nor desirable, unless those invested with the sovereignty are gifted with angelic virtues (*id.*); the demonstrations of Hobbes hold only for the state whose sovereign is god, whom alone one can trust in all things (*id.*). In the *New method of teaching and learning jurisprudence*, he writes: history is the mother of observation; and on its nature, constitution, and classification Bacon is excellent in his *Growth of the sciences* and the not less brilliant *New organon* (I, 32a).

In *A new physical hypothesis* Leibniz formulates the principle of sufficient reason for the first time explicitly: there is nothing without a reason (*id.*). The corollaries for arithmetical disputation include: III. physics. But why physics is not clear either. Also, this first venture of Leibniz into

physics is – after Galileo, Kepler, Descartes, and at the time of Huygens – not mathematical.

Why mathematics is clear. The art of combination is to rest on the fundamental arithmetic.

Leibniz's first mathematical treatise is *An addition to the arithmetic of infinities, in which it is shown that a greatest number or a number of all numbers is impossible or nothing, and it is shown by examples that what is taken to be axioms can be demonstrated*, in 1672.

He writes: although *The art of combination* is a childish work, I do not scorn it entirely; I point out in it that all propositions of the pure sciences do nothing but propound definitions; I point out as well that this fact gives rise to a fertile art of construction of theorems; but let us show by the deed that the axioms can be demonstrated (*id.*). And he shows by the deed that all of Euclid's axioms can be demonstrated.

The fifth, for instance:

the whole is greater than the part;
for, if the parts are a , b , definition 1,
the whole is $a + b$, definition 2,
if a is less, definition 3,
 $c = a + b$ is greater, definition 4,
and if we order the definitions, the demonstration is complete;
the whole = $a + b$, definition 2,
 $a + b = c$, definition 4,
 c = greater, definition 4,
the part = a , definition 1,
and a = less, definition 3 (*id.*).

Leibniz has taken a decisive step from logic to mathematical logic. He has discovered also that the step poses the problem of the foundations of mathematics.

In the *History and origin of the differential calculus*, in 1714, Leibniz reports about that earlier time: at that time, in the first flower of youth, he added to his study of history and jurisprudence more profound reflections for which he had a natural inclination (*id.*).

So, after the *Addition to the arithmetic of infinities*, he wrote between 1673 and 1677 a dozen fragmentary manuscripts which attest that, having a natural inclination for these more profound reflections, Leibniz mastered the entire earlier development of mathematics, through Viète, Descartes, Cavalieri, Fermat, Wallis,

Pascal, Barrow, Newton. And there are three major new results in these manuscripts. In the *New method to investigate the tangents of curves*, in 1673, Leibniz speaks of: functions (*id.*). In the *Second part of the analysis of quadrature*, in 1675, he discovered the notation for the integral. He writes:

$$\int x^2 = \frac{x^3}{3} \text{ (id.)}.$$

And in the *Examples of the inverse method of tangents*, also in 1675, he discovered the notation for the differential. He writes:

$$\int y dy = \frac{y^2}{2} \text{ (id.)}.$$

The years around 1679.

Having advanced in mathematics as far as mathematics has advanced, Leibniz focused on the philosophical calculus. He wrote the *Beginning and specimens of a general science for the instauration and growth of the sciences*, the *Calculus of consequences*, and the manuscript beginning *Cum animadverterem plerosque*. He wrote also the *Elements of the universal characteristic*, the *Elements of calculus*, the *Elements of the universal calculus*, the *Investigations into the universal calculus*, the *Method of examination of inferences by numbers*, the *Rules with which the validity of inferences and the forms and moods of categorical syllogisms can be judged by numbers*.

Frege writes: Leibniz's conception of a general characteristic, or *calculus philosophicus*, was so gigantic, the attempt to realize it could not go beyond the preparation.

That the attempt to realize the conception could not go beyond the preparation explains the fragmentary character of the manuscripts in which the attempt is made.

That the success of the attempt to realize the conception is nevertheless as gigantic as the conception the fact attests that logic is mathematics, and it is Leibniz's attempt to realize the conception of a general characteristic, or *calculus philosophicus*, that established this fact.

The names of the manuscripts suffice to indicate this. The philosophical calculus is the universal calculus, or simply calculus. The philosophical calculus, or universal calculus, or simply calculus, is the calculus of consequences. It is the method of examination of inferences by numbers.

It is the rules with which the validity of inferences can be judged by numbers.

It is the general science for the instauration and growth of the sciences. So that: there will be no more need for disputes between two philosophers than between two calculators; it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate.

The success of the attempt to realize the philosophical calculus, or simply calculus, as Leibniz called it, is the foundation of the success of the differential calculus, as Leibniz called it, and integral calculus, or simply calculus. It is with the development of the differential and integral calculus, or simply calculus, that in much of philosophy, and in all of its better parts, it has come to suffice to take a pen, sit down to an abacus, and calculate.

In general, the success of the attempt to realize the philosophical calculus is the foundation of all of Leibniz's subsequent successes. The great advance of mathematics, and hence also of physics, and hence also of philosophy in all its parts.

Also in 1679 Leibniz wrote *On dyadic number*. This is the discovery of the binary notation. Leibniz begins by listing the numbers:

1	1,
2	10,
3	11,
4	100,
5	101,

all the way to:

98	1111110,
99	1111111,

then notes: the sequence can easily be continued (*id.*), then shows addition, subtraction, and multiplication.

Also in 1679 he wrote *On the analysis of situation*. The analysis of situation anticipates topology.

1682 to 1693.

Leibniz continued to work on the philosophical calculus. He wrote *A specimen of the universal calculus*, in 1685, and the *General investigations into the analysis of concepts and truths*, in 1686.

He wrote jurisprudence. He wrote the preface to the collection of laws *Code of the law of nations*, in 1693.

He wrote politics. He wrote the *Perpetual irony, a most Christian Mars, or an apology for the wars of the most Christian king against the Christians*, in 1683. The most Christian king is Louis XIV.

Those do not break new ground. The following all do.

Pascal had constructed a calculating machine. Leibniz constructed a more advanced calculating machine, and described it in *An arithmetical machine, which performs not only addition and subtraction, but also multiplication without any and division nearly without any animate labor*, in 1685.

More generally, these are the years in which Leibniz greatly advanced mathematics. He published the following. *On the true proportion of the circle to the circumscribed square, expressed in rational numbers*, in 1682, in which he showed that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$. *On discovering the dimensions of figures*, in 1684. *A new method for maxima and minima, as well as tangents, impeded by neither fractional nor irrational quantities, and a singular kind of calculus for it*, in 1684, in which he formulated the rules, or algorithm, as he called it, of the differential calculus, as he called it. *A new meditation on the nature of the angles of contact and osculation, and on their use in practical mathematics for substituting easier figures for more difficult ones*, in 1686. *On a recondite geometry and the analysis of indivisibles and infinites*, in 1686, in which he formulated the rules, or algorithm, of the integral calculus. *On optical lines, and other things*, in 1689. *On the isochronic line in which a heavy body without acceleration descends*, in 1689. *On the line in which a body curves by its own weight, and its remarkable use in discovering any number of mean proportionals and logarithms*, in 1691. *On the line formed by an infinite number of lines drawn ordinatewise which meet each other and are all tangent to it, and on the consequent new use of the analysis of infinites*, in 1692, in which he introduced the technical term function. *A supplement to practical geometry extending to transcendental problems with the help of a new very general method of infinite series*, in 1693. *A supplement to dimensional geometry, or the most*

general production of all quadratures by motion, as well as the construction of many lines from given conditions of the tangents, in 1693, in which he proved the fundamental theorem of calculus.

With these publications calculus became what it is. Euler writes: calculus, considered before Leibniz various individual techniques, Leibniz organized into a discipline, collected its rules into a system, and gave it a crystal clear explanation (*Differential*, pr.). Thus also mathematics became what it is. Euler writes: this higher analysis has penetrated throughout all branches of mathematics to such an extent that what can be explained without it can be judged next to nothing (*id.*).

In these years Leibniz began greatly advancing physics as well. It is now also clear why physics.

Because mathematics is physics.

The titles of the mathematical treatises indicate this. They concern themselves with optical lines, with the isochronic line in which a heavy body without acceleration descends, with the line in which a body curves by its own weight.

With Galileo: the world is written in mathematics. With Descartes: all of corporeal nature is the object of pure mathematics. And: I admit in physics no other principles than in geometry or abstract mathematics. With Newton: the principles of natural philosophy are mathematical.

From Galileo through Leibniz the preestablished harmony of mathematics and physics is shown ever more clearly and distinctly. Physical problems are formulated and solved ever more explicitly mathematically. The laws of nature are formulated ever more explicitly mathematically. And the search for the principles of mathematics such as they are the general principles of nature, and thus of physics, is ever more explicit, and successful.

Leibniz's great advance of physics begins with his success in the attempt to formulate these most general principles. This success is moreover twofold. It is success in the attempt to ground these most general principles immediately in calculus, and ultimately in the philosophical calculus. And it is success in the attempt to reach this greatest generality substantially more explicitly than it has been.

Leibniz wrote *On a general principle useful*

for the explanation of the laws of nature through the divine wisdom, in 1687.

He writes: the principle of general order has its origin in the infinite, and is absolutely necessary in geometry, but is useful in physics as well, because the sovereign wisdom, which is the source of all things, acts as a perfect geometer, observing a harmony to which nothing can be added (*id.*).

The principle is useful in physics because the sovereign wisdom, which is the source of all things, is nature. And nature acts as a perfect geometer.

Leibniz continues: the principle can be formulated thus; when the difference between two cases can be diminished less than any given quantity in what is given, or what is posed, it must be possible to diminish it less than any given quantity in what is sought, or what is the result, as well; or to put it more easily, when the cases, or what is given, approach one another continuously and at last disappear one into the other, it must be that their results or consequences, or what is sought, do so as well; and this, in turn, depends on the more general principle, the order of the given is the order of the sought (*id.*).

The principle of general order explains the laws of nature. The order of the given is the order of the sought. It explains also the discovery of the laws of nature.

It explains also Leibniz's further advances.

He wrote *A brief demonstration of a memorable error of Descartes and others concerning a natural law, according to which god would always conserve the same quantity of motion, misused also in mechanics*, in 1686. The error of Descartes is that god does not always conserve quantity of motion, where quantity of motion is the product of volume and speed, god always conserves motive force, where motive force is the product of mass and the square of the velocity.

The product of mass and the square of the velocity is twice the kinetic energy.

In 1687 Newton's *Principles* appeared. All subsequent treatises, Leibniz's and any other, reflect this fact.

Leibniz wrote *An examination of the causes of the heavenly motions*, in 1689, and the untitled manuscript beginning *Cum geometrias demon-*

strationibus, also in 1689. In the latter he notes that Copernicus has been confirmed by Kepler: who first revealed to mortals the laws of the heaven, the truth of things, the laws of the gods (*id.*), and Kepler has been confirmed by the demonstration that: areas described by bodies with respect to a center are proportional to the times whatever the law governing their motion toward or away from the center (*id.*). And concludes: we have thus achieved what the ancients could hardly envision in their prayers, we have reduced by geometrical analysis the primary phenomena of the world to the simplest and clearest principles, and that is, to the best and truest hypotheses (*id.*).

He wrote the *Preliminary specimen of dynamics, on the law of nature relating to the power of bodies*, in 1691. He writes in the *Preliminary specimen*: I lay down the true elements of a new science of power and action, which can be called dynamics (*id.*).

Also in 1691, he wrote the *Essay on dynamics concerning the laws of motion, in which it is demonstrated that conserved is not the same quantity of motion but the same absolute force, or the same quantity of moving action*. He writes in the *Essay on dynamics*: I was the first to make known two laws, the law of conservation of absolute force, and the law of continuity, that every change takes place by indefinitely small transitions and never by a leap (*id.*). Absolute force is the product of mass and the square of the velocity. The law of continuity follows from the principle of general order.

In the *Essay on dynamics* Leibniz achieved his greatest advance of the new science of dynamics. The development of the new science of dynamics is his greatest advance of physics.

Dynamics is the new science of power and action. Hence the further development of dynamics. Hence the newer sciences of thermodynamics, electrodynamics, quantum electrodynamics.

He wrote the *Protoearth, or treatise on the first state of the earth and the vestiges of its ancient history in the monuments of nature*, in 1693. With the *Protoearth* the history of the earth became an empirical investigation.

Hence the further development of geology, Buffon, Hutton, Werner, Lyell. And hence the

further development of biology. As the investigation of the history of the earth advanced, it became the foundation of the investigation of organisms. The investigation of organisms became the investigation of the history of organisms, Buffon, Lamarck. And Darwin.

In these years, finally, Leibniz began the explicit work at the synthesis of all knowledge, and more generally of everything, into a unitary and uniform description.

It is clear why. Logic is mathematics, mathematics is physics, physics is metaphysics, metaphysics is logic.

He wrote the following. The *Meditations on knowledge, truth, and ideas*, in 1684. The untitled manuscript beginning *Prima veritates sunt*, in 1685. The *Discourse on metaphysics*, in 1686. And the *Animadversions on the general parts of Descartes's Principles*, in 1692.

Part I of Descartes's *Principles* is *On the principles of human knowledge*, part II is *On the principles of the material things*. The *Animadversions* analyzes these parts of Descartes's *Principles* section by section.

The principles are the principles of the acquisition of knowledge and the principles of things. And the great success of Leibniz's attempt at the synthesis of all knowledge, and more generally of everything, is in showing substantially more explicitly than it has been how and why the principles of things and the principles of the acquisition of knowledge are both principles. In showing how and why the principles of things are principles of the acquisition of knowledge. And in thus showing which are the principles of all principles. Which are the principles.

The principle of contradiction is the principle of all theorems, the principle of sufficient reason is the principle of all observations. These are principles of the acquisition of knowledge.

The principle of general order explains the laws of nature. The order of the given is the order of the sought. It is also a principle of knowledge. But it cannot be a principle of knowledge if it is not a principle of things. The order of the sought cannot be the order of the given if it is not order. The principle has its origin in the infinite, and this is why it is absolutely necessary in geometry, and is useful in physics as well. This is why it

explains not only the discovery of the laws of nature. It explains the laws of nature.

In the same way the principle, there is nothing without a reason, cannot be a principle of knowledge if it is not a principle of things. That there is nothing without a reason cannot be without a reason. The reason is god. Nature.

And this is why the principle, when the cases approach one another continuously it must be that their consequences do so as well, depends on the principle of general order. Why the laws of conservation and continuity follow from the principle of general order. Why, in general, physics is metaphysics. The laws of nature depend on, and follow from, the most general principles, the principles of the world, or nature, or god, in its entirety.

Leibniz writes in the *Discourse on metaphysics*: god is an absolutely perfect being (1); he acts always in the most perfect way possible (4); he does nothing without order (6); not only is there nothing absolutely irregular in the world, such a thing cannot even be conceived (*id.*); god has chosen the most perfect world, and that is, the one which is at the same time the simplest in hypotheses and the richest in phenomena (*id.*).

Concerned with hypotheses and with the phenomena, god is a mathematician and physicist. Concerned with the simplest, with the richest, and especially with the simplest and at the same time the richest, he is a metaphysician.

Leibniz continues: although all particular phenomena of nature can be explained mathematically or mechanically, the general principles of corporeal nature and of mechanics are nevertheless metaphysical rather than geometrical (18); the principle of all existences and of the laws of nature must be sought in god, because he always proposes the best and the most perfect (19); if god's wisdom is recognized in the mechanical structure of particular bodies, it must be that it is manifested also in the general economy of the world and in the constitution of the laws of nature (21).

And Leibniz illustrates with the principle of least time which: Snel and Fermat have most ingeniously applied to refraction (22). Refraction shows god's wisdom in structuring particular bodies mechanically, the principle of least time

shows that he always proposes the best in the general economy of the world and the constitution of the laws of nature.

Physics is metaphysics, and metaphysics is logic.

Leibniz writes in the manuscript beginning *Primae veritates sunt*: the primary truths are those which affirm the same or deny the opposite of its opposite; for instance,

A is A,

A is not not A,

if it is true that *A is B*, it is false that *A is not B*,

everything is as it is,

everything is equal to itself,

nothing is greater than itself,

and so on (*id.*); the axiom that nothing is without a reason follows immediately; otherwise there would be truths which could not be proved *a priori*, and that is, which could not be resolved into identities, contrary to the nature of truth, which is always an explicit or implicit identity (*id.*).

Truth is identity. The principle of sufficient reason follows immediately. Conversely, the principle of all things is god. The principle of sufficient reason follows immediately.

And considered in themselves, all disciplines are theoretical, considered in their uses, they are all practical. Leibniz concludes in the *Discourse on metaphysics*: and morality must be joined to metaphysics, and that is, god must be considered not only the principle and cause of all substances and all beings, but also the leader of all intelligent substances, and the absolute monarch of the most perfect city (35). Why justice is now clear.

1694 to 1716.

In these final years of Leibniz's labors many of his treatises are dominated by the difficulties of enlightenment. Descartes writes: whoever walks in the dark long enough to become used to it can no longer bear the light of day. A half century later, it has become clear that the cause of the difficulties lies deeper. Leibniz writes: men have aversion to light, and love being returned to darkness (*Antibarbarian*).

Leibniz continued his mathematical investigations. He published *A new application of the differential calculus and its use for multiple constructions of lines from a given condition of*

tangents, in 1694, *The proper construction of the problem of the paracentric isochronic curve, as well as some general facts about the nature and differential calculus of osculates, and on the constructions of transcendental lines, one entirely geometrical, another mechanical but very general; with a supplement on a way to render the discovery of transcendental lines universal, so that they include any case whatever and pass through a given point*, in 1694, the *Communication of his own solution of the problem of the curve of fastest descent, proposed to the geometers by Mr Johann Bernoulli, and of two other solutions, communicated to him first by Mr Johann Bernoulli, then by marquis de l'Hôpital, as well as his own solution of another problem proposed by the same*, in 1697, the *New specimen of analysis for the science of the infinite concerning sums and quadratures*, in 1702.

He returned one more time to the philosophical calculus. In the years around 1695 he wrote *A mathematics of reason, On the mathematical determination of the forms of the syllogism*, and *A not inelegant specimen of demonstration in abstracts*.

He continued his labors of laying down the true elements of the new science of dynamics. He wrote *A specimen of dynamics for the discovery of the admirable laws of nature of the forces of bodies and their actions on one another, and for their reduction to their causes*, in 1695. This is his most comprehensive physical treatise.

He wrote *An anagogical essay in the research of causes*, in 1695, and *On nature itself, or on the inherent force and actions of created things, to confirm and illustrate their dynamics*, in 1698.

The more successful the attempt at the synthesis of all knowledge, and of everything, into the system of nature and the principles of philosophy has become, the more it has met with objections, and the more has Leibniz found himself responding to the objections. He reaffirms in the *Anagogical essay*: all natural phenomena can be explained mechanically, but the principles of mechanics cannot be explained geometrically, they depend on the more sublime principles showing the wisdom of the author in the order and perfection of his work (*id.*).

That it follows that the world is the most

perfect has remained an unpalatable difficulty. Leibniz explains in *On the radical origination of things*: perfection is nothing but the quantity of essence (*id.*); you still object, we experience the contrary in the world; that so it appears at first glance I admit; but a closer inspection forces the contrary view (*id.*); and, as the jurists say, it is not just to judge without inspecting all the law (*id.*); it can also be objected, if this were so, the world would have long ago become a paradise; the response is easy; because the continuum is infinitely divisible, there are always parts asleep in the abyss of things, yet to be roused and brought to further and greater advancement; progress never comes to an end (*id.*).

That the foundations of dynamics are metaphysical has remained an unpalatable difficulty as well. Leibniz writes in *On nature itself*: a force to act and react is the very substance of things (*id.*).

He wrote the *Clarification of the difficulties which Mr Bayle has found in the new system of the union of soul and body*, in 1698, and the *Reply to the reflections contained in the second edition of the Critical dictionary of Bayle, article Rorarius, on the system of preestablished harmony*, in 1702. A generation before Berkeley, asleep recently in the abyss of things, scepticism was once again roused to greater and better things by Bayle: a mind so great and profound it cannot but instruct (*Clarification*); an adversary just as well as profound, and so fair he foresees my replies (*Reply*).

Against its ever greater power and scope, the lack of rigor of calculus remained an unpalatable difficulty as well. Leibniz wrote the *Justification of the calculus of infinitesimals by that of the ordinary algebra*, in 1701. As Newton is clear that the resolution of the difficulty is limit so is Leibniz. He writes: although it is not rigorously true that rest is a species of motion, or that equality is a species of inequality, just as it is not rigorously true that the circle is a species of the regular polygon, nevertheless it can be said that rest, equality, and the circle terminate the motions, inequalities, and regular polygons which by continuous change reach them and vanish in them; and although these terminations are not comprised rigorously in the variations which they limit, they nevertheless have the same properties;

otherwise the law of continuity is violated (*id.*).

He wrote the untitled manuscript beginning *Nullum quidem librum*, in 1702. It is in this that he reports: with Aristotle and Descartes, against Democritus and Gassendi, I admit no vacuum; against Aristotle, with Democritus and Descartes, I consider all rarefaction and condensation only apparent; with Democritus and Aristotle, against Descartes, I think that there is something passive in bodies next to extension; with Plato and Aristotle, against Democritus and Descartes, I recognize an active force in bodies; with Democritus and Descartes, against the multitude of scholastics, I hold that the phenomena of bodies can always be explained mechanically (*id.*).

He wrote the *Explication of binary arithmetic, which employs only the characters 0 and 1, with remarks on its utility, and on how it makes sense of the ancient Chinese figures of Fu Xi*, in 1703. He writes: binary calculation is the most fundamental for science (*id.*).

He wrote the *Manifesto for the defense of the rights of Charles III, king of Spain*, in 1703. The Spanish succession was contested between the Bourbon Philip and the Habsburg Charles.

He wrote the *Opinion on the principles of Pufendorf*, in 1706. He writes: as Grotius justly observes, there would be a natural obligation to others even on the hypothesis, impossible as it is, that god does not exist; for the care for his own preservation and well-being imposes on man many requirements to care for others, as even Hobbes admits (4); thus, even if nothing is superior to god, there must be a supreme justice in him; the essence of the just does not depend on his free decision, but on eternal truths, which constitute the essence of divinity itself (*id.*).

He published the *Essays of theodicy on the goodness of god, the freedom of man, and the origin of evil*, in 1710. He writes: there are two famous labyrinths in which our reason is very often lost; one concerns the great question of the free and the necessary, and above all the production and origin of evil, the other consists in the discussion of continuity and the indivisibles which appear to be its elements, and in which the consideration of the infinite must also enter (*pr.*). The *Theodicy* explores the first labyrinth.

He wrote *The antibarbarian physicist, for real*

philosophy, against the revival of the qualities of the scholastics and chimerical intelligences, in 1710. Leibniz writes: those who think gravity essential to matter drive physics back to barbarism and the occult qualities of the scholastics (*id.*). This is where Hegel learned that Newton is a barbarian. How complete a barbarian he established himself.

At this time calculus was beset by a most discreditable and disgraceful distribution of credit and disgrace. Leibniz had to write the *History and origin of the differential calculus*, in 1714.

In 1715 he wrote *The metaphysical foundations of mathematics*. The metaphysical foundations are the philosophical calculus, and this is Leibniz's last attempt to show that.

Those do not break new ground. The following do.

Leibniz completed the most advanced specimen of the philosophical calculus.

This is in the manuscript beginning *Defin. 1. Eadem seu*, in 1695.

He greatly advanced history.

He wrote the *Brunswick annals of the western empire*, in 1716. He had been working on it since 1685. Along the way he had published the *Letter on the connection of the houses of Brunswick and Este*, in 1695, the *Dissertation on the origin of the Germans*, in 1697, and the preface to the *Additions to history, writers on German matters collected from manuscripts*, in 1698. The *Annals* deals with the empire from Charlemagne, prince 768-814, crowned emperor in 800, through Henry II, prince 1002-24, crowned emperor in 1014. In 1716, when he died, Leibniz was nearing completion. Parts of the *Annals* appeared in 1749-53. It was published in its entirety in 1843-6, in six volumes, some 2 300 pages.

He discovered the importance of China.

This is in the preface to *The latest on China*, in 1697, and the *Discourse on the natural theology of the Chinese*, in 1716.

He undertook his most comprehensive investigation of knowledge.

This is in the *New essays on human understanding*, in 1704. The *New essays* is in the form of a dialogue between Philalethes and Theophilus. Locke published *An essay concerning human understanding*, in 1690. In the *New essays* Philal-

ethes introduces passages from Locke's *Essay*, Theophilus comments on the passages, and advances greatly beyond them.

And he completed his labors at the synthesis of all knowledge, and of everything, into the system of nature and the principles of philosophy.

This is in the *New system of the nature and of the communication of substances, as well as of the union which there is of the soul and the body*, in 1695, *On the radical origination of things*, in 1697, the *Principles of nature and grace founded on reason*, in 1714, and the *Principles of philosophy, or monadology*, also in 1714.

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Next, some of Leibniz's most important results can be shown more particularly, under the heads, the philosophical calculus, calculus, dynamics, the history of the earth, history, China, the principles.

The philosophical calculus.

The philosophical calculus is the general characteristic is the general science is the universal calculus is calculus.

Leibniz's labors began with the conception of the philosophical calculus, in 1666, with *The art of combination*. He mastered all mathematics, and in the years around 1679 the philosophical calculus became his central concern. He began advancing mathematics, and returned to the philosophical calculus in the years around 1685. He continued advancing mathematics, began advancing physics, and returned to the philosophical calculus one more time in the years around 1695.

And to show his success, two specimens of the calculus can be shown, one from 1679, a specimen of early success, and one from 1695, the most advanced specimen. Then how Leibniz describes the calculus in general can be shown. And then the various names he gave the calculus can be sorted out.

A specimen of early success is the *Calculus of consequences*, in 1679. Leibniz begins:

two things have to be distinguished in all arguments, the form and the matter; for there can be an argument which succeeds with respect to a certain matter yet cannot be applied to all instances of the same form; for instance,

all triangles are trilateral,

some triangles are not equilateral,
therefore some equilaterals are not trilateral;
the conclusion is good, but by virtue of the matter, not the form;

so, a calculus which deals with the matter can be separated from a formal calculus (*id.*).

In the *Beginning and specimens of a general science* he writes: to construct the general characteristic nothing more is necessary than to provide its grammar and to compile the dictionary of its most frequent terms, or what is the same, to give the characteristic numbers of all ideas (11).

He continues in the *Calculus of consequences*:
and although I have not yet discovered the true characteristic numbers of things, a calculus which deals with the form of inferences can be demonstrated with fictitious numbers standing in place of the true characteristic numbers; and this is what I propose to do here;

in all categorical propositions the terms are two, the subject and the predicate; to these are added the copula, or is, affirmation or negation, or quality, and a sign, all or some, generality or particularity, or quantity; for instance, in the proposition, the pious are happy, the pious and happy are the terms, the pious is the subject, happy is the predicate, and are is the copula; the proposition, the pious are happy, affirms, the proposition, the wicked are not happy, denies; and the propositions, all the pious are happy, and, no one wicked is happy, are general, affirmative and negative, respectively;

in all propositions the predicate is in the subject, or the notion of the predicate is contained in the notion of the subject; in the general affirmative proposition, all men are animals, the concept of animal is contained in the concept of man; similarly, a negative proposition, by denying that the predicate is in the subject, affirms that the negation of the predicate is in the subject; no one wicked is happy, is the same as, all the wicked are not happy;

moreover, every composite notion is composed of other notions, some positive, some negative; for instance, prime number is number not divisible by another number greater than unity; hence, in general, every notion can be expressed by two characteristic numbers, one with the sign +, the other with the sign -; for instance,

prime is number not divisible

+22 -17;

also, when positive terms are in the relation of genus and species, their negations are in the relation of species and genus; if body is the genus of which animal is a species, not animal is the genus of which not body is a species; body is more general than animal, but not animal is more general than not body; for all not bodies are not animals, but not conversely; hence, we construct for every notion its characteristic number by multiplying the numbers of the notions of which it is composed;

thus, if animal rational
 +13 -5 +8 -7,

then for this term man

the characteristic number is +(13×8) - (5×7),

or +104 -35;

we must only be careful that no number is contained in both the positive and the negative parts, and that is, that the positive and the negative parts are not divisible by the same number; for if we write

 animal rational

 +13 -5 +10 -7,

 man

 +130 -35,

this is absurd; the notion signified by +5 is contradictory to that signified by -5, but 5 is contained in 10, animal is thus contained in rational, and this is absurd;

and now we see how the quantity, quality, and truth of propositions can be distinguished;

 the pious are wretched

 +10 -3 +14 -5;

it is clear that +10 and -5 are incompatible, signifying a contradiction, and the proposition in which these characteristic numbers are found is false, and its contradictory is true, by virtue of the terms alone;

 all the wise are pious

 +70 -33 +10 -3;

it is clear that the predicate must be in the notion of the subject, because the characteristic numbers of the subject are divisible by the characteristic numbers of the predicate;

 all men are rational animals

 +130 -35 +10 -7 +13 -5;

it is clear that +130 is divisible by +10 and by

+13, and -35 is divisible by -7 and by -5 (*id.*).

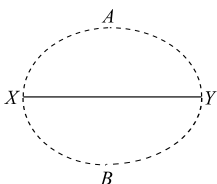
Leibniz has not discovered the true characteristic numbers of things, but he has a calculus which deals with the form of inferences. We see how the quantity, quality, and truth of propositions can be expressed formally. We thus also see that truth or falsehood can be judged by form alone. And it is a calculus.

The most advanced specimen is in the manuscript beginning *Defin. 1. Eadem seu*, in 1695.

It is axiomatic. From 6 definitions, 2 postulates, and 2 axioms, Leibniz deduces 24 propositions, and 2 corollaries. He illustrates as well with 9 scholia.

The definitions, postulates, and axioms are:
definition 1. same or coincident are these, which can everywhere be substituted for one another without the loss of truth; for instance, triangle and trilateral;

$A=B$ signifies that A and B are the same; as of the straight line XY and the straight line YX we say that $YX=XY$, or that the shortest path from X to Y coincides with that from Y to X ;



definition 2. diverse are these, which are not the same; for instance, circle and triangle;

$A \neq B$ signifies that A and B are diverse;

definition 3. A is in L , or L contains A , is the same as that a plurality, which includes A , can be added together so that it coincides with L ;

definition 4. all these, in which there is what is in L , are together said to be components of that L which is composed or constituted of them;

$B \oplus N=L$ signifies that B is in L , or L contains B , and that B and N constitute or compose L ;

definition 5. subalternates I call these, of which one is in the other; as A and B , if either B is in A or A is in B ;

definition 6. disparate are these, of which neither is in the other;

axiom 1. $B \oplus N=N \oplus B$, or transposition changes nothing;

postulate 1. given anything, there can be something diverse from it, and if one pleases, disparate, or such that one is not in the other;

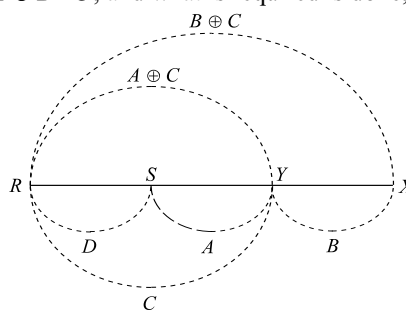
postulate 2. any plurality, as A and B , can be

added together to compose one, $A \oplus B$, or L ;
axiom 2. $A \oplus A=A$; if nothing new is added, nothing new results, or repetition changes nothing; for 4 coins and 4 other coins are 8 coins, but 4 coins and the same 4 coins are not (*id.*).

And this is one of the propositions:

proposition 22. given two disparates A and B , to find a third diverse from them C , which composes with them subalternates $A \oplus C$ and $B \oplus C$, and that is, although A and B are neither in the other, $A \oplus C$ and $B \oplus C$ are one of them in the other;

solution. if we want $A \oplus C$ to be in $B \oplus C$, although A is not in B , this is how it can be done; assume, by postulate 1, some D such that it is not in A ; let, by postulate 2, $A \oplus D=C$; and what is required is done;



for $A \oplus C=A \oplus A \oplus D$, by construction, $=A \oplus D$, by axiom 2;

similarly, $B \oplus C=B \oplus A \oplus D$, by construction;

but $A \oplus D$ is in $B \oplus A \oplus D$, by definition 3;

therefore, $A \oplus C$ is in $B \oplus C$; Q.E.F.

SY and YX are disparate; let $RS \oplus SY=YR$; and $SY \oplus YR$ will be in $XY \oplus YR$;

let A be equilateral, B parallelogram, D equiangular, C equilateral equiangular, or regular; it is clear that, although equilateral and parallelogram are disparate, and neither is in the other, a regular equilateral is in regular parallelogram, or square;

but will this prescribed construction, you ask, succeed in all cases? let, for instance, A be trilateral, B quadrilateral; is it then not impossible to find a notion such that A and B are both in it, and thus $A \oplus C$ which is in $B \oplus C$? for A and B are incompatible?

I reply, our general construction depends on postulate 2, that any two can be added together

as components; god, soul, body, point, and heat compose the aggregate of these five things; in the same way quadrilateral and trilateral can be the components of an aggregate, and the problem is solved;

for assume that D is anything not contained in trilateral, say, circle; then $A \oplus D$ is trilateral and circle; call it C ; $C \oplus A$ is then nothing but trilateral and circle, too; and whatever is in $C \oplus B$ is in trilateral, circle, and quadrilateral; if one wishes to apply this general calculus of compositions to a special mode of composition, if, for instance, one wishes trilateral, circle, and quadrilateral not only to compose an aggregate, but to be all a notion in the same subject, he has to see whether they are compatible; immovable straight lines at a distance from one another compose an aggregate, but do not compose a continuum (*id.*).

Leibniz has taken a decisive step beyond subject and predicate. A decisive step toward function and argument. A decisive step also toward sets. And it is a calculus *par excellence*.

18 manuscripts are collected in the *Beginning and specimens of a general science for the instauration and growth of the sciences*. The largest manuscript, comprising one third of the whole, is a collection of fragments. The other 17 range from a couple of pages to about a dozen pages each. 10 of the 18 are untitled. The other 8 are titled. 1 title is *Preparation for an encyclopedia or universal science*. 1 is *Specimen of a universal calculus*. 1 is *Discourse on THE METHOD OF CERTAINTY AND THE ART OF DISCOVERY, to end disputes and make great progress in a short time*. 5 are, or contain, *Beginning and specimens of a GENERAL SCIENCE for the instauration and growth of the sciences*. Leibniz can hardly have been clearer what the title of the prospective treatise was to be. He can hardly have been clearer that it is Bacon's labors that he proposes to continue, too.

And it is in the *Beginning and specimens of a general science* that he describes the general science in general.

He writes:

it is an old saying, god made all things by weight, measure, and number; there are things, however, that cannot be weighed, those that have

no force or power, there are things also that have no parts and cannot be measured; but there is nothing that cannot be numbered; number is, as it were, the metaphysical form, and arithmetic is the statics of the world, by which the powers of things are explored;

that the greatest mysteries lie hidden in number men have known since Pythagoras;

but I do not know of any mortal who has yet seen the true principle by which each thing can be assigned its own characteristic number;

no one has devised a universal language, or universal characteristic, which is simultaneously an art of discovery and an art of judgment, whose marks, or characters, do in discovery and judgment what the marks of arithmetic do in numbers and the marks of algebra do in abstract magnitudes;

I stumbled necessarily on this admirable observation, that an alphabet of human thought is possible, such that by the combination of its letters and the analysis of the words built from them alone all things can be both discovered and judged;

I published this miraculous discovery in *The art of combination* (11);

the observation of mathematical analysis then showed me the truest and most beautiful examples of this general analysis of human thought (14);

all human reasoning is perfected by signs, or characters; neither the things themselves nor the ideas of things can always be observed distinctly by the mind; nor need they be; because signs are substituted for them;

under signs I comprehend words, letters, chemical, astronomical, and Chinese figures, hieroglyphs, musical, cryptographic, arithmetical, and algebraic marks, and all others which stand in thoughts for things; when the signs are written, drawn, or carved, they are called characters;

the vulgar languages are indispensable in reasoning; they are nevertheless hampered by innumerable equivocations and cannot serve as a calculus; otherwise errors in reasoning could be detected as solecisms and barbarisms from the very form and construction of the words; the marks of arithmetic and algebra alone have until now had this admirable advantage (15);

even geometry has been treated analytically only to the degree to which it has been reduced to numbers by algebra, in which numbers are designated generally by letters (14);

in arithmetic and algebra alone all reasoning consists in the use of characters, and an error of the mind is the same as an error of calculation;

having truly applied myself to this matter, I saw that all human thoughts can be resolved into a few primitives; if characters are assigned to these, characters can then be formed for notions derivative from them, and it will always be possible to discover from the derivative notions the primitives which are their components;

it will be possible to find definitions, or values, and then to demonstrate the properties implied by the definitions (15);

and it will be possible to arrange propositions in their natural order;

to construct the general characteristic, then, nothing more is necessary than to provide the grammar of this miraculous language and to compile the dictionary of its most frequent terms, or what is the same, to give the characteristic numbers of all ideas;

once the characteristic numbers of most notions are determined, the human race will have a new organon so much more powerful than microscopes and telescopes as reason is than vision; for given anything else men may deteriorate rather than improve; correct reasoning alone is always salutary; and who will doubt that reasoning will always be correct if it is everywhere as clear and certain as arithmetic has been;

disputes are now won by desires, not by reason; in controversies the Gordian knot is cut, not the end of the rope found; and the more so whenever life is at issue (11);

controversies will never end, silence will never be imposed on the sects, until complex reasonings are reduced to simple calculations, and words of vague and uncertain meaning are reduced to determinate characters (14);

our true characteristic will reduce controversies to calculation with numbers; all reasons will be weighted on a balance; for even probabilities can be calculated (11);

all paralogisms will be nothing but errors of calculation; all sophisms will be shown solecisms

or barbarisms and will be easily corrected by the laws of this philosophical grammar;

there will be no more need for disputes between two philosophers than between two calculators; it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate;

it is manifest that the art of discovery cannot establish truths of fact or what depends on chance or accident; so it may seem to some that the art will be of little use in matters which depend on conjecture, in civil or natural history, in the examination of natural bodies or of intelligent persons, in the life of communities, in medicine, the law, war, the government of republics;

I reply, as powerful as reason is in all these, and it is the most powerful, so powerful and much more will be this art; for it is nothing but the ultimate elevation of human reason and its most comprehensive employment by symbols and marks;

by this analysis it will be possible to reach a conclusion where without it no result at all can be either determined or expressed from the available data; it will be possible either to attain an infinite approximation or to determine the degree of probability (14);

the characteristic art will contain the true organon of the general science of all things under human reasoning, always being at the same time a calculus of evident demonstrations;

moreover, the characteristic will exhibit also the order of the sciences, and it will become clear that the elements of arithmetic are prior and simpler than the elements of the logical calculus of the figures and modes of reasoning (15).

The philosophical calculus is the general characteristic is the general science is the universal calculus is calculus.

And it still seems to some that the philosophical calculus is of little use in matters which depend on conjecture, in civil or natural history, in the examination of natural bodies or of intelligent persons, in the life of communities, in medicine, the law, war, the government of republics. It still only seems so. By the philosophical calculus it is possible to reach a conclusion where without it no result at all can be obtained from the available data. It is often possible to attain an infinite

approximation and it is always possible to determine the degree of probability.

Mathematical analysis shows the truest and most beautiful examples of the philosophical calculus, Leibniz reports two centuries before Frege. Frege reports: the formula languages of arithmetic, geometry, and chemistry are realizations of the philosophical calculus for particular areas, the concept writing adds the area in the middle, adjoining all others, and the work can continue, with the greatest prospect of success, at filling the gaps in the existing formula languages, connecting their separate areas into a single domain, and extending this domain to areas which lack such a language.

Moreover, the philosophical calculus exhibits the order of the sciences, and it is clear that the elements of arithmetic are the prior and simple.

It is the philosophical calculus, then, because the philosophers all can, even if not all will, take their pens, sit down to their abaci, and calculate.

It is the general characteristic because it is the general language.

It is the general science because it is the science of all sciences, or synonymously, the knowledge of all knowledges.

It is the universal calculus because it is the calculus of all things.

And it is calculus, because it is mathematical, the general language is the language of the world and the language of the world is the general language, mathematics is the language of the world and the language of the world is mathematics, and mathematics is calculation.

clxxxiii

Calculus.

Taking mathematics from the beginning of the world to the time of Newton and Leibniz, what Newton and Leibniz have done is all mathematics.

If this had to be qualified, it would because in the previous couple of generations Desargues has laid the foundations of projective geometry and Pascal has developed it further, Fermat has developed the theory of numbers, Huygens has developed the theory of probability, and what Newton and Leibniz have done has not encompassed all of these results explicitly. So it could

be concluded that what Newton and Leibniz have done is nearly all mathematics, not all.

It is yet better to conclude that what they have done is all mathematics.

Two generations later, Euler reports: calculus has penetrated throughout all branches of mathematics to such an extent that what can be explained without it can be judged next to nothing. Next to nothing excludes projective geometry, the theory of numbers, and the theory of probability, and calculus is very much what Newton and Leibniz have done. Newton has discovered the whole of which his predecessors have discovered parts, and Leibniz, with Euler again: has organized it into a discipline, collected its rules into a system, and given it a crystal clear explanation.

Moreover, mathematics is part of physics, logic is part of mathematics. Mathematics, such as it investigates form as separated from matter, is distinct from the rest of physics. Logic, such as it investigates the form of form, is distinct from the rest of mathematics. And taking physics such as it is distinct from mathematics, ignoring animals and plants, Newton has done all of it. Taking logic such as it is distinct from mathematics, Leibniz has done all of it.

Moreover, considering the power of what they have done, the scope of what they have done, and the unity and uniformity of what they have done, as the unity and uniformity of what they have done is attested by *The mathematical principles*, mathematical *par excellence*, and by the philosophical calculus, calculus *par excellence*, what they have done is all mathematics, not nearly all. Positively what they have done approaches the limit which all is.

Moreover, what Newton and Leibniz have done is a consequence of the true way. Of the fact that in the 3 generations of acquisition of knowledge of which they are the culmination the true way was followed more resolutely than either in the previous 157 or in the following 11.

And this fact cannot not be as important as there are facts before the investigation of knowledge.

To show more particularly a few of Leibniz's most important results in mathematics such as it is distinct from both logic and physics, two of his treatises can be presented. The one anticipating

topology. And the first of the dozen which organized calculus into a discipline, the one which organized the differential calculus into a discipline.

Topology is the investigation of continuity. It was organized into a discipline by Poincaré. More particularly, such as Poincaré developed it, topology is concerned with invariance under continuous transformation. This has since come to be called general topology, or combinatorial topology, or algebraic topology, to distinguish it from point set topology, developed in the generation after Poincaré by Hausdorff. Point set topology is concerned with continuity and limit in sets. It builds on the labors of Cantor. General topology, or combinatorial topology, or algebraic topology, and in general topology, builds on the labors of Möbius, Lobachevsky, Riemann, and Klein. It is anticipated earlier by Euler. And that it is anticipated earlier still by Leibniz the fact that Poincaré organized it into a discipline under the title *Analysis situs*, in 1895, suffices to attest.

In *De analysi situs*, or *On the analysis of situation*, in 1679, Leibniz writes:

the celebrated mathematical analysis is analysis of magnitude, not of situation; this is why it performs arithmetic directly and immediately, but geometry only circuitously; to reduce a problem of geometry to algebra, the determination of figures to equations, is often difficult, and it is difficult also to return from the equations to the constructions, from algebra to geometry;

the true analysis of situation remains to be supplied; and this the fact also shows that all analysts have assumed many things from elementary geometry, which are deduced not from magnitudes but from figures, and which are not sufficiently determined;

figure in general contains next to quantity quality, or form; thus, equal are those whose magnitude is the same, but similar are those whose form is the same;

and the true cause why the geometers have not considered sufficiently similarity is that they have not had a general notion of it distinct enough to treat mathematically;

to explain quality, or form, comes down to this, similar are those which cannot be distinguished when observed singly;

imagine two temples built according to the same law; there is nothing in one that cannot be observed in the other; the material is everywhere the same, white Parian marble, if you like, the proportions of walls, columns, and all the rest are everywhere the same, the angles are everywhere the same; if someone is led into these temples with his eyes closed, then opens his eyes and walks about, first in one of the temples, then in the other, he will find nothing to distinguish them; if they differ only in magnitude, however, they can be distinguished if observed simultaneously, or if an object is carried from one to the other and compared with each; in the same way, the body of the observer, or any of its members, can be compared with the temples, and their magnitudes will then appear different; but if the observer is considered a seeing mind concentrated at a point, without magnitude, and discerning only what the intellect can, numbers, proportions, angles, then no distinction will appear; the temples are thus called similar, because they cannot be distinguished when observed singly, but only when observed together, or each together with something else;

but let us come to examples;

Euclid shows that the sides of similar triangles are proportional only after many detours first in book V; with our notion of similarity this can be done directly;

we demonstrate first that equilateral triangles are similar;

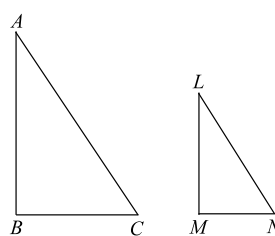
let the angles of triangles ABC and LMN , A , B , C , and L , M , N , respectively, be equal;

I say that the triangles are similar;

I use this new axiom, what cannot be distinguished from the determinants, and that is, from what is given, cannot be distinguished at all;

given base BC and angles B and C , and hence also A , triangle ABC is given; in the same way, given base MN and angles M and N , and hence also L , triangle LMN is given;

but from these data the triangles, observed singly,



cannot be distinguished; in each triangle the base and adjacent angles are given, but the base cannot be compared with the angles; there is nothing else to examine from the determinants but the ratio of each angle to a right angle, and that is, its magnitude; and since this is the same in both triangles, the triangles cannot be distinguished singly, and are therefore similar;

the converse is manifest, similar triangles are equiangular;

otherwise, if there is an angle A in ABC to which no angle in LMN is equal, this suffices to distinguish ABC , observed singly, from LMN ; this shows that the sides of similar triangles are proportional;

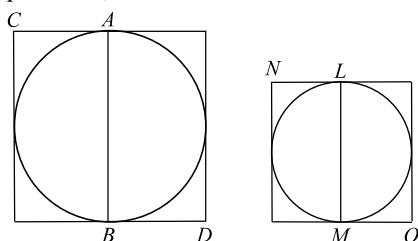
for if two sides are given, as AB and BC , which have a ratio which no two sides of LMN have, then each triangle, observed singly, can be distinguished from the other;

and finally, if the sides are proportional, the triangles are similar;

for if the sides are given, the triangles are given, and it suffices, by our axiom, that from the ratio of the sides the triangles cannot be distinguished, nor can they be, observed singly, from anything else that is given;

in the same way, Euclid shows that circles are as the squares of the diameters only in book X and by a *reductio*; from our notion of similarity this can be shown directly;

let about circle with diameter AB square CD be circumscribed, about circle with diameter LM square NO ;



the determinants are similar in both figures, circle to circle, square to square, and the relation of square to circle;

hence, by the axiom, figures $ABCD$ and $LMNO$ are similar;

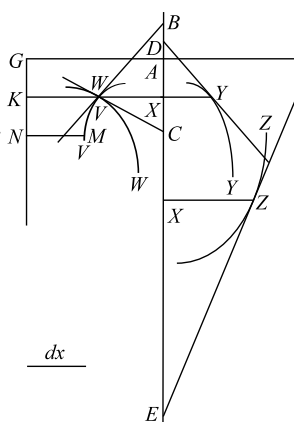
therefore, by the definition of similarity, circle AB is to square CD as circle LM is to square

NO ; therefore, circle AB is to circle LM as square CD is to square NO ; Q.E.D.;

these considerations set forth a new kind of calculus entirely different from the algebraic; I like to call it analysis of situation, because it explains situation directly and immediately; what empirical imagination understands from figures, this calculus derives by certain demonstrations, and from these follow things which the imagination cannot penetrate at all (*id.*).

The organization of calculus into a discipline begins with *A new method for maxima and minima, as well as tangents, impeded by neither fractional nor irrational quantities, and a singular kind of calculus for it*, in 1684. Leibniz writes:

let axis AX be given, as well as several curves, VV , WW , YY , ZZ , of which the ordinates perpendicular to the axis, VX , WX , YX , ZX , are called respectively v , w , y , z , and AX itself, the abscissa, is called x ; the tangents VB , WC , YD ,



ZE intersect the axis at B , C , D , E ; next, some straight line assumed arbitrarily is called dx , and the straight line which is to dx as v , or w , or y , or z , is to XB , or XC , or XD , or XE , is called dv , or dw , or dy , or dz , or the difference of these v , or w , or y , or z ;

positing this, the rules of the calculus are the following;

let a be a given constant, then $da=0$ and $d(ax)=a dx$;

if $y=v$, and that is, if any ordinate of the curve YY is equal to any corresponding ordinate of the curve VV , then $dy=dv$;

addition and subtraction; if $z-y+w+x=v$, then $d(z-y+w+x)=dv=dz-dy+dw+dx$;

multiplication; $d(xv)=x dv+v dx$; or, positing $y=xv$, $dy=x dv+v dx$;

we note that in this calculus we treat x and dx in the same way as y and dy , or as any other indeterminate quantity with its differential;

we note also that we cannot always go back from a differential equation to an ordinary equation unless we are cautious; but more about this elsewhere;

division; positing $\frac{v}{y} = z$, $d\frac{v}{y} = dz = \frac{\pm v dy \mp y dv}{y^2}$;

we note this about signs; when we simply substitute for a quantity its differential, the signs are preserved, for v we write dv , for $-v$ we write $-dv$; but when it comes to explaining the values, to considering the relation of v to x , we have to decide whether dv is positive or negative; dv is negative when the ordinates v decrease as x increases; when v neither increases nor decreases, $dv = 0$, at this point the ordinate is a maximum or minimum and the tangent is parallel to the axis; when dv is infinite with respect to dx , the tangent is perpendicular to the axis, and that is, it is the ordinate itself; when, as the ordinates v increase, their differences dv also increase, and that is, when dv is positive, $d dv$, the difference of the differences, is also positive, and when dv is negative, $d dv$ is also negative, then the curve is concave toward the axis; in the other case it is convex; where the increment is a maximum or minimum, or where the increments turn from decreasing to increasing, or conversely, there is an inflection point; this occurs when neither $v = 0$ nor $dv = 0$, but $d dv = 0$; this shows also that the problem of finding a maximum, has not two, but three roots;

powers; $dx^a = ax^{a-1} dx$; e.g., $dx^3 = 3x^2 dx$;
 $d\frac{1}{x^a} = \frac{a dx}{x^{a+1}}$; e.g., $d\frac{1}{x^3} = -\frac{3 dx}{x^4}$;

roots; $d\sqrt[b]{x^a} = \frac{a}{b} dx \sqrt[b]{x^{a-b}}$; e.g., $d\sqrt{y} = \frac{dy}{2\sqrt{y}}$;

but the rule for integral powers also suffices for fractional powers and roots, because a power is a fraction when the exponent is negative, and changes into a root when the exponent is a fraction;

knowing thus the algorithm, as I call it, of this calculus, which I call differential, all other differential equations can be discovered by the common calculus, minima and maxima can be found, as well as tangents;

dx , dy , dv , dw , dz can be considered proportional to the momentary differences, or the incre-

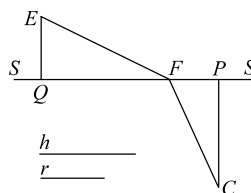
ments and decrements, of the corresponding x , y , v , w , z ; and given any equation, its differential equation can be written;

it is clear that our method extends to transcendental lines, those of an indefinite power, with which the algebraic calculus cannot deal, and it thus holds in the most general way, without any particular suppositions which are not always satisfied; we have to keep in mind only that to find a tangent is to draw the line connecting two points at an infinitely small distance on the curve, or the side of the polygon with an infinite number of angles which is equivalent to the curve; and this infinitely small distance can always be expressed by a differential, like dv , or by a relation to it, and that is, by a tangent; in particular, if y is a transcendental quantity, for instance the ordinate of a cycloid, and it is a part of a calculation in which z , the ordinate of another curve, is determined, and sought is dz , or by it the tangent of this latter curve, we can by all means determine dz by dy , because we have the tangent of the cycloid;

let me propose examples of the calculus (*id.*).

Four examples follow. Much the simpler are the second and the fourth. In particular:

given two points C and E , and line SS in the same plane, sought is the point F on SS , such that when CF and EF are drawn, and two



other lines h and r are given, $CF \cdot h + EF \cdot r$ is a minimum; or, if SS is the line separating two media, and h is the density of one, like water, r is the density of the other, like air, sought is the least path from C to E ;

we assume that all possible sums of rectangles, or all possible paths, are represented by the ordinates KV of the curve VV perpendicular to GK in the earlier figure; we call these ordinates w ; sought is their minimum, NM ;

since C and E are given, their perpendiculars to SS , CP and EQ , are given;

CP we call c , EQ e , PQ p , QF or AX x , CF f , and EF g ;

$FP = p - x$, and $f = \sqrt{c^2 + p^2 - 2px + x^2}$, or, abbreviating, $f = \sqrt{l}$;

and $g = \sqrt{e^2 + x^2}$, or, abbreviating, $g = \sqrt{m}$;

hence, $w = h\sqrt{l} + r\sqrt{m}$;

and by the rules of our calculus, $dw = 0$ in the case of a minimum, and the differential equation is $0 = \frac{h dl}{2\sqrt{l}} + \frac{r dm}{2\sqrt{m}}$;

but $dl = -2(p-x)dx$, and $dm = 2x dx$;

hence, $\frac{h(p-x)}{f} = \frac{rx}{g}$;

and if we posit that f and g , or CF and EF , are equal, as refraction at point F is the same whatever the length of CF ,

$$h(p-x) = rx;$$

and that is, the sines of the angles of incidence and refraction are inversely as the densities of the media;

we thus have a demonstration in our calculus of the general foundation of optics, catoptrics, and dioptrics;

I add as an appendix the solution of a problem which de Beaune proposed to Descartes, and he did not solve it;

sought is a line WW of such a nature that when the tangent to the axis WC is drawn, XC is always equal to the constant straight line a ;

now, XW , or w , is to XC , or a , as dw is to dx ;

hence, if dx , which can be assumed arbitrarily, is assumed constant, like b , and that is, if x , or

AX , increases uniformly, then $w = \frac{a}{b} dw$;

the ordinates w are thus proportional to their dw , to their own increments or differences, and that is, if the x are in an arithmetical progression, the w are in a geometrical progression, or, if the w are numbers, the x are logarithms; the line WW is therefore logarithmic (*id.*).

clxxxiv

Dynamics.

In 1684: we have a demonstration in our calculus of the general foundation of optics, catoptrics, and dioptrics. The general foundation is Fermat's principle of least time. In the following years we have many more demonstrations of physical propositions. Among them the following. *The isochronic line in which a heavy body without acceleration descends. The line in which a body curves by its own weight.* Galileo posed the problem of this line, the catenary. In 1691

Huygens, Leibniz, and Johann Bernoulli solved it. *The curve of fastest descent.* Johann Bernoulli posed the problem of this curve, the brachistochrone. In 1697 Newton, Leibniz, Jakob Bernoulli, Johann Bernoulli, and l'Hôpital solved it. Johann Bernoulli writes: you will be completely astounded when I tell you that the very same cycloid, the isochrone of Huygens, is our brachistochrone (*Brachistochrone*).

But Leibniz's greatest advance of physics is the discovery of mv^2 – or motive force, as he calls it in 1686, or absolute force, or absolute living force, as he calls it in 1691, or living force, as he calls it in 1695 – and its conservation, and hence the development of dynamics.

Descartes discovered that: god will never make a miracle. Instead: he always conserves the quantity of matter in the world, and he always conserves the quantity of motion in the world.

And the entire subsequent development of physics can be described as labors to establish what, exactly, he conserves when he always conserves the quantity of matter in the world, and always conserves the quantity of motion in the world.

First what the quantity of matter is had to be established.

Descartes defined matter as extension. If matter is extension, the quantity of matter will be volume. Huygens showed that the quantity of matter is not volume. And Newton showed what it is. With the first definition of *The principles* the problem is solved. The quantity of matter is mass, the product of volume and density.

Next the laws of the conservation of matter had to be formulated.

The first philosopher, Thales, said that water is the principle of all things, and four generations later Empedocles said that earth, water, air, and fire are the material principles of all things, or the elements of all things. It was in the century before Descartes, beginning with Paracelsus, that the doctrine of the four elements was challenged, and in the generation after Descartes that it was destroyed, by Boyle. Matter began being analyzed into materially indeterminate particles. Descartes analyzes it into particles of three kinds distinguished by size and rapidity of motion (*Principles*, III, 52). Newton writes: all bodies seem to

be composed of hard particles; the smallest may cohere by the strongest attractions and compose bigger particles of weaker virtue, these may cohere and compose bigger particles whose virtue is still weaker, and so on, until the progression ends in the biggest particles on which the operations of chemistry and the colors of natural bodies depend, and which compose bodies of sensible magnitude. To formulate the laws of the conservation of matter, however, the particles had to be determined materially. Or what is the same, the elements had to be discovered. In the century-odd from Boyle through Lavoisier they were. Lavoisier, in the *Elementary treatise on chemistry*, in 1789, lists 33 simple substances, or elements. And Lavoisier formulated the law of the conservation of matter. He writes: nothing is created in the operations of either art or nature; and it can be laid down as a principle of all operations that the quantity of matter before and after an operation is the same; the quality and the quantity of the elements are the same, and nothing takes place but changes and modifications of the combinations of the elements; on this principle the entire art of making experiments in chemistry is founded (I, 13).

In the same way, first what the quantity of motion is had to be established.

If matter is extension, the quantity of matter will be volume, and the quantity of motion will be the product of volume and speed. Huygens showed that the quantity of motion is not the product of volume and speed. And Newton showed what it is. With the second definition of *The principles* this problem is solved as well. The quantity of motion is momentum, the product of mass and velocity.

In the same way, next the laws of the conservation of motion had to be formulated.

In 1656 Huygens solved the problem of collisions, and the solution implies the conservation of momentum. In 1669 Huygens states the law of the conservation of momentum explicitly: the quantity of motion of two bodies can increase or decrease when they collide, but it remains always the same quantity of motion in the same direction sustaining the quantity of motion in the opposite direction.

In 1687, in *The principles*, Newton deduced

the law of the conservation of momentum from the laws of motion. Corollary 3 to the laws is: the quantity of motion, determined by taking the sum of the motions of bodies in one direction and the difference of those in the opposite direction, does not change from the action of the bodies among themselves (*id.*).

And had the law of the conservation of momentum been the only law of the conservation of motion, that would have been that, with the formulation of the law of the conservation of momentum the problem of the formulation of the laws of the conservation of motion would have been conclusively solved.

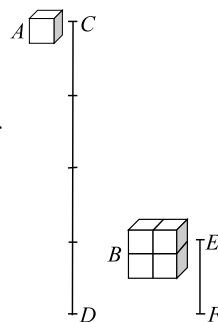
In 1686, however, Leibniz published *A brief demonstration of a memorable error of Descartes and others concerning a natural law, according to which god would always conserve the same quantity of motion, misused also in mechanics*.

Leibniz writes: Descartes, taking it that motive force and quantity of motion are equivalent, asserted that god conserves the same quantity of motion in the world; but there is a great difference between motive force and quantity of motion;

to show this, I assume, first, that a body falling from a certain height, if nothing external impedes it, acquires the same force which will raise it to the same height;

I assume also, second, that the same force will raise body *A* of 1 pound to height *DC* of 4 fathoms as will raise body *B* of 4 pounds to height *FE* of 1 fathom;

it follows that body *A*, in falling from height *CD*, acquires the same force as body *B* in falling from height *EF*; for in falling from *C* to *D* body *A* acquires, by the first assumption, the force required to raise a body of 1 pound, in particular itself, to *C*, and that is, the force required to raise it to a height of 4 fathoms; in the same way, in falling from *E* to *F* body *B* acquires, by the first assumption, the force required to raise a body of 4 pounds, in particular itself, to *E*, and that is, the force required



to raise it to a height of 1 fathom;
let us now see whether the quantities of motion
are the same;

Galileo has proved that the speed acquired in the
fall *CD* is twice the speed acquired in the fall
EF; we multiply therefore body *A*, which is 1,
by its speed, which is 2, and the product, or
the quantity of motion, is 2; but when we
multiply body *B*, which is 4, by its speed,
which is 1, the product, or the quantity of
motion, is 4;

there is therefore a great difference between
motive force and quantity of motion (*id.*).

There is a great difference between motive
force and quantity of motion. And if the quantity
of motion is taken to be the product of volume
and speed, and Descartes asserted that god con-
serves this, Descartes's is a memorable error. God
does not conserve this. If two generations after
Descartes, however, the quantity of motion is
taken to be the product of mass and velocity, god
does conserve this. As Huygens has shown. That
motive force is the product of mass and the
square of the velocity is only implicit in 1686, in
the *Brief demonstration*. In 1691, in the *Essay on
dynamics*, Leibniz is explicit that absolute force,
or absolute living force, is the product of mass
and the square of the velocity. He is explicit also
that god conserves this.

And god does conserve this, too. Hence, Huy-
gens's solution of the problem of collisions
implies, both, the conservation of momentum and
the conservation of kinetic energy in elastic
collisions.

To establish the fact that god does not con-
serve either mv or mv^2 , but conserves both, mv
and mv^2 , took a couple of generations.

In 1736, in the *Mechanics, or the science of
motion presented analytically*, Euler first formu-
lated mechanics analytically.

In 1743, in the *Treatise on dynamics*, d'Alembert
formulated mechanics analytically. D'Alembert,
like Newton, formulates three laws: of the
force of inertia, of the composition of motion,
and of equilibrium (*pr.*). The third, the law of
equilibrium, is the law of the conservation of
momentum. In d'Alembert's formulation: if two
bodies, whose velocities are inversely propor-
tional to their masses, move in opposite direc-

tions so that one cannot move without displacing
the other, they are in equilibrium (46).

And d'Alembert resolved the difficulty. Not
either mv or mv^2 , both mv and mv^2 .

He writes: the question whether the force of a
body in motion is proportional to the product of
mass and velocity or to the product of mass and
the square of the velocity has divided the geome-
ters for thirty years (*pr.*); when one speaks of the
force of bodies in motion one either does not
attach a clear idea to the word force or has to
understand the property of bodies in motion to
overcome obstacles or to resist them; therefore,
the force is judged not by the distance a body
traverses in uniform motion, or by the time it
takes to traverse a distance in uniform motion,
but by the obstacles it encounters and the resis-
tance they offer it (*id.*); it is clear that the motion
of a body can be opposed by three kinds of
obstacles; such as destroy its motion altogether,
such as have precisely the resistance necessary to
destroy its motion in an instant, which is the case
of equilibrium, and such as destroy its motion
gradually, which is the case of retarded motion;
the first kind is of no use; it is in equilibrium and
in retarded motion that the measure of force is
sought (*id.*); and in equilibrium the product of
mass and velocity, or the quantity of motion,
represents the force; but in retarded motion the
number of overcome obstacles is as the square of
the velocity; for instance, a body which with a
certain velocity compresses a certain spring will
with twice that velocity compress four such
springs (*id.*); and the effects are different, but
there is nothing more in the body in the one case
than in the other (*id.*); in the one case, the effect
is the distance traversed uniformly in a given
time, and this effect is as the velocity; in the other
case, the effect is the distance traversed before the
motion is consumed, and this effect is as the
square of the velocity (*id.*).

Or what is the same, integration of the force
over the time of its action on a mass – the tempo-
ral integral – yields the change in the momentum
of the mass. Integration of the force along the line
of its action – the spatial integral – yields the
change in the kinetic energy of the mass.

D'Alembert concludes: thus, if a problem of
mechanics is proposed to two geometers, one of

whom is an adversary of living forces and the other is their advocate, their solutions, if they are correct, will be in perfect agreement (*id.*); the rest is futile metaphysical discussion or, even less worthy of philosophers, a dispute about words (*id.*).

The solutions of the two geometers, if they are correct, will be in perfect agreement. The rest, however, is, pace d'Alembert, not about words. It is about the foundation of dynamics, and d'Alembert's is a *Treatise on dynamics*, too. It is also about what is absolute.

In 1689, in the manuscript beginning *Cum geometrias demonstrationibus*, Leibniz writes: we have achieved what the ancients could hardly envision in their prayers, we have reduced by geometrical analysis the primary phenomena of the world to the simplest and clearest principles, and that is, to the best and truest hypotheses (*id.*).

Through Galileo to Newton – through *The two new sciences dealing with mechanics and locomotion* to *The mathematical principles of natural philosophy* – we have.

The mathematical principles appeared in 1687.

In 1704, Newton writes: *vis inertiae* is a passive principle by which bodies persist in their motion or rest; by this principle alone there could never have been any motion in the world; some other principle was necessary to put bodies in motion; and now that they are in motion, some other principle is necessary to conserve the motion; it seems probable that god in the beginning formed matter in solid, massy, hard, impenetrable, movable particles; it seems further that these particles had not only *vis inertiae*, with such passive laws of motion as naturally result from it, but that they also are moved by active principles, such as that of gravity, and that which causes fermentation, and the cohesion of bodies.

The passive principles have been discovered. The discovery of the passive principles has posed the problem of the active principles. And Leibniz has begun the explicit search for the active principles in 1691.

The new sciences of mechanics and locomotion have achieved what the ancients could hardly envision in their prayers. And the success of the new sciences of mechanics and locomotion has posed the problems which the new science of

dynamics has begun to formulate and solve.

In 1691, in the *Preliminary specimen of dynamics, on the law of nature relating to the power of bodies*, Leibniz writes: I lay down the true elements of a new science of power and action, which can be called dynamics (*id.*).

His greatest advance of the new science of dynamics is also in 1691, in the *Essay on dynamics concerning the laws of motion, in which it is demonstrated that conserved is not the same quantity of motion but the same absolute force, or the same quantity of moving action*. His most comprehensive treatment of the active principles is in 1695, in *A specimen of dynamics for the discovery of the admirable laws of nature of the forces of bodies and their actions on one another, and for their reduction to their causes*.

In the *Specimen*, in 1695, he writes:

in corporeal things there is something other than extension, and indeed prior to extension, namely, the force of nature implanted everywhere by the creator;

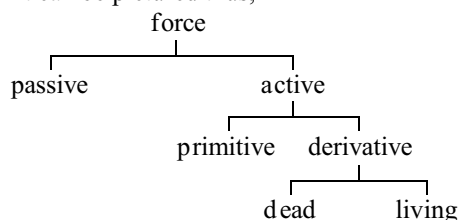
active force is twofold, primitive and derivative; derivative is a limitation of primitive, resulting from the collision of bodies with one another, and it is of different degree in different bodies; primitive, which is nothing but *ἐντελέχεια ἡ πρώτη*, corresponds to soul, or substantial form;

passive force is also primitive and derivative; primitive passive force, or force of resistance, corresponds to primary matter;

the investigation of derivative force and resistance treats the different degree of striving of different bodies, or their different degree of resistance; it is of this that the laws of action have to account;

force in which there is no motion I call dead force; force joined with actual motion I call living force; living force arises from an infinity of continuous impressions of dead force (1).

It can be pictured thus,



Primitive passive and primitive active corre-

spond to matter and form.

Living force, being derivative, is a limitation of primitive active force. Of different degree in different bodies, living force is the different degree of striving of bodies, or their different degree of resistance. A force joined with actual motion, arising from an infinity of continuous impressions of force without motion, it cannot not be a problem of calculus.

Leibniz next shows that: the true measure of living force is the product of mass and the square of the velocity (*id.*), and continues:

the entire effect is thus always equal to the full cause; hence, we can substitute living forces for one another in our equations without any fear that they might imply perpetual mechanical motion; for instance, if it were true that body *A* of mass 2 and velocity 1 and body *B* of mass 1 and velocity 2 are equal in force, we could substitute one for the other; but it is not true; for if *A* descends from a height of 1 foot and we substitute for it *B*, it will ascent to a height of 4 feet; we will thus end up with 2 times the force with which we began, and that is, we will create perpetual mechanical motion, and this is absurd (*id.*).

Perpetual motion is absurd. Leibniz discovered why. Perpetual motion is absurd because energy is conserved. In the relation of living forces the entire effect is always equal to the full cause.

Leibniz continues: marvelous theorems in dynamics follow (2). He does not prove them. Among them are the following five:

motion is a pure relation; and the laws of nature must be so formulated that the relative nature of motion is preserved, the phenomena before and after a collision must not be distinguished by rest or absolute motion;

what is in a substance can be understood from itself and its order;

no change is by a leap; rest is a special case of motion, the case of vanishing motion; equality is vanishing inequality;

there are no elements of bodies;

there is no action without reaction, and the two are equal and in opposite directions (*id.*).

The first is the principle of relativity, and its first formulation with respect to the laws of nature in general anticipates the generality of Einstein's formulation. The second is a conse-

quence of the fact that in the three generations through Leibniz substance has quit being inaccessible to observation and experiment. The third is the law of continuity, which Leibniz has discovered, and which entails the unconditional dependence of the formulation and solution of physical problems on calculus. The fourth anticipates the fact that, once organized into Mendeleev's table, the elements have been broken down into electrons, protons, and neutrons, and protons and neutrons have been broken down into quarks. The fifth is Newton's third law of motion.

Leibniz's most important physical treatise is the *Essay on dynamics*, in 1691. He writes:

the mathematicians have long established that the relative speeds of bodies before and after a collision are the same; but it is possible that these speeds remain the same, while the true speeds and absolute forces of the bodies change in an infinite number of ways; the conservation of relative speed thus does not concern something absolute in bodies;

another conservation is that of progress, but this is not the conservation of something absolute either; I call progress the quantity of motion with its direction (*id.*).

Progress is momentum. Leibniz continues:

progress does not concern something absolute either, because it is possible that, while the speeds, quantities of motion, and forces of bodies are very large, their progress is 0;

what has contributed the most to the confusion of force with quantity of motion is the misuse of statics; for in statics two bodies are in equilibrium when their speeds are inversely as their masses, or when they have the same quantity of motion;

absolute force, however, must be calculated by the violent effect it can produce; I call violent effect that which consumes the force of the agent, which, for instance, gives a body a speed, or raises it to a height; and the force of a body can be calculated by the product of its mass and the height to which it can rise;

and it is discovered by reason and by experiment that not the quantity of motion, but absolute living force, or that calculated by the violent effect it can produce, is conserved; for if this living force could ever be increased, the effect would be more powerful than the cause, there

would be perpetual mechanical motion, and this is absurd;

I can show the conservation of another quantity as well, what I call the conservation of moving action; the rule is that among the same colliding bodies there must always be the same quantity of moving action in a given time;

to calculate moving action, first the formal effect of the motion has to be calculated; the formal effect consists in what is changed by the motion, and it is thus the product of mass and distance; moving action is then calculated by the vigor of the formal effect;

for instance, 100 pounds are to be transported from one place to another; this is the formal effect; one transports the weight in 1 hour, another in 2 hours; I say that the action of the first is 2 times that of the second, being 2 times prompter in producing the same effect; thus, when a mass of 3 is transported 5 feet in 75 minutes, the action is the same as when a mass of 1 is transported 1 foot in 1 minute (*id.*).

The formal effect is the product of mass and distance, md . Vigor is velocity, v . Moving action is the vigor, or velocity, of the formal effect, mdv . But $d = vt$. Hence $mdv = mv^2t$. Moving action is the product of living force and time.

Leibniz continues:

now for the verification of the conservation of moving action; moving action is the exercise of force over time; and as the force is conserved and moving action is the product of force and time, the same action is conserved in equal times;

I will now reduce the whole to three simple and beautiful equations; if the masses of two bodies are a and b , and their speeds before and after collision are, of the first body v and x , of the second y and z , these are the equations;

1. the linear equation, which expresses the conservation of relative speed,

$$v - y = z - x;$$

the signs can vary in particular cases; as they can in the next equation;

2. the plane equation, which expresses the conservation of progress,

$$av + by = ax + bz;$$

3. the solid equation, which expresses the conservation of absolute force, or moving action,

$$avv + byy = axx + bzz;$$

this equation is excellent in this also that all variations of signs depending on directions are eliminated by the squaring of the velocities; and because of this, too, this equation gives something absolute, independent of relative velocity or progress (*id.*).

Leibniz concludes:

the conservation of the same force in the world is an inviolable truth; what is absorbed in the particles of bodies is lost from the total force of colliding bodies, but it is not lost absolutely from the universe (*id.*).

Work is the product of force and distance, Fd . Energy is potential and kinetic, and kinetic energy is one half the product of mass and the square of the velocity, $\frac{1}{2}mv^2$. Leibniz has taken

a major step toward the discovery of work. He has taken the decisive step toward the discovery of kinetic energy. And he has taken a major step toward the discovery of the relation of work and kinetic energy, the ground on which the principle of conservation of energy will be discovered.

On Newton's authority there is absolute motion and rest. Leibniz argues that motion and rest are always relative, argues it repeatedly, and explicitly against Newton.

That something is absolute, and it is a problem of physics to show what it is, there is no disagreement between Newton and Leibniz. On Leibniz's authority the conservation of twice the kinetic energy is absolute.

Helmholtz discovered the principle of conservation of energy, in 1847, in *On the conservation of force*. In his formulation: the sum of the living forces and the forces of tension is constant; and in this most general form we can describe this law as the principle of conservation of force.

Clausius discovered the laws of thermodynamics in 1850. In 1865, in *On different convenient for application forms of the main equations of the mechanical theory of heat*, he formulates the first law thus: the energy of the world is constant.

Maxwell discovered the laws of electrodynamics in 1864. In 1873, in *A treatise on electricity and magnetism*, he writes: kinetic energy appears so fundamental a fact of nature we can hardly conceive the possibility of resolving it into anything else (568).

Thermodynamics is a theory of energy. Electrodynamics is a theory of energy. The theory of relativity is a theory of energy. Quantum theory is a theory of energy.

In 1905, in *Does the inertia of a body depend on its energy content?* Einstein discovered that mass and energy are convertible into one another. That

$$E = mc^2,$$

where E is energy, m is mass, c is a constant, the velocity of light. Einstein comments: the mass of a body is a measure of its energy content (*id.*).

The conservation of matter, too, is a form of the conservation of energy.

If something is absolute, and it is, and it is a problem of physics to show what it is, and it is, the conservation of energy is absolute.

clxxxv

The history of the earth.

The investigation of minerals begins with Theophrastus, in *On stones*. It advanced beyond Theophrastus, and greatly beyond him, eighteen and a half centuries later, with Agricola, in *On metals, in which the uses, instruments, machines, and all other things concerning metals are observed, and described not only in the clearest way, but with images in the appropriate places*, in 1556. Agricola wrote also *On the origin and causes of subterranean things; on the nature of things flowing out of the earth; on the nature of fossils; on the old and new metals*, in 1546.

Not quite one and a half centuries later Leibniz wrote the *Protoearth, or treatise on the first state of the earth and the vestiges of its ancient history in the monuments of nature*, in 1693.

The investigation of the first state of the earth had begun two generations earlier, this, too, with Descartes, in *The principles of philosophy*, in 1644. The investigation of the vestiges of the earth's ancient history in the monuments of nature had begun in the previous generation, with Steno, in the *Harbinger of a dissertation on solids enclosed by nature in solids*, in 1669.

What had not begun before Leibniz, and begins with him, is the empirical investigation of the first state of the earth. Leibniz grounded the investigation of the first state of the earth in the investigation of minerals, including the vestiges

of the earth's ancient history in the monuments of nature.

Part IV of *The principles of philosophy* is *On the earth*. And Descartes begins with the first state of the earth: the earth which we inhabit was formerly a star like the sun, in the center of its own vast vortex (2). In part III Descartes has analyzed matter into three kinds of particles distinguished by size and rapidity of motion. In its first state: the earth was composed entirely of matter of the first element (*id.*). The smallest and fastest particles constitute the first element, of which the sun also was and still is composed. But: the vortex of the earth was smaller (*id.*); and it ultimately descended into the larger vortex of the sun (*id.*). As motion degenerates and particles aggregate, the elements degenerate as well, the first kind into the second, the second kind into the third. And: if we consider the earth a short time after its descent into the vortex of the sun, we can distinguish three regions, innermost, composed of matter of the first element (3); intermediate, composed of matter of the second element (4); and outermost, composed of matter of the third element; and this is the matter from which all bodies surrounding us originated (5).

The earth has a core, an intermediate region, and a crust. The core is matter like that of the sun. From the crust all bodies surrounding us originated.

As the elements continued to move through one another: fissures appeared in the crust, and became larger in time (41); the crust broke into various parts (42); and so the mountains, valleys, oceans, and all other features on the surface of the earth were formed (44).

Movements of the crust formed the features on the surface of the earth.

With minerals Descartes deals briefly (60-3), noting: I would have described them in detail, had I had the opportunity to make the necessary experiments (63).

Steno made the necessary observations and experiments, and in the *Harbinger of a dissertation on solids enclosed by nature in solids* advanced greatly the investigation of the earth.

He laid the foundations of crystallography. Steno discovered that: crystals are matter in hexagonal shape, growing by the addition of

matter in the shape of hexagonal pyramids (*id.*).

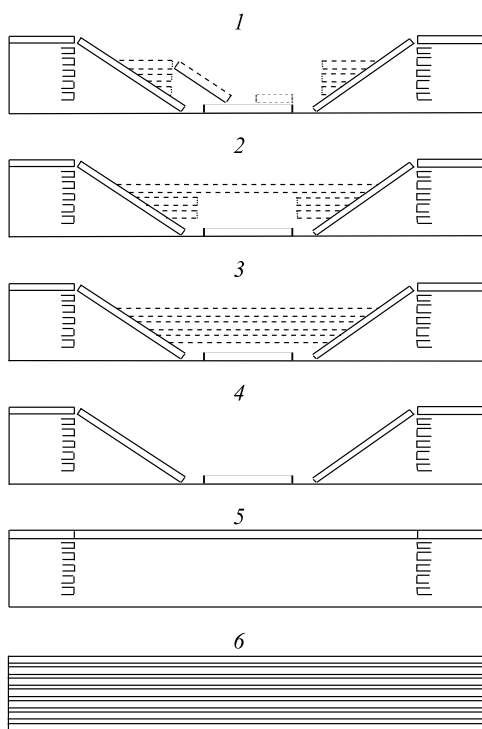
He showed what fossils are.

Glossopetrae found in Malta showed solids enclosed by nature in solids. Steno writes: the question is whether the glossopetrae of Malta were once the teeth of sharks; and this is clearly the same question as the general question whether bodies similar to marine bodies found far from the sea were once produced in the sea (*id.*). And not only the glossopetrae of Malta were once the teeth of sharks: what has been said about the teeth of sharks holds for the teeth of rays, the vertebrates of fishes, whole fishes of every kind, the crania, horns, teeth, femurs, and other bones of land animals (*id.*); and what has been said about animals and their parts holds also for plants, and whether dug out of the strata of the earth or hidden in rocky substances (*id.*).

Moreover: the bones of animals and other solids of this kind had become solid when the earth and the rock containing them were still fluid; not only did not the earth and the rock enclosing these solids produce these solids, the earth and the rock enclosing these solids did not exist when these solids were produced (*id.*).

With Steno also begins the investigation of the stratification of the earth. Steno begins with: Descartes's account of the origin of the strata of the earth (*id.*). Further, however, he has made the necessary observations and experiments. The *Harbinger* concludes with the picture below.

Steno explains: these figures show how from the present appearance of Tuscany we infer six distinct stages of development, and they thus also illustrate what we have said about the strata of the earth in general; dotted lines represent strata of sand and clay, solid lines represent strata of rock; figure 6 shows a vertical section of Tuscany at the time when the rocky strata were still whole and parallel to the horizon; figure 5 shows the huge crevices dug out by the forces of fire and water while the upper strata remained unbroken; figure 4 shows the mountains and valleys formed by the breakage of the upper strata; figure 3 shows the new strata made by the sea in the valleys; figure 2 shows a portion of the lower strata in the new beds destroyed while their upper strata



remain unbroken.

figure 1 shows the hills and valleys formed by the breakage of the upper strata in the new beds (*id.*).

In the *Protoearth* Leibniz does not report as many observations and experiments as Agricola, but he does many more than Steno. And while Steno leaves the first state of the earth alone, Leibniz does not.

He writes:

wise men have developed the hypothesis that there were once vast globes like the fixed stars, produced by the light of the sun, or ejected by the sun itself; their matter boiled until they were covered with the slags extruded during the fusion; the accretion of material extinguished the internal heat and a cooled crust hardened all around; and thus an opaque star reflecting light was born, like the planets;

this hypothesis is supported by the vestiges of the first face of the earth; for all slags produced by fusion are a kind of glass, and just as slag forms on metals in a blast furnace, so the crust must have been like a slag covering the molten

earth; and how could rocks and sands, which all change into glass, have been formed, if not by the melting of matter by the great first force of the fire of nature? nor is it strange that the power of this fire surpassed greatly in duration and intensity the power of our furnaces, even if our art advances every day (3);

because burnt materials attract moisture as they cool, and this is how chemists make oils, we can assume that in the beginning, before light was separated from darkness, when the globe still burned, the fire thrust moisture in the air;

then, just as in distillation, the generated moisture condensed into watery vapors;

then, as the cooled surface of the earth swallowed the moisture, it reverted to water, which began washing the face of the earth and drawing into itself the vast vestiges of tar and salt;

then, it is plausible that, just as metals and other substances become porous by melting, so the crust, shrinking as it cooled, left behind huge bubbles, hollows under vaults, which enclosed air and moisture;

then, the crust separated into sheets and came together into uneven clumps, according to the differences of material and heat; it ruptured in places, and the broken fragments tumbled into sloping valleys, while the harder places remained in the highest places, forming mountains;

then, the weight of the water began seeking a bed into the soft ground; from its own weight, and as air burst from the vaults, water flowed down the mountains; great floods followed, which deposited sediments; and when these hardened, new strata were formed over the old;

the face of the still tender globe changed in these ways until conditions gradually subsided and reached an equilibrium;

thus also the twofold origin of solid bodies is explained, first they cooled after melting by fire, second they hardened again after dissolving in water (4);

this description of the cradle of our globe is truly plausible, and it contains the seeds of a new science, called natural geography;

and if the vestiges of the old earth conform to the present appearance of things, when the curiosity of mortals advances far enough to describe the kinds of the grounds and strata in the various

regions, our descendants will be able to explain everything better (5).

The following is a sample of the field investigations, by Leibniz himself or others, recorded in the *Protoearth*:

springs near the peak of Brocken, in the Harz (7); copper bearing schist at Osterode, in the Harz (8); shafts in the silver mines in Rammelsberg, near

Goslar, in the Harz (9);

veins of cinnabar in Idria, near Trieste (10);

ammoniac breathing out of the ground in Campania (12);

the Scaras cliffs near Stockholm (15);

stalactites in the Baumann cave, in the Harz (16);

a stream forming the gray stone covering mill wheels in Langenholtensen, near Alfeld, in Hesse (*id.*);

shapes of fishes on schistous stone in Eisleben, in the Harz (18);

springs of liquid bitumen near Burgdorf, in the Harz (19);

rocks in the salt mines near Krakow (21);

petrified shells near Grund, in the Harz (23);

petrified shells near Volterra, in Tuscany (25);

glossopetrae in the alum mines in Lüneburg, in Lower Saxony (30);

the glossopetrae of Malta (31);

animal bones in the Scharzfeld cave, in the Harz (36);

animal bones in the Baumann cave (37);

hot springs in Bellilus, in France (38);

amber dug out in Blumenau, near Hannover (38);

the strata around the fountains of Modena (42); oak petrified with its roots and branches 300 m

underground in a tunnel in Joachimsthal, in Bohemia (45);

the strata in a well in Amsterdam (48).

Most of the investigations are in the Harz, because from 1680 to 1685 Leibniz labored, in the field, to introduce wind power in the operations of the local silver mines. More particularly, to capture the water draining from higher elevations, use windmills to pump the captured water back up to higher elevations, then use the power of that water let fall as needed. More generally, to harness the conservation of energy.

In these five years engineering was Leibniz's primary occupation. Among his secondary occupations were organizing calculus into a discipline

and discovering the conservation of living force.
And so the history of the earth became an empirical investigation.

Geology has not occupied the philosophers as much as man has. The consequences, when it has, including for man, Darwin attests. Darwin did much geology. He also added a *Historical sketch* to the third edition of *The origin*. The sketch notes Aristotle, then begins with Buffon and Lamarck, and Darwin eventually counts: of the 34 authors named in this sketch, who believe in the modification of species, or at least disbelieve in separate acts of creation, 27 have written on special branches of natural history or geology (*id.*).

clxxxvi

History.

Leibniz entered the service of the dukes of Brunswick-Lüneburg in 1676 and remained in their service until his death in 1716. Appointed a privy counselor, from 1680 to 1685 his chief duty was the engineering project in the silver mines of the dukes. From 1685 to 1716 his chief duty was the writing of the history of dukes.

Johann Friedrich was the duke of Brunswick-Lüneburg from 1665 to 1679. Ernst August was the duke of Brunswick-Lüneburg from 1679 to 1692, when the duchy was raised to the ninth electorate of the empire, the electorate of Hanover, and Ernst August became the elector of Hanover from 1692 to 1698. Georg Ludwig was the elector of Hanover from 1698 to 1727, and appointed third in succession to the British throne by the settlement act of the English parliament in 1701, became the king of Great Britain, George, from 1714 to 1727.

The engineering project came to an end, when, as expenditure mounted, the duke canceled it. The difficulty with the writing of the history was similar. The dukes wanted the history of the dukes. Leibniz wanted more.

The empire in the west begins with

Charlemagne prince 768-814.

Charlemagne's domain was divided by his successors and in the following century the empire declined. Then it rose again, and the Saxon emperors,

Henry prince 919-36,

Otto prince 936-73,
Otto II prince 973-83,
Otto III prince 983-1002,
Henry II prince 1002-24,
the Salian emperors,
Conrad II prince 1024-39,
Henry III prince 1039-56,
Henry IV prince 1056-106,
Henry V prince 1106-25,
and the Hohenstaufen emperors,
Conrad III prince 1138-52,
Frederick prince 1152-90,
Henry VI prince 1190-7,
Frederick II prince 1197-250,
were the most powerful succession of princes on earth this side of China.

The dukes of Brunswick were the descendants of the Welfs. The Welfs acquired the duchy of Bavaria in 1070 and the duchy of Saxony in 1137. Henry the Lion held the two duchies from 1142 to 1180, and, in the service of Frederick, enlarged substantially his domains. In 1176 he abandoned the emperor. By 1180 Frederick had deprived him of Bavaria and of much of Saxony, confining him to the lands around Lüneburg. The hostility of the Lion's successors against the emperors accordingly lingered. Until 1235 when Frederick II enfeoffed them with the duchy of Brunswick-Lüneburg.

None of his Welf ancestors is as important as Henry the Lion, even as they can be traced back to the time of Charlemagne's successor,

Louis the Pious prince 814-40.

That Henry the Lion himself is not important, the couple of facts above suffice to show. That the dukes of Brunswick-Lüneburg were never important, the fact suffices to show that, within the empire, they became the ninth electors, and had to wait to become the ninth electors until the time of Leibniz, when, without, the empire was not as important as France, or England, or Spain, or the Ottoman empire, or Russia. Even as the duchy of Brunswick-Lüneburg was more important than the other Brunswick principalities, Grubenhagen, or Göttingen, or Wolfenbüttel, or Calenberg.

The dukes of Brunswick-Lüneburg wanted the history of the dukes.

What Leibniz wanted Gibbon describes. Gibbon completed *The history of the decline and*

fall of the Roman empire in 1788. In the following years he began work on the *Antiquities of the house of Brunswick*. Gibbon died in 1794. His drafts for the *Antiquities* were published in 1796. Gibbon writes: with a genius accustomed to draw lines of communication between the most distant sciences, Leibniz traced in his introduction the revolutions of the country and its inhabitants, of the country from the natural remains of fossils and petrifications, of the inhabitants from the national vestiges of language and manners; the story of a province and a family swelled in his capacious mind into the annals of the western empire; the origins of the Welfs in Bavaria and the marquises of Este would have been interwoven in their proper place; and the narrative would have been deduced from the reign of Charlemagne in 768 to the last emperor of the Saxon line in 1024; but the term of an antediluvian life would have been scarcely adequate to the labors and projects of Leibniz (1).

Pace Gibbon, the term of three scores and ten was adequate to the labors and projects of Leibniz. Kant, when he comes to describe genius in the *Anthropology*, writes: genius is the man, not of wide range of mind, but of intense greatness of mind, who makes an epoch in everything he undertakes, like Newton and Leibniz (I, 59).

What the dukes wanted, although contaminated by what Leibniz wanted, appeared in 1749-53. After the labors of his successor as a historian of the electors of Hanover, then the labors of his successor, and then the labors of his successor, the *Origins of the Welfs* appeared in 1749-53 in six volumes – four volumes on the origins of the Welfs, preceded by one on the origins of the Germans, preceded by the *Protoearth*.

The gigantic conception of tracing the revolutions of the country from the natural remains of fossils and petrifications, of the inhabitants from the national vestiges of language and manners, and of deducing the origins of the Welfs from the annals of the empire, is Leibniz's. The *Protoearth* is Leibniz's as well. The rest Gibbon describes: the hands of several workmen are apparent, the bold and original spirit of Leibniz, the crude erudition and hasty conjectures of Eckhart, the useful annotations of Gruber, and the critical disquisitions of the principal editor Scheidt

(*Antiquities*, 1).

Leibniz's own deduction of the origins of the Welfs from the annals of the empire from Charlemagne to Henry II, the 2 300 pages of the *Brunswick annals of the western empire*, was first published in 1843-6.

In the course of his researches Leibniz established that the Welfs are related to the Este, and reported the fact in the *Letter on the connection of the houses of Brunswick and Este*, in 1695. He published the brief *Dissertation on the origin of the Germans*, in 1697. And he published the collection of laws, *Code of the law of nations*, in 1693, and the collections of chronicles, *Additions to history, writers on German matters collected from manuscripts*, in 1698, and *Writers serving to illustrate Brunswick matters*, in 1707-11.

Frege writes: Leibniz's conception of a philosophical calculus was gigantic. And the success of Leibniz's labors at realizing it is as gigantic as the conception.

The conception of a history reaching from the first state of the earth to the Saxon emperors is gigantic itself. And if the success of his labors at realizing it is not as gigantic as the conception, Leibniz's clarity as to how it will be realized is.

It will be by following the true way.

Grandiose histories, reaching before the first state of the earth, and after the Saxon emperors, too, there have been. What there has not been is such a history, which begins with the collection of the particulars, marshals them in their series and order, and proceeds by continuous and gradual ascent, in order to end with the most general principles.

Minute histories, steeped in the particulars, there have been very many. What there has not been is such a history, which establishes stages of certainty, ascends from observations to principles, descends from these to new observations, ascends from these to higher principles, descends again, ascends, and until it reaches the law with its clauses.

Grandiose histories not inconvenienced by the particulars have beset history especially after Hegel.

Minute histories not inconvenienced by importance have always beset history.

Leibniz writes: I do not despise the combing of

antiquities for the tiniest trifles, because the knowledge which critics extract from them can sometimes lead to more important things (*New essays*, IV, 16); but one reads a big book of history, erudite, well written, it has accomplished its author's goal, it is excellent of its kind, and yet it contains nothing useful; and by this I do not mean simply morality, with which the theater of human life is replete, but the skills and knowledge which not everyone has when he needs them (*id.*); it is astonishing that so many useful things remain to be done, yet men always turn to what has already been done, or to the utterly useless, or in any case to the least important (*id.*).

It is not astonishing, not really. Leibniz adds: I see no remedy for this until, in calmer times, society involves itself in the matter (*id.*). In the meantime, the utterly useless and the least important have dominated history. And have come to do so even more after Leibniz than before.

With his researches in archives and publication of old documents Leibniz is one of the pioneers of the critical reexamination of old documents. The most important in this is Spinoza, with the *Theologico-political treatise*, in 1670. At this time also Mabillon laid the foundations of paleography. With *On diplomatic matters, in which whatever pertains to the materials, writing, and style of old documents, whatever pertains to seals, monograms, subscripts, and chronological notes, whatever pertains to antiquities and forensic historical disciplines, is explained and illustrated*, in 1681, Mabillon established rules for the determination of the dates and authenticity of old documents.

Leibniz writes in the preface to the *Additions to history*: truly excellent documents, those from which we can learn something which is not easily explained by reason alone, I find rare, even among good writers, and much more so among those inept and barbarous writers whom the centuries condemned to ignorance produced (*id.*). Truly excellent documents, those from which we can learn something, are rare. Hence, as historians have devoted themselves to archives and old documents, archives and old documents have become an inexhaustible source of the utterly useless and the least important. The more so after the passage of Voltaire, Hume, and Gibbon, the

last historians to do both, with Gibbon: examine carefully all original materials, and with Voltaire: concern themselves only with what merits the attention of all times.

Nor has it helped that historians have been human. Leibniz writes in the preface to the *Code of the law of nations*: history is unfaithful when it does not rely on the testimony of great men or of public acts (*id.*). The greater the men, and the more public the acts, the more their testimony has been scorned. Hence, as Leibniz continues: men seldom believe flatterers, but they are all too ready to lend an ear to those who malign; there are historians who represent men as worse than they are; and the malignity of men and their envy of the powerful confers authority on these writers (*id.*). Gibbon concurs: there are many historians who put us in mind of the admirable saying of the great Condé, *ces coquins nous font parler et agir comme ils auraient fait eux-mêmes à notre place* (*History*, 14).

Toward the realization of the gigantic conception of a history reaching from the first state of the earth to the Saxon emperors by following the true way Leibniz took four major steps.

At the far end, he transformed the history of the earth into an empirical investigation.

At the near end, he collected the particulars about the Saxon emperors and began marshaling them in their series and order.

He also pointed the way toward bridging the abyss between the two. He writes: the names of rivers usually come from the greatest antiquity and indicate the best the oldest language and the most ancient inhabitants of a country; for this reason they deserve special investigation; in general, languages are the most ancient monuments of peoples, older than writing or the arts, and they indicate the best the origin, kinships, and migrations of peoples; for this reason etymologies, well understood, are curious and consequential (*New essays*, III, 2); most investigations of European origins, customs, and antiquities have been confined to the Teutonic; I wish scholars would do the same for the Walloon, Basque, Slavonic, Finnish, Turkish, Persian, Arabic, Georgian, and others, in order to discover what their relations are, which will explain the origin of nations the best (*id.*).

And he pointed the way toward the most general principles. He writes: I have said more than once that a new kind of logic is needed, to treat the degrees of probability (IV, 16); the investigation of the degrees of probability is very important; for when a question cannot be decided absolutely, nevertheless the degree of probability can always be determined from the available data, and hence a rational judgment can be made of what is most likely (2); the critics of historical matters have great regard for contemporary witnesses (16); but probability is drawn from the nature of things; when Copernicus was almost alone in his opinion, his opinion was yet incomparably more probable than the opinion of all the rest of the human race (2).

Moreover: it may seem to some that the philosophical calculus will be of little use in civil history; I reply, as powerful as reason is in it, so powerful and much more powerful will be the philosophical calculus; it will make it possible to reach a conclusion where without it no result at all can be obtained from the available data; it will make it possible either to attain an infinite approximation or to determine the degree of probability.

History is not doomed to either the utterly useless and the least important or world-views, historicism, historicity, historicity, and the like. History is a part of physics.

clxxxvii

China.

Organisms, by being organisms, are opportunistic. Men have accordingly always harvested one another.

Leibniz writes in the preface to *The latest on China*: certainly the greatest evils men themselves do and inflict on one another; it is a true saying, a man to a man is a wolf; our folly is great and universal (*id.*).

As long as knowledge has been acquired men have been harvesting knowledge, too.

The acquisition of knowledge and the opportunism of men must be separated sharply from one another.

This is how the acquisition of knowledge has proceeded.

Ptolemy put China on the map of the earth. In

latitude and longitude, knowledge of the surface of the earth reaches in the *Geography* from the Shetland Islands in the north to cape Delgado in the south and from the Canary Islands in the west to the mouth of the Red River in the east. The last is: a port of the Chinese (VII, 3). Further north Ptolemy lists: five inland cities of the Chinese (*id.*). Further north is: the land of Seres (*id.*). Seres is Xinjiang. And: further east and south lie unknown lands (*id.*).

Eleven and a half centuries later, from 1271 to 1295 Polo voyaged from Venice to China and back. From 1318 to 1330 Odoric voyaged from Venice to China and back. From 1325 to 1354 ibn Battuta voyaged from Tangier to China and back. Polo wrote the *Description of the world*. Odoric wrote the *Description of the eastern part of the world*. Ibn Battuta wrote the *Travels*. All three describe China. Odoric describes it briefly and vaguely. Ibn Battuta describes it not so briefly but also vaguely. Polo describes it neither briefly nor vaguely. He describes

Kublai great khan and Yuan emperor

1260-94,

the court, wars, rule, and domain of the great khan and Yuan emperor (76-105), the many cities which he visited and of which he heard, the people and their customs (106-58). In the west, knowledge of what China is, other than the land furthest east, begins with Polo.

Not quite one and a half centuries after Polo, prince Henry began sending his knights to sea. In 1434 Eanes rounded cape Bojador. In 1456 Ca' da Mosto reached the Bissagos Islands. In 1482 Cão crossed the equator. In 1488 Dias rounded the cape of Good Hope. In 1498 da Gama reached India. In 1511 Albuquerque captured Malacca. Two years later a Portuguese ship reached China. In 1519-22 Magellan and El Cano circumnavigated the earth. And in 1526 de Torenó drew the first map of the earth showing the Pacific, and hence the entire circuit of the earth, in its true proportions. The problem of where China belongs on the map of the earth was solved conclusively.

In 1582 Ricci came to China. He learned Chinese, styled himself Li Madou, and remained in China until his death in 1610. Ricci wrote in Chinese on god and Christianity, and on the astrolabe, arithmetic, and measures. He rendered

Euclid into Chinese. Into Latin he rendered the four books – the foundation of Confucianism – the *Great learning*, *The mean*, the *Analects*, and *Mencius*. And in 1615 his journals were edited and published, *On the Christian expedition among the Chinese undertaken by the Society of Jesus, from the commentaries of father Ricci, in which the customs, laws, and institutions of the kingdom of China, as well as the most difficult beginnings of the new church there, are described accurately and with the greatest fidelity*.

Ricci laid the foundations of Sinology.

As to opportunism, a Portuguese trading station was established in China in 1517. The mission of the Society of Jesus was established in 1579. Ricci was its second head.

In the newly discovered lands in the west the harvesting of bodies was facilitated by their relative sparsity, and facilitated greatly by the rapid collapse of the two native states, of the Aztecs and of the Incas, which had hitherto been doing the harvesting themselves. The harvesting of souls was facilitated greatly by the lack of native doctrines.

In China the harvesting of bodies was hampered by their great number, and hampered severely by the presence of a powerful native state, with powerful institutions holding the population together, which had for more than two millennia been doing the harvesting themselves, were rather better at it than the newcomers, and were positively not going to surrender it to the newcomers. The newcomers' harvesting of bodies had to be confined to trade. The harvesting of souls was hampered by Confucius, and hampered severely by Confucianism. Two millennia of development of philosophy, and three centuries of institutionalization of this philosophy into a state doctrine, equipped with worship and rites, confronted the prospective harvesters of souls. As the title of Ricci's journals accurately and with the greatest fidelity attests, the beginnings of the Christian expedition among the Chinese were most difficult.

Hence, in the couple of generations after Ricci, Sinology was dominated by the question, what is Confucianism? as the new church there was dominated by the question, what to do about it? Are the rites of Confucianism civic, and thus

secular, and then no harm done in permitting converts to continue in them? or are the rites religious, and thus idolatrous, and then prospective converts have to renounce them before baptism? To evangelize, or not, was not at issue. Among those who weighed in on the matter were Arnauld and Mabillon.

Leibniz wrote the preface to the collection *The latest on China*, in 1697, and the *Discourse on the natural theology of the Chinese*, in 1716.

He begins in the preface:

I judge it a singular decision of the fates that the highest development of civilization by the human race should be gathered at the two extremes of our continent, in Europe, and in China, which adorns the east as Europe does the opposite end of the earth; supreme providence seems to have arranged things so that, when the two most cultivated and most remote peoples stretch their arms toward one another, those between them will be led to better life as well (*id.*).

The latest on China begins with this, then. Civilization is one. Its highest development on earth adorns the far end of the earth as it does the near. And supreme providence has ordained that, when the two ends of the highest development of civilization on earth stretch their arms toward one another, better life will follow.

Leibniz continues:

the Chinese empire challenges Europe in cultivated land, and certainly surpasses it in population; and in many other ways the Chinese meet us in an almost equal combat, so that now we win, now they win;

in the useful arts of life and the experience of natural things we are, all things considered, equal; each has practical knowledge which it can profitably communicate to the other;

in profound meditations and the theoretical sciences we win; in logic and metaphysics, and the knowledge of incorporeal things, we merit the victory; in the understanding of forms abstracted from matter, and that is, in the mathematical sciences, we excel by far, as the comparison of their astronomy with our, for instance, clearly shows; they are ignorant of that great light of the mind, the art of demonstration, and have remained content with that empirical geometry which among us even the artisans have;

they cede to us also the military sciences, not out of ignorance, but by their own decision; because they despise everything that makes men ferocious and, as if in emulation of the highest doctrine of Christ, are averse to war; and they would have been very wise in this, had they been alone on earth; as it is, even the good have to cultivate the arts of war, lest the evil prevailed in everything;

in these things we are superior;

but who will have thought, that, superior as we believe ourselves in all matters of morality, there is a people on earth who will vanquish us in the precepts of civil life? for this is what we discover in the Chinese, as we come to learn the facts;

we are their equal in the productive arts; we win in the theoretical sciences; but, shameful as it is, we are vanquished in practical philosophy, and that is, in the precepts of ethics and politics; it is indeed hard to describe how beautifully all the laws of the Chinese, unlike those of other peoples, aim at public peace and order in the community so that men inconvenience one another the least;

certainly the greatest evils men themselves do and inflict on one another; it is a true saying, a man to a man is a wolf; our folly is great and universal;

is it evil, then, that someone has found the remedy? for certainly the Chinese have attained a higher standard; in a vast society of men they have accomplished more than the founders of our religious orders have accomplished within their narrow ranks;

the state of our affairs being what it is, descending as we are into ever greater corruption, we need missionaries from the Chinese to teach us the practice of natural religion, just as we have sent them teachers of revealed religion; for I believe that a wise judge, not of the beauty of goddesses, but of the excellence of peoples, will award the golden apple to the Chinese; unless we win by a superhuman thing, by the divine gift of the Christian religion (*id.*).

That we will not win by that thing either, the fact guarantees that, while we have sent them teachers of revealed religion, they are yet to send us missionaries to teach us the practice of natural religion.

More important, Leibniz discovered the importance of China.

Civilization is one. And the fundamental problem, which its independent development in the several millennia through the fifteenth Christian century in the west and in the east poses, is of its differential development. How and why the problems of the development of the productive arts, of the theoretical sciences, and of politics and ethics at one end of the development of civilization inform the problems of the development of the productive arts, of the theoretical sciences, and of politics and ethics at the other end of the development of civilization. In general, and most important, how and why, while the productive arts developed equally, at one end of civilization the theoretical sciences advanced, at the other end of civilization politics and ethics advanced.

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The principles.

The true way ends with the most general principles.

And the most general principles are the two. The principle of contradiction. And the principle of sufficient reason.

Aristotle discovered that the principle of contradiction is the principle of all principles. Leibniz discovered that the two, the principle of contradiction and the principle of sufficient reason, are the most general principles.

Leibniz writes: our reasonings are founded on two great principles, that of contradiction, by which we judge that what involves a contradiction is false, and what is contradictory to the false is true; and that of sufficient reason, by which we consider that no fact can be found true or existent, no assertion true, without a sufficient reason why it is thus and not otherwise (*Monadology*, 31-2). And: the great foundation of mathematics is the principle of contradiction; this one principle suffices to demonstrate all arithmetic and all geometry, and that is, all mathematical principles; but to proceed from mathematics to physics another principle is also necessary, and this is the principle of sufficient reason (to Clarke, Dec 1715).

The foundation of mathematics is the principle

of contradiction. To proceed from mathematics to physics the principle of sufficient reason is also necessary. And our reasonings are founded on these two great principles.

These facts are so general, the acquisition of knowledge in its entirety, the development of philosophy in its entirety, and all our reasonings cannot but corroborate these most general facts.

And among the very general facts, if not quite so general, which corroborate these most general facts are the following. The discovery that the principle of contradiction is the principle of all principles is a consequence of the development of mathematics. The discovery that the principle of sufficient reason is the other principle of the greatest generality is a consequence of the development of physics. And the most far reaching consequences of the two discoveries are, in turn, as far reaching as consequences can be. The restoration of nature to being nature is a consequence of the first. The greatest advance of the investigation of knowledge is a consequence of the second.

Counting back to the 3 oldest calculations of area, mathematics begins twenty-seven centuries before Aristotle. Then, twenty-five centuries after it has begun, two centuries before Aristotle, the development of mathematics accelerates greatly. Aristotle is preceded by two centuries of great advance of mathematics, and succeeded by two centuries of continuing great advance of mathematics.

In the two centuries before him, Pythagoras discovered the incommensurability of the side and diagonal of the square, and thus demonstration. Hippias discovered the quadratrix, solving the problem of trisecting the angle conditionally. Theodorus of Cyrene proved that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational. Hippocrates of Chios squared lunes, solving the problem of squaring the circle partially, and wrote the first *Elements*. Archytas doubled the cube, solving this problem completely. Theaetetus developed the theory of the irrationals. Eudoxus developed the method of exhaustion, and formulated the theory of proportion.

Parmenides, two generations after Pythagoras, four before Aristotle, discovered the principle of contradiction. He formulates the principle as

explicitly as this:

there are two roads,
what is is and cannot not be,
which is the road of truth and conviction,
and what is not is not and must not be,
which you will never know or utter.

Aristotle writes: the principle of contradiction is the most certain of all principles; it is the principle of all principles.

The two centuries after Aristotle are the time of Euclid, Archimedes, and Apollonius. Menaechmus discovered the conic sections in the generation before Euclid. These centuries are the time also of Chrysippus and the propositional calculus. Eudoxus formulated the theory of the motions of the heavenly bodies in concentric spheres as well, and these two centuries are the time also of the first great advances in the mathematical treatment of physical problems. The time of the physical treatises of Euclid. Of those of Archimedes. Of Aristoxenus and harmonics. Autolycus and the heavenly spheres. Aristarchus and the distances of the moon and the sun. Eratosthenes and the radius of the earth. Diocles and burning mirrors. Hipparchus and the transformation of astronomy and geography into mathematical investigations.

Thus, there is the quantity of mathematics in the two centuries before Aristotle, and the two centuries after him.

Then there is the quality. Demonstration, that great light of the mind, was discovered. It advanced to the first *Elements*. To the solution of the problems of trisecting the angle, squaring the circle, and doubling the cube. To the theory of the irrationals. To the method of exhaustion. To the theory of proportion. Aristotle followed. Demonstration advanced to the *Elements*. To Archimedes. To Apollonius. To Hipparchus.

Then there is the fact that, with Aristotle: knowledge is acquired only by demonstration.

Then there is the fact that Aristotle developed logic, including the predicate calculus.

The fact also that Chrysippus discovered the propositional calculus.

The conclusion is unavoidable. The discovery that the principle of contradiction is the principle of all principles is a consequence of the development of mathematics.

The steps in the development of mathematics are the particulars. The true way begins with the collection of the particulars, marshals the particulars in their series and order, proceeds by continuous and gradual ascent, and ends with the most general principles.

The discovery that the principle of contradiction is the principle of all principles is an empirical generalization over the development of mathematics.

Aristotle writes: if contradictories were true, all things would be one; the same thing would be a trireme and a wall and a man; it is what follows from what Protagoras says.

A few centuries later, contradictories became true. All things became one. The same thing became a father and a son and a holy spirit. It was what followed from scriptures.

The darkest of dark ages also followed.

Seven and a half centuries after Augustine, five and a half before Spinoza, Abelard writes: god can do and do not only what he does and does not, and only in the way and at the time in which and at which he does it and does it not, and no other.

So a trial of strength began between god and the principle of all principles.

The principle won.

One and a half centuries after Abelard, Aquinas writes: all confess that god is omnipotent; but if we consider the matter carefully, power is with respect to possible things, and the proposition, god can do all things, is correctly understood to mean that god can do all things which are possible; all things which do not imply a contradiction, therefore, are among all possible things these with respect to which god is called omnipotent; while whatever implies a contradiction does not come within the scope of his omnipotence because it cannot have the character of possibility.

If god had to submit to the principle, all other things had. Including the things that had been especially eager to be triremes and walls and men.

Two generations after Aquinas, Occam is the most important logician in the nineteen centuries between Chrysippus and Leibniz.

Occam discovered Occam's razor. In its formulation in the *Summa of all logic*: what can be

done with less is done in vain with more (I, 12). In its formulation in the *Seven quodlibets*: plurality must not be posited without necessity (IV, 27). In the *Seven quodlibets* Occam is explicit also that the razor follows from the principles of the philosopher, and that the excision which it demands concerns god, too. He writes: according to the principles of the philosopher plurality must not be posited without necessity (*id.*), and: you may object that producing infinitely many things is not repugnant to god; I reply, it is repugnant (III, 1).

Occam answers 170 questions in the *Seven quodlibets*. Many are concerned with logical matters, terms, propositions, inferences, truth, falsehood, generality, particularity, definition, predication, the categories. Many are concerned with matters which would rather be left alone by logical matters. The following is a sample of the latter: can an angel see our thoughts and desires (IV, 9)? can an angel move (I, 5)? can an angel move through vacuum (8)? does one angel speak to another (6)? is the proposition, this is my body, when uttered by the priest at mass literally true (II, 19)? can Christ as he is in the eucharist see and be seen (IV, 13)? can Christ's soul move Christ's body in the eucharist (14)? can Christ's soul move Christ's body under the host both part by part and as a whole (15)? can Christ's body under the host not have a position which is a place (18)? is Christ's body, when it begins to be sacramentally under the appearance of the bread on the altar, truly changed (VI, 3)? is Christ's being the son of the blessed virgin not absolute (IV, 11)? can the blessed virgin have been in the state of original sin (III, 10)? does the blessed virgin have an impulse toward sin (11)? is the extended material substance of Christ's body composed of substantial parts of the same nature and distinct in position (IV, 19)? can the three divine persons assume the same nature numerically (8)? is his paternity distinct from the father (I, 3)? is the divine essence of the father impossible simultaneously with the son (VI, 7)?

In the next two centuries these questions were answered, and again, and again, and conclusively. Even if mostly still only tacitly. There was nothing to do but to praise their folly.

Not quite two centuries after Occam, Erasmus

writes in *The praise of folly*: here are some questions worthy of the great and illuminated, as they are called, theologians; is the proposition, the father hates the son, possible? can god have taken the likeness of a woman? the likeness of the devil? of an ass? of a gourd? and how would the gourd have preached the gospel? performed the miracles? been nailed to the cross (28)? and is it a lesser crime to cut the throats of one thousand than to stitch a poor man's shoe on the sabbath? to let the world perish than to tell a lie? to render these subtlest of subtleties still more subtle is what our scholastics, realists, nominalists, Albertists, Thomists, Scotists, and Occamists, do (*id.*).

Still, with all due respect to Erasmus, and never mind cutting the throats of one thousand and letting the world perish, can god have taken the likeness of a gourd? and how would the gourd have preached the gospel? performed the miracles? been nailed to the cross?

In the next century, having gathered momentum since the time of Abelard, mathematics began advancing greatly once again, physics became mathematical physics, and Occam's questions, and Erasmus's, too, began being answered overtly.

One century-odd after Erasmus, Descartes, having greatly advanced mathematics and physics himself, began closing that matter, too: even if some malignant spirit of the utmost power and cunning employs all his industry to deceive me, he can never make true anything in which I recognize a manifest contradiction. Hence: god cannot on pain of contradiction be false. Hence: nature is nothing but god himself.

And in the next generation, Spinoza closed the matter: god is nature.

You may object that producing infinitely many things is not repugnant to god. I reply, it is repugnant to nature. Excising god, then, nature is nature.

The conclusion is again unavoidable. The principle of contradiction cleared philosophy of supernature. And better, considering that nature was native to philosophy, then supernature invaded it, the principle of contradiction restored nature to being nature.

The true way ascends and descends, from experiments to principles and from the principles

to new experiments. The discovery that the principle of contradiction is the principle of all principles is fundamentally inductive. The abolition of supernature is fundamentally deductive.

clxxxix

The anticipations of the principle of sufficient reason can be traced back to Plato.

In his earlier dialogues Plato investigates piety, courage, friendship, temperance, beauty. In the *Meno* he investigates virtue. In the *Phaedo* he investigates the immortality of the soul. In the *Symposium* he investigates love. In the *Phaedrus* he investigates the journey of the soul to the realm beyond the heaven. In the *Republic* he investigates justice. In the *Theaetetus* he investigates knowledge. In the *Timaeus* he investigates the world.

He writes in the *Timaeus*: since Timaeus is the best at astronomy and has devoted himself to the study of the nature of all things, let him speak about the origin of the world (27a). And so Timaeus does: we must begin with this distinction, there is that which always is and never becomes and there is that which always becomes and never is; one is grasped by thought, the other by perception; and everything that becomes must come out of some cause, because it is impossible for anything to come to be without a cause; now, if a maker, in order to make something, looks for his paradigm at what always is and never becomes, what he makes is necessarily beautiful, but if he looks for his paradigm at what always becomes and never is, what he makes is not beautiful; as to the whole heaven, or the world, we must first of all ask, has it always been, or has it come to be? it has come to be; because it is visible, tangible, corporeal, and all such things are perceptible; but what comes to be must have a cause; and which was the paradigm at which the father and maker of all things looked in order to make the world? did he look at what always is or at what always becomes? clearly, if the world is beautiful and the maker was good, he looked at what always is (27e-9a); and why did he make the world? because he was good (29d-e).

It is impossible for anything to come to be without a cause.

The importance of Plato's result is as great as

its generality, the greatest. And the difficulties which remained to be resolved were just as great.

One of the difficulties was, indeed, greater than the greatest. It is impossible for anything to come to be without a cause. Including the world. Excluding, however, what always is and never becomes. Where is it, then, what always is and never becomes? in the not world? For even if it is excluded, for the sake of the argument, from having to have a cause, if it is in relation to the world, to what always becomes and never is – and the fact that Plato has much to say about it, while in the world, presumably, guarantees that it is in relation to the world – its relation to the world cannot be excluded from having to have a cause. Not if it is impossible for anything to come to be without a cause. If it is impossible for anything to come to be without a cause, it is impossible for anything to come to be without a cause. This difficulty is, in fact, Plato's main concern in the *Parmenides*.

The same difficulty besets the maker. Granted, for the sake of the argument, that he is in the not world, his making of the world cannot not be in relation to the world, cannot not be what became and was not never. This is why he made the world because he was good, to address this difficulty. But as the maker so his goodness. If it is impossible for anything to come to be without a cause, what is the cause of his goodness coming to make the world? Hence also another difficulty. Whatever the cause of his goodness, what is its cause? and its cause? and its cause? In general, if it is impossible for anything to come to be without a cause, it is impossible for this impossibility to come to be without a cause, and for this impossibility, and so on endlessly.

These are among the lesser difficulties. They are formal, matters of consistency, and the principle of contradiction, Aristotle, in particular, will resolve them.

The fundamental difficulty is material, however, the principle of sufficient reason will have to resolve it, and will not for two millennia.

The fundamental difficulty is that *Timaeus* may be the best at astronomy but the *Timaeus* is not, and *Timaeus* may have devoted himself to the study of the nature of all things but the *Timaeus* has not. The *Timaeus* is a sizable treatise,

and Plato has much to say about the things in the world, those that always become and never are, the elements, stars and planets, plants and animals, flesh and bone, pain and pleasure, health and disease, thought and perception. In particular, it is impossible for anything to come to be without a cause, and Plato finds the cause of everything. The cause is always beautiful and good. What it is not is the outcome of the study of what always becomes. Plato is the successor of two centuries of great advance of physics, including in the couple of generations before him Anaxagoras, Empedocles, Oenopides, Leucippus, Meton, Hippocrates, Democritus. Plato's causes are not contaminated by the results of his predecessors. Nor are they contaminated by the collection of the particulars. The *Timaeus* is beautiful and its maker was good. Clearly, he did not look for his paradigm at what always becomes and never is, he looked at the paradigm at which the maker of the world looked.

The difficulty is, which are, in fact, the causes of things?

It remained a difficulty for two millennia. Descartes still has to write: you say that nothing is easier than to fit some cause to an effect; but even if it were true that there are effects to which it is easy to fit various causes, one cause to each effect, it is not always so easy to fit the same cause to many different effects unless it is the true cause which produces them.

The difficulty is that nothing is easier than to fit some cause to an effect. But that a cause has been fit to an effect will not make it the true cause which produces the effect. Nothing but the principle of sufficient reason will make it the true cause which produces the effect.

Aristotle resolved all matters of consistency in this case, as he did in so many other.

He writes in the *Physics*: the world is one, there is nothing next to the world, or outside the world.

Moreover: philosophy searches for the principles of the sensible things, and to argue, while searching for their essence, that there is some other kind of essence, is prattle.

He writes in the *Posterior analytics*: we know something when we know that its cause is its cause and cannot be otherwise; we know it by

demonstration; demonstration always depends on true principles; and these principles cannot themselves be demonstrated; to ask for their demonstration is to ask for demonstration to infinity.

And why there can be no demonstration to infinity, and the causes of things cannot be an infinite series, and we can thus know that the cause of something is its cause and cannot be otherwise, is the central problem of book II of the *Metaphysics*.

Aristotle writes: clearly, there is a first principle, and the causes of things are neither infinite in number nor infinite in kind; material causes cannot be an infinite series, flesh from earth, earth from air, air from fire, and so on without end; efficient causes cannot be an infinite series either, man is moved by air, air by the sun, the sun by strife, and so on to infinity; the same with final causes, walking is for the sake of health, health for the sake of happiness, happiness for the sake of something else, and one thing always for the sake of another; and the same with formal causes; for every intermediate term has to be intermediate between a first term and a last term, and the first term has to be the cause; for the last term cannot be the cause, because it cannot be the cause of either the intermediate or the first; and the intermediate cannot be the cause, because it can be the cause only of the last; and it makes no difference whether there is one intermediate term or there are more; and if the series were infinite, all terms would be intermediate; if there were no first term, there would be no cause at all (2); those who ask for another definition before every definition destroy knowledge; for there is no knowledge until an end is reached (*id.*).

Conversely, if knowledge has been acquired, an end has been reached, and if an end has been reached, there is an end.

Aristotle resolved the formal difficulties conclusively. We know something when we know that its cause is its cause and cannot be otherwise. And there is a first principle, the causes of things are not an infinite series.

He took a major step as well toward the resolution of the material difficulty.

The world is one. And Aristotle described it.

In the *Physics*, *On the heaven*, *On generation and corruption*, *Meteorology*, *On the soul*, *His-*

tory of animals, *Parts of animals*, *Movement of animals*, *Progression of animals*, *Generation of animals*, *On the senses and the sensible*, *On memory and recollection*, *On youth, old age, life and death*, and *respiration*. Collecting and building on the results of his predecessors paradigmatically. Collecting the particulars, marshaling them in their series and order. Always assigning causes, too. And it is with Aristotle's physical treatises that the labors of the philosophers to discover which are, in fact, the causes of things attained their first substantial successes.

Only their first substantial successes, however. The description of the world is vague in Aristotle. It quit being vague with Ptolemy.

In book II of the *Metaphysics* Aristotle begins by distinguishing theoretical philosophy and practical philosophy. Theoretical philosophy is the search for truth. And: we do not know the true if we do not know the cause (1). Hence the argument that there is a first principle, and the causes of things are not an infinite series. We can thus know the cause, and the true. Mathematics and physics are theoretical philosophy. Mathematics and physics both search for the causes, and attain truth. But not both with rigor. Aristotle concludes in book II: the rigor of mathematics should not be demanded in all things, but only in those that have no matter; it is not the way of physics, for all nature has matter (3).

It is the way of physics. Four and a half-odd centuries later, Ptolemy writes: mathematics is concerned with form and locomotion; and it can deal with the objects of physics because everything material can be understood by its locomotion.

And if it had not been the way of physics before Aristotle, it became the way of physics in the century after him. In the generation after him Autolycus wrote *On the moving sphere* and *On the risings and settings*, Euclid wrote the *Phenomena* and *Optics*, in the next generation Aristarchus wrote *On the sizes and distances of the sun and moon*, and these are all mathematical investigations. In the next generation Archimedes wrote *On the equilibrium of planes* and *On floating bodies*, and these are mathematical investigations *par excellence*, Archimedes treats weights and fluids axiomatically. And Leibniz's

standing illustration of the fact that to proceed from mathematics to physics, next to the principle of contradiction, the principle of sufficient reason is also necessary, is Archimedes.

Mathematics is the way of physics, and proceeding from mathematics to physics led to the resolution of the material difficulty. It showed which are, in fact, the causes of things.

The mathematical treatment of physical problems advanced in the four and a half-odd centuries from Aristotle through Ptolemy. Then it did not advance for a long time. Nothing did. It began advancing again, counting from Buridan and Oresme, two and a half-odd centuries before Bacon, Galileo, and Kepler. Then the advance accelerated. Then the 3 generations beginning with Bacon, Galileo, and Kepler followed.

In these 3 generations philosophy advanced greatly. Philosophy such as it is mathematics and physics.

Mathematics advanced greatly.

And physics advanced greatly. Physics such as it is mathematical exclusively.

Aristotle writes: we know something when we know that its cause is its cause and cannot be otherwise.

Bacon writes: true knowledge is knowledge by causes. Galileo writes: to recognize the cause removes the marvel. Kepler writes: no other way can succeed but grounding the motions in their causes. Descartes writes: to be perfect knowledge must be deduced from first causes. Huygens writes: in the true philosophy the cause of all effects is sought. Spinoza writes: we understand god to the degree to which we know the causes of nature.

Bacon writes: although there is nothing in nature but individual bodies performing individual acts, they do so by law, and the investigation, discovery, and explanation of this law with its clauses is the foundation of knowledge.

Stevin discovered the law of the inclined plane.

Galileo discovered the law of falling bodies, the law of the pendulum.

Snel discovered the law of refraction.

Pascal discovered the law of pressure of confined fluids.

Boyle discovered the law of volume and

pressure of gases.

Hooke discovered the law of elasticity.

Kepler discovered the laws of the motions of the heavenly bodies.

Huygens discovered the principle of wave fronts.

Fermat formulated the principle of least time.

Galileo discovered the principle of relativity.

Descartes formulated the laws of nature, and discovered that the laws of nature are fundamentally laws of conservation.

Huygens discovered the principle of conservation of momentum.

Leibniz discovered the principle of conservation of living force.

Newton formulated the laws of motion. The discovery of the first began with Galileo and Kepler, and was complete with Descartes. The discovery of the second began with Descartes and Huygens, and was complete with Newton. The discovery of the third began and was complete with Newton.

Newton discovered the law of gravity.

The world is one. The world is the system of the world. And Kepler grounded the system of the world in the laws of the motions of the heavenly bodies. Newton grounded it in the more general laws of motion and law of gravity.

Newton begins in *The principles*: because the recent successors of the ancients have undertaken to reduce the phenomena of nature to mathematical laws, this treatise proposes to perfect mathematics as it relates to philosophy.

As physics became mathematical, laws were discovered. And the laws showed which are, in fact, the causes of things.

Hence, as physics became steadily more explicitly mathematical, and steadily more general laws were discovered, the discovery that the principle of sufficient reason is the other, next to the principle of contradiction, most general principle was steadily more explicitly anticipated.

Aristotle writes: there is a first principle, the causes of things are not an infinite series.

Aristotle's is a formal argument, not an empirical generalization.

Occam writes: what can be done with less is done in vain with more.

Occam's is also a formal argument, not an

empirical generalization.

Bacon writes: the law with its clauses is the foundation of knowledge.

Galileo writes: nature is inexorable and immutable, never transgresses against the laws imposed on her, and does not care whether her reasons and modes of operation are understood by men or not.

Kepler writes: that the motions of the planets are ordered according to a fixed and immutable law is beyond controversy; otherwise there would have been no astronomy.

And Kepler's, now, is an empirical generalization. The fact of astronomy shows that the motions of the planets are ordered according to a fixed and immutable law, not the other way around.

Descartes writes in the *Rules*: the whole method consists in the ordering and arrangement of things; in this one rule all human industry is contained. In *The world*: the laws follow from this alone that god is immutable, and, acting always in the same way, produces always the same effect. In the *Replies*: there is no existent thing of which it cannot be asked which is the cause why it exists; and this can be asked even of god; not because he needs a cause in order to exist, but because the very immensity of his nature is the cause or the reason why he needs no cause in order to exist. In *The principles*: god is the primary cause of motion; he always conserves the quantity of motion in the universe; and from the immutability of god we know the rules or laws of nature, which are the secondary causes of motion. Also in *The principles*: although all truths which I include in my principles have always been known to everyone, no one until this time has recognized them as the principles of philosophy, and that is, that from them the knowledge of all other things in the world can be deduced.

With Descartes the anticipation has become as explicit as this: there is no existent thing of which it cannot be asked which is the cause why it exists. The formal difficulties are resolved explicitly: it can be asked which is the cause why it exists even of the first cause, not because it needs a cause in order to exist, but because its nature is the cause why it needs no cause in order to exist. The generality is indicated: from the truths in-

cluded in the principles the knowledge of all other things in the world can be deduced. It is clear also that the laws point to the first cause: we know the laws of nature from the immutability of the first cause.

Huygens writes: the geometers prove their propositions from certain and incontestable principles; here the principles are verified by the conclusions drawn from them; the nature of things will not have it otherwise; it is nevertheless always possible to attain a degree of probability which falls short of complete evidence in nothing; for if the things demonstrated from the assumed principles correspond perfectly to the phenomena which experience has manifested, the more so if the number of these phenomena is great, and the more so if new phenomena have been foreseen from the hypotheses, and effects have been found to correspond to the expectations, this must be a grand confirmation of the success of the investigation.

The principles are verified by the conclusions drawn from them. The conclusions correspond perfectly to the phenomena, and a degree of probability is attained which falls short of complete evidence in nothing. The causes are, in fact, the causes. The nature of things will not have it otherwise.

Spinoza writes: nothing exists of which it cannot be asked which is the cause or the reason why it exists. And: it is ultimate folly to ascribe to the power of god something of which we do not know the natural cause.

In *The mathematical principles* the causes and effects are the forces. The true forces, and the absolute forces. Newton writes: the causes which distinguish true motion from relative are the forces impressed on bodies to generate motion; the effects which distinguish absolute motion from relative are the forces of recession from the axis of circular motion. And Newton anticipates the discovery that the principle of sufficient reason is the other, next to the principle of contradiction, most general principle yet more explicitly. Rule 1 of philosophizing is: to admit no more causes of things than are true and suffice to explain the phenomena. Newton comments: this is why the philosophers say, what can be done with less is done in vain with more; nature is

simple, and things do not luxuriate in superfluous causes. Rule 2 is: therefore, to assign to effects of the same kind the same causes.

The discovery that the principle of sufficient reason is the other, next to the principle of contradiction, most general principle is an empirical generalization. The true way ascends from observations to principles, descends from these to new observations, ascends from these to higher principles, descends again, ascends, and until it reaches the most general principles. The discovery is an empirical generalization over the development of physics, most particularly over the discovery of the law with its clauses.

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Leibniz concerns himself with the two most general principles beginning to end.

In 1666, in *The art of combination*, he writes: the primary propositions are two, the principle of all theorems or necessary propositions, what is thus is or is not thus and conversely, and the principle of all observations or contingent propositions, something exists (app.).

In 1671, in *A new physical hypothesis*, he reformulates the principle of all observations: there is nothing without a reason (*id.*).

In 1679, in the *Beginning and specimens for a general science*, he writes:

I use two principles in demonstration; one is, what implies a contradiction is false; the other is, a reason can be given for every truth, and that is, the notion in the predicate is always in the notion in the subject either explicitly or implicitly, and this holds for contingent truths not less than it does for necessary (14).

Also in 1679, in the manuscript beginning *Cum animadverterem plerosque*, he writes:

proof cannot go on to infinity, some things have to be assumed without proof; not tacitly and stealthily, however, dissimulating our intentions, as the philosophers usually do, but as the geometers do, stating in good faith the assumed axioms at the beginning, so that it can be known that all conclusions are demonstrated from the posited hypotheses;

first of all, then, I assume that every assertion is either true or false, if the affirmation is true, the negation is false, if the negation is true, the

affirmation is false, what is denied to be true is false, what is denied to be false is true, what is denied to be affirmed, or affirmed to be denied, is to be denied, what is affirmed to be affirmed, or denied to be denied, is to be affirmed, it is false that what is false is true and what is true is false, it is true that what is true is true and what is false is false; all these are comprehended under the one name, the principle of contradiction;

we thus see that the primary true propositions are those called identities;

but every true proposition, whether it is an identity or not, can be proved *a priori* from axioms, from propositions which are identities and definitions; all truths, therefore, even the most contingent, can be demonstrated *a priori*, or a reason can be shown why they are rather than are not; and this is why it is said, nothing occurs without a cause, or nothing is without a reason;

this axiom, nothing is without a reason, is among the greatest and most fruitful in all human knowledge, on it stands a great part of metaphysics, physics, and moral science; nor can the existence of god be proved from creatures without it, nor can an argument proceed either from causes to effects or from effects to causes, nor can any conclusion be drawn in civil matters; whatever is not of mathematical necessity stands on it; Archimedes, for instance, when he assumes that two equal weights are in equilibrium with respect to a center, this is a corollary of our axiom; for, when everything is posited the same on either side, there is no reason why the weights will incline to one side rather than the other; and with this assumption Archimedes demonstrates the matter with mathematical necessity (*id.*).

The law of the lever is a corollary of the principle of sufficient reason. And so are all laws.

By 1679 Leibniz has established that the fact of demonstration, the fact that all truths, even the most contingent, can be demonstrated *a priori*, shows that there is a sufficient reason. The principle of contradiction, on which demonstration stands, entails the principle of sufficient reason. He has established also the generality of the principle of sufficient reason. It is among the greatest and most fruitful in all knowledge. Whatever is not of mathematical necessity stands on it.

Leibniz has established that the principle of contradiction and the principle of sufficient reason are the two most general principles of the acquisition of knowledge. Descartes separated sharply the principles, on one side, from all other things in the world, on the other, and among the principles the principles of the acquisition of knowledge, on one side, from the principles of all other things in the world, on the other. In the following years Leibniz will establish that the principle of contradiction and the principle of sufficient reason are the two most general principles of the acquisition of knowledge because they are the two most general principles of all other things in the world.

In 1686, in the *Discourse on metaphysics*, he writes:

although all particular phenomena of nature can be explained mathematically or mechanically, the general principles of corporeal nature and of mechanics are nevertheless metaphysical rather than geometrical (18);

the principle of all existences and of the laws of nature must be sought in god, because he always proposes the best and the most perfect (19); if god's wisdom is recognized in the mechanical structure of particular bodies, it must be that it is manifested also in the general economy of the world and in the constitution of the laws of nature (21).

Leibniz illustrates with the principle of least time which: Snel and Fermat have most ingeniously applied to refraction (22). The principle of least time shows that god's wisdom is manifested in the general economy of the world and in the constitution of the laws of nature. And so do all laws.

God's wisdom is the principle of all existences and of the laws of nature. It is manifested in the general economy of the world and in the constitution of the laws of nature. And it is manifested in the constitution of the laws of nature because it is manifested in the general economy of the world. It is the principle of the laws of nature because it is the principle of all existences. Earlier in the *Discourse* Leibniz writes:

god is an absolutely perfect being (1);
he acts always in the most perfect way possible (4);

he does nothing without order (6); not only is there nothing absolutely irregular in the world, such a thing cannot even be conceived (*id.*).

There is nothing irregular in the world. Hence the laws of nature.

In 1687, in *On a general principle useful for the explanation of the laws of nature through the divine wisdom*, he is more explicit. He writes:

the principle of general order has its origin in the infinite, and is absolutely necessary in geometry, but is useful in physics as well, because the sovereign wisdom, which is the source of all things, acts as a perfect geometer, observing a harmony to which nothing can be added;

the principle can be formulated thus; when the difference between two cases can be diminished less than any given quantity in what is given, or what is posed, it must be possible to diminish it less than any given quantity in what is sought, or what is the result, as well; or to put it more easily, when the cases, or what is given, approach one another continuously and at last disappear one into the other, it must be that their results or consequences, or what is sought, do so as well; and this, in turn, depends on the more general principle, the order of the given is the order of the sought (*id.*).

The definition of limit shows that the sovereign wisdom observes a harmony to which nothing can be added. And so do all mathematical and physical principles.

The relation is now explicitly of cause and effect. The principle of general order is absolutely necessary in geometry and useful in physics because the sovereign wisdom observes a harmony to which nothing can be added. The order of the given is the order of the sought. And not the other way around. God does nothing without order, and this is the order of the sought. Hence the order of the given is the order of the sought.

God's wisdom is the principle of the laws of nature because it is the principle of all existences. The order of the given is the order of the sought because god does nothing without order. And in the following years Leibniz will reduce these very general propositions to the two most general.

In 1697, in *On the radical origination of things*, he writes:

beyond the world, or the aggregate of finite

things, there is one that is dominant, and not as the soul is in me, or as I rule my body, but as a much higher reason; the one dominant being of the universe not only rules the world, but makes it and is superior to it, and is, therefore, to speak extramundanelly, the ultimate reason of things;

for a sufficient reason to exist can be found neither in any one thing nor in the entire aggregate and series of things; however far the regress reaches into previous states, there is never in these states a complete reason why there is the world, and it is the way it is;

even in eternal things, if there is no cause, a reason must be understood; in things that persist the reason is their necessity, or essence; in things that change the reason is their prevailing direction, for when reasons do not necessitate, so that the contrary implies a contradiction, they yet direct; it follows that even if the world is supposed eternal, the ultimate extramundane reason of things, or god, cannot be escaped;

the present world is necessary physically, or hypothetically, but not absolutely, or metaphysically;

and in order to explain how from eternal, or essential, or metaphysical truths temporal, or contingent, or physical truths follow, we must first acknowledge that something rather than nothing exists; there is thus a direction toward existence; essence itself is directed toward existence; and perfection is nothing but the quantity of essence;

it thus marvelously becomes intelligible how a certain divine mathematics, or metaphysical mechanism, is employed in the very origination of things and the greatest possible determination occurs;

and we have thus derived physical necessity from metaphysical, and shown that, even if the world is not metaphysically necessary, in that its contrary implies no contradiction, it is physically necessary, or determined, in that its contrary implies imperfection, or moral absurdity (*id.*).

A certain divine mathematics, or metaphysical mechanism, is employed in the very origination of things. Thus the greatest possible determination occurs. The divine mathematics determines the world. And the divine mathematics and the world determine mathematics and physics. The

two most general principles are the principles of the divine mathematics and the world, and hence the same two most general principles are the principles of mathematics and physics.

So it is clear also why: I have said more than once that a new kind of logic is needed, to treat the degrees of probability (*New essays*, IV, 16). It is needed to determine the quantity of essence, or perfection. And as the greatest possible determination occurs, it is clear that probability is certainty, not uncertainty.

Leibniz also makes it possible to remove, at long last, the vagueness from the relation of *causa* and *ratio*, *cause* and *raison*, cause and reason, *Ursache* and *Grund*. Vaguely synonymous before him, and hence the source of an indefinite amount of confusion before him as well as after, after Leibniz 'cause' can refer to a relation in the divine mathematics and the world, exclusively, 'reason' to a relation in mathematics and physics, exclusively.

On the other hand, the ultimate reason of things cannot be extramundane. With Kant: there is nothing other than the world. Nor need anything be extramundane, not even the divine mathematics. With Kant again: the world is the mathematical whole of all phenomena, and the very same world is called nature when considered a dynamical whole. Nothing either can be or need be extramundane because of Hume's principle. Still with Kant: not to carry the use of reason beyond the field of all possible experience is Hume's principle.

Other than that, other than a tendency to speak extramundanelly of the ultimate reason, Leibniz is unwavering in his empiricism.

Hegel writes: as he is the opposite of Newton in another way, so Leibniz is philosophically against Locke and his empiricism, as well as against Spinoza (*Hist. philosophy*, III, 2.1.C.1). Leibniz is not the opposite of Newton in any way, he is philosophically with Spinoza, not against him, and if he is philosophically against Locke and his empiricism, he is in that Locke wavers in his empiricism, Leibniz does not in his, and Leibniz criticizes Locke for wavering in his.

In 1704, in the *New essays on human understanding*, Leibniz writes:

what I find least to my liking in your definition

of truth is that it looks for truth in words; so the same sense expressed in Latin, German, English, and French is not the same truth;

it is better to find truths in the relations of the objects of ideas, which make them included or not included in one another; this does not depend on languages, and we have it in common with god and the angels;

we can thus distinguish truths, which are independent of our good pleasure, from expressions, which we invent as we see fit;

let us be content to look for truth in the correspondence of propositions, which are in the mind, with the things the propositions are about (IV, 5).

Truth is independent of our good pleasure. We have this in common with god and the angels, that truth is independent of their good pleasure as well. Truth is the correspondence of propositions with the things the propositions are about. This definition of truth Leibniz has in common with Parmenides, Aristotle, Occam, Descartes, Hume, Kant, Frege. Clearly, he has it in common with god and the angels as well.

Leibniz identifies the reason for the wavering, too:

it is easier to reason demonstratively in mathematics in large part because experience can guarantee the reasoning at every moment, as it does in the figures of the syllogism; but in metaphysics and morality the parallelism of reasons and experience is no longer there; while in physics experiments demand labor and expense; and men allow their attention to waver, and are consequently led astray, as soon as they are deprived of that faithful guide experience, which aids and sustains their steps like the little wheeled devices which keep small children from falling over (2).

Given that Locke did much metaphysics and morality, so did Hegel, Locke did no mathematics, Hegel did not either, Locke left the labor and expense of experiments in physics to other philosophers, so did Hegel, and that in all these ways the two are representative, the reason is the sufficient reason.

In 1714, Leibniz's last major treatises are the two sets of *Principles*, the *Principles of nature and grace founded on reason* and the *Principles of philosophy, or monadology*. The most general principles are central to both.

In the *Principles of nature and grace* Leibniz writes:

the last reason of things is called god (8);

and the supreme wisdom of god has made him choose above all the laws of motion best fit to abstract, or metaphysical, reasons; he conserves the same quantity of absolute force, or action, the same quantity of respective force, or reaction, and the same quantity of directive force; moreover, action is always equal to reaction, and the entire effect is always equal to its full cause; and what is surprising is that a reason for these laws of motion – discovered in our time, and some of which I have discovered myself – cannot be given by consideration of efficient causes, or matter, alone; I have discovered that recourse to final causes is necessary, that these laws depend not on the principle of necessity, as logical, arithmetical, and geometrical truths do, but on the principle of fitness, and that is, on the choice of wisdom; and this is one of the most efficacious and sensible proofs of the existence of god for those who can go deeply in these things (11);

because all is regulated in things once and for all with as much order and correspondence as possible (13).

In the *Principles of philosophy*, finally:

our reasonings are founded on two great principles;

that of contradiction, by which we judge that what involves a contradiction is false, and what is contradictory to the false is true (31);

and that of sufficient reason, by which we consider that no fact can be found true or existent, no assertion true, without a sufficient reason why it is thus and not otherwise (32).

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The discovery that the principle of sufficient reason is, next to the principle of contradiction, the other principle of the greatest generality is an empirical generalization over the development of physics. Most particularly over the discovery of the laws.

Conversely, the laws are the corollaries of the principle of sufficient reason.

The laws discovered by Archimedes. Galileo's laws. Kepler's. Newton's. The laws discovered by Leibniz. And the laws discovered after Leib-

niz. The laws of thermodynamics. And Helmholtz writes: the theoretical part of physics seeks to comprehend the processes of nature according to the law of causality; and we are urged in this task and justified in it by the principle that every change in nature must have a sufficient cause (*Conservation*, intr.). The laws of electrodynamics. And Maxwell writes: the general maxim of physical science is, the same causes will always produce the same effects. The laws of relativity. And to show that the principle of relativity has to be generalized, Einstein writes: Newtonian mechanics does not actually satisfy the demands of causality (*Foundation*, 2). The laws of quantum theory. And against the supposition that quantum theory is not a theory of causes, Bohr defined complementarity, and Bohr writes: complementarity can be regarded as a rational generalization of the very ideal of causality (*Epistemological*).

But the most far reaching consequence of the discovery that the principle of sufficient reason is, next to the principle of contradiction, the other principle of the greatest generality is the advance of the investigation of knowledge.

Hume and Kant are two-odd generations after Newton and Leibniz.

Hume writes: all objects of human inquiry may be divided into two kinds, relations of ideas and matters of fact; of the first kind are geometry, algebra, arithmetic, and, in short, every affirmation which is either intuitively or demonstratively certain; but matters of fact are not ascertained in the same manner; the contrary of every matter of fact is possible, because it can never imply a contradiction; and if we anatomize reasonings concerning matters of fact, we shall find that they are all founded on the relation of cause and effect; by means of this relation alone we can go beyond the evidence of our memory and senses.

And: knowledge of the relation of cause and effect arises entirely from experience; in inferences concerning matters of fact animals are not guided by reasoning, neither are children, nor are the generality of mankind, nor are the philosophers; experimental reasoning, which we possess in common with the beasts, and on which the whole conduct of life depends, is nothing but a species of instinct, or mechanical power.

Kant writes: whatever their source, and what-

ever their form, judgments are with respect to their content either analytic or synthetic; judgments from experience are always synthetic; mathematical judgments are in their entirety synthetic; so are metaphysical judgments; thus, the problem on which everything depends is, how are synthetic *a priori* propositions possible?

And: what agrees with the formal conditions of experience is possible; when the connection is determined in accordance with the general conditions of experience, it is necessary; the principle of contradiction is the principle of all analytic knowledge; the possibility of experience is the principle of all synthetic judgments; the principle of sufficient reason is the reason of possible experience; and here is the solution of the puzzle, synthetic *a priori* propositions are related to nothing but the possibility of experience; the possibility of experience is what gives all knowledge *a priori* its objective validity.

Hume has proposed: to establish the foundation on which the sciences can stand with any security. Kant has proposed: to change the ways of metaphysics by undertaking, following the example of the geometers and the investigators of nature, its complete revolution.

The foundation on which the acquisition of knowledge stands is experience.

Experience is the same in beast and philosopher.

Experience has form.

And form gives the acquisition of knowledge its objective validity.

Darwin will show that experience is the outcome of the development of all organisms.

Frege will show the uniformity of the acquisition of knowledge.

The great advance of the investigation of knowledge with Hume and Kant is the unconditional consequence of the great advance of the acquisition of knowledge. The acquisition of knowledge posed the problems, and supplied the solutions of the problems, of the investigation of knowledge. More particularly, it is the unconditional consequence of the great advance of mathematics and physics. More particularly still, it is the unconditional consequence of the great advance of physics such as it became mathematical exclusively. Yet more particularly, it is the un-

conditional consequence of the discovery of the laws. Yet more particularly, it is the unconditional consequence of the discoveries which showed ever more clearly and distinctly that the laws are, in fact, one law with its clauses. And most particularly, it is the unconditional consequence of the discovery, anticipated very nearly in the rules of philosophizing formulated by Newton, established explicitly and conclusively by Leibniz, that the two principles, of contradiction and of sufficient reason, are the most general principles which are the sufficient reason of the laws, of physics, of mathematics, and of the acquisition of knowledge.

The acquisition of knowledge advanced greatly by following the true way. And the investigation of knowledge advanced greatly by following the true way.

The two principles, of contradiction and of sufficient reason, are the principles, on one side, of the world, and hence, on the other side, of the

acquisition of knowledge.

On one side, the laws are their corollaries.

On the other side, the rules of the method, to include in judgments nothing but what is so clear and distinct that it cannot be doubted, to divide difficulties into parts such as will lead to their better solution, to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex, and always to make enumerations so complete and reviews so general as to omit nothing, are their corollaries.

Beginning with the collection of the particulars, marshaling them in their series and order, proceeding by continuous and gradual ascent, ending with the most general principles.

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And this is how the method has been followed when it has been followed as staunchly in the search for truth as the thread of Theseus must be followed in the labyrinth.

enumeration and review

Below are the particulars collected in parts one and two of the investigation. Their arrangement is explained in part one.

important

3120 ^{BC}	3 calculations of area	the century around <i>c.</i> 3080 ^{BC}
3040	Narmer	prince <i>c.</i> 3000
3000	Djer	prince <i>c.</i> 2960
2720	Djoser	prince <i>c.</i> 2680
2720	Enmebaragesi	prince <i>c.</i> 2680
2640	Sneferu	prince <i>c.</i> 2600
2620	Khufu	prince <i>c.</i> 2580, son of Sneferu
2620	division of grain among 164 571 men	– the century around <i>c.</i> 2580
2620	table of squares with sides from 10×60 to $\frac{1}{12} \times 60$	– the century around <i>c.</i> 2580
2580	Khafre	prince <i>c.</i> 2540, son of Khufu
2575	Ur-Nanshe	prince <i>c.</i> 2535
2545	Akurgal	prince <i>c.</i> 2505, son of Ur-Nanshe
2515	Eanatum	prince <i>c.</i> 2475, son of Akurgal
	inscriptions	
2480	Neferirkare	prince <i>c.</i> 2440
2480	Palermo stone	under Neferirkare, prince <i>c.</i> 2440
2380	Urukagina	prince <i>c.</i> 2340
	inscriptions	
2362 - 2281	Sargon	prince <i>c.</i> 2337-281
	inscriptions	
2360	Unas	prince <i>c.</i> 2320
	tomb inscription	
2246	quadrilateral with sides 2, $1\frac{1}{6}$, 2, and $2\frac{5}{6}$	– the century before <i>c.</i> 2156
2246	width of a rectangle with length $67\frac{1}{2}$ and area 100	– the century before <i>c.</i> 2156
2161 - 2103	Ur-Nammu	prince <i>c.</i> 2121-03
	code	
2128 - 2055	Shulgi	prince <i>c.</i> 2103-055, son of Ur-Nammu
2102 - 2046	Amar-Sin	prince <i>c.</i> 2055-46, son of Shulgi
2082 - 2037	Shu-Sin	prince <i>c.</i> 2046-37, son of Amar-Sin
2077	Sumerian prince list	under Ibbi-Sin, prince <i>c.</i> 2037-13
2062 - 2013	Ibbi-Sin	prince <i>c.</i> 2037-13, son of Shu-Sin
2046 - 1976	Amenemhet	prince <i>c.</i> 2006-1976
2016 - 1932	Senwosret	prince <i>c.</i> 1976-32, son of Amenemhet
2016	account of work on a building including dimensions of chambers	– under Senwosret, prince <i>c.</i> 1976-32
1972 - 1897	Amenemhet II	prince <i>c.</i> 1932-897, son of Senwosret
1937 - 1878	Senwosret II	prince <i>c.</i> 1897-78, son of Amenemhet II
1918 - 1845	Senwosret III	prince <i>c.</i> 1878-45, son of Senwosret II
1911	temple register from Lahun	– observation in 1871 $\pm$ 5
1885 - 1800	Amenemhet III	prince <i>c.</i> 1845-00, son of Senwosret III

1885	Kudur-Mabuk	prince <i>c.</i> 1845
1885	book of 87 problems	under Amenemhet III, prince <i>c.</i> 1845-00
1861	Larsa prince list	under Rim-Sin, prince <i>c.</i> 1821-761
1854	Isin prince list	under Damaqilishu, prince <i>c.</i> 1814-791
1846 - 1761	Rim-Sin	prince <i>c.</i> 1821-761, son of Kudur-Mabuk
1839 - 1791	Damaqilishu	prince <i>c.</i> 1814-791
1837 - 1792	Sinmuballit	prince <i>c.</i> 1812-792
1826	Moscow papyrus	the century around <i>c.</i> 1786
1817 - 1749	Hammurabi code	prince <i>c.</i> 1792-49, son of Sinmuballit
1789 - 1711	Samsuiluna	prince <i>c.</i> 1749-11, son of Hammurabi
1784	286 problems	the century before <i>c.</i> 1694
1784	15 Pythagorean triples	the century before <i>c.</i> 1694
1784	calculation of the diagonal of the square	– the century before <i>c.</i> 1694
1784	calculation of the area of the circle	– the century before <i>c.</i> 1694
1751 - 1683	Abieshuh	prince <i>c.</i> 1711-683, son of Samsuiluna
1723 - 1646	Ammiditana	prince <i>c.</i> 1683-46, son of Abieshuh
1686 - 1625	Ammisaduqa	prince <i>c.</i> 1646-25, son of Ammiditana
1686	Venus tablet of Ammisaduqa	– under Ammisaduqa, prince <i>c.</i> 1646-25
1670	Hattusili	prince <i>c.</i> 1630
1665 - 1594	Samsuditana	prince <i>c.</i> 1625-594, son of Ammisaduqa
1665	Babylonian prince list	under Samsuditana, prince <i>c.</i> 1625-594
1634	When on high	the century around <i>c.</i> 1594
1630	Apepi	prince <i>c.</i> 1590
1630	Mursili	prince <i>c.</i> 1610, grandson of Hattusili
1585 - 1521	Amenhotep	prince <i>c.</i> 1545-21
1561 - 1509	Thutmose	prince <i>c.</i> 1521-09
1534 - 1491	Thutmose II	prince <i>c.</i> 1509-491, son of Thutmose
1510	Karnak prince list	under Thutmose III, prince <i>c.</i> 1470-37
1496 - 1437	Thutmose III	child <i>c.</i> 1491 - prince <i>c.</i> 1470-37, son of Thutmose II
1490	map of Nippur	the century before <i>c.</i> 1400
1450 - 1408	Ashur-bel-nisheshu	prince <i>c.</i> 1417-08
1415 - 1363	Eriba-Adad	prince <i>c.</i> 1390-63, son of Ashur-bel-nisheshu
1390 - 1327	Ashur-uballit	prince <i>c.</i> 1363-27, son of Eriba-Adad
1366 - 1302	Ramesses	prince <i>c.</i> 1304-2, father of Seti
1360 - 1317	Enlil-nirari	prince <i>c.</i> 1327-17, son of Ashur-uballit
1346 - 1291	Seti	prince <i>c.</i> 1302-291, son of Ramesses, father of Ramesses II
1342	Abydos prince list	under Seti, prince <i>c.</i> 1302-291
1331	Saqqara prince list	under Ramesses II, prince <i>c.</i> 1291-24
1330 - 1273	Adad-nirari	prince <i>c.</i> 1305-273, son of Enlil-nirari
1316 - 1224	Ramesses II	prince <i>c.</i> 1291-24, son of Seti
1281	Turin papyrus	the century after <i>c.</i> 1291
1160 - 1115	Ashur-resh-ishi	prince 1133-15
1140 - 1076	Tiglath-pileser inscriptions	prince 1115-076, son of Ashur-resh-ishi
1028 - 963	David	prince <i>c.</i> 1003-963
988 - 923	Solomon	prince <i>c.</i> 963-23, son of David
970 - 912	Ashur-dan II	prince 935-12
950 - 891	Adad-nirari II	prince 912-891, son of Ashur-dan II
930 - 884	Tukulti-Ninurta II	prince 891-84, son of Adad-nirari II
921 - 771	Omri	prince <i>c.</i> 881-71
910 - 859	Ashur-nasirpal II	prince 884-59, son of Tukulti-Ninurta II
896 - 853	Ahab	prince <i>c.</i> 871-53, son of Omri
881	Li	deposed in 841

860	Yahwistic document	the century before c. 791/50
	parts of <i>Genesis, Exodus, and Numbers</i>	
810	Amos	prophecy between c. 791 and c. 750
807 - 750	Uzziah	prince at 16 - prince c. 791-50
790	Isaiah	prophecy began c. 750
785 - 727	Tiglath-pileser III	prince 745-27
767	Assyrian prince list	under Shalmaneser V, prince 727-2
766 - 726	Ahaz	prince c. 738-26
765 - 722	Shalmaneser V	prince 727-2, son of Tiglath-pileser III
764	Homer	the century before 676/3
	<i>Iliad</i>	
	<i>Odyssey</i>	
763	Micah	prophecy between c. 750 and c. 697
761	observation of lunar eclipses from 721 to 720	
760 - 705	Sargon II	prince 722-05, younger son of Tiglath-pileser III
746 - 697	Hezekiah	prince 726-697, son of Ahaz
740 - 680	Sennacherib	prince 705-680, son of Sargon II
734	Hesiod	one generation after Homer
	<i>Work and days</i>	
717	<i>When Anu and Enlil</i>	the century before 627
715 - 668	Esarhaddon	prince 680-68, son of Sennacherib
694	Psalms	from the time of the Yahwistic document to that of the Priestly code
690 - 627	Ashurbanipal	prince 668-27, younger son of Esarhaddon
689 - 610	Psamtik	prince 664-10
667	Jeremiah	prophecy began in 627
662	Deuteronomy	the law promulgated in 622
653	Epimenides	purified Athens between c. 632 and 594
	fragments	
649 - 578	Pittacus	died in 578 having lived more than 70 years
648 - 609	Josiah	prince at 8 - prince 640-09
634	Solon	archon 594
633	Ezekiel	prophecy began in 593
625	Thales	predicted an eclipse which occurred in 585
611 - 541	Anaximander	611 - the decade after 546
599	Pherecydes	under Cyrus, prince 559-30
597	Joshua, Judges, Samuel, Kings	– the two generations after 587
595	Proverbs	between <i>Deuteronomy</i> and the Priestly code
590	Anaximenes	between Anaximander and Xenophanes
584 - 530	Cyrus	prince 559-30
578 - 522	Polycrates	tyrant c. 538-22
570 - 495	Pythagoras	c. 570 - the decade after c. 500
570 - 473	Xenophanes	c. 570 - the decade after 478
	fragments	
562	Hecataeus	under Darius, prince 522-486
560	Haggai	prophecy began in 520
555 - 522	Cambyses	prince 530-22, son of Cyrus
551 - 479	Confucius	c. 551 - c. 479
	<i>Analects</i>	
550 - 486	Darius	c. 550 - 486
	Behistun inscription	
540	Heraclitus	one generation after Pythagoras
	fragments	
532	Alcmaeon	between Pythagoras and Empedocles
529	Priestly code	the century before 445/33

	parts of <i>Genesis, Exodus, and Numbers</i>	
	<i>Leviticus</i>	
528	<i>Job</i>	the century after 538
520 - 465	Xerxes	c. 520 - 465
519	<i>Great learning, The mean</i>	the century around c. 479
519	Hippasus	between Pythagoras and Philolaus
515 - 445	Parmenides	c. 515 - the decade after c. 450
	fragments	
500 - 428	Anaxagoras	c. 500 - c. 428
495 - 435	Empedocles	c. 495 - c. 435
495 - 429	Pericles	c. 495 - 429
490 - 420	Protagoras	c. 490 - c. 420
490	Oenopides	a little younger than Anaxagoras
490	Zeno of Elea	born c. 490
485 - 380	Gorgias	c. 485 - c. 380
484 - 425	Herodotus	484 - the decade after 430
	<i>Histories</i>	
483 - 424	Artaxerxes	
481	Melissus	general in 441
480 - 402	Hellanicus	c. 480 - the decade after 407
480	Leucippus	between Anaxagoras and Democritus
479	Ezra	great assembly in 445/33
472	Meton	observation in 432
470 - 394	Hippias	younger than Protagoras - the decade after 399
469 - 399	Socrates	
469	Hippocrates	contemporary with Socrates
	Hippocratic corpus, including	
	<i>Epidemics I</i>	
	<i>Epidemics III</i>	
	<i>Precepts</i>	
469	Philolaus	contemporary with Socrates
467	Thrasymachus	parodied in 427
464 - 399	Thucydides	general in 424 - the decade after 404
460 - 394	Theodorus of Cyrene	one generation after Oenopides - the decade after 399
460	Democritus	40 years younger than Anaxagoras
460	Hippocrates of Chios	one generation after Oenopides
453 - 405	Darius II	prince 424-05, son of Artaxerxes
453 - 403	Critias	c. 453 - 403
	fragments	
449	Cratylus	younger than Socrates
448	Hicetas	between Philolaus and Plato
447 - 364	Euclid of Megara	older than Plato - the decade after 369
445	Ctesias	under Artaxerxes II, prince 405-359
443 - 365	Camillus	tr. mil. 403 - 365
436 - 338	Isocrates	
430 - 359	Artaxerxes II	prince 405-359, son of Darius II
430 - 350	Xenophon	c. 430 - the decade after 355
	<i>Cavalry commander</i>	
429	<i>Chronicles, Ezra, Nehemiah</i>	– the century after 445/33
427 - 347	Plato	
	<i>Apology</i>	
	<i>Euthyphro</i>	
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Phaedo
Symposium
Phaedrus
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Cratylus
Parmenides
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- 427 **Archytas** contemporary with Plato
- 414 - 369 **Theaetetus** c. 414 - 369
- 410 - 362 **Epaminondas** c. 410 - 362
- 408 - 356 **Eudoxus** c. 408 - c. 356
- 407 Aeneas Tacticus general in 367
- On the defense of fortified positions*
- 405 - 335 **Ephorus** c. 405 - the decade after 340
- 396 - 324 Anaximenes of Lampsacus under Philip, prince 356-36 - the decade after 329
- 395 - 317 **Ponticus** contemporary with Xenocrates - the decade after 322
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- 395 - 314 Xenocrates
- 391 Metrodorus of Chios between one and two generations after Isocrates
- 384 - 322 **Aristotle**
- book I of the *Metaphysics*
- book XII of the *Metaphysics*
- books XIII-XIV of the *Metaphysics*
- Physics*
- On the heaven*
- On generation and corruption*
- Meteorology*
- On the soul*
- History of animals*
- Parts of animals*
- Movement of animals*
- Progression of animals*
- Generation of animals*
- On the senses and the sensible*
- On memory and recollection*
- On sleep and waking*
- On dreams*
- On divination in sleep*
- On length and brevity of life*
- On youth, old age, life and death, and respiration*
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- Posterior analytics*
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Poetics
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382 - 336 **Philip**

378 **Menaechmus**

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375 Anaxarchus

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371 - 289 **Mencius**

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Mencius

371 - 287 **Theophrastus**

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On stones

371 **Aristoxenus**

contemporary with Theophrastus

Elements of harmonics

371 **Dicaearchus**

contemporary with Theophrastus

371 **Eudemus**

contemporary with Theophrastus

371 **Pytheas**

contemporary with Dicaearchus

370 - 327 Callisthenes

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366 - 282 **Ptolemy**

360 - 270 **Pyrrho**

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356 - 323 **Alexander**

354 Callippus

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350 - 259 Timaeus

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346 **Autolycus**

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346 **Euclid**

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346 **Herophilus**

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346	Manetho	under Ptolemy
341 - 270	Epicurus	
335 - 263	Zeno of Citium	
335	Timocharis	observations from 295 to 272
331 - 232	Cleanthes	
327 - 269	Strato	scholarch 287-69
322	Zenodotus	under Ptolemy II, prince 282-46
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320	Aristarchus	observation in 280
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316 - 242	Arcesilaus	
316	Ariston of Chios	contemporary with Arcesilaus
316	Erasistratus	one generation after Herophilus
312	Dionysius	observations from 272 to 241
309	<i>Ecclesiastes</i>	between <i>Chronicles</i> , <i>Ezra</i> , <i>Nehemiah</i> and <i>Daniel</i>
308 - 246	Ptolemy II	
305 - 215	Hiero II	c. 305 - 215
287 - 212	Archimedes	c. 287 - 212
	<i>On the sphere and cylinder</i>	
	<i>Measurement of the circle</i>	
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274 - 194	Eratosthenes	276/3 - 196/3
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257	Diocles	one generation after Archimedes
247 - 183	Hannibal	
242 - 187	Antiochus III	c. 242 - 187
240 - 152	Diogenes the Babylonian	c. 240 - c. 152
240 - 151	Critolaus	contemporary with Diogenes the Babylonian - the decade after 156
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233	Apollonius	born 246/21
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216 - 144	Aristarchus of Samothrace	c. 216 - c. 144
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189	Daniel	the generation after 164
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145 - 86	Sima Qian	c. 145 - the decade after 91
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140 - 82	Mucius	c. 140 - 82
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- 106 - 43 Servius
- 100 - 44 **Caesar**
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- 89 - 16 **Siculus** under Caesar - the decade after 21
- 83 - 43 Hirtius cos. 43 - 43
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- 79 - Alfenus cos. 39
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- 65 - 29^{AD} **Strabo** 68/3 - the decade after 24^{AD}
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- 63 - 14 **Augustus**
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- 59 - 17 **Livy**
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- 58 - 21 **Labeo** pr. 18^{BC} - c. 21
- 52 - Aenesidemus one generation before Philo Judaeus
- 42 - 37 **Tiberius**
- 24 - Cleomedes between Posidonius and Ptolemy
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- 22^{BC} - Philo Judaeus old in 39^{AD}
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- 22 - **Hero** observation in 62
- 24 - 79 **Pliny**
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- 25 - *Gospel of Mark* the decade before 70
- 37 - 98 Josephus 37 - the decade after 93
- 43 - 102 **Clement of Rome** bishop 93-102
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- 43 - *Gospel of Matthew* between 75 and 90
- 48 - 125 **Plutarch** 45/50 - the decade after 120
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50 - 135	Akiva	c. 50 - 135
50	Gospel of Luke, Acts of the apostles	– between 80 and 100
50	mishna	from two generation before Akiva through two after
53 - 117	Trajan	
55 - 135	Epictetus	50/60 - 130/40
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55	Gospel of John, Apocalypse	– between 90 and 100
57	Marinus	one generation before Ptolemy
58	Menelaus	observations in 98
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87	Ptolemy	observations from 127 to 141
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87	Theon of Smyrna	observations from 127 to 132
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89	Aëtius	between Plutarch and Galen
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102 - 173	Julian	c. 102 - the decade after 168
121 - 180	Marcus	
121 - 195	Irenaeus	under Marcus, prince 161-80 - the decade after 190
127	Gellius	born 125/8
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129 - 208	Galen	129 - 199/216
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135	ha-Nasi	born 135
146 - 211	Severus	
146 - 212	Papinian	contemporary with Severus - 212
150 - 214	Clement of Alexandria	c. 150 - 211/6
159	Empiricus	one generation after Galen
	<i>Outlines of Pyrrhonism</i>	
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160 - 240	Tertullian	c. 160 - c. 240
164	Alexander of Aphrodisias	wrote between 198 and 209
170 - 235	Hippolytus	c. 170 - 235
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176 - 223	Ulpian	contemporary with Paul - 223
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176	Paul	one generation after Papinian
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200 - 258	Cyprian	c. 200 - 258
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205 - 270	Plotinus	
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219	Laertius	two generations after Empiricus
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234 - 308	Porphyry	234 - 305/10
	<i>On the life of Plotinus and the arrangement of his works</i>	
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240 - 325	Iamblichus	c. 240 - c. 325
	<i>On the common mathematical knowledge</i>	
	<i>On the life of Pythagoras</i>	
244 - 313	Diocletian	
250 - 325	Lactantius	c. 250 - c. 325
255 - 336	Arius	250/60 - 336
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275 - 337	Constantine	c. 275 - 337
280	Pappus	observation in 320
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306	talmud	from two generations after ha-Nasi through two after Ashi
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330 - 379	Basil of Caesarea	c. 330 - 379
330 - 390	Gregory of Nazianzus	c. 330 - 390
335 - 394	Gregory of Nyssa	c. 335 - c. 394
339 - 397	Ambrose	c. 339 - 397
340 - 420	Jerome	331/48 - 420
347 - 395	Theodosius	
353 - 423	Pelagius	350/5 - the decade after 418
354 - 430	Augustine	
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	<i>Confessions</i>	
	<i>On the trinity</i>	
	<i>On the literal reading of Genesis</i>	
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	<i>The city of god against the pagans</i>	
357 - 427	Ashi	died in 427
380 - 461	Leo	pope 440-61
380	Capella	wrote between 410 and 429
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394	Vincent of Lérins	wrote in 434
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398	Theodosian code	promulgated in 438
401 - 450	Theodosius II	
411 - 485	Proclus	410/2 - 485
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431 - 518	Anastasius	c. 431 - 518
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433 - 493	Odoacer	c. 433 - 493
443	Dionysius the Areopagite <i>Celestial hierarchy</i>	the century before 533
480 - 524	Boethius	c. 480 - 524
480 - 543	Tribonian <i>Digest</i>	c. 480 - c. 543
483 - 565	Justinian	
488 - 585	Cassiodorus	485/90 - the decade after 580
489	Simplicius commentary on <i>On the heaven</i>	left Athens in 529
489	Eutocius	contemporary with Simplicius
489	Zosimus <i>New history</i>	two generations after 408/50
540 - 604	Gregory <i>Dialogues</i>	c. 540 - 604
570 - 632	Muhammad koran	c. 570 - 632
581 - 656	Uthman	conversion c. 611 - 656
587 - 644	Umar	581/92 - 644
601 - 680	Muawiyah	595/607 - 680
671	Tariq ibn Ziyad	general in 711
673 - 735	Bede	
711 - 775	al-Mansur	709/13 - 775
718 - 775	Constantine V	
735 - 816	Leo III	pope 795-816
742 - 814	Charlemagne	
767 - 820	al-Shafii <i>Treatise on the sources of the law</i>	
778 - 840	Louis the Pious	
786 - 833	al-Mamun	
795 - 852	al-Khwarizmi <i>Algebra</i>	790/800 - the decade after 847
810 - 870	al-Bukhari	
831 - 901	Thabit ibn Qurra	826/36 - 901
860 - 930	al-Razi <i>Book of secrets</i>	died in 925/35
861 - 929	al-Battani <i>Astronomical treatise</i>	one generation after Thabit ibn Qurra - 929
865 - 950	al-Farabi <i>The views of the inhabitants of the ideal city</i>	860/70 - c. 950
876 - 936	Henry	c. 876 - 936
905 - 959	Constantine VII	
912 - 973	Otto	
935	<i>Suda</i>	one generation after Constantine VII
955 - 983	Otto II	
958 - 1025	Basil II	
965 - 1040	ibn al-Haytham <i>Optics</i>	965 - c. 1040
973 - 1020	Leif Eriksson	970/5 - c. 1020
973 - 1024	Henry II	
973 - 1055	al-Biruni <i>The monuments of past generations</i>	973 - the decade after 1050
980 - 1002	Otto III	
980 - 1037	ibn Sina <i>Canon</i>	

12*	enumeration and review
990 - 1039 Conrad II	c. 990 - 1039
1017 - 1056 Henry III	
1020 - 1085 Gregory VII	c. 1020 - 1085
<i>The dictate of the pope</i>	
1033 - 1109 Anselm	
<i>Monologue</i>	
<i>Prologue</i>	
1035 - 1099 Urban II	c. 1035 - 1099
1048 - 1123 Roscelin	1045/50 - 1120/5
1048 - 1127 Khayyam	1048 - 1123/31
<i>Demonstration of problems of algebra</i>	
1050 - 1106 Henry IV	
1058 - 1111 al-Ghazali	
<i>The incoherence of the philosophers</i>	
<i>The revival of the religious sciences</i>	
<i>Deliverance from error and union with the almighty</i>	
1059 - 1128 Fulcher	c. 1059 - 1127/9
<i>The deeds of the Franks on their pilgrimage to Jerusalem</i>	
1079 - 1142 Abelard	
<i>Yes and no</i>	
<i>Theology</i>	
<i>History of calamities</i>	
1086 - 1125 Henry V	
1090 - 1153 Clairvaux	
1093 - 1152 Conrad III	
1096 - 1141 Hugh of St Victor	c. 1096 - 1141
<i>Instruction, on the study of reading</i>	
1100 - 1160 Lombard	c. 1100 - 1160
1113 - 1158 Otto of Freising	1111/5 - 1158
<i>The history of the two cities</i>	
1114 - 1187 Gerard	c. 1114 - 1187
1123 - 1190 Frederick	c. 1123 - 1190
1126 - 1198 ibn Rushd	
<i>The incoherence of the incoherence</i>	
1133 - 1202 Joachim of Fiore	1130/5 - 1202
1135 - 1204 Maimonides	
<i>The guide of the perplexed</i>	
1138 - 1193 Salah al-Din	
1158 - 1220 David of Dinant	under Innocent III, pope 1198-216 - the decade after 1215
1161 - 1216 Innocent III	
letters to Acerbus	
1161 - 1227 Genghis Khan	1155/67 - 1227
1165 - 1197 Henry VI	
1165 - 1223 Philip II	
1168 - 1241 Gregory IX	1165/70 - 1241
<i>Mother of the sciences, 1231</i>	
letters to William of Auxerre	
1169 - 1253 Grosseteste	1168/70 - 1253
<i>On the motion of bodies and light</i>	
1170 - 1245 Fibonacci	c. 1170 - the decade after 1240
<i>Book of calculation</i>	
<i>Book of squares</i>	
1187 - 1241 Ögedei	1185/9 - 1241
1194 - 1250 Frederick II	
constitutions of Melfi, 1231	

- On the art of hunting with birds*
 1200 - 1280 **Albert** 1193/207 - 1280
On minerals
On plants
On animals
- 1206 - 1255 Batu 1205/7 - 1255
 1209 - 1259 Möngke
 1214 - 1270 **Louis IX**
 1215 - 1286 William of Moerbeke c. 1215 - c. 1286
 1215 - 1294 **Kublai**
 1217 - 1265 Hülegü c. 1217 - 1265
 1217 - 1274 Bonaventure c. 1217 - 1274
Discourses on the six days
- 1217 - 1292 **Roger Bacon** 1214/20 - 1292
The great work
- 1225 - 1274 **Aquinas** 1224/6 - 1274
Theological summa
- 1233 Witelo born 1230/5
Perspective
- 1239 - 1307 **Edward**
 1254 - 1324 **Polo** c. 1254 - 1324
Description of the world
- 1260 - 1328 **Eckhart** c. 1260 - 1328
 1266 - 1308 **Scotus** c. 1266 - 1308
 1280 - 1342 Marsilius c. 1280 - c. 1342
 1281 - 1360 **Orhan**
 1285 - 1349 **Occam** 1280/90 - 1349
Summa of all logic
Seven quodlibets
- 1286 - 1331 Odoric c. 1286 - 1331
Description of the eastern part of the world
- 1295 - 1349 Bradwardine 1290/300 - 1349
 1295 - 1358 **Buridan** c. 1295 - c. 1358
Questions on the Physics
Questions on On the heaven
- 1295 Heytesbury wrote in 1335
 1304 - 1373 **ibn Battuta** 1304 - the decade after 1368
Travels
- 1312 - 1377 **Edward III**
 1320 - 1382 **Oresme** c. 1320 - 1382
Questions on the geometry of Euclid
Treatise on the configuration of qualities and motions
The book on the heaven and the world
- 1332 - 1406 **ibn Khaldun**
Introduction to history
- 1357 - 1433 **John**
 1370 - 1444 **Bruni**
The history of the Florentine people
- 1394 - 1460 **prince Henry**
 1395 - 1450 Eanes c. 1395 - the decade after 1445
 1403 - 1461 **Charles VII**
 1407 - 1457 **Valla**
 1420 - 1482 **Benincasa**
 atlas, 1462
 1423 - 1483 **Louis XI**

- 1432 - 1481 **Mehmed II**
 1432 - 1488 Ca' da Mosto
 1433 - 1499 **Ficino**
 1436 - 1476 **Regiomontanus**
 On triangles of all kinds, 1463
 1440 - 1505 **Ivan III**
 1450 - 1486 **Cão** c. 1450 - 1486
 1450 - 1498 Cabot c. 1450 - 1498
 1450 - 1500 **Dias** c. 1450 - 1500
 1451 - 1504 **Isabella**
 1451 - 1506 **Columbus**
 report, 15 Feb 1493
 letter to Isabella and Ferdinand V, 7 Jul 1503
 1452 - 1516 **Ferdinand V**
 1453 - 1515 **Albuquerque**
 1454 - 1512 Vespucci
 letter to Lorenzo il Popolano, Mar 1503
 1457 - 1509 **Henry VII**
 1460 - 1510 **de la Cosa**
 map, 1500
 1460 - 1524 **da Gama** c. 1460 - 1524
 1468 - 1520 Cabral 1468 - c. 1520
 1468 - 1526 **de Torenó** drew maps in 1508 - 1526
 map, 1526
 1468 - 1536 **Erasmus** 1466/9 - 1536
 Encomium of custom, or the praise of folly, 1511
 1469 - 1527 **Machiavelli**
 The prince, 1513
 Discourses on the first ten books of Livy, 1517
 1470 - 1520 **Waldseemüller** c. 1470 - 1518/21
 map, 1507
 Introduction to cosmography, with certain principles of geometry and astronomy necessary for the matter, 1507
 1473 - 1543 **Copernicus**
 Little treatise on the hypotheses concerning the heavenly motions, 1514
 Letter against Werner, 1524
 On the revolutions of the heavenly spheres, 1543
 1475 - 1541 **Pizarro** c. 1475 - 1541
 1476 - 1526 **El Cano** c. 1476 - 1526
 1478 - 1535 More
 A truly golden little book, no less beneficial than entertaining, on the best state of the republic and the new island of Utopia, 1516
 1480 - 1521 **Magellan** c. 1480 - 1521
 1483 - 1540 **Guicciardini**
 The history of Italy, 1540
 1483 - 1546 **Luther**
 commentary on the *Letter to the Galatians*, 1531
 table talk 1531-46
 1485 - 1547 **Cortés**
 1489 Pigafetta sailed with Magellan in 1519
 The voyage around the world, 1526
 1491 - 1547 **Henry VIII**
 1493 - 1541 **Paracelsus**
 1494 - 1555 **Agricola**

On the origin and causes of subterranean things; on the nature of things flowing out of the earth; on the nature of fossils; on the old and new metals, 1546

On metals, in which the uses, instruments, machines, and all other things concerning metals are observed, and described not only in the clearest way, but with images in the appropriate places, 1556

1495 - 1566 **Suleiman**

1499 - 1557 Tartaglia

1500 - 1558 **Charles V**

1501 - 1576 **Cardano**

The great art, or the rules of algebra, 1545

The great art of arithmetic, 1550

1503 - 1566 Nostradamus

1514 - 1564 **Vesalius**

On the structure of the human body, 1543

1517 - 1590 Zarlino

1527 - 1598 Philip II

1530 - 1596 **Bodin**

1533 - 1584 William

1533 - 1603 **Elizabeth**

1540 - 1603 **Viète**

Mathematical canon, or on the triangle, 1579

Introduction in the analytic art, 1591

Preliminary notes on logistic by species, 1593

Two treatises on the examination and emendation of equations, 1593

Supplement to geometry, 1593

Responses to various mathematical matters, 1593

Celestial harmony, 1600

1540 - 1609 Scaliger

1544 - 1603 **Gilbert**

On the magnet, magnetic bodies, and the great magnet of the earth, new physiology demonstrated by many arguments and experiments, 1600

New philosophy of our sublunar world, 1603

1546 - 1601 **Tycho**

On the new and never in the memory of any age seen star, first observed in the month of November of the year of Christ 1572, a mathematical survey, 1573

On the recent phenomena in the aethereal world, 1588

Introductory exercises for the instauration of astronomy, 1602

1548 - 1600 **Bruno**

The Ash Wednesday supper, described in five dialogues, by four interlocutors, with three reflections, on two subjects, 1584

On infinity, the universe, and the worlds, 1584

1548 - 1620 **Stevin**

On the decimal, teaching an unheard-of light, how all human calculation in need of fractions can be done with whole numbers, 1585

The principles of the art of weighs, 1586

The course of the heaven, 1608

1550 - 1617 **Napier**

1552 - 1610 Ricci

On the Christian expedition among the Chinese undertaken by the Society of Jesus, from the commentaries of father Ricci, in which the customs, laws, and institutions of the kingdom of China, as well as the most difficult beginnings of the new church there, are described accurately and with the greatest fidelity, 1615

1561 - 1626 **Bacon**

An advertisement touching controversies of the church of England, 1589

Essays or councils, civil and moral, 1597

Religious meditations, 1597
On the colors of good and evil, 1597
A declaration of the practices and treasons attempted and committed by Robert, late earl of Essex, and his accomplices, 1601
A brief discourse touching the happy union of the kingdoms of England and Scotland, 1603
Certain considerations touching the better pacification and edification of the church of England, 1604
Apology in certain imputations concerning the late earl of Essex, 1604
On the dignity and growth of the sciences, 1605
On the wisdom of the ancients, 1609
The charge touching duels, 1614
Second part of the great instauration, new organon, or directions for the interpretation of nature, 1620
The history of the reign of king Henry VII, 1622
Natural and experimental history to ground philosophy, or phenomena of the universe, which is the third part of the instauration of the sciences, part one, history of winds, 1622
History of life and death, 1623
Apophthegms new and old, 1625
New Atlantis, 1627
Forest of forests, or a natural history, 1627
Maxims of the law, 1630

1564 - 1642 **Galileo**

The little balance, 1586
On motion, 1592
Treatise on fortifications, 1593
On mechanics, 1600
Dialogue on the new star, 1605
Treatise on the sphere, or cosmography, 1606
The operations of the geometric and military compass, 1606
Defense against the calumnies and impostures of Baldesar Capra in his Astronomical consideration on the new star of 1604, as well as, and much more, in publishing as his invention the construction and uses of the geometric and military compass under the title Usus et fabrica circini proportionis, 1607
Starry messenger, unfolding great and most wonderful sights and displaying for the inspection of everyone, and especially of the philosophers and astronomers, what with a telescope built by himself Galileo Galilei observed about the face of the moon, the fixed stars, the Milky Way, the nebulous stars, and first of all about four planets circling Jupiter's star at unequal intervals and periods and with marvelous speed, which, unknown to anyone until now, he discovered and decided to name the Medicean stars, 1610
Discourse on bodies floating on water or moving in it, 1612
History and demonstrations concerning sunspots and their appearances, contained in three letters to Mark Welser, 1613
Letter to Mme Christina of Lorraine, grand duchess of Tuscany, 1615
Discourse on the ebbs and flows of the sea, 1616
Discourse on the comets, 1618
The assayer, in which on a delicate and precise scale are weighted the matters contained in The astronomical and philosophical balance of Lothario Sarsi, 1623
Letter to Francesco Ingoli in response to the Disputations on the location and rest of the earth, 1624
Report on flood control, 1630
Dialogue on the two main systems of the world, Ptolemaic and Copernican, 1632
Astronomical operations, 1637
Discourses and mathematical demonstrations in two new sciences dealing with mechanics and locomotion, 1638
Letter to prince Leopold of Tuscany, concerning chapter fifty of the Litheosphorus of Fortunio Liceti, 1641

On Euclid's definitions of proportion, 1642
 letters to Giuliano de' Medici; letters to Liceti

1567 - 1625 **Maurice**

1571 - 1630 **Kepler**

Harbinger of cosmographic treatises, containing the cosmographic mystery, on the marvelous proportion of the heavenly spheres, and on the true and proper causes of the number, magnitude, and periods of the heavens, demonstrated by the five regular geometric bodies, 1596

On the more certain foundations of astrology, new brief treatise toward a theory of the world, together with a physical forecast for the year from the birth of Christ 1602, 1602

Additions to Witelo, in which the optical part of astronomy is treated, and first of all artificial observation and estimation of the diameters and eclipses of the sun and the moon, with examples of important eclipses, 1604

Forecast for the year from the gracious birth of our lord and savior Jesus Christ 1605, together with a thorough report on the appearance of an unusual new star, 1604

On the new star in the foot of Serpent-bearer, book full of disquisitions astronomical, physical, metaphysical, meteorological, and astrological, ἐνδόξιος and παρὰδόξιος, 1606

On the star of third magnitude in Swan which was unknown until the year 1605 and has not yet been extinguished, 1606

On the true year of the birth of our savior Jesus Christ, examination of the recent proposal of Suslyga that the customary epoch is four years short, 1606

Exhaustive report on the new bearded star, or comet, which appeared in September and October of 1607, and what it means; together with an entirely new and peculiar yet well grounded discourse on what comets actually are, whence they come, by what their motion is governed, and with which of their shapes they have something to indicate for mankind, 1608

New astronomy ἀπαστολογητός, or celestial physics derived from commentaries on the motions of the star of Mars, from the observations of Tycho, 1609

A singular phenomenon, or Mercury in the sun, 1609

Answer to Dr Röslin's Discourse on the constitution of the present, and how it develops into the future, 1609

Conversation with the Starry messenger sent recently to mankind by Galileo, 1610

Tertius interveniens, or warning to some theologians, physicians, and philosophers, and especially Dr Feseli, that by cheap condemnation of the stargazer's superstition they may throw out the child with the bath-water and thus act inadvertently against their profession, 1610

Dioptrics, or demonstration of the consequences for seeing and for what is seen of the newly discovered telescope, 1611

A new year's gift, or on the hexagonal snowflake, 1611

Report on observations of the four wandering satellites of Jupiter, which Galileo with the right of discoverer named the Medicean stars, 1611

Repeated more exhaustive report that our lord and savior Jesus Christ was born not just one year before the beginning of our present time-reckoning as Röslin, Bünting, and others hold, nor just two years as Scaliger and Calvisius together with many old church writers hold, but full five years; the time of the birth of Christ determined from correct harmony and comparison of heathen and Jewish histories, as well as from agreement with the course of the heaven, 1613

On the true year in which the eternal son of god assumed human nature in the womb of the blessed virgin Mary, 1614

New stereometry of wine casks, especially those most in shape like the Austrian, and the best use in them of a cubical gauging rod, with a supplement to Archimedean stereometry, 1615

Harmonics of the world, in five books, I. geometrical, II. architectonic, III. harmonic, IV. metaphysical, psychological, and astrological, V. astronomical and metaphysical, 1619

Epitome of Copernican astronomy, written in the form of questions and answers, 1618-21

On comets, in three books, I. astronomical, II. physical, III. astrological, 1619

New ephemerides of the heavenly motions beginning with the year of the common era 1617, from observations mainly of Tycho, physical hypotheses, and the Rudolphine tables, 1619-30

Kanones pueriles, or chronology from Adam to this current year of Christ 1620, divided in six parts and confirmed from the holy scripture of the Old and New testaments, with support from heathen

antiquities and astronomical calculation, 1620

Astronomical report on two great and peculiar lunar eclipses seen in the past year 1620, and on the great solar eclipse which will occur in the current year 1621 on 11/21 May, with a brief recapitulation of all great solar eclipses seen in Europe in eighty years and what in each case the state of spiritual and secular affairs before and after has been, 1621

Defense of my Harmonics of the world against the Analytical demonstrations of Dr Fludd, 1622

Discourse on the great conjunction, or meeting, of Saturn and Jupiter in the fiery sign of Lion which occurs in July of the year 1623, as also on the many and multifarious prognostications about this year, 1623

One thousand logarithms of as many round numbers; legitimate premise of the demonstration of their origin and use; a new arithmetic or compendium, of which nothing is more admirable or useful for the solution of nearly all problems of calculation, especially those in the study of triangles, without multiplication, 1624

Supplement to the One thousand logarithms, containing precepts on their use, 1625

Shield bearer for the Dane Tycho against Chiaramonti's Antitycho, 1625

Rudolphine tables, with which the restoration of astronomical science, long collapsed, is continued, first conceived by that phoenix of astronomers Tycho in 1564, from most accurate observations of the stars especially after 1572, 1627

On the rare and marvelous phenomena of the year 1631, incursions of Venus and Mercury in the sun, an admonition to astronomers and students of the things in the heaven, 1629

On the letter of Dr Bartsch about the computation and edition of ephemerides, 1629

Dream, or on lunar astronomy, 1634

1575 - 1624 **Boehme**

1578 - 1657 **Harvey**

1579 - 1644 **van Helmont**

Rise of medicine, or beginnings of a new physics; new progress in medicine, for the destruction of decease, for long life, 1648

1580 - 1626 **Snel**

1583 - 1645 **Grotius**

1588 - 1648 **Mersenne**

Physico-mathematical reflections, in which the nature and art of admirable effects are explained by most certain demonstrations, 1644

1588 - 1679 **Hobbes**

Leviathan, or the matter, form, and power of the commonwealth ecclesiastical and civil, 1651

Six lessons to the professors of mathematics, one of geometry, the other of astronomy, in the university of Oxford, 1656

1591 - 1661 **Desargues**

1592 - 1655 **Gassendi**

Mercury seen in the sun, Venus not seen, 1632

1594 - 1632 **Gustav II**

1596 - 1650 **Descartes**

A compendium of music, 1618

Treatise on clear and useful rules for the direction of the mind in the search for truth, 1628

The world, or a treatise on light and the other principal objects of sense, 1633

Discourse on the method to conduct one's reason well and search for truth in the sciences, as well as the Optics, Meteorology, and Geometry, which are essays in this method, 1637

Meditations on the first philosophy, in which the existence of god and the distinction between man's soul and body are demonstrated, 1641

Replies to objections raised by some men of learning against the Meditations, 1641

The principles of philosophy, 1644

The description of the human body and all its functions, those that do not depend on the soul and those that depend on it, as well as the principal cause of the formation of its members, 1648

Comments on a broadsheet published in Belgium at the end of 1647, entitled An account of the human mind, or rational soul, which explains what it is and what it can be, 1648

The passions of the soul, 1649

The search for truth by the natural light, 1650
 letters to Mersenne; letters to Morin

1598 - 1647 **Cavalieri**

Geometry of indivisibles of continua developed by a new method, 1635
Six exercises in geometry, 1647

1599 - 1658 **Cromwell**

1601 - 1665 **Fermat**

Method for the investigation of maxima and minima, 1638
On the transformation and simplification of equations of location for the comparison of all forms of areas under curves, and on the use of geometrical progression for the quadrature of parabolas and hyperbolas to infinity, 1659

1602 - 1675 Roberval

1602 - 1686 **Guericke**

1608 - 1647 **Torricelli**

1608 - 1679 Borelli

1611 - 1675 Turenne

1611 - 1687 Hevelius

1612 - 1694 Arnauld

1616 - 1703 **Wallis**

Arithmetic of infinities, or a new method of investigation of the quadrature of curves and other difficult mathematical problems, 1656
Treatise of algebra, both historical and practical, showing its origin, progress, and advancement, from time to time, and by what steps it has attained to the height at which it now is, 1685

1618 - 1663 **Grimaldi**

1619 - 1683 Colbert

1620 - 1682 **Picard**

1621 - 1686 Condé

1623 - 1662 **Pascal**

1625 - 1712 **Cassini**

Astronomical observations made in different regions of the kingdom during the year 1672, 1673
The elements of astronomy, verified by Mr Cassini from the relation of his tables to the observations made by Mr Richer on the island of Cayenne, 1684

1626 - 1697 **Redi**

1627 - 1691 **Boyle**

New physico-mechanical experiments, touching on the spring of air, and its effects, made for the most part in a new pneumatic engine, 1660
The sceptical chemist, or chemico-physical doubts and paradoxes, touching the experiments whereby vulgar spagyrist are wont to endeavor to evince their salts, sulfur, and mercury, to be the true principles of things, 1661
The origin of forms and qualities according to the corpuscular philosophy, illustrated by considerations and experiments, 1666

1627 - 1704 Bossuet

1627 - 1705 Ray

1628 - 1694 **Malpighi**

Anatomical observations on the lungs, 1661

1629 - 1695 **Huygens**

Theorems on the quadrature of the hyperbola, ellipse, and circle when the center of gravity of segments is given, to which is appended 'Εξέτασις of the measurement of the circle of Gregory of St Vincent, 1651
On the magnitude of the circle, with an appendix on the constructions of some famous problems, 1654
On the motion of colliding bodies, 1656
Treatise on calculation in games of chance, 1656
New observation of a moon of Saturn, 1656
The clock, 1658

The system of Saturn, or on the causes of Saturn's marvelous phenomena and its satellite, a new planet, 1659

Short instruction on the use of the clock to find longitude, 1665

Rules for the motion of bodies in collision, 1669

The pendulum clock, or geometrical demonstrations concerning the motion of pendula applied to clocks, 1673

Treatise on light, in which are explained the causes of what occurs in reflection, in refraction, and particularly in the strange refraction of Iceland crystal, 1678

Compendium on star observation freed from difficulties by optical tubes, 1684

Memoirs on the grinding of lenses for telescopes, 1685

Discourse on the cause of gravity, 1690

Κοσμοθεωρός, or conjectures on the countries and inhabitants of the heavenly bodies, 1698
letters to the *Journal of scientists*

1630 - 1677 **Barrow**

1630 - 1696 **Richer**

1632 - 1677 **Spinoza**

Treatise on the correction of the understanding, and on the way in which it is best directed toward true knowledge of things, 1662

Parts I and II of Descartes's Principles of philosophy demonstrated in the geometrical manner, 1663

Metaphysical thoughts, in which difficulties arising in the general as well as special parts of metaphysics are explained, 1663

Theologico-political treatise, 1670

Ethics demonstrated in geometrical order, 1677

Political treatise, in which it is demonstrated how the society in which monarchy rules, as well as that in which aristocracy rules, needs to be ordered so as not to lapse into tyranny and to preserve inviolate the peace and freedom of the citizens, 1677

Compendium of the grammar of the Hebrew language, 1677

letters to Oldenburg

1632 - 1694 **Pufendorf**

1632 - 1704 **Locke**

Two treatises of government, in the former the false principles and foundation of Sir Robert Filmer and his followers are detected and overthrown, the latter is an essay concerning the true origin, extent, and end of civil government, 1690

An essay concerning human understanding, 1690

1632 - 1707 **Mabillon**

On diplomatic matters, in which whatever pertains to the materials, writing, and style of old documents, whatever pertains to seals, monograms, subscripts, and chronological notes, whatever pertains to antiquities and forensic historical disciplines, is explained and illustrated, 1681

1632 - 1723 **Leeuwenhoek**

1633 - 1707 **Vauban**

1635 - 1702 **Hooke**

1635 - 1715 **Burnet**

1637 - 1680 **Swammerdam**

1638 - 1675 **James Gregory**

1638 - 1686 **Steno**

Harbinger of a dissertation on solids enclosed by nature in solids, 1669

1638 - 1715 **Louis XIV**

1638 - 1715 **Malebranche**

1639 - 1691 **Louvois**

1642 - 1727 **Newton**

Enumeration of lines of the third order, 1668

On analysis by equations of infinite number of terms, 1669

A treatise on the method of fluxions and infinite series, with its application to the geometry of

curves, 1671
New theory of light and colors, 1672
An account of a new catadioptrical telescope, 1672
A series of queries propounded by Mr Newton, to be determined by experiments, positively and directly concluding his new theory of light and colors, and here recommended to the industry of the lovers of experimental philosophy, 1672
A treatise on the quadrature of curves, 1676
A treatise on the system of the world, 1685
The mathematical principles of natural philosophy, 1687
A historical account of two notable corruptions of scripture, 1691
On the nature of acids, 1692
The origin of monarchies, 1693
A scale of the degrees of heat; description and signs of heat, 1701
Optics, or a treatise on the reflection, refraction, inflection, and colors of light, 1704
Universal arithmetic, or a treatise on arithmetical composition and resolution, 1707
The chronology of ancient kingdoms amended, to which is prefixed a short chronicle from the first memory of things in Europe to the conquest of Persia by Alexander, 1728
Observations on the prophecies of Daniel and the Apocalypse of st John, 1733
 letters to Hooke; letters to Bentley

1644 - 1710 **Römer**

1646 - 1716 **Leibniz**

Dissertation on the art of combination, in which from the fundamental arithmetic new precepts are extracted for the theory of combinations and transpositions, and their uses for all sciences are shown; new seeds sown for the art of meditation or the logic of discovery, 1666
New method of teaching and learning jurisprudence, from the principles of the art of teaching and the light of experience, 1667
Elements of natural law, 1671
Project for the conquest of Egypt, presented to Louis XIV, 1671
A new physical hypothesis, by which most phenomena of nature are derived from one universal motion supposed about our globe, without disdaining either the Tychonians or the Copernicans, 1671
An addition to the arithmetic of infinities, in which it is shown that a greatest number or a number of all numbers is impossible or nothing, and it is shown by examples that what is taken to be axioms can be demonstrated, 1672
New method to investigate the tangents of curves, 1673
Second part of the analysis of quadrature, 1675
Examples of the inverse method of tangents, 1675
Jurisdiction of the emperor, on the supreme and delegated rights of the German princes, 1677
Beginning and specimens of a general science for the instauration and growth of the sciences, 1679
Calculus of consequences, 1679
 manuscript beginning *Cum animadverterem plerosque*, 1679
Elements of the universal characteristic, 1679
Elements of calculus, 1679
Elements of the universal calculus, 1679
Investigations into the universal calculus, 1679
Method of examination of inferences by numbers, 1679
Rules with which the validity of inferences and the forms and moods of categorical syllogisms can be judged by numbers, 1679
On dyadic numbers, 1679
On the analysis of situation, 1679
On the true proportion of the circle to the circumscribed square, expressed in rational numbers, 1682
Perpetual irony, a most Christian Mars, or an apology for the wars of the most Christian king against the Christians, 1683
On discovering the dimensions of figures, 1684

- A new method for maxima and minima, as well as tangents, impeded by neither fractional nor irrational quantities, and a singular kind of calculus for it*, 1684
- Meditations on knowledge, truth, and ideas*, 1684
- A specimen of the universal calculus*, 1685
- An arithmetical machine, which performs not only addition and subtraction, but also multiplication without any and division nearly without any animate labor*, 1685
- manuscript beginning *Primae veritates sunt*, 1685
- General investigations into the analysis of concepts and truths*, 1686
- A new meditation on the nature of the angles of contact and osculation, and on their use in practical mathematics for substituting easier figures for more difficult ones*, 1686
- On a recondite geometry and the analysis of indivisibles and infinities*, 1686
- A brief demonstration of a memorable error of Descartes and others concerning a natural law, according to which god would always conserve the same quantity of motion, misused also in mechanics*, 1686
- Discourse on metaphysics*, 1686
- On a general principle useful for the explanation of the laws of nature through the divine wisdom*, 1687
- On optical lines, and other things*, 1689
- On the isochronic line in which a heavy body without acceleration descends*, 1689
- An examination of the causes of the heavenly motions*, 1689
- manuscript beginning *Cum geometrias demonstrationibus*, 1689
- On the line in which a body curves by its own weight, and its remarkable use in discovering any number of mean proportionals and logarithms*, 1691
- Preliminary specimen of dynamics, on the law of nature relating to the power of bodies*, 1691
- Essay on dynamics concerning the laws of motion, in which it is demonstrated that conserved is not the same quantity of motion but the same absolute force, or the same quantity of moving action*, 1691
- On the line formed by an infinite number of lines drawn ordinatewise which meet each other and are all tangent to it, and on the consequent new use of the analysis of infinities*, 1692
- Animadversions on the general parts of Descartes's Principles*, 1692
- A supplement to practical geometry extending to transcendental problems with the help of a new very general method of infinite series*, 1693
- A supplement to dimensional geometry, or the most general production of all quadratures by motion, as well as the construction of many lines from given conditions of the tangents*, 1693
- Protoearth, or treatise on the first state of the earth and the vestiges of its ancient history in the monuments of nature*, 1693
- preface to the *Code of the law of nations*, 1693
- A new application of the differential calculus and its use for multiple constructions of lines from a given condition of tangents*, 1694
- The proper construction of the problem of the paracentric isochronic curve, as well as some general facts about the nature and differential calculus of osculates, and on the constructions of transcendental lines, one entirely geometrical, another mechanical but very general; with a supplement on a way to render the discovery of transcendental lines universal, so that they include any case whatever and pass through a given point*, 1694
- A mathematics of reason*, 1695
- On the mathematical determination of the forms of the syllogism*, 1695
- A not inelegant specimen of demonstration in abstracts*, 1695
- manuscript beginning *Defin. 1. Eadem seu*, 1695
- A specimen of dynamics for the discovery of the admirable laws of nature of the forces of bodies and their actions on one another, and for their reduction to their causes*, 1695
- New system of the nature and of the communication of substances, as well as of the union which there is of the soul and the body*, 1695
- An anagogical essay in the research of causes*, 1695
- Letter on the connection of the houses of Brunswick and Este*, 1695
- Communication of his own solution of the problem of the curve of fastest descent, proposed to the*

geometers by Mr Johann Bernoulli, and of two other solutions, communicated to him first by Mr Johann Bernoulli, then by marquis de l'Hôpital, as well as his own solution of another problem proposed by the same, 1697

On the radical origination of things, 1697

preface to The latest on China, 1697

Dissertation on the origin of the Germans, 1697

On nature itself, or on the inherent force and actions of created things, to confirm and illustrate their dynamics, 1698

Clarification of the difficulties which Mr Bayle has found in the new system of the union of soul and body, 1698

preface to the Additions to history, writers on German matters collected from manuscripts, 1698

Justification of the calculus of infinitesimals by that of the ordinary algebra, 1701

New specimen of analysis for the science of the infinite concerning sums and quadratures, 1702

Reply to the reflections contained in the second edition of the Critical dictionary of Bayle, article

Rorarius, on the system of preestablished harmony, 1702

manuscript beginning Nullum quidem librum, 1702

Explication of binary arithmetic, which employs only the characters 0 and 1, with remarks on its utility, and on how it makes sense of the ancient Chinese figures of Fu Xi, 1703

Manifesto for the defense of the rights of Charles III, king of Spain, 1703

New essays on human understanding, 1704

Opinion on the principles of Pufendorf, 1706

Essays of theodicy on the goodness of god, the freedom of man, and the origin of evil, 1710

The antibarbarian physicist, for real philosophy, against the revival of the qualities of the scholastics and chimerical intelligences, 1710

Principles of nature and grace founded on reason, 1714

Principles of philosophy, or monadology, 1714

History and origin of the differential calculus, 1714

The metaphysical foundations of mathematics, 1715

Discourse on the natural theology of the Chinese, 1716

Brunswick annals of the western empire, 1716

letters to Huygens; letters to Clarke; letters to Wagner

1646 - 1719 **Flamsteed**

letters to Cassini

1647 - 1706 **Bayle**

A historical and critical dictionary, 1697

1650 - 1702 **William III**

1655 - 1705 **Jakob Bernoulli**

The art of conjecture, to which are added a treatise on infinite series and a letter written in French on the game of tennis, 1713

1656 - 1708 **Tournefort**

1656 - 1742 **Halley**

To that glory of our time and people, the mathematico-physical treatise of the most excellent Isaac Newton, 1687

review of The mathematical principles of natural philosophy, 1687

An estimate of the degree of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslau, with an attempt to ascertain the price of annuities upon lives, 1693

A synopsis of the astronomy of comets, 1705

A singular method to determine the parallax of the sun, or its distance from the earth, by observing Venus in the sun, 1716

Considerations on the change of the latitudes of some of the principal fixed stars, 1717

1660 - 1727 **George**

1660 - 1734 **Stahl**

Fundamental zymotechnic, or general theory of fermentation, in which the causes and effects of this noblest art and most useful and most subtle part of chemistry are presented from their

mechanico-physical principles, 1697

1666 - 1736 **Gray**

1667 - 1733 **Saccheri**

1667 - 1748 **Johann Bernoulli**

The curvature of a ray of light in a nonuniform medium, the solution of the problem, proposed in these Acta in 1696, of finding the brachistochrone, or the line in which a heavy particle descends in the shortest time, and on the construction of the synchrone, or the wave front of rays of light, 1697

1668 - 1744 **Vico**

Principles of a new science dealing with the common nature of nations, 1725

1672 - 1725 **Peter**

1672 - 1731 **Geoffroy**

1679 - 1754 **Wolff**

1685 - 1731 **Taylor**

1685 - 1740 **Charles VI**

1685 - 1753 **Berkeley**

Philosophical commentaries, 1708

An essay toward a new theory of vision, 1709

A treatise concerning the principles of human knowledge, wherein the chief causes of error and difficulty in the sciences, with the grounds of scepticism, atheism, and irreligion, are inquired into, 1710

Three dialogues between Hylas and Philonous, the design of which is plainly to demonstrate the reality and perfection of human knowledge, the incorporeal nature of the soul, and the immediate providence of a deity, in opposition to sceptics and atheists, 1713

On motion, or the principle and nature of motion, and on the cause of the communication of motions, 1721

The theory of vision, or visual language showing the immediate presence and providence of a deity, vindicated and explained, 1733

The analyst, or a discourse addressed to an infidel mathematician, wherein it is examined whether the object, principles, and inferences of the modern analysis are more distinctly conceived, or more evidently deduced, than religious mysteries, 1734

Siris, a chain of philosophical reflections and inquiries concerning the virtues of tar-water, and divers other subjects connected together and arising one from another, 1744
letters to Prior

1689 - 1755 **Montesquieu**

On the spirit of the laws, or on the relation which the laws must have with the constitution of every government, with customs, climate, religion, commerce, etc., 1748

1693 - 1762 **Bradley**

An account of a newly discovered motion of the fixed stars, 1728

1694 - 1768 **Reimarus**

Apology, or treatise in defense of the rational worshiper of god, 1767

1694 - 1778 **Voltaire**

The century of Louis XIV, 1751

The philosophy of history, or a historical dissertation on the origin, manners, customs, and religions of the different nations of antiquity, with a concise exposition of their religious superstitions, 1765

1698 - 1746 **Maclaurin**

1698 - 1759 **Maupertuis**

1700 - 1782 **Daniel Bernoulli**

1706 - 1790 **Franklin**

1707 - 1778 **Linnaeus**

The system of nature, according to the three kingdoms of nature, by classes, orders, genera, and species, 1735

The genera of plants, as well as their natural characters, by number, shape, place, and proportion of all fruit bearing parts, 1737

Botanical philosophy, in which the foundations of botany are explained, with definitions of parts, examples of terms, and observations of rarities, 1751

The species of plants, showing the properly recognized plants, related by genera, with specific differences, common names, selected synonyms, and native locations, arranged by the sexual system, 1753

1707 - 1783 **Euler**

Mechanics, or the science of motion presented analytically, 1736

Introduction to the analysis of infinities, 1748

Theory of the motion of the moon, accounting for all inequalities, 1753

Instruction in the differential calculus, with its application in finite analysis and the doctrine of series, 1755

Theory of the motion of solid or rigid bodies, established from the first principles of our knowledge and adapted to all motions of bodies of this kind, 1765

Instruction in the integral calculus, 1768-70

1707 - 1788 **Buffon**

Natural history, general and particular, with a description of the cabinet of the king, 1749-89, including

Experiments on the development of heat in bodies, 1774

Further experiments on the development of heat in different mineral substances, 1774

Researches on the cooling of the earth and the planets, 1775

1708 - 1778 **Pitt**

1711 - 1776 **Hume**

A treatise of human nature, being an attempt to introduce the experimental method of reasoning into moral subjects, 1739

An abstract of a book lately published, entitled A treatise of human nature, etc., wherein the chief argument of that book is farther illustrated and explained, 1740

Essays, moral, political, and literary, 1741

Of the middle station of life, 1742

A letter from a gentleman to his friend in Edinburgh, containing some observations on a specimen of the principles concerning religion and morality, said to be maintained in a book lately published, entitled A treatise of human nature, etc., 1745

An inquiry concerning human understanding, 1748

An inquiry concerning the principles of morals, 1751

The history of England, 1754-62

Four dissertations, I. The natural history of religion, II. Of the passions, III. Of tragedy, IV. Of the standard of taste, 1757

A concise and genuine account of the dispute between Mr. Hume and Mr. Rousseau, with the letters that passed between them during their controversy, 1766

My own life, 1776

Dialogues concerning natural religion, 1779

1712 - 1778 **Rousseau**

Discourse on the question, whether the restoration of the sciences and the arts has contributed to the purification of the morals, 1750

Discourse on the origin and foundations of inequality among men, 1755

Julie, or the new Héloïse, letters of two lovers, inhabitants of a small town at the foot of the Alps, 1761

On the social contract, or principle of political right, 1762

Émile, or on education, 1762

Rousseau judge of Jean-Jacques, a dialogue, 1780-2

Reveries of a solitary walker, 1782

The confessions, 1782-9

1712 - 1786 **Frederick II**

The general principles of war, applied to the tactics and discipline of the Prussian troops, 1747

1713 - 1784 **Diderot**

Encyclopedia, or analytic dictionary of the sciences, arts, and trades, by a society of men of letters,

1751-72

*Detailed explanation of the system of human knowledge, in the Encyclopedia, 1751**Encyclopedia, in the Encyclopedia, 1755*

1715 - 1780 Condillac

1717 - 1768 Winckelmann

1717 - 1783 **d'Alembert***Treatise on dynamics, 1743**Preliminary discourse of the editors, in the Encyclopedia, 1751**Limit, in the Encyclopedia, 1765*

1718 - 1783 Morelly 1718 - the decade after 1778

The code of nature, or the true spirit of the laws neglected or unknown in all times, 1755

1723 - 1780 Blackstone

*Commentaries on the laws of England, 1765-9*1723 - 1790 **Adam Smith***An inquiry into the nature and causes of the wealth of nations, 1776*

letters to Strahan

1724 - 1804 **Kant***Thoughts on the true estimation of living forces and assessment of the proofs used on this controversial matter by Leibniz and other mechanics, together with some preliminary observations on the force of bodies in general, 1747**Investigation of the question whether the earth in its rotation on its axis, by which it brings about the alteration of day and night, has undergone any change since the times of its origin, and how this can be established, 1754**The question whether the earth ages, considered physically, 1754**General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws, 1755**A new elucidation of the first principles of metaphysical knowledge, 1755**Brief exposition of some reflections on fire, 1755**History and natural description of the most remarkable occurrences of the earthquakes which struck a large part of the earth at the end of the year 1755, 1756**On the causes of the earthquakes on the occasion of the disaster which befell the western countries of Europe toward the end of last year, 1756**Further observations on the earthquakes experienced for some time, 1756**The use of metaphysics combined with geometry in natural philosophy, whose first test is the physical monadology, 1756**New remarks toward an elucidation of the theory of winds, 1756**A brief consideration of the question whether the west winds in our regions are humid because they pass over a large sea, 1757**New doctrine of motion and rest, and its consequences for the first principles of natural science, 1758**An attempt at some observations on optimism, 1759**The false subtlety of the four syllogistic figures, 1762**The only possible argument for demonstration of the existence of god, 1763**An attempt to introduce the concept of negative magnitudes in philosophy, 1763**Investigation on the clarity of the principles of natural theology and morality, 1764**Observations on the feeling of the beautiful and sublime, 1764**Essay on the diseases of the head, 1764**Dreams of a seer of spirits, explained by dreams of metaphysics, 1766**On the first ground of the difference of directions in space, 1768**On the form and principles of the sensible and intelligible world, 1770**On the different races of men, 1775**Critique of pure reason, 1781**Prolegomena to any future metaphysics which will be able to stand as science, 1783**Idea for a general history with a cosmopolitan purpose, 1784**Answer to the question, what is enlightenment? 1784*

Groundwork for the metaphysics of morals, 1785
 review of Herder's *Ideas for the Philosophy of the history of humanity*, 1785
On the volcanoes on the moon, 1785
On the wrongfulness of the unauthorized publication of books, 1785
Determination of the concept of a human race, 1785
The presumed beginning of human history, 1786
Metaphysical elements of natural science, 1786
What does it mean to orient oneself in thinking? 1786
On the philosophers' medicine of the body, 1786
Critique of practical reason, 1788
On the use of teleological principles in philosophy, 1788
Critique of judgment, 1790
On a discovery by which any new critique of pure reason will be made dispensable by an older one, 1790
On the failure of all philosophical attempts at theodicy, 1791
Religion within the boundaries of reason alone, 1793
On the common saying, this may be correct in theory but it is of no use for practice, 1793
Something on the influence of the moon on the weather, 1794
The end of all things, 1794
Toward perpetual peace, a philosophical draft, 1795
On a recently arisen superior tone in philosophy, 1796
Proclamation of the near conclusion of a treaty of perpetual peace in philosophy, 1796
The metaphysics of morals, 1797
On a supposed right to lie from philanthropy, 1797
The quarrel of the faculties, 1798
Anthropology considered in pragmatic respect, 1798
On the power of the mind to be master of its feeling sick by sheer resolution, 1798
On turning out books, 1798
Declaration concerning Fichte's theory of science, 1799
Logic, a manual for lectures, 1800
Physical geography, edited from manuscripts, 1802
Which is the true progress metaphysics has made in Germany since the time of Leibniz and Wolff? 1804
 letters to Beck

1726 - 1797 **Hutton**

Abstract of a dissertation concerning the system of the earth, its duration, and stability, 1785
The theory of the earth, with proofs and illustrations, 1795

1728 - 1777 **Lambert**

Preliminary advice for those who seek to square and rectify the circle, 1766
Memoir on some remarkable properties of circular and logarithmic transcendental quantities, 1768
Notes and comments on the drawing of maps of the earth and heaven, 1772

1728 - 1779 **Cook**

1728 - 1799 **Black**

Experiments on magnesia alba, quicklime, and some other alkaline substances, 1756

1729 - 1786 Mendelssohn

1729 - 1797 Burke

1731 - 1810 **Cavendish**

Three papers containing experiments on factitious air, 1766

1732 - 1799 **Washington**

1732 - 1807 Lalande

Memoir on the parallax of the sun, deduced from observations in the South Sea, in the kingdom of Astrakhan, and in China, 1772

1735 - 1826 John Adams

1736 - 1793 Bailly

1736 - 1806 **Coulomb**

1736 - 1813 **Lagrange**

Analytic mechanics, 1788

1737 - 1794 **Gibbon**

The history of the decline and fall of the Roman empire, 1776-88

Antiquities of the house of Brunswick, 1796

1737 - 1798 Galvani

1738 - 1822 **Herschel**

On nebulous stars properly so called, 1791

On the nature and construction of the sun and fixed stars, 1794

1740 Montgolfier

Joseph 1740 - 1810; Étienne 1745 - 1799

1741 - 1790 **Joseph II**

1743 - 1794 **Lavoisier**

Physical and chemical essays, 1774

Reflections on phlogiston to serve as a continuation of the theory of combustion and calcination
published in 1777, 1783

Elementary treatise on chemistry, presented in a new order and according to the modern
discoveries, 1789

1743 - 1822 Haüy

1744 - 1803 Herder

Extract from an exchange of letters about Ossian and the songs of ancient peoples, 1773

Also a philosophy of history, for the education of mankind, 1774

Ideas for the philosophy of the history of mankind, 1784-91

with Goethe and Möser

On the German kind and art, 1773

1744 - 1829 **Lamarck**

Zoological philosophy, or exposition of considerations on the natural history of animals, on the
diversity of their organization and the faculties they obtain from it, on the physical causes which
maintain life in them and give rise to the movements they make, as well as on what produces
feeling in some of them and intelligence in those endowed with it, 1809

1745 - 1827 **Volta**

1748 - 1822 Berthollet

Researches on the laws of affinity, 1801

1748 - 1836 **Sieyès**

What is the third estate? 1789

1749 - 1817 Werner

1749 - 1827 **Laplace**

Exposition of the system of the world, 1796

Treatise on celestial mechanics, 1799-825

Philosophical essay on probabilities, 1814

1749 - 1832 **Goethe**

Götz von Berlichingen, 1773

The sufferings of young Werther, 1774

An intermaxillary bone in the upper jaw is to be attributed to man as well as animals, 1784

An attempt to explain the metamorphosis of plants, 1790

Contributions to optics, 1791-2

Faust, 1808-32

Toward the theory of colors, 1810

letters to Lavater

with Herder and Möser, *q. v.*

1751 - 1836 Madison

with Hamilton and Jay, *q. v.*

1753 - 1823 Lazare Carnot

1756 - 1804 Alexander Hamilton

1755 or 1757 - 1804

with Madison and Jay

The federalist, a collection of essays written in favor of the new constitution as agreed by the federal

convention on 17 Sep 1787, 1788

1756 - 1836 **Godwin**

An inquiry concerning political justice and its influence on morals and happiness, 1793

1758 - 1794 **Robespierre**

1758 - 1805 Nelson

1759 - 1805 **Schiller**

The robbers, 1781

The stage considered as a moral institution, 1784

History of the separation of the United Netherlands from the Spanish government, 1788

What is called and to what end is studied universal history? 1789

History of the thirty years' war, 1791-3

On the tragic art, 1792

On the esthetic education of man, in a series of letters, 1795

On the sublime, 1801

Wallenstein's camp, The Piccolomini, and Wallenstein's death, 1801-4

Wilhelm Tell, 1804

1762 - 1814 Fichte

Foundation of the entire theory of science, 1794

First introduction in the theory of science, 1797

Second introduction in the theory of science, for readers who already have a philosophical system, 1797

The system of the theory of morals according to the principles of the theory of science, 1798

1766 - 1817 Mme de Staël

On Germany, 1810

1766 - 1834 Malthus

An essay on the principle of population, or a view of its past and present effects on human happiness, with an inquiry into the prospects respecting the future removal or mitigation of the evils which it occasions, 1798

1766 - 1844 **Dalton**

New system of chemical philosophy, 1808-27

1767 - 1794 Saint-Just

On nature, on the civil state, on the city, or the rules of independence, of government, 1792

Republican institutions, 1794

1768 - 1830 **Fourier**

Analytic theory of heat, 1822

1768 - 1834 Schleiermacher

The Christian faith, presented in connection with the principles of the evangelical church, 1821-2

1769 - 1821 **Napoleon**

Napoleonic code, 1804

1769 - 1832 **Cuvier**

1769 - 1852 **Wellington**

1770 - 1831 **Hegel**

The system of science, part one, the phenomenology of spirit, 1807

Announcement, 1807

The science of logic, 1812-6

Encyclopedia of the philosophical sciences in outline, 1817

Natural right and political science in outline, elements of the philosophy of right, 1821

Lectures on the history of philosophy, 1833-6

Lectures on the philosophy of history, 1840

1773 - 1829 **Young**

1774 - 1862 Biot

1775 - 1836 **Ampère**

Essay on the philosophy of the sciences, or analytic exposition of a natural classification of all human knowledge, 1834-43

1775 - 1854 Schelling

1776 - 1856 **Avogadro**

1777 - 1851 **Ørsted**

1777 - 1855 **Gauss**

General principles of the theory of the shape of fluids in the state of equilibrium, 1829

1778 - 1829 **Davy**

1778 - 1850 **Gay-Lussac**

1779 - 1848 **Berzelius**

1780 - 1831 **Clausewitz**

On war, 1832

1784 - 1846 **Bessel**

Determination of the axes of the elliptic spheroid of revolution corresponding best to the available measurements of meridian arcs, 1837

1786 - 1853 **Arago**

1788 - 1860 **Schopenhauer**

The world as will and representation, 1819

On the freedom of man's will, 1839

1789 - 1857 **Cauchy**

Course in analysis at the royal polytechnic school, part I, algebraic analysis, 1821

Summary of the lessons given at the royal polytechnic school on the infinitesimal calculus, 1823

1790 - 1832 **Champollion**

1790 - 1868 **Möbius**

1791 - 1867 **Faraday**

Experimental researches in electricity, 1839-55, including

Some points of magnetic philosophy, 1855

1792 - 1856 **Lobachevsky**

On the principles of geometry, 1829

New principles of geometry, with a complete theory of parallels, 1836-8

Geometrical investigations on the theory of parallels, 1840

1792 - 1866 **Hincks**

1792 - 1876 **von Baer**

1796 - 1832 **Carnot**

Reflections on the motive power of fire and on the proper engines to develop this power, 1824

1797 - 1875 **Lyell**

Principles of geology, or the modern changes of the earth and its inhabitants considered as illustrative of geology, 1830-3

1798 - 1857 **Comte**

Course in positive philosophy, 1830-42

System of positive politics, or treatise on sociology, instituting the religion of humanity, 1851-4

Positive catechism, or summary exposition of the universal religion, in thirteen systematic conversations between a woman and a priest of humanity, 1852

1803 - 1873 **Liebig**

Chemistry in its application to agriculture and physiology, 1840

1804 - 1881 **Schleiden**

1805 - 1865 **Hamilton**

On a general method in dynamics, by which the study of the motions of all free systems of attracting or repelling points is reduced to the search and differentiation of one general relation, or characteristic function, 1834

1807 - 1873 **Agassiz**

1808 - 1873 **Napoleon III**

1809 - 1882 **Darwin**

On the formation of mold, 1837

On the connection of certain volcanic phenomena in South America; and on the formation of mountain chains and volcanoes as an effect of the same power by which continents are elevated, 1838

Journal of researches into the geology and natural history of the various countries visited by H.M.S.

Beagle, under the command of captain FitzRoy, from 1832 to 1836, 1839
Observations on the parallel roads of Glen Roy, and of other parts of Lochaber in Scotland, with an attempt to prove that they are of marine origin, 1839
On the distribution of the erratic boulders and on the contemporaneous unstratified deposits of South America, 1841
The structure and distribution of coral reefs, 1842
Notes on the effects produced by the ancient glaciers of Caernarvonshire, and on the boulders transported by floating ice, 1842
Geological observations on the volcanic islands visited during the voyage of H.M.S. Beagle, together with some brief notices of the geology of Australia and the cape of Good Hope, 1844
Geological observations on South America, 1846
On the geology of the Falkland Island, 1846
Geology, in John Herschel's Manual of scientific inquiry, prepared for the use of her majesty's navy, and adapted for travelers in general, 1849
A monograph of the sub-class cirripedia, with figures of all species, 1851-4
Extract from an unpublished work on species, consisting of a portion of a chapter entitled, On the variation of organic beings in a state of nature; on the natural means of selection; on the comparison of domestic races and true species, 1858
The origin of species by means of natural selection, or the preservation of favored races in the struggle for life, 1859
On the two forms, or dimorphic condition, in the species of primula, and on their remarkable sexual relations, 1861
The various contrivances by which orchids are fertilized by insects, 1862
On the sexual relations of the three forms of lythrum salicaria, 1864
The power of movement in plants, 1865
The variation of animals and plants under domestication, 1868
The descent of man, and selection in relation to sex, 1871
The expression of emotions in man and animals, 1872
Insectivorous plants, 1875
The effects of cross- and self-fertilization in the vegetable kingdom, 1876
Recollections of the development of my mind and character, 1876
The different forms of flowers on plants of the same species, 1877
Biographical sketch of an infant, 1877
The formation of vegetable mold, through the action of worms, with observations on their habits, 1881
The action of carbonate of ammonia on the roots of certain plants, 1882
The action of carbonate of ammonia on chlorophyl-bodies, 1882
notes in the Athenaeum
letters to Nägeli

1810 - 1861 **Cavour**

1810 - 1882 **Schwann**

1810 - 1895 Rawlinson

The Persian cuneiform inscription at Behistun, deciphered and translated; with a memoir on Persian cuneiform inscriptions in general, and on that of Behistun in particular, 1846-51

The cuneiform inscriptions of western Asia, 1861-75

1811 - 1832 **Galois**

1814 - 1876 **Bakunin**

1815 - 1864 **Boole**

The mathematical analysis of logic, 1847

An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities, 1854

1815 - 1897 **Weierstrass**

Introduction in the theory of analytic functions, Berlin lectures, 1878

1815 - 1898 **Bismarck**

1818 - 1883 **Marx**

*Economic-philosophical manuscripts from 1844
ad Feuerbach, 1845*

The class struggles in France from 1848 to 1850, 1850

The eighteenth Brumaire of Louis Bonaparte, 1852

Contribution to the critique of political economy, 1859

The capital, critique of political economy, 1867

letters to Engels; letters to Lassalle; letters to Weydemeyer

with Engels

*The German ideology, critique of the newest German philosophy in its representatives Feuerbach,
B. Bauer, and Stirner, and of the German socialism in its different prophets, 1846*

Manifesto of the communist party, 1848

1819 - 1868 Foucault

Experimental determination of the speed of light, and the parallax of the sun, 1862

1820 - 1895 Engels

Herr Eugen Dühring's upheaval in science, 1878

Dialectic of nature, 1882

*The origin of the family, private property, and the state, in connection with the investigations of
Lewis H. Morgan, 1884*

Ludwig Feuerbach and the end of classical German philosophy, 1886

Contribution to the history of early Christianity, 1895

letters to Mehring

with Marx, q. v.

1821 - 1894 **Helmholtz**

On the conservation of force, 1847

On Goethe's natural-scientific works, 1853

On the relation of the natural sciences to all science, 1862

The new advances in the theory of vision, 1868

On the goal and advances of natural science, 1869

On the origin of the planetary system, 1871

1821 - 1902 Virchow

Cellular pathology grounded in physiological and pathological histology, 1858

1822 - 1884 **Mendel**

Experiments on plant hybrids, 1865

letters to Nägeli

1822 - 1888 **Clausius**

On the moving force of heat and the laws which can be derived from it for the theory of heat, 1850

*On different convenient for application forms of the main equations of the mechanical theory of
heat, 1865*

1822 - 1895 Pasteur

1824 - 1887 Kirchhoff

1824 - 1907 **Kelvin**

On the secular cooling of the earth, 1862

The age of the earth as an abode fitted for life, 1897

1825 - 1905 Oppert

Sumerian or Akkadian? 1875

1826 - 1866 **Riemann**

On the hypotheses which lie at the foundation of geometry, 1854

1831 - 1879 **Maxwell**

On Faraday's lines of force, 1855

On physical lines of force, 1861

A dynamical theory of the electromagnetic field, 1864

Address to the mathematical and physical sections of the British association, 1870

Theory of heat, 1871

On the mathematical classification of physical quantities, 1871

A treatise on electricity and magnetism, 1873

Matter and motion, 1877

1831 - 1916 **Dedekind**

What are and what should the numbers? 1888

1832 - 1920 Wundt

1833 - 1911 Dilthey

Introduction to the spiritual sciences, 1883

The types of world-view and their formation in the metaphysical systems, 1911

1834 - 1907 **Mendeleev**

Correlation of the properties and atomic weights of the elements, 1869

1843 - 1910 Koch

1844 - 1900 **Nietzsche**

Untimely observations, second part; on the usefulness and disadvantage of history for life, 1874

Beyond good and evil, prelude to a philosophy of the future, 1886

Toward the genealogy of morality, a polemic, 1887

1844 - 1906 **Boltzmann**

1844 - 1918 Wellhausen

Prolegomena to the history of Israel, 1878

1845 - 1918 **Cantor**

On a property of the totality of all real algebraic numbers, 1874

Contributions to the foundation of the transfinite theory of sets, 1895-7

1848 - 1925 **Frege**

On a geometrical representation of the imaginary forms in the plane, 1873

Methods of calculation grounded in an extension of the concept of quantity, 1874

Concept writing, a formula language of pure thinking modeled on that of arithmetic, 1879

On applications of the concept writing, 1879

On the goal of the concept writing, 1882

On the scientific justification of the concept writing, 1882

Geometry of pairs of points in the plane, 1883

The foundations of arithmetic, a logico-mathematical investigation of the concept of number, 1884

On formal theories in arithmetic, 1886

Function and concept, 1891

On the law of inertia, 1891

On concept and object, 1892

On sense and reference, 1892

The laws of arithmetic, derived by the concept writing, 1893-903

The whole number, 1895

On the concept writing of Mr Peano and my own, 1896

On the foundations of geometry, 1903-6

What is a function? 1904

The thought, a logical investigation, 1919

Negation, a logical investigation, 1919

letters to Russell

1849 - 1925 **Klein**

Elementary mathematics from a higher standpoint, 1908

1852 - 1912 **Meiji**

1853 - 1928 Lorentz

1854 - 1912 Poincaré

Analysis situs, 1895

1856 - 1940 **Thomson**

1857 - 1894 Hertz

On the fundamental equations of electromagnetics for bodies at rest, 1890

On the fundamental equations of electromagnetics for bodies in motion, 1890

The principles of mechanics, in new interdependence, 1894

1858 - 1932 **Peano**

The principles of arithmetic presented by a new method, 1889

- 1858 - 1947 **Planck**
Lectures on thermodynamics, 1897
On the theory of the law of the distribution of energy in the normal spectrum, 1900
The unity of the physical picture of the world, 1908
Eight lectures on theoretical physics, 1909
- 1861 - 1947 Whitehead
Process and reality, an essay in cosmology, 1929
Adventures of ideas, 1933
 with Russell, q. v.
- 1862 - 1915 Boveri
On multipolar mitosis as a means of analysis of the cell nucleus, 1902
- 1862 - 1943 **Hilbert**
Foundations of geometry, 1899
Mathematical problems, 1900
Axiomatic thinking, 1918
The new grounding of mathematics, first statement, 1922
The logical foundations of mathematics, 1923
On the infinite, 1925
Knowledge of nature and logic, 1930
- 1864 - 1909 **Minkowski**
Space and time, 1908
- 1864 - 1920 Max Weber
The Protestant ethics and the spirit of capitalism, 1905
- 1866 - 1925 **Sun Yat-sen**
- 1866 - 1952 Croce
History reduced to the general concept of art, 1893
- 1867 Wright Wilbur 1867 - 1912; Orville 1871 - 1948
- 1868 - 1942 Hausdorff
- 1870 - 1924 **Lenin**
- 1871 - 1937 **Rutherford**
The radiation and emanation of radium, 1904
Radium – the cause of the earth's heat, 1905
Some cosmical aspects of radioactivity, 1907
- 1871 - 1953 **Zermelo**
Investigations on the foundations of the theory of sets, 1908
- 1872 - 1970 **Russell**
The principles of mathematics, 1903
On denoting, 1905
Mathematical logic as based on the theory of types, 1908
Knowledge by acquaintance and knowledge by description, 1911
The problems of philosophy, 1912
Our knowledge of the external world as a field for scientific method in philosophy, 1914
The philosophy of logical atomism, 1918
Introduction to mathematical philosophy, 1919
How to read and understand history, the past as the key to the future, 1943
My mental development, 1944
A history of western philosophy, 1945
Is there a god? 1952
My philosophical development, 1959
 letters to Frege
 with Whitehead
Principia mathematica, 1910-3
- 1873 - 1967 **Hertzprung**
On the radiation of stars, 1905-7
- 1877 - 1947 **Hardy**

- Mendelian proportions in a mixed population*, 1908
On the zeroes of Riemann's $\zeta(s)$ function, 1914
A mathematician's apology, 1940
- 1878 - 1956 **Lukasiewicz**
Aristotle's syllogistic from the standpoint of modern formal logic, 1951
- 1879 - 1955 **Einstein**
On the electrodynamics of moving bodies, 1905
Does the inertia of a body depend on its energy content? 1905
On the inertia of energy required by the principle of relativity, 1907
On the principle of relativity and the conclusions drawn from it, 1907
The principle of relativity and its consequences in modern physics, 1910
On the influence of gravitation on the propagation of light, 1911
The speed of light and the statics of the gravitational field, 1912
On the theory of the static gravitational field, 1912
On the present state of the problem of gravitation, 1913
The formal foundation of the general theory of relativity, 1914
On the general theory of relativity, 1915
The field equations of gravitation, 1915
The foundation of the general theory of relativity, 1916
Hamilton's principle and the general theory of relativity, 1916
Cosmological considerations on the general theory of relativity, 1917
On the special and the general theory of relativity, a popular account, 1917
Geometry and experience, 1921
The problem of space, aether, and the field in physics, 1930
 with Grossmann
Outline of a generalized theory of relativity and of a theory of gravitation, 1913
- 1880 - 1930 Wegener
- 1880 - 1959 **Chetverikov**
On certain aspects of the evolutionary process from the point of view of contemporary genetics, 1926
- 1881 - 1966 Brouwer
Intuitionism and formalism, 1912
- 1882 - 1944 Eddington
The internal constitution of the stars, 1926
 with Dyson and Charles Davidson
A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of 29 May 1919, 1920
- 1883 - 1946 Keynes
- 1885 - 1962 **Bohr**
Discussion with Einstein on epistemological problems in atomic physics, 1949
- 1887 - 1961 **Schrödinger**
- 1888 - 1925 **Friedmann**
On the curvature of space, 1922
- 1889 - 1945 **Hitler**
My struggle, 1925-7
- 1889 - 1951 Wittgenstein
Notebooks 1914-6
Logico-philosophical treatise, 1921
Lecture on ethics, 1929
Philosophical investigations, 1953
- 1889 - 1953 **Hubble**
Cepheids in spiral nebulae, 1925
Extragalactic nebulae, 1926
A relation between distance and radial velocity among extragalactic nebulae, 1929
The realm of the nebulae, 1936
- 1889 - 1976 Heidegger

- Being and time*, 1927
What is metaphysics? 1929
Introduction to metaphysics, 1935
The question about the thing, contribution to Kant's theory of the transcendental principles, 1935
- 1891 - 1970 Carnap
The logical structure of the world, 1928
Pseudoproblems in philosophy, the extrapsychological and the realism controversy, 1928
Elimination of metaphysics through logical analysis of language, 1932
- 1892 - 1987 de Broglie
- 1894 - 1966 Lemaître
A homogenous universe of constant mass and increasing radius accounting for the radial velocity of extragalactic nebulae, 1927
The primeval atom hypothesis and the problem of the clusters of galaxies, 1958
- 1901 - 1976 **Heisenberg**
On the visual content of quantum theoretical kinematics and mechanics, 1927
The physical principles of quantum theory, 1930
On the history of the physical explanation of nature, 1932
Physics and philosophy, the revolution in modern science, 1958
- 1901 - 1983 Tarski
The semantic conception of truth and the foundations of semantics, 1944
- 1902 - 1984 Dirac
The principles of quantum mechanics, 1930
- 1902 - 1985 Braudel
The Mediterranean and the Mediterranean world in the age of Philip II, 1949
Material civilization, economy, and capitalism, 15th-18th century, 1968-79
- 1902 - 1994 Popper
The logic of scientific discovery, 1934
On the sources of knowledge and of ignorance, 1960
Epistemology without a knowing subject, 1967
- 1903 - 1989 Lorenz
Observations on the recognition of instinctive behavior characteristic of the species in birds, 1932
The companion in the environment of the bird, 1935
Kant's theory of the a priori in the light of contemporary biology, 1941
- 1904 - 1968 Gamow
The creation of the universe, 1952
My world line, 1970
- 1906 - 1978 Gödel
Some metamathematical results on completeness and consistency, 1930
On formally undecidable propositions of Principia mathematica and related systems, 1931
Russell's mathematical logic, 1944
- 1910 - 1976 Monod
Chance and necessity, an essay on the natural philosophy of modern biology, 1970
 with Jacob
Genetic regulatory mechanisms in the synthesis of proteins, 1961
- 1912 - 1954 Turing
- 1916 - 2004 Crick
 with Watson
Molecular structure of nucleic acids, a structure for deoxyribose nucleic acid, 1953
- 1918 - 1988 Feynman
Lectures on physics, 1963-5
QED, the strange theory of light and matter, 1985
- 1920 - 2013 Jacob
 with Monod, q. v.
- 1928 - 2025 Watson
 with Crick, q. v.

1929 - 2019 Gell-Mann

poets

688	Archilochus	sings of an eclipse which occurred in 648
684	Alcman	the two generations after 676/3
619	Alcaeus	one generation after Pittacus
578	Anacreon	contemporary with Polycrates
525 - 456	Aeschylus	c. 525 - 456
518 - 441	Pindar	518 - the decade after 446
496 - 406	Sophocles	497/5 - 406
494 - 428	Phidias	under Pericles, general in 454 - the decade after 433
450 - 386	Aristophanes	c. 450 - 386
402	Praxiteles	worked in 364/1
370	Apelles	worked in 332/29
70 - 19	Virgil	
65 - 8 ^{BC}	Horace	
43 ^{BC} - 17 ^{AD}	Ovid	
120 ^{AD} - 190	Lucian	115/25 - c. 190
	<i>Dialogues of the dead</i>	
1474 - 1533	Ariosto	
	<i>The furious Orlando</i> , 1516	
1475 - 1564	Michelangelo	
1508 - 1580	Palladio	
1547 - 1616	Cervantes	
	<i>The ingenious hidalgo Don Quixote de la Mancha</i> , 1605-15	
1660 - 1731	Defoe	
	<i>The life and strange surprising adventures of Robinson Crusoe, of York, a mariner</i> , 1719	
1729 - 1781	Lessing	
1756 - 1791	Mozart	
1770 - 1827	Beethoven	
1788 - 1824	Byron	

not important

1630	Ahmose	under Apepi, prince c. 1590
788 - 734	Nabonassar	prince 748-34
678	<i>Nahum</i>	prophecy between 664 and 612
628	<i>Ruth</i>	the century before 538
620	Orphic poems	the century before Pythagoras
616	Jehoiachin	prince at 18 in 598
560	<i>Zechariah</i>	prophecy began in 520
523	Acusilaus of Argos	between Hecataeus and Herodotus
523	Cadmus of Miletus	between Hecataeus and Herodotus
489 - 418	Laches	older than Socrates - 418
485	Nehemiah	governor of Judea from 445 to 433
485	Phrynis	won a contest c. 445
472	Euctemon	contemporary with Meton
455	<i>Esther</i>	the century after 465
451 - 361	Timotheus	died at 90 in 365/57
450	Pherecydes of Athens	one generation after Hellanicus
449 - 406	Phaedrus	younger than Socrates - the decade before 401
440 - 403	Charmides	c. 440 - 403
421 - 400	Meno	very young in 401 - 400
417	Phaedo	born c. 417

335	Aristyllus	contemporary with Timocharis
236	Mago	the century before 146
191	Cassius Dionysius	two to three generations before Varro
171 - 133	Attalus III	
161	Diophanes	one to two generations before Varro
82 - 46 ^{BC}	Vercingetorix	led revolt in 52 - 46
27 ^{BC} - 36 ^{AD}	Thrasyllus	under Tiberius, prince 14-37 - 36
6 ^{AD}	st Peter	contemporary with st Paul
77	Basilides	under Hadrian, prince 117-38
91	Tatian	one generation before Irenaeus
98	Valentinus	under Pius, prince 138-61
104	Marcion	excommunicated in 144
121	Montanus	under Marcus, prince 161-80
154 - 222	Bardaisan	
170	Sabellius	contemporary with Hippolytus
171	Beryllus	under Caracalla, prince 211-7
200	Paul of Samosata	bishop c. 260-70
255 - 326	Alexander of Alexandria	contemporary with Arius - 326
273 - 355	Donatus	consecrated bishop in 313 - 355
295 - 367	Victorinus	290/300 - the decade after 362
313 - 398	Didymus the Blind	c. 313 - 398
315 - 368	Hilary	c. 315 - 368
315 - 403	Epiphanius	310/20 - 403
346	scholia in the <i>Elements</i>	reach back to the time between Pappus and Proclus
351 - 407	Chrysostom	347/54 - 407
359 - 383	Gratian	
372	Hesychius of Jerusalem	priest c. 412
376 - 444	Cyril of Alexandria	c. 376 - 444
393 - 466	Theodoret	c. 393 - 466
419 - 455	Valentinian III	
465 - 533	Fulgentius	462/8 - 533
597	ibn Abi Wakkas	general in 637
786 - 863	Paschasius	786 - 860/5
827 - 869	Cyril	
1096 - 1164	Héloïse	1092/100 - 1164
1100 - 1170	al-Bayhaqi	died in 1170
<i>Addition to The keepers of wisdom</i>		
1130 - 1195	Henry the Lion	1129/31 - 1195
1130 - 1202	Eugenius of Sicily	c. 1130 - 1202
1135 - 1205	Amalric	died in the decade before 1210
1150 - 1231	William of Auxerre	c. 1150 - 1231
1161	Acerbus	contemporary with Innocent III
1170	decree of the archbishop of Sens, the bishop of Paris, and other bishops, gathered in Paris, on the expurgation of heresy and on the different books of Aristotle, 1210	
1175	decree prescribing the way of reading in the arts and in theology, indicating which books the scholars are not allowed to read, arranging the disciplines of the sciences and in general the university, Aug 1215	
1436 - 1517	Jiménez de Cisneros	
1444 - 1522	de Nebrija	
1449 - 1515	Manutius	
1453 - 1536	Lefèvre d'Étaples	1450/5 - 1536
1463 - 1503	Lorenzo il Popolano	
1465 - 1526	Ferro	
1468 - 1522	Johann Werner	
1490 - 1573	de Sepúlveda	

- 1495 - 1552 Apian
 1499 Fiore contemporary with Tartaglia
 1500 - 1547 Bunel
 1502 - 1585 Gregory XIII
 1503 - 1576 de Carranza
 1504 - 1575 Bullinger
 1507 - 1577 Viglius
 1517 - 1586 Granvelle
 1519 - 1605 Beza
 1522 - 1568 count of Egmont
 1523 - 1576 Hoppers
 1524 - 1568 count of Horn
 1531 - 1598 Mocenigo
 deposition against Bruno, 23 Mar 1592
 1535 - 1612 van Meteren
 1545 - 1606 Bünting
 1545 - 1616 Röslin
 1550 - 1602 van Reyd
 1552 - 1612 Rudolph II
 1553 - 1617 de Thou
 1556 - 1615 Calvisius
 1558 - 1614 Welser
 1560 - 1613 Rittershausen
 1562 - 1647 Longomontanus
 1565 - 1637 Christina of Lorraine
 1565 - 1652 Chiaramonti
 1567 - 1601 earl of Essex
 1570 - 1640 Suslyga
 1572 - 1649 Strada
 1574 - 1636 Giuliano de' Medici
 1574 - 1637 Fludd
 1576 - 1649 Schopp
 letter to Rittershausen, 17 Feb 1600
 1576 report of the committee appointed to examine the propositions, the sun is in the center of the world and
 does not move, and the earth is not in the center and moves, to the holy office, 24 Feb 1616
 1577 - 1657 Liceti
 1578 - 1649 Ingoli
 1579 - 1639 Meursius
 1579 - 1644 Bentivoglio
 1580 - 1626 Capra
 1580 - 1667 Wendelin
 *Lunar eclipses observed from the year 1573 to the year 1643, based on Atlantic tables, their idea
 explained, 1644*
 1581 - 1656 Ussher
 *The annals of the old testament deduced from the first origin of the world, with a chronicle of Asian
 and Egyptian affairs, 1650*
 1583 - 1634 Wallenstein
 1583 - 1654 Grassi
 1583 - 1656 Morin
 1584 - 1653 Dinet
 1584 - 1667 Saint-Vincent
 1585 - 1619 Vanini
 1586 - 1649 Burgundius
 1588 - 1653 Filmer
 1590 - 1621 Cosimo II

- 1593 sentence in the trial of Galileo, 22 Jun 1633
 1598 - 1671 Riccioli
New Almagest, embracing the old and the new astronomy, from the observations of others and one's own, proposing new theorems, problems, and tables, 1651
- 1599 - 1656 Ottavio Piccolomini
 1600 - 1633 Bartsch
 1601 - 1652 de Beaune
 1605 - 1694 Boulliau
 1615 - 1660 van Schooten
 1617 - 1675 Leopoldo de' Medici
 1618 - 1677 Oldenburg c. 1618 - 1677
 1620 - 1687 Nicolas Mercator
 1622 - 1685 de Sluse
 1622 - 1689 Streete
 1625 - 1679 Johann Friedrich
 1628 - 1704 Hudde
 1629 - 1698 Ernst August
 1629 - 1707 Towneley
 1632 - 1723 Wren
 1635 - 1682 Becher
Chemical laboratory, or subterranean physics, 1669
- 1660 - 1717 Gabriel Wagner c. 1660 - 1717
 1661 - 1704 l'Hôpital
 1661 - 1719 Garth
 1661 - 1742 de Polignac
 1662 - 1742 Bentley
 1664 - 1730 Johann von Eckhart
 1668 - 1705 Sophie Charlotte
 1669 - 1724 Pound
 1672 - 1719 Addison
 1675 - 1729 Clarke
 1680 - 1751 Thomas Prior
 1680 - 1753 de Boze
Eulogy for the cardinal de Polignac, 1742
- 1682 - 1716 Cotes
 preface to the second edition of *The mathematical principles of natural philosophy*, 1713
- 1683 - 1746 Philip V
 1683 - 1760 George II
 1686 - 1748 Gruber
 1688 - 1737 Conduitt
 Keynes manuscripts
- 1709 - 1761 Scheidt
 1715 - 1785 Strahan
 1720 - 1794 Möser
History of Osnabrück, a general introduction, 1768
 with Herder and Goethe, q. v.
- 1735 - 1788 John Brown
 1736 - 1796 Macpherson
 1740 - 1813 Stock
An account of the life of George Berkeley, late bishop of Cloyne in Ireland, 1776
- 1741 - 1801 Lavater
 1743 - 1819 Friedrich Jacobi
 1745 - 1792 Hearne
 1745 - 1829 Jay
 with Hamilton and Madison, q. v.

1752 - 1840 Blumenbach
 1755 - 1830 Soemmerring
 1761 - 1840 Beck
 1776 - 1841 Herbart
 1780 - 1838 Antommarchi

Memoirs, or the last days of Napoleon, 1825

1785 - 1871 Bekker
 1791 - 1865 Encke
 1792 - 1871 John Herschel
 1802 - 1870 Botta
 1804 - 1872 Feuerbach
 1805 - 1865 FitzRoy
 1806 - 1856 Stirner
 1807 - 1853 Laurent
 1809 - 1882 Bruno Bauer
 1812 - 1876 Kuhlmann
 1816 - 1856 Charles Gerhardt
 1816 - 1899 Carl Gerhardt
 1817 - 1887 Grün
 1817 - 1891 Nägeli
 1818 - 1866 Weydemeyer
 1818 - 1881 Lewis Morgan
 1825 - 1864 Lassalle
 1826 - 1910 Rassam
 1833 - 1921 Dühring
 1835 - 1910 Schiaparelli
 1835 - 1924 Hensen
 1842 - 1910 William James
 1843 - 1904 Tannery
 1846 - 1919 Mehring
 1847 - 1922 Favaro
 1851 - 1916 Silvanus Thompson

The life of William Thomson, baron Kelvin of Largs, 1910

1857 - 1940 Adam
 1859 - 1941 Bergson
 1859 - 1952 Dewey
 1862 - 1929 Kugler
 1868 - 1914 Couturat
 1868 - 1939 Dyson
 with Eddington, *q. v.*

1873 - 1958 Moore
Principia ethica, 1903

1875 - 1970 Charles Davidson
 with Eddington, *q. v.*

1878 - 1936 Grossmann
 with Einstein, *q. v.*

1880 - 1936 Spengler
The decline of the west, outlines of a morphology of world history, 1918

1889 - 1975 Toynbee
A study of history, 1934-61

1900 Bourbaki began publishing in 1940
Elements of the history of mathematics, 1960

1903 - 1950 Orwell
1984, 1949

1922 - 1996 Kuhn

- The structure of scientific revolutions*, 1962
 1926 - 1984 Michel Foucault
The archeology of knowledge, 1969
History of sexuality, 1976-84
 1930 *Dictionary of scientific biography*, 1970-90
 1949 - 1984 Laporte
History of shit, 1978

not dated (with their earliest source in the text)

Enlil (Eanatum)
 Re (Unas)
 Ishtar (Venus tablet of Ammisaduqa)
 Apsu, Tiamat, Anu, Ea, Marduk (*When on high*)
 Adam, Cain, Seth, Noah, Abraham, Moses, Joshua (Yahwistic document)
 Amos (*Amos*)
 Isaiah (*Isaiah*)
 Achilles, Zeus, Apollo, Odysseus, Menelaus, Theseus, Ocean, Hephaestus, Hermes, Aphrodite,
 Heracles, Euphorbus, Ariadne, Oedipus, Scylla (Homer)
 Micah (*Micah*)
 Hesperus, Phosphorus (Hesiod)
 Adad (*When Anu and Enlil*)
 Jeremiah (*Jeremiah*)
 Ezekiel (*Ezekiel*)
 Gideon, Samuel, Saul, Abigail, Elisha (*Joshua, Judges, Samuel, Kings*)
 Haggai (*Haggai*)
 Job (*Job*)
 Lycurgus, Aristaeas (Herodotus)
 Euthyphro, Lysis, Timaeus (Plato)
 Aethalides, Hermotimus, Pyrrhus (Ponticus)
 Nicomachus (Aristotle)
 Daniel (*Daniel*)
 Fu Xi (Sima Qian)
 Jupiter, Mercury, Mars, Minerva, Venus (Varro)
 Numa (Livy)
 Jesus, Antichrist, Timothy (st Paul)
 Mark, Mary (*Gospel of Mark*)
 Matthew (*Gospel of Matthew*)
 Luke, Dionysius the Areopagite (*Gospel of Luke, Acts of the apostles*)
 John (*Gospel of John, Apocalypse*)
 Simon, Menander, Cerinthus, Nicolaus, Saturninus, Carpocrates, Cerdo, Marcus, Thebuthis, Simon,
 Cleobius, Dositheus, Gorthaeus, Severus, Apelles, Potitus, Basilicus, Syneros, Priscilla, Maximilla,
 Florinus, Blastus, Miltiades, Themiso, Alexander, Alcibiades, Theodotus, Artemon, Theodotus,
 Asclepiodotus, Theodotus, Natalius, Symmachus, Ambrose, Proclus, Nepos, Coracion (Eusebius)
 Lucifer (Augustine)
 Philology, Rhetoric, Geometry (Capella)
 Feselius (Kepler)
 Phoebadius, Faustinus, Gregory of Elvira, Philastrius, Idacius, Eucherius, Arnobius Junior, Eugenius
 of Carthage, Cerealis, Vigilus of Thapsus, Victor Vitensis, Facundus, Oecumenius, Nicetas, de
 Zúñiga, de Colines, Edward Lee, Hessels, Montano, Canter, Franciscus Lucas, Caryophilus,
 Chiffletius, Brian Walton, Richard Simon, Gilbert Burnet (Newton)
 Theophilus, Philalethes (Leibniz)
 Hylas, Philonous (Berkeley)
 Julie, Émile, Sophie (Rousseau)
 Götz von Berlichingen, Werther, Lotte, Faust (Goethe)

Bor, Hooft, Brandt, Le Clerc, Dinoth, Spittler, Watson, Wilhelm Tell, Max Piccolomini (Schiller)

Charon (Lucian)
Orlando (Ariosto)
Don Quixote, Sancho Panza (Cervantes)
Robinson Crusoe (Defoe)

Nahum (*Nahum*)
Ruth (*Ruth*)
Orpheus (Orphic poems)
Zechariah (*Zechariah*)
Esther (*Esther*)
Ossian (Macpherson)

the numbering of inscriptions and fragments follows these editions

Eanatum, Urukagina – Cooper

Sargon – Gelb/Kienast

Tiglath-pileser – Grayson

Epimenides, Xenophanes, Heraclitus, Parmenides, Critias – Diels

Ponticus – Wehrli

Aristotle – Rose

And counting centuries from the year of birth assigned to the 3 calculations of area, these are the distribution of the particulars and their totals,

	important and writings		poets and writings		not important and writings	
3120-3021	2	–	–	–	–	–
3020-2921	1	–	–	–	–	–
2920-2821	–	–	–	–	–	–
2820-2721	–	–	–	–	–	–
2720-2621	3	–	–	–	–	–
2620-2521	6	–	–	–	–	–
2520-2421	3	1	–	–	–	–
2420-2321	3	3	–	–	–	–
2320-2221	2	–	–	–	–	–
2220-2121	2	1	–	–	–	–
2120-2021	5	–	–	–	–	–
2020-1921	4	–	–	–	–	–
1920-1821	11	–	–	–	–	–
1820-1721	8	1	–	–	–	–
1720-1621	8	–	–	–	1	–
1620-1521	3	–	–	–	–	–
1520-1421	4	–	–	–	–	–
1420-1321	8	–	–	–	–	–
1320-1221	2	–	–	–	–	–
1220-1121	2	1	–	–	–	–
1120-1021	1	–	–	–	–	–
1020- 921	5	–	–	–	–	–
920- 821	4	1	–	–	–	–
820- 721	14	3	–	–	1	–
720- 621	13	1	2	–	2	–
620- 521	18	6	3	–	5	–
520- 421	40	25	4	–	10	–
420- 321	36	67	2	–	2	–
320- 221	22	8	–	–	1	–

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enumeration and review

220- 121	11	3	—	—	3	—
120- 21	21	17	3	—	2	—
20 ^{BC} - 80 ^{AD}	18	15	—	—	2	—
81- 180	19	20	1	1	7	—
181- 280	16	10	—	—	3	—
281- 380	14	10	—	—	9	—
381- 480	10	5	—	—	3	—
481- 580	7	4	—	—	—	—
581- 680	5	—	—	—	1	—
681- 780	6	1	—	—	—	—
781- 880	8	4	—	—	2	—
881- 980	11	3	—	—	—	—
981-1080	11	11	—	—	—	—
1081-1180	20	10	—	—	9	1
1181-1280	18	10	—	—	—	—
1281-1380	12	11	—	—	—	—
1381-1480	32	16	2	1	7	—
1481-1580	34	104	2	1	39	3
1581-1680	64	168	1	1	41	4
1681-1780	83	187	3	—	20	5
1781-1880	77	190	1	—	36	3
1881-1980	28	47	—	—	7	7
	755	964	24	4	213	23

and the not dated are 177.