

An investigation of knowledge
part three of eight

Radoslav Datchev

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In part one the problems which knowledge poses have been collected into one problem. A fundamental part of the problem is to distinguish the acquisition of knowledge and knowledge. There can be no access to knowledge but through its acquisition, and the problem of knowledge cannot be solved, if the problem of the acquisition of knowledge is not solved first. A fundamental part of the problem of the acquisition of knowledge is to collect the acquisition of knowledge in its entirety, and the problem of the acquisition of knowledge cannot be solved, if the

problem of collecting the acquisition of knowledge in its entirety is not solved first.

In part two the method has been presented.

And in this part three the ground is established on which the acquisition of knowledge can be collected in its entirety, and how firm the ground is is shown.

In parts one and two the argument has been analytic, proceeding from effects to their causes, and in this part three the analysis continues. The attempt at synthesis, assuming the causes discovered and explaining the phenomena from them, should follow in the subsequent parts four, five, six, seven. Knowledge is a difficult thing.

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**In order to formulate the problems we are to solve we call the council
of our predecessors so that we can adopt the sound in what
they have said and avoid the erroneous**

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To review the problem.

The problem is knowledge.

The world is one, there is nothing next to the world, or outside the world. To speak of many worlds, of an intelligible world next to a sensible world, a Greek world and a German world, a world of physics and a world of ethics, a spiritual world next to a material world, my world and your world, is empty peddling of words.

Truth is truth, truth is not falsehood, falsehood is falsehood, falsehood is not truth. Nor can truth develop. Truth is the same in all space and all time. The same in all philosophers. The same for all men and all Martians. The same for all men and all Martians, too, such as the philosophers have found it.

And knowledge cannot be knowledge if it is not in relation to the world, and in relation to truth. Knowledge is one, knowledge is knowledge, knowledge cannot develop, knowledge is the same in all space and all time, the same in all philosophers, the same for all men and all Martians, and for all men and all Martians such as the philosophers have acquired it. The labors to acquire knowledge cannot not have finality. All results that will be knowledge cannot not be, if they will be knowledge, conclusive.

Knowledge cannot develop. Knowledge is acquired. The acquisition of knowledge is attested by the fact of the acquisition of knowledge. The acquisition of knowledge is the development

in relation to knowledge, truth, and the world. And there can be no access to the world, or to truth, or to knowledge but through the acquisition of knowledge.

The problem of knowledge has to be divided into two parts, knowledge and the acquisition of knowledge, the problem of knowledge and the problem of the acquisition of knowledge. And the first part, the problem of knowledge, cannot be formulated or solved unless the second part, the problem of the acquisition of knowledge, is formulated and solved first.

A development, the acquisition of knowledge cannot not have a beginning. It cannot not be a development preceded by no inkling of knowledge or its acquisition. In particular, the beginning of the acquisition of knowledge on earth cannot not have a beginning of the acquisition of knowledge on earth. Also, a development from no inkling to finality, the acquisition of knowledge must be possible to collect in its entirety. In particular, the acquisition of knowledge on earth must be possible to collect from its beginning through a certain time in its entirety.

These two, the problem of the beginning of the acquisition of knowledge and the problem of collecting the acquisition of knowledge in its entirety, have to be the decisive parts of the problem of the acquisition of knowledge.

For the acquisition of knowledge either can be and has been produced by selection, or has not been and cannot be produced by selection. And nothing but the solution of the problems of the

beginning of the acquisition and of collecting it in its entirety can show either, that it can be and has been produced by selection, or that it has not been and cannot be produced by selection. For if it has not been and cannot be produced by selection, a principle distinct from selection has produced it. And the problems of the beginning of the acquisition and of collecting it in its entirety will be solved only if the solution, both, stands on the theory of evolution, and points as its principle either to selection or to that principle distinct from selection.

Beginning with the first philosopher, knowledge has been acquired by the labors of the philosophers. The acquisition of knowledge, however, predates the first philosopher. The development of philosophy has also included more than the acquisition of knowledge.

The other two parts of the problem of the acquisition of knowledge have to be the relation of the acquisition of knowledge and the development of philosophy and the kinds of the acquisition of knowledge.

The fact that the development of philosophy has included the acquisition of knowledge has to be crucial.

If the development of philosophy is the labors of the philosophers, exclusively, and beginning with the first philosopher knowledge has been acquired by the labors of the philosophers, exclusively, before the first philosopher knowledge has been acquired by the labors of these predecessors of the philosophers of whom the philosophers are the successors, exclusively. And if so, and the labors of their predecessors and the philosophers can be distinguished from everything else, there can be no reason, in order to distinguish the acquisition of knowledge from everything else, to contaminate the labors of their predecessors and the philosophers with anything else. With men, for instance.

If knowledge has been acquired by the labors of their predecessors and the philosophers, exclusively, and the labors of their predecessors and the philosophers are collected, from the first to the last, the acquisition of knowledge will thus be collected, included in its entirety in the labors of their predecessors and the philosophers from the first to the last.

The development of philosophy has included more than the acquisition of knowledge. As the labors of their predecessors and the philosophers are collected, in order to distinguish the acquisition of knowledge from everything else, the labors of their predecessors and the philosophers have to be sorted out, into these that are the acquisition and these that are not.

And both, collecting the labors of their predecessors and the philosophers and sorting them out into these that are the acquisition and these that are not, can depend on the labors of their predecessors and the philosophers, exclusively.

The labors are cumulative. If experience is the foundation of the acquisition of knowledge, all earlier acquisition has to be part of the foundation of all later acquisition. The more successful the earlier acquisition, too, the more it has to be the foundation of the later acquisition exclusively. And if so, to collect the labors of their predecessors, and to sort them out into these that are the acquisition of knowledge and these that are not, cannot not be an essential part of the labors of their predecessors and the philosophers.

These are the particulars, the labors of their predecessors and the philosophers.

Little is to be hoped for unless all particulars are collected, and marshaled in their series and order.

The attempt to solve the problem must begin by collecting all the labors of their predecessors and the philosophers, and marshaling them in their series and order.

In order to proceed by continuous and gradual ascent, establish stages of certainty, ascend from observations to principles, descend from these to new observations, ascend from these to higher principles, descend again, ascend, and until it reaches the law with its clauses.

Clearly, it all follows from the labors of their predecessors and the philosophers.

Aristotle describes the development:
all animals have perception,
from perception memory develops in some of them,
from memory experience develops, of which animals other than man have little,
from experience art develops, and first the arts aiming to procure the necessities of life devel-

oped,
 then the arts aiming to give pleasure developed,
 when all such arts were established, knowledge
 which is neither useful nor pleasurable was
 discovered, thus mathematics was first devel-
 oped,
 and the first philosophers began asking questions
 about the motions of the moon, the sun, and
 the stars, and about the origin and develop-
 ment of the world.

The later stages Aristotle describes also in more detail:

first, men were forced by necessity to discover
 the useful arts, like sowing and the grinding of
 corn,
 next, they discovered the arts aiming not at neces-
 sity but at beauty,
 next, they turned to politics, and discovered laws
 and all the things that keep a city together,
 next, they went further and turned to things
 themselves and their nature,
 finally, they turned to divine and unchangeable
 things.

That perception is knowledge was argued
 before Plato already, and still would be. If per-
 ception is knowledge, memory is knowledge,
 experience is knowledge, the grinding of corn is
 knowledge, the arts aiming at beauty are knowl-
 edge, mathematics is knowledge, politics is
 knowledge, physics is knowledge, philosophy is
 knowledge. Everything is knowledge, or what is
 the same, nothing is knowledge.

But everything is not knowledge. The develop-
 ment, perception, memory, experience, the grind-
 ing of corn, the arts aiming at beauty, mathemat-
 ics, politics, physics, philosophy, cannot be the
 development of knowledge, and not only because
 knowledge cannot develop. Lest everything be
 knowledge, it cannot be the development which
 the acquisition of knowledge is either. It can
 include the development which the acquisition of
 knowledge is. But if so, it has to be separated
 sharply into two, not acquisition of knowledge,
 first, on one side, acquisition of knowledge, later
 on, on the other. This is the problem of the begin-
 ning of the acquisition of knowledge.

Later on, everything is not knowledge either.
 Later on, once the acquisition of knowledge has
 begun, all of the acquisition has to be separated

sharply, on one side, from everything else, from
 all that is not the acquisition, on the other. This is
 the problem of collecting the acquisition in its
 entirety.

And in Ptolemy there is a solution of one part
 of the problem of the beginning of the acquisition
 and of one part of the problem of collecting the
 acquisition in its entirety. In each case of one part
 only, but in each case of that part a complete
 solution.

Ptolemy writes: mathematics alone is certain
 and unshakable knowledge. And in order to
 acquire certain and unshakable knowledge Ptol-
 emy collected observations. He writes: the longer
 the time between observations, the greater the
 accuracy with which periods of revolution can be
 determined; we must thus endeavor to increase
 the accuracy of our propositions to the degree to
 which the length of time between us and the most
 ancient accurate observations allows; we must not
 neglect to examine all such records. So Ptolemy
 did not neglect to examine all such records, and
 collected all accurate observations.

Collected the labors of his predecessors.

And thus solved one part of the problem of the
 beginning of the acquisition of knowledge com-
 pletely. He found the beginning of accurate
 observation. Accurate observation begins with the
 predecessor of the philosophers who made 3
 lunar eclipse observations in Babylon from 721 to
 720.

Ptolemy thus solved one part of the problem of
 collecting the acquisition of knowledge in its
 entirety completely as well. He collected all
 accurate observations from the first through his
 own. All accurate observations from the first
 through Ptolemy's own are 94, made by these 17
 predecessors of the philosophers and philoso-
 phers,

3 lunar eclipse observations from 721 to 720,
 1 lunar eclipse observation in 621,
 3 lunar eclipse observations from 523 to 491,
 1 by Meton and Euctemon in 432,
 3 lunar eclipse observations from 383 to 382,
 6 by Timocharis from 295 to 272,
 1 by Aristarchus in 280,
 8 by Dionysius from 272 to 241,
 3 observations of Mercury and Saturn from 245
 to 229,

4 lunar eclipse observations from 201 to 174,
 3 autumn equinox observations from 162 to 158,
 14 by Hipparchus from 147 to 127^{BC},
 1 by Agrippa in 92^{AD},
 2 by Menelaus in 98,
 1 lunar eclipse observation in 125,
 4 by Theon of Smyrna from 127 to 132,
 36 by Ptolemy from 127 to 141.

The first accurate observation cannot not attest the acquisition of knowledge. Whether it attests the beginning of the acquisition as well, or rather, points to the beginning of the acquisition earlier.

All accurate observations from the first through Ptolemy's cannot not attest the acquisition of knowledge either. Whether all 94 attest the acquisition, or rather, only some of them, those that advanced the acquisition, do.

The solution of the two problems stands on the labors of Ptolemy as they stand on the labors of his predecessors, exclusively. It stands on the fact of the acquisition of knowledge, and the fact that the acquisition of knowledge is cumulative.

Hence, as Ptolemy solved the problems of the beginning of the acquisition and of collecting it in its entirety with respect to one part of mathematics, so the problems can be solved with respect to all parts of mathematics. As they can be solved with respect to mathematics, so they can be solved with respect to physics, and politics, and philosophy. And negative as the solution with respect to those will have to be, so the problems can be solved with respect to the arts aiming at beauty, and the grinding of corn, and experience, and memory, and all the way back to perception.

Beginning with the particulars, and that is, with the labors of the philosophers and their predecessors, and that is, with the success of these labors. And that is, not with an organism, but with the fact of the acquisition of knowledge.

Hume and Kant show why.

Everything of which organism and philosopher, both, are aware terminates in experience. Everything of which oak, elm, Adam, Aristotle, Ptolemy, and Hume are aware terminates in experience. Hume writes: the most ignorant and stupid peasants, nay infants, nay brute beasts, improve by experience and learn the qualities of natural objects by observing the effects which

result from them; experimental reasoning itself, which we possess in common with the beasts, is nothing but a species of instinct, or mechanical power. When brute beast infers from the fluidity and transparency of water that it will suffocate him, and Adam infers from the light and warmth of fire that it will consume him, these are inferences from experience, and what Aristotle has collected in his treatises, Ptolemy in his, and Hume in his, are inferences from experience.

The entire development, and every part of it, perception, memory, experience, the grinding of corn, the arts aiming at beauty, mathematics, politics, physics, philosophy, resolves itself into experience, and hence into inferences from experience.

At the far end: all perceptions resolve themselves into impressions and ideas; and all ideas are copied from impressions. Given experience, impressions are nothing but part of experience. At the near end: mathematicians pretend that their ideas must be comprehended by pure and intellectual view of which the superior faculties of the soul alone are capable; and philosophers are fond of this notion of spiritual and refined perception, by which they cover their absurdities, and refuse to submit to the decision of clear ideas by appealing to such as are obscure and uncertain; but to destroy this artifice, we need but reflect on the principle that all ideas are copied from impressions. And given experience, impressions are nothing but part of experience.

Perception through philosophy, the development is in its entirety the development of inferences from experience.

Thus, when Ptolemy collected the accurate observations of his predecessors, he collected inferences from experience. And he thus separated these inferences from experience from all other inferences from experience. He separated them from all earlier inferences from experience, and from all other contemporary inferences from experience. These 94 inferences from experience, on one side, all other inferences from experience made by oaks, elms, brute beasts, Adam, infants, the most ignorant and stupid peasants, mathematicians, and philosophers, on the other side.

And as these inferences from experience can be separated from all other inferences from

experience, so all inferences from experience can be divided into these that are part of the acquisition of knowledge, on one side, and those that are not part of it, on the other.

Because Ptolemy's collection of earlier accurate observations is part of the success of his labors to make more accurate observations. The earlier collection is part of the later success. His collection of earlier mathematics is part of the success of his labors to continue the advance of mathematics. His collection of earlier acquisition of knowledge is part of the success of his labors to continue the acquisition of knowledge.

If experience is the foundation of the acquisition of knowledge, all earlier acquisition of knowledge has to be part of the foundation of all later acquisition of knowledge, and the more successful the earlier acquisition of knowledge the more it has to be the foundation of the later acquisition of knowledge exclusively. Conversely, the later the acquisition of knowledge, and the more successful it is, the more unambiguously, and the more exclusively, it has to point to all earlier acquisition of knowledge.

It follows from Kant's discovery, that experience has form.

In order to solve the problem on which everything depends, how are synthetic *a priori* propositions possible? Kant solves the problems, how is mathematics possible? and how is physics possible? In order to investigate knowledge, he investigates mathematics and physics.

He writes: whether the labor of knowledge travels the secure path of science can be judged by success. And he judges by success. Logic entered the secure path of science with the labors of one particular philosopher, mathematics entered it with the labors of one particular philosopher, physics entered it with the labors of one particular philosopher, and metaphysics will enter it with the labors of one particular philosopher. To judge by success is to judge formally. And to judge formally is to judge propositions and inferences. In each case the labors of the particular philosopher are propositions and inferences.

Hence: the first beginning of human history does not have to be invented, it can be taken from experience. The first beginning of the acquisition of knowledge does not have to be invented either.

It can be taken from experience. To do so is to take it from the acquisition of knowledge itself. To do so is to judge by success. To do so is to judge formally. And to do so is to judge propositions and inferences.

Propositions and inferences, by being propositions and inferences, are in connection with one another. Given a number of propositions and inferences, they are in connection with other propositions and inferences, which are in connection with other propositions and inferences, and all the way to all propositions and inferences.

If knowledge is one, the acquisition of knowledge cannot not be one development. If this development occurs independently in different places and in different times, the facts of these particular developments cannot not corroborate the general fact that the acquisition of knowledge is one development, and one and the same development.

Given a number of propositions and inferences, then, the collection of all propositions and inferences has to do no more than follow the connection of the propositions and inferences with one another. And if it is possible to separate sharply the propositions and inferences which the 94 accurate observations collected by Ptolemy are, on one side, from everything else, on the other, it cannot be impossible to separate sharply the propositions and inferences which the acquisition of knowledge is, on one side, from everything else, on the other.

Darwin formulated the theory of evolution.

The theory of the origin and development of organisms cannot decide what the acquisition of knowledge is, but cannot not decide what the acquisition of knowledge is not. If the acquisition of knowledge is not produced by selection, it is produced by a principle distinct from selection, and the end of an organism cannot be the end of the acquisition of knowledge and the end of the acquisition of knowledge cannot be the end of an organism.

The theory of evolution is thus the decisive negative test of the formulation and solution of the problems of the beginning of the acquisition of knowledge and of collecting the acquisition in its entirety. Selection cannot not decide where the development from perception through philosophy

has to be divided into two, not acquisition of knowledge, on one side, and hence acquisition of knowledge, on the other. Where inferences from experience have to be divided into two, those which precede the acquisition, on one side, and hence those with which the acquisition begins, on the other. And selection cannot not decide where all subsequent development, once the acquisition has begun, has to be divided into two, not part of the acquisition, on one side, and hence part of it, on the other. Where all subsequent inferences from experience, after the first inferences attesting the acquisition, have to be divided into two, those which are not part of the acquisition, on one side, and hence those which are, on the other.

Thus, selection shows why observation is not looking or seeing. Why the grinding of corn is not part of the acquisition. Why oak and elm instance parricide and ingratitude. Why the thought gives no knowledge.

Frege discovered that the elements of a proposition are function and argument, showed that the firmest proof is free of gaps, formalized truth, generality, and existence, distinguished the reference and the true from the thought and the sense, showed that logic is a part of mathematics.

Frege's results thus have to be the decisive affirmative test of the formulation and solution of all problems before the investigation of knowledge.

Thus, if the formulation and solution of the problems has to begin with the collection of propositions and inferences, and there is always the danger of contaminating the collection with thoughts and sense, nothing can guide the collection better in the effort to follow the connection of the propositions and inferences with one another, and avoid the contamination, than the distinction of the reference and the true from the thought and the sense.

The acquisition of knowledge is a path. From the thought and the sense to the reference and the true.

And the reference and the true can guide the collection because proof can guide it. Proof is to bring a proposition in connection with the propositions which are its consequences and of which it is the consequence. And proof develops, from surmises from an insufficient number of particu-

lar cases to firmer proof, and firmer, and firmer, and to chains of inferences free of gaps. From the 3 oldest calculations of area to the book of 87 problems to the collection of 286 problems to Pythagoras to Euclid to Archimedes to Viète to Descartes to Newton to Leibniz to Euler to Gauss. From the Venus tablet of Ammisaduqa to Eudoxus to Hipparchus to Ptolemy to Copernicus to Galileo to Kepler to Newton to Laplace to Einstein to Friedmann to Hubble. From Thales to Empedocles to Boyle to Lavoisier to Clausius to Mendeleev to Maxwell to Planck to Bohr to Heisenberg. From Aristotle to Theophrastus to Linnaeus to Lamarck to Darwin to Mendel. From Aristotle to Chrysippus to Leibniz to Frege. All labors of bringing propositions in connection with the propositions which are their consequences and of which they are the consequences.

If the elements of a proposition are function and argument, the hypothesis that the acquisition of knowledge is itself a function has to be viable. It will be the function of one argument

x acquires knowledge of y .

The main difficulty with respect to the function itself will be its extent. Is it part of it? or not? To resolve this difficulty, the difficulties with respect to the value of the function will have to be resolved first. The main difficulty with respect to the value will be reality. Is it knowledge? or not? To resolve this difficulty, the difficulties with respect to the argument of the function will have to be resolved first. The main difficulty with respect to the argument will be relevance. Has it acquired knowledge? or not? To resolve this difficulty will be to collect all the relevant, and only the relevant, objects which are the argument. And if these objects are propositions and inferences, and all the propositions and inferences, and only the propositions and inferences, which constitute the acquisition of knowledge are collected, and thus separated sharply from everything else, this will have to make the hypothesis plausible.

The problem of knowledge, then, divides into two parts, knowledge and the acquisition of knowledge.

The parts of the problem of the acquisition of knowledge are the beginning of the acquisition, the collection of the acquisition in its entirety, the

relation of the acquisition and the development of philosophy, and the kinds of the acquisition.

The world cannot not be the terminal object of the acquisition of knowledge.

The reference of all true propositions is the same, the true.

The parts of the problem of knowledge are the relation of knowledge and truth, and the relation of knowledge and the world.

The problem of the acquisition of knowledge is first. And of its parts, the problems of its beginning and of collecting it in its entirety are first.

With Maxwell: not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it.

If the problems of the beginning of the acquisition of knowledge and of collecting it in its entirety are solved, how much there is of it will have been answered.

Beginning with the first philosopher, the acquisition of knowledge has been part of the development of philosophy. But the acquisition of knowledge predates the development of philosophy by millennia. And the development of philosophy has included more than the acquisition of knowledge.

The problem of the kinds of the acquisition is not which they are. The kinds are mathematics, physics, history, politics, and their subdivisions. The problem is of their relations. It is to describe how each kind has first developed, and how all kinds have developed from one another. The problem is to describe the system of the kinds.

If the problems of the relation of the acquisition of knowledge and the development of philosophy and of the kinds of the acquisition are solved, of what kind it is will have been answered.

Then, finally, the attempt to solve the problem of knowledge, of its relation to truth and to the world, can be made.

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The task, then, is to collect all particulars, and marshal them in their series and order.

And first of all, to establish the ground on which all particulars can be collected, and marshaled in their series and order.

Given a number of propositions and inferences, the collection of more propositions and inferences, and more, and more, and all the way to all propositions and inferences has to do no more than follow the connection of the propositions and inferences with one another.

But to follow the connection of the propositions and inferences with one another is difficult. Nor are any propositions and inferences just given. Propositions and inferences have had to be discovered, and their connection with one another has had in each case to be discovered. And if discovery has made it easier to show in any one case that a discovered proposition is a proposition and a discovered inference is an inference, rather than, say, a prayer, or an image saturated with feelings, discovery has not made the connection of propositions and inferences with one another much easier to follow, not even when the connection has been long discovered. The connection of images saturated with feelings with one another is easy to follow, and the connection of propositions and inferences with one another is not easy to follow, because the connection of images saturated with feelings with one another is easy to follow.

If propositions and inferences have been discovered, and the labors of the philosophers attest the discovery, the discovered propositions and inferences have to be recovered from the labors of the philosophers. The labors of the philosophers have to be sorted out into propositions and inferences, and the propositions and inferences sorted out into these that constitute the acquisition of knowledge and these that do not.

These are the particulars, the labors of the philosophers and their predecessors.

The collection of the particulars has to begin with the labors of the philosophers and their predecessors.

And this, now, cannot be so difficult.

Indeed, the collection can begin with any philosopher. There is the danger, given how ubiquitously the appellation philosopher has been claimed, that the collection may then begin with what appears to be a philosopher, but is not, in fact, a philosopher. If, however, the cumulative labors of the philosophers decide, by their accumulation, who is a philosopher, who is not, as the

collection proceeds from the labors of one philosopher, whether only apparent or real, to another, and to another, and to another, if it ever hits upon one real philosopher, the preceding errors in the collection will begin to correct themselves, and will eventually correct themselves entirely.

Nor does the collection have to begin with any philosopher. Having accumulated for some millennia, the labors of the philosophers have long decided who is more likely, and who is very much more likely, to be, in fact, a philosopher.

Moreover, the second rule is, to divide difficulties into parts such as will lead to their better solution, the third is, to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex.

The collection should begin with an important philosopher or predecessor of the philosophers, and the more important the better, and it should begin early rather than late, and the earlier the better.

Thus, clearly, Aristotle can be an excellent beginning. In fact, Aristotle began the collection himself. Aristotle discovered the review of the earlier labors of formulation and solution of problems as the ground of the formulation and solution of problems. He reviews his predecessors in all his treatises, and reviews them systematically not less than 16 times. Each of his reviews is a collection of propositions and inferences. Each of his reviews is thus also a partial solution of the problems of the beginning of the acquisition of knowledge and of collecting it in its entirety.

Aristotle writes: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

Aristotle's discovery that there is the council of our predecessors, and the council of our predecessors can instruct us into adopting the sound and avoiding the erroneous, makes it possible to begin earlier. The collection can begin as early as there are anticipations of the council.

Thales is the first philosopher, Anaximander is the second, Anaximenes is the third, Pythagoras is the fourth, Xenophanes is the fifth. The fifth philosopher, Xenophanes is the first to leave his own words behind. He has left behind 120-odd

lines, 40-odd fragments. And in these he notes, and judges, a number of his predecessors. The first philosopher to leave his own words behind, Xenophanes is also the first to judge his predecessors explicitly.

The collection can thus begin with Xenophanes, two centuries before Aristotle, very early in the development of philosophy.

But it can begin earlier still, much earlier, and long before the development of philosophy.

The method is not new, Descartes writes. Twenty centuries before Descartes, all of Aristotle's successes are successes of the method. The convocation of the council of our predecessors is itself a success of the method.

The successes of the method begin with the Sumerian prince list. And the Sumerian prince list is a collection of its predecessors. A collection of the labors of its predecessors.

It is with the Sumerian prince list that the collection should begin, seventeen centuries before Aristotle, fifteen before Xenophanes, fourteen and a half before the first philosopher.

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The ground, then, on which all particulars can be collected, and marshaled in their series and order, is established in three steps.

The predecessors of the philosophers and philosophers are propositions. These are the particulars. Some of them are important. And some of these are the steps of advance in the acquisition of knowledge.

The first step is thus grounded in the first great success of the method, the Sumerian prince list. The particulars collected by the Sumerian prince list are sorted out. And on this ground it is shown that all the particulars can be collected.

The second step is grounded in the first philosophers who judge their predecessors. These are the first philosophers who have left behind their own words, the fifth philosopher, the seventh, and the eighth, Xenophanes, Hecataeus, and Heraclitus. The particulars collected by Xenophanes, Hecataeus, and Heraclitus are sorted out. And on this ground it is shown that importance is a fact, and all the important can be collected.

The third step is grounded in the philosopher who discovered the systematic review of our

predecessors, Aristotle. The particulars collected by Aristotle in the course of the most important session of the council of our predecessors are

sorted out. And on this ground it is shown that all the steps of advance in the acquisition of knowledge can be collected.

The first complete enumeration of our predecessors

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The method is not new, Descartes writes, for nothing is older than truth. Nothing is older than truth, truth is as old as the world. The search for truth is not so old. It is yet older than Descartes, older than Aristotle, and older than the first philosopher. It is older than the Sumerian prince list, too, but not much older in this case. Success in the search for truth is not as old as the search for truth. And older than Descartes, and Aristotle, and the first philosopher, success in the search for truth is not older than the Sumerian prince list.

Excavation in Mesopotamia began in 1842, with Botta. In 1853 Rassam found the ruins of the palace of Ashurbanipal in Nineveh, including the royal library. From the royal library more than 30 000 tablets were recovered. By 1857 Rawlinson, Hincks, and Oppert had deciphered cuneiform. By 1875 Hincks and Oppert had distinguished the oldest language written in cuneiform, Sumerian, from the many languages which later borrowed cuneiform from it. Sumerian began being written five and a half millennia ago. The most important of the languages which learned to write from Sumerian are, a millennium and a half after Sumerian, Akkadian, and, in the following millennium, Babylonian and Assyrian. And in the century after Rawlinson, Hincks, and Oppert, the Sumerian record, the Akkadian record, the Babylonian record, the Assyrian record, and the entire Mesopotamian record, some millions of excavated tablets, was published.

Ashurbanipal prince 668-27, was the most powerful prince on earth. Counting back to

Enmebaragesi prince c. 2680, the first prince to leave his inscriptions behind, Ashurbanipal was also the successor of twenty centuries of princes, some 67 generations of princes. More immediately, in the seven centuries

before him, beginning with

Ashur-uballit prince c. 1363-27, the Assyrian princes dominated Mesopotamia, and in the century through Ashurbanipal, beginning with

Tiglath-pileser III prince 745-27, the Assyrian princes were the undisputed masters of all Mesopotamia, and became the masters of Egypt, too. In the century between Tiglath-pileser III and Ashurbanipal, the Assyrian was the most powerful state the earth had seen. The most powerful state the earth had seen was the successor of two millennia of development. In these two millennia mathematics had also greatly developed, history had greatly developed, physics had greatly developed.

Counting from Ashurbanipal to Rawlinson, Hincks, and Oppert, these are twenty-five centuries of nearly complete ignorance of Mesopotamian antiquity.

The recovery of Egyptian antiquity began with Napoleon's expedition to Egypt in 1798-9. Among the philosophers who accompanied Napoleon were Monge, Berthollet, Fourier, Geoffroy Saint-Hilaire, Malus. Among the finds of the expedition was the Rosetta stone. By 1824 Young and Champollion had deciphered hieroglyphic. In the following century-odd Egypt was excavated, and the Egyptian record was published.

The development of the state in Egypt, and the first prince

Narmer prince c. 3000, predate Enmebaragesi by three-odd centuries.

The centuries of nearly complete ignorance of Mesopotamian antiquity are centuries also of nearly complete neglect of Egyptian antiquity.

Two centuries after Ashurbanipal, Herodotus wrote the *Histories*. And in the *Histories* Herodotus established the general fact of Egyptian antiquity, and collected some of the relevant

particular facts. The investigation of Egyptian antiquity begins with Herodotus, and begins with him most successfully. One and a half centuries later, Manetho wrote the *Egyptian history*. Its excellence, the great number of relevant particular facts, and the great advance in the investigation of Egyptian antiquity, are all attested by the fact that Manetho's history remains, and will forever remain, the foundation of the investigation of Egyptian antiquity. Herodotus did not establish the general fact of Mesopotamian antiquity, and collected but a few of the relevant particular facts. He yet writes: there were many princes of Babylon, and I will report on them in my Assyrian book (I, 184), even as this book remained unwritten. One and a half-odd centuries later, in the generation after Manetho, Berossus wrote the *Babylonian history*. It is not as successful as Manetho's history, concerned as Berossus is with divinities more than with princes. It is nevertheless the beginning of the investigation of Mesopotamian antiquity.

Counting from Manetho to Young and Champollion and from Berossus to Rawlinson, Hincks, and Oppert, these are twenty-one and a half centuries of complete neglect of Egyptian antiquity and complete ignorance of Mesopotamian antiquity.

The exceptions are two. One is Ptolemy. The other are the philosophers who undertook to defend the scriptural account of the antiquity of humanity against: Manetho, who is a ridiculous liar (Josephus, *Apion*, I, 29), and: Berossus, who is a shameless liar (Syncellus, *Collection*, 25). The most important among these philosophers are Josephus, Eusebius, and Syncellus, who, by way of exposing the lies, preserved substantial fragments from Manetho and Berossus.

Thus, all that time Herodotus was there. Manetho and Berossus were there. The pyramids, the many lesser tombs, the inscriptions and papyri in the tombs, the remains of the palace of Ashurbanipal, the royal library, the millions of buried tablets, were all there. All waiting for the investigation of antiquity to continue.

The neglect and ignorance are thus not difficult to explain. Leibniz writes: it is astonishing that so many useful things remain to be done, yet men always turn to what has already been done,

or to the utterly useless, or in any case to the least important. It is not astonishing. It is the explanation of neglect and ignorance.

And the importance of Mesopotamian and Egyptian antiquity cannot be overstated. The 3 oldest calculations of area predate the first prince by not quite one century. Counting back to either, the 3 oldest calculations of area or the first prince, the facts of Mesopotamian and Egyptian antiquity are the facts of one half of the development of everything that postdates the development of organisms.

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The first tablet containing a fragment of what turned out to be the Sumerian prince list was published in 1906, by Hilprecht. In the following decades more tablets were published in which the list was recognized as a prince list. And in 1939, in *The Sumerian king list*, Jacobsen collected all the tablets on which the list was recorded, analyzed them into fourteen manuscripts, established that the manuscripts are all derived from a single original, edited them, and published the principal edition of the list.

The list has since gone by the name Sumerian king list. So have its successors, the Larsa king list, the Babylonian king list, the Assyrian king list. But the fact alone that Enmebaragesi, the first prince in the Sumerian list attested independently of the list, was forty-seven centuries ago, and even Shalmaneser V, the last prince in the Assyrian list, was twenty-seven and a half centuries ago, should suffice to show that Enmebaragesi and Shalmaneser V are not kings. If the great kings and the Spartan kings, the Roman emperors and the Roman pontiffs, the caliphs and the sultans, the French kings and the Russian tsars, are, each in particular, all princes, in general, and not the other way around, Enmebaragesi and Shalmaneser V are kings not more than they are popes. The lists are prince lists.

The name is not very important. The following is very important.

The list has since been published and investigated such as Jacobsen published it. Such as Jacobsen published it, however, the list is not one list, it is three lists.

The concerns of Jacobsen are primarily philo-

logical, and so have been the concerns of the few who have since investigated the list. And important as the philological concerns may have been, they have obscured two very important facts established by Jacobsen himself.

Such as Jacobsen published it, the list is a list of 144 princes of 16 cities.

It begins:
when princeship descended from heaven, it came to Eridu;

Eridu,

Alulim was prince, he reigned 28 800 years,

Alalgar, he reigned 36 000 years,

2 princes, they reigned 64 800 years;

princeship was then taken to Badtibira;

Badtibira,

Enmeluana, he reigned 43 200 years,

Enmegalana, he reigned 28 800 years,

divine Dumuzi, he was a shepherd, he reigned 36 000 years,

3 princes, they reigned 108 000 years;

princeship was then taken to Larsa;

Larsa,

Ensipaziana, he reigned 28 800 years,

1 prince, he reigned 28 800 years;

princeship was then taken to Sippar;

Sippar,

Enmedurana, he reigned 21 000 years,

1 prince, he reigned 21 000 years;

princeship was then taken to Shuruppak,

Shuruppak,

Ubartutu, he reigned 18 600 years,

1 prince, he reigned 18 600 years;

5 cities, 8 princes, they reigned 241 200 years;

then the flood swept over (*id.*).

This is one list. The antediluvian prince list.

Such as Jacobsen published it, the list ends with 15 princes of Isin.

This is another list. The Isin prince list.

The rest is a list of 121 princes of 10 cities. In this order. 23 princes of Kish, 12 princes of Uruk, 4 princes of Ur, 3 princes of Awan, 8 more princes of Kish, 1 prince of Hamazi, 3 more princes of Uruk, 3 more princes of Ur, 1 prince of Adab, 6 princes of Mari, 1 more prince of Kish, 6 princes of Akshak, 7 more princes of Kish, 1 more prince of Uruk, 11 princes of Akkad, 5 more princes of Uruk, 20 princes of the Gutian horde, 1 more prince of Uruk, 5 more princes of

Ur.

And this is the list. The Sumerian prince list.

The fourteen manuscripts edited by Jacobsen are all derived from a single original. Four of the manuscripts begin with the princes of Kish. Three begin with the antediluvian princes. Seven are too fragmentary to warrant a judgment. On philological grounds Jacobsen determined that the list of antediluvian princes is a later addition to the original list. He determined also that the original list predates the princes of Isin.

The last five princes in the Sumerian prince list, with their dates, are

Ur-Nammu	prince c. 2121-03,
Shulgi	prince c. 2103-055,
Amar-Sin	prince c. 2055-46,
Shu-Sin	prince c. 2046-37,
Ibbi-Sin	prince c. 2037-13.

The Sumerian prince list presumes that princeship is one, and at any one time the princes of one city have it. And under these princes princeship was, in fact, one, and these princes of Ur, in fact, had it. These princes dominated the valley of the two rivers and the cities in it, unquestionably under Ur-Nammu, Shulgi, Amar-Sin, and Shu-Sin. The time of these princes is the time also of the composition of the bulk of the Sumerian record, by far. And this is the time of the composition of the Sumerian prince list. In particular, given that Ibbi-Sin is the last of these princes, and the last prince in the list, the Sumerian prince list was composed under Ibbi-Sin.

Under Ibbi-Sin the princes of Ur lost their dominion over the valley. The one princeship was divided among the cities. A general of Ibbi-Sin,

Ishbi-Erra	prince c. 2027-1994,

became the independent prince of Isin. Ishbi-Erra is the first prince in the Isin prince list. The Larsa prince list reaches back to

Naplanum	prince c. 2024-04.

The Assyrian prince list reaches back to: 17 princes who lived in tents (*id.*), followed by: 10 princes who were ancestors (*id.*), followed by: 6 princes whose names were written on bricks (*id.*). The fourth of the princes whose names were written on bricks is

Puzur-Ashur	prince c. 1980.

The Babylonian prince list reaches back to

Sumuabum	prince c. 1894-80.

The Isin, the Larsa, the Assyrian, and the Babylonian are each a list of the one succession of the princes of one city. They are all four local. Three of them, the Isin, the Larsa, and the Babylonian, are each also contemporary with the succession. Given the precedent of the Sumerian list, and the steadily growing abundance of the record of the princes, these lists are each an extract from the local contemporary records.

The Sumerian is a list of the several successions of the princes of 10 cities. It is not local, and it is not contemporary with the successions. In space, it reaches from Mari in the northwest to Awan in the southeast across 730 km along the great circle, $\frac{1}{55}$ of the circumference of the earth.

In time, it reaches back to Enmebaragesi, six and a half centuries before the time of composition of the list, $\frac{1}{9}$ of the time since the 3 oldest calculations of area and the first prince.

The Sumerian prince list is an investigation.

And the importance of the fact that the Sumerian prince list is an investigation its successors show. They learned from it immediately to record the local princes, and arrange them in their series and order. They learned from it immediately as well, as the antediluvian prince list attests, to reduce old traditions to lists. How well this last lesson was learned the Assyrian prince list shows, concerned as it is with princes who lived in tents and princes who were ancestors. Later on the Hebrew scriptures show it.

To investigate, the successors of the Sumerian prince list took twelve centuries to learn. The oldest document, after the Sumerian prince list, which attests investigation of the past is the synchronistic chronicle. The synchronistic chronicle correlates the successions of the Assyrian and Babylonian princes. It was composed under

Adad-nirari III prince 811-783.

Similarly, five centuries separate the 3 oldest calculations of area, from the century around *c.* 3080, and the next advance in mathematics, the problem of division of grain among 164 571 men, and the table of squares with sides from 10×60 to $\frac{1}{12} \times 60$, from the century around *c.* 2580, and not quite four centuries separate this advance and the

next, the problem of the area of a quadrilateral with sides 2, $1\frac{1}{6}$, 2, and $2\frac{5}{6}$, and the problem of

the width of a rectangle with a length $67\frac{1}{2}$ and an

area 100, from the century before *c.* 2156. Nine centuries also separate the discovery of the planets, the Venus tablet of Ammisaduqa, and the next observation of the planets. Venus was discovered under

Ammisaduqa prince *c.* 1646-25.

The oldest accurate observation recovered by Ptolemy is of lunar eclipses from 721 to 720. The oldest observation of the planets recovered from the millions of tablets excavated in the last couple of centuries, besides the Venus tablet of Ammisaduqa, is one generation older, of lunar eclipses from 747 to 746.

Other than mathematical, the Sumerian prince list is the first, and for a long time the only, investigation.

First, then, the magnitude of the success of the investigation can be shown. Then the particulars can be collected and sorted out.

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To begin with, below is the list.

There are variants in a few of the numbers of the reign years in the different manuscripts. These are ignored. None of them concerns the reign years of the later princes, those of Akkad or their successors, which are corroborated abundantly by the inscriptions of the princes. Also, fond as the list is of addition, a couple of its totals have had to be corrected. As has had to be, above, the total of the antediluvian prince list. Also, there are six lacunae, four names of princes are lacking in all manuscripts, and two numbers of reign years. None of these is very important. Finally, on the presumption of corruption in the transmission, five of the names in the list have been corrected into the names attested by the inscriptions of the princes, Elulu of Ur into Elili, Lugalkinshedudu of Uruk into Lugalkignedudu, Elulu of Akkad into Eluldan, and Laerabum and Yarlaganda of the Gutian horde into Laerab and Yarlagan.

And this is the list:

when princship descended from heaven, it came to Kish;

Kish,

Gashur was prince, he reigned 1 200 years,
Kullassinabel, he reigned 900 years,
Nangishlishma, he reigned 670 years,
Endaraana, he reigned 420 years, 3 months,
and $3\frac{1}{2}$ days,

Babum, he reigned 300 years,
Puannum, he reigned 840 years,
Kalibum, he reigned 960 years,
Kalumum, he reigned 840 years,
Zuqaqip, he reigned 900 years,
Atab, he reigned 600 years,
Mashda, son of Atab, he reigned 840 years,
Arwium, son of Mashda, he reigned 720 years,
Etana, he was a shepherd, he went to heaven,
he made all the lands firm, he reigned 1 500
years,

Balih, son of Etana, he reigned 400 years,
Enmenuna, he reigned 660 years,
Melamkish, son of Enmenuna, he reigned 900
years,

Barsalnuna, son of Enmenuna, he reigned
1 200 years,

Samug, son of Barsalnuna, he reigned 140
years,

Tizqar, son of Samug, he reigned 305 years,

Ilku, he reigned 900 years,

Itasadum, he reigned 1 200 years,

Enmebaragesi, he smote Elam, he reigned 900
years,

Aka, son of Enmebaragesi, he reigned 625
years,

23 princes, they reigned 17 920 years, 3
months, and $3\frac{1}{2}$ days;

Kish was smitten, princesship was taken to Uruk;
Uruk,

Meskiaggasher, son of Utu, he reigned 324
years, he went into the sea and came out in
the mountains,

Enmerkar, son of Meskiaggasher, he built
Uruk, he reigned 420 years,

divine Lugalbanda, he was a shepherd, he
reigned 1 200 years,

divine Dumuzi, he was a fisherman, his city
was Kuara, he reigned 100 years,

divine Gilgamesh, his father was a ghost, he
was the lord of Kulaba, he reigned 126

years,

Ur-Nungal, son of divine Gilgamesh, he
reigned 30 years,

Utulkalama, son of Ur-Nungal, he reigned 15
years,

Labashir, he reigned 9 years,

Enundarana, he reigned 8 years,

Meshe, he was a smith, he reigned 36 years,

Melamana, he reigned 6 years,

Lugalkigin, he reigned 36 years,

12 princes, they reigned 2 309 years;

Uruk was smitten, princesship was taken to Ur;

Ur,

Mesanepada, he reigned 80 years,

Meskiagnuna, son of Mesanepada, he reigned
36 years,

Elili, he reigned 25 years,

Balulu, he reigned 36 years,

4 princes, they reigned 177 years;

Ur was smitten, princesship was taken to Awan;

Awan,

. . . , he reigned . . . years,

. . . , he reigned . . . years,

. . . , he reigned 36 years,

3 princes, they reigned 356 years;

Awan was smitten, princesship was taken to Kish;

Kish,

Susuda, he was a fuller, he reigned 200 years,

Dadasig, he reigned 81 years,

Mamagal, he was a boatman, he reigned 360
years,

Kalburn, son of Mamagal, he reigned 195
years,

Tuge, he reigned 360 years,

Menuna, son of Tuge, he reigned 180 years,

Enbi-Eshtar, he reigned 290 years,

Lugalumu, he reigned 360 years,

8 princes, they reigned 2 026 years;

Kish was smitten, princesship was taken to
Hamazi;

Hamazi,

Hatanish, he reigned 360 years,

1 prince, he reigned 360 years;

Hamazi was smitten, princesship was taken to
Uruk;

Uruk,

Enshakushana, he reigned 60 years,

Lugalkiginedudu, he reigned 120 years,

Argandea, he reigned 7 years,

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3 princes, they reigned 187 years;
Uruk was smitten, princesship was taken to Ur;
Ur,
Nane, he reigned 54 years,
Meskiagnuna, son of Nane, he reigned 48 years,
... , son of Meskiagnuna, he reigned 2 years,
3 princes, they reigned 104 years;
Ur was smitten, princesship was taken to Adab;
Adab,
Lugalanemundu, he reigned 90 years,
1 prince, he reigned 90 years;
Adab was smitten, princesship was taken to Mari;
Mari,
Anbu, he reigned 30 years,
Anba, son of Anbu, he reigned 17 years,
Bazi, he was a leather worker, he reigned 30 years,
Zizi, he was a fuller, he reigned 20 years,
Limer, he was a priest, he reigned 30 years,
Sharumiter, he reigned 9 years,
6 princes, they reigned 136 years;
Mari was smitten, princesship was taken to Kish;
Kish,
Ku-Baba, she was an innkeeper, she made
Kish firm, she reigned 100 years,
1 prince, she reigned 100 years;
Kish was smitten, princesship was taken to Akshak;
Akshak,
Unzi, he reigned 30 years,
Undalulu, he reigned 6 years,
Uruk, he reigned 6 years,
Puzur-Nirah, he reigned 20 years,
Ishuil, he reigned 24 years,
Shu-Sin, son of Ishuil, he reigned 7 years,
6 princes, they reigned 93 years;
Akshak was smitten, princesship was taken to Kish;
Kish,
Puzur-Sin, son of Ku-Baba, he reigned 25 years,
Ur-Zababa, son of Puzur-Sin, he reigned 400 years,
Simudar, he reigned 30 years,
Usiwatar, son of Simudar, he reigned 7 years,
Eshtarmuti, he reigned 11 years,
Ishme-Shamash, he reigned 11 years,
Nannia, he was a stonecutter, he reigned 7

years,
7 princes, they reigned 491 years;
Kish was smitten, princesship was taken to Uruk;
Uruk,
Lugalzagesi, he reigned 25 years,
1 prince, he reigned 25 years;
Uruk was smitten, princesship was taken to Akkad;
Akkad,
Sargon, his father was a gardener, he was a cupbearer of Ur-Zababa, he built Akkad, he reigned 56 years,
Rimush, son of Sargon, he reigned 9 years,
Manishtushu, older brother of Rimush, son of Sargon, he reigned 15 years,
Naram-Sin, son of Manishtushu, he reigned 37 years,
Sharkalisharri, son of Naram-Sin, he reigned 25 years;
then who was prince? who was not? Igigi was prince, Nanum was prince, Imi was prince,
Eluldan was prince, 4 princes in 3 years,
then Dudu, he reigned 21 years,
Shudurul, son of Dudu, he reigned 15 years,
11 princes, they reigned 181 years;
Akkad was smitten, princesship was taken to Uruk;
Uruk,
Ur-Nigin, he reigned 7 years,
Ur-Gigir, son of Ur-Nigin, he reigned 6 years,
Kuda, he reigned 6 years,
Puzur-Ili, he reigned 5 years,
Ur-Utu, he reigned 6 years,
5 princes, they reigned 30 years;
Uruk was smitten, princesship was taken to the Gutian horde;
the Gutian horde,
they had no prince at first,
then Imta, he reigned 3 years,
Inkishush, he reigned 6 years,
Sarlagab, he reigned 6 years,
Shulme, he reigned 6 years,
Silulumesh, he reigned 6 years,
Inimabakesh, he reigned 5 years,
Igeshaush, he reigned 6 years,
Yarlagab, he reigned 15 years,
Ibate, he reigned 3 years,
Yarla, he reigned 3 years,
Kurum, he reigned 1 year,

Apilkin, he reigned 3 years,
 Laerab, he reigned 2 years,
 Irarum, he reigned 2 years,
 Ibranum, he reigned 1 year,
 Hablum, he reigned 2 years,
 Puzur-Sin, son of Hablum, he reigned 7 years,
 Yarlagan, he reigned 7 years,
 Sium, he reigned 7 years,
 Tirigan, he reigned 40 days,
 20 princes, they reigned 91 years and 40 days;
 the Gutian horde was smitten, princship was
 taken to Uruk;

Uruk,
 Utuhegal, he reigned 7 years, 6 months, and 7
 days,
 1 prince, he reigned 7 years, 6 months, and 7
 days;

Uruk was smitten, princship was taken to Ur;
 Ur,

Ur-Nammu, he reigned 18 years,
 divine Shulgi, son of Ur-Nammu, he reigned
 48 years,
 divine Amar-Sin, son of divine Shulgi, he
 reigned 9 years,
 Shu-Sin, son of divine Amar-Sin, he reigned 9
 years,
 Ibbi-Sin, son of Shu-Sin, he reigned 24 years,
 5 princes, they reigned 108 years (*id.*).

121 princes.

But this number can be corrected to 120.

The first two princes in the first series of
 princes of Ur are:

Mesanepada,

Meskiagnuna, son of Mesanepada.

The first two princes in the second series of
 princes of Ur are:

Nane,

Meskiagnuna, son of Nane.

Mesanepada, Aanepada, and Meskiagnuna are
 among the princes who left their inscriptions
 behind. Mesanepada reports:

Mesanepada, prince of Ur (ins. 1).

Aanepada reports:

Aanepada, prince of Ur, son of Mesanepada,
 prince of Ur (ins. 3).

Meskiagnuna reports:

Meskiagnuna, prince of Ur (ins. 1).

Nearly five centuries separate these princes and
 the composition of the list. It can be concluded

that Aanepada has been corrupted to Nane, and
 that the one series of three consecutive princes
 has become two series of two princes each in the
 list.

120 princes, then.

2 were shepherds, 1 was a smith, 2 were
 fullers, 1 was a boatman, 1 was a leather worker,
 1 was a priest, 1 was an innkeeper, 1 was a stone-
 cutter, 1 was a cupbearer. 1 went to heaven, 1
 went into the sea and came out in the mountains.
 Of 1 the city was Kuara, 1 was the lord of
 Kulaba. 1 was the son of Utu, 1 was the son of a
 ghost, 1 was the son of a gardener, 32 were the
 sons of earlier princes. 1 made all the lands firm.
 1 smote Elam. 1 built Uruk. 1 made Kish firm. 1
 built Akkad. The length of their reigns varied,
 from a maximum of 1 500 years to a minimum of
 40 days.

Their cities were Kish, Uruk, Ur, Awan,
 Hamazi, Adab, Mari, Akshak, Akkad, and the
 Gutian horde.

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This is the list. Then there is all the independ-
 ent evidence about the same objects of investiga-
 tion. The two, the list and the independent evi-
 dence, can be examined against one another, and
 conclusions can be drawn, about the princes,
 about their cities, about reality, about the list,
 about the independent evidence, and about the
 beginning of investigation.

The main difficulties which the list raises are
 two.

One is chronology. The solution of the prob-
 lem of the chronology of the Sumerian princes,
 however, is inseparable from the solution of the
 problem of the chronology of Mesopotamian
 antiquity in general, which is inseparable from
 the solution of the problem of the chronology of
 Egyptian antiquity, and these are both inseparable
 from the solution of the problem of the chronol-
 ogy of antiquity in general. The problem of
 chronology demands special treatment, and this
 is best postponed. Here the solution of the prob-
 lem of chronology, in general, and of the chronology
 of the Sumerian princes, in particular, is taken for
 granted.

The other main difficulty is reality. Some of
 the princes in the list reigned long, longer than

princes more recently have reigned. Or lived. This is not unusual. On the authority of *Genesis*: Adam, he lived 930 years; Seth, he lived 912 years; Enosh, he lived 905 years (5). On the other hand, however, not all the princes in the list reigned that long, some reigned not longer than princes more recently have reigned. Or lived. Some of the princes in the list were the sons of the sun and of a ghost, respectively. This is not unusual either, considering whose sons Adam or Jesus were. Again, however, on the other hand, some of the princes were the sons of gardeners and of earlier princes, respectively. Some of the princes in the list went to heaven and into the sea. On the other hand, again, however, some of the princes smote Elam, built Uruk, made Kish firm, built Akkad. The difficulty is, what is real? what is not? who was, in fact, a prince? who was not?

The independent evidence is, first of all, and most important, by far, the inscriptions which the princes left behind.

And given the list and the inscriptions of the princes, three kinds of princes can be identified. First, there are the princes attested both by the list and by the inscriptions, and either their own inscriptions or the inscriptions of other princes. Next, there are the princes not attested by the list, but attested either by their own inscriptions and the inscriptions of other princes or by the inscriptions of other princes only. Finally, there are the princes attested by their own inscriptions only.

Of the first kind there are 30 princes.

27 of the princes in the list left their inscriptions behind. In the order of the list, Enmebaragesi, Mesanepada, Aanepada, Meskiagnuna, Elili, Enshakushana, Lugalkignedudu, Lugalzagesi, Sargon, Rimush, Manishtushu, Naram-Sin, Sharkalisharri, Eluldan, Dudu, Shudurul, Ur-Nigin, Ur-Gigir, Laerab, Yarlagan, Sium, Utuhegal, Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin.

Enmebaragesi is the first of the princes in the list to leave his inscriptions behind. Enmebaragesi is also the first prince to leave his inscriptions behind. In particular, he left 1 inscription behind. Enmebaragesi reports:

Enmebaragesi, prince of Kish (ins. 1).

Mesanepada is the second of the princes in the list to leave his inscriptions behind. One century

after Enmebaragesi, Mesanepada is also the third prince to leave his inscriptions behind. Accordingly, he left 3 inscriptions behind, and he has more to report. Mesanepada reports:

Mesanepada, prince of Ur, son of Meskalamdug, prince of Kish (ins. 1).

Mesanepada is also the second prince attested both by his own inscriptions and by the inscriptions of another prince. Aanepada reports:

Aanepada, prince of Ur, son of Mesanepada, prince of Ur (ins. 3).

Aanepada left 6 inscriptions behind, Meskiagnuna left 1 inscription behind, Elili left 1 inscription behind, Enshakushana left 4 inscriptions behind.

One century after Mesanepada, Lugalkignedudu left 6 inscriptions behind. He reports:

Lugalkignedudu, prince of Kish, prince of Uruk, prince of Ur (ins. 2).

Thus, if Mesanepada does not yet, Lugalkignedudu does attest that, long before the list presumes that princship is one, the princes themselves presume that princship is one.

One century after Lugalkignedudu, Lugalzagesi left 1 inscription behind. He reports:

Lugalzagesi, prince of Uruk, son of Uu, prince of Umma; Enlil gave Lugalzagesi the lordship of the land, put all lands at his feet, from the east to the west; from the lower sea, along the two rivers, to the upper sea, Enlil gave him no rival; the princes of Sumer submitted to him at Uruk (ins. 1).

Sargon, for his part, reports:

Sargon, prince of Akkad, prince of Kish, lord of the land; he conquered Uruk and razed its walls; he brought Lugalzagesi, the prince of Uruk, in a dog collar to the temple of Enlil (ins. C-1).

Sargon left behind more than 2 dozen inscriptions, his successor Rimush left behind 2 dozen, his successor Manishtushu 1 dozen, his successor Naram-Sin more than 8 dozen, his successor Sharkalisharri more than 7 dozen. These princes reigned, as the list attests, and the inscriptions corroborate, 142 years. And if the princes before them labored to show that princship is, in fact, one, in these 142 years these princes succeeded in showing that princship is, in fact, one. To Sargon, Rimush, Manishtushu, Naram-Sin, and

Sharkalisharri Enlil, in fact, gave no rival. From the east to the west, from the lower sea, along the two rivers, to the upper sea. From the Persian Gulf, along the two rivers, to the Mediterranean. Sargon reports the cause:

Sargon, lord of the earth, won 34 battles (ins. C-2),

as well as the cause of the cause:

5 400 men ate bread daily before Sargon (*id.*).

After Sharkalisharri princship fell apart:

who was prince? who was not?

Eluldan, Dudu, Shudurul, Ur-Nigin, Ur-Gigir, Laerab, Yarlagan, and Sium left behind from 1 to 4 inscriptions each.

Beginning with Utuhegal, two generations after Sharkalisharri, the labors to restore the unity of princship began succeeding again. Utuhegal left a half dozen inscriptions behind. Under Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin princship was one. And their numerous inscriptions are but a small fraction of the contemporary record.

3 of the princes in the list left no inscriptions behind but are attested by the inscriptions of other princes. The inscriptions of the other princes also explain why they left no inscriptions behind. Enshakushana reports:

Enshakushana, lord of Sumer, sacked Kish and captured Enbi-Eshtar, prince of Kish (ins. 1).

Sharkalisharri reports:

Sharkalisharri, prince of Akkad, captured Sarlagab, prince of the Gutians (ins. D-27).

Utuhegal reports:

Utuhegal, prince of Uruk, vanquished Tirigan, captured his men; Tirigan, prince of the Gutians, ran away; Utuhegal's men captured Tirigan, his wife, and his children (ins. 4).

All in all, these are the 30 princes attested both by the list and by the inscriptions, and either their own inscriptions or those of other princes. In chronological order, Enmebaragesi, Mesanepada, Aanepada, Meskiagnuna, Elili, Enshakushana, Enbi-Eshtar, Lugalkignedudu, Lugalzagesi, Sargon, Rimush, Manishtushu, Naram-Sin, Sharkalisharri, Sarlagab, Eluldan, Dudu, Shudurul, Ur-Nigin, Ur-Gigir, Laerab, Yarlagan, Sium, Utuhegal, Tirigan, Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin.

Their cities are Kish, Ur, Uruk, Akkad, and the

Gutian horde.

And these 30 princes are all real. Each of these 30 princes was, in fact, a prince. Across forty-seven centuries to forty-odd centuries, from

Enmebaragesi prince *c.* 2680, to

Ibbi-Sin prince *c.* 2037-13, the evidence about these princes is diamond. In how firm it is, and in how valuable, both.

Next, there are the princes not attested by the list, but attested either by their own inscriptions and the inscriptions of other princes or by the inscriptions of other princes only.

Enmebaragesi is the first prince to leave his inscriptions behind. Mesanepada is the third. The second is Meskalamdug. Meskalamdug left 1 inscription behind. He reports:

Meskalamdug, prince (ins. 1).

The second prince to leave his inscriptions behind, Meskalamdug is also the first prince attested both by his own inscriptions and by the inscriptions of another prince. Mesanepada reports: Mesanepada, prince of Ur, son of Meskalamdug, prince of Kish.

Then there are the 9 princes of Lagash, Ur-Nanshe, Akurgal, Eanatum, Enanatum, Enmetena, Enanatum II, Enentarzi, Lugalanda, and Urukagina.

Ur-Nanshe reports:

Ur-Nanshe, prince of Lagash (ins. 1),

Akurgal reports:

Akurgal, prince of Lagash, son of Ur-Nanshe (ins. 1),

Eanatum reports:

Eanatum, prince of Lagash, son of Akurgal (ins. 4),

Enanatum reports:

Enanatum, prince of Lagash, son of Akurgal, brother of Eanatum (ins. 2),

Enmetena reports:

Enmetena, prince of Lagash, son of Enanatum (ins. 5),

Enanatum II reports:

Enanatum, prince of Lagash, son of Enmetena (ins. 1),

Lugalanda reports: Lugalanda, prince of Lagash, son of Enentarzi, prince of Lagash (ins. 2),

Urukagina reports:

Urukagina, prince of Lagash (ins. 1).

The series of the princes follows.

Enmetena also reports:

Enmetena, prince of Lagash, and Lugalkiginedudu, prince of Uruk, swore to their brotherhood (ins. 3),

and, not quite so propitiously, Urukagina reports: the prince of Umma sacked Lagash; Urukagina, prince of Lagash, did not transgress; Lugalzagesi, prince of Umma, bears the guilt (ins. 5).

Given the two synchronisms, the dates of the princes of Lagash follow.

Ur-Nanshe already reports:

Ur-Nanshe, prince of Lagash, son of Gunidu; he went to war against the prince of Ur and the prince of Umma; he vanquished the prince of Ur; he vanquished the prince of Umma; he made burial mounds for them (ins. 6).

In the century from

Ur-Nanshe prince c. 2535,
through

Enmetena prince c. 2440,
the princes of Lagash were the most powerful princes in the valley of the two rivers. Ur-Nanshe left behind 2 and a half dozen inscriptions, Akurgal left behind a half dozen, Eanatum 1 dozen, Enanatum 1 and a half dozen, Enmetena 2 and a half dozen, Enanatum II 1 inscription, Enentarzi 1 inscription, Lugaland 3 inscriptions, Urukagina 1 dozen inscriptions.

Then there are the 6 more princes of other cities whom the princes of Lagash attest in their inscriptions.

Eanatum (ins. 2) and Enmetena (ins. 1) report: Mesalim, prince of Kish, erected a monument. Mesalim himself left behind 3 inscriptions.

Eanatum reports: Eanatum, prince of Lagash, vanquished Zuzu, prince of Akshak; he destroyed Akshak (ins. 5). Zuzu, accordingly, left no inscriptions behind. Nor did any other prince of Akshak.

Ur-Nanshe reports: Ur-Nanshe vanquished the prince of Umma; he made burial mounds for it. But the princes of Umma would not submit. The princes of Lagash had to continue to vanquish the princes of Umma, and make burial mounds for it. Accordingly, the princes of Lagash attest in their inscriptions 4 princes of Umma. Enmetena attests: Ush (ins. 1). Enmetena attests: Enakale

(*id.*). Enanatum (ins. 2) and Enmetena (ins. 1) attest: Ur-Luma. Enmetena attests also: Il (*id.*). Ur-Luma himself left behind 2 inscriptions, Il left behind 1 inscription. Ur-Luma reports:

Ur-Luma, prince of Umma, son of Enakale, prince of Umma (ins. 1).

Il reports:

Il, prince of Umma, grandson of Enakale, prince of Umma (ins. 1).

And there is this 1 inscription of Gishakidu:

Baraimun, wife of Gishakidu, prince of Umma, daughter of Ur-Luma, prince of Umma, daughter-in-law of Il, prince of Umma (ins. 1).

The series and dates of the 5 princes of Umma follow, Ush, Enakale, Ur-Luma, Il, and Gishakidu.

Then there is the successor of Lugalkiginedudu, Lugalkisalsi. Lugalkisalsi left 4 inscriptions behind. He reports:

Lugalkisalsi, prince of Uruk and prince of Ur, son of Lugalkiginedudu, prince of Uruk and prince of Ur (ins. 1).

Then there is the predecessor of Lugalzagesi, Uu. Uu left no inscriptions behind. Lugalzagesi, however, reports:

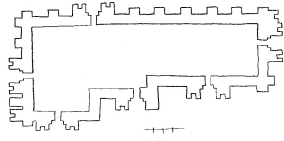
Lugalzagesi, prince of Uruk, son of Uu, prince of Umma (ins. 1).

Urukagina reports: Lugalzagesi sacked Lagash. Lugalzagesi concurs: Enlil gave Lugalzagesi the lordship of the land; the princes of Sumer submitted to him. Then Sargon reports: Sargon brought Lugalzagesi in a dog collar to the temple of Enlil. The following not quite one and a half centuries are the time of Sargon, Rimush, Manishtushu, Naram-Sin, and Sharkalisharri. And in this time there are no princes not attested by the list, but attested either by their own inscriptions and the inscriptions of other princes or by the inscriptions of other princes only.

In the two-odd generations after Sharkalisharri the unity of princeliness was lost. Who was prince? who was not? In this time there are 3 princes not attested by the list, but attested either by their own inscriptions and those of other princes or by those of other princes only, Ur-Baba, Gudea, and Puzur-Inshushinak.

Ur-Baba and his younger brother Gudea were princes of Lagash. They left behind many inscriptions. Gudea is, in fact, the Sumerian prince who

left behind the largest number of his own statues. More important, on one of his statues the Eninnu is drawn. It is the decisive step toward the discovery of maps. Under the Eninnu, the temple of Ningirsu, a graduated ruler is drawn, showing that the problem of the determination of distance and orientation across a building has been solved (statue B). Seven centuries later the problem of the determination of distance and orientation across a city, across 2 km, will be solved, and this will be the first map. The map of Nippur.



Puzur-Inshushinak left behind 1 dozen inscriptions. Puzur-Inshushinak reports:

Puzur-Inshushinak, prince of Susa, lord of Elam, son of Shimpishuk (ins. 1).

Then Ur-Nammu reports:

Ur-Nammu, prince of Ur, prince of Sumer and Akkad . . . ; . . . marched against Elam; Puzur-Inshushinak, prince . . . ; . . . brought the booty to Enlil (ins. 29).

Fragmentary as the inscription is, it is clear who was booty, who brought the booty to Enlil. The following is the century of Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin, and in this time there are again no princes not attested by the list, but attested either by their own inscriptions and the inscriptions of other princes or by the inscriptions of other princes only.

All in all, through the time of Ibbi-Sin, there are 22 such princes in the valley of the two rivers. In chronological order, Meskalamdug, Mesalim, Ur-Nanshe, Akurgal, Ush, Eanatum, Enakale, Zuzu, Enanatum, Ur-Luma, Enmetena, Il, Enanatum II, Lugalkisalsi, Gishakidu, Enentarzi, Uu, Lugalandu, Urukagina, Ur-Baba, Gudea, and Puzur-Inshushinak.

Their cities are Lagash, Kish, Umma, Uruk, Ur, Akshak, and Susa.

And these 22 princes are all real as well. Each of these princes was, in fact, a prince as well. The evidence about these princes is diamond as well.

$30 + 22 = 52$. 52 princes in the valley of the two rivers through the time of Ibbi-Sin who were, in fact, princes.

Of each of these princes the fact that he was a prince is established independently of that prince.

Hence, the princes determine one another's position in space. The prince of Lagash vanquished the prince of Umma. The prince of Akkad vanquished the prince of Uruk. The prince of Ur vanquished the prince of the Gutians. They determine one another's position in time. Mesanepada was the son of Meskalamdug. Il was the grandson of Enakale. Enmetena and Lugalkiginedudu swore to their brotherhood. They determine their series and order spatially and temporally, both. In Ur, Meskalamdug, his son Mesanepada, his son Aanepada. In Umma, Uruk, Lagash, Akkad, and Kish, Lugalzagesi vanquished Urukagina, Sargon vanquished Lugalzagesi. They determine one another's motions. Vanquished, swore to their brotherhood, marched against, erected a monument, captured, conquered, sacked, razed, brought in a dog collar to the temple of Enlil, won 34 battles, brought the booty to Enlil.

With Kepler: the position, order, and motion of the parts of the world is the system of the world.

In the same way, the position, order, and motion of the parts of princship is the system of princship.

The princes are a system.

And the Sumerian prince list discovered the system.

The 52 princes can be divided into earlier and later, into those who reigned in the three and a half centuries from Enmebaragesi through Lugalzagesi and those who reigned in the three centuries from Sargon through Ibbi-Sin. Hence, tabulating, this is how the princes divide,

	earlier	later	total
in the list	9	21	30,
not in the list	19	3	22,
	28	24	52.

Of the later princes, reaching three centuries into its past, the list recovered 87.5 %. Of the earlier princes, reaching six and a half centuries into its past, it recovered 32.1 %. Of all the princes it recovered 57.7 %.

Today, three centuries into the past would be right after Louis XIV, before Walpole, before Maria Theresa, the time of Frederick William, of Peter, long before the United States, or the seven years' war. Six and a half centuries into the past

would be right after Frederick II, the time of Edward, of Louis IX, of Alfonso X, before Osman, the time of the seventh crusade, long before the hundred years' war.

On this ground alone it can be concluded that the Sumerian prince list is the perfect investigation.

But there are further grounds, too.

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The third kind of princes are those attested by their own inscriptions only.

The evidence is in each case meager.

Here is a sample of 8.

The princes of Kish are very important in the list. They are the first in the list, and there are 39 of them arranged in 4 groups. The independent evidence corroborates the list. The importance of the princes of Kish is attested by the inscriptions of Enmebaragesi, Meskalamdug, Mesanepada, Mesalim, and by the inscriptions of all the other princes who styled themselves prince of Kish, including Sargon.

3 princes of Kish not attested otherwise left inscriptions behind, 1 inscription each:

Lugalatu, prince (ins. 1),

Uhub, prince of Kish (ins. 1),

Lugalsilasi, prince of Kish (ins. 1).

The princes of Adab are not very important in the list. There is but 1 of them. The independent evidence again corroborates the list. 4 princes of Adab left behind 1 to 2 inscriptions each:

Medurba, prince of Adab (ins. 1),

Eiginimpae, prince of Adab (ins. 1-2),

Barahenidu, prince of Adab (ins. 1-2),

Lugaldalu, prince of Adab (ins. 1).

None of the 4 is attested otherwise.

There are no princes of Lullubum in the list. If this is a judgment, *ex silentio*, that the princes of Lullubum are not important at all, the independent evidence again corroborates the list. 1 prince of Lullubum left 1 inscription behind:

Anubanini, powerful prince, prince of Lullubum (ins. 1).

Powerful as he was, Anubanini was not powerful enough to have another prince attest him. Nor was any other prince of Lullubum powerful enough to have another prince attest him.

The sample is not representative in that the

inscriptions of these 8 princes are less fragmentary than most.

All in all, 3 to 4 dozen princes of this kind can be assigned on paleographic grounds to the time before Ib-bi-Sin. Clearly, the evidence about them being what it is, the princes of this kind cannot be counted exactly.

Clearly, the time of none of the princes of this kind is determined exactly. Given the system of the princes, it is determined of none of the princes of this kind where exactly in the temporal series and order of the system he would belong. In most cases, but not all, the cities of the princes of this kind are determined. When they are, the cities are Kish, Uruk, Ur, Umma, Mari, Adab, the Gutian horde, Elam, Lagash, Shuruppak, Lullubum, Simurru. Nevertheless, where exactly in the spatial series and order of the system any of the princes of this kind would belong is not determined either. Because the spatial and temporal order depend on one another. And this, in turn, because, other than that they each made a votive inscription, the motions of the princes of this kind are not determined at all.

The conclusion is unavoidable. The princes of this kind are not, in fact, parts of the system of the princes.

That every prince of this kind is attested meagerly, and that he is attested only by himself, cannot not be a cause and effect. Because he is attested meagerly every prince of this kind is not attested independently of himself. And it cannot not be a cause of both that every prince of this kind is not important at all. Because he is not important at all every prince of this kind is attested meagerly and not attested independently of himself.

It will be objected that the record is suspect. That there is no knowing what is lost. But the record is not suspect. The record is suspected, wantonly. The record is perfect. There is knowing what is lost. The not important is lost. The important is not lost. Enmebaragesi already is attested independently. Less than a century after Enmebaragesi, Meskalamdug is attested both by his own inscriptions and those of another prince. One and a half centuries after Enmebaragesi, Ur-Nanshe is attested, between his own and those of other princes, by several dozen inscriptions. And

assigned on paleographic grounds to the time before Ib-bi-Sin, the 3 to 4 dozen princes attested each a prince by no prince but himself have to be all assigned to the end of that time, when the evidence of the inscriptions of the princes, those attested each a prince by princes other than himself, is massive.

The further conclusion is unavoidable. None of the princes of this kind is, in fact, a prince.

That each of them is, in fact, a man need not be doubted. That any of them is, in fact, a prince has to be doubted. Lest there were no standard, and who was prince? who was not? were answered *ad hoc*, and at will. And on the standard, a prince is, in fact, a prince, if, at a minimum, he is attested a prince by 1 other prince, none of the princes of this kind is, in fact, a prince. That it is not a demanding standard, Zuzu and Uu attest, whose only determined motions are vanquished by Eanatum and succeeded by Lugalzagesi, respectively.

And the list discovered this, too, that there cannot not be a standard. The list writes:

Sharkalisharri, son of Naram-Sin, he reigned 25 years;

then who was prince? who was not? Igigi was prince, Nanum was prince, Imi was prince, Eluldan was prince, 4 princes in 3 years.

Still 52 princes, then.

30 of them are in the list, out of the 120 in the list. And given that there is a standard, who of the remaining 90 was prince? who was not? can be answered as well.

Next to the inscriptions of the princes, there is the other independent evidence.

Of this, the most important is the oldest chronicle.

It is one of the early compositions about Gilgamesh, *Gilgamesh and Aka*. The name is descriptive, and new. The composition is old, from the century-odd of the princes from Ur-Nammu to Ib-bi-Sin.

And this oldest chronicle – this forty-one-centuries-old chronicle – has been suspected of not being a chronicle. Because it is one of the many compositions about Gilgamesh, and the other compositions about Gilgamesh abound in divinities.

Like the Hebrew scriptures and the Homeric

epics, the epic *Gilgamesh* is the product of a long development, and much the longest of the three. The major steps in this development are also three. The final step is the eleven-tablet epic, the late Akkadian *Gilgamesh*, composed by Sin-liqe-uninni in the century before c. 1206, four centuries before the earliest of the writings collected in the Hebrew scriptures, five centuries before Homer, and eight centuries after the first step. The middle step is the early Akkadian *Gilgamesh*, composed under the early Babylonian princes, in the century before c. 1594, four centuries before the final step, and four centuries after the first. The first step is the Sumerian *Gilgamesh*, composed in the century before c. 2013, four centuries before the middle step, eight centuries before the final step, twelve centuries before the earliest of the writings collected in the Hebrew scriptures, thirteen centuries before Homer.

The Sumerian *Gilgamesh* comprises five short compositions. Four of the five were assimilated into the later Akkadian versions. One, *Gilgamesh and Aka*, was left alone. In the four that were assimilated into the later Akkadian versions, Gilgamesh battles the guardian of the forest of cedar, battles the bull of heaven, descends to the netherworld, and dies. Among the divinities he encounters are Enlil and Utu, Inanna and Ereshkigal, Dumuzi. In the one that was left alone, Gilgamesh encounters no divinities, battles no monsters, does not descend to the netherworld, and is very much alive.

In particular, he vanquishes Aka. *Gilgamesh and Aka* is short, but 110-odd lines. And these are the main events it reports: the envoys of Aka, son of Enmebaragesi, came from Kish to Gilgamesh in Uruk; the envoys of Aka told Gilgamesh, submit; the old men of Uruk told Gilgamesh, let us submit; the young men of Uruk told Gilgamesh, let us wage war; Gilgamesh told Enkidu, ready the arms; Aka laid siege to Uruk; Gilgamesh climbed the walls of Uruk; myriad fell; Aka submitted; Gilgamesh let Aka go free back to Kish (*id.*).

Not only is *Gilgamesh and Aka* the oldest chronicle. On the ground alone of the plausibility of what it does not report and what it reports, both, it is a much better chronicle than all too many of its successors, including the *Chronicles*

in the Hebrew scriptures.

What it reports is what the princes report in their inscriptions: submit, let us submit, let us wage war, laid siege, myriad fell, submitted.

More particularly, *Gilgamesh and Aka* and the list explain one another. The list begins with a group of princes of Kish. The last two are: Enmebaragesi, and Aka, son of Enmebaragesi. Then: Kish was smitten, princeship was taken to Uruk. And the fifth prince of Uruk in the list is Gilgamesh.

The two, *Gilgamesh and Aka* and the list, report the same events, and events such as the princes report. Whether one of the two is a source of the other, or their sources are independent, and among the sources of the list cannot not be the inscriptions of the princes, the chronicle and the list enhance one another's authority.

The inscription which Enmebaragesi left behind, the chronicle, and the list corroborate each other. Given the position of Enmebaragesi in the system of the princes, the inscription, the chronicle, and the list determine the positions of Aka and Gilgamesh exactly. Aka and Gilgamesh are parts of the system of the princes.

And in the same way as the compositions about Gilgamesh can be divided into *Gilgamesh and Aka*, on one side, all other compositions about Gilgamesh, on the other – into evidence about the princes, on one side, not evidence about the princes, on the other – all the rest of the evidence can be divided. Beginning with the evidence which the list itself is.

The list begins with 23 princes of Kish, Kish was then smitten, princeship was taken to Uruk, and 12 princes of Uruk follow. There are 21 princes of Kish before Enmebaragesi, and 4 princes of Uruk before Gilgamesh.

Of 5 of these 25 the list has more than their names, descent, and reign years to report. These 5:

Etana, he was a shepherd, he went to heaven, he made all the lands firm, he reigned 1 500 years,

Meskiaggasher, son of Utu, he reigned 324 years, he went into the sea and came out in the mountains,

Enmerkar, son of Meskiaggasher, he built Uruk, he reigned 420 years,

divine Lugalbanda, he was a shepherd, he reigned 1 200 years,

divine Dumuzi, he was a fisherman, his city was Kuara, he reigned 100 years,

All 5 are attested independently of the list. None of the 5 with anything good, if he is to be a prince.

The best attested is either Etana or Dumuzi, depending on whether Dumuzi, he was a fisherman, is Dumuzi, he was a shepherd. If the fisherman is the shepherd, Dumuzi is the best attested. Otherwise Etana is.

Etana is attested with the five-tablet composition *Etana*. It is Akkadian, contemporary with the early Akkadian *Gilgamesh*, not with the Sumerian. The main events it reports are these: the gods build a city for men, the greater gods directing the work, the lesser gods doing it; they choose Etana to be the prince of the city; but Etana has no heir; in the meantime, an eagle and a serpent swear to their brotherhood; but the eagle devours the children of the serpent; to punish the perfidy, the gods throw the eagle into a pit to die of hunger; Etana saves the eagle; he mounts the eagle, and flies to heaven in search of the plant that will give him an heir (*id.*).

Dumuzi is one of the divinities whom Gilgamesh encounters in *Gilgamesh and the netherworld*. But he is well attested independently, as is his wife. The compositions are in this case Sumerian, contemporary with the Sumerian *Gilgamesh*. And these are the main events which three of the compositions about Inanna and Dumuzi, *The marriage of Dumuzi and Inanna*, *Inanna's descent to the netherworld*, and *The death of Dumuzi*, report: Dumuzi, the shepherd, courts Inanna, the lady of the sky; he marries her; but Inanna is the lady of love and war, her older sister Ereshkigal is the lady of death and the netherworld, and the two are bitter enemies; Inanna wants to be the lady of the netherworld as well; she descends there; Ereshkigal outwits her younger sister and turns her into a corpse; but Inanna has asked Enki, the lord of wisdom, to help her if she does not return; the lord of wisdom revives the lady of love and war; but she cannot leave the netherworld unless someone else takes her place; she offers Dumuzi, who is now the prince of Uruk; and demons drag the recalcitrant prince of

Uruk to the netherworld (*id.*).

As befits the god of fertility, Dumuzi is attested also in the Akkadian record, and the Babylonian as well as the Assyrian, and attested by god himself, too. *Ezekiel* reports: the lord took me to the north gate of his house, women sat there bewailing Tammuz, and the lord said, there are even greater abominations (8).

In the antediluvian prince list there is the prince of Badtibira:

Dumuzi, he was a shepherd, he reigned 36 000 years.

Dumuzi, then, the one of the Sumerian prince list, on the ground that he was a fisherman, he was not a shepherd, he was not the husband of Inanna. On the ground that he was the prince of Uruk, he was not the prince of Badtibira, he was the husband of Inanna.

Etana and Dumuzi, and any Dumuzi, need not be burdened with the presumption of reality. Or the presumption that any of them is, in fact, a prince.

In general, none of the 25 princes of Kish and Uruk in the list before Enmebaragesi and Gilgamesh, respectively, need be burdened with the presumption of reality. Or the presumption that he is, in fact, a prince.

These 25, Gashur, Kullassinabel, Nangishlishma, Endaraana, Babum, Puannum, Kalibum, Kalumum, Zuqaqip, Atab, Mashda, Arwium, Etana, Balih, Enmenuna, Melamkish, Barsalnuna, Samug, Tizqar, Ilku, Iltasadum, Meskiaggasher, Enmerkar, Lugalbanda, Dumuzi.

120 – 25 = 95. 30 of the 95 are attested by the inscriptions of the princes. 2 more are attested by the oldest chronicle.

Finally, there are the second oldest and the third oldest chronicles, the Tummal chronicle and the Esagil chronicle. They are the records of the motions of princes with respect to the two temples, the Tummal and the Esagil. Dating them from the last prince noted in each, the Tummal chronicle was completed under

Ishbi-Erra prince c. 2027-1994,
in the generation after the list. The Esagil chronicle was completed under

Sumulael prince c. 1880-44,
postdating the list by one and a half centuries.

And these second and third oldest chronicles

— these four-millennia-old chronicles — have been suspected of not being chronicles, too. The Tummal chronicle because it is monotonous. The Esagil chronicle because it has a lesson to teach.

The Tummal chronicle is monotonous: Aka made the Tummal preeminent; then the Tummal fell into ruin; Meskiagnuna made the Tummal preeminent; then the Tummal fell into ruin (*id.*), and three more times a prince made the Tummal preeminent, then the Tummal fell into ruin. One is reminded of Wellhausen's description of the Hebrew *Kings*. Wellhausen writes: one is reminded of the Hegelian thesis, antithesis, synthesis, when one hears the monotonous beat with which history advances here, rebellion, affliction, conversion, peace, rebellion, affliction, conversion, peace, the keynote being, they did evil in the eye of the lord, went whoring after strange gods (*Prolegomena*, VII, 1.1).

The Esagil chronicle has a lesson to teach. The lesson is: the princes who gave fish to the priests of Marduk fared well, the princes who failed to give fish to the priests of Marduk fared poorly (*id.*). A Hegelian thesis and antithesis, clearly, and a lesson all too many priests other than those of Marduk have had to teach.

On the ground alone of the plausibility of what they do not report and what they report, both, these two chronicles are much better chronicles than all too many of their successors, too.

Other than Ishbi-Erra, under whom it was completed, the Tummal chronicle records the motions of 12 princes. It has the series:

Gilgamesh,

Ur-Lugal, son of Gilgamesh (*id.*).

The list has the series:

Gilgamesh,

Ur-Nungal, son of Gilgamesh.

It can be concluded that Ur-Lugal is Ur-Nungal.

The chronicle has the two series:

Mesanepada,

Meskiagnuna, son of Mesanepada (*id.*),

and:

Nane,

Meskiagnuna, son of Nane (*id.*),

just as the list has them. It can be concluded that the corruption is the same, Nane is Aanepada, and the one series of three consecutive princes has become two series of two princes each in the

chronicle as in the list.

The princes attested by the chronicle are thus 11, and they are all in the list as well. 8 are attested also by the inscriptions of the princes, Enmebaragesi, Mesanepada, Aanepada, Meskiagnuna, Ur-Nammu, Shulgi, Amar-Sin, and Ibbi-Sin. 2 are attested also by the oldest chronicle, Aka and Gilgamesh. 1 is Ur-Nungal.

Other than Sumulael, under whom it was completed, the Esagil chronicle records the motions of 14 princes, even if lacunae have robbed posterity of knowing in each case whether the prince gave fish, or failed to give fish, to the priests of Marduk. All 14 are in the list. 1 is Enmerkar, however, one of those who need not be burdened with the presumption of reality.

The princes attested by the chronicle are thus 13, and they are all in the list as well. 9 are attested also by the inscriptions of the princes, Enmebaragesi, Sargon, Naram-Sin, Utuhegal, Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin. 1 is attested also by the other chronicles, Aka. The remaining 3 are Puzur-Nirah, Ku-Baba, and Ur-Zababa.

What the two chronicles report is what the princes report in their inscriptions: building temples, making temples preeminent, providing for the priests of the temples. The list is a source of the two chronicles, taking it for granted that as all later prince lists learned from the first prince list, so the chronicles learned from the first prince list to arrange the princes in their series and order. The list and the chronicles enhance each other's authority. The inscriptions greatly enhance the authority of the list and the chronicles.

Hence, Ur-Nungal, son of Gilgamesh, is a part of the system of the princes.

So is Puzur-Nirah, prince of Akshak.

As to Ku-Baba and Ur-Zababa, the list reports: Ku-Baba, she was an innkeeper, she made Kish firm, she reigned 100 years, Puzur-Sin, son of Ku-Baba, he reigned 25 years, Ur-Zababa, son of Puzur-Sin, he reigned 400 years, Sargon, his father was a gardener, he was a cup-bearer of Ur-Zababa, he built Akkad, he reigned 56 years.

It can be concluded that not only Ku-Baba and Ur-Zababa are parts of the system of the princes,

but so is Puzur-Sin. He is not attested independently of the list. On the authority of the list, however, he is the successor of Ku-Baba, she made Kish firm, and the predecessor of the predecessor of Sargon, he built Akkad. Puzur-Sin is a part of a series of very important determined motions of the princes. Even if, pace the list, Ku-Baba, she did not reign 100 years, Ur-Zababa, he did not reign 400 years.

37 of the princes in the list, then, are, in fact, princes.

$95 - 37 = 58$.

These 58, Utulkalama, Labashir, Enundarana, Meshe, Melamana, Lugalkigin, Balulu, . . . , . . . , Susuda, Dadasig, Mamagal, Kalbum, Tuge, Menuna, Lugalmu, Hatanish, Argandea, . . . , Lugalnemundu, Anbu, Anba, Bazi, Zizi, Limer, Sharumiter, Unzi, Undalulu, Urur, Ishuil, Shu-Sin, Simudar, Usiwatar, Eshtarmuti, Ishme-Shamash, Nannia, Igigi, Nanum, Imi, Kuda, Puzur-Ili, Ur-Utu, Imta, Inkishush, Shulme, Silulumes, Inimabakesh, Igeshaush, Yarlagab, Ibate, Yarla, Kurum, Apilkin, Irarum, Ibrarum, Hablum, Puzur-Sin.

And it can be concluded that these 58 are one and all not important at all. And, hence, that none of them is, in fact, a prince.

That there are in the system of the princes more objects that are not important at all than there are important objects is characteristic of systems. Thus, in the solar system, for instance, there are the 1 sun, the 8 planets, the several dozen satellites, the many more asteroids, the very many more comets, and the yet indefinitely larger number of smaller debris, the meteoroids. It is characteristic also that, unless a piece of debris, in either the solar system or the system of the princes, is coming directly toward one, the debris are not important at all.

The fact that the list's collection of the princes includes so many objects that are not important at all also shows the perfection of the collection. Bacon writes: we must begin without either premature speculation or any great subtlety. And so the list did.

The princes do not have to be all suns, or even planets. But they cannot be meteoroids. To be a prince a prince has to be, at a minimum, a comet. There cannot not be a standard.

The standard, moreover, cannot not be the standard of the objects themselves. It cannot not be the standard of the evidence about the objects because it is the standard of the objects themselves.

In 1742, Hume writes: of the English kings, from the conquest to the succession of his present majesty, we may reckon 28 sovereigns, omitting those who died minors; and 8 of them are princes of great capacity, William, Henry II, Edward, Edward III, Henry V, Henry VII, Elizabeth, and William III (*Station*).

It is 661 years from the conquest to the succession of his then present majesty. Much as it is from Enmebaragesi to the list.

Thirty-eight centuries after the list, the evidence about the princes has become mountainous. Hume nevertheless faces, and solves, the same problems. Is it a prince? is it not? To reckon 28 sovereigns, Hume omits not only the 2 who died minors. He omits also Mary II who gave one of the princes of great capacity the sovereignty. He omits Cromwell, too. But, pace Hume, was the lord protector not, in fact, a sovereign? a prince, in fact, and a prince of great capacity?

8 of the 28 are princes of great capacity, 20 are not. And these 20, are they all princes of any capacity? All 20, in fact, princes? And if this question is difficult to answer with respect to each of these princes, in particular, because they are the English kings, and the English kings are, with the French kings, the two most important series of princes in these six and a half centuries, there is the indefinitely larger number of other contemporary princes with respect to each of whom, in particular, the question is not difficult to answer at all. The predecessor of his then present majesty, for instance, before he became 1 of the 28, was 1 of the princes with whom Leibniz had to concern himself. Those of Lüneburg, and Grubenhagen, and Göttingen, and Wolfenbüttel, and Calenberg.

The importance of the solution of these problems should need no recommendation.

Hume writes: had Henry IV, Richelieu, and Louis XIV been Spaniards, and Philip II, Philip III, and Philip IV Frenchmen, the history of these two nations would have been entirely reversed (*Essays*, I, 14).

The solution of these problems begins with the list.

In the valley of the two rivers through the time of Ibbi-Sin, then, there are 59 princes. Dividing them into earlier and later, into those in the three and a half centuries from Enmebaragesi through Lugalzagesi and those in the three centuries from Sargon through Ibbi-Sin, they divide thus,

	earlier	later	total
in the list	16	21	37,
not in the list	19	3	22,
	35	24	59.

The list recovered 45.7 % of the earlier, 87.5 % of the later, 62.7 % of the total.

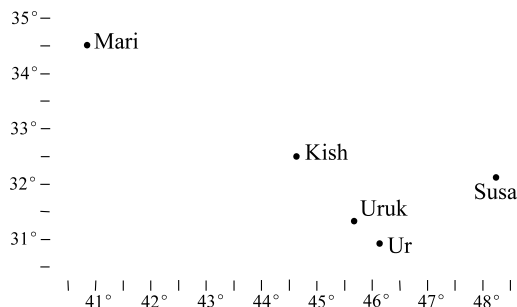
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The princes in the list are the princes of 10 cities. In the order of the list, Kish, Uruk, Ur, Awan, Hamazi, Adab, Mari, Akshak, Akkad, and the Gutian horde.

A state is, with Hobbes: a power contrary to our natural passions, keeping men in awe, reducing their wills unto one will. And all 10 of the cities in the list are states, in Latin *civitates*, in Greek πόλεις. Next to the one in the valley of the Nile, they are the first states, too.

But they are also, all 10 of them, localities on the surface of the earth. Some of them are cities, in Latin *urbes*, in Greek ἄσται, some of them are countries, one of them is a horde.

And the success of the list in collecting the latter kind of objects, localities on the surface of the earth, can be assessed. Then its success in collecting the former kind of objects, states, can be assessed. And hence, finally, its success in collecting the principal objects of investigation, the princes, can be assessed.



This plot can anchor the description of the

localities. The 5 plotted cities have all been excavated. Hence their latitudes and longitudes.

The distances are, along the great circle, Mari to Ur 630 km, Ur to Susa 250 km, Mari to Susa 730 km. Mari, Kish, Uruk, and Ur all lie on the Euphrates. Not far north of the Euphrates, the Tigris runs northwest to southeast as well. The two rivers empty together in the Persian Gulf. Downstream from Ur, further southeast, four and a half millennia ago the Persian Gulf was close to the city, it is today some 250 km from the city as the rivers have continued to silt the gulf. Upstream from Mari, further northwest, the Euphrates approaches the Mediterranean. The sea is some 500 km from the city. The lower valley of the two rivers, with the cities of Kish, Uruk, and Ur in it, is the country of Sumer. In the east, the valley of the two rivers is bounded by the Zagros range, rising to a height of 4.4 km. Susa lies at the foot of the Zagros.

The plot represents the portion of the surface of the earth which, given a map, can be recovered from the list.

The first 3 localities in the list, then, Kish, Uruk, and Ur, are each a city. Each is abundantly attested. Each has also been unearthed.

Awan is a country, not a city. And countries are seldom as clearly defined as cities. Awan is the country north of Susa. But it can include Susa, and extend further south. Puzur-Inshushinak, for instance, styles himself indiscriminately: prince of Susa, lord of Elam (ins. 1-5), and: prince of Awan (ins. 6-7). Elam is the country south of Susa. But it can include Susa, and extend further north. It is from these mountainous countries in the east that disorder tended to descend on the cities in the valley. Susa has been unearthed. Awan and Elam are well attested, if not as well as Kish, Uruk, or Ur.

Hamazi is also in the east, north of Awan. It is not so well attested, and it is not clear whether it is a country, or a city, or both.

Adab and Mari are cities. Both have been unearthed. Adab is 70 km north of Uruk.

Akshak is a city. It is not so well attested, and it has not been unearthed. It was not far from Kish.

Akkad is a country and a city. The country of Akkad is the middle valley of the two rivers,

north of the country of Sumer. The well attested city of Akkad has not been unearthed. It has disappeared under Baghdad. It is thus 100 km north of Kish.

The Gutian horde, finally, also came from the mountains in the east. And a horde though it is, it indicates a locality as definitely, more or less, as the countries of Awan and Elam. Or the countries of Sumer and Akkad, for that matter.

The portion of the surface of the earth with which the list is concerned is thus about two thirds of that represented in the plot. It is the portion north and east of the line connecting Mari and Ur.

It is a substantial portion.

Some dozens of names of localities can be recovered from the Sumerian record earlier than the list. The large majority are but names. It is clear that a locality is at issue, but it is not clear where the locality is. It is clear why. The first map, the map of Nippur, is a map of a city, and it postdates the list by six centuries. The first description of the valley of the two rivers explicit enough to allow the extraction of a map from that description alone is in Herodotus, sixteen centuries after the list. The first map of the valley is in Ptolemy, twenty-one and a half centuries after the list.

Of those localities not noted in the list, of which it is clear where they are, most are within the portion of the surface of the earth extracted from the list.

Thus, there are the two cities whose princes the list failed to recover, Lagash and Umma. Both are well attested, and both have been unearthed. Lagash is 60 km northeast of Ur, Umma is 50 km northwest of Lagash.

Princes of Lullubum, Simurru, and Shuruppak are attested by their own inscriptions only. Lullubum and Simurru are not well attested. Both are countries in the mountains in the east, north of Awan. The city of Shuruppak is better attested, and it has been unearthed. It is 50 km north of Uruk.

There is also the city of Nippur. Never the city of its own state, a cluster of temples, it was contested by the states in the valley. Very well attested, it has been unearthed. Nippur is 70 km southeast of Kish.

A few of the localities not noted in the list, of which it is clear where they are, are outside the portion of the surface of the earth extracted from the list. These are all attested by the inscriptions of the princes of Akkad, Sargon, Rimush, Manishtushu, Naram-Sin, and Sharkalisharri, to whom Enlil gave no rival from the lower sea to the upper sea.

The portion of the surface of the earth with which the list is concerned is substantial quantitatively. North to south, from Mari to Ur, the list reached across not quite 4° of latitude along the meridian, west to east, from Mari to Susa, taking Susa to stand for either Awan or the Gutian horde, it reached across 7° of longitude along the parallel. From Mari to Susa it reached across 730 km along the great circle, $\frac{1}{55}$ of the circumference of the earth.

The princes of Akkad reached further.

The portion is substantial qualitatively as well. The list took the second step toward the discovery of maps.

Ptolemy writes: to describe the earth distance and orientation must be determined.

The princes took the first step. With their campaigns they began collecting localities on the surface of the earth, and arranging them, if for a long time implicitly more than explicitly, in their series and order.

The list collected localities, and arranged them in their series and order, also implicitly rather than explicitly, by collecting and arranging in their series and order the resident states.

It is the more decisive step of the two. The determination of distance and orientation will begin on the map of Nippur with the determination of distance and orientation across a city, and the determination of the distances and orientations of cities and states will precede the determination of the distances and orientations of the natural features on the surface of the earth, and will remain the better determination of the two for a long time. It is in Herodotus, and it is in Ptolemy still.

And the success of the list in collecting states, the first states, is very great.

The princes in the list are gathered in groups, of Kish, Uruk, Ur, Awan, Kish again, Hamazi,

Uruk again, Ur again, Adab, Mari, Kish again, Akshak, Kish again, Uruk again, Akkad, Uruk again, the Gutian horde, Uruk again, Ur again.

That it is the best way to arrange the princes, any attempt to arrange the princes of the last six and a half centuries, for instance, will show. The attempt to arrange them well will show it conclusively.

That the princes are the best way to identify the states can be taken on Aristotle's authority. He writes in the *Politics*: what makes a state one and the same is the same constitution, whether the name stays the same or not, and whether the people remain the same or change entirely (III, 3), and: constitution is the arrangement of magistracies and especially the highest (6).

So Brandenburg is Prussia is Germany, for instance, the same state identified by the one series of electors of Brandenburg, kings of Prussia, and emperors of Germany.

And to identify the states in the valley of the two rivers through the time of Ibbi-Sin, the 59 princes collected from the list and the inscriptions of the princes can once again be divided into earlier and later, those in the three and a half centuries from Enmebaragesi through Lugalzagesi and those in the three centuries from Sargon through Ibbi-Sin.

Arranging the cities in the order of the first prince in each, and counting the princes, the earlier are,

	in the list	not in the list	total
Kish	6	1	7,
Uruk	4	1	5,
Ur	5	1	6,
Lagash	—	9	9,
Umma	—	6	6,
Akshak	1	1	2,
	16	19	35.

There is complexity in the relations of the states. The third prince to leave his inscriptions behind describes himself: Mesanepada, prince of Ur, son of Meskalamdug, prince of Kish. One century later, Enshakushana, one of the princes of Uruk in the list, describes himself: Enshakushana, lord of Sumer. One generation later, Lugalkiginedudu, also one of the princes of Uruk in the list, describes himself: Lugalkiginedudu, prince of Kish, prince of Uruk, prince of Ur. One cen-

tury later, Urukagina, a prince of Lagash, writes of Lugalzagesi: Lugalzagesi, prince of Umma. Lugalzagesi himself, also one of the princes of Uruk in the list, describes himself: Lugalzagesi, prince of Uruk, son of Uu, prince of Umma.

That the complexity is the result of the labors of the princes to establish that princship is one both their inscriptions and the list attest abundantly. The inscriptions attest just as abundantly that these princes failed to do so.

In these three and a half centuries the very important states are Kish, Uruk, Ur, and Lagash. Kish, Uruk, and Ur are the first states. Then Lagash rose. Lagash is the first state of 9 consecutive important princes, the first 5 of them all very important. Umma is also an important state, but not so important. Akshak is less important. Mari, Adab, Awan, and Hamazi, whose princes the list also collected, are less important still. And there are no other states of any importance.

Ur-Nanshe, the first of the 9 princes of Lagash, predates the list by four and a half centuries, Urukagina, the last of the 9, predates it by two and a half centuries. The list failed to recover Lagash, and this is the one major error of the list. There is a trace of Umma in the list. It has Lugalzagesi. But it has neither the city of Umma nor any of the other princes attested by the inscriptions. The omission of Umma is also an error.

Of the very important earlier states, then, the list recovered 3 out of 4. Of all states, very important and less important, it recovered 8 out of 10.

Arranged in the same way, the later are,

	in the list	not in the list	total
Akkad	8	—	8,
Gutian horde	5	—	5,
Lagash	—	2	2,
Uruk	3	—	3,
Elam	—	1	1,
Ur	5	—	5,
	21	3	24.

There is less complexity now. Sargon abolished the complexity. And there is after him less complexity in two ways. The number of very important states is smaller, and the states of lesser importance are very much less important.

The very important states in these three centuries are Akkad and Ur. Akkad is the first state which succeeded in subjugating all other states

and becoming the dominant power on earth. Ur repeated the feat. Ur also became, however, the first state of massive writing, and of writing of many kinds. In the two-odd generations between Akkad and Ur disorder descended on the state from the mountains in the east. The state which brought the disorder from the mountains is the Gutian horde. The Gutian horde is important in being the first state of this kind. Elam, or Susa, is another of the states in the east during that time of disorder. The states of Lagash and Uruk are minor local principalities during that time. And there are no other states of any importance. The states of Lullubum, Simurru, and Shuruppak, which left behind one or two inscriptions each, and failed to be noticed by any other state each, attest this fact.

The later Lagash, Uruk, and Ur are states different from the earlier Uruk, Ur, and Lagash. This follows from Aristotle's definition.

The omission of the later Lagash and of Elam by the list, if it is an error at all, is a minor error.

Of the very important later states, then, the list recovered 2 out of 2. Of all later states, 4 out of 6.

All in all, the list recovered 5 out of the 6 very important states, 12 out of all 16 states.

These are, next to the one in the valley of the Nile, the first states. This is the beginning of the state.

Hobbes, Locke, Rousseau, Marx, all concern themselves with the beginning of the state. All four deduce the beginning of the state from general principles. Collection of the particulars, or continuous and gradual ascent from the particulars, does not hamper the obtainment of the principles.

But there are the particulars. The Sumerian prince list collected them.

Finally, the success of the list in collecting the principal objects of investigation, the princes, can be assessed.

The list has correlated the princes to localities on the surface of the earth. To do so, it has collected 10 of these localities, reaching across $\frac{1}{55}$ of the circumference of the earth.

Other than those in the valley of the Nile, it has collected 62.7 % of all princes. To do so, it has reached six and a half centuries into its past, $\frac{1}{9}$

of the time since the 3 oldest calculations of area and the first prince.

It has arranged the princes in their series. In states. It has collected, other than the one in the valley of the Nile, 83.3 % of the very important states, 75.0 % of all states.

It has discovered the system of the princes. And it has discovered the system of the system of the princes. The system of the states. Kish, Uruk, and Ur rose – then minor states appeared – then Akkad rose – then the Gutian horde brought disorder – then Ur restored the order. When the rise of Lagash is added, this general result, established by the list, is conclusive. Nothing will ever be subtracted from it, nothing will ever be added to it.

There are errors in the list. But there is no confusion. Bacon writes: truth will emerge sooner from error than from confusion (*Organon*, II, 20). The list discovered that there is a standard. Who was prince? who was not? And it is because there are facts in the list, because there are many facts in the list, that the errors in the list have been corrected, and an indefinite number of other errors has been corrected. Because, in general, the list is the first investigation other than mathematical, and, directly or indirectly, all subsequent investigation rests on it. In particular, the determination of all facts of early Mesopotamian antiquity depends on the facts established by the list.

The last and most important rule of the method is to make enumerations so complete and reviews so general as to omit nothing. And the list is not trying to make enumerations so complete and reviews so general as to omit nothing. It is succeeding.

Implicitly if not explicitly, yet essentially and substantially, the list is a complete enumeration.

It is the first complete enumeration of the princes.

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It remains to show that the princes are the predecessors of the philosophers, and to draw conclusions from this fact.

That the princes are the predecessors of the philosophers can be shown in two ways. Given that the list is the predecessor of the philosophers,

it can be shown that the princes of the Sumerian cities are the predecessors of the list. And given that the princes of the Sumerian cities are the predecessors of the great kings, it can be shown that the great kings are the predecessors of the philosophers.

The success of the list can be analyzed into the solutions of four problems. A mathematical problem, a physical problem, a historical problem, and a political problem.

A collection of princes, of localities on the surface of the earth, and of states, the list is a collection also of additions. Nor are the additions elementary. What is the sum, for instance,

$$1\,200 + 900 + 670 + 420 + 300 + 840 + 960 + 840 + 900 + 600 + \\ + 840 + 720 + 1\,500 + 400 + 660 + 900 + 1\,200 + 140 + \\ + 305 + 900 + 1\,200 + 900 + 625 = ?$$

Traced back to the 3 oldest calculations of area, the beginning of mathematics predates the list by ten and a half centuries. And if the 3 calculations of area are the first inkling of mathematics, the accumulation of more and more sophisticated inklings in these ten and a half centuries is attested best by the princes.

One and a half centuries after Enmebaragesi, Ur-Nanshe reports: Ur-Nanshe, prince of Lagash, vanquished Ur; he vanquished Umma; he made burial mounds for them. Two generations later, Eanatum counts the mounds: Eanatum, prince of Lagash, vanquished Umma; he made 20 burial mounds for it (ins. 6). Not quite two centuries later, Sargon counts objects much harder to count: Sargon, lord of the earth, won 34 battles. He counts also many more objects: 5 400 men ate bread daily before Sargon. And his successor Rimush was rather a mathematician. Rimush reports: Rimush, lord of the earth, won three times the fight for Sumer; he destroyed 11 322 men; he captured 2 520 men; when Kazallu rebelled, he destroyed 12 052 men; he captured 5 862 men; altogether he captured 54 016 men (ins. C-1); he vanquished Ur and Umma; he destroyed 8 040 men; he captured 5 460 men (ins. C-2); he vanquished Adab and Zabalam; he destroyed 15 718 men; he captured 14 576 men (ins. C-4); he vanquished Umma and Kian; he destroyed 8 960 men; he captured 3 540 men (ins. C-5); he vanquished Barahshum, Zahara, and Elam; he destroyed 16 212 men; he captured

4 216 men (ins. C-6).

Two and a half centuries after Rimush, the list records the princes, and adds their numbers. It records also their reign years, and adds these numbers, too.

And this last addition attests the first solution of a problem of mathematical sophistication as great as any yet solved. The problem of time reckoning.

The rotation of the earth on its axis is periodic, and so is the revolution of the earth around the sun. The period of the first is the day, the period of the second is the year. And selection has taught organisms to tell these periods. To tell when it is day, when night, and when it is spring, when summer, when autumn, when winter.

The explicit distinction of these periods, on the other hand, is not attested in the valley of the two rivers before Sargon. The explicit distinction of one year from another is first attested in the valley of the Nile on the Palermo stone, compiled under Neferirkare, prince c. 2440. In the valley of the two rivers it is first attested by Sargon.

Sargon is the first prince to leave behind names of the years of his reign. He has left behind the names of 4 of the years of his reign: the year Sargon went against Simurru (ins. D-1),

the year Sargon destroyed Urua (ins. D-2), the year Sargon destroyed Elam (ins. D-4), the year Sargon destroyed Mari (ins. D-5).

Reckoning years by the names bestowed on them by the deeds of a prince remained the only way of reckoning years for the following two millennia, and remained the main way of reckoning years for the following four millennia.

Rimush has left behind the name of 1 of his years. Manishtushu has not left behind the name of any of his years. Naram-Sin has left behind the names of 14 of his years. Sharkalisharri has left behind the names of 17 of his years.

Then the Gutian horde brought disorder. Then the princes of Ur restored the order. Ur-Nammu reigned 18 years, and has left behind the names of 3 of them. Shulgi reigned 48 years, and has left behind the names of all 48 of them. Amar-Sin reigned 9 years, and has left behind the names of all 9 of them. Shu-Sin reigned 9 years, and has left behind the names of all 9 of them. Ibbi-Sin

reigned 24 years, and has left behind the names of all 24 of them. The princes of Ur did restore order.

And that Sargon discovered the reckoning of years by the naming of each after his deeds the list corroborates.

It reports:

Sargon, he was a cupbearer of Ur-Zababa, and:

Ur-Zababa, he reigned 400 years.

Some of the earliest princes reigned longer still:

Enmebaragesi, he reigned 900 years,

Aka, he reigned 625 years.

And Ur-Zababa is not the only prince close to the time of Sargon blessed with a reign of antediluvian length:

Lugalkiginedudu, he reigned 120 years,

Ku-Baba, she reigned 100 years.

It is beginning with Sargon that all reigns stabilize at the catastrophically low, by antediluvian standards, lengths such as the reigns of princes since have had, from the maximum of:

Sargon, he reigned 56 years,

to the minimum of:

Tirigan, he reigned 40 days.

The conclusion is unavoidable. No data before Sargon. Data beginning with Sargon.

And the list generalized Sargon's result vastly. It collected the years, counted them, and totaled the counts.

The difficulty of the problem Hume shows. A mountain of evidence has been amassed, the problem of chronology has long been solved, and in order to arrange in their series and order the princes of England from William to his then present majesty, to arrange one succession of princes, and a succession as orderly as there have been successions of princes, Hume still faces the difficulty, who was prince? who was not? Having to discover that there is evidence, and to take the first step toward the solution of the problem of chronology, the list faces the difficulty also of having to arrange in their series and order several successions, and the difficulty also that some of these successions are orderly but some are not. The succession from Sargon to Sharkalisharri is orderly. The succession from Ur-Nammu to Ibbi-Sin is orderly as well. In between, however, who was prince? who was not?

The list arranges the princes in their series and order in each state, arranges the states in their series and order, and totals the reign years for each state.

Totaling the totals, from the beginning of the reign of Sargon to the end of the reign of Ibbi-Sin the total of the list is 417 years, 6 months, and 47 days.

It is an excellent result.

From the inscriptions of the princes the three synchronisms can be recovered, of Ur-Nammu and Utuhegal, of Utuhegal and Tirigan, and of Sarlagab and Sharkalisharri. The time of disorder is thus reduced to order, and the error of the list can be corrected. It can be corrected, given the inscriptions of the princes – and the list. The error of the list cannot be corrected without the list. Without its record of the reign years of the princes, and its totals for the states.

Corrected, the dates are,

Sargon prince c. 2337-281,

this date is certain within 20 years, and

Ibbi-Sin prince c. 2037-13,

and this date is certain within 10 years. From the beginning of the reign of Sargon to the end of the reign of Ibbi-Sin the total is 324 years $\pm$ 15 years.

The importance of the problem Einstein shows. In *On the electrodynamics of moving bodies* Einstein deduces the theory of special relativity from two postulates and one definition. The definition is: the definition of simultaneity, or synchronicity, or time (1). It is the last improvement on the solution of the problem of time reckoning. Special relativity stands on the solution of the problem of time reckoning, on special relativity general relativity stands, on general relativity the big bang theory stands.

As the last improvement on the solution of the problem of time reckoning is the outcome of the labors of the philosophers, so the first solution of the problem is the outcome of the labors of the princes and the list. As the last improvement on the solution is the discovery of propositions, so the first solution is the discovery of propositions: the year Sargon went against Simurru, the year Sargon destroyed Elam, Sargon, he reigned 56 years, 11 princes of Akkad, they reigned 181 years.

And if the first solution of the problem of time

reckoning attests the acquisition of knowledge, the question of what is relevant to the acquisition of knowledge can be answered.

On the evidence of this one instance, the acquisition of knowledge – the development from no inkling to time reckoning – is not attested by beast, or man, or god. Not even the 5 400 who ate bread daily before Sargon, not even Enlil.

The acquisition of knowledge is attested by the princes and the list. Not by all or any princes either, never mind any list. It is attested by particular princes and this particular list. Nor can it be attested by more than these 10 particulars, Sargon, 3 of the 4 princes of Akkad who succeeded him, Rimush, Naram-Sin, and Sharkalisharri, and the 5 princes of Ur, Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin – by the 9 princes who first left behind names of the years of their reigns – and the Sumerian prince list, which first collected the reign years of the princes.

And the acquisition of knowledge is attested, rather, by 4 of these 10 particulars. Sargon, Naram-Sin, Shulgi, and the Sumerian prince list.

Sargon, who first left behind names of the years of his reign, the names of 4 of the 56 years of his reign. After him Rimush did what Sargon did, left behind the name of 1 of the 9 years of his reign.

Naram-Sin, who first left behind many of the names of the years of his reign, the names of 14 of the 37 years of his reign. After him Sharkalisharri did what Naram-Sin did, left behind the names of 17 of the 25 years of his reign. Ur-Nammu left behind the names of 3 of the 18 years of his reign.

Shulgi, who first left behind all the names of the years of his reign, the names of 48 of the 48 years of his reign. After him Amar-Sin, Shu-Sin, and Ibbi-Sin did what Shulgi did, left behind all the names of the years of his reign each.

And the Sumerian prince list, which first collected the years of the reigns of the princes, totaled them, and hence reckoned time across not less than three centuries, across 417 years, 6 months, and 47 days, on its own reckoning, across 324 years $\pm$ 15 years, in fact.

On the evidence of this one instance, the acquisition of knowledge is the steps of advance in the acquisition of knowledge, exclusively.

The success of the list can be analyzed into the solutions of four problems, and if time reckoning is the mathematical, the description of the surface of the earth is the physical.

Enmebaragesi attests 1 locality: Kish.

One century later, Mesanepada attests 2 localities: Ur and Kish.

One half century later, Ur-Nanshe attests 4 localities. He reports: Ur-Nanshe, prince of Lagash; he vanquished Ur; he vanquished Umma. And: Ur-Nanshe, prince of Lagash; he had ships of Dilmun bring timber (ins. 2). Dilmun is somewhere in the Persian Gulf. Where exactly is not clear. Ur-Nanshe's successors continued to have ships of Dilmun bring timber, and the locality is well attested, but it is not more exactly attested. On the other hand, the excitement of the fact that the exploration of the surface of the earth is reaching ever further is detectable in the labors of Ur-Nanshe already. His victory over Ur and Umma is recorded on 1 inscription, his feat of having ships of Dilmun bring timber is recorded on 6 inscriptions (ins. 2, 5, 17, 20, 22-3).

Two generations later, the successor of Ur-Nanshe's successor Eanatum attests 16 localities. He reports: Eanatum, prince of Lagash, unleashed a deluge on Umma; he vanquished Elam and Subartu; he vanquished Susa; he vanquished Urua; he vanquished Arua; he vanquished Sumer; he vanquished Ur (ins. 1); he vanquished Uruk; he vanquished Kiutu; he sacked Uruaz; he sacked Mishime; he destroyed Akshak; Inanna gave him the princeship of Kish next to the lordship over Lagash; he vanquished Mari (ins. 5).

Clearly, Eanatum's was a more propitious relationship with Inanna than her husband's.

Of some of the localities it is clear where exactly they are, of some it is not. It is clear where exactly Mari and Susa are. The limits of the exploration of the surface of the earth attested by Eanatum are thus those of the portion of the surface of the earth in the list, 730 km apart.

In the two centuries from Enmebaragesi to Eanatum, then, the development is from no inkling to localities 730 km apart.

The rise of Lagash attests 16 localities. One and a half centuries later Akkad rose.

Sargon attests 27 localities:

Akkad, Kish, Uruk, Ur, Eninmar, Lagash,

Umma, the upper sea, the lower sea, Mari, Elam, Meluhha, Magan, Dilmun, Jarmuti, Ebla, Nippur, Barahshum, Urua, Shalamu, Kardede, Heni, Bunban, Sapum, Awan, Susa, Simurru (ins. C-1, C-2, C-5, C-7, D-1).

Rimush attests 12 localities. 6 are not attested by Sargon:

Sumer, Kazallu, Adab, Zabalam, Kian, Zahara (ins. C-1, C-4/6).

Manishtushu attests 5 localities. 2 are not attested by his predecessors:

Anshan, Sherihum (ins. 1).

Naram-Sin attests 38 localities. 23 are not attested by his predecessors:

Eanna, Tuttul, Kesh, Eridu, Sippar, Kutha, the source of the Tigris, the source of the Euphrates, Lullubum, Armanum, Tiwa, Girtab, Apilak, Amurru, Subartu, Talhad, Ullisum, Marad, Tutu, Niquim, Maridaban, Sabunum, Senaminda (ins. 1, 2, 4, 11, C-1, C-3, C-5, A-1, A-3, B-6, D-9, D-10, D-14).

Sharkalisharri attests 10 localities. 2 are not attested by his predecessors:

Akshak, the Gutians (ins. D-25, D-27).

The rise of Akkad attests 60 localities.

Clearly, Sargon, prince of Akkad, lord of the earth, was a great explorer. Naram-Sin, prince of Akkad, lord of the four quarters (ins. 3); the god of Akkad (ins. B-1), was an even greater explorer. Naram-Sin is clear, too, that his labors are reaching ever further: Naram-Sin, lord of the four quarters; since man was created, no prince of all princes had conquered Armanum and Ebla (ins. C-5); Naram-Sin, prince of Akkad; when he took the path to Talhad, no prince of all princes had taken this path (ins. C-3).

Again, of some of the localities it is clear where exactly they are, of some it is not. It is clear where exactly the upper sea, the lower sea, the source of the Tigris, and the source of the Euphrates are. The upper sea and the lower sea are 1 300 km apart. From its source to its mouth the Tigris is 1 900 km long, from its source to its mouth the Euphrates is 2 800 km long.

In the two-odd centuries from Eanatum to Naram-Sin the development is from localities 730 km apart to localities not less than 1 300 km apart.

The exploration of the surface of the earth will not reach further until the rise of the Assyrian princes in the century-odd from Tiglath-pileser III through Ashurbanipal, fifteen centuries after Naram-Sin, and will not reach as dramatically further as it did under the princes of Lagash and the princes of Akkad until the great kings, Cyrus, Cambyses, and Darius, two centuries later still.

In the nineteenth Christian century the interior of Africa was opened, the Arctic was penetrated, Antarctica began being penetrated, and the surface of the earth was thus explored in its entirety. And the forty-four centuries from Ur-Nanshe through the nineteenth Christian century can be divided into two parts. The twenty-five centuries from Ur-Nanshe through Caesar, when conquest was the exclusive means of the exploration of the surface of the earth. And the nineteen centuries after Caesar, when conquest followed immediately upon the exploration of the surface of the earth.

With their campaigns the princes of Lagash and of Akkad took the first step toward the discovery of maps. By arranging states in their series and order the list took the second and more decisive step.

Ptolemy writes in the *Geography*: the earth, since it is very large and does not surround us, no single man can see in whole or even in part.

In particular, if a man sees the part, the part is small and irrelevant, if the part is large and relevant, a man does not see the part. To map it, the part has to be observed, not seen. And mountains and valleys, rivers and seas, are not readily accessible to observation. No natural feature on the surface of the earth, but very small lakes and very small islands, is. States, on the other hand, as soon as they appeared in any numbers made themselves more readily accessible to observation. As soon as they appeared in any numbers, states began defining themselves spatially as unambiguously each as its neighbors would be made to tolerate, first by the city in which each was resident, then by the country which each conquered, including in that country as many of the neighboring states as the neighboring states would be made to tolerate. The princes thus collected more localities, much further apart. Collecting localities by collecting the resident

states, the list is more explicit. The determination of the distances and orientations of cities and states will thus precede that of the natural features on the surface of the earth, and will remain for a long time the better determination.

The solution of the problem of the description of the surface of the earth is the outcome of the labors of the princes and the list. It is the discovery of propositions:

Ur-Nanshe, he had ships of Dilmun bring timber,
to Sargon Enlil gave no rival from the upper sea
to the lower sea,

Naram-Sin, he reached the source of the Tigris
and the source of the Euphrates,
Akkad was smitten, princeship was taken to
Uruk.

And if the first solution of the problem of the description of the surface of the earth attests the acquisition of knowledge, the question of what is relevant to the acquisition can be answered again, and it has to be answered in the same way.

On the evidence of this second instance, the acquisition of knowledge – the development from no inkling to the description of a substantial portion of the surface of the earth – is not attested by the men or any other of the organisms which populated this or any other portion of the surface of the earth.

It is attested by the princes and the list. Not by all or any princes or any list either. It is attested by particular princes and this particular list. Nor can it be attested by more than these 10 particulars, Enmebaragesi, Mesanepada, Ur-Nanshe, Eanatum, Sargon, Rimush, Manishtushu, Naram-Sin, and Sharkalisharri – the 9 princes who first left behind the record of a growing number of localities – and the Sumerian prince list, which first collected localities by collecting the resident states.

And it is attested, rather, by 5 of these 10 particulars. Ur-Nanshe, Eanatum, Sargon, Naram-Sin, and the Sumerian prince list.

Ur-Nanshe, who first attests 4 localities. Before him Enmebaragesi attests 1 locality, and Mesanepada attests 2 localities. But the description of the surface of the earth cannot begin with 1 or 2 localities. Ur-Nanshe attests also the excitement of exploration.

Eanatum, who first attests 16 localities.

Sargon, who first attests 27 localities. After him Rimush and Manishtushu did as Sargon did.

Naram-Sin, who first attests 38 localities, distributed from the source of the Euphrates in the north to the Persian Gulf in the south and from the Mediterranean in the west to the foot of the Zagros in the east. Naram-Sin attests also the clarity of reaching ever further afield. After him Sharkalisharri did as his predecessors did.

And the Sumerian prince list, which first attests the collection of localities by the collection of 10 resident states, and the arrangement of the 10 resident states in their temporal, if not yet spatial, series and order.

On the evidence of this second instance, too, the acquisition of knowledge is the steps of advance in the acquisition of knowledge, exclusively.

cciv

The historical problem.

Voltaire wrote *The century of Louis XIV*. Gibbon wrote *The history of the decline and fall of the Roman empire*. Gibbon writes: diligence and accuracy are the only merits which a historical writer may ascribe to himself, if any merit indeed can be assumed from the performance of an indispensable duty; I may therefore be allowed to say that I have carefully examined all original materials that could illustrate the subject which I have undertaken to treat. Voltaire writes: this history concerns itself only with what merits the attention of all times.

Both, the performance of the indispensable duty of examining all original materials and its performance to the end of what merits the attention of all times, begin with the list.

The list's most successful attempt to examine all original materials is attested by the fact that not more than 59 princes of any importance can be recovered from the Mesopotamian record through Ibbi-Sin and the list collected 37 of them.

As to how, the grandfather of history will have had to do it the way the father of history had to do it. The father of history traveled, by boat and on foot, from Olbia in the north (IV, 16-8) to Elephantine in the south (II, 29), and from Cyrene in the west (32) to Susa in the east (V, 52-3), across 23° of latitude and 27° of longitude. To

collect the princes of 10 cities 4° of latitude and 7° of longitude apart, the grandfather of history will have had to travel, with only the two rivers in between, mostly on foot.

In this, too, the princes are the predecessors of the list. The greatest advance was achieved by Enmetena.

The successor of Ur-Nanshe is Akurgal, his successor is Eanatum, his successor is Enanatum, his successor is Enmetena. And Enmetena informs posterity:

Enlil, father of the gods, drew the border between Lagash and Umma; Mesalim, prince of Kish, measured the border and set up a monument there;

Ush, prince of Umma, smashed the monument and marched into the plain of Lagash; Lagash defeated Umma, made burial mounds for it;

Eanatum, prince of Lagash, uncle of Enmetena, marked the border with Enakale, prince of Umma; he restored the monument of Mesalim and did not march into the plain of Umma;

Ur-Luma, prince of Umma, diverted the water, set fire to the monuments, brought foreigners in the land; Enanatum, prince of Lagash, made war on him; Enmetena, son of Enanatum, defeated him; Ur-Luma was killed; Enmetena made 5 burial mounds for Umma;

Il took the princship of Umma; he diverted the water; Enmetena, prince of Lagash, sent envoys to Il, prince of Umma; he said, the border is mine; and he set up the monument (ins. 1).

In this one inscription Enmetena informs posterity of the labors of 8 princes of 3 cities across 3 generations.

Herodotus is the father of history, the Sumerian prince list is its grandfather, and unlike organisms, history does not have a great-grandfather. If it will nevertheless have one, however, Enmetena will be the great-grandfather of history. Enmetena attests the first unambiguous inkling of history.

As to what merits the attention of all times, the princes do.

That the list presumes that the princes merit the attention of all times the fact of the list shows. That in this as well the princes are the predecessors of the list their inscriptions show. And that the princes merit the attention of all times should be superfluous to argue.

Herodotus proposes to concern himself with: the great and magnificent deeds of Greeks and barbarians (I, 1), and concerns himself with the great and magnificent deeds of the great kings, Cyrus, Cambyzes, Darius, Xerxes, and the generals who stopped the great kings, Miltiades, Themistocles, Pausanias.

Thucydides proposes to concern himself with: the war of the Athenians and the Spartans, because it was a great war (I, 1), and concerns himself with the generals who fought the great war, Pericles, Archidamus, Brasidas, Alcibiades.

Polybius proposes to solve the problem: how did the Romans succeed in bringing nearly the entire known earth under a single government (I, 1)? and in order to do so concerns himself with the generals who stopped Hannibal and then brought nearly the entire known earth under a single government, Fabius Maximus, Marcellus, Scipio, Cato, Flamininus, Aemilius Paullus, Scipio Aemilianus.

Voltaire concerns himself with Louis XIV.

Hume's *History of England* is the history of the princes of England, from those before William (1-3), through William (4), through James II (70-1).

The history of the decline and fall of the Roman empire is the history of the decline and fall of the Roman princes.

That the princes merit the attention of all times should be superfluous to argue. Polybius writes: no one can be so petty or apathetic as not to want to know how did the Romans succeed in bringing under a single government, for the first time ever, nearly the entire known earth (I, 1).

Then again, no one? Leibniz writes: it is astonishing that men always turn to the utterly useless, or in any case the least important.

If the examination of all original materials to the end of what merits the attention of all times is a problem, the first solution of this problem is the outcome of the labors of the princes and the list as well. It is the discovery of propositions.

And if the first solution of this problem attests the acquisition of knowledge as well, the question of what is relevant to the acquisition can be answered once again, and once again has to be answered in the same way.

On the evidence of this third instance as well,

the acquisition of knowledge – the development from no inkling to the first successful attempts to examine all original materials to the end of what merits the attention of all times – is attested by no one, and more generally nothing, but the princes and the list as well, and not by all or any princes or any list, but by particular princes and this particular list as well.

Nor can it be attested by more than these 22 particulars, Enmebaragesi, Mesanepada, Ur-Nanshe, Eanatum, Enanatum, Enshakushana, Enmetena, Lugalkignedudu, Lugalzagesi, Urukagina, Sargon, Rimush, Manishtushu, Naram-Sin, Sharkalisharri, Utuhegal, Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, and Ib-bi-Sin – the 21 princes who first left behind the growing number of substantial inscriptions, and substantial in either quality or quantity – and the Sumerian prince list, which first collected the princes.

And in this case, as in the previous two, it is attested, rather, by some, not all, of these 22 particulars.

In this case, however, it is more difficult to separate sharply the particulars that attest the acquisition from the particulars that do not.

The mathematical problem, time reckoning, and the physical problem, the description of the surface of the earth, have long been formulated explicitly and solved completely and conclusively. It is thus clear which the steps of advance in the acquisition of knowledge, exclusively, are. The first 4 names of reign years – the first 14 years out of 37 – the first 48 years out of 48 – the first reckoning of time across not less than three centuries. The first collection of 4 localities – the first collection of 16 localities – the first collection of 27 localities – the first collection of 38 localities, and distributed from the source of the Euphrates to the Mediterranean to the Persian Gulf to the Zagros – the first collection of 10 states. The historical problem, presuming that it has been formulated explicitly, has not been solved completely or conclusively. It has not been solved in general at all, and it has not been solved in particular but on a few, and all too few, occasions. It is thus more difficult to achieve clarity and distinction with respect to this problem.

In any case, the acquisition of knowledge is attested by 2 of the 22 particulars. The first

unambiguous inkling of history, and the grandfather of history.

Enmetena, who records the labors of 8 princes of 3 cities across 3 generations.

And the Sumerian prince list, which collects 120 princes of 10 cities, and arranges them in their series and order, reaching six and a half centuries into its past.

In any case as well, it is the remaining 20 particulars, and nothing but these 20 particulars, that have to be sorted out into these, if any, who acquired knowledge and those who did as their predecessors did. These, if any, who attest the acquisition and those who do not.

In any case, too, on the evidence of this third instance as well, the acquisition of knowledge is the steps of advance in the acquisition of knowledge, exclusively.

Finally, the political problem which the list solved is the end of the state.

Implicit rather than explicit as it is, a solution of this problem is indicated unambiguously by the following.

Of the 59 princes in the valley of the two rivers through Ibbi-Sin the list recovered 37. Of all 37 it records the names and the reign years. Of 18 it records also the ancestry, in 16 cases the father or the mother, in 2 cases the kind of the father: Gilgamesh, his father was a ghost; Sargon, his father was a gardener. Of 3 it records also deeds which are not in direct relation to a state: Gilgamesh, he was the lord of Kulaba; Ku-Baba, she was an innkeeper; Sargon, he was a cupbearer of Ur-Zababa. Kulaba is a district of Uruk. And of 3 it records also deeds which are in direct relation to a state: Enmebaragesi, he smote Elam; Ku-Baba, she made Kish firm; Sargon, he built Akkad.

It can be inferred, a prince builds a state, makes it firm, smites another state.

More generally, the origin of princeship is heaven: when princeship descended from heaven, it came to Kish.

Once princeship has descended from heaven, however, the mechanics of taking it from one state to another are earthly and uniform:

Kish was smitten, princeship was taken to Uruk, Uruk was smitten, princeship was taken to Ur,

Ur was smitten, princeship was taken to Awan, Awan was smitten, princeship was taken to Kish, and as long as there are states, one state is smitten, princeship is taken to another state. And princeship is one and only one. The one and only one princeship is taken from Kish to Uruk, to Ur, to Awan, back to Kish, and as long as there are states, the one and only one princeship is taken from one state to another state.

It can be inferred, a prince builds a state, and the end of the state is to smite all other states and bring them under itself, under the one and only one state which the state is.

It is a solution of the problem of the end of the state.

And there is the problem of the end of the state. On the authority of Aristotle, Hobbes, Locke, Rousseau, Hegel, Marx. All of whom have also obtained solutions of the problem different from that of the list.

Aristotle writes: when several villages unite in a single community large enough to be more or less self-sufficient, the state has come into being, originating in the bare needs of life, and continuing for the sake of the good life.

Hobbes writes: the final cause, end, or design of men in the introduction of that restraint upon themselves to live in commonwealths is the foresight of their own preservation, and of a more contented life thereby (*Leviathan*, 17).

Locke writes: the great and chief end of men's uniting into commonwealths, and putting themselves under government, is the preservation of their property.

Rousseau writes: each of us in common puts his person and all his power under the supreme direction of the general will; this act of association produces a moral and collective body which receives from this very act its unity, self, and will; this public person was formerly called city, it is today called state when passive, sovereign when active (*Contract*, I, 6); the general will tends toward equality (II, 1); it is always right, and it always tends toward public utility (3).

Hegel writes: the state is the reality of the ethical idea, ethical spirit as the revealed, clear to itself, substantial will (*Right*, 257); it is the reality of the substantial will, which it has in the raised to its generality particular self-consciousness; this

substantial unity is an absolutely unmoved end in itself (258).

Expounding Marx, Engels writes: the state serves the need to hold class conflicts in check.

The state is for the sake of the good life. Its final cause, or end, or design is men's preservation and a more contented life. Its great and chief end is the preservation of property. The general will produces it and it tends toward equality and always toward public utility. Its absolutely unmoved end is the unity of the particular self-consciousness in the generality of the reality of the ethical spirit as substantial will. It serves the need to hold class conflicts in check.

And here are two series of facts which do not corroborate Aristotle. Or Hobbes. Or Locke. Or Rousseau. Or Hegel. Or Marx. But do corroborate the list.

First, the following are only the most successful attempts, and only at their most successful, to smite all other states and bring them under one state.

Lagash under
Eanatum prince c. 2470.
Akkad in the century-odd from
Sargon prince c. 2337-281,
through
Naram-Sin prince c. 2257-20.
Ur in the not quite one century from
Ur-Nammu prince c. 2121-03,
through
Shu-Sin prince c. 2046-37.
Babylon under
Hammurabi prince c. 1792-49.
Egypt in the one and a half centuries from
Thutmose prince c. 1521-09,
through
Amenhotep III prince c. 1402-364.
Ashur, the second most successful attempt, in the century from
Adad-nirari prince c. 1305-273,
through
Tukulti-Ninurta prince c. 1243-05,
again under
Tiglath-pileser prince 1115-047,
and again under
Ashur-nasirpal II prince 884-59,
and yet again in the century-odd from
Tiglath-pileser III prince 745-27,

through
Ashurbanipal prince 668-27.
The great kings in the two-odd generations from
Cyrus prince 559-30,
through
Darius prince 522-486.
Macedon, the most meteorically successful attempt, under
Philip prince 359-36,
and
Alexander prince 336-23.
Rome, the most successful attempt, and successful indiscriminately as a republic and as an empire, in the three-odd centuries from
Scipio consul 205^{BC},
through
Trajan prince 98-117^{AD}.
The prophet and the caliphs, the other meteoric success, in the two-odd generations from
Muhammad hijra 622,
through
Muawiyah prince 661-80.
The western empire under
Charlemagne prince 768-814.
The great khans in the three generations from
Genghis Khan prince 1206-27,
through
Kublai prince 1260-94.
The sultans in the not quite two and a half centuries from
Orhan prince 1326-60,
through
Suleiman prince 1520-66.
The western empire again under
Charles V prince 1516-56.
The revolution under
Napoleon prince 1799-815,
and in 1807 it did appear that only perfidious Albion still stands between Napoleon and one and only one princeship under Napoleon. Perfidious Albion, however, had become the dominant power on earth in the previous century, and was to remain the dominant power on earth for another century after Napoleon.

Second, Sargon built Akkad, and after all the building in between, forty-one centuries after Sargon, Washington, Hamilton, and Adams built the United States, three generations later Cavour built Italy, in the next generation Bismarck built

Germany, not quite two generations later Lenin built the Soviet Union. And however successful the United States may have been at the preservation of property, it was rather more successful at quadrupling in not quite three generations, sometimes by purchase, always by smiting, the territory under itself. As soon as it was built, Germany proposed to catch up with perfidious Albion, and overtake it, and win a world war to this end, if need be, and precipitate another world war, if need be. Italy, too, did its best, such as it was, in the wars. And however successful the Soviet Union may have been at holding class conflicts in check during its brief existence, it was rather more successful at both smiting and bringing states under itself.

The first attempts to solve the problem of the end of the state, then, predate the philosophers. Counting back to Eanatum, they predate the first philosopher by eighteen and a half centuries.

And once again, if the end of the state is a problem, the first solution of this problem is the outcome of the labors of the princes and the list. It is the discovery of propositions.

And if the first solution of this problem attests the acquisition of knowledge, the question of what is relevant to the acquisition can be answered, and has to be answered in the same way.

On the evidence of this fourth instance, the acquisition of knowledge – the development from no inking to the first successful attempts to establish what the end of the state is – is not attested by men or gods, not even Enlil.

It is attested by the particular princes who labored successfully at building the state, making it firm, and smiting all other states, and the particular list which collected these labors of the princes and generalized over them.

Nor can the acquisition be attested by more than these 18 particulars, the 5 princes under whom Lagash rose, Ur-Nanshe, Akurgal, Eanatum, Enanatum, and Enmetena, the 3 princes who successfully emulated them, Enshakushana, Lugalkignedudu, and Lugalzagesi, the 5 princes under whom Akkad conquered and dominated the valley of the two rivers, Sargon, Rimush, Manishtushu, Naram-Sin, and Sharkalisharri, the 4 princes under whom Ur rose, Ur-Nammu, Shulgi, Amar-Sin, and Shu-Sin, and the Sumerian prince

list, which established that princship is one and only one, and that the one and only one way of taking it from one city to another is smiting.

In this case, as in the previous three, the acquisition of knowledge is attested by some, not all, of these 18 particulars.

But in this case as well, as in that of the historical problem, it is more difficult to separate sharply the particulars that attest the acquisition from those that do not than it is in the cases of the mathematical and the physical problems, and it is rather more difficult than in the case of the historical. That there is the problem of the end of the state Aristotle, Hobbes, Locke, Rousseau, Hegel, and Marx attest unambiguously. They attest just as unambiguously, however, that the problem remains to be formulated explicitly, never mind solved completely or conclusively. To achieve clarity and distinction with respect to this problem is very difficult.

Nevertheless, in this case as in that, the acquisition of knowledge is attested by 4 of the 16 particulars.

Eanatum, who attests the first attempt to build the dominant power on earth, his campaigns reaching across 730 km.

Sargon, who attests the first successful attempt to build the dominant power on earth, his dominion reaching across 1 300 km.

Naram-Sin, who attests the first successful attempt to give permanence to the dominant power on earth, his dominion reaching across 1 300 km, and not less than 1 000 km east to west and 1 000 km north to south.

And the Sumerian prince list, which attests the first attempt to gather the end of the state from the end of the state, its success grounded in the collection of 37 real princes of 10 real states, reaching back to the first real prince of the first real state.

In this case, too, it is the remaining 14 particulars, and nothing else, that have to be sorted out into these, if any, who acquired knowledge and those who did as their predecessors did. These, if any, who attest the acquisition of knowledge and those who do not.

And in this fourth case, too, the acquisition of knowledge is the steps of advance in the acquisition of knowledge, exclusively.

ccv

The princes of the Sumerian cities are the predecessors of the list, the list is the predecessor of the philosophers, and *ipso facto* the princes of the Sumerian cities are the predecessors of the philosophers.

Fourteen and a half centuries separate the list and the first philosopher. But the princes of the Sumerian cities are the predecessors of the great kings as well, and the great kings are the immediate predecessors of the philosophers.

The greatest of the great kings are the first four,

Cyrus	prince 559-30,
Cambyses	prince 530-22,
Darius	prince 522-486,
Xerxes	prince 486-65.

The first philosopher, Thales, predicted an eclipse which occurred in 585. The eclipse dates Thales, and hence the beginning of philosophy. Plato founded the Academy *c.* 385. The rise of the great kings thus coincides in time substantially with much of the first two centuries of the development of philosophy.

Xenophon is a contemporary of Plato. Like Plato, he is a student of Socrates. Like Plato, he writes on Socrates. Like Plato, he is concerned with education. Like Plato, he enlists Socrates in the service of education. The outcome is in Xenophon, as it is in Plato, an encomium of Socrates. Xenophon's encomium is contained in the four books of *Memorabilia* and the briefer *Apology*, *Home management*, and *Symposium*. Unlike Plato, Xenophon writes also on Cyrus. Cyrus also he enlists in the service of education. The outcome is also an encomium. This encomium is contained in the eight books of *The education of Cyrus*.

Xenophon begins: we considered how cattle obey the cowherd and never conspire against him, while men always conspire against those who try to rule them; we concluded that it is easier for a man to rule other creatures than to rule men; but then we considered Cyrus and we changed our mind; because a vast number of men, cities, and peoples obeyed him willingly, even if some of them lived many months away from him and knew that they will never see him; with a little

band of Persians he became the ruler of the Medes and the Hyrcanians, then he conquered the Syrians, the Assyrians, the Arabs, the Cappadocians, the Phrygians, the Lydians, the Carians, the Phoenicians, and the Babylonians; and he ruled also the Bactrians, the Indians, the Cilicians, the Sacae, the Paphlagonians, the Magadidae, and many other peoples, including the Greeks in Asia, the Cypriots, and the Egyptians; and among them all he inspired such fear that no one dared to oppose him, and everyone wanted to please him and to do his bidding (I, 1.2-5).

So much so, god wanted to please him and to do his bidding. *Isaiah* reports: thus said the lord to Cyrus, leading him by the hand to subdue peoples and undo kings; I will go before you and I will level the hills, I will shatter the bronze gates, I will cut the iron bars; you do not know me, but it is I who has given you your title; you do not know me, but it is I who has given you your strength, so that through you all men from the rising sun to the setting sun will know that there is no one but me (45).

On the authority of *Isaiah*, all men could learn from Cyrus that there is no one but the lord. On the authority of Xenophon, they could learn from Cyrus that there are the Persians, Medes, Hyrcanians, Syrians, Assyrians, Arabs, Cappadocians, Phrygians, Lydians, Carians, Phoenicians, Babylonians, Bactrians, Indians, Cilicians, Sacae, Paphlagonians, Magadidae, Greeks in Asia, Cypriots, and Egyptians. They could learn also that as cattle obey the cowherd and never conspire against him, so can men. And, in fact, the philosophers, if not all men, not only could learn from the great kings. They did. The philosophers did not learn from the great kings mathematics. But they did learn from the great kings geography, and history, and more generally physics, and politics, and more particularly ethics, too.

Geography.

Beginning from nothing, in the two centuries after Enmebaragesi, the expeditions of the princes reached with Eanatum from Mari in the north to Ur in the south and from Mari in the west to Susa in the east, across 4° of latitude along the meridian and 7° of longitude along the parallel.

From this base, two centuries later, the expeditions of the princes reached with Naram-Sin from

the source of the Euphrates in the north to its mouth in the Persian Gulf in the south and from the mouth of the Orontes in the Mediterranean in the west to Susa in the east, across 10° of latitude and 13° of longitude. The portion of the earth brought under exploration was thus quadrupled.

The expeditions of the princes reached further fifteen and a half centuries later, when the rise of the Assyrian princes reached its peak. Tiglath-pileser III, his successors Shalmaneser V, Sargon II, and Sennacherib established their dominion over the entire domain over which the Akkadian princes had established theirs. And from this base, Sennacherib's successor

Esarhaddon prince 680-68, first broke out of the valley of the two rivers and conquered the lower valley of the Nile. His successor Ashurbanipal conquered Thebes. The expeditions of the Assyrian princes thus reached from the source of the Euphrates in the north to Thebes in the south and from the delta of the Nile in the Mediterranean in the west to Susa in the east, across 14° of latitude and 18° of longitude. The portion of the earth brought under exploration was nearly doubled.

From this base, one and a half centuries later, the expeditions of the great kings reached from the northernmost shores of the Black Sea in the north to the southernmost shores of the Red Sea in the south and from the strait of Gibraltar in the west to the mouth of the Indus in the Indian Ocean in the east, across 33° of latitude and 73° of longitude. The portion of the earth brought under exploration was increased by an order of magnitude, some 10 times.

The following development occurred under the great kings, too. Like Eanatum, Naram-Sin, and Esarhaddon before him, Cyrus conquered further than any of his predecessors. His successor Cambyses conquered further than any of his predecessors and for the sake of conquering further than any of his predecessors. His successor Darius conquered further than any of his predecessors and for the sake of doing so distinguished exploration from conquest. His successor Xerxes did not conquer further than any of his predecessors, but he was a philosopher *par excellence*, and one of the many ways in which he was, was to undertake exploration for the sake of

exploration.

Assyria fell after Ashurbanipal, and four kingdoms rose, the Babylonian, the Egyptian, the Mede, and the Lydian.

Cyrus conquered three of them, the Mede, the Lydian, and the Babylonian. He then reached further east than any of his predecessors, and fell fighting the Massagetae beyond the Amu Darya.

Cambyses conquered the fourth kingdom, the Egyptian. He then sent an expedition further southwest and himself led an expedition further south than any of his predecessors. On the expedition in the southwest, Herodotus reports:

Cambyses sent 50 000 men against the Ammonians (III, 25); they never came back; it was said that the entire army was buried by a sandstorm (26).

On the expedition in the south, he reports:

the king himself led an expedition against the Ethiopians; but on this campaign to the uttermost ends of the earth, the army ran out of provisions before it had completed one fifth of its journey; the men began to eat the train animals; but these ran out, too, and the king still did not abandon the expedition; the men began to eat grass; but then the army entered the desert; the men began drawing lots to eat one out of ten men; and the king then turned back (25).

Darius led expeditions to the uttermost ends of the earth in the north and in the east. On the expedition in the north, Herodotus reports:

the people living in these parts are the stupidest of all, except for the Scythians; indeed, the Scythians have made the greatest discovery, they have discovered how no one can conquer them; they have no cities, carry their houses with them, and live not from tillage but off their herds; so how can they not be invincible (IV, 46)? and when the king advanced against them, the Scythians had carried their houses away; the king advanced further, but the Scythians had again retreated; the king advanced further, but the Scythians had again retreated (121-5); tired of the endless pursuit, Darius sent a herald to tell them, fight or submit (126); the Scythians did neither, and the king abandoned the expedition (134-5).

On the expedition in the east, Herodotus reports:

most of Asia was discovered by Darius; the king wanted to know where the Indus issues into

the sea, and sent ships to explore the river; they set out from Caspatyrus, sailed down the river toward the sea, then west through the ocean, and in the thirteenth month returned through the Red Sea; Darius then conquered the Indians and sailed the ocean; thus it was discovered that, except to the east, Asia is bounded by the sea (44).

And this was the domain of the great kings. Herodotus reports: the Persians called Darius the shopkeeper, because he kept accounts for everything (III, 89). What the shopkeeper had, then, the shopkeeper inventoried. Keeping pace with his conquests and the developing administrative organization of his realm, Darius's Behistun inscription lists 23 countries and peoples (6), his Susa inscription lists 27 (*id.*), his Naqsh-e Rostam inscription lists 29 (*id.*). Quoting a tax register, Herodotus lists 77 countries and peoples. He writes:

Darius set up twenty satrapies, assigned each people to a satrapy, and determined the yearly tribute of each satrapy;

satrapy I; Ionians, Magnesians in Asia, Aeolians, Carians, Lycians, Milyans, and Pamphylians; 400 Babylonian silver talents;

satrapy II; Mysians, Lydians, Lasonians, Cabalians, and Hytennians; 500 talents;

satrapy III; Hellespontines, Phrygians, Bithynians, Paphlagonians, Mariandyni, and Syrians; 360 talents;

satrapy IV; Cilicians; 500 talents and 360 horses; satrapy V; Posideium, Phoenicia, Palestinian Syria, and Cyprus; 350 talents;

satrapy VI; Egypt, the parts of Libya neighboring Egypt, Cyrene, and Barca; 700 talents and 120 000 bushels of grain;

satrapy VII; Sattagydae, Gandarians, Dadicae, and Aparytae; 170 talents;

satrapy VIII; Susa and the rest of Cissia; 300 talents;

satrapy IX; Babylon and the rest of Assyria; 1 000 talents and 500 boys to be eunuchs;

satrapy X; Ecbatana and the rest of Media, Paricanians, and Orthocorybanthians; 450 talents;

satrapy XI; Caspians, Pausi, Pantimathi, and Daritae; 200 talents;

satrapy XII; Bactrians as far as the Aegli; 360 talents;

satrapy XIII; Pactyes, Armenians, and their

neighbors as far as the Black Sea; 400 talents; satrapy XIV; Sagartians, Sarangae, Thamanaei, Utians, Mycians, and islanders in the Persian Gulf; 600 talents;

satrapy XV; Sacae and Caspians; 250 talents;

satrapy XVI; Parthians, Chorasmians, Sogdians, and Arians; 300 talents;

satrapy XVII; Paricanians and Ethiopians in Asia; 400 talents;

satrapy XVIII; Matieni, Saspies, and Alarodians; 200 talents;

satrapy XIX; Moschi, Tibareni, Macrones, Mosynoeci, and Mares; 300 talents;

satrapy XX; Indians; 360 talents of gold dust; and, subtracting 140 talents spent on the

Cilician cavalry, and reckoning that one Babylonian talent is worth one and one sixth Euboic talents and that gold is worth thirteen times its weight in silver, the yearly tribute amounts to 14 560 Euboic talents; and in this total I disregard amounts of 10 talents or less;

this tribute came to Darius from Asia and Africa; later, however, tribute was collected also from the Aegean islands and from Europe as far as Thessaly;

Persia is the only country exempt from tribute; and the following peoples were ordered not to pay tribute but to give gifts, as they still do in my time;

Ethiopians; 2 choenixes of unrefined gold and 200 ebony logs every two years;

Colchians and their neighbors in the Caucasus; 100 boys and 100 girls every four years;

Arabs; 1 000 talents of frankincense every year (III, 89-97).

1 Euboic talent equaled 60 minae, 1 mina equaled 100 drachmae, 1 drachma equaled 6 obols. Recorded by Thucydides, a treaty was sworn by Athens and Argos in 420, stipulating that: when one of the cities is helped by troops from the other, it will maintain them at the rate of 3 obols for one infantryman for one day and 1 drachma for one cavalryman for one day (V, 47).

At this rate, on their yearly income of 14 560 talents – disregarding amounts of 10 talents or less, the 120 000 bushels of grain collected from Egypt, the 360 horses collected from Cilicia, the 1 choenix of unrefined gold and 100 ebony logs collected from Ethiopia, and the 1 000 talents of

frankincense collected from Arabia, as well as the tribute collected later from the Aegean islands and Europe as far as Thessaly, never mind the boys and the girls – the great kings could maintain 478 684 infantrymen for one year in the field. Or 239 342 cavalrymen for one year in the field. Or, say, 400 000 infantrymen and 40 000 cavalrymen for one year in the field.

This was the domain of the great kings. Beyond this domain, Xerxes, the philosopher, reached further than any of his predecessors. Herodotus reports:

Xerxes was about to impale Sataspes for the rape of Zopyrus's daughter, but agreed to spare him if he sailed around Africa; so Sataspes sailed west, passed the strait of Gibraltar and then sailed south, but after many months there was always more sea to cross, and he turned back; to Xerxes he said that he had reached a land of dwarfs, and had turned back because his ship would not sail further; Xerxes did not believe him, and impaled him for the old offense (IV, 43).

It was nineteen centuries before another ship, sent out by another prince, by prince Henry, sailed further. In *The chronicle of the discovery and conquest of Guinea*, in 1453, de Zurara reports: prince Henry wanted to know the lands beyond the Canary Islands and cape Bojador; it seemed to him that if he or some other prince did not endeavor to gain this knowledge, sailors and merchants never will, because they never trouble without the certain prospect of profit; and prince Henry began sending his ships out to those parts (7); and although he sent out many times some of his foremost men, no one dared to pass cape Bojador; beyond that cape, men said, there are no people, the land is sand, there is no water, no tree, no herb, and the currents in the sea are so violent no ship can ever return; for twelve years the prince sent out ships every year, and no one dared to pass cape Bojador (8); at last he sent out Eanes, one of his squires, whom he afterward knighted; Eanes went to the Canary Islands, and touched by the same terror, turned back; this was in 1433; but the next year the prince sent him out again, charging him to strain to the utmost; and on this voyage Eanes rounded cape Bojador (9).

The domain of the great kings, then, reached from the southern slopes of the Caucasus, where

the Colchians and their neighbors were taxed, in the north, to the southern shores of the Red Sea, where the Ethiopians were taxed, in the south, 3 200 km apart, and from the city of Barca, in satrapy VI, in the west, to the valley of the upper Indus, in satrapy XX, in the east, 4 900 km apart.

77 countries and peoples large enough to owe more than 10 talents were arranged in their series and order in that domain.

Darius explored the Indian Ocean, Xerxes explored the Atlantic Ocean. The strait of Gibraltar and the mouth of the Indus are 6 900 km apart, $\frac{1}{6}$ of the circumference of the earth.

And the great kings organized this portion of the earth into a unity, all of it by the still loose bond of exploration, most of it by the always tight bond of taxation.

Thus the great kings.

As to the philosophers, on the authority of Aristotle, philosophy begins with Thales, and because Thales first proposed solutions of two problems, the problem of the principles of all things and the problem of the earth.

Thales said that the earth is at rest because it floats on water, like wood (Aristotle, *Heaven*, II, 13). Anaximander, the second philosopher, said that the earth is at rest because of equilibrium (*id.*), and it is shaped like a drum (Hippolytus, *Refutation*, I, 6). Anaximenes, the third philosopher, said that the earth is flat, and at rest because it floats on the air below, like a lid (Aristotle, *Heaven*, II, 13). Thales and Anaximander predate Cyrus by one-odd and one generation, respectively, Anaximenes is contemporary with Cyrus.

Two-odd generations after Anaximenes, Parmenides discovered that the earth is a sphere (Laertius, IX, 21-2). Parmenides is contemporary with Xerxes.

In the meantime drawing the map of the earth had become part of the solution of the problem of the earth.

In the *Geography*, on the authority of Eratosthenes, Strabo writes: first Anaximander drew a map, and Hecataeus wrote a geographical treatise (I, 1.11).

There is no more in the record about the map Anaximander drew. There is much in the record, however, from the treatise Hecataeus wrote, as

well as about the map he drew. Hecataeus is the seventh philosopher, and contemporary with Darius.

Also, one generation after Parmenides, there is the *Histories*. Herodotus is contemporary with the successor of Xerxes Artaxerxes, and by this time the philosophers have become so many, counting them consecutively has quit being illuminating.

From the first geographical treatise there are some 330-odd fragments in the record. Hecataeus notes in them 300-odd localities. The localities are distributed from Scythia in the north (fr. 185-90) to Ethiopia in the south (fr. 325-7), and from the strait of Gibraltar in the west (fr. 39) to the Indus in the east (fr. 296). Coinciding precisely with the reach of the expeditions of the kings.

Hecataeus was from Miletus. Part of the domain of the great kings since Cyrus, the Greek cities of Asia Minor resolved to revolt against Darius. Aristagoras, the leader of Miletus, urged them to revolt. On the other side, Herodotus writes: Hecataeus urged them strongly not to wage war on the great king; and in order to persuade them, he pointed to the power of the great king, and listed for them all the peoples ruled by the king (V, 36).

With a list of peoples his means of persuasion, Hecataeus failed to persuade them. Aristagoras thus went to Sparta and Athens to solicit help for the revolt. He took with him, Herodotus reports: a map of the entire earth, drawn on a bronze tablet, showing all the seas and all the rivers (49). No one but Hecataeus will have drawn the map of the entire world, showing all the seas and all the rivers.

In Sparta, Aristagoras pointed to the map and said to the king: Cleomenes, here, next to the Ionians are the Lydians, next to the Lydians are the Phrygians, next to the Phrygians are the Cappadocians, next to the Cappadocians are the Cilicians, next to the Cilicians are the Armenians, next to the Armenians are the Matieni, next to the Matieni is Cissia, and in Cissia is Susa where the great king has his palace; capture Susa and you will rival Zeus in wealth (*id.*).

Qualitatively, then, if not yet quantitatively, Hecataeus has solved the problem of orientation from the Ionians to Susa.

Herodotus continues: how far from the sea is

Susa? Cleomenes asked; three months, Aristagoras replied (*id.*).

Hecataeus has solved the problem of distance from the Ionians to Susa, too. In this case quantitatively, too.

Moreover, it was not Hecataeus alone who appreciated the importance of the quantitative determination of distance. The Spartan king did. Herodotus continues: my Milesian friend, Cleomenes said, there is no argument that can take the Spartans on a three months' march from the sea (*id.*). The Athenian assembly of the people, on the other hand, did not. Herodotus continues: so Aristagoras went to Athens, and came before the assembly of the people; he said the same things he had said in Sparta, and because it is easier, it seems, to fool many than to fool one, he did not fool Cleomenes, but he fooled 30 000 Athenians; so they voted to send 20 ships to Ionia (97).

Two-odd generations after Hecataeus, Herodotus describes in the *Histories* 1 012 localities. West to east, they are distributed from the Atlantic (I, 203) to the Indus (IV, 44), coinciding precisely with the reach of the expeditions of the kings. North to south, they are distributed from the problem of the source of the Dnieper (53) to the problem of the source of the Nile (II, 31-3). In the north, where the people are the stupidest of all, except for the Scythians, Herodotus reached a little further than Darius, in the south, where there is not even grass to eat, he reached a little further than Cambyses. North to south and west to east, the explicit determination of distance and orientation begins with the *Histories*. Like the shopkeeper, Herodotus keeps accounts for everything. In particular, he keeps arranging cities, countries, and peoples in their series and order. And much more than one half of the portion of the earth on which Herodotus arranged localities in their series and order is within the tight unity of the domain which the great kings taxed.

Geography begins with Hecataeus and Herodotus, and Hecataeus and Herodotus learned from no one more than they did from the great kings.

History.

To the fact that Herodotus has been quoted, and will be, and because he cannot not be, as often as he has been, and will be, it suffices to add this one fact. Beginning to end, the *Histories*

is concerned with:

the antecedents of the rise of Cyrus (I, 1-74),
the expeditions of Cyrus (I, 75-216),
the expeditions of Cambyses (II, 1-III, 66),
the expeditions of Darius (III, 67-VII, 4),
the expeditions of Xerxes (VII, 5-IX, 122).

Herodotus is the father of history, and Herodotus learned from no one more than he did from the great kings.

Physics.

On their annual income of 14 560 talents the great kings could maintain 478 684 infantrymen for one year in the field.

And they did.

In 480, Herodotus reports: Xerxes invaded Europe with 1 700 000 men (VII, 60).

It is one of Herodotus's many reports about the great kings which have met with scepticism. Surely no one can have gathered such an army. And surely no one can have known in any case how large the army is.

Leibniz writes: the malignity of men and their envy of the powerful confers authority on such writers. Gibbon quotes Condé: *ces coquins nous font parler et agir comme ils auraient fait eux-mêmes à notre place.*

On their annual income of 14 560 talents the great kings could maintain 478 684 infantrymen for one year in the field. When Alexander captured Susa he found in the kings' palace: 40 000 talents in coin, next to other treasures of incalculable value (Plutarch, *Alexander*, 36). Darius began preparing the expedition in 488, Herodotus reports (VII, 1). When Xerxes embarked on it, the expedition had been in preparation for eight years.

Then there is the fact of opportunism. When the great king proposed to march to the end of Europe, how many able-bodied men this side of India failed to respond to the call to plunder all Europe? Herodotus writes: the further the king advanced more and more men followed him (VIII, 66).

Then there is the method which Xerxes, the philosopher, employed to count his men. Herodotus writes: they packed together as tightly as possible 10 000 men, drew a circle around them, then let them go and built a wall to the height of a man's navel on the circumference of the circle;

next they packed others inside the wall, next others, and so until they had counted them all (VII, 60).

His men counted, Xerxes reviewed them: Persians, Medes, Cissians, Hyrcanians, Assyrians, Chaldeans, Bactrians, Sacae, Indians, Arians, Parthians, Chorasmians, Sogdians, Gandarians, Dadicae, Caspians, Pactyes, Utians, Mycians, Paricanians, Arabians, Ethiopians who live above Egypt, Ethiopians of the east, Libyans, Paphlagonians, Ligyans, Matieni, Mariandyni, Cappadocians, Phrygians, Lydians, Mysians, Thracians, . . . , Cabalees, Milyae, Moschi, Tibareni, Macrones, Mossynoeci, Mares, Colchians, Alarodians, Saspises, islanders from the Red Sea, Sagartians, Phoenicians, Palestinian Syrians, Egyptians, Cypriots, Cilicians, Pamphylians, Lycians, Asian Dorians, Carians, Ionians, Aegean islanders, Hellespontines (VII, 61-95), taxed jointly by the king, marched and sailed jointly in the army and navy of the king.

Clearly, the rule was, and for the great kings before it was for the historian, always to make enumerations as complete and reviews as general as to omit nothing.

The human population of the earth was some 115 million. The great kings ruled more than one fifth, some 25 million.

The great kings established the fact that humanity is large. And Xerxes made this fact accessible to observation.

The great kings established the fact as well that the earth is large.

Cyrus said:

I am the lord of the earth (cylinder),

Cambyses left no great inscriptions behind,

Darius said:

I am the lord of the earth (Susa ins.),

Xerxes said:

I am the lord of the earth (Persepolis ins.),

Artaxerxes said:

I am the lord of the earth (Persepolis ins.),

and they were the lords of the earth.

But it is once again Xerxes, the philosopher, in Herodotus, the historian, who said it best.

Before embarking on the expedition, Xerxes addressed his council: Persians, the great king said, it is not a new law introduced by me, it is a law handed down to me by my predecessors, that

we never rest; you know all the peoples Cyrus, Cambyses, and Darius conquered; so when I succeeded to the throne, I considered how I can be worthy of my predecessors and how I can give the Persians as much power as they did; I decided to lead my army in Europe, against Greece; if we conquer the Greeks, we will make the limits of Persia the same as the limits of heaven are for Zeus; I will march to the end of Europe, and when the sun looks down, it will see no country bordering on ours, because I will make them all one country; there will be no city or people left among men to fight against us (VII, 8).

The expedition failed. At Salamis and at Plataea, led by Themistocles and by Pausanias, the Athenians and the Spartans stopped the great king.

Xerxes did not march to the end of Europe. He did not make all countries one country. There remained cities and peoples among men to fight against the Persians. The great king did not make the limits of Persia the same as the limits of heaven are for Zeus. When the sun looked down, it saw countries bordering on the country of the great kings.

In particular, this is what the sun saw.

If it revolved around the earth once every day with its customary speed, that of the rotation of the earth on its axis, if it was equipped to see not better than the human eye, if it was, in order to see, at the cruising altitude of a large airplane, and if it fixed its eye on the parallel 30° north of the equator, the sun saw for $3\frac{1}{2}$ hours every day the country of the great kings. It saw the Pacific for $8\frac{1}{2}$ hours. It saw the Atlantic for $4\frac{1}{2}$ hours. It saw America for $2\frac{1}{2}$ hours. In order to see the

countries of the Spartans and of the Athenians, the sun had to shift its eye to the parallel 38° north of the equator, and it could then see the Aegean, the sea which the Athenians dominated, for 12 minutes, and the Peloponnese, the peninsula which the Spartans dominated, for 8 minutes. It could then also see the city of Athens for about 30 seconds. 1° further south, it could see the city of Sparta for about 10 seconds.

The great kings established the fact that the

earth is large, and they made this fact as well accessible to observation.

All early philosophers are reported to have traveled. Where, except in that he traveled further, Herodotus is representative. By boat and on foot, he traveled from Olbia in the north to Elephantine in the south and from Cyrene in the west to Susa in the east. He traveled the domain of the great kings.

In the *Two new sciences*, Galileo begins by showing that a large boat built the way a small boat is built and a large column built the way a small column is built will fall apart under their own weight. He then generalizes: one cannot reason from the small to the large because many things which are successful in the small cannot exist in the large (1), and with this result the two new sciences of mechanics and locomotion begin.

Conversely, many things which exist in the large do not exist in the small. One such thing is humanity. Another is the earth.

Thus, at first the earth floated on water, like wood. It was at rest because of equilibrium, and it was shaped like a drum. It was flat, and it floated on the air below, like a lid.

Then the great kings established the fact that the earth is large. From the strait of Gibraltar to the mouth of the Indus it is 6 900 km, 1.1 earth radii.

Then Parmenides discovered that the earth is a sphere.

Another thing which exists in the large and not in the small is the sun. This fact, in the generation after Parmenides, Anaxagoras began discovering. Anaxagoras said that the sun is a piece of red-hot metal larger than the Peloponnese (Aëtius, II, 21).

ccvi

Politics.

All the many facts about the state of the great kings that have already been noted can be gathered into two general facts.

Size. The most powerful states next to that of the great kings were those of the Spartans and of the Athenians. And if the sun saw more of the state of the great kings than of the continent of America, it saw also 15+ times more of it than of the state of the Athenians, and 25+ times more of it than of that of the Spartans.

Power. Xenophon considered how cattle obey the cowherd, then he considered all the peoples which wanted to please Cyrus and do his bidding.

So, the philosophers could consider the cattle. They could consider the one dozen cities which wanted to please the Spartans and do their bidding. They could consider the two dozen cities which at the peak of their power wanted to please the Athenians and do their bidding. And the six dozen peoples which wanted to please the great kings and do their bidding Xerxes brought to Athens for the philosophers to consider.

Another fact is permanence.

With Aristotle: what makes a state one and the same is the same constitution, and constitution is the arrangement of magistracies and especially the highest.

At the time of Cyrus, Sparta was under the dual kings Anaxandridas and Ariston, Athens was under the tyrant Pisistratus.

In the two centuries beginning with Anaxandridas and Ariston Sparta remained one and the same state.

In *The constitution of Athens* Aristotle counts 8 changes of constitution in Athens from Pisistratus through his own time (41). In the two centuries beginning with Pisistratus Athens was 8 different states.

The Spartans and the Athenians defeated Xerxes. Having thus failed to follow the example of Cyrus, Cambyeses, and Darius, Xerxes provided his successors with another example to follow. He consolidated the domain conquered by his predecessors. Xerxes was succeeded by

Artaxerxes	prince 465-24,
Darius II	prince 423-04,
Artaxerxes II	prince 404-358,
Artaxerxes III	prince 358-38,
Darius III	prince 336-0,

and in the two centuries beginning with Cyrus the state of the great kings, unifying one fifth of humanity, remained one and the same. Moreover, when

Alexander prince 336-23, conquered that state and became its highest magistrate, it remained the same state. Alexander's successors destroyed that state.

Himself, Alexander was also a philosopher. Arrian reports: when we march to the ocean in the

east, the king said, all Asia will be ours; when we next sail south from the Persian Gulf and come back through the strait of Gibraltar, all Africa will be ours; and the limits of our empire will then be the same as the limits Zeus set on the earth (*Alexander*, V, 26).

One last fact.

The state is a democracy, an aristocracy, or a monarchy. Thus Plato (*Republic*, VIII, 544d-5a), Aristotle (*Politics*, III, 7), Zeno of Citium (Laertius, VII, 131), Polybius (VI, 3), Cicero (*Republic*, 42-3), Marsilius (*Defender*, I, 8.2), Machiavelli (*Discourses*, I, 2), Grotius (*Law*, I, 3.8), Hobbes (*Leviathan*, 19), Locke (*Government*, II, 10), Montesquieu (*Laws*, II, 2), Rousseau (*Contract*, III, 3).

Herodotus reports: after the death of Cambyeses, seven Persians gathered to discuss what form of government to establish; Otanes proposed democracy, Megabyzus proposed aristocracy, Darius proposed monarchy; one man gave us freedom, Darius said, one man will preserve our freedom (III, 80-2).

Surely no Persian proposed that Persia should be a democracy.

Herodotus writes: when Mardonius came to Ionia, a wonderful thing happened, wonderful, that is, for those who do not believe that Otanes proposed that Persia should be a democracy; Mardonius brought down the tyrants and established democracies in the cities (VI, 43).

For those who do not doubt that Otanes proposed that Persia should be a democracy, the analysis of the state into democracy, aristocracy, and monarchy predates Plato by four generations. For those who do not doubt that Otanes proposed that Persia should be a democracy and for those who do, both, Herodotus predates Plato by two generations.

Whether they wanted to or not, when the philosophers began concerning themselves with politics, they learned from no one more than they did from the great kings.

Ethics.

Plato writes in the *Laws*: Cyrus granted a degree of freedom to the peoples he ruled, he also valued those who contributed with their advice to the government of the state; and by their freedom, friendship, and sharing of ideas the Persians were

successful in everything; but the king neglected the education of Cambyses, and Cambyses was driven mad by debauchery; the usurper Darius then introduced a measure of equality for all, with his generosity he created a feeling of community and friendship among the Persians, and he added to the lands bequeathed him by Cyrus as many lands again; but Darius also neglected the education of Xerxes, and Xerxes committed the same mistakes as Cambyses; indeed, none of the great kings after Darius has been truly great (III, 694a-5e).

Like Xenophon, and like the lord, Plato appreciates the success of Cyrus and Darius. But he also disapproves of them. Cyrus neglected the education of Cambyses, Darius neglected the education of Xerxes. Hence their debauchery and mistakes. Indeed, none of the great kings after Darius was truly great.

There is another reason for Plato's disapproval. Earlier in the *Laws* he writes: had the Persians won the war, we would have been all mixed together, Greek with Greek, Greek with barbarian, barbarian with Greek, as we now see the peoples ruled by the Persians all mixed together (693a).

Had the Persians won the war, we would have been all mixed together.

There is yet another reason. To answer the question, what is justice? Plato begins in the *Republic* with a call to arms: are we, Socrates asks, to join ranks with the wise and blessedly happy Simonides, Pittacus, and Bias, who said that justice is to give everyone his due, and fight against Periander, Perdiccas, Xerxes, and Ismenias, who said that justice is to do good to your friends and harm to your foes? yes, Polemarchus replies (I, 335e-6a).

Justice is to give everyone his due. Justice is not to do good to your friends and harm to your foes.

And it ought to be said that Plato is in the wrong, morally wrong, on all counts, education, the mixing of peoples, and justice, all three.

To start with the last, justice.

The wise and blessedly happy Simonides, Pittacus, Bias, Socrates, and Polemarchus as well as the unwise and cursedly unhappy Periander, Perdiccas, and Ismenias can be – here – ignored.

It suffices to consider the philosopher and the great king. In the *Republic*, the philosopher, Plato, has done the doctrine, justice is to give everyone his due, the justice it deserves. In his inscriptions, the great king, Darius, who neglected the education of Xerxes, has done the doctrine, justice is to do good to your friends and harm to your foes, the justice it deserves.

On Plato's authority, then, god made men of three kinds, gold, silver, and iron or bronze (III, 415a-b). So it is their due to be educated into, and then continue to be, philosophers, guardians, and producers, respectively (IV, 420c-7d). It is the philosophers' due to rule all three kinds (V, 473c-4c). More particularly, the philosophers' due is to distinguish the gold, silver, and iron or bronze from one another, to educate the gold into themselves, into philosophers, to educate the silver into guardians, and to neglect the education of the iron or bronze so it becomes producers (II, 374e-III, 412a). The guardians' due is to be educated by the philosophers into guardians, the producers' due is to have their education neglected so they become producers (*id.*). Education over, everyone's due is to give everyone his due, the philosophers the philosophers', the guardians the guardians', the producers the producers' (IV, 433a-d).

Clearly, it is everyone's due as well to be the friend of the doctrine, justice is to give everyone his due. This is liable to have the consequence that outside the city someone is still the foe of the doctrine, inside the city everyone is its friend. Accordingly, it is the philosophers' due, too, to impart to the guardians this their due: to guard the city against its external foes and its internal friends, so that the former lack the power to harm the city, the latter lack the will (III, 414a-b).

The Behistun inscription begins:

I am Darius, the great king, the king of kings, the king of Persia, the king of countries (1).

The Susa inscription begins:

I am Darius, the great king, the king of kings, the king of all kinds of peoples of all kinds of origin; I am the lord of the earth, far and wide (*id.*).

The Behistun inscription then lists the peoples of all kinds and all kinds of origin (7). The Susa inscription lists the growing number of peoples (*id.*). The Behistun inscription continues:

king Darius says, within these countries whoever was a friend, I firmly protected him, whoever was a foe, I utterly destroyed him (8), and concludes:

king Darius says, I ruled in accordance with justice; I did no wrong to the weak and no wrong to the strong; whoever helped my rule, I favored him, whoever was its foe, I utterly destroyed him (63).

The Susa inscription explains:

king Darius says, the peoples were in commotion, all peoples fighting each other; by the grace of Ahura Mazda I brought an end to the fighting; they no longer fight each other at all, all are at peace; they fear the law; the strong do not fight or destroy the weak (*id.*).

And can it be better, morally better, to divide humanity into gold, silver, and iron than divide it into the strong and the weak? to impart the doctrine that justice is to give everyone his due, and especially the philosopher his due, than the doctrine that justice is to do no wrong to the weak and no wrong to the strong, so that the strong do not fight or destroy the weak? to guard the philosopher's doctrine against its external foes and its internal friends than firmly to protect the friend of the king's doctrine and utterly to destroy its foe?

To take education next.

On the one hand, Cyrus neglected the education of Cambyses, Darius neglected the education of Xerxes, and Plato neglected the education of the producers. On the other hand, Plato educated the philosophers and the guardians, and Plato's successors have educated the producers, too.

Kant, for his part, urges you: have the courage to use your own understanding.

For if you lack the courage to use your own understanding, someone will use your own understanding for you. And he will begin by educating you.

Plato writes in the *Protagoras*: the sophists are peddlers of nourishment for the soul, and if you do not know what you are buying, they may well deceive you; so if you know what you are buying, buy safely from Protagoras or anyone else, but if you do not know what you are buying, you are rolling the dice; because if you buy food or drink, you can take them home and ask someone knowledgeable whether they are good, but teaching you

cannot take home in a jar, you are stuck with it in your soul (313c-4b).

Plato is not a peddler of nourishment for the soul. He is giving you the nourishment free of charge. And do you know what you are given? If you know, take safely from Plato or anyone else. But if you do not know, you are rolling the dice. In fact, Plato is rolling it for you. Because if you buy food or drink, you can take them home and ask someone knowledgeable, but teaching you cannot take home in a jar, you are stuck with it in your soul. And if you could take it home in a jar to ask someone knowledgeable, to whom would you be taking the jar but to Plato?

And humanity has been stuck in its soul with much worse than the doctrine that men are gold, silver, and iron and the doctrine that it is the philosophers' due to impart to the guardians their due to guard the city against its external foes and its internal friends.

Education may be better than neglect of education, politically. Morally, can neglect of education be worse than education? worse than Plato's education, for instance?

Finally, Darius writes: the peoples no longer fight each other at all, all are at peace; they fear the law; the strong do not fight or destroy the weak.

Two generations later, the great kings had built the royal road, Sardis to Susa. Herodotus reports: the road is 450 parasangs long, a ninety-day journey from one end to the other, protected by 111 guard stations (V, 53). 450 parasangs is 2 700 km. Herodotus describes the road in some detail, as it begins in Sardis, in Lydia, passes through Phrygia, Cappadocia, and Cilicia, then across the Euphrates, then through Armenia, then across the Tigris, then through the country of the Matieni, then across the Diyala, then through Cissia, and then on the Karkheh is Susa, where the palace of the great kings is. And adds: it is safe along its entire length (52). After the fall of the great kings, in the twenty-three centuries since, the times when the road from Sardis to Susa has been safe along its entire length have to be counted in days, not years.

Two generations after Herodotus, Plato writes: had the Persians won the war, we would have been all mixed together, Greek with Greek, Greek

with barbarian, barbarian with Greek, as we now see the peoples ruled by the Persians all mixed together.

How many wars of Greek with Greek – beginning with the Peloponnesian – of Greek with barbarian, of barbarian with Greek, and of barbarian with barbarian would Greeks and barbarians have been spared had the Persians won the war?

Darwin writes: it seems to me highly probable that any animal with well-marked social instincts would inevitably acquire a moral sense as soon as its intellectual powers had become as well developed as in man; this view of the origin and nature of the moral sense accords well with what we see of the early undeveloped condition of this faculty in mankind; the virtues which must be practiced are those which are still recognized as the most important; but they are practiced almost exclusively in relation to the men of the same tribe.

The generalization from the tribe to humanity was achieved by the princes.

And a most important step in the generalization was taken by the great kings, who mixed the peoples from Thrace to India to Ethiopia to Libya for two centuries, and forced the mixture to fear the law.

The conclusive step was taken by the Roman consuls, generals, and then emperors, who mixed the peoples from Britain to Mesopotamia to Nubia to Mauritania for five and a half centuries, and cemented the mixture's fear of the law by the development of jurisprudence.

The generalization began with the princes of the Sumerian cities.

A decisive step was taken by the Akkadian princes, Sargon, Rimush, Manishtushu, Naram-Sin, and Sharkalisharri, under whom the valley of the two rivers from the Mediterranean to the Persian Gulf remained one and only one state for one and a half centuries.

In the generation before Sargon, Urukagina is the first prince to leave behind inscriptions concerned with laws.

Urukagina reports:

there were the old ways;

but when Ningirsu gave the princship of Lagash to Urukagina, Urukagina changed the old ways (ins. 1).

Among Urukagina's changes of the old ways

are these:

when a good ass is born to the weak, and the strong says, I want to buy it from you, and the weak says, pay me the price I want, the strong shall not strike him;

when the house of the weak is next to the house of the strong, and the strong says, I want to buy it from you, and the weak says, pay me the price I want, the strong shall not strike him (*id.*).

Three generations after Sharkalisharri, Ur-Nammu promulgated the oldest code of laws.

Among Ur-Nammu's laws are these:

if a man kills a man, he shall be killed;

if a man breaks the bone of another, he shall pay 60 shekels;

if a man knocks out the tooth of another, he shall pay 2 shekels;

if a man is a witness in a case and refuses to take the oath, he shall pay whatever is at stake in the case (*id.*).

The code begins with a preamble:

Ur-Nammu, the mighty, prince of Ur, prince of Sumer and Akkad;

when An and Enlil gave the princship of Ur to Nanna, Nanna gave it to Ur-Nammu so that justice and truth prevail;

I did not let the strong man take the orphan;

I did not let the man of 60 shekels take the man of 1 shekel (*id.*).

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The princes in the valley of the two rivers, then, are the predecessors of the philosophers. With the princes in the valley of the Nile, whose series begins three-odd centuries earlier, they are the earliest predecessors of the philosophers.

The labors of the princes attest the beginning of time reckoning, the beginning of the exploration of the surface of the earth, the beginning of the examination of all original materials to the end of what merits the attention of all times, the beginning of the attempts to establish what the end of the state is. They attest the acquisition of knowledge.

So does the list, the first investigation, the perfect investigation, with which success in the search for truth begins.

If knowledge has been acquired by the labors of their predecessors and the philosophers, exclu-

sively, and these labors are collected, the acquisition of knowledge will thus be collected, included in its entirety in the collection.

And the princes and the list show that knowledge has been acquired by their predecessors and the philosophers, exclusively, that these labors can be collected, and that the collection will include the acquisition of knowledge in its entirety.

They show that the collection depends on their labors, exclusively, too.

The princes in the valley of the two rivers from Enmebaragesi through Ibbi-Sin are 59.

Dividing them – because the rule is to divide difficulties into parts such as will lead to their better solution – into important and not important, the important, with the very important in bold, are these 53,

Enmebaragesi	prince c. 2680,
Aka	prince c. 2650,
Gilgamesh	prince c. 2650,
Ur-Nungal	prince c. 2620,
Meskalamdug	prince c. 2600,
Mesanepada	prince c. 2570,
Mesalim	prince c. 2555,
Aanepada	prince c. 2540,
Ur-Nanshe	prince c. 2535,
Meskiagnuna	prince c. 2510,
Akurgal	prince c. 2505,
Ush	prince c. 2495,
Elili	prince c. 2490,
Enatum	prince c. 2475,
Enakale	prince c. 2475,
Enanatum	prince c. 2470,
Enshakushana	prince c. 2460,
Ur-Luma	prince c. 2445,
Enmetena	prince c. 2440,
Lugalkignedudu	prince c. 2440,
Puzur-Nirah	prince c. 2420,
Il	prince c. 2415,
Enanatum II	prince c. 2410,
Lugalkisalsi	prince c. 2410,
Ku-Baba	prince c. 2400,
Gishakidu	prince c. 2395,
Enentarzi	prince c. 2390,
Puzur-Sin	prince c. 2370,
Lugalanda	prince c. 2360,
Lugalzagesi	prince c. 2340,
Urukagina	prince c. 2340,

Ur-Zababa	prince c. 2340,
Sargon	prince c. 2337-281,
Rimush	prince c. 2281-72,
Manishtushu	prince c. 2272-57,
Naram-Sin	prince c. 2257-20,
Sharkalisharri	prince c. 2220-195,
Dudu	prince c. 2192-71,
Shudurul	prince c. 2171-56,
Ur-Baba	prince c. 2165,
Ur-Nigin	prince c. 2158-1,
Laerab	prince c. 2152-0,
Ur-Gigir	prince c. 2151-45,
Gudea	prince c. 2145,
Yarlagan	prince c. 2138-1,
Sium	prince c. 2131-24,
Utuhegal	prince c. 2128-1,
Ur-Nammu	prince c. 2121-03,
Puzur-Inshushinak	prince c. 2110,
Shulgi	prince c. 2103-055,
Amar-Sin	prince c. 2055-46,
Shu-Sin	prince c. 2046-37,
Ibbi-Sin	prince c. 2037-13.

The not important are these 6,

Zuzu	prince c. 2475,
Enbi-Eshtar	prince c. 2460,
Uu	prince c. 2370,
Sarlagab	prince c. 2206-0,
Eluldan	prince c. 2192,
Tirigan	prince c. 2124.

To establish that these 59 are the princes in the valley of the two rivers from Enmebaragesi through Ibbi-Sin, some 180 particulars have been sorted out, the 120 collected by the list, and some 60 collected from the inscriptions of the princes, other prince lists, and the oldest chronicles.

The collection of the 59, out of some 180, depends on the list, on the inscriptions of the princes – and 52 of the 59 are attested either by their own inscriptions and the inscriptions of other princes or by the inscriptions of other princes only – and on the three chronicles, *Gilgamesh and Aka*, the Tummal chronicle, and the Esagil chronicle.

And as the collection of the particulars has begun so the collection of the particulars can continue.

If it includes the acquisition of knowledge in its entirety, however, the collection does not include only the acquisition of knowledge. If the

acquisition of knowledge is the steps of advance in the acquisition of knowledge, exclusively, the collection includes more than the steps of advance in the acquisition of knowledge.

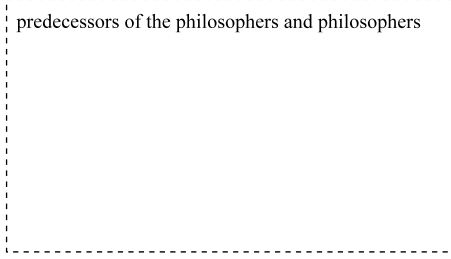
Thus, in order to distinguish the acquisition of knowledge from everything else, the task is, while continuing to collect their predecessors and the philosophers, to divide them also into these that are the acquisition of knowledge and those that are not. As this division depends on the labors of their predecessors and the philosophers, exclusively, too.

To picture it. Euler writes: because a general concept encloses an indefinite number of individual objects, we may regard it as a space in which

all these objects are enclosed (*Princess*, II, 34).

The enumeration of the 59 princes has begun to enclose this space.

predecessors of the philosophers and philosophers



The task is to enclose within this space the space of the acquisition.

The discovery of the important among our predecessors

ccviii

With years of birth and, where warranted, of death assigned – while postponing once again the treatment of the problem of chronology, in general, as well as of the chronology of the first philosophers, in particular – these are the first, the second, the third, the fourth, the fifth, the seventh, and the eighth philosophers,

625	Thales,
611 - 541	Anaximander,
590	Anaximenes,
570 - 495	Pythagoras,
570 - 473	Xenophanes,
562	Hecataeus,
540	Heraclitus.

The eighth philosopher is not quite three generations after the first. And among the many developments which the seven attest is that through the fourth the philosophers have not left their own words behind, but beginning with the fifth the philosophers have left their own words behind, and will continue leaving ever more of them behind.

Thales, Anaximander, Anaximenes, and Pythagoras are very well attested. Not quite a century after Pythagoras, Herodotus writes about Thales and Pythagoras. Two generations later,

Plato writes about Thales and Pythagoras. One generation later, Aristotle writes about all four, Thales, Anaximander, Anaximenes, and Pythagoras. In the next generation Aristotle's students Theophrastus and Eudemus wrote the first histories of physics and of mathematics, respectively, and thereafter the evidence is massive. As Plato, for instance, attests, to whatever else they have paid attention, the philosophers have paid the closest attention to themselves, to the philosophers.

Xenophanes, Hecataeus, and Heraclitus are just as well attested. Herodotus writes about Hecataeus, Plato and Aristotle write about Xenophanes and Heraclitus, and thereafter the evidence about them is massive as well. But they have also left behind their own words, Xenophanes some 40-odd fragments, Hecataeus some 360-odd fragments, Heraclitus some 130-odd fragments.

Aristotle is not quite two and a half centuries after the first philosopher, one and a half centuries after the eighth. His most extensive review of his predecessors is in book I of the *Metaphysics*. He writes about Xenophanes: he said nothing clearly (5). About Alcmaeon: all he said was vague hints (*id.*). About Anaxagoras and Empedocles: they did not quite know what they were

saying, fighting the way untrained men fight (4), even if Anaxagoras: was like a mature man among babblers (3). About Melissus: we can ignore him as too naive (5). About Leucippus and Democritus: the problem of the beginning of change they, like the others, lazily neglected (4). About Hippo: we can ignore him because of the paltriness of his thought (3). About Plato: what he said is prattle (9). In general: still young, making its first steps, philosophy was on this, as on all other matters, so to say, stammering (10).

In their own words, Xenophanes, Hecataeus, and Heraclitus exhibit philosophy still young, making its first steps, not quite knowing what it was saying, fighting the way untrained men fight, stammering, babbling, prattling. Philosophy *in statu nascendi*.

The 40-odd fragments of Xenophanes amount to some 120-odd lines. Most of the 360-odd fragments of Hecataeus are but the name of a locality each. The 130-odd fragments of Heraclitus amount to a couple of hundred lines.

And about whatever else it began stammering, babbling, and prattling, philosophy began stammering, babbling, and prattling about its predecessors. The first philosopher to leave his words behind, Xenophanes notes in his fragments 6 particular predecessors of his. Later in the same generation, Hecataeus notes 1 particular predecessor of his. In the next generation, Heraclitus notes 11 particular predecessors of his. Like the princes who began noting the princes as soon as there were princes in any numbers, the philosophers began noting the philosophers as soon as there were philosophers in any numbers.

The collection of the philosophers can begin with Xenophanes, Hecataeus, and Heraclitus. And having begun with the Sumerian prince list, the collection of their predecessors and the philosophers should continue with Xenophanes, Hecataeus, and Heraclitus.

ccix

Like Homer and Hesiod before him, Xenophanes was a rhapsode, singing his own songs (Laertius, IX, 18). Like Homer and Hesiod before him, and like Parmenides and Empedocles after him, he left behind verse, and distinguishing his own words from the context in which they are

quoted is seldom problematic.

In particular, his own words amount to some 120-odd lines, some 820-odd words.

It is not too many words, compared with how many later philosophers and not philosophers have tended to leave behind.

It will be taken to be another instance of the deficiency of the record.

Scepticism and worse have long made it normative to suspect the record. To suspect its integrity, and not just to question its veracity, but to assume its mendacity. There is no knowing what is lost. And what has survived is the outcome not just of accidents, decay and fires, rodents and mildew, the utility of clay tablets for building and of parchment for kindling, neglect and ignorance. What has survived is the outcome of the deliberate choices of those who quoted and copied their predecessors, and thus, no doubt, the outcome of prejudice and bias.

Against all that, the record is perfect. There is knowing what is lost. Nothing important is lost, not 1 thing.

It is important to show this in particular, however, and not only because of the longevity and ubiquity of the supposition that the record is suspect. Like chronology, the integrity of the record demands special treatment, and this is also best postponed.

Here, suffice it to note this. If of the princes in the valley of the two rivers through Ibbi-Sin the list collected 37 princes attested independently, 25 more princes who, whether they are attested independently or not, need not be presumed real, and yet 58 more princes who are not attested independently and, although they can be presumed real, cannot be presumed important at all, how likely can it be that the Sumerian record has lost even 1 prince?

In his 820-odd words Xenophanes has left behind results concerning the following. The gods and god. The principles of all things. The earth. Rain, clouds, wind, and rivers. Knowledge. What is worthy and what is not. His predecessors. Four of the seven results, those concerning the gods and god, knowledge, what is worthy and what is not, and his predecessors, are a very great advance each.

First, then, Xenophanes's results can be

shown. And then the particulars can be collected and sorted out.

ccx

The gods enter the record twenty-four and a half centuries before Xenophanes.

They enter the record with the inscriptions of the princes, and as the inscriptions of the princes attest the growing complexity of the relations of the princes with one another and with their cities over a steadily larger portion of space and time, so the inscriptions of the princes attest the growing complexity of the relations of the princes with the gods, and of the gods with the princes, with their cities, and with one another, over a steadily larger portion of space and time.

Thus in the valley of the Nile, and thus, and more explicitly, in the valley of the two rivers.

In the valley of the two rivers Mesanepada, prince *c.* 2570, first attests a dedication to a god: for Lugalkalama, Mesanepada, prince of Ur (ins. 1). One generation later, Aanepada and Ur-Nanshe attest that the princes were concerned for the well-being of the gods: Aanepada, prince of Ur, built the temple of Ninhursag (ins. 3), and: Ur-Nanshe, prince of Lagash, built the temple of Ningirsu, built the temple of Nanshe (ins. 2). Two generations later, Eanatum campaigned from Mari to Susa. And he attests that as the princes were concerned for the well-being of the gods, so the gods were concerned for the well-being of the princes: Eanatum, prince of Lagash, given strength by Ningirsu, chosen by Nanshe, nursed by Ninhursag, loved by Inanna (ins. 5). Eanatum attests also that as the cities had a lord, so heaven and earth had a lord: Enlil, lord of heaven and earth (*id.*). One generation later, Enmetena recorded the deeds of three generations of princes. And he attests that as the princes had fathers, so the gods had a father, their father was the lord of heaven and earth, and he, the father, the lord of heaven and earth, was concerned for the borders of the cities: Enlil, father of the gods, drew the border between Lagash and Umma (ins. 1). One century later, Lugalzagesi attests that the father, the lord of heaven and earth, was concerned for all lands, east to west, lower sea to upper sea, and concerned in some detail: Enlil gave Lugalzagesi the lordship of the land, put all lands at his feet,

from the east to the west; from the lower sea to the upper sea, Enlil gave him no rival (ins. 1). Later in the same generation, however: Sargon brought Lugalzagesi in a dog collar to the temple of Enlil (ins. C-1), and: to Sargon, lord of the earth, Enlil gave no rival from the upper sea to the lower sea (*id.*), attesting the growing difficulty of ascertaining what exactly the father, the lord of heaven and earth, gave to whom, and who exactly, the lord of heaven and earth or Sargon, was the lord of the earth. Two generations later, Naram-Sin campaigned from the source of the Euphrates to its mouth. And he attests that he was a god: Naram-Sin, the god of Akkad (ins. B-1). One century later, Ur-Nammu attests that the gods had begun being concerned for justice and truth: when An and Enlil gave the princeship of Ur to Nanna, Nanna gave it to Ur-Nammu so that justice and truth prevail (code).

Princeship was one under the princes of Akkad, again under the princes of Ur, and again under the princes of Babylon for two centuries beginning with Hammurabi, prince *c.* 1792-49. The acquisition of knowledge advanced greatly under the princes of Ur, and again under the princes of Babylon, the advance culminating in the collection of 286 problems and the Venus tablet of Ammisaduqa. And also under Ammisaduqa, prince *c.* 1646-25, not quite five centuries after they had begun being concerned for justice and truth, the gods undertook to make men.

The oldest account of the making of men is the three-tablet composition *When the gods were men*. It begins: when the gods were men, they did all the work, the work was very heavy, and the misery of the gods was very great (1); let us slaughter one god, Enki proposes, let us mix clay with his flesh and blood, and make men to do the work (*id.*). And so the gods do. In the following century the gods undertook to make everything.

The oldest account of creation is the seven-tablet composition *When on high*. It begins: when on high the heaven had no name, and down below the earth had no name, Apsu was the begetter, Tiamat bore all (I, 1-4). And the main events of creation are these: Apsu begets and Tiamat bears the gods; but this progeny of theirs is very noisy and Apsu cannot sleep; let us do away with them, Apsu decides; but the gods hear of the decision,

and Ea kills Apsu; in the middle of Apsu Ea begets Marduk; Tiamat, to avenge her husband, charges Qingu to lead an army of serpents, lions, scorpions, fish, bulls against the gods (I); the gods acknowledge Marduk the greatest among them, his command shall be truth, his word shall never be wrong, no god shall ever go beyond the limits he sets; Marduk leads the army of the gods against Tiamat and Qingu; Marduk is victorious; he splits Tiamat in two, like a fish, and sets up one of her halves to be heaven (IV); he arranges the stars, the sun, and the moon; marks the year, the months, and the days; from the eyes of Tiamat he lets the Euphrates and the Tigris flow; he sets up the heaven above and the netherworld below; he builds Babylon (V); the gods bring Qingu before Ea, they slaughter him, and from his blood Ea makes men; Ea charges the men with the work of the gods (VI).

Marduk built Babylon, David built Israel. David was prince c. 1003-963. David was succeeded by Solomon. After Solomon the principality split into the two feeble and steadily feeble principalities of Israel and Judah. And one century after Solomon, seven and a half centuries after *When on high*, the Yahwistic document corrected its results. The Babylonian had been great princes and theirs a great state, the Israelite were not great princes and theirs not a great state. The greatest among the gods, his command shall be truth, his word shall never be wrong, became concerned for one narrow and steadily narrower line of creation.

Early on: the serpent talks to the woman; the woman eats of the fruit in the middle of the garden; she gives the man to eat as well; god calls out to the man, where are you? I was afraid because I was naked, the man replies; who told you that you were naked? god asks the man; what have you done? he asks the woman (*Genesis*, 3).

A little later: the sons of the gods see that the daughters of the men are very beautiful, and take them for wives; god sees how great the wickedness of men is; he says to Noah, I will put an end to all flesh, I will do away with everything on earth (6).

A little later still: god says to Abram, I will make you a great nation (12); raise your eyes, look to the north and to the south, to the east and

to the west, all the land that you see I give to you and your descendants forever (13).

The difficulty of ascertaining what exactly god gives to whom persisted, however. The Assyrian princes began rising greatly again under Ashurnasirpal II, prince 884-59, and princesship was once again one from Tiglath-pileser III, prince 745-27, through Ashurbanipal, prince 668-27. Israel fell to Tiglath-pileser III's successor Sargon II. Judah fell one generation after Ashurbanipal to the Babylonian Nebuchadnezzar, prince 605-565. In the century after the Yahwistic document, the first prophets *Amos*, *Hosea*, and *Isaiah* are contemporary with the ruin of Israel. *Amos* reports: the lord roars from Zion, thunders from Jerusalem (1); listen, Israelites; because of all the peoples on earth I was concerned for you alone, I will punish you for your wickedness (3); can misfortune befall a city if the lord did not send it (*id.*)? One century-odd after the first prophets, *Deuteronomy* is contemporary with the ruin of Judah. And *Deuteronomy* generalized the results obtained in the earlier Hebrew writings, proved the most important result, and drew the appropriate political and moral conclusions.

The lord god speaks through Moses: is there a god in heaven or on earth who can match the deeds of the lord god (3)? search the ages since the day god created man on the earth, from one end of the heaven to the other, has a people heard a god speak out of the fire and remained alive? has a god by his deeds, signs, portents, and war taken a people from amidst another people? you have before you clear proof that the lord alone is god, there is no other (4).

From the proof the most important result follows: you shall have no other gods beside me (5). Everything else that you shall follow as well. Nanna had through Ur-Nammu regulated a few of your relations with the other members of your tribe. The lord god through Moses regulates them all. The rewards and punishments are correspondingly greater: if you obey the laws, you will be blessed above all other peoples, you will destroy all the peoples whom the lord god delivers to you (7); if you forget the lord god, he will destroy you (9).

Homer and Hesiod predate the rise of the Greek cities. They postdate the Yahwistic docu-

ment by one century. Homer wrote the *Iliad* and *Odyssey*, Hesiod wrote the *Theogony* and *Work and days*. With these the Greek record begins. And as the Hebrew record begins – and ends – with god, so the Greek record begins with the gods. These are the main events in book I of the *Iliad*: the Achaeans are fighting the Trojans; Agamemnon, their chief, offends the priest of Apollo; the priest prays to Apollo to avenge him; Apollo sends a plague on the Achaeans; Achilles, the greatest warrior among the Achaeans, calls an assembly to deal with the plague; Agamemnon placates the priest, but offends Achilles; Achilles calls Thetis, his mother, from the depths of the sea, and asks her to ask Zeus, the greatest among the gods, to avenge him; the gods, led by Zeus, return from their feast among the Ethiopians at the Ocean; Thetis asks Zeus, Zeus nods ascent; Zeus returns home, on mount Olympus, to Hera, his wife; Hera asks, what have you done, my lord? Zeus replies, sit down in silence and do as I say, my lady, for all the gods can do nothing when I lay my hands upon them (*id.*). These are some of the main events in the *Theogony*: Chaos is first; he gives birth to Earth, Tartarus, and Love; he then gives birth to Darkness and Night; Earth then gives birth to Heaven, Mountains, and Sea; Earth then lies with Heaven, and conceives three cyclopes, three one-hundred-handed creatures, and twelve titans; one of the titans is Cronus; but Heaven hates his terrible progeny and will not let it come out of its mother's womb; Earth makes a sickle of adamant, gives the sickle to Cronus, and when Heaven lies with Earth, Cronus castrates his father; blood drops on Earth and from the drops furies, giants, and nymphs are born; Cronus throws his father's member in the sea, and from it Aphrodite is born (116-210).

And then there is Xenophanes.

Xenophanes sings:

Homer and Hesiod ascribe to the gods
everything shameful among men,
stealing, cheating, lying to one another (fr. 11).

As Homer and Hesiod, so men:

men think that the gods are born, are attired,
speak, have bodies like themselves (fr. 14).

As Homer, Hesiod, and men, so the Ethiopians
and the Thracians:

the Ethiopians say the gods are snub-nosed and

black,

the Thracians say they are blue-eyed and red-
haired (fr. 16).

As Homer, Hesiod, men, the Ethiopians, and the
Thracians, so oxen, horses, lions, and every kind:

if oxen, horses, and lions had hands
and could draw and make statues like men,
horses would draw the gods like horses,
oxen would draw them like oxen,
and every kind like itself (fr. 15).

Not only is Xenophanes's result of perfect generality. Hume writes: anatomists join their observations and experiments on human bodies to those on beasts, and from the agreement of these experiments derive an additional argument for any particular hypothesis; let us, therefore, apply this method of inquiry to our anatomy of the mind. Xenophanes applies this method of inquiry to our anatomy of the mind twenty-four centuries before Hume. Two centuries before Herophilus, too, before the first anatomist.

And as it is of perfect generality negatively, so is Xenophanes's result of perfect generality affirmatively. He sings:

one god, greatest among gods and men,
in no way like men in body or thought;
all of him sees, all of him thinks, all of him hears;
he shakes all things without toil, by thought
alone;

always in the same place, he does not move,
it does not befit him to go here or there (fr. 23-6).

This is twenty-one and a half centuries before
Descartes's result: nature is god, twenty-two
centuries before Spinoza's result: god is nature.
With all the alternative results which more or less
all philosophers and not philosophers obtained in
the meantime.

ccxi

Unlike the gods, who enter the record twenty
centuries before Xenophanes, the problems of the
principles of all things and of the earth enter it
with the first philosopher, two generations before
Xenophanes.

Thales said that water is the principle. Anax-
imenes said that air is prior to water and it is the
principle. Pythagoras said that number is the
principle.

Xenophanes sings:

we are all born from earth and water (fr. 33),
and:

all things that are born and grow are earth and
water (fr. 29).

Thales said that the earth floats on water.
Anaximander said that it is at rest because of
equilibrium. Anaximenes said that it floats on the
air below.

Xenophanes sings:

the upper limit of the earth we see at our feet,
touching the air,
downward it stretches without limit (fr. 28).

Neither of these results of Xenophanes's is an
advance on his predecessors. These results of his
nevertheless attest the fact that from the first the
philosophers have been philosophers jointly by
jointly laboring to improve on the solutions of the
problems solved by their predecessors.

This result of his is an advance:

rain, clouds, winds, and rivers come from the sea
(fr. 30),

as rain, clouds, winds, and rivers do, and Xe-
nophanes first says that they do. Two centuries
later, in the *Meteorology*, Aristotle will explain
how they do (I, 9, 13).

And what Xenophanes says about knowledge
is a very great advance. Xenophanes first speaks
about knowledge explicitly. More particularly, he
first distinguishes knowledge from belief.

He sings:

no man knows the clear; no man knows
about the gods and all the things I speak about;
even if one stumbles onto something,
he does not know it; belief clings to all things
(fr. 34).

It has not failed to suggest scepticism. Sceptics
deny that Pyrrho was the first sceptic – Laertius
reports – they say that Democritus already was a
sceptic, and before him Heraclitus was a sceptic,
and before him Xenophanes was a sceptic, and
before him the seven wise men were sceptics, and
before them Homer was a sceptic. And before
him Enmebaragesi was a sceptic.

No man knows about the gods. Men think that
the gods are born, are attired, speak, have bodies
like themselves. The Ethiopians say the gods are
snub-nosed and black, the Thracians say they are
blue-eyed and red-haired. Belief clings to the
gods. More generally, no man knows about all the

things Xenophanes speaks about. Belief clings to
all the things he speaks about. More generally
still, no man knows the clear. Even if one stum-
bles onto something, he does not know it. Belief
clings to all things.

For his part, Xenophanes does not think that
the gods are born, are attired, speak, have bodies
like men. He does not say that the gods are snub-
nosed and black, or blue-eyed and red-haired. He
says: one god, in no way like men in body or
thought. Xenophanes knows about the gods.
More generally, Xenophanes knows about all the
things he speaks about. Other than the gods, he
positively knows about rain, clouds, winds, and
rivers, for instance. More generally still, if he has
stumbled onto something, he knows it. Even if he
does not know the clear, he knows the clearer.
Even if belief clings to all the things he speaks
about, belief cannot cling to them exclusively any
longer. Even if belief clings to all things, too, it
cannot cling to them exclusively any longer
either. Knowledge has begun clinging to them as
well.

Nor can Xenophanes believe that knowledge is
belief. Nor can he believe that knowledge is not
belief. Nor can he know that knowledge is belief.
He can only know that knowledge is not belief.

If knowledge is not belief, belief cannot cling
to all things exclusively, it cannot cling exclu-
sively to knowledge, and cannot cling exclusively
to any of the things to which knowledge as well
clings. Belief can cling to all things, including
knowledge, exclusively, if knowledge is belief.
But if knowledge is belief, belief cannot be
knowledge, and it can only be a purgative. As
Timon discovered: overthrowing all assertions,
the sceptic assertion overthrows itself as well,
like a purgative which, throwing out of the bow-
els everything else, throws out itself as well.

One century before Timon, one and a half
centuries after Xenophanes, Plato's fundamental
result in the *Theaetetus*, in the first investigation
of knowledge, is that knowledge is not belief.

That belief clings to all things nothing attests
better than the fact that two and a half millennia
later, incoherence notwithstanding, belief contin-
ues to cling to the proposition that knowledge is
belief.

Xenophanes discovered that belief clings to all

things. Not exclusively, however. Xenophanes also discovered that knowledge is not belief.

If no man knows about the gods and all the things Xenophanes speaks about, but Xenophanes knows about the gods and all the things he speaks about, Xenophanes excludes himself from the collection of all men.

And there is further evidence for this in the fragments.

Xenophanes sings:

the gods have not revealed all things to mortals
from the beginning,
but in time, by searching, they find the better

(fr. 18).

If 'mortals' refers to all men, such as the gods have not revealed all things to them from the beginning, whether Xenophanes again excludes himself from the collection of all men depends on what 'they' refers to. And if belief clings to all things, it cannot be likely to refer to all men, such as they cling to the belief which clings to all things. It has to refer to those who, unlike all men, but like Xenophanes, by searching, find the better.

In any case, the following fragment is unambiguous:

they honor the winners of horse races and foot
races,
of boxing matches and wrestling contests,
but those are less worthy than I am;
better than the strength of men and horses is our
wisdom;
it is not just to put strength above good wisdom,
it does not bring good government to the city,
nor does it fill the storehouses with goods (fr. 2)

Philosophy was young, making its first steps, not quite knowing what it was saying, fighting the way untrained men fight, stammering, babbling, and prattling.

Young as it was, stammering, babbling, and prattling, philosophy already knows that our wisdom is good wisdom. Our good wisdom is better than the strength of men and horses, it is above strength, it will bring good government to the city and fill its storehouses with goods.

Also, our good wisdom already fights the way philosophy fights. Those are less worthy than I am.

ccxii

And Xenophanes notes 6 particular predecessors of his, and this is a very great advance, too.

If philosophy is a development, this development, like any development, has steps, these steps are particular, and it is nothing but these particular steps in relation to one another that constitutes the development. And it is with Xenophanes that the development which philosophy is becomes steps which are explicitly in relation to the earlier steps in the development, and hence it becomes a development which identifies itself explicitly as such, as a development, and hence, if the development is an advance, identifies itself explicitly as such as well, as an advance. If the acquisition of knowledge is a development, too, this development has steps, these steps are particular, and nothing but these particular steps in relation to one another constitutes this development. And it is again with Xenophanes that this development, too, becomes steps explicitly in relation to the earlier steps in the development, and hence a development which identifies itself explicitly as a development, and as the acquisition of knowledge cannot not be an advance, identifies itself explicitly as an advance as well.

Thus, with respect to Xenophanes, when Xenophanes is judged to be results concerning the gods, the principles of all things, the earth, rain and clouds, knowledge, what is worthy, and his predecessors, rather than any other results, he is so judged on the authority of his successors. When some of his results, and not any other, are judged an advance each, they are so judged on the authority of his successors as well. There is the authority of Plato and Aristotle, for instance. In each case, Plato and Aristotle, either or both, are concerned with the gods, the principles of all things, the earth, rain and clouds, knowledge, what is worthy, and their predecessors, concerned with their predecessors' results concerning those, and concerned to improve on their predecessors' results concerning those. When Xenophanes is judged, this is in each case his successors' judgment of his relation to his predecessors. And this relation of judgment of successors and predecessors – in which every successor is also a predecessor and every predecessor, except the first, is also a successor – and hence it is a development,

and hence an advance, is for the first time explicit in Xenophanes.

Xenophanes has left behind his own words in some 40-odd fragments. 31 of these render his own verses. In the rest his successors report what he said.

Xenophanes notes the 6 particular predecessors of his whom he notes in 8 fragments. 4 are his own verses, 4 are reports of his successors. And in the chronological order of the 6 predecessors of his whom he notes, with the successors of his who note him noting his predecessors, these are the 8 fragments.

Herodian wrote the 20 books of the *General prosody*, among the parts that survive are some on dichronous meters, and in one of these (296) Herodian illustrates meter once with Xenophanes:

all learned from Homer from the first (fr. 10).

Empiricus wrote the 11 books of *Against the mathematicians*. And Empiricus reports (I, 289): about Homer and Hesiod Xenophanes says,

they tell of many lawless deeds of the gods,
stealing, cheating, lying to one another (fr. 12),

and again (IX, 193): about Homer and Hesiod Xenophanes says,

Homer and Hesiod ascribe to the gods
everything shameful among men,
stealing, cheating, lying to one another (fr. 11).

Gellius wrote the 20 books of the *Attic nights*. The collection of *varia* had by Gellius's time become a well-established genre, the genre is at its best with the *Attic nights*, and one of the bits of *varia* in it is this (III, 11): as to the time of Homer and Hesiod, some, like Xenophanes and Philochorus, have written that Homer was older than Hesiod, others have written that he was younger (fr. 13).

Laertius wrote the 10 books of *The lives and doctrines of the philosophers*. And writing about Epimenides, Laertius reports (I, 111): the Cretans say that Epimenides lived 299 years, but Xenophanes had heard that he lived 154 years (fr. 20).

Laertius again, writing about Thales, reports (I, 23): Thales was the first astronomer, the first to predict eclipses and to determine the solstices, so writes Eudemus in the history of astronomy, and Thales was admired for that by Xenophanes and Herodotus and noted by Heraclitus and Democritus (fr. 19).

Laertius again, writing about Pythagoras, reports (VIII, 36): that Pythagoras was different men at different times Xenophanes confirms, when he says this about Pythagoras,

once, they say, he passed by someone,
who was whipping a puppy; he took pity,
and said, stop, do not whip him, it is the soul
of my friend, I can tell by the yelp (fr. 7).

An anonymous scholiast, finally, glossing this exchange in Aristophanes's *Peace*: how is Sophocles? he is now Simonides; how so? he would put to sea on a sieve for money (697-9), comments: Xenophanes, too, called Simonides a money-grubber (fr. 21).

The particulars, then, with years of birth and death assigned, and the number of fragments in which each is noted, are

764	Homer	4,
734	Hesiod	3,
653	Epimenides	1,
625	Thales	1,
570 - 495	Pythagoras	1,
556 - 466	Simonides	1.

The task is to distinguish the acquisition of knowledge from everything else, and in order to do so, to collect their predecessors and the philosophers, and divide them into these whose labors are the acquisition and those whose labors are not. As the collection and division depend on the labors of their predecessors and the philosophers, exclusively.

And the task is substantially easier when it depends on the labors of Xenophanes than when it depends on the labors of the Sumerian prince list.

There are the same difficulties, but they are minor now.

One is again chronology. But to fix the dates of Homer and Hesiod is substantially easier than to fix the dates of the early princes. To fix the dates of Pythagoras or Simonides is not hard at all.

Reality is again a difficulty as well. Is Homer real? is he not? was Homer, in fact, a predecessor of the philosophers? was he not?

Then again, Xenophanes sings:

the sacred field of Zeus

is near Pisa, in Olympia, by the spring (fr. 2),

and:

she whom men call Iris is a cloud,
 purple, red, and yellow to behold (fr. 32),
 and is Zeus real? is Iris real? and do Zeus and Iris
 belong in the collection of the predecessors of the
 philosophers, too? Considering, for that matter,
 what *Deuteronomy* has to say about him, if not
 Spinoza, does:

one god, greatest among gods and men,
 belong in the collection, too?

Suffice it to say that, unlike Zeus, Iris, and one
 god, greatest among gods and men, Homer is the
Iliad and *Odyssey*, and the *Iliad* and *Odyssey* are
 real enough to count. The *Iliad* is 15 691 lines,
 the *Odyssey* is 12 110 lines.

Xenophanes's judgments of his predecessors
 constitute the collection, and these judgments are
 all readily shown to attest the development of
 philosophy, and all readily shown either to attest
 the acquisition of knowledge or to pose the
 problem of distinguishing the acquisition from
 the not acquisition.

The collection includes the first and the fourth
 philosophers. Thales predicted eclipses and
 determined the solstices and Xenophanes admired
 him for that. Pythagoras taught the transmigration
 of the soul – it is the soul of my friend, I can tell
 by the yelp – and Xenophanes ridiculed him for
 that. The philosophers will continue, some of
 them to predict eclipses and determine the sol-
 stices, some of them to teach the transmigration
 of the soul, some of them to admire the ones, or
 the others, some of them to ridicule the others, or
 the ones. The labor of predicting eclipses and
 determining the solstices attests the acquisition,
 and if so, the labor of admiring the prediction and
 determination may attest it as well, if the labor of
 admiring it is a way of examining the acquisition.
 It is harder to know whether the labor of teaching
 the transmigration of the soul attests the acquisi-
 tion, but if so, for this very reason the labor of
 ridiculing the transmigration may well attest it, as
 the labor of ridiculing it should be an excellent
 way of examining the not acquisition.

The collection includes Homer and Hesiod,
 and Xenophanes does not admire Homer and
 Hesiod. Homer and Hesiod tell of many lawless
 deeds of the gods, ascribe to them everything
 shameful among men, and the philosophers will
 continue to tell of many lawful deeds of the gods,

ascribe to them everything praiseworthy among
 men, and they will admire one another for that,
 and sometimes ridicule one another. Also, all
 learned from Homer from the first, and Xenophanes
 does not admire Homer for that either. Plato
 writes: Homer taught all Greece (*Republic*, X,
 606e), and from whom all learn, and from whom
 they ought to learn, and who teaches them, and
 who ought to teach them, will continue to occupy
 the philosophers. Whether any of these labors, of
 telling of the gods, ascribing to them one thing or
 another, of learning and of teaching, attests the
 acquisition of knowledge is harder to know as
 well. But if so, again, the labor of ridiculing these
 labors may well attest it. Also, Xenophanes said
 that Homer was older than Hesiod. The philoso-
 phers will solve the problem of chronology. And
 while the labors of Homer of being older than
 Hesiod and of Hesiod of being younger than
 Homer do not attest the acquisition, the labor of
 Xenophanes of saying so may.

The collection includes also Epimenides and
 Simonides. Epimenides lived 154 years, and
 Xenophanes does not admire Epimenides for that
 either. The labor of living 154 years does not
 attest the acquisition. But if Adam, he lived 930
 years – and Enmebaragesi, he reigned 900 years
 – and the philosophers will concern themselves
 with Adam, if not so much with Epimenides or
 Enmebaragesi, the examination of the proposi-
 tion, Adam lived 930 years, may well attest the
 acquisition, and if so, so may Xenophanes's
 proposition that Epimenides lived 154 years, not
 299. Simonides was a moneygrubber, and Xe-
 nophanes does not admire Simonides for that
 either. The philosophers will continue to disap-
 prove of moneygrubbing. And neither the labor of
 moneygrubbing nor the labor of disapproving of
 moneygrubbing need attest the acquisition, but
 the philosophers will eventually explain money-
 grubbing, too, and this labor attests the acquisi-
 tion. With Adam Smith: there is a certain propen-
 sity in human nature to truck, barter, and ex-
 change (*Wealth*, I, 2), and: a man must be per-
 fectly crazy who does not employ all the stock
 which he can command (II, 1).

If Xenophanes admired Thales, and ridiculed
 Homer, Hesiod, Epimenides, Pythagoras, and
 Simonides, the ratio of admiration to ridicule of

1:5 itself attests the development of philosophy, and may well attest the acquisition of knowledge as well.

Most important, the fragments in which Xenophanes notes his predecessors can be counted. He notes Homer in 4 fragments, Hesiod in 3 fragments, Epimenides, Thales, Pythagoras, and Simonides in 1 fragment each. And the conclusion can be drawn immediately. Homer is the most important, followed by Hesiod, and the 2 are more important than the other 4.

Importance can be the ground on which the labors of their predecessors and the philosophers can be divided into the small number of these who may attest the acquisition of knowledge, on the side of importance, and the large number of those who may not attest the acquisition, on the side of not importance.

Taking it for granted, clearly, that the acquisition of knowledge is important.

That the ground is firm the fact shows that in these 8 fragments Xenophanes judges his predecessors explicitly, but he cannot not judge his predecessors in all his fragments, implicitly, if not explicitly, and in his other fragments he cannot not judge, implicitly, the same predecessors of his. He cannot not judge Homer and Hesiod in the fragments concerned with the Ethiopians and the Thracians, oxen and lions, one god. Thales in the fragments concerned with earth and water, rain and clouds.

How firm the ground is the fact shows that 8 fragments, 20-odd lines, from 1 philosopher, and the first philosopher to leave his words behind, suffice to warrant the conclusion.

The fact also shows it that the same 8 fragments, as they suffice to warrant the conclusion with respect to Xenophanes's predecessors, so they suffice to warrant it with respect to his successors.

With years assigned – though not to the anonymous scholiast – and the number of fragments in which each notes Xenophanes, the successors of his who note him noting his predecessors are

121 ^{AD}	Herodian	1,
127	Gellius	1,
159	Empiricus	2,
219	Laertius	3,
	scholiast on the <i>Peace</i>	1.

And counting the fragments, the conclusion can be drawn immediately. The most important is Laertius, followed by Empiricus, and the 2 are more important than the other 3.

As, in fact, Laertius is the most important authority, by far, very far, on the earlier development of philosophy. In *The lives and doctrines of the philosophers* Laertius not only notes 1 273 predecessors of his. He investigates 102 of them under his own head each, and does not investigate the other 1 171 under his own head each.

Other than the 11 books of *Against the mathematicians*, Empiricus left behind the 3 books of the *Outlines of Pyrrhonism*. Empiricus is a sceptic, and in his treatises he argues that knowledge is impossible, as a sceptic would. He does so systematically, however – knowledge is impossible with respect to one problem, and another, and another, and another, and impossible on one ground, and another, and another, and another – as a sceptic should not. Lest the sceptic assertion, instead of overthrowing itself as it overthrows all dogma, were itself a dogma. Hence, whatever the coherence of the doctrine he expounds, Empiricus is an important source on the earlier development of philosophy, if not nearly as important as Laertius.

Seven to eight centuries separate these writers from Xenophanes, and Empiricus is representative in that some of his reports on his predecessors are more important than the doctrine he expounds. Some of the bits of *varia* in Gellius's collection are more important than Gellius's collection. Some of the illustrations in Herodian's investigations of meter are more important than the investigations of meter. And some of their glosses are more important than the altogether unimportant anonymous scholiasts.

The ground is firm, and it is firm, too, because it is clear why it is firm, and ever firmer. It is firm, and ever firmer, because it is established by the cumulative labors of the philosophers, exclusively.

ccxiii

Later in the generation of Xenophanes, Hecataeus is the seventh philosopher.

Hecataeus has left behind his own words in some 360-odd fragments. The large majority are

just a name each. Hecataeus also wrote prose, not verse. Strabo reports: Cadmus, Pherecydes, and Hecataeus first began imitating poetry by writing prose (I, 2.6). Hecataeus wrote prose, and in the fragments which are more than a name each, his own words are sometimes difficult to distinguish from the context in which they are quoted. And if in all these fragments his successors report what he said, not many of them attest more than a clause of his own words. Some 12 fragments attest more than a clause of his own words.

Hecataeus wrote two treatises.

On the authority of Eratosthenes, Strabo writes: Hecataeus wrote a geographical treatise. It is the first geographical treatise.

The exploration of the surface of the earth can be traced back to Eanatum. Eanatum campaigned from Mari in the north to Ur in the south and from Mari in the west to Susa in the east.

Twenty centuries later, the expeditions of the great kings reached from the northernmost shores of the Black Sea in the north to the southernmost shores of the Red Sea in the south and from the strait of Gibraltar in the west to the mouth of the Indus in the Indian Ocean in the east.

Hecataeus is contemporary with Darius.

And of the 360-odd fragments he has left behind, 330-odd are from the geographical treatise. Of 280-odd of these the source is Stephan of Byzantium. Stephan of Byzantium wrote the 60 books of the *Peoples*. A monument to grammar, the *Peoples* is not a monument to geography. Which the fact suffices to show that, writing four centuries after Ptolemy, Stephan of Byzantium has no use for Ptolemy. The following are representative of the fragments whose source is the *Peoples* (s. v.): Matyceti, a Scythian people, Hecataeus in Europe (fr. 189); Miletus, a notable city of the Ionians in Caria, Hecataeus in Asia (fr. 240). Europe and Asia were the two parts of Hecataeus's treatise.

All in all, the fragments attest some 300-odd localities. They cannot be counted more precisely. A large number is cities, some are islands and capes, and these can be counted precisely. But a large number as well is peoples, and these are more difficult to count. As in the entire earlier record so in Hecataeus countries are identified with the resident peoples, and resident peoples

are not among the stable features on the surface of the earth. Thus, it is clear where exactly Miletus, the notable city of the Ionians in Caria, is, but it is not clear where exactly the Matyceti, a Scythian people, are. Worse, among the peoples in the fragments are peoples like the Pelasgians (fr. 127, 133), and Halizones (fr. 217), who are peoples also fighting with the Trojans against the Achaeans in the *Iliad* (II, 840-3, 856-7), and for all the labors of Herodotus to find out where the Pelasgians are (I, 57-8; VI, 137; VIII, 44), and of Strabo still to find out where the Halizones are (XII, 3.20-3), it remains unclear where the Pelasgians are, should they be burdened with the presumption of reality, and it is clear that the Halizones need not be burdened with the presumption of reality.

On the other hand, the following all shows that Hecataeus's is the first geographical treatise.

The number of localities, more than 300 of them, many more than in any earlier collection of localities.

Their distribution, from Scythia in the north (fr. 185-90) to Ethiopia in the south (fr. 325-7), from the strait of Gibraltar in the west (fr. 39) to the Indus in the east (fr. 296), across 33° of latitude, 73° of longitude, coinciding precisely with the reach of the expeditions of the great kings.

Most important, there is also: the map of the entire earth, drawn on a bronze tablet, showing all the seas and all the rivers, which Aristagoras took with him to Sparta to show to Cleomenes. At least from the Ionians to Susa, Hecataeus has solved the problem of orientation qualitatively, if not quantitatively, and has solved the problem of distance quantitatively.

Two-odd generations later, in the *Histories*, Herodotus has advanced geography greatly. He has advanced the solutions of the problems of distance and orientation, both. The distribution of the localities he describes reaches substantially further in the north and in the south. He describes 1 012 localities. These include cities, islands, capes, rivers, seas, countries, they can be counted precisely, and they do not include peoples. It is in the *Histories* that the distinction between countries and the resident peoples is for the first time clear. And in all of this the success of Herodotus

depends substantially on the success of Hecataeus.

The other treatise Hecataeus wrote is attested variously as the *Genealogies*, the *Histories*, and the *Stories of heroes*. It is not the first historical treatise.

Hecataeus has left behind some 30-odd fragments from this treatise. They attest heroes and their genealogies: the Deucalionids (fr. 13-6), the Argonauts (fr. 17-8), the Danaids (fr. 19-22), Heracles and the Heraclidae (fr. 23-30). They attest heroes and their genealogies well attested by Homer and Hesiod.

This is how the treatise began:

I will write down what I believe is the truth, because many stories are told among the Greeks which seem to me ridiculous (fr. 1).

The first historical treatise is Herodotus's. A main reason is that the ridiculous stories told among the Greeks are not his main concern. Nor are, more generally, heroes and their genealogies his main concern. His main concern are the great and magnificent deeds of Greeks and barbarians, and first and foremost the great and magnificent deeds of the great kings. Hence, Herodotus first succeeds in separating truth, on one side, from the ridiculous stories told among the Greeks, on the other. More generally, it is for the first time in the *Histories* that the distinction between the great and magnificent deeds of Greeks and barbarians, such as these have to be burdened with the presumption of reality, and heroes and their genealogies, such as these need not be burdened with the presumption of reality, is clear.

Still, Hecataeus is the one predecessor of his, next to Homer, whose errors Herodotus explicitly proposes to correct (II, 143; VI, 137).

And Hecataeus notes explicitly 1 particular predecessor of his.

Concerned with localities which the great kings discovered, Hecataeus cannot not judge the labors of the great kings, implicitly, if not explicitly. Concerned with heroes and peoples with whom Homer and Hesiod are concerned, he cannot not judge the labors of Homer and Hesiod, implicitly, if not explicitly. And some of the reports of what he said relate what he said to what they said (fr. 14, 18, 23-4).

In 1 fragment the judgment is explicit.

An anonymous scholiast is glossing this report of Euripides in the *Orestes*: this is the hill where Danaus, accused by Aegyptus, first gathered the people to hear the case (871-2). The case was difficult, concerning as it was the betrothal of the 50 daughters of Danaus to the 50 sons of his brother Aegyptus and their flight to Argos. The scholiast clarifies: Hecataeus writes, Aegyptus himself did not go to Argos, but his sons did; and Hesiod makes them 50, but I say they were not even 20 (fr. 19).

Ignoring Aegyptus and his sons, whatever their number, the 1 particular predecessor of his whom Hecataeus notes explicitly in his fragments, with the number of fragments, is

734 Hesiod 1.

The conclusion can be drawn that the 1 predecessor of his whom Hecataeus notes in his fragments explicitly is 1 of the 6 predecessors of his whom Xenophanes notes in his fragments explicitly.

The successor of Hecataeus who notes him noting his predecessor is

scholiast on the *Orestes* 1.

The conclusion can be drawn that scholiasts found the philosophers useful for the glossing of the poets.

ccxiv

In the generation after Xenophanes and Hecataeus, Heraclitus is the eighth philosopher.

Heraclitus has left behind his own words in some 130-odd fragments, a couple of hundred lines. He has also left behind prose, not verse. He has left behind superb prose, however. Also, before Plato, or Timon, long before Voltaire, or Nietzsche, Heraclitus has elevated scorn, in general, and the ridicule of his predecessors, in particular, into a high art. Distinguishing his own words from the context in which they are quoted is thus seldom difficult.

Heraclitus notes explicitly 11 particular predecessors of his.

First, though, his other results should be shown.

It will be objected that Heraclitus should not, and cannot, be reduced to results.

Upon reading Heraclitus, Socrates said: what I understood is excellent, and so is no doubt what

I did not understand, but it needs a diver to get to the bottom of it (Laertius, II, 22). Timon called Heraclitus: the riddler (Laertius, IX, 6). Cicero reports: Heraclitus is called the obscure (*Ends*, II, 15). Strabo corroborates: Heraclitus is called the obscure (XIV, 1.25). Timon called him also: cuckoo (Laertius, IX, 6).

Long before Descartes formulated the first rule: clarity and distinction, philosophy had developed a predilection for obscurity and vagueness. Sceptics denied that Pyrrho was the first sceptic, and said that Heraclitus, too, was a sceptic. More generally, obscurity and vagueness have been eager to find themselves in Heraclitus.

Three generations after the first philosopher philosophy was still young. It was still stammering, babbling, and prattling. But it was not stammering, babbling, and prattling toward obscurity and vagueness. Not yet. Not in Heraclitus. Philosophy had to grow up some more, to mature some more, before it learned to do that. Before it discovered the vices of clarity and distinction and the virtues of obscurity and vagueness.

It needs a diver to get to the bottom of what Heraclitus says, Socrates said. And in the century after Socrates the two divers, Plato and Aristotle, got to the bottom of what Heraclitus says.

Even if some of what Heraclitus says is obscure, some of what he says is clear, and some of what he says is crystal clear. Laertius writes: some of what Heraclitus says is so clear and distinct even the dullest can understand it; and for brevity and gravity of exposition he is incomparable (IX, 7).

Heraclitus can be analyzed into results concerning the following. Λόγος. All things. Change. Fire. They. Digging. His predecessors. And all seven results are a very great advance each.

ccxv

If what Heraclitus says will be obscure, it will be most of all because of what he says about λόγος. And he says much about λόγος.

He says:

λόγος is forever; yet men do not understand it after they have heard about it just as before they have heard about it; all things pass according to λόγος; yet men do not understand it even when I explain each thing according to its nature (fr. 1);

the common must be followed, and λόγος is the common; yet the rabble live as if each had his own understanding (fr. 2);

fire turns into sea, of which one half is earth, one half burns, and the same λόγος measures the sea before and after (fr. 31);

Bias, the son of Teutames, was of more λόγος than the rest (fr. 39);

you will not find the limits of the soul down any road, so deep is its λόγος (fr. 45);

listening not to me, but to λόγος, it is wise to agree that all things are one (fr. 50);

they set themselves against what they are the most in touch with, λόγος, which orders the whole; what they meet every day seems foreign to them (fr. 72);

a fool is excited by every λόγος (fr. 87);

of those whose λόγοι I have heard not one understands that the wise is set apart from all things (fr. 108);

the soul is λόγος which enlarges itself (fr. 115).

Clearly, Heraclitus says something deep about λόγος. Hence the possibility, not so clearly, that what he says about λόγος is as deep as the λόγος of the soul. But you will not find the limits of the soul down any road. Hence the possibility, altogether unclearly now, that you will not find the limits of λόγος down any road either.

It is better, however, not to be excited by this possibility. For, if a fool is excited by every λόγος, it is better not to be excited by any λόγος. Not even by the λόγος that you will not find the λόγος of λόγος.

First of all, then, granted that philosophical usage has long rendered Heraclitus's λόγος untranslatable, it should nevertheless be clear, even if it has not been, that because usage is vague, because much philosophical usage is vaguer than usage, and because philosophical usage has rendered nothing vaguer than it has λόγος, it does not follow that Heraclitus, when he introduced λόγος in philosophical usage, introduced it vaguely.

Philosophical usage, never mind usage, has had λόγος be anything and everything from word through speech through story and account through reason through idea and form through measure and definition through law and principle

through logic through ratio through the word of god, the son of god, and god himself. The early historians, Plato and Aristotle, the mathematicians after Euclid, and the *Gospel of John* can illustrate.

Herodotus notes Hecataeus on four occasions, and on three of these he calls him λογοποιός (II, 143; V, 36, 125), λόγος-maker. Thucydides (I, 21) and Polybius (VII, 7) call their early predecessors λογογράφοι, λόγος-writers. Arrian calls Herodotus himself λογοποιός (*Alexander*, III, 30). Herodotus himself calls λογοποιός Aesop, too (II, 134). The *Stories of heroes* is the *Ἡρωλογία*. In the early historians λόγος tends to be in the narrow range between story and account.

In Plato and Aristotle λόγος is everywhere (*passim*), and it ranges over anything from word through principle.

Definition 3 in book V of the *Elements* is: λόγος is a measurable relation of two magnitudes of the same kind (*id.*). In the mathematicians after Euclid λόγος is unambiguous, it is ratio.

The *Gospel of John* begins: in the beginning was λόγος, and λόγος was with god, and λόγος was god (1). By this time, six centuries after Heraclitus, the range of λόγος had become unlimited.

What Heraclitus says about λόγος can be sorted out into four parts.

One. Men do not understand λόγος after they have heard about it just as before they have heard about it. Men do not understand λόγος even when Heraclitus explains each thing according to its nature. Λόγος must be followed, yet the rabble live as if each had his own understanding. They are the most in touch with λόγος, yet they set themselves against it. They meet λόγος every day, yet it seems foreign to them. A fool is excited by every λόγος. And of those whose λόγοι Heraclitus has heard not one understands that the wise is set apart from all things. Listening not to Heraclitus, but to λόγος.

This cannot be too difficult to understand. It cannot be too difficult to understand that men are the rabble, the rabble are men, they do not understand λόγος, and live as if each had his own understanding. Xenophanes already has established that they prefer the strength of men and horses to our good wisdom. And the less difficult

it should be to understand that they do not understand, if Heraclitus explains each thing according to its nature, and they are to listen not to Heraclitus, but to λόγος.

It depends on what λόγος is. Whatever λόγος is, however, it is more about them, whether they understand – they do not – and how they live, than it is about λόγος.

Two. Bias was of more λόγος than the rest.

Clearly, Bias was not one of them.

But this is more about Bias than about λόγος.

Three. You will not find the limits of the soul down any road, so deep is its λόγος. The soul is λόγος which enlarges itself.

This is difficult, as difficult as it is deep.

And it also depends on what λόγος is, but it is also more about the soul than about λόγος.

Four. Λόγος is forever. All things pass according to λόγος. Λόγος is the common. Fire turns into sea, and the same λόγος measures the sea before and after. Listening to λόγος, it is wise to agree that all things are one. Λόγος orders the whole.

This now is about λόγος. And it can itself be divided into three parts.

A. Λόγος is forever, λόγος is the common.

B. All things pass according to λόγος, listening to λόγος, it is wise to agree that all things are one, λόγος orders the whole.

C. Fire turns into sea, and the same λόγος measures the sea before and after.

And to get to the bottom of λόγος has to be to get to the bottom of the second and the third of these, the relation of λόγος and all things, and the relation of λόγος and fire.

Because this is the deepest.

And because it is to the bottom of this that the divers' divers, Plato and Aristotle, got. Plato and Aristotle are both excited by Heraclitus. Plato examines him in some detail in the *Cratylus* and *Theaetetus*. Aristotle examines him in some detail in his physical and logical treatises. And excited as Plato and Aristotle both are by Heraclitus, and excited as they both are by λόγος, neither of them is excited by Heraclitus's λόγος. Neither Plato nor Aristotle ever notes that Heraclitus says anything about λόγος. Plato is excited by what Heraclitus says about all things. Aristotle is excited by what Heraclitus says about all things

and by what he says about fire.

The 10 fragments above are all Heraclitus says about λόγος. In 3 of them he speaks about all things. And he has more to say about all things.

In the fragments above he says:

all things pass according to λόγος,
it is wise to agree that all things are one,
the wise is set apart from all things.

He says also:

wholes and not wholes, together apart, in tune out of tune, one thing from all things and all things from one thing (fr. 10);

the best renounce all things for one, eternal glory among men; but the rabble stuff themselves like cattle (fr. 29);

one is the wise, to know how all things are steered through all things (fr. 41);

war is the father of all things, the king of all things, he shows that some are gods, others men, he makes some slaves, others free (fr. 53);

all things steers the thunderbolt (fr. 64);

fire will judge all things (fr. 66);

one must know that war is common, justice is strife, and all things are by strife and necessity (fr. 80);

all things are in exchange for fire, fire in exchange for all things, like goods for gold and gold for goods (fr. 90).

This can be divided into six parts.

One. The rabble stuff themselves like cattle.

This is about them.

Two. The best renounce all things for one, eternal glory among men. One is the wise, to know how all things are steered through all things. It is wise to agree that all things are one. The wise is set apart from all things.

This is not about them. It is about the best, and the wise. It is about knowledge, too. It is wise to agree that all things are one. The wise is set apart from all things, however. And one is the wise, to know how all things are steered through all things.

Three. War is the father of all things, the king of all things, he shows that some are gods, others men, he makes some slaves, others free. One must know that war is common, justice is strife, and all things are by strife and necessity.

This is how all things are steered through all things. By war. By strife and necessity.

And this result cannot have been more unambiguously affirmative. Wise is to know how all things are steered through all things. And wise, Heraclitus knows how all things are steered though all things. All things are steered through all things by strife and necessity.

Four. Wholes and not wholes, together apart, in tune out of tune, one thing from all things and all things from one thing. All things steers the thunderbolt.

Moreover, some are gods, some are men, some are slaves, some are free, some are Bias, some are rabble, some are the best, some stuff themselves like cattle, λόγος is the common, war is common, all things are by strife, justice is strife, war is the father of all things, war is the king of all things, all things are by necessity, the thunderbolt is Zeus, too, and:

one, the solely wise, will and will not be called Zeus (fr. 32),

and will and will not be called all too many other things besides.

This is what would give plausibility to the supposition that Heraclitus is stammering, babbling, and prattling toward obscurity and vagueness. This is the ground on which the doctrine, all things change, nothing abides, will be read in Heraclitus.

Five. Fire will judge all things. All things are in exchange for fire, fire in exchange for all things, like goods for gold and gold for goods.

This is what shows that Heraclitus is stammering, babbling, and prattling toward clarity and distinction. This is the ground on which a more coherent doctrine than, all things change, nothing abides, will be read in Heraclitus.

Six. All things pass according to λόγος.

And if Heraclitus is stammering, babbling, and prattling toward clarity and distinction, the relations of all things, fire, and λόγος are becoming clearer. All things are steered through all things by strife and necessity, all things are in exchange for fire, and all things pass according to λόγος. All things are steered through all things by strife by being in exchange for fire, and all things are steered through all things by necessity by passing according to λόγος.

Plato read the doctrine, all things change, nothing abides, in Heraclitus.

Aristotle reports: in his youth Plato became acquainted with Cratylus and the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge of them, and Plato held this view in his later years as well (*Metaphysics*, I, 6).

Among Heraclitus's fragments are these 2:

on those who step in the same river different and different waters flow (fr. 12);

one cannot step in the same river twice (fr. 91).

The source of the second fragment is Plutarch, in *On the E at Delphi* (18). The source of the first fragment is Eusebius, in the *Preparation for the gospel*, quoting Arius Didymus quoting Zeno of Citium quoting Heraclitus (XV, 20).

Plato writes in the *Cratylus*: Heraclitus said that all things change, nothing abides, and you cannot step in the same river twice (402a).

All things change, nothing abides, are either Heraclitus's own words or, more probably, to judge by the lack of either scorn or picturesqueness, Plato's reading of Heraclitus.

Either way, Heraclitus discovered that all things pose the problem of their change. Either way, Plato established this fact, Plato first investigated the problem in all its generality, and Plato formulated its first solution such as it has remained the ground of all subsequent improvement on its solution.

Plato writes: there is the beautiful itself, the good itself, and the same for all things that there are; and the beautiful itself is always what it is; if it always passed away, we would have to say that it were now this, now that; at the very instant we spoke of it, it would become something different; and if it were never the same, how could it be anything at all? or be ever known by anyone (439c-e)?

With Plato, all things do not change, something abides.

Pace Plato, however, and whether the words, all things change, nothing abides, are Heraclitus's own or not, the doctrine, all things change, nothing abides, does not have to be ascribed to Heraclitus.

Aristotle writes: we can take Heraclitus to say that all things change except for one thing which does not change and is the principle of all change

(*Heaven*, III, 1). We can take Heraclitus otherwise, and this is the Heraclitean doctrine that all things change, nothing abides. All things are always in flux. Cratylus, in the generation before Plato, first took Heraclitus in this way. Aristotle reports: Cratylus said that you cannot step in the same river even once, and stopped talking altogether, thenceforth only pointing with his finger (*Metaphysics*, IV, 5). Showing a respect for consequences, too, which, regrettably, none of the philosophers who have since maintained that doctrine has emulated. Aristotle writes: if what Parmenides and Melissus say is that all things are one in essence, then they maintain the Heraclitean doctrine that the same thing is good and bad, and good and not good, and man and horse, and this is to affirm not that all things are one, but that they are nothing (*Physics*, I, 2).

We do not have to take Heraclitus to say that the same thing is man and horse. We can take him to say that all things change except for one thing which does not change and is the principle of all change.

This is how Aristotle takes him. The principle of all change is fire.

Heraclitus speaks about fire in 3 of the fragments above. And he has more to say about it.

In the fragments above he says:

fire turns into sea, of which one half is earth, one half burns, and the same λόγος measures the sea before and after,

fire will judge all things,

all things are in exchange for fire, fire in exchange for all things, like goods for gold and gold for goods.

He says also:

this world, the same for all, was made by neither god nor man, it always was and is and will be, an everlasting fire kindled in measures and extinguished in measures (fr. 30);

god is day night, winter summer, war peace, satiety hunger, just as fire, when mixed with spices, is named after their savor (fr. 67).

In book I of the *Metaphysics* Aristotle writes: the earliest philosophers took the principles of all things to be only material, principle they identified with primary matter; and the originator of this kind of philosophy is Thales who said that water is the principle (3); Thales said that water

is the principle, Anaximenes said that air is prior to water and it is the principle, Heraclitus said that fire is prior and it is the principle (*id.*).

All three results, of the first philosopher, of the third, and of the eighth, are a very great advance each.

With Thales and the first solution of the problem of the principles of all things philosophy begins.

Anaximenes not only discovered that to solve the problem is to establish what is prior to what. Anaximenes discovered that to solve the problem in general is to solve it in particular. Thales said that water is the principle. He said also that the earth is at rest because it floats on water, like wood (Aristotle, *Heaven*, II, 13). Anaximenes said that air is prior to water and it is the principle. He said also that air is infinite (Simplicius, comm. *Physics*, 24). That air is always in motion (*id.*). That air is invisible, and made visible by cold, hot, wet, and motion (Hippolytus, *Refutation*, I, 7). That all things come from air through rarefaction and condensation, rarefied air becomes fire, condensed it becomes wind, condensed further it becomes cloud, condensed further it becomes water, condensed further it becomes earth, and from these all other things come (*id.*). That the soul is air and it holds us together, as air encloses the world and holds it together (Aëtius, I, 3). That the planets are moved by gusts of thick air (II, 23). That the earth is flat, and it is at rest because it floats on the air below, like a lid (Aristotle, *Heaven*, II, 13). With Newton: the entire difficulty in philosophy is from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena. The resolution of the entire difficulty begins with Anaximenes, with his valiant reduction of the phenomena to air.

Heraclitus's advance is not in the pictures. Fire will judge all things. All things are in exchange for fire, fire in exchange for all things, like goods for gold and gold for goods. God is day night, winter summer, war peace, satiety hunger, just as fire, when mixed with spices, is named after their savor. Excellent as the pictures are, if they are to understand when Heraclitus explains each thing according to its nature.

The great advance is in the greater particularity

and greater generality of the result, both.

On the side of particularity, it is in the first inkling that the solution of the problem will have to be quantitative. That fire will have to be measured. Fire turns into sea, of which one half is earth, one half burns, and the same λόγος measures the sea before and after.

On the side of generality, it is in the first explicit relation of the principles of all things to the world. This world, the same for all, was made by neither god nor man, it always was and is and will be, an everlasting fire kindled in measures and extinguished in measures.

It is also in the first inkling that in relation to the world the problem of the principles of all things is a problem of development. The world is an everlasting fire kindled in measures and extinguished in measures. Aristotle writes: Heraclitus said that at some time all things become fire (*Physics*, III, 5), and: most philosophers seem to think that the world is generated, and generation over, some of them think that it is eternal, others that it will be destroyed like any natural thing, others again, like Heraclitus and Empedocles, that it only changes, it is sometimes as it is now, sometimes in the process of destruction or regeneration, and so without end (*Heaven*, I, 10).

Hence, with respect to change, the advance is the discovery that all things pose the problem of their change, and the greater the advance is in that all things pose the problem of their change as well as the problem of their stability, and the two in relation to one another. With Aristotle: all things change except for one thing which does not change and is the principle of all change. And the fragments of Heraclitus attest not only the principle of all change, fire. They attest that this world, the same for all, was made by neither god nor man, it always was and is and will be.

Hence, with respect to all things, Aristotle analyzes the principles of all things into material, efficient, formal, and final.

Thales discovered that all things have principles. Material principles.

One generation-odd later, Anaximenes first anticipates that the principles of all things cannot be only material. Air is the material principle, but also, if only implicitly, the efficient. Air is the principle of all things. And all things come from

air through rarefaction and condensation.

In the next generation, Pythagoras first anticipates that the principles of all things cannot not be also formal. Number is the principle of all things. And even if it was first proposed as a material principle, number has ever since been a formal principle.

One generation later, Heraclitus first anticipates that the principles of all things cannot not be all three, material, efficient, and formal. All things are in exchange for fire. By strife, and by necessity. All things also pass according to λόγος. The same λόγος measures fire before it becomes sea and after. Λόγος orders the whole.

Hence, finally, with respect to λόγος, λόγος anticipates logic.

ccxvi

About them Heraclitus speaks in 4 of the fragments above, and he has more to say about them, too.

In the fragments above he says:

men do not understand λόγος after they have heard about it just as before they have heard about it, they do not understand it even when I explain each thing according to its nature,

the rabble live as if each had his own understanding,

the rabble stuff themselves like cattle,

they set themselves against what they are the most in touch with, λόγος, what they meet every day seems foreign to them.

He says also:

the rabble do not understand those things, not even when they stumble on them (fr. 17);

they know neither how to listen nor how to speak (fr. 19);

when they hear, they yet do not understand; present, they are yet absent (fr. 34);

what mind or sense do they have? they trust poets, learn from the mob, do not know that the rabble are bad, the few are good (fr. 104).

Nor is Heraclitus judging the dogs when he says:

dogs bark at what they do not know (fr. 97), or judging the asses when he says:

asses prefer rubbish to gold (fr. 9).

Philosophy was young. It was so young, it did not yet know what to call itself. It calls itself our wisdom and good wisdom in Xenophanes. It calls

itself the best, the wise, and the few in Heraclitus.

Young as it was, not yet knowing what to call itself, philosophy was already crystal clear that it is not them. That it is in opposition to them. Philosophy, in fact, cannot be any clearer on the matter than it already is in Heraclitus.

And they are bad. The few are good. It will be irresistible to take it as a practical judgment, and a moral judgment, in particular. And so taken, a very censorious moral judgment it is. Not only are they bad. They live as if each had his own understanding. They stuff themselves like cattle. They are rabble. They are dogs. They are asses. And the last may well be unfair to the asses.

And taking it as a practical judgment, and a moral judgment, in particular, will obscure the fact that, whether it is a practical judgment, and a moral judgment, in particular, or not, and it is not, it is a theoretical judgment, and it is a theoretical judgment, even if it were a practical judgment, and a moral judgment, in particular, before it is, and in order to be, if it would be, a practical judgment, and a moral judgment, in particular. It is a judgment of truth before it is, and in order to be, if it would be, a judgment of action.

It is all about understanding and knowledge.

Men do not understand λόγος, they do not understand it even when Heraclitus explains each thing according to its nature. The rabble do not understand those things, not even when they stumble on them. They know neither how to listen nor how to speak. When they hear, they yet do not understand. What mind or sense do they have? they trust poets, learn from the mob, do not know that the rabble are bad, the few are good. Dogs bark at what they do not know.

In opposition, the common must be followed, and λόγος is the common. Listening not to me, but to λόγος, it is wise to agree that all things are one. One is the wise, to know how all things are steered through all things. One must know that war is common, justice is strife, and all things are by strife and necessity.

It is the acquisition of knowledge that sets philosophy apart.

Moreover, Heraclitus is clear about the cause. The best, the wise, and the few prefer gold to rubbish, asses prefer rubbish to gold.

He is clear also why the best, the wise, and the

few will all too often themselves prefer rubbish to gold. He says:

nature hides itself (fr. 123).

Wherefore:

much digging, little gold (fr. 22).

ccxvii

Heraclitus notes the 11 particular predecessors of his whom he notes in 11 fragments. 8 or 9 of the fragments are his own words, 2 or 3 are, to judge by their civility, his successors' reports of what he said. And in the chronological order of the 11 predecessors of his whom he notes, with the successors of his who note him noting his predecessors, these are the fragments.

Hippolytus wrote the 10 books of the *Refutation of all heresies*, to argue that all heresies are rooted in the doctrines of the philosophers. And writing about Heraclitus, Hippolytus reports (IX, 9): Heraclitus said that men are deceived even by the obvious, as even Homer, wiser than all Greeks, was; Homer saw some boys walking with their fishing rods; he asked them, what did you catch? and the boys replied, what we caught we left behind, what we did not catch we take with us; the boys had been delousing themselves (fr. 56).

An anonymous scholiast on the *Iliad*, glossing the line: he was born on the same night as him (XVIII, 251), notes: from this line, as also from the line, no mortal has escaped fate, Heraclitus concluded that Homer, too, was an astronomer (fr. 105). The second line is VI, 488.

Laertius, in *The lives and doctrines of the philosophers*, writing about Heraclitus, reports (IX, 1): Heraclitus said, Homer should have been thrown out of the contests and flogged, and so should have been Archilochus (fr. 42).

Plutarch, in the *Camillus*, writes (19): Heraclitus scolded Hesiod for saying that days are good and bad, for by nature days are all the same (fr. 106).

Hippolytus again, in the *Refutation*, writing about Heraclitus, reports (IX, 10): Heraclitus said, Hesiod is the teacher of most men, they think he knew the most, he who did not know that day and night are one (fr. 57).

Laertius again, writing about Heraclitus, reports (IX, 1): Heraclitus said, much learning

does not teach sense, or it would have taught Hesiod and Pythagoras, even Xenophanes and Hecataeus (fr. 40).

Laertius again, writing about Bias (I, 88): Heraclitus said, Bias, the son of Teutames, was of more λόγος than the rest (fr. 39).

Laertius again, writing about Thales (I, 23): Thales was the first astronomer, the first to predict eclipses and to determine the solstices, so writes Eudemus in the history of astronomy, and Thales was admired for that by Xenophanes and Herodotus and noted by Heraclitus and Democritus (fr. 38).

Laertius again, writing about Pythagoras (VIII, 6): Heraclitus said, Pythagoras, the son of Mnesarchus, researched things more than anyone and made himself a wisdom, much learning, a sham (fr. 129).

Philodemus wrote the 7 books *On rhetoric*, substantial parts of which survive, and in two passages from book VII Philodemus writes: all rhetoric aims to deceive, it is according to Heraclitus the prince of cheats (*id.*). The word rendered cheats, κόπιδοι, is rare. And an anonymous scholiast on Euripides's *Hecuba*, glossing a line in which the word occurs (131), notes: Heraclitus accused Pythagoras of being a cheat. It can be inferred, from the rarity of the word and the likelihood that Heraclitus will have continued abusing Pythagoras rather than suddenly begun abusing rhetoric, that Heraclitus said that Pythagoras was the prince of cheats (fr. 81).

Strabo, finally, in the *Geography*, writing about Ephesus, reports (XIV, 1.25): Heraclitus said, the Ephesians should leave their city to the boys and all hang themselves, they who exiled their best man, Hermodorus, saying, let none of us excel, and if anyone would, let him excel elsewhere (fr. 121).

With the number of fragments in which each is noted, the 11 particular predecessors of his whom Heraclitus notes explicitly are

764	Homer	3,
734	Hesiod	3,
688	Archilochus	1,
652	Teutames	1,
625	Thales	1,
622	Bias	1,
600	Mnesarchus	1,

570 - 495	Pythagoras	3,
570 - 473	Xenophanes	1,
562	Hecataeus	1,
540	Hermodorus	1.

Counting the fragments, the conclusion can be drawn immediately. Homer, Hesiod, and Pythagoras are the most important, and they are more important than the other 8.

And meager as the ground of the conclusion is quantitatively, the same conclusion can be drawn qualitatively.

Heraclitus's judgments of his predecessors constitute the collection of 11, as Xenophanes's judgments of his predecessors constitute the collection of 6 extracted from his fragments, and Hecataeus's judgments of his predecessors constitute the collection of 1 extracted from his fragments.

The collection extracted from Heraclitus, then, includes Homer and Hesiod, so does the collection extracted from Xenophanes, and the predecessor of his extracted from Hecataeus is Hesiod. And all three, Xenophanes, Hecataeus, and Heraclitus, censure Homer and Hesiod, censure them more severely than they do their other predecessors, and for graver and more general errors. Xenophanes censures them for what they have said about the gods, and the gods are a central concern of his fragments. Hecataeus corrects the error of Hesiod concerning the sons of Aegyptus, as Xenophanes corrects the errors of both Homer and Hesiod concerning the gods. Xenophanes censures Homer because all learned from him from the first. Heraclitus censures him, he who, wiser than all Greeks, was deceived by boys. He censures Hesiod, the teacher of most men, who think he knew the most, he who did not know that day and night are one. Homer, too, was an astronomer. Clearly, Homer should have been thrown out of the contests and flogged. And never mind learning, much learning did not teach Hesiod sense. The concern is teaching, learning, much learning, sense, wisdom, knowledge, as it is a central concern of the fragments of both Xenophanes and Heraclitus. Much learning did not teach Xenophanes and Hecataeus sense either, but on so much Heraclitus agrees with them. Qualitatively as well as quantitatively, Homer and Hesiod are the most important.

The collection extracted from Heraclitus includes Pythagoras, as does the collection extracted from Xenophanes. Xenophanes and Heraclitus both ridicule Pythagoras, feeble as ridicule is to describe Heraclitus's judgments of Pythagoras. Xenophanes ridicules him for teaching the transmigration of the soul. Heraclitus ridicules him for graver and more general errors. Much learning did not teach him sense either. Worse, he researched things more than anyone and made himself a wisdom, much learning, a sham. He was the prince of cheats. The wisdom which Pythagoras made himself, the sham, would be philosophy. And the concern is the same, teaching, learning, much learning, sense, wisdom, knowledge. Qualitatively as well as quantitatively, Heraclitus has discovered that Pythagoras is as important as Homer and Hesiod.

Of the remaining 8, the only 1 whom the collections extracted from Heraclitus and Xenophanes both include is Thales. Thales first predicted eclipses, he was admired for that by Xenophanes and noted by Heraclitus, so writes Eudemus in the history of astronomy, so writes Laertius in the history of philosophy. Cumulatively, then, judged by Xenophanes and Heraclitus, both, Thales is more important than the other 7. And again, qualitatively as well as quantitatively. Little in Heraclitus suggests concern with astronomy, but his judgments of his predecessors do. Homer, too, was an astronomer. Hesiod said that days are good and bad, but by nature they are all the same. They think Hesiod knew the most, he who did not know that day and night are one.

The collection extracted from Heraclitus includes Xenophanes and Hecataeus. The collection extracted from Xenophanes does not include Xenophanes even if: they are less worthy than I am. The collection extracted from Hecataeus does not include Hecataeus even if: the stories told among the Greeks seem to me ridiculous. The collection extracted from Heraclitus does not include Heraclitus even if: I explain each thing according to its nature. A philosopher cannot establish his own importance. Only his successors can, and, if he is important, do. Thus, Heraclitus began establishing the importance of Xenophanes and Hecataeus. And once again, qualitatively as well as quantitatively. Indeed, the quality, failure

to be taught sense, is as likely as not quintessentially philosophical, considering how frequently and vigorously the philosophers have found it in their predecessors.

The collection extracted from Heraclitus includes also Archilochus, Teutames, Bias, Mnesarchus, and Hermodorus. Archilochus, too, should have been flogged, but not as much as Homer, clearly, considering the lack of specific reasons. Bias was of more λόγος than the rest, but this cannot be high praise considering of how little λόγος the rest were. Hermodorus was the best man of the Ephesians, but this cannot be high praise either, considering that the Ephesians should all hang themselves. As to the labors of Teutames and Mnesarchus of fathering Bias and Pythagoras, respectively, these cannot be more important qualitatively than they are quantitatively either.

With the number of fragments in which each notes Heraclitus – dividing the contribution of Philodemus and the scholiast on the *Hecuba* in half, and arranging the scholiasts in the order of the books they are glossing – the successors of Heraclitus who note him noting his predecessors are

110 - 37 ^{BC}	Philodemus	0.5
65 ^{BC} - 29 ^{AD}	Strabo	1,
48 ^{AD} - 125	Plutarch	1,
170 - 235	Hippolytus	2,
219	Laertius	5,
	scholiast on the <i>Iliad</i>	1,
	scholiast on the <i>Hecuba</i>	0.5.

The conclusion can be drawn immediately. Laertius is the most important, by far, followed by Hippolytus, and the 2 are more important than the other 5.

And qualitatively as well as quantitatively.

Laertius investigates the doctrines of the philosophers, and in order to investigate the doctrines of the philosophers.

Hippolytus investigates the doctrines of the philosophers in order to show that all heresies are rooted in them. Much like Empiricus, who investigates the doctrines of the philosophers in order to show that knowledge is impossible. Much like Empiricus as well, whose reports on his predecessors are more important than the doctrine he expounds, Hippolytus's reports on his predeces-

sors are more important than the doctrine he expounds.

Strabo notes what Heraclitus said in the *Geography*, writing about Ephesus, and about notable Ephesians, in particular.

In Philodemus and in Plutarch the doctrines of the philosophers are yet further removed from what is at issue. What Heraclitus said provides Philodemus with a picturesque illustration. In the *Camillus*, Plutarch writes about the sack of Rome by the Gauls, and about lucky and unlucky days, in particular, and what Heraclitus said provides him with an opportunity to scold him for scolding Hesiod.

The two scholiasts, finally, much like any other scholiasts, glossing the poets mostly with the poets, found occasionally the philosophers as well useful for the glossing of the poets.

Importance can be the ground on which the labors of their predecessors and the philosophers can be divided into the small number of these who may attest the acquisition of knowledge and the large number of those who may not attest the acquisition of knowledge.

ccxviii

No claim involving importance will fail to be met with suspicion. Who says it is important? important for what? for whom? can it be but a vested interest? and worse?

Scepticism and worse have made it a norm to suspect importance, and made the norm ubiquitous. The world is nowhere but in the ideas of the individual. There is no truth, only world-views. Such cannot tolerate importance.

All that needs to be said about all that is what Heraclitus has said about them.

Importance is important. It is, in fact, the most important.

To begin with, there is, for instance, the solar system. The sun, the planets, the satellites, the asteroids, the comets, and the meteoroids. And it may be doubted that the sun is, with Kepler, the most beautiful, even as it is the most beautiful. But it may not be doubted, it may only be believed doubted, that, again with Kepler, the sun is the source of the light and heat from which all life flows into the plants. Also with Kepler, the sun is the greatest. It is the largest, by far. It is what

makes the system a system. It determines the position, order, and motion of the parts. It is in the focus of the conics which other bodies describe, not the other way around. They describe equal areas in equal times with respect to it, not the other way around. They revolve around it, not the other way around. It is the source of life, also, because it is the source of energy. It is the most powerful, by far. It is the efficient cause. It is the material cause, too, the other bodies are its debris. It is the substantial in the system. As even its name indicates, it is the essential in the system, too. It is of the greatest value, too, however value be judged, in energy, in life, in light, in truth. It deserves the greatest consideration, practically as well as theoretically.

And if all of this were not in relation to one another, to describe the sun as the most important could be avoided. But all of this is in relation to one another, and it is all relations of dependence on one another. And the interdependence of beauty, light, heat, life, size, system, position, order, motion, parts, cause, power, energy, matter, substance, essence, value, consideration, practice, theory, cannot not be described, implicitly if not explicitly, if the sun is described. And it cannot be described explicitly but in terms of importance.

Nor is it an accident that the controversy over the Copernican hypothesis, including the burning of Bruno and Galileo's crime of vehement suspicion of heresy, involved scriptures, faith, and god. Nor could the controversy not involve scriptures, faith, and god, if these were to be important, let alone the most important.

And as the sun, so anything of any importance. Anything of multiple and complex relations of interdependence. And that is, anything.

Importance and not importance, if they describe multiple and complex relations of interdependence, cannot not be general, like general and particular, whole and part, genus and species, cause and effect, substantial and accidental, essential and inessential. They cannot not be relations of all things.

Thus, a house cannot not have more and less important parts, and in relation to beauty, light, heat, life, size, system, position, order, motion, cause, power, energy, matter, substance, essence,

value, consideration, practice, theory. And the analogy with the sun holds relation by relation because, with Aristotle: if a house had been made by nature, it would have been made in the same way as it is now made by art; and if things made by nature could be made also by art, they would be made in the same way as they are now made by nature (*Physics*, II, 8).

The more so the more houses. The greater the multiplicity and complexity of relations of interdependence, the more some are more important than other, and some of the more important very important, some of the less important altogether unimportant.

The more so a city. The more so the more cities.

The more so a state. The more so the more states. And the greater the multiplicity and complexity of relations of interdependence have been, the more but a few states have been very important, most states have been altogether unimportant. And much like the bodies of the solar system, states have excelled at sorting out their relations of importance and not importance. Kish and Lagash were more important than Akshak and Mari, which were more important than Lullubum and Simurru. Athens and Sparta were more important, by far, than any of the couple of hundred other Greek states, the most important among whom were Miletus in the east, Syracuse in the west, Thebes further north. The princes of Akkad built a state more important than any earlier state. One and a half millennia later the Assyrian princes built a more important state, and, with all due respect to the Hebrew scriptures, a far more important state than Judah and Israel. The great kings built a more important state, and, with all due respect to Marathon, Salamis, and Plataea, a far more important state than Athens and Sparta. The Roman generals and princes built a more important state, and the more important it was in that it became the vehicle of Christianity, too. And when, one and a half millennia later, still more important states were built, but two, Britain and France, undertook to determine which was the most important, did determine it, after the congress of Vienna the concert of the great powers included but a handful of these, the concert yet fell apart, and but one, Germany, undertook to

show the rest which was the most important. That the attempt involved world wars, of which the second was fought with atomic weapons, too, should suffice to indicate the multiplicity and complexity of relations of interdependence.

Nor is ethics possible but on the supposition that what you do is important, indeed important enough to decide your fate for all eternity.

The more so theory. The search for truth.

And the more so because the multiplicity and complexity of relations of interdependence cannot not include the multiple and complex relations of development, which in turn cannot not include the multiple and complex relations of advance.

Hence, in philosophy, in physics, in mathematics, in the acquisition of knowledge, it has never, over time, been difficult to tell – it has, in fact, been obvious – that the very important is very important. Plato is very important. Aristotle is very important. Euclid. Archimedes. Ptolemy. Augustine. Copernicus. Bacon. Galileo. Kepler. Descartes. Huygens. Newton. Leibniz. Euler. Hume. Kant. Hegel. Gauss. Darwin. Maxwell. Frege. Einstein. In each case there are many results of many and great consequences, and of many and great consequences in relation to the philosopher's predecessors, whose results the philosopher has advanced, and of many and great consequences in relation to the philosopher's successors, whose results, by advancing his results, have established his importance.

In history, too, even as the utterly useless and the least important dominate it, the great and magnificent deeds of Greeks and barbarians are the great and magnificent deeds of Greeks and barbarians and what merits the attention of all times is what merits the attention of all times. All attempts to make the important not important and the not important important are doomed by definition. Herodotus is very important. Polybius. Spinoza. Voltaire.

Importance is a fact.

And with Maxwell once again: not good or bad, noxious or beneficial, but of what kind it is, and how much there is of it.

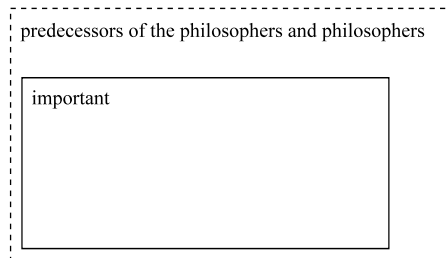
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If knowledge has been acquired by the labors of their predecessors and the philosophers, exclu-

sively, the collection of their predecessors and the philosophers will include the acquisition of knowledge in its entirety. The collection will include more than the acquisition, however. The task is thus to collect their predecessors and the philosophers, and to separate within that collection the smaller collection of these whose labors are the acquisition.

But the development of philosophy has not only included more than the acquisition of knowledge. It has included very much more. There is also the variety of the kinds of what it has included. And importance cannot coincide with the acquisition of knowledge. Importance, however, can be the ground on which within the large collection of their predecessors and the philosophers the very much smaller collection of the important among them can be separated, and if the acquisition of knowledge is important, this smaller collection now will include the acquisition of knowledge in its entirety.

And the acquisition of knowledge is important. To picture it.



As the larger space of the predecessors of the philosophers and the philosophers is enclosed, within it the smaller space of the important will be enclosed, within which the yet smaller space of the acquisition of knowledge can be enclosed.

Xenophanes, Hecataeus, and Heraclitus show that it should be done, that it can be done, and how it has to be done.

Young as philosophy was, not yet knowing what to call itself, it concerned itself with the gods and god, the principles of all things, the earth, rain, clouds, wind, and rivers, what is worthy and what is not, the localities on the surface of the earth, mapping the surface, the stories told among the Greeks, λόγος, fire, how λόγος measures fire, strife, necessity, how all

things are by strife and necessity, change, the everlasting fire which this world is, asses and what they prefer. Our wisdom, good wisdom, the best, the wise, and the few, philosophy was already also much learning which does not teach sense, a sham, and cheating. As soon as it appeared, philosophy posed the problem of what is it, what is not it, what it is, what it is not. Posed the problem to itself, and concerned as it was, and is, with all things, and one as the wise was, and is, to know how all things are steered through all things, it posed the problem to all things. Clearly, steered as all things are through all things, and one as the wise is, it posed the problem in terms of importance, too.

The same with knowledge. But a couple of thousand words, the couple of thousand words are all about knowledge, belief, truth, understanding, how to listen, how to speak, trust, learning, much learning, teaching, teaching them all, mind, sense, wisdom, good wisdom, sham wisdom. As soon as it appeared, philosophy posed the problem of knowledge, too, what is it, what is not it, what it is, what it is not. In terms of importance, too.

And if it should be asked what is philosophy, and what philosophy is, and what is knowledge, and what knowledge is, it should be clearly and distinctly.

And it can be.

Xenophanes established the particularity of his predecessors.

Homer and Hesiod are what they said about the gods. Epimenides is the question of how long he lived. Thales is the prediction of eclipses and determination of the solstices. Pythagoras is the teaching of transmigration. Simonides is money-grubbing. The judgments of Xenophanes separate the 6 sharply from everything else.

Including oxen, horses, and lions.

Including gods. Zeus and Iris do not belong in the collection of the particulars. One god, greatest among gods and men, does not belong in it either.

Including men. The Ethiopians and the Thracians do not belong in the collection. The winners of horse races, foot races, boxing matches, and wrestling contests do not belong in it. Those who honor the winners do not belong in it either.

Separated sharply from everything else, the particulars can be arranged in their series and

order. And Xenophanes has taken a step toward the solution of this problem as well, when he said that Homer was older than Hesiod.

Hecataeus, meager as the evidence is, corroborates Xenophanes. His judgments separate Hesiod sharply from everything else. Including Aegyptus who did not have even 20 sons. Including the Greeks among whom many ridiculous stories are told.

With Heraclitus the evidence is not meager. The particulars are 11, nearly twice the number in Xenophanes. And their separation cannot have been sharper. They are not cattle, or dogs, or asses. Zeus does not belong in their collection. Dionysus (fr. 15), Dike (fr. 28), the Sibyl (fr. 92), the lord whose oracle is in Delphi (fr. 93), Helios and the Erinyes (fr. 94) do not belong in it either. Λόγος does not belong in it either. As to the separation of the particulars from them, it will never be sharper. The sheer increase in the number of the particulars is an advance in the solution of the problem of arranging them in their series and order as well. Much learning did not teach Hesiod, Pythagoras, Xenophanes, and Hecataeus sense, and it did not, apparently, in chronological order.

Most important, the judgments are cumulative. The collection of 11 constituted by the judgments of Heraclitus overlaps with the collection of 1 constituted by the judgments of Hecataeus and the collection of 6 constituted by the judgments of Xenophanes. The later judgments are thus also judgments of the earlier judgments. The later collection is an examination of the earlier collections, quantitatively and qualitatively. Heraclitus notes some of his predecessors and not others, as his predecessors note some of their predecessors and not others, and he notes some of those they note. He notes some of those he notes more than others, as they note some of those they note more than others, and some of those he notes more are some of those they note more. He ridicules and admires, and mostly the former, some more than others, as they ridicule and admire, and mostly the former, some more than others, and some of those he ridicules and admires more are some of those they ridicule and admire more. Quantitatively and qualitatively, the judgments cannot not determine importance, and, examining one an-

other, cannot not over time determine it conclusively.

Xenophanes discovered that some of his predecessors are more important than others. Heraclitus corroborated and generalized the discovery. Some of the predecessors of the philosophers are more important than others.

The judgments are cumulative. The relation of Heraclitus to his predecessors is a relation of Heraclitus's successors to their predecessors, including Heraclitus, his predecessors, and his relation to his predecessors, it is a relation of their successors to their predecessors, including the earlier successors, their predecessors, and their relation to their predecessors, including Heraclitus, his predecessors, and his relation to his predecessors, and so on, for as long as there are philosophers.

Hence, Homer, Hesiod, Thales, and Pythagoras are very important, Teutames and Mnesarchus are altogether unimportant, and having accumulated for twenty-five centuries after the initial accumulation, some 83 generations of philosophers, these judgments have long been established conclusively, and for as long as there are philosophers.

With reference to the Euler spaces, particularity encloses the larger, importance encloses the smaller. And particularity and importance have, in fact, long enclosed both spaces implicitly. The task is only to make this fact explicit.

Bacon writes: let no one be alarmed at the multitude of the particulars. Let no one be alarmed because there is importance. As they have been drawn, the spaces cannot be to scale. The smaller has to be very much smaller.

And how to enclose the spaces explicitly is clear. The judgments of the successors of Xenophanes, Hecataeus, and Heraclitus have to be ascertained.

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Collecting all the particulars extracted from Xenophanes, Hecataeus, and Heraclitus, with the number of fragments in which each is noted by any of the 3, the particulars are

764	Homer	7,
734	Hesiod	7,
688	Archilochus	1,

653	Epimenides	1,
652	Teutames	1,
625	Thales	2,
622	Bias	1,
600	Mnesarchus	1,
570 - 495	Pythagoras	4,
570 - 473	Xenophanes	1,
562	Hecataeus	1,
556 - 466	Simonides	1,
540	Hermodorus	1.

13 particulars. The task is to show which are the important, which are the not important.

In 8 cases this is trivial.

All learned from Homer from the first. Homer was wiser than all Greeks. Homer ascribed to the gods everything shameful among men. Homer should have been thrown out of the contests and flogged. In the *Histories* Herodotus corrects the errors of two predecessors of his explicitly, Homer and Hecataeus. In his dialogues Plato quotes Homer more often than any of his predecessors, by far. In his treatises Aristotle quotes him more often than any of his predecessors as well, if not by far.

The cumulative judgment of Xenophanes, Heraclitus, Herodotus, Plato, and Aristotle suffices, and more than suffices, to show that Homer is important. Their judgment of Homer, moreover, is the cumulative judgment of all their successors who have judged them, all who have judged Xenophanes, all who have judged Heraclitus, all who have judged Herodotus, all who have judged Plato, and all who have judged Aristotle, and have thus judged their judgment of their predecessors.

Also, the judgment of Homer is a judgment of his labors, and his labors are concerned with propositions, as Xenophanes's judgment of Homer already attests. The propositions are collected in the *Iliad* and *Odyssey*.

Mutatis mutandis, Hesiod, Thales, Pythagoras, Xenophanes, and Hecataeus. All 5, Xenophanes, Heraclitus, Herodotus, Plato, and Aristotle, judge Hesiod. All 5 judge Thales. All 5 judge Pythagoras. Heraclitus, Plato, and Aristotle judge Xenophanes. Heraclitus and Herodotus judge Hecataeus. And to note but the most important labors of each, Hesiod wrote the *Theogony* and *Work and days*. Thales is the first philosopher. Pythag-

oras discovered that number is the principle of all things, and the incommensurability of the side and diagonal of the square. Xenophanes discovered that knowledge is not belief, and that some of his predecessors are more important than others. Hecataeus wrote the first geographical treatise.

Homer, Hesiod, Thales, Pythagoras, Xenophanes, and Hecataeus are important.

Teutames was the father of Bias. Mnesarchus was the father of Pythagoras. And this is the only labor each left behind for his successors to judge. Even if the labor of Mnesarchus, if its outcome transmigrated, may have been unusually strenuous.

Teutames and Mnesarchus are not important.

In the other 5 cases – Archilochus, Epimenides, Bias, Simonides, and Hermodorus – to ascertain how their successors have judged their importance is more difficult. Because, on the one hand, they are not as important as Homer, Hesiod, Thales, Pythagoras, Xenophanes, or Hecataeus, but, on the other hand, they are not as not important as Teutames or Mnesarchus either.

More difficult, but not too difficult.

To begin with the easiest, Epimenides.

The Cretans say that Epimenides lived 299 years, but Xenophanes had heard that he lived 154 years. More reassuringly, Plato also notes Epimenides (*Laws*, I, 642d). Aristotle reports that Epimenides was a seer (*Rhetoric*, III, 17), and notes him in *The constitution of Athens* (1) as well. In *The lives and doctrines of the philosophers* Laertius treats 102 predecessors of the philosophers and philosophers under his own head each, and Epimenides is 1 of the 102 (I, 10).

And the following is decisive. Epimenides is the Cretan who said: all Cretans are liars (fr. 1). This one proposition suffices to show that Epimenides is important.

The source of the fragment is st Paul. If not exactly st Paul. One half of the letters in the Pauline corpus are spurious, one of the spurious is the *Letter to Titus*, and the *Letter to Titus* reports: it was a Cretan prophet who said, all Cretans are liars, vicious beasts, lazy gluttons (1). One and a half centuries later, Clement of Alexandria writes: the Cretan of whom the apostle Paul speaks in the letter to Titus is Epimenides

(*Miscellanies*, I, 59).

Eighteen and a half centuries later, in 1931, in *On formally undecidable propositions of Principia mathematica and related systems*, Gödel comments on Gödel's theorem: its argument is closely related to the liar antinomy (1). Gödel does not refer the antinomy explicitly to Epimenides, but earlier, in 1908, in the *Mathematical logic as based on the theory of types*, Russell does.

Russell discovered the paradox of the set of all sets that are not members of themselves in 1902. Frege replied: your discovery has caused me the greatest surprise and, I would almost say, consternation. Russell collected more paradoxes, and in 1908, in the *Mathematical logic as based on the theory of types*, he begins by listing seven of them, beginning with the earliest: Epimenides the Cretan said that all Cretans were liars; was this a lie? if he was lying, he was speaking the truth, and vice versa (1).

Epimenides is important.

Bias was of more λόγος than the rest. Herodotus notes Bias twice (I, 27, 170). Plato notes him in the *Greater Hippias* (281c), in the *Protagoras* (343a), and in the *Republic* (I, 335e). Aristotle notes him in the *Ethics* (V, 1), and in the *Rhetoric* (II, 13). In Laertius he is 1 of the 102 under his own head each (I, 5).

In the *Republic*, Bias marches in one of the armies waging the war for justice: are we, Socrates asks, to join ranks with the wise and blessedly happy Simonides, Pittacus, and Bias, who said that justice is to give everyone his due, and fight against Periander, Perdiccas, Xerxes, and Ismenias, who said that justice is to do good to your friends and harm to your foes? yes, Polemarchus replies.

In the *Protagoras*, Plato writes: philosophy had its first beginning among the Cretans and the Spartans (342a-b). He then provides the evidence: the Spartans have the best education in philosophy; to be a Spartan is to be a philosopher more than it is to be a fighter; and one of the seven wise men, next to Thales, Pittacus, Bias, Solon, Cleobulus, and Myson, was the Spartan Chilon (342d-3a).

And the last is decisive. Pace Plato, philosophy did not have its first beginning among the

Cretans and the Spartans. With Plato, however, the seven wise men are an important part of the problem of the beginning of philosophy. How important, Laertius attests in *The lives*, when, after a brief introduction, he turns to the first philosopher, and then to ten more of the seven wise men (I, 40-2). By the time of Laertius, six and a half centuries after Plato, the seven wise men had become twenty-two. Lists of the seven had multiplied, variable in content, to accommodate more and more variable wisdom. Epimenides and Pythagoras are among the twenty-two. Plato's is the oldest list of the seven. Counted among the seven by all lists, four were the wisest, Pittacus, Solon, Thales, and Bias. And the sayings and doings of these wise men, first attested in the case of Bias by Heraclitus, Herodotus, Plato, and Aristotle, are an important part of the problem of the beginning of philosophy.

Bias is important.

The Ephesians should all hang themselves except Hermodorus, their best man.

Five centuries later, Strabo is the source of the fragment, and after quoting Heraclitus, Strabo adds: it is said that Hermodorus wrote some laws for the Romans (XIV, 1.25).

Not quite one century later, describing the statues in the forum, Pliny reports: the statue of Hermodorus the Ephesian, who interpreted the laws copied by the decemviri, was erected in the comitium by the people (XXXIV, 11).

Two-odd generations later, writing on the origin and development of the law, and specifically on the origin of the Twelve tables, Pomponius notes: decemviri were appointed to study the laws of the Greek cities and then give laws to their city (*Digest*, I, 2.2), and later adds: some have written that a certain Hermodorus of Ephesus, then in exile in Italy, assisted the decemviri (*id.*).

And this is all there is about Hermodorus in the Greek and Roman record.

To establish what the connection between the development of the law in the Greek cities and the early development of Roman law is as important as Roman law is important, and Roman law is as important as the fact that on the development of Roman law the development of all subsequent law depends. Hermodorus, however,

is not that connection.

He might have been, had Strabo had more to say. Had Pliny had more to report than the judgment of the Romans, which, whether it is expressed by erecting a statue or by exiling their best man, there is no reason to judge better than the judgment of the Ephesians. Most of all, had Pomponius, one of the finest jurists and the ultimate authority on the early development of Roman law, had more to say.

Had Polybius noted Hermodorus, or Cicero, or Siculus, or Dionysius of Halicarnassus, or Plutarch, writing as they all are on the relations of Rome and the Greek cities, on the Roman constitution and laws, and specifically on the Twelve tables.

Had Herodotus noted the Ephesians' best man, or Plato, or Aristotle, or Theophrastus, writing as they all are close to the time of Hermodorus, and concerned as they all are with laws.

Most of all, had Livy noted Hermodorus. Livy, the ultimate authority on the early development of Rome, describing the quarrels of patricians and plebeians which led to the appointment of decemviri and the drawing and enactment of the Twelve tables, has this to report about the relation of the Twelve tables to the laws of the Greek cities: the patricians sent to Athens three of their number, Manlius Vulso, Postumius Albus, and Sulpicius Camerinus, to copy Solon's famous laws and study the institutions, customs, and laws of the other Greek cities (III, 31).

Hermodorus is not important.

Simonides was a moneygrubber.

Either way, Simonides is a very fine poet. Herodotus (VII, 228) records his epitaph for the 300 who fell at Thermopylae:

stranger, tell the Spartans,
we lie here, obedient to their word.

In the canon of the Alexandrian grammarians Simonides is one of the 9 lyric poets (Quintilian, X, 1.61-4), next to Alcaeus, Sappho, Anacreon, Pindar.

Plato writes about Simonides in the *Protagoras* and in the *Republic*.

In the *Republic*, Simonides is wise and blessedly happy, marching shoulder to shoulder with Pittacus, Bias, Socrates, and Polemarchus, against Periander, Perdiccas, Xerxes, and Ismenias.

Earlier in the dialogue, Polemarchus, unaware perhaps that he is broaching a ten-book-long dialogue with his haste, says, justice is to speak the truth and pay one's debts; Socrates muses, so this is what justice is; it is, Polemarchus confirms, if we are to trust Simonides who says that it is just to give each what is owed him; Socrates then muses further, Simonides is no doubt a wise and divine man, but what exactly does he mean (I, 331b-e)?

In the *Protagoras*, Plato writes: the sophists are peddlers of nourishment for the soul, and if you do not know what you are buying, they may well deceive you; so if you know what you are buying, buy safely from Protagoras or anyone else, but if you do not know what you are buying, you are rolling the dice. Having thus introduced the problem of the peddling of nourishment for the soul, Plato proceeds to depict Protagoras, surrounded by a throng of admirers, in the act of peddling nourishment for the soul. And Protagoras says: when in a poem of his Simonides says,

it is hard for a man to become truly good,

and later in the same poem criticizes Pittacus for saying,

it is hard to be good,

Simonides contradicts himself; the admirers applaud; not one of the admirers, Socrates turns to Prodicus and asks, tell me, Prodicus, are becoming and being the same (339a-d)?

Earlier in the dialogue, while introducing the problem of the peddling, Plato adds: the art of the sophists is very ancient; the men who practiced it in ancient times, however, fearing the odium attached to it, disguised it sometimes as poetry, as Homer, Hesiod, and Simonides did, sometimes as mystery or prophecy, as Orpheus and Musaeus did, sometimes even as athletics, as Iccus and Herodicus did; Agathocles, a great sophist, disguised it as music, and so did Pythocles (316d-7e).

Plato has Socrates say this also: Simonides was eager to be known as a philosopher (343c).

In which, to be known as a philosopher, if he was eager to be known as a philosopher, Simonides failed. Simonides is not known as a philosopher.

He might have succeeded, could either the doctrine, it is just to give each what is owed him,

which Plato has Socrates ascribe to Simonides, or the distinction between becoming good and being good, which Plato has Protagoras fail to read in the verses of Simonides and Socrates read in them, be, in fact, ascribed to Simonides. Could either of these be, in fact, ascribed to Simonides, the question of his part in the development of that doctrine or in the discovery of that distinction might have been important. As it is, it is not. The evidence about Hermodorus suffers from lack of specificity. Plato's evidence about Simonides suffers from surfeit of specificity. Simonides has to share that doctrine with the motley army of Pittacus, Bias, Socrates, and Polemarchus. As the motley army of Periander, Perdiccas, Xerxes, and Ismenias has to share the alternative doctrine. That distinction Simonides has to share with the peddler Socrates against the peddler Protagoras. The peddling itself he has to share also not only with Homer and Hesiod, and Iccus and Herodicus, and Agathocles and Pythocles. He has to share it with Orpheus and Musaeus, neither of whom need be burdened with the presumption of reality.

Aristotle has an anecdote: Hiero's wife asked Simonides whether it is better to be rich or wise; rich, he replied, it is the wise who while away their days at the doors of the rich (*Rhetoric*, II, 16).

Cicero has another: Hiero asked Simonides about the nature of the gods; Simonides requested a day to consider; when the next day Hiero asked for an answer, Simonides requested two more days; when two days later Hiero asked again, Simonides requested four more days; and so on, until Hiero asked him why; the more I ponder the matter, Simonides replied, the more obscure it seems (*Gods*, I, 60).

Plutarch, in *Whether war or art brought Athens more fame*, has an apothegm: Simonides said, painting is silent poetry, poetry is painting that speaks (3).

Simonides is not among the 102 under his own head each in Laertius.

Seventeen centuries after Plutarch, in 1766, Lessing wrote the *Laocoön, or on the limits of painting and poetry*. The apothegm of Simonides provides Lessing with his starting point and his central argument.

The proposition of Epimenides excited the philosophers after Russell. The apothegm of Simonides did not excite the philosophers after Lessing, excited as they were at the time by the possibility of reading poetry in the highest realms of spirituality. The culmination of this excitement is Schopenhauer. In *The world as will and representation*, he writes: the poet grasps the idea, the being of mankind, the adequate objectivity of the thing in itself in its highest degree outside all relation and all time (I, 51). In the four volumes of *The world* Schopenhauer finds no occasion to note Simonides.

Simonides is not important.

That a poet, nay, a very fine poet, is judged not important will cause distress. Needlessly.

It will cause distress because it will be taken to say that the poet is not important. It need not be so taken. Not yet anyway. Homer and Hesiod are judged important. And it need not cause distress even if it turns out, eventually, that the poet is not important.

Importance has to be a relation of the particulars. If so, and the particulars constitute kinds, and importance is one of the relations constituting the kinds, and some of the particulars are important, some of the particulars are not important, it has to be also that some of the kinds are important, some of the kinds are not important. If so, however, if some of the kinds are important, some of the kinds are not important, importance can only be a relation of the kinds such as they are constituted by the particulars. If importance is established by the philosophers' cumulative judgment of their predecessors, the importance and not importance of the kinds can only be a consequence of the importance and not importance of the particulars. And if so, the importance and not importance of the kinds can only be determined after the importance and not importance of the particulars is determined. And if so, kinds had better not be supposed important or not important after flying from the particulars to most general principles and, taking them to be settled, proceeding to judgment.

Thus, if Epimenides is important, and he was a seer, this will not make the kind seer, if there is such a kind, important. More seers, more important seers, and more kinds in relation to the kind

seer, given that some of these kinds are themselves important, some of these kinds are themselves not important, can only do that, can only make the kind seer important. In the same way, if Bias is important, and he was a wise man, this will not make the kind wise man important. If Hermodorus is not important, this will not make his kind not important, whatever his kind was, the Ephesians' best man, perhaps. And in the same way, if Homer and Hesiod are important, Simonides is not important, and their kind is poet, if there is such a kind, this will not make this kind two thirds important one third not important. More poets, more important and not important poets, and more kinds, some of them important, some of them not important, can only do that.

And either way, whether it turns out, eventually, that the poet is important, or very important, or not important, or very very very not important, if what makes the poet a poet, and the more so what makes the fine poet a fine poet, is grasping the idea, the being of mankind, the adequate objectivity of the thing in itself in its highest degree outside all relation and all time – and Schopenhauer's is as good an explanation of poetry as there is – can the poet, and the more so the fine poet, grasping the thing in itself, nay, in the highest degree, nay, outside all relation and all time, be beholden to anything in the world, including importance?

Archilochus should have been thrown out of the contests and flogged.

Either way, Archilochus is a very fine poet, too, finer even than Simonides. Aristotle (*Rhetoric*, III, 17) records his verses on the occasion of a solar eclipse:

think nothing impossible any more,
do not swear that anything will not befall.

In the canon of the Alexandrian grammarians Archilochus is one of the 3 iambic poets (Quintilian, X, 1.60).

Archilochus, however, is not even a soldier in Plato's motley armies, and not even the source of a *bon mot* that has inspired a poet to write a book on poetry.

Archilochus is not important.

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Collecting all the particulars from which the

fragments from which the particulars extracted from Xenophanes, Hecataeus, and Heraclitus are extracted, with the number of fragments, these particulars are

110 - 37 ^{BC}	Philodemus	0.5,
65 ^{BC} - 29 ^{AD}	Strabo	1,
48 ^{AD} - 125	Plutarch	1,
121	Herodian	1,
127	Gellius	1,
159	Empiricus	2,
170 - 235	Hippolytus	2,
219	Laertius	8,
	scholiast on the <i>Iliad</i>	1,
	scholiast on the <i>Hecuba</i>	0.5,
	scholiast on the <i>Orestes</i>	1,
	scholiast on the <i>Peace</i>	1.

12 particulars. The task is the same, to show which are the important, which are the not important.

The particulars in this collection of 12 are substantially later than the particulars in the earlier collection of 13. With respect to the first philosopher, those in the earlier collection range from one and a half centuries before him through not quite one century after him. Those in the later collection, ignoring the scholiasts, range from five-odd centuries after him through eight and a half centuries after him. At the time of the earlier particulars philosophy was yet to be born, was born, and, born, was still young. At the time of the later particulars philosophy was not young any more. The record was also younger at the time of the earlier, older at the time of the later.

Hence, there are, in general, two substantial differences.

Ceteris paribus, early is more important than late. Because, with Aristotle: in all investigations labor either takes over from those who have gradually advanced the investigation, or makes the first discovery, and the first discovery usually advances the investigation only a little, even if it is more important than the discoveries that spring from it; for it may well be, as the saying is, that the beginning is the most important; it is also the most difficult; once the beginning is discovered, it is easier to develop the rest.

The two collections illustrate. The 13 earlier particulars include the first philosopher, the discovery of proof, the discovery that knowledge

is not belief, the beginning of geography. There is no new beginning among the 12 later particulars.

Also, *ceteris paribus* again, to ascertain importance late is easier than early. For the same reason. As the record is amassed, it becomes in part steadily more explicit, in part steadily more exclusively derivative. The steadily more explicit part becomes also an ever smaller part, the steadily more exclusively derivative part becomes an ever larger part.

The two collections again illustrate. The later 12 have left behind substantially more than the earlier 13. The later 12 are also in part more explicit, in part more exclusively derivative. The more explicit is a small part, the more exclusively derivative a large part.

To ascertain the judgment of his successors in the case of Laertius is trivial.

The lives and doctrines of the philosophers is the second very important step in the advance of the history of philosophy. Six centuries before Laertius, Theophrastus and Eudemus took the first. Eighteen and a half centuries after Laertius, a third remains to be taken.

Laertius notes 1 273 predecessors of his. He investigates 102 of them under his own head each. And he arranges the 102 into kinds. In book I Laertius begins with the problem of the origin of philosophy, then investigates the first philosopher and ten of the seven wise men. One of the divisions of philosophy is into: Ionian and Italian, the Ionian began with Anaximander, the Italian with Pythagoras, and they are so called because Thales, who taught Anaximander, was from Ionia, while Pythagoras philosophized mostly in Italy (I, 13). In book II Laertius investigates first Anaximander and his immediate successors (II, 1-17). But the main division of philosophy is into schools, and he investigates next in book II Socrates, his followers, and the Socratic schools (18-144), including among these Aristippus and the Cyrenaics (65-104). In books III and IV he investigates Plato and the Academics. In book V Aristotle and the Peripatetics. In book VI Antisthenes and the Cynics. In book VII Zeno of Citium and the Stoics. In book VIII Pythagoras and the Pythagoreans. In book IX he investigates first the philosophers who had no school (1-60), beginning with Heraclitus (1-17) and Xenophanes

(18-20), then turns to Pyrrho and the sceptics (61-116). In book X he investigates Epicurus and the Epicureans. And the history of philosophy continues to be analyzed into kinds according to where the philosophers were from, where they mostly philosophized, and schools.

Hegel's restoration of spirit to the substantiality and purity of its being concluded with the *History of philosophy* and *Philosophy of history*. Part I of the *History of philosophy* is: Greek philosophy, comprising more than one half of its bulk, and Hegel's most important source on Greek philosophy, next to Plato and Aristotle, is Laertius (*passim*).

Laertius is important.

To ascertain the judgment of their successors in the cases of Strabo and Plutarch is not difficult.

Strabo has left behind the 17 books of the *Geography*. He writes two centuries after Eratosthenes, and one century-odd after Hipparchus. Strabo reports: Eratosthenes grounded geography in the principles of mathematics and physics (I, 4.1), and: Hipparchus showed that geographical investigation is impossible without astronomical observation (1.12). Strabo agrees: the geographer must rely on geometry and astronomy (II, 5.2). But not too much: amputation at the joints is right, haphazard amputation is wrong, and Eratosthenes must be criticized for haphazard amputations; the geographer does not need geometrical definitions, countries are well enough defined by their rivers, mountains, and the peoples inhabiting them (II, 1.30). So there are no haphazard amputations in the *Geography*. Strabo describes more than 6 000 localities, but it remained for Ptolemy, one and a half centuries later, to determine their longitudes and latitudes. On the other hand, Strabo reports in some detail on most of the 6 000 localities, and associates men of note with many of them.

It was thirteen-odd centuries after Ptolemy that maps took a step beyond him. In 1459, Fra Mauro drew a map of the earth, still grounded essentially and substantially in Ptolemy, but informed also by the voyages of Polo in the east and the voyages of prince Henry's knights down the African coast. Fra Mauro writes about the different features on the map on the very features on the map, and on India he writes: those who want to know

about the customs and practices of these people should read Arrian and Strabo who write copiously about India (*id.*).

Strabo is an important, if not a very important, step in the development of geography. He is also a very important source and authority, both, on his predecessors.

Strabo is important.

Plutarch wrote 66 lives. Through the judgment of his successors, 50 of the 66 have remained in the record. Through the same judgment, Plutarch has left behind six dozen-odd other treatises. These are larger in bulk than the lives, and have acquired the collective name *moralia*. The name suits them.

It suits also the lives. Plutarch writes: I write lives, not history; and the most magnificent deeds do not always show the virtues and vices of a man the best; indeed, a word or a jest sometimes reveals a man's character better than winning a battle in which thousands fall, marshaling a great army, or investing a city (1); my task is to dwell on what illuminates the movements of the soul, and thus portray a man's life, leaving it to others to tell of his great deeds and battles (*id.*). Plutarch writes this in the life of Alexander, no less.

Hence, writing lives, not history, dwelling on what illuminates the movements of the soul, Plutarch wrote the lives indiscriminately of Theseus and Romulus, for instance, who need not be burdened with the presumption of reality, and Alexander and Caesar, for instance, who need very much be burdened with that presumption. On the other hand, dwelling though he is on those movements, in the 44 lives of those who need be burdened with that presumption Plutarch did not leave it to others to tell of their great deeds and battles. He did it himself.

In the generation before Bacon, Galileo, and Kepler, among their important predecessors – next to Viète, Gilbert, Tycho, and Bruno – is Bodin. Bodin wrote the *Method for the easy knowledge of history*, in 1566. Bodin writes: history has four parts, human, which is uncertain and confused, natural, which is definite but sometimes uncertain because it is concerned with matter, mathematical, which is certain because it is free of concern with matter, and divine, which is the most certain and by nature immutable (1).

This suffices to indicate the generality of Bodin's treatise, and hence the fact that with his concern with the method he anticipates Bacon and Descartes. In the *Method*, Plutarch is Bodin's second most important authority (*passim*), after Aristotle (*passim*). And in this Bodin is representative of the philosophers in the preceding century.

The *moralia* is important, if not very important. The 50 lives are an important, if not a very important, step in the development of history. Plutarch is also, like Strabo, a very important source and authority, both, on his predecessors.

Plutarch is important.

To ascertain the judgment of their successors in the cases of Philodemus, Herodian, Hippolytus, Gellius, and Empiricus is more difficult. Here as above, because neither are they as important as Laertius, Strabo, or Plutarch, nor are they altogether unimportant. Here as above as well, more difficult, but not too difficult.

Empiricus, in the 14 books of the *Outlines of Pyrrhonism* and *Against the mathematicians*, argues scepticism systematically. The development of scepticism from Pyrrho through Timon through Aenesidemus is important, and while Empiricus is not a further step in this development, he is the most important source for this development. Also, arguing against the dogmatics one dogmatic after another after another, he is an important source, if not an important authority, on the entire earlier development of philosophy. Empiricus is an important source of Hegel, too.

Empiricus is important.

Hippolytus wrote the 10 books of the *Refutation of all heresies*. He writes: we must not neglect the figments of those whom the Greeks call philosophers; we must examine them, because of the great madness of the heretics (I, pr.); for all the doctrines of the heretics are rooted in the wisdom of the Greeks and the doctrines of the philosophers (*id.*). Hippolytus is thus an important source on the figments.

Hippolytus is a younger contemporary of Clement of Alexandria and an older contemporary of Origen. One and a half centuries after st Paul, Clement of Alexandria began the assimilation of pagan philosophy into Christian doctrines, and thus the transformation of Christian doctrines into philosophy. In the next generation, Origen

began the subjection of scriptures to philological and antiquarian scholarship, wrote exegesis and commentaries, wrote *On the principles*, and began the transformation of Christian philosophy into orthodoxy. Hippolytus is a part of this very important development.

One great madness, rooted in the doctrines of Heraclitus, was that of Noetus (IX, 7). Another great madness, rooted in the doctrines of Pythagoras, was that of the Elcasaites (14). More important, to those two great madresses, and indeed to other great madresses as well, one after another the two bishops of Rome, Zephyrinus and Callistus, succumbed (11-2). As to Hippolytus: we resisted them, we did not permit that many shall fall victim to their delusions, and we had the people on our side (13). With Hippolytus thus begins the venerable tradition of schism in the see of Rome. The bishops of Rome will become the popes, and the venerable tradition of antipopes next to the popes will also be traced back to Hippolytus.

One century after Hippolytus, Eusebius lists half a dozen writings of his (*History*, VI, 22). As orthodoxy kept becoming ever less patient with great madresses, however, it turned out that Hippolytus's had been a great madness itself, and his writings fell into oblivion. The *Refutation of all heresies* was rescued from oblivion in 1842, when a copy was unearthed in a convent on mt Athos.

Hippolytus is an important, if not a very important, part of the early development of Christianity. He is also an important, if not a very important, source, and not an authority, on his predecessors.

Hippolytus is important.

Gellius wrote the 20 books of the *Attic nights*. He begins: in order to provide some amusement for my children, whenever a Greek or Latin book came into my hands, I jotted down what I liked, of all and any kinds, without any plan or order (pr. 1-2). What I like, of all and any kinds, without any plan or order, had by Gellius's time become a well-established genre. Gellius continues: unlike the many who have made such collections without any discrimination, however, I, having at heart the noble saying of the illustrious Ephesian, much learning does not teach sense,

wrote down only a few things and only such as can instill in a quick mind the desire to learn for itself (pr. 11-2). Gellius thus came close to achieving the impossible, to producing a respectable specimen of that genre.

These are the first three bits of what Gellius liked: how Plutarch says Pythagoras measured the height of Hercules (I, 1), how Herodes Atticus used a passage from Epictetus in reply to a boastful youth (2), how Chilon saved a friend, and what Theophrastus and Cicero say about friendship (3). Gellius is an important source on his predecessors. He is also, like Plutarch and Laertius, amusing.

Gellius is a minor authority of Bodin in the *Method* (1, 4, 6, 9), and Bodin is representative of the philosophers in the preceding century in this as well.

Gellius is important.

But if so, barely so. If Gellius is important, with him importance has to be very close to the line separating it from not importance.

Philodemus is noted once by Strabo, twice by Laertius. Writing on Gadara, Strabo notes: Philodemus the Epicurean was from Gadara (XVI, 2.29). Writing on Epicurus, Laertius quotes: the book of Philodemus the Epicurean on the philosophers (X, 3, 24). Cicero notes Philodemus (*Ends*, II, 119), and epigrams of his found their way in anthologies.

Philodemus's treatises, though, fell into oblivion. Then, in 1738 the excavation of Herculaneum, buried in 79 by the eruption of mt Vesuvius, began. In 1750 a villa was unearthed with a large library with a large number of charred papyri in it. In the following century the papyri were restored and published. Philodemus's treatises were thus rescued from oblivion. Fragments from a dozen treatises were recovered, and substantial parts from three treatises, *On signs*, *On rhetoric*, and *On piety*.

And if the record had been trying to consign Philodemus's treatises to oblivion, and with the recovery of the Herculaneum papyri was prevented from doing so, it is very hard to see that anything important has thus been achieved.

On signs defends inference by analogy. It is beginning to end: they say (*passim*); but we say (*passim*). They are the Stoics, we are the Epicu-

reans, and the quarrel over signs had been going on for a century. *On rhetoric* defends an earlier book against an unnamed but: very foolish (II, 2.b) opponent who failed to appreciate the earlier book's defense of the Epicurean tenet that rhetoric is an art. All three, *On signs*, *On rhetoric*, and *On piety*, and all fragments, are not only explicitly derivative, they are explicitly derivative not from the ultimate master, Epicurus, but from Philodemus's immediate master, Zeno of Sidon, who is himself derivative of the ultimate master.

Epicurus is very important, but the Epicureans are not. The longer the four schools, the Academy, the Lyceum, the Garden, and the Stoa, produced philosophers, the better they exemplified the fact that, with Bacon: in the schools everything was adverse to the advancement of knowledge. In the development of Epicurean doctrine, to the degree to which there was a development of Epicurean doctrine, Epicurus's student Metrodorus of Lampsacus can be judged important. Two and a half centuries later, Lucretius is important. In the intervening centuries, approaching perilously the line separating importance from not importance, Zeno of Sidon can be judged important.

Philodemus is not important.

Herodian wrote the 20 books of the *General prosody*, investigating the accentuation of 60 000 words. Against the better judgment of the record, parts of this have also been rescued from oblivion.

Herodian is not important.

To ascertain the judgment of their successors in the cases of the four scholiasts, finally, is trivial.

The scholiasts attest the fact that the record is not severe in its judgment, not severe at all. There is nothing in the record more than there are anonymous scholiasts, glossing the books, glossing the glosses, glossing the glosses of the glosses, and so on indefinitely.

The four scholiasts, the two on the *Iliad* and on the *Hecuba* noting Heraclitus noting Homer and Pythagoras, the one on the *Orestes* noting Hecataeus noting Hesiod, and the one on the *Peace* noting Xenophanes noting Simonides, are, like all scholiasts, with barely an exception, altogether unimportant.

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If importance is a fact, established in any particular case by the cumulative judgment of the philosophers, the longer the time of the accumulation in any particular case, the more likely it has to be that the judgment is conclusive. And whether the judgment has been explicit or implicit, and whether the accumulation has been explicit or implicit.

Thus, with respect to the princes of the Sumerian cities the judgment should in every particular case be conclusive. Even as the judgment and the accumulation have been through all the centuries when the philosophers were ignorant of the particulars of Mesopotamian antiquity only implicit.

With respect to the predecessors of the philosophers between Homer and Hecataeus the judgment should be conclusive as well. The judgment and the accumulation have in these cases been mostly explicit in the centuries of their more immediate successors, through Plato, Aristotle, Theophrastus, Eudemus, then mostly implicit as judgment and accumulation have been over their successors' explicit judgment of Plato, Aristotle, Theophrastus, Eudemus and hence over their judgment of their predecessors, and over their successors' successors' explicit judgment of the judgment of their successors, and over their successors' successors' successors' judgment.

With respect to the predecessors of the philosophers between Strabo and Laertius, too, the judgment should be conclusive.

With respect to the princes of the Sumerian cities the accumulation has been over 135+ generations, counting from Ibbi-Sin, with respect to the predecessors of the philosophers between Homer and Hecataeus it has been over 85+ generations, counting from Hecataeus, with respect to those between Strabo and Laertius it has been over 59+ generations, counting from Laertius. Thus quantitatively. Qualitatively, it has continued when philosophy and the acquisition of knowledge have flourished, when they have not flourished, when they have advanced greatly, when they have not advanced at all, when dark ages have flourished, when they have flourished greatly, and again, and again.

In the cumulative judgment of the philosophers, over time, there need be no error.

There may be errors in ascertaining the judgment of the philosophers.

Such errors need be neither many nor gross, however. The likelihood of such errors can be minimized, too. The more of the record is examined, the smaller the likelihood of error should be. It should be very small if the entire record is examined. And it should be infinitesimal if the record is examined again against its prior examination, and examined again against its prior examinations, and until all errors are eliminated.

Moreover, it can be indicated clearly where the errors, if there are errors, may be.

Importance, if it is many and complex relations of interdependence, cannot not admit of degree.

Hence, the dichotomy of important and not important can be reiterated. The important can be divided into very important and important if not very important. The not important can be divided into not important and very not important.

Further, given that errors can be either of omission or of commission, those of commission should be innocuous, those of omission the dangerous. Importance need not be impaired if a not important particular is judged important, but may be if an important particular is judged not important. If so, judgment should in every particular case be as permissive as possible. If there is an error, then, it will be in judging an important particular very important, a not important particular important, or a very not important particular not important, rather than the other way around.

Hence, in that the very important are important, and the very not important not important, the likelihood of error should be infinitesimal. Errors, if there are errors, should be confined to the particulars close to the line separating importance from not importance, and should be all on the side of importance.

Thus, when Bias is judged important, just, he may be not important. When Simonides is judged not important, the likelihood that he may be important should be minimal. When Gellius is judged important, just, he may also be not important. When Philodemus is judged not important, the likelihood that he may be important should be

minimal as well. When Herodian is judged not important, just, rather than very not important, this cannot itself be but very not important.

Collecting all 25 particulars, the 13 predecessors of theirs whom Xenophanes, Hecataeus, and Heraclitus note explicitly, and the 12 successors of theirs who note them noting their predecessors, the important are these 14, with the very important in bold,

764 **Homer**,
 734 **Hesiod**,
 653 Epimenides,
 625 **Thales**,
 622 Bias,
 570 - 495 **Pythagoras**,
 570 - 473 **Xenophanes**,
 562 **Hecataeus**,

65^{BC}- 29^{AD} **Strabo**,
 48^{AD}- 125 **Plutarch**,
 127 Gellius,
 159 Empiricus,
 170 - 235 Hippolytus,
 219 **Laertius**.
 The not important are these 5,
 688 Archilochus,
 556 - 466 Simonides,
 540 Hermodorus,
 110 - 37^{BC} Philodemus,
 121 Herodian.

And the very not important are these 6, Teutames, Mnesarchus, scholiast on the *Iliad*, scholiast on the *Hecuba*, scholiast on the *Orestes*, scholiast on the *Peace*. Nor need these even be arranged in their series and order.

The formal convocation of the council of our predecessors

ccxxiii

Philosophy was young. It was making its first steps. It did not quite know what it was saying. It was fighting the way untrained men fight. It was stammering, babbling, and prattling.

The difficulty with the organic simile is that it is not picturesque enough. Is philosophy like a man? or an ass perhaps? a sparrow? a shark? an oak? a butterfly? a fungus? a paramecium? In any case, who were its mother and father? or where did the egg come from? or the seed? or the spore? or was it just fission? but where did the mother cell come from then? In any case, too, what is its life span? Older as it was with Aristotle, was it not still stammering, babbling, and prattling? for that matter, is it not still stammering, babbling, and prattling?

The difficulty with the martial simile is the same. What are the men training for? war? boxing matches? brawls? In any case, to fight singly? or in armies? with fists? or with aircraft carriers? And one way or another, who is the enemy?

All these questions can be answered. They do not have to be, and the answers will spoil the similes, if they are. But they can be answered,

one and all, because what is philosophy, what is not philosophy, what philosophy is, what philosophy is not, are one and all matters that have been examined so assiduously by the philosophers that it is not difficult to answer, one way or another, any question involving philosophy.

Philosophy posed the questions of what is it, what is not it, what it is, what it is not, as soon as it appeared. Not knowing yet what to call itself, it was our wisdom and good wisdom, the best, the wise, and the few, and also much learning which does not teach sense, a sham, and cheating. And what is it, what is not it, what it is, what it is not, have occupied Plato and Aristotle, Epicurus and Zeno of Citium, Cicero, Augustine, al-Farabi and al-Ghazali, Abelard, Aquinas and Occam, Bacon and Galileo, Descartes and Leibniz, Hume and Kant, Hegel, Marx.

Here are two more recent illustrations.

In 1914, in *Our knowledge of the external world as a field for scientific method in philosophy*, Russell begins: philosophy has from the earliest times made greater claims and achieved fewer results than any other branch of learning; ever since Thales said that all is water, philosophers have been ready with glib assertions about

the sum-total of things, and equally glib denials have come forward from other philosophers ever since Thales was contradicted by Anaximander (1).

Needless to add, a philosopher himself, Russell glibly continues: I believe that the time has now arrived when this unsatisfactory state of things can be brought to an end (*id.*).

In 1955, *What is it – philosophy?* Heidegger asks, and replies: philosophy is φιλοσοφία (*id.*); φιλοσοφία goes back to φιλόσοφος; and Heraclitus, presumably, coined φιλόσοφος (*id.*); but Heraclitus and Parmenides were not yet “philosophers”; why not? because they were the greater thinkers; and this is not an assessment of performance; it rather points to another dimension of thinking, to being in harmony with λόγος, and that is, with ἐν πάντα (*id.*); the step toward “philosophy” was prepared by the sophists, and was taken by Socrates and Plato (*id.*); with Descartes certainty became the standard of truth; and the mood of confidence in always obtainable absolutely certain knowledge remains the πάθος of modern philosophy (*id.*); but the expressly adopted and unfolding itself correspondence, corresponding to the call of the being of being, is philosophy (*id.*).

Needless to add, Heidegger is himself not a “philosopher”, and not in the mood of confidence in always obtainable absolutely certain knowledge. Why not? Because he is expressly adopting the unfolding itself correspondence, corresponding to the call of the being of being.

With both Russell and Heidegger, then, with both a “philosopher” and a greater thinker, both glibly asserting and glibly denying, as with Xenophanes and Heraclitus, knowledge and philosophy are involved with one another. Also, whatever is knowledge, and whatever knowledge is, what is philosophy is philosophy, and what philosophy is philosophy is, from the earliest times. Since Thales and Anaximander, and Heraclitus and Parmenides. And whether, with Heidegger, Heraclitus coined φιλόσοφος, or, pace Heidegger, Pythagoras coined it.

To complete establishing its ground, the collection of their predecessors and the philosophers, and the separation within it of those who attest the acquisition of knowledge from those

who do not, should turn next to Aristotle.

And before that, before collecting and sorting out the particulars from Aristotle, it can be shown how, two and a half centuries after philosophy was born, or hatched, or sprouted, or moulted, or developed from a spore, or split from a mother cell, Aristotle answered the questions of what is philosophy, and what philosophy is, all appearances to the contrary, conclusively.

ccxxiv

To begin with the words. Because philosophy is φιλοσοφία, φιλοσοφία goes back to φιλόσοφος, words are λόγοι, and λόγοι are always exciting.

Φιλόσοφος enters the record with this fragment of Heraclitus:

many things must research loving wisdom men (fr. 35).

Loving wisdom is φιλόσοφος, φιλόσοφος is philosophical, and philosophical thus enters the record as a term of abuse. Of abuse of Pythagoras, in particular.

There is no context. But there is no doubt that the fragment refers to Pythagoras.

Because Pythagoras researched things more than anyone and made himself a wisdom, much learning, a sham, much learning did not teach Pythagoras sense, and Pythagoras was the prince of cheats.

And because the successors of Heraclitus are unambiguous in ascribing the coining of φιλόσοφος to Pythagoras.

One and a half centuries after Heraclitus, in the generation between Plato and Aristotle, Ponticus wrote the first history of Pythagoras and his followers (Laertius, V, 88). And on the authority of Ponticus, Cicero reports: Pythagoras came to Phlius once, and it happened that the tyrant of the city, Leon, engaged him in a conversation; at length, amazed at his ingenuity, Leon asked Pythagoras what his art is; he has no art, Pythagoras replied, he is a philosopher; amazed now at the word he had never heard, Leon asked what a philosopher is; life is like the Olympic games, Pythagoras replied, some go there to compete, some to make money by selling and buying, some to just watch what and how is done, and those last, a very few, call themselves lovers

of wisdom (*Tusculan*, V, 8-9).

Also on the authority of Ponticus, Laertius reports that Pythagoras's reply to Leon was: he calls himself lover of wisdom because no one but the god is wise (I, 12).

Lover of wisdom is also φιλόσοφος, φιλόσοφος is philosopher, and that Pythagoras did not coin the word to abuse himself, the moral of the story shows, in either Cicero's or Laertius's version. And the philosophers, beginning with Plato, not prone to abusing themselves, have been fond of the moral of the story, very fond of it in Laertius's version.

Four generations separate Heraclitus and Plato. In the first three of these the word remained very rare.

Two generations after Heraclitus, Zeno of Elea wrote *Against the philosophers*. This the *Suda* reports (*s. v.*), thirteen-odd centuries after the fact. Not himself a Pythagorean, Zeno wrote against the Pythagoreans, presumably.

In the same generation Herodotus wrote the *Histories*, or what is the same, to go by the word, ἱστορίαι being researches, the *Researches*. It is not the first book. But it is the first book of researches. It is not the first book from which facts can be recovered. But it is the first book which is itself the ground of the recovery of facts. It is a book essentially and substantially of facts, and of facts of great and magnificent quantity and quality. So much so, it is a test. If it is great and magnificent, Herodotus has noted it, if Herodotus has not noted it, it is not great and magnificent.

And in Herodotus but once a cognate verb occurs. Solon came to Croesus once, and the king said to him: my Athenian friend, I have heard much about your great wisdom and wide travels; loving wisdom, they say, you have traveled much of the earth looking for it (I, 30).

To love wisdom is φιλοσοφῶ, wisdom is σοφία, and while φιλοσοφία does not occur in the *Histories*, σοφία is common. Herodotus notes also Pythagoras. He writes: Pythagoras was not the least of the wise men among the Greeks (IV, 95). Wise man renders σοφιστής, and this is also common. To go by the words, then, Solon was great and magnificent, Pythagoras was great and magnificent, and the words σοφιστής and σοφία were great and magnificent. But the words φιλό-

σοφος and φιλοσοφία were not yet great and magnificent.

They became great and magnificent two generations later, with Plato.

It is in these two generations that the Greek record becomes voluminous. From this time there is Thucydides's *History*. Then there is the corpus of Lysias's 30-odd speeches. Then there are the corpora of Isocrates's 20-odd discourses, of Xenophon's dozen-odd treatises, and of Plato's two dozen-odd dialogues. There are also numerous fragments from other philosophers and other historians. From this time as well are the earliest treatises in the Hippocratic corpus. There are all the poets, too, tragic, comic, lyric.

The verb, φιλοσοφῶ, occurs in Thucydides (II, 40). It occurs also in Lysias (VIII, 11). All three, φιλόσοφος, φιλοσοφία, and the verb, are common, but not frequent, in Isocrates. They are very infrequent in Xenophon. In Plato they are very frequent, increasingly frequent, and as frequent as any words in the later dialogues, in the *Gorgias*, *Phaedo*, *Phaedrus*, *Republic*, *Theaetetus*, *Sophist*, *Statesman*. These are Plato's words.

Thus the words.

As to their references, the rule is Frege's: about the reference of words must be asked in their connection in propositions, not in their isolation.

Also with Frege, the reference must be distinguished sharply from the sense. The reference of a proposition is the true or the false. The sense of a proposition is the thought. The references of the words are the objects themselves, whose relations in propositions are the true or the false. The senses of the words are saturated with the ideas and feelings which thoughts would arouse.

The reference must be distinguished sharply from the sense, because: given a sense, it does not follow that a reference corresponds to it.

And it can be concluded that when 'philosopher' was coined it referred to Pythagoras. The sense varied, from a prince of cheats to a lover of wisdom. Then the sense broadened, broad as it had been from the first. Then in Plato 'philosopher' and 'philosophy' acquired the boundless sense they have since had, but began acquiring their references as well, philosopher and philoso-

phy, respectively. And then Aristotle fixed the references and for good.

To show this, the connection of words in propositions in Herodotus can be shown, then the connection of words in propositions in Plato, and then the connection of words in propositions in Aristotle.

ccxxv

Xenophanes writes: better than the strength of men and horses is our wisdom; it is not just to put strength above good wisdom.

Wisdom is σοφία.

One generation later, Heraclitus writes: one, the solely wise, will and will not be called Zeus; one is the wise, to know how all things are steered through all things; the wise is set apart from all things. Also: Homer, wiser than all Greeks, was deceived by boys; Pythagoras researched things more than anyone and made himself a wisdom, much learning, a sham. And: many things must research loving wisdom men.

The wise is τὸ σοφόν, wiser is σοφώτερος, wisdom is σοφία, loving wisdom is φιλόσοφος.

The words are related, σοφία, σοφός, φιλόσοφος. And given the other propositions in which Pythagoras occurs in Heraclitus, it can be concluded that 'philosopher' refers to Pythagoras. But if it refers to no other object, it cannot be clear what kind of object Pythagoras is. If 'wiser than all Greeks' refers to Homer, given the connections of the words, it can be concluded that the references of the two, 'philosopher' and 'wiser than all Greeks', not just overlap, but are the same. It is then clearer what kind of objects the two, Pythagoras and Homer, are. But not too clear. The lover of wisdom is the prince of cheats, his wisdom is much learning is a sham, the wiser than all Greeks was deceived by boys, all well saturated with ideas and feelings. Clearly, Xenophanes and Heraclitus disapprove of these objects. On the other hand, however, wisdom is ours, it is good, it is not the strength of men and horses, the wise will and will not be called Zeus, it is to know how all things are steered through all things, it is set apart, all well saturated with ideas and feelings as well. Clearly, Xenophanes and Heraclitus approve of wisdom. And given that 'philosopher', or synonymously 'wiser than all

Greeks', is a term of abuse, it is not too clear what kind of object it abuses.

Two generations after Heraclitus, the words are common in Herodotus. There are more connections of the words in propositions. And here are 8 of these connections, in the chronological order of the objects to which the words are related.

Herodotus writes:

Melampus introduced Dionysus among the Greeks, the name, the sacrifices, and the phallic procession; not all of the cult, for other wise men advanced it further after him; but I think that Melampus was a wise man who devoted himself to divination and, having learned them in Egypt, introduced many new things to the Greeks, including the worship of Dionysus (II, 49).

Melampus is σοφός, those who advanced the cult further are σοφισταί. The time of Melampus, and rather timelessness, is established by the fact that he is a seer in Homer already (*Odyssey*, XI, 291).

Next:

at the time of Croesus all wise men came to Lydia on one occasion or another; and so did Solon, who had made laws for the Athenians and had then gone abroad for ten years, to avoid, he said, being forced by the Athenians to change the laws; so on his way back home from Egypt, Solon came to Lydia, Croesus entertained him in his palace, and said to him, my Athenian friend, I have heard much about your great wisdom and wide travels; loving wisdom, they say, you have traveled much of the earth looking for it; so let me ask you, have you seen someone who is the most fortunate of all (I, 29-30)?

All wise men who came to Croesus are σοφισταί, what Solon has is σοφία, and what he does is φιλοσοφείω. Solon was archon in 594. Also, Solon does have an answer to that question: Tellus the Athenian is the most fortunate of all (*id.*).

Next:

Cleisthenes of Sicyon wanted to find the best man for his daughter, Agariste; he made a proclamation throughout Greece that whoever wanted to be his son-in-law should come to Sicyon; thirteen suitors came; for one year Cleisthenes entertained them, watching their valor, temper, education,

and character; he liked Hippoclidides best; but on the night Cleisthenes was to announce his decision, Hippoclidides so far excelled all others in drinking and dancing that Cleisthenes could not stand his revelry any longer and said, you danced your marriage away; Hippoclidides said, nor does Hippoclidides care (VI, 126-9).

The thirteen suitors were men of distinction. One was: the brother of Titormus, who excelled all Greeks in strength (*id.*). Another was: the son of Pheidon, the king of Argos, who introduced weights and measures in the Peloponnese (*id.*). Another was: the son of Amyris, who was called the wise (*id.*). Amyris was called ὁ σοφός. Cleisthenes was tyrant of Sicyon c. 585. Also, he gave Agariste to Megacles, the Athenian Cleisthenes was their son, and Pericles was their great-grandson.

Next:

the deposed Spartan king Demaratus advised Xerxes during his expedition against the Athenians and the Spartans; after Thermopylae Demaratus advised the great king to blockade the Peloponnese with his navy; he told the king, Chilon, the wisest among us, said that the Spartans should rather drown at sea than float on it (VII, 235).

So the Spartans did not build a navy. Chilon is σοφώτατος. He was ephor in 556. Also, Herodotus does not mention the entity of the seven wise men, but he notes five of the seven, such as their list is in Plato. Other than Solon and Chilon, he notes Pittacus, Thales, and Bias.

Next:

the Getae believe that they are immortal, they say that they do not die but go to Zalmoxis; and I heard from some Greeks that, when he was a man, this Zalmoxis was a slave of Pythagoras, who was not the least of the wise men among the Greeks (IV, 94-5).

Pythagoras is, like those who advanced the cult of Dionysus and all wise men who came to Croesus, σοφιστής. Pythagoras moved to Italy when Asia Minor fell to Cyrus in 546. Also: the Getae live hard, and are rather stupid (*id.*).

Next:

the wisest custom of the Babylonians is, I think, this; once a year, in every town, they would gather all girls ripe for marriage and a herald

would put them up for sale; he would put up first the best-looking, the men who wanted to marry her would bid, then he would put up the second best, and so on; when he had gone through the good-looking, he would put up the worst-looking and ask, who will take the least money to marry her? they would bid, and the money for her would come from that paid for the good-looking girls; then he would put up another, and so on (I, 196).

The custom is, like Chilon, σοφώτατος. Sadly: the custom lapsed after the fall of Babylon (*id.*). Babylon fell to Cyrus in 539.

Next:

after the death of Cambyses, seven Persian nobles gathered to decide what form of government to institute, democracy, aristocracy, or monarchy; Darius argued for monarchy, and won the debate; then the seven decided that the next day, as the sun rose, whoever's horse neighed first shall be king; now Darius had a squire, Oebares, who was a wise man; there was a mare the stallion of Darius loved best, Oebares tied this mare outside the city, and led the stallion around her for a long time before letting him mount her; next morning the nobles came out of the city to meet the rising sun, and when they came to the place where the mare was tied, the stallion of Darius plunged forward and neighed; there was at the same time a flash of lightning; the others jumped from their horses and did obeisance to Darius (III, 80-6).

Oebares is σοφός. Darius became the great king in 522.

Finally:

the Athenians thought Themistocles wise before the victory at Salamis already, after the victory they thought him the wisest (VIII, 110); indeed, all Greece thought him the wisest of all Greeks; so the Spartans gave the victor's prize to the Spartan Eurybiades, but gave Themistocles the finest chariot for his wisdom (124).

Themistocles is σοφός, then σοφώτατος, and the finest chariot is the prize for σοφία. Themistocles won the battle at Salamis in 480.

Much digging, little gold. So also, much sense, little reference. But there is more of it now.

If, for all the abundance of sense, 'wise', 'the wisest', and 'loving wisdom' have a reference each, the reference of 'wise' is Melampus, those

who advanced the cult of Dionysus further, all wise men who came to Croesus, Amyris, Pythagoras, and Oebares. The reference of 'the wisest' – and whether among the Spartans, the customs of the Babylonians, or all Greeks – is Chilon, the custom of putting up all girls ripe for marriage for sale, and Themistocles. The reference of 'loving wisdom' is Solon.

The object Melampus is hampered by the lack of reality. The objects those who advanced the cult of Dionysus further and all wise men who came to Croesus are hampered, and the object the custom of putting up all girls ripe for marriage for sale is hampered severely, by the lack of particularity. The object Amyris, even if it is real and particular enough, by being not more than the father of a suitor of a daughter of Cleisthenes of Sicyon, is hampered by insignificance. The remaining 5, however, are fine objects, objects of some number, and connected through the propositions. They are connected also with the propositions in Xenophanes and Heraclitus.

Given, then, that the reference of 'wise' is Pythagoras and Oebares, of 'the wisest' Chilon and Themistocles, and of 'loving wisdom' Solon, vague as it still is what kind of objects these are, it is less so. They indicate, both, that the reference of 'philosopher' will be fixed, and the direction in which it will be fixed.

Pythagoras taught Zalmoxis immortality. Oebares won his kingdom for Darius by a stratagem. Chilon gave the Spartans the advice that made them preeminent. Themistocles won the battle at Salamis by a grand stratagem. Solon made laws for the Athenians, traveled much of the earth, and knew who is the most fortunate of all.

Vaguely as it is, the objects hampered by indefiniteness indicate the same direction.

And vaguely as it is, the direction is the same as it is in the propositions of Xenophanes and Heraclitus. Teaching, learning, much learning, sense, wisdom, knowledge.

Frege writes: the striving for truth is what forces us to advance from the sense to the reference.

This is more clearly indicated in Herodotus. In that wisdom is ascribed to more particulars. In that it is ascribed more clearly to the labors of

each. And in that there is a gradation in the judgment of the labors, wise, the wisest, loving wisdom. The ground on which it can be presumed that the reference of 'philosopher' and the reference of 'the wisest' is the same is firmer in Herodotus than it is in Xenophanes and Heraclitus.

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The striving for truth which forces us to advance from the sense to the reference is indicated unambiguously in Plato.

To begin with, his treatises can be compared with those of Xenophon.

Xenophon is a contemporary of Plato, but a couple of years older. Like Plato, he is a student of Socrates. Like Plato, he is a philosopher, both are among the 102 under his own head each in Laertius. Like Plato, he has left behind many treatises, and the concerns of his treatises are in many ways the same as those of Plato's treatises.

On the other hand, Xenophon is not one of the most important philosophers, Plato is. One of the reasons is that Plato endowed philosopher and philosophy with the boundless sense they have since had. Another is that Plato's is the decisive step toward fixing their references.

The corpus of Xenophon includes the following. The *Greek history*, in which Xenophon continues Thucydides, recording events from 411 to 362. *The march of Cyrus*, in which Xenophon records the attempt of the pretender Cyrus, brother of the great king Artaxerxes II, to seize the Persian throne in 401, in which Xenophon took part as one of the leaders of the 12 900 mercenaries in Cyrus's army (I, 7). *The education of Cyrus*, in which, having concluded at first that it is easier for a man to rule cattle than men, and having then considered the great king and changed his mind, Xenophon undertakes: to study the origin, nature, and education of Cyrus (I, 1). The *Agesilaus*, in which, under the name of the Spartan king, Xenophon provides a list of the virtues of the leader of the city. The *Hiero*, in which, under the name of the tyrant of Syracuse, Xenophon extols the pleasures of the citizen against those of the tyrant. The *Apology*, in which Xenophon describes Socrates defending himself at his trial, and the *Memorabilia*, *Home manage-*

ment, and *Symposium*, in which he describes Socrates discussing virtue after virtue after virtue, and whether they are knowledge: tell me, Critobulus, is home management the name of a kind of knowledge, like medicine, smithery, and carpentry? I think so, Socrates, Critobulus replies (*Home*, 1). The *Spartan constitution*, in which Xenophon extols the virtues of that constitution. The *Revenues*, in which Xenophon proposes: a plan to relieve Athens of its poverty (1), the linchpin of the plan being: rather than hire out the public silver mines, the city should buy 1 200 slaves to work the mines, and gradually increase their number to 6 000, then 10 000, and so on, until there are three public slaves at work for every citizen (4). The *Cavalry commander*, in which Xenophon again concerns himself with the relevant virtues, yet the *Cavalry commander* is also the first treatise concerned specifically with the conduct of war. And *On horsemanship* and *On hunting*, in which Xenophon once again concerns himself with the relevant virtues.

Plato did not continue Thucydides, did not take part in an attempt to seize the Persian throne, and neglected horsemanship and hunting. Other than that, in their concern with education, with whether it is easier to rule cattle than men, with the virtues of the leader of the city, with the pleasures of the tyrant and with how inadequate they are, with Socrates's defense at his trial, with Socrates's virtues and with those of his disquisitions, with virtue after virtue after virtue, and whether they are knowledge, like medicine, smithery, and carpentry – and I think so, Socrates – with Sparta and the virtues of its constitution, with Athens, with its misfortunes, and with relieving them, and with the virtues of the commander of the philosophers, if not with those of the cavalry commander, Plato's treatises are hard to distinguish from Xenophon's. Had philosophy been but ethics and politics, had it been but practical, Xenophon may well have been as important a philosopher as Plato. In fact, in times when philosophy has been predominantly ethics and politics, concerned predominantly with the search for action, the philosophers have concerned themselves with Xenophon not less than with Plato.

Plato's treatises can be divided into early,

middle, and late. Ignoring the dozen or so spurious.

And Plato's early treatises are but ethics. In the *Apology* Socrates defends himself at his trial. In the *Crito* Socrates is in prison, condemned to death, and Crito urges him to escape, but the laws of Athens urge him to obey the laws. In the *Euthyphro* Socrates asks Euthyphro: what is piety? In the *Laches* he asks Laches: what is courage? In the *Lysis* he asks Lysis: what is friendship? In the *Charmides* he asks Charmides: what is temperance? In the *Greater Hippias* he asks Hippias: what is beauty? In the *Lesser Hippias* he asks Hippias: who is the better man, Achilles or Odysseus? In the *Ion* he explains to Ion, himself a rhapsode, that the rhapsode does not know what he says about Homer, what he says is inspired.

Had Plato left behind but his early treatises, he would not have been a more important philosopher than Xenophon. He would have been a less important philosopher. The *Greek history*, *The march of Cyrus*, and the *Cavalry commander* are a more important treatise each than any of the early Platonic dialogues.

The words, philosopher and philosophy, are common in these treatises. The objects, however, to which the words would refer, if indeed they have references, have not yet become an explicit concern for Plato. Thus, Socrates, too, is not yet so much a philosopher as he is the wisest. In the *Apology*, for instance, to the Athenians, as they are about to condemn him to death, Socrates reports: you know my friend Chaerephon; he once went to Delphi and ventured to ask the Pythia, is anyone wiser than Socrates? and the Pythia replied, no; so I wondered what the god meant; I began to examine my fellow citizens to find out whether they were wise; and I found out that they were all lacking in wisdom; I knew that I was lacking in wisdom myself; but they did not know that they were; I knew that I was; and I then understood the sign; only the god is wise, man's wisdom is of little or no account (20e-3b).

It is in the following treatises, the middle, that the philosopher and philosophy became not only an explicit but a most fundamental concern for Plato, and it is in these treatises that he endowed the philosopher and philosophy with the bound-

less sense they have since had.

Like the early, these treatises are Socratic dialogues as well. But this is more a matter of style now than of substance. In the early dialogues, those to whom Socrates talks are lacking in any wisdom whatsoever. It is beginning with the two *Hippias* and the *Protagoras* that Plato gives someone other than Socrates, Hippias and Protagoras, in particular, a chance, even if it is not yet a very fair chance, to present what he takes to be wisdom before having Socrates explain to him that it is not wisdom. More important, Plato's concerns are now much more general than this virtue or that, and much more substantial.

In the *Protagoras* Plato discovered the sophists, and discovered that: the sophists are peddlers of nourishment for the soul, and if you do not know what you are buying, they may well deceive you. It is not only, then, that those other than Socrates are lacking in wisdom. It is that some of them have made themselves a wisdom, a sham. There are many of them, too, they have a leader, Protagoras, and since you do not know what you are buying, they will deceive you. The danger is very great, for: teaching you cannot take home in a jar, you are stuck with it in your soul. The sophists are the enemy, then. And they are an enemy, and it is a war, and Plato will continue to wage the war against the sophists in a dialogue after a dialogue after a dialogue. And Plato will win that war. More important, in the *Protagoras* Plato has generalized the concern with virtue. The question now is: can virtue be taught (320a-b)? and Plato does not have Protagoras say, yes, Socrates, or, no, Socrates, he has him defend at some length what he takes to be wisdom before having Socrates explain to him that it is not wisdom. More important still, what Socrates explains is grounded explicitly in the distinction of becoming and being.

In the *Meno* the concern with virtue is generalized further, the question is: what is virtue? More important, Plato has discovered the conundrum: one cannot search for what one knows, for he already knows it, and cannot search for what he does not know, for how will he search without knowing what to search for. More important still, Plato has solved the conundrum: the soul is

immortal; there is nothing it has not seen in this world or the netherworld, there is nothing it does not know; all learning and discovery is recollection. Hence, yet more important, it is in the *Meno* that Plato first affirms: knowledge is not belief. And with the doctrine of recollection he has taken the decisive step toward the theory of the forms.

In the *Euthydemus* the concern with virtue is generalized again. The question is: are Euthydemus and his brother Dionysodorus teaching philosophy and virtue not only to those who are persuaded that they should learn philosophy and virtue from them, but to those also who are not, and either because they doubt that philosophy and virtue can be taught or because they doubt that Euthydemus and Dionysodorus can teach them (274d-5a)? Euthydemus and his brother Dionysodorus are then annihilated. But the generalization is not in this success in the war against the sophists. The generalization is in the presumption that the teaching of virtue is the teaching of philosophy. And Plato will never abandon this presumption.

In the *Gorgias* the question is: if Gorgias is an orator, as he claims to be, what is it that he does (449a-b)? The answer is: oratory is flattery; it is like cosmetics and sophistry; and it is, like them, shameful (463a-d). The *Gorgias* is thus also negative. It annihilates Gorgias, however, and it is thus not a minor success in the war against the sophists, like the *Euthydemus*, it is a major success, like the *Protagoras*, for if Protagoras is the enemy's leader, Gorgias is the second-in-command. The negative result: oratory is shameful, also implies an affirmative result, and Plato will make this result more explicit in the *Phaedrus*: no one can speak well on any matter unless he has pursued philosophy properly (261a).

In the *Menexenus*, accordingly, Plato has Socrates deliver an eulogy on the dead in the Corinthian war (236d-49c), in 387. There was the eulogy Pericles delivered on the dead early in the Peloponnesian war, in 430 (Thucydides, II, 35-46). Clearly, having pursued philosophy properly, the philosopher can speak on the dead better than Pericles. There was also Socrates's younger contemporary Lysias, and Lysias was a great orator. In the *Phaedrus*, accordingly, Plato introduces a speech of Lysias on love (230e-4c), then

has Socrates deliver not one better speech on love, but two better speeches on love (237c-41d, 244b-57b).

In the *Phaedo* Plato argues that the soul is immortal. He presents four arguments, other than having Socrates drink the hemlock. More important, whatever the practical considerations demanding the doctrine of the immortality of the soul, theory, the search for truth, also demands the doctrine. Plato has established that knowledge is not belief. Hence the difficulty: how does one search for knowledge? And hence the resolution of the difficulty: knowledge is recollection. Knowledge, such as it is not belief, demands the doctrine of recollection, and the doctrine of recollection demands the doctrine of the immortality of the soul. Hence, most important, knowledge, such as it is not belief, demands also the theory of the forms. And in the *Phaedo* Plato develops the theory: the equal itself, the beautiful itself, and every thing itself, are the real, every thing itself is uniform, remains always the same, and never in any way admits of any change whatsoever; the equal itself is not among the sensible things; there are two kinds of essence, the sensible things and the not sensible things; if there is anything beautiful among the sensible things, it is beautiful only because it participates in the beautiful itself; and all things that there are are by participation in these realities, the equal itself, the beautiful itself, the good itself, and all other things of this kind.

In the *Symposium*, on the example of the beautiful, Plato describes the ascent of the soul from the sensible things to the forms, or what is the same, the acquisition of knowledge: one goes always upward for the sake of the form of beauty; beginning from beautiful things and using them like a ladder, he goes from one body to two bodies, and from two bodies to all beautiful bodies, then from beautiful bodies to beautiful laws, then from laws to learning beautiful things, and then from this lesson to the final lesson, to the end, which is beauty itself, and he then knows what is the beautiful (211c).

In the *Phaedrus* Plato describes in more detail the end of the ascent, or what is the same, of the acquisition of knowledge: beyond the heaven there is a realm where what is is without color,

without shape, without solidity, a being which is what it is; and this being is the object of all true knowledge, accessible to the mind alone, the pilot of the soul. Plato describes also how: the pilot, in a chariot drawn by two horses, one good, one bad, journeys to that realm beyond the heaven to catch a glimpse of the being which is what it is.

And in the *Republic* Plato brings it all together and develops it into one invincible whole. He has established that there are two kinds of essence, the sensible things and the not sensible things. He has established that the not sensible are the object of all true knowledge, accessible to the mind alone. He has established that one goes always upward for the sake of the not sensible. And he now completes the picture. He reiterates twice the dichotomy of sensible and not sensible: the sensible things are images of bodies and bodies; the not sensible things are the triangles and the like which the mathematicians study and the forms; this is also their gradation from the most obscure to the clearest, from images to bodies to the objects the mathematicians study to the forms (VI, 509d-11e). And one thing is indeed the clearest: what in the realm of the visible things the sun is to sight and the visible things, in the realm of the intelligible things the good itself is to the mind and the intelligible things (508b). It follows: the most important thing to learn is the form of the good and that it is by their relation to the form of the good that things are beneficial; if we do not know this, even the most perfect knowledge of anything else is of no use to us (505a). It follows further: it is the form of the good that gives, both, truth to what one knows and one's power to know it; although an object of knowledge, the form of the good is also the cause of knowledge and truth; it is thus right to say that knowledge and truth are good, but wrong to say that either of them is the good; the good is higher (508e-9a). This is the ultimate theoretical result. The ultimate practical result follows: men are fettered in a cave (VII, 514a); nor do they want to be dragged out of the cave (515c-e). Yet: our task is to compel the best among them to make the ascent out of the cave to the good (519c). And in the rest of the *Republic* Plato accomplished the task. He showed what justice is, and ought to be. What the city is, and ought to be. What education

is, and ought to be. And the subjugation of knowledge and truth to the good, and the assimilation of ethics, politics, and education into one another, and all into the good, for if we do not know the good, even the most perfect knowledge of anything else is of no use to us, has since been the unconditional foundation of invincibility. It is in Augustine later on. Later still in Hegel.

Had Plato left behind only his early and middle treatises, and not the late, he would have still been one of the most important philosophers. Because of all the doctrines above.

And because it is in these treatises that he made the philosopher and philosophy the preoccupation of the philosopher and philosophy the philosopher and philosophy have since been.

First of all, Plato discovered that there is an enemy, and the danger the enemy poses is very great.

The enemy Plato discovered, the sophists, Plato annihilated. Plato discovered also, fortunately, that the enemy is everywhere.

He writes in the *Protagoras*: the art of the sophists is very ancient; the men who practiced it in ancient times, however, fearing the odium attached to it, disguised it sometimes as poetry, as Homer, Hesiod, and Simonides did, sometimes as mystery or prophecy, as Orpheus and Musaeus did, sometimes even as athletics, as Iccus and Herodicus did; Agathocles, a great sophist, disguised it as music, and so did Pythocles.

Never mind Protagoras and Gorgias. Never mind Euthydemus and his brother Dionysodorus. Agathocles and Pythocles are sophists. Nay, Iccus and Herodicus are sophists. Nay, Homer, Hesiod, and Simonides are sophists. Nay, Orpheus and Musaeus are sophists. The enemy is *ad hoc*, at will, and at the will of the philosopher. And the philosopher has never since lacked in an enemy. And having an enemy has never since failed to provide the philosopher with the goal of being the philosopher.

If the enemy is everywhere, lurking in music and athletics and mystery and prophecy and poetry, the challenge before the philosopher may appear frightening. But it does not frighten the philosopher. The philosopher is himself everywhere.

Also in the *Protagoras*, Plato writes: philoso-

phy had its first beginning among the Cretans and the Spartans; and philosophy is still practiced most among them, and it is they that have the best education in philosophy; indeed, it is a great secret, though well known to the wise, that the true virtue of the Spartans is not fighting prowess but wisdom; indeed, to be a Spartan is to be a philosopher more than it is to be a fighter, as the sayings of the Spartans show; because to put things into a brief saying which men remember is the mark of the man who knows; as it was the mark of the seven wise men, Thales, Pittacus, Bias, Solon, Cleobulus, Myson, and Chilon; their Spartan wisdom is shown by their sayings, know yourself, for instance, and, nothing too much; and their sayings, now on everybody's lips, the wise men dedicated to Apollo at Delphi (342a-3c).

If the Cretans and the Spartans are philosophers, indeed they have the best education in philosophy, and the seven wise men are philosophers, clearly Socrates can recruit in the *Republic* in his own army not only two of the seven wise men but also the sophist disguised as a poet Simonides, and in his enemy's army whoever he will: are we, Socrates asks, to join ranks with the wise and blessedly happy Simonides, Pittacus, and Bias, and fight against Periander, Perdiccas, Xerxes, and Ismenias? And, yes, we are.

The philosopher is also better than the orator. In the *Phaedrus*: no one can speak well on any matter unless he has pursued philosophy properly.

The philosopher is also better than the king. In the *Republic*: until the philosophers become kings or the kings become true philosophers, the states will have no respite from evil.

More generally, and most generally, in fact, the philosopher is better than anyone. Also in the *Republic*: men are gold, silver, and iron; gold is the philosopher; hence, it is his due, the philosopher's, to distinguish gold, silver, and iron, to educate the gold into himself, into the philosopher, and to educate everyone else into his due, including most particularly that it is his due to be educated and ruled by the philosopher.

It may appear that there is a difficulty in that you may not want to be silver or iron. You may want to be the philosopher. But it is not a difficulty, not really. If you want to be the philoso-

pher, you can be the philosopher. Plato has indicated the key in the *Protagoras* already. It is the wise saying, know yourself.

In the *Phaedrus*, before he tells you about the realm beyond the heaven where your soul can catch a glimpse of the being which is what it is, Plato has Socrates tell you: know yourself, the Delphic inscription orders, so I saw that it is ridiculous for me to search for other things before I know myself (229e-30a).

It may appear to you that other things are beautiful, so you would rather search for them than know yourself. But this is not a difficulty either.

In the *Theaetetus*, before turning to the investigation of knowledge, Plato has Socrates reassure you: I am barren of wisdom myself, but those who associate with me I help discover many beautiful things within themselves.

And associating with Plato's dialogues will do just as beautifully.

Thus, there has since been no end, nor can there be an end, to the beautiful things you, and I, and everyone, and anyone, have discovered, and will continue to discover, within myself. That it is all philosophy, *ad hoc*, at will, and at anyone's will, goes without saying.

Or what is the same, the sense of philosopher and philosophy has since been boundless, boundlessly saturated with any ideas and any feelings any thoughts would arouse.

Nor have you, or I, or everyone, or anyone, suffered unduly from the striving for truth which forces us to advance from the sense to the reference.

With these treatises of his, in general, and with what he does in them to the philosopher and philosophy, in particular, Plato illustrates paradigmatically one of the two ways to search for truth. And not the one Bacon recommends.

In the *Theaetetus*, too, Plato describes the philosopher. This is one part of the description: on the wings of his reason, the philosopher flies through the world, deep below the earth and high above the heaven, as Pindar says; he investigates in every way the entire nature of each thing and never condescends to notice things close at hand (173e-4a).

It is a better description of that way than Ba-

con's. Positively in that it is very much better saturated with ideas and feelings.

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Had Plato not left behind his late treatises Aristotle's treatises would have been a miracle. As it is, they are not a miracle. Aristotle's treatises build essentially and substantially on Plato's late treatises. And not on the middle, or the early.

In the *Cratylus*, having in the earlier dialogues developed the theory of the forms in some detail and drawn many conclusions from it, mostly practical, Plato finally states explicitly and at its most general, and most theoretical, the problem which the theory solves. The problem, posed by the doctrine which Plato reads in Heraclitus, is: all things change, nothing abides. Plato objects: but there is the beautiful itself, the good itself, and the same for all things that there are; and the beautiful itself is always what it is; if it always passed away, we would have to say that it were now this, now that; at the very instant we spoke of it, it would become something different; and if it were never the same, how could it be anything at all? or be ever known by anyone? This is the problem. If all things change, nothing abides, we would have to say of anything that it were now this, now that, we could not speak of anything, nothing could be anything at all, and nothing could ever be known by anyone. And the theory of the forms is a very successful solution of the problem. As successful as the theory of the four elements formulated by Empedocles two generations before Plato is of the problem of the material principles of all things.

In the *Parmenides* Plato investigates the difficulties of the theory. Which all by itself shows that it is a theory, and a successful theory. That it presents difficulties, these can be identified, and the attempt can be made to resolve them. Also, in the *Parmenides* Plato does not have Socrates explain to Parmenides that what Parmenides takes to be wisdom is not wisdom, he has Parmenides explain to Socrates that what Socrates takes to be wisdom is not wisdom. The most fundamental difficulty of the theory is participation. In the *Phaedo*: Socrates says, if there is anything beautiful among the sensible things, it is beautiful only because it participates

in the beautiful itself. In the *Parmenides*: Parmenides says, if what makes large things large is largeness itself, above this largeness and the things it makes large another form of largeness has to appear to make them both large, and above all these then another form has to appear to make them all large, and above these another, and another; Socrates says, perhaps, then, these forms are thoughts and are only in the mind; Parmenides says, then all things would be thoughts (132a-d).

The *Theaetetus* is not only the first investigation of knowledge. It is a most successful investigation. Plato shows that knowledge is not perception. That knowledge only comes after a long development. That knowledge is not belief, and belief is not knowledge. Knowledge and belief are exclusive of one another. And not only is man not the measure of all things. In geometrical proofs the geometer alone is the measure. In general, whoever has acquired knowledge alone is the measure.

In the *Sophist*, pace its title, the sophist is not Plato's main concern. Plato's main concern is investigation by dichotomy. Hence, in the course of the investigation Plato anticipates the definition of truth. He writes: falsehood is to say of what is that it is not and of what is not that it is (241a), and: truth is to say of what is that it is, falsehood is to say of what is not that it is (263a-b). Hence, also, he formulates and solves the problem of the highest genera of all things. The highest genera are: being, motion, rest, difference, and sameness (254c-9e). Taking another step toward the explicit formulation and solution of the problem of the relation of all things and their principles.

In the *Statesman*, too, Plato's main concern is not the statesman. His main concern is still investigation by dichotomy. Hence, he anticipates this most fundamental result as well: all knowledge is either theoretical, like arithmetic, or practical, like carpentry (258c-e).

Not quite one century before him Parmenides discovered the principle of contradiction. In the generation after him Aristotle developed logic. And in this development from Parmenides to Aristotle Plato's, in the *Cratylus*, *Parmenides*, *Theaetetus*, *Sophist*, and *Statesman*, is the most

important step.

All five are concerned with knowledge. All five are investigations. All five are theoretical investigations.

Also, as a matter of style, in the *Parmenides*, *Sophist*, and *Statesman*, Plato has Socrates in the background. Not the one who imparts wisdom, Socrates is one of those to whom wisdom is imparted. And whatever the style, none of the five, the *Cratylus*, *Parmenides*, *Theaetetus*, *Sophist*, and *Statesman*, is a Socratic dialogue as a matter of substance. They deal with problems posed by Heraclitus, in the *Cratylus*, by Protagoras, in the *Theaetetus*, by Parmenides, in the *Parmenides*, *Sophist*, and *Statesman*. And Plato does not deal with the problems by joining ranks with the wise and blessedly happy Parmenides against the unwise and cursedly unhappy Heraclitus and Protagoras. He examines Heraclitus not less than Parmenides, and to Protagoras as well he gives a fair chance in the *Theaetetus*, as he does not in the *Protagoras*, to present what he takes to be wisdom. It can be concluded that Plato has found out that the Socratic disquisition, for all its pedagogical virtues, and rather because of all its pedagogical virtues, is all too much like oratory, which is flattery and, like cosmetics and sophistry, shameful. Investigation, the search for truth, and knowledge, need another way.

In the *Timaeus* Plato investigates the world. Some of the results are practical, like carpentry. For instance: what is made beautifully has a maker, therefore the world has a maker. And: why did he make the world? because he was good. But some of the results are theoretical, like arithmetic. The first anticipation of the principle of sufficient reason, for instance: it is impossible for anything to come to be without a cause. Or the description of the planets as a system: time came into being with the world when the maker brought into being the sun, the moon, and the five other planets; the planets run in circles with different speeds, and so that there is a conspicuous measure of their speeds, the maker kindled the light of the sun; hence, a month is when the moon has completed its cycle, a year is when the sun has completed its cycle, and a perfect year is when all have together completed their cycle; time is the wanderings of these bodies; the maker

made them of fire to be the brightest; he made them round to be like the world; and he gave each two motions, rotation in the same and revolution with the same (38b-40a). Nay: he made the earth rotate on its axis (40b).

The *Critias* Plato proposes as a continuation of the *Timaeus* (106a-c). In particular, he has Critias report that: there is in the Atlantic a large island, Atlantis, and some 9 000 years ago there was a war between Athens and Atlantis (108e). And by way of introducing the story of the war, Plato has Critias describe the origin and development of the Athenians. This is their origin: the gods apportioned the earth among themselves by lot; Hephaestus and Athena, however, because they pursued philosophy and the arts, received the land suitable to them in virtue and intelligence; and in this land they made good men spring from the earth and taught them to rule themselves (109b-d). In one of his fragments, one generation before Plato, Critias himself writes: at first men lived like beasts; next laws were established so that justice may rule mankind, and the laws held men back from doing evil in the open, but not from doing it in secret; so a wise man discovered the fear of gods, to frighten evildoers even in their thoughts (fr. 25).

In the last two dialogues, the *Philebus* and *Laws*, Plato's main concerns are again ethical and political.

In the *Philebus* the question is: is pleasure the good? or wisdom is the good? or something else is the good (14b)? In the course of the dialogue Plato nevertheless obtains some results which are more like arithmetic than like carpentry. He returns, for instance, to the problem of the highest genera of all things. The genera are: the limited, the unlimited, the mixture of the two, and the cause of the mixture (23c-7c). Anticipating form, matter, that all things are form and matter, and that they have causes.

In the *Laws* Plato rebuilds the best city. He obtains the result: god is the measure of all things. The result also: men are puppets of the gods (I, 644d). But in the course of this dialogue as well he obtains some results which are more arithmetical. It is in the *Laws* that he for the first time judges the great kings with some, if not too much, discretion. Also, Simplicius reports:

Eudemus reports that Eudoxus first formulated the hypotheses of regular circular motion to which the apparent irregular motions of the planets can be reduced; and it was Plato who had posed the problem of finding hypotheses which will show that the apparently irregular motions of the planets are regular (comm. *Heaven*, 488). And one of the passages corroborating Eudemus's report is in the *Laws*. Plato writes: the planets do not wander; each planet perpetually describes one fixed orbit; it is only an appearance that its path is always changing (VII, 822b).

Much in these treatises, and especially in the *Cratylus*, *Parmenides*, *Theaetetus*, *Sophist*, and *Statesman*, suggests the other way, the one Bacon recommends. And so does, in particular, what Plato does in them to the philosopher and philosophy.

To begin with, here is more fully the passage in which Plato describes the philosopher in the *Theaetetus*: the philosopher has no concern for the affairs of his city or for anything else his fellow citizens do; only his body lives and sleeps in the city; his reason spurns such things; on the wings of his reason he flies through the world, deep below the earth and high above the heaven, as Pindar says; he does geometry on earth and astronomy in heaven; he investigates in every way the entire nature of each thing and never condescends to notice things close at hand; so Thales, once, while gazing at the stars, fell into a well; a clever Thracian slave-girl was passing by and drew the moral, eager to know what was in the heaven, Thales did not know what was in his feet; and the joke applies to all philosophers (173e-4a).

If you still want to be the philosopher, then, it is not as easy as it used to be. Knowing yourself and discovering many beautiful things within yourself will not do. You have to have no concern for the affairs of your city or for anything else your fellow citizens do. You can still live and sleep in the city, but you have to spurn such things. On the wings of your reason you can fly through the world, deep below the earth and high above the heaven, as Pindar says, but you also have to do geometry and astronomy. You can investigate in every way the entire nature of each thing, but you also have to gaze at the stars and

fall into a well while gazing at the stars.

You, I, everyone, and anyone are not the philosopher any longer.

In the *Sophist*, moreover, Theodorus brings along a stranger: is he a guest, Socrates asks, or a god? he is not a god, Theodorus replies, but he is divine, like all philosophers; indeed, Socrates says, the true philosopher, unlike the false, looks from on high on those below; but those below, in their ignorance, see the philosopher in all sorts of different ways, now as worthless, now as worth everything in the world, now as a statesman, now as a sophist, now as completely mad (216a-d).

Worthless, worth everything in the world, a statesman, a sophist, completely mad, are appearances. Reality is that there is the true philosopher, and the false philosopher. Also in the *Sophist*: truth is to say of what is that it is, falsehood is to say of what is not that it is. The true philosopher is the philosopher. The false philosopher is the not philosopher.

And if the task is to find the philosopher, as distinct from the not philosopher, Plato has already found him.

Protagoras is Plato's central concern both in the *Protagoras* and in the *Theaetetus*. 'Protagoras', however, does not refer to the same object in the *Protagoras* and in the *Theaetetus*.

In the *Protagoras* the goal is to show that: the sophists are peddlers of nourishment for the soul, and if you do not know what you are buying, they may well deceive you. The means is a depiction of Protagoras. The archsophist Protagoras is surrounded by a throng of admirers, including the secondary sophists Hippias and Prodicus. Through much of the dialogue the archsophist peddles nourishment for the soul. None of his throng of admirers knows what he is buying. But Socrates is also present, and he knows what he is buying.

In the *Theaetetus* the goal is: knowledge. The means is a proposition affirmed by Protagoras. Plato reports: Protagoras said, man is the measure of all things, of those that are that they are, of those that are not that they are not; thus his *Truth* began. And most of the treatise is an examination of this proposition and its premises and consequences.

What Plato says about Protagoras in the *Pro-*

tagoras is all *ad hominem*. What Plato says about Protagoras in the *Theaetetus* is all *ad philosophum*. The proposition, man is the measure of all things, the foundation of the argument in the *Theaetetus*, is not even mentioned in the *Protagoras*. The peddler, the foundation of the argument in the *Protagoras*, is not even said to be a peddler in the *Theaetetus*. The reference of 'Protagoras' in the *Protagoras* is the man. The reference of 'Protagoras' in the *Theaetetus* is the proposition, man is the measure of all things, with its premises and consequences.

And that the philosopher is propositions is corroborated by what Plato does to Parmenides and Heraclitus.

Parmenides is Plato's central concern in the *Parmenides*. And 'Parmenides' refers to the man in a couple of lines. In the rest of the treatise, as well as in Plato's other treatises, 'Parmenides' refers to propositions. In the *Parmenides* Plato begins by depicting Parmenides: venerable, distinguished, about sixty-five, accompanied by Zeno, about forty, tall and handsome, rumored to have been the object of Parmenides's affections as a boy (127a-b). Plato then has Socrates tell Parmenides: you say in your poem that all things are one (128a-b). And the rest of the treatise is in two parts, first, on the ground of the proposition, all things are one, the difficulties of the theory of the forms are shown, second, the premises and consequences of the proposition, all things are one, are examined. Plato notes Parmenides explicitly in three other treatises as well. In the *Theaetetus* he again ascribes to him the doctrine, all things are one (179d-81a). And once in the *Symposium* (178b), three times in the *Sophist* (237a, 244e, 258c), he quotes him, fr. 13, 7, 8, and 7 again, respectively. Other than the couple of lines of depiction, propositions one and all.

Plato writes in the *Cratylus*: Heraclitus said that all things change, nothing abides, and you cannot step in the same river twice. Then the examination of this proposition with its premises and consequences is the ground on which Plato formulates the problem which the theory of the forms solves. Plato notes Heraclitus explicitly in four other treatises as well. In the *Theaetetus*, as he again ascribes to Parmenides the doctrine, all things are one, so he again ascribes to Heraclitus

the doctrine, all things change (179d-81a). And in the *Greater Hippias* (289a), *Symposium* (187a-b), and *Republic* (VI, 498a-b), he quotes him, fr. 82, 51, and 6, respectively. Propositions one and all. Plato never says anything about the man, and a good thing, too, considering what he has to say about Protagoras and Parmenides.

That the philosopher is propositions is corroborated also, feebly as it is, by the fact that in the late dialogues even Plato's motley armies have become less motley. He writes in the *Theaetetus*: with the exception of Parmenides, all wise men in the past, Protagoras and Heraclitus and Empedocles and the masters of the two kinds of poetry, Epicharmus in comedy, Homer in tragedy, have said that nothing ever is and all things are always becoming (152d-e).

If the philosopher is propositions, not men, on the authority of Plato's treatises the philosopher is, in chronological order, Heraclitus, Parmenides, and Protagoras.

And if the philosopher is propositions, the men are appearances, always becoming and never the philosopher, more of the philosopher can be recovered from Plato's treatises, strictly on Plato's authority.

Theodorus of Cyrene, other than that he brings along a stranger or a god in the *Sophist*, discovered that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational, Plato reports in the *Theaetetus*. Besides, Theodorus of Cyrene diligently listens to the general conversation in the *Theaetetus*, then to the general conversation in the *Sophist*, then to the general conversation in the *Statesman*, and Plato finds occasions to note that: he is a master of geometry, astronomy, arithmetic, and music; that Theaetetus is his student (*Theaetetus*, 145a); that Protagoras is his friend (162a); and that since youth he has inclined away from general conversations (165a). The discovery that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational, on one side. Everything else, he brings along a stranger or a god, he is a master of geometry, astronomy, arithmetic, and music, Theaetetus is his student, and Protagoras is his friend, as well as that since youth he has inclined away from general conversations, commendable as this last is, on the other side. Not incidentally, the discovery that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational, is one of the two most important discoveries which Plato

reports in his treatises.

In book I of the *Republic* Thrasymachus is annihilated. Before Socrates explains to him that what he takes to be wisdom is not wisdom, however, Thrasymachus is allowed to say: justice is what is useful to the stronger; for, whether in a tyranny, or in a democracy, or in an aristocracy, the rulers make the laws, they make them useful to themselves, and proclaim what they have made just; so justice is the same in all cities, it is what is useful to the established rule; and since the established rule is the stronger, justice is the same everywhere, it is what is useful to the stronger. And so much suffices to establish that the philosopher includes Thrasymachus. Moreover, Socrates objects: no one who rules, to the degree to which he is a ruler, seeks his own advantage, he seeks the advantage of those he rules. And when Thrasymachus retorts: do you still have your wet nurse, Socrates? this may be *ad hominem*. But when Thrasymachus adds: you seem to think that shepherds and cowherds seek the advantage of the sheep and the cattle, this is *ad philosophum*.

Dramatically, the philosopher includes even Anaxagoras. In the *Phaedo* Socrates informs his teary followers: I heard that Anaxagoras taught that mind orders all things and is the cause of all things; I was delighted, thinking that I had found someone to teach me what I wanted to know; but then I read the books of Anaxagoras, found that they were all about causes like air, aether, water, and other such things, and realized that he had nothing to teach me; I began to study the causes of things by myself; and I discovered that earth and air and other such things are not the true causes of things; the true cause of things is what is for the best (97b-9b). On the other hand, Plato reports in the *Cratylus*: Anaxagoras said that the moon shines with the reflected light of the sun (409a-c). It is not what Socrates wanted to know, but it is the other of the two most important discoveries which Plato reports in his treatises.

Even with respect to Socrates the Herculean labor of disentangling the philosopher from the man can be undertaken. And it is a Herculean labor, the one of the stables. Because in the *Phaedo* Plato has Socrates tell his teary followers this also: take my advice and think little of Socrates, much of truth. But the evidence of the Pla-

tonic treatises that Plato took this advice, where Socrates is concerned, if not other matters, is far from unambiguous. Fortunately, that Aristotle took this advice, including where Socrates is concerned, the evidence of the Aristotelean treatises is unambiguous.

In general, if the striving for truth forcing us to advance from the sense to the reference is very much in evidence in Plato, and in particular with respect to the philosopher and philosophy, the much more powerful striving to keep saturating the sense with ideas and feelings is very much in evidence as well.

This can explain why those are the two most important discoveries which Plato reports in his treatises. The majority, by far, of the many important discoveries of his predecessors, including the very important, Plato does not report in his treatises.

It can explain also why the majority of the philosophers among his predecessors, including the very important, cannot be recovered from his treatises.

Pythagoras, for instance, Plato notes but once. In book X of the *Republic* Homer is annihilated: Socrates asks, is any city governed well because of the laws which Homer gave it, as Sparta is because of the laws of Lycurgus, Sicily because of the laws of Charondas, Athens because of the laws of Solon? no, Glaucon replies; was any war won because of Homer's generalship or advice? no, Glaucon replies; is Homer famous for useful discoveries, as Thales and Anacharsis are? no, Glaucon replies; did Homer have any followers whom he taught a distinct way of life, as Pythagoras had followers whom he taught a distinct way of life? no, Glaucon replies (599d-600b).

Xenophanes, also for instance, Plato also notes but once. Plato has described in the *Theaetetus* the war of Heracliteans and Eleatics: to everything the Heracliteans say the Eleatics say the opposite. In the *Sophist* the stranger or god whom Theodorus brings along is an Eleatic, he leads the general conversation, and on one occasion reports: our Eleatic tribe originated with Xenophanes, if not earlier (242d).

Much sense, no reference. Not where Pythagoras and Xenophanes are concerned.

Nevertheless, in Plato's treatises 'philosopher'

has begun acquiring its reference. The reference is, in chronological order, Heraclitus, Parmenides, Anaxagoras, Protagoras, Thrasymachus, Theodorus of Cyrene. Hence, 'philosophy' has begun acquiring its reference as well. The reference is the respective propositions.

Thus, from Pythagoras, through Xenophanes and Heraclitus, through Herodotus, to Plato, and from his early treatises, through his middle treatises, to his late treatises, the direction in which the references will be fixed is indicated unambiguously.

And Aristotle will fix them.

Hence, Aristotle also shows that all particulars pertaining to the matter can be collected and marshaled in their series and order. That their predecessors and the philosophers can be separated sharply, on one side, everything else, on the other. Among their predecessors and the philosophers the important can be separated sharply, on one side, everything else, on the other. And among the important among their predecessors and the philosophers the steps of advance in the acquisition of knowledge can be separated sharply, on one side, everything else, on the other.

ccxxviii

Aristotle writes in book I of the *Metaphysics*: the principles are of four kinds, material, efficient, formal, and final, and we have investigated them sufficiently in the *Physics*; but let us now call on our predecessors who have investigated being and searched for truth and see what they have said about the principles, for they can help us either find another kind or reassure ourselves that we have determined the kinds correctly (3). And he reviews 20 predecessors of ours who have investigated being and searched for truth.

In the *Physics*: whether the principles are one or more the physicists differ (I, 4). And he reviews 9 physicists.

Later in the *Physics*: all who have studied nature in a way worth considering have spoken about the infinite and have without exception made it a principle of things (III, 4). And he reviews 5 who have studied nature in a way worth considering.

In *On the heaven*: we now turn to the earth, its

position, whether it is at rest or in motion, and its shape; and on this there are different judgments (II, 13). And he reviews 9 different judgments.

Later in *On the heaven*: whether there is generation, earlier judgments differ from one another as well as from truth (III, 1). And he reviews 10 earlier judgments.

Later in *On the heaven*: the following predecessors of ours have discussed heavy and light (IV, 2). And he reviews 3 who have.

In *On generation and corruption*: whether alteration is the same as generation, the philosophers before us disagree (I, 1). And he reviews 4 philosophers.

Later in *On generation and corruption*: how action takes place, this is what the philosophers before us think (8). And he reviews what 4 philosophers think.

In the *Meteorology*: let us now explain comets, after first reviewing the judgments of others (I, 6). And he reviews the judgments of 5 others.

Later in the *Meteorology*: let us now explain the origin, cause, and nature of the Milky Way, again beginning with the judgments of others (8). And he reviews the judgments of 3 others.

Later in the *Meteorology*: about earthquakes 3 explanations have been proposed (II, 7). And he reviews the 3 explanations.

In *On the soul*: beginning now the investigation of the soul, in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous (I, 2). And 13 of our predecessors participate in the following session of the council.

In the *History of animals*: on the veins the following have written (III, 2). And he reviews 3 who have.

In *On youth, old age, life and death, and respiration*: few of the physicists before us have concerned themselves with respiration (7). And he reviews 5 who have.

In the *Sophistical refutations*, concluding his logical investigations: of this investigation it was not that something was done before us, something was not done; nothing at all was done (34). And he yet reviews 4 steps in the development of the teaching of contentious arguments.

In the *Politics*: in order to consider which is

the best state, we must examine constitutions, and not just those of well-governed states, but those also conceived in speech and held in high esteem (II, 1). And he reviews 25 predecessors of ours who have given laws to states or conceived them in speech.

Aristotle discovered the council of our predecessors and called it into permanent session. So that we can adopt the sound in what they have said and avoid the erroneous. He discovered the review of the earlier labors of formulation and solution of problems as the ground of the formulation and solution of problems. The ground exclusively, too, if the ground is to be sound and not erroneous.

And the 16 reviews above are only the most systematic. Aristotle reviews his predecessors in all his treatises, *passim*. No philosopher has followed the fourth rule more staunchly, including, most particularly, with respect to his predecessors.

In the 16 reviews above alone Aristotle notes 56 predecessors of his. 35 are important, 21 are not important. And among the important the steps of advance in the acquisition of knowledge can be separated sharply as well.

And Aristotle supplies the ground of all three, the collection of their predecessors and the philosophers, the separation of the important among them, and the separation of the steps of advance in the acquisition of knowledge among them.

That these are all propositions, judgments, problems and their solutions, Aristotle himself can hardly have been more explicit. This is what he collects in the 16 reviews. What our predecessors who have investigated being and searched for truth have said about the principles. What the physicists have said about the number of the principles. What all who have studied nature in a way worth considering have said about the infinite. Judgments on the earth, its position, whether it is at rest or in motion, and its shape. Judgments on generation, and in their relation to one another and to truth. Those who have discussed heavy and light. The philosophers' disagreement whether alteration is generation. How the philosophers think action takes place. Judgments on comets. Judgments on the Milky Way. Explanations of earthquakes. The sound and erroneous

said in the council of our predecessors about the soul. The physicists' concerns with respiration. What has been written on the veins. How contentious arguments have been taught. Constitutions, and whether of states or conceived in speech.

The task is to distinguish the acquisition of knowledge from everything else. To this end, to collect the particulars, and marshal them in their series and order. And to this end, first, to establish the ground on which to do so.

And to show that Aristotle supplies the ground of all three, the collection of their predecessors and the philosophers, the separation of the important among them, and the separation of the steps of advance in the acquisition of knowledge among them – and thus the ground of the collection of the particulars, and marshaling them in their series and order – the most important review, by far, the one on the principles of all things in book I of the *Metaphysics*, can be reviewed, and this in six steps. The review can be presented. It can be shown how Aristotle discovered the first philosopher. It can be shown that in the same way he discovered many other philosophers. It can be shown that he fixed the reference of 'philosopher' and 'philosophy'. The 20 particulars in the review can then be sorted out, the important among them separated, and among them the steps of advance in the acquisition of knowledge. And it can be shown that the review is complete, that these are the particulars, all important among them, and all steps of advance in the acquisition of knowledge among them.

And this should conclude the attempt to establish the ground on which all particulars can be collected, and marshaled in their series and order.

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In book I of the *Metaphysics* Aristotle begins with his most general investigation of knowledge.

He describes the development: by nature all animals have perception; from perception memory develops in some of them; from memory experience develops; from experience art develops; and first the arts aiming to procure the necessities of life developed; then the arts aiming to give pleasure; and when all such arts were established, knowledge which is neither useful nor pleasurable was discovered.

He defines this last: knowledge which is neither useful nor pleasurable is knowledge of the principles of things. And: knowledge of the principles of things implies knowledge of the principles of all things, and to know all things is the furthest removed from perception.

The principles of all things are the end of the development, and these, the principles of all things, Aristotle adds: are the concern of the present investigation.

In this way: the principles are of four kinds, material, efficient, formal, and final, and we have investigated them sufficiently in the *Physics*; but let us now call on our predecessors who have investigated being and searched for truth and see what they have said about the principles, for they can help us either find another kind or reassure ourselves that we have determined the kinds correctly.

And the session of the council of our predecessors on the principles of all things follows.

Aristotle writes:

the earliest philosophers took the principles of all things to be only material, principle they identified with primary matter; and the originator of this kind of philosophy is Thales who said that water is the principle;

perhaps Thales said so because he saw that the nourishment of everything is moist, or that heat is generated from moisture, or that the seeds of everything are moist; some even think that the ancients, who lived long before us and first spoke about the gods, held the same opinion about nature, for they made Ocean and Tethys the parents of creation and made the gods swear by the water called Styx;

it may be uncertain that this opinion is that ancient; but Thales said that water is the principle; this is why he also said that the earth floats on water;

Hippo we can ignore because of the paltriness of his thought;

Anaximenes and Diogenes of Apollonia said that air is prior to water and it is the principle;

Hippasus and Heraclitus said that fire is prior and it is the principle;

Empedocles then added earth to these and said that all four, earth, water, air, and fire, are the principles;

Anaxagoras, older than Empedocles but working after him, said that an infinite number of particles are the principles, as each thing is generated from its own particles and destroyed into its own particles;

from what these thinkers said it could appear that the principles are only material; but as they advanced, the investigation itself showed them the way and forced them to go further; wood does not make beds and bronze does not make statues; there is a reason for change, there are efficient principles, as we will say;

and this was discovered when someone, like a mature man among babblers, said that there is reason in nature, as there is reason in animals, and it is the cause of the world and its order; we know that Anaxagoras said so; some say that Hermotimus said so earlier (3).

and it may be that the first to search for this kind of thing was Hesiod, or whoever first spoke of love or desire as a principle, as Parmenides does when he describes the genesis of all things,

first of all gods she made Love,

and Hesiod says,

first of all Chaos came into being;

then big-bosomed Earth,

and Love, the fairest of the immortals,

implying that there must be a cause of the motion and combination of things;

later Empedocles made love and strife principles;

clearly, all these thinkers grasped in some way two of the kinds of principles which we have distinguished in the *Physics*, the material and the efficient; they expressed themselves vaguely, however, not quite knowing what they were saying, fighting, as it were, the way untrained men fight;

Empedocles, then, first introduced the division of this principle, positing not one source of motion but two; as he was the first also to speak of four material principles;

Leucippus and Democritus, next, made atoms and void the elements, and said that the modification of all things is due to the shape, order, and position of these elements; the problem of the beginning of change they, like the others, lazily neglected;

and this is how far the earlier thinkers have

advanced the investigation of these two kinds of principles (4);

at the same time and even earlier the Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this science, and from its study they came to think that its principles are the principles of all things; and since among these principles numbers are by nature first, they saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water; so they said that numbers are the principles of all things, and the whole world is a harmony and number;

our object is to see whether what these thinkers said are the principles falls under the kinds which we have found; and it is clear that these thinkers take number to be the principle both as the matter of things and as the form of their modifications and changes;

other Pythagoreans said that the principles are ten, and arranged them in pairs, limit-unlimited, odd-even, one-many, right-left, male-female, rest-motion, straight-curved, light-darkness, good-bad, square-oblong;

and Alcmaeon borrowed his doctrines from these thinkers, or they borrowed theirs from him; for he said that most human things go in pairs, white and black, sweet and bitter, good and bad, large and small; but all Alcmaeon said was vague hints;

these thinkers tried to explain change, others said that there is no change in the world; of these Parmenides said that all is one in form, Melissus said that it is one in matter, while the first of them, Xenophanes, said nothing clearly, just looked at the sky and said that the one is god;

Xenophanes and Melissus we can ignore as too naive;

Parmenides spoke better, he said that only what necessarily is is and nothing else is; forced to account for the phenomena, however, he said that, even though all that is is one necessarily, it appears as many and as such has two principles, fire and earth, associating fire with what is, earth with what is not (5);

after these philosophers came Plato; in his youth Plato became acquainted with Cratylus and the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge

of them, and Plato held this view in his later years as well;

in the meantime Socrates had abandoned physics altogether, and devoted himself to ethics; and while searching for the general in ethical matters, he first concerned himself with definitions;

Plato followed Socrates; he presumed, however, that definitions cannot be of the sensible things because the sensible things are always changing; definitions have to be of entities of another kind; these entities he called forms; the sensible things are named after the forms, and the many sensible things called by the same name as a given form are by participation in that form; and since the forms are the principles of all things, he took their elements to be the elements of all things; as matter, the great and the small are the principles; as form, the one is the principle; because from the great and the small, by participation in the one, numbers come; and Plato agrees with the Pythagoreans that all other things are by participation in numbers; but what this participation is remains unclear;

Plato's verdict, then, on the matter which we are investigating, is that the principles are of two kinds, formal and material, as the forms are the principles of everything else, and the one is the principle of the forms (6);

thus, our review of those who have searched for the principles and truth has been brief; but so much we have learned;

none of our predecessors spoke of a kind of principle other than the four which we have distinguished in the *Physics*; at the same time, though, they all had an inkling of these four kinds, vague as it was; in particular, they grasped the material; no one grasped clearly the formal, but some of them grasped it in some way; the efficient they also grasped in some way; and those who spoke of reason and of love as principles grasped the final in some way as well; even if no one expressed these kinds clearly as principles;

we can conclude that these thinkers testify that we have determined correctly how many and of what kind the kinds of the principles are (7).

Aristotle next indicates which are the four most important solutions of the problem of the

principles of all things. By criticizing at some length those of Empedocles, Anaxagoras, and the Pythagoreans (8). And by criticizing severely and at greater length Plato's theory of the forms (9). In particular, he criticizes Plato's formulation of the theory in the *Phaedo* by elaborating on the difficulties to which Plato has pointed in the *Parmenides* (*id.*). In the course of the criticism he notes in passing one more of our predecessors: Eudoxus also argued that the forms make the sensible things what they are in the way white gives whiteness to a mixture (*id.*). And concludes with his verdict: philosophy searches for the principles of the sensible things, and to argue, while searching for their essence, that there is some other kind of essence, is prattle (*id.*).

Then he concludes the book:
still young, making its first steps, philosophy was on this matter, as on all other matters, so to say, stammering (10).

ccxxx

One of the many very important results which the review attests is the discovery of the first philosopher.

Aristotle postdates Thales by two and a half centuries. And let us call on our predecessors who have investigated being and searched for truth and see what they have said about Thales.

Here is, first, once again the report of Laertius: Thales was the first astronomer, the first to predict eclipses and to determine the solstices, so writes Eudemus in the history of astronomy, and Thales was admired for that by Xenophanes and Herodotus and noted by Heraclitus and Democritus.

And here are the dates of the 6 predecessors of ours in the report as well as of Laertius and Plato and Aristotle.

625	Thales,
570 - 473	Xenophanes,
540	Heraclitus,
484 - 425	Herodotus,
460	Democritus,
427 - 347	Plato,
384 - 322	Aristotle,
371 ^{BC}	Eudemus,
219 ^{AD}	Laertius.

The report attests the great importance of

Eudemus. Eudemus wrote the first history of mathematics, including astronomy, and five centuries later the first history of mathematics is one of the two indispensable sources and ultimate authorities on the early philosophers for Laertius, as it remains eighteen centuries later still, and will remain forever, one of the two indispensable sources and ultimate authorities on the early philosophers.

The other is the first history of physics, which Theophrastus wrote.

Aristotle divided philosophy into physics and mathematics. Aristotle also began the systematic research of our predecessors who have investigated being and searched for truth. And his students Theophrastus and Eudemus wrote the first histories of physics and mathematics.

And if Aristotle established that Thales was the first philosopher, this explains the conclusion which Eudemus drew, that Thales was the first astronomer. As well as the conclusion that he first introduced mathematics among the Greeks. The last Proclus attests. Also on the authority of Eudemus, Proclus writes: Thales traveled to Egypt, and first introduced mathematics among the Greeks (comm. *Elements*, 65). Recalling also, apparently, Aristotle's report that the priests in Egypt had leisure.

It explains also the conclusion which Theophrastus drew, that Thales introduced physics among the Greeks. Simplicius reports: Thales first introduced physics among the Greeks; he had many predecessors, Theophrastus says, but he surpassed them all so much as to overshadow them completely (comm. *Physics*, 23).

We are to adopt the sound in what our predecessors have said and avoid the erroneous, however, and we should adopt the facts which Eudemus reports about Thales but avoid some of his general conclusions. And because we should adopt everything which Aristotle and Theophrastus say about Thales.

On the eclipse that Thales predicted Herodotus reports in the *Histories*. It is the first prediction of an eclipse. A solar eclipse, no less. It is, like everything else in the *Histories*, a great and magnificent deed.

More particularly, Thales predicted an eclipse four and a half centuries before Hipparchus

formulated the theory of the sun and the theory of the moon, and hence the theory of eclipses. Nay, he predicted an eclipse one and a half centuries before Anaxagoras discovered the cause of eclipses. Nay, he predicted an eclipse one century-odd before Parmenides discovered that the earth is a sphere. A solar eclipse, no less. Thales's is a very great and magnificent deed.

As to the solstices, on the one hand, Hesiod already, one century-odd before Thales, sings in the *Work and days* of both the winter solstice (564-7) and the summer solstice (663-5). On the other hand, when Ptolemy collected all accurate observations, the oldest observation of a solstice which he found was that of Meton, one and a half centuries after Thales.

There is another report of Eudemus on Thales. Proposition 26 in book I of the *Elements* is: if two triangles have two angles equal to two angles, and one side equal to one side, respectively, the remaining angle and sides are equal (*id.*). And Proclus comments: in the history of geometry Eudemus attributes this theorem to Thales, because, he says, the method which Thales used to measure the distance of ships from the shore shows that he must have used it (comm. *Elements*, 352).

If this is to say that Thales proved that theorem, in order to prove it Thales would have had not only to prove, as Euclid does, the preceding twenty-five theorems. He would have had to prove them three centuries before Euclid. He would have had to prove them also two generations before Pythagoras discovered proof.

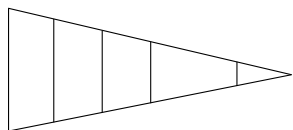
And if these are the facts, we should avoid Eudemus's general conclusions on pain of losing in vagueness the fundamental fact, that Thales is the first philosopher.

Where the solstices are concerned, if the fact is that Thales determined them better than Hesiod, but not as well as Meton, on this ground, the determination of the solstices, the first astronomer – and if the first astronomer is the first philosopher, the first philosopher – would be Hesiod or Meton rather than Thales. If the ground is the record of the solstices, the first astronomer would be Hesiod. If the ground is the observation of the solstices, the first astronomer would be Meton. If the latter is the better ground, and it is,

the first astronomer would postdate the discoveries, among others, that the earth is a sphere and of the cause of eclipses. If it is the better ground, too, it is not because it is the observation of the solstices, but because it is observation. But if so, Aristotle writes: the observations of the Egyptians and the Babylonians have been kept for many years and from them much of our evidence is derived. And Ptolemy attests the oldest accurate observation, in Babylon, of a lunar eclipses in 721, one and a half centuries before Thales. Not to mention the Venus tablet of Ammisaduqa, nine centuries earlier still.

Thales's is the first prediction of an eclipse. More particularly, Herodotus reports: in the middle of the battle of Alyattes and Cyaxares, in the sixth year of their war, a solar eclipse occurred, day became night before their eyes, the kings stopped their battle and made peace; and this was the eclipse that Thales had predicted to the Ionians, he had said that it will occur not later than that year (I, 74). A very great and very magnificent deed. Thales was admired for that by Xenophanes, noted by Heraclitus, admired by Herodotus, noted by Democritus. On this ground, however, having said that a solar eclipse will occur not later than that year – four and a half centuries before the formulation of the theories of the sun, moon, and eclipses, one and a half centuries before the discovery of the cause of eclipses, one century-odd before the discovery that the earth is a sphere – Thales would be the first prophet rather than the first astronomer. And on this ground, now, prophecy, Thales would find it very difficult to be the first. The earlier record abounds in prophets. *Amos*, for one, predates him by two centuries.

Eudemus does not attest the method which Thales used to measure the distance of ships from the shore. But Pliny reports: Thales measured the height of the pyramids by measuring their shadows at that time of day when our shadows are as long as ourselves (XXXVI, 82). It can be presumed that the method is the same. To picture it, the base and side of the largest triangle can be the height of the pyramid and its shadow, the base



and side of the smallest triangle ourselves and our shadows. The ship at sea can be at the vertex, and the two largest bases and the distance between them measured on land. And that, pace Eudemus, Thales need not have used any Euclidean proposition to accomplish either of these great and magnificent deeds, the fact shows that the picture is borrowed from the mathematician *par excellence*, his labors recorded on the tablet Strasbourg 364, who solves 14 problems involving the sides and areas of all the figures in the picture eleven and a half centuries before Thales, twelve centuries before Pythagoras discovered proof.

Thales first predicted an eclipse. He determined the solstices. He measured the distance of ships from the shore. He measured the height of the pyramids. And he is on none of these grounds the first astronomer, or the first mathematician, or the first philosopher.

Going back, then, to these predecessors of ours who were predecessors also of Aristotle, this is what they have said about Thales.

Two generations after Thales, Xenophanes admired him. Xenophanes also said that our wisdom is better than the strength of men and horses.

One generation later, Heraclitus noted Thales. Heraclitus also said that the best, the wise, and the few are not like the asses who prefer rubbish to gold. He said also that loving wisdom men must research many things.

Two generations later still, one and a half centuries after Thales, Herodotus notes him three times in the *Histories*.

Herodotus proposes to begin the *Histories* with the deeds of Croesus, and in order to do so begins rather with the deeds of his predecessors on the Lydian throne. He reaches his immediate predecessor Alyattes, and while reporting on his war with the Mede king Cyaxares, writes: in the middle of a battle in the sixth year of their war, a solar eclipse occurred, day became night before their eyes, the kings stopped their battle and made peace; and this was the eclipse that Thales had predicted to the Ionians, he had said that it will occur not later than that year.

Croesus succeeded Alyattes on the Lydian throne, Astyages succeeded Cyaxares on the Mede throne, then Cyrus overthrew Astyages and

became the king of Persians and Medes. Croesus resolved to wage war on Cyrus, marched east, and reached the Kizilirmak. Herodotus writes: and I think that Croesus crossed the river by bridges that were already there; people say that there were no bridges, Croesus wondered how to take his army across, and Thales diverted the river from its course for him; but I think there must have been bridges, for how did they cross the river on their way back (I, 75)?

Cyrus defeated Croesus and threatened the Ionian cities of Asia Minor. Herodotus writes: Thales, whose remote ancestors were Phoenicians, advised the Ionians to set up a joint council to decide what to do (170). The Ionians ignored the advice.

Thales predicted an eclipse, was said to have diverted a river from its course, indeed the largest river in Asia Minor, and advised the Ionians, and the advice was excellent, too.

Herodotus also says that Melampus introduced Dionysus among the Greeks and he was wise. Solon made laws for the Athenians, and loving wisdom traveled much of the earth looking for it. The father of one of the suitors of the daughter of Cleisthenes of Sicyon was called the wise. Chilon, the wisest among the Spartans, advised them not to build a navy. Pythagoras, not the least of the wise men among the Greeks, taught Zalmoxis immortality. The wisest custom of the Babylonians had, sadly, lapsed. Oebares won his kingdom for Darius and he was wise. Themistocles won at Salamis, all Greece thought him the wisest, and the Spartans gave him the finest chariot for his wisdom.

Herodotus notes 804 predecessors of his in the *Histories*. And if there were philosophers among them, his research did not establish this fact. Thales is one of the 804. And if he belongs to a kind, it can be presumed that the kind is the wise, the wisest, and loving wisdom, even if Herodotus does not explicitly assign Thales to this kind. Specimens of the kind then are, in chronological order, Solon, Thales, Chilon, Pythagoras, Oebares, Themistocles. And – even as it ignores Melampus, the father of the suitor, and the custom – the kind is multifarious. Making laws, traveling much, predicting an eclipse, diverting a river, advising the Ionians, advising the Spartans,

teaching immortality, winning a kingdom, winning a battle. Nor is Thales the first of this kind.

Two generations after Herodotus, two centuries after Thales, Plato notes him – ignoring a mention in one of the spurious letters (II, 310e-1a) – four times.

Once in the early treatises.

In the *Greater Hippias* Socrates encounters Hippias, Hippias reports that he is busy with the affairs of his city, and Socrates muses: so this is what it is to be truly wise; but tell me, Hippias, why is it then that Pittacus, Bias, the followers of Thales, and Anaxagoras, though they were famous for wisdom, all kept away from the affairs of the city (281c)?

Twice in the middle treatises.

In the *Protagoras*, that philosophy had its first beginning among the Cretans and the Spartans, it is still practiced most among them, and they have the best education in it, part of the evidence are the seven wise men: Thales, Pittacus, Bias, Solon, Cleobulus, Myson, and Chilon, whose Spartan wisdom is shown by their brief sayings, know yourself, for instance, and, nothing too much, which are now on everybody's lips.

In book X of the *Republic*, part of the evidence for the deficiency of Homer is: is Homer famous for useful discoveries, as Thales and Anacharsis are? no, Glaucon replies.

And once in the late treatises.

In the *Theaetetus* Plato describes the philosopher: the philosopher has no concern for the affairs of his city or for anything else his fellow citizens do; on the wings of his reason he flies through the world, deep below the earth and high above the heaven. And the description concludes: so Thales, once, while gazing at the stars, fell into a well; a clever Thracian slave-girl was passing by and drew the moral, eager to know what was in the heaven, Thales did not know what was in his feet; and the joke applies to all philosophers.

Plato has established the fact that there are philosophers among his predecessors. He has established also that Thales is a philosopher, indeed the paradigmatic philosopher. Plato's general descriptions, however, have made the kind philosopher ubiquitous, ranging from the Cretans and the Spartans through the plan to win all kingdoms for the philosophers to flying

through the world. But even if the general descriptions are ignored, the kind philosopher in Plato is not less multifarious than the kind the wise, the wisest, and loving wisdom in Herodotus. From the passages involving Thales alone, specimens of the kind are, in chronological order, Pittacus, Solon, Cleobulus, Thales, Anacharsis, Bias, Myson, Chilon, Anaxagoras. And from what Plato says about Thales alone, the kind includes keeping away from the affairs of the city, showing Spartan wisdom by brief sayings which are on everybody's lips, being famous for useful discoveries, falling into a well while gazing at the stars. Nor is Thales the first of this kind.

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One generation-odd after Plato, two and a half centuries after Thales, Aristotle notes him seven times.

In the review in book I of the *Metaphysics*.

Aristotle says: the earliest philosophers took the principles of all things to be only material, principle they identified with primary matter; and the originator of this kind of philosophy is Thales who said that water is the principle. Also: Anaximenes said that air is prior to water and it is the principle; Heraclitus said that fire is prior and it is the principle; Empedocles added earth to these and said that all four, earth, water, air, and fire, are the principles.

The kind is what our predecessors took the material principles of all things to be. Specimens are Thales, Anaximenes, Heraclitus, Empedocles. Thales is explicitly the first of the kind.

In the review of the earlier judgments on the earth in *On the heaven*.

Aristotle says: the oldest judgment is that of Thales, he said that the earth is at rest because it floats on water, like wood. Also: Anaximander said that it is at rest because of equilibrium; Anaximenes said that it is flat, and it is at rest because it floats on the air below, like a lid; Xenophanes said that it stretches infinitely downward, to save himself the trouble of looking for causes.

The kind is judgments on the earth. Specimens are Thales, Anaximander, Anaximenes, Xenophanes. Thales is again explicitly the first.

Most explicitly Aristotle describes the kind when he introduces the review in *On the soul*: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous. The kind is the formulation and solution of problems. Specimens are Thales, Anaximander, Anaximenes, Xenophanes, Heraclitus, Empedocles. And Thales is the first of this kind.

Aristotle discovered the first philosopher.

And it is a discovery like any other discovery of an object not known before the discovery to be the object it is, known after the discovery to be the object it is.

It is a discovery like the discoveries of America, of Uranus, of hydrogen, of Sumerian, of the electron. Columbus crossed the Atlantic and reported that he has reached Asia. Vespucci proposed that it is not Asia but a new continent. Waldseemüller drew the map of the earth showing that it is the fourth part of the earth. The kind is the continents, or the parts of the earth. The series is that of discovery, America was discovered fourth. Herschel reported the observation of a new comet. Lexell calculated its orbit and showed that it is not a comet but a planet. Laplace calculated the eccentricity of its orbit and proved that it is a planet, the seventh planet of the solar system. This is the kind, the planets of the solar system, and the series is both that of discovery and that of the objects themselves. Cavendish demonstrated that inflammable air is 11 times lighter than fixed air, and thus that it is a distinct air. Lavoisier called inflammable air hydrogen, and proposed that hydrogen is one of 33 simple substances. Mendeleev arranged 63 elements in their general system in which hydrogen is the first. The kind is the elements, the series is their series. Rawlinson, Hincks, and Oppert deciphered cuneiform, then Hincks and Oppert disentangled the languages written in it from one another and identified Sumerian as the first. The kind is the languages committed to writing, the series is their series. Thomson, in the *Cathode rays*, shows that: the ratio of mass to charge for cathode rays is 10^{-7} , compared to 10^{-4} for hydrogen ions in electrolysis, the smallest value of this quantity previously known (*id.*), and concludes: in cathode

rays matter is in a new state, this matter being the substance from which all chemical elements are built up (*id.*). The substance is the electron. The kind is the quantity, the ratio of mass to charge. The values of this quantity are the series. It is the series of the objects. And two generations after Thomson it was shown also what the kind of the electron is, it is lepton.

In each case the discovery has assigned an object to a kind. Given the relation of the objects which makes it a kind, the discovery has shown that the object is in this relation to the other objects of the kind. Given that their series and order cannot not be part of the relation of the objects, the discovery has shown the place of the object in their series and order.

The discovery of the first philosopher is a discovery like every other discovery, too, by being the final in a series of steps.

In general, the labors to distinguish the philosopher from everything else begin with Xenophanes. Our wisdom is better than the strength of men and horses. Then there is Heraclitus. The best, the wise, and the few, on one side, the asses, on the other. Then there are the wise, the wisest, and loving wisdom in Herodotus. Then Plato distinguished the philosopher sharply from everything else. Not with his multifarious general descriptions. With the doctrines he read in Heraclitus, Parmenides, and Protagoras, and his labors to improve on them. Finally, Aristotle described the kind, the formulation and solution of problems.

Hence, in particular, there is a first philosopher, and Thales is the first philosopher. The first philosopher is not one of the wise men, Solon, Thales, Chilon, Pythagoras, Oebares, Themistocles. Or one of the wise men philosophers Pittacus, Solon, Cleobulus, Thales, Anacharsis, Bias, Myson, Chilon, Anaxagoras. The first philosopher is the first in the series Thales, Anaximander, Anaximenes, Xenophanes, Heraclitus, Empedocles.

The kind is the formulation and solution of problems. The kind thus includes propositions, and not sentences. Prayers, for instance. Most particularly, the kind includes propositions in their relation with one another, propositions of which they are the consequences and propositions

which are their consequences.

Frege writes: the knowledge of a scientific truth goes as a rule through several stages of certainty; first surmised perhaps from an insufficient number of particular cases, a general proposition is more and more certainly established when through chains of inferences it is brought in connection with other truths as either consequences are derived from it or conversely it is recognized as the consequence of already firmly established propositions; this is the path by which a proposition is gradually won.

In book I of the *Metaphysics* Aristotle supplies the facts on which Frege's description of the path stands. Aristotle describes the initial stages of the path paradigmatically.

He writes: Thales said that water is the principle, Anaximenes said that air is prior and it is the principle, Heraclitus said that fire is prior and it is the principle, Empedocles added earth and said that all four, earth, water, air, and fire, are the principles.

Also: Thales said that water is the principle; this is why he also said that the earth floats on water.

Also: perhaps Thales said that water is the principle because he saw that the nourishment of everything is moist, or that heat is generated from moisture, or that the seeds of everything are moist; some even think that the ancients, who lived long before us and first spoke about the gods, held the same opinion about nature, for they made Ocean and Tethys the parents of creation and made the gods swear by the water called Styx.

And on these three descriptions and the one in the review in *On the heaven* the fact that Thales is the first philosopher and the fact that Aristotle discovered that Thales is the first philosopher both stand unshakably.

Aristotle showed that Thales proposed the first solution of the material principle of all things.

In addition, he showed that Thales proposed the first solution of the problem of the earth.

In addition, he showed that the second proposition can be brought through chains of inferences in connection with the first. Aristotle says: Thales said that the earth floats on water because he said that water is the principle.

In addition, he showed that the first proposition can be brought through chains of inferences in connection with earlier propositions. He assigns two kinds of reasons why Thales said that water is the principle. He says: perhaps Thales said that water is the principle because he saw that the nourishment of everything is moist, or that heat is generated from moisture, or that the seeds of everything are moist. These are Aristotle's surmises from an insufficient number of particular cases as to what Thales's surmises from an insufficient number of particular cases may have been. Aristotle says also, however: the ancients, who lived long before us and first spoke about the gods, held the same opinion about nature, for they made Ocean and Tethys the parents of creation.

And so the ancients, who lived long before us and first spoke about the gods, did. In particular, so did Homer, one and a half centuries before Thales. In the *Iliad*: Ocean is the begetter of the gods, Tethys is their mother (XIV, 201-4). And if Homer did not first speak about the gods, eight and a half centuries before him, in *When on high*: Apsu is the begetter, Tiamat bears all. Apsu is freshwater, Tiamat is seawater. And if *When on high* did not first speak about the gods, it is the first account of creation. And the connection of these propositions, presuming that they are propositions, explains why Thales said, not Ocean and Tethys are the parents of creation, but water is the principle of all things. The connection also shows how great the advance which Thales made is. It thus also poses the problem of explaining better why and how Thales made the advance. And that is, it poses the problem of bringing Thales's proposition through chains of inferences in connection with more propositions affirmed earlier as either consequences are derived from it or it is recognized as their consequence.

Thales is the first philosopher. The kind is the formulation and solution of problems.

And, hence, all other reports about Thales can be sorted out. What belongs to the philosopher, on one side, everything else, on the other.

Beginning with the five other occasions on which Aristotle notes Thales.

He notes him in another of the systematic reviews of his predecessors, in *On the soul*. The

following are – ignoring the Pythagoreans – the four earliest judgments on the soul in the review, in chronological order: Thales seems to have thought that the soul is a cause of motion, for he said that the lodestone has a soul because it moves iron; Heraclitus said that the principle is soul, a warm exhalation from which all things come, incorporeal and in ceaseless flux; Alcmaeon said that the soul is immortal, because it is in ceaseless motion and all divine things, the moon, the sun, the stars, and the whole world, are in ceaseless motion; Anaxagoras said that soul is the cause of the motion of things, although what exactly he said is not clear; in many places he said that thought is the cause of order and beauty, in other places that soul is the cause.

Thales's is a fine proposition, the lodestone has a soul because it moves iron. But it belongs in philosophy not more than Homer's line: Ocean and Tethys are the parents of creation. Given only the four judgments above, Thales's, Heraclitus's, Alcmaeon's, and Anaxagoras's, the formulation and solution of the problem of the soul begins with Heraclitus, not with Thales. Because of the many connections, amply emphasized by Aristotle, of what Heraclitus said with what Alcmaeon and Anaxagoras said. And because of the many connections, amply attested by his fragments and corroborated by Aristotle, of what Heraclitus said about the soul with what he said about λόγος, fire, and all things. The connection which Aristotle proposes of what Thales said with what his successors said is too tenuous. And there is no connection at all of what he said about the soul with his two other propositions, those about water. Thus about the soul. As to the lodestone, the formulation and solution of the problem of the lodestone begins with Gilbert, twenty-one and a half centuries after Thales, nineteen after Aristotle. In *On the magnet*, Gilbert begins with a review: of the opinions and delusions of ancient and modern writers (I, 1). In particular, he has one opinion to review: the ancient writers record that the lodestone attracts iron (*id.*). And only one: but its other properties were all hid, and lest the story of the lodestone be too jejune and too brief, to this sole property fictions and falsehoods were appended and fed to mankind in the early times not less than they are

today (*id.*). The proposition, the lodestone has a soul because it moves iron, does not belong in philosophy, it belongs among the fictions and falsehoods fed to mankind in the early times not less than they are today.

Later in *On the soul*, continuing the examination of what our predecessors have said, Aristotle writes: some have thought that soul is intermingled with everything in the world, and perhaps for this reason Thales said that all things are full of gods (I, 5).

It is a fine proposition, too, all things are full of gods. This proposition has been fed to mankind rather more resolutely than the proposition about the lodestone. This proposition yet belongs in philosophy not more than that. The formulation and solution of the problem of the gods begins with Xenophanes.

In the *Politics*, discussing property, and the means of amassing wealth, in particular, Aristotle illustrates with the anecdote: reproached for poverty, Thales predicted a good season for olives through his knowledge of the stars, hired all olive presses in Chios and Miletus in advance, and, cornering the oil market, made a fortune; the moral that goes with this story is that the philosophers can become rich if they want to; but the story shows also that to establish a monopoly is one of the means of amassing wealth (I, 11).

Edifying as the anecdote is, including in that its moral contradicts the moral of the anecdote of the well, it belongs in philosophy less even than the fictions fed to mankind. In any case, the formulation and solution of the problem of property begins with Aristotle.

In the *Nicomachean ethics*, deep into the compendium of virtues which books III-VII are, in order to distinguish wisdom from prudence, in particular, Aristotle has the illustration: men like Anaxagoras and Thales are said to be wise but not prudent because they show no concern for their own interest, and people may admire them for knowing marvelous and even divine things but would say that their knowledge is useless as it is not of what is good for men (VI, 7).

That Thales is said to be wise but not prudent, not even edifying as it is, belongs in philosophy even less than the anecdote of the fortune-making Thales. In any case, the formulation and solution

of the problem of virtues and virtue cannot predate Socrates.

And in the *Politics*, in yet another of the systematic reviews, Aristotle reports also this: some say that Onomacritus was the first to have a special skill in lawgiving, Thales became his companion, and Lycurgus and Zaleucus were students of Thales, and Charondas was a student of Zaleucus; but all this confuses the times of these men (II, 12).

It confuses the times of the real among these men, and confuses the real among them with not real. Whether such confusion belongs in philosophy should be clear. In any case, while lawgivers predate the first philosopher by seventeen and a half centuries, counting back to Urukagina, the first prince to leave behind inscriptions concerned with laws, the attempts of the philosophers to formulate and solve problems of lawgiving – even if, clearly, to formulate and solve them better than statesmen have, or ever will – do not predate Protagoras.

Thus Aristotle.

Plato reports that Thales kept away from the affairs of the city, that his Spartan wisdom is shown by the brief saying, know yourself, that he is famous for useful discoveries, and that he fell into a well.

The saying, know yourself, now on everybody's lips, became philosophy, even if it became philosophy irreversibly, first with Socrates. The rest, keeping away from the affairs of the city, being famous for useful discoveries, and falling into a well, belongs to the wise man. To the philosopher other things belong.

Herodotus reports that Thales predicted an eclipse, that he was said to have diverted the largest river in Asia Minor from its course, and that he advised the Ionians.

All three belong to the wise man, not to the philosopher. Including the great and magnificent prophetic deed of the eclipse.

Finally, with respect to the conclusions which Theophrastus and Eudemos drew from the fact that Thales is the first philosopher, Thales introduced physics among the Greeks, and, with Theophrastus, so Thales did. With the two problems, of the material principles and of the earth. Thales also introduced mathematics among the Greeks,

and, with Eudemus, so Thales did. Pace Eudemus, however, not with the prediction of eclipses, or the determination of the solstices. With the measurement of the distance of ships from the shore and the measurement of the height of the pyramids. And the formulation and solution of mathematical problems goes back to the 3 oldest calculations of area, twenty-five centuries before Thales.

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It is clear why Aristotle discovered the first philosopher. He was looking for him.

It is clear also how he discovered him. He was following the true way. He writes: we shall try, to the best of our abilities, to omit no member of their kingdom, however lowly. And he collected 600 kinds of animals. He writes: we will study in the constitutions we have collected what kinds of influences preserve and destroy particular constitutions, then we will be better able to judge which constitution is the best. And he collected the constitutions of 158 states, not less than one half of all contemporary states. He writes: in order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous. And he collected, nearly enough, all members of the council.

Aristotle thus discovered the first philosopher, and Aristotle thus discovered many other philosophers.

Neither of the two propositions, water is the principle, and the earth floats on water, like wood, is noted before Aristotle.

And none of these 9 philosophers is noted before Aristotle,

611 -	541	Anaximander,
590		Anaximenes,
532		Alcmaeon,
519		Hippasus,
480		Leucippus,
479		Cleidemus,
470		Hippo,
460		Democritus,
460		Hippocrates of Chios.

There is nothing at all in the record before Aristotle about any of the 9, not even about the respective men, and the record is by that time

voluminous.

More particularly, and more important, there is nothing about any of the 9 in Herodotus or Plato. The first 4 predate Herodotus by more than one generation. All 9 predate Plato by more than one generation. And 6 of the 9 are very important philosophers.

Why and how Aristotle discovered them the council of our predecessors shows. Counting each of the 16 systematic reviews 1 session of the council, Aristotle calls on Anaximander in 3 sessions of the council, on Anaximenes in 3 sessions, on Alcmaeon in 2, on Hippasus in 1, on Leucippus in 5, on Hippo in 2, on Democritus in 12, on Hippocrates of Chios in 1.

And below is some of what Aristotle discovered, and of what his successors discovered. And what his successors discovered is of two kinds. Most of it is derived explicitly from Theophrastus and Eudemus. The rest is to be presumed derived directly or indirectly from Theophrastus and Eudemus.

Anaximander said that the earth is at rest because of equilibrium (Aristotle, *Heaven*, II, 13), and it is shaped like a drum (Hippolytus, *Refutation*, I, 6). He drew a map of the earth (Strabo, I, 1.11). He discussed the sizes and distances of the planets (Simplicius, comm. *Heaven*, 471). He said that the stars originated in a ball of fire (Hippolytus, *Refutation*, I, 6), that the earth was wet at first, then began to dry up (Alexander of Aphrodisias, comm. *Meteorology*, 67), that the first living creatures were born from moisture (Aëtius, V, 19), that living creatures came from the moisture evaporated by the heat of the sun, and that man came from such a creature, and that is, from fish (Hippolytus, *Refutation*, I, 6). The second philosopher improved substantially on the first philosopher's solution of the problem of the earth. Hence, also, the formulation and solution of the problems, both, of the origin and development of the world and of the origin and development of living creatures, including man, begins with the second philosopher. And the second philosopher already anticipates the big bang theory and the theory of evolution, both.

Anaximenes said that air is prior to water and it is the principle (Aristotle, *Metaphysics*, I, 3). He said also that air is infinite (Simplicius,

comm. *Physics*, 24), that air is always in motion (*id.*), that air is invisible, and made visible by cold, hot, wet, and motion (Hippolytus, *Refutation*, I, 7), that all things come from air through rarefaction and condensation, rarefied air becomes fire, condensed it becomes wind, condensed further it becomes cloud, condensed further it becomes water, condensed further it becomes earth, and from these all other things come (*id.*), that the soul is air and it holds us together, as air encloses the world and holds it together (Aëtius, I, 3), that the planets are moved by gusts of thick air (II, 23), that the earth is flat, and it is at rest because it floats on the air below, like a lid (Aristotle, *Heaven*, II, 13). The third philosopher improved substantially on the first philosopher's solution of the problem of the material principles of all things. Hence, also, what principle is is clear with the third philosopher already. And the first three philosophers already show how greatly the acquisition of knowledge accelerated with the development of philosophy.

Alcmaeon said that the soul is immortal, because it is in ceaseless motion and all divine things, the moon, the sun, the stars, and the whole world, are in ceaseless motion (Aristotle, *Soul*, I, 2). But he also said that the senses are connected to the brain (Theophrastus, *Senses*, 26), and that the ruling power is in the head (Aëtius, V, 17). It is the discovery of the function of the brain. Alcmaeon also said that health is the equal rule of opposites, of moist and dry, cold and hot, bitter and sweet, and the cause of disease is the rule of one of them (Aëtius, V, 30). It is the oldest attempt to assign cause to disease. Alcmaeon also excised eyes to investigate their nature (Calcidius, comm. *Timaeus*, 279). It is the beginning of dissection.

Hippasus said that fire is the principle (Aristotle, *Metaphysics*, I, 3). Iamblichus also reports: the Pythagoreans were of two kinds, listeners and students, and the listeners did not recognize the students as Pythagoreans because, the listeners said, the students had received their doctrines not from Pythagoras but from Hippasus (*Pythagoras*, 18), and: Hippasus first revealed the construction of the dodecahedron to the uninitiated, and he was drowned at sea for that (*id.*). Iamblichus

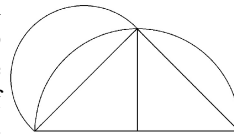
reports also: the listeners follow precepts as divine revelations, say nothing of their own, think it wrong to do so, and consider whoever knows the greatest number of precepts the wisest (*Knowledge*, 25; *Pythagoras*, 18). Hippasus, then, is associated with the construction of the dodecahedron. He is also the first philosopher who had the opportunity to be a listener, and did not avail himself of the opportunity. The first also to be drowned at sea for that.

Cleidemus said that lightning is only an appearance, just as the sea appears to shine when struck with a rod at night (Aristotle, *Meteorology*, II, 9). He said also that plants are composed of the same elements as animals (Theophrastus, *History*, III, 1).

Hippo said that the soul is water (Aristotle, *Soul*, I, 2). He was also an atheist (Plutarch, *Notions*, 31).

Hippocrates of Chios said that comets are planets, the tail is only an appearance due to the reflection of the sun's light by the moisture which a comet attracts when it moves through the northern regions (Aristotle, *Meteorology*, I, 6). He also squared lunes (Aristotle, *Refutations*, 11). The different cases are recorded by Simplicius. The simplest is this: given that circles, and so also semicircles, are to one another in the ratio of the squares of their diameters, in the right isosceles triangle the semicircle on the side is equal to one half the semicircle on the base, and, subtracting the common area, the lune is equal to one half the triangle (comm. *Physics*, 56-7). It is the first solution, partially, of the problem of squaring the circle. Hippocrates showed also that to double the cube is to find two mean proportionals: for, if $\frac{a}{b} = \frac{b}{c} = \frac{c}{2a}$, then $\frac{a^3}{b^3} = \frac{1}{2}$ (Eutocius, comm. *Sphere*, II). Hippocrates also wrote the first *Elements* (Proclus, comm. *Elements*, 66). The first treatise in which theorems were deduced from axioms. Hippocrates of Chios is the most important mathematician in the one and a half centuries-odd from Pythagoras to Eudoxus.

Finally, Leucippus formulated the theory of atoms and void. Aristotle writes: in order to save



the phenomena against those who argued that all is one and abolished the multiplicity, motion, generation, and corruption of things, Leucippus formulated the theory of atoms and void (*Corruption*, I, 8). Democritus developed the theory. The importance of the theory of atoms and void cannot be exaggerated. And Aristotle discovered the importance of the theory of atoms and void. He discusses Leucippus in 5 of the 16 systematic reviews, Democritus in 12 of the 16. Plato does not note Democritus. Aristotle discusses him 60+ times.

Even more dramatic than the discovery of these philosophers not noted before him, however, are Aristotle's discoveries among the philosophers noted before him. Because they are noted before him.

Empedocles formulated the theory of the material principles of all things which remained the theory of the material principles of all things for two millennia. He also first distinguished sharply the efficient principles of all things, love and strife, from the material, earth, water, air, and fire. Plato notes Empedocles 2 times. In the *Meno*: Empedocles said that there are effluences from things (76c), and to enlist him in one of his armies in the *Theaetetus*: with the exception of Parmenides, all wise men in the past, Protagoras and Heraclitus and Empedocles and the masters of the two kinds of poetry, Epicharmus in comedy, Homer in tragedy, have said that nothing ever is and all things are always becoming. Aristotle discusses Empedocles in 9 of the 16 systematic reviews, and discusses him all in all 120+ times.

And yet more dramatic are Aristotle's discoveries among the lawgivers. Because his predecessors have very much to say about lawgivers and lawgiving.

Aristotle discovered Draco. The oldest Athenian lawgiver is not noted before Aristotle.

Solon is noted. Herodotus reports: Solon made laws for the Athenians. Plato – the great-great-grandson of Solon's brother – notes him often. He notes Solon's laws: is any city governed well because of the laws which Homer gave it, as Sparta is because of the laws of Lycurgus, Sicily is because of the laws of Charondas, Athens is because of the laws of Solon? no, Glaucon re-

plies. He notes his wisdom: Solon was the wisest of the seven wise men (*Timaeus*, 20d). And if, know yourself, is Thales's saying now on everybody's lips, nothing too much, is Solon's saying now on everybody's lips. So it remained for Aristotle to establish that: Solon introduced democracy in Athens (*Politics*, II, 12), and to describe Solon's laws (*Constitution*, 5-12).

More generally, in *The constitution of Athens* Aristotle describes: the earliest Athenian magistracies (1-3), Draco's laws (4), Solon's laws (5-12), the tyranny of Pisistratus and his son Hippias (13-9), Cleisthenes's laws (20-2), and through the time of writing. The history of Athens, too, depends unconditionally on Aristotle.

As does the history of poetry. In the *Poetics* once again Aristotle first researched the origin and development of poetry (3-5). And established that: Aeschylus increased the number of actors to two; Sophocles increased the number to three (4).

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The striving for truth is what forces us to advance from the sense to the reference.

And so it did to the reference of 'philosopher' and 'philosophy'.

The striving is very much in evidence in the advance from Xenophanes to Heraclitus to Herodotus to Plato to Aristotle. And in Aristotle, at the end of the path, it can hardly have been more explicitly in evidence.

Philosophy is the search for truth.

And this is an empirical generalization. Counting from the first philosopher, over two and a half centuries of the search for truth.

We know something when we know that its cause is its cause and cannot be otherwise, we know it by demonstration, and demonstration always depends on true principles.

This is also an empirical generalization. Counting from the first philosopher, over two and a half centuries of the discovery of causes, by demonstration, from true principles.

We have dealt with the first principles, with natural motion, with the heavenly bodies and their order, and with the elements, we have to deal now with the phenomena in the region closest to the heavenly bodies, and when we next deal with animals and plants, our task will be

complete.

In each case by first calling the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

The council itself is an empirical generalization.

Thales is the first solution of the problem of the material principles. Anaximenes is the next and better solution. Heraclitus is the next and better solution. Empedocles is the next and better solution. Thales is also the first solution of the problem of the earth. Anaximander is the next and better solution. Anaximenes is the next and better solution. Xenophanes is the next solution, although he saved himself the trouble of looking for causes. Thales is also the proposition that the lodestone has a soul because it moves iron. Heraclitus is the proposition that the soul is exhalation, incorporeal and in ceaseless flux. Alcmaeon is the proposition that the soul is immortal because it is in ceaseless motion. Anaxagoras is the proposition that soul is the cause of the motion of things, although what exactly he said is not clear.

The knowledge of a scientific truth goes through stages of certainty. First surmised from an insufficient number of particular cases, a proposition is more and more certainly established when through chains of inferences it is brought in connection with other truths as either consequences are derived from it or it is recognized as the consequence of already established propositions. This is the path by which a proposition is gradually won.

And this is also an empirical generalization. Counting from the first philosopher, over twenty-four centuries of the winning of propositions.

All true propositions have the same reference, the true.

Philosophy is the search for truth.

The philosophers have not only found truth, they have amassed it.

Those who search for truth find singly little or nothing, but jointly they amass much, and from those who have done it better we have the solutions of problems.

The reference of 'philosopher' is the objects Thales, Anaximander, Anaximenes, Pythagoras, Xenophanes – the first philosopher, the second,

the third, the fourth, the fifth – all of them propositions, the propositions in their connection with one another the formulation and solution of problems – and all the way to the last philosopher.

The reference of 'philosophy' is the series of these objects.

The analogy with how the references of 'continent', 'planet' – including 'the third planet' – 'element', 'language committed to writing', 'elementary particle' have been fixed holds. In each case an empirical generalization has fixed the reference.

And in this case as in those the reference has been fixed for good.

'Philosopher' and 'philosophy' were endowed with boundless sense by Plato already, and have continued to be saturated with ideas and feelings.

Whenever philosophy has advanced, however, synonymously the search for truth has advanced. The philosophers have won propositions, have formulated and solved problems.

This is how philosophy advanced through Ptolemy.

This is how philosophy emerged from the darkest of dark ages. From Abelard through Roger Bacon, Aquinas, Occam.

And this is how philosophy has continued to advance.

Copernicus.

Viète, Gilbert, Tycho.

The brightest period in the acquisition of knowledge is the brightest period in the search for truth, the winning of propositions, the formulation and solution of problems. Bacon, Galileo, Kepler, Descartes, Huygens, Spinoza, Newton, Leibniz.

Hume, Kant.

Lavoisier, Laplace.

Darwin.

Thermodynamics, electrodynamics.

Frege.

Relativity, quantum theory, the big bang theory.

And this is also an empirical generalization. Counting from the first philosopher, over twenty-six centuries of the acquisition of knowledge. Counting from the predecessors of the philosophers, from the 3 oldest calculations of area, over

fifty-one centuries of the acquisition of knowledge.

ccxxxiv

The development of philosophy has included the acquisition of knowledge.

The task is to collect the particulars, and marshal them in their series and order, and to this end, first, establish the ground on which to do so.

And the review in book I of the *Metaphysics* is suited perfectly to the task. We have to adopt the sound in what Aristotle has said and nearly all in what he has said is sound, but not all, there is a little that is erroneous and we have to avoid that.

In particular, the task is to collect their predecessors and the philosophers in the review, separate the important among them, and the steps of advance in the acquisition of knowledge among these.

And to begin with the collection of the particulars, or what is the same, of the members of the council participating in the session on the principles of all things, there are two possible misunderstandings to clear away first, and two difficulties to resolve, one minor, one major.

While criticizing the theory of the forms late in the review, Aristotle writes: if the forms are numbers, how can they be principles? man is one number, Socrates another number, Callias another number (9)?

And Socrates is a member of the council not because he is one of the placeholders in the proposition, man is one number, Socrates another number, Callias another number, but because earlier in the review Aristotle writes: Socrates abandoned physics, devoted himself to ethics, and while searching for the general in ethical matters, first concerned himself with definitions. Nothing but a placeholder, Callias is not a member of the council.

Nor is Hermotimus.

Aristotle writes: wood does not make beds and bronze does not make statues; there are efficient principles; and this was discovered when someone, like a mature man among babblers, said that there is reason in nature, and it is the cause of the world and its order; we know that Anaxagoras said so; some say that Hermotimus said so earlier.

The mature man among babblers who discov-

ered that wood does not make beds is Anaxagoras. Hermotimus is not even a man, never mind a mature man among babblers. Even if this jibe of Aristotle's, unlike most of his other jibes, may have since lost some of its punch.

There is little about Hermotimus in the record. What there is, however, is, to the degree to which anything involving transmigration can be unambiguous, unambiguous. One thing Hermotimus did was this: his soul would leave his body, wander about, then come back and tell him what it had seen (Pliny, VII, 52). Another, if it is another, was that he was Pythagoras. Or Pythagoras was him. Ponticus reports: Pythagoras was first Aethalides, a son of Hermes, next he was Euphorbus and was wounded as such by Menelaus at Troy, next he was Hermotimus, next he was the Delian fisherman Pyrrhus, and only then he became Pythagoras (fr. 89).

More important, there are two entities in the review, the ancients and the Pythagoreans, which will each claim a seat in the council, but neither of which must be allowed in the council.

Aristotle writes: the earliest philosophers took the principles of all things to be only material, and the originator of this kind of philosophy is Thales who said that water is the principle; some think that the ancients, who lived long before us and first spoke about the gods, held the same opinion about nature, for they made Ocean and Tethys the parents of creation and made the gods swear by the water called Styx.

Entities committed to no particular place and time have been eager to populate philosophy ever since Plato began enlisting his predecessors in motley armies. So he brought into being the entities of the Heracliteans, and the Eleatics, and especially the entity of peddlers, the sophists. So since have been brought into being the entities of the Academics and the Stoics, of an Ionian philosophy and an Italian philosophy, of an Egyptian mathematics and a Babylonian mathematics, and especially the largest of all entities, the ancients and the moderns. For the more indefinitely larger an entity has been the better it has served the other way, the one of obscuring, and obfuscating, the particulars and their series and order. The fact of development of philosophy, the facts of steps in the development, the facts of advance in the

development, the facts of resistance to the advance, are one and all facts about the particulars and their series and order.

Entities committed to no particular place and time must not be allowed in the council. All such entities must be discarded.

In one of two ways. From some, like the Pythagoreans, the particulars which such entities obscure must be recovered, then the entities discarded. From others, like the ancients, nothing but confusion can be recovered, and such entities must be discarded out of hand.

As Aristotle's report attests, the ancients were ready to obscure the fact of the beginning of philosophy at Aristotle's time already. Thales said that water is the principle. But some thought, and still think, that the ancients, who lived long before us, held the same opinion about nature.

The two lines which Aristotle provides as evidence can be recovered from Homer. Homer said that Ocean and Tethys are the parents of creation (*Iliad*, XIV, 201-4), and that the gods swear by the water called Styx (270-1). The entity ancients can thus be replaced by the particular Homer.

It is very much better, however, not to dignify the ancients with the supposition that they have anything to do with the particulars. And the other way around, not to contaminate any particular with the supposition that it has anything to do with the ancients.

Such entities not only thrive on uncertainty and breed uncertainty, they thrive on confusion and breed confusion, with the attendant obfuscation. Never mind that Aristotle has become ancient, and all moderns, and all postmoderns, will become ancients. The entity ancients has never lost its appeal as a means of analysis of the past because it not only confuses young and old with old and new, it readily substitutes the latter for the former. So that the young be old and the old be new. Bacon writes: the old age of the world is the true antiquity, and such is our own age, not the earlier age in which the ancients lived; that age is only with respect to us the older, with respect to the world it is the younger (*Organon*, I, 84).

The ancients must be discarded out of hand.

The Pythagoreans, on the other hand, present

the major difficulty before the collection of the particulars in the review. Because Aristotle did not resolve this difficulty. To borrow from him, the problem of the beginning of change Leucippus and Democritus, like the others, lazily neglected, the problem of breaking apart the entity Pythagoreans and recovering the particulars from it Aristotle, like Plato, lazily neglected.

Others did not, though. Ponticus, in the generation of Aristotle, first wrote on Pythagoras and the Pythagoreans. Another of Aristotle's students, Aristoxenus, wrote on Pythagoras and the Pythagoreans. There are all the reports derived from Theophrastus and Eudemus. There are independent reports on Pythagoras and the Pythagoreans, beginning with Xenophanes, Heraclitus, and Herodotus. Laertius, Porphyry, and Iamblichus wrote on Pythagoras and the Pythagoreans. The entity Pythagoreans can be broken apart and the particulars recovered.

This is done explicitly below.

Here, the only particular that has to be recovered before the entity is discarded is Pythagoras.

Aristotle writes: the Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this science.

Pythagoras, not the Pythagoreans, discovered proof, and Pythagoras, never mind the Pythagoreans, greatly advanced mathematics.

On the authority of Eudemus, Proclus reports: Pythagoras discovered the incommensurability of the side and diagonal of the square (comm. *Elements*, 65). And the incommensurability of the side and diagonal of the square is inaccessible to anything but proof. Aristotle writes: that the side and diagonal of the square are incommensurable is demonstrated by showing that, if they are supposed commensurable, even numbers turn out to be odd (*Pr. analytics*, I, 23).

Thus:

if they are commensurable, their ratio is rational;

let it be $\frac{p}{q}$ in least terms,

hence, $2p^2 = q^2$,

q^2 is thus even, q is thus even, and as $\frac{p}{q}$ is in

least terms, p is odd,

q is even, so let $q = 2r$,

then, $2p^2 = 4r^2$, and $p^2 = 2r^2$,

p^2 is thus even, and p is thus even, but it is also odd.

Pythagoras discovered proof, and he greatly advanced mathematics.

Aristotle continues: from the study of mathematics they came to think that its principles are the principles of all things; and since among these principles numbers are by nature first, they saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water; so they said that numbers are the principles of all things, and the whole world is a harmony and number.

Pythagoras, not the Pythagoreans, discovered the most important proposition, number is the principle of all things.

Iamblichus lists the precepts which Pythagoras gave to his followers, and among the precepts is: what is the wisest? number (*Pythagoras*, 82). Drawing on Theophrastus, Aëtius reports: Pythagoras said that of the principles 1 is god and good, 2 is a demon and bad (I, 7).

Pythagoras discovered that number is the principle of all things. And the rest of Aristotle's report corroborates this conclusion. Pythagoras first advanced mathematics, and from its study came to think that number is the principle of all things, and that the whole world is a harmony and number.

Aristotle continues: it is clear that these thinkers take number to be the principle both as the matter of things and as the form of their modifications and changes.

It is clear that Pythagoras takes number to be both the material principle and the formal principle of all things.

Aristotle continues: other Pythagoreans said that the principles are ten, and arranged them in pairs, limit-unlimited, odd-even, one-many, right-left, male-female, rest-motion, straight-curved, light-darkness, good-bad, square-oblong; and Alcmaeon borrowed his doctrines from these thinkers, or they borrowed theirs from him; for he said that most human things go in pairs, white and black, sweet and bitter, good and bad, large and small; but all Alcmaeon said was vague hints.

These other Pythagoreans can be ignored. The more so if Alcmaeon borrowed his doctrines from them or they borrowed theirs from him. The more

so if all Alcmaeon said was vague hints.

Aristotle writes also: Plato agrees with the Pythagoreans that all other things are by participation in numbers; but what this participation is remains unclear.

If Plato agrees with the Pythagoreans that all other things are by participation in numbers, Plato agrees with Pythagoras that all other things are by participation in numbers. The more so if the Pythagoreans would agree with Pythagoras. The more so if what this participation is remains unclear.

Finally, when late in the review Aristotle criticizes four solutions of the problem of the principles at greater length, those of Empedocles, Anaxagoras, the Pythagoreans, and Plato, thus indicating which the most important solutions of the problem are, the Pythagoreans are not important, important is Pythagoras.

The remaining 19 particulars in the review are indicated clearly and distinctly. Including Pythagoras in place of the Pythagoreans, in chronological order, all 20 are,

734	Hesiod,
625	Thales,
590	Anaximenes,
570 - 495	Pythagoras,
570 - 473	Xenophanes,
540	Heraclitus,
532	Alcmaeon,
519	Hippasus,
515 - 445	Parmenides,
500 - 428	Anaxagoras,
495 - 435	Empedocles,
481	Melissus,
480	Leucippus,
470	Hippo,
469 - 399	Socrates,
463	Diogenes of Apollonia,
460	Democritus,
449	Cratylus,
427 - 347	Plato,
408 - 356	Eudoxus.

All 20 are important, but not all 20 because of the facts which Aristotle reports about them in this review. And 14 of the 20 are very important but again not all 14 because of the facts which Aristotle reports about them in this review.

Aristotle begins: let us call on our predeces-

sors who have investigated being and searched for truth and see what they have said about the principles, for they can help us either find another kind or reassure ourselves that we have determined the kinds correctly.

He concludes: they grasped the material; no one grasped clearly the formal, but some of them grasped it in some way; the efficient they also grasped in some way; and those who spoke of reason and of love as principles grasped the final in some way as well; even if no one expressed these kinds clearly as principles.

And the review divides into three parts. The first shows how they grasped the material principles. The second shows how they grasped the efficient principles. The third shows how they grasped the formal principles. And there is no fourth part. That they grasped the final principles nothing but Aristotle's concluding remark indicates.

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The material principles.

Aristotle writes: Thales said that water is the principle; Hippo we can ignore because of the paltriness of his thought; Anaximenes and Diogenes of Apollonia said that air is prior to water and it is the principle; Hippasus and Heraclitus said that fire is prior and it is the principle; Empedocles then added earth to these and said that all four, earth, water, air, and fire, are the principles.

The order is qualitative, from the first solution, water, to the better solution, air, to the better, fire, to the best, earth, water, air, and fire.

This explains why Hippo is noted after Thales. He said that water is the principle (Hippolytus, *Refutation*, I, 16). He said so one and a half centuries after Thales, one generation after Empedocles, and we can ignore him on this ground, because of the paltriness of his thought. Why Hippo said so, however, can also be explained. Aristotle writes in the review in *On the soul*: Hippo, among the more naive thinkers, said that the soul is water (I, 2). And, with all due respect to Aristotle, this thought of Hippo's is a very fine thought, considering what all too many other philosophers have thought about the soul. Nor was Hippo among the more naive thinkers in this way: he was an atheist.

Ignoring Hippo nevertheless, and arranging them in their chronological order, the 6 solutions of the problem of the material principles, which Aristotle notes in this part of the review, are,

Thales said that water is the principle,

Anaximenes said that air is prior to water and it is the principle,

Heraclitus said that fire is prior and it is the principle,

Hippasus said that fire is the principle,

Empedocles added earth to these, and said that all four, earth, water, air, and fire, are the principles,

Diogenes of Apollonia said that air is the principle.

In book II of the *Physics* Aristotle defines material principle: that out of which a thing comes to be and which persists.

This is how Lavoisier defines it: if to the name elements, or principles of bodies, we attach the idea of the last terms which analysis has reached, then all substances which we have not yet been able to decompose by any means are elements (*Treatise*, pr.).

It is on this ground that Thomson concludes: in cathode rays matter is in a new state, this matter being the substance from which all chemical elements are built up.

Lavoisier's test, the last terms which analysis has reached, is anticipated in the relation, prior to.

Anaximenes said that air is prior to water. Heraclitus said that fire is prior. And when Empedocles added earth and said that all four are the principles, the four are prior to all other things, but none of the four is prior to any of the other three.

Empedocles sings:

the four roots of all things are

bright Zeus, life-giving Hera, Hades, and

Nestis whose tears make springs for men (fr. 6),

more explicitly:

the sun, the earth, the sky, and the sea (fr. 22),

and still more explicitly:

fire, water, earth, and the immense height of air
(fr. 17).

Also:

there is nothing beside these to become or to
perish,

for if they perish they will no longer be;
can anything increase the all? where will it come
from?
and how will it perish? no, nothing can be void
of them;
there is nothing beside them, they run through
one another,
become now this now that, yet always remain the
same (*id.*).

Can anything increase the all? where will it come from? and how will it perish? No, nothing can be void of them. Empedocles anticipates the conservation of matter, no less.

And Empedocles's remained the last terms which analysis has reached for the following two millennia, until Paracelsus, van Helmont, and Boyle. Not to mention that Hegel found his solution of the problem of the material principles of all things more philosophical than Lavoisier's.

Himself, Aristotle did not improve on this solution of a problem by his predecessors, as he did on so many other, very substantially. He yet improved on it. In *On generation and corruption* he writes: the elements are four; not fire, air, water, and earth, but simpler bodies similar to them. Adopting the sound in what Empedocles said as well as what, after Empedocles, Leucippus and Democritus said. In *On the heaven* Aristotle also writes: because the primary motion is circular, but the motion of the composite bodies below the sphere of the fixed stars is determined by the elements prevailing in them, we conclude that there is another element beyond the four we see on earth, aether.

Thus, on the ground of what they have said about the material principles of all things, it can be concluded that all 6 – Thales, Anaximenes, Heraclitus, Hippasus, Empedocles, and Diogenes of Apollonia – are important.

Hippasus affirmed that fire is the principle in the same generation as Heraclitus.

Diogenes of Apollonia made the attempt to restore the principles to greater simplicity. Three generations before Empedocles, Anaximenes said: all things come from air through rarefaction and condensation, rarefied air becomes fire, condensed it becomes wind, condensed further it becomes cloud, condensed further it becomes water, condensed further it becomes earth, and from these all other things come. One generation

after Empedocles, Diogenes of Apollonia said: all things are modifications of the same thing and are the same thing, otherwise air, fire, water, and earth could not mix and nothing could come to be (fr. 2); air has mind and steers all things, it is god and in all things (fr. 5); without mind it could not have been divided so as to keep the measure of all things; all things are arranged in the best possible way (fr. 3).

And it is clear that the remaining 4 are steps of advance in the acquisition of knowledge. Thales, Anaximenes, Heraclitus, and Empedocles.

The efficient principles.

Aristotle concludes the first part of the review, showing that our predecessors have grasped the material principles, with Anaxagoras: Anaxagoras said that the principles are an infinite number of particles, as each thing is generated from its own particles and destroyed into its own particles.

And continues: from what these thinkers said it could appear that the principles are only material; but as they advanced, the investigation itself showed them the way and forced them to go further; wood does not make beds; there are efficient principles; and this was discovered when Anaxagoras said that reason is the cause of the world and its order; and it may be that the first to search for this kind of thing was Hesiod, or whoever first spoke of love as a principle, as Parmenides also does, implying that there must be a cause of the motion and combination of things; later Empedocles made love and strife principles, positing not one source of motion but two, as he was the first also to speak of four material principles; Leucippus and Democritus next made atoms and void the elements, and said that the modification of all things is due to the shape, order, and position of these elements; and this is how far the earlier thinkers have advanced the investigation of these two kinds of principles.

It is a more difficult problem, the efficient principles, substantially more difficult. Aristotle thus has to investigate what our predecessors said about them at greater length. As they advanced in the investigation of the material principles, the investigation itself showed them that there are efficient principles. But as the investigation had shown them which the material principles are so it had to show them also which the efficient

principles are. It had to show them also what the relation of the efficient principles and the material is. And it is more difficult for Aristotle, and it remains more difficult, to show clearly what the investigation showed them about the efficient principles than it is to show clearly what it showed them about the material principles.

There are thus 6 anticipations and solutions of the problem of the efficient principles, which Aristotle notes in this part of the review, 2 anticipations,

Hesiod spoke of love as a principle, implying that there must be a cause of the motion and combination of things,

Parmenides spoke of love as a principle, and, in their relation to those of the problem of the material principles, 4 solutions of the problem of the efficient,

Anaxagoras said that an infinite number of particles are the principles, as each thing is generated from its own particles and destroyed into its own particles, and reason is the cause of the world and its order,

Empedocles said that earth, water, air, and fire are the principles, and love and strife are the sources of motion,

Leucippus and Democritus said that atoms and void are the principles, and the modification of all things is due to their shape, order, and position.

In book II of the *Physics* Aristotle defines efficient principle: the beginning of the change or rest of a thing.

And on the solution of this problem by his predecessors Aristotle improved very substantially. He writes in the *Physics*: the earth is in the water, the water is in the air, the air is in the aether, and the aether is in the world; but the world is one, there is nothing next to the world, and we cannot continue and say that the world is in something else. Also: everything in motion must be moved by something; and since an infinite series is impossible, there must be a first mover, itself unmoved. And in book XII of the *Metaphysics*: we conclude that the first mover is at the circumference of the world, moves the fixed stars, and thus communicates motion to everything further below as well.

It remained the solution of the problem for two

millennia, until Galileo, Kepler, Descartes, Huygens, and Newton.

It distinguishes clearly the efficient principle from the material principles. The first mover, on one side, the four bodies simpler than but similar to earth, water, air, and fire, and the fifth, aether, on the other side. The solution is very explicit also about the relation of the two. It packs the world, with the elements stacked on one another, from the side of the material principles, and with the motion communicated by the first mover to everything below in the order of the elements, from the side of the efficient principles.

Gravity is a very much better solution. It rests very much more on observation. It rests on laws. It is quantitative. Qualitatively as well, it is very much more explicit about the dependence of gravity and matter on one another. The law of gravity depends on the laws of motion, the first law of motion is the law of inertia, and inertia is inherent in matter. It is vaguer, however, qualitatively, as to which the efficient principle is, or the efficient principles are, gravity, or the forces. There is gravity in bodies universally. But gravity is a force, and the more general laws of motion are laws of forces. And it is much vaguer, qualitatively, as to which the material principles are. Galileo's, Descartes's, and Newton's search for the subtlest particles remained inconclusive. It will not be until quantum theory that it will become possible to point unambiguously to quarks and leptons, or the material principles, on one side, and the gravitational, strong, weak, and electromagnetic forces, or the efficient principles, on the other.

Thus, on the ground of what they have said about the efficient principles, of the 6 – Hesiod, Parmenides, Anaxagoras, Empedocles, Leucippus, and Democritus – 5 are important.

Hesiod says,

first of all Chaos came into being;
then big-bosomed Earth,
and Love, the fairest of the immortals,

implying that there must be a cause of the motion and combination of things. Elsewhere, turning to the problem of space in book IV of the *Physics*, Aristotle writes: Hesiod was right to begin with Chaos and say that first of all things was Chaos, implying that space, in which things are, must be

first (1). In general, Aristotle quotes Homer very often in order to make a point picturesquely. He does not quote Hesiod as often, but he does, as the two instances attest, in order to adopt the sound in what he said. One generation after Homer, Hesiod is a major step toward the first philosophers. The *Theogony* and *Work and days* are both investigations as much as they are poetry, of the descent of the gods from one another and of the times favorable and unfavorable for different work, respectively. In particular, one century-odd before Thales, Hesiod spoke of love implying that there must be a cause of the motion and combination of things.

Parmenides spoke of love implying that there must be a cause of the motion and combination of things one century-odd after Thales. Parmenides is not important on this ground, what he said about the efficient principles.

And 4 of the 5 are steps of advance in the acquisition of knowledge. Anaxagoras, Empedocles, Leucippus, and Democritus.

Anaxagoras discovered that there are efficient principles. He did not quite know what he was saying, and what exactly he said is not clear. But he said that reason is the cause of the world and its order. In many places he also said that thought is the cause of order and beauty, and in other places that soul is the cause. Moreover, his solution of the problem of the material principles, an infinite number of particles, as each thing is generated from its own particles and destroyed into its own particles, is a step toward the development of the theory of atoms and void.

Empedocles first distinguished clearly the efficient principles from the material:

fire, water, earth, and the immense height of air,
cursed strife separates them, equal in every way,
love unites them, equal in length and breadth

(fr. 17).

Equal in every way, equal in length and breath. Empedocles does anticipate conservation.

And the formulation of the theory of atoms is a very high point in the development of philosophy, and in particular of physics. In *On generation and corruption* Aristotle writes on Leucippus: in order to save the phenomena against those who argued that all is one and abolished the multiplicity, motion, generation, and corruption

of things, Leucippus formulated the theory of atoms and void (I, 8), and on Democritus: all earlier thinkers investigated generation and corruption superficially except Democritus; Democritus has not only thought about all problems, he is distinguished also by his method; and no one except him said anything clear about change (2). And in book I of the *Metaphysics*: Leucippus and Democritus made atoms and void the elements, and said that the modification of all things is due to the shape, order, and position of these elements, as A differs from N in shape, AN differs from NA in order, and N differs from Z in position (4).

Like the others, Leucippus and Democritus lazily neglected the problem of the beginning of change. Unlike the others, what Democritus said about change is clear. Aristarchus anticipates the heliocentric theory eighteen centuries before Copernicus. Democritus anticipates the atomic theory twenty-two centuries before Dalton.

The formal principles.

Aristotle writes: the Pythagoreans devoted themselves to mathematics; they were the first to advance this science, and from its study they came to think that its principles are the principles of all things; and since among these principles numbers are by nature first, they saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water; so they said that numbers are the principles of all things; and it is clear that these thinkers take number to be the principle both as the matter of things and as the form of their modifications and changes; Alcmaeon said that most human things go in pairs, white and black, sweet and bitter, good and bad, large and small; but all Alcmaeon said was vague hints; these thinkers tried to explain change, others said that there is no change in the world; of these Parmenides said that all is one in form, Melissus said that it is one in matter, while the first of them, Xenophanes, said nothing clearly, just looked at the sky and said that the one is god; Xenophanes and Melissus we can ignore as too naive; Parmenides spoke better, he said that only what necessarily is is and nothing else is; forced to account for the phenomena, however, he said that, even though all that is is one necessarily, it appears as many and as such

has two principles, fire and earth, associating fire with what is, earth with what is not; after these philosophers came Plato; in his youth Plato became acquainted with Cratylus and the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge of them, and Plato held this view in his later years as well; in the meantime Socrates had abandoned physics altogether, and devoted himself to ethics; and while searching for the general in ethical matters, he first concerned himself with definitions; Plato followed Socrates; he presumed, however, that definitions cannot be of the sensible things because the sensible things are always changing; definitions have to be of entities of another kind; these entities he called forms; the sensible things are named after the forms, and the many sensible things called by the same name as a given form are by participation in that form; and since the forms are the principles of all things, he took their elements to be the elements of all things; as matter, the great and the small are the principles; as form, the one is the principle; because from the great and the small, by participation in the one, numbers come; and Plato agrees with the Pythagoreans that all other things are by participation in numbers; but what this participation is remains unclear; Plato's verdict, then, on the matter which we are investigating, is that the principles are of two kinds, formal and material, as the forms are the principles of everything else, and the one is the principle of the forms.

While criticizing Plato later in the review Aristotle notes also: Eudoxus also argued that the forms make the sensible things what they are in the way white gives whiteness to a mixture.

The formal principles are a still more difficult problem, substantially more difficult than the efficient principles. Aristotle thus has to investigate what our predecessors said about them at still greater length. As they advanced in the investigation of the material principles, and the investigation itself showed them that there are also efficient principles, it showed them that there are formal principles as well. As it had shown them which the material principles are, and it had to show them also which the efficient principles are, it had to show them as well which the formal principles are. As it had to show them also what

the relation of the efficient principles and the material principles is, it had to show them as well what the relation of the formal principles and the efficient and the material principles is. And it is more difficult for Aristotle, and it remains more difficult, to show clearly what the investigation showed them about the formal principles than it is to show clearly what it showed them about the efficient principles.

Anaxagoras said that reason or thought or soul is the efficient principle. But an infinite number of particles, as each thing is generated from its own particles and destroyed into its own particles, implies also the particles themselves as an efficient principle.

Similarly, it is because the modification of all things is due to their shape, order, and position that atoms and void are the efficient principles as well as the material.

Number is both the material and the formal principle. But also the efficient, if it is a principle as the form of the modifications and changes of things.

Indeed, all those thinkers tried to explain change. But the difficulty extends to these who said that there is no change in the world.

Parmenides said that only what necessarily is is and nothing else is. All is one in form.

Parmenides spoke better. But Melissus then said that all is one in matter. The material principle is the formal principle.

Socrates found the general in definitions.

But Cratylus argued that all sensible things are always in flux and there can be no knowledge of them.

So Plato presumed that definitions cannot be of the sensible things, they have to be of entities of another kind, and these he called forms. Plato demands discussion and criticism at greater length and in more detail than any of our predecessors. And what Plato said about the different kinds of principles proves the most difficult to reduce to clarity and distinction. Aristotle concludes: the forms are the principles of all things, the great and the small as material principles, the one as a formal principle, and the principles are thus of two kinds, formal and material, the forms are the principles of everything else, the one is the principle of the forms. And appearances to the

contrary, it is not all inconsistent, only difficult. The forms of the great and the small are the material principles, the form of the one is the formal principle, and thus the forms are the formal principles of everything else, the form of the one is the material principle of the forms, and in this way it is the formal principle, and in this way the forms are yet the principles of all things.

Ignoring, then, Xenophanes, Alcmaeon, and Eudoxus, but not Melissus, and replacing the Pythagoreans with Pythagoras, there are the labors of 6 predecessors of ours at grasping the formal principles, and in relation to their labors at grasping the efficient and the material principles, which Aristotle notes in this part of the review, Pythagoras advanced mathematics, from its study came to think that its principles are the principles of all things, saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water, and said that numbers are the principles of all things, and all other things are by participation in numbers,

Parmenides said that all is one in form, only what necessarily is is and nothing else is, and even though all that is is one necessarily, it appears as many and as such has two principles, fire and earth,

Melissus said that all is one in matter, Socrates first concerned himself with definitions, Cratylus argued the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge of them,

Plato presumed that definitions cannot be of the sensible things because the sensible things are always changing, definitions have to be of entities of another kind, the forms, the sensible things are by participation in the forms, and since the forms are the principles of all things, their elements are the elements of all things, as matter the great and the small are the principles, as form the one is the principle.

The solution of this problem, the formal principles, Aristotle advanced very far beyond his predecessors. And not so much with the definition of formal principle in book II of the *Physics*: the form and paradigm of a thing, and that is, the definition of what it is and its genus.

With his logical investigations. In which: it

was not that something was done before us, something was not done, nothing at all was done. Aristotle established the fundamental dichotomy of matter and form. He distinguished the two sharply. Matter is no determination whatsoever, form is any determination whatsoever. He established that form must be analyzed into propositions. He established what a proposition is. A proposition is a sentence affirming what is true or false. He established what the formal relation of propositions is. The formal relation of propositions is the inference. He established what demonstration is. Demonstration is an inference grounded in what is true.

All this conclusively. And – through Chrysippus, Abelard, and Occam – it remained the best solution of the problem for more than two millennia, until Leibniz, Kant, and Frege.

Hammer and chisel are good for working raw lumber, copperplate requires an etching needle.

And etching began with Aristotle. It was not that some etching was done before him, some etching was not done, no etching at all was done. Much hammer and chisel work was done, however, and etching could not begin before much hammer and chisel work had been done.

The grasping of the formal principles begins with the advance of mathematics. And the advance of mathematics will continue to lead their better grasping. In particular, the grasping of the formal principles begins with Pythagoras. Pythagoras discovered the most important proposition, number is the principle of all things. Water is not the principle of all things, and even if it is prior to water, air is not the principle of all things either. And the problem of the relation of number and air is solved affirmatively as well. There are more analogies with the things that are and become in number than in air. All other things are by participation in numbers, and what this participation is is not very clear, but it is a solution of the problem of the relation of all things to the principles.

Parmenides discovered the principle of contradiction. If the principle of contradiction is the principle of all principles, it cannot not be the principle of all things. If the principle is form, it cannot not be that all is one in form. But if all is one, there cannot be many. And if form does not change, there cannot be change. Change and that

there are many are only appearances, and only as such have different principles. With the discovery of the principle of contradiction Parmenides has taken a major step as well toward the discovery of propositions and of their relation of following from one another. He has discovered, too, that the principle of contradiction demands, come what may, consistency. The distinction between real and apparent is also a major advance toward the explicit formulation and solution of the problem of the relation of form and all things.

The fragments of Melissus attest, come what may, consistency. He says: it always was and will be; otherwise it is nothing; and nothing comes out of nothing (fr. 1); everlasting, it is unlimited; if it is unlimited, there is no void; if there is no void, it is full; if it is full, it does not move (fr. 7); if it is divided, it moves; if it moves, it cannot be (fr. 10); if it is, it is one (fr. 9). With Aristotle: Melissus said that all is one in matter. Melissus is a step in the generalization of the formulation of the problems of all things. All is one in form. Or all is one in matter. Hence, rather, how is all one in both form and matter?

Socrates first concerned himself with definitions. It is the decisive step toward the discovery of propositions and of their relation of following from one another.

Cratylus argued the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge of them. Xenophanes discovered that knowledge is not belief. Parmenides showed that knowledge depends on the principles:

there are two roads,
what is and cannot not be,
which is the road of truth and conviction,
and what is not is not and must not be,
which you will never know or utter.

Cratylus read in Heraclitus the doctrine that all things are in flux and there can be no knowledge of them. But there is the fact of knowledge, and knowledge depends on the principles. Hence Plato presumed that there are entities of another kind. He writes: if the beautiful itself always passed away, how could it be known by anyone? Aristotle writes: to argue, while searching for the principles of the sensible things, that there is some other kind of essence, is prattle.

Before that, however, Plato formulated the theory of the forms, and investigated its difficulties, too.

And Aristotle's advances in logic all depend on the theory of the forms and the investigation of its difficulties.

It can be concluded, on the ground of what they have said about the formal principles, that all 6 – Pythagoras, Parmenides, Melissus, Socrates, Cratylus, and Plato – are important.

Steps of advance in the acquisition of knowledge are 4 of the 6. Pythagoras, Parmenides, Socrates, and Plato.

The final principles.

The review divides into three parts, on the material principles, on the efficient principles, and on the formal principles. And there is no fourth part, on the final principles.

The final principles are different.

The definition of final principle in book II of the *Physics* is: the end of a thing or what it is for.

Bacon writes: final causes corrupt rather than advance the sciences; they are in relation to the nature of man, not in relation to the world.

One way or another, there is nothing in particular in the review that our predecessors said about these principles, not sound, not erroneous. There is, in fact, nothing at all, but Aristotle's concluding remark: they grasped the material principles; some of them grasped the formal in some way; the efficient they also grasped in some way; and those who spoke of reason and of love as principles grasped the final in some way as well.

It has to be concluded that with respect to the final principles of all things there are no particulars. Never mind important. Never mind steps of advance in the acquisition of knowledge.

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Aristotle examines the solutions of the problem of the principles in the chronological order of the discovery of the kinds of the principles. First the solutions of the problem of the material principles, then the solutions of the problems of the efficient and the formal principles. He examines the solutions of the problem of the material principles in the chronological order of the improvements on the solutions. He then examines the solutions of the problem of the efficient

principles in their chronological order, and then the solutions of the problem of the formal principles in their chronological order.

If the solutions are examined in their strict chronological order, however, more explicit conclusions can be drawn about the discovery of the kinds of the principles and their relations. Hence the relations of the solutions to the kinds of the principles can be made more explicit. And hence which are the steps of advance in the acquisition of knowledge can be determined more precisely.

First, then, there is the anticipation,
Hesiod love,
representing, here and below, the anticipations and the solutions in a line or two each, taking it that the line or two stand for the proposition, this is the principle, or these are the principles, and all the propositions with which this proposition has been brought in connection explicitly.

One century-odd after Hesiod, the problem of the principles of all things was discovered.

The first two solutions of the problem are,
Thales water,
Anaximenes air.

Given that the principles are of different kinds, and one of the kinds is material, Thales's and Anaximenes's are solutions of the problem of the material principles. But the discovery that the principles are of different kinds, and one of the kinds is material, was yet to be made. Pending this discovery, Thales's and Anaximenes's are solutions of the problem of the principles whatever their kinds, if they have kinds.

The next solution is,
Pythagoras number.

The discovery that the principles are of different kinds, and one of the kinds is material, another is formal, was still to be made. Pending this discovery, Pythagoras's is also a solution of the problem of the principles whatever their kinds, if they have kinds. Moreover, if it is an improvement on the earlier solutions, it is because it is a solution of the same problem as they are. Not water, because air is prior to water. Neither water nor air, because number is prior to both. Aristotle writes: the Pythagoreans saw in numbers more analogies with the things that are and become than in fire or earth or water. And if water and air

are solutions of the problem of the material principles, number is a solution of the problem of the material principles as well.

Aristotle writes also: it is clear that the Pythagoreans take number to be the principle both as the matter of things and as the form of their modifications and changes. Given that the principles are of different kinds, and one of the kinds is material, another is formal, Pythagoras's discovery that number is the principle of all things rather anticipates the solution of the problem of the formal principles. The more so if the discovery of proof led to it. Pythagoras's is the first inkling that one of the kinds of the principles is formal. Nor is the anticipation too remote, if, with Galileo, mathematics is the language of the world. Even if nothing in what Pythagoras said indicates an inkling that the principles are of different kinds.

Philosophy was young, two generations old with Pythagoras. Our predecessors did not quite know what they were saying, fighting the way untrained men fight.

The investigation itself, however, was showing them the way.

The next two solutions are,
Heraclitus fire,
Hippasus fire.

Then there is a major advance. The next solution is threefold,
Parmenides love,
 one,
 and fire and earth.

Philosophy was four generations old with Parmenides. And love is not an advance. It is an anticipation of the efficient principles, but as remote as Hesiod's two-odd centuries earlier. The major advance are the other two. This is the discovery that the principles are of different kinds. The kinds are real and apparent. In reality all is one. Only what necessarily is is and nothing else is. In appearance it is many and as such has two principles, fire and earth. The discovery of the principle of contradiction is a further anticipation that one of the kinds of the principles is formal. The kinds real and apparent anticipate the kinds formal and material as well.

In the next generation there is again a major advance. Mature men appeared among the bab-

blers. The solutions are,

Anaxagoras	an infinite number of particles, and reason,
Empedocles	earth, water, air, and fire, and love and strife.

Anaxagoras and Empedocles are, within a decade, exact contemporaries.

Anaxagoras discovered that there are efficient principles. He distinguished the efficient principles from the material. Empedocles, by proposing more explicit solutions of both problems, distinguished sharply the two kinds, the material principles and the efficient. And yet the greatest advance are their solutions, and both their solutions, of the problem of the material principles.

Here are a few of the propositions which Anaxagoras brought in connection with one another, too.

He writes: all things were together, infinite in number and size, for size was also infinite (fr. 1); when all things began separating, earth hardened first, just as water comes from the clouds, earth comes from water, and from earth stones harden (fr. 16); all things thus have a portion of all things; only reason is infinite, rules itself, and is mixed with nothing and is all by itself (fr. 12); when reason begins motion, it separates things from all that is moved, and all things are moved and divided (fr. 13); and since the portions of the great and the small are equal in number, all things have a portion of all things (fr. 6); in all things there is a portion of all things except reason; and in some things there is reason, too (fr. 11); we conclude that there are as many seeds of all things as there are kinds of things (fr. 4).

On Empedocles's line:

the sea is the sweat of the earth (fr. 55),

Aristotle notes: metaphors may be what is expected in poetry but should not be in physics (*Meteorology*, II, 3). Anaxagoras wrote prose, not verse, but it holds in general, sense may be what is expected in either poetry or prose but should not be in physics. Nevertheless, in particular, how powerful the striving for truth is the fact shows that 'reason' and 'love and strife' can each be presumed to have a reference. Anaxagoras's reason cannot help sharing all too many of the ideas and feeling which have saturated reason ever since. The more so, if: what exactly he said

is not clear, he said that reason is the cause, in other places that thought is the cause, in other places that soul is the cause. Unlike all that, however, Anaxagoras's reason can be presumed to share reference with Leibniz's reason. And Empedocles's love and strife cannot help sharing all too many of the ideas and feelings which have saturated love and strife ever since. Unlike all that as well, however, Empedocles's love and strife can be presumed to share reference with Newton's attraction and repulsion. Needless to add, Anaxagoras, Empedocles, Newton, and Leibniz share all too many ideas and feelings, too.

Anaxagoras's is the first solution of the problem of the efficient principles of all things. Aristotle states it thus: Anaxagoras said that reason is the cause of the world and its order. And this proposition anticipates the principle of sufficient reason. With Anaxagoras thus also the anticipation that one of the kinds of the principles is formal continues.

Empedocles distinguished the efficient principles sharply from the material. And here are a few of the propositions concerning the efficient principles which he brought in connection with one another:

when strife gathers strength in her limbs and rises
to her power (fr. 30),
one by one the god's limbs begin to tremble
(fr. 31).

and:

when strife founders to the bottom of the whirl,
love reaches the middle of the eddy,
and all things gather around her into one (fr. 35),

With Anaxagoras and Empedocles philosophy was not quite five generations old. The investigation itself was showing them the way. And the accumulation of solutions, and most particularly the accumulation of solutions improving on one another, is how the investigation itself was showing them the way. Anaxagoras's and Empedocles's are the first two solutions of the problem of the efficient principles. With them the accumulation of solutions of this problem was just beginning. But the investigation of the material principles had already accumulated 6 solutions before them, improving on one another. 4 of these solutions also, Anaximenes's, Pythagoras's,

Yet Anaxagoras's solution of this problem is an even greater advance. Aristotle states it thus: Anaxagoras said that an infinite number of particles are the principles, as each thing is generated from its own particles and destroyed into its own particles. Aristotle describes them also as: homogeneous particles (*id.*).

Number does not belong in the series water, air, fire, fire, fire and earth, earth, water, air, and fire, it belongs in a different series. But Anaxagoras's solution does not clearly belong in the series water, air, fire, fire, fire and earth, earth, water, air, and fire, either. An infinite number of particles is rather like number, and not only in that it assigns a number to the particles. In that also that the particles are homogeneous. There are as many seeds of all things as there are kinds of things.

The next solutions are,

Melissus	one,
Leucippus	atoms and void,
Socrates	definitions,
Diogenes of Apollonia – air,	
Democritus	atoms and void,
	and their shape, order, and
	position.

If Diogenes of Apollonia's is an attempt to restore the material principles to greater simplicity come what may, so is Melissus's earlier, when he said that all is one in matter. And if Diogenes's solution is like Empedocles's, Melissus's is like Anaxagoras's. It is a more formal and more general solution. Hence it is a step also in the generalization of the formulation of the problems of all things.

In the review Aristotle speaks not of Pythagoras but of the Pythagoreans, and not of Leucippus singly and Democritus singly but of the two jointly. And ever since Leucippus and Democritus have tended to populate philosophy as an entity. The tendency has been reinforced by the

appearance of the larger entity of Democriteans soon after Aristotle. There have been other pairs of philosophers who have labored jointly. Marx and Engels, for instance. And if the task is to attack or to defend the entity Marxists, and the more so the entity Marxism, it is better not to distinguish the labors of Marx from the labors of Marx and Engels from the labors of Engels. If the task is to investigate, and the more so to investigate development, the labors of Marx, the labors of Marx and Engels, and the labors of Engels had better be distinguished from one another. Similarly, Leucippus and Democritus are two philosophers, not one. And while Aristotle speaks throughout his treatises of the Pythagoreans, and speaks most often of Leucippus and Democritus jointly, he speaks also of Leucippus singly and of Democritus singly. From Aristotle's reports alone the two philosophers can be distinguished. The more important this is because the evidence about them depends on Aristotle more than that about any other philosophers, including Anaximander and Anaximenes. And that it is important to distinguish them the fact shows that Aristotle's reports are unambiguous in distinguishing the two solutions, first Leucippus solved the problem of the material principles, then Democritus solved the problem of the efficient principles.

Leucippus's solution of the problem of the material principles is a very great advance. Atoms are, by being atoms, indivisible. Atoms thus anticipate more clearly than Aristotle's definition of material principle Lavoisier's definition of element. Lavoisier's definition is: elements are all substances which we have not yet been able to decompose by any means.

Like Anaxagoras's, Leucippus's is a more formal and more general solution of the problem of the material principles. If Empedocles's solution affirms that earth, water, air, and fire are the elements, Leucippus's solution, improving on Anaxagoras's, affirms that atoms and void are the kind of the elements. And this distinction, the elements, on one side, the kind of the elements, on the other, anticipates the later development of the investigation of the material principles of all things, too. As earth, water, air, and fire eventually succumbed to decomposition and the search for the elements – hydrogen, nitrogen, oxygen –

accelerated, on one side, it was Avogadro's discovery of the distinction of atoms and molecules, on the other, that made it possible for Mendeleev to formulate the law of the elements. And when Thomson showed that cathode rays are a means to decompose the chemical elements, quantum theory had to solve both parts of the problem, to show which are the elements and to show what their kind is.

Democritus advanced Leucippus's solution of the problem of the material principles. One of his results reported by Aristotle is: Democritus says that the larger an atom is the heavier it is, and hence also the hotter (*Corruption*, I, 8). Theophrastus elaborates: Democritus distinguishes heavy and light by size; the lighter contains more void, the heavier less (*Senses*, 61).

Democritus's solution of the problem of the efficient principles is: the modification of all things is due to the shape, order, and position of the elements. It is a very great advance. It is incorporated in Aristotle's better solution of the problem. The source of all motion is one, at the circumference. And the shape, order, and position of everything below, spheres concentric with respect to the same circumference, suffice for the communication of motion to everything below. Democritus's solution also anticipates, as Aristotle's does not, Dalton. Once atoms entered philosophy, many philosophers have had many things to say about atoms. It is the proposition: the modification of all things is due to the shape, order, and position of the elements, on which the theory developed by Democritus rests, which shows that, unlike most of what other philosophers have said about atoms, the theory developed by Democritus anticipates the theory developed by Dalton.

The last two solutions are,

Cratylus	flux,
Plato	the forms,
	and the one.

Cratylus is important. But his is not a solution of the problem of the principles. As he asserted, if all things are in flux, there can be no knowledge of them. If flux would be a principle, this principle could not not be that there could be no principle.

And after two centuries of accumulation of

solutions of the problem of the material principles improving on one another, and then of solutions of the problem of the efficient principles improving on one another, and of anticipations that the principles of all things cannot be only material, and then of anticipations that the principles of all things cannot be only material and efficient, Plato discovered that the principles of all things are also formal. The investigation itself showed them the way.

Thus Aristotle's predecessors in the review.

And it can all be tabulated, adding, in order to complete the review through Aristotle, Aristotle's solutions.

The solutions of the problem of the material principles divide into two parts. One part is,

Thales	water,
Anaximenes	air,
Heraclitus	fire,
Hippasus	fire,
Parmenides	fire and earth,
Empedocles	earth, water, air, and fire,
Diogenes of Apollonia – air,	
Aristotle	earth, water, air, fire, and aether.
The other part is,	
Anaxagoras	an infinite number of particles,
Melissus	one,
Leucippus	atoms and void,
Democritus	atoms and void,
Plato	the one,
Aristotle	simpler bodies similar to the elements.

The anticipations and solutions of the problem of the efficient principles are,

Hesiod	love,
Parmenides	love,
Anaxagoras	reason,
Empedocles	love and strife,
Democritus	the shape, order, and position of the elements,
Aristotle	the first mover.

The anticipations and solutions of the problem of the formal principles are,

Pythagoras	number,
Parmenides	one,
Anaxagoras	reason,
Socrates	definitions,
Plato	the forms,
Aristotle	propositions.

And the steps in the solution of the three problems, and thus the steps of advance in the acquisition of knowledge, are these.

Of the problem of the material principles,

Thales	water,
Anaximenes	air,
Heraclitus	fire,
Parmenides	fire and earth,
Anaxagoras	an infinite number of particles,
Empedocles	earth, water, air, and fire,
Leucippus	atoms and void,
Aristotle	earth, water, air, fire, and aether, simpler bodies similar to the elements.

Of the problem of the efficient principles,

Anaxagoras	reason,
Empedocles	love and strife,
Democritus	the shape, order, and position of the elements,
Aristotle	the first mover.

Of the problem of the formal principles,

Pythagoras	number,
Parmenides	one,
Anaxagoras	reason,
Socrates	definitions,
Plato	the forms,
Aristotle	propositions.

This is how far, through Aristotle, the investigation itself led them.

The principles of all things are material, efficient, and formal.

The material principles, or the elements, of all things are earth, water, air, and fire. The kind of the elements is simpler bodies similar to the elements. Or better, the kind of the elements is atoms and void.

The efficient principle, or the cause, of all things is the first mover. Or the efficient principles, or the causes, of all things are the shape, order, and position of the elements.

The formal principles, or the principles, of all things are propositions.

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Aristotle concludes in the review: none of our predecessors spoke of a kind of principle other than the four which we have distinguished in the *Physics*.

Nothing but the completeness of the review

can warrant the conclusion.

And to show the completeness of the review, two things can be shown.

First, that the three kinds, material, efficient, and formal, are all the kinds of the principles of all things. That the principles of all things are of these three kinds and no other.

Second, that the solutions in the review are all the solutions of the problem of the principles of all things, material, efficient, and formal. That, through Aristotle, there are no other solutions of the problem.

And, first, then, to show that the three, material, efficient, and formal, are all the kinds of the principles of all things, and there are no other kinds, the final principles of all things are singularly qualified. Because the attempts to establish that there are this kind of principles of all things, too, include Plato. Include Aristotle himself. Include god. Include Leibniz, too.

The review begins: the principles are of four kinds, material, efficient, formal, and final, and we have investigated them sufficiently in the *Physics*.

In particular, the argument that the principles are of four kinds is in book II of the *Physics*.

The *Physics* begins: in physics our task is to find the principles (I, 1). And first, in book I, Aristotle deals with the problem of: whether the number of the principles is one or more than one (2). Once again by adopting the sound and avoiding the erroneous in what our predecessors have said on the matter, and in particular, what Anaximander, Parmenides, Anaxagoras, Empedocles, Melissus, Democritus, Plato have said on the matter. He concludes: the principles cannot be one in number (3), rather: they are two or three (6), and rather: they are three (7).

And rather, he argues in book II, they are four.

He begins in book II with a general description of nature: nature is the principle and cause of motion and rest, and substantially and not accidentally (1), and: nature is the primary underlying matter of what has in itself the principle of motion and change, and also the kind or the form affirmed in the definition (*id.*).

The description implies the three, material, efficient, and formal.

However: there is also the end, or that for the

sake of which, for nature is also the end or that for the sake of which; for if a thing changes toward an end, the end of the motion is that for the sake of which (2).

On this ground, pace Aristotle, the end cannot be a principle distinct from the principle of motion. The end of the motion cannot be substantially different from the motion, including the beginning of the motion. On this ground the final principle is the efficient principle.

Nevertheless, Aristotle affirms that the principles are of four kinds, material, efficient, formal, and final, defines the kinds, and exemplifies (3).

And the rest of book II is an argument that there is, next to the other three kinds of principles, this fourth kind, the final principles.

Aristotle first argues that chance is not a principle (4-6). Then reaffirms: the principles are four, the matter, the form, the mover, and the end (7). He notes: the last three often coincide (*id.*).

And then writes: we must now explain why to nature belongs also the kind of principle which is for the sake of something, and explain it in relation to necessity and its place in nature (8); thus, why should nature not work not for the sake of something, or because it is better so, but just as it rains, not to make the corn grow, but of necessity? vapor goes up, cools, falls down, and incidentally the crop grows; similarly, if the crop is spoiled, the rain does not fall to spoil it; it falls, and incidentally the crop is spoiled; and why should it not be the same with all parts of nature? that teeth, for instance, grow by necessity, the incisors sharp and fit to tear, the molars broad and fit to grind, not for the sake of this, but by chance? and so in all cases in which it appears that there is a purpose? whenever the parts came together as if for an end, they survived, having come out for an end by chance, and whenever they came out otherwise, they perished, and continue to perish, as Empedocles says of his man-faced oxen (*id.*).

Darwin quotes this passage in the historical sketch with which *The origin* opens, and comments: we here see the principle of natural selection shadowed forth, but how little Aristotle fully comprehended the principle is shown by his remarks on the formation of teeth (*id.*).

Aristotle continues: but this cannot be the

case; for teeth and all other natural things come to be either always or for the most part in a certain way, and this cannot be by chance; it may be by chance that it rains often in summer, but not that it rains often in winter, it may be by chance that it is hot in winter, but not that it is hot in summer; if these things are not by chance, then, they are for the sake of something; and these things are natural; it follows that there is an end in the things that come to be and are by nature (*id.*).

Granted, then, that it is not by chance that it is hot in summer, for the sake of what is it hot in summer? Granted that it is not by chance that it rains often in winter, for the sake of what does it rain often in winter? If it rains neither to make the corn grow nor to spoil the crop, for the sake of what does it rain? It rains by necessity. But these things are natural. Vapor goes up, cools, falls down, and this is the end, the end of the motion, and the final principle is the efficient principle. Or rain has no end, but there is an end in the things that come to be and are by nature, and rain is not by nature.

The argument has failed to separate the final principles from the efficient. Which explains why Aristotle has to argue that the final principles often coincide with the efficient. That they often coincide with the formal, too. Indeed: since nature is twofold, matter and form, the form is the end (*id.*).

It has to be concluded that nature is matter, motion, and form. And there is no end distinct from motion.

The question is thus rather what is the end of the argument? And that is, what is the efficient principle? Or what is the same, what is the cause? The cause of the motion such as it ends in the final principles?

The proximate cause is that in this case, as in but a very few other, Aristotle failed to separate theory from practice. He is explicit: as in action, so in nature, and as in nature, so in action, if nothing interferes (*id.*).

As in nature, so in action, if nothing interferes. Much interferes, however. Hence, not as in action, so in nature on pain of annihilating truth.

The following have been the chief interferences.

Aristotle begins in the *Nicomachean ethics*:

every art and every investigation, every action and every choice appears to aim at some good; it is thus well said that the good is that at which all things aim (I, 1).

It is well said by Plato that the good is that at which all things aim. All things, including truth. And if that is the good, that is the good, but it has not been too good for truth.

Aristotle writes: choice makes action excellent (III, 2); and choice cannot relate to the impossible; if one said that he chooses the impossible, he would be thought foolish; but one can desire the impossible, immortality, for instance (*id.*).

And it may well be foolish, but Plato does not just desire the impossible. He does not just choose it either. He teaches you, too, in a dialogue after a dialogue after a dialogue, an argument after an argument after an argument, to choose it, too. Not to mention two millennia of Christianity and one and a half of Islam.

Choice can not only relate to the impossible, it has been exceedingly eager to relate to the impossible. And this has not been good for truth either. Not good at all.

Aristotle writes: in action the end is the first principle (VII, 8). And the end has been the first principle in action not as in: vapor goes up, cools, falls down. As in: to make the corn grow and to spoil the crop, if you are good and if you are bad, respectively.

Hence the fundamental importance of the separation of theory from practice, of the search for truth from the search for action. And in Aristotle, and before him, and after him, there has been, and there can be, success in the search for truth only when the search has been, and the search is, exclusive of the search for action.

Theoretically, then, the final principles are the efficient principles. The end is the end of motion. Hence, better, there are no final principles. And practically, the final principles cannot be principles. Not if action can ignore truth. Which, that action can ignore truth, the choice of the impossible suffices to show. The end is the end of action. Which is some good. Whether it is possible or impossible. Whether it is true or false.

If the failure to separate theory from practice is the proximate cause, the good points unambiguously at the ultimate cause, the organism.

It points also, also unambiguously, at the beginning of the motion in philosophy.

Not quite one century before Aristotle, after one and a half centuries of physics, counting from the first philosopher: Socrates abandoned physics altogether, and devoted himself to ethics. Plato has Socrates report: I read the books of Anaxagoras, found that they were all about causes like air, aether, water, and other such things, and realized that he had nothing to teach me; I began to study the causes of things by myself; and I discovered that earth and air and other such things are not the true causes of things; the true cause of things is what is for the best.

In the next generation, Plato writes: it is the form of the good that gives, both, truth to what one knows and one's power to know it; although an object of knowledge, the form of the good is also the cause of knowledge and truth; it is thus right to say that knowledge and truth are good, but wrong to say that either of them is the good; the good is higher.

Higher, it has had no difficulty making the true false, the false true, the possible impossible, the impossible possible.

In the next generation, Aristotle argues that there are, next to the material, efficient, and formal, final principles. The search for truth advanced very greatly with Aristotle, however. And other than in book II of the *Physics*, the search is everywhere in his theoretical treatises a search for material, efficient, and formal principles, and nowhere for final principles. In his logical treatises. And in his physical treatises. In the rest of the *Physics*, in book I and in the investigation of motion in books III-VIII. In *On the heaven*. In *On generation and corruption*. In the *Meteorology*. In *On the soul*. In the treatises on animals. Aristotle begins in the *Parts of animals*: there is the end, and there is the beginning of motion, and we must decide which is first, which is second; and clearly, the end is first, that for the sake of which (I, 1). The end is yet very difficult to find in the 21 books of the *History of animals*, *Parts of animals*, *Movement of animals*, *Progression of animals*, and *Generation of animals*. Aristotle continues in the *Parts*: turning to the investigation of animals, we shall try, to the best of our abilities, to omit no member of their king-

dom, however lowly (5). And never mind the end, he describes 600 kinds of animals.

The search is for final principles in his practical treatises. In book I of the *Politics* Aristotle investigates the state, but in the following books II-VIII he does not investigate the state, he investigates the best state. And the end, of action, not of motion, is the end of the *Ethics* beginning to end.

It was at that time that schools began producing the philosophers, and the philosophers in the schools, including the Lyceum, abandoned physics altogether, and devoted themselves to ethics.

Three-odd centuries after Plato and the foundation of the Academy, in *On the good and bad ends*, Cicero reviews the labors of the philosophers in the schools. He writes: nothing is more important in life than philosophy, and nothing is more important in philosophy than to ask what is the end (I, 11).

Cicero's older contemporary Varro wrote *On philosophy*. Augustine reports: in *On philosophy*, by a diligent and subtle study, Varro shows that all possible schools on the supreme good are 288; Varro says that by nature men desire pleasure, or tranquility, or both, or what is first by nature, for a total of 4; in each case for its own sake, or for the sake of virtue, or virtue for its sake, for a total of 12; in each case for oneself or for the community, for a total of 24; in each case certainly, like the Stoics, or only probably, like the New Academics, for a total of 48; in each case like the other philosophers or like the Cynics, for a total of 96; and in each case for a life of leisure, or for a life of study, or for a life of action, for a total of 288 (XIX, 2).

With this report begins Augustine's concluding argument in *The city of god against the pagans*, in books XIX-XXII, on: the merited ends of the two cities, earthly and heavenly (1). And Augustine continues: but what does any of that matter to us? we ought to judge by truth, not by what some men have thought (3); if we are asked, therefore, what the city of god believes about the supreme good and evil, we shall reply, eternal life is the supreme good, eternal death is the supreme evil (4); moreover, we do not see our own good, and we must therefore seek it by faith (*id.*); the philosophers have supposed that the supreme

good and evil are to be found in this life; with amazing vanity they have wished to be happy here and now, to achieve blessedness by their own efforts; but truth has mocked them; in the words of the prophet, the lord knows the hearts of men, and the apostle Paul has said, the lord knows that the thoughts of the wise are in vain (*id.*).

The darkest of dark ages eventually came to an end. Its merited end, presumably. The philosophers resumed the search for truth, including with respect to the principles of all things, and for a long time, rather, the principle of all things. For it was clear who the principle is. The principle nevertheless began encountering difficulties being the principle. Being the material principle. David of Dinant said that god is the primary matter. The books of David of Dinant had to be brought to the bishop of Paris and burned. Being also the formal principle. Master Amalric said that god is the formal principle of all things. The body of master Amalric had to be removed from the cemetery and cast into unconsecrated ground. Aquinas argues: god is not the primary matter, he is its creator. And: he is not the formal cause of all things, he is the exemplar cause of all things. Being the efficient principle and the final principle he encountered no difficulty yet. Aquinas argues: he is the efficient cause of all things. And: he is the final cause of all things. The final cause and the efficient cause often coincide.

Three and a half centuries after Aquinas, the instauration of the sciences began.

Bacon writes: final causes corrupt rather than advance the sciences; they are in relation to the nature of man, not in relation to the world.

Two-odd generations after Bacon, Spinoza explained both, how they are in relation to the nature of man, and hence why they corrupt the sciences.

In the *Ethics demonstrated in geometrical order*, Spinoza writes: I assume what should be admitted by all; namely, that all men are born ignorant of the causes of things, and that they all have the desire for what is useful to them and are conscious of this desire; from this it follows, first, that men believe themselves free, being conscious of this desire, and in their ignorance never even dream of the causes which incline them to have

this desire; second, that men do all things for an end, for what is useful to them and what they desire; hence it follows further that men seek to know in all things only the final causes, and as soon as they have found them they are content and never see any cause to doubt them (I, app.); men are thus bound to consider all of nature as a means of their ends (*id.*); and they are bound to conclude that there are rulers of nature endowed with freedom like their own; they judge these rulers by themselves and assert that these rulers ordained everything for the use of man in order to subjugate man and to have him honor them (*id.*); thus it follows finally that everyone invented for himself a way of worshiping god so that god would love him more than others and would direct the entire course of nature toward the satisfaction of his blind cupidity and insatiable avarice (*id.*).

Eternal life and blessedness for me, eternal death and damnation for others.

It is a consequence of this result of Spinoza's, too, that: god is nature.

As Newton writes: god without dominion, providence, and final causes is nothing but fate and nature.

Leibniz writes: god conserves the same quantity of absolute force, or action, the same quantity of respective force, or reaction, and the same quantity of directive force; moreover, action is always equal to reaction, and the entire effect is always equal to its full cause; and what is surprising is that a reason for these laws of motion cannot be given by consideration of efficient causes, or matter, alone; I have discovered that recourse to final causes is necessary, that these laws depend not on the principle of necessity, as logical, arithmetical, and geometrical truths do, but on the principle of fitness, and that is, on the choice of wisdom; and this is one of the most efficacious and sensible proofs of the existence of god for those who can go deeply in these things.

It is surprising that a reason for these laws of motion – the conservation of living force, the conservation of momentum, the equality of action and reaction – cannot be given by consideration of efficient causes, or matter, alone. Leibniz discovered that these laws depend not on the principle of necessity, as logical, arithmetical,

and geometrical truths do, but on the principle of sufficient reason.

And, pace Leibniz, the principle of sufficient reason is not a final principle. Even if final principles and formal principles often coincide. It suffices that the principle of sufficient reason is a formal principle. With Kant: the principle of contradiction is the principle of all analytic judgments, the principle of sufficient reason is the principle of all synthetic judgments.

There are no final principles. The end is the end of motion. The end of rain, for instance. Vapor goes up, cools, falls down. Or the end of teeth. Natural selection. Or more generally the end of the organism.

Or the end of the acquisition of knowledge. Or the end of the search for truth. As Frege describes it: the striving for truth forces us to advance from the sense to the reference. And that it is the end of motion, just like the end of rain and the end of teeth, the fact should suffice to show – even if, to judge by past experience, two and a half millennia of past experience now, nothing will suffice to show – that it forces us against not only the blind cupidity and insatiable avarice of the organism. It forces us against the reinforcement of the cupidity and avarice by a philosopher after a philosopher after a philosopher for two and half millennia now.

So also the end of action, as in nature, so in action, if nothing interferes. Much interferes, however.

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To show that the solutions in the review are, through Aristotle, all the solutions of the problem of the principles of all things, material, efficient, and formal, Empiricus, writing five and a half centuries after Aristotle, is singularly qualified.

For three reasons, one general, two more particular.

In general, because Empiricus is a sceptic. And the supposition that Aristotle's review is not complete, and it can be amplified, against Aristotle, when Aristotle presumes explicitly that the review is complete, such a supposition cannot not rest, ultimately, on scepticism.

More particularly, because Empiricus is not only a sceptic, he argues scepticism, and argues it

systematically. Hume writes: sceptical reasonings, were they possible, would be both strong and weak, the enemy of reason is obliged to take shelter under her protection. Empiricus takes shelter under her protection systematically. He argues that knowledge is impossible with respect to one problem, then with respect to another problem, then another, and another, and through the 3 books of the *Outlines of Pyrrhonism*, then in more detail through the 11 books of *Against the mathematicians*. And when the turn of the problem of the material principles of all things comes in the *Outlines*, Empiricus introduces it with a review (III, 30-2). When in *The mathematicians* the turn of the problem of the principles of all things comes, Empiricus again introduces it with a review (IX, 4-12), and when later the turn of the problem of the elements comes, he introduces it once again with a review (359-65). Empiricus's reviews are richer than Aristotle's review. And their abundance is in all cases representative of how, and to what ends, Aristotle's review has to be amplified.

Also, no amplification of Aristotle's review of any plausibility has since been proposed, or can be proposed, other than those of Empiricus.

The first of the three reviews, the one in the *Outlines*, is of: what the dogmatics have said about the material principles of all things (III, 30). And Empiricus lists 23 distinct solutions of the problem.

The next review, the earlier of the two in *The mathematicians*, is of: what the physicists have said about the principles of all things; some have said that the principles are efficient, others that they are material (IX, 4). Empiricus then lists 7 solutions of the problem of the principles. The last is: the Stoics also say that there are two principles, god and undifferentiated matter (11). He concludes: we see that the best physicists have said that these two are the principles (12), and then proceeds to annihilate in some detail these two principles: the gods (13-193), and: matter (194-330).

The last review, the later of the two in *The mathematicians*, is of: what they have said about the primary elements; the main opinions are two, that the elements are bodies, and that they are incorporeal (359). And Empiricus then lists 22

solutions of the first kind, and 3 of the second.

Clearly, Empiricus's reviews are an extensive examination of Aristotle's review of the solutions of the problem of the material principles, if not quite so extensive of those of the efficient and the formal principles.

First the material principles, then.

The review in the *Outlines* lists 23 distinct solutions. And the later of the two reviews in *The mathematicians*, of the two main opinions on the elements, that they are bodies, and that they are incorporeal, lists 22 solutions of the first kind. 17 of these are the same as 17 of the solutions in the review in the *Outlines*. The other 5 are not. The later review lists also 3 solutions of the second kind. 2 of these are also among the 23 in the review in the *Outlines*. The other 1 is not. This yields the following total. 3 solutions of the problem of the incorporeal principles, 2 of which are solutions also of the problem of the material principles. And 28 solutions of the problem of the material principles, 2 of which are solutions also of the problem of the incorporeal principles.

To arrange the 28 solutions of the problem of the material principles in their series and order, there are the usual difficulties.

The Pythagoreans are not in this case a difficulty, for the solution, number, is supplied in one of the reviews indeed by the Pythagoreans, but in the other review by Pythagoras. 3 further solutions, however, are also supplied by entities, 2 by the Stoics, 1 by the mathematicians.

Empiricus writes: the Stoics say that earth, water, air, and fire are the principles; and why even mention the mythical matter without quality of some of them (*id.*)?

Laertius reports: there are two principles of the whole, active and passive; the passive is matter; the active is the inherent in matter law, or god, which is everlasting and the producer of everything; this is what Zeno says in *On substance*, Chrysippus in book I of the *Physics*, and Posidonius in book II of the *Physical law* (VII, 134); the world comes into being when its substance is transformed from fire to air, air to water, water to earth; to this process of condensation of the elements corresponds a process of rarefaction; animals, plants, and all other kinds are formed from the four elements by mixture; this genera-

tion and destruction of the world Zeno describes in *On the whole*, Chrysippus in book I of the *Physics*, and Posidonius in book I of *On the world* (142).

The 2 solutions, earth, water, air, and fire, and matter without quality, are thus supplied by Zeno of Citium and corroborated by Chrysippus and Posidonius. The entity Stoics can be discarded. Counting Zeno in place of the entity, the total remains 28.

Empiricus writes in both reviews: the mathematicians say that the limits of bodies are the principles (*id.*).

The entity mathematicians brought into being in *Against the mathematicians* has to be discarded out of hand. This brings the total to 27.

Empiricus writes: Democritus and Epicurus say that atoms are the principles, but this opinion is more ancient, for Posidonius says that its originator is a certain Phoenician called Mochus (*id.*).

What to do about Mochus is clear. Even if this jest of Posidonius's may also have lost some of its punch. All the punch can be recovered from Strabo: Posidonius said that this Mochus was born before Trojan times (XVI, 2.24). This brings the total to 26.

Finally, Heraclitus supplies 2 solutions of the problem. Empiricus writes: some say that Heraclitus said that air is the principle, some say that he said that fire is the principle (*id.*). Counting this 1 solution, air or fire, the final total is 25.

The philosophers supplying the 25 solutions are 24, as Zeno of Citium supplies 2. And the 24 can now be arranged in their series and order.

625	Thales,
611 - 541	Anaximander,
599	Pherecydes,
590	Anaximenes,
570 - 495	Pythagoras,
570 - 473	Xenophanes,
558	Onomacritus,
540	Heraclitus,
519	Hippasus,
500 - 428	Anaxagoras,
495 - 435	Empedocles,
490	Oenopides,
484	Archelaus,
470	Hippo,

463	Diogenes of Apollonia,
460	Democritus,
450	Idaeus,
395 - 317	Ponticus,
384 - 322	Aristotle,
355	Diodorus Cronus,
341 - 270	Epicurus,
335 - 263	Zeno of Citium,
327 - 269	Strato,
130 ^{BC}	Asclepiades.
19 through Aristotle, 5 after him.	

And the solutions can now be tabulated in their chronological order, dividing them into the two parts of the problem.

Of the first part, which are the elements, there are 15 solutions, 14 through Aristotle, 1 after him.

Thales	water,
Pherecydes	earth,
Anaximenes	air,
Xenophanes	earth and water,
Onomacritus	earth, water, and fire,
Heraclitus	air or fire,
Hippasus	fire,
Empedocles	earth, water, air, and fire,
Oenopides	air and fire,
Archelaus	air,
Hippo	water and fire,
Diogenes of Apollonia	air,
Idaeus	air,
Aristotle	earth, water, air, fire, and
	aether,

Zeno of Citium earth, water, air, and fire.

Of the second part, the kind of the elements, there are 10 solutions, 5 through Aristotle, 5 after him.

Anaximander	the infinite,
Pythagoras	number,
Anaxagoras	homogeneous particles,
Democritus	atoms,
Ponticus	seamless particles,
Diodorus Cronus	minute indivisible bodies,
Epicurus	atoms,
Zeno of Citium	matter without quality,
Strato	qualities,
Asclepiades	seamless particles.

Two general conclusions can be drawn immediately.

One. Empiricus comments: so large is the

disagreement among the dogmatics (*id.*). Disagreement? Large disagreement?

Two. Philosophy was by that time eight centuries old. In the first two and a half centuries it had advanced greatly in the solution of the problem of the material principles of all things. In the following five and a half centuries it had not advanced greatly at all.

As to the particulars, those after Aristotle are the ground of the two general conclusions. About the first part of the problem, which are the elements, Zeno of Citium, who supplies the only solution in the list, shows – and the solutions of Chrysippus and of Posidonius can corroborate – that Empedocles's solution had established itself as the solution of this part of the problem. About the second part, the kind of the elements, Diodorus Cronus, Epicurus, Zeno, Strato, and Asclepiades show that the philosophers either reaffirmed Leucippus's solution of this part of the problem, or their attempts to improve on it were neither more substantial nor more successful than Aristotle's: simpler bodies similar to the elements. Which is itself not very substantial.

The 18 particulars before Aristotle can be divided into three parts.

These 8 are in both Empiricus's reviews and Aristotle's review, and are in Empiricus as they are in Aristotle,

Thales	water,
Anaximenes	air,
Pythagoras	number,
Hippasus	fire,
Anaxagoras	homogeneous particles,
Empedocles	earth, water, air, and fire,
Diogenes of Apollonia	air,
Democritus	atoms.

Next, these 3 are in both Empiricus and Aristotle, but not in Empiricus as they are in Aristotle,

Xenophanes	earth and water,
Heraclitus	air or fire,
Hippo	water and fire.

To ascribe the solution, water and fire, to Hippo is an error. Hippasus, as Empiricus also reports, said that fire is the principle. Hippo said that water is the principle. Neither is a very important philosopher, and by the time of Empiricus, when Hippasus and Hippo are noted at all,

they are as often as not confused and conflated. They are conflated also in Empiricus's contemporary Tertullian (*Soul*, 5), for instance, later on in Mamertus (*Soul*, II, 7).

Similarly, when: some say that Heraclitus said that air is the principle, some say that he said that fire is the principle, the former are confused.

These are minor matters, and, in particular, exceptions. Empiricus is an important source on the earlier development of philosophy and he is seldom in error in what he reports on his predecessors.

Correcting the solutions,

Xenophanes	earth and water,
Heraclitus	fire,
Hippo	water,

the major matter concerns Aristotle's judgment. On Aristotle's authority Heraclitus belongs in the review, but: we can ignore Xenophanes, and: we can ignore Hippo. The major matter is whether Aristotle's judgment is sound, and we should adopt it, or erroneous, and we should avoid it.

Hippo said that water is the principle. He said also that the soul is water. Also: all sweet water comes from the sea, for it is deeper than any well (fr. 1). And there is no evidence that he brought the proposition, water is the principle, in connection with other propositions. Thus five generations after Thales, one generation after Anaxagoras and Empedocles, in the generation of Democritus. With respect to the problem of the material principles of all things, we not only can ignore Hippo, we should ignore him.

Xenophanes sings:

we are all born from earth and water,

and Empiricus himself is the source of this fragment (*Mathematicians*, X, 314). Also:

all things that are born and grow are earth and
water.

The proposition:

rain, clouds, winds, and rivers come from the sea,

can also be presumed remotely connected with those two propositions. And Xenophanes brought in connection with those propositions no other propositions. Xenophanes is early, two generations after Thales. By this time, however, Anaximenes had already said that air is prior to water and it is the principle, and brought not less than a dozen propositions in connection with this propo-

sition. Xenophanes is a very important philosopher, but not because of what he said about earth and water. With respect to the problem of the material principles of all things, we should ignore Xenophanes as well.

Finally, these 7 are in Empiricus's reviews and not in Aristotle's review,

Anaximander	the infinite,
Pherecydes	earth,
Onomacritus	earth, water, and fire,
Oenopides	air and fire,
Archelaus	air,
Idaeus	air,
Ponticus	seamless particles.

And if there are errors of omission in Aristotle's review, these would be Aristotle's errors.

To take them in reverse chronological order, as this is the order, more or less, of increasing plausibility of omission on Aristotle's part.

Ponticus is an older contemporary of Aristotle, a decade or so older. He is a very important philosopher. He said that the earth rotates on its axis (Aëtius, III, 13). Hicetas had first said so not quite two generations earlier. But Ponticus first said also that Mercury and Venus revolve around the sun and this can explain their retrogradations, retardations, and stations (Vitruvius, IX, 1.6; Calcidius, comm. *Timaeus*, 176). He said also that the world is infinite (Aëtius, II, 1), and that each star is a whole world containing earth and air (13). He first wrote on Pythagoras and the Pythagoreans. As to the material principles of all things, however, that he said that they are seamless particles is one of but two reports in the record. The other is that he wrote *Against Democritus* (Laertius, V, 87). Clearly, the seamless particles differ from atoms in name. It cannot be concluded that they differ from atoms in anything but name. Aristotle never notes Ponticus. Whether Aristotle ignored him or not, with respect to the problem of the material principles of all things, we should ignore him.

Idaeus is not not important, he is very very not important. There is nothing at all about him in the record other than this one report of Empiricus, that he said that air is the principle. Given that vaguely chronological order is detectable in Empiricus's list, Idaeus can be dated to the century after Anaxagoras. If so, he said that air is

the principle at a time when to say that air is the principle had apparently become routine. Which should in turn lend some support to dating him to that time.

Archelaus is not very very not important, only very not important. He said that air is the principle. Other than that, as Empiricus notes in *The mathematicians*: he was a teacher of Socrates (IX, 360). Other than that, also, Xenophon never notes this teacher of Socrates, Plato never notes him, Aristotle never does.

Oenopides is very important. On the authority of Eudemus, Theon of Smyrna reports that Oenopides discovered the obliquity of the ecliptic (III, 40). Also on the authority of Eudemus, Proclus reports on a couple of Euclidean propositions, on I, 12: Oenopides studied it because it is useful in astronomy (comm. *Elements*, 283), and on I, 23: Oenopides first studied it (333). With respect to the problem of the material principles of all things, however, Oenopides, a younger contemporary of Anaxagoras and Empedocles, said nothing that had not already been said.

Onomacritus Aristotle notes once: some say that Onomacritus was the first to have a special skill in lawgiving, that although a Locrian by birth, he was trained in Crete and practiced prophecy there (*Politics*, II, 12). Onomacritus had another special skill. Herodotus reports: Onomacritus was banished from Athens when he was caught inserting his own oracle among the oracles of Musaeus (VII, 6). Indeed, inserting his own oracle among the oracles of Musaeus will seem a special skill in lawgiving, too, even if being caught will not. In any case, Empiricus reports: Onomacritus says in the Orphic books that earth, water, and fire are the principles (*id.*). Those would be the books in which Onomacritus inserted his own oracle. And that he did so is not a reason to banish him from the problem of the principles of all things. But that those are the Orphic books is. Philosophy was still young, but not that young.

Pherecydes is important, just. Strabo reports: Cadmus, Pherecydes, and Hecataeus first began imitating poetry by writing prose (I, 2.6). Pherecydes is the earliest of the three. More particularly, he imitated Hesiod. Aristotle notes him twice. Discussing lice in the *History of animals*, he

notes: Pherecydes also suffered from lice (V, 31). And criticizing the Pythagoreans and Plato in books XIII-XIV of the *Metaphysics*, and in particular the doctrine that the principle is good, he writes: some of the writers of myth seem to say that the first is not the best when they make Night and Heaven, or Chaos, or Ocean first, but then have Zeus reign; others, like Pherecydes, seem to make the first the best (XIV, 4). In particular, Laertius reports that Pherecydes's book began thus: Zeus and Time and Earth always were (I, 119). And there she is, Earth, the principle of all things. Which explains also how the principles of all things would be found in the Orphic books.

Anaximander, finally, is a very important philosopher. Anaximander is the second philosopher. And Anaximander is the only philosopher other than those collected by Aristotle who has been ubiquitously supposed, because he said that the infinite is the principle, to belong in the review of the philosophers who proposed solutions of the problem of the principles of all things.

Once Aristotle discovered Anaximander, Aristotle's successors recovered more of what Anaximander said. He said that the earth is shaped like a drum. He drew the first map of the earth. He discussed the sizes and distances of the planets. He said that the stars originated in a ball of fire, the earth was wet at first, then began to dry up, the first living creatures were born from the moisture evaporated by the heat of the sun, and man came from such a creature, and that is, from fish.

Aristotle, for his part, notes Anaximander explicitly four times.

He calls on him in the session of the council of our predecessors on: the earth, its position, whether it is at rest or in motion, and its shape. And he reports that: Anaximander said that the earth is at rest because of equilibrium.

Book XII of the *Metaphysics* is a general survey of physics. Aristotle discusses substance, matter and form, actual and potential, and motion, concluding with the general description of the world. He begins with the solution of the problem of substance: substance is of three kinds, matter, form, and the particulars composed of

matter and form, like Socrates or Callias (3). And one of the arguments on which this result stands is: a thing can only accidentally come out of what is not; all things must come out of what is, but is only potentially, and not actually; and this is the one of Anaxagoras; for it is better to say not that all things were together, or that they were a mixture, as Empedocles and Anaximander say, or what Democritus says, but to say that all things were one potentially but not actually; which shows also that these thinkers had some inkling of matter (2).

The central problem of the *Physics* is motion. Aristotle deals with it in books III-VIII. In preparation for this, he deals in book II with the kinds of the principles, material, efficient, formal, and final. And in preparation for this, he begins in book I with the problem of the number of the principles. To formulate and solve the problem he calls the council of our predecessors. In particular, he examines first in some detail those who have said that all is one, Parmenides and Melissus (2-3), and concludes: to say that all things are one and there is nothing beside it is absurd (3). He continues: of the physicists some make the underlying matter one, fire, or air, or water, and generate everything from it by condensation or rarefaction, others say that opposites are contained in the one and emerge from it by separation, like Anaximander and all those who say that what is is both one and many, like Empedocles and Anaxagoras (4). He then examines in some detail Empedocles and Anaxagoras (4-6), and concludes: it is better to say that the principles are a finite number, like Empedocles, than to say that they are an infinite number, like Anaxagoras (6).

Turning to motion in book III, Aristotle begins with the formulation of the problem. He writes: there can be no motion over and above what moves (1); it thus has to be possible for a thing to cause motion without being in motion (*id.*); motion, then, is the actualization of what is only potential (*id.*). He affirms: it is not absurd that the actualization of one thing is in another (3). To establish that it is not, however, we have to call the council back into session. And the council has to deliberate on the infinite. Aristotle writes: it is incumbent on those who investigate nature to treat of the infinite; and all who have studied

nature in a way worth considering have spoken about the infinite and have without exception made it a principle of things (4). And he notes 5 who have studied nature in a way worth considering. Among them Anaximander. Aristotle writes: everything is either a principle or derived from a principle; but the infinite cannot be determined, and it can thus depend on nothing as its principle; and if it is a principle, it can have neither a beginning nor an end; and as it cannot be derived from any other principle, the infinite itself would be taken as a principle, embracing and steering all things, as those say who do not recognize other principles, like reason or love; the infinite then would be the divine itself, immortal and indestructible, as Anaximander and most of the other physicists say (*id.*).

Thus Aristotle.

On the other hand, that Anaximander said that the infinite is the principle there is the authority of Theophrastus. In fact, on the authority of Theophrastus Simplicius reports not only that: Anaximander said that the infinite is the principle (*comm. Physics*, 24), but also that: Anaximander first called it principle (150), ἀρχή. And Theophrastus's successors all agree. Aëtius says that Anaximander said that the infinite is the principle (I, 3), Empiricus, Hippolytus (*Refutation*, I, 6), Laertius (II, 1), Simplicius.

At issue, then, is whether Anaximander said that the infinite is the principle. And this issue cannot not be part of the issue of whether the review in book I of the *Metaphysics* is complete.

And it cannot be about words. It can only be about propositions. It can only be about the connections of words in propositions, and the connections of propositions with one another.

And to decide the issue, first of all, it may be that the good Homer sometimes nods, but it may not be that the good Aristotle ever nods. There are errors in Aristotle, but there is no confusion. At issue is not whether Anaximander is an omission, at issue is whether Aristotle's is an error of omission.

Also, Anaximander, the second philosopher, did not quite know what he was saying, fighting the way untrained men fight. And although no later philosopher quite knows what he is saying either, some know better than other, and some

know better than other what these other are saying. Some fight the way trained men fight. At issue is who does.

And these are all the propositions brought in connection with one another by either Anaximander or his successors and either explicitly or implicitly:

all things were a mixture, as Empedocles and Anaximander say;

opposites are contained in the one and emerge from it by separation, as Anaximander, Empedocles, and Anaxagoras say;

and as it cannot be derived from any other principle, the infinite itself would be taken as a principle, embracing and steering all things, as those say who do not recognize other principles, like reason or love; the infinite then would be the divine itself, immortal and indestructible, as Anaximander and most of the other physicists say.

Thus Aristotle. It can be added that Simplicius reports:

as Anaximander poetically said, things come to be and perish according to necessity, justice, and time (comm. *Physics*, 24).

What Anaximander said about the infinite is very important. With respect to the problem of the principles of all things, however, it is very much better to adopt Aristotle's judgment, and avoid the opposite.

And it is, in fact, more important to adopt Aristotle's judgment with respect to Anaximander than with respect to Pherecydes, Onomacritus, Archelaus, or Idaeus. The latter annihilate the distinction between real and fictional bluntly, the way sceptics fight. The former annihilates it more subtly, the way idealists fight. Because Anaximander said that the infinite is the principle, Hegel writes in the *Lectures on the history of philosophy*: Anaximander's principle is not material, and it can be taken to be thought (I, 1.1.A.2). Worse, it is the way those fight, to quote Gödel again, whose realistic attitude is stronger in theory than in practice, and once they start on a concrete problem, the objects to be analyzed turn out into fictions. Because Anaximander said that the infinite is the principle, Russell writes in *A history of western philosophy*: Anaximander is much more interesting than

Thales (I, 1.2); Anaximenes is not quite as interesting as Anaximander (*id.*).

Pace Theophrastus and all his successors, Anaximander did not say that the infinite is the principle. The material principal, or the efficient principle, or the formal principle. Never mind the final principle.

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Thus the material principles.

About the efficient and the formal principles less needs to be said.

The earlier of the two reviews in *Against the mathematicians*, is of: what the physicists have said about the principles of all things; some have said that the principles are efficient, others that they are material.

Empiricus then lists what 7 physicists have said. He writes: the originator is Homer, and he was followed by Anaxagoras and Empedocles and a vast number of others; for the poet speaks about the principles when he allegorically calls the first cause Proteus and the substance Eidothea; Anaxagoras says that all things were together, then reason put them in order, assuming that reason, which he takes to be god, is the efficient principle, and homogeneous particles are the material principle; as Aristotle says, Hermotimus, Parmenides, and Hesiod said so earlier, as when Hesiod says,

first of all Chaos came into being;
then big-bosomed Earth, the foundation of all,
and Love, the fairest of the immortals,

and Parmenides says,

first of all gods she made Love;

Empedocles holds the same opinion, for next to the four elements he makes love and strife principles; the Stoics also say that there are two principles, god and undifferentiated matter; so we see that the best physicists have said that these two are the principles (IX, 4-12).

The body Pythagoras was before he became the Delian fisherman Pyrrhus we can discard out of hand. The Stoics we can discard after replacing them with Zeno of Citium. The list of the best physicists then becomes: Homer, Hesiod, Parmenides, Anaxagoras, Empedocles, and Zeno of Citium.

The other two lists of Empiricus's are to show:

how large the disagreement of the dogmatics is. This list is to show their agreement: that these two are the principles. With respect to the efficient principles, in particular, it is to show that they are gods: Proteus, reason, which Anaxagoras takes to be god, Love, the fairest of the immortals, Love, first of all the gods, love and strife, god. Hence, Empiricus next turns to annihilating in detail the gods, or what is the same, the efficient principles.

This review is thus not a good examination of Aristotle's review of the formulations and solutions of the problem of the efficient principles. It shows well, however, why Aristotle's review was not examined better.

Through Aristotle, the investigation of the efficient principles had advanced to the formulation of the theory of atoms and void by Democritus and the theory of the first mover by Aristotle.

After Aristotle, the schools flourished. Philosophy became an orchard. The schools taught logic, physics, and ethics. Logic became the fence, physics the earth and the tree, ethics the fruit. In the schools, accordingly, everything was adverse to the advancement of knowledge. A couple of generations after their foundation the philosophers in the schools had abandoned the fence and the earth and the tree altogether and devoted themselves to the fruit.

One of the four major schools was the Garden. And Epicurus argued atoms and void. Unlike any of his successors at the Garden, Epicurus is a very important philosopher. He advanced ethics. On the ground of a doctrine of atoms and void, his is the first elaborate argument that there are no gods, and explanation of the belief in gods. In physics, and in the theory of atoms and void, in particular, Epicurus made no advance other than to assimilate it into ethics. After him there was one further advance, the doctrine of atoms and void, Epicurus's, not Democritus's, was rendered into Latin verse. This advance Lucretius made.

Another of the four major schools was the Stoa. Zeno of Citium is also very important. He said: there are two principles of the whole, active and passive; the passive is matter; the active is the inherent in matter law, or god, which is everlasting and the producer of everything. He said also: all things are determined; determination

is the chain of causes whereby things happen, or the law whereby the world develops (Laertius, VII, 149). Also: nothing incorporeal is capable of action, everything that acts or is acted upon is body (Cicero, *Academic*, I, 39). Also: the soul is a warm breath (Laertius, VII, 157). And his successors at the Stoa could have advanced physics, had they tried to disentangle the inherent in matter law from god the producer of everything. Or, given that the chain of causes whereby things happen is the law, tried to determine some of the causes in the chain with some particularity. As it was, they did not.

Aristotle's immediate successors at the Lyceum advanced physics. Theophrastus advanced it substantially with his investigations of plants, fire, stones, winds, fish, and the senses. But they did not advance the theory of motion. Then the Lyceum also became a school of ethicists.

By the time of Empiricus the assimilation of scriptures into philosophy and of philosophy into scriptures had begun. Empiricus writes in the generation before Origen.

The question of what the efficient principles of all things are had been answered. They are the gods. Then who precisely they are became a question of orthodoxy. And as such the question was answered, too.

By the time of Empiricus the philosophers had abandoned the acquisition of knowledge very generally. In particular, the problem of the material principles of all things the philosophers had abandoned more or less completely. The problem of the efficient principles of all things the philosophers had not abandoned, very far from it. Accordingly, in the solution of the problem of the material principles of all things philosophy did not advance. In the solution of the problem of the efficient principles of all things philosophy did not not advance. In the solution of this problem philosophy regressed very greatly.

And so, more or less completely, until Bacon, Galileo, and Kepler.

Thus, other than Homer who allegorically calls the first cause Proteus, there are no counterexamples against which to examine the completeness of Aristotle's review of the solutions of the problem of the efficient principles.

There is yet the affirmative evidence of the

fact of the difficulty of the problem. Aristotle's review of the solutions of the problem of the material principles is briefer, and it can be, because clarity has been achieved by our predecessors: they grasped the material principles. The efficient principles, however: they grasped in some way, but no one expressed them clearly as principles. The review of the solutions of the problem of the efficient principles thus has to be longer, to be in strict chronological order, and to involve the detailed examination of the more important results.

And the formal principles are a yet more difficult problem. Of our predecessors: no one grasped clearly the formal principles; some of them grasped them in some way; but no one expressed them clearly as principles either. Including Plato.

The later of the two reviews in *Against the mathematicians*, is of: what they have said about the primary elements; the main opinions are two, that the elements are bodies, and that they are incorporeal. And Empiricus lists 3 solutions of the second kind. 2 of them are among the 23 in the review of solutions of the problem of the material principles in the *Outlines* as well.

1 of the 3 solutions is supplied by the mathematicians. Discarding that, the other 2 are, Pythagoras number, Plato the forms.

One century after Aristotle Chrysippus advanced logic greatly with his investigation of the modes of inference and discovery of *modus ponens*.

Chrysippus is an exception. As the schools assimilated physics into ethics, so they did logic. So they did everything. Logic they assimilated indirectly, though. They assimilated it directly into rhetoric and grammar. Rhetoric advanced greatly, grammar advanced greatly. And logic thus regressed further than physics.

In particular, in the solution of the problem of the formal principles of all things philosophy did not not advance either. In the solution of this problem as well philosophy regressed very greatly.

And so, more or less completely, until Bacon, Galileo, and Kepler.

Thus, there are no counterexamples against

which to examine the completeness of Aristotle's review of the solutions of the problem of the formal principles either.

There is yet again the affirmative evidence of the fact of the difficulty of the problem. Aristotle's review of the solutions of the problem of the formal principles is longer still, in strict chronological order as well, and involves the still more detailed examination of the more important results.

Finally, the problem of the principles of all things begins with Thales.

Before Thales Aristotle notes one anticipation of the problem, in Hesiod.

And in this, too, that Hesiod rather than Homer is the particular which belongs among the anticipations of the problem of the principles of all things, we should adopt Aristotle's judgment. And for the same reason, not to endanger the distinction between real and fictional.

Propositions are at issue, their connections with one another, and nothing else can be at issue.

Hesiod sings:

first of all Chaos came into being,
then big-bosomed Earth, the foundation of all
immortals,

and dark Tartarus below,
and Love, the fairest of the immortals;
from Chaos came Darkness and black Night;
Night lay with Darkness and bore Aether and
Day;

Earth bore starry Heaven, equal to herself,
and Mountains, and barren Sea;
then she lay with Heaven and bore Ocean
(*Theogony*, 116-33),

and she bore eleven other titans, three cyclopes, and three one-hundred-handed creatures, then this progeny of hers bore more, then the progeny of this progeny of hers bore yet more.

With their multitude and proximity to one another these sentences show clearly the striving from the sense to the reference. With respect to love. And with respect to love because also with respect to chaos, to what came into being first of all, to earth, to the foundation of all, to what is under the earth, to night and day, to brightness and aether, to heaven, to the equal to earth, to mountains, sea, and ocean. Saturated as it all is

with ideas and feelings.

In its scarcity and isolation Homer's sentence, Ocean and Tethys are the parents of creation, shows clearly no striving, only saturation with ideas and feelings. The more so, the more clearly the sentence shows no striving, only saturation with ideas and feelings, when it is ascribed to the ancients. The more so the sentence, the gods swear by the water called Styx, when it is gathered from Homer, and the more so still, when it is gathered from the ancients. Even if sense, unlike reference, is boundless, and that among the ideas and feelings in Homer are both the first cause and the substance can be taken on the authority of either Proteus and Eidothea or Empiricus.

In Hesiod there is an inkling of the problem of the principles.

If there is an inkling of the problem in Homer it is not in those sentences.

And similarly in the earlier record.

One century before Homer, if there is an inkling of the problem in the Yahwistic document it is very remote.

Seven and a half centuries before the Yahwistic document, *When on high* reports of Marduk:

he set up the stations of the great gods, one star
for each;

he set up the year, and marked its limits,

twelve months of three stars each;

he made the days of the year,

fixed the paths of the stars, lest they go astray;

he allotted their domains to Enlil and to Ea;

in the belly of Tiamat he fixed the zenith;

he made the moon, to shine in the night (V, 1-12),

The multitude and proximity to one another of these sentences shows the striving from the sense to the reference. In *When on high*, nine centuries before Hesiod, there is an inkling of the problem of the principles.

And there is no inkling before *When on high*.

The book of Pherecydes began: Zeus and Time and Earth always were. This oldest writer of prose imitating Hesiod postdates Thales by one generation. And this fact suffices to attest that there is no further inkling of the problem between Hesiod and Thales.

There are no formulations or solutions of the problem of the principles of all things in the solar system and the immediately surrounding region

void of stars from the beginning of the solar system to Thales. There is no inkling of the problem before *When on high*.

The review in book I of the *Metaphysics* is complete. Adding Aristotle, those are all the steps in the formulation and solution of the problem of the principles of all things from the beginning of the problem through Aristotle.

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The problem is the acquisition of knowledge.

But little is to be hoped for unless all particulars are collected, and marshaled in their series and order.

The task is thus to collect all particulars, and marshal them in their series and order.

And first of all to establish the ground on which to do so.

The labors of their predecessors and the philosophers are the particulars.

The task is thus to collect their predecessors and the philosophers. To separate the important among them. And to separate the steps of advance in the acquisition of knowledge among them.

And Aristotle's review in book I of the *Metaphysics* is suited perfectly to the task. It shows that the task can be accomplished. And it shows how it can be accomplished.

The predecessors of the philosophers and philosophers are their labors, and their labors are the winning of propositions. Aristotle can hardly have been more explicit. Thales said that water is the principle. Anaximenes said that air is prior to water and it is the principle. Plato said that definitions cannot be of the sensible things, that they have to be of entities of another kind, that the sensible things are by participation in these entities.

The predecessors of the philosophers and philosophers are propositions, and they are distinguished sharply from everything else.

In particular, there are 20 predecessors of the philosophers and philosophers in the review. Hesiod, Thales, Anaximenes, Pythagoras, Xenophanes, Heraclitus, Alcmaeon, Hippasus, Parmenides, Anaxagoras, Empedocles, Melissus, Leucippus, Hippo, Socrates, Diogenes of Apollonia, Democritus, Cratylus, Plato, Eudoxus. One and all propositions.

These are the particulars.

Propositions follow from one another. They are brought in connection with one another through chains of inferences. Consequences are derived from them and they are recognized as consequences of other propositions. And some of the predecessors of the philosophers and philosophers have not brought many propositions in connection with one another, or derived many consequences from them or recognized them as the consequences of many other propositions. Others have, and twofold, by drawing many consequences from their predecessors, as Anaximenes already does from Thales, and by pursuing far the consequences themselves, as Pythagoras already does.

In particular, 16 of the 20 are of the latter kind. Hesiod, Thales, Anaximenes, Pythagoras, Heraclitus, Hippasus, Parmenides, Anaxagoras, Empedocles, Melissus, Leucippus, Socrates, Diogenes of Apollonia, Democritus, Cratylus, Plato.

These are the important.

And it is into steps in the formulation and solution of problems that Aristotle analyzes some of the important. In the formulation and solution of a problem many propositions are brought in connection with one another, many consequences are derived from them and they are recognized as the consequences of many other propositions. Further, and more important, the gaps in the chains of inferences have been narrowed substantially. This alone can make it possible to subordinate many propositions to one another and gather them into the formulation and solution of a distinct problem. As the problem of the material principles of all things is shown in the review more clearly a distinct problem than the problems of the efficient principles of all things and of the formal principles of all things, and as these three more particular problems are shown more clearly a distinct problem each than the more general problem of the principles of all things. Just as important, the narrowing of the gaps in the chains of inferences has been gradual. This alone can make it possible to identify steps in the development of the formulation and solution of a problem.

In particular, 11 of the 16 are steps in the formulation and solution of the three problems, of

the material, efficient, and formal principles. Thales, Anaximenes, Pythagoras, Heraclitus, Parmenides, Anaxagoras, Empedocles, Leucippus, Socrates, Democritus, Plato.

And these are the steps of advance in the acquisition of knowledge.

The acquisition is uniform. Propositions.

It is a unity. One development, Hesiod through Thales through Pythagoras through Empedocles through Democritus through Plato.

Cumulative development. Thales to Empedocles, Hesiod to Democritus, Pythagoras to Plato, Hesiod to Plato.

If one proposition leads to another, it can lead to all propositions. And one proposition leads to another from the first. Thales said that water is the principle, Anaximenes said that air is prior to water and it is the principle. In the formulation and solution of a problem many propositions are subordinated to one another. If one problem leads to another, it can lead to all problems. And one problem leads to another from the first. Thales said that water is the principle, and this is why he also said that the earth floats on water.

In order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

Aristotle thus announces the formal convocation of the council of our predecessors. And it is formal.

How great an advance the formal convocation of the council is the fact shows that, in order to formulate the problems we are to solve, to call the council of our predecessors implicitly into session has been indispensable before Aristotle, to call the council explicitly into session has been unconditional after him.

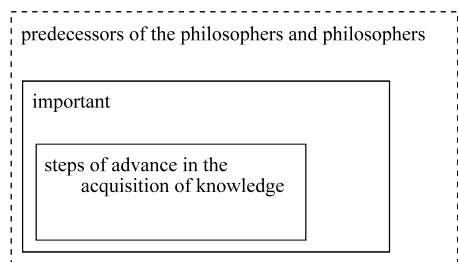
Why, once again the review shows. In this one review Aristotle reduces one major part of the two and a half centuries of the earlier acquisition of knowledge to clarity and distinction. Very little of that clarity and distinction can be recovered from all of Plato. None of it can be recovered from all the philosophers before Plato.

And if from this one session of the council of our predecessors the particulars can be collected, the important among them can be separated, and the steps of advance in the acquisition of knowl-

edge among them can be separated, and thus one part of the acquisition of knowledge can be separated sharply from everything else, all parts of the acquisition of knowledge can be.

This, then, is the ground of the collection of all particulars, and marshaling them in their series and order. The collection of their predecessors and the philosophers. The separation of the important among them. And the separation of the steps of advance in the acquisition of knowledge among them.

It is pictured below. In order to collect the acquisition of knowledge in its entirety, those spaces have to be enclosed.



Clearly, all steps of advance in the acquisition of knowledge are important, and not the other way around, not all important are steps of advance in the acquisition of knowledge. And all important are predecessors of the philosophers and philosophers, and not the other way around, not all predecessors of the philosophers and philosophers are important.

Clearly, also, the 11 steps of advance in the acquisition of knowledge, the 16 important, and the 20 predecessors of the philosophers and philosophers collected from Aristotle's review are in each case a small fraction of all steps of advance in the acquisition, all important, and all predecessors of the philosophers and philosophers, respectively.

Not equally small, however. However small a fraction of all steps of advance in the acquisition of knowledge the 11 are, and they cannot be too small a fraction, the 16 are a smaller fraction of all important. And however small this fraction, and it cannot be too small either, the 20 are a very much smaller fraction of all predecessors of the philosophers and philosophers.

The number of objects in the largest space

cannot be infinite, not even on the supposition that every man, past, present, and future, is a philosopher. It is nevertheless large, even given that not every man, past, present, and future, is a philosopher.

Fortunately, there is the fact of importance. Some are important, the large majority, by far, very far, are not important. And the only importance of the not important can be precisely that, the negative, to show the importance of the important by showing the not importance of the not important.

The task thus does not require the collection of all the predecessors of the philosophers and philosophers. It requires the collection of all the important. Among the not important it requires the collection of only so many of them as will suffice to show that the line separating the important from the not important is drawn sharply. This is the line enclosing the middle space and this line should be drawn sharply. The line enclosing the largest space should not be drawn sharply. To try to do so would not only be, as Leibniz describes it, to turn to the utterly useless, or in any case the least important. It would be, the important being the task, to annihilate the task. By the collection of all the objects belonging in the middle space it should be enclosed sharply. But the largest space should not be enclosed sharply, it should be left to merge indefinitely into whatever is outside it, men, past, present, and future, asses and cattle, and everything else.

The line enclosing the smallest space should be drawn sharply as well. All steps of advance in the acquisition of knowledge should be distinguished sharply among all the important.

Thus, from the collection of some 180 predecessors of the philosophers in the course of the examination of the Sumerian prince list 53 important have been separated sharply. From the collection of 25 predecessors of the philosophers and philosophers in the course of the examination of Xenophanes, Hecataeus, and Heraclitus 14 important have been separated sharply. From the collection of 20 predecessors of the philosophers and philosophers in the course of the examination of Aristotle's review 16 important have been separated sharply.

In the first two cases the steps of advance in

the acquisition of knowledge have not been separated sharply. In the third case they have been, they are 11.

This is the ground.

On this ground the collection of all particulars, and marshaling them in their series and order, can proceed apace. And thus also the solution, even if still only implicitly, of the problems of the beginning of the acquisition of knowledge, and of collecting the acquisition in its entirety, can proceed apace.

ccxli

To show how firm the ground is there can be no better way than to build on it.

As with respect to the problem of the principles of all things the predecessors of the philosophers and philosophers, all the important among them, and all the steps of advance in the acquisition of knowledge among them, can be collected from the first through Aristotle, so with respect to the same problem they can be collected after Aristotle.

As they can be collected with respect to this problem, so they can be collected with respect to another problem. And another, and another, and another.

With Descartes: a problem is everything in which there is truth or falsehood.

If one part of the acquisition of knowledge can be analyzed into propositions, and hence into a distinct problem, all parts of the acquisition can be.

Counting only the 16 most systematic reviews of his predecessors, Aristotle analyzes, next to the principles of all things, the following 15 parts of the acquisition into a distinct problem each. The number of the principles. The infinite. The earth, its position, whether it is at rest or in motion, and its shape. Whether there is generation. Heavy and light. Whether alteration is the same as generation. How action takes place. Comets. The origin, cause, and nature of the Milky Way. Earthquakes. The soul. Respiration. The veins. The teaching of contentious arguments. Which is the best state.

Because the gaps in the chains of inferences have been narrowed substantially it is possible to subordinate many propositions to one another and gather them into the formulation and solution of

a problem. Because the narrowing of the gaps in the chains of inferences has been gradual it is possible to identify steps in the development of the formulation and solution of the problem.

And all the way back to the first step, and to all the anticipations before the first step.

Aristotle thus can trace the solution of the problem of the material principles back to Thales, even if in what Thales said the connections of the proposition, water is the principle, with any other propositions about the principles are only implicit. He can trace the solution of the problem of the efficient principles back to Hesiod, even if what Hesiod said about love can have no reference, and can only indicate the striving which forces us to advance from the sense to the reference. He can establish also the connection of the two propositions, water is the principle, and, the earth floats on water, even if nothing but the word, water, connects explicitly the two sentences.

In general, the finality in their formulation and solution is what makes it possible to analyze the acquisition of knowledge into problems. At least some of the gaps in the chains of inferences have been eliminated completely, and the gradual ascent has reached that step.

Thus, the problem of the material principles, or the elements, of all things was solved conclusively by Mendelev. The gradual ascent can thus be traced back to Lavoisier, to Boyle, to Empedocles, to Thales. Quantum theory has solved the problem of the material principles, or the elements, of all things better, and it does not even have to be shown that the solution is conclusive in order to trace it back to Mendelev, and further. Quantum theory has solved the problem of the efficient principles, or the causes, of all things as well, and again it does not even have to be shown that the solution is conclusive in order to trace it back to Maxwell, Clausius, Newton, Aristotle, Anaxagoras. The problem of the formal principles, or the principles, of all things has been solved by Frege, and again it does not even have to be shown that the solution is conclusive in order to trace it back to Leibniz, Chrysippus, Aristotle, Parmenides.

The problem of the earth, its position, whether it is at rest or in motion, and its shape, has also

been solved. The part about its shape Parmenides and Eudoxus already solved conclusively. The part about whether it is at rest or in motion Copernicus, Galileo, and Kepler solved conclusively. The part about its position Copernicus, Galileo, and Kepler solved very much better than Ptolemy, who had solved it very much better than Aristotle, and it has since been solved very much better than Copernicus, Galileo, and Kepler, and in fact conclusively, by the big bang theory.

The problem of the soul has also been solved. Conclusively, too. And again this does not even have to be shown, and it does not even have to be shown which is the best solution, in order to trace any one of its several solutions back to its predecessors.

Even the problem of which is the best state has been solved.

In the same way, Aristotle, to begin with, has to be the successor of the two and a half centuries of development which has led to the formulation of the theory of the four elements, the theory of atoms and void, and the theory of the forms, theories *par excellence* all three, in order to trace the problems of which the theories are formulations and solutions back to the formulations and solutions on which the theories improve.

And it is the steps of advance in the acquisition of knowledge, exclusively, that pose the problem of the acquisition of knowledge.

And in this, too, Aristotle can hardly have been clearer in the review. Who the important are is determined by the steps in the formulation and solution of the problems. It is in relation to one another that the 11 steps – Thales, Anaximenes, Pythagoras, Heraclitus, Parmenides, Anaxagoras, Empedocles, Leucippus, Socrates, Democritus, Plato – are important. And it is in relation to them, exclusively, that the other 5 important are also important. Hesiod in relation to the steps in the formulation and solution of the problem of the efficient principles, Hippasus and Diogenes of Apollonia in relation to the steps in the formulation and solution of the problem of the material principles, Melissus and Cratylus in relation to the steps in the formulation and solution of the problem of the formal principles. And who the predecessors of the philosophers and philosophers are is determined by the important. It is in

relation to one another that the 16 important are members of the council of our predecessors. And it is in relation to them, exclusively, that the remaining 4 are members of the council as well. Xenophanes said nothing clearly about what Parmenides spoke better. All Alcmaeon said was vague hints where Pythagorean doctrines are concerned. Hippo we can ignore because of the paltriness of his thought on the material principles. And Eudoxus said what Plato had said. Mercifully, also, 4 not important suffice to separate the important sharply from the not important. For there can be no end to obscurity, vagueness, paltriness, and repetition.

To show how firm the ground is, then, and continue the attempt at solution of the problems of the beginning of the acquisition and of collecting it in its entirety, we should call the council of our predecessors into session, and again, and again, and again, and again.

Clearly, we should call the council into session on the principles of all things. Aristotle has described the session of the council on this problem through his time, and the session of the council on this problem after Aristotle should be described. The problem of the principles of all things is of all three, the material principles, or the elements, of all things, the efficient principles, or the causes, of all things, and the formal principles, or the principles, of all things. The problem is of great generality, the greatest, in fact, lest there were generality greater than all things. Because of its great generality, the collection and marshaling of the particulars in their series and order with respect to this problem has to capture much of the acquisition. The problem is also difficult, as difficult as only all things can be. And if the particulars are collected and marshaled in their series and order with respect to this problem, the ground on which they are collected and marshaled has to be firm.

But the problem of the principles of all things is difficult. We should begin by calling the council into session on a problem which is not so difficult. Thales said that water is the principle, and this is why he also said that the earth floats on water. Aristotle's review of what our predecessors have said about the earth, although not complete, is one of his more comprehensive. The

problem is, as Aristotle formulates it, the earth, its position, whether it is at rest or in motion, and its shape. This problem is not so difficult. Not twenty-three and a half centuries after Aristotle. Also, given that one part of the problem is the position of the earth, and that is, its position in the world, this problem is of the same generality as that of the principles of all things, the greatest. Lest there were generality greater than the world. The collection and marshaling of the particulars in their series and order with respect to this problem should also capture much of the acquisition.

We should call the council into session also on the soul. The problem is, as Aristotle formulates it, what is the soul? Aristotle's review of what our predecessors have said about the soul, although not complete either, is also one of his more comprehensive.

The problem of the soul is in Aristotle, and the problem of the soul is after Aristotle as well, a theoretical problem, a search for truth, a physical problem, in particular.

This problem, the physical problem of the soul, is more difficult than the problem of the earth, if not as difficult as the problem of the principles. And one reason for the difficulty is that there is theory, but also practice. Search for truth, but also search for action. Problems are theoretical, but also practical. And next to the theoretical problem, the physical problem of the soul, in particular, there is in this case also the practical problem, the ethical problem of the soul, in particular. And as Aristotle's review suffices to show that there is the physical problem of the soul, that there is the ethical problem of the soul Plato's concern to teach you what you should do about the soul suffices to show.

In Plato already the search for truth is subjugated to the search for action. And Aristotle already has to disentangle the physical problem of the soul from the ethical. Some centuries after Aristotle the search for action subjugated the search for truth so decisively as to arrest it altogether. And when at long last the council resumed its session on the soul, once again the physical problem of the soul had to be disentangled from the ethical.

The collection and marshaling of the particu-

lars in their series and order with respect to the physical problem of the soul should thus not only capture more of the acquisition. It should show that the ground is firm enough to separate the acquisition of knowledge sharply not just from the not acquisition, but from its deliberate subjugation to the not acquisition.

Also, the ethical problem of the soul cannot be part of the acquisition. But as the physical problem has to be separated from the ethical one way or another, the development of the ethical problem can be described explicitly, and hence the relation of the two problems, the physical and the ethical, can be described explicitly as well. If the ethical problem is not part of the acquisition, there can be no steps of advance in the acquisition to collect from its development. The description of its development can nevertheless show that the ground is firm enough to capture practice as well as theory.

Practical problems cannot be more difficult to formulate and solve than theoretical, if only because, in whatever else it be grounded, action cannot not be grounded ultimately in selection. Practical problems – political problems, and the more so ethical problems – have yet been liable to be supposed very much more difficult to formulate and solve, let alone solve conclusively. If, indeed, the matters at issue – political matters, and the more so ethical matters – would be allowed to be accessible to analysis at all, including analysis into problems. Commitment to one solution against all others, and that is, practical commitment, commitment to action, not to truth, has been eager to maintain the impossibility of improving on the solution, and to this end the inaccessibility of the matter at issue to analysis, let alone analysis into problems, let alone steps in the formulation and solution of the problems.

The description of the development of the ethical problem of the soul can show this, that the ethical problem of the soul is a problem, that the problem has been discovered, when and why it has been, that there are steps in its development, and which are the steps and why.

We should call the council into session on the best state as well.

Aristotle writes: in order to consider which is the best state, we must examine constitutions, and

not just those of well-governed states, but those also conceived in speech and held in high esteem.

The problem of the best state is thus also, like the problem of the soul, twofold, there is the theoretical problem of the best state and there is the practical problem of the best state. The theoretical problem is about the well-governed states. The practical problem is about those conceived in speech and held in high esteem. On Plato's authority, for instance, Sparta is a well-governed state. In Plato's *Republic*, on the other hand, there is one conceived in speech and held in high esteem.

The theoretical problem, the search for truth, is a historical problem, in particular. Which is the best among the well-governed states.

The practical problem, the search for action, is a political problem, in particular. Which is the best among those conceived in speech and held in high esteem.

These problems are both more difficult than the two problems of the soul, the theoretical problem of the best state is more difficult than the theoretical problem of the soul, and the practical problem of the best state is more difficult than the practical problem of the soul.

Aristotle's review of our predecessors who have concerned themselves with the best state is, with respect to those conceived in speech and held in high esteem, comprehensive, if not quite complete. With respect to the well-governed states, it is not comprehensive or complete at all.

This points at the main cause of the difficulty of the two problems.

Ever since they began concerning themselves with the state, the philosophers have with great determination ignored all real states and devoted themselves to conceiving states in speech instead and holding them in high esteem. This includes Aristotle, too. In the investigation of the state the philosophers have followed the false way with great determination. Hence, with respect to the historical problem, whether the labors of building the real states, including the well-governed among them, including the best among them, are part of the acquisition of knowledge, and which among these labors are, is very difficult to determine. It is very important to determine, too.

Hence, also, with respect to the political

problem, whether the labors of the philosophers of building states in speech and holding them in high esteem are part of the acquisition or not, is difficult to determine as well. It is important to determine as well.

For ethics cannot be part of the acquisition of knowledge. But politics cannot not be.

The collection and marshaling of the particulars in their series and order with respect to the historical problem of the best state should show that the labors of building the real states, including the best among them, are part of the acquisition of knowledge, and which among these labors are, and thus capture more of the acquisition. With respect to the political problem of the best state it should show whether the labors of building states in speech are part of the acquisition, and if they are, which of these labors are. The collection and marshaling of the particulars in their series and order with respect to the two problems in relation to one another should also inform further the relation of theoretical and practical problems.

Finally, although the sessions on the earth and on the principles, if not so much those on the soul and on the best state, have to involve much mathematics, we should call the council into session on one more strictly mathematical problem as well. This cannot be gathered from Aristotle's reviews of what our predecessors have said, as the other four problems are. It can yet be gathered from Aristotle. He concerns himself with mathematics, and not just its part logic. Discussing *reductio* proofs in the *Prior analytics*, for instance, he writes: that parallels meet follows from the assumption that the sum of the angles of the triangle is greater than two right angles (II, 17). Twenty-one and a half-odd centuries later Lobachevsky showed that there is nothing absurd in the assumption that the sum of the angles of the triangle is less than two right angles. In the next generation Riemann showed that there is nothing absurd in the assumption that the sum is greater than two right angles either. We can call the council into session also on this problem, the sum of the angles of the triangle, and the assumptions on which the sum depends.

The task is to collect the predecessors of the philosophers and philosophers, all the important

among them, and all the steps of advance in the acquisition of knowledge among them.

And to this end, next, we call the council of our predecessors in five sessions on these problems, in order of increasing difficulty.

The earth, its position, whether it is at rest or in motion, and its shape.

The soul, and both, what it is, and what you should do about it.

The best state, and both, the well-governed state, and the one conceived in speech.

The sum of the angles of the triangle, and the assumptions on which the sum depends.

The principles of all things, and all three, the material principles, or the elements, of all things, the efficient principles, or the causes, of all things, and the formal principles, or the principles, of all things.

Five sessions of the council

The earth, its position, whether it is at rest or in motion, and its shape	751
The soul, and both, what it is, and what you should do about it.	816
The best state, and both, the well-governed state, and the one conceived in speech.	926
The sum of the angles of the triangle, and the assumptions on which the sum depends	986
The principles of all things, and all three, the material principles, or the elements, of all things, the efficient principles, or the causes, of all things, and the formal principles, or the principles, of all things	1119

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And first of all, before turning to the sessions, several general matters have to be addressed. Concerning the following. The references of 'all' and 'entirety'. The record. Chronology. Reality. And the last two centuries.

The references of 'all' and 'entirety'.

The task is to collect the predecessors of the philosophers and philosophers, all the important among them, and all the steps of advance in the acquisition of knowledge among them, with respect to those five problems, in particular, and to collect the acquisition of knowledge in its entirety, in general. Clearly, 'all' has to refer to all and 'entirety' has to refer to entirety. And they can be so restricted as to refer, 'all' to all and 'entirety' to entirety.

Like all observation, observation of the acquisition of knowledge is from a point in space and time. That this is how all observation is is an

inductive truth inferred from many, but far from all, and forever far from all, observations.

In particular, spatially the point has been the solar system. And the solar system does have nearly enough the ratio of a point to the world. Counting to the aphelion of Eris and to the limit of observation, respectively, the ratio is, in kilometers, $3 \times 10^{10} : 4.4 \times 10^{23}$. Or about $1 : 15\,000\,000\,000\,000$. Temporally the point has been the acquisition of knowledge. And the acquisition of knowledge also has nearly enough, if not as nearly, the ratio of a point to the world. Counting from the 3 oldest calculations of area and from the big bang, respectively, the ratio is, in years, $5 \times 10^3 : 1.4 \times 10^{10}$. Or about $1 : 3\,000\,000$.

Inference from observation has reached spatially to the end of observation, temporally from the beginning of the world to its end.

And this is inference from the acquisition of knowledge.

Inference about the acquisition of knowledge, however, should not reach that far. Neither observation of the relevant objects nor inference from it has been pursued quite as diligently as observation of other objects and inference from it has been.

All inference about the acquisition of knowledge should be limited spatially to the region immediately surrounding the point of observation. To the solar system and the immediately surrounding region void of stars. And it should be limited temporally accordingly. Within those spatial limits, it should be limited temporally in one direction from the point of observation to the beginning of the solar system, in the other direction to the point of observation itself. Lest the inference infringed on the present, never mind the future.

All inference about the acquisition of knowledge should be limited to this portion of space and time. The solar system and the immediately surrounding region void of stars from the beginning of the solar system to the time immediately preceding all current inference.

Hence, all references of all parts of all propositions about the acquisition of knowledge should be presumed restricted to objects within this portion of space and time, whether any of the references includes objects without this portion of space and time or not. The reference of 'all' in all such propositions and of 'entirety' in the entirety of these propositions should be presumed to be all objects within this portion of space and time and the entirety of these objects, whether the reference includes objects without this portion of space and time or not.

Thus, the hypotheses, for instance, that there were philosophers on earth in the Mesozoic, or that there were at that time, or that there are now, philosophers on Uranus, can be examined. But the hypotheses that there were philosophers anywhere in the world 5 billion years after the big bang, or that there were at the time of the Mesozoic, or that there are now, philosophers in Proxima Centauri, cannot be examined.

Thus also, to collect all the important among the predecessors of the philosophers and philosophers, and all the steps of advance in the acquisition of knowledge among them, with respect to

those five problems, in particular, and to collect the acquisition of knowledge in its entirety, in general, is to collect them within this portion of space and time. The solar system and the immediately surrounding region void of stars from the beginning of the solar system to the time immediately preceding all current inference.

The record.

Aristotle discovered the first philosopher. He discovered 6 of the 20 predecessors of the philosophers and philosophers in the review of the formulation and solution of the problem of the principles of all things. He discovered 4 of the 16 important among them. He discovered 3 of the 11 steps of advance in the acquisition of knowledge among them. Reaching centuries into his past.

And the suspicion has been irresistible, how much more was there that was lost before it was recovered? There is no knowing what is lost.

The same difficulty besets, or appears to beset, the more recent record as well. The record has long lost the ability to lose anything. As it has also become ever more voluminous, though, the stronger the suspicion has become that to examine a substantial part of it, let alone all of it, is impossible.

Moreover, what Aristotle recovered Aristotle recovered. The stronger the suspicion has been that what survives is the outcome of prejudice and bias. Needless to add, any examination, of anything, is the outcome of prejudice and bias.

If the record has lost even 1 important predecessor of the philosophers or philosopher, nothing can be affirmed about all important predecessors of the philosophers and philosophers. All that may be affirmed could at best be about all who survive.

Against all that, the record has lost nothing important. Not 1 thing.

And the examination of all of the record is possible because it is of the essence of the record to examine itself, to examine all of itself, and always to examine itself. The council of our predecessors is how the record does it, how it examines itself, all of itself, and always. And because the council does it, examine itself, all of itself, always, the record has lost nothing important, not 1 thing.

It is important to show this in particular,

however. The record demands special treatment. And this is best postponed.

Here, in the five following sessions of the council, the integrity of the record is taken for granted. The council of our predecessors is not petulant. The council of our predecessors is wise. It has not lost any of its members, not even I.

Chronology.

The power of Aristotle's analysis of his predecessors rests on the marshaling of the particulars in their series and order. Most particularly, their temporal series and order. First the material principles, then the efficient and the formal. First water, then air, then fire, then all four, earth, water, air, and fire. First Anaxagoras and Empedocles, then Leucippus and Democritus. First the Pythagoreans, then Socrates, then Cratylus, then Plato.

This is the ground also of the most fundamental proposition: the investigation itself showed them the way.

Aristotle determined the temporal series and order of the particulars only relatively, however, and not in all cases precisely either. And the temporal series and order of the particulars has to be fixed absolutely, and in all cases precisely.

The solution of the problem of chronology begins with the Palermo stone in the valley of the Nile, and with Sargon in the valley of the two rivers. The Palermo stone and Sargon's inscriptions first attest the explicit distinction of one year from another.

Two and a half millennia later Ptolemy fixed chronology absolutely. He correlated from observation of the heavenly bodies the years from 748^{BC} to 137^{AD} with the years of the reigns of 54 princes – Babylonian princes, Assyrian princes, great kings, successors of Alexander, and Roman princes.

The most important steps in the formulation and solution of the problem of chronology between Sargon and Ptolemy are five.

Three centuries after Sargon, the Sumerian prince list.

Seven and a half centuries after the Sumerian prince list, the series of Assyrian princes from Adad-nirari through, six and a half centuries later, Ashurbanipal, whose inscriptions are steadily more explicit chronicles, and under whom the

Assyrian prince list was compiled.

Two centuries after Ashurbanipal, Herodotus.

In the generation after Herodotus, Hippias, who compiled the list of the former victors at the Olympic games.

And not quite one century after Hippias, four and a half before Ptolemy, Aristotle. Aristotle with his student Callisthenes compiled the list of victors at the Pythian games (Plutarch, *Solon*, 11; Callisthenes, fr. 1, T23). This is not very important. Very important are Aristotle's reviews. Herodotus fixed chronology relatively massively, arranging in their series and order 804 predecessors of his in the *Histories*. With the reviews Aristotle extended the reach of relative chronology to the acquisition of knowledge.

And chronology should be fixed absolutely back to the beginning of the acquisition of knowledge, and before the beginning of the acquisition of knowledge, and determined in all cases precisely.

Investigation of the earlier acquisition has suffered greatly from being populated with entities like Egyptian mathematics, Babylonian astronomy, the Sumerians, the Greeks, and worse. Fixing chronology is the ground on which all that can be cleared away.

Investigation of all the acquisition, earlier and later, has suffered disgracefully from the distribution of credit and disgrace. Cardano wantonly claimed the discovery of the method for solving cubic equations. Leibniz wantonly claimed the discovery of calculus. Lobachevsky did not discover non-Euclidean geometry. Helmholtz did not discover the principle of conservation of energy. Darwin did not discover selection. Chronology is the ground on which all that can be cleared away as well.

Chronology is a *sine qua non* of history, and chronology is a *sine qua non* of the investigation of the acquisition of knowledge.

The temporal series and order of the predecessors of the philosophers and philosophers has to be fixed absolutely, and determined in all cases precisely, in order to determine the relations of all steps of advance in the acquisition of knowledge, and all the way back to the first, clearly and distinctly.

In order also to establish the ground of the

proposition, the investigation itself showed them the way, unshakably.

Chronology, like the record, demands special treatment. And this is also best postponed.

Here, in the five following sessions of the council, the resolution of all chronological difficulties is taken for granted. Hence also the certainty of chronology, its absolute certainty, is taken for granted.

Reality.

Reality is always a difficulty. Never nearly as grave, however, as the labors at reinforcing the barriers before the acquisition of knowledge and the investigation of knowledge, and again, and again, and again, would make it.

The difficulty can be divided into three parts.

One. Anaxagoras said that reason is the cause; some say that Hermotimus said so earlier.

Is Hermotimus real? More important, is Homer real? The earlier acquisition abounds in such problems. Is Sin-liqe-unninni real? Is Gilgamesh real? For much of its development philosophy has rested on scriptures, too. And is Moses real? Is Amos? Hosea? Isaiah? And Jesus? Mark? Matthew? Luke? John? Such problems have seldom bedeviled the more recent acquisition. But they have. Ossian fooled Gibbon, Herder, Goethe, Schopenhauer.

The resolution of this part of the difficulty is, with labor, straightforward. In each case as soon as the question, real? or not? arises, all the relevant evidence should be examined, and the question answered explicitly. It has thus in many cases already been answered. It has been answered, for instance, nor has it been too difficult to answer, in the cases of the 120 princes collected in the Sumerian prince list. 37 are real. 25 are not real. The remaining 58 are not important at all, *ipso facto* the question, real? or not? with respect to any of them is not important at all, and to ask it, never mind answer it, can have no other end but, with Leibniz once again, the utterly useless, or in any case the least important.

Two. Thales said that water is the principle; some think that the ancients held the same opinion about nature. Aristotle also lazily neglected to break apart the entity Pythagoreans and recover the particulars from it.

More recently, next to the ancients and the

moderns, and the postmoderns, too, entities of choice have been the scientists, European and any other culture, western and any other civilization, and worse.

The difficulty is that all such entities are fictional. They entered philosophy with Plato's wars. And as Plato abundantly attests, it is a peculiarity of philosophical wars, unlike the real, that they are easier to fight with fictional armies against fictional armies. Accordingly, ever since Plato entities committed to no particular place and time have obfuscated the particulars and their series and order.

The resolution of this part of the difficulty is, also with labor, also straightforward. All the relevant evidence should be examined. And all entities committed to no particular place and time discarded. Those from which nothing but confusion can be recovered discarded out of hand. The rest discarded after the recovery of the particulars.

With respect to especially pernicious entities, like Egyptian mathematics or Babylonian astronomy, and especially recalcitrant entities, like the Pythagoreans or the sophists, this should be done explicitly. And it is below. In any case, however, as soon as such entities are encountered, all the relevant evidence should be examined, the particulars recovered, if there are particulars to recover, and the entities discarded.

Three. The elements are earth, water, air, and fire. The elements are hydrogen, nitrogen, and oxygen. The elements are quarks and leptons. The causes are love and strife. The cause is the first mover. The first mover is god. The cause is gravity. The causes are the gravitational, strong, weak, and electromagnetic forces. The elements of a proposition are subject and predicate. The elements of a proposition are function and argument.

Are those all real? And can, if any of them is not real, any of them be real?

But this gravest part of this gravest of difficulties does not even have to be addressed. If all other difficulties have been resolved, and all particulars have been collected and marshaled in their series and order, this part of this difficulty will have been resolved. If only because the series and order of the particulars cannot not be estab-

lished by the labors of their predecessors and the philosophers, exclusively, and the labors of their predecessors and the philosophers cannot not be cumulative. And if all steps of advance in the acquisition of knowledge are collected, they cannot not show that quarks and leptons stand on hydrogen, nitrogen, and oxygen, which stand on earth, water, air, and fire, and how and why. That the gravitational, strong, weak, and electromagnetic forces stand on gravity, which stands on the first mover, which stands on love and strife, and how and why. That function and argument stand on subject and predicate, and how and why.

Finally, the last two centuries.

Like the record, and chronology, the last two centuries demand special treatment.

For two reasons, one more general and affirmative, one more particular and negative. Because the present is always difficult to judge. And because the present present is the highest, and particularly difficult to judge.

The present is always difficult to judge because it is the present. Judgment is cumulative, it needs time to accumulate, and with respect to the present cannot have had much time to accumulate.

The present present is the highest because spirit has been restored to the substantiality and purity of its being. Judgment has begun to accumulate, but nothing sound has begun to accumulate.

Being the highest, the present has increasingly appeared to itself to encompass not more than the last two years, if not the last two days, or the last two hours. Certainly not the last two decades, never mind the last two centuries.

But the present cannot not encompass unconditionally the last 2 human generations, including the philosophers in them, if any, all still alive, all still eagerly judging. The past cannot reach closer to the present than 4 human generations, so that all philosophers, and all their judgments, are safely dead. And the present can only come out of the past, and slowly, and gradually.

Counting back to the 3 oldest calculations of area, the acquisition of knowledge is 171 human generations old. Counting back to Narmer, the state is 169 human generations old.

The last two centuries encompass the last not

quite 7 human generations.

Counting back to the beginning of the solar system, too, the ratio of the last two centuries to the time before that is, in years, $2 \times 10^2 : 4.6 \times 10^9$. Or about 1 : 20 000 000.

And if there have been many philosophers in the last two centuries, and many very important among them, there have been many more earlier in these two centuries than later, and this distribution severely aggravates the difficulty, too.

In the last two centuries thermodynamics has been developed, in 1850 Clausius formulated its laws. Electrodynamics has been developed, in 1864 Maxwell formulated its laws. The kinetic theory has been developed, by Maxwell and Boltzmann. The theory of relativity has been developed, by Einstein, the special in 1905, the general in 1916. Quantum theory has been developed, between 1900 and 1935, by Planck, Einstein, Rutherford, Bohr, de Broglie, Schrödinger, Heisenberg, Pauli, Dirac, Fermi, and Yukawa. The big bang theory has been developed, beginning with Friedmann's solution of the field equations of general relativity, in 1922, and Hubble's discovery of the expansion of the world, in 1929. These are one and all, thermodynamics, electrodynamics, the kinetic theory, relativity, quantum theory, and the big bang theory, one development, and an essential part of this development is a great advance in the solution of the problem of the principles of all things, of the material principles of all things and of the efficient principles of all things, and a great advance in the solution of the problem of the earth, of its position in the world. This development, however, is difficult to describe in particular, and either with respect to the problem of the principles of all things, or with respect to the problem of the earth, or with respect to any other problem.

One reason is that thermodynamics is not quite 6 generations old, the big bang theory is not quite 2 generations old. Judgment needs time to accumulate.

But the main reason is the other. Judgment has begun to accumulate, but nothing sound has begun to accumulate.

Physics is not a part of philosophy any longer. It is a science. And science follows the ruinous path of misinterpreting spirit and depriving it of

its power. History does not learn from physics and mathematics any longer. The younger sister has broken the fetters in which the older sister held her since Descartes, Spinoza, and Hobbes. And all has become world-views. Or discourses.

To the degree to which they have been investigated thermodynamics, electrodynamics, the kinetic theory, relativity, quantum theory, and the big bang theory have been investigated in isolation each. But they are chains of inferences, one and all long chains of inferences, and one and all one another's premises and consequences. None is a premise without its consequences and none is a consequence without its premises. Each is in isolation a gap in the chains of inferences. Indeed, a world-view. Or a discourse. Relativity and quantum theory have attracted more attention than the rest. In order to elevate them into a revolution in modern science, against the earlier, classical, science. But there is no discontinuity, nor can there be discontinuity, between relativity and quantum theory and their premises, thermodynamics, electrodynamics, and the kinetic theory. Nor is there discontinuity, nor can there be discontinuity, between thermodynamics, electrodynamics, and the kinetic theory and their premises, the classical science of Newton and Leibniz, Euler, Lagrange, and Laplace. Relativity and quantum theory are impossible without their premises, thermodynamics, electrodynamics, and the kinetic theory, which are impossible without Newton and Leibniz, Euler, Lagrange, and Laplace. Just as their joint consequence, the big bang theory, is impossible without them both, and their premises, and the premises of their premises.

In the last two centuries mathematics has advanced greatly as well. This is the time of Cauchy, Lobachevsky, Hamilton, Boole, Weierstrass, Riemann, Cantor, Frege, Peano, Hilbert, Zermelo. It has been a joint development, too, of physics and its part mathematics. An essential part of this development is a great advance in the solution of the problem of the sum of the angles of the triangle and the assumptions on which it depends, and a great advance in the solution of the problem of the principles of all things, of the formal principles of all things. But this development is even more difficult to describe in particu-

lar, and either with respect to the problem of the assumptions on which the sum of the angles of triangle depends, or with respect to the problem of the principles of all things, or with respect to any other problem.

Non-Euclidean geometry is 6-odd generations old, the theory of sets is 3-odd generations old.

Nor is mathematics a part of philosophy any longer. It is not even a science. It is a free creation of the human mind. Like art, if more poorly saturated with ideas and feelings.

In 1859 Darwin formulated the theory of evolution. In 1865 Mendel discovered the laws of heredity. The advance of population biology and genetics followed. This development has been part of the advance of physics, including mathematics. And this development has informed substantially and essentially the problem of the soul, and both the physical problem and the ethical. But this development is yet more difficult to describe in particular, and either with respect to the problem of the soul, or with respect to any other problem.

The theory of evolution is 5-odd generations old.

Nor is biology a part of physics any longer. Nor is it just another of the sciences. It is the science which most ruinously follows the path of misinterpreting spirit and depriving it of its power.

The revolution was defeated in 1815. In the following two centuries Italy was built, Germany was built, a world war was fought, in the course of which the Soviet Union was built, another world war was fought, the Soviet Union fell apart. In the meantime the entire surface of the earth was brought under the state, at first under a few powerful states, then under many, steadily more, and steadily feebler states. If this development is not part of the development of physics, yet the two developments, of physics, including mathematics, and of the state, cannot not inform one another substantially and essentially. And this development informs substantially and essentially the problem of which is the best state, and both the historical problem and the political. But this development is the most difficult of all to describe in particular, and either with respect to the problem of which is the best state, or with

respect to any other problem.

Germany is 5 generations old. The last world war is not yet 3 generations old.

Philosophy has always tended to ignore the real states in favor of those it would itself conceive in speech and hold in high esteem and the real wars in favor of those it would itself fight and win, the philosophical. And rid of physics and mathematics now, unabashedly a vehicle for the combination, permutation, and transposition of words, philosophy has had no time at all for, or indeed any patience with, real states and real wars. History, the unfettered younger sister now, has had no time at all for, or any patience with, states and wars either. It has been busy with coffee and tobacco, sexuality, and shit.

The last two centuries demand special treatment. And this treatment should come, like themselves, last.

Here, in the five following sessions of the council, following the second rule, to divide difficulties into parts such as will lead to their better solution, time is divided into two parts. From the beginning of the solar system to about two centuries ago. And the last two centuries or so. In each case where exactly the division should occur the steps of advance in the acquisition of knowledge decide. And in four of the five cases with respect to the first of these parts all particulars are collected and marshaled in their series and order, with respect to the second it is, here, only indicated how they can be. In one case, that of the best state, all particulars are collected and marshaled in their series and order with respect to four centuries only, with respect to the centuries before and after these four centuries it is, here, only indicated how they can be.

Again, the five sessions are on these problems.

The earth, its position, whether it is at rest or in motion, and its shape.

The soul, and both, what it is, and what you should do about it.

The best state, and both, the well-governed state, and the one conceived in speech.

The sum of the angles of the triangle, and the assumptions on which the sum depends.

The principles of all things, and all three, the material principles, or the elements, of all things, the efficient principles, or the causes, of all

things, and the formal principles, or the principles, of all things.

The earth, its position, whether it is at rest or in motion, and its shape

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The task is to show that the acquisition of knowledge can be collected in its entirety. And to this end, in particular, to collect the predecessors of the philosophers and philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them, with respect to this problem, the earth, its position, whether it is at rest or in motion, and its shape.

At issue are propositions. The winning of propositions.

A general proposition is first surmised from an insufficient number of particular cases. It is more and more certainly established when through chains of inferences it is brought in connection with other propositions as either consequences are derived from it or conversely it is recognized as the consequence of already established propositions. This is the path by which a proposition is gradually won.

Because the gaps in the chains of inferences have been narrowed substantially it is possible to collect propositions into the formulation and solution of a distinct problem. Because the narrowing of the gaps in the chains has been gradual it is possible to identify steps in the development of the formulation and solution of the problem. And both depend on the finality in the formulation and solution of the problem. At least some of the gaps in the chains of inferences have been eliminated completely, and the gradual ascent has reached that step.

The task is thus to collect all propositions brought in connection with one another in the formulation and solution of this problem, the earth.

The formulation of the problem is Aristotle's. It shows finality at its most unambiguous. The problem is divided into the three parts such as will lead to its better solution, the position of the earth, whether it is at rest or in motion, and its shape. Aristotle's solution of the problem shows

finality at its most unambiguous, too. The solution is: in the center of the world, at rest, a sphere.

And in order to formulate and solve the problem Aristotle calls the council of our predecessors into session.

The collection can thus begin with Aristotle's review. This review of his is not complete. The review can be completed. The predecessors of the philosophers and philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them through Aristotle can be collected. And then the predecessors of the philosophers and philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them since Aristotle can be collected.

Aristotle traces the formulation and solution of the problem back to Thales, who said that the earth floats on water, two and a half centuries before himself. Four and a half centuries after Aristotle, Ptolemy measured the world, and hence advanced greatly the solution of the problem of the earth. The oldest result on which Ptolemy's advance stands is the oldest accurate observation, of lunar eclipses in 721-0, eight and a half centuries before himself, one and a half centuries before Thales. The oldest observation is eleven and a half centuries older than the oldest accurate observation, predating Thales by thirteen centuries, Ptolemy by twenty centuries. The temple register from Lahun records an observation of Sirius in the 7th year of Senwosret III, prince c. 1878-45. This is the first inkling with respect to the problem of the earth. After Ptolemy, the main advances are Copernicus, who discovered that the earth revolves around the sun, fourteen centuries after Ptolemy, Newton, who grounded the system of the world in the laws of motion and the law of gravity, one and a half centuries after Copernicus, and Einstein, who developed the theory of general relativity, on the solution of whose equations the big bang theory rests, the theory of the origin and development of the world, two and a half centuries after Newton.

These are the main steps in the development of the problem,

1911	temple register from Lahun,
625	Thales,

384 - 322 ^{BC}	Aristotle,
87 ^{AD}	Ptolemy,
1473 - 1543	Copernicus,
1642 - 1727	Newton,
1879 - 1955	Einstein.

The steps thus divide the development of the problem into eight parts,
all the centuries before the temple register from Lahun,
the thirteen centuries from the temple register from Lahun to Thales,
the two and a half centuries from Thales to Aristotle,
the four and a half centuries from Aristotle to Ptolemy,
the fourteen centuries from Ptolemy to Copernicus,
the one and a half centuries from Copernicus to Newton,
the two and a half centuries from Newton to Einstein,
the century-odd since Einstein.

And the collection of the particulars proceeds in this order. Beginning with Aristotle's review, the review is completed, and the predecessors of the philosophers and philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them from Thales to Aristotle are collected. Then the earlier particulars are collected, those from the temple register from Lahun to Thales. Then the question of the beginning of the problem is addressed. Then the later particulars are collected. Those from Aristotle to Ptolemy. Those from Ptolemy to Copernicus. Those from Copernicus to Newton. The particulars from Newton to Einstein are divided into two parts, those through

1784 - 1846 Bessel,
who in 1838 measured the distance of 61 Cygni,
and those after Bessel. Those through Bessel are collected. And it is indicated how those after Bessel can be collected, those through Einstein, as well as those since Einstein.

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The two and a half centuries from Thales to Aristotle.

It is in the four books of *On the heaven* and book XII of the *Metaphysics* that Aristotle de-

scribes the world in its entirety. The world is one, it is spherical, the primary motion is circular, at the circumference is the first mover moving directly the sphere of the fixed stars and indirectly everything inside, inside the sphere of the fixed stars are the spheres of the planets, Saturn, Jupiter, Mars, the sun, Venus, Mercury, and the moon, and further inside the spheres of the elements, fire, air, water, and earth.

The foundation of the system is in book II of *On the heaven*. It is here that Aristotle establishes that: the world is spherical (4); at the circumference are the fixed stars; the planets are inside (8); the motions of the fixed stars are simple, those of the planets composite (10).

Having established this, at the end of book II he writes: we now turn to the earth, its position, whether it is at rest or in motion, and its shape (13). And he calls the council of our predecessors into session. He reviews what our predecessors have said about the earth.

What our predecessors have said about the principles of all things Aristotle reviews twice. In book I of the *Physics* he reviews what our predecessors have said about the number of the principles. Then in book II he solves the problem of the kinds of the principles. And then in book I of the *Metaphysics* he reviews what our predecessors have said about the principles. The review in book I of the *Physics* leads to the solution of the problem. The review in book I of the *Metaphysics* follows the solution, it examines the solution. Hence also its completeness.

And the review of what our predecessors have said about the earth is like the first of the two reviews of what they have said about the principles. It leads to the solution of the problem. We are to adopt the sound in what our predecessors have said and avoid the erroneous, and it turns out that we are mostly to avoid. The problem of the earth is more like those of the efficient and the formal principles than like that of the material. Our predecessors have grasped in some way the position of the earth, whether it is at rest or in motion, and its shape. But the problem has remained to be formulated and solved explicitly. And this is what Aristotle does. He formulates the problem explicitly in the course of the review, then solves the formulated problem after the

review.

He has analyzed the problem into the three parts, position, at rest or in motion, and shape, and the review proceeds accordingly.

He writes:

as to its position, there are different judgments;

most, and in fact all who presume that the world is finite, say that the earth is in the center;

but the Italian philosophers called Pythagoreans take the opposite view; they say that fire is in the center, and the earth is one of the stars, creating the appearance of night and day by its revolution around the center; they posit also another earth in opposition to ours, calling it counter-earth; not looking to explain the phenomena with all this, but to fit the phenomena to their opinions;

as to rest or motion, there is no general agreement either;

those who deny that the earth is in the center say that it revolves around the center, and not just the earth, so does the counter-earth;

others say that the earth is in the center but rotates on its axis; so it stands in the *Timaeus*;

nor is there an agreement about its shape; some say it is spherical, some say it is flat, some say it is like a drum;

those who say it is flat, say that it has to have this shape because it is at rest; indeed, the difficulty must strike everyone; free in the air, a piece of earth falls, a larger piece falls faster, a larger piece faster still, yet the entire earth, free in midair, appears to move not at all;

hence some, like Xenophanes, say that it stretches infinitely downward, to save themselves the trouble of looking for causes;

others say that it rests on water; this is, in fact, the oldest judgment on the matter, that of Thales, who said that the earth is at rest because it floats on water, like wood; as if it could not be asked, and on what does the water float?

Anaximenes, Anaxagoras, and Democritus say that it is at rest because it is flat; it floats on the air below, like a lid; this seems to be what flat things do, they say, even the wind cannot move them; so the air supports the earth like water in a clepsydra;

Empedocles says that it is at rest because the

very fast motion of the heaven prevents it from moving, like water in a cup, when the cup is spun fast, and the water is thus prevented from moving downward, as is natural for it;

Anaximander says that it is at rest because of equilibrium; and this is ingenious, but not true; for on this argument everything which reaches the center must stay in the center; and we observe not only many things which stay in the center, but many things also which move toward the center;

and these are the judgments about the shape of the earth, its position, and whether it is at rest or in motion (13).

These, then, are the 8 particulars noted explicitly in the review,

- 625 Thales,
- 611 - 541 Anaximander,
- 590 Anaximenes,
- 570 - 473 Xenophanes,
- 500 - 428 Anaxagoras,
- 495 - 435 Empedocles,
- 460 Democritus,
- 427 - 347 Plato.

Aristotle has noted also the Italian philosophers called Pythagoreans.

The latest result in the review is Plato's. And one generation-odd after Plato, this is Aristotle's solution of the problem. He continues:

let us first decide whether it is at rest or in motion;

if it were in motion, changes in the risings and settings of the fixed stars would be observed; but the same stars always rise and set in the same positions on the horizon;

moreover, the natural motion of earth, and of pieces and whole alike, is toward the center;

we conclude that the earth does not move, and is nowhere but in the center;

and its shape must be spherical;

for every bit of earth has weight until it reaches the center;

further, how else can the outline in lunar eclipses be always circular?

further, observation of the stars shows not only that the earth is spherical but that it is not large; for even a small change in position on its surface results in a large change in the horizon; there are stars which are seen in Egypt but not further north;

nor should it seem incredible that the ocean continues beyond the strait of Gibraltar all the way to India;

indeed, the mathematicians calculate the circumference of the earth at 400 000 stades;

and all this shows not only that the earth is a sphere, but that it is not a large one either (14).

In the solution of this problem, of the earth, as in the solution of the problem of the principles of all things, Aristotle's advance over his predecessors is very great. In the solution of this problem, however, unlike that of the principles of all things, in which there was little advance in the centuries after Aristotle, the advance continued in the centuries after him, his successors advanced the solution of the problem of the earth very greatly over him, and all the way to Ptolemy.

It is clear why. In the first two and a half centuries of its development from Thales through Aristotle philosophy was very much the development of physics and its part mathematics jointly. Then the schools flourished. Philosophy became an orchard. Physics such as the schools taught it became the earth and the tree of the fruit. But the schools did not teach mathematics. Mathematics developed greatly, all of it outside the schools. And as mathematics developed greatly, physics such as it is not the earth and the tree of the fruit developed greatly.

The problem of the principles of all things was not to become a mathematical problem for a long time. But that the problem of the earth had already become a mathematical problem Aristotle's conclusive argument, the first measurement of the circumference of the earth, suffices to show.

Observation is measurement and calculation. Advance in observation is part of the advance of mathematics.

And as all of Aristotle's arguments show, the foundation of Aristotle's advance in the solution of the problem of the earth is the advance in observation. The foundation of the continuing advance in the solution of the problem after Aristotle and all the way to Ptolemy is the advance in observation. And when at length the darkest of dark ages came to a close, and the advance resumed, the foundation of the great advance in the solution of the problem by Copernicus, and the more so the great further advance

in the solution of the problem after him, is the advance in observation.

The advance in observation should be the ground on which to sort out and complete Aristotle's review.

Ptolemy collected all earlier accurate observations. 94 of them. And all earlier accurate observations indicate clearly and distinctly three steps in the development of observation. The oldest accurate observation is of a lunar eclipse in 721. The oldest accurate observation of a solstice is by Meton in 432. And the oldest accurate observation of a planet or star is by Timocharis in 295.

Tabulating,

761 observation of lunar eclipses from
721 to 720,

472 Meton,

335 Timocharis.

Thus, before Meton observation has succeeded in reducing to accuracy only the most spectacular heavenly events, eclipses. Beginning with Meton, it has succeeded in reducing to accuracy the solstices. Beginning with Timocharis, it has succeeded in reducing to accuracy the planets and stars.

And these steps in the advance in observation are illustrated and corroborated by what our predecessors have said about the earth.

8 particulars are noted explicitly in Aristotle's review. 6 of them, Thales, Anaximander, Anaximenes, Xenophanes, Anaxagoras, and Empedocles, predate Meton. The latest of the 6, Empedocles, predates him by not quite one generation. The review attests the concern of the 6 with the phenomena, pieces of falling earth, floating water, floating lids, flat things in the wind, water in a clepsydra, water in a spinning cup. But it attests no observation of the heavenly bodies. Nor does any of their fragments, and their fragments are many, attest observation of the heavenly bodies.

The other 2 are Democritus and Plato. They postdate Meton.

Aristotle himself is about halfway between Meton and Timocharis. He postdates the oldest accurate observation of a solstice by three generations and predates the oldest accurate observation of a planet or star by two generations.

And all of Aristotle's arguments show that the

foundation of the advance in the solution of the problem of the earth is the advance in observation. Because they are all concerned with the phenomena. And because three of them – the outline in lunar eclipses is circular, the same stars always rise and set in the same positions on the horizon, and there are stars seen in Egypt but not further north – rest on observation. None of the three is an observation as Aristotle has it, they are all three too vague as Aristotle has them. But they are all three observations. The outline in lunar eclipses had been observed, lunar eclipses had been observed accurately for four centuries by that time. And the observation of risings and settings of fixed stars and the observation of fixed stars at different latitudes are attested in the next generation by Autolycus and Euclid.

In the generation before Aristotle Eudoxus wrote the first *Phenomena*, in which he described 32 northern and 15 southern constellations. This is on the authority of Hipparchus who wrote a commentary on Eudoxus's *Phenomena*. And Ptolemy reports: Hipparchus found very few observations of fixed stars from before his time, and none earlier than those of Timocharis. Eudoxus predates Timocharis by two-odd generations.

There are 3 observations of the heavenly bodies elsewhere in Aristotle himself. 1 earlier in *On the heaven*: observation shows that the moon is closer to the earth than the other planets; for we have seen it pass under Mars. And 2 in the *Meteorology*: Mercury rises only a little above the horizon, and: we have ourselves observed Jupiter coincide with one of the stars of Gemini. But none of the 3 is an accurate observation yet.

The development of the problem of the earth in the two and a half centuries from Thales to Aristotle can thus itself be divided into two parts.

The one century-odd from Thales to Anaxagoras, when there is very little observation and such as there is is very crude.

And the following one century-odd from Anaxagoras to Aristotle, when there is more observation and it is not so crude. It has to be to this time, too, before Timocharis but after Meton, that Ptolemy refers when he writes: the oldest observations of the planets are useless because they are too crude.

And on this ground, more particularly, Aris-

tole's review can be sorted out, and completed.

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First.

The main difficulty which Aristotle's review poses can be resolved. This are once again the Italian philosophers called Pythagoreans.

In a vain, as it turned out, attempt to disclaim novelty, Copernicus writes in *On the revolutions of the heavenly spheres*: I reread the works of all philosophers I could find, and having learned from Cicero and Plutarch that Philolaus, Hicetas, Ecphantus, and Ponticus have argued that the earth moves, I began myself to consider that the earth moves (ded.).

Copernicus failed in that attempt. Even as the name Pythagorean hypothesis was attached to the Copernican theory, it was assailed for wanton novelty. With that attempt, however, Copernicus resolved the difficulty which Aristotle lazily neglected. He recovered the particulars from the entity Pythagoreans. The entity can thus be discarded, and the evidence about the particulars collected.

What Copernicus takes to be Plutarch has since been shown to be Aëtius. And the source of Aëtius for the early philosophers is Theophrastus.

Aëtius reports: Philolaus the Pythagorean said that fire is in the center, and he calls it the hearth of the world; next around the hearth revolves the counter-earth; and next is the earth we inhabit, which is always in opposition to the counter-earth and this is why we never see it (III, 11); Philolaus said that there is another fire at the circumference; but the center is primary and around it 10 bodies revolve, the heaven, the 5 planets, the sun, the moon, the earth, and closest to the hearth the counter-earth (II, 7).

Laertius reports: Philolaus first said that the earth moves; but others say that Hicetas first said so (VIII, 85).

Cicero reports: Theophrastus says that Hicetas said that the heaven, the sun, the moon, and the stars are all at rest, and nothing in the world moves except the earth; the earth creates the appearance that all those move by turning on its axis (*Academic*, II, 123).

Laertius reports: the doctrines of the Pythagoreans were secret down to the time of Philolaus;

he first brought them out in the three famous books which Plato bought for 100 minae; I have this from Aristoxenus, in book X of the *Rules of education* (VIII, 15).

Plato himself has two followers of Philolaus among those attending to the drinking of the hemlock in the *Phaedo* (61d).

First, then, Philolaus said that the earth moves. Next, Hicetas discovered that the earth's rotation on its axis can explain the apparent revolution of the heaven. The sequence should be probable in any case. First: they fit the phenomena to their opinions. Next, they tried to explain the phenomena. And the authority of Theophrastus and Aristoxenus settles the matter.

Philolaus is a contemporary of Socrates.

Dating Hicetas between Philolaus and Plato, he made the very important discovery of the earth's rotation on its axis in the next generation.

Aëtius reports: Ecphantus and Ponticus said that the earth rotates on its axis (III, 13).

Hippolytus reports: Ecphantus said that atoms are the principles (*Refutation*, I, 15).

Dating Ecphantus between Democritus and Ponticus, he said that the earth rotates on its axis at the time when Plato also said so, in the generation after Hicetas. When Plato and Ecphantus said so, then, they corroborated Hicetas.

When in the next generation Ponticus said so, he thus also corroborated Hicetas. Ponticus, however, first said as well that Mercury and Venus revolve around the sun and this can explain their retrogradations, retardations, and stations. This can be inferred from the following reports. Vitruvius writes: the paths of Mercury and Venus wreath around the sun, and this is why they appear sometimes to go back, sometimes to slow down, sometimes to stop (IX, 1.6). Vitruvius records this important result, but does not record who obtained it. In the fourth Christian century Calcidius writes: when Ponticus drew the circles of Lucifer and the sun, he assigned a single center to both, thus showing why Lucifer is sometimes a morning star sometimes an evening star (comm. *Timaeus*, 176). Hence, Ponticus obtained the result recorded by Vitruvius. One century after Calcidius, Capella records the result in more detail also without recording who obtained it (*Philology*, VIII, 879-82). That the result

was obtained after Hicetas, but before Aristotle resolved the question of at rest or in motion – at rest – should be probable in any case.

Arranging the 4 particulars in their series and order,

469 Philolaus,
448 Hicetas,
427 Ecphantus,
395 - 317 Ponticus,

it should be probable in any case that they all belong in the second of the two parts into which the two and a half centuries from Thales to Aristotle divide, when there is more observation and it is not so crude. When there are observations of the planets, useless and too crude as they are.

And no other of the large number of particulars that can be recovered from the entity Pythagoreans belongs in the development of the problem of the earth. And the number is large, Iamblichus alone has a list of 234 (*Pythagoras*, 266-7).

Next.

Aristotle's reports on Anaximander, Anaximenes, Anaxagoras, and Democritus can be made more explicit.

Aristotle writes: some say the earth is spherical, some say it is flat, some say it is like a drum.

Hippolytus reports: Anaximander said that the earth is shaped like a drum (*Refutation*, I, 6).

Aristotle writes: Anaximenes, Anaxagoras, and Democritus say that the earth is at rest because it is flat; it floats on the air below, like a lid.

The first philosopher said that the earth floats on water, like wood. It can be concluded that the third philosopher said that it floats on the air below, like a lid.

On Anaxagoras and Democritus there are further reports.

Hippolytus: Anaxagoras said that the earth is at rest carried by the air below because there is no void (8).

Aëtius: Democritus said that the earth moved about at first because it was small and light, but as it became larger and heavier came to rest in the center (III, 13).

Next.

The glaring omission of Aristotle's in the review, the discovery that the earth is a sphere, can be remedied. Parmenides made this most

important discovery.

Laertius has these four reports:

Anaximander said that the earth is spherical (II, 1);

Alexander Polyhistor in *The succession of the philosophers* says that among the doctrines of the Pythagoreans is that the earth is spherical (VIII, 24-5);

some say that Pythagoras first said that the earth is spherical, but Theophrastus says that it was Parmenides, and Zeno of Citium says that it was Hesiod (VIII, 48);

Parmenides first said that the earth is spherical; thus Theophrastus in the history of the physicists where he records nearly all the doctrines (IX, 21-2).

The dates are,

734 Hesiod,
611 - 541 Anaximander,
570 - 495 Pythagoras,
515 - 445 Parmenides.

And that Hesiod said that big-bosomed Earth, the foundation of all immortals, is spherical, illuminates only Zeno of Citium's sense of humor.

That Anaximander said that the earth is spherical is ingenious, but not true. Anaximander said that the earth is shaped like a drum.

This leaves Pythagoras and Parmenides.

On one side, the authority cannot be firmer than Alexander Polyhistor, on the supposition that the report about Pythagoras is gathered from his account of the doctrines of the Pythagoreans. Alexander Polyhistor is a minor grammarian at the time of Caesar (Suetonius, *Grammarians*, 20). On the other side the authority is Theophrastus in the history of the physicists where he records nearly all the doctrines. This alone settles the matter.

But there is also the fact that Pythagoras predates Philolaus, in particular, and the time when observation is not very little and very crude any longer, in general, by one century. In general, Parmenides is two-odd generations closer to this time. And, in particular, there is the fact also of the rediscovery that the evening star is the morning star. Laertius reports: Parmenides also first said that the evening star is the morning star; thus Favorinus in book V of the *Memorabilia*; others

say that it was Pythagoras in a poem which Callimachus says, however, is not by Pythagoras (IX, 23). On one side, the authority again cannot be firmer than Alexander Polyhistor, on the same supposition. On the other side, the authorities are Callimachus and Favorinus. Callimachus is a major grammarian – and poet – at the time of Ptolemy II. The authority of Favorinus can be judged by the fact that he is quantitatively the most important source of Laertius. Laertius quotes him explicitly 45+ times. His second most important source is Hermippus whom he quotes explicitly 38+ times. Alexander Polyhistor he quotes explicitly 10+ times.

Parmenides discovered that the earth is a sphere. That the evening star is the morning star he rediscovered. This discovery predates him by eleven and a half-odd centuries, attested on the Venus tablet of Ammisaduqa.

Parmenides discovered that the earth is a sphere, and three generations later Plato first corroborates the result. In the *Phaedo* Plato argues that the soul is immortal. In the course of the argument he develops the theory of the forms. And in conclusion he describes the earth. He writes: I am convinced that if the earth is a sphere in the middle of the heaven, it needs neither air nor anything else to keep it from falling; the uniformity of the heaven and the equilibrium of the earth suffice; for a thing in equilibrium in the middle of something uniform will have no tendency to incline more or less in any direction, it will have a tendency to remain unmoved (109a).

Next.

The most difficult of the three parts into which Aristotle analyzes the problem is the position of the earth, and that is, its position in the world. Hence, proceeding deductively, Aristotle establishes first, and in this order, that the world is spherical, at the circumference are the fixed stars, the planets are inside, and the motions of the fixed stars are simple, those of the planets composite. Then he turns to the earth.

And these results can also be reduced to particularity.

Theon of Smyrna writes: Oenopides discovered the obliquity of the ecliptic (III, 40).

Pliny writes: Anaximander is said to have discovered the obliquity of the ecliptic (II, 31).

On one side, Theon of Smyrna is a contemporary of Ptolemy, and left behind the *Mathematical matters useful for the study of Plato*, the source of the report. On 4 of the observations in the *Almagest* Ptolemy notes: they were given to us by the mathematician Theon (IX, 9; X, 1, 2). The name Theon is common, but the philosophers who concerned themselves with either mathematical matters or accurate observation are not common at all, and are of increasing scarcity by Ptolemy's time. Occam's razor demands: plurality must not be posited without necessity. And it can be concluded, in respect for Occam's razor, that the mathematician Theon is Theon of Smyrna. On the authority of Eudemus also: Oenopides first studied Euclidean propositions because they are useful in astronomy. On the other side, the rule seems to have been, to ascribe everything to the first philosopher may be too much, but why not to the second philosopher?

490 Oenopides

discovered the obliquity of the ecliptic.

With the discovery of the obliquity of the ecliptic the planets quit being just wanderers. They became wanderers, back and forth, if not on its own fixed path each yet, all in the same fixed plane.

Meton observed a solstice. Ptolemy writes: to examine the accuracy of our determination of the summer solstice we can compare it first with that of the associates Meton and Euctemon, even if it is crude, because of its antiquity; they observed a summer solstice on 27 June 432 at dawn (*Almagest*, III, 1). It is one of the comparisons which allows Ptolemy to conclude that the length of the solar year is $365 + \frac{1}{4} - \frac{1}{300}$ days. While: on the reckoning of the associates Meton and Euctemon its length is $365 + \frac{1}{4} + \frac{1}{76}$ days (*id.*).

Meton discovered also the Metonic cycle, of 235 lunar months, very nearly equal to 19 solar years, after which the new moon occurs on the same day of the year. This result was improved by Callippus, in the generation after Aristotle, into the Callippic cycle, equal to 4 Metonic cycles plus 1 day. Ptolemy uses both (*id.*). It can be concluded, in respect for Occam's razor again, that while

472 Meton

is a very important philosopher, his associate

472 Euctemon

is not important.

Crude as it is, Meton's observation of a solstice is an observation, and belongs in the development of the problem of the earth. Earlier looking at the solstices, Thales's, or earlier still Hesiod's in the *Work and days*, does not belong in the development of the problem.

With the observation of the solstices, the wanderers obtained fixed points with respect to which to wander.

Simplicius writes: Anaximander first discussed the sizes and distances of the planets, thus Eudemus writes, adding, however, that the Pythagoreans first determined the order of the planets (comm. *Heaven*, 471).

It need not be doubted that Anaximander, although the second philosopher, first discussed the sizes and distances of the planets.

It need not be doubted either that Pythagoras, the fourth philosopher, first discussed the order of the planets.

As to determination, the determination of their size begins, one century-odd after Anaximander, with Anaxagoras. Aëtius reports: Anaxagoras said that the sun is a piece of red-hot metal larger than the Peloponnese (II, 21). Even as the determination of the size of the Peloponnese took longer, apparently. One century after Anaxagoras, Eudoxus left behind a description of the Peloponnese which is strictly qualitative (Strabo, IX, 1.1). It is Polybius, two centuries later still, who left behind the oldest number: cape to cape the circumference of the Peloponnese is 4 000 stades (XXXIV, 12).

The determination of the order of the planets begins, two-odd generations after Pythagoras, also with Anaxagoras. Hippolytus reports: Anaxagoras said that the moon is beneath the sun and closest to us (*Refutation*, I, 8).

The position of the moon in the order was thus fixed. The determination of the size of the planets begins with Anaxagoras, and the determination of their order begins with him. Hence, Anaxagoras discovered the cause of eclipses, too. Hippolytus reports: Anaxagoras said that lunar eclipses occur when the earth blocks the sun's light from falling

on the moon, solar eclipses occur when the moon hides the sun from view (*Refutation*, I, 8). Plato first notes the discovery (*Cratylus*, 409a-c).

One century-odd before Anaxagoras, Thales predicted a solar eclipse. And Thales belongs in the development of the problem of the earth, indeed there would be no such development if it were not for Thales. But not because of his prophetic prediction, one century-odd before Anaxagoras.

In the generation of Anaxagoras, one century after Pythagoras, the determination of the order of the planets continues with Philolaus. Aëtius again: Philolaus said that fire is in the center; he said that there is another fire at the circumference; but the center is primary and around it 10 bodies revolve, the heaven, the 5 planets, the sun, the moon, the earth, and closest to the hearth the counter-earth.

The counter-earth, completing the number to the perfect 10, fit the phenomena to their opinions. The other 9, however, rather explain the phenomena.

One generation later, Plato wrote the *Republic*. The *Republic* shows what justice is, and ought to be. It ends with the story of Er. Er was killed in battle, but twelve days later came back to life, to tell of what he saw on the other side. And one thing he saw was: the spindle of necessity, spinning around all things in the heaven; there were 8 whorls; the innermost, the eighth, spun the fastest; the seventh, the sixth, and the fifth spun together with the same speed; the fourth was slower, the third slower still, and the second yet slower; the spindle itself turned on the lap of necessity (X, 616b-7b).

Even if he had to go to the other side to show it, Plato shows that the positions of Mars, Jupiter, and Saturn in the order – slower, slower still, and yet slower – were fixed.

To fix the positions of the sun, Mercury, and Venus in the order – spinning together with the same speed – remained difficult. Aristotle is vague on their order. Ptolemy has to examine it at some length. Copernicus will eventually fix it, when he fixes the position of the earth in the order.

The sequence, first the discovery of the fixed plane of revolution of the planets and the determi-

nation of the position of the moon in the order of the planets, then the determination of the fixed points in the paths of the planets, then the determination of the positions of the other planets in their order, should be probable in any case. It should be probable in any case as well that with the first two results, the discovery of the fixed plane of revolution and the determination of the position of the moon, the first of the two parts into which the two and a half centuries from Thales to Aristotle divide, when there is very little observation and such as there is is very crude, comes to an end, and with the next two results, the determination of the fixed points, and then of the positions of the other planets, the second part begins, when there is more observation and it is not so crude. When there are also observations of the planets, useless and too crude as they are.

With the determination of their order, the wanderers became a wanderer, back and forth, on its own fixed path each, with fixed points on the path with respect to which to wander, all in the same fixed plane. Hence: Plato posed the problem of finding hypotheses which will show that the apparently irregular motions of the planets are regular (Simplicius, comm. *Heaven*, 488). And: Eudoxus formulated the hypotheses of regular circular motion to which the apparent irregular motions of the planets can be reduced (*id.*).

The determination of the distances of the planets begins with Aristarchus, two generations after Aristotle.

Next.

There is the fact: the mathematicians calculate the circumference of the earth at 400 000 stades.

Not the mathematicians. The singular mathematician who calculated the circumference of the earth is Eudoxus.

408 - 356 Eudoxus

is a couple of decades younger than Plato, a couple of decades older than Aristotle.

Eudoxus formulated the theory of proportion.

He developed the method of exhaustion.

He formulated the theory of the motions of the heavenly bodies in concentric spheres. And Aristotle notes the theory in book XII of the *Metaphysics*: to explain the motions of the planets Eudoxus assigned 3 spheres each to the moon

and the sun, and 4 spheres to each of the other planets (8).

He described 32 northern and 15 southern constellations.

He wrote a *Description of the earth*, of which there are substantial fragments in Strabo.

It can be concluded that it would have taken a miracle to prevent Eudoxus from calculating the circumference of the earth.

With the calculation of the circumference of the earth Eudoxus did not only solve the problem of the shape of the earth conclusively. Stades varied from about .2 km to about .15 km. Eudoxus thus calculated the radius of the earth at not more than about 12 700 km and not less than about 9 500 km. The equatorial radius of the earth is 6 378 km. Eudoxus did not only show: that the earth is not a large sphere. He solved the problem of the order of magnitude of the size of the sphere.

Finally.

One of Aristotle's arguments for the sphericity of the earth is: how else can the outline in lunar eclipses be always circular?

The observation of eclipses belongs in the development of the problem of the earth.

Earlier in *On the heaven*, while examining the order of the planets, Aristotle reports: we have ourselves seen the moon pass under Mars; and there are similar reports about the other planets from the Egyptians and the Babylonians whose observations have been kept for many years and from whom much of our evidence is derived.

More generally, observation belongs in the development of the problem. In fact, the development which leads to the formulation and solution of the problem of the earth, beginning with Thales, is substantially and essentially the development of observation.

The reports of the Egyptians and the Babylonians have to be collected. The reports of the Greeks, too. And these, the reports of the Egyptians, Babylonians, and Greeks, are all the reports that can be collected. There are no other reports from the first through the time of Aristotle.

The accurate observations can be collected from Ptolemy, and then all other observations can be collected. This should be next, the collection of the particulars constituting the earlier part of

the development of the problem, the thirteen centuries from the temple register from Lahun to Thales.

Here, taking the results of that collection for granted in order to complete the collection with respect to this part, the two and a half centuries from Thales to Aristotle, there are the following particulars.

No particulars are attested by the reports of the Egyptians from this time, the two and a half centuries from Thales to Aristotle. 3 particulars are attested by Ptolemy and the reports of the Babylonians. 1 important,

566 observation of Jupiter from 526 to 490,

and 2 not important,

563 observation of lunar eclipses from 523 to 491,

463 observation of Mars and Saturn from 423 to 400.

The particulars have to be arranged in their series and order. Clearly, postdating Thales,

566 observation of Jupiter from 526 to 490,

belongs in the series of the philosophers, after

570 - 473 Xenophanes

and before

562 Hecataeus.

Thus, clearly as well, as Xenophanes is the fifth philosopher, the observation of Jupiter from 526 to 490 is the sixth philosopher, Hecataeus is the seventh philosopher.

And, finally, no observation other than those enumerated above is attested by the reports of the Greeks.

Looking and seeing is attested.

One and a half centuries before Thales, Hephaestus carved on the shield of Achilles: all the constellations that adorn the heaven, the Pleiades and the Hyades and Orion and the Great Bear who looks at Orion and who alone never bathes in the Ocean.

Thales himself fell into a well while gazing at the stars.

Besides, the closest looking and seeing comes to observation is the little there is about Cleostratus. And it is not close at all.

The spurious *On weather signs* in Theophrastus's corpus reports: there have been good astron-

omers in different places, Matricetas in Methymna who looked from mt Lepetymnos, Cleostratus in Tenedos who looked from mt Ida, Phaeinos in Athens who looked from mt Lycabettus; Meton, who discovered the 19-year cycle, was a student of Phaeinos, and while Phaeinos was an alien in Athens, Meton was an Athenian; and others have also looked (4). Pliny as well writes: Anaximander is said to have discovered the obliquity of the ecliptic, Cleostratus is said to have explained the signs in it beginning with Aries and Sagittarius (II, 31).

Dating him between Anaximander and Meton, 541 Cleostratus is, given the two reports, not very not important. Matricetas and Phaeinos are very not important. Cleostratus is not important.

And with this the collection of the particulars with respect to the problem of the earth in the two and a half centuries from Thales to Aristotle is complete.

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At issue are propositions. The winning of propositions.

A general proposition is first surmised from an insufficient number of particular cases. It is more and more certainly established when through chains of inferences it is brought in connection with other propositions as either consequences are derived from it or conversely it is recognized as the consequence of already established propositions. This is the path by which a proposition is gradually won.

Because the gaps in the chains of inferences have been narrowed substantially propositions can be collected into the formulation and solution of a distinct problem. Because the narrowing of the gaps in the chains has been gradual steps in the development of the formulation and solution of the problem can be identified. Both depend on finality. At least some of the gaps in the chains of inferences have been eliminated completely, and the gradual ascent has reached that step.

And the collection is complete not because, implicitly, it points to all propositions brought in any connection whatsoever with the propositions brought in connection with one another essentially in the formulation and solution of the

problem of the earth. The collection is complete because, explicitly, it contains all propositions brought in connection with one another essentially in the formulation and solution of the problem, and thus separates them sharply from everything else.

Propositions follow from one another. One proposition leads to another and can lead to all propositions.

And each of the results noted explicitly in the course of the collection will lead to many propositions. All too many of the results will lead to all too many propositions. They will lead not only to propositions brought in connection with other propositions through chains of inferences as one another's consequences. They will lead to surmises from an insufficient number of cases, and to surmises from no cases at all. They will lead to endless saturation with ideas and feelings, too.

Thales said that the earth floats on water because he said that water is the principle.

Anaximander, if he did not discover the sphericity of the earth or the obliquity of the ecliptic, yet said that the stars originated in a ball of fire and man came from fish.

Anaximenes said that the earth floats on air, and that air is the principle, and brought not less than a dozen propositions in connection with this last proposition.

Plato's results about the sphericity of the earth and the place of Mars, Jupiter, and Saturn in the order of the planets are very small parts of very large disquisitions on the immortality of the soul and the theory of the forms and what justice is and ought to be, and the just reward awaiting the unjust on the other side, too.

The results of Eudoxus entail the entire earlier development of mathematics.

The results of Aristotle can each be followed, proposition by proposition, consequences and surmises, to any other proposition in the Aristotelean corpus.

If the collection is complete, however, it is not because, implicitly, it points to all propositions brought in any connection whatsoever with the propositions brought in connection with one another essentially in the formulation and solution of the problem of the earth. If the collection is complete, it is because, explicitly, it contains

all propositions brought in connection with one another essentially in the formulation and solution of the problem, and thus separates them sharply from everything else. And what is essential can be made explicit.

The finality in the formulation and solution of a problem, the elimination of at least some of the gaps in the chains of inferences, makes it possible to collect propositions into the formulation and solution of a distinct problem. The gradual narrowing of the gaps in the chains makes it possible to identify steps in the development of the formulation and solution of the problem.

And finality Aristotle's formulation and solution of the problem of the earth shows at its most unambiguous. The problem of the earth is its position, at rest or in motion, and its shape. Each of the three parts is so clear and distinct, one of them has by Aristotle's time already succumbed, and the other two will eventually also succumb, all three as formulated by Aristotle, to calculation.

Aristotle's solution of the problem is: in the center, at rest, a sphere. It is not perfect. Aristotle's solution yet not only remained the solution of the problem, improved substantially by his successors through Ptolemy, until Copernicus. Aristotle's solution is as clear and distinct as his formulation of the problem is.

The gradual narrowing of the gaps in the chains the true way shows unambiguously. The solution of the problem begins with the boldest of flights. The earth floats on water because water is the principle. Flying continues, and remains the main way of the solution through Plato. Spinning around all things in the heaven, the spindle of necessity spins Mars, Jupiter, and Saturn. But it is not the way exclusively any longer. How the spindle spins them – Jupiter more slowly than Mars, Saturn more slowly than Jupiter – Baconian labors have decided. The labors of Eudoxus of collecting 32 northern and 15 southern constellations – some hundreds of stars – are Baconian paradigmatically. And Aristotle solves the problem deductively, he describes the world, hence concludes: at rest, and: in the center, and hence concludes: a sphere. Yet the result: a sphere, rests firmly, never mind the deduction, on observations, equipped with a number, too.

In Ptolemy the solution is inductive, not deductive. And Aristotle's conclusion, a sphere, has become a premise. Ptolemy argues from: the observed circular motions of the heavenly bodies (*Almagest*, I, 3), and: the observations showing that the earth is a sphere (4), to the conclusions: in the center (5), and: at rest (6-7).

In Copernicus the solution is also inductive, not deductive. And the premises are the same as in Ptolemy, but Copernicus has inverted their order, and, a sphere, has become the first premise in order to draw different conclusions. Copernicus argues from: the observations showing that the earth is a sphere (*Revolutions*, I, 2-3), to: the observed circular motions of the heavenly bodies (4), to the conclusions: in motion (5), and: not in the center (6).

The investigation itself showed them the way, and it showed them the true way, not the false.

The essential is the true way. The gradual winning of propositions. The gradual narrowing of the gaps in the chains of inferences. The gradual accumulation of all that is won.

And this is what the collection does. Beginning with Aristotle's formulation and solution, and with observation as the firmest guide, the collection follows the propositions brought by Aristotle in connection with one another back to the propositions on which they rest, to the propositions on which these rest, to the propositions on which these rest, and all the way back to Thales.

Which should all be probable in any case, resting as it is on the true way, in general, and, in particular, on Aristotle's analysis of the problem, Ptolemy's analysis of observation, and Frege's analysis of the path by which a proposition is gradually won.

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In the two and a half centuries from Thales to Aristotle, then, these 19 are all important philosophers who are part of the development of the problem of the earth,

625	Thales,
611 - 541	Anaximander,
590	Anaximenes,
570 - 495	Pythagoras,
570 - 473	Xenophanes,
566	observation of Jupiter from 526 to

515 - 445	Parmenides,
500 - 428	Anaxagoras,
495 - 435	Empedocles,
490	Oenopides,
472	Meton,
469	Philolaus,
460	Democritus,
448	Hicetas,
427 - 347	Plato,
427	Ecphantus,
408 - 356	Eudoxus,
395 - 317	Ponticus,
384 - 322	Aristotle.

And to show that the line between important and not important is drawn sharply, these 4 are some of the more important among the not important, observation of lunar eclipses from 523 to 491,

541	Cleostratus,
472	Euctemon,
463	observation of Mars and Saturn from 423 to 400.

The formulation and solution of the problem of the earth begins with the first philosopher, with Thales. There is no formulation or solution of the problem of the earth before the first philosopher, before Thales.

Thales said that the earth is at rest because it floats on water, like wood.

It is a solution of one of the three parts of the problem explicitly, and of another of the three implicitly. At rest. And the floating on water implies that this is its position in the world.

Anaximander said that it is at rest because of equilibrium, and it is shaped like a drum. He discussed the sizes and distances of the planets.

It is a solution of all three parts of the problem, of two explicitly, of the third still implicitly, yet more explicitly than the earlier solution. At rest. A drum. And equilibrium and the question of the sizes and distances of the planets imply more immediately its position in the world.

Anaximenes said that it is at rest because it is flat, it floats on the air below, like a lid.

The solution, at rest, is more firmly grounded, resting on the solution, flat.

Pythagoras discussed the order of the planets. That the problem of its position is the problem

of its position in the world is becoming clear.

Xenophanes said that it stretches infinitely downward.

The part about its position is becoming the ground on which to solve the other two parts.

From 526 to 490 Jupiter was observed.

The accumulation of observation continues.

Parmenides discovered that the earth is a sphere. He rediscovered that the evening star is the morning star.

The problem of its shape has been solved. It remains to establish this fact.

Anaxagoras said that it is at rest because it is flat, carried by the air below because there is no void. He said that the moon is beneath the sun and closest to us. He said that the sun is a piece of red-hot metal larger than the Peloponnese. He discovered the cause of eclipses, lunar eclipses occur when the earth blocks the sun's light from falling on the moon, solar eclipses occur when the moon hides the sun from view.

The determination of the order of the planets has begun. The determination of their size has begun. The problem of the position of the earth is clearly of its position in the world. It is clear also that the problem of the earth is in all its parts a problem of the determination of order and size. The solution of the problem is in all its parts more explicit, and more firmly grounded. Anaxagoras is a very decisive step.

Empedocles said that it is at rest because the very fast motion of the heaven prevents it from moving, like water in a cup, when the cup is spun fast, and the water is thus prevented from moving downward.

A step toward experiment has been taken.

Oenopides discovered the obliquity of the ecliptic.

Measurement has begun.

Meton observed a summer solstice on 27 June 432 at dawn. He calculated the length of the solar year at $365 + \frac{1}{4} + \frac{1}{76}$ days. He discovered the cycle synchronizing the periods of the moon and the sun.

Very sophisticated measurement has begun.

Philolaus said that fire is in the center, there is another fire at the circumference, but the center is primary and around it 10 bodies revolve, the

heaven, the 5 planets, the sun, the moon, the earth, and closest to the hearth the counter-earth.

The determination of the order of the planets continues. And the solution of the part of the problem, at rest, has been questioned.

Democritus said that it is at rest because it is flat, it moved about at first because it was small and light, but as it became larger and heavier came to rest in the center.

It is the first inkling that the development of the earth can inform the problem.

Hicetas said that the heaven, the sun, the moon, and the stars are all at rest, and nothing in the world moves except the earth, the earth creates the appearance that all those move by turning on its axis.

Hicetas's result will be corroborated by Plato, by Ecphantus, and by Ponticus. Aristotle will examine it in order to establish the solution, at rest. Ptolemy will examine it. Buridan will examine it. Oresme will examine it. And Copernicus will then reaffirm it, the earth rotates on its axis. When Aristarchus proposed that the earth revolves around the sun, no philosopher in the eighteen centuries between him and Copernicus examined this proposition. Aristarchus's is a remote anticipation. Hicetas's is not a remote anticipation. Hicetas discovered the earth's rotation on its axis.

Plato said that it rotates on its axis. He said that it is a sphere in the middle of the heaven, and it needs neither air nor anything else to keep it from falling, the uniformity of the heaven and its equilibrium suffice. He said that the spindle of necessity, spinning around all things in the heaven, spins 8 whorls, the innermost, the eighth, with the greatest speed, the seventh, the sixth, and the fifth together with the same speed, the fourth more slowly, the third more slowly still, and the second yet more slowly. He posed the problem of finding hypotheses which will show that the apparently irregular motions of the planets are regular.

The determination of the positions of the planets is substantially complete. The solution of the problem is in all its parts more explicit, and more firmly grounded. That the problem is a mathematical problem has been proposed.

Ecphantus said that the earth rotates on its

axis.

Eudoxus described 32 northern and 15 southern constellations. He formulated the theory of the motions of the heavenly bodies in concentric spheres, assigning 3 spheres each to the moon and the sun, and 4 spheres to each of the other planets. He calculated the radius of the earth at between 12 700 km and 9 500 km.

The advance in observation has taken a major step. A mathematical theory has become the solution of the problem. The part about its shape has been solved conclusively. Eudoxus is a very decisive step.

Ponticus said that the earth rotates on its axis. He said that Mercury and Venus revolve around the sun and this explains their retrogradations, retardations, and stations.

Aristotle follows.

In the center. At rest. A sphere.

The main arguments are seven.

Two are remote inferences from observation. The natural motion of earth, and of pieces and whole alike, is toward the center. Every bit of earth has weight until it reaches the center.

Three are grounded directly in observation. No changes in the risings and settings of the fixed stars are observed, the same stars always rise and set in the same positions on the horizon. The outline in lunar eclipses is always circular. Even a small change in position results in a large change in the horizon, there are stars seen in Egypt but not further north.

One is Eudoxus's number. 400 000 stades.

One anticipates Columbus. The ocean continues beyond the strait of Gibraltar all the way to India.

And to tabulate the results, separating them into the explicit formulation and solution of the problem, divided into its three parts, and the grounds on which the formulation and solution of the problem rest.

Its position.

Thales	floats on water,
Anaximander	in equilibrium,
Anaximenes	floats on the air below,
Xenophanes	stretches infinitely downward,
Anaxagoras	below the moon,
Philolaus	second from the center,
Democritus	in the center,

Plato	in the center,
Eudoxus	in the center,
Aristotle	in the center.

In the century-odd from Thales to Anaxagoras several solutions are examined, all surmises from an insufficient number of particular cases. Then in the century-odd from Anaxagoras to Aristotle, as through chains of inferences more and more propositions are brought in connection with one another, the problem is solved. In the center. And this will remain the solution of this part of the problem of the earth for the eighteen and a half centuries until Copernicus.

At rest or in motion.

Thales	at rest,
Anaximander	at rest,
Anaximenes	at rest,
Anaxagoras	at rest,
Empedocles	at rest,
Philolaus	in motion,
Democritus	moved at first, came to rest,
Hicetas	in motion,
Plato	in motion,
Ecphantus	in motion,
Eudoxus	at rest,
Ponticus	in motion,
Aristotle	at rest.

In the century-odd from Thales to Anaxagoras the philosophers agree, at rest. Then in the century-odd from Anaxagoras to Aristotle the problem is vigorously examined – at rest, in motion, moved at first, came to rest, in motion, in motion, in motion, at rest, in motion. Then Aristotle solves the problem. At rest. And this will remain the solution of this part of the problem until Copernicus.

Its shape.

Anaximander	like a drum,
Anaximenes	flat,
Parmenides	a sphere,
Anaxagoras	flat,
Democritus	flat,
Plato	a sphere,
Eudoxus	a sphere 400 000 stades in circumference,
Aristotle	a sphere.

In the century-odd from Thales to Anaxagoras several solutions of this part of the problem are examined as well. Then in the century-odd from

Anaxagoras to Aristotle it is shown that Parmenides has solved the problem. A sphere. And – qualified qualitatively inessentially by Newton, twenty-one and a half centuries after Parmenides – this will remain the conclusive solution of this part of the problem. The earth is a sphere.

Grounds.
Anaximander discussed the sizes and distances of the planets,
Pythagoras discussed the order of the planets,
tablet observed Jupiter,
Anaxagoras discovered the cause of eclipses, proposed that the sun is larger than the Peloponnese,
Oenopides discovered the obliquity of the ecliptic,
Meton measured the solstices, calculated the length of the year, synchronized the periods of the moon and the sun,
Hicetas discovered the earth's rotation on its axis,
Plato determined the order of the planets,
Eudoxus described 47 constellations, formulated the theory of the motions of the heavenly bodies,
Ponticus proposed that Mercury and Venus revolve around the sun,

In the century-odd from Thales to Anaxagoras the grounds are few, and next to crude observation, not firmer than discussion. In the century-odd from Anaxagoras to Aristotle the grounds are many, firm, and increasingly as firm as measurement and calculation. Beginning with Eudoxus the ground is a theory. And the theory of the motions of the heavenly bodies in concentric spheres will remain the theory of the motions of the heavenly bodies for the nineteen and a half centuries until Kepler.

And the steps of advance in the acquisition of knowledge are the 14, Thales, Anaximander, Anaximenes, Pythagoras, Parmenides, Anaxagoras, Oenopides, Meton, Democritus, Hicetas, Plato, Eudoxus, Ponticus, Aristotle. In particular, Thales the earth is at rest, floats on water,

Anaximander discussed the sizes and distances of the planets, the earth is in equilibrium, at rest, like a drum,
Anaximenes the earth floats on the air below, flat,
Pythagoras discussed the order of the planets,
Parmenides the earth is a sphere,
Anaxagoras discovered the cause of eclipses, proposed that the sun is larger than the Peloponnese, the earth is below the moon, discovered the obliquity of the ecliptic,
Oenopides discovered the obliquity of the ecliptic,
Meton measured the solstices, calculated the length of the year, synchronized the periods of the moon and the sun,
Democritus the earth is in the center, moved at first, came to rest,
Hicetas the earth rotates on its axis,
Plato determined the order of the planets,
Eudoxus described 47 constellations, formulated the theory of the motions of the heavenly bodies, the earth is a sphere 400 000 stades in circumference,
Ponticus proposed that Mercury and Venus revolve around the sun,
Aristotle the earth is in the center, at rest, a sphere.

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The thirteen centuries from the temple register from Lahun to Thales.

There is no formulation or solution of the problem of the earth before Thales. But Thales is not *ex nihilo*, and the problem of the earth is not *ex nihilo*. And what the problem of the earth is from, Aristotle and Ptolemy, both, indicate unambiguously. It is from observation. From the observation of the heavenly bodies, in particular.

And all observation predating Thales can be collected. First all accurate observations can be collected from Ptolemy. Then from the earlier record all crude observations can be collected as

well.

Of the 94 accurate observations in the *Almagest* 8 predate Aristotle. 1 of the 8 is the observation of solstice by Meton. The other 7 are older. All 7 are of lunar eclipses, all 7 made in Babylon.

They divide into three parts.

1 observation was made in 721, 1 was made in 720, and 1 more was made later in 720 (*Almagest*, IV, 6). All 3 were made within a couple of years, and Occam's razor demands that this is 1 predecessor of the philosophers. Nor can it be probable at all, given the scarcity of observations, that there were suddenly several predecessors of the philosophers not just making observations, making accurate observations.

1 observation was made in 621 (V, 14). One century after the first, this is 1 more predecessor of the philosophers.

And 1 observation was made in 523, 1 in 502, and 1 in 491 (IV, 9; V, 14). These 3 observations were made within one generation, and Occam's razor demands that this is 1 particular as well. One century after the second, and postdating Thales, clearly a philosopher in this case, not a predecessor of the philosophers.

The first of the three is the beginning of accurate observation, close to one and a half centuries before the first philosopher. This predecessor of the philosophers is very important. The second, one century after the first, one generation before the first philosopher, is important, if not very important. The third, one century later still, two centuries after the first, at the time of Pythagoras and Xenophanes, although himself a philosopher, is not important.

Tabulating, assigning years of birth for the sake of uniformity, the important are,

761 observation of lunar eclipses from
721 to 720,

661 observation of a lunar eclipse in
621,

the not important is,

563 observation of lunar eclipses from
523 to 491.

The 1 not important, the philosopher, has been listed above among the particulars belonging to the two and a half centuries from Thales to Aristotle. The 2 important, the predecessors of the philosophers, belong among the particulars

predating Thales.

These are the accurate observations. And as the accurate observations can be collected, so all observations can be collected.

That all observations are, in fact, collected depends on the presumption, again, that the record has lost nothing important, and this is, again, argued below. It depends also on the examination of the entire record and the recovery of the particulars. In particular, as later on the entity Pythagoreans has to be broken apart and the particulars recovered, so earlier on the entities Babylonian astronomy and Egyptian astronomy, Babylonian mathematics and Egyptian mathematics, and the like, brought to life in the last century-odd with the excavation and publication of the Mesopotamian and Egyptian records, have to be broken apart and the particulars recovered. This is done explicitly below as well. Here, the resolution of this difficulty is also taken for granted.

All in all, then, 2 important particulars can be recovered from the Egyptian record. From the Babylonian record 9 important particulars can be recovered, including the 2 attesting accurate observations recovered by Ptolemy.

Not incidentally, these numbers are all by themselves an argument that these are, in fact, all the relevant particulars predating Thales. From the one and a half centuries after Thales and before Meton, Ptolemy recovered the accurate observations of 1 particular. From the following one and a half centuries beginning with Meton and before Timocharis, he recovered the accurate observations of 2 particulars. From the following four-odd centuries beginning with Timocharis and before himself, he recovered the accurate observations of 11 particulars.

Aristotle writes: the observations of the Egyptians and the Babylonians have been kept for many years and from their reports much of our evidence is derived.

Ptolemy examined the reports and collected the accurate observations. And these are all from the Babylonians, none from the Egyptians.

From the Egyptian record 2 observations of a star can be collected, both very crude, and no observation of an eclipse or of a planet, crude as little as accurate, corroborating Ptolemy.

From the Mesopotamian record more observations can be collected, of eclipses, of planets, and of stars, all crude, all corroborating Ptolemy's result as well, that the most ancient accurate observation is from 721.

In particular, one generation before the most ancient accurate observation, the oldest crude observation of an eclipse is from 747, the first in a series of crude observations of lunar eclipses recorded on the tablets BM 41985 and BM 32238. This is 1 predecessor of the philosophers.

One generation-odd later, in the generation after the most ancient accurate observation, another series of crude observations of lunar eclipses begins in 703. This series is recorded on the tablets BM 45640, BM 35789, and BM 35115. This is 1 more predecessor of the philosophers.

The earliest, these 2 are important, the earlier of the 2 very important.

Two generations later, two-odd generations after the most ancient accurate observation, another series of crude observations of lunar eclipses begins in 647. The tablet is BM 32363. This and all later observations of lunar eclipses in the Babylonian record, of which there is an overabundance, eclipses being now a means of divination, are not important.

Tabulating, the important are,
787 observation of lunar eclipses
beginning in 747,
743 observation of lunar eclipses
beginning in 703,
the not important is,
687 observation of lunar eclipses
beginning in 647.

The oldest observation of a planet is the Venus tablet of Ammisaduqa. Ammisaduqa was prince c. 1646-25.

Nine centuries later, there is a very crude observation of Saturn from 647 to 633. The tablet is BM 76738+76813

Two generations later, and just predating Thales, there is a crude observation of Venus from 587 to 586. The tablet is BM 35196.

Two generations later, there is a crude observation of Jupiter from 526 to 490. The tablet is BM 36823.

The earliest observations of the planets, these 4 – the first 3 predecessors of the philosophers, the last a philosopher – are important, and the first 2 of the 4 very important.

Next, contemporary with Meton, there is a crude observation of Mars and Saturn from 423 to 400. The tablet is BM 32209+41854. This and all later observations of the planets in the Babylonian record, of which there is an abundance, if not an overabundance, the planets now also being a means of divination, are not important.

Tabulating, the important are,
1686 Venus tablet of Ammisaduqa,
687 observation of Saturn from 647 to
633,
627 observation of Venus from 587 to
586,
566 observation of Jupiter from 526 to
490,
the not important is,
463 observation of Mars and Saturn
from 423 to 400.

The last important and the not important have been listed above among the particulars belonging in the two and a half centuries from Thales to Aristotle.

Next, one of Aristotle's arguments for the immobility of the earth is: the same stars always rise and set in the same positions on the horizon, and another of his arguments for its sphericity is: there are stars which are seen in Egypt but not further north.

The observations of the fixed stars also have to be collected.

From the Babylonian record 2 particulars can be recovered, *The plough* and *When Anu and Enlil*.

The plough is a 3-tablet collection of phenomena appropriate for divination. And among these there are observations of the morning risings of three dozen constellations and stars. Among the constellations are Pegasus, Pisces, Perseus, Orion, Hydra, Canus Major, Ursa Major, Corvus, Centaurus, Serpens, Cygnus, Lyra, and Aquila, among the stars are Sirius, Arcturus, and Procyon. That they are observations that they are dated shows. That they are very crude observations that calculation cannot reduce them to errors smaller than ± 10 days, and in order to do even so,

it has to presume them to be risings three to five centuries before the date of composition of *The plough*, shows.

When Anu and Enlil is a 70-tablet collection of omens, some 7 000 omens, into which the observation of Venus recorded on the Venus tablet of Ammisaduqa was, nearly one millennium after the observation, incorporated. And as the observation of Venus can be disentangled from the omens, so observations of fixed stars can be. Unlike the observation of Venus, these are all crude like those in *The plough* and cruder.

The dates are,

727 *The plough,*

717 *When Anu and Enlil.*

Early, these are both important, *The plough* very important.

And those are all important particulars predating Thales in the Babylonian record. 9 of them. 4 attesting observations of eclipses, 2 of these accurate, 3 attesting observations of the planets, and the 2 collections, *The plough* and *When Anu and Enlil*, attesting observations of the fixed stars.

Finally, observation begins in Egypt, thirteen centuries before Thales, two-odd centuries before the Venus tablet of Ammisaduqa and the beginning of observation in Babylon. 2 observations, and only 2, can be recovered from the Egyptian record.

Both are observations of Sirius. Of its heliacal rising, in particular. The first, recorded in the temple register from Lahun, was made, the register reports: in the 7th year of Senwosret III (*id.*). The second, recorded in the list of feasts beginning in the 9th year of Amenhotep, was made, the list of feasts reports: in the 9th year of Amenhotep (*id.*). The Egyptian civil year of 365 days lagged behind the true year with a period of 1 460 years, very nearly. Censorinus reports that the civil new year coincided with the heliacal rising of Sirius in 139^{AD} (*Birthday*, 21). Hence the reports of the two risings can be fixed from observation. The temple register from Lahun fixes the 7th year of Senwosret III in 1871 ± 5 , and the list of feasts beginning in the 9th year of Amenhotep fixes the 9th year of Amenhotep in 1536 ± 10 . The uncertainty is due to the fact that, as Ptolemy writes: the observation of horizon phenomena, and of

first and last visibility, is particularly uncertain. In the second case it is due also to the uncertainty of the place of observation.

The dates are,

1911 temple register from Lahun,

1576 list of feasts beginning in the 9th
year of Amenhotep.

The earliest, these are very important.

There is nothing else in the Egyptian record. There are no observations of the planets, no observations of eclipses.

And with this the collection of the particulars with respect to the problem of the earth in the thirteen centuries from the temple register from Lahun to Thales is complete.

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The problem of the earth is from observation. And in the thirteen centuries from the temple register from Lahun to Thales, these 11 are all important predecessors of the philosophers who are part of the development of observation,

1911	temple register from Lahun,
1686	Venus tablet of Ammisaduqa,
1576	list of feasts beginning in the 9 th year of Amenhotep,

787 **observation of lunar eclipses**
beginning in 747,

761 **observation of lunar eclipses**
from 721 to 720,

743 observation of lunar eclipses
beginning in 703,

727 *The plough,*

717 *When Anu and Enlil,*

687 **observation of Saturn from 647**
to 633,

661 observation of a lunar eclipse in
621,

627 observation of Venus from 587 to
586.

To show that the line between important and not important is drawn sharply, this is 1 of the more important among the not important,

687 observation of lunar eclipses
beginning in 647.

The 11 important divide the thirteen centuries from the temple register from Lahun to Thales clearly and distinctly into two parts.

The first eleven-odd centuries, when there are

the first 3 particulars, 2 of them in Egypt, 1 of them in Babylon, themselves distributed over three-odd centuries, and followed by eight centuries in which there is nothing.

And the following one and half-odd centuries, when there are the following 8 particulars, all 8 in Babylon, distributed evenly over these one and a half-odd centuries, and followed by the first philosopher.

At first, then, there is no inkling.

Then, in Egypt, there is the first observation. The observation of a star.

It is not a miracle. The first observation is contemporary, within a generation or so, with the book of 87 problems. And not only are 87 problems solved in the book. The book begins: the rules of calculation, the gateway to the knowledge of all things and the uncovering of all secrets.

Then, in Babylon, there is the Venus tablet of Ammisaduqa.

It is not a miracle either. The discovery of Venus, attested clearly and distinctly by the record of its synodic period, and *ipso facto* the discovery of the planets, is part of the first of the three brightest periods in the development of mathematics, other parts of which are the mathematician who left behind 286 problems, including problems implying quadratic equations, the mathematician who left behind a table of 15 Pythagorean triples, and the mathematician who calculated the value of $\sqrt{2}$ at 1.414 212 963 ...

Then, again in Egypt, three-odd centuries after the first, there is another observation of the star.

Then there is nothing, in either Egypt or Babylon, for eight centuries.

This is not a miracle either. The times fertile in the sciences and favorable to their development have not been many. Fourteen centuries separate Ptolemy and Copernicus.

Then, and only in Babylon now, continuous observation begins.

A series of observations of lunar eclipses begins in 747.

The observation of lunar eclipses from 721 to 720 follows.

It is accurate enough to begin the series of observations on which Ptolemy's results explicitly rest.

A series of observations of lunar eclipses

begins in 703.

The sustained observation of the stars begins.

The plough records the morning risings of three dozen stars.

When Anu and Enlil records more stars.

The observation of the planets begins anew.

From 647 to 633 Saturn is observed.

In 621 a lunar eclipse is observed.

It is the next in the series of observations on which Ptolemy's results explicitly rest.

The observation of Venus resumes.

It is observed from 587 to 586.

Observation has been accumulating for one and a half centuries. Observation of eclipses, observation of the planets, observation of stars. Observation has become accurate, too.

The philosophers follow. And with them the formulation and solution of the problem of the earth.

And to tabulate the results, separating them by place and time.

In Egypt, earlier.

register observed Sirius,

list of feasts observed Sirius.

A very important result, the first observation is repeated three-odd centuries later, and so the two observations remain.

In Babylon, earlier.

Venus tablet discovered Venus, and *ipso facto* the planets.

A most important result, the discovery of the planets is not followed by other results for nine centuries.

In Babylon, later.

787 observed lunar eclipses,

761 observed lunar eclipses

accurately,

743 observed lunar eclipses,

Plough observed three dozen stars,

Anu and Enlil observed stars,

687 observed Saturn,

661 observed a lunar eclipse

accurately,

627 observed Venus,

Clearly, the investigation of the heavenly bodies has begun.

The accumulation of propositions has begun. Nor are the propositions surmised from an insufficient number of particular cases. More and more

propositions are brought in connection with one another, more and more propositions are brought in more and more connections with one another.

Propositions are won, and conclusively.

The true way is indicated unambiguously.

And the steps of advance in the acquisition of knowledge are the 6, the temple register from Lahun, the Venus tablet of Ammisaduqa, the observation of lunar eclipses beginning in 747, the observation of lunar eclipses from 721 to 720, *The plough*, the observation of Saturn from 647 to 633. In particular,

register	observed Sirius,
Venus tablet	discovered Venus, and <i>ipso facto</i> the planets,
787	observed lunar eclipses,
761	observed lunar eclipses
	accurately,
<i>Plough</i>	observed three dozen stars,
687	observed Saturn.

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The problem of the earth is from observation. The development of observation is the development which leads to the formulation and solution of the problem of the earth.

And to show this more explicitly, two other developments predating Thales can be considered, the development of the accounts of creation, including the creation of the earth, and the development of the description of the surface of the earth.

Concerned with the earth, these developments are both in relation to the subsequent development of the problem of the earth. The relation is remote in the case of the accounts of creation, even if these have involved god, and have for this reason been liable to suggest themselves as the source of everything, in general, and of the problem of the earth, in particular. The relation is not remote in the case of the description of the surface of the earth. And both developments point, the development of the accounts of creation negatively, the development of the description of the surface of the earth affirmatively, to observation as the development which leads to the formulation and solution of the problem of the earth. Corroborating Aristotle and Ptolemy, too. Substantially and essentially, it is the development of

observation which leads to the problem of the earth.

3 accounts of creation predate Thales,

1634 *When on high*,

860 Yahwistic document,

734 Hesiod.

When on high is the first account of creation. It begins: when on high the heaven had no name, and down below the earth had no name, Apsu was the begetter, Tiamat bore all. Creation then proceeds in this way: Apsu begets and Tiamat bears the gods; but this progeny of theirs is very noisy and Apsu cannot sleep; let us do away with them, Apsu decides; but the gods hear of the decision, and Ea kills Apsu; in the middle of Apsu Ea begets Marduk; Tiamat, to avenge her husband, charges Qingu to lead an army of serpents, lions, scorpions, fish, bulls against the gods; the gods acknowledge Marduk the greatest among them, his command shall be truth, his word shall never be wrong, no god shall ever go beyond the limits he sets; Marduk leads the army of the gods against Tiamat and Qingu; Marduk is victorious; he splits Tiamat in two, like a fish, and sets up one of her halves to be heaven; he arranges the stars, the sun, and the moon; marks the year, the months, and the days; from the eyes of Tiamat he lets the Tigris and the Euphrates flow; he sets up the heaven above and the nether-world below; he builds Babylon; the gods bring Qingu before Ea, they slaughter him, and from his blood Ea makes men.

Seven and a half centuries later, the Yahwistic document is the second account of creation. It begins: when god made the earth and the heaven, there was no shrub and no grass on the earth, because god had not yet sent rain; nor was there a man to till the earth; water welled up from below and covered the earth; and god made man from the dust of the earth. Creation is thus over, and the Yahwistic document turns to what happened in the garden of Eden, how the woman and the man ate the fruit of the tree of the knowledge of good and bad, and what happened after that and because of that.

One century-odd later, the *Theogony* is the third account of creation. Hesiod sings:

first of all Chaos came into being,

then big-bosomed Earth, the foundation of all

immortals,
and dark Tartarus below,
and Love, the fairest of the immortals;
from Chaos came Darkness and black Night;
Night lay with Darkness and bore Aether and

Day;

Earth bore starry Heaven, equal to herself,
and Mountains, and barren Sea;
then she lay with Heaven and bore Ocean,

and she bore eleven other titans, three cyclopes,
and three one-hundred-handed creatures, then this
progeny of hers bore more, the progeny of the
progeny more, and the progeny of the progeny of
the progeny yet more.

At first the earth had no name. In the middle of
Apsu Ea begot Marduk. Marduk split Tiamat in
two, and set up one of her halves to be heaven.
He arranged the stars, the sun, and the moon,
marked the year, the months, and the days, and let
the Tigris and the Euphrates flow. He set up the
heaven above and the netherworld below. He
built Babylon.

When god made the earth and the heaven,
there was no shrub and no grass on the earth,
because god had not yet sent rain. Water welled
up from below and covered the earth.

First Chaos came into being. Then Earth,
Tartarus, Love, Darkness, and Night. Night bore
Aether and Day. Earth bore Heaven, Mountains,
Sea, and Ocean. Earth bore also eleven other
titans, three cyclopes, and three one-hundred-
handed creatures.

The thought gives no knowledge, the reference
gives knowledge. The acquisition of knowledge
is the advance from the thought and the sense to
the reference and the true.

And there is much thought and sense in the
accounts of creation. Reference and true there is
little. If there is any. The striving which forces us
to advance from the thought and the sense to the
reference and the true is there. Strong in *When on
high*, feeble in the Yahwistic document, stronger
in Hesiod. The striving is attested by the effort to
bring objects in connection with one another.
Where the earth is concerned, in *When on high*
with the heaven, the stars, the sun, the moon, the
year, the months, the days, the Tigris, the Euphra-
tes, Babylon. In the Yahwistic document with the
heaven, shrub, grass, rain, water. In Hesiod with

night, day, heaven, mountains, sea, ocean. That
the striving has not forced us too far, though, the
means of the connection attest. First the labors of
Marduk provide the connection. Then the labors of
god. Then the labors of the gods, mostly of
procreation now.

In the development of the accounts of creation
before Thales no proposition has been won.

In the development of observation many
propositions have been won.

Then there is the fact of accumulation. Very
few of its successors build on the labors of *When
on high*. Seven and a half centuries after *When on
high* there is the Yahwistic document. Nor is the
Yahwistic document an advance over *When on
high*, far from it, not as an account of creation.
Then the successors of the Yahwistic document
build on its labors. Build prophecy, with *Amos*,
Hosea, *Isaiah*, *Micah*, *Jeremiah*, and the law,
with *Deuteronomy*.

Not quite three and a half centuries after the
Yahwistic document, the Priestly code builds on
its labors. Where creation is concerned: god
separates light from darkness, makes heaven,
makes the earth and the sea, fills the earth with
plants, makes the sun, the moon, and the stars,
fills the sea and the heaven with living creatures,
fills the earth with animals, and makes man in his
own image.

The Priestly code is contemporary with Par-
menides.

One century-odd after the Yahwistic document
there is Hesiod. Hesiod's account of creation is
an advance over that of *When on high*. One
century-odd then separates Hesiod from Thales.
It is one century-odd of many fine poets emulat-
ing Hesiod. And nothing in these poets connects
big-bosomed Earth, the foundation of all immor-
tals, with, the earth is at rest because it floats on
water, like wood.

7 of the predecessors of the philosophers
attesting the development of observation before
Thales, on the other hand, belong in this one
century-odd.

The development of the accounts of creation
attests the accumulation of few propositions, and
of none of these is the reference the true.

The development of observation attests the
accumulation of many propositions, and of many

of these the reference is the true.

Finally, if the accounts of creation are in relation to the problem of the earth, they are not in relation to the part whether it is at rest or in motion, or the part about its shape. They are in relation to the part about its position.

Its position relative the heaven, the stars, the sun, the moon, the year, the months, the days, the Tigris, the Euphrates, Babylon – as well as Apsu, Tiamat, Ea, Marduk, Qingu, serpents, lions, scorpions, fish, bulls, the netherworld, and men. Relative the heaven, shrub, grass, rain, water – as well as god, tilling the earth, the garden of Eden, the fruit of the tree of the knowledge of good and bad, the man, and the woman. Relative night, day, heaven, mountains, sea, ocean – as well as chaos, tartarus, darkness, aether, titans, cyclopes, and one-hundred-handed creatures.

If observation is in relation to the problem of the earth, it is in relation to the same part of the problem, too, its position.

Its position relative the moon, specifically in lunar eclipses, the planets, specifically Venus and Saturn, and the stars, specifically Pegasus, Pisces, Perseus, Orion, Hydra, Canus Major, Ursa Major, Corvus, Centaurus, Serpens, Cygnus, Lyra, Aquila, Sirius, Arcturus, and Procyon.

The relation of the development of the accounts of creation to the problem of the earth is remote.

The relation of the development of observation to the problem is not remote. It is the relation of propositions. Of the accumulation of propositions brought in connection with one another through chains of inferences. Observation is the ground on which the formulation and solution of the problem will advance. And if it does not pose the problem of the earth explicitly, the accumulation of observation does pose implicitly the part of the problem about its position just as this part of the problem will be solved, its position relative the moon, the planets, and the stars.

A cause is an object followed by another, and where all the objects similar to the first are followed by objects similar to the second, or, in other words, where, if the first object had not been, the second never had existed.

The development of observation is a cause of the formulation and solution of the problem of

the earth.

The development of the description of the surface of the earth can be traced back to Ur-Nanshe, nineteen and a half centuries before Thales.

The inscriptions of Ur-Nanshe, prince *c.* 2535, attest 4 localities.

Two generations later, the inscriptions of Eanatum, prince *c.* 2475, attest 16 localities.

One and a half centuries later, the inscriptions of Sargon, prince *c.* 2337-281, attest 27 localities.

Not quite three generations later, the inscriptions of Naram-Sin, prince *c.* 2257-20, attest 38 localities.

Ur-Nanshe and Eanatum are princes of Lagash. Lagash is the first state in the valley of the two rivers to rise. The description of the surface of the earth begins with the first state in the valley of the two rivers to rise.

Sargon and Naram-Sin are princes of Akkad. Akkad is the first state to rise very greatly. The advance in the description of the surface of the earth continues with the great rise of the state.

Coinciding with the reach of the expeditions of the princes, and hence with their domain, the 38 localities attested by Naram-Sin are distributed from the Mediterranean to the sources of the Euphrates and the Tigris to the Persian Gulf.

Nine centuries later the state rises very greatly again with the Assyrian princes, beginning with Ashur-uballit, prince *c.* 1363-27. The rise culminates in the one century-odd from Tiglath-pileser III, prince 745-27, through Ashurbanipal, prince 668-27. And the description of the surface of the earth advances once again.

An inscription of Tiglath-pileser III begins: Tiglath-pileser, the great prince, the true prince, the lord of the earth, the prince of Ashur, the prince of Babylon, the prince of Sumer and Akkad, the lord of the four quarters, the prince who brought under his sway the lands from the sea of the rising sun to the sea of the setting sun (ins. I, 787/804). And in this one inscription, recording his campaigns, Tiglath-pileser III notes 149 localities, not counting those, some dozens of them, whose names are lost in lacunae.

One century later, under Ashurbanipal, the expeditions of the princes, and hence their domain, have reached from the Persian Gulf to Asia

Minor to the lower valley of the Nile. And so has the distribution of the growing number of localities attested by the inscriptions of the princes.

Ashurbanipal predates Thales by two generations.

Cyrus postdates him by one generation-odd. With Cyrus, prince 559-30, the state rises very greatly once again, and continues to rise very greatly through Darius, prince 522-486. The expeditions of the princes, and hence their domain, reach from the northern shores of the Black Sea to the Indus to the southern shores of the Red Sea to the gulf of Sidra. Under Xerxes, prince 486-65, the expeditions of the princes reach beyond the strait of Gibraltar.

The expeditions of the princes, and most particularly the expeditions of the great kings, are a cause of the development of geography.

On the authority of Eratosthenes, Strabo reports: first Anaximander drew a map, and Hecataeus wrote a geographical treatise.

The fragments of Hecataeus attest 300-odd localities, their distribution coinciding precisely with the reach of the expeditions of the great kings. Two-odd generations later Herodotus describes 1 012 localities, their distribution coinciding precisely with the reach of the expeditions of the great kings as well. Hecataeus also improved on the map of the earth, and Herodotus also improved on the map of the earth. With Hecataeus also, and decisively with Herodotus, the determination of the distance and orientation of the localities on the surface of the earth begins.

The relation of the development of the description of the surface of the earth before Thales and its development after Thales is not remote. It is causal. The advance in the description by the princes, and most particularly its advance by the great kings, is a cause of the development of geography.

The relation of the development of the description of the surface of the earth before Thales and the problem of the earth is not remote either. The expeditions of the princes, and their collection of localities – and the localities are collected in propositions – first showed how large the earth is.

And the relation of the development of the formulation and solution of the problem of the earth and the development of the description of

the surface of the earth after Thales – the relation of the development of the problem of the earth and the development of geography – is causal, too. The development of the formulation and solution of the problem of the earth is a cause of the development of geography.

The dates are,

611 - 541 Anaximander,

562 Hecataeus,

484 - 425 Herodotus.

Anaximander is the second philosopher. He said that the earth is at rest because of equilibrium, and shaped like a drum, and discussed the sizes and distances of the planets. And he drew the map of the earth.

Hecataeus postdates also Anaximenes who said that the earth is at rest because it is flat, it floats on the air below, like a lid, Pythagoras, who discussed the order of the planets, and Xenophanes, who said that the earth stretches infinitely downward. And he described 300-odd localities, improved on the map of the earth, and began determining distance and orientation.

Herodotus postdates also Parmenides who discovered that the earth is a sphere and rediscovered that the evening star is the morning star, Anaxagoras who said that the earth is at rest because it is flat, carried by the air below because there is no void, that the moon is beneath the sun and closest to us, discovered the cause of eclipses, and said that the sun is a piece of red-hot metal larger than the Peloponnese, Empedocles who said that the earth is at rest because the very fast motion of the heaven prevents it from moving, like water in a cup, when the cup is spun fast, and is contemporary with Oenopides who discovered the obliquity of the ecliptic. And he described 1 012 localities, improved on the map of the earth, and continued the determination of distance and orientation.

Eudoxus, too, in the generation before Aristotle, formulated the theory of proportion, developed the method of exhaustion, formulated the theory of the motions of the heavenly bodies in concentric spheres, described 47 constellations, and measured the earth. And he wrote also the *Description of the earth*, improving on the map of the earth, and continuing the determination of distance and orientation.

The development of geography is inseparable from observation. And if the development of the problem of the earth is a cause of the development of geography, it is because the development of observation before Thales is the development which leads to the development of the problem of the earth.

Substantially and essentially so.

Substantially, by accumulating the matter of the problem, eclipses, the planets, the stars. In general, the heavenly bodies. One of which the earth will turn out to be.

Essentially, by following the true way. It is the accumulation of propositions brought in connection with one another through chains of inferences, and more propositions brought in connection with one another through chains of inferences more free of gaps, by far, than any other in the centuries before Thales.

And the particulars collected above are not only all observations of the heavenly bodies predating Thales. They are all observations predating Thales.

There are no other observations in the record. And there is but 1 other result which, although not attesting observation explicitly, cannot not be grounded in observation. The map of Nippur. The date is,

1490 map of Nippur.

On the map of Nippur distance and orientation are determined across 2 km.

It is clear why the heavenly bodies are the first objects reduced to observation. Bright spots of light against the darkness of the night sky, if any objects are given clearly and distinctly to the senses, the heavenly bodies are.

Of the 11 important particulars 2 are from the valley of the Nile, 9 are from the valley of the two rivers. The particulars thus attest two independent parts of the development of observation, in the valley of the Nile, and in the valley of the two rivers.

In the valley of the Nile, there is 1 particular, attesting one observation, of the brightest fixed star. Three-odd centuries later there is 1 particular again, attesting one observation, of the same fixed star. Then there is no observation.

Counting back to the first prince, Narmer, development in the valley of the Nile can be

traced back twenty-four centuries before Thales. And in these twenty-four centuries, first there are eleven-odd centuries in which there is no observation, then there is 1 observation, then there are three-odd centuries in which there is no observation, then there is 1 observation, then there are nine and a half centuries in which there is no observation.

The difficulty of observation can hardly have been attested more dramatically.

In the valley of the two rivers, there is 1 particular, attesting spectacular results, an observation of Venus over twenty-one years, through twelve of its synodic periods, *ipso facto* the discovery of the fact that Venus is a planet, and *ipso facto* the discovery of the planets. Then there is no observation for nine centuries. Then there are 8 particulars in the following one and a half centuries, attesting the observation of a series of lunar eclipses, the accurate observation of another series of lunar eclipses, the observation of another series of lunar eclipses, the observation of three dozen constellations and stars, including Pegasus, Pisces, Perseus, Orion, Hydra, Canus Major, Ursa Major, Corvus, Centaurus, Serpens, Cygnus, Lyra, Aquila, Sirius, Arcturus, and Procyon, again the observation of stars, the observation of Saturn over 16 years, another accurate observation of a lunar eclipse, and the observation of Venus again over 2 years.

Counting back to the 3 oldest calculations of area, development in the valley of the two rivers can be traced back twenty-five centuries before Thales. In these twenty-five centuries, first there are fourteen and a half centuries in which there is no observation, then there is the Venus tablet of Ammisaduqa, then there are nine centuries in which there is no observation.

Again attesting the difficulty of observation.

But then in the following one and a half centuries sustained observation begins, continues, becomes accurate, continues, reaches dozens of the heavenly bodies.

Thales and the problem of the earth follow.

Attesting the decisive importance of observation in the acquisition of knowledge.

And once sustained observation begins, continues, becomes accurate, continues, reaches dozens of the heavenly bodies, the attempts to

formulate and solve the problem of the earth begin, and continue, it can confidently be presumed that, with Bacon, the whole thing is done by machinery.

The more so if observation is not *ex nihilo* either. And it is clear what observation is from.

The first observation of Sirius is contemporary with the book of 87 problems, the highest peak in the development of mathematics in the valley of the Nile.

The Venus tablet of Ammisaduqa is part of the first of the three brightest periods in the development of mathematics. Within two centuries or so there are, including the Venus tablet of Ammisaduqa, more than a dozen important mathematicians. One of whom solves 286 problems. Another finds 15 Pythagorean triples. Another calculates the value of $\sqrt{2}$ at 1.414 212 963 ...

The development of observation in the one and a half centuries preceding Thales is not accompanied by other mathematical results. Yet there is the Venus tablet of Ammisaduqa nine centuries earlier. Just as there is Ptolemy fourteen centuries before Copernicus. There is also the first of the three brightest periods in the development of mathematics nine centuries earlier. Just as there is the second of the three brightest periods in the development of mathematics, the two and a half centuries from Hippocrates of Chios through Hipparchus, sixteen centuries before Copernicus.

Observation is from mathematics. The development of observation is a consequence, and a part, of the development of mathematics.

ccli

All the centuries before the temple register from Lahun.

Aristotle traces the development of the problem of the earth back to Thales. But the problem is not *ex nihilo*. And it can be traced back to the preceding development of observation. But observation is not *ex nihilo* either. And it can be traced back to the preceding development of mathematics. But mathematics is not *ex nihilo* either.

Also, if the relation of the development of the description of the surface of the earth before Thales and the development of the problem of the earth is not remote, if the expeditions of the

princes first showed how large the earth is, the development of the problem can be traced back also to the preceding labors of the princes. But the princes are not *ex nihilo* either.

The attempt to collect the development of the problem of the earth in its entirety – to collect the predecessors of the philosophers and the philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them, with respect to this problem – cannot not involve the attempt to trace the development of the problem back to its beginning.

The development of the problem is a part of the acquisition of knowledge. The attempt to collect the acquisition of knowledge in its entirety cannot not involve the attempt to trace the acquisition back to its beginning.

And the development of the problem of the earth informs, as it cannot not inform, if it is a part of the acquisition of knowledge, the problem of the beginning of the acquisition of knowledge. In particular, it indicates where and when the formulation and solution of the problem of the beginning of the acquisition should be sought.

If the problem of the earth is from observation, and observation is from mathematics, the beginning of mathematics can be traced back to the 3 oldest calculations of area. And no further.

If the problem is also from the labors of the princes, the labors of the princes can be traced back to the first prince, Narmer. And no further.

The dates are,

3120	3 calculations of area,
3040	Narmer.

Hence all the centuries before the temple register from Lahun can be divided into two.

And all the centuries, including all the centuries before the temple register from Lahun, are many.

The centuries since the formation of the solar system are some 46 000 000.

The centuries since the beginning of life on earth are some 40 000 000.

The centuries since the beginning of man, counting back to *homo* rather than *australopithecus*, and to *homo habilis*, in particular, and the oldest tools, the Oldowan tools, are some 26 000.

The centuries since the oldest city, at Çatal Hüyük, are 85.

The centuries since the temple register from Lahun are 39.

The centuries since the Venus tablet of Ammisaduqa are 37.

And all the centuries before the temple register from Lahun divide into all the centuries before the 3 calculations of area and Narmer, on one side, and the eleven to twelve centuries after, on the other.

This is the division which poses the problem of the beginning of the acquisition of knowledge.

In the valley of the two rivers fourteen and a half centuries separate the 3 calculations of area and the Venus tablet of Ammisaduqa.

In the valley of the Nile eleven-odd centuries separate Narmer and the temple register from Lahun.

In the valley of the two rivers the princes cannot be traced back further than Enmebaragesi, prince *c.* 2680. Enmebaragesi postdates the 3 calculations of area by four centuries. The first prince postdates the beginning of mathematics by four centuries.

In the valley of the Nile the remotest beginnings of mathematics cannot be traced back further than the Palermo stone, under Neferirkare, prince *c.* 2440, which records the measurement of the level of the high Nile. The Palermo stone postdates Narmer by five and a half centuries. The remotest beginnings of mathematics postdate the first prince by five and a half centuries.

In the valley of the Nile, eleven-odd centuries after the first prince, five and a half centuries after the remotest beginnings of mathematics, observation begins.

In the valley of the two rivers, fourteen and a half centuries after the beginning of mathematics, ten and a half centuries after the first prince, observation begins later, but begins spectacularly. In this valley also, nine centuries later, observation resumes, continues, becomes sustained, continues, becomes accurate, continues.

The problem of the beginning of the acquisition of knowledge has to be fundamentally of the differential development in the two valleys. The development of mathematics in each. The development of the labors of the princes and hence of the state in each. The development of the two, mathematics and the state, in relation to one

another in each. And the relation of the two parts of the one and the same development which the development in the valley of the two rivers and the development in the valley of the Nile are.

That the development is one and the same in the two valleys, the development of the problem of the earth, too, traced back to observation, and mathematics, and to the labors of the princes, shows.

ccli

The four and a half centuries from Aristotle to Ptolemy.

In the two and a half centuries from Thales to Aristotle the advance in the formulation and solution of the problem of the earth was very great.

And in the four and a half centuries from Aristotle to Ptolemy the advance continued to be very great.

To collect all particulars and marshal them in their series and order with respect to these four and a half centuries – and with respect to all subsequent centuries – is substantially easier. The problem has been formulated, and solved, and better, and better. Many propositions have been won, and more, and more, they have been brought in more connections with one another through chains of inferences, and more, and more, the gaps in the chains of inferences have been narrowed, and more, and more. It is thus substantially easier the follow the chains.

These 21, then, are all important philosophers who are part of the development of the problem of the earth in the four and a half centuries from Aristotle to Ptolemy,

371	Dicaearchus,
354	Callippus,
346	Autolycus,
346	Euclid,
335	Timocharis,
320	Aristarchus,
312	Dionysius,
287 - 212	Archimedes,
285	observation of Mercury and Saturn from 245 to 229,
274 - 194	Eratosthenes,
233	Apollonius,
203	Hypsicles,

187	Hipparchus,
187	Theodosius of Bithynia,
135 - 51	Posidonius,
117	Geminus,
24 ^{BC}	Cleomedes,
24 ^{AD} - 79	Pliny,
58	Menelaus,
87	Theon of Smyrna,
87	Ptolemy.

To show that the line between important and not important is drawn sharply, these 2 will do,

335	Aristyllus,
202	observation of autumn equinoxes from 162 to 158.

Aristyllus made observations at the time of Timocharis. The observation of autumn equinoxes from 162 to 158 is another of those collected in the *Almagest*.

The 21 important divide the four and a half centuries into two parts. The two centuries from Aristotle to Hipparchus. And the two and a half centuries from Hipparchus to Ptolemy.

In the two centuries from Aristotle to Hipparchus, the great acceleration of the acquisition of knowledge, which began in the previous two and a half centuries, continued. It is at this time that philosophy began being produced by the schools. Outside the schools, however, it is the time of the second of the three brightest periods in the development of mathematics. And in particular, the power of mathematics was increasingly brought to bear on the problem of the earth, the problem was formulated and solved mathematically, and the formulation and solution was grounded in a mathematical theory of great sophistication.

In the two and a half centuries from Hipparchus to Ptolemy the acquisition of knowledge decelerated. The philosophers in the schools had devoted themselves unwaveringly to ethics. Rhetoric flourished. Mathematics advanced less and less frequently. And in particular, there was no advance in the formulation and solution of the problem of the earth for two and a half centuries after Hipparchus. Yet then there was Ptolemy. Ptolemy achieved the last great advance in the acquisition of knowledge for a long time to come, and in particular, the last great advance in the formulation and solution of the problem of the earth until Copernicus.

And these are the results of the philosophers. Many of them have been presented above in some detail.

Dicaearchus improved substantially the result for the radius of the earth.

Archimedes reports in *The sand-reckoner*: some have tried to prove that the circumference of the earth is about 300 000 stades.

Not some. Dicaearchus. Eudoxus wrote a *Description of the earth*. One generation later Aristotle's student Dicaearchus wrote a *Description of the earth*. There are substantial fragments from both Eudoxus and Dicaearchus in Strabo, and from no other philosopher from the century-odd between Eudoxus and Archimedes. The fragments from Dicaearchus include the oldest measurement of the distance between the Peloponnese and the strait of Gibraltar at 10 000 stades (II, 4.2), too.

Eudoxus calculated the circumference of the earth at 400 000 stades. Presuming uniformity of the unit of measurement from Eudoxus through Dicaearchus to Eratosthenes, Eudoxus calculated the radius of the earth at 10 200 km, Dicaearchus calculated it at 7 600 km.

Callippus improved the result for the cycle synchronizing the periods of the moon and the sun.

Meton discovered the Metonic cycle, of 235 lunar months, very nearly equal to 19 solar years. Callippus discovered the Callippic cycle, equal to 4 Metonic cycles plus 1 day.

Autolycus and Euclid are contemporary. They took the first step after Eudoxus in the mathematical development of the theory of the motions of the heavenly bodies in concentric spheres.

Autolycus proved 42 propositions in *On the moving sphere* and *On the risings and settings*, beginning with the definition of uniform motion.

Euclid proved 684 propositions of the *Elements*, *Data*, *Phenomena*, and *Optics*. In the *Phenomena*, in particular, he proved 18 propositions. The first is the proof that the earth is in the center of the world.

Autolycus and Euclid defined the circles of the equator, the ecliptic, the horizon, and the meridian.

With Timocharis the accurate observation of the planets and stars began.

6 of the 94 dated observations in the *Almagest* are from him.

Aristarchus discovered the method of measuring the distances of the moon and the sun, measured the distances of the moon and the sun, and proposed that the earth revolves around the sun,

The method, from dichotomies of the moon and from lunar eclipses, is formulated in *On the sizes and distances of the sun and moon*. It remained the method of measuring the distances of the moon and the sun until Huygens and Cassini. Aristarchus measured the distance of the moon at 20 earth radii, the distance of the sun at 380 earth radii. As to the revolution of the earth around the sun, as Aristarchus anticipated Copernicus, so his critics anticipated Copernicus's. The scholar of the Stoa Cleanthes said that Aristarchus should be sued for impiety for disturbing the hearth of the world.

Dionysius made accurate observations of Mercury, Mars, and Jupiter.

8 of the dated observations in the *Almagest* are from him.

Archimedes packed the world with sand.

In the *Sand-reckoner* he calculated that on the hypothesis that the world is the sphere whose center is the earth and radius is equal to the distance from the center of the earth to the center of the sun 10^{51} grains of sand suffice to pack the world. On Aristarchus's hypothesis 10^{63} grains of sand suffice.

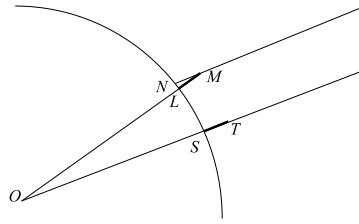
The philosopher who has to go under the name observation of Mercury and Saturn from 245 to 229 made the first accurate observation of Saturn.

3 of the dated observations in the *Almagest* can be attributed to him.

Eratosthenes measured the earth.

The ground of the method of measuring the radius of the earth which Eratosthenes discovered is the observation of the positions of the heavenly bodies at different latitudes, hence the determination of the latitudes of localities on the same meridian, and hence the determination of the distance between these localities. It has remained the ground of the methods of measuring the radius of the earth.

Cleomedes, in *The circular theory of the heavenly bodies*, describes the discovery: the rays of the sun striking different points on the



surface of the earth are parallel;

Aswan, *S*, and Alexandria, *L*, are on the same meridian;

a sundial in Aswan, *TS*, casts no shadow at summer solstice;

at the same hour a sundial in Alexandria, *ML*, casts a shadow, *NL*;

the straight lines through the sundials, *TO* and *MO*, intersect in the center of the earth;

the circumference of the earth is so great that the two triangles, *OLS* and *MLN*, are very nearly rectilinear, and hence similar;

the distance between Aswan and Alexandria, *SL*, is 5 000 stades;

the angle at the tip of the sundial in Alexandria, *LMN*, is $\frac{1}{50}$ of the circumference of the circle;

hence, the distance between Aswan and Alexandria is $\frac{1}{50}$ of the circumference of the earth;

and the circumference of the earth is 250 000 stades (I, 10).

Strabo (II, 5.7) and Pliny (II, 247) record the result as 252 000 stades.

The rays of the sun striking different points on the surface of the earth are parallel. Because the sun is far. The straight lines through vertical sundials intersect in the center of the earth. Because the earth is a sphere. The two triangles, *OLS* and *MLN*, are very nearly rectilinear. Because the earth is large, and hence its curvature small. The approximation takes the side *OL* to be 250 050 instead of 250 000, a difference of 0.02%. The accuracy of observation is attested by the report: no shadow was observed at summer solstice in Aswan in an area 300 stades in diameter (Cleomedes, I, 10). Two centuries after Meton and the beginning of accurate observation of the sun, two generations after Timocharis and the beginning of accurate observation of the planets

and stars, the determination of latitude, and hence of the distance of localities on the same meridian, became a trivial problem with Eratosthenes. The determination of longitude was not to become a trivial problem until Cook's voyages in 1768-79. Alexandria is thus not on the meridian of Aswan, it is 3° west of it. But the difference in the length of the shadow at the same time in two localities on the same latitude 3° of longitude apart is very much less than 1%. Eratosthenes's older contemporary Archimedes proved that $3\frac{10}{71} < \pi < 3\frac{1}{7}$.

And the accurate measurement of the angle at the tip of a sundial had been a trivial problem at least since Meton.

The method is impeccable.

Hence so is the result.

Stades differed. Pliny, however, has these two reports. He writes: 252 000 stades are in our Roman reckoning 31 500 miles (II, 247). 1 Roman mile = 1.48 km, and on this reckoning Eratosthenes measured the radius of the earth at 5 010 km. But Pliny writes also: Eratosthenes reckoned 1 schoenus = 40 stades (XII, 53). 1 schoenus $\approx$ 6.3 km, and thus 1 stade $\approx$ 0.16 km. On this reckoning the measurement of the circumference at 250 000 stades is of the radius at 6 370 km, of the circumference at 252 000 stades is of the radius at 6 420 km. And this is Eratosthenes's reckoning. The method shows that it is. And the measurement of the distance between Aswan and Alexandria corroborates that it is. On this reckoning Eratosthenes measured the distance at 800 km. It is, in fact, 840 km.

Eratosthenes measured the radius of the earth at 6 400 km.

Apollonius greatly advanced the theory of the motions of the heavenly bodies in concentric spheres.

He showed that motion on an eccentric circle and motion on an epicycle moving on a circle can reduce the motions of the planets to uniformity. A body observed to move not uniformly can be shown to move uniformly about a center which is not the point of observation. A body observed to move not uniformly can also be shown to move uniformly on a circle whose center moves uniformly as well about the point of observation in its center. The two models can also be superim-

posed on one another. And the eccentric and the epicyclic models of planetary motions remained the foundation of the theory of the planets until Kepler.

Hypsicles wrote the *Rising times*.

In the *Rising times* Hypsicles proposes to establish the different lengths of the times of rising of the constellations in the twelve divisions of the ecliptic. The attempt is hampered by the lack of both trigonometry and accurate observations. The *Rising times* is the first treatise dividing the ecliptic into 360° .

Then the great advance in the previous two centuries culminated in Hipparchus.

Hipparchus developed trigonometry, vastly improved observation, formulated the theory of the motions of the sun and of the moon, and determined the distance of the moon.

The development of trigonometry, the calculation of lengths from angles and of angles from lengths, first made it possible to calculate the sizes of the eccentric circles, epicycles, and circles on which the epicycles revolve, and the speeds with which the bodies on them revolve uniformly, such as their uniform motion will correspond to their observed not uniform motion.

The perfection of observation two results can illustrate. Hipparchus discovered the precession of the equinoxes, the change in their position in relation to the fixed stars. Ptolemy reports: by comparing his own observations of fixed stars with the observations of the same stars by his predecessors, Hipparchus found that the stars move in relation to the equinoxes 1° in about 100 years (*Almagest*, VII, 2).

Hipparchus also determined the length of the synodic month. Ptolemy reports: Hipparchus showed that the smallest constant period of solar and lunar months is 126 007 days plus 1 equinoctial hour; he found that this period consists of 4 267 synodic months, 4 573 returns in anomaly, and 4 612 revolutions less about $7\frac{1}{2}^\circ$; and hence,

dividing the number of days by the number of synodic months, he found that the mean length of the synodic month is $29 + \frac{31}{60} + \frac{50}{60^2} + \frac{8}{60^3} + \frac{20}{60^4}$ days (IV, 2).

Synodic month is the interval between succes-

sive occurrences of the same lunar phase, return in anomaly is the return of the heavenly body to its position of the same apparent velocity, revolution is its revolution to the same point on the ecliptic. And as Hipparchus found the mean length of the synodic month to be 29 days, 12 hours, 44 minutes, and 3 seconds, so the mean length of the synodic month is 29 days, 12 hours, 44 minutes, and 3 seconds.

Hipparchus formulated the theory of the sun on the eccentric hypothesis. Ptolemy found no reason to correct it, the theory of the sun in book III of the *Almagest* is the theory of the sun as Hipparchus formulated it, and so the theory of the sun remained until Copernicus. The theory of the moon Hipparchus formulated on the epicyclic hypothesis. Ptolemy presents it in book IV, and in book V corrects it and improves on it substantially. Given the theory of the sun and the moon, Hipparchus calculated eclipses for the following 600 years.

He measured the mean distance of the moon at $67\frac{1}{3}$ earth radii. The distance of the sun he measured at 490 earth radii.

It is with the labors of Hipparchus that the investigation of the heaven conclusively became a mathematical investigation. Hence so did also the investigation of the surface of the earth. Strabo reports: there can be no geography without astronomy, Hipparchus said.

Theodosius of Bithynia wrote the *Spherics*, *On habitations*, and *On days and nights*.

The *Spherics* is the first attempt to prove theorems about the arcs and chords of the celestial circles. *On habitations* deals with the variation of the heavenly phenomena at different latitudes, *On days and nights* with the variation of the daily arcs of the ecliptic, in particular, at different latitudes. Both are elementary treatises, such treatises demanded apparently by the growing sophistication of astronomical theory.

Posidonius calculated the distance of the sun.

Strabo reports: Posidonius calculated the circumference of the earth at 180 000 stades (II, 2.2). Cleomedes reports: Posidonius calculated the circumference at 240 000 stades (I, 10).

Cleomedes reports also: at Aswan at summer solstice things cast no shadow in an area with a

diameter of about 300 stades; and given this fact, Posidonius assumed that the circle of the motion of the sun around the earth is 10 000 times greater than the circumference of the earth; and hence he showed that the diameter of the sun is 3 000 000 stades (II, 1).

If the circumference of the earth is 240 000 stades, the ratio of the radius of the area in which the sun casts no shadow and the radius of the earth is $\pi : 800$. On what grounds Posidonius assumed that the ratio of the circumference of the earth and the orbit of the sun around the earth is $1 : 10\,000$ is not clear.

Whatever the grounds, and even if there are no grounds, it is equivalent to assuming a distance of the sun of 10 000 earth radii. Which is a better number, it was to turn out, than any of the numbers obtained on better and very much better grounds through and including Kepler. And if the grounds are not, the number itself is well attested. Pliny also reports: Posidonius said that winds and clouds reach to a height of 40 stades; above that the air is perfectly luminous; the distance between the clouds and the moon is 2 000 000 stades, and the distance between the moon and the sun is 500 000 000 stades (II, 85). If the circumference of the earth is 240 000 stades, the distance between the clouds and the moon is 52 earth radii, the distance between the moon and the sun is 13 100 earth radii.

Geminus wrote the *Introduction to the phenomena*.

It is another elementary treatise.

Cleomedes wrote *The circular theory of the heavenly bodies*.

It is yet another elementary treatise. This elementary treatise, however, is an important source on Cleomedes's predecessors.

Pliny wrote the *Natural history*, in 37 books, and in book II he described the world.

The *Natural history* is a vast compendium of facts. Pliny himself reports: we have read 2 000 books (pr.17); we have collected from them 20 000 facts, and have also added from our experience (*id.*). And so is the description of the world in book II, in particular, it contains, by Pliny's count: 417 facts, investigations, and observations (*id.*), collected from 44 writers, and Pliny lists the 44 writers (*id.*). The *Natural his-*

tory, in general, and the description of the world, in particular, is an important source on Pliny's predecessors. It is also elementary.

Menelaus defined the spherical triangle, and developed spherical trigonometry.

He wrote the *Spherics*, in 3 books. Definition 1 in book I is: a triangle on the surface of the sphere is the area enclosed by three intersecting arcs of great circles, these arcs always being smaller than semicircles (*id.*). In book I, in 39 propositions, he proves for spherical triangles results proved by Euclid for plane triangles, as well as results which hold for spherical triangles and not for plane triangles. In book II, in 21 propositions, he proves more rigorously results about arcs and chords first obtained by Theodosius of Bithynia. And in book III, in 25 propositions, he develops spherical trigonometry. Ptolemy's development of trigonometry in book I of the *Almagest* continues the labors of Menelaus. 2 of the dated observations in the *Almagest* are also from Menelaus.

If the angles of the spherical triangle are measured by the tangents at its vertices, the sum of the angles of the spherical triangle is greater than two right angles. Menelaus measures the angles by the inclination of the planes containing the great circles.

Theon of Smyrna made observations, and wrote the *Mathematical matters useful for the study of Plato*.

4 of the dated observations in the *Almagest* are from him. Book III of the *Mathematical matters useful for the study of Plato* is astronomical and appropriately elementary.

Then there is Ptolemy. Ptolemy developed trigonometry further, further improved observation, corroborated the theory of the sun, improved the theory of the moon, formulated the theory of the planets, catalogued 1 022 stars, measured the distances of the moon and of the sun, hence calculated the distances of the planets, hence the minimal distance of the fixed stars, and thus measured the world. He also calculated the latitudes and longitudes of 6 345 localities, and fixed chronology absolutely.

The mean distance of the moon is 59 earth radii. The mean distance of the sun is 1 210 earth radii. With the radius of the sphere of the earth

and water as a unit, the radius of the sphere surrounding the air and fire is 33, the radius of the sphere of the moon 64, the radius of the sphere of Mercury 166, the radius of the sphere of Venus 1 079, the radius of the sphere of the sun 1 260, the radius of the sphere of Mars 8 820, the radius of the sphere of Jupiter 14 187, and the radius of the sphere of Saturn 19 865.

The sphere of the fixed stars is thus at least 19 865 earth radii away. The radius of the sphere of the world is thus at least 19 865 earth radii as well.

And to tabulate the results with respect to the problem of the earth, separating them into grounds of the solution and solution of the problem.

The grounds are twofold, the improvement of observation and the advance in the development of the theory of the motions of the heavenly bodies. The two, the improvement of observation and the advance of the theory, are very much one another's grounds, too.

Observation.

Callippus	improved the cycle synchronizing moon and sun,
Timocharis	began the accurate observation of the planets and stars,
Dionysius	observed accurately the planets,
245 to 229	observed accurately the planets,
Hipparchus	vastly improved observation,
Ptolemy	further improved observation, catalogued 1 022 stars.

The theory.

Autolycus	began the mathematical development of the theory,
Euclid	began the mathematical development of the theory,
Aristarchus	discovered the method of measuring the distances of the moon and sun,
Apollonius	discovered the eccentric and the epicyclic models,
Hipparchus	developed trigonometry, formulated the theory of the sun, formulated the theory of the moon,
Theodosius	proved theorems about celestial circles,

Menelaus developed spherical
trigonometry,
Ptolemy developed trigonometry further,
improved the theory of the
moon,
formulated the theory of the
planets.

And in all of its three parts the problem was solved very much better.

Its shape.

This part was, in fact, solved conclusively, very nearly. The radius of the earth is,

Dicaearchus 7 600 km,
Eratosthenes 6 400,
correction 6 378.

Its position.

The part of where the earth is in relation to the moon was also solved conclusively. The mean distance of the moon is,

Aristarchus 20 earth radii,
Hipparchus $67\frac{1}{3}$,
Ptolemy 59,
correction 60.2.

The part of where it is in relation to the sun was solved on increasingly firmer grounds increasingly better. The mean distance of the sun is,

Aristarchus 380 earth radii,
Hipparchus 490,
Ptolemy 1 210.

On unascertainable grounds an excellent number was obtained, too,

Posidonius 13 100.
Correction 23 455.

The part of where it is in relation to the world in its entirety was also solved. Not yet by Archimedes when he packed the world with sand.

Archimedes 10^{63} grains of sand suffice to
pack the world.

But, after the measurement of the distance of the moon, and sun, and calculation of the distances of the planets, and fixed stars, by Ptolemy.

Ptolemy the radius of the world is at least
19 865 earth radii.

The world is,

Ptolemy 2×10^8 km across,
correction 9×10^{23} .

At rest or in motion.

Aristarchus in motion around the sun.

Not only all other philosophers either argued or took it for granted that it is at rest. Aristarchus himself in *On the sizes and distances of the sun and moon* takes it for granted that it is at rest.

The hypothesis that the earth revolves around the sun remained unexamined until Copernicus. Why none of the philosophers in the not quite one and a half centuries between Aristarchus and Hipparchus, a time of continuous great advance, nor Ptolemy later on, examined the hypothesis, can be explained by the great and steadily greater success of astronomical theory, and of the theory of concentric spheres, in particular, grounded firmly in, corroborating, and continuing to corroborate, the hypothesis, at rest. Why none of the philosophers in the fourteen centuries after Ptolemy examined the hypothesis, can be explained as well. It was not the examination of hypotheses, that hypothesis or any other, that occupied the philosophers in these centuries, when there were philosophers in these centuries.

Finally, there are also the elementary treatises, Hypsicles, Theodosius of Bithynia, Geminus, Cleomedes, Pliny's compendium, Theon of Smyrna.

It can be presumed that the growing sophistication of astronomical theory was a cause of the growing abundance of elementary treatises. It can also be presumed, however, that a more powerful cause was the deceleration of the acquisition of knowledge, and that the fondness for the elementary is a concomitant of the devotion to ethics.

It can be presumed as well that the growing abundance of elementary treatises, too, prepared the ground for, and is a cause of, the impending advent of the darkest of dark ages.

The steps of advance in the acquisition of knowledge are the 10, Dicaearchus, Autolycus, Euclid, Timocharis, Aristarchus, Eratosthenes, Apollonius, Hipparchus, Menelaus, Ptolemy. In particular,

Dicaearchus	the earth's radius is 7 600 km,
Autolycus	began the mathematical development of the theory,
Euclid	began the mathematical development of the theory,
Timocharis	began the accurate observation of the planets and stars,
Aristarchus	discovered the method of

	measuring the distances of the moon and sun, the moon is 20 earth radii away, the sun is 380, the earth revolves around the sun,
Eratosthenes	the earth's radius is 6 400 km,
Apollonius	discovered the eccentric and the epicyclic models,
Hipparchus	vastly improved observation, developed trigonometry, formulated the theory of the sun, formulated the theory of the moon, the moon is $67\frac{1}{3}$ earth radii away, the sun is 490, developed spherical trigonometry,
Ptolemy	improved observation further, catalogues 1 022 stars, developed trigonometry further, improved the theory of the moon, formulated the theory of the planets, the moon is 59 earth radii away, the sun is 1 210, the radius of the world is at least 19 865 earth radii.

ccliii

The fourteen centuries from Ptolemy to Copernicus.

There was no advance until Regiomontanus, in the generation before Copernicus.

There was for a long time a great retreat instead. And this is all too readily shown.

Two centuries after Ptolemy, Pappus wrote the *Collection*. In book VI he begins: the judgment of those who teach astronomy is lacking where the treasury of astronomy is concerned (1). Not where the *Almagest* is concerned. The *Almagest* is the greatest. Their judgment is lacking where what to teach in preparation for the *Almagest* is

concerned. This is the judgment which Pappus proposes to correct. And does so by presenting and discussing in book VI *On the moving sphere* of Autolycus, the *Phenomena* and *Optics* of Euclid, *On the sizes and distances of the sun and moon* of Aristarchus, and the *Spherics* and *On days and nights* of Theodosius of Bithynia. This is the treasury of astronomy.

It was not exactly stocked with new treasures. Autolycus and Euclid were by the time of writing six-odd centuries old, Aristarchus was six centuries old, Theodosius of Bithynia four and a half centuries. Nor was the treasury overstocked. With, say, Apollonius, or Hipparchus, or Menelaus. The treasures in the treasury were only for preparation for the *Almagest*. Not too strenuous a preparation either.

Nor, indeed, with all due respect to Pappus, was the treasury of astronomy a treasury. All god's treasures of wisdom and knowledge were hidden in Christ.

Thus, eight and a half centuries after Pappus, Gerard had to learn Arabic in order to translate the pagan philosophers for the Christian philosophers to read, should the Christian philosophers venture to read them.

13 of the 71 treatises which Gerard translated are astronomical. 1 is *The greatest*, 5 are predecessors of *The greatest*, 7 are successors of *The greatest*.

The predecessors are *On the moving sphere* of Autolycus, the *Rising times* of Hypsicles, the *Spherics* and *On habitations* of Theodosius of Bithynia, and the *Spherics* of Menelaus.

The successors are *On the science of the motion of the orb* of Mashaallah, the *Elements of the science of the motions of stars* of al-Farghani, *On the motion of the eighth sphere* and *The things which need exposition before reading the Almagest* of Thabit ibn Qurra, the *Tables* and *On the dawn* of al-Jayyani, and the *Correction of the Almagest* of Jabir ibn Aflah.

The greatest was now ten-odd centuries old.

Autolycus was fourteen-odd centuries old, Hypsicles and Theodosius of Bithynia thirteen centuries, Menelaus ten and a half centuries. All still for preparation for *The greatest*.

The successors span the four centuries before Gerard. And they attest, with al-Farghani and

Thabit ibn Qurra, that the successors did not venture beyond preparation for *The greatest* either. As al-Farghani has it, beyond the elements, as Thabit ibn Qurra has it, beyond the things which need exposition before reading *The greatest*. With al-Jayyani and Jabir ibn Aflah, they did not venture beyond preparation for the preparation, with Mashaallah, beyond preparation for the preparation for the preparation.

And so the philosophers continued the preparation. Until, with Peurbach and Regiomontanus, three centuries after Gerard, they did at long last conclude the preparation.

These 20 are all important philosophers who are part of the development of the problem of the earth in the fourteen centuries from Ptolemy to Copernicus,

280	Pappus,
324	Theon of Alexandria,
354 - 415	Hypatia,
773 - 866	al-Farghani,
789 - 889	Habash al-Hasib,
831 - 901	Thabit ibn Qurra,
861 - 929	al-Battani,
950 - 1009	ibn Yunus,
965 - 1040	ibn al-Haytham,
1022 - 1092	al-Zarqali,
1114 - 1187	Gerard,
1181 - 1256	Sacrobosco,
1201 - 1274	al-Tusi,
1221 - 1284	Alfonso X,
1295 - 1358	Buridan,
1320 - 1382	Oresme,
1324	ibn al-Shatir,
1423 - 1461	Peurbach,
1436 - 1476	Regiomontanus,
1473 - 1543	Copernicus.

To show that the line between important and not important is drawn among the particulars, these 5 will do,

722	Mashaallah,
990 - 1084	al-Jayyani,
1105	Jabir ibn Aflah,
1287 - 1360	John of Saxony,
1430 - 1504	Walther.

John of Saxony revised the *Alfonsine tables*. And 3 of the dated observations in Copernicus's *On the revolutions* are from Walther.

The 20 important divide the fourteen centuries

into five parts.

In the two and a half centuries from Ptolemy to Hypatia astronomy lingered on in a few commentaries, then it, too, was stopped.

Hence, in the four-odd centuries from Hypatia to al-Farghani there is nothing.

In the three and a half centuries from al-Farghani to Gerard the Muslim philosophers began the attempts to revive astronomy. The attempts were neither too frequent nor too vigorous.

In the three centuries from Gerard to Peurbach the Christian philosophers began the attempts to revive astronomy. These were neither too frequent nor too vigorous either.

Until in the one half century from Peurbach to Copernicus the philosophers did at long last revive astronomy.

Little thus needs to be said more particularly.

Pappus wrote a commentary on the *Almagest*.

Theon of Alexandria wrote a commentary on the *Almagest*.

Hypatia, the daughter of Theon of Alexandria, made additions to his commentary on the *Almagest*.

The mathematical competence of these commentaries remained unsurpassed until Peurbach and Regiomontanus.

Hypatia then shared the fate of astronomy. Socrates Scholasticus, in the *History of the church*, reports: in 415, during lent, the Christians in Alexandria learned that the pagan philosopher Hypatia is spreading calumnies against the bishop, Cyril; and as she once passed by Cyril's church, she was dragged out of her carriage and into the church, stripped naked, and torn to pieces by the congregation led by the reader, Peter, who first tore a piece with his own hands; her flesh was then scraped from the bones with oyster-shells, her limbs delivered to the flames (L, 7).

Hypatia is contemporary with Augustine.

Ibn Khaldun reports: Muslim rule flourished, the Muslims settled, became versed in the crafts and sciences, and began to desire to study philosophy; so al-Mansur sent envoys to the Roman emperor to ask for books, and the emperor sent him back Euclid's book and some writings on physics.

Two generations after al-Mansur, al-Farghani wrote the *Elements of the science of the motions*

of stars. It is an elementary introduction to Ptolemy uncontaminated by mathematics.

Habash al-Hasib wrote the *Compendium*.

Thabit ibn Qurra wrote *On the motion of the eighth sphere* and *The things which need exposition before reading the Almagest*.

Al-Battani wrote the *Astronomical treatise*.

From Habash al-Hasib to Thabit ibn Qurra to al-Battani the preparation for the *Almagest* is of increasing mathematical sophistication. Al-Battani began also the attempts to improve on Ptolemy's parameters with new observations.

Ibn Yunus wrote the *Astronomical handbook with tables*. Ibn Yunus begins with new observations.

Ibn al-Haytham wrote *On the configuration of the world*. He wrote also the *Optics*.

Al-Zarqali wrote the *Tables*, the most comprehensive astronomical tables since Ptolemy.

Then, in the generation after al-Zarqali, al-Ghazali did for Islam what Augustine had done for Christendom, ushered in its dark age.

Gerard translated *The greatest* and 12 other astronomical treatises.

Sacrobosco wrote *On the sphere*. It is an elementary introduction to the *Almagest*, like al-Farghani's, and following al-Farghani more than Ptolemy.

Al-Tusi wrote the *Memorandum*.

Ibn Rushd, two-odd generations earlier, had argued that the eccentric circle and the equant mar the purity of the hypothesis of circular motion. Al-Tusi thus undertook to eliminate the eccentric and the equant from the theory of the planets. He proves in the *Memorandum*: if one circle rolls inside the periphery of another circle, and the radius of the first circle is one half of the radius of the second, any point on the circumference of the first circle describes a straight line, and this straight line is a diameter of the second circle (13). Hence, mounting the planet on a circle, and rolling this circle inside an epicycle, al-Tusi disposed of the eccentric and the equant.

Alfonso X was the prince of Castile 1252-84. Under him, and under his direction, the *Alfonsine tables* were compiled. The *Alfonsine tables* are, with but a few changes, the tables of the *Almagest*. They were revised in 1327 at Paris by John of Saxony, became standard throughout Christen-

dom, and remained standard until Kepler. The *Alfonsine tables* are an important step in the return from introductions, epitomes, and commentaries to the mathematical competence of the master.

Buridan wrote the *Questions on On the heaven*. In the *Questions* he resumed the discussion of the possibility that the earth moves, that its revolution on its axis can explain the apparent revolution of the heavens.

Buridan also, against ibn Rushd, argued for eccentric circles. In the *Questions on the Metaphysics* he writes: on the authority of the commentator, who rejects eccentric circles in the theory of the planets, some argue against them (XII, 11); the appearances can be saved, however, without epicycles, with eccentric circles alone (10).

Oresme wrote *The book on the heaven and the world*. In *The book* he also discusses the possibility that the revolution of the earth on its axis can explain the apparent revolution of the heavens.

Ibn al-Shatir wrote the *Final investigation concerning the correction of the planetary theory*. His correction as well concerns the eccentric and the equant. Ibn al-Shatir disposes of them by mounting the planet on an epicycle mounted on an epicycle.

In that Copernicus, too, dispensed with the equant, the labors to remove the equant, or the eccentrics, or the equant and the eccentrics, or the epicycles, from the theory of the planets, because they violate the hypothesis of circular motion, these labors of al-Tusi, Buridan, and ibn al-Shatir anticipate those labors of Copernicus. Kepler, for his part, restored the equant to the theory.

Buridan and Oresme anticipate Copernicus with the resumption of the discussion of the possibility that the earth moves, too.

Then, thirteen and a half centuries after Ptolemy, Peurbach began, and in 1462 Regiomontanus completed, the *Epitome of the Almagest*, with new observations, corrections of Ptolemy's computations, and critical notes. It is the first in the long series of introductions, epitomes, and commentaries of greater mathematical sophistication than Pappus's.

And Regiomontanus took the first step beyond Ptolemy.

He advanced trigonometry. He wrote *On triangles of all kinds*, in which he formulates the laws of sines and cosines for spherical triangles (V, 1-2),

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

and

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

where a , b , and c are the sides of the spherical triangle opposite its angles A , B , and C . Hence Regiomontanus computed also the *Tables of directions*, including sines, cosines, and tangents, with a base of $\sin 90^\circ = 6\,000\,000$ (*id.*).

Regiomontanus also prepared the *Ephemerides*, for the years from 1475 to 1506, published in 1474. Their excellence Columbus attests. Columbus's son, with him on his fourth voyage, reports: the natives, believing to have us at their mercy, were bringing us less food every day; but god does not forget those who trust in him; the admiral recalled that there would be a total eclipse of the moon; the day before the eclipse he summoned the chiefs of the natives, and told them that god is angry with them for not bringing us food, will punish them with famine and pestilence, and will give them a clear sign of it; to see the sign, they should look at the rising moon that night; and when the complete eclipse began, the natives came running to the ships, laden with provisions, begging the admiral to ask god to forgive them; the admiral promised to speak to god (*Life*, 103). The admiral had Regiomontanus's *Ephemerides* at hand.

Copernicus published the *Little treatise on the hypotheses concerning the heavenly motions* in 1514, *On the revolutions of the heavenly spheres* in 1543.

Copernicus's was a revolution.

The revolution was yet not *ex nihilo*.

In general, counting back to Abelard and Gerard, the acquisition of knowledge had been accelerating for four centuries by the time of Copernicus. It accelerated steadily in the first two of these four centuries, and after slowing down in the following century, accelerating steadily more in the following century.

Hence, in particular, in the two generations before Copernicus, with Peurbach and Regiomontanus, the investigation of the heaven had for

the first time since Ptolemy begun advancing again.

In particular, too, in the century before Copernicus the investigation of the surface of the earth had begun advancing for the first time since Ptolemy as well, and by the time of Copernicus had advanced very greatly. The great advance in the investigation of the surface of the earth and the great advance in the investigation of the heaven were inseparable in the seven centuries culminating in Ptolemy. And the very great advance in the investigation of the surface of the earth in the century before Copernicus has to be the proximate cause of the Copernican revolution.

Prince Henry began sending his knights to sea. In 1434 Eanes rounded cape Bojador. In 1456 Ca' da Mosto reached the Bissagos Islands. In 1482 Cão crossed the equator. In 1488 Dias rounded the cape of Good Hope. In 1492 Columbus crossed the Atlantic. In 1498 da Gama reached India. In 1519-22 Magellan and El Cano circumnavigated the earth.

In 1526 de Torenó drew the first map of the earth in its true proportions.

In 1543 Copernicus discovered the solar system.

Those 20 philosophers, then, are part of the development of the problem of the earth in the fourteen centuries from Ptolemy to Copernicus. To the degree to which there is a solution of the problem in Ptolemy, and those philosophers examine Ptolemy, and examine his solution of the problem, explicitly or implicitly, and those 20 philosophers one and all examine Ptolemy, they are part of the development of the problem.

Grounds.

Some new observations were added to the old. Not many. Of the 78 dated observations in *On the revolutions*, 45 are from the *Almagest*, 27 are by Copernicus himself. 6 were made after Ptolemy and before Copernicus, 3 of them by al-Battani.

The theory remained the same until Regiomontanus.

Regiomontanus developed trigonometry.

And Copernicus then solved two of the three parts of the problem dramatically better.

At rest or in motion.

Copernicus revolves on its axis and around the sun.

sun.

In *The two main systems* as well, he calculated the distance of a star. He writes: if the sun is 1 208 earth radii away, and its apparent diameter is $\frac{1}{2}^\circ = 30' = 1\,800'' = 108\,000'''$, and the apparent diameter of a star of the sixth magnitude is $50'''$, the apparent diameter of the sun is 2 160 times larger, and if that star is equal to the sun and not larger, it is at a distance 2 160 times the distance of the sun.

He began and advanced greatly the development of the theory of motion and the world, developed and advanced it mathematically, the theory having learned the language of the world, which is mathematics.

Kepler transferred all of astronomy from fictitious circles to natural causes.

He discovered the laws of the motions of the heavenly bodies.

He described the system of the world. The position, order, and motion of the parts of the world is the system of the world.

He calculated the distance of the sun at $3\,469\frac{1}{3}$ earth radii.

He wrote on the inhabitants of the moon.

Snel, in 1617, in the *Batavian Eratosthenes*, began the measurement of the length of 1° of the meridian.

Descartes advanced greatly the theory of motion and the world, explicitly the theory of the world now, explicitly grounded in the laws of nature.

He first described the world beyond the solar system. The world is a system of stellar systems, of which the solar is one. The sphere of the stars, having long bounded the world, was resolved into stellar systems.

He first described the development of the world, and within the development of the world, the development of the earth. The earth was formerly a star like the sun, in the center of its own vast vortex. The elements of all matter are of three kinds, distinguished by size and rapidity of motion. In its first state the earth was composed entirely of matter of the first element, the smallest and fastest, of which the sun also was and still is composed. Smaller, the vortex of the earth de-

scended ultimately into the larger vortex of the sun. As motion degenerates and the elements aggregate, the elements degenerate as well, the first kind into the second, the second into the third. And a short time after its descent into the vortex of the sun, the earth had degenerated into three regions, innermost, composed of matter of the first element, intermediate, composed of matter of the second element, and outermost, composed of matter of the third element. This last is the matter from which all bodies surrounding us originated. As the elements continued to move through one another fissures appeared in the crust, and became larger in time. The crust broke into various parts, and so the mountains, valleys, oceans, and all other features on the surface of the earth were formed.

Not endowed with history before their creation, having then long been created in a few verses each, the world and the earth, both, acquired a history. The problem of the position of the earth, having long been of its position in space alone, became the problem of its position in time as well.

Hevelius drew a map of the moon, discovered the libration of the moon in longitude, and catalogued 1 564 stars.

Picard, in 1671, in the *Measurement of the earth*, completed the measurement of the length of 1° of the meridian. He measures the length of 1° of the meridian at: 57 060 toises (11), and hence the diameter of the earth at: 6 538 594 toises (*id.*). 1 toise = 1.949 m. The radius of the earth is 6 372 km.

Cassini, between 1671 and 1684, discovered four of the five major satellites of Saturn. The first Huygens discovered in 1656. In 1684 Cassini calculated the parallax of Mars at $25\frac{1}{2}''$,

hence the parallax of the sun at $9\frac{1}{10}''$, and hence

the distance of the sun at 21 600 earth radii.

Huygens greatly improved the telescope.

He greatly improved the measurement of time.

He discovered Titan, discovered the ring of Saturn, calculated, in 1659, the distance of the sun at 25 086 earth radii, and hence determined the true proportions of the solar system.

He advanced greatly the theory of motion and

the world.

He described the inhabitants of the planets.

In the *Κοσμοθεωρὸς* as well, in 1698, he calculated the distance of a star. It is the fourth such calculation. The third Newton made in 1685, the second James Gregory in 1668. Huygens writes: according to Cassini, from very exact observations of the parallax of Mars, the distance from the earth to the sun is about 22 000 earth radii, according to a very probable conjecture of mine it is about 24 000 earth radii (2); it was a pretty fancy of Hesiod's that an anvil dropped from the top of heaven will reach the earth in 10 days, and in 10 more days will arrive at the bottom of hell; I will not use an anvil, but a good cannonball of a great gun which travels 100 toises in 1 second; this cannonball will spend 25 years in its passage from the earth to the sun (*id.*); and seeing that the stars are so many suns, if we suppose one of them to be equal to ours, it will follow that its distance from us is as much greater than the distance of the sun as its apparent diameter is smaller than the apparent diameter of the sun (*id.*); I studied how to lessen the diameter of the sun so as to make it appear not larger than that of Sirius; so I closed one end of my twelve-foot tube with a very thin plate with a hole in it with a diameter $\frac{1}{144}$ of an inch; I turned that end

to the sun, and preventing all other light from reaching my eye, I could see a part of the sun whose diameter was about $\frac{1}{182}$ of the whole; but

this still appeared much brighter than Sirius; so I placed against the hole a glass which I use in my microscopes, and I found it to appear of much the same brightness as Sirius; and according to the rules of optics, I found that the diameter now was $\frac{1}{152}$ of $\frac{1}{182}$, and that is, $\frac{1}{27\,664}$ of the diameter of the sun (*id.*); the ratio of the distance of the sun to the distance of Sirius is thus 1 to 27 664; and multiplying this number by 25, the cannonball will spend almost 700 000 years in its journey from us to the nearest fixed star (*id.*).

Before Copernicus, in 25 years the cannonball would have left the world. Considering also that the world was at that time 5 500 years old, the cannonball would have had to travel to Sirius, on

Huygens's reckoning, more than 120 durations of the world.

Richer, in 1672, crossed the Atlantic to make observations in Cayenne, and published his results in the *Astronomical and physical observations made on the island of Cayenne*, in 1679. The important results are two.

One is the measurement of the position of Mars. Cassini calculated the parallax of Mars, hence the parallax of the sun, and hence its distance, from the simultaneous measurements of the position of the planet by himself in Paris and Richer in Cayenne, with the distance across the Atlantic as a base.

The other is the discovery that: a pendulum beating seconds is shorter in Cayenne than in Paris (*Observations*, 10). This discovery of Richer's, as Newton points out in *The principles*, verified experimentally, and was explained by, the fact that the earth is not a perfect sphere but an elliptic spheroid, flattened at the poles and elongated at the equator by gravity.

Hooke, in 1674, in the *Attempt to prove the motion of the earth from observations*, formulated: three suppositions; first, all celestial bodies whatsoever have an attraction, or gravitating power, toward their own center by which they attract not only their own parts, and keep them from flying away, as we observe in the earth, but attract also all other celestial bodies that are within the sphere of their activity; second, all bodies whatsoever put into a direct and simple motion will continue to move forward in a straight line until they are deflected and bent by some other powers into a motion describing a circle, ellipse, or some other curve; third, these attractive powers are the more powerful in operation the nearer the body on which they act is to their center (*id.*).

Hooke adds: but what the several degrees of the powers are I have not yet experimentally verified (*id.*). Newton verified them, $F \propto \frac{1}{r^2}$.

Burnet wrote *The sacred theory of the earth, containing an account of the origin of the earth, and of all the general changes which it has already undergone, or is to undergo, until the consummation of all things*, in 1681. He borrowed the sacred from *Genesis*, the rest from

Descartes.

Steno, in 1669, began the investigation of the stratification of the earth.

James Gregory improved the telescope. In 1668 he calculated the distance of Sirius at 83 190 distances of the sun.

Newton greatly improved the telescope.

He discovered the laws of motion.

He discovered the law of gravity.

He formulated the theory of motion and the world.

He described the system of the world.

He deduced Kepler's laws from the laws of motion.

He showed why the earth revolves around the sun, and why it is a sphere.

He writes: I have shown the two major cases of attraction, when the centripetal force decreases as the square of the distance and when it increases as the distance; the effects in both cases, that bodies revolve in conic sections, and that centripetal forces compose spherical bodies, are effects of the same law; which is worthy of note.

The revolution of the earth around the sun is the effect of gravity. Its sphericity is the effect of gravity as well. Which is worthy of note.

He showed that the earth is not a perfect sphere, and why it is not.

He proves: the axes of the planets are smaller than the diameters perpendicular to them; because of the rotation of the planets on their axes their parts, by receding from the axes, endeavor to ascend at the equator.

In particular, he showed that the length of a pendulum beating seconds at the equator is shorter than the length of a pendulum beating seconds at Paris by $1 \frac{87}{1000}$ lines. The diameter of

the earth at the equator is to its diameter through the poles as 230 to 229. The earth is an elliptic spheroid of eccentricity 0.004.

He showed that the mass of the earth is $\frac{1}{169\,282}$ of that of the sun, the force of attraction

on its surface $\frac{435}{10\,000}$ of that on the surface of the sun.

He showed that the moon let fall to the earth with all the force with which it is otherwise

maintained in its orbit will fall in 1 minute $15 \frac{1}{12}$ Paris feet.

He showed that the mass of Jupiter is $\frac{1}{1\,067}$ of that of the sun, the force of attraction on its surface $\frac{943}{10\,000}$ of that on the surface of the sun, the mass of Saturn $\frac{1}{3\,021}$ of that of the sun, the force of attraction on its surface $\frac{529}{10\,000}$ of that on the surface of the sun.

He calculated the orbits of comets.

In 1685, in *A treatise on the system of the world*, he writes: the apparent diameter of the disc of Saturn is $17''$ or $18''$; Saturn therefore receives but about $\frac{1}{2\,100\,000\,000}$ of the sun's

light, for so much smaller is the surface of its disc than the surface of a sphere at the distance at which Saturn is from the sun (*id.*). – The ratio of the two surfaces is $\frac{\pi r^2}{4 \pi R^2}$, where r is the radius

of Saturn, R is the radius of a sphere whose surface is at the distance at which Saturn is from the sun, and the ratio of $\frac{r}{R}$ is, very nearly, the

sine of the apparent radius of the disc of Saturn.

– Newton continues: only the illuminated hemisphere of Saturn receives light from the sun; and if we suppose that Saturn reflects about $\frac{1}{4}$ of this

light, the light reflected from its illuminated hemisphere will be about $\frac{1}{4\,200\,000\,000}$ of the

whole light emitted by the sun's hemisphere; and, therefore, since light is rarefied in the ratio of the square of the distance from the luminous body, if the sun were at a distance $10\,000 \sqrt{42}$ greater than that of Saturn, it would appear as bright as Saturn now does without its ring, and that is, somewhat brighter than a fixed star of the first magnitude; we can therefore suppose that the distance from which the sun would shine as a fixed star exceeds that of Saturn about 100 000 times (*id.*).

Had the world remained 5 500 years old, that cannonball would have had to travel to the distance from which the sun would shine as a fixed

star more than 4 300 durations of the world.

In *The principles*, he writes: a globe of incandescent iron one inch wide will scarcely lose all its heat in one hour; a larger globe will preserve its heat longer in the ratio of its diameter, for its surface is smaller in this ratio with respect to the heat it contains; so a globe of incandescent iron equal to the earth, and that is, about 40 000 000 feet wide, will scarcely cool in as many days, or about 50 000 years.

It will scarcely cool in 9 of those durations of the world.

And before tabulating the results, how the problem of the duration of the world arose, and the problem of the position of the earth became a problem of its position in time as well as in space, should be considered.

cclv

When on high is the first account of creation. Seven and a half centuries later, the Yahwistic document is the second.

And *When on high* is much the better account. It needs 66 verses to accomplish creation. The Yahwistic document makes do with 7. Marduk arranges the stars, the sun, and the moon, marks the year, the months, and the days, lets the Tigris and the Euphrates flow, builds Babylon. In the Yahwistic document god makes heaven and earth, and without further ado proceeds to make man from the dust of the earth. In *When on high* the very beginning is not incoherent. When the heaven had no name, Apsu was the begetter, Tiamat bore all. In the Yahwistic document, when there was no shrub and no grass on the earth, and water welled up from below and covered the earth, god made the earth and the heaven.

The virtue of the Yahwistic document is simplicity. Little as god does, he does all by himself.

Not quite three and a half centuries later, the Priestly code incorporated the Yahwistic document into itself.

To accomplish creation the Priestly code needs 27 verses. The account is not of greater specificity than that of *When on high*. It is not only of greater coherence, however. It is very much better ordered. God separates light from darkness, then makes, in this order, heaven, the earth and the

sea, plants, the sun, moon, and stars, the living creatures of the sea and the heaven, animals, man in his own image.

And the Priestly code has thus answered explicitly the questions, both, of when the world was made and when the earth was made.

The Priestly code incorporated into itself the stories of the patriarchs told by the Yahwistic document. It amplified the stories with the doctrine that god gave the Mosaic law to the patriarchs from the first. But the Priestly code is also fond of numbers, and not just the 1st day, the 2nd, the 3rd, the 4th, the 5th, the 6th, and the 7th. It amplified the stories of the patriarchs with numbers, too.

And its answers to the questions of when the world was made and when the earth was made are thus not just explicit, they are quantitative.

The striving for truth which forces us to advance from the thought and the sense to the reference and the true is seldom in evidence in the Priestly code. Where there are numbers, however, and the more so where there is an abundance of numbers, the reference and the true cannot not be there.

With Descartes: a problem is everything in which there is truth or falsehood.

And if the Priestly code did not, its successors formulated and solved the problems of when the world was made and when the earth was made explicitly.

Here, then, are the Priestly code's answers to the two questions, when the world was made, and when the earth was made, with the earlier numbers on whose ground the two problems were formulated and solved explicitly. And as the Priestly code is fond of questions of origins not less than it is of numbers, the numbers show its answers to three more questions of origins along the way. When man was made, when the first city was built, and when the first man of might on earth was born.

First:

when god began making heaven and earth, the earth was without form and void; and god made light; the 1st day (*Genesis*, 1); then he made heaven, the 2nd day (*id.*).

This is when the world was made, heaven was complete by the 2nd day.

Next:

god gathered the water on one side, the earth on the other; the 3rd day (*id.*).

This is when the earth was complete, 1 day after the world.

Next:

god said, let us make man in our image; and god made man in his image; the 6th day (*id.*).

This is when man was made, 3 days after the earth was made.

Next:

Adam begot Cain;

Cain begot Enosh;

and Cain built a city and named it after his son, Enosh (4).

This is when the first city was built. As god was not fond of Cain and his descendants, the Priestly code does not supply their succession with numbers. But:

Adam begot also Seth (*id.*),

god was fond of Seth and his descendants, and the Priestly code supplies their succession with numbers:

Adam was 130 years old when he begot Seth;

Seth was 105 years old when he begot Enosh;

Enosh was 90 years old when he begot Kenan;

Kenan was 70 years old when he begot Mahalalel;

Mahalalel was 65 years old when he begot Jared;

Jared was 162 years old when he begot Enoch;

Enoch was 65 years old when he begot Methuselah;

Methuselah was 187 years old when he begot Lamech;

Lamech was 182 years old when he begot Noah;

Noah was 500 years old when he begot Shem, Ham, and Japheth (5).

This yields an average of 156 years, and presuming that Cain was begotten not long before Seth, and given that he built a city after he begot Enosh, the first city was built some 300 years after man was made.

Next:

Ham begot Cush (10);

Cush begot Nimrod, he was the first man of might on earth, he was a mighty hunter, the mainstays of his kingdom were Babylon, Uruk, and Akkad in the land of Shinar (*id.*).

And this is when the first man of might on earth

was born. Again, god was not fond of the descendants of Ham, but he was fond of the descendants of Shem through Arpachshad, and it is these that the Priestly code supplies with numbers:

Shem was 100 years old when he begot Arpachshad;

Arpachshad was 35 years old when he begot Shelah (11).

Presuming that Shem and Ham begot their firstborns at about the same time, and so did their firstborns – as Noah also begot Shem, Ham, and Japheth at the same time – the first man of might on earth was born some 1 400 years after the first city was built.

In years after creation,

the world made	0,
the earth made	0,
man made	0,
the first city built	300,
the first man of might on earth	1 700.

Clearly, the dates are absolute. *Anno mundi*.

And these are 9 of the philosophers who formulated and solved explicitly the problem of when the world was made, and thus also, explicitly or implicitly, the other four problems as well. Their dates are,

246 ^{BC}	Demetrius,
80 ^{AD}	ben Halafta,
265 - 339	Eusebius,
354 - 430	Augustine,
539 - 594	Gregory of Tours,
673- 735	Bede,
973 - 1055	al-Biruni,
1540 - 1609	Scaliger,
1581 - 1656	Ussher.

Demetrius computed the numbers first. In the Alexandrian library, soon after its foundation, two-odd centuries after the Priestly code, the earlier Hebrew holy writings were translated into Greek. Into the septuagint. Two-odd generations later – at the time of Archimedes, Eratosthenes, and Apollonius – Demetrius, working from the septuagint, computes: from creation to the flood 2 264 years; from the flood to the birth of Abraham 1 070 years (fr. 1).

Ben Halafta first computed the numbers from the Hebrew holy writings themselves. Ben Halafta is contemporary with Ptolemy. He is also part of the scholarship which, having in the

previous century begun commenting on the holy writings with the mishna, was preparing itself to begin in the following centuries commenting on the commenting with the talmud. In *The order of the world*, ben Halafta computes: from creation to the flood 1 656 years, from the flood to the birth of Abraham 292 years (1); from the birth of Abraham to the exodus 500 years (1-9); from the exodus to the destruction of the first temple 890 years (9-27); from the destruction of the first temple to the destruction of the second temple 490 years (27-30). For a total of 3 828 years from creation to the destruction of the second temple. The second temple was destroyed in 70^{AD}.

Eusebius is contemporary with Pappus. This is also the time, and it is because of the labors of Eusebius, too, of the triumph of Christianity. Eusebius wrote *The history of the church* and the *Chronicle*. In the *Chronicle* he correlated the history recorded by the pagans with the history recorded in the holy scriptures. And in the *Chronicle*, he computes: 2 242 years from creation to the flood; then 942 years to the birth of Abraham; then 505 years to the exodus; then 479 years, or 600 years, to the beginning of the building of the temple by Solomon; then 512 years to the restoration of the temple under Darius; then 548 years to the ministry of our lord Jesus Christ down to the 15th year of Tiberius (II, pr.). For a total of 5 228 years, or 5 349 years, from creation to the ministry of our lord Jesus Christ down to the 15th year of Tiberius. The 15th year of Tiberius is 29^{AD}.

There was thus the difficulty, were the years from creation to the flood, and then to the birth of Abraham, to be computed from the writings in the Hebrew? or their translation into the Greek?

Augustine resolved this difficulty, as he did so many others. In *The city of god against the pagans*, he writes: we reckon 2 262 years, against the 1 656 years of the Hebrews, even we then have to believe that the men of old refrained from begetting sons for 1 400 years (XV, 20). And Augustine computes: 2 262 years to the flood (*id.*); 1 072 to Abraham (XVI, 10); about 1 200 to the foundation of Rome (17); 787 to the resurrection (XVIII, 54). For a total of about 5 321 years from creation to the resurrection. The resurrection, although a resurrection, if it occurred at the end of the ministry of our lord Jesus Christ,

occurred in the 15th year of Tiberius, 29^{AD}.

Gregory of Tours, in the *History of the Franks*, computes: 5 454 years from creation to the resurrection (X, 31).

Bede, in *On the reckoning of time*, computes: 3 952 years from creation to the birth of Christ in the 42nd year of Augustus (66). Bede counts the years of Augustus from the death of Caesar in 44^{BC} (*id.*).

Al-Biruni, in *The monuments of past generations*, computes: 4 334 years from creation to the birth of Muhammad (6-7). Muhammad was born c. 570^{AD}.

Scaliger writes in the generation before Galileo and Kepler. In the *New work for the improvement of chronology*, in 1583, he computes creation to 3950^{BC} (VII).

Ussher is an older contemporary of Descartes. In *The annals of the old testament deduced from the first origin of the world*, in 1650, he computes creation to 4004^{BC}. More precisely: creation occurred 4 003 years, 70 days, and 6 hours before midnight on 1 January 1^{AD} (read.).

To tabulate the solutions of this problem as well, rounding to the nearest century, and recalculating, for the sake of uniformity, with respect to the birth of Descartes, rather than that of Christ, or Muhammad, the world was made,

Demetrius	6 900 ^{BD} ,
ben Halafta	5 400,
Eusebius	6 900,
Augustine	6 900,
Gregory of Tours	7 000,
Bede	5 600,
al-Biruni	5 400,
Scaliger	5 500,
Ussher	5 600.

It can be concluded, with the Priestly code, ben Halafta, Bede, al-Biruni, Scaliger, and Ussher, against the septuagint, Demetrius, Eusebius, Augustine – the men of old did not refrain from begetting sons for 1 400 years, apparently – and Gregory of Tours, that the world was made in 5 500^{BD}.

So was the earth. So was man. The first city was built in 5 200^{BD}. The first man of might on earth was born in 3 800^{BD}.

Descartes described the development of the world, and within it, the development of the

earth. In the next generation Steno began the investigation of the stratification of the earth, and Burnet wrote *The sacred theory of the earth*. The world and the earth acquired a history. The problem of the position of the earth became the problem of its position in time as well as in space.

And if this part of the problem is not *ex nihilo*, it is from the problem of when the world was made, attesting by this time not less than eighteen and a half centuries of vigorous development, formulation, solution, examination, reexamination, correction, corroboration.

More generally, in the one and a half centuries from Copernicus to Newton, as he was finally leaving the acquisition of knowledge alone, god made several substantial contributions to it.

Also with Descartes: the laws of nature follow from this alone that god is immutable.

Also with him: god always conserves the quantity of matter in the world, and always conserves the quantity of motion in the world.

And the numbers extracted from the Priestly code not only have the virtue of being numbers. They remained the only numbers for the age of the world and for the age of the earth until Buffon, one century-odd after Descartes.

Moreover, on the age of the world, the age of the earth, and the age of man, the Priestly code is in error by many orders of magnitude. But on the age of the first city and the age of the first man of might on earth its numbers are excellent.

The first city is Çatal Hüyük, with a radioactive date for its lowest level *c.* 6500^{BC}. The first city was thus built,

Priestly code	5 200 ^{BD} ,
correction	8 100.

The first man of might on earth is Sargon, prince *c.* 2337-281^{BC}. The mainstay of his kingdom was Akkad, his kingdom included Uruk and the site on which Babylon was to be built, and it need not be doubted that he was a mighty hunter. The first man of might on earth was thus born,

Priestly code	3 800 ^{BD} ,
correction	4 000.

The world and the earth acquired a history with Descartes.

Descartes's successors will write the history better, and better, as it extends over millions of

years, and billions of years.

Hegel, for his part, will write: to determine how the earth was millions of years ago, and one can be as generous with years as one pleases, is not interesting; the interesting is confined to what is now (*Encyclopedia*, II, 339); and this the Mosaic history of creation captures best, when it quite naively says, today appeared the plants, today the animals, and today man (*id.*).

cclvi

To tabulate the results of the philosophers in the one and a half centuries from Copernicus to Newton with respect to the problem of the earth.

Grounds.

Tycho	improved observation,
Galileo	built a telescope,
Huygens	improved the telescope,
James Gregory	improved the telescope,
Newton	improved the telescope.

Kepler also built a telescope. Hevelius built a telescope 150 feet long. In the generation between Huygens and Newton Cassegrain also improved the telescope.

As the fact that it rested now on the telescope and its improvement all by itself shows, the advance in observation was inseparable from the advance of the theory of motion and the world.

Gilbert	the earth is a magnet, the earth and the moon attract and repel one another,
Galileo	greatly advanced the theory,
Kepler	transferred all of astronomy to natural causes, discovered the laws of the motions of the heavenly bodies,

Descartes	greatly advanced the theory, formulated the laws of nature,
Huygens	greatly advanced the theory,
Hooke	all celestial bodies attract all other celestial bodies,
Newton	discovered the laws of motion, discovered the law of gravity, formulated the theory in all its generality and unity.

The great advance of the theory was inseparable from the great advance of mathematics, and most particularly the discovery and development of

calculus.

The problem of the earth was solved in two of its parts conclusively.

At rest or in motion.

Galileo proved that it revolves around the sun,

Newton the revolution is the effect of gravity.

Hicetas had discovered that it rotates on its axis.

Laplace will show that the cause of the rotation is the same, gravity.

Foucault will demonstrate the rotation experimentally.

In 1851, in *A physical demonstration of the motion of rotation of the earth by means of a pendulum*, Foucault reports: a pendulum with a 5 kg bob was suspended from a steel thread 2 m long and let swing through arcs of 15° to 20° (*id.*); the point of oscillation was observed to move constantly to the left of the observer, indicating the gradual deviation of the plane of oscillation in the same direction as the apparent motion of the celestial sphere (*id.*).

And this is the terminal solution of this part of the problem. In motion, rotates on its axis and revolves around the sun, and the cause is gravity.

Its shape.

Snel began the measurement of 1° of the meridian,

Picard completed the measurement of 1° of the meridian,

the earth's radius is 6 372 km,

Richer a pendulum beating seconds is shorter at the equator than

further north,

Newton not a perfect sphere, it is a spheroid of eccentricity 0.004,

its shape is also the effect of gravity.

Maupertuis will corroborate from observation Newton's deduction of the shape.

And Bessel will bring the measurement of the earth to its end.

In 1737, in *On the measure of a degree of the meridian at the polar circle*, Maupertuis measures the length of 1° of the meridian at the latitude of Lapland, corrects this measurement and that of Picard of the length of 1° at the

latitude of northern France for precession, refraction, and aberration, and obtains for the lengths of the two arcs: in toises, 56 925 and 57 437 (*id.*).

In 1837, in the *Determination of the axes of the elliptic spheroid of revolution*, Bessel concludes: the semiaxes of the spheroid through the equator and the poles are, in toises, 3 271 954 and 3 261 073 (5).

The entire development of this part of the problem is thus this.

Parmenides said that it is a sphere. Its radius is,

Eudoxus 10 200 km,

Dicaearchus 7 600,

Eratosthenes 6 400,

Picard 6 372,

Bessel 6 377 and 6 356.

Bessel's numbers have been refined to 6 378 and 6 357. Its eccentricity is,

Newton 0.004,

Bessel 0.003.

This number of Bessel's cannot be refined.

And this is the terminal solution of this part of the problem. Very nearly a sphere, of equatorial radius 6 378 km, eccentricity 0.003, and the cause is gravity.

Its position.

This part of the problem was not solved conclusively. The advance in it was yet the greatest. In space, its position was determined in what became explicitly the system of the world – explicitly the solar system, with Kepler, explicitly the system of stellar systems of which the solar is one, with Descartes, within this system the true proportions of the solar were determined, with Huygens, and all parts of the system began being weighed and measured, with Newton. In time, the determination of its position in time began.

In space.

Tycho showed that the fixed stars are not fixed,

Bruno the world is infinite,

Galileo discovered the four major satellites of Jupiter,

a star of the sixth magnitude is 2 160 distances of the sun

away,

Kepler the sun is $3\,469\frac{1}{3}$ earth radii

	away,
Descartes	described the solar system, described the system of stellar systems,
Hevelius	catalogued 1 564 stars,
Cassini	discovered four of the major satellites of Saturn, the sun is 21 600 earth radii away,
Huygens	discovered Titan and the ring of Saturn, the sun is 25 086 earth radii away,
	described the solar system in its true proportions, Sirius is 28 000 distances of the sun away,
James Gregory	Sirius is 83 190 distances of the sun away,
Newton	began weighing and measuring all parts of the system, the earth's mass is $\frac{1}{169\,282}$ of the sun's, attraction on its surface is $\frac{435}{10\,000}$ of that on the sun's, calculated the orbits of comets, a star of the first magnitude is 950 000 distances of the sun away,
In time.	
Descartes	described the development of the earth,
Steno	began the investigation of stratification,
Burnet	described the changes the earth has undergone,
Newton	a globe of incandescent iron equal to the earth will not cool in 50 000 years.

The magnitude of the advance the determination of the two distances, of the sun and of the stars, indicates most dramatically.

This is the development of the problem of the distance of the sun through this time,

Aristarchus	380 earth radii.
Hipparchus	490,
Ptolemy	1 210,

Kepler 3 469 $\frac{1}{3}$,

Huygens 25 086,
Cassini 21 600,
correction 23 455.

Ptolemy obtained the distance of the sphere of the fixed stars from the calculation of the sizes of the eccentrics and epicycles of the planets. Galileo obtained the distance of a star of the sixth magnitude, James Gregory the distance of Sirius, the brightest star, Newton of a star of the first magnitude, Huygens of Sirius, all four from the calculation of the brightness of the star compared with that of the sun. The classification of the stars by magnitude, where the first is the brightest, the sixth the dimmest, is also from Ptolemy (*Almagest*, VII, 4).

And this is the development of the problem of the distance of the stars through this time,

Ptolemy 17 distances of the sun,
Galileo 2 160,
James Gregory 83 190,
Newton 950 000,
Huygens 28 000,
correction 270 000.

The correction is from the nearest star, Proxima Centauri. And the results of James Gregory, Newton, and Huygens are all three within an order of magnitude.

And if such was the drama of the reference and the true, the drama of the thought and the sense Fontenelle captured.

Fontenelle wrote the *Conversations on the plurality of worlds*, in 1686. In the previous two generations it had become a well-established genre to write on the inhabitants of the plurality of worlds. Fontenelle describes the inhabitants of the moon, Mercury, Venus, Mars, Jupiter, and Saturn, then also: the inhabitants of the infinite number of the systems of fixed stars (5). And the marquise, to whom he describes the inhabitants, is moved to exclaim: but they are nearly annihilated by the way you speak of them! you are forced to call them, inhabitants of one of the planets of one of the infinite vortices; and ourselves, to whom this phrase applies as well, you scarcely know how to distinguish among so many worlds; the earth becomes so frighteningly small, I believe I will never be impressed by anything

else (*id.*).

Fontenelle offers in consolation: if roses, which live but a day, were to write histories and leave their memoirs behind, they would describe their gardener in a certain way; and after 15 000 generations those who leave a description to their descendants would change nothing; and they would say, we have always seen the same gardener, in all the memory of roses we have seen only him, and he has always been exactly as he is, certainly he does not die like us, certainly he is immutable (*id.*).

Alas, it was more than 41 years that the roses had been seeing the gardener. It could not be too long before the phenomena challenged the hypothesis of his immutability.

The steps of advance in the acquisition of knowledge are the 10, Gilbert, Tycho, Galileo, Kepler, Descartes, Picard, Cassini, Huygens, Steno, Newton. In particular,

Gilbert	the earth is a magnet, the earth and the moon attract and repel one another,
Tycho	improved observation, showed that the fixed stars are not fixed,
Galileo	built a telescope, greatly advanced the theory of motion and the world, proved that the earth revolves around the sun, discovered the four major satellites of Jupiter, a star of the sixth magnitude is 2 160 distances of the sun away,
Kepler	transferred all of astronomy to natural causes, discovered the laws of the motions of the heavenly bodies, the sun is $3\,469\frac{1}{3}$ earth radii away,
Descartes	described the solar system, greatly advanced the theory of motion and the world, formulated the laws of nature, described the system of stellar

Cassini

Huygens

Steno

Newton

described the development of the earth, discovered four of the major satellites of Saturn, the sun is 21 600 earth radii away, improved the telescope, greatly advanced the theory of motion and the world, discovered Titan and the ring of Saturn, the sun is 25 086 earth radii away, described the solar system in its true proportions, Sirius is 28 000 distances of the sun away, began the investigation of stratification, improved the telescope, discovered the laws of motion, discovered the law of gravity, formulated the theory of motion and the world in all its generality and unity, the earth is a spheroid of eccentricity 0.004, the revolution of the earth is the effect of gravity, so is its shape, began weighing and measuring all parts of the system, the earth's mass is $\frac{1}{169\,282}$ of the sun's, attraction on its surface is $\frac{435}{10\,000}$ of that on the sun's, calculated the orbits of comets, a star of the first magnitude is 950 000 distances of the sun away, a globe of incandescent iron equal to the earth will not cool in 50 000 years.

cclvii

The one and a half centuries from Newton to

Bessel.

Two and a half centuries separate Newton and Einstein. These centuries can be divided into two parts.

In the one and a half centuries from Newton to Bessel, the theory of the world developed in the previous one and a half centuries, and culminating in Newton, was greatly developed further, including through its analytic formulation by Lagrange in 1788, and remained the foundation of the formulation and solution of all problems. Including the problem of the earth.

In the one century from Bessel to Einstein, thermodynamics was developed, Clausius formulated its laws in 1850, electrodynamics was developed, Maxwell formulated its laws in 1864, and these became, next to mechanics and its laws – such as the Newtonian theory and its laws became now – the foundation of the formulation and solution of all problems variously. Including the problem of the earth. The incorporation of mechanics, thermodynamics, and electrodynamics into a unitary theory of the world began with Einstein and the theory of relativity.

And all particulars with respect to the problem of the earth from the one and a half centuries from Newton to Bessel can be collected, and thus the collection of all particulars from the first through that time completed.

For the one century from Bessel to Einstein it can be only indicated how they can be collected. It is the first of the last two centuries, and the last two centuries demand special treatment.

These 20, then, are all important philosophers who are part of the development of the problem of the earth in those one and a half centuries,

1644 - 1710 **Römer**,
 1646 - 1716 **Leibniz**,
 1646 - 1719 Flamsteed,
 1656 - 1742 **Halley**,
 1693 - 1762 **Bradley**,
 1698 - 1759 **Maupertuis**,
 1707 - 1783 **Euler**,
 1707 - 1788 **Buffon**,
 1717 - 1783 **d'Alembert**,
 1724 - 1804 **Kant**,
 1726 - 1797 Hutton,
 1732 - 1807 Lalande,
 1736 - 1813 **Lagrange**,

1738 - 1822 **Herschel**,
 1740 - 1784 Lexell,
 1749 - 1817 Werner,
 1749 - 1827 **Laplace**,
 1769 - 1839 **William Smith**,
 1777 - 1855 **Gauss**,
 1784 - 1846 **Bessel**.

These 5 are some of the more important among the not important,

1711 - 1786 Thomas Wright,
 1730 - 1817 Messier,
 1746 - 1834 Piazzi,
 1747 - 1826 Bode,
 1758 - 1840 Olbers.

Thomas Wright proposed that the Milky Way is a layer of stars one of which the sun is, in 1750. Messier published a catalogue of 45 nebulae in 1774, extended to 103 nebulae in 1784. The discovery of the asteroids began in 1801, when Piazzi discovered Ceres. Olbers discovered Palas in 1802. And Bode published in 1772, in the *Introduction to the knowledge of the starry heaven*, the series 4+0, 4+3, 4+6, 4+12, 4+24, 4+48, 4+96, 4+192, 4+384, approximating the series of the distances of the planets, rounded to the nearest unit, Mercury at 4+0, Venus at 4+3, the earth at 4+6, Mars at 4+11, Jupiter at 4+48, Saturn at 4+91. In 1781 the approximation deteriorated when Uranus was found at 4+188. In 1801 it improved when Ceres was found at 4+24. In 1846 Neptune was found at 4+297.

Many of the results of these philosophers have been presented above in some detail as well.

Römer measured the velocity of light, at 2.14×10^8 m/s.

With Leibniz, in the *Protoearth*, the history of the earth became an empirical investigation.

Flamsteed catalogued 2 935 stars.

Halley discovered Halley's comet, calculated transits of Venus, and discovered the proper motion of the stars.

Comets had been atmospheric phenomena before Tycho. Newton first calculated orbits of comets in *The principles*, on the supposition that their orbits are parabolic, very nearly.

In 1705, in *A synopsis of the astronomy of comets*, Halley writes: on the supposition that the orbits of comets are parabolic it would follow that, impelled toward the sun by a centripetal

force, comets descend from spaces infinitely distant, and by their falls acquire such a velocity as to run off again into the remotest parts of the universe, moving upward with a perpetual tendency so as never to return again; but since they appear frequently enough, and since none can be found to move with hyperbolic motion, or motion swifter than such as they may acquire by their gravity to the sun, it is highly probable that they rather move in very eccentric orbits and make their returns after long periods of time; if so, their number will be determinate and perhaps not so great; besides, the space between the sun and the fixed stars is so immense that there is room enough for a comet to revolve (2).

And if so, if comets move in eccentric orbits

		ascending node	inclination of orbit		perihelion of orbit	distance of perihelion
1337	Gemini	24° 21' 00''	32° 11' 00''	Taurus	7° 59' 00''	.40666
1472	Capricorn	11° 46' 20''	5° 20' 00''	Taurus	15° 33' 30''	.54273
1531	Taurus	19° 25' 00''	17° 56' 00''	Aquarius	1° 39' 00''	.56700
1607	Taurus	20° 21' 00''	17° 02' 00''	Aquarius	2° 16' 00''	.58680
1682	Taurus	21° 16' 30''	17° 56' 00''	Aquarius	2° 52' 45''	.58328

And in 1758 Halley's comet returned as foretold.

Halley adds in the *Synopsis*: one more thing may perhaps not be improper or unpleasant to advertise to the astronomical reader; some of these comets have their nodes so very near the annual orbit of the earth that, if it shall so happen that the earth is near the node of such a comet, while the comet passes by, what might be the consequences of so near an approach, or contact, or shock, of the celestial bodies, I leave to the studious of physical matters to discuss (*id.*).

In 1716, in *A singular method to determine the parallax of the sun, or its distance from the earth, by observing Venus in the sun*, Halley calculates transits of Venus due in 1761 and 1769. And adds: therefore again and again we recommend it to the curious observers of the stars to whom, when our lives are over, we entrust these observations, that, mindful of our advice, they strenuously apply themselves to these observations.

And in 1769 they did.

and make their returns: we are able to determine the period and axis of revolution of a comet, and hence to foretell its return; indeed, I believe that the comet which Apian observed in 1531, the one which Kepler and Longomontanus observed in 1607, and the one which I observed in 1682, are one and the same (*id.*); and I dare venture to foretell that it will return in 1758 (*id.*).

A historian *par excellence*, Halley examines the record, and collects 24 observations of comets accurate enough to allow the calculation of their orbits. Below are 5 of the 24, including the 3 earliest. The one from 1531 is the observation of Apian. The one from 1607 is the observation of Kepler and Longomontanus. The one from 1682 is Halley's observation.

In 1717, in the *Considerations on the change of the latitudes of some of the principal fixed stars*, Halley reports: I took pains to compare the declinations of the fixed stars delivered by Ptolemy in the third chapter of book VII of the *Almagest*, as observed by Timocharis nearly 300 years before Christ and by Hipparchus about 170 years later, with what we now find; Ptolemy places Aldebaran 20' more northerly in declination than we now find it, Sirius 22' more northerly, and Arcturus 33' more northerly. And adds: what shall we say? it is scarcely credible that the ancients could be deceived in so plain a matter, three observers confirming one another; the most conspicuous in the heaven, these stars are in all probability the closest to the earth; if stars have motion of their own, it is most likely to be perceived in these; and though utterly imperceptible in the space of a century, it may show itself by the alteration of their places in so long a time as 1 800 years.

Bradley discovered the aberration of light,

hence calculated its velocity, and hence determined the distance of the stars.

In 1728, in *An account of a newly discovered motion of the fixed stars*, Bradley reports that in 1725 he began: observations to determine the parallax of the fixed stars (*id.*), and in particular: of γ Draconis (*id.*). He reports: on 3 Dec 1725 the star was observed for the first time to pass near the zenith (*id.*); observations were made on 5 Dec, on 11 Dec, and on 12 Dec (*id.*); on 17 Dec I perceived the star to pass a little more southerly than before; no other cause of this appearance was suspected than the uncertainty of the observations (*id.*); on 20 Dec the star passed still more southerly; satisfied now that it could not be owing entirely to the want of exactness in the observations, we began to think that some regular cause produced this gradual increase of the star's distance from the pole; we thus took care to examine how much it was (*id.*); about the beginning of Mar 1726 the star was found to be $20''$ more southerly than at the time of the first observation; and it now seemed to have arrived at its utmost limit southward, for by the middle of April it appeared to move back toward the north; we concluded that it would now continue northward; and it happened as conjectured, the star moved northward until September, when it became stationary again, being $20''$ more northerly than in June and not less than $39''$ than in March (*id.*).

Bradley reports on the careful examination of his telescope, on the continued observations on γ Draconis as well as on other stars, on his several conjectures in the search for the regular cause, and concludes: at last I conjectured that the phenomena proceed from the progressive motion of light and the annual motion of the earth in its orbit; for I perceived that, if light is propagated in time, the apparent place of a fixed object would not be the same when the eye is at rest as when it is moving in any direction other than that passing through the eye and the object (*id.*). Bradley perceived the aberration of light.

From the aberration of light he calculates its velocity, at 2.80×10^8 m/s.

And draws the conclusion: this hypothesis gives the true law of the variation of the declination in stars (*id.*); and if so, it must be granted

that the parallax of the fixed stars is much smaller than has until now been supposed; I am of the opinion that, if it were $1''$, I should have perceived it in the great number of observations that I made, and especially of γ Draconis; and if the parallax is not as great as $1''$, the star is above 400 000 times further from us than the sun (*id.*).

Maupertuis corroborated from observation that the earth is a spheroid flattened at the poles.

Euler reformulated mechanics analytically, in the *Mechanics, or the science of motion presented analytically*, in 1736. He improved observation, in the *Theory of the motions of planets and comets, containing an easy method of determining the orbits of planets and comets from a few observations*, in 1744. He improved the theory of the moon, in the *Theory of the motion of the moon, accounting for all inequalities*, in 1753, and the *Theory of the motions of the moon, treated by a new method*, in 1772.

Buffon formulated and solved the problem of the origin of the solar system, calculated the age of the earth, and determined the epochs in its development.

He published the first volume of the *Natural history* in 1749. It contains three treatises, *On the manner of study and treatment of natural history*, the *History and theory of the earth*, and the *Proofs of the theory of the earth*.

In the *History and theory of the earth* he presents for the first time the experiments on rates of cooling from which he will determine the age of the earth.

In the *Proofs of the theory of the earth* he formulates and solves the problem of the origin of the solar system. He begins: nothing is better proved in physics than the existence of gravity in every material substance; and this general cause being known, its effects would be easily deduced, had the action of the forces producing them not been so complex (1). Complexity notwithstanding, he deduces from this general cause the origin of the solar system. A comet striking the sun obliquely projected in space a liquid mass equal to $\frac{1}{650}$ of the sun's mass, the liquid diffused, and

then reassembled into spheres revolving around the sun. Showing also that it is not improper or unpleasant to advertise the collision of a comet

into the sun as well as the earth.

In 1775, in the *Researches on the cooling of the earth and the planets*, he calculates the age of the earth at at least 74 832 years.

In 1779, in the *Epochs of nature*, he determines the epochs in the development of the earth. The epochs are seven:

1st epoch, when the earth and the planets acquired their form;

2nd epoch, when matter hardened and formed the interior rock of the globe and the great vitreous masses on its surface;

3rd epoch, when the waters covered the continents;

4th epoch, when the waters withdrew and the volcanoes became active;

5th epoch, when elephants and other animals of the south inhabited the lands in the north;

6th epoch, when the separation of the continents occurred;

7th epoch, when the power of man joined that of nature (*id.*).

D'Alembert reformulated mechanics analytically, in the *Treatise on dynamics*, in 1743.

Kant proposed that the Milky Way is the system of stellar systems of which the solar is one, and made the second attempt, in 1755, to solve the problem of the origin of the solar system. The solar system originated from the cooling and contraction of a nebula.

Hutton wrote the history of the earth, continuing the collection of observation and experiment, in the *Abstract of a dissertation concerning the system of the earth, its duration, and stability*, in 1785, and *The theory of the earth, with proofs and illustrations*, in 1795. The age of the earth, and of the world, became indefinite. Hutton writes in the *Abstract*: the world has neither a beginning nor an end. In *The theory*: we find no vestige of a beginning, no prospect of an end.

Lalande, in the *Memoir on the parallax of the sun, deduced from observations in the South Sea, in the kingdom of Astrakhan, and in China*, in 1772, calculated the parallax of the sun at $8''.6$ (*id.*), and hence its distance at 23 984 earth radii (*id.*). The observations are of the transit of Venus in 1769. The curious observers of the stars had not been mindful of Halley's advice in 1761, but had been in 1769. The observations in the South

Sea Cook made.

Lagrange wrote the *Analytic mechanics*, in 1788. It is the culmination of the analytic reformulation of mechanics. Lagrange deduces mechanics from a single principle, the principle of virtual velocities, a generalization of the principle of least action, itself a generalization of Newton's second law.

Herschel discovered Uranus. He discovered the proper motion of the solar system, mapped the Milky Way, and began and greatly advanced the investigation of nebulae. Herschel took the decisive step in the investigation of the world beyond the solar system.

In 1781 Herschel reported the observation of a new comet. Lexell proposed that it is the seventh planet. Laplace proved that it is, and calculated its distance at 19.2 distances of the sun.

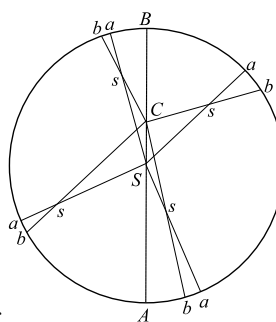
In *On the proper motion of the sun and solar system*, in 1783, Herschel writes:

that several of the fixed stars have proper motion is now so well confirmed that it admits of no further doubt; from the time of Halley we have had continued observations that show Arcturus, Sirius, Aldebaran, Procyon, Castor, Rigel, Altair, and many more actually in motion;

this alone would lead us strongly to suspect that there is not one fixed star in the heavens; there hardly remains any doubt in the general motion of all starry systems, and consequently of the solar among the rest;

I shall now point out the method of detecting the direction and quantity of the supposed proper motion of the sun by a few geometrical deductions;

let the sun be at S , and the fixed stars dispersed in all possible directions and distances at s, s, s, s ; let the sun move in the direction from A to B ; suppose it now arrived at C ; it is evident from the mere inspection of the figure that the stars s, s, s, s , which were seen at a, a, a, a , will now be seen at b, b, b, b , and that is, every star will appear to have receded

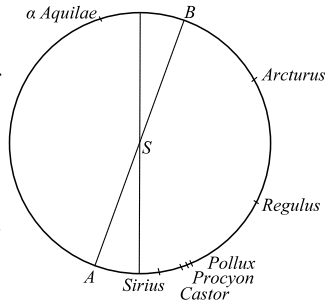


more or less from point *B*;

from Dr Maskelyne we find the proper motion of 7 stars,

	right ascension	declination
Sirius	- 0 ^{''} .63	- 1 ^{''} .20,
Castor	- 0 ^{''} .28,	
Procyon	- 0 ^{''} .80,	
Pollux	- 0 ^{''} .93,	
Regulus	- 0 ^{''} .41,	
Arcturus	- 1 ^{''} .40	- 2 ^{''} .01,
α Aquilae	+ 0 ^{''} .57;	

let the next figure represent an equatorial zone with the solar system in the center and the 7 stars referred to it according to their right ascensions; suppose the direction *AB* to be from a point not far from 77° of right ascension to its opposite 257°, and the sun to move in the direction from *S* toward *B*; then this one motion

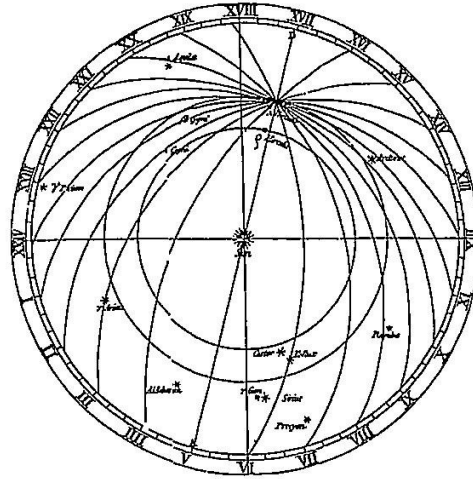


will account for all the stars, for Arcturus, Regulus, Pollux, Procyon, Castor, and Sirius should appear to decrease in right ascension, α Aquilae should appear to increase; moreover, if the sun at the same time ascends toward some point in the northern hemisphere, toward the constellation of Hercules, for instance, then the observed change in the declinations of Sirius and Arcturus will also be resolved into this single motion of the solar system;

Mr Lalande gives us the proper motion of 12 stars in 50 years,

Arcturus	- 1' 11 ^{''}	- 1' 55 ^{''} ,
Sirius	- 37 ^{''}	- 52 ^{''} ,
β Cygni	- 3 ^{''}	+ 49 ^{''} ,
Procyon	- 33 ^{''}	- 47 ^{''} ,
ϵ Cygni	+ 20 ^{''}	+ 34 ^{''} ,
γ Arietis	- 14 ^{''}	- 29 ^{''} ,
γ Geminorum	- 8 ^{''}	- 24 ^{''} ,
Aldebaran	+ 3 ^{''}	- 18 ^{''} ,
β Geminorum	- 48 ^{''}	- 16 ^{''} ,
γ Piscium	+ 53 ^{''}	+ 7 ^{''} ,
α Aquilae	+ 32 ^{''}	- 4 ^{''} ,
α Geminorum	- 24 ^{''}	- 1 ^{''} ;

the next figure represents the 12 stars projected on the equatorial plane (3).



This is Herschel's original figure. In the projection of λ Herculis, in the right top quadrant, the projections of the great circles intersect, dividing the celestial sphere into 24 hours. Herschel continues:

Regulus being added, and Castor being double, we have 14 stars; the motion of every star, except Regulus, is assigned in declination as well as in right ascension, so that we have to account for not less than 27 motions; and by assuming a point somewhere near λ Herculis, and supposing the sun to have a proper motion toward that part of the heaven, we shall satisfy 22 of these proper motions; for on the supposed motion of the sun β Cygni, α Aquilae, ϵ Cygni, γ Piscium, and Aldebaran ought to have an apparent progression according to the hour circle XVIII, XIX, XX, and so on, or an increase in right ascension, while Arcturus, Regulus, the two stars of α Geminorum, Pollux, Procyon, Sirius, and γ Geminorum ought to have an apparent regression according to the hour circle XVI, XV, XIV, and so on, or a decrease in right ascension, and according to Mr Lalande's table, excepting β Cygni and γ Arietis, all these motions really take place; with regard to the change in declination, every star ought to go toward the south; and we find but three exceptions in β and ϵ Cygni, and γ Piscium; and all these exceptions must be resolved into the real proper motions of the stars (*id.*).

And so the solar system moves, toward Hercules, toward a point within 10° of λ Herculis, with a velocity of not less than 13 km/s.

In *On the construction of the heavens*, in 1785, Herschel writes:

an observer enclosed in a globular cluster of stars, and not far from the center, will never be able to see with the naked eye to the end of it; for it cannot be expected that his eye should reach the borders of the cluster which has perhaps not less than 50 stars in depth everywhere around him;

if he is placed in an extended cluster of stars, however, he will perceive a shining zone, or milky way, to surround the whole sphere of the heavens, owing to the combined light of the stars which are too remote to see; he will imagine this single collection of stars to be the whole contents of the heavens;

allowing him now the use of a common telescope, he begins to suspect that the milky way of the bright path which surrounds the sphere may be owing to stars; he perceives a few clusters in various parts of the heavens, and finds also nebulous patches;

he now increases his power of vision, and applying himself to a close observation, he finds that the milky way is indeed a collection of very small stars; he perceives that the objects called nebulae are nothing but clusters of stars; he forms the idea of immense strata of fixed stars, clusters of stars, and nebulae, and that all these appearances must naturally arise from the confined situation in which we are placed; and what before appeared the whole region of fixed stars has now assumed the shape of an extended nebula, not indeed one of the least, but perhaps far from being the most considerable of the numberless clusters which enter into the construction of the heavens (*id.*).

Not indeed one of the least, but perhaps far from being the most considerable, the Milky Way is an extended nebula, one of the numberless clusters which enter into the construction of the heaven. Herschel continues:

that the Milky Way is a most extensive stratum of stars of various sizes admits no longer of the least doubt, and that our sun is one of the heavenly bodies belonging to it is just as evident; I

have now viewed and gauged this shining zone in almost every direction, and find it composed of stars whose number, by the account of these gauges, increases and decreases in proportion to their apparent brightness to the naked eye (*id.*).

The number of stars in the Milky Way increases and decreases in every direction from the point of observation in proportion to their combined apparent brightness. Herschel continues:

the stars of the first magnitude, being in all probability the nearest, furnish us with a step to begin our scale; setting off, therefore, the distance of Sirius or Arcturus as unity, we suppose that the stars of the second magnitude are at twice the distance, those of the third magnitude at three times the distance, and so forth;

and to proceed to particulars, I shall begin by giving the following table of gauges (*id.*).

In the table of gauges Herschel divides the heavens into fields, and counts the stars in each field. The fields are 683. The stars in the 683 fields are 40 599. Herschel continues:

problem; the stars being supposed to be nearly equally scattered, and their number in a field of view of a known angular diameter being given, to determine the length of the visual ray;

suppose a rectangular cone cut by equidistant planes perpendicular to the axis; then, if 1 star is placed at the vertex, and another in the axis at the first intersection, 6 stars may be set around it to be equally distant from one another and from the central star; carrying on these positions in the same way, the series of the number of stars in the several sections is 1, 7, 19, 37, . . . ; by the differential method the sum of the series is

$$na + n \frac{n-1}{2} d' + n \frac{n-1}{2} \frac{n-2}{3} d'' + \dots,$$

where a is the first term, d' , d'' , d''' , . . . the differences, and since $a = 1$, $d' = 6$, $d'' = 6$, $d''' = 0$, the sum is n^3 ; let s be the number of stars, and b the diameter of the rectangular cone; since the field of view of a telescope is a cone, its volume is to that of the rectangular cone of stars formed by the above construction in the ratio of the squares of their diameters, the heights being the same; the stars in the cones are in the ratio of the volumes; and if the diameter of the base of the cone of the field of view is 1, we have $n = \sqrt[3]{b^2 s}$,

where $n - 1$ is the length of the visual ray (*id.*).

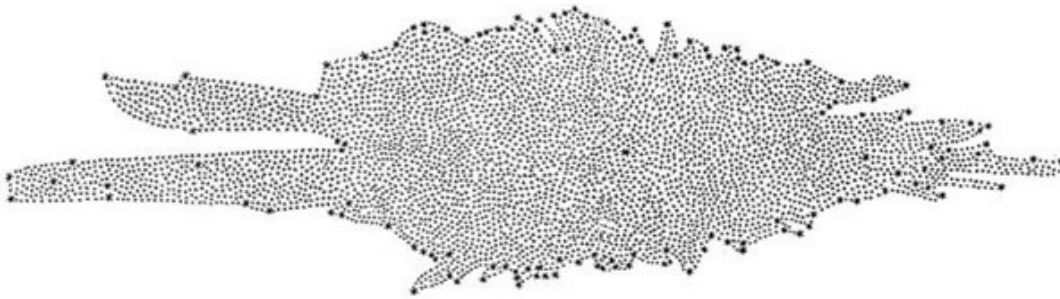
The formula has extracted from the number of stars in a field a length along which they are distributed. Herschel continues:

a delineation of our nebula may now be attempted; for this purpose in the following table the visual ray is calculated for any number of stars in a field from 0.1 to 100 000 (*id.*).

The visual ray is so calculated. Herschel

continues:

and by taking out of this table the visual rays which answer to the gauges, and applying lines proportional to them around a point, according to right ascensions and north pole distances, we may delineate by the ends of these lines a solid; I shall, however, content myself at present with only a section; it is the one at an angle of 35° to our equator;



we may see that our nebula is a very extensive branching compound of many millions of stars, which most probably owes its origin to many remarkably large stars as well as closely scattered small stars which may have drawn together the rest;

and to have some idea of the wonderful extent of this system, I must observe that the section is drawn on a scale where the distance of Sirius is not more than $\frac{1}{80}$ of an inch; all stars which we are able to distinguish with the naked eye on the finest nights may be comprehended within a sphere around the large star in the middle, representing our situation in the nebula, of less than $\frac{1}{8}$ of an inch radius (*id.*).

On the scale of the copy of the map on this page the distance of Sirius is not more than $\frac{1}{110}$ of an inch, the radius of the sphere comprehending all stars which we are able to distinguish with the naked eye on the finest nights is less than $\frac{1}{11}$ of an inch.

In 1786 Herschel published the *Catalogue of one thousand new nebulae and clusters of stars*. By 1802 he had catalogued 2 500 nebulae.

Lexell proposed that the seventh planet of the

solar system has been discovered.

Werner wrote the history of the earth, continuing the collection of observation and experiment, in *On the external signs of fossils*, in 1774, the *Short classification and description of the different kinds of mountains*, in 1786, *An attempt at an explanation of the origin of volcanoes by the kindling of great strata of coal, as a contribution to the natural history of basalt*, in 1789, and the *New theory of the origin of veins, with an application to the constitution of mountains*, in 1791.

Laplace greatly advanced mechanics, solved all outstanding problems concerning the solar system, solved the problem of its origin conclusively, and solved the problem of the origin of comets.

He published the *Theory of the attractions of spheroids and of the shape of the planets* in 1785. The theory rests on Laplace's equation.

He published the *Memoir on the secular inequalities of the planets and satellites* in 1787, the *Theory of Jupiter and of Saturn* in 1788, *On the secular equation of the moon* in 1788, the *Memoir on the theory of the ring of Saturn* in 1789, the *Memoir on the secular variations of the orbits of the planets* in 1789, and *On the theory of the satellites of Jupiter* in 1790. He showed that the current acceleration of Jupiter and deceleration of Saturn are an effect of their mutual gravi-

tation with a period of 929 years, that the acceleration of the moon is an effect of the eccentricity of the earth's orbit with a period of millions of years, that, in general, all perturbations in the orbits of the bodies in the solar system are periodic and self-correcting, and the system is therefore stable. And in 1799-825, in the *Treatise on celestial mechanics*, he systematized celestial mechanics analytically.

The theory of the origin of the solar system he formulated in the *Exposition of the system of the world*, in 1796. He writes:

all planets revolve around the sun, from west to east, and nearly in the same plane; the satellites revolve around their planets, in the same direction, and nearly in the same plane; and the sun, the planets, and these satellites in which rotation has been observed all rotate in the same direction and nearly in the same plane; a phenomenon so extraordinary cannot be the effect of chance, it points to a general cause;

an equally remarkable phenomenon is that the orbits of the planets and satellites are of small eccentricity, but those of the comets are very elongated; the orbits of the system present no nuance between great and small eccentricity; we are again forced to recognize the effect of a regular cause;

what is this primitive cause?

whatever its nature, this primitive cause must have embraced all the planets, and considering the prodigious distances separating them, it could have been nothing but a fluid of immense extent; in order to give them nearly circular motion in the same direction around the sun, the fluid must have surrounded this star like an atmosphere; we are thus led to think that, due to its excessive heat, the atmosphere of the sun extended primordially beyond the orbits of all the planets and retreated successively to its present limits;

in the primitive state in which we suppose it, the sun resembles the nebulae composed of a nucleus and surrounding nebulosity;

Herschel has observed the earlier states of nebulae, not in one of them, but in their ensemble, as in a vast forest the growth of trees can be observed in the individuals of different age;

if we conceive, by analogy, that all stars are formed in this way, we can imagine their earlier

state of nebulosity; and going back as far as possible, we arrive at nebulosity so diffuse that its existence can barely be conceived;

from the progress of condensation of nebulous matter we arrive at the same conclusion, that the sun was once surrounded by a vast atmosphere;

if bodies penetrated deeply into the solar atmosphere, its resistance would have made them fall on the sun; we may therefore conjecture that the planets were formed at the successive limits of the atmosphere by the condensation of zones of vapors which the atmosphere, as it was cooling, abandoned in the plane of its equator;

the sun's atmosphere cannot extend indefinitely; its limit is the point where the centrifugal force due to its rotation balances the gravity; but as cooling contracts the atmosphere and condenses the particles on the surface, by the law of areas rotation increases; thus the centrifugal force increases; the point at which gravity is equal to it is closer to the center; supposing therefore that the atmosphere extends at any one epoch to that limit, it must, as it cools, abandon particles at the limit and at the successive limits produced by the increasing rotation; the abandoned particles continue to revolve around the star because their centrifugal force is balanced by their gravity;

by their mutual attraction the particles of the successively abandoned zones of vapors must form distinct concentric rings revolving around the sun;

each ring of vapors must nearly always break into several masses; because their interior particles have smaller velocity, these masses must acquire spheroidal form with rotation in the direction of their revolution; they have therefore formed planets in the state of vapor;

in the center of each a nucleus begins to grow by the condensation of the atmosphere around it; in this state the planet resembles perfectly the sun in its nebulous state; the cooling must therefore produce at the different limits of the atmosphere phenomena similar to those described, rings and satellites revolving around the center in the direction of its rotation, and rotating in the same direction;

the small eccentricity of the orbits of the planets and satellites, their small inclination to the plane of the solar equator, and the identity in

the direction of rotation and revolution of all these bodies thus follow from our hypothesis with great probability.

The cause of the earth's revolution around the sun is gravity. The cause of its shape is also gravity. The cause of its rotation on its axis is also gravity. The cause of its origin is also gravity. Which is worthy of note.

Laplace notes: such has been the lot of Newton's brilliant discovery that every difficulty which it has encountered has provided it with an opportunity for a new triumph. Lambert notes: the law of gravity some genie revealed to Newton in a moment of rapture (*Letters*, 5).

It has remained the theory of the origin of the solar system.

Laplace concludes the formulation of the theory by addressing the problem of the origin of comets. He writes:

on our hypothesis the comets are strangers to the planetary system; we can conceive of them as formed of nebular matter outside this system; when they arrive in the part of space where the attraction of the sun predominates, the sun forces them to describe elliptic or hyperbolic orbits; the great eccentricity of their orbits follows from our hypothesis; if the orbits are elliptic, they are very elongated because the major axes are at least equal to the radius of the sphere of activity of the sun (V, 6).

And Laplace thus solved this problem, too. Neptune is at 30 times the distance of the earth from the sun. The part of space where the attraction of the sun predominates extends to about 100 000 times that distance, a little more than one third of the distance to the nearest other star. This region beyond the planetary system contains remains of the nebula from which the solar system developed. And from this region, formed of nebular matter, comets continue to originate. When they arrive in the part of space where the attraction of the sun predominates, the sun forces them to describe elliptic or hyperbolic orbits, and if the orbits are elliptic, they are very elongated because the major axes are equal to the radius of the sphere of activity of the sun.

William Smith drew the first map of strata, *A delineation of the strata of England and Wales, with part of Scotland*, in 1815.

Gauss greatly advanced mechanics. In 1809 he wrote the *Theory of the motion of the heavenly bodies moving in conic sections around the sun*. The central problem is the calculation of the orbit of a body from as few as 3 observations. Piazzi discovered Ceres in 1801. Then the asteroid disappeared behind the sun, and attempts to find it again appeared futile. As Gauss describes the situation: for the resolution of this crisis of urgent necessity, and the rediscovery of the planetary atom among the innumerable stars in the heavens after the lapse of nearly one year, all hope rested solely on a sufficiently accurate knowledge of its orbit grounded in only a very few observations (pr.). And as few as 3. Gauss writes: for the complete determination of the motion of a heavenly body in its orbit 7 elements are required; if, however, the mass is known or can be neglected, the number can be reduced to 6, and hence, since every observation provides two elements, longitude and latitude, or right ascension and declination, it is certainly simplest to adopt 3 geocentric observations, which will suffice for the determination of the 6 unknowns (115). In the *Theory* Gauss develops also the method of least squares, for minimizing the errors of observation (174-89), resting on: the principle that the sum of the squares of the differences between the observed and the computed quantities must be a minimum (186). In 1818 Gauss wrote the *Determination of the attraction when, in place of a planet at a given point, its mass is distributed through its orbit in proportion to the time it takes to describe any one part*.

Bessel brought the measurement of the earth to its end, and measured the distance of 61 Cygni.

In the *Determination of the axes of the elliptic spheroid of revolution*, in 1837, he determined the axes of the elliptic spheroid of revolution.

He determined the distance of a star in the *Determination of the distance of the 61st star of Swan*, in 1838.

In the letter announcing the discovery, he reports: I undertook to make this investigation on 61 Cygni because of its great proper motion (to J. Herschel, 23 Oct). In particular, he measures the angular distance of the star from two nearby stars of the ninth and tenth magnitude, respectively, over the fifteen months from Aug 1837 to Oct

1838 (*id.*), corrects the measurements for refraction and aberration (*id.*), and concludes: the most probable value of the annual parallax of 61 Cygni is $0''.3136$ (*id.*), and: since the mean error is only $\pm 0''.0202$, and that is, not $\frac{1}{15}$ of the computed

value, and all comparisons show that the influence of the parallax on the observations follows the theory as nearly as can be expected considering its minuteness, we can no longer doubt that this parallax is sensible; and assuming it to be $0''.3136$, we find the distance of 61 Cygni from the sun to be 657 000 times the mean distance of the earth from the sun; to traverse this distance light takes 10.3 years (*id.*).

The parallax and distance of 61 Cygni are,
Bessel $0''.31$ 660 000 distances of the sun,
correction $0''.29$ 720 000.
To traverse this distance light takes 11.4 years.

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To tabulate these results, too.

Römer	measured the velocity of light, at 2.14×10^8 m/s,
Flamsteed	catalogued 2 935 stars,
Halley	measured a difference of $20'$ over 1 800 years,
Bradley	measured a difference of $39''$ over 6 months, discovered the aberration of light, the velocity of light is 2.80×10^8 m/s,
Herschel	gauged 40 599 stars, catalogued 2 500 nebulae,
Bessel	measured a difference of $0''.31$ over 1 year.

The advance in observation was great quantitatively. The number of stars catalogued by Flamsteed, in the generation of Newton, was not substantially greater than the number catalogued by his predecessors. Three generations later the number of stars gauged by Herschel was greater by more than an order of magnitude. The advance was greater still qualitatively. In the generation after Newton Halley measured minutes. One generation later Bradley measured seconds. Three generations later Bessel measured fractions of seconds. Nebulae were also distinguished, and

their collection began.

The unit of distance had been the radius of the earth. As measurement reached further, the distance of the sun became the unit. As measurement reached still further, with Bessel already the velocity of light was becoming the unit.

The advance in observation was inseparable from the advance in the theory. All advances in observation show this, and the greatest advances in observation, Halley's, Bradley's, Herschel's, Bessel's, show it dramatically. Bessel argues as explicitly as this: the observations follow the theory.

The theory was the one revealed to Newton by some genie in a moment of rapture, and the rapture continued.

Euler	advanced greatly the theory,
d'Alembert	advanced the theory,
Lagrange	advanced greatly the theory,
Laplace	advanced greatly the theory,
Gauss	advanced greatly the theory.

With Euler, Laplace, and Gauss the theory also aimed specifically at the improvement of observation. With Gauss, indeed: at the resolution of this crisis of urgent necessity, the rediscovery of the planetary atom among the innumerable stars in the heavens.

The advance in the solution of the problem of the earth was correspondingly great as well.

The solutions of the two parts of the problem solved conclusively in the previous one and a half centuries were terminally refined.

Maupertuis	confirmed its shape from observation,
Bessel	the semiaxes of the elliptic spheroid are 6 377 and 6 356 km.

It was shown also that the elliptic spheroid, as it moves on its axis, and moves around the sun, moves toward λ Herculis.

The third part of the problem, its position, was still not solved conclusively. Its position in space was determined substantially better, however. The first steps were taken toward the determination of its position in time. And the decisive step was taken toward the solution of the problem of its position in space and in time jointly, as these cannot not depend on one another substantially and essentially, with the formulation of the theory

of its origin and development.

The determination of its position with respect to the sun entered its final stages.

Lalande the sun is 23 984 earth radii away.

In 1862 Foucault will measure with a rotating mirror the velocity of light at 2.98×10^8 , hence the parallax of the sun at $8''.86$, and hence its mean distance at 23 280 earth radii.

And this is the entire development of the problem of the distance of the sun in its conclusive stages.

Huygens 25 086 earth radii,

Cassini 21 600,

Lalande 23 984,

Foucault 23 280.

Foucault's number has been refined to 23 455.

Thus, also, as the smaller unit of distance, the radius of the earth, had been calibrated conclusively in the previous century, this larger unit, the distance of the sun, was calibrated conclusively at this time. And at this time not only was the yet larger unit, the velocity of light, discovered, its calibration entered in its final stages, too.

This is the entire development of this problem,

Rømer 2.14×10^8 m/s,

Bradley 2.80×10^8 ,

Foucault 2.98×10^8 ,

This number of Foucault's has been refined to 3.00×10^8 .

The inventory of the solar system continued, and hence the better determination of its extent.

Halley the comet observed in 1531 will return in 1758,

Herschel observed Uranus,

Lexell proposed that it is the seventh planet,

Laplace it is at 19.2 distances of the sun,

the period of perturbation in the orbits of Jupiter and Saturn

is 929 years,

the period of perturbation in the orbit of the moon is millions

of years,

all perturbations are periodic and self-correcting,

the comets are formed outside the planetary system.

The advance beyond the solar system was greater yet.

Halley Aldebaran, Sirius, and

Arcturus move,

Bradley γ Draconis is more than

400 000 distances of

the sun away,

Kant the Milky Way is the system of stellar systems of which

the solar is one,

Herschel the solar system moves toward

λ Herculis,

mapped the Milky Way,

began the investigation of

nebulae,

Bessel 61 Cygni is 657 000 distances

of the sun away.

Given the distance of 61 Cygni, and the distance of Uranus, if the distance of Sirius is of the same range as that of 61 Cygni, and the extent of the planetary system is indicated by the distance of Uranus, then on the map of the Milky Way on which the distance of Sirius is not more than $\frac{1}{80}$

of an inch, the planetary system extends not more than $\frac{1}{2\,400\,000}$ of an inch. To borrow Gauss's

description, never mind the asteroids, the planetary system is an atom.

The earth had acquired a history in the previous one and a half centuries, and the philosophers continued to collect observations and write the history.

Leibniz began the empirical

investigation of the earth,

Buffon the earth is at least 74 832

years old,

described the seven epochs of

its development,

Hutton wrote the history of the earth,

Werner wrote the history of the earth,

William Smith drew the first map of strata.

Leibniz, Buffon, Hutton, Werner, and William Smith are only the main steps in the development of geology in these one and a half centuries. The investigation of the earth developed in these one and a half centuries, but developed slowly, substantially more so than the contemporary investigation of the heaven.

In the *Principles of geology*, in 1833, Lyell writes of his predecessors: we hear of sudden and violent revolutions (III, 1); we are told of general catastrophes and a succession of deluges (*id.*).

Predilection for sudden and violent revolutions, general catastrophes, and a succession of deluges is one reason for the slow development of the investigation of the earth.

Another reason, the counterpart of the first, is the dearth of numbers. Where the age of the earth is concerned, Buffon's remained the only number for three generations. It was in 1862 that Kelvin improved on it: not less than 20 million years, nor more than 400 million.

In the meantime, the number extracted from the Priestly code was not abandoned in favor of Buffon's number and attempts to improve on it. It was abandoned in favor of: the world has neither a beginning nor an end; we find no vestige of a beginning, no prospect of an end.

Finally, the greatest advance was the development of the first theory of origins, the theory of the origin and development of the solar system, including the earth.

Buffon originated from the collision of a comet into the sun,
Kant originated from the cooling and contraction of a nebula,
Laplace formulated the theory of its origin from the cooling and contraction of a nebula conclusively.

The steps of advance in the acquisition of knowledge are the 13, Rømer, Leibniz, Halley, Bradley, Euler, Buffon, Kant, Lagrange, Herschel, Laplace, William Smith, Gauss, Bessel. In particular,

Rømer measured the velocity of light, at 2.14×10^8 m/s,
Leibniz began the empirical investigation of the earth,
Halley the comet observed in 1531 will return in 1758, Aldebaran, Sirius, and Arcturus move,
Bradley discovered the aberration of light, the velocity of light is 2.80×10^8 m/s,

Euler
Buffon

Kant

Lagrange
Herschel

Laplace

Gauss
William Smith
Bessel

The one century from Bessel to Einstein, and the century-odd since Einstein.

To measure the parallax of 61 Cygni, Bessel employed a 16-cm micrometer with a divided lens, designed for the observation of small angles, designed and built by Fraunhofer.

In 1814 Fraunhofer built the first spectrometer. Investigating the dispersive power of lenses, he

γ Draconis is more than 400 000 distances of the sun away,
advanced greatly mechanics, the earth is at least 74 832 years old,
described the seven epochs of its development, the solar system originated from the collision of a comet into the sun,
the Milky Way is the system of stellar systems of which the solar is one,
the solar system originated from the cooling and contraction of a nebula,
advanced greatly mechanics, discovered Uranus, the solar system moves toward λ Herculis,
mapped the Milky Way, catalogues 2 500 nebulae, advanced greatly mechanics, Uranus is at 19.2 distances of the sun,
all perturbations in the orbits of the bodies in the solar system are periodic and self-correcting,
the comets are formed outside the planetary system, formulated the theory of the origin and development of the solar system,
advanced greatly mechanics, drew the first map of strata, 61 Cygni is 657 000 distances of the sun away.

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noticed dark lines in the spectrum of sodium flame. He found similar lines in the spectra of other elements, measured the wavelengths of the lines, and established that their relative positions are constant whether the sources of light are heated gases, or metals, or the sun, or the moon, or Venus, or Sirius. He published the discovery in 1817, in the *Determination of the refractive and color-dispersing power of different types of glass, in relation to the improvement of achromatic telescopes*.

In 1860, in the *Chemical analysis by the observation of spectra*, Kirchhoff and Bunsen showed that the Fraunhofer lines are the absorption lines in the spectrum, and in particular, in the spectra of stars produced by the absorption of the elements in their atmospheres.

The Fraunhofer lines became the foundation of the observation of the heaven.

In 1905-7, in *On the radiation of stars*, Hertzsprung solved the problem of the relation of intrinsic brightness and spectral type of stars, classified stars, and established the fact of their evolution.

In 1905, in *On the electrodynamics of moving bodies*, Einstein formulated the special theory of relativity.

In 1908, in the *Space and time*, Minkowski generalized the theory.

Minkowski begins: the views on space and time, which I would like to develop, have grown on the soil of experimental physics; in this is their strength; and their tendency is radical; from now on, space for itself and time for itself should shrink into shadows, and only a kind of union of the two should preserve independence (pr.).

In 1916, in *The foundation of the general theory of relativity*, Einstein formulated the general theory of relativity.

In 1922, in *On the curvature of space*, Friedmann solved the field equations of general relativity for: a world whose curvature is constant with respect to space and dependent on time.

In 1925, in the *Cepheids in spiral nebulae*, Hubble measured the distance of Andromeda.

In 1926, in the *Extragalactic nebulae*, he classified galaxies.

In 1929, in *A relation between distance and radial velocity among extragalactic nebulae*, he

formulated the law relating the recession velocities of galaxies to their distances.

In the next two generations the big bang theory was developed. The big bang theory is an inference from the labors of Einstein, Friedmann, and Hubble.

In the generation after Einstein, Friedmann, and Hubble, the most important are the labors of Lemaître and Gamow. Lemaître united the solution of the field equations with the evidence for recession of the galaxies into a hypothesis of a big bang. Gamow elaborated the hypothesis by uniting the solution of the equations with the evidence for development of radiation into particles into stars into galaxies. In the next generation, as the observational evidence steadily grew, the most important is the development of quantum theory into the standard model, in which the most important are the labors of Gell-Mann. The standard model describes the strong and the electroweak forces, the last unifying the weak and the electromagnetic, and the particles, quarks and leptons.

The big bang was followed by the decoupling of the forces, gravitational, strong, weak, and electromagnetic, then the formation of quarks and leptons, then of elements, then of stars, then of galaxies.

And the problem of the position of the earth has been solved conclusively as well.

The Milky Way is 100 000 light years across. There are 2×10^{11} stars in it. The sun is 26 000 light years from its center.

There are 10^{11} galaxies in the world.

The big bang occurred 1.4×10^{10} years ago.

This entails the limit of observation at 4.6×10^{10} light years away.

The earth is 1.4×10^{10} years after the big bang, 4.6×10^9 years after the formation of the solar system, revolving around the sun, revolving with the sun around the center of the Milky Way, and receding with the Milky Way from all other galaxies in the world, which is 9.3×10^{10} light years across to the limit of observation.

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Little is to be hoped for unless all particulars are collected, and marshaled in their series and order.

And the following conclusions can be drawn from the attempt to collect all particulars, and marshal them in their series and order, with respect to the problem of the earth.

Knowledge has been acquired. A large amount of knowledge. And the development of the problem shows this clearly and distinctly.

Propositions have been won, very many propositions, the propositions have been brought in connection with one another in chains of inferences, very many chains of inferences, and the chains have become ever more free of gaps.

From the thought and the sense to the reference and the true.

The earth, which god made when there was no shrub and no grass on the earth, because god had not yet sent rain, and water welled up from below and covered the earth, big-bosomed Earth, who bore Heaven, Mountains, Sea, Ocean, eleven other titans, three cyclopes, and three one-hundred-handed creatures, is a sphere, not a large one either, of a circumference of 400 000 stades, and indeed not a perfect sphere, but an elliptic spheroid of eccentricity 0.004, and rather 0.003, and it is also not at rest, it rotates on its axis and revolves around the sun, and revolves also around the center of the Milky Way, and recedes also from all other galaxies in the world.

In the heaven 1 star is observed. At the time when god made heaven as well, and big-bosomed Earth bore Heaven, three dozen constellations and stars are observed. Then 47 constellations are described, 1 022 stars are catalogued, 40 599 stars are gauged, and until it is inferred that the stars in the Milky Way are 2×10^{11} , and presuming the Milky Way an average galaxy, the stars in the world are 2×10^{22} .

Just as clearly and distinctly the development of the problem shows how it has been done. The council of our predecessors has done it.

In order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

In order to collect the development of the problem, the predecessors of the philosophers and philosophers have been collected, all important among them, and all steps of advance in the acquisition of knowledge among them.

In particular, from the thirty-seven centuries from the temple register from Lahun through Bessel 108 important predecessors of the philosophers and philosophers have been collected, and 55 steps of advance in the acquisition of knowledge among them. Counting centuries from the year of birth assigned to the register, their distribution is,

1911-1812	1	1,
1811-1712	—	—,
1711-1612	1	1,
1611-1512	1	—,
1511-1412	—	—,
1411-1312	—	—,
1311-1212	—	—,
1211-1112	—	—,
1111-1012	—	—,
1011- 912	—	—,
911- 812	—	—,
811- 712	5	3,
711- 612	4	2,
611- 512	6	4,
511- 412	9	6,
411- 312	10	8,
311- 212	4	2,
211- 112	5	1,
111- 12	1	—,
11 ^{BC} - 89 ^{AD}	4	2,
90- 189	—	—,
190- 289	1	—,
290- 389	2	—,
390- 489	—	—,
490- 589	—	—,
590- 689	—	—,
690- 789	2	—,
790- 889	2	—,
890- 989	2	—,
990-1089	1	—,
1090-1189	2	—,
1190-1289	2	—,
1290-1389	3	—,
1390-1489	3	2,
1490-1589	6	4,
1590-1689	15	9,
1690-1789	16	10.

The beginning is the most difficult, it is also the most important.

In particular, the beginning is small, halting, and slow.

The very beginning is 1 observation of 1 star, Sirius, the brightest fixed star, the sixth brightest heavenly body, in the valley of the Nile.

In the following century there is nothing.

In the following century, the beginning in the other valley, the valley of the two rivers, is the observation of Venus, the third brightest heavenly body, over twenty-one years, through twelve of its synodic periods.

In the following century, there is 1 more observation of Sirius, in the valley of the Nile, and thus, with this predecessor of the philosophers already, the third in the series, the second in the valley of the Nile, the council of our predecessors is in evidence. The second in this valley, three-odd centuries after the first, he does what the first has done. Clearly, adopting the sound.

In the following seven centuries there is nothing, in the valley of the Nile, in the valley of the two rivers, or in any other valley.

Then the beginning is over. Five centuries of acceleration follow. In the first two of the five observation is resumed, becomes sustained, reaches more heavenly bodies, becomes accurate, continues, and at the end of the second of these centuries the problem of the earth is discovered by the first philosopher. In the following two of the five the attempts of the philosophers to improve on the solution of the problem accumulate, and are increasingly successful. In the last of the five a theory is formulated, the theory of the motions of the heavenly bodies in concentric spheres, and the attempts to build the theory mathematically begin.

The council is in evidence throughout.

Of why observation resumes and becomes sustained in the valley of the two rivers, the spectacular success of the Venus tablet of Ammisaduqa has to be, at least in part, the answer. Nine centuries after the Venus tablet of Ammisaduqa, its successors resume doing what it has done. Clearly, again, adopting the sound. Then they continue doing what their predecessors have done.

Among the philosophers the council is explicitly in session with the second philosopher already, as the second philosopher solves better the problem solved by the first philosopher, and the council is in session ever more explicitly, as the

philosophers adopt the sound and avoid the erroneous in what their predecessors have said. Rather more the latter than the former, too, as the chains of inferences are still far from free of gaps. Aristotle then convokes the council formally.

Once the theory is formulated, and the labors to build it mathematically begin, the council is in session as explicitly as propositions are one another's premises and consequences explicitly now, and the philosophers adopt the sound and avoid the erroneous increasingly by demonstration. Rather more the former than the latter, too, as the chains of inferences are now mathematical.

Nor is there any discontinuity in the work of the council, because some of its sessions are at one time, some at another, some in one place, some in another, or because some are not mathematical, some mathematical.

The gap between the Venus tablet of Ammisaduqa and the observation of lunar eclipses beginning in 747 need not be filled with the rise of divination. There is the analogy instead with Archimedes and Galileo, or Apollonius and Kepler. Nine centuries separate the observation of lunar eclipses beginning in 747 from the Venus tablet of Ammisaduqa. Eighteen and a half centuries separate Galileo from: the superhuman Archimedes, whom I cannot mention without feeling awe. Nor were these eighteen and a half centuries less of a gap in the work of the council than those nine. Eighteen centuries separate Kepler from Apollonius, the oracle who dictated ellipses to his pen. The same eighteen centuries.

The gap between Babylon and Thales need not be filled with travels. The observation of lunar eclipses, the observation of series of lunar eclipses, the accurate observation of lunar eclipses, the observation of three dozen constellations and stars, the observation of Saturn, the observation of Venus – all attested in the one and a half centuries before the first philosopher – each singly implies, and all jointly imply strongly, the problem of the position of the earth.

And Plato, no less – next to all that there is in the dialogues about immortality and virtue – Plato argued that the earth revolves on its axis, and determined the place of Mars, Jupiter, and Saturn in the order of the planets, and: Plato posed the problem of finding hypotheses which

will show that the apparently irregular motions of the planets are regular; and Eudoxus formulated the hypotheses of regular circular motion to which the apparent irregular motions of the planets can be reduced.

Thereafter, substantially mathematical now, the work of the council continues steadily through all gaps in time, all gaps in place, and all gaps due to the loss of all mathematical competence. When the acquisition of knowledge decelerates, when it stops altogether, and when it greatly accelerates once again.

Thus, the following four centuries are of deceleration. The sessions of the council are less frequent. The attempts to build the theory mathematically are nevertheless increasingly successful, and culminate in Ptolemy.

In the following thirteen centuries there is no advance. After the first three of the thirteen the sessions cease altogether for the following three. Resumed in the following seven, the sessions are infrequent, and not too vigorous. When there are sessions, they examine the Ptolemaic theory.

The next four centuries are once again of great acceleration. In the first of the four the examination of the Ptolemaic theory is concluded with the formulation of the Copernican hypothesis. In the following two the attempts to reformulate the theory of the world are increasingly successful, and culminate in Newton. In the following century the Newtonian theory is generalized.

In general, over all the thirty-seven centuries, observation begins, then observations are accumulated, then the problem is formulated, then a theory becomes the ground of the solution of the problem. The theory is developed in two major steps. Through Ptolemy, followed by the examination of the theory, then through Newton, followed by the generalization of the theory.

The council is one and only one. As tightly bound in its unity and uniformity as this, the observation of lunar eclipses in 721-0 is part of the *Almagest*, the *Almagest* is part of Halley's discovery of the proper motion of stars.

Thus, the reference of 'the earth' in the proposition, the earth floats on water, like wood, in the proposition, the earth is at rest, in the center, in the proposition, the earth revolves around the sun, and in the proposition, the earth moves toward

λ Herculis, is the same. The sense is different, but the reference is the same. The four propositions are bound inextricably in one and only one body of chains of inferences, and it is this fact, that they are bound inextricably in one and only one body of chains of inferences, that shows, and nothing else can show, that the reference of the first is the false, the reference of the second is the false, the reference of the third is the true, and the reference of the fourth is the true. From the sense to the reference. The reference is discovered. And it is thus also these propositions in relation to one another that show, and nothing else can show, that 'the earth' has a reference. The reference of 'the earth' is the earth. The third planet of the solar system.

The reference of 'sphere' in the propositions, the earth is a sphere, the earth is a sphere of radius 10 200, the earth is a sphere of radius 6 372, and the earth is not a sphere, but an elliptic spheroid of eccentricity 0.003, is the same. The sense is different, but the reference is the same. The body of chains of inferences is the same, this fact shows, and nothing else can show, that the reference of the first proposition is very nearly the true, the reference of the second proposition is the true well within an order of magnitude, the reference of the third proposition is very nearly the true, and the reference of the fourth proposition is the true. The reference is discovered. And it is thus also these propositions in relation to one another that show, and nothing else can show, that 'sphere' has a reference. The reference of 'sphere' in these propositions, in all propositions about spheres in Euclid, and all propositions about spheres in Riemann, is also the same. And it is those propositions, the earth is a sphere, of radius 10 200, of radius 6 372, and not a sphere, but an elliptic spheroid, that show, and nothing else can show, that 'sphere' has a reference in Riemann as in Bessel as in Euclid as in Parmenides. The reference is sphere. A surface equidistant from a point.

And the reference of 'motion' is the same in the proposition, it does not move, in the proposition, it moves, in the Aristotelean doctrine of motion, and in the Newtonian theory of motion. The reference of the first proposition is the false, the reference of the second proposition is the true,

the reference of the Aristotelean doctrine is mostly the false, the reference of the Newtonian theory is the true to a first approximation. And the reference of 'motion' is motion. Change in position with respect to time.

The work of the council abounds in error.

And the work of the council abounds very much more in finality. From the first, too. Sirius was in 1871 ± 5 where the temple register from Lahun says it was. Venus was in either 1646-25 or 1638-17, and very probably in 1646-25 rather than 1638-17, where the Venus tablet of Ammisaduqa says it was.

Finally, the problem of knowledge divides into two parts, the acquisition of knowledge and knowledge.

The parts of the problem of the acquisition of knowledge are the beginning of the acquisition, the collection of the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition.

The problem of the earth is from observation. Observation begins with the temple register from Lahun and the Venus tablet of Ammisaduqa.

It is clear why observation begins with the observation of the heavenly bodies. If any objects are given clearly and distinctly to the senses these are.

Observation is from mathematics. In both valleys observation succeeds great advance in mathematics. The relation of the development of mathematics and the beginning of observation is causal.

The relation of the development of the description of the surface of the earth by the labors of the princes and the development of the problem of the earth is not remote.

It can also be presumed that the relation of the two developments, the description of steadily more localities across a steadily larger portion of the earth and hence the beginning of the observation of the heavenly bodies, is not remote.

And, hence, on the evidence of the problem of the earth, the acquisition of knowledge has to be traced back to the beginning of mathematics, and to the beginning of the labors of the princes. In particular, to the 3 oldest calculations of area, and to Narmer, in the valley of the two rivers, and in the valley of the Nile. And pending further evi-

dence, no further. The problem of the beginning of the acquisition of knowledge is of the beginning of mathematics, and in particular, the 3 oldest calculations of area, and the beginning of the labors of the princes, and in particular, Narmer. If not *ex nihilo*, what are these two predecessors of the philosophers from, each singly, and both jointly. The problem of the beginning of the acquisition is fundamentally of the differential development in the two valleys, too.

The problem of the earth has yielded 108 important predecessors of the philosophers and philosophers, 55 steps of advance in the acquisition of knowledge. And it can confidently be presumed that these are not a small fraction of the acquisition in its entirety. One of the parts of the problem is of its position in the world, the problem is thus explicitly of the greatest generality, and the collection of the particulars which are part of its development cannot not capture a substantial fraction of the acquisition. The 55 steps are thus also some of them smaller, some of them larger, some of them very large. Eudoxus, Aristotle, Hipparchus, Ptolemy, Copernicus, Galileo, Kepler, Descartes, Huygens, Newton, Leibniz, Halley, Buffon, Herschel, Laplace are all 1 step each.

Philosophy begins with Thales, and the problem of the earth is one of the two problems with which philosophy begins. The acquisition of knowledge begins earlier. Observation predates the problem. And the discovery of the problem of the earth is a vast generalization over the earlier observation. And if how vast is not sufficiently clear and distinct with the single proposition of the first philosopher, the earth floats on water, like wood, it is crystal clear and distinct with the second philosopher. Anaximander said that the earth is at rest because of equilibrium, and shaped like a drum, discussed the sizes and distances of the planets, and drew the map of the earth.

The generalization is over observation. Over the entire observation accumulated earlier. Philosophy is an empirical generalization.

A vast generalization, too, on the evidence of this problem, as soon as it begins, philosophy encompasses the acquisition of knowledge in its entirety. The result that the earth is a sphere has not been obtained outside the one and only one

development of philosophy from Thales through Bessel. Nor has any of the results built on this result. Nor could it have been.

As to its kind, on the evidence of this problem, the acquisition of knowledge is theory, exclusively. Search for truth uncontaminated by any concern for action. Nor is the fact that use has been found for results obtained in the course of the development of the problem evidence to the contrary. And not only because, in general, men are organisms, organisms are opportunistic, and will use, or try to use, anything. Because, in particular, the main use has been of observation for divination. *When Anu and Enlil* is a collection of omens, Ptolemy wrote astrology, Kepler did. How close the relation of observation and divination is, however, can be judged by the fact that in the same twenty-three centuries from *When Anu and Enlil* to Kepler for divination was used the word of god, not hampered by any observation, and for most of these centuries itself resolutely hampering observation. As to other uses, never mind the use the Copernican hypothesis was found to have. This is the use Lactantius, one and a half centuries after Ptolemy, found for the sphericity of the earth: is there anyone foolish enough to believe that the antipodes walk upside down? and how do they explain that all things on the other side do not fall in the sky (*Institutes*, III, 24)?

On the evidence of this problem, the acquisition of knowledge is theory. Theory is physics, and mathematics is a part of physics.

Also, if the standard is the performance of the indispensable duty, the examination of all original materials, Aristotle, Ptolemy, Copernicus, Galileo, Kepler, Newton, Leibniz, Halley, Buffon, Herschel, Laplace are historians *par excellence*. History is a part of physics, too.

The parts of the problem of knowledge are the relation of knowledge and truth, and the relation of knowledge and the world.

The development of the problem of the earth shows the acquisition of knowledge.

It shows the accumulation of very many propositions whose reference is the false, very many more propositions whose reference is the true, and the sharp and conclusive separation of the true from the false.

It shows that the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world have been described, large parts, small, larger, smaller, and all of them, and all in relation to one another.

And the relation of the accumulation of propositions whose reference is the true and the acquisition of knowledge cannot be exclusion. And the relation of the accumulation of propositions whose reference is the true and the description of the system of the world cannot be exclusion. And the relation of the description of the system of the world and the acquisition of knowledge cannot be exclusion.

And the relation of the reference of all true propositions, or what is the same, truth, and the system of the world, or what is the same, the world, and knowledge cannot be exclusion.

And all these propositions cannot not entail one another.

Thus, also, scepticism and idealism being what they are, being normative since the restoration of spirit to the substantiality and purity of its being, too, all being world-views and discourses, it may seem that it may be doubted that the development of the problem of the earth shows that knowledge has been acquired.

That the development of the problem shows the accumulation of propositions whose reference is the true may not be doubted. It may no doubt be believed doubted, but it may not be doubted. That the system of the world has been described may not be doubted either, even as this, too, may no doubt be believed doubted. And if the relation of truth, the world, and knowledge cannot be exclusion, that the development of the problem of the earth shows that knowledge has been acquired may not be doubted either, even as this, too, may no doubt be believed doubted.

**The soul,
and both, what it is,
and what you should do about it**

cclx

Aristotle begins in *On the soul*: knowledge of the soul contributes greatly to the advance of

truth, and most of all physics, because the soul is the principle of life (I, 1). He notes: the problem, what is the soul? is one of the most difficult (*id.*). He presents the difficulties which the problem poses. He reaffirms: the investigation of the soul is a part of physics (*id.*).

And he calls the council of our predecessors into session.

The problem is, what is the soul?

The solution of the problem is, the soul is the principle of life.

The problem is a part of physics. The solution of the problem contributes greatly to the advance of truth, and most of all physics.

The problem is also one of the most difficult.

The problem of the soul is a physical problem, like that of the earth.

Unlike that of the earth, it is also one of the most difficult.

The problem of the earth Aristotle can formulate as explicitly as this, its position, at rest or in motion, and its shape. What is it? is very much part of the problem, and the solution of the problem answers this question. It is the sphere at rest in the center of the world. And rather, it is the third planet of the solar system. Yet Aristotle can formulate the problem more explicitly, and as explicitly as that, because the formulation and solution of the problem has already been grounded firmly in observation, and calculation. The formulation and solution of the problem of the soul has not been grounded firmly in observation, or calculation. The problem of the soul is thus just that, what is it? After Aristotle the formulation and solution of the problem of the earth will be grounded in a mathematical theory. The formulation and solution of the problem of the soul will not be grounded in a theory for a long time, the preliminary difficulty will persist, which, if any, are the relevant observations, and calculations, and although Aristotle's formulation and solution of the problem is very explicit, for a long time the problem will remain just as it had been, what is it?

There have been other physical problems which have long resisted reduction to more explicit formulation. What is motion? What is time? What is light? There have been such mathematical problems, too. What is number? What is

equality? More recently, for the first one and a half centuries of its development the investigation of electricity was dominated by, what is it? Still more recently, those particles, or waves, or particle-waves, what are they?

The soul, however, what is it? has had to labor under, and against, the peculiar difficulty that there is also the ethical problem of the soul.

That there is the ethical problem of the soul, and, both, that it is a problem and that it is ethical, Plato suffices to show. Plato investigates the soul relentlessly, most explicitly searching for truth. And he teaches you what you should do about it just as relentlessly, just as explicitly searching for action.

The claim of ethics to the problem of the soul is older than Aristotle. Aristotle separates the physical problem sharply from all contamination by ethics. After him again, however, the physical problem, what is it? has had to labor hard – when it has – to extricate itself from the ethical, what should you do about it?

But there is a virtue in this adversity.

The task is to collect all particulars and marshal them in their series and order with respect to the problem of the soul, in particular. In general, to show that the acquisition of knowledge can be collected in its entirety.

And if the collection of the development of the physical problem of the soul has to separate it from the development of the ethical problem of the soul one way or another, this can be done explicitly, and the development of the ethical problem can be described explicitly as well.

The collection and marshaling of the particulars in their series and order with respect to the physical problem can thus show that the ground on which the particulars are collected and marshaled is firm enough to separate the acquisition of knowledge sharply not just from the not acquisition, but from its deliberate subjugation to the not acquisition.

Also, the ethical problem cannot be part of the acquisition, and there can be no steps of advance in the acquisition to collect from its development. But the description of its development can show that the ground is firm enough to capture practice as well as theory. It can show that the ethical problem of the soul is a problem, that the prob-

lem has been discovered, when and why it has been, that there are steps in its development, and which are the steps and why.

And if the ethical problem of the soul has not been part of the acquisition of knowledge, its development has yet been part of the development of philosophy, and the description of its development should inform the problem of the relation of the acquisition of knowledge and the development of philosophy. The development of philosophy has included more than the acquisition of knowledge, and the description of the development of the ethical problem of the soul should show what and why.

The problem of the earth is formulated by Aristotle.

The problem begins with Thales.

The grounds on which the problem is formulated and solved indicate the development which leads to the problem before Thales.

After Aristotle the problem is seldom addressed explicitly as Aristotle formulates it. A part of a theory, it is formulated and solved as the theory is built, together with many other larger and smaller problems. The development of the theory and accumulation of results, however, indicate the development of the problem clearly and distinctly.

Mutatis mutandis the problem of the soul.

With the physical problem, not the ethical, guiding the collection.

The physical problem of the soul, what is it? begins with the third philosopher, Anaximenes. After Aristotle, the main advances are Descartes, and Darwin. These are the main steps in the development of the problem,

590 Anaximenes,

384 - 322^{BC} Aristotle,

1596 - 1650^{AD} Descartes,

1809 - 1882 Darwin.

The steps divide the development into five parts, all the centuries before Anaximenes, the two centuries from Anaximenes to Aristotle, the twenty centuries from Aristotle to Descartes, the two centuries from Descartes to Darwin, the two centuries since Darwin.

And the collection of the predecessors of the philosophers and philosophers, all important among them, and all steps of advance in the

acquisition of knowledge among them with respect to the problem of the soul proceeds in this order. Aristotle's review is completed, and the particulars from Anaximenes to Aristotle with respect to the physical problem are collected. Then the development of the ethical problem in that time is described. Then the question of the beginning of the physical problem is addressed. Then the question of the beginning of the ethical problem. Then the later particulars are collected, those from Aristotle to Descartes with respect to the physical problem. Then the development of the ethical problem in that time is described. Then the particulars from Descartes to Darwin with respect to the physical problem are collected. Then the development of the ethical problem in that time is described. And finally the relevant developments, with respect to the physical problem and with respect to the ethical problem, in the two centuries since Darwin are indicated.

cclxi

The two centuries from Anaximenes to Aristotle.

Aristotle reviews what our predecessors have said about the principles of all things twice. The review in book I of the *Physics* leads to the problem, with this review Aristotle formulates the problem. The review in book I of the *Metaphysics* follows the solution of the problem, with this review Aristotle examines the solution. Hence also its completeness.

The review of what our predecessors have said about the earth is like the former, it leads to the problem, with it Aristotle formulates the problem.

And the review of what our predecessors have said about the soul is like the former as well, it leads to the problem, and with it Aristotle formulates the problem. But it is also like the latter, the one with which Aristotle examines the solution, in that it is of considerable size, and it is in two parts. Aristotle first reviews what our predecessors have said more generally (2), then examines in detail the more important among them (3-5). Clearly, the difficulty of the problem demands the exertion of the council.

The review is thus comprehensive. And it is not complete, but provides the grounds on which

to complete it.

First of all, then, the particulars from the review can be collected.

On this occasion, too, Aristotle lazily neglected to break apart the entity Pythagoreans. He has three reports:

some of the Pythagoreans say that the motes in the air are the soul, others say that what moves the motes is the soul (2);

the Pythagorean story that any soul can be in any body is ridiculous (3);

by far the most ridiculous opinion is that the soul is number which moves itself (4).

By far the most ridiculous opinion Aristotle does not ascribe explicitly to the Pythagoreans here, but in book I of the *Metaphysics* ascribes to them explicitly the opinion: the soul is number (5).

The ridiculous story that any soul can be in any body has to be ascribed to Pythagoras himself on the authority of Xenophanes.

That Pythagoras himself taught that the soul is immortal Herodotus reports. Pythagoras himself said that number is the principle. It has to be concluded that Pythagoras himself also drew the ridiculous inference, the soul is number.

As to the Pythagoreans who said that the motes in the air are the soul, and the others who said that the soul moves the motes, none of them said anything important, including because Democritus compared the atoms which the soul is to the motes in the air.

With Pythagoras in place of the Pythagoreans, Aristotle notes these predecessors of ours,

764	Homer,
625	Thales,
620	Orphic poems,
570 - 495	Pythagoras,
540	Heraclitus,
532	Alcmaeon,
500 - 428	Anaxagoras,
495 - 435	Empedocles,
480	Leucippus,
470	Hippo,
463	Diogenes of Apollonia,
460	Democritus,
453 - 403	Critias,
427 - 347	Plato.

And, correcting only the report about Pythagoras, below is what Aristotle reports they said

about the soul. It is corroborated by their fragments. It is corroborated by the reports of Aristotle's predecessors, as, indeed, his reports about the Pythagoreans are corroborated by Xenophanes and Herodotus. In Plato's case it is corroborated by the *Timaeus*. It is corroborated everywhere by Aristotle's successors. All of which should be a matter of course, given that this problem, too, Aristotle analyzed, organized, and systematized, including most particularly what out predecessors said.

In chronological order:

Homer was commended by Democritus for saying,

Hector lay with thought distraught;

Thales said that the lodestone has a soul because it moves iron (2); he also said that all things are full of gods;

in the Orphic poems it is said that soul comes in from the whole with every breath (5);

Pythagoras said that the soul is number, it is immortal, and any soul can be in any body;

Heraclitus said that the principle is soul, a warm exhalation from which all things come, incorporeal and in ceaseless flux;

Alcmaeon said that the soul is immortal, because it is in ceaseless motion and all divine things, the moon, the sun, the stars, and the whole world, are in ceaseless motion;

Anaxagoras said that soul is the cause of the motion of things; in many places he said that thought is the cause of order and beauty, in other places that soul is the cause; he said also that there is soul in all animals, great and small, high and low, but thought there is not in all animals, nor indeed in all men;

Empedocles said that the soul is formed from all the elements, each of them being soul; he says,

by earth we see earth, by water water,

by aether divine aether, by fire destructive fire,

and by love love, by hatred cruel hatred (2);

he said also that each of the parts of the body is a ratio of the elements (4); of the bones, for instance, he says,

loving earth in its big-bosomed womb

won from clear water 2 parts out of 8,

from fire 4, and so white bones were formed (5);

it can be asked, is the soul this ratio of the elements (4)?

what Leucippus said is similar to what Democritus said;

Hippo said that the soul is water;

Diogenes of Apollonia said that the soul is air, because air is the finest, it is the principle, and, being the finest, it can originate motion;

Democritus said that the soul is a kind of fire; atoms are infinite in number; and spherical atoms he calls fire or soul; he compares them to the motes in the air which we see in shafts of light; spherical atoms are the soul because atoms of this shape are best suited to permeate everywhere and, being themselves in motion, to move all other atoms; this is why he considers respiration the distinctive mark of life; as its surroundings compress the body of the animal, it extrudes atoms which impart motion to other atoms; the spherical atoms within are themselves never at rest, but their motion needs to be reinforced by the motion of similar atoms from without in the act of respiration;

Critias said that the soul is blood;

Plato also fashions the soul from the elements in the *Timaeus*; similarly, in his lectures on philosophy he said that the animal itself is constituted from the form and the one, like everything else (2); in the *Timaeus* he says also that the soul is in motion and, being connected to the body, moves the body; the maker of the world makes the soul from the elements, divides it by the harmonic numbers so that it be capable of harmony, bends the straight line into a circle, divides this circle into 2 circles, and these 2 circles into 7 circles; all this implying that the motion of the soul is the same as the locomotion of the heaven (3).

How sharply the physical problem of the soul is separated from the ethical the fact shows that Aristotle examines the *Timaeus* and not a line of the rest of the Platonic dialogues, and when in the second part of the review he examines in detail the more important of his predecessors, he examines Plato in detail, but Empedocles in greater detail, and Democritus in yet greater detail.

These are the particulars in the review.

Next, these predecessors and contemporaries of Aristotle answered the question, what is the soul? explicitly as well, and belong in the development of the problem,

590 Anaximenes,

519 Hippasus,

515 - 445 Parmenides,

395 - 317 Ponticus,

395 - 314 Xenocrates.

Anaximenes said that the soul is air and it holds us together, as air encloses the world and holds it together (Aëtius, IV, 3);

Hippasus said that the soul is fire (*id.*);

Parmenides said that the soul is fire (*id.*);

Ponticus said that the soul is not a substance, the body has all the faculties ascribed to the soul (fr. 72);

Xenocrates said that the soul has no form or substance, it is number and the power of number (Cicero, *Tusculan*, I, 20).

In all five cases the ultimate source of the reports is Theophrastus, the history of the physicists.

Next, there is

384 - 322 Aristotle.

Book I of *On the soul* is the formulation of the problem. Books II-III are the solution. The formulation is in two steps. Aristotle presents the difficulties which the problem poses (I, 1), then calls the council of our predecessors (2-5). And the solution is in two steps as well. Aristotle presents it in outline (II, 1), then in detail (II, 2-III, 13).

He begins by affirming:

the soul is the principle of life.

These are then the difficulties:

first, we have to determine which are the genera of the soul and what it is;

next, does it exist only potentially or actually? the answer to this question is very important;

next, does it have parts?

next, those who investigate the soul today seem to confine themselves to the soul of man; but we must not evade the question whether the soul can be defined generally, for all animals, or has to be defined separately for horse, dog, man, and god;

next, are there many souls, or only parts of one soul?

finally, are all affections of the soul shared by the body, or some of them are peculiar to the soul? for if some are peculiar to it, the soul can exist separately from the body, but if none are peculiar to it, the soul cannot exist separately

from the body (I, 1).

This is the solution in outline:
 substance is matter, or form, or the compound
 of the two;
 matter is potentiality, form is actuality;
 substance are first of all bodies;
 some bodies have life;
 life is self-nourishment, growth, and decay;
 it follows that a body which has life is a com-
 pound substance;
 the soul thus must be the form of a body which
 has life;
 it is thus the actuality of such a body;
 the soul and the body together are the animal;
 and the soul is inseparable from the body (II,
 1).

In detail, Aristotle elaborates on all these
 propositions (2-3), reaffirms:

life is fundamentally nourishment, which
 implies growth and decay (2),
 and concludes:

the soul is the principle of life, the efficient
 principle as well as the formal principle as well as
 the final principle (4).

He discusses the senses (5-12), hence perception,
 imagination, thinking, how the soul knows (III, 1-
 9), and locomotion (10), affirming throughout
 that all affections of the soul are shared by the
 body. And concludes with another generalization:

the soul is nutritive in all things alive, plants as
 well as animals, in some of them, the animals, it
 is also sensitive, and in some of these it is also
 rational (11-2).

The difficulties are all resolved.

The soul is the principle of life. The character-
 istic mark of life is nourishment. The genera of
 the soul are efficient principle, formal principle,
 and final principle. It exists actually. It has no
 parts, it is, however, nutritive, sensitive, and
 rational, inclusive of one another. It can be, and
 has been, defined generally, for horse, dog, man,
 and god as well as plant. There are many souls,
 not only parts of one soul. And all affections of
 the soul are shared by the body, the soul cannot
 exist separately from the body.

Aristotle's solution of this problem, too, is a
 great advance.

These are all philosophers through Aristotle
 who answered the question, what is the soul?

explicitly.

Hence, next, the grounds on which they an-
 swered the question can be ascertained.

The grounds are of two kinds.

Aristotle notes Homer, Thales, and the Orphic
 poems. But the formulation and solution of the
 problem of the soul does not begin with Homer,
 or with Thales, or with the Orphic poems. It
 begins with Anaximenes. The third philosopher
 first answered the question, what is the soul? The
 soul is air and it holds us together, as air encloses
 the world and holds it together. The formulation
 and solution of the problem of the soul begins
 with Anaximenes.

And why the soul is air and it holds us to-
 gether, as air encloses the world and holds it
 together, is clear. Because air is the principle of
 all things. Anaximenes said that the principle of
 all things is air, and drew the inference, the soul
 is air.

The ground is not the true way, not yet.

And the successors of Anaximenes in the
 following three generations did as he did. Pythag-
 oras said that the principle of all things is num-
 ber, and drew the inference, the soul is number.
 Heraclitus said that the principle is fire, and the
 soul is a warm exhalation from which all things
 come. Alcmaeon said that the soul is immortal
 because all divine things are in ceaseless motion.
 Hippasus said that the principle is fire, and so is
 the soul. Parmenides said that the material prin-
 ciples are fire and earth, and the soul is fire.

The investigation itself, however, was showing
 them the way.

Anaxagoras, like a mature man among bab-
 blers, saw that wood does not make beds and
 bronze does not make statues. There are efficient
 principles next to the material. And like a mature
 man among babblers, Anaxagoras also said that
 there is soul in all animals, great and small, high
 and low.

The philosophers continued to follow the
 faster way. Hippo said that the principle is water,
 and so is the soul. Diogenes of Apollonia said
 that the principle is air, and so is the soul. De-
 mocritus said that the principles are atoms and
 void, and the soul is spherical atoms.

Nevertheless, beginning with Anaxagoras the
 solution of the problem begins to acquire com-

plexity, and the other ground is increasingly in evidence.

There is soul in all animals, great and small, high and low. Empedocles discussed the composition of the parts of the body. Democritus considered the body of the animal and its surroundings. Even Plato considered how the animal is made. In Aristotle life is nourishment, growth, and decay. And there is soul not only in horse and dog, next to man and god. There is soul in all things alive, plants as well as animals.

The investigation itself was showing them the way, and it was leading them toward observation. Not, indeed, with any haste or urgency. It was yet pointing unambiguously at animals and plants. The advance in the formulation and solution of the problem in the second of the two centuries from Anaximenes to Aristotle points unambiguously at the first steps toward the firmer ground of the investigation of animals and plants. The investigation of animals is attested spectacularly by the 600 kinds of animals which Aristotle describes, at the very end of these two centuries. The investigation of plants is attested spectacularly by the 550 kinds of plants which Theophrastus describes, in the generation after Aristotle. But the beginnings of these investigations, small as they are, can be traced back before Aristotle and Theophrastus. Also, the investigation of that particular animal, man, begins one and a half centuries before Aristotle, with the beginning of theoretical medicine. Practical medicine, a search for action, predates the philosophers by millennia. Theoretical medicine, however, a search for truth, begins with Alcmaeon. And in the one and a half centuries from Alcmaeon to Aristotle theoretical medicine advanced substantially.

These, the investigation of animals, the investigation of plants, and theoretical medicine, are the firmer ground on which the formulation and solution of the problem of the soul advanced.

The investigation of animals is attested spectacularly by the five treatises, the *History of animals*, *Parts of animals*, *Movement of animals*, *Progression of animals*, and *Generation of animals*. Animals, though, had found it difficult, more difficult than the heavenly bodies, for instance, to attract the attention of the philosophers before Aristotle. Animals will find it diffi-

cult, much more difficult than virtues and vices, for instance, to attract the attention of the philosophers for a long time after him, too. Thus, none of the five treatises opens with a review of what our predecessors have said about the animals. Nor are there many quotes. In all the 21 books which the five treatises comprise Aristotle quotes Democritus 13 times, Homer 12 times, Empedocles 11 times, Anaxagoras 5 times, Ctesias 4 times, Alcmaeon 3 times, Herodotus 3 times.

Here as elsewhere Aristotle quotes Homer because he is fond of making a point picturesquely.

Ctesias and Herodotus are sources of reports from faraway places.

The other 4 are the beginnings of the investigation of animals. If the investigation of animals begins before Aristotle, this is the evidence for its beginnings. And its beginnings are as small qualitatively as they are quantitatively. The following is representative:

Alcmaeon said that goats inspire through their ears (*Hist. animals*, I, 11);

Anaxagoras said that male and female are differentiated in the seed, Empedocles and Democritus said that they are differentiated in the uterus, and Empedocles said that they are because of heat and cold, Democritus said that they are because the seed of one of the parents prevails (*Gen. animals*, IV, 1);

Empedocles said that many traits of animals are accidental, like the backbone, which is shaped the way it is because the foetus turns in the womb (*Part. animals*, I, 1);

Democritus said that all animals have viscera, even if in bloodless animals they are too small to see (III, 4).

All the philosophers constituting the development of zoology through Aristotle are thus these,

532	Alcmaeon,
500 - 428	Anaxagoras,
495 - 435	Empedocles,
460	Democritus,
384 - 322	Aristotle.

All 5 are among the philosophers who answered the question, what is the soul? explicitly.

The investigation of plants is attested spectacularly by the two treatises, the *History of plants* and *Causes of plants*. Plants, though, had found

it more difficult than animals even to attract the attention of the philosophers before Theophrastus. They will also find it more difficult than animals even to attract their attention for a long time after him, too. Neither of the two treatises opens with a review of what our predecessors have said about the plants. The quotes are fewer still. In all the 15 books which the two treatises comprise Theophrastus quotes Menestor 7 times, Democritus 6 times, Hesiod 4 times, Empedocles 4 times, Cleidemus 4 times, Alexander's expedition 3 times.

Theophrastus is fond of making a point pictorially with Hesiod rather than Homer.

His source of reports from faraway places is Alexander's expedition rather than Ctesias or Herodotus.

The other 4 are the beginnings of the investigation of plants. If the investigation of plants begins before Theophrastus, this is the evidence for its beginnings. And its beginnings are also as small qualitatively as they are quantitatively. This is representative:

Cleidemus said that plants are made of the same elements as animals, and they fall short of being animals according as their constitution is less pure and colder (*History*, III, 1.4);

Menestor said that these plants are the hottest which live the most in water, like the rush, reed, and galingale (*Causes*, I, 21.6).

All the philosophers constituting the development of botany through Theophrastus are thus these,

495 - 435 Empedocles,
479 Cleidemus,
460 Democritus,
433 Menestor,
371 - 278 Theophrastus.

4 of the 5 belong in the two centuries from Anaximenes to Aristotle, and 2 of these 4 are among the philosophers who answered the question, what is the soul? explicitly.

Practical medicine predates the philosophers. One of the treatises in the Hippocratic corpus is *On the art*, and this is how *On the art* defines medicine: I define medicine as the removal of distress in the sick, the alleviation of violent disease, and the refusal to treat cases in which the disease has already won, knowing that not every-

thing is possible in medicine (3). If this is medicine, this is practical medicine, a search for action, not a search for truth. And the third part, the all too practical, the refusal to treat cases in which the disease has already won, need not be presumed older than the Hippocratic corpus. But the other two parts, the removal of distress in the sick and the alleviation of violent disease, predate the philosophers by millennia.

Thus, closer to the philosophers, Homer attests doctors.

Machaon, although a warrior like the rest of the Homeric heroes, is recalled from battle to mend the wounds of Menelaus (*Iliad*, IV, 192-219), and when he is later wounded himself, he is quickly taken to the safety of the camp because, Idomeneus informs Nestor:

a doctor is worth many warriors
when he cuts out an arrow and applies a soothing
medicine (XI, 505-20).

Homer also reports:

in Egypt every man is a doctor
and wiser than men anywhere else (*Odyssey*, IV, 231-2).

Theoretical medicine, on the other hand, does not predate the philosophers. It begins three generations after the first philosopher, with Alcmaeon.

Alcmaeon said that the senses are connected to the brain (Theophrastus, *Senses*, 26), and that the ruling power is in the head (Aëtius, V, 17). Alcmaeon excised eyes to investigate their nature (Calcidius, comm. *Timaeus*, 279). Alcmaeon said that health is the equal rule of opposites, of moist and dry, cold and hot, bitter and sweet, and the cause of disease is the rule of one of them (Aëtius, V, 30).

Alcmaeon discovered the function of the brain. Dissection begins with him. His is the oldest attempt to assign cause to disease. The fact that theoretical medicine begins with Alcmaeon – the fact that the search for truth with respect to the distress in the sick and violent disease, never mind their removal or alleviation, begins with Alcmaeon – is unassailable.

One generation-odd later, Empedocles said what the ratio of the elements in the bones is. He said what the ratio of the elements in the blood is (fr. 98). He assigned a cause to breathing:

when blood rushes away, bubbling air rushes in,
when blood rushes back, air is breathed out
(fr. 100).

The discovery of the function of the brain, like the discovery of the sphericity of the earth, took time and labor to corroborate. This is how Empedocles solved the problem of the seat of thought:

nourished in a sea of blood, rushing back
and forth, is the seat of thought;
for the blood around the heart is thought (fr. 105).

In the next generation, Hippocrates is a contemporary of Democritus. One of the *Letters* in the Hippocratic corpus relates: the fellow citizens of Democritus asked Hippocrates to come and cure Democritus of madness; Hippocrates came, and found Democritus sitting under a tree, surrounded by the bodies of the animals he had been dissecting (17).

Hippocrates himself only laid the foundations of the Hippocratic corpus. Writings were added to the corpus six centuries after him still. And most of the 70+ writings in the corpus are not just practical, they show no interest in theory whatsoever.

But not all of them. And among the oldest, dating back to the time of Hippocrates himself, are the *Epidemics I* and *Epidemics III*. Neither shows any interest in practice whatsoever, in neither is there a hint of an attempt at treatment. The *Epidemics I* is the record of three epidemics, the *Epidemics III* of one more. 42 individual cases are recorded, 25 of them terminal. For instance:

9th day, cold, babbles, cannot sleep; 10th day, pain in the legs, all symptoms stronger, delirious; 11th day, died (I, 27.12);

3rd day, restless; 4th day, convulsions; 5th day, died in the morning (III, 1.4);

3rd day, intense pain; 4th day, delirious; 7th day, died (1.10).

There is theory in the Hippocratic corpus. But, again, not too much. The corpus attests the search for truth. But it attests also the much more powerful resistance of the search for action to the search for truth. The resistance of medicine to becoming theoretical the fact alone suffices to show that treatises continued to be added to the one steadily older and hence increasingly venera-

ble corpus of medical treatises for six centuries. The Hippocratic corpus did not begin becoming obsolete until Galen. And if not exclusively, the corpus is practical through and through. More than theory, there is in it etiquette: dress simply (*Decorum*, 1); do not gossip (7). There is wisdom: life is short, the art endures (*Aphorisms*, 1). There are exhortations: medicine is the noblest art (*Law*, 1); the doctor who is a philosopher is like a god (*Decorum*, 5). There is much prudence: avoid hopeless cases (*Art*, 3); avoid risky ones, too (*Fractures*, 36). There is even decency: do not begin by asking for your fee (*Precepts*, 4). And there is much explicit resistance to theory. In *On ancient medicine*: I do not think medicine needs new hypotheses (1); medicine is not like the study of the heaven or the study of the earth, where one can speak without anyone ever knowing whether what he says is true (*id.*); medicine has long had everything it needs to be knowledge (2); and it seems to me of the greatest importance that one confine himself to things which everyone knows, for the matters to study are the diseases of which he himself suffers (*id.*). In *On the nature of man*: this treatise is not for those who wish to study the constitution of man more deeply than is profitable for medicine (1).

Thus, from about the time of Hippocrates the record begins to abound in doctors. Two instances. Plato notes Herodicus, the brother of Gorgias (*Gorgias*, 448b), and Plato has the sophist Gorgias say: when my brother or some other doctor has a patient who refuses to take his medicine or to submit to treatment, it is I who go to persuade him (456b). In the *History of animals* Aristotle quotes a passage from Polybus on the veins (III, 3). Polybus is the son-in-law of Hippocrates, and can be presumed, on the ground of the language and subject matter of the passage quoted by Aristotle, to be the writer of *On the nature of man*, the treatise which is not for those who wish to study the constitution of man more deeply than is profitable for medicine.

The record begins to abound in doctors, but not in theoretical doctors. After its first advance, theoretical medicine will for a very long time, against the resistance of practical medicine, advance neither too often nor too much.

Thus, from the three generations from Hippoc-

rates through Aristotle, Aristotle is the only philosopher who can be added to the list of theoretical doctors.

Among his treatises are the brief *On the senses and the sensible*, *On memory and recollection*, *On sleep and waking*, *On length and brevity of life*, and *On youth, old age, life and death, and respiration*. These are not very important, they add little to the five treatises on animals and *On the soul*.

Very important is the spurious *Problems*. It is a collection of 891 problems. A collection of physical problems, it is either by Theophrastus or Strato, or derived from Aristotle, Theophrastus, or Strato.

The *Problems* is in 38 books, under these heads: medicine (I), perspiration (II), wine and drinking (III), intercourse (IV), fatigue (V), posture (VI), sympathetic action (VII), chill and shivering (VIII), bruises and scars (IX), dispositions (X), voice (XI), pleasant odors (XII), unpleasant odors (XIII), the effect of locality on temperament (XIV), mathematics (XV), inanimate things (XVI), animate things (XVII), rhetoric (XVIII), music (XIX), vegetables (XX), barley (XXI), fruit (XXII), the sea (XXIII), hot water (XXIV), air (XXV), winds (XXVI), courage (XXVII), temperance (XXVIII), justice (XXIX), wisdom (XXX), eyes (XXXI), ears (XXXII), nose (XXXIII), mouth (XXXIV), touch (XXXV), face (XXXVI), the whole body (XXXVII), the color of flesh (XXXVIII).

The problems can thus be gathered into four kinds. Problems involving learning and its objects, in books XV-XIX. Problems of meteorology, in books XXIII-XXVI. Problems involving virtues, in books XXVII-XXX. And problems of medicine, the human body, and nutrition, in books I-XIV, XX-XXII, and XXXI-XXXVIII. These last are 621 of all the 891 problems.

With these last 621 the *Problems* is a medical treatise, and a theoretical medical treatise. This is a sample of the problems concerning the human body: why does rubbing the eye stop sneezing (XXXI, 1)? why do the eardrums of divers burst in the sea (XXXII, 2)? why does one usually sneeze twice rather than once or many times (XXXIII, 3)? why are the teeth more sensitive to cold than heat (XXXIV, 3)? why can one not

tickle oneself (XXXV, 6)? And if these problems are not too theoretical, the following are unambiguously theoretical. This is a sample of the problems of medicine: why is disease caused by great excess (I, 1)? but why is then disease often cured by excess (2)? why is death more likely in the 100 days after solstice (26)? why is disease rarer in winter than in summer, but more often fatal (28)? why do some drugs relax the stomach, but not the bladder, others relax the bladder, but not the stomach (40)?

Three and a half centuries after Aristotle, Celsus wrote the compendium *On medicine*, in 8 books. In the preface Celsus has a list of the earliest physicians: Pythagoras, Empedocles, Hippocrates, Democritus, Praxagoras, Chrysippus of Cnidus, and Diocles of Carystus (*id.*).

Praxagoras, Chrysippus of Cnidus, and Diocles of Carystus are all 3 from the generation after Aristotle. None of the 3 is important either.

The list begins with Pythagoras. No medical results can be ascribed to him.

Alcmaeon is not in the list. He discovered the function of the brain, and dissection and etiology begin with him.

Empedocles is in the list. He said that bone is a ratio of the elements, blood is a ratio of the elements, assigned a cause to breathing, said that the blood around the heart is thought.

Alcmaeon and Empedocles are both associated with Pythagorean brotherhoods, Alcmaeon with the brotherhood at Croton under the master himself (Laertius, VIII, 83). Writing about Darius, and thus about the time of Pythagoras, Herodotus reports: at this time the Crotoniates became famous as the best doctors (III, 131).

No medical results can be ascribed to Pythagoras. Pythagoras, however, discovered incommensurability, discovered proof, said that number is the principle of all things.

It can be concluded that the advance in mathematics is a cause of the beginning of theoretical medicine.

Pythagoras also said that the soul is number, it is immortal, and it transmigrates.

It can be concluded that the advance in the investigation of the soul is a cause of the beginning of theoretical medicine as well.

Alcmaeon corroborates both conclusions.

Theoretical medicine begins with him. He also said that the soul is immortal, because it is in ceaseless motion and all divine things, the moon, the sun, the stars, and the whole world, are in ceaseless motion. Aristotle also writes: Alcmaeon borrowed his doctrines from the Pythagoreans, or they borrowed theirs from him.

Empedocles corroborates both conclusions as well. Theoretical medicine continues with him. He also said that the soul is a ratio of the elements.

All the philosophers constituting the development of theoretical medicine through Aristotle are thus these,

532 Alcmaeon,
495 - 435 Empedocles,
469 Hippocrates,
460 Democritus,
384 - 322 Aristotle.

4 of the 5 are among the philosophers who answered the question, what is the soul? explicitly.

Merging the enumerations above, of the philosophers who answered the question, what is the soul? explicitly, of the philosophers constituting the development of zoology, of those constituting the development of botany, and of those constituting the development of theoretical medicine, yields the complete enumeration of the philosophers constituting the development of biology, including the investigation of the animal man, through Aristotle.

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In the two centuries from Anaximenes to Aristotle, then, these 20 are all important philosophers who are part of the development of the physical problem of the soul,

590 Anaximenes,
570 - 495 Pythagoras,
540 Heraclitus,
532 Alcmaeon,
519 Hippasus,
515 - 445 Parmenides,
500 - 428 Anaxagoras,
495 - 435 Empedocles,
480 Leucippus,
479 Cleidemus,
470 Hippo,

469 Hippocrates,
463 Diogenes of Apollonia,
460 Democritus,
453 - 403 Critias,
433 Menestor,
427 - 347 Plato,
395 - 317 Ponticus,
395 - 314 Xenocrates,
384 - 322 Aristotle.

To show that the line between important and not important is drawn among the particulars, the 2 doctors will do,

485 Herodicus,
439 Polybus.

The formulation and solution of the problem of the soul begins with the third philosopher, with Anaximenes.

Aristotle notes Homer in the review, or rather notes that Democritus notes Homer. Aristotle writes: Democritus takes soul and mind to be the same, because he takes what appears and what is true to be the same, and for this reason he commends Homer for saying, Hector lay with thought distraught (I, 2).

Aristotle notes also the Orphic poems. Criticizing his predecessors for failing to consider animals and plants, he writes: thus in the Orphic poems it is said that soul comes in from the whole with every breath borne by the wind; but plants do not breathe (5).

Aristotle notes twice also Thales. Arguing that his predecessors have all associated the soul with motion, he writes: Thales also seems to have thought that the soul is a cause of motion, for he said that the lodestone has a soul because it moves iron (2). And noting that: some have thought that soul is intermingled with everything in the world (5), he adds: perhaps for this reason Thales also said that all things are full of gods (*id.*).

Hector lay with thought distraught. Soul comes in from the whole with every breath borne by the wind. And the lodestone has a soul, and all things are full of gods.

The problem of the soul, on the other hand, begins with Anaximenes.

Anaximenes not only first answered the question, what is the soul? explicitly. He brought the answer in connection with other propositions of

which it is the consequence and which are its consequences in a chain of inferences.

The soul is air. The principle of all things is air. And the soul holds us together as air encloses the world and holds it together.

Philosophy begins with the two problems, of the principles of all things, and of the earth. The second philosopher improved on the formulation and solution of one of them, and brought the solution in connection with more propositions in longer chains of inferences. The third philosopher improved on the formulation and solution of both problems, brought the solutions in connection with more propositions in longer chains of inferences, bringing the solutions of the two problems in connection with one another in these chains of inferences, too, and formulated and solved the problem of the soul, and brought the solution of this problem in connection with the solutions of the other two problems in chains of inferences.

The problem of the soul is not *ex nihilo* either. And it is not from thought distraught, or soul borne by the wind, or the soul of the lodestone or the gods with whom all things are full. The problem of the soul is from the development of philosophy, and most particularly the development of the problem of the principles of all things.

The third philosopher discovered the problem of the soul, and the fourth philosopher, in the next generation, advanced the formulation and solution of the problem substantially.

By far the most ridiculous opinion is that the soul is number which moves itself. The story that any soul can be in any body is ridiculous. The opinion, or story, that the soul is immortal cannot be ridiculous, not after what Plato has done to it.

Either way, in Anaximenes the gaps in the chains of inferences are very wide, from the principle of all things to the soul to the world, with but a dozen propositions about the principle and the world in between. In Pythagoras the gaps are just as wide, if not wider. Nevertheless, with number Pythagoras has discovered the term which can close all gaps in all chains of inferences. With immortality he has discovered the term which will never lose its appeal as a way of filling any gaps, in chains of inferences and otherwise. And with transmigration he has begun

filling the gaps nearer the soul. When he recognized the soul of his friend in a puppy, the soul of his friend and the body of the puppy have begun filling these gaps. When he was first a son of Hermes, then a Homeric hero, then Hermotimus, then a Delian fisherman, then himself, five bodies have begun filling these gaps.

In the next two-odd generations the philosophers continued to solve the problem in the same way, by flying to the most general principle and, taking it to be settled, proceeding to judgment of the soul.

One generation after Pythagoras, what Heraclitus said about soul, flux, fire, λόγος, all things, Aristotle reduced to coherence thus: the principle is soul, a warm exhalation from which all things come, incorporeal and in ceaseless flux.

In the same generation, there is a great advance with Alcmaeon. There is the first indication of the firmer ground on which the problem will be solved. The ruling power is in the brain, excision of eyes, the cause of disease, goats. But the step was not taken by Alcmaeon himself. The soul is immortal, because it is in ceaseless motion, as all divine things are.

In the next generation, Hippasus said that as the principle so the soul.

Parmenides said that as one of the principles so the soul.

Then, in the next generation, the mature man among babblers appeared.

For the first century of the development of the problem the soul had been either material or immortal. The soul is air. It is immortal. It is in ceaseless flux. It is immortal. It is fire. It is fire.

Then Anaxagoras said: the soul is the cause of the motion of things.

Even more maturely, he said: there is soul in all animals, great and small, high and low.

He discussed animals.

The result: nor is there thought in all men, is an important advance, too.

With Anaxagoras the problem of the soul is explicitly about all animals.

And with Empedocles, in the same generation, the first step toward the formulation and solution of the problem on the firmer ground of the true way is in evidence.

Empedocles discussed both animals and

plants.

The soul is formed from all the elements. Each of the parts of the body is a ratio of the elements. Bone, for instance, is 2 parts water, 4 parts fire, and 2 parts earth and air. Aristotle writes: it can be asked, is the soul the ratio of the elements which each of the parts of the body is? And the answer can only be, yes. The soul is the ratio of the elements which each of the parts of the body is.

In the next generation, the attempts at the problem are already routine. The grounds on which the solution of the problem will be improved are also slowly accumulating. And with Democritus the step toward the true way is more decisive.

Leucippus said that the soul is atoms.

Cleidemus discussed plants.

Hippo said that the soul is water.

Hippocrates studied disease.

Diogenes of Apollonia said that the soul is air.

Democritus said that the soul is spherical atoms, which are also what fire is. These atoms permeate everywhere and, themselves in motion, move all other atoms. The spherical atoms within are never at rest, but their motion needs to be reinforced by the motion of similar atoms from without in the act of respiration.

The solution of the problem has begun acquiring complexity.

Democritus discussed animals and plants. He also performed dissections, the madness from which his fellow citizens wanted him cured.

He said that its surroundings compress the body of the animal, and it extrudes atoms which impart motion to other atoms. Respiration is thus the distinctive mark of life.

The problem is about the animal and its surroundings.

And Democritus's is the first explicit answer to the question, what is life? Respiration is the distinctive mark of life.

The investigation itself was showing them the way.

In the same generation, Critias said that the soul is blood. Critias is also the philosopher who said that a wise man discovered the fear of gods, to frighten evildoers even in their thoughts.

In the next generation, Menestor discussed

plants.

Plato said that the animal is constituted from the form and the one, like everything else. The maker of the world makes the soul from the elements, and divides it by the harmonic numbers so that it be capable of harmony, all this implying that the motion of the soul is the same as the locomotion of the heaven. The soul moves the body.

In the next generation, Ponticus generalized what several of his predecessors had said. The soul is not a substance, the body has all the faculties ascribed to the soul.

So did Xenocrates. The soul has no form or substance, it is number and the power of number.

And there is Aristotle.

He investigated 600 kinds of animals.

The *Problems* asks 621 medical questions.

The soul is the principle of life.

The characteristic mark of life is nourishment.

There is soul in all plants and animals, including man.

The soul has no parts, yet it is nutritive, sensitive, and rational, inclusive of one another.

All affections of the soul are shared by the body. The soul cannot exist separately from the body.

Thus, in the first two and a half centuries of the development of philosophy the advance in the formulation and solution of the problem of the soul was very great.

It was not as great as the advance in the formulation and solution of the problem of the earth. Fewer propositions were won. The gaps in the chains of inferences remained substantially wider. The chains of inferences were not collected into a theory.

This may be explained by the greater difficulty of this object, the soul, than that, the earth.

But it can also be explained by the fact that the philosophers found it more difficult to resist the appeal of the false way, and follow the true way, with respect to this object, the soul, than with respect to that, the earth, whether greater difficulty is inherent to this object than to that. The attempts to formulate and solve this problem were grounded very much less in observation, and calculation, than the attempts to formulate and solve that problem. And that this need not be due

to inherent greater difficulty of this object of investigation, the soul, than that, the earth, the numbers obtained by the first sustained attempts to follow the true way with respect to this problem, 600 kinds of animals, 550 kinds of plants, 621 medical questions, show.

Aristotle writes: those who investigate the soul seem to confine themselves to the soul of man. And this can explain why those who investigated the soul found it difficult to resist the appeal of the false way, and follow the true. As it explains why those who investigated the soul had already discovered, and formulated and solved, the ethical problem of the soul. Even in Aristotle the investigation of 600 kinds of animals is not too much in evidence in the investigation of the soul. Not too many of the very many results obtained in the five treatises on animals inform the results obtained in *On the soul*.

The advance in the formulation and solution of the problem of the soul was not as great as the advance in the formulation and solution of the problem of the earth. But the advance in the formulation and solution of no other problem was nearly as great as the advance in the formulation and solution of the problem of the earth.

And in the formulation and solution of the problem of the soul the advance was very great. The problem was discovered. It was formulated and solved, and better, and better, and on firmer grounds, and firmer. All the following proposition were won.

There is soul in all animals.

The soul is the ratio of the elements which each of the parts of the body is.

Respiration is the distinctive mark of life.

There is soul in all plants and animals.

The soul is the principle of life.

The characteristic mark of life is nourishment.

The soul is nutritive, sensitive, and rational, inclusive of one another.

All affections of the soul are shared by the body. The soul cannot exist separately from the body.

And to tabulate the results. Among the grounds only the firmer are included. The inferences, as the principle of all things so the soul, and, as the world so the soul, drawn as they were after their discovery by Anaximenes as a matter

of course, can be taken for granted.

Anaximenes	the soul is air,
Pythagoras	it is number, it is immortal, it transmigrates,
Heraclitus	it is a warm exhalation, incorporeal and in ceaseless flux,
Alcmaeon	the ruling power is in the brain, excised eyes, assigned cause to disease, discussed animals, the soul is immortal,
Hippasus	it is fire,
Parmenides	fire,
Anaxagoras	discussed animals, the soul is the cause of motion, there is soul in all animals, discussed animals and plants, the parts of the body are ratios of the elements, the soul is the ratio of the elements of the parts of the body,
Leucippus	it is atoms,
Cleidemus	discussed plants,
Hippo	it is water,
Hippocrates	studies disease,
Diogenes of Apollonia	– it is air,
Democritus	discussed animals and plants, performed dissections, the soul is spherical atoms, its surroundings compress the animal and it extrudes atoms, respiration is the distinctive mark of life,
Critias	the soul is blood,
Menestor	discussed plants,
Plato	the maker of the world makes the soul from the elements and divides it by the harmonic numbers, it moves the body, the body has all the faculties ascribed to the soul,
Ponticus	it is number and the power of number,
Xenocrates	investigated 600 kinds of
Aristotle	

animals,
621 medical problems in the
Problems,
the soul is the principle of life,
the characteristic mark of life is
nourishment,
there is soul in all plants and
animals,
it is nutritive, sensitive, and
rational inclusively,
its affections are shared by the
body, it cannot exist
separately.

The steps of advance in the acquisition of
knowledge are the 7, Anaximenes, Pythagoras,
Alcmaeon, Anaxagoras, Empedocles, Democri-
tus, Aristotle. In particular,

Anaximenes the soul is air,
Pythagoras it is number,
it is immortal,
it transmigrates,
Alcmaeon the ruling power is in the brain,
excised eyes,
assigned cause to disease,
discussed animals,
Anaxagoras the soul is the cause of motion,
there is soul in all animals,
Empedocles discussed animals and plants,
the parts of the body are ratios
of the elements,
the soul is the ratio of the
elements of the parts
of the body,
Democritus the soul is spherical atoms,
its surroundings compress
the animal and it extrudes
atoms,
respiration is the distinctive
mark of life,
Aristotle investigated 600 kinds of
animals,
621 medical problems in the
Problems,
the soul is the principle of life,
the characteristic mark of life is
nourishment,
there is soul in all plants and
animals,
it is nutritive, sensitive, and

rational inclusively,
its affections are shared by the
body, it cannot exist
separately.

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Thus the physical problem of the soul.
Then there is the ethical problem of the soul.
Aristotle formulates and solves the physical
problem of the soul explicitly, and formulates it
with a review of our predecessors. The attempt to
describe the development of the physical problem
of the soul, and to collect the particulars and
marshal them in their series and order, cannot
begin more propitiously than with Aristotle. And
the attempt to describe the development of the
ethical problem of the soul cannot begin more
propitiously than with Plato. Plato does not
review our predecessors. He is not as systematic
as Aristotle. But he is not less explicit. And he
tells you what you should be told not just with
clarity and distinction, he tells you what you
should be told with irresistible and seldom re-
sisted force. And what you should be told is
much of it explicitly, and all of it implicitly,
about the soul.

And that any consideration of the soul, and
positively of the physical problem of the soul,
demands consideration of the ethical problem of
the soul, and positively an answer to the question
of what the relation of the two is, the physical
problem and the ethical problem, Aristotle's
studied separation of what Plato says about the
soul into what is part of the physical problem,
and what is not, shows. As part of the physical
problem Aristotle examines the *Timaeus* and not
a line of the rest of the Platonic dialogues.

And there is very much about the soul in the
rest of the Platonic dialogues. The soul is a
central problem of the *Protagoras*. It is a central
problem of the *Meno*. It is the central problem of
the *Phaedo*. It is a problem of the *Symposium*. It
is a central problem of the *Phaedrus*. It is the
problem in whose solution the *Republic* culmi-
nates. It is a problem of the *Laws*.

In the development of the physical problem of
the soul Plato is not very important. In the devel-
opment of the ethical problem of the soul Plato is
all-important. The question of what the relation

of the two problems is, is, with respect to the physical problem, very important. With respect to the ethical problem, it should be, even as it is unlikely that it would be, all-important.

In the *Protagoras*, Plato has an excited young man wake Socrates before dawn, to ask Socrates to take him to Protagoras, because he wants to become a student of Protagoras. Socrates agrees, but first warns the young man:

the sophists are peddlers of nourishment for the soul, and if you do not know what you are buying, they may well deceive you; so if you know what you are buying, buy safely from Protagoras or anyone else, but if you do not know what you are buying, you are rolling the dice; because if you buy food or drink, you can take them home and ask someone knowledgeable whether they are good, but teaching you cannot take home in a jar, you are stuck with it in your soul.

And there it is, the first formulation and solution of the ethical problem of the soul.

Your soul will be nourished. Teaching is how it will be. So, if you know what you are buying, buy safely from anyone. But do you know what you are buying? Is it good? You need someone knowledgeable to teach you what to buy.

Socrates takes the young man to Protagoras, and they find Protagoras surrounded by a throng of admirers. Protagoras peddles nourishment for the soul. None of his admirers knows what he is buying. But Socrates is not one of the admirers. He knows what he is buying.

You need someone knowledgeable to teach you what to buy. Socrates will teach you.

A problem is everything in which there is truth or falsehood.

And that truth and falsehood are at stake the concern with knowledge attests. Even if it is here only implicit that knowledge is the outcome of the search for truth. Do you know what you are buying? They may well deceive you. You need someone knowledgeable.

Practice is the search for action.

And that action is at stake not just the concern with food and drink and their storage attests. Not just the concern with selling and buying either. That at stake is whether the food is good for you attests it unambiguously. Yet more strongly,

however, that it is all about teaching you attests it. Even if it is here only implicit as well that the essence of the search for what is good for you – indeed its soul – is teaching you what is good for you. You will be taught in any case. But will Protagoras deceive you? Or Socrates teach you what is good for you?

The tension between the search for truth and the search for action is thus in evidence from the first as well. They may well deceive you. You need someone knowledgeable. He will not deceive you.

In the following dialogues Plato will make it steadily more explicit that the concern is with truth. He will make it also steadily more explicit that the concern is with what is the best for you, and that the essence of this concern is teaching you what is the best for you. And the tension between the two, truth and teaching you what is the best for you, will also steadily mount.

In the *Meno*, Plato has another young man ask Socrates: tell me, Socrates, can virtue be taught? Socrates counters: but what is virtue?

Neither question is answered explicitly in the dialogue. In the course of the *Meno*, however, Plato for the first time affirms unambiguously:

the soul is immortal.

In the course of the *Meno* as well he develops the doctrine of recollection:

the soul is immortal; there is nothing it has not seen in this world or the netherworld, there is nothing it does not know; all learning and discovery is recollection.

It can be presumed that the doctrine is the outcome of the search for truth. On the ground, too, that Plato has Socrates illustrate the doctrine by helping a slave boy recall how to double the square (82b-5b).

But the presumption is under some strain. The doctrine is developed in the course of a quest for virtue and the teaching of virtue. And neither of the questions, what is virtue? and can it be taught? is answered explicitly. But the outcome of the quest is nevertheless affirmative. When Socrates first asks, what is virtue? the young man confidently answers the question. Socrates then shows the young man that he thought he knew, but did not know what virtue is. Himself, Socrates does not know either, but he knows that he

does not know. So, can the young man know that the young man does not know? that Socrates knows that the young man does not know? and that Socrates knows that Socrates knows that Socrates does not know? Yes. There is nothing the soul of the young man has not seen in this world or the netherworld. It is only, clearly, that the young man is not quite as good at recollection as Socrates is.

The young man can buy safely from Socrates.

In the *Phaedrus*, Plato has yet another excited young man recite to Socrates a speech of Lysias on love. Then urged by the young man, Socrates recites a better speech on love, then another yet better speech on love.

In the course of the *Phaedrus* Plato develops further the theory of the forms. He describes:

the realm beyond the heaven where what is is without color, without shape, without solidity, a being which is what it is; and this being is the object of all true knowledge, accessible to the mind alone, the pilot of the soul.

He describes also how:

the pilot of the soul, in a chariot drawn by two horses, one of them good, the other one bad, journeys to that realm beyond the heaven to catch a glimpse of the forms.

The theory of the forms is the outcome of the search for truth. Not because the theory is developed in the *Phaedo* in the course of proofs of the immortality of the soul, or developed further in the *Phaedrus* in the course of recitation of speeches on love, or developed further in the *Republic* in the course of a disquisition on justice, the greater part of which is a disquisition on education. Because of all the propositions, which Plato brings in connection with one another through chains of inferences in the theory, many are not about virtue, love, immortality, justice, or education. Because, also, in the *Parmenides* Plato examines the theory by subjecting it to severe criticism. As he never does the doctrine of recollection, or the proofs of the immortality of the soul, or the doctrine of justice and education.

Either way, Plato is explicit, at stake is all true knowledge.

All true knowledge is accessible to the mind alone, the pilot of the soul, in that realm beyond the heaven.

The soul is analyzed into three parts, the mind and the passions, good and bad, a pilot, a good horse, and a bad horse. It is thus clear why you are liable to deception. You are liable to yield to the bad horse. But it is clear also that you can be taught, you can be trained to drive the chariot as you should.

It is in the *Phaedrus* also that Plato has Socrates teach you: know yourself, the Delphic inscription orders, so I saw that it is ridiculous for me to search for other things before I know myself.

It is thus becoming steadily clearer as well how you can avoid deception. Keep buying from Socrates. There is nothing in this world or the netherworld your soul has not seen. Your soul has journeyed to that realm beyond the heaven where all true knowledge is. And it is ridiculous to search for other things before you know yourself.

In the *Phaedo* Plato has Socrates, condemned to death, in jail, on the eve of his execution, surrounded by his followers. At issue is the immortality of the soul. Socrates proves it. Four times:

opposites come from one another, so life must come from death;

man knows the equal itself, the beautiful itself, the good itself, and these are not among the sensible things, so the soul must have seen them before it came into this life;

the simple is indestructible, and the soul must be simple;

the forms are eternal, and the soul must be what it is by participation in the form of life.

The immortality of the soul proved, Plato has Socrates drink the hemlock.

It is in the *Phaedo* also that Plato has Socrates teach you: I discovered that earth and air and other such things are not the true causes of things; the true cause of things is what is for the best.

The *Phaedo* should leave no doubt that the search is for truth. The result, the soul is immortal, is the outcome of proof, no less, of four proofs, no less.

Then again, the true cause of things is what is for the best.

After the four proofs, and before he has Socrates drink the hemlock, Plato has him teach you:

take my advice and think little of Socrates, much of truth.

Then again, right after that he has him teach you:

that the soul is immortal everyone should keep repeating to himself like an incantation (114d).

Q.E.D. You yet keep repeating it to yourself like an incantation.

In the *Republic* Plato shows what justice is. Shows what the state is. Shows what education is. Divides men into three kinds, and charges the philosopher with their education and neglect of education, respectively. Charges him also with the rule of the city. Develops also the theory of the forms at its finest.

As the investigation proceeds the presumption that the search is for truth is under increasing strain. The philosopher is too busy with action, dividing men into kinds, educating them, ruling them.

Then Plato annihilates the presumption explicitly. He writes:

the most important thing to learn is the form of the good and that it is by their relation to the form of the good that things are beneficial; if we do not know this, even the most perfect knowledge of anything else is of no use to us;

and it is the form of the good that gives, both, truth to what one knows and one's power to know it; although an object of knowledge, the form of the good is also the cause of knowledge and truth; it is thus right to say that knowledge and truth are good, but wrong to say that either of them is the good; the good is higher.

If we do not know this, even the most perfect knowledge of anything else is of no use to us.

And Plato spares no effort to teach us this. Education is a central concern of seven of the ten books of the *Republic*, II-VII and X. Children demand education, men and women demand education, the guardians of the state demand education, the philosopher demands education, the ruler of the state demands education, that it is just to give everyone his due and everyone to give the philosopher his due demands education, that the philosopher be the ruler of the state and the ruler of the state be the philosopher demands education. Education is the essence of justice, the essence of the state, the essence of the philosopher, the essence of the state that would be just, the essence of the state that would be just under

the philosopher.

And this is how the *Republic* ends:

Socrates says, it remains to consider the rewards for virtue;

and asks, have you realized that the soul is immortal?

the young man is transfixed, are you sure?

Socrates replies, I would be wrong not to be, and so would you;

because virtue benefits the soul, vice corrupts it;

and virtue is rewarded, vice is punished (X, 608c-9d).

That virtue and vice are rewarded and punished, respectively, Plato argues explicitly or implicitly everywhere. Most explicitly here. He describes briefly the rewards for virtue and the punishments for vice which the gods bestow here on earth. Then he describes at some length those awaiting the virtuous and the vicious after death. The rewards after death, the greatest awaiting the philosopher, he describes somewhat vaguely, but the punishments after death, the greatest awaiting the tyrant, he describes with some specificity:

savage men grab the criminals, bound their feet and arms, flay them, lacerate them on thorn bushes (615e-6a).

Then he concludes the dialogue:

the soul is altered inevitably by the life it chooses;

everyone therefore must consider the nature of the soul, decide which life is good, which life is bad, and choose accordingly (618b-c);

and if we are persuaded by what I say, we will believe that the soul is immortal, that every evil and every good can befall it, and we will always hold to the right path, practicing justice in every way; in this way we will be the friends of ourselves and of the gods here on earth and after death, and like the victors in the games, will earn our rewards; here and in the hereafter we will be happy (621c-d).

And there it is. The ultimate formulation and solution of the ethical problem of the soul.

The problem is: have you realized that the soul is immortal?

The solution is: are you sure? I would be wrong not to be, and so would you.

More particularly: you should consider the

nature of the soul, you should conclude that the soul is immortal, you should believe that your immortal soul is yet altered by the life you choose, and you should choose accordingly, so that you avoid being flayed and lacerated on thorn bushes, earn your rewards, and here and in the hereafter be happy.

A problem is everything in which there is truth or falsehood. And the search for truth has been relentless. All true knowledge. The true cause of things. The most perfect knowledge. What gives truth to what one knows. The cause of knowledge and truth.

It is not a theoretical problem, however. The subjugation of the search for truth to the search for action has been just as relentless. Is it good? Is it beneficial? Is it of use? Is it the good?

It is an ethical problem, in particular. Ethics is in Plato paradigmatically all about teaching, you, and the best, jointly and inseparably, beginning to end all about teaching you what is the best for you. Do you know whether what you are buying is good? Plato will teach you. Have you realized that the soul is immortal? Are you sure? Plato has taught you that you would be wrong not to be.

And not only is there the danger of being flayed and lacerated on thorn bushes. And not only, if you avoid that danger, you can be happy. You can be happier than the tyrant, and not 3 times happier, or 9 times happier, or 27 times happier, or 81 times happier, or 243 times happier, you can be 729 times happier. For: the life of the philosopher is 729 times happier than the life of the tyrant (*Republic*, IX, 587b-e).

Concern with action is not new with the philosophers. Concern with what is good and what is bad is as old as organisms. As concern with nourishment, and with storing it in jars, can illustrate. Concern with teaching is not quite as old, it is yet as old as organisms capable of transmitting newly acquired experience to the young, not younger than birds and mammals. Concern with rewards and punishments is older than the philosophers, too, even if in this case no organisms older than men need be implicated.

New with the philosophers is the generalization. The concern with what is the best, and for all of you. The concern to teach you what is the best for you, and again all of you. And reward

and punish you, and again all of you.

Most of all new is the enlistment of the search for truth in the service of the search for action. Hence the generalization, too.

Arguments, books full of arguments, more books more full of arguments, nay, full of proofs, collected in doctrines, nay, resting on theories.

Every sentence means something, but not every sentence is a proposition. A proposition is a sentence which affirms what is true or false. A prayer, for instance, is neither true nor false. We are thus concerned only with propositions, the rest we can ignore, they belong to rhetoric or poetry.

With all due respect to Aristotle, they belong most of all to ethics. Nor can we ignore them.

A prayer is not a proposition. An incantation is not a proposition either. An exhortation is not a proposition either. More generally, no sentence involving should can be a proposition, and not only because it cannot be either true, or false, and *tertium non datur*. Because any attempt to fix the reference of 'should', should there have been a reference of 'should', will annihilate all and any of the sense should should have had. No sentence involving you can be a proposition either, and not only because you are not distributed, and either as a subject or as a predicate. Because any attempt to distribute you, and fix your reference, cannot not annihilate your sense, precluding the possibility that the reference of 'you', should there have been a reference of 'you', be the sense of you.

Nevertheless, neither in ethics, in general, nor in the ethical problem of the soul, in particular, just prayers, incantations, and exhortations are collected. There are all the arguments, and proofs, and doctrines. There are all the propositions, and more propositions, brought in connection with one another in chains of inferences, and more chains of inferences. And the sentences, prayers, incantations, and exhortations, are brought in connection with the propositions in the chains of inferences, and if they are brought in connection with the propositions in the chains of inferences not through the reference and the true, but through the thought and the sense, they are thus brought in connection, as far as the thought and the sense is concerned, as firm as the connection through the reference and the true can never be.

One way or another, given only prayers, the development of prayers cannot be described. Given in addition propositions, however, the development of prayers can be described. Accepting the risk that, concerned as it will be with prayers, the description will be liable to be taken to be itself a prayer.

As the development of the physical problem of the soul can be traced back before Aristotle, so the development of the ethical problem of the soul can be traced back before Plato, or, if the ethical problem of the soul is Plato's, and it is, the development leading to the formulation and solution of the problem can be traced back before Plato. As that development can be traced forward after Aristotle, also, so this development can be traced forward after Plato.

The problem is: have you realized that the soul is immortal?

The solution is: are you sure? I would be wrong not to be, and so would you.

More particularly: you should consider the nature of the soul, you should conclude that the soul is immortal, you should believe that your immortal soul is yet altered by the life you choose, and you should choose accordingly, so that you avoid being flayed and lacerated on thorn bushes, earn your rewards, and here and in the hereafter be happy.

It is an essential part of the problem that you are taught what the problem is and what the solution of the problem is.

cclxiv

Restricting the description to the two centuries from Anaximenes to Aristotle, the development leading to, and through, the formulation and solution of the ethical problem of the soul can be considered twofold. The development leading to, and through, what you should do about it. And that leading to, and through, teaching you that.

First what you should do about it.

In the generation after Anaximenes, Pythagoras is the fourth philosopher. And Pythagoras said that the soul is number, it is immortal, and it transmigrates.

The immortality of the soul thus enters philosophy with Pythagoras.

It does not enter ethics with Pythagoras,

however. It enters physics with him. Nor could it enter ethics with him, because there would be no ethics for another century.

There is Aristotle's authority. The Pythagorean proposition, the soul is immortal, as well as the ridiculous story, any soul can be in any body, are part of the development of the physical problem, and no Platonic proposition or sentence on the matter, ridiculous or otherwise, is part of this development.

There is no ethics in Pythagoras, because Pythagoras will not have anything to do with you. He unquestionably has established what is good and what is bad. But he will not tell you. He will, in fact, make every effort to keep it secret from you.

The Pythagorean brotherhood was sworn to secrecy. So much so, when Hippasus revealed to the uninitiated how to construct the dodecahedron, never mind how to transmigrate, he was drowned at sea for that. On the authority of Aristoxenus Laertius reports: the doctrines of the Pythagoreans were secret down to the time of Philolaus; he first brought them out in the three famous books which Plato bought for 100 minae. Philolaus is one century after the master. 1 mina equaled 100 drachmae, 1 drachma equaled 6 obols, one infantryman was paid 3 obols for one day in the field. Instead of buying the three famous books, Plato could have maintained 500 infantrymen for 40 days in the field.

The generalization from what is good to what is the best for you remained to be made. The subjugation of the search for truth to the search for action remained to be undertaken. The first philosophers expressed themselves vaguely, not quite knowing what they were saying, fighting the way untrained men fight. It is not clear that Pythagoras distinguished the search for truth and the search for action. Yet, given the distinction, he subjugated the search for action to the search for truth, not the other way around. By far the most ridiculous opinion that the soul is number indicates this. And the following indicates it rather unambiguously.

Plato notes Pythagoras but once. He writes: Pythagoras had followers and taught them a distinct way of life.

Pythagoras taught them proof and how to

construct the dodecahedron, the immortality of the soul and how to transmigrate.

He taught them also the following. Laertius reports:

do not stir the fire with a knife;
do not walk on the main road;
do not shake hands too eagerly;
do not keep swallows under the roof;
do not make water on nail- and hair-trimmings;

do not look back when crossing a boundary;
do not eat red mullet;
do not eat the heart of an animal;
do not touch beans (VIII, 17-9).

The ban on beans was the foundational. So much so, as the death of Socrates became a lesson for his followers, so did the death of Pythagoras for his. Laertius reports: the Crotoniates set the houses of the Pythagoreans on fire; the master himself escaped; but a field of beans barred his way; the master halted rather than touch beans; and his pursuers cut his throat (39).

Do not touch beans. Pythagoras thus deprived his followers of a good source of protein, iron, and vitamins B₁ and B₂.

Why? remains a secret.

Aristotle tried to uncover it. He writes: Pythagoras enjoined abstention from beans because beans are like the genitals, or because beans are like the gates of Hades, or because beans are destructive, or because beans are like the world, or because beans are oligarchic (fr. 195).

Excellent reasons all. Excellent reasons all also not to stir the fire with a knife, not to keep swallows under the roof, not to make water on nail- and hair-trimmings, not to eat red mullet. Excellent reasons all also to construct the dodecahedron, or to transmigrate, or both.

One generation after Pythagoras, what Heraclitus said about soul, flux, fire, λόγος, all things, Aristotle reduced to clarity in the review of the development of the physical problem of the soul thus: the principle is soul, a warm exhalation from which all things come, incorporeal and in ceaseless flux.

Without Aristotle's help, this can be read about the soul in the fragments of Heraclitus:

the soul is λόγος which enlarges itself;
you will not find the limits of the soul down

any road, so deep is its λόγος;

death for souls is to become water, death for water is to become earth, earth comes from water, and from water comes soul (fr. 36).

a drunk man is led by a boy, stumbling and not knowing his way, because his soul is wet (fr. 117);

a dry soul is the wisest and the best (fr. 118);

A dry soul is the best, a wet soul is not good. It is a step toward ethics. But a small step and implicit rather than explicit. It would have been larger and more explicit, had Heraclitus not disapproved so vigorously of so much else. Had he also said less about them. For he has little to say about you, but much about them, and if you are one of them, you do not understand even when Heraclitus explains each thing according to its nature, you do not understand even when you stumble on it, you know neither how to listen nor how to speak, what mind or sense do you have? you stuff yourself like cattle, you prefer rubbish to gold.

Given the distinction between the search for truth and the search for action, Heraclitus, too, like Pythagoras, subjugated the search for action to the search for truth, not the other way around.

Heraclitus did not have followers to whom to leave his death as a lesson. With the hindsight of ethics, his death should nevertheless be a lesson, and a better one than most. Heraclitus ordered his slaves to cover him with cow dung and so expired (Laertius, IX, 4).

One generation-odd after Heraclitus, Anaxagoras greatly advanced physics. And no step toward ethics, not small, not implicit, can be imputed to him. Even if it will yet turn out that he is very important in the development of ethics.

In the same generation, Empedocles also greatly advanced physics. And with Empedocles there is a step toward ethics. Still implicit rather than explicit, but a more decisive step.

With Anaxagoras and Empedocles the philosophers do not express themselves so vaguely any longer, they know better what they are saying, they are beginning to fight the way trained men fight.

And, with Aristotle, what Anaxagoras and Empedocles said about the soul is all part of the physical problem of the soul. Empedocles, in

particular, said that each of the parts of the body is a ratio of the elements, and the soul is the ratio of the elements which each of the parts of the body is.

The step toward ethics does not involve the soul. The step is in that Empedocles has a doctrine of punishment:

it is an oracle of fate that those who err
be exiled from the gods for 30 000 years,
the sky pursues them to the sea,
the sea pursues them to the bottom of the earth,
the earth pursues them to the shining sun,
the sun pursues them to the circling sky,
one god chases to another what they all abhor;
and I am now one of those exiles from the gods,
punished for having put my trust in strife

(fr. 115).

On this ground the step is also toward the generalization from what is good to what is the best for you.

Unfortunately, Empedocles did not have followers to teach, like Pythagoras, was not concerned too much with them, like Heraclitus, was a physicist *par excellence*, like Anaxagoras, and has very little to say to you. If you would nevertheless know what is the best for you, it would have to be this:

wretches, utter wretches,
keep your hands away from beans (fr. 141).

Clearly, if you wretches, utter wretches, do not, you will be exiled from the gods for 30 000 years.

Empedocles has taken a step toward the generalization from what is good to what is the best for you. No step, however, toward the subjugation of the search for truth to the search for action can be imputed to him either. Given the distinction, Empedocles, too, subjugated the search for action to the search for truth.

Thus the following are all theoretical advances, practical they cannot be.

Empedocles advanced medicine. Accordingly, you will learn from him not only all remedies for illness, and for old age:

you will learn from me all remedies for illness,
and for old age, and how to bring back
from Hades the strength of the dead (fr. 111).

Five centuries later on, the feat of bringing back from Hades the strength of the dead will become routine. The feat will become routine on

the same ground, too, on which Empedocles accomplished it:

my friends,

I am an immortal god amongst you (fr. 112).

Empedocles also reports:

I was once a boy and a girl
and a bush and a bird and a fish (fr. 117).

All Pythagoras had been had been a son of Hermes, a Homeric hero, and three different men. Empedocles was a bush and a bird and a fish.

Dying, finally, the philosophers clearly took to be an opportunity not only to teach their followers, or humanity, as the case may be, a lesson, but an opportunity also to perform an experiment. And if Empedocles's lesson is not the best, and it may well be that it should be, his experiment is unquestionably the best: he jumped into Etna (Laertius, VIII, 69). Even if Strabo tries to spoil the experiment by arguing that: it is impossible to reach the edge of Etna's crater and throw anything into it because of its fiery gusts (VI, 2.8).

In the following generation the army which Plato will unflinchingly fight arrayed itself under the general, Protagoras, the second-in-command, Gorgias, and the lieutenants, Hippias and Prodicus. The dates are,

490 - 420 Protagoras,
485 - 380 Gorgias,
470 - 394 Hippias,
470 - 394 Prodicus.

The four are important in the development of the form, of teaching. In the development of the content they are not important. As Plato assessed them: they teach nothing but the common opinions of the rabble and this is what they call wisdom (*Republic*, VI, 493a).

And then there is Socrates. And ethics begins with Socrates. What is good is generalized most explicitly by Socrates into what is the best for you, and all of you. The subjugation of the search for truth to the search for action cannot be more explicit than it is in Socrates. And of teaching Socrates remains the paradigm. All three.

Aristotle reports: Socrates abandoned physics altogether, and devoted himself to ethics.

Plato has Socrates report: I heard that Anaxagoras taught that mind orders all things and is the cause of all things; I was delighted, thinking that I had found someone to teach me what I wanted

to know; but then I read the books of Anaxagoras, found that they were all about causes like air, aether, water, and other such things, and realized that he had nothing to teach me; I began to study the causes of things by myself; and I discovered that earth and air and other such things are not the true causes of things; the true cause of things is what is for the best.

Xenophon reports: Socrates disapproved of curiosity about how the god arranged the heaven; whoever meddles with such things, he said, risks going mad like Anaxagoras, so madly proud of his explanation of the divine machine (*Memorabilia*, IV, 7.6).

Anaxagoras first. Then Socrates. Socrates such as has realized that Anaxagoras has nothing to teach him, discovers the true cause of things, and begins teaching you.

The generation after Socrates is the generation of his followers. It is in this generation, and with his followers, that ethics becomes ubiquitous.

Five followers of Socrates had his own followers each. The first three are important,

447 - 364 Euclid of Megara,

444 - 361 Antisthenes,

437 - 361 Aristippus.

The fifth is not important,

417 Phaedo,

whose hair, as a young man, Socrates caresses in the *Phaedo* (89b).

Euclid of Megara taught: the one is the good (Cicero, *Academic*, II, 129).

Antisthenes taught: exertion is the good (Laertius, VI, 2).

Aristippus taught: pleasure is the good (Cicero, *Duties*, III, 116).

That Phaedo, too, had his own followers is attested by Strabo (IX, 1.8). What Phaedo taught, however, the record has mercifully consigned to oblivion.

The fourth is Plato.

Ethics begins with Socrates, and ethics becomes ubiquitous with Plato's fellow-followers of Socrates not less than with Plato. But the ethical problem of the soul is Plato's.

In the *Apology* Plato has Socrates say about death not more than this: death is one of two things, either the dead are nothing and perceive nothing, or it is, as some say, a migration of the

soul from here to some other place. And about the soul not more than also this: I will continue exhorting you, my good friends, are you not ashamed of caring more for wealth and honor and reputation than for wisdom and truth and making your soul as good as possible (29d)?

In Xenophon's 7 books of Socratic disquisitions Socrates has nothing whatsoever to say about the soul.

Nor do Euclid of Megara, or Antisthenes, or Aristippus have anything to say about the soul.

The soul is Plato's.

The next is the generation of the followers and students of the followers. It is the generation of

410 - 322 Diogenes,

a follower of Antisthenes. And of

407 - 339 Speusippus,

Xenocrates, and Aristotle, all three students of Plato. And as the Academy has already come into being, they are not just followers, they are students. Speusippus also succeeded Plato as second head of the Academy, Xenocrates succeeded Speusippus as its third head. Aristotle founded his own school, the Lyceum.

And none of the four, the three students of Plato's as little as Diogenes, has anything to teach you about the soul. Ethics has become ubiquitous. But not the soul. Not yet, not for several centuries. For the time being Plato had apparently exhausted the ethical problem of the soul.

Speusippus and Xenocrates have each little to say about the soul, and what each has to say about it belongs in the physical problem.

Speusippus said that the substances are four, the one, number, spatial magnitude, and the soul (Aristotle, *Metaphysics*, VII, 2).

Xenocrates said that the soul has no form or substance, it is number and the power of number.

Diogenes has nothing at all to say about the soul. And what Aristotle has to say about it belongs in the physical problem.

Diogenes and Aristotle are each a very important step in the development of ethics.

Diogenes called Plato's lectures a waste (Laertius, VI, 24); Plato called him Socrates gone mad (54).

Diogenes was eating figs when he met Plato and offered him some; Plato ate them all; Diogenes said, I offered them to you to share, not to eat

them all (25).

Plato saw him washing lettuces and said, had you paid court to Dionysius, you would not be washing lettuces; Diogenes said, had you washed lettuces, you would not be paying court to Dionysius (58).

Plato invited friends to his house; Diogenes trampled on his carpets and said, I trample on Plato's vanity; Plato said, with a vanity of your own (26).

Plato proposed a definition, man is a featherless biped; Diogenes plucked a chicken and brought it to the Academy; Plato fixed the definition, with broad nails (40).

Demosthenes was dining in a tavern; a stranger asked Diogenes to point the orator to him; Diogenes stretched out his middle finger, there is the leader of the Athenians (34).

Alexander came to him and said, I am Alexander the great king; Diogenes said, I am Diogenes the dog (60).

Perdiccas sent an order for Diogenes to appear before him on penalty of death; a spider can threaten me with death, Diogenes said, Perdiccas I would expect to threaten me that he would be miserable without my company (44).

To fortune he opposed courage, he said, to the law nature, to passion reason (38); one needs reason to learn to live, or a halter (24).

He said that children should be common; there is no impropriety in stealing from a temple; there is no impropriety in eating the flesh of any animal, including man, as the customs of many tribes show (72-3). He tried to eat raw meat but could not digest it (34).

Asked what is best in man, he replied, freedom (69). He lives his life, he said, as Heracles lived his, preferring nothing to freedom (71). Asked of what city he is a citizen, he replied, of the world (63); only of the world is one properly a citizen (72).

He followed the example of the trainers of choruses, he said, who set the note a little too high to make sure everyone hit the right note (35).

He was eating in the street; people gathered and someone called out, you are a dog; you are dogs, he told them, standing there, watching me eat (61).

Guests at a feast threw the bones at him; he lifted a leg and pissed on them (46).

Everyone was leaving the theater, he was going in (64).

Aristotle develops in the *Nicomachean ethics* a doctrine *par excellence* of what is the best for you, and teaches it to you. Book I is on ethics in general and its relation to politics. Book II is on virtue in general. Books III-VII are a compendium of virtues. Aristotle informs you along the way: man is not the best thing in the world (VI, 7). The highest virtue is friendship, and Aristotle dwells on it in books VIII-IX. Book X is on pleasure and happiness. Aristotle concludes: the happiest life is the theoretical (7).

In all the 10 books he notes the soul twice. The main division of virtues is into virtues of character and virtues of mind, and Aristotle introduces the division by restating the theoretical result: the soul is nutritive, sensitive, and rational inclusively (I, 13). And he introduces the compendium of virtues of mind by proposing: we said that the soul is irrational and rational; let it be now assumed that the rational is also twofold, knowing, which deliberates on what cannot be otherwise, and calculating, which deliberates on what can be otherwise (VI, 1).

Had you let yourself be taught what is the best for you by Diogenes, you would have preferred nothing to freedom. You could have had the courage to go in the theater when everyone is leaving.

Had you let yourself be taught what is the best for you by Aristotle, you could have known that you are not the best thing in the world. Your life would have been the happiest searching for truth.

cclxv

Teaching is as old as organisms capable of transmitting newly acquired experience to the young.

Teaching among the philosophers, in particular, begins with Pythagoras.

Iamblichus reports: Pythagoras did not accept those who came to become his students immediately, he asked each about his family and kin, watched how he laughed, how he talked, listened, walked, and if he found him worthy, he allowed him to be neglected for three years; during this

time the neglected was not admitted to the brotherhood, but the master watched him constantly in secret, and if at the end of the third year he still seemed steadfast and studious, he was admitted to the brotherhood for a period of five years of silence; and if at the end of the fifth year he had shown himself with his silence sluggish or dull, he was expelled, but if he had shown himself otherwise, he became a full-fledged member of the brotherhood (*Pythagoras*, 17).

Laertius adds: during the five years of silence the student was not allowed to look at the master, after passing an examination at the end of the fifth year he was allowed (VIII, 10).

With Pythagoras, then, teaching among the philosophers begins with great circumspection, on the part of the teacher and of the student, both, the one exerting himself to neglect and keep silent, the other to be neglected and kept silent.

As to the efficacy of the teaching, Iamblichus reports: the Pythagoreans were of two kinds, listeners and students, and the listeners did not recognize the students as Pythagoreans because, the listeners said, the students had received their doctrines not from Pythagoras but from Hippasus (*Pythagoras*, 18).

Listeners renders ἀκουσματικοί, students renders μαθηματικοί.

The philosophy of the students, or mathematicians, consisted, presumably, of mathematics. Of what the philosophy of the listeners consisted, Iamblichus is explicit: the philosophy of the listeners consists of undemonstrated precepts ordering them to do certain things; they follow these precepts as divine revelations, say nothing of their own, think it wrong to do so, and consider whoever knows the greatest number of precepts the wisest (*Knowledge*, 25; *Pythagoras*, 18).

The efficacy of the teaching, then, if it was to produce both mathematics and precepts followed as divine revelations, was great. If it was to produce either mathematics or precepts followed as divine revelations, it was not great.

Teaching with no circumspection at all, and on the part of either the teacher or the student, begins three generations later, with Protagoras.

Protagoras told the young man: young man, if you come to study with me, from the day you

come, you will go home every day a better man than the day before (Plato, *Protagoras*, 318a-b).

Accordingly, Protagoras was followed by students from every city through which he passed (315a-b).

Accordingly, Protagoras was the first to charge a fee for teaching (349a). Aristotle reports: Protagoras let the student assess the value of being taught and accepted what the student decided (*Nic. ethics*, IX, 1). Laertius reports: Protagoras was the first to collect a fee of 100 minae (IX, 52). Someone, clearly, instead of maintaining 500 infantrymen for 40 days in the field, went home every day a better man than the day before.

Accordingly, Protagoras was emulated. Gorgias, Hippias, and Prodicus could each go to any city and persuade the young to pay him to teach them (Plato, *Apology*, 19e-20a). Protagoras was emulated and surpassed. Socrates tells Hippias: Protagoras, Gorgias, and Prodicus made more money out of wisdom than any craftsman out of any skill; not so, Hippias replies, I think I have made more money than any other two teachers combined (*Gr. Hippias*, 282d-e).

For the lack of circumspection Socrates blames the teachers: they teach nothing but the common opinions of the rabble and this is what they call wisdom. Hippias explains: my listeners do not want to hear about the motions of the planets, or geometry, or arithmetic, or harmony, they want to hear about the genealogies of heroes and the foundation of cities (285b-e).

As to the efficacy of the teaching of Protagoras, young men went home every day better men than the day before. Of that of Gorgias, Aristotle reports: they still teach like Gorgias, who gave speeches to his students to learn by heart, like giving them shoes rather than teaching them how to make shoes. Of that of Hippias, he would have taught the motions of the planets, or geometry, or arithmetic, or harmony, as it was he taught the genealogies of heroes and the foundation of cities. Of that of Prodicus: he taught that first of all the right use of the word should be learned (*Euthydemus*, 277e).

Again, then, if teaching was to produce better men, speeches like shoes, the genealogies of heroes, and the right use of the word, any or all of these, its efficacy was great. If it was to produce

the motions of the planets, geometry, arithmetic, or harmony, its efficacy was not great.

Next, Socrates did not charge a fee. He began teaching all of you instead, and it is a decisive step in the development of teaching. How decisive, the fact attests that Hippias may have made more money than any other two teachers combined, but Socrates had more students than all earlier philosophers combined. To note only the important, Critias, Alcibiades, Euclid of Megara, Antisthenes, Aristippus, Xenophon, and Plato are all students of his.

As to the efficacy of the teaching of Socrates, Plato taught the immortality of the soul. Euclid of Megara taught the one. Antisthenes taught exertion. Aristippus taught pleasure. Xenophon was a very important historian.

Xenophon reports: at the trial of Socrates the prosecution charged, Critias and Alcibiades were followers of Socrates, and in the days of the oligarchy Critias bore the crown for greed and violence, in the days of the democracy Alcibiades exceeded everyone in insolence and licentiousness (*Memorabilia*, I, 2.12-4). Himself a loyal follower of Socrates, Xenophon then appeals to a number of circumstances in extenuation of Socrates's crime of having taught Critias and Alcibiades.

Ignoring the charges of the prosecution, Critias said that the soul is blood, and that a wise man discovered the fear of gods. Alcibiades was the finest general of the Athenians throughout the Peloponnesian war.

If teaching, then, was to produce a great variety, the soul is immortal and also blood, the good is exertion and also pleasure, piety and impiety, an important historian and a fine general, any or all of these, its efficacy was very great. If it was to produce one of these and not the others, or at least some of these but not the rest, its efficacy was none.

Next, Plato's is the most decisive step in the development of teaching.

Plato teaches you in not less than 27 dialogues, not counting the spurious. In one of the 27, too, the greatest, he teaches you in great detail how to make philosophers: selected in childhood (II, 374e-6c), watched at all ages to ensure that neither duress nor magic spells will make them

forget that they must always do what is best for the city (III, 412e), they will study through their thirtieth year arithmetic, geometry, astronomy, and harmony (VII, 521c-31d), these will prepare them for the truths of dialectic, which are not suitable for immature minds of under thirty (537d-9a), they will study these truths for five years (531e-7c), then they will be sent out to acquire experience in the arts of war and peace for fifteen years (539b-40a), and these who have excelled throughout, at fifty, full-fledged philosophers now, will join their betters in the task of ruling the city (540a-c).

Nor did Plato just teach you to make the ruler of the city a philosopher. He sailed twice to Syracuse to make himself its tyrant Dionysius II a philosopher.

Plato also founded the Academy. He founded it c. 385, Aristotle emulated the precedent in 335, founded the Lyceum, and philosophy has ever since been a product of schools.

As to the efficacy of teaching, the conclusion that Plato himself has grave doubts about it is difficult to avoid. Through their thirtieth year the prospective philosophers will study arithmetic, geometry, astronomy, and harmony. Alas, the listeners of Hippias did not want to hear about the motions of the planets, or geometry, or arithmetic, or harmony, they wanted to hear about the genealogies of heroes and the foundation of cities. And Plato has clearly learned this lesson from Hippias. There is no geometry, or arithmetic, or astronomy, or harmony in the *Republic*, there is the foundation of a city. There is some geometry, or arithmetic, or astronomy, or harmony in the *Timaeus*. There is none in the remaining 25 dialogues either.

The labors at making Dionysius II a philosopher ended with Plato's imprisonment by the prospective philosopher (Plutarch, *Dion*, 20). Archytas saved Plato's life (*id.*).

Plato is the teacher of Aristotle. The ethicist *par excellence* is the teacher of the physicist *par excellence* and logician *par excellence*.

The efficacy of teaching you the immortality of the soul, and what you should do about it, will prove eventually very great. For the time being, as far as Plato's teaching of that doctrine is concerned, there is no evidence of any efficacy

whatsoever. Diogenes trampled on Plato's carpets. Aristotle has no use for that doctrine whatsoever. Plato taught also Speusippus and Xenocrates. The second head of the Academy left behind no evidence of any use for that doctrine either. The third head of the Academy left behind no evidence of any use for that doctrine either.

On the evidence of these first centuries of development of philosophy through Aristotle, then, the efficacy of teaching in producing more teaching, and more, and more, was very great.

The efficacy of teaching in producing philosophy was none.

Apropos of the success of the emperor-philosopher Marcus in teaching his son the emperor-gladiator Commodus, Gibbon writes: the power of instruction is seldom of much efficacy except in those happy dispositions where it is almost superfluous (*History*, 4).

Writing at the time of enlightenment, Gibbon is overly optimistic.

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Anaximenes discovered the physical problem of the soul. The formulation and solution of this problem begins with him. In the following two centuries through Aristotle the formulation and solution of this problem advanced greatly. And one and a half centuries after Anaximenes the ethical problem of the soul was discovered. This problem is Plato's. The formulation and solution of this problem begins with Plato.

This problem is not *ex nihilo* either.

The first step toward it is Pythagoras. It is a small step. The soul is immortal. It is a solution of the physical problem. The soul is also number. And it has to be rather more plausible, too, that because the soul is number, it is also immortal, than the other way around, because the soul is immortal, it is also number.

The next step is Empedocles. It is also a small step. You will learn from me how to bring back from Hades the strength of the dead. Empedocles solved the physical problem. The soul is a ratio. Empedocles was also a doctor. And again, it has to be rather more plausible that because you will learn from me all remedies for illness and old age, you will learn from me also how to bring back from Hades the strength of the dead, than

the other way around, because you will learn from me how to bring back from Hades the strength of the dead, you will learn from me also all remedies for illness and old age. Empedocles was an immortal god. An immortal god because a doctor. The other way around, a doctor because an immortal god, is a later development.

The large steps are the growth of teaching and the development of ethics.

Teaching begins among the philosophers with Pythagoras.

Then teaching grows greatly with Protagoras, Gorgias, Hippias, and Prodicus.

Then ethics begins with Socrates.

And teaching cannot not be a cause of the development of ethics. Selection will guarantee that.

Protagoras was sending the young home every day better than the day before. Gorgias was giving them speeches. Hippias was telling them about heroes and cities. Prodicus was telling them the right use of the word. If the young are taught, selection will guarantee, and the more the young are taught, the more will selection guarantee, that the young are taught what is good for the young. The transmission of newly acquired experience to the young will, by being what it is, and having been what it is for millions of generations, guarantee that what is good for the young is transmitted to the young.

To teaching, and to the growth of teaching, a cause does not have to be assigned. Selection is the cause.

Ethics is the generalization from what is good to what is the best for you. All of you. And not just good. The best. The good.

To the search for what is good a cause does not have to be assigned either. The cause is the same, selection.

To the generalization a cause has to be assigned. Selection will not do this. The generalization cannot be from selection.

Socrates realized that Anaxagoras has nothing to teach him. He began to study the causes of things by himself. He discovered that earth and air and other such things are not the true causes of things. And he discovered that the true cause of things is what is for the best.

Anaxagoras thus yet taught Socrates both

intensively and extensively. He taught Socrates that there are the causes of things. He taught Socrates, and a very important lesson it is, that earth and air and other such things may be the true causes of things. He taught Socrates, and a most important lesson it is, that there is the true cause of things.

With Plato, the most important thing to learn is the form of the good. If we do not know this, even the most perfect knowledge of anything else is of no use to us.

There is thus knowledge. There is the most perfect knowledge, too. And the form of the good is not just good. The form of the good is also an object of knowledge. The form of the good is also the cause of knowledge and truth. The form of the good is also higher than knowledge and truth.

And that there is knowledge, that there is the most perfect knowledge, that there is an object of knowledge, that there is cause, that there is truth, and that there is the cause of knowledge and truth, too, are all from Anaxagoras, and if they are also from Socrates, they are because in Socrates they are from Anaxagoras.

The development of philosophy, and most particularly the development of physics is the cause of the generalization.

The search for truth is first. Then there is ethics. Ethics is not just a search for action. Ethics is a search for action after the search for truth.

Most particularly, ethics is the subjugation of the search for truth to the search for action.

And to the subjugation the development of physics cannot be the cause. Once again, however, selection can be.

Maxwell writes: it was a great step in science when men became convinced that they must begin by asking not whether a thing is good or bad, noxious or beneficial, but of what kind it is, and how much there is of it.

The great step was taken decisively in the one and a half centuries beginning with the first philosopher.

Then the next step was taken. Of what kind it is, and how much there is of it. But of what kind it is, and how much there is of it, is it good or bad, noxious or beneficial?

Socrates asks, will it teach me what I want to

know? is it for the best?

Plato asks, is it good? is it beneficial? is it of use? is it the good?

Selection is the cause of teaching, and of the growth of teaching, and teaching, and the growth of teaching, is a cause of the development of ethics.

But the chief causes of the development of ethics are, negatively, the advance of physics, and then, affirmatively, reasserting itself, selection.

This can explain the great efficacy of teaching and its lack of any efficacy whatsoever, both. Its great efficacy in producing ever more teaching, and its lack of any efficacy whatsoever in producing philosophy, including in producing ethics.

The ethical problem of the soul cannot be part of the acquisition of knowledge. The subjugation of the search for truth to the search for action precludes this possibility. The ethical problem of the soul is a step of advance in the development of prayers, incantations, and exhortations.

And the relation of the two problems, the physical problem of the soul and the ethical problem of the soul, can only be exclusion.

Thus, with respect to the physical problem, Aristotle ignores the *Protagoras*, *Meno*, *Phaedo*, *Phaedrus*, *Republic*. He examines the *Timaeus*, and examines more Empedocles, and more Democritus. And that the relation of the two problems is exclusion, is, with respect to the physical problem, very important, because it is the substance and the essence of the ethical problem to subjugate the physical to itself, and if it succeeds, as in a few centuries it will, it will arrest the development of the physical.

With respect to the ethical problem, Plato ignores the physicists as resolutely as Aristotle ignores the ethicists. And that the relation of the two problems is exclusion, should be, with respect to the ethical problem, all-important, because what you should do about it is good, and beneficial, but, if the development of the physical problem continues, as it eventually will, it will continue asking of what kind the soul is, and how much there is of it, and the answers will have to be again subjugated, and again.

Moreover, if the subjugation falters, it may even be asked of what kind what you should do about it is, and how much there is of it.

The ethical problem of the soul begins with Plato. And considering the two centuries through Aristotle, there is after Plato no further development in the formulation and solution of the problem.

cclxvii

All the centuries before Anaximenes.

The physical problem of the soul is from the principle of all things. Anaximenes said that the principle is air, and the soul is air. And his successors continued to solve the problem in the same way. The principle is number, the soul is number. The principle is fire, the soul is fire. The principles are the four elements, the soul is a ratio of the elements. The principles are atoms, the soul is atoms. Aristotle, too, begins the solution of the problem with: substance is matter, or form, the soul is form.

The firmer ground on which the philosophers began improving on the formulation and solution of the problem are the investigation of animals, the investigation of plants, and theoretical medicine. The investigation of animals does not begin before Alcmaeon. The investigation of plants does not begin before Empedocles. Theoretical medicine begins with Alcmaeon. The beginnings of all three are small, too.

The problem of the principles of all things begins with Thales.

Besides, there is nothing in the centuries before Anaximenes which may lead to the physical problem of the soul. The physical problem of the soul is all from the development of philosophy.

Where the problem of the earth is concerned, there are earlier the two observations of Sirius in the valley of the Nile. There is the discovery of Venus, and there is the beginning of the sustained observation of the heavenly bodies in the valley of the two rivers.

Where the physical problem of the soul is concerned, there is nothing remotely comparable.

There are animals and plants in the earlier record. There are depictions of animals and plants. There is the millennia-old fact of their domestication. But there is nothing which may suggest their examination, let alone observation, let alone investigation. The closest that there is to

a development which may suggest examination is medicine. And it is not close at all. The physical problem of the soul, the investigation of animals, the investigation of plants, and theoretical medicine, all four, are from the development of philosophy, and the development of physics, in particular.

This is what there is in the earlier record about medicine.

Herodotus writes one generation-odd after Alcmaeon, in the generation after Empedocles and before Hippocrates. In the *Histories* Herodotus researches medicine as he does everything else, and these are his findings.

He writes three times about medicine.

One. Describing the customs of the Babylonians, he describes: their wisest custom; once a year, in every town, they gather all girls ripe for marriage and a herald puts them up for sale.

Then he describes:

their second wisest custom; they bring their sick to the marketplace, because they have no doctors at all; in the marketplace, those who pass by come to the sick and give them advice and comfort; when someone has suffered from the same sickness or has known someone who has, he tells them what to do; or he tells them how he has recovered or how someone else has; no one may pass by the sick without asking what ails them (I, 197).

Then he describes their burial customs.

Two. Describing the customs of the Egyptians, he describes their divination.

Then he describes:

how they do medicine; every doctor is a doctor for one disease only; the whole country is full of doctors; some are for the eyes, some are for the head, some are for the teeth, some are for the belly, some are even for diseases one cannot see (II, 84).

Then he describes their burial customs.

Three. Writing at length about Darius, he reports:

once, while hunting, Darius jumped from his horse, and twisted his ankle; the king had Egyptian doctors, because the Egyptian doctors were thought the best, but they made his trouble worse; after a week of pain and sleepless nights, someone told the king about Democedes, and the king

ordered Democedes brought before him at once;

Democedes was from Croton; he had gone first to Aegina and had showed such skill that after one year the Aeginetans had hired him for 60 minae; the following year the Athenians had hired him for 100 minae; the year after that Polycrates of Samos had hired him for 120 minae; it was at that time, and not least because of Democedes, that the Crotoniates became famous as the best doctors;

Democedes cured Darius's ankle, and was lavishly rewarded; when the king ordered the Egyptian doctors impaled, Democedes begged the king for their lives and saved them (III, 129-32).

Later on, Democedes cured a growth on the breast of Darius's wife Atossa (133).

The Babylonians brought their sick to the marketplace. This second wisest Babylonian custom anticipates the Hippocratic: it seems to me of the greatest importance that one confine himself to things which everyone knows, for the matters to study are the diseases of which he himself suffers.

There are in the Mesopotamian record some dozens of tablets concerned with distress and disease.

A few are early, from the time of the princes of Ur. The most sophisticated is
2103 list of 15 remedies.

For what exactly they are remedies is not a concern of the writer. In any case, the remedies are of two kinds, to be administered internally and externally, respectively. This is of one the first kind:

pour strong beer over resin, heat it on the fire, put it in bitumen oil, let the sick drink it (9).

This is one of the second kind:

pour water over dried ground snake, mix it with ground root of fir and feces of bat, heat it, wash the sick with it, then rub him with oil (13).

Most are late, close to the time of which Herodotus writes. Among the most sophisticated are

666 30-odd prognoses,

666 100-odd prognoses.

These are some of the prognoses in the first:

if the right breast is red, he will live, if the left breast is red, he will die, if both breasts are red, it does not matter;

if the right breast is yellow, he will get worse, if both breasts are yellow, it is not good;

if both breasts are black, he will die quickly;

if the right breast is dark, he will get worse, if the left breast is dark, he will die painfully, if both breasts are dark, he will die (*id.*).

The tablet is Sm 232, in the British museum.

These are some of the prognoses in the second:

if he has burning pain in the stomach on the left, and vomits blood for thirty-one days, it is the hand of Nergal, he will die;

if he has burning pain in the stomach on the right, and vomits all the time, it is the hand of Ishtar, he will live;

if he has burning pain in the stomach on the left, and does not vomit, it is the hand of Ishtar, he will die;

if he has burning pain in the stomach on the left, and drips blood, it is the hand of Adad, he will die (*id.*).

The tablet is A 3506, also in the British museum.

In Egypt the whole country was full of doctors. The Egyptian doctors were thought the best. Homer earlier corroborates: in Egypt every man is a doctor and wiser than men anywhere else.

There are in the Egyptian record a half dozen papyri concerned with distress and disease. They corroborate Homer and Herodotus, attesting the greater sophistication of medicine in this valley.

The most sophisticated is

1560 48 treatments of injuries.

The 48 treatments of injuries is the peak in the development of medicine before the philosophers.

Of the 48 injuries, 27 are to the head, 6 to the neck or the throat, 5 to the arm, 10 to the breast, the shoulder, or the spine. The concern is surgical. Three instances:

if you see a gaping wound in the head splitting the skull, you should touch the wound; if you find something bad in the wound, he shudders, the swelling is large, he bleeds from the nose or the ears, his neck is stiff, he cannot look at his shoulders or breast, you should say, I will treat him; you should not bind him; his treatment is sitting; make two bricks to support the head, and wait for the turning point (4);

if you see a wound in the forehead reaching the skull, you should prepare egg from an ostrich, mix it with grease, and put it in the wound; the

spell for the remedy is this, repelled be the enemy who is in the wound, cast out be the evil who is in the blood, this temple will not crumble, Isis is my guardian, the son of Osiris will rescue me; after that cook figs, grease, and honey, and apply this to the wound (9);

if you see crushed vertebrae in the neck, one fallen into the other, if he will not speak, his head drops, he does not feel his arms or his legs, there is no treatment (33).

Mummification of cats, crocodiles, ibises, and most of all of the princes and their retinue, is attested over some two millennia. The preparation of the mummy required the removal of the viscera. The viscera were stored in jars. There is no evidence that the removal of the viscera, or their storage in jars, over two millennia, suggested their examination, never mind observation, never mind investigation.

Democedes was from Croton. It was at that time, and not least because of Democedes, that the Crotoniates became famous as the best doctors.

That time is,

562 Democedes.

Herodotus does not report that Democedes was a Pythagorean. Iamblichus does (*Pythagoras*, 257). Herodotus reports that Democedes was betrothed to the daughter of his fellow Crotoniate the wrestler Milo (III, 137). Iamblichus reports that the six times Olympic champion Milo was also a Pythagorean (*id.*).

The next is the generation of Alcmaeon. Alcmaeon was also from Croton, also a Pythagorean.

The physical problem of the soul is all from the development of philosophy. The relation of the formulation and solution of the problem to any earlier development is very remote.

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The ethical problem of the soul is also from the development of philosophy. But the relation of the formulation and solution of this problem to earlier developments is not remote. There are in this case two earlier developments which lead to the problem.

One is the discovery of the immortal soul of man in the valley of the Nile. The other is the

great expansion of the state and attendant mixing of peoples, and hence the development of the law, in the valley of the two rivers.

The immortality of the soul enters philosophy with Pythagoras. But it predates Pythagoras by eighteen centuries.

Herodotus reports that Pythagoras taught the immortality of the soul.

He reports also: the Egyptians first said that the soul of man is immortal, that when the body dies it creeps out and enters into some other animal born at this time, and that in this way it goes through all things living on land, in the sea, and in the air, until it creeps back into a human body; the cycle, they say, is 3 000 years.

And the immortal soul of man pervades the Egyptian record.

The development of the state in the valley of the Nile divides into the seven and a half centuries of the old kingdom, followed by two centuries of disorder, then the three centuries of the middle kingdom, then again two centuries of disorder, then the four centuries of the new kingdom, then again five centuries of disorder. Then the Assyrian princes conquered the valley of the Nile.

The old kingdom begins with

Narmer prince c. 3000.

And the third prince already,

Djer prince c. 2960,

leaves behind his tomb. His successors do as Djer does, and all leave behind their tombs. Then, six and a half centuries after Djer, in the tomb of

Unas prince c. 2320,

228 spells are inscribed which will take Unas to heaven. Unas reports:

Unas, an indestructible spirit, is coming to Re in heaven.

In heaven:

Unas is the servant of the gods,

Unas squats before Re, Unas opens his coffers, Unas unseals his decrees, Unas seals his dispatches,

Unas does what Unas is told.

Also:

Unas eats the gods,

Unas eats their magic,

the big ones are his morning meal, the middle ones are his midday meal, the little ones are his

evening meal.

Also:

Unas sheds his seed,
 Unas has any woman Unas wants.
 The successors of Unas are,
 Teti prince c. 2300,
 Pepi prince c. 2280,
 Merenre prince c. 2260,
 Pepi II prince c. 2240.

Teti, Pepi, Merenre, and Pepi II, an indestructible spirit each, each does as Unas does. Each copies and multiplies the spells which will take him to Re in heaven, for a total of 759 spells.

Then the old kingdom is destroyed, and for two centuries disorder follows. The immortal soul of man survives. The middle kingdom rises. The immortal soul of man flourishes. The middle kingdom is destroyed, and greater disorder follows. The immortal soul of man flourishes. The new kingdom rises. The immortal soul of man flourishes. The new kingdom is destroyed, and yet greater disorder follows. The immortal soul of man flourishes. The Assyrian princes conquer the valley, then the great kings conquer it, then Alexander conquers it, then the Romans conquer it. The immortal soul of man flourishes.

The immortal soul of man pervades the Egyptian record. Just four instances.

2022 poem beginning, I opened my
 mouth to my soul.

The poem is from the time early in the middle kingdom. In particular:

I opened my mouth to my soul,
 to answer what it had said,
 it is too much if my soul will not talk to me (*id.*).

The outcome is beneficial. Having threatened to leave me, my soul talks to me, and stays (*id.*).

1690 Horemkhauf.

Horemkhauf was a priest and official at the time of disorder after the middle kingdom. On a funeral stele, Horemkhauf describes his deeds, and concludes:

I was an excellent official on earth, I will be an excellent spirit in the netherworld (*id.*).

Anticipating Plato. Excellent on earth, excellent in the netherworld.

938 Nebneteru.

Nebneteru was a priest and minor official at the time of disorder after the new kingdom. On a

funeral statue, Nebneteru describes his deeds, then implores his successors:

priests! divine fathers of Amun! give water to my statue and you will want for nothing; for my soul rises early, the spirit of the priest and royal official, every day, without end (*id.*).

Finally,

27^{BC} hymn to Imhotep.

Inscribed in the temple of Ptah at Karnak, the hymn implores:

your priests offer you all good things,
 wine, beer, and milk, and incense at night;
 may your soul come from heaven to this house of
 yours every day;

when your soul hears the song of the priest,
 may you hear the chants of your servant as he sets
 all good things for your spirit;

for you are the healer,
 you bring back to life;

may your soul accept all that you love from your
 son Caesar Augustus (*id.*).

The hymn implores Imhotep. Imhotep had been the chief official of

Djoser prince c. 2680^{BC}.

Djoser's is the oldest pyramid. Imhotep had built this oldest pyramid, and by the time of that hymn he had long been deified. Caesar Augustus, on whose behalf the hymn implores Imhotep, is

Tiberius prince 14-37^{AD}.

The fathers of the church will date the resurrection of our lord and savior Jesus Christ to the 15th year of Tiberius.

The Egyptian record attests little development of the immortal soul of man. But it does rather attest its indestructibility.

The immortal soul of man pervades the Egyptian record.

In the Mesopotamian record – very much more massive than the Egyptian – on the other hand, the immortal soul of man is nowhere to be found. Not even in the writings comprising the Hebrew scriptures. Immortality – and the immortality of the body, not the immortality of the soul – enters the writings comprising the Hebrew scriptures with the latest of them,

189^{BC} *Daniel*.

Daniel postdates Pythagoras by four centuries, Plato by two and a half centuries.

Unas *Daniel* postdates by twenty-two centu-

ries.

Pervading the one, nowhere to be found in the other, the immortal soul of man cannot have been shown more clearly and distinctly discovered. It was discovered in one of the two valleys, and not in the other. Just as the sustained observation of the heavenly bodies was discovered in one of the two valley, and not in the other.

In the same way, the state began expanding greatly in one of the two valley, and hence the law developed in one of the two valleys, and not in the other.

Ethics is the generalization from what is good to what is the best for all of you. And the steps toward this generalization, extensively, with the mixing of peoples under the expanding state, and intensively, with the development of the law, are a development which leads to ethics, in general, and to the ethical problem of the soul, in particular.

The great expansion of the state begins with Sargon prince c. 2337-281 and his successors. Sargon and his successors conquered the entire valley of the two rivers and maintained it under the one and only one state of Akkad for one and a half centuries. To do so, they campaigned annually from one end of the valley to the other. And thus with Sargon and his successors begins the mixing of peoples extensively, by campaigning over them for one and a half centuries.

Disorder followed the princes of Akkad.

Then Ur-Nammu and his successors brought and maintained under the one and only one state of Ur much of the valley.

The oldest code of laws is the code of Ur-Nammu prince c. 2121-03.

And Ur-Nammu's code attests the first attempt at the mixing of peoples intensively, by bringing them under one and only one law. Ur-Nammu and his successors maintained the valley, and the peoples in it, under one state, and under one code of laws, for one century.

Ur-Nammu's code has some 40 laws.

Next is the code of

Lipit-Ishtar prince c. 1933-22,
with some 50 laws.

Next is the code of

Dadusha prince c. 1785,

with some 60 laws.

Next is the code of

Hammurabi prince c. 1792-49,
with some 280 laws. The greatest, the code of Hammurabi was copied, and again, and again, for the following twelve-odd centuries.

Lipit-Ishtar and Dadusha are each the prince of his own state only, Lipit-Ishtar of Isin, Dadusha of Eshnunna. Their inscriptions yet attest, and the inscriptions of the princes of the neighboring states amply corroborate, the continuing labors of the state to bring all neighboring states under itself.

Hammurabi completed these labors, and was once again the lord of the entire valley. Under Hammurabi and his successors the valley remained under the one state of Babylon, under one code of laws, mixing the peoples, for two centuries.

After a hiatus of two and a half centuries, the rise of the Assyrian princes began with

Ashur-uballit prince c. 1363-27,
and the development continued and accelerated both extensively and intensively.

Tukulti-Ninurta prince c. 1243-06,
conquered the entire valley,

Tiglath-pileser prince 1115-076,
conquered nearly the entire valley,

Ashur-nasirpal II prince 884-59
conquered nearly the entire valley, and

Tiglath-pileser III prince 745-27
and his successors through

Ashurbanipal prince 668-27
brought under the one state of Assyria the peoples of the entire valley, much of Asia Minor, and the valley of the Nile, too. Not less than one tenth of humanity. The mixing of one tenth of humanity continued under the Assyrian princes vigorously, and increasingly vigorously. The vigor is attested by the Hebrew prophets, the Hebrews being one of the objects of the mixing.

Intensively, this is the time when god himself began giving you the law. It is the time of the first two of the four steps in the development of the law of the Hebrew scriptures,

860 Yahwistic document,

662 Deuteronomy.

The steps are four, counting the three steps in the development of the torah as well as Ezra. The

torah is the foundation of the Hebrew scriptures, the torah is the law, and the torah developed in three steps, the Yahwistic document, *Deuteronomy*, and the Priestly code. The law is not a primary concern of the Yahwistic document. But it is the primary concern of *Deuteronomy* and of the Priestly code. All three collect the law in codes. The first step, the code of the Yahwistic document, is in *Exodus* (20-3). The second step, and the most important, is the code of *Deuteronomy* (12-26). The third step, the final redaction of the law by the Priestly code, is *Leviticus*. Ezra then enforced the law as the constitution of the Hebrews.

The code of the Yahwistic document is not as sophisticated as the code of Hammurabi. The laws are fewer, regulating fewer relations, and the relations they regulate are simpler. The relations they regulate are simpler, yet also much the same, from killing, eyes, and teeth: life for life, eye for eye, tooth for tooth (21), to widows, orphans, and asses: you shall not molest the widow or the orphan (22), and what you shall do: when a man gives his ass to another man (*id.*). The major developments are two. The code begins:

you shall have no other gods beside me (20). The laws are given to you explicitly by god, other than whom you shall have no gods, and if you do in this way as you shall, it is god himself who gives you the laws. Also, he gives them to you, in particular:

you shall not,
you shall,
you shall not,
you shall (*passim*),

and not less than 50 times.

The major developments with *Deuteronomy* are also two. The code is now substantially more sophisticated than the code of Hammurabi. The number and complexity of what you shall not, and shall, has increased greatly. Hence, also, so have your rewards and punishments:

if you obey the law, you will be blessed above all other peoples, you will destroy all the peoples whom the lord god delivers to you (7);

if you forget the lord god, he will destroy you (9).

He gives the law to you, in particular. Not to all of you, however. You have long been taught,

not by the Hebrew scriptures, by those to whom the Hebrew scriptures are the old testament, that he gives the law to all of you. That it is not to all of you that he gives the law, however, your rewards and punishments cannot have been more explicit. He gives the law to you, if you are a you of his people. If you are a you of not his people, he does not give the law to you. So much so, and so explicitly so, this can explain why, when you have been a you of not his people, you have tended to do what you can to show that his people do not obey the law.

In the following two and a half centuries – the centuries from Thales to Aristotle – the development accelerated still more both extensively and intensively.

Extensively, the expansion of the state reached a very high peak with the great kings, and the great kings are a decisive step in the mixing of peoples. Beginning with

Cyrus prince 559-30,
they mixed the peoples from Asia Minor to Libya to Ethiopia to India for two centuries. Mixed for two centuries one fifth of humanity, one fifth of all of you then alive.

Not only extensively either. King Darius says: king Darius says, the peoples were in commotion, all peoples fighting each other; by the grace of Ahura Mazda I brought an end to the fighting; they no longer fight each other at all, all are at peace; they fear the law.

Intensively, the development of the law of the Hebrew scriptures was completed. The dates are,
529 Priestly code,
479 Ezra.

With the Priestly code and its final redaction the codification of the law became more elaborate than any earlier, comprising 613 laws now. 613 you shall not, and shall. Why 613, the amoraim, in the talmud, explained: 365 prohibitions, because this is the number of days in the year, and 248 injunctions, because this is the number of parts in the body (*Lashes*, 23b).

Ezra then enforced the law. In the great assembly of the people under Ezra, after twenty-four days of reading of the law, celebration, and prostration, the people bound themselves in writing to obedience to the law. In preparation for the great assembly: all foreign wives and all

children born to them were expelled (*Ezra*, 10). It was also decreed: you shall not give your daughters to their sons in marriage, you shall not let their daughters marry your sons; and you shall do nothing for their well-being or advantage (9).

It was not a symmetrical relation. The great assembly was convoked under the auspices of the great king, organized by the cupbearer of the great king.

The great assembly counted the people: the entire congregation of the people numbered 42 360, not counting their slaves, male and female, which were 7 337; they also had 245 singers, male and female, 736 horses, 245 mules, 435 camels, and 6 720 asses (*Nehemiah*, 7).

Without the horses, mules, camels, and asses, the people, their slaves, and their singers were 49 942. With the horses, mules, camels, and asses they were 57 888. Either way, they were not more than one four hundredth of only the peoples currently mixed by the great kings.

The mixing of one fifth of humanity under the great kings, the enforcement of the law such as, if you obey it, you will be blessed above all other peoples, if you forget the lord god, he will destroy you, and the first couple of centuries of the development of philosophy are contemporary.

The immortal soul of man entered philosophy with the fourth philosopher. One century later ethics began with Socrates. In the next generation ethics became ubiquitous. In the same generation Plato formulated and solved the ethical problem of the soul.

To find the cause of the immortal soul of man cannot be difficult. Its discoverer already, the indestructible spirit, indicates it rather unambiguously. He eats, morning meal, midday meal, evening meal. And he sheds his seed, has any woman he wants.

The same cause can be assigned to the celerity with which the immortal soul of man found itself in ethics. Ethics is the generalization from what is good to what is the best for all of you. And but one generation after the discovery of the generalization, while Diogenes was exerting himself to teach you to prefer nothing to freedom, Plato taught you to consider the nature of the soul, and conclude that it is immortal.

To assign a cause to the expansion of the state,

and hence the mixing of peoples and the development of the law, is more difficult.

It can be presumed that of the mixing of peoples and the development of the law the expansion of the state is the cause. This is indicated strongly by the development of the state through the Akkadian princes through the princes of Ur through the Babylonian princes through the Assyrian princes, including the incorporation of the state in the valley of the Nile into their one and only one state, never mind the incorporation of the petty principalities of the Hebrews into their one and only one state, through the great kings.

And of the expansion of the state, and hence the mixing of peoples and the development of the law, selection cannot be the cause.

To begin with, selection cannot produce the state. Not if the state is a power contrary to our natural passions, keeping men in awe, reducing their wills unto one will.

Selection cannot produce the expansion of the state either. 5 400 ate bread daily before Sargon. And Sargon, the lord of the earth, won 34 battles. Opportunism will make 5 400 eat bread daily before Sargon. But it will not make Sargon, never mind the 5 400, try to win, let alone win, 34 battles. Nor can selection produce the generalization implicit in the expansion of the state over steadily more of the surface of the earth. And that the expansion of the state is such a generalization – that the expansion of the state is exploration of the surface of the earth – Naram-Sin is already clear. Naram-Sin, lord of the four quarters, since man was created, no prince of all princes had conquered Armanum and Ebla. Naram-Sin, prince of Akkad, when he took the path to Tal-had, no prince of all princes had taken this path.

Selection cannot produce the generalization implicit in bringing all states under one and only one state either.

Nor can it produce the generalization implicit in bringing steadily more peoples, steadily more of humanity, under one and only one state.

Or under one and only one law.

That selection cannot produce these generalizations is indicated strongly by the resistance against them, too.

The one and only one state has had to be

rebuilt, and again, and again.

Under the auspices of the great kings, the great assembly excluded itself from the great kings' mixing of peoples.

Plato as well, teaching though he was all of you, disapproved of the great kings' mixing of peoples, Greek with Greek, Greek with barbarian, barbarian with Greek, all mixed together.

More generally, in place of all states on earth Plato built one for you in heaven. In that state as justice in the state so justice in the soul, as justice in the soul so justice in the state.

Aristotle, for his part, writes: even when the end is the same for man and for the state, the end of the state is greater and more complete.

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The twenty centuries from Aristotle to Descartes.

Beginning again with the physical problem.

In these twenty centuries, these 30 are all important philosophers who are part of the development of the physical problem of the soul,

371 - 287 **Theophrastus**,
 371 **Dicaearchus**,
 346 **Herophilus**,
 341 - 270 **Epicurus**,
 335 - 263 **Zeno of Citium**,
 327 - 269 Strato,
 316 **Erasistratus**,
 130 Asclepiades,
 27^{BC} Celsus,
 24^{AD} - 79 **Pliny**,
 77 Dioscorides,
 129 - 208 **Galen**,
 860 - 930 **al-Razi**,
 980 - 1037 **ibn Sina**,
 1098 - 1179 Hildegard,
 1194 - 1250 **Frederick II**,
 1200 - 1280 **Albert**,
 1225 - 1274 **Aquinas**,
 1270 - 1326 Mondino,
 1478 - 1553 Fracastoro,
 1488 - 1534 Brunfels,
 1493 - 1541 **Paracelsus**,
 1498 - 1554 Bock,
 1514 - 1564 **Vesalius**,
 1516 - 1565 **Gesner**,
 1517 - 1564 **Belon**,

1519 - 1603 **Cesalpino**,
 1578 - 1657 **Harvey**,
 1585 - 1630 Cesi,
 1596 - 1650 **Descartes**.

To show that the line between important and not important is drawn among the particulars, these are 9 of the more important among the not important,

354^{BC} Meno,
 58^{AD} Rufus of Ephesus,
 773 - 868 al-Jahiz,
 1501 - 1566 Fuchs,
 1507 - 1566 Rondelet,
 1510 - 1590 Paré,
 1523 - 1562 Fallopius,
 1533 - 1619 Fabricius,
 1561 - 1636 Sartorio.

Meno, also a student of Aristotle, wrote the first history of medicine, as Theophrastus and Eudemus wrote the first histories of physics and of mathematics. Rufus of Ephesus has left behind *On the names of the parts of the human body*. Al-Jahiz wrote *On animals*, drawing a little on Aristotle, more on traditional stories, a bestiary focused firmly on the moral of the stories, and the closest the Muslim philosophers came to concerning themselves with animals. Fuchs wrote *On the history of plants*, describing some 500 kinds, the third of the treatises, after those of Brunfels and Bock, with which the restoration of botany began. Rondelet wrote *On marine fishes, with their images, and universal history of aquatic animals*, describing some 300 kinds, the third of the treatises, after those of Gesner and Belon, which laid the foundations of ichthyology. The last 4 are part of the acceleration of the development of medicine after Vesalius. Paré wrote *The method of treating wounds caused by harquebuses and other guns*, in 1545. Fallopius wrote the *Anatomical observations*, in 1561. Fabricius wrote *On the formation of the foetus*, in 1604. Sartorio wrote *On the art of measurement in medicine*, in 1614.

In the first of these twenty centuries the investigation of plants advanced greatly, theoretical medicine advanced greatly, and there was a great advance in the formulation and solution of the problem of the soul. Then the investigation of plants did not advance for nineteen centuries. The investigation of animals, with but one exception,

did not advance in these nineteen centuries either. Theoretical medicine advanced again four and a half centuries later, and there was also a substantial advance in the formulation and solution of the problem of the soul. Then theoretical medicine did not advance for fourteen centuries. Nor was there an advance in these fourteen centuries, nor could there be, in the formulation and solution of the problem of the soul. Then, in the last of these twenty centuries, at long last, theoretical medicine advanced greatly, the investigation of animals advanced, the investigation of plants advanced, and there was a great advance in the formulation and solution of the problem of the soul.

The twenty centuries can be divided into three parts. The five centuries from Aristotle to Galen. The fourteen centuries from Galen to Vesalius. And the not quite one century from Vesalius to Descartes.

Theophrastus wrote the *History of plants* and *Causes of plants*, in 15 books, describing 550 kinds of plants. Aristotle had advanced the investigation of animals from more or less 0 to 600. Theophrastus advanced the investigation of plants from more or less 0 to 550.

Theophrastus wrote also the brief *On fish*. He considers two kinds of fish: fish that venture on dry land and live in the air (1-6), and: fish that burrow in the earth and live there (7-11). He concludes by asking: is water the natural place of these fish (12)?

Aristotle had proposed that the fundamental kinds of animals are aquatic and terrestrial. The importance of Theophrastus's concern is thus clear. It is clear also that, where the fish are concerned, the collection of the particulars has not yet begun, nor has the examination of the fundamental kinds of animals on this ground.

Where Aristotle and Theophrastus left the investigation of animals and plants, there the investigation of animals and plants remained for nineteen centuries. And this proposition demands but three qualifications. There was one advance on Aristotle, and but one, in these nineteen centuries, the emperor, Frederick II, investigated birds. There was one addition to Theophrastus, and but one, in these nineteen centuries, Dioscorides described the medicinal virtues of plants. And

there were three advances in theoretical medicine, and but three, in these nineteen centuries.

Dicaearchus said that the soul is nothing.

Cicero reports: Dicaearchus wrote a treatise, in 3 books, in which he argued that the soul is nothing; the word soul means nothing, animals and plants are said to have a soul without any reason; neither in man nor in beast is there such a principle, immaterial or material; all capacity for action and all sensation are diffused throughout the body and cannot be separated from the body; there is nothing apart from the body, which is built in such a way that all its activities and powers of sensation are due entirely to the natural combination of the parts (*Tusculan*, I, 21).

The soul is nothing. There is nothing apart from the body, which is built in such a way that all its activities and powers of sensation are due entirely to the natural combination of the parts.

It is a spectacular result, anticipating developments two millennia into the future. Much like Aristarchus's result, the earth revolves around the sun. Much like Aristarchus's result, too, Dicaearchus's result was ignored by his successors. Cicero adds: but let us ignore Dicaearchus (41). And so the philosophers did, ignored Dicaearchus. Empiricus twice notes in passing: Dicaearchus said that the soul is nothing (*Outlines*, II, 31; *Mathematicians*, VII, 349). And that was that.

The first two of the three advances in theoretical medicine are Herophilus and Erasistratus.

One generation after Theophrastus and Dicaearchus, Herophilus identified the origin of the nerves in the brain (Galen, *Books*, 3), distinguished sensory from motor nerves (Galen, *Parts*, X, 12), described the paths of cranial nerves (*id.*), distinguished the cerebrum from the cerebellum (Galen, *Procedures*, IX, 5), distinguished the optical nerve (Galen, *Parts*, X, 12), identified the cornea, retina, iris, and choroid coat (Rufus of Ephesus, *Body*, 153).

Herophilus also said that disease is caused by imbalances of the four humors (Celsus, I, pr. 15).

One generation after Herophilus, Erasistratus distinguished the veins from the arteries (Galen, *Doctrines*, VI, 6), identified their origin in the heart and their principal branches (*id.*), inferred the capillaries (Galen, *Parts*, VI, 17), showed that air from the lungs is carried to the heart (Galen,

Procedures, VII, 4), described the heart valves and identified their function in ensuring the irreversibility of blood flow (Galen, *Doctrines*, I, 10).

Erasistratus also said that disease is the morbid seepage of blood in the vessels carrying pneuma and of pneuma in the vessels carrying blood (Celsus, I, pr.15).

With Herophilus and Erasistratus anatomy and physiology begin.

Also, with Herophilus and Erasistratus theory is distinguished from practice at its sharpest.

Herophilus and Erasistratus performed human vivisections (Celsus, I, pr.23-4).

Harvey, in *On the movement of the heart and blood in animals*, in 1628, begins: when in many vivisections – and all results, which Harvey reports in *On the movement of the heart and blood in animals*, Harvey reports established during the vivisection of animals. For, he writes: the error of all anatomists is that they only dissect dead men (6). Harvey performed no human vivisections. In the *Second essay to Riolan*, however, he reports: I have sometimes opened the chest and pericardium of a strangled human cadaver within two hours of its hanging, before the redness of the face had disappeared (*id.*). The fresh human cadavers the dispensation of justice provided. As it had the specimens investigated by Herophilus and Erasistratus.

Where Herophilus and Erasistratus left theoretical medicine, there theoretical medicine, with but the one exception of Galen, remained for eighteen and a half centuries.

And where Aristotle left the physical problem of the soul – never mind Dicaearchus – there the physical problem of the soul remained for nineteen centuries, too.

In the five centuries from Aristotle to Galen the chief cause are the schools.

Plato founded the Academy c. 385, Aristotle founded the Lyceum in 335, Epicurus founded the Garden in 307, Zeno of Citium founded the Stoa in 301. In the following century all other schools dwindled – the followers of Euclid of Megara, the Megarians, the followers of Aristipus, the Cyrenaics, the followers of Diogenes, the cynics, the followers of Pyrrho, the sceptics. The Academy, Lyceum, Garden, and Stoa flourished,

and all four well into Christian times.

What more particularly flourished, two of the schools, Aristotle's and Epicurus's, can illustrate here. The other two schools, Plato's and Zeno of Citium's, can illustrate later on, when the ethical problem of the soul is considered.

At the Lyceum: Theophrastus had 2000 students (Laertius, V, 37).

1 of the 2000, Strato, was a physicist, the rest were not. And the ratio of 1:1999 indicates clearly and distinctly the direction in which the Lyceum was to continue developing.

The first four scholars of the Lyceum, with the years when the school was under each, are,

Aristotle	335 - 322,
Theophrastus	322 - 287,
Strato	287 - 269,
Lyco	269 - 225.

Of Aristotle Laertius has 156 treatises to record, counting the 158 constitutions which Aristotle collected 1 treatise (V, 22-7). Of Theophrastus he has 226 treatises to record (42-50). Of Strato he has 46 treatises to record (59-60).

Of Lyco, scholar for forty-four years, he has no treatise to record. He has Lyco's wise sayings to record (65-6), and Lyco's taste in dress (67), this last philosophical concern of Lyco's, too, reaching Laertius across the five centuries separating the two. The Lyceum had by Lyco's time become not just a school of not writers either. It had become a school of not readers. Strabo reports: Aristotle left his books to Theophrastus; Theophrastus left his books to Neleus, and Neleus took them to Asia Minor and left them to his heirs; the result was that after that time the Peripatetics had no books at all, with the exception of only a very few (XIII, 1.54). Plutarch corroborates the fact (*Sulla*, 26). Nor is there a hint in the record that in the following two centuries anyone, at the Lyceum or in Asia Minor or anywhere else, found any use for the books of Aristotle or his successors.

Epicurus writes: nothing comes out of nothing; nothing goes into nothing; the whole is always the same, and it is nothing but bodies and void (*Herodotus*, 39); bodies are composed of the elements (40); the elements are atoms (41); atoms have no qualities, but shape, weight, and size; all bodies are configurations of atoms (54). This is

not new with Epicurus.

Cicero summarizes Epicurus thus: he considered the logic of the Stoics of no use in bettering one's life and mind; he considered physics of the highest importance; it is in the course of the study of physics that we learn the meaning of terms, the nature of discourse, and the rules of inference; knowledge of the nature of things frees us from superstition and fear of death, knowing what nature requires we escape from ignorance and become better (*Ends*, I, 63).

That in the course of the study of physics we learn logic is new with Epicurus. That physics frees us from superstition and fear of death, and so we become better, is also new with him. The goal of making us better is also not new with him.

The most important Epicurean is Lucretius, two and a half centuries after the master. Lucretius sings of the master:

he ventured
far beyond the flaming ramparts of the world,
his mind and soul traversed the all's immensity
(I, 72-4);

and to the master:

you are the father, the discoverer of things,
you give us precepts, so that we,
bees sipping at every blossom,
eagerly feed on your golden sayings (III, 9-12).

Granted, for the sake of the argument, that the master ventured far beyond the flaming ramparts of the world, his followers avoided such perilous undertakings. They kept to sipping at every blossom.

About the soul Epicurus said:

the soul is corporeal, composed of particles, dispersed over the whole body, resembling very nearly wind with an admixture of heat (*Herodotus*, 63); it scatters after death (66).

Zeno of Citium said:

the soul is a warm breath.

Strato said:

all activities of the soul are motions; the soul can think nothing which it has not first seen (Simplicius, comm. *Physics*, 965).

Physics advanced greatly in these centuries, with Autolycus, Euclid, Timocharis, Aristarchus, Archimedes, Eratosthenes, Hipparchus, Hero, Ptolemy. None of it in the schools. None of it is about animals or plants either.

Theoretical medicine did not advance in the four and half centuries from Erasistratus to Galen either.

The chief cause in this case is practical medicine.

Treatises kept being added to the Hippocratic corpus. I do not think medicine needs new hypotheses. Medicine has long had everything it needs to be knowledge. And it seems to me of the greatest importance that one confine himself to things which everyone knows, for the matters to study are the diseases of which he himself suffers.

The most important doctor in these four and a half centuries is, two centuries after Erasistratus, two and a half before Galen, Asclepiades.

Galen wrote books on 3 particular physicians among his predecessors, Hippocrates, Erasistratus, and Asclepiades.

Asclepiades said that the principles are seamless particles. He said that disease is caused by the impediment of the motion of the seamless particles in the body (Celsus, I, pr.15); he said that his aim is to cure safely, swiftly, and pleasantly (III, 4.1-2); he used no drugs for treatment, only diet (V, 1.2).

Pliny writes: the men of the greatest eminence in medicine are Hippocrates, who foretold a plague and dispatched his pupils throughout the cities to help, Cleombrotus, who saved the life of king Antiochus, and Critobulus, who extracted an arrow from the eye of king Philip; yet the highest renown belongs to Asclepiades, who founded a new school, disdained the calls of Mithridates to serve him, discovered a way to prepare wine for the sick, took a man from his burial and brought him back to life, and most of all because he made a wager with fortune that he should not be deemed a physician if he was ever in any way sick, and he won the wager, for he died in an extreme old age by falling off a staircase (VII, 123-4).

A new school, disdain for Mithridates, wine for the sick, a man brought back to life, a wager with fortune, pleasant cure – it is difficult to disagree with Pliny.

One century after Asclepiades, Celsus wrote a treatise on agriculture, rhetoric, the military arts, and medicine. The part on medicine survives. It

is in 8 books. Celsus writes: medicine has three parts, cure by diet, cure by drugs, and cure by hand (I, pr.9). Books I-IV deal with the cure by diet, books V-VI with the cure by drugs, book VII with the cure by hand. Book VIII is an elementary description of the body.

Not quite two generations later, Pliny wrote the *Natural history*, in 37 books. He reports: we have read 2 000 books; we have collected from them 20 000 facts, and have also added from our experience. How much Pliny has added from his experience, the number of the facts on animals, plants, and drugs suffices to show. These are the relevant books with Pliny's count of the facts in the them:

VIII-XI	animals	4 931,
XII-XIX	plants	7 685,
XX-XXXII	drugs	13 820.

The quantity is excellent. The quality varies. For instance, book IX is on aquatic animals. The facts are 650. Among them:

the largest number of animals and the largest of them live in the Indian ocean; there are whales 4 iugera in size and sharks 200 cubits long; the lobsters there grow to 6 feet and the eels in the Ganges to 300 feet (IX, 4-5).

1 iuger is 2 500 m², 1 cubit is about $1\frac{1}{2}$ feet, 1 foot is 0.3 m. Also:

an embassy from Lusitania came to Tiberius to report that a triton had been seen and heard singing in a cave, with exactly the well-known shape (9).

The well-known shape was half man half fish. And still among the facts on aquatic animals:

the legate of Gaul wrote to the divine Augustus to report that a large number of dead nereids had been washed on the shore (*id.*).

The well-known shape was that of woman.

A half century later, Dioscorides wrote *On the materials of medicine*, in 5 books, describing some 600 plants, 35 animal substances, and 90 minerals, focused firmly on their medicinal virtues.

A half century later, Galen advanced medicine greatly. Galen wrote 115 treatises, and these superseded the Hippocratic corpus. Galen advanced theory. The fact that his treatises superseded the Hippocratic shows that he advanced

practice. And he advanced practice rather more than theory.

One of the 115 treatises is *On my own books*, and in it Galen analyzes his own books into the following 17 kinds:

1. written during my first stay in Rome;
2. written before going to Rome;
3. on anatomy;
4. on therapy;
5. on prognosis;
6. commentaries on Hippocrates;
7. expressing differences with Erasistratus;
8. about Asclepiades;
9. expressing differences with the doctors of the Empiricist school;
10. expressing differences with the doctors of the Methodist school;
11. on logical proof;
12. on ethics;
13. on the philosophy of Plato;
14. on the philosophy of Aristotle;
15. on the philosophy of the Stoics;
16. on the philosophy of Epicurus;
17. on language and rhetoric (*id.*).

Galen's concerns were clearly general. They were also, also clearly, not too theoretical.

One of Galen's fundamental results is the explicit grounding of the doctrine of the four humors in the theory of the four elements.

Empedocles formulated the theory of the four elements.

The doctrine of the four humors can be traced back to the Hippocratic *On the nature of man*: the human body contains blood, phlegm, yellow bile, and black bile; and health is the right proportion of these four (4).

Herophilus said that disease is caused by imbalances of the four humors.

In *On mixtures*, Galen writes: animal bodies are a mixture of hot, cold, wet, and dry, just as all bodies are mixtures of fire, air, water, and earth (I, 1); and, determined by observation, the four humors can be described in terms of hot, cold, wet, and dry; thus, black bile is colder than blood, yellow bile is hotter, phlegm is the coldest (II, 4).

On this ground, with Galen's successors alchemy and medicine will become joint concerns – the labors at changing the mixtures of the elements and at changing the mixtures of the

humors will become fundamentally the same. As Roger Bacon writes: the same medicine which would remove all impurities and corruptions of baser metals so that they become the purest gold is believed by the philosophers capable of removing also the corruptions of the human body and prolonging life for many ages (*Work*, VI, 12).

With and without the theory of the four elements, the doctrine of the four humors dominated therapy, counting from Hippocrates to Virchow, for twenty-three centuries. In 1858, in the *Cellular pathology*, Virchow writes: the humoral schools have generally had the greatest success in medicine because they have offered the most plausible interpretation of morbid processes (1).

And on the same ground, Galen improves on Aristotle's solution of the problem of the soul substantially.

Animal bodies are a mixture of hot, cold, wet, and dry, just as all bodies are mixtures of fire, air, water, and earth. And in *That the affections of the soul depend on the mixtures in the body*, Galen argues what the title proposes: the affections of the soul depend on the mixtures in the body (1). In particular: the soul has three parts, nutritive, sensitive, and rational (3); the seat of the rational soul is the brain (2); the mixture in the heart is the sensitive soul; and the mixture in the liver is the nutritive soul (4).

Galen improves on Aristotle's solution of the problem of the soul with respect only to animals, and with respect only to these among them with a brain, heart, and liver. With respect to these animals, however, he not only assigns a seat to the rational soul, and affirms what the sensitive soul is, and what the nutritive soul is. Aristotle argues that the soul is the efficient, formal, and final principle of life. Galen argues that the formal principle is material. He writes: we must take Aristotle's form to be the mixture of the hot, cold, wet, and dry (3). Also, Plato argues that the soul is immortal. Galen writes: Plato says that the rational part of the soul is immortal, but I can neither affirm nor deny this (*id.*).

Also, throughout *On anatomical procedures* and *On the use of the parts* Galen discusses the anatomy of man in comparison to that of apes.

In the five centuries from Aristotle to Galen the formulation and solution of the problem of

the earth advanced very greatly. The grounds on which the problem was solved, the steadily more sophisticated mathematical theory part of which were steadily more and better observations, became steadily firmer, and the solution of the problem advanced all the way to Ptolemy. The formulation and solution of the problem of the soul advanced as well, but not nearly as greatly. The advance in the grounds, on which the problem was to be solved better, was relatively both very small and very infrequent, and the advance in the solution of the problem was correspondingly but the one step taken by Galen.

In the fourteen centuries from Galen to Vesalius the acquisition of knowledge in general fell on hard times. And still, hard as the times were on all problems, they were particularly hard on the physical problem of the soul. The philosophers looked at the stars, once in a long while. At animals and plants they barely looked. Even as they looked at the soul with great intensity.

In particular, as in general, the cause is orthodoxy, first Christian, then also Muslim.

In Islam, the closest the philosophers came to concerning themselves with animals is the bestiary of al-Jahiz. The closest they came to concerning themselves with plants are collections of drugs. Medicine flourished. Practical medicine. The greatest doctors are al-Razi and ibn Sina.

Seven and a half centuries after Galen, al-Razi advanced therapy. Where theory is concerned, al-Razi advanced alchemy, he is an important advance in the investigation of the elements.

One century-odd later, ibn Sina wrote the *Canon*. It is the most substantial contribution to therapy since Galen. Where theory is concerned, ibn Sina as well advanced alchemy, he is an important advance in the investigation of the elements as well.

In Christendom, the darkest of dark ages came to a close with Abelard and Gerard. Where animals, plants, and the problem of the soul are concerned, it came to a close very gradually.

Hildegard is a younger contemporary of Abelard, an older contemporary of Gerard, one century-odd after ibn Sina.

Hildegard had visions since she was fifteen. A nun in a Benedictine cloister, she became its prioress, then established her own convent. A

committee under the archbishop of Köln examined her visions, concluded that they are true, and appointed a monk to help her record them. Clairvaux was an enthusiastic supporter. Next to the books recording the visions, Hildegard wrote *On simple medicines* and *On compound medicines*. In the *Simple* she describes herbs, trees, animals, precious stones, and metals, focused firmly on their curative powers. In the *Compound* she describes diseases and their causes, drawing on Galen's humoral theory, and more on traditional stories.

One century later, Frederick II, *stupor mundi*, was emperor 1212-50. He fought throughout to restore the supremacy of the empire over the papacy. In 1231 he promulgated the constitutions of Melfi, the most important codification of the law between Justinian and Napoleon. And he laid the foundations of ornithology with *On the art of hunting with birds*.

On the art of hunting with birds is the one, and only one, advance in the investigation of animals between Aristotle and Gesner.

It is in 6 books. Book I is a survey of general ornithology. Book II deals with the rearing and training of falcons, book III with the use of the lure, books IV-VI with the techniques of falconry.

The advance is book I. The emperor deals with the classification of birds, and then describes their feeding habits, migration, mating, nesting, anatomy and physiology, flight, and molting. The description rests on observation and experiment, pursued by the emperor with greater determination and greater success than any of his predecessors since Aristotle. Two instances. The emperor reports:

like the cuckoo, the ostrich does not incubate its own eggs; in the case of the ostrich we ascribe the failure to perform this important task to its fear of breaking the eggs by the weight of its large body; moreover, the extreme heat of the sun is sufficient to hatch them; in Egypt even the eggs of barnyard fowl are kept warm and the young hatched in the sun, without the mother; we have observed this in Egypt, and have successfully repeated it here in Apulia (23g);

birds are instinctively aware of the proper date to begin migration so as to avoid rains and

storms; they are usually able to choose also a period of favorable winds; they prefer lateral or rear winds, and they wait for such winds with the same sagacity with which sailors wait for them (20); we have observed that, when a favorable wind begins, migrating birds will hasten to take advantage of it, neglecting even food and sleep for this important purpose; we have observed migrating birds, which have begun to devour the food we have thrown to them, abandoning the food and flying away as soon as a favorable wind begins; they will endure hunger and will travel day and night rather than miss such an advantageous opportunity (*id.*).

In the same generation, Albert wrote *On animals*, in 26 books, *On plants*, in 7 books, and *On minerals*, in 5 books. None of the three is an advance on Aristotle or Theophrastus. It is yet with these treatises that, having for a millennium concerned itself with little but punishments and rewards after the world, philosophy returned to concerning itself with the creatures in the world.

On animals is in 26 books.

Its bulk, books I-XIX, is an exposition of the *History of animals*, *Parts of animals*, and *Generation of animals*, in which Albert has little to add to Aristotle.

Books XX and XXI are Albert's generalization. In book XX he deals with the principles of animals:

the material principles, water (1.1); earth (1.2); air (1.3); fire (1.4); whether the fifth element is substantially present in animals (1.5); and how the elements are mixed in their bodies (1.8);

the power of the first principle (2.1);

how the formal principle, which comes from the soul and is in the seed, shapes the body (2.3); and how the formal principle is present in the body, which is a mixture of the four elements (2.4).

In book XXI Albert arranges the animals in a hierarchy: according to the modes of perfection (1). Highest to lowest, the animals are:

man, who alone is the most perfect animal (*id.*);

the pigmy, who is midway between man, who has divine intellect, and the mute animals, in whom the divine light cannot be perceived (2);

the monkeys, which are especially capable of

instruction (3);

the quadrupeds, who also can be taught (4);
and the birds, who also can be taught (5);

the aquatic animals, who, although they cannot be taught, are clever and shrewd (6);

the serpents, who are also clever and shrewd, if less so (7);

and the worms and insects, who are also clever and shrewd, if less so (8).

All three are important steps in the restoration of the investigation of animals, the focus on the material principles, the inclusion of man in a hierarchy including also the worms and insects, and the uniform criterion on which the hierarchy is arranged, the capacity to be taught and shrewdness.

Books XXII-XXVI are an alphabetically arranged bestiary. Albert describes in

book XXII	113 quadrupeds,
book XXIII	113 birds,
book XXIV	140 aquatic animals,
book XXV	62 serpents,
book XXVI	49 worms and insects.

In book XXIII there is a large section of falcons and falconry, in much of which, Albert reports: we have followed the expert findings of the emperor Frederick (20). The total of the animals in the bestiary is 477 kinds.

On plants is less important. It is in 7 books. Its bulk, like that of *On animals*, is an exposition. While the exposition in *On animals* is of Aristotle, however, that in *On plants* is not of Theophrastus. In the Aristotelean corpus there is the brief spurious *On plants*, and it is of this that books I-IV of Albert's *On plants* are an exposition. Book V is Albert's generalization. Book VI is an alphabetically arranged description of trees, gathered under 37 heads, and herbs, gathered under 22 heads. Book VII is on agriculture and horticulture.

In the next generation, Aquinas assimilated Aristotle into Christian philosophy. Assimilated Aristotle, in general, and Aristotle's solution of the problem of the soul, in particular. Aquinas thus shows particularly well why there was not, nor could there be, an advance in the solution of the physical problem of the soul.

The *Theological summa* begins with god (I, 2-49), then turns to the angels (50-64), then to the

work of the six days (65-74). Then to man (75-102), and:

because man is composed of both spiritual and corporeal substance, we have to ask in the first place what belongs to the essence of the soul (75).

And we ask, and answer:

is the soul a body (75.1)? no; the soul is the first principle of life; and the principle of life cannot be a body, otherwise every body would be either alive or a principle of life (*id.*);

does the human soul subsist (75.2)? yes; a rational principle cannot be a body; the incorporeal subsists; the human soul, which is also reason and mind, is incorporeal, and hence subsists (*id.*);

do the souls of the brute animals subsist (75.3)? no (*id.*);

is the human soul incorruptible (75.6)? yes; there is corruption only where there is contrary; in the heavenly bodies there is no contrary and they are incorruptible, in the soul there is no contrary either, and it is therefore also incorruptible (*id.*);

are the soul and the angels of the same species (75.7)? no (*id.*);

is the soul united to the body as its form (76.1)? yes (*id.*);

are there in man other souls essentially different from the rational (76.3)? no; Plato says that there are several souls in the body, nutritive and sensitive next to the rational, but Aristotle rejects this, in man these are parts of one soul (*id.*);

is there in man a form other than the soul (76.4)? no (*id.*);

is the whole soul in each part of the body (76.8)? yes (*id.*);

was the soul made by god (90.1)? yes (*id.*);

was it made at creation (90.2)? yes (*id.*);

was it made before the body (90.4)? no (*id.*).

Hence, clearly, we have to ask about the first man:

did the first man see god in his essence (94.1)? no (*id.*);

did he see the angels in their essence (94.2)? no (*id.*);

did he know all things (94.3)? yes (*id.*).

Hence, we have to ask also about the state of innocence:

was Adam in the state of innocence the lord of all animals (96.1)? yes (*id.*);

was he the lord of all other creatures (96.2)? yes (*id.*);

would in the state of innocence all men have been equal (96.3)? no (*id.*);

would one man have been lord over other men (96.4)? yes (*id.*);

would man have been immortal (97.1)? yes (*id.*);

would there have been generation (98.1)? yes (*id.*);

would generation have been by sexual intercourse (98.2)? yes (*id.*);

would men have been born righteous (100.1)? yes (*id.*);

would children have been born with perfect knowledge (101.1)? no (*id.*).

And Aquinas resolves the relevant geographical difficulty, too:

is paradise, which is the abode of man, a corporeal place (102.1)? yes; everything scripture tells us about paradise it tells us in a way of history, and everything scripture tells us in this way, we must hold true; moreover, Isidore says in the *Etymologies* that paradise is in the east; the philosopher says in *On the heaven* that east is the right side of heaven, and the right side is the nobler; it is therefore fitting that god should place paradise in the east (*id.*).

One generation-odd later, Mondino is the most important doctor in the five and a half centuries from ibn Sina to Vesalius. His *Anatomy* remained the fundamental book on anatomy for two and a half centuries. And why, not quite eleven and a half centuries after Galen, Mondino is not an advance on Galen, the following can show. This is how Mondino begins the instruction on dissection in the *Anatomy*:

we place the dead body of the beheaded or hanged in the supine position; and we ask first about the whole, then about the parts;

considering the whole, we should first of all know how man differs from the beasts; he differs in three ways, in the form or position of his members, in his habits and arts, and in that he has some parts which they do not have;

and first of all we note that man is of upright stature; and man is made in this way for four

reasons;

first, because he is made of more aether and air than the other animals, his matter is thus lighter, and the lighter tends upward;

second, compared with other animals of the same size, man has more heat in him, and heat also tends upward;

third, man has the most perfect form, which he shares with the angels, and, ruling the world, the angles are above;

fourth, man is made so that he can understand, and to this end he is made upright, because the senses and the brain are in the head (I, 1).

To the reasons why man is made upright the angels have contributed rather more than the beheaded or hanged.

Two centuries on, it was at long last the time that was to be called renaissance.

And renaissance it was. Art was reborn, education was reborn, the state was reborn, scholarship was reborn. Religion was reborn, too.

And philosophy was reborn. In two kinds. Providing the specimens over which Bacon's description of the two ways, and only two, is an empirical generalization.

In particular, Fracastoro and Paracelsus exemplify one of the two ways. Brunfels and Bock exemplify the other.

Fracastoro wrote the *Syphilis, or the Gallic disease*, in 1530, in which he first describes that disease. Nor is the *Syphilis* just a medical treatise, it is a poem, in hexameter, singing of the shepherd Sifilo punished by the sun god for his infidelity. Fracastoro's most important treatise is *On contagion, contagious diseases, and their cure*, in 1546, in which he argues, three and a half centuries before Pasteur and Koch, that disease is caused by minute particles, transmitted by touch, or by clothing and sheets, or by the air, leprosy being of the first kind, for instance, plague of the third. This argument is part of the more general argument, which Fracastoro develops in *On the sympathy and antipathy of things*, in 1542, that motion is caused by the transmission of minute particles between things.

The atomic theory, for its part, was transmitted from Leucippus and Democritus to Epicurus, from Epicurus to Lucretius, and after a hiatus of not quite sixteen centuries, it was with its trans-

mission from Lucretius to Fracastoro that it began rising once again.

Paracelsus was a doctor and a seer. He called himself Beyond-Celsus, and concerned himself with things beyond generally. Among his treatises, for instance, are *The book paragranum* and *The books of the opus paramirum*, beyond the seed and beyond the miracle, respectively. Where theory is concerned, Paracelsus, like al-Razi and ibn Sina earlier, advanced alchemy. Paracelsus proposed that the elements are not the four, earth, water, air, and fire. The elements are the three, mercury, sulfur, and salt.

Brunfels wrote the *Images of living plants, drawn from nature with great accuracy and skill, as well as their effects*, in three volumes, in 1530-6. Brunfels describes not more than some 230 kinds. He also draws the verbal descriptions from Theophrastus and Dioscorides. For the first time, however, Brunfels has the plants drawn from nature. Conceived by Brunfels, the images are executed by Hans Weiditz the younger. With great accuracy and skill.

Bock wrote the *New book of plants, on the differences, effects, and names of the plants growing in the German lands*, in 1539. Bock describes some 700 kinds. And for the first time since Theophrastus, or Dioscorides, he draws the verbal descriptions from nature. With great accuracy and skill, too.

Counting from Thales to Theophrastus, it took the philosophers two and a half centuries to discover that they can describe plants. Counting from Theophrastus to Dioscorides, it took them four and a half centuries to discover that they can try, more cautiously, to do it again. Counting from Dioscorides to Hildegard, it took them ten-odd centuries to discover that they can try, very cautiously, to do it again. Counting from Hildegard to Brunfels and Bock, it took them four centuries to discover that they can try once again, and begin doing it, this time, valiantly, as a matter of routine.

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The next is the generation of Vesalius.

Vesalius published *The structure of the human body* in 1543.

With *The structure of the human body* anat-

omy and physiology began advancing very greatly once again.

Also in 1543 Copernicus published *On the revolutions of the heavenly spheres*. In 1545 Cardano published *The great art*. Theory very generally, the search for truth very generally, began advancing very greatly once again.

The structure of the human body is in 7 books. Vesalius reports in the preface:

in book I I have described the nature of all bones and cartilages, which, since all other parts are supported by them and must be described in relation to them, the students of anatomy must learn first;

book II treats of the ligaments, by which the bones and cartilages are linked with one another, and of the muscles, which execute the movements that depend on our will;

book III deals with the network of the veins, which carry to the muscles, bones, and other parts of the body the ordinary blood by which they are nourished, and of the arteries, which control the mixture of innate heat and vital spirit;

book IV treats of the network of nerves, which carry the vital spirit to the muscles, and of all other nerves;

book V explains the structure of the organs performing the nutrition, which is effected through food and drink, as well as, because of their proximity in position, of the instruments designed by our most high creator for the propagation of the species;

book VI is devoted to the heart, the matter of the vital faculty, and the parts that maintain it;

book VII describes the harmony between the structure of the brain and the organs of sense, without, however, repeating the description of the network of nerves, which begins in the brain (*id.*).

Vesalius adds: in the arrangement of the material I have followed Galen (*id.*). *The structure of the human body* is a massive advance on Galen. Including most particularly with its illustrations, designed by Vesalius, executed by students from the studio of Titian.

What Vesalius has to say about the soul is little, but portentous.

Chapter 15 in book VI is *On the operations of the heart and its parts*, and Vesalius begins by reviewing briefly what some of the ancients have

said about the soul. He then writes:

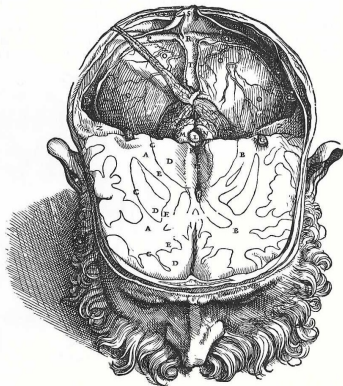
but lest I annoyed some censor of heresy, I shall refrain altogether from considering the divisions of the soul and their seats; for today there are very many censors of our holy and true religion; and when they hear someone whisper about what Plato, or Aristotle, or the commentators, or Galen, have said about the soul, even in anatomy where most of all these matters must be examined, they immediately judge him suspect in his faith and in his belief in the immortality of the soul; they do not understand that the physician, just to perform his art properly, must examine these matters (*id.*).

Pace Vesalius, they understand perfectly.

In anatomy most of all these matters must be examined.

Descartes, three generations later, did more damage to the soul than any other philosopher. And great as the damage is in the *Meditations*, where Descartes demonstrates the soul by the arguments of philosophy rather than theology, it is in *The passions of the soul*, where he finds that: the pineal gland is the principal seat of the soul, that the damage is irremediable.

Below is figure 7 in book VII. Vesalius explains: this figure differs considerably from the previous three, because much of the brain has been removed, the portion of the brain attached to the cerebellum has been cut away, and we have also opened the sinuses of the dural membrane (*id.*). In the center, midway between the ears, marked by a tiny L, is the pineal gland.



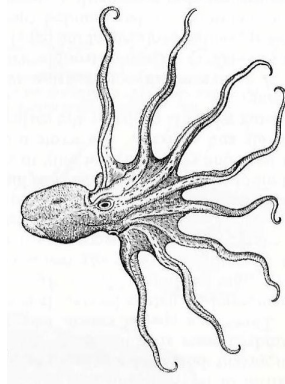
In the same generation, Gesner compiled the *Universal library, or most complete catalogue of*

all books in the three languages, Latin, Greek, and Hebrew, extant and not extant, ancient, modern, and through this day, in 4 volumes, in 1545-55. The writers of all the books in the three languages are not less than 2 000. In the *Mithridates, or on the different languages*, in 1555, Gesner described some 130 languages.

And Gesner wrote the *History of animals*, in 5 massive volumes. Volume I is *On viviparous quadrupeds*, volume II *On oviparous quadrupeds*, volume III *On birds*, volume IV *On fishes and aquatic animals*, volume V *On serpents*. Volumes I-IV Gesner published in 1551-8, the unfinished volume V was published in 1587.

With the *History of animals*, nineteen centuries after Aristotle, zoology is restored.

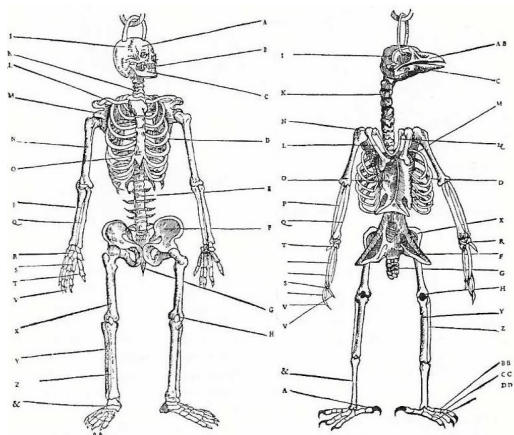
There are among the animals which Gesner describes some of the monsters of the bestiaries of his more immediate predecessors. Gesner, however, has collected all animals described before him, not less than 1 500 kinds. He has described many of them from nature, the five volumes are amply illustrated, by Gesner himself, with great skill and accuracy, and many of the illustrations have been drawn from nature. As the heads of the volumes show, Gesner has improved the systematic organization of the animals as well. And the most important is volume IV. With Frederick II the investigation of birds advanced over Aristotle. With Gesner the investigation of fishes and aquatic animals advanced substantially over Aristotle.



The drawing is from volume IV, under the head *Polypus*. *Polypus* is octopus. Gesner reports: I have drawn it from a sketch sent to me by Rondelet (*id.*).

In the same generation, Belon wrote the *Natural history of strange marine fishes, with a true depiction and description of the dolphin, and of several others of its kind*, in 1551, and the *Natural history of birds, with their description and portraits drawn from nature*, in 1555.

In the *Natural history of birds* Belon describes some 200 kinds.



The two drawings are from chapter 12, *The anatomy of the bones of birds compared with that of the terrestrial animals and man*. The lines indicate the twenty-seven points of comparison considered by Belon between the anatomy of the bones of man and that of birds.

Later in the same generation, Cesalpino wrote *On plants*, in 1583.

With *On plants*, nineteen centuries after Theophrastus, botany is restored

On plants is in 16 books. In book I Cesalpino deals with the kinds of the plants, and their parts, structures, and functions in general. In books II–XVI he describes some 1 500 kinds. In books II–XV he describes the flowering plants, in book XVI he describes the ferns, duckweed, mosses, lichens, fungi, and algae. Linnaeus, in the *Botanical philosophy*, begins with the classification of his predecessors, and classifies Cesalpino as: the first systematic botanist (54). In *On plants* Cesalpino writes: however accurate a description may be, if it gives no knowledge of the genus, it cannot indicate a definite species, and it is usually deceptive (pr.). In the *Botanical philosophy* Linnaeus quotes this proposition under the fundamental principle: a plant is described completely

when it is given its generic name and its specific name (256).

One generation-odd later, Cesi published the *Apiarium*, in 1625, with the first drawings of objects observed under a microscope, of bees magnified 5 times. Galileo built the microscope for Cesi. In the *Pythosophical tables*, in 1630, Cesi made the first attempt at the classification of plants based on their morphology and physiology.

Later in the same generation, Harvey discovered the circulation of the blood. In 1628, in *An anatomical essay on the movement of the heart and blood in animals*, he writes:

blood permeates from the right ventricle into the lungs, from the lungs into the left auricle, and from the left auricle into the left ventricle (6);

the function of the heart is to expel blood into the arteries (17);

the heart is a muscle (5); it is like a machine in which, when one wheel moves another, all moves at once (*id.*).

And concludes:

the heart is the tutelary deity of the body, the foundation of life, the source of all things, performing the functions of nourishing, warming, and activating the body as a whole (9); we must agree with Aristotle that the heart is preeminent (17); it is created first, to be the seat of blood, life, sensation, and movement (*id.*); like the king in the state it rules the whole body (*id.*); it is the source and root of all powers of the animal (*id.*).

In 1649, in the *Second essay to Riolan in which many objections to the circulation of the blood are refuted*, he writes:

the heart is the beginning of all things in the body; it is the creator, the fount, and the spring; it is the first cause of life (*id.*).

Aristotle concludes: the soul is the principle of life. Harvey concludes: the heart is the source of all things, the creator, the fount, and the spring, the first cause of life. Aristotle concludes: the soul is the efficient, formal, and final principle of life. Never mind formal and final principles. The heart is the efficient principle of life. Harvey concludes: a muscle, a machine, the heart performs the functions of nourishing, warming, and activating the body as a whole, it is the seat of life, sensation, and movement, it is the source and root of all powers of the animal.

And if it is not the principle of life of plants, it is the principle of life of all animals. Harvey writes:

if oysters, mussels, and zoophytes of all kinds are devoid of hearts, it is because the whole body of such an animal is nothing but a heart (*Heart*, 17).

Harvey does not affirm explicitly that the soul is not the principle of life. Even as he can hardly have affirmed it, implicitly as it is, more unambiguously.

Later in the same generation, Descartes affirms it explicitly.

In 1633, in *The world, or a treatise on light and the other principal objects of sense*, he writes:

the body is nothing but a machine made of earth; and all functions of this machine follow entirely naturally from the mere disposition of its organs, neither more nor less than the motions of a clock or another automaton follow from those of its counterweights and wheels; it is thus not necessary to suppose that there is in it a soul, vegetative or sensitive, or any other principle of motion and life than its blood and its spirits agitated by the heat of the fire burning continuously in its heart, or to suppose that this fire is not of the same nature as all the fires in inanimate bodies.

It is not necessary to suppose that there is a soul in the body, vegetative or sensitive, or any other principle of motion and life than its blood and its spirits. The body is nothing but a machine made of earth, and all functions of this machine follow entirely naturally from the mere disposition of its organs.

The soul is not the principle of life. There is no other principle of motion or life than the blood and spirits of the body.

It remained to establish these propositions firmly by following the true way as staunchly as the thread of Theseus must be followed in the labyrinth. And so in the following years Descartes did.

In 1644, in *The principles of philosophy*, he concludes:

I will not add, as I previously intended, two more parts to these principles, a fifth on living things, on animals and plants, and a sixth on man;

I have not yet examined perfectly all the things I would want to discuss in these parts;

but I will say a few things about the objects of the senses; even though the human soul informs the whole body, it has its principal seat in the brain; it is here that it not only understands and imagines, but also senses;

the motions excited in the brain by the nerves affect the soul, or mind, which is closely conjoined to the brain, in different ways corresponding to the different motions; and these different affections, or thoughts, consequent to the motions, are called perceptions.

In 1648, in *The description of the human body and all its functions*, he writes:

the human body is a machine; and the heat in the heart is the great spring and the principle of all motions in the machine.

And in 1649, in *The passions of the soul*, he writes:

what is a passion in the soul is an action in the body; everything we experience in us must be attributed to our body; it is an error to believe that the soul gives motion and heat to the body;

the soul is united to all parts of the body; but there is a little gland in the brain where the soul exercises its functions more particularly than in the other parts of the body; this gland is the principal seat of the soul; and I am persuaded that the soul can have no other place in the entire body, where it exercises its functions immediately, but this gland, because I have observed that all other parts of our brain are double, just as we have two eyes, two hands, two ears, and generally all our organs of external sense are double; but since we can have only one and simple thought of one and the same thing at the same time, it is necessary that there is a place where the two images from any two impressions can be assembled into one.

The pineal gland is the principal seat of the soul. The soul is united to all parts of the body. It is an error, however, to believe that the soul gives motion and heat to the body. Everything we experience must be attributed to the body.

The soul is not the principle of life.

In anatomy most of all, Vesalius writes, the divisions of the soul and their seats must be examined.

Galen already has concluded that the seat of the rational soul is the brain, the mixture in the heart is the sensitive soul, the mixture in the liver is the nutritive soul. Also, we must take Aristotle's form to be the mixture of the hot, cold, wet, and dry.

Harvey concludes that the heart is the beginning of all things in the body, the creator, the fount, and the spring, the first cause of life.

Descartes concludes that the pineal gland is the principal seat of the soul, and it is united to all parts of the body, but it is not what gives motion and heat to the body. It is not the principle of life. There is no other principle of motion or life than the blood and spirits of the body.

The advance in the formulation and solution of the problem of the soul has thus been very great.

It remains to show what exactly the soul is.

However, Aristotle concludes that the soul is the efficient, formal, and final principle of life. And the problem has been liberated from the formal and the final principles, both. It has become clear that the problem is about the efficient principle of life. About the first cause of life. About what gives motion to the body. About the principle of motion or life.

And the soul is not the efficient principle of life. It remains to establish this affirmatively. To show conclusively that the heart is the principle. Or that the blood and spirits of the body are the principles. Or to find the principle of the heart and of the blood and spirits of the body.

And thus show also, as Dicaearchus has said nineteen and a half-odd centuries earlier, that the soul is nothing.

cclxxi

In the twenty centuries from Aristotle to Descartes as well the investigation of animals and plants and theoretical medicine are the ground of the advance in the formulation and solution of the physical problem of the soul.

12 of the 30 particulars collected from these centuries are part of the development of the investigation of animals and plants, Theophrastus, Dioscorides, Pliny, Hildegard, Frederick II, Albert, Brunfels, Bock, Gesner, Belon, Cesalpino, Cesi. 4 of these advanced the investigation, Theophrastus, Frederick II, Gesner, Cesalpino.

12 of the 30 are part of the development of medicine, Herophilus, Erasistratus, Asclepiades, Celsus, Galen, al-Razi, ibn Sina, Mondino, Fracastoro, Paracelsus, Vesalius, Harvey. 5 of these advanced anatomy and physiology, Herophilus, Erasistratus, Galen, Vesalius, Harvey. And 2 of these advanced also the formulation and solution of the physical problem of the soul, Galen, Harvey.

6 of the 30 proposed solutions of the problem as well, Dicaearchus, Epicurus, Zeno of Citium, Strato, Aquinas, and Descartes. And 2 of these advanced the formulation and solution of the problem, Dicaearchus and Descartes.

The steps of advance in the acquisition of knowledge are thus the 12, Theophrastus, Dicaearchus, Herophilus, Erasistratus, Galen, Frederick II, Vesalius, Gesner, Belon, Cesalpino, Harvey, Descartes. In particular,

Theophrastus	described 550 kinds of plants,
Dicaearchus	the soul is nothing,
Herophilus	advanced anatomy and physiology,
Erasistratus	advanced anatomy and physiology,
Galen	advanced anatomy and physiology, the seat of the rational soul is the brain, the mixture in the heart is the sensitive soul, the mixture in the liver is the nutritive,
Frederick II	advanced the investigation of birds,
Vesalius	advanced anatomy and physiology,
Gesner	described 1 500 kinds of animals,
Belon	compared the anatomy of the bones of man and bird,
Cesalpino	described 1 500 kinds of plants,
Harvey	advanced anatomy and physiology, the heart is the first cause of life,
Descartes	the pineal gland is the principal seat of the soul, the soul is not the principle of

life,
there is no other principle than
the blood and spirits of
the body.

cclxxii

And the ethical problem of the soul in the twenty centuries from Aristotle to Descartes.

The second rule is to divide difficulties into parts such as will lead to their better solution.

Thus, with respect to the physical problem, the twenty centuries are divided into the parts indicated by the major steps in the development of the problem, the five centuries from Aristotle to Galen, the fourteen centuries from Galen to Vesalius, and the not quite one century from Vesalius to Descartes.

With respect to the ethical problem, the twenty centuries have to be divided differently. The major steps in the development of the state indicate the parts. This follows from the fact that, with Aristotle: even when the end is the same for man and for the state, the end of the state is greater and more complete.

In particular, in 202^{BC} the Romans won the second Punic war, and Rome became the dominant power on earth. In the following century Rome extended its dominion to the Euphrates. The dominion of Rome remained undisputed through the reign of

Theodosius prince 379-95^{AD}.

In the previous century the Christian church had become ecumenical, the church of the empire. In the century after Theodosius the empire began retreating before the barbarians. The church continued to grow.

The twenty centuries thus divide into these three parts. The one and a half centuries before the second Punic war. The six centuries from the second Punic war to Theodosius. And the twelve and a half centuries after Theodosius.

Formulated and solved by Plato, the ethical problem of the soul is: have you realized that the soul is immortal?

The solution is: are you sure? I would be wrong not to be, and so would you.

More particularly: you should consider the nature of the soul, you should conclude that the soul is immortal, you should believe that your

immortal soul is yet altered by the life you choose, and you should choose accordingly, so that you avoid being flayed and lacerated on thorn bushes, earn your rewards, and here and in the hereafter be happy.

Teaching you that, the problem and its solution, is an essential part of the problem.

And in the one and a half centuries before the second Punic war, and before Roman dominion, the ethical problem of the soul is nowhere to be found. Where teaching is concerned, this is the time of the flourishing of the schools.

In the first three of the six centuries from the second Punic war to Theodosius, the first three of the six centuries of Roman dominion, the schools decline as teaching spreads throughout the Roman domain. This is the time also when immortality enters the Hebrew writings and thence the earliest Christian writings. In the following three of these six centuries, teaching is everywhere in the Roman domain. This is the time also of the triumph of the Christian church. And with the labors of the fathers of the church the solution of the ethical problem of the soul triumphs as well, and becomes orthodoxy.

In the twelve and a half centuries after Theodosius, and after Roman dominion, and the time of continuing growth of Christianity, and then also of the rise and growth of Islam, teaching at first declines very greatly, then at length revives and grows very greatly. The solution of the ethical problem of the soul remains orthodoxy throughout.

There is no development until Spinoza, one generation after these twelve and a half centuries.

More particularly.

The development of the schools can be described in two steps.

One. Early in the Peloponnesian war Pericles praised the Athenians: our city is the schoolmistress of all Greece (Thucydides, II, 40-1). The Athenians lost the war. But the schoolmistress of all Greece their city was. The philosophers all came to be taught, then to teach, and to establish their courts, in their city. Of the heads of the minor schools one is Antisthenes of Athens, the rest are Euclid of Megara, Aristippus of Cyrene, Diogenes of Sinope, Pyrrho of Elis. The first three heads of the Academy are Plato of Athens,

Speusippus of Athens, Xenocrates of Chalcedon. Of the Lyceum – Aristotle of Stagira, Theophrastus of Eresus, Strato of Lampsacus. Of the Stoa – Zeno of Citium, Cleanthes of Assos, Chrysippus of Soli. Of the Garden the first two are Epicurus of Samos, Hermarchus of Mytilene, the city of the third, Polystratus, is not recorded.

In the one and a half centuries before the second Punic war philosophy came out of Athens, and out of the Athenian schools, in particular. Only toward the end of this time the kings of Egypt challenged the monopoly of the schoolmistress when they founded the library and attendant school at Alexandria.

Two. Roman conquest reached and included the schoolmistress, then also Alexandria. The conquerors developed a taste for philosophy, the philosophers developed a taste for following the conquerors. Philosophers spread along the routes of the Roman domain, with the philosophers teaching spread, and schools, and rhetorical, grammatical, and medical more than philosophical, became commonplace in the large cities, and common in the small cities, too.

One and a half centuries after the second Punic war, by the time of Cicero and Caesar, the Athenian schools had become ordinary, and the successions of scholars at all four lapse into obscurity. Everyone now, more or less, was a philosopher. Brutus, for instance, was a Stoic, Cassius was an Epicurean. Cato Uticensis was a Stoic, too. Cicero, for his part, was not an Epicurean, he was now an Academic now a Peripatetic now a Stoic.

At the schools ethics flourished, and very soon after their foundation ethics exclusively.

At the Lyceum, Aristotle taught that the happiest life is the theoretical.

Not long after that, and rather because of that, the Lyceum became a school of not readers.

At the Garden, Epicurus taught:

pleasure is the first and innate good, it is the beginning and the end of every choice (*Menoeceus*, 129); but pleasure is not what the ignorant believe it to be; pleasure is the absence of pain in the body and of trouble in the soul; pleasure is not achieved by eating, drinking, revelry, or love, pleasure is achieved by sober reasoning, searching for the ground of every choice, and banishing

from the soul all that troubles it (132);

he who understands this will direct his every choice toward securing the health of the body and the tranquility of the soul; this is the sum and the end of the blessed life; to be free from pain and fear is the end of all our actions (128).

The doctrine was collected into the 40 *Principal maxims*, its essence into the first 4, the four-fold medicine:

1. a blessed and eternal being will neither trouble about any other being nor bring trouble on it;

2. death is nothing to us, for, resolved into its elements, the body can have no feeling, and what we do not feel is nothing to us;

3. the ultimate pleasure is the removal of all pain;

4. pain, if continuous, does not last long, if acute, is very short (*id.*).

The sum and the end of the blessed life is the health of the body and the tranquility of the soul.

After that, the high point in the development of Epicurean ethics is:

you give us precepts, so that we,
bees sipping at every blossom,
eagerly feed on your golden sayings.

At the Academy, Plato taught that you should believe that your soul is immortal, and choose accordingly, so that here and in the hereafter be happy.

This is the succession of scholars at the Academy,

Plato	c. 385 - 347,
Speusippus	347 - 339,
Xenocrates	339 - 314,
Polemon	314 - 270,
Crates of Athens	270 - soon after,
Arcesilaus	soon after 270 - 240,
Lacydes	240 - 215,
Telecles	215 -
Euandrus	after Telecles,
Hegesinus	after Euandrus - c. 155,
Carneades	c. 155 - 136,
Carneades the younger	136 - 130,
Crates of Tarsus	130 - 126,
Clitomachus	126 - 109,
Philo of Larissa	109 -

Antiochus of Ascalon after Philo of Larissa. Then the succession lapses into obscurity.

Of the 13 scholarchs after Xenocrates, not less than 10 are not important, very not important, and very very not important. Not more than 3 are of any importance, Arcesilaus, Carneades, and Antiochus of Ascalon.

Cicero reports: Arcesilaus did not say what he thought, but argued against everyone else (*On the orator*, III, 67); he contradicted everything (*Ends*, V, 10). Empiricus concurs: the doctrine of Arcesilaus and our doctrine are virtually the same (*Outlines*, I, 232). Arcesilaus said: the wise man does not assent to anything (Cicero, *Academic*, II, 66). Arcesilaus yet assented to teach you: men, like adulterers, envy the fame and fortune of others, while their own lives can give them pleasurable contemplation (Plutarch, *Tranquility*, 9).

Cicero again: Carneades examined all opinions on the highest good, and found that the philosophers all say that the highest good is pleasure, or that the highest good is freedom from pain, or that the highest good is to follow what is first by nature, and in each case either its pursuit or its attainment; there are thus 6 possible opinions on the highest good (*Ends*, V, 16-21).

Empiricus reports: Antiochus of Ascalon brought the Stoa in the Academy, he showed that the Stoic doctrines are in Plato (*Outlines*, I, 235). Antiochus also taught: wisdom is not sufficient for a happy life, health, wealth, and success are also necessary (Cicero, *Ends*, V, 81).

The philosophers concerned themselves with your immortal soul, and what you should do about it, as little at the Academy as at any of the other schools.

This can be taken to indicate the importance of this Platonic doctrine. The Lyceum forgot Aristotle, and Theophrastus. The Garden sipped at every blossom. At the Academy under Plato mathematicians had gathered. Plato had developed the theory of the forms. Plato had investigated knowledge. Plato had built the ideal city. His successors forgot all this, too.

The paltriness which the schools achieved was as general as it was profound.

After Antiochus of Ascalon the Academy, too, disappears from the record.

The next is the generation of Cicero. And in the previous century immortality had entered the

Hebrew writings. Other than that, Cicero is the philosopher who, three centuries after Plato, recalled your immortal soul, if not quite yet what you should do about it.

Cicero wrote some two dozen philosophical treatises. And in the two books of the *Academic matters* your immortal soul is nowhere among the academic matters. In the five books of *On the good and bad ends* your immortal soul is nowhere among the good and bad ends. Then, in the *Tusculan disputations*, your immortal soul is brought to life again.

The *Tusculan disputations* is in five books. In book I Cicero teaches you contempt for death, in book II fortitude against pain, in books III-IV fortitude against misfortune and distress, and in book V that virtue is all by itself sufficient for a happy life.

In book I he proposes to show that: death is not an evil (15-7). He begins with a review of: what all the philosophers have said about death and the soul (18-23), and concludes: you need a god to decide which of all these opinions is true (24). Then, pending the appearance of a god, Cicero considers the two possibilities: the soul either is immortal (26-81), or perishes at death (82-116). The arguments for immortality are from the *Phaedo*, *Phaedrus*, *Republic*, and *Timaeus*. Clearly, if the soul is immortal, death is not an evil. Nor is it an evil, however, if the soul perishes:

if one lives well and scorns human vicissitudes, he can die with dignity (95-108);

it is indeed easy to meet death with equanimity, when one is solaced by the honor he has won (109-20);

in any case, can what is necessary for all men be an evil for one of them (119-20)?

Cicero is teaching you. Not quite yet what Plato is teaching you, though. Cicero is teaching you that death is not an evil, and what you should do about that.

And as the death of the philosopher has to be an argument, too, Cicero fell in the battle against Augustus and Antony to rule Rome, and so also much of the earth, and fell with dignity by his own hand. He had won much honor, too.

Antiochus of Ascalon taught that wisdom is not enough, health, wealth, and success are also

necessary. Accordingly, he brought the Stoa in the Academy, to provide the success.

The Stoa was by far the most successful school.

This is the succession of scholars,	
Zeno of Citium	301 - 263/1,
Cleanthes	263/1 - 233/1,
Chrysippus	233/1 - 208/5,
Zeno of Tarsus	208/5 -
Diogenes the Babylonian	- c. 152,
Antipater of Tarsus	c. 152 - c. 129,
Panaetius	c. 129 - c. 109,
Dardanus and	
Mnesarchus	c. 109 -

Then this succession of scholars of the Athenian Stoa also lapses into obscurity. But the Stoa continued to be the most successful school. Stoic schools in other cities appeared, at Rhodes under Posidonius, for instance. And the philosophers continued to be much more likely to profess Stoic than any other allegiance through the time when they all began professing the single allegiance allowed by the church.

Of the 9 scholars, Zeno of Citium, Chrysippus, and Panaetius are very important, Cleanthes, Diogenes the Babylonian, and Antipater of Tarsus are important. The other 3 are not important.

The very important among the later Stoics are,
 4^{BC} - 65 Seneca,
 55 - 135 Epictetus.

Zeno of Citium said that there are two principles of the whole, active and passive. The passive is matter. The active is the inherent in matter law, or god, which is everlasting and the producer of everything. The substance of god is the whole world (Laertius, VII, 148). All things are determined. Determination is the chain of causes whereby things happen, or the law whereby the world develops. Nothing incorporeal is capable of action, everything that acts or is acted upon is body. The soul is a warm breath.

Zeno was a physicist.

Laertius reports also: Zeno and Cleanthes, like the older philosophers generally, dealt with ethics more simply (84). More particularly: Zeno first spoke of duty in actions (108); and in *On the nature of man* he also first said that the end is to live in agreement with nature (87).

Theophrastus had 2000 students. Zeno said:

his chorus is larger, but mine is more harmonious (Plutarch, *Progress*, 6).

One of his choristers was Cleanthes. He had no aptitude for physics, and his fellow students called him an ass; he did not mind, Cleanthes told them, for he alone was strong enough to carry Zeno's burden (Laertius, VII, 170).

Cleanthes said: fate leads the willing, drags the unwilling (Seneca, *Letters*, 107).

Cleanthes has left behind the *Hymn to Zeus*. It begins:

most glorious of gods, known by many names,
 all-powerful, lord of all change in nature,
 lawgiving pilot of the world, I hail you,

and ends:

bountiful Zeus, rescue mankind from wretched
 ignorance,
 give men the wisdom with which you steer all
 things in justice;
 thus honored, we will honor you,
 for there is no greater glory for a man or god
 than to sing forever of the shared law (*id.*).

When Chrysippus came to the school, he told Cleanthes: just give me the doctrines, I will find the proofs (Laertius, VII, 179).

Later on Chrysippus discovered the modes of inference.

It was said that, if the gods take to logic, they will to no other but Chrysippus's (180).

If the gods take to logic.

Diogenes the Babylonian and Antipater of Tarsus developed grammar. Laertius reports: in *On speech* Diogenes distinguished five parts of speech, proper name, common name, verb, conjunction, and article (57); in *On words and their meaning* Antipater added adverb (*id.*).

Diogenes was lecturing on anger once; a student walked up to him and spat in his face; no, Diogenes said, I am not angry, but I am not sure I should not be (Seneca, *Anger*, III, 38).

Zeno was a physicist. He and Cleanthes, like the older philosophers generally, dealt with ethics more simply. Chrysippus was a logician. Diogenes and Antipater were grammarians.

It is with Panaetius that Stoic ethics begins to excel.

Cicero reports: Panaetius rejected the depressing harshness of the earlier Stoics, he had no use for their severity or their tortured arguments, his

doctrine was gentler (*Ends*, IV, 78-9).

Panaetius said:

when men deliberate on an action, they consider, first, whether the action is honorable, second, whether it is useful, and third, whether honorable and useful are in conflict (Cicero, *Duties*, I, 9).

The honorable? Or the useful? A depressingly harsh choice. Requiring tortured arguments.

Panaetius said:

the useful is never in conflict with the honorable, nothing useful is not honorable and nothing honorable is not useful; indeed, the greatest curse on human life has been the separation of the two (III, 34).

So much for the depressing harshness. And never mind tortured arguments, no deliberation at all is needed any longer. Nothing useful is not honorable and nothing honorable is not useful.

For instance:

virtue depends on perceiving what is true, on restraint of passion, and on treating others with moderation so that we may through the cooperation of others secure for ourselves fully and amply what nature requires (II, 18).

Usefully and honorably, fully and amply, for ourselves.

Zeno first spoke of duty. Clearly, the new gentle doctrine posed difficult questions about duty. Panaetius wrote *On duties*. Cicero's *On duties* is an exposition of Panaetius's *On duties*. And along the way Cicero exemplifies with a student of Panaetius's, Hecaton, who considered, for instance, the following duties:

should the good man feed his slaves when corn becomes very expensive?

when a storm threatens a ship and cargo must be discarded, should an expensive horse or a cheap slave be thrown overboard?

and if the ship capsizes and a fool grabs a plank, should the wise man wrest it from him (III, 89)?

It was at the time of Panaetius, and through his labors, that the conquerors of the earth began learning philosophy.

Two generations earlier, Cato had said: the Greeks are a vile and meddlesome race; they will corrupt everything if they give us their books and their doctors (Pliny, XXIX, 14). Scipio, at the

height of his victories in the second Punic war, had been harangued in the senate for wearing a Greek dress and reading books (Livy, XXIX, 19).

Two generations after Cato and Scipio, Scipio Aemilianus won the third Punic war. Cicero reports: Scipio Aemilianus had learned Greeks with him at all times and in public (*On the orator*, II, 154). Not only was Panaetius one of the learned Greeks. Cicero describes Aemilianus in the *Republic* and *Laelius* surrounded by a circle of friends. Four of those in Scipio's circle, one future tribune and three future consuls, Tubero, tr. c. 130, Fannius, cos. 122, Scaevola Augur, cos. 117, and Rutilius, cos. 105, were students of Panaetius (Cicero, *Brutus*, 101, 114; *On the orator*, I, 75; Pomponius, *Digest*, I, 2.2). These are the first Romans reported taught by a philosopher.

Of none of these four students of Panaetius's is there a hint in the record of anything that they may have learned from Panaetius. In particular, it can confidently be presumed that the conquerors of the earth were in no need of being taught that the honorable is useful and the useful honorable.

Yet philosophy, or in any case ethics, the conquerors of the earth did learn.

Two generations later, Cicero and Lucretius are philosophers, or in any case ethicists, *par excellence*.

It was at this time also that it became *de rigueur* to maintain a domestic philosopher. Of broad philosophical interests, Cicero maintained several, the Stoic Diodotus (*Academic*, II, 115) and the Peripatetic Tyrannio (*Atticus*, IV, 4a), among them. Cato Uticensis maintained Athenodorus Cordylion (Plutarch, *Cato Ut.*, 10, 16). Cordylion, to distinguish him from Athenodorus Calvus, the domestic of Augustus (Suetonius, *Caesars*, II, 89). Of broad interests himself, Augustus maintained also Arius Didymus (Plutarch, *Antony*, 80-1).

Seneca is three generations after Cicero and Lucretius.

Seneca was the tutor and then close advisor of Nero. Nero, as Tacitus reports: undertook to annihilate virtue itself (*Annals*, XVI, 21). After an unsuccessful conspiracy against the princeps, Seneca died with dignity by his own hand (XV, 60-4).

Seneca wrote many tragedies. He wrote also the *Gourdification*, recording the session of the council of the gods in which Augustus, a god for a half century already, carries a successful motion to deport the new god Claudius from mt Olympus, pursuant to which Mercury hauls Claudius to the underworld (11-2). He wrote also *To Marcia on consolation*, *To Helvia on consolation*, *To Polybius on consolation*, *On anger*, *On the happy life*, *On the brevity of life*, *On clemency*, *On the constancy of the wise man*, *On leisure*, *On providence*, *On the tranquility of mind*, *On benefits*. Clearly, if you need consolation, you will find it in Seneca. *On clemency* is addressed to Nero.

Seneca wrote also the *Natural questions*. In its seven books he writes on the rainbow, reflection and mirrors, clouds, rain, snow, wind, earthquakes, lightning and thunder, rivers and seas, comets. However:

it is salutary to mix everything with a moral lesson (II, 59).

Accordingly, the discussion of lightning, for instance, ends:

why fear lightning, is it more honorable to die of diarrhea than to be struck by lightning (*id.*)? The discussion of earthquakes, also for instance, ends:

earthquakes are truly terrifying, they devour homes, families, whole cities, submerge nations and regions; yet, it does not matter whether I am struck dead by a rock or crushed by a mountain; let us then have courage in the face of this great disaster (VI, 1).

These are more particular moral lessons. The general moral lesson is:

the highest good is to live in agreement with nature (*Leisure*, 5);

by living as they said one should, Zeno and Chrysippus accomplished more than they would have if they had commanded armies, held offices, or passed laws (6).

In *On the happy life*, however:

happy is the life in agreement with its own nature (3).

With nature? Or with its own nature?

Seneca concludes:

I am saying one thing, living another, you say; oh, spiteful creatures made the same reproach to Plato, and Epicurus, and Zeno; but I speak of

virtue, not of myself, I quarrel with all vice, especially my own; when I am able, then I will live as I should (18).

With its own nature. When I am able, then I will live as I should.

Seneca writes in the *Moral letters*:

I like what I read in Hecaton; what progress did I make today? I became a friend of myself; such a man will be a friend of everyone (6).

Clearly, if the friends, everyone and myself, quarrel, as friends do, and as these two friends, everyone and myself, rather always do, when I am able, then I will be a friend of everyone.

Also, it was Hecaton who considered the duty, should the good man feed his slaves when corn becomes very expensive?

Two generations later, Epictetus was a slave of a freedman of Nero's. Freed, he established his own school in Epirus.

Epictetus, like Socrates, did not write. Arrian wrote down the *Discourses*.

Epictetus is a high point in the development of ethics.

The *Discourses* is in four books, unflinching in purpose from the first to the last.

Right from the start Epictetus tells you – the philosopher – precisely what you should:

I must die? must I also die whining? I must be fettered? must I also wail? I must go into exile? must I go without a smile? you threaten to fetter me? me? you can fetter my leg, but my moral purpose not even Zeus can fetter; these are the lessons the philosopher should rehearse, to write down every day, and to exercise (I, 1).

He gives you a model:

Epictetus will not be better than Socrates, but if he is not worse than Socrates, this suffices; I will not be Milo either, but this is no reason to neglect my body (2).

He provides you with the necessary help:

if a man gives his heart and soul, as he should, to the belief that we are begotten by god and god is the father of men and gods, this man will never have dishonorable or mean thoughts about himself (3).

He gives you the standard of progress:

so who makes progress in virtue? the man who reads Chrysippus? great progress that! if you have withdrawn from external things, if you have

turned your attention to your moral purpose, cultivating it so that it agrees with nature, to become elevated, free, unfettered, faithful, honorable, if you become these things and shun what others crave, and if you rise in the morning and observe these things, if you bathe like a faithful man, if you eat like a dignified man, now this is a man who truly makes progress (4).

Finally, if you are still beholden to the prejudice that there is something paltry about progress measured by how you bathe and how you eat, nothing is paltry:

in everything in the world there is a reason to praise providence (6);

your nose is running? are you not given hands to wipe it (*id.*)?

nothing is more useless than the hair on your chin; nature yet uses even this in the most suitable way, for is it not by this that we tell a man from a woman (16)?

In the rest of the *Discourses* the lessons are, as is appropriate, rehearsed, and rehearsed. To facilitate the rehearsal still further, Arrian prepared also a digest of the *Discourses*, the *Manual*.

The useful is honorable.

When I am able, then I will live as I should.

Chrysippus? great progress that! if you bathe like a faithful man, if you eat like a dignified man, then you truly make progress.

Treat others with moderation so that you secure for yourself fully and amply what nature requires.

Become a friend of yourself, so you will be a friend of everyone.

Nor did Epictetus neglect your duty to others: always ask yourself, what is the kind of thing? so, when you like some jug, tell yourself, it is a jug, and you will not be upset when the jug is broken; when you kiss your child or wife, tell yourself, it is a human being, and you will not be upset when it dies (*Manual*, 3).

It is a jug.

Five-odd centuries after Socrates, ethics has developed. It has come a long way from the simple ethics of the older philosophers. It has prepared itself perfectly to be taken over by the fathers of the church.

And rather the other way around, the development of ethics is a main cause of the fathers of

the church.

Cicero recalled your immortal soul, if not yet to teach you what you should do about it. In the following century the philosophers forgot it again.

Epictetus, however, recalled that other lesson, if you give your heart and soul, as you should, to the belief that we are begotten by god and god is the father of men and gods, you will never have dishonorable or mean thoughts about yourself.

He recalled also that it is all very well to exhort you, but it is better to punish you:

so you call yourself a Stoic? those who falsely claim Roman citizenship are severely punished; so should those who falsely claim that great and dignified title go free? or you think they cannot be punished? the divine law imposes the greatest penalty on those guilty of the greatest crimes; so what are the terms of the penalty? let him who falsely pretends to be a vainglorious braggart, let him who disobeys the divine providence be an abject slave, suffer grief, envy, and pity, in a word be miserable and lament (*Discourses*, III, 24).

Epictetus is contemporary with the earliest fathers of the church.

Before turning to the fathers, however, the description of the development of ethics which leads to the fathers cannot be complete without the emperor,

Marcus prince 161-80.

One and a half centuries after Marcus, his life in the *Augustan history* begins: Marcus devoted himself to philosophy throughout his life, and he excelled all princes in the purity of his character (IV, 1). Sixteen and a half centuries after Marcus, Hegel writes: nothing more excellent can be read with respect to morality, the power of the good will, or meditations to oneself than those written by Marcus, the emperor of the entire then known civilized earth (*Hist. philosophy*, I, 2.A.3).

The meditations to oneself are under the title *To myself*. As Marcus himself reports, he wrote *To myself* in the camp, fighting the Quadi (II, 1; III, 1).

Here is what Marcus has to say in *To myself* about his predecessors:

Scipio and Cato and Augustus and Hadrian and Pius are dead (IV, 33);

Augustus and Hadrian are no more (VIII, 5);
Hadrian and Pius are gone (25);
is anyone still sitting by Hadrian's bier (37)?
all things pass, the court of Pius is gone, and
the court of Hadrian, and the courts of Alexander,
and Philip, and Croesus (X, 24).

It is representative. There is nothing more in
To myself than there is morbidity.

It is brief, in twelve very brief books. In book
I Marcus thanks the gods for giving him his
grandfather (1), father (2), mother (3), great-
grandfather (4), many teachers (5-15), and his
predecessor Pius (16). Then, beginning in book
II, he writes:

I walk in nature until I have to lie down and
rest, breathing my last (4); brief is the life of
every one of us (6).

And so it continues, except that the lists of dead
princes are supplemented with lists of dead not
princes:

think constantly that all men die, that grave
philosophers like Pythagoras, Heraclitus, Socra-
tes, and acute minds like Eudoxus, Archimedes,
Hipparchus, were all laid in the ground a long
time ago (VI, 47).

And so through the end of book XII:

mortal man, you have been a citizen of a great
city, what does it matter whether it is for five
years or fifty (36)?

It is not an accident that the decline of Rome
begins with the philosopher who wrote *To myself*.

Marcus is contemporary with Irenaeus, the
next is the generation of Clement of Alexandria
and Tertullian, the next of Hippolytus, Africanus,
and Origen.

Marcus tried to render another service to
philosophy.

In Athens in 176, Marcus endowed four chairs
in philosophy, Academic, Peripatetic, Epicurean,
and Stoic, with a salary of 10 000 drachmae each,
paid by the imperial treasury.

There is no trace of the Athenian schools in
the record in the previous two centuries. And
unfortunately, or fortunately, Marcus's attempt to
resurrect the schools came to nothing. The
schools emerge from obscurity – in Lucian (*Eu-
nuch*, 3), Cassius Dio (LXXII, 31), and Philo-
stratus (*Sophists*, II, 2) – to acknowledge receipt
of the money, then revert right back to obscurity.

cclxxiii

The six centuries from the second Punic war to
Theodosius, the time of Roman dominion, are the
decisive in the development of the church, of
holy scriptures, and of the doctrine of immortal-
ity.

The six centuries divide clearly into three
parts. The two centuries from the second Punic
war to

Augustus	prince 27 ^{BC} -14 ^{AD} ,
The two centuries from Augustus to	
Marcus	prince 161-80.
And the two centuries from Marcus to	
Theodosius	prince 379-95.

In the two centuries from the second Punic war
to Augustus the Romans conquered the earth
from the Atlantic in the west to the Euphrates in
the east and from the mouth of the Rhine in the
north to the fifth cataract of the Nile in the south.
Rome became the greatest power humanity has
seen, before or after. Augustus reports: 500 000
Roman citizens served under my command
(*Deeds*, 3). Rome began assimilating the peoples
it conquered, in the second of these centuries
increasingly by admitting them into citizenship.
In the census in 14^{AD}, Augustus reports: I counted
4 937 000 citizens (8). The human population of
the Roman domain was some 50 million, one
quarter of humanity. One tenth were citizens, one
tenth of these had served under Augustus.

In these two centuries the Athenian schools
declined. The philosophers spread through the
domain unified by Rome, and developed the
appropriate ethics. Teaching grew greatly.

In these two centuries the development of the
church, of holy scriptures, and of the doctrine of
immortality remained confined to Judea.

Judea continued to be what it had long been, a
tiny part of a great domain. The great kings had
conquered much of the earth, and begun mixing
the peoples in their domain. To exclude his own
people from the mixing, Ezra had cleansed it, and
sworn it in writing to obey the law. Then the
successors of Alexander had divided his domain,
Ptolemy and Seleucus had divided Africa and
Asia, and under them and their successors the
mixing of peoples had continued and accelerated.
Sworn in writing though they had been, the

Hebrews had again begun succumbing to assimilation. To exclude his own people from the mixing and assimilation, in 168 Judah Maccabee rose in revolt against the successor of Seleucus Antiochus IV. Victorious in the field, Judah purified the temple in 164. The *Second book of the Maccabees* reports: those who bravely fought for Judaism restored the temple, freed Jerusalem, and reaffirmed the law when it was in danger of being abolished (2). It is the first time it is called Judaism. Judah Maccabee continued to fight, and although he fell in 160, after him Judea remained relatively independent for one and a half-odd centuries, and for most of this time under the dynasty, the Hasmoneans, which begins with Judah, first as high priests, then as kings. Relatively, but not too independent. The mixing and assimilation, increasingly under Rome now, continued and accelerated, and purified though the temple was, the law reaffirmed, the Hebrews continued to succumb to assimilation.

Clearly, neither swearing in writing nor bravely fighting for Judaism was enough.

Thus, immortality enters the Hebrew writings at this time with the latest of those which will be incorporated into the Hebrew holy scriptures, *Daniel*. And while the philosophers after Plato forgot immortality, the successors of *Daniel* did not, and soon found it in the fount, in Plato, too.

The dates are,

189	<i>Daniel</i> ,
164	<i>Second book of the Maccabees</i> ,
114	<i>Wisdom of Solomon</i> .

Daniel reports:

I heard a voice, Gabriel, make this man understand; and he came to me and said, man, understand that this oracle is about the end (8).

And the oracle about the end which the angel makes this man understand is:

at that time many of those who sleep in the dust of the earth will awake, some to eternal life, some to eternal abhorrence (12).

Immortality thus enters the Hebrew writings, and, in particular, resurrection enters them, and resurrection rather of the body. Many of those who sleep in the dust of the earth will awake. The earlier Hebrew writings never distinguish body and soul, and *Daniel* does not distinguish them either.

Some to eternal life, some to eternal abhorrence.

The *Second book of the Maccabees* and the *Wisdom of Solomon* were composed in Alexandria, in Greek. The large community of Hebrews there had succumbed severely to assimilation. In the previous century the earlier Hebrew writings had also been rendered into Greek – into the septuagint – in Alexandria.

In the generation after *Daniel*, the *Second book of the Maccabees* reports:

the king of the world will raise us up again to eternal life (7).

Us, and you:

the creator of the world, in his mercy, will give you back your life and breath again (*id.*).

It is still resurrection, of your life and breath.

Two generations or so later, the *Wisdom of Solomon* distinguishes body and soul:

wisdom will not enter a deceitful soul, nor dwell in a body given to sin (1).

It distinguishes also the sources of immortality and death:

god created man for immortality (2); the envy of the devil brought death in the world (*id.*).

It is explicit also that it is all about justice:

the souls of the just are in god's hand; they may be punished in the sight of men, but theirs is the certain hope of immortality (3);

the just will live forever (5).

The writer of the *Wisdom of Solomon* recommends himself as Solomon (9). Solomon has read Plato. At the same time, Antiochus of Ascalon read the Stoic doctrines in Plato. Nor could Solomon have failed to read Panaetius in Plato.

When one century-odd later the Hebrew holy writings – the septuagint, in particular – began becoming the old testament, the *Second book of the Maccabees* and the *Wisdom of Solomon* both began becoming parts of that testament as well.

Back in Judea, two sects began vying for the stewardship of the law, the pharisees and the sadducees. Josephus describes the sects:

the pharisees are thought to know the law the best and to be the leading sect; they say that everything happens by fate, and that is, by god; men decide whether to do good or evil, but in every deed there is also fate; the soul is imperishable, and the souls of the good pass into other

bodies, the souls of the evil are punished forever; the sadducees utterly deny fate, and say that god is incapable of being part of sin; they say that men are free to choose good or evil, and everyone must decide for himself; the eternity of the soul, and punishments and rewards in the hereafter, they deny utterly as well (*War*, II, 8).

The philosophers in the schools forgot Plato. The pharisees and the sadducees, however, were steeped in Plato.

Later on, st Paul, before he became st Paul, was a pharisee. At the same time, as st Paul was becoming st Paul, the pharisees also composed the mishna.

In the two centuries from Augustus to Marcus *pax Romana* flourished. The Roman domain reached its greatest extent, incorporated Britain, and expanded north of the Danube, and east of the Euphrates. Assimilation flourished. One generation after Marcus, in 212, the process of extension of citizenship was completed. Ulpian reports: everyone in the Roman domain has been made Roman citizen by a rescript of Caracalla (*Digest*, I, 5.17). Peace and prosperity flourished.

And so did teaching.

Modestin records a rescript of Pius, the predecessor of Marcus:

in lesser cities 5 doctors, 3 rhetoricians, and 3 grammarians may be exempt from public duty; in the largest cities 10 doctors, 5 rhetoricians, and 5 grammarians may be exempt; beyond this number no one may be exempt; and the number does not include philosophers, because only a few study philosophy, and the wealthy can voluntarily choose to give the benefit of philosophy to their cities (XXVII, 1.6).

Of the 6 345 localities to which Ptolemy assigns latitudes and longitudes not less than one half are cities, not less than 2 000 are cities in the Roman domain. Ignoring the doctors, as well as the philosophers whom the benevolence of the wealthy may have given to their cities, and presuming, however, that no opportunity to be exempt from taxes would be missed, the total would be not less 12 000 teachers of rhetoric and grammar. Theophrastus taught 2 000. Even if his prowess was exceptional, the average would not be less than, say, 1 000. If so, the total would be not less than 12 000 000 taught. Not less than one

fifth of the human population of the Roman domain.

In these two centuries the development of the church, of holy scriptures, and of the doctrine of immortality came out of Judea, and spread through the Roman domain.

It came out of Judea with

6^{AD} - 62 st Paul.

And st Paul teaches you, in the *First letter to the Thessalonians*:

do not remain ignorant about the dead, brothers; do not grieve about them like the men who have no hope; we believe that Jesus died and rose again, and so will those who die Christians, god will bring them back to life with Jesus; these are the lord's words, we tell you;

we who are left alive will not go first; at the word of command, at the sound of the archangel's voice and of god's trumpet-call, the lord will descend from heaven himself, and first the Christian dead will rise; then we who are left alive will join them, to meet the lord in the clouds; and we shall always be with the lord;

console yourselves with these words (4).

In the *Letter to the Romans*:

you have no defense, you whoever you are, you who sit in judgment; you who judge are equally guilty (2); do you imagine that you will escape the judgment of god (*id.*)?

all alike have sinned; all alike are justified by god alone through Jesus Christ (3); through one man sin entered the world, and through sin death, and death pervaded the human race; for all men have sinned (5);

shall we all then persist in sin? no! no! baptized into Jesus Christ we are on the new path of life (6); confess that Jesus is the lord, and you will find salvation (10).

All alike have sinned. Do you imagine that you will escape the judgment of god? Confess that Jesus is the lord, and you will find salvation.

First the Christian dead will rise. Then we who are left alive will join them.

St Paul is teaching you resurrection, and the resurrection rather of the body. The pharisees, to judge by Josephus's report, distinguished body and soul. This former pharisee, to judge by his letters, does not. His is the more straightforward doctrine of *Daniel* and the *Second book of the*

Maccabees. In any of his letters, and in the spurious as little as in his own, st Paul has very little to say about your soul. Nor does he distinguish your soul from your body. In the *First letter to the Corinthians*, for instance: if there is an ensouled body, there is also an enspirited body (15). And when he teaches you, in the *Letter to the Romans*: every soul must submit to the authority above it (13), clearly, every ensouled body and every enspirited body, both, must.

That st Paul is teaching you resurrection, and the resurrection rather of the body, the early fathers of the church corroborate clearly and distinctly.

To collect the fathers, they have to be divided into three kinds first.

Two and a half-odd centuries after st Paul, Eusebius wrote *The history of the church*. He begins: the chief matters treated in this work are the following (I, 1). The chief matters are five, and the first two are:

1. the lines of succession from the holy apostles, and the times that elapsed in each from the time of our savior to our own; the most important things that happened in the church; the pastors of the famous congregations; and the men of each generation who, by preaching and writing, spread the divine word;

2. the names and times of those who, driven by passion for novelty, strayed very far from the truth, and proclaiming themselves founts of knowledge, falsely so called, like savage wolves wreaked havoc on the flock of Christ (*id.*).

The first kind of fathers are the pastors of the famous congregations. The second kind are the men who, by preaching and writing, spread the divine word. The third kind are the savage wolves. The three kinds overlap very substantially. Many pastors were also men who spread the word. Most of the men who spread the word were also pastors. And many pastors and men who spread the word turned out to be also savage wolves. The three kinds can nevertheless be distinguished, as Eusebius already, in fact, distinguishes them. Hence, the pastors can be collected later. The savage wolves can be collected later as well. Here, the men who spread the word can be collected. Again, most of them were also pastors, and quite a few of the them turned out to be also

savage wolves.

These 23 are all important fathers of the church who, by preaching and writing, spread the divine word, through Vincent of Lérins,

43 - 102	Clement of Rome,
56	Ignatius,
58	Polycarp,
98 - 166	Justin Martyr,
121	Irenaeus,
150 - 214	Clement of Alexandria,
160 - 240	Tertullian,
170 - 235	Hippolytus,
181	Africanus,
185 - 254	Origen,
200 - 258	Cyprian,
250 - 325	Lactantius,
251 - 356	st Antony,
265 - 339	Eusebius,
295 - 373	Athanasius,
330 - 379	Basil of Caesarea,
330 - 390	Gregory of Nazianzus,
335 - 394	Gregory of Nyssa,
339 - 397	Ambrose,
340 - 420	Jerome,
354 - 430	Augustine,
384	Orosius,
394	Vincent of Lérins,

These 5 are some of the more important among the not important,

58	Papias,
98	Hegesippus,
187 - 265	Dionysius of Alexandria,
315 - 368	Hilary,
351 - 407	Chrysostom.

One generation after st Paul, Clement of Rome, in the *Letter to the Corinthians*, teaches you:

think, my friends, how the lord offers us proof after proof that there is going to be resurrection, of which he made the first fruit, by raising him from the dead, Jesus Christ;

think, my friends, how there is resurrection at this very instant; night sinks to rest, day rises, day passes away, night comes again; and how, in what way, does the crop rise? the power of the lord raises it from the dead, so that the fruit springs up from the seed (24);

look at that strange bird in the east, the phoenix (25).

Clement describes that strange bird in the east in some detail, and concludes:

now, if the creator of all things makes use of a bird to show the magnificence of his promise, can it be strange that he has resurrection in store for those who serve him (26)?

Later in the same generation, Ignatius, in the *Letter to the Romans*, reports:

I am writing to all churches to assure them that I am in earnest, truly, about dying for god;

pray, let me be a meal for the beasts, they are my way to god; I am his wheat, ground fine by the lions' teeth to be the bread of Christ (4);

there is no pleasure for me in meats that perish, there is no delight for me in this life; I long for the bread of the lord, I long for the flesh of Jesus Christ, I crave to drink of his blood, which is imperishable life (7).

By that time Unas had been having the big ones for his morning meal, the middle ones for his midday meal, the little ones for his evening meal, for twenty-four centuries.

Christianity thus also began flourishing greatly in these two centuries.

Judaism was canonized. The torah was closed. From the couple of generations before

50 - 135 Akiva

through the couple of generations after him, the mishna was written. The mishna is not just a commentary on the torah. The torah is the written law. And the mishna is the oral law. The mishna is the law on the law. The mishna was then also closed. From the couple of generations after

135 ha-Nasi

through the couple of generations after

357 - 427 Ashi

the talmud was written. The talmud is not just a commentary on the mishna either. The entire canon of Hebrew holy scriptures was closed, torah, neviim, and kethuvim. And the talmud is the law on the law on the law, as well as the law on the entire law.

Ashi is contemporary with Augustine.

In the two centuries from Marcus to Theodosius the power of Rome and the unity of its domain held firm. Under continuous and steadily heavier assault now. Barbarian incursions began. The barbarians developed a fondness for plundering the wealthy state, and armed hordes began

coming from the ends of Eurasia to claim their share of the plunder. The state retreated to the firmer frontiers of the Danube and the Euphrates, still holding firm. It continued its labors at assimilation as well, of the barbarian hordes now. In this, however, the state was now increasingly ineffectual. In this the church was now increasingly effectual.

In these two centuries teaching accordingly also diminished, but not too much yet either. The men who, by preaching and writing, spread the divine word, for instance, were most of them also teachers of rhetoric and grammar.

And in these two centuries the church, holy scriptures, and the doctrine of immortality triumphed.

Cyprian established the fact:

he cannot have god for his father who does not have the church for his mother.

Eusebius attests the closure of the canon of Christian holy scriptures (*History*, III, 25).

In 325, at Nicaea, the first ecumenical council of bishops gathered, under the emperor, Constantine. The bishops decreed Christianity a dogma, the Nicene creed.

In 381 Theodosius decreed the surrender of all churches to Nicene bishops. In 391 he decreed the closure of all pagan temples and interdiction of all pagan cults.

In the generation after Theodosius the state began retreating before the barbarians.

It is the generation also of Augustine. As he himself lists them in the *Reconsiderations*, Augustine wrote 93 treatises, in 254 books.

And in the next generation Vincent of Lérins concluded the development:

every care should be taken to hold fast to what is believed everywhere, always, by all.

The fathers of the church one and all vigorously taught at first resurrection, then the immortality of the soul. In the development of the doctrine by the fathers, however, there are only two steps, Origen and Augustine, with one important contribution by the pagan Plotinus in between.

Not quite two centuries after st Paul, Origen's most important treatise is *On the principles*.

On the principles is in four books.

In book I Origen treats of the father, the son,

the holy spirit, and the rational creatures.

The rational creatures are demons, angels, and men. Origen explains:

god made all souls pure and obedient to god;
but some of them sinned badly and became demons;

others sinned more lightly and became angels;
still others sinned not as badly as the demons,
but not as lightly as the angels either; and for
these god made the world and bound them in
bodies for punishment (8).

In book II Origen treats of the world and the creatures in it. He writes:

we should now inquire whether after this world there will be healing and improvement, very severe and full of pain for those who have refused to obey the word of god, but an instruction and training for those who in the present life have devoted themselves to the word of god, and having become purer in spirit and attained the capacity for divine wisdom, may advance to a richer understanding of the truth;

for there is no doubt that the savior promises something more glorious and more splendid than the present world, and exhorts all who believe in him to direct their lives toward it (3).

Origen inquires whether after this world there will be healing and improvement in some detail, and concludes:

there is resurrection of the dead;
and there is also punishment;

but the punishment is not forever; for when the body is punished, the soul is purified, and it is thus restored to its original state; the punishment of the wicked and of the demons thus has an end, too, the wicked and the demons will be restored to their original state as well (10).

In book III Origen treats of the struggle of the soul, bound in a body for punishment, helped by angels, hindered by demons, to choose salvation.

In book IV he treats of the divinity of the holy scriptures.

With Origen the doctrine of immortality is restored to its Platonic substantiality and purity.

God made all souls pure and obedient to god.
But they all sinned.

Those that sinned not as badly as the demons,
but not as lightly as the angels either, god bound
in bodies for punishment.

There is resurrection.

There is also punishment.

To those who in the present life have devoted themselves to the word of god, and having become purer in spirit and attained the capacity for divine wisdom, the savior promises something more glorious and more splendid than the present world.

The punishment of the wicked is very severe and full of pain, but it is not forever, the wicked will be restored to their original state as well.

Augustine will make two fundamental corrections.

In Origen punishment is very severe and full of pain, but not forever. Augustine prefers punishment very severe, full of pain, and forever.

In Origen to those who in the present life have devoted themselves to the word of god the savior promises something glorious and splendid. Augustine prefers salvation gratuitous and not for merit.

In the generation after Origen

205 - 270 Plotinus

writes:

beyond all thought and all knowledge, ineffable and incomprehensible, is the one (*Enneads*, V, 4.1); the one is not all things, the one is prior to all things (III, 8.9); the one is the good (VI, 9.6); the one begets all things (9.3); all things proceed from the one (V, 4.1).

the highest procession is thought (1.3);

next is the soul (*id.*);

the soul creates the world (II, 9.7);

a soul always exists in the soul, and proceeds from the soul (IV, 3.3);

but a soul commits two evils; the first is its descent in a body, the second is the evil it does after its arrival here (8.5);

a soul cleanses itself of everything bodily by practicing virtue (I, 2.3-7); it ascends toward the one, it reaches the one, and it is restored to true life; it embraces all that it loves with its entire being; there is no part of it that does not touch the one (VI, 9.9).

Soul and body are distinguished sharply in Origen. They are distinguished yet more sharply in Plotinus.

In Origen the soul is bound in a body for punishment. Plotinus prefers its descent in a body

to be an evil which the soul commits.

One and a half-odd centuries after Origen, one and a half after Plotinus, Augustine writes in *On the Christian doctrine*:

as the abandonment of earlier life, which comes by repentance, is a death of the soul, so the death of the body is the dissolution of earlier existence; and just as the soul is reformed after repentance, so we must believe and hope that after death, to which we are all given by the bondage of sin, the body is reformed into something better at the resurrection; which is not to say that flesh and blood take over the kingdom of heaven; it is to say that the corruptible becomes incorruptible, the mortal becomes immortal, and it is moved into supreme tranquility by its blessed and perfect soul (I, 18).

In *On nature and grace*:

in the beginning man's nature was created without any fault or any sin (3);

human nature such as we are all born in it from Adam, however, requires a physician (*id.*); the defect which darkens all natural goods in it is from that original sin; that criminal nature, therefore, deserves the justest punishment (*id.*);

the grace of Christ, therefore, without which neither child nor man can be saved, is given gratuitously and not for merit (4);

those who are not saved through grace by receiving the bath of regeneration, therefore, are justly condemned (*id.*); and the whole thing must be punished (*id.*);

who, therefore, can be so completely mad not to be moved to ineffable gratitude for the mercy of god who saves whomever he wishes (5)?

And in *The city of god against the pagans*, Augustine devotes the last four of its twenty-two books, XIX-XXII, to:

the merited ends of the two cities,

the one before last, XXI, to:

the eternal punishment (1),

the last, XXII, to:

the eternal blessedness (1).

Human nature requires a physician. That criminal nature deserves the justest punishment. And the whole thing must be punished.

The soul is reformed after repentance.

Those who are not saved through grace, therefore, are justly condemned. Punishment is eternal.

So we must believe and hope that after death the body is reformed into something better at the resurrection. The corruptible becomes incorruptible, the mortal becomes immortal, moved into supreme tranquility by its blessed and perfect soul. Blessedness is also eternal.

Who, therefore, can be so completely mad not to be moved to ineffable gratitude for the mercy of god who saves whomever he wishes?

Plato formulated and solved the ethical problem of the soul.

Have you realized that the soul is immortal?

Are you sure? I would be wrong not to be, and so would you.

You should consider the nature of the soul, you should conclude that the soul is immortal, you should believe that your immortal soul is yet altered by the life you choose, and you should choose accordingly, so that you avoid being flayed and lacerated on thorn bushes, earn your rewards, and here and in the hereafter be happy.

Teaching you that is an essential part of the problem.

Eight centuries after Plato, the problem has developed.

The formulation of the problem has been given irresistible urgency. Have you realized? has become, who, therefore, can be so completely mad?

The solution has been honed to perfection. You should consider, has been removed, all consideration has been done for you. Your rewards are still eternal. But the flaying and laceration has also been extended to all eternity.

Eight centuries after Plato, the problem is nevertheless unambiguously Plato's problem.

In *On the Christian doctrine*, Augustine reports:

a scandalous calumny was leveled by readers of Plato, who had the temerity to say that our lord Jesus Christ learned all his ideas from the writings of Plato, for Plato undeniably lived long before our lord came in the flesh; but my good friend Ambrose examined secular history and found out that Plato went to Egypt at the time when Jeremiah was there, and thus demonstrated that it is surely more likely that Plato was shown our books by Jeremiah and so learned and wrote the things for which he is justly praised.

Ambrose examined secular history and demonstrated that it is surely more likely that Plato was shown our books by Jeremiah and so learned and wrote the things for which he is justly praised.

Surely less likely as it may be, the examination of history, secular and otherwise, shows the pharisees steeped in Plato, st Paul before becoming st Paul a pharisee, Origen steeped in Plato, Plotinus steeped in Plato.

In *The city of god against the pagans*, Augustine writes in conclusion:

if he who, according to Plato, promised to do the impossible, to raise up the flesh so that it becomes incorruptible, immortal, and spiritual, why do our adversaries still insist that what god has promised is impossible? the world has long believed in god's promise; the god of whom we proclaim that he will do the impossible is the very god of whom Plato declares that he can do the impossible (XXII, 26);

Plato and Porphyry agree with us; Plato says that holy souls will return to bodies; Porphyry says that they will not return to evil; and it follows from these premises that the soul will receive a body in which it can live forever in felicity without any evil; which is precisely what the Christian faith preaches (28).

The pagan
234 - 308 Porphyry
was a follower of Plotinus.

It is Plato's problem. But the problem has developed. The formulation has been given irresistible urgency, the solution has been honed to perfection.

The most important development, however, is the more general.

In Plato already the essential are the sentences, the prayers, incantations, and exhortations. Plato has yet brought many propositions in connection with one another in the formulation and solution of the problem. And he has brought these propositions in connection with many other propositions. He has brought the problem in connection with the doctrine of recollection. He has grounded the problem in arguments, one from opposites, one from knowledge, one from the simple, one from the forms, before the one from the hemlock. He has brought it in connection with the theory of the forms. He has incorporated it in

the ideal city, and in the life-long education of its guardians, who will study through their thirtieth year arithmetic, geometry, astronomy, and harmony.

The fathers have divested the problem of all that. All that they have replaced with holy scriptures.

It is still a problem, more or less. A problem is everything in which there is truth or falsehood. And st Paul: I speak the truth (*Romans*, 9). Origen: we hold that this alone which in no way disagrees with the church and the apostles is to be believed as the truth (*Principles*, I, pr.). Augustine: my soul is like a house, small for you to enter; but I pray to you, enlarge it; for you are the truth (*Confessions*, I, 5).

Truth, too, has been reduced to prayers, incantations, and exhortations.

Nothing shows more clearly and distinctly that the efficacy of teaching in producing philosophy is none. The fathers liberated teaching, and you, from all philosophy.

The true philosopher is the lover of god.

And when, conversely, the lover of god became the true philosopher, all philosophy, including that of the fathers, their labors completed, stopped.

The cause is the same, the organism.

The ethical problem of the soul is not part of the acquisition of knowledge.

The relation of the ethical problem of the soul and the physical problem of the soul, and of the ethical problem of the soul and any physical problem, is exclusion.

cclxxiv

The twelve and a half centuries after Theodosius.

Little has to be said about these centuries.

The first generation in these centuries is Augustine's.

The next is Vincent of Lérins's.

Every care should be taken to hold fast to what is believed everywhere, always, by all.

It was.

And in the following twelve centuries there is no development in the ethical problem of the soul.

Four instances,

1217 - 1292 Roger Bacon,
1260 - 1328 Eckhart,
1483 - 1546 Luther,
1596 - 1650 Descartes.

At length philosophy revived.

Counting from Abelard and Gerard, Roger Bacon is the successor of four generations of philosophers who have been taking steps, cautiously, but steadily, to divest themselves of the fathers. And in the series of these steps, Roger Bacon's is the boldest.

He wrote *The great work*, in seven parts. Part I is *The causes of error*, part II *Philosophy*, part III *The study of tongues*, part IV *Mathematics*, part V *Optical science*, part VI *Experimental science*, part VII *Moral philosophy*.

While the philosophers still avoided experience and mathematics not less than they did the devil, Roger Bacon's central argument in *The great work* is that there can be no knowledge without either experience or mathematics.

Where moral philosophy is concerned, however, he writes:

the soul is immortal;

and in the other life felicity is reserved for some, but misery for the evil (1).

The felicity is:

purification of the soul from sin, withdrawal of the soul from the natural desires of the body, elevation of the mind from this world of sense, witnessing things of which in this life the human mind is incapable of forming judgment (*id.*).

The misery is:

the unbelievers will be compelled to believe by actual suffering (*id.*);

and the wicked will suffer in hell with the devil himself, to see the very savage and furious author of death whom they have followed in their useless and harmful desires (*id.*).

Even if Roger Bacon does introduce something new in the solution of the problem. All these doctrines he finds not in the fathers, but in:

Aristotle, Cicero, the Hermetic corpus, and ibn Sina (*id.*).

One generation-odd later, the greatest departure from orthodoxy in these centuries on the matter of the soul is Eckhart's.

Eckhart preached in both Latin and German. The following is from the German sermons:

there is nothing in heaven or on earth, nothing among all the marvelous creatures which god has brought into being, so similar to god as the temple which the human soul is; here alone god is at home;

not even the highest angels are as perfectly like god as the human soul (1);

if someone asks me, why do we pray? why do we fast? why do we do the devotions? why are we baptized? why did god take the flesh? I reply, so that god be born in the soul and the soul be born in god; this is why scripture was written; this is why god created the world; this is why god made the angles; so that god be born in the soul and the soul be born in god (38);

god creates the world perfectly and entirely now; for god is in all things, but as divine and rational, god is in the innermost and highest part of the soul more properly than he is in anything else; and it is here that god creates the world; it is here that god made everything he made 6 000 years ago and it is here that god will make everything he will make 6 000 years hence, if the world continues that long;

god creates everything in the innermost and highest part of the soul; it is here that the father gives birth to the son, and it is here that with his only begotten son he gives birth to you (30);

god makes himself beautiful for the soul and gives himself to the soul; god has made the whole godhead to make the soul desire him, and he will have no rival (41);

Plato, that great priest, speaks of great things; he speaks of purity which is neither in the world nor outside the world, neither in time nor in eternity, and it is from this purity that god, the eternal father, pours the abundance and depth of his godhead, and it is toward this purity that he gives birth to the only begotten son of his and makes us this same only begotten son of his (28);

the more we turn inward, the more we are transformed into the son, the more the son is born in us, the more we are born in the son, and the more we become the son (41);

I have often spoken of a spark, a light in the soul which is not created and cannot be created; it is this light that apprehends god without a medium, without concealment, naked, as he is in himself;

untouched by space and time, opposed to all creatures, this spark desires nothing but god, naked, as he is in himself; this spark is not satisfied with the father, the son, and the holy spirit, not even with the three together; this light is not satisfied even with achieving union with the divine nature; nay, I say something even more astonishing; I declare, by the eternal truth itself, that this spark is not satisfied with the divine being; it wants to know where the divine being comes from, it wants to penetrate its foundation, it wants to penetrate into the desert where no distinction has ever been glimpsed, not the father, not the son, not the holy spirit; only here, in this most inward place, where everything is a stranger, in the stillness and immobility more inward than itself, the spark is satisfied (48);

this the wise should understand, those with less understanding should believe (39).

Clearly, Eckhart has neglected your punishment. And he has vastly exaggerated your rewards. You may have eternal blessedness. But your soul is more perfect than the highest angels? The father gives birth to his only begotten son in your soul? Your soul is his only begotten son? Your soul is not satisfied with the father, the son, and the holy spirit, not even with the three together?

In 1329 a committee under John XXII judged all this heretical.

As Eusebius reports, savage wolves had wreaked havoc on the flock of Christ from the first. And two centuries after the savage wolf Eckhart, the savage wolf who wreaked the greatest havoc on the flock of Christ appeared. The wars to exterminate that wolf and bring the flock of Christ back into the fold continued for four generations. The wolf won.

Nevertheless, in *On the freedom of a Christian man*, in 1520, Luther writes:

let us consider the benefits;
Christ is full of grace, life, and salvation;
the soul is full of sin, death, and damnation;
now let faith come between them; sin, death, and damnation will be Christ's, grace, life, and salvation will be the soul's (*id.*).

One century after Luther, the last generation in these twelve and a half centuries is Descartes's.

The great instauration of the sciences had

begun in the previous generation with the great advances of Bacon, Galileo, and Kepler. Descartes himself greatly advanced mathematics and physics, including most particularly the formulation and solution of the physical problem of the soul. With practical philosophy Descartes concerned himself once.

In 1637, in *The method*, Descartes writes:

I formulated for myself provisional morals, consisting of only three of four maxims.

The first maxim is:

to obey the laws and customs of my country, retaining constantly the religion in which god had given me the grace to be instructed since my childhood, and governing myself in all other things by following the most moderate and furthest removed from excess opinions of the most sensible among those with whom I had to live.

If Descartes is thus teaching you, he is to govern yourself in all other things by the most moderate and furthest removed from excess opinions. In that one thing, however, to retain constantly the religion in which god has given you the grace to be instructed.

There is no development in the ethical problem of the soul until the next generation, with Spinoza.

Thus in Christendom.

In these centuries Islam also rose and grew very greatly.

Where immortality is concerned,

570 - 632 Muhammad teaches you the older and more straightforward doctrine:

for the unbelievers we have prepared chains and fetters and a blazing fire; robes of fire we have prepared for them; scalding water will rain on their heads, melting their skins and bellies; they will be lashed with iron rods; and when they try to run away, they will be dragged back to hell, and will be told, have now the taste of fire (22.19-22);

the blessed will recline on carpets lined with thick brocade, fruit of every kind will hang within their reach; which of the blessings of your lord will you deny?

bashful virgins will be theirs, untouched by either man or jinn; which of the blessings of your lord will you deny?

virgins as fair as corals and rubies; which of the blessings of your lord will you deny?

two other gardens will be there, with a gushing spring in each; which of the blessings of your lord will you deny?

each will be planted with fruits and dates and pomegranates; which of the blessings of your lord will you deny?

in each there will be virgins chaste and fair; which of the blessings of your lord will you deny?

dark-eyed virgins will be there sheltered in tents, whom neither man nor jinn has touched; which of the blessings of your lord will you deny (55.54-75).

By that time Unas had been shedding his seed for twenty-nine-odd centuries.

And the Muslim philosophers tended to stay further away from discussions of immortality than they did from animals. As

1058 - 1111 al-Ghazali

informs you:

about the joys of heaven and the pains of hell the believers are informed sufficiently in the koran (*Happiness*, 4).

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The *Acts of the apostles* reports:

the apostle came to Athens and the philosophers went to hear what he had to say; and when he brought up the raising of the dead, some scoffed, others said, we will listen to this some other time.

Aristotle is four centuries before the apostle.

The first two of these four centuries are one of the three brightest period in the acquisition of knowledge.

In these two centuries the schools also flourished. The Academy gradually became a school of sceptics. From a school of physicists the Lyceum became a school of ethicists. At the Garden bees sipped at every blossom from the first. From a school of physicists, logicians, and grammarians the Stoa became the school of Panaetius who taught that the useful is honorable.

In the next two centuries, the two before the apostle, the acquisition of knowledge decelerated. The schools sank into obscurity. Teaching spread.

A century before the apostle, Cicero taught

that death is not an evil. A contemporary of the apostle, Seneca taught, when I am able, then I will live as I should.

One century after the apostle, after Ptolemy and Galen the acquisition of knowledge stopped. Teaching flourished. In lesser cities 5 doctors, 3 rhetoricians, and 3 grammarians were exempt from taxes, in the largest cities 10 doctors, 5 rhetoricians, and 5 grammarians. No philosophers were exempt, because only a few studied philosophy, and the wealthy could voluntarily choose to give the benefit of philosophy to their cities.

Not quite two generations after the apostle, Epictetus taught: Chrysippus? great progress that! if you bathe like a faithful man, if you eat like a dignified man, then you truly make progress.

Two generations later, the emperor thought constantly, think constantly that all men die.

The fathers of the church flourished.

No one scoffed, all listened.

The degeneration of philosophy is a main cause of the rise of the church, of holy scriptures, and of the doctrine of immortality.

cclxxvi

The two centuries from Descartes to Darwin.

In these two centuries, these 25 are all important philosophers who are part of the development of the physical problem of the soul,

1626 - 1697 **Redi**,
 1627 - 1705 Ray,
 1628 - 1694 **Malpighi**,
 1632 - 1723 **Leeuwenhoek**,
 1635 - 1702 **Hooke**,
 1637 - 1680 **Swammerdam**,
 1638 - 1686 **Steno**,
 1656 - 1708 Tournefort,
 1660 - 1734 **Stahl**,
 1665 - 1721 **Camerarius**,
 1683 - 1757 Réaumur,
 1707 - 1778 **Linnaeus**,
 1707 - 1788 **Buffon**,
 1708 - 1777 Haller,
 1720 - 1793 Bonnet,
 1730 - 1799 **Ingen-Housz**,
 1744 - 1829 **Lamarck**,
 1769 - 1832 **Cuvier**,
 1772 - 1844 Geoffroy Saint-Hilaire,
 1773 - 1858 **Robert Brown**,

1788 - 1868 **Boucher de Perthes,**
 1792 - 1876 **von Baer,**
 1804 - 1881 Schleiden,
 1809 - 1882 **Darwin,**
 1810 - 1882 **Schwann.**

These are 6 of the more important among the not important,

1635 - 1672 Willughby,
 1641 - 1712 Grew,
 1734 - 1794 Caspar Wolff,
 1752 - 1840 Blumenbach,
 1801 - 1858 Müller,
 1801 - 1867 Lartet.

Willughby coauthored two of Ray's treatises. Grew wrote *The anatomy of plants, with an idea of a philosophical history of plants*, in 1682. Caspar Wolff, in *The theory of generation*, in 1759, formulated the theory of epigenesis. Blumenbach wrote *On the genesis of the native varieties of man*, in 1776. Müller wrote the *Handbook of the physiology of man*, in 1834-40. Lartet discovered the first anthropomorphic fossil ape, *pliopithecus*, in 1836.

The true way begins with the collection of the particulars, proceeds by continuous and gradual ascent, and ends with the most general principles.

And the two centuries divide clearly into two parts.

In the first of these two centuries, through Linnaeus and Buffon, having begun in the previous century with Vesalius, Gesner, Belon, Cesalpino, Harvey, Descartes, the collection of the particulars continued, the true way was followed as staunchly in the investigation of animals and plants as the thread of Theseus must be followed in the labyrinth, observation and experiment accumulated, and with Linnaeus and Buffon the true way began reaching the most general principles. The grounds of the formulation and solution of the problem of the soul became firmer and firmer.

In the second of these two centuries, through Darwin, the collection of the particulars continued and accelerated, observation and experiment mounted, and the theory of organisms began being formulated. Beginning with Buffon, the investigation of organisms was grounded in the investigation of the earth. Beginning with Ingen-Housz, the investigation of organisms began

being grounded in the investigation of the material constitution of bodies as well. The theory of organisms and the collection of the particulars continued to examine one another, and the theory of organisms was formulated conclusively. Selection is the principle of life. The problem of the soul was also solved conclusively.

The investigation of nature became fundamentally experimental with Galileo. Two generations later, its part the investigation of organisms became fundamentally experimental, too, with Malpighi. Telescopic investigation also begins with Galileo in 1610. Two generations later, microscopic investigation begins with Malpighi in 1661, Hooke in 1665, Redi in 1668.

Redi showed that there is no spontaneous generation.

In the *Experiments on the generation of insects*, in 1668, Redi concludes:

although an infinite number of insects is seen daily produced in the dead bodies of animals and decayed plants, they are all generated by insemination, and the putrefied matter in which they are formed has no other office than to be the place, or nest, where the animals lay their eggs and where they also find nourishment;

no animal of any kind is ever generated in dead flesh unless there is in it a previously laid egg (*id.*).

Ray established species as the fundamental unit of the classification of organisms.

He wrote the *Ornithology*, with Willughby, in 1678, the *Method of plants*, in 1682, the *History of fish*, with Willughby, in 1686, the *History of plants*, in 1686-704, the *Synopsis of quadruped and serpentine animals*, in 1693, the *History of insects*, in 1710. And Ray established species as the fundamental unit of the classification of organisms not with the general arguments of the *Method of plants*. With the description of species of quadrupeds, birds, fish, serpents, and insects. And most of all with the description of 18 655 species of plants in the three massive volumes of the *History of plants*.

With Malpighi the investigation of organisms became experimental. Next to anatomy and physiology, histology became a part of the investigation of organisms, next to the study of the structure and functions of organs, the study of the

structure and functions of tissues.

With Malpighi as well the investigation of organisms became microscopic.

In 1611 already Galileo used a telescope as a microscope, at close range, to observe insects. In the following years, as he continued to build telescopes and to perfect them, he began building also microscopes. In 1624 he reports: I have observed many tiny animals with great admiration; the flea is horrible, but the gnat and the moth are very beautiful; I have observed also how flies walk upside down on a mirror (to Cesi, 23 Sep). In the same year Galileo built a microscope for Cesi, and in 1625, in the *Apiarium*, Cesi published the first drawings of objects observed under a microscope, of bees magnified 5 times.

After it was first built, the telescope had to wait for Galileo to discover what it is for. Galileo indicated also what the microscope is for. The microscope yet had to wait another generation—odd, for Malpighi in 1661, Hooke in 1665, Redi in 1668, Swammerdam in 1669, Leeuwenhoek in 1676, to discover what it is for.

In *On the lungs*, in 1661, Malpighi demonstrated the capillaries.

In *The anatomy of plants*, in 1675, he writes: the nature of things is enveloped in darkness, and it can be uncovered only by analogy; hence the need to follow it entirely in order to analyze the more complex mechanisms by means of the simpler and more readily accessible to the experience of the senses;

enveloped in their own darkness, the highest animals remain obscure; hence the need to study them by the analogies provided by the simpler animals; it is for this reason that I was led to the study of insects; but the study of insects has difficulties of its own; and for this reason I was led to the study of plants (intr.).

Following the third rule, in particular, as staunchly as the thread of Theseus must be followed in the labyrinth. Nor have the highest animals – the more complex mechanisms which they are – remained obscure.

Leeuwenhoek discovered protozoa.

Like Galileo, Descartes, Huygens, and Spinoza, Leeuwenhoek ground his own lenses. In particular, he ground some 550 lenses. One of those, which he has left behind, is of magnifying

power of 270 and resolving power of $1.4\ \mu$. Leeuwenhoek's descriptions imply magnifying power of 500 and resolving power of $1.0\ \mu$. More powerful lenses were not to be ground for two centuries. $1\ \mu = .001\ \text{mm}$.

Writing to the Royal Society, in 1676, Leeuwenhoek reports:

these animalcules are more than 10 000 times smaller than the animalcules which Swammerdam described and named water fleas;

in one drop of water there are not less than 1 000 of them (to Oldenburg, 9 Oct).

Hooke discovered the cell.

In the *Micrographia, or some physiological descriptions of minute bodies*, in 1665, he begins:

it is the great prerogative of mankind above other creatures that we are not only able to behold the works of nature, or to sustain our lives by them; we have also the power of considering, comparing, altering, assisting, and improving them to various ends; and as this is the peculiar privilege of human nature in general, so it is capable of being so far advanced by the helps of art and experience as to make some men excel others in their observations and deductions almost as much as they do the beasts (pr.).

The last result, the one concerning the observations and deductions of some men, was at that time, as it could not be at others, the result of direct observation.

Hooke describes and depicts the cells of cork, then counts the cells:

I observed 60 of these cells in the 18th part of 1 inch; whence I conclude that there must be nearly 1 100 of them in 1 inch, and therefore 1 166 400 in 1 square inch, and 1 259 712 000 in 1 cubic inch (18).

Swammerdam described the development of insects.

In the *General history of insects*, in 1669, he writes:

the pupa does not suddenly change into a flying animal; rather, the same insect, having taken the form of a pupa by shedding its skin, gradually becomes a flying animal; its changes are nothing but the very changes that occur in a chick, which does not suddenly become a hen, but becomes a hen by gradually growing its parts; or also, it is like the young of a frog, which does

not suddenly become a frog, but becomes gradually a frog by the development of its parts;

the limbs which an insect acquires, when it changes from a state without limbs into a state with limbs, do not appear suddenly, they grow with the insect at their proper places under the skin, as the animal adds one after another, and that is, by epigenesis (2);

the parts of the animal are not born suddenly, they grow slowly, one after another, and they increase and are born as the animal gradually projects new limbs, gradually adds new parts, by epigenesis, and not by sudden transformation, not by metamorphosis; and this is the foundation of all changes of the bloodless animals (3).

Steno showed what fossils are. Fossils are not just the remains of animals and plants. Fossils are the remains of animals and plants more ancient than the rocks in which they are embedded.

Twenty-two centuries before Steno, Xenophanes first noticed the glossopetrae of Malta (Hippolytus, *Refutation*, I, 14), Herodotus described fossils (II, 12), and fossils continued to fascinate the philosophers. In the century-odd before Steno, on fossils wrote Leonardo, Fracastoro, Agricola, Cardano, Mattioli, Palissy, Cesalpino, Fallopius, Mercati.

In the *Harbinger of a dissertation on solids enclosed by nature in solids*, in 1669, Steno writes:

the glossopetrae of Malta were once the teeth of sharks;

what has been said about the teeth of sharks holds for the teeth of rays, the vertebrates of fishes, whole fishes of every kind, the crania, horns, teeth, femurs, and other bones of land animals;

and what has been said about animals and their parts holds also for plants, and whether dug out of the strata of the earth or hidden in rocky substances.

Moreover:

the bones of animals and other solids of this kind had become solid when the earth and the rock containing them were still fluid; not only did not the earth and the rock enclosing these solids produce these solids, the earth and the rock enclosing these solids did not exist when these solids were produced.

Steno has discovered also why fossils, the solids enclosed by nature in solids, will provide the chronology of the stratification of the earth, and hence of the earth.

It had been a miraculous dozen of years, 1665 to 1676.

There are 1 259 712 000 cells in 1 cubic inch. There is no spontaneous generation. Solids enclosed by nature in solids are the remains of animals and plants. There are animalcules more than 10 000 times smaller than the smallest animalcules previously observed, not less than 1 000 of them in one drop of water.

Moreover, the development of insects is like the development of frogs and the development of birds. The study of man leads to the study of insects which leads to the study of plants.

The investigation of animals, the investigation of plants, and the investigation of man, too, were becoming the one uniform investigation of organisms.

Tournefort established genus as the fundamental unit of the collection of species in the classification of organisms.

He published the *Elements of botany, or method to know the plants*, in three massive volumes, two of them with illustrations, in 1694. And Tournefort established genus as the fundamental unit of the collection of species, like Ray, not with general arguments, but with the description of 698 genera in the *Elements*. Linnaeus writes in the *Botanical philosophy*: Tournefort was the first to establish generic characters (209).

And Linnaeus collected the species in the genera,

Ray	18 655 species,	
Tournefort		698 genera,
Linnaeus	5 944 species of	1 239 genera.

Stahl then reaffirmed, the soul is the principle of life.

Stahl formulated the theory of phlogiston, between 1697, in the *General theory of fermentation*, and 1723, in the *Foundations of dogmatic and experimental chemistry*.

In the generation before Stahl, Boyle had destroyed the theory of the four elements. He had shown that no substance had been decomposed into earth, water, air, and fire. Three generations after Stahl, Lavoisier will destroy the theory of

phlogiston. He will compile the table of 33 simple substances, and show that a great number of substances have been decomposed into the 33 simple substances and recomposed from them. In the meantime, in the century-odd from Boyle to Lavoisier, the decompositions and recompositions were gathered. On the ground of the theory of phlogiston.

Phlogiston is a substance. It is the principle of combustion. It is present in all three kingdoms of nature, mineral, vegetable, and animal.

In minerals the expulsion and reception of phlogiston are reversible. Stahl demonstrated the reversibility. And as Kant writes: when Stahl changed metal into calx and calx back into metal, new light dawned on all investigators of nature.

In vegetables and animals, however, the expulsion of phlogiston is irreversible, once its phlogiston is lost, the original compound cannot be recomposed. The principle of life is soul.

Stahl showed that the discoveries of the last two generations, no spontaneous generation, cell, fossils, microscopic animalcules, development of insects which is like that of frogs and birds, investigation of man by the investigation of insects and plants, tens of thousands of species, hundreds of genera, each posed, and the more so all jointly posed, acutely the problem of generalizing over them, and thus, most particularly, the reexamination of the problem of the principle of life.

Descartes had proposed, it is not necessary to suppose that there is a soul in the body, or any other principle of life than its blood and its spirits, and the body is nothing but a machine made of earth, and all functions of this machine follow entirely naturally from the mere disposition of its organs.

It will be labeled mechanism.

Stahl proposed, the soul is the principle of life.

It will be labeled vitalism.

And the discoveries continued, and continued to pose ever more acutely the problem of generalizing over them, and thus the reexamination of the problem of the principle of life.

Camerarius discovered the same two sexes in plants as in animals.

Whether plants are male and female, like animals, Theophrastus asks, then Cesalpino, then

Malpighi. Camerarius demonstrated that they are experimentally.

In *On the sex of plants*, in 1694, Camerarius recognizes the anther as the male organ in the flower, the ovary with the style as the female organ. Hence he recognizes the difference between dioecious plants and monoecious plants, these with male and female flowers on separate plants and these with male and female flowers on the same plant. Hence he shows, both, that isolated dioecious male plants and that monoecious plants in which the anthers are removed produce fruit, but the fruit is seedless. And hence Camerarius concludes:

we are thus justified to regard the anther as the male part, the ovary with the style as the female part (*id.*).

Réaumur demonstrated regeneration.

When the Academy of Sciences was founded in 1666, Colbert charged it with the task of studying and improving the industries of the realm. Two generations later, Réaumur undertook enthusiastically this task. Over two decades he studied the production of steel, then the production of malleable cast iron, then the production of porcelain, and made substantial improvements to all three. He improved substantially also the thermometer.

Réaumur also wrote the *Memoirs to serve as a history of insects*, in 6 volumes, in 1734-42. The kind insect remained all-inclusive, and Réaumur treats in the *History of insects* not only of spiders, arthropods, worms, polyps, mollusks, crustaceans, but of amphibians and reptiles as well. He writes in the introductory *Memoir on the history of insects in general, and on the view according to which they are treated in this work*:

the part of the history of insects, to which I am most sensitive, concerns their ingenuity and their industries; and their industries have often decided the order in which I treat them; I believe, for instance, that one would like to see together all the insects that clothe themselves, and especially those that are very remarkable in this, rather than disperse them into different classes as they necessarily would be according to the method of Swammerdam (*id.*).

Clearly, continuing the study of the industries of the realm.

Réaumur demonstrated regeneration in 1741. He divided a hydra into parts, and the parts not only continued to live, but after several days each part regenerated the whole animal. In 1745 Bonnet observed the 26 parts, into which a worm had been cut, grow into 26 perfectly constituted worms.

Linnaeus and Buffon then each took a decisive step toward the most general principles.

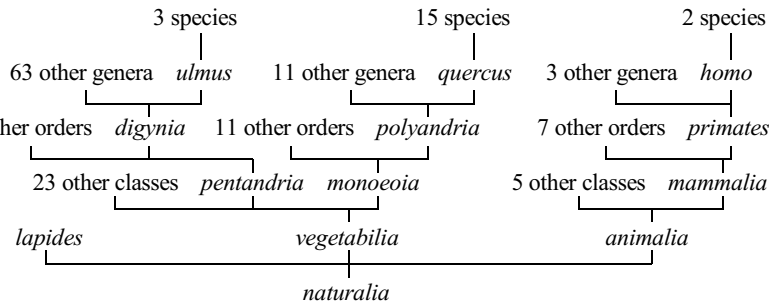
Linnaeus unified all organisms into their one and only one classification.

From *The system of nature, according to the three kingdoms of nature, by classes, orders, genera, and species*, in 1735, through *The species of plants, showing the properly recognized plants, related by genera, with specific differences, common names, selected synonyms, and native locations, arranged by the sexual system*, in 1753, and the fifth edition of *The genera of plants, as well as their natural characters, by number, shape, place, and proportion of all fruit bearing parts*, in 1754, to the tenth edition of *The system of nature*, in 1758.

In the fifth edition of *The genera of plants* and the tenth edition of *The system of nature* Linnaeus arranges the two kingdoms of the empire nature, the kingdom of animals into its 6 classes, their 34 orders, their 312 genera, their 5 897 species, and the kingdom of plants into its 25 classes, their 112 orders, their 1 105 genera, their 5 944 species, for a total in the 2 kingdoms of 31 classes, 146 orders, 1 417 genera, and 11 841 species.

Darwin writes: the arrangement of species, genera, and families into what is called the natural system some authors regard as merely a scheme, or an artificial method; but the famous expression of Linnaeus that the characters do not make the genus, the genus gives the characters, seems to me to imply that some deeper bond is included in our classification than mere resemblance.

The plants are arranged by the sexual system.



The same two sexes in plants as in animals imply the deeper bond. And their one and only one classification implies it clearly and distinctly.

For instance.

Linnaeus is contemporary with Hume. Hume writes: anatomists join their observations and experiments on human bodies to those on beasts, and from the agreement of these experiments derive an additional argument for any particular hypothesis; let us, therefore, apply this method of inquiry to our anatomy of the mind. And as, in fact, anatomists join their observations and experiments on human bodies to those of plants as well, Hume applies this method of inquiry to our anatomy of the mind as well: to put the affair to trial, let us suppose that by the dropping of its seed, an oak or elm produces a sapling below it, which, springing up by degrees, at last overtops and destroys the parent tree; I ask, if in this instance there be any relation wanting, which is discoverable in parricide or ingratitude?

And above is pictured the deeper bond of the cousins, elm, oak, and man. Elm is *ulmus*, oak is *quercus*, man is *homo*. The 3 species of elm are *americana*, *campestris*, and *pumila*, the 15 species of oak are *aegilops*, *alba*, *cerris*, *coccifera*, *esculus*, *gramuntia*, *ilex*, *molucca*, *nigra*, *phellos*, *prinus*, *robur*, *rubra*, *smilax*, and *suber*, the 2 species of man are *sapiens* and *silvestris orang outang*.

Buffon published the first 3 volumes of the *Natural history* in 1749. By his death in 1788 he completed 36 volumes, volumes I-XXIX and the supplementary volumes I-VII. In the order of publication,

volume I deals with the manner of study and treatment of natural history, and formulates the theory of the earth,

volume II deals with the general history of animals,
 volumes III deals with the natural history of man,
 volumes IV-XV deal with the history of quadrupeds,
 volumes XVI-XVIII deal with the history of birds,
 supplementary volumes I-II revisit the theory of the earth,
 supplementary volume III deals with the general history of quadrupeds,
 supplementary volume IV revisits the natural history of man,
 volumes XIX-XXI continue the history of birds,
 supplementary volume V deals with the epochs of nature,
 volumes XXII-XXIII continue the history of birds,
 supplementary volume VI continues the history of quadrupeds,
 volume XXIV continues the history of birds,
 volumes XXV-XXIX deal with the history of minerals,
 supplementary volume VII continues the history of quadrupeds.

In 1788-804 Lacépède published 8 more volumes, on reptiles, fishes, and cetaceans.

In 1749, Buffon concludes the formulation of the theory of the earth with the chapter *General views on nature*. The general views are two. The first is:

nature is an immense living power subordinated to the supreme being, who animates the world by his command; all natural actions depend on, and are sustained by, his concurrence; manifested to mankind, nature is part of the divine omnipotence, which is its cause and effect, mode and substance, design and execution (*id.*).

The second is:

all matter is attracted as the square of the distance inversely; and this law admits of no variations in any particular attractions other than such as arise from the configuration of the constituent particles of each substance;

and I view and understand nature in this way; the phenomena exhibited by crude matter are caused by this single force;

and from this force, combined with that of heat, originate the living particles which give rise

to, and sustain, all the various actions of organisms (*id.*).

From the view of nature the manner of study and treatment of natural history follows.

Buffon thus begins the formulation of the theory of the earth by affirming:

the general history of the earth must precede the history of its productions; it is a necessary investigation for those who want to study nature in all the variety of her manifestations, who want to study the life and habits of animals, or the growth and surroundings of plants (*id.*).

The history of the earth must precede the history of its productions. And the history of its productions has since been preceded by the history of the earth.

Accordingly, Buffon formulates first the theory of the origin of the earth. A comet striking the sun obliquely projected in space a liquid mass equal to $\frac{1}{650}$ of the sun's mass, the liquid diffused, and then reassembled into spheres revolving around the sun.

The theory of the earth is thus the theory of its development. It is thus necessary to establish the age of the earth. The earth is a debris of the sun, and its development is determined fundamentally by cooling. And Buffon began the experiments on rates of cooling, suggested by Newton in *The principles*, in 1749. He concluded them in 1775. The earth is at least 74 832 years old. The other view objected. In 1779, in the *Epochs of the earth*, Buffon reports: according to you, I am told, the earth is 75 000 years old; is it even possible to form an idea of a series of centuries so long? my only reply is to present the evidence and my deductions from it; and the evidence shows that, far from having unnecessarily increased the age of the earth, I may have shortened it far too much (*id.*).

The *Epochs of nature* is Buffon's final formulation of the theory of the development of the earth.

The epochs are seven:

1st epoch, when the earth and the planets acquired their form;

2nd epoch, when matter hardened and formed the interior rock of the globe and the great vitreous masses on its surface;

3rd epoch, when the waters covered the continents;

4th epoch, when the waters withdrew and the volcanoes became active;

5th epoch, when elephants and other animals of the south inhabited the lands in the north;

6th epoch, when the separation of the continents occurred;

7th epoch, when the power of man joined that of nature.

And this is how organisms developed:

the 3rd epoch began some 30 000 to 40 000 years after the formation of the earth;

the seas covered the earth almost entirely, to a height of at least 2 000 fathoms, for this is the height at which shells cease to appear;

and from this we can infer that the animals to which these remains belong are the first inhabitants of the globe;

judging by the immense quantity of these remains, their population was innumerable; for these remains have formed the beds of limestone, marble, chalk, and tufa, which constitute our hills and stretch over all parts of the earth;

we must also presume that these first inhabitants of the still boiling sea can only have been shellfish and fish of kinds different from those today; we can expect that these species existed in the earlier part of this epoch, some 30 000 to 40 000 years after the formation of the earth, continued for some 10 000 or 15 000 years, and then perished;

all living creatures which exist today would have been able to exist in the 5th epoch, when the temperature became similar to the present;

the first men appeared in the 7th epoch;

naked in mind and body, exposed to the elements, the prey of ferocious animals, would they not have sought to unite, to defend themselves by their numbers, to work together to build dwellings and protect themselves?

and they began to sharpen hard stones, jades, and flints into those hand axes, which have long been thought fallen from the sky or formed by lightning, but are, in fact, the first relics of the art of man in the state of nature (*id.*).

With Linnaeus and Buffon the investigation of organisms advanced very greatly.

And in the next century the theory of organ-

isms was formulated, examined, and formulated conclusively.

cclxxvii

Haller wrote the *Elements of the physiology of the human body*, in 8 volumes, in 1757-66. It is the most important medical treatise after Vesalius, grounded extensively in experiment, the first to describe in detail the correlations of functions and structures. Haller demonstrated irritability. The action of muscles and nerves is not by subtle fluids. Muscles have the capacity for contraction with stimulus, or irritability. Irritability has since been shown the property of all organic matter.

Haller wrote also the *Practical medical library*, in 4 volumes, in 1776-88, listing some 52 000 titles.

Bonnet demonstrated parthenogenesis.

Two-odd generations earlier, Leeuwenhoek observed female aphids producing offspring without eggs. In 1740 Bonnet observed one isolated female aphid producing 95 offspring without insemination.

Bonnet also formulated, in the *Considerations on organized bodies*, in 1762, and *The philosophical palingenesis*, in 1769, the two theories, the theory of a series of catastrophes in the development of the earth, and the theory of preformation in the development of organisms.

In the next two generations catastrophes explained fossils, in particular, and the development of the earth, in general. The investigation of the earth advanced with Werner and Hutton. The followers of Werner preferred inundations, they were Neptunists. The followers of Hutton preferred conflagrations, they were Plutonists.

Darwin writes: as we do not see the cause, we invoke cataclysms (*Origin*, 3).

The theory of preformation is the first attempt to build a general theory of the organism. It is the theory that each organism develops from a germ cell in which all its parts are preformed. The organism is differentiated from the first.

Preformation can be traced back to Leeuwenhoek and Swammerdam, and more immediately to Réaumur. Bonnet formulated the theory explicitly. Haller lent the theory his authority.

The organism is preformed.

Or it is not. This is the theory of epigenesis.

Each organism develops its parts in the course of its growth. The differentiation of the organism develops from an undifferentiated structure.

Caspar Wolff formulated this theory in *The theory of generation*, in 1759. Bonnet and Haller criticized the theory. In the next edition of his treatise, in 1764, Caspar Wolff formulated the theory explicitly in opposition to the theory of preformation. The most prominent proponent of epigenesis was Blumenbach.

The theories of preformation and epigenesis are not very sophisticated. Nor are they too successful. At the time when the theory of the motions of the heavenly bodies was advancing from Euler to Lagrange and Laplace, the theory of the elements was about to advance to Lavoisier, with preformation and epigenesis the theory of organisms has not advanced nearly as far as the theory of the motions of the heavenly bodies had with Hipparchus or the theory of the elements had with Boyle.

Nevertheless, that these are not only attempts to build the theory of organisms, but attempts to build the theory of evolution, in particular, Kant can show.

The *Critique of judgment*, in 1790, is in two parts. Part I is the *Critique of esthetic judgments*, and its central concern is taste, and hence also art. Part II is the *Critique of teleological judgments*, and its central concern is the investigation of organized nature, and thus first and foremost organic nature.

Kant writes:

it is well worthwhile to go, with the help of comparative anatomy, through the great creation of organized natures in order to see whether there may not be in them something similar to a system;

and the correspondence of so many genera of animals to a common plan, which seems to be fundamental in the structure not only of their bones but of all their other parts as well, allows us to hope that something can be achieved here on the principle of the mechanism of nature, without which there can be no natural science at all; the analogy of forms, which for all their differences seem to have been produced according to a common original type, strengthens the suspicion of a real relationship in their production

from a common parent, from the form in which the principle of purpose seems authenticated the best, and that is, from man, down to polyp, and from polyp down to mosses and lichen, and from these down to the lowest stage of organized nature which we observe, and that is, crude matter;

we are thus compelled to think of the whole of organized nature as derived only from matter and its powers according to mechanical laws, like those working in the formation of crystals;

hence, the archeologist of nature can allow himself to derive that great family of creatures, and we must so represent them, as a family; if their relationship is to have a foundation, he can allow himself to derive that great family from the surviving traces of the oldest revolutions through the mechanisms known or supposed by him;

he can suppose the bosom of mother earth, as she emerges from her chaotic state, like a great animal, to give birth in the beginning to creatures of less purposive form; these creatures then to give birth to others adapted better to their place of birth and to their relations to one another, and so forth, until this great womb becomes ossified and limited to giving birth to definite species, so that these species remain as they are born at the time when the more fruitful initial formative powers end;

and if the variation, to which certain individuals of organized genera are accidentally subjected, becomes hereditary and is incorporated into the generative faculty, we cannot judge this variation as anything but the development of the purposive capacities preformed in the species, with a view to the preservation of the race (80).

The archeologist of nature will derive the great family of creatures from the surviving traces of the oldest revolutions through mechanisms. According to mechanical laws, like those working in the formation of crystals.

There is a real relationship in their production from a common parent of man, polyp, mosses, and lichens.

And it is a development with a view to the preservation of the race grounded in variation which becomes hereditary.

Kant concludes by judging the two theories, preformation and epigenesis:

even if the great superiority of the theory of epigenesis over that of preformation in explaining the empirical evidence is ignored, it yet finds peculiar favor with reason;

because this theory regards nature as producing itself, not merely unfolding itself; it requires a lesser expenditure of the supernatural; it leaves it to nature to produce everything after the first beginning; as, indeed, the first beginning is always barred to physics, however it may try to include it in a chain of causes (81).

Ingen-Housz discovered photosynthesis.

In the *Experiments on vegetables, discovering their great power of purifying the common air in sunshine, and of injuring it in the shade and at night*, in 1779, Ingen-Housz shows that:

dephlogisticated air oozes out of the leaves of plants (4);

the production of dephlogisticated air from the leaves is due not to the warmth of the sun, but to its light (6),

and concludes:

it is an unfavorable circumstance that air is not an object of our sight; if it was, we should see that plants have respiration as animals do; that leaves are its organs; that they have pores which absorb air, and others which throw it out like the excretory ducts of animals; that the secreted air, being dephlogisticated air, is thrown out as noxious to the plant, as Dr Priestley and Mr Scheele have demonstrated; and that in most plants the greatest part of inhaling pores are on the upper side of the leaf, the excretory ducts on the underside (7).

Dephlogisticated air, as Dr Priestley and Mr Scheele demonstrated, is, as Lavoisier christened it, oxygen.

The investigation of organisms began being grounded in the investigation of the material constitution of bodies.

Lavoisier and Laplace, in the *Memoir on heat*, in 1783, demonstrated that:

respiration is combustion, moreover perfectly similar to that of carbon (4).

Galvani, in the *Commentary on the forces of electricity in muscular motion*, in 1791, demonstrated that:

electricity is present in the animal itself; and we are confirmed in this view by the assumption

that during the occurrence of the phenomenon a very fine nervous fluid flows from the nerves to the muscles like the electric current from a Leyden jar (3).

Lamarck formulated the theory of evolution of animals.

And the theory of animals now, if not yet the theory of organisms, is very sophisticated and very successful.

Among the many treatises of Lamarck, the most important are these four. The *Hydrogeology, or researches on the influences which waters have on the surface of the terrestrial globe; on the causes of the existence of the basins of the seas, their shift and their successive movement to different points on the surface of the earth, as well as the changes which living bodies exert on that surface*, in 1802. The *Researches on the organization of living bodies and particularly on their origin, on the cause of their development and progress in their composition, and on that which, tending continually to destroy it in each individual, brings about their death*, also in 1802. The *Zoological philosophy, or exposition of considerations on the natural history of animals, on the diversity of their organization and the faculties they obtain from it, on the physical causes which maintain life in them and give rise to the movements they make, as well as on what produces feeling in some of them and intelligence in those endowed with it*, in 1809. And the *Natural history of invertebrate animals, presenting the general and particular characters of these animals, their distribution, their classes, their families, their orders, and citation of the principal reported species; preceded by an introduction offering to determine the essential characters of the animal, its distinction from vegetable and other natural bodies, as well as exposition of the fundamental principles of zoology*, in 7 volumes, in 1815-22.

What Lamarck's concerns are, his titles make clear. And the generality of his concerns explains why he coined biology, in 1802. Biology is the investigation of the organization of living bodies, their origin, the cause of their development and progress, the changes they exert on the surface of the earth. Biology is the science of life. In the previous one and a half centuries, counting back

to Descartes, the investigation of animals, the investigation of plants, anatomy, physiology, and all medicine, and the problem of the principle of life, too, had increasingly become biology, the one uniform investigation of life.

Lamarck formulated the theory of evolution in 1802, in the *Researches on the organization of living bodies and particularly on their origin*, and developed it in 1809, in his most important treatise, the *Zoological philosophy*.

The *Zoological philosophy* is in 3 parts. And as Lamarck is fond of the explicit title generally, the titles of the three parts make clear what the main concerns of the treatise are. Part I is *Considerations on the natural history of animals, their characters, affinities, organization, classification, and species*. Part II is *An inquiry into the physical causes of life, the conditions required for its existence, the exciting force of its movements, the faculties which it confers on the bodies possessing it, and the results of its presence in these bodies*. Part III is *An inquiry into the physical causes of feeling, the force which produces actions, and the origin of the acts of intelligence observed in the various animals*.

The theory of evolution is developed in part I. In the detailed contents Lamarck outlines the argument:

it is not true that species are as old as nature, and have all existed as long; it is true that they have been formed one after another, have only a relative stability, and are invariable only temporarily (3);

it is a positive fact that, when the scale of the animals is traced in the usual direction from the most perfect to the most imperfect, an increasing degradation and simplification of organization is observed; consequently, when the scale is traced in the opposite direction, and that is, in the order of nature, an increasing complexity in the organization of the animals is observed, and a complexity which would have increased gradually and regularly, if the conditions of the environment, the mode of life, and so on, had not produced many irregularities in the scale (6);

their environment acts on the organization, the general form, and the structure of the animals; changes in their environment, their mode of life, and so on, cause corresponding changes in the

activities of the animals; and when the activities of the animals become permanent, they in turn cause, both, the more frequent use of some parts of the animals, and hence the development and enlargement of these parts, and the less frequent use of other parts and sometime the cessation of their use, and hence the unfavorable development of these parts, their reduction, and ultimately their disappearance (7).

And in chapter 7, *On the influence of the environment on the activities and habits of the animals, and the influence of the activities and habits of these living bodies on the modification of their organization and structure*, Lamarck reviews the argument:

the true principles are these;

1. every fairly considerable and permanent change in the environment of any race of animals produces a real change in the needs of this race;

2. every change in the needs of the animals necessitates new activities from them for the satisfaction of these needs, and hence new habits;

3. every new need, necessitating new activities for its satisfaction, requires that the animal should either make more frequent use of some of its parts which it used less before, and thus develop and enlarge these parts, or make use of entirely new parts to which the new need has imperceptibly given birth by working on the inner feelings of the animal (*id.*).

And hence Lamarck formulates:

two laws of nature, always verified by observation;

first law, in every animal, which has not passed the limit of its development, the more frequent and continuous use of any organ gradually strengthens, develops, and enlarges this organ, giving it power proportional to the length of time of its use, while the permanent disuse of any organ imperceptibly weakens and deteriorates this organ, and progressively diminishes its functional capacity until it ultimately disappears;

second law, all acquisitions or losses which nature produces in the individuals through the influence of the environment in which their race is placed for a long time, and hence through the influence of the predominant use or permanent disuse of any organs, are passed on by reproduction to new individuals, provided that the ac-

quired modifications are common to both sexes or at least to the individuals producing the young (*id.*).

Three illustrations:

the giraffe, the largest mammal, is known to live in the interior of Africa in places where the soil is nearly always arid and barren; the animal is thus obliged to browse on the leaves of trees and to make constant efforts to reach them; and from this habit, maintained in the entire race, the forelegs of the animal have become longer than its hind legs, and its neck is lengthened to such a degree that, without standing up on its legs, the giraffe reaches to a height of 6 meters;

the bird, which is drawn to the water by its need to find there its prey, separates the digits of its feet when trying to strike the water and move on its surface; by the continuous and repeated separations of the digits the skin connecting them at the base will acquire the habit of being stretched; and thus in course of time the large webs will be formed, which unite the digits of ducks, geese, and other birds, such as we actually observe them; similarly, from the efforts to swim, and that is, to push against the water and move in it, have developed the membranes between the digits of frogs, sea tortoises, otters, beavers, and so on (*id.*);

suppose that the seed of a grass or any other plant, which grows normally in a damp meadow, is carried to the slopes of a neighboring hill, where the ground is still moist enough for the plant to maintain its existence; suppose further that after living and reproducing many times in this new place, the plant reaches little by little the dry and almost barren ground of the mountain side; if this plant succeeds in living there and perpetuating itself for a number of generations, it will change so much that the botanists who find it will make it into a separate species (3).

In the *Historical sketch* added to the third edition of *The origin*, Darwin writes:

Lamarck first did the eminent service of arousing attention to the probability that all change in the organic world, as well as the inorganic, is the result of law, and not of miraculous interposition (*id.*).

And the eminent service is very extensive.

Darwin solved the problem of life. The prob-

lem of the origin and development of all organisms. And the formulation of the problem of life as the problem of the origin and development of all organisms is a consequence most particularly of the labors of Linnaeus and Lamarck. Linnaeus solved the problem of the classification of all organisms, and the solution of this problem posed explicitly, and nothing else could pose explicitly, the problem of life as the problem of the relations of all organisms. The theory of evolution formulated by Lamarck posed explicitly, and nothing else posed nearly as explicitly, the problem of the relations of all organisms as the problem of the origin and development of all organisms.

And although a theory of animals, the theory formulated by Lamarck posed the problem of the relations of all organisms as the problem of the origin and development of all organisms very explicitly.

Lamarck writes: it is not true that species are as old as nature; it is true that they have been formed one after another.

The problem is of how species have been formed one after another.

Lamarck writes: when the scale of animals is traced from the most imperfect to the most perfect, and this is, in the order of nature, an increasing complexity in the organization of the animals is observed.

The problem is of how species of increasing complexity have been formed one after another.

Lamarck writes: it is a complexity which would have increased gradually and regularly, if the conditions of the environment, the mode of life, and so on, had not produced many irregularities in the scale. Also: I do not propose that the existing animals form a simple series, graded regularly throughout; but I do propose that the animals form a branching series which, even if graded irregularly, is free from discontinuity, or at least has been at one time free from discontinuity (3).

The problem is of how species of increasing complexity have been formed one after another in a branching and free from discontinuity series.

Lamarck writes: new needs imperceptibly give birth to new parts by working on the inner feelings of the animal.

It is a very great advance. The problem of the

formation of new species is a problem of the formation of new parts of the animal. There can be no new species without the formation of new parts of the animal from the old.

What the mechanism of formation of new parts from the old is – what the working on the inner feelings of the animal is – will not be discovered until Mendel. What will mediate the relation of new needs and new parts will not become clear until it becomes clear what mutations are. And what the mechanism of the relation of new needs and new parts is will not be established until Chetverikov shows how Darwin stands on Mendel.

Remotely as it is, Lamarck anticipates this. He writes: everything combines to prove my proposition, it is not the shape of the body of the animal or of its parts that gives rise to the habits of the animal and its mode of life; on the contrary, it is by the habits, mode of life, and all influences of the environment that nature has, little by little, in the course of time, succeeded in building the shape of the body of the animal and of its parts, and hence also its faculties (7).

Lamarck is clear also about the crucial character of heredity. It is part of the second law that: all acquisitions or losses which nature produces in the individuals are preserved by reproduction to new individuals, provided that the acquired modifications are common to both sexes or at least to the individuals producing the young.

He is clear also that the problem can only be solved at the level of the individual. All acquisitions or losses which nature produces in the individuals are preserved by reproduction to new individuals. And more generally still: among living bodies nature contains nothing but individuals which succeed one another by reproduction and spring from one another (3).

And one way or another, all change in the organic world, as well as the inorganic, is the result of law.

With Cuvier the investigation of fossils became the investigation of organisms.

Cuvier's most important treatise is the *Researches on the fossil bones of quadrupeds, in which are established the characters of the several species of animals which seem to have been destroyed in the revolutions of the globe*, in

1812, in 4 volumes. As Cuvier reports in the preface: in these 4 volumes will be found the entire series of the author's application of comparative anatomy to the history of the globe (*id.*). And the *Preliminary discourse* of the *Researches* Cuvier expanded in subsequent editions, and published eventually separately as the *Discourse on the revolutions of the surface of the globe, and on the changes which they have produced in the animal kingdom*.

Cuvier wrote also *The kingdom of animals according to its organization, to serve as a foundation of the natural history of animals and as an introduction to comparative anatomy*, in 1817.

He wrote also the *History of fishes*, in 1828-49, of which he published 9 volumes through his death in 1832, and 22 volumes were published through 1849.

Linnaeus, in *The system of nature*, divides the animals into 6 classes:

- mammals,
- birds,
- reptiles,
- fishes,
- insects,
- worms.

Two generations later, Cuvier, in *The kingdom of animals*, divides the animals into 4 branches subdivided into 19 classes:

- vertebrates
 - mammals,
 - birds,
 - reptiles,
 - fishes,
- mollusks
 - cephalopods,
 - pteropods,
 - gastropods,
 - acephals,
 - brachiopods,
 - cirripeds,
- articulated
 - annelids,
 - crustaceans,
 - arachnids,
 - insects,
- zoophytes
 - echinoderms,

helminths,
acalephes,
polyps,
infusorians.

Most important, by far, are Cuvier's labors in paleontology.

He begins in the *Discourse on the revolutions of the surface of the globe*:

as a new species of antiquarian I have had to learn to decipher and restore these old monuments, to recognize and reassemble in their original order the scattered and mutilated fragments of which they are composed, to reconstruct the ancient beings to which the fragments belonged, to reproduce them in their proportions and their characters, and to compare them to the beings living on the surface of the earth today; this is a new art, and it presupposes a new science, namely, the science of the laws governing the coexistence of the different parts of the organism (*id.*).

Cuvier laid the foundations of paleontology, and this threefold. Extinct organisms. The investigation of extinct organisms became a part, and a most important part, of the investigation of organisms. The development of organisms. Hence, more generally, the investigation of the development of extinct organisms became a part, and a most important part, of the investigation of the development of organisms. The development of the earth. And hence, still more generally, the investigation of the development of extinct organisms became a part of the investigation of the development of the earth. It became its chronological foundation, in particular.

Extinct organisms.

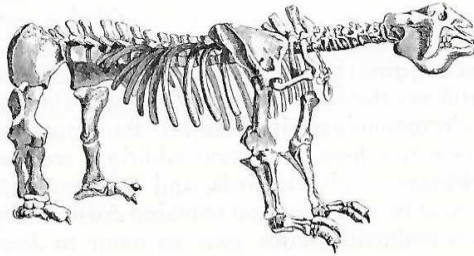
Cuvier writes in the *Discourse on the revolutions*:

the least piece of bone has a determinate character in relation to the class, to the order, to the genus, and to the species to which it belongs, and to such a degree that every time one has one well-preserved piece of bone, one can, with labor and a little help from analogy and comparison, determine all these things, the class, the order, the genus, and the species, as certainly as when one has the entire animal before him (*id.*).

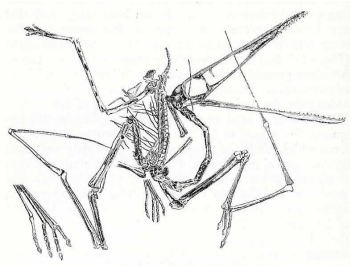
Two instances.

In *A skeleton found in Paraguay*, in 1796,

Cuvier reports: the fossil was deposited 100 feet below the surface of a sandy formation near La Plata (*id.*); the animal is 12 feet long and 6 feet high (*id.*); it belongs to the family of sloths, armadillos, pangolins, anteaters, and armadillos (*id.*); the genus I can name *megatherium*, the species *megatherium americanum* (*id.*).



A fossil found by Collini, was identified by Blumenbach as a bird. In 1801, in the *Species of quadrupeds*, Cuvier writes: this is a very remarkable reptile endowed apparently with the ability to fly (*id.*). He classified the animal, and named it, in 1809, in the *Memoir on the fossil skeleton of a flying reptile from the region of Eichstadt, which some naturalists have taken to be a bird, with which we constitute the genus saurians, and which we name pterodactyl*.



The development of organisms.

Cuvier writes in the *Discourse on the revolutions*:

my most important concern, and the chief object of my researches, constituting also their connection with the theory of the earth, has been to ascertain the particular strata in which each species is found, and hence to inquire whether general laws can be established, and in relation either to the zoological divisions or to the greater or lesser resemblance of fossil species to the species which still exist on earth;

and the laws which have already been established are very distinct and satisfactory;

first, it is clear that the oviparous quadrupeds are found considerably earlier, and that is, in more ancient strata, than the viviparous;

the earliest appearance of fossil bones seems to indicate that dry lands and fresh waters must have existed before the formation of the chalk strata; yet neither at that early epoch, nor during the formation of the chalk strata, nor even for a long time after that, do we find any fossil remains of mammiferous land quadrupeds;

we find bones of mammiferous sea animals in the coarse shell limestone which covers immediately the chalk strata in the neighborhood of Paris; but no bones of mammiferous land animals are found in that formation; yet immediately on reaching more recent formations, the bones of land quadrupeds are very abundant;

as it is reasonable to suppose that neither shell nor fish existed at the time of formation of the primary rocks, we are led to conclude that the oviparous quadrupeds began to exist along with the fishes at the beginning of the period which produced the secondary formations; while land quadrupeds did not appear until a long time after that, until the coarse shell limestone had already been deposited; and these last are the greater part of our genera of shells, even if they are species quite different from the species found in the state of nature today (*id.*).

In the order of their first appearance, the chronological series is:

shellfish,
fishes,
oviparous quadrupeds,
viviparous sea animals,
viviparous land animals.

Shellfish, fishes, oviparous quadrupeds, viviparous sea animals, and viviparous land animals have appeared one after the other. But not one from the other.

Cuvier writes in 1812:

but can the present races of animals, it will be said, not be modifications of those found among the fossils, can they not have been produced by changes in their circumstances and environment, can they not have been modified in the course of time (*id.*)?

As Lamarck has said in 1809.

Yes.

The fundamental reasons are two, negative and affirmative, respectively.

The negative reason is the lack of fossils.

Cuvier writes:

the entrails of the earth have preserved no monuments whatsoever of such a curious genealogy (*id.*), including:

no fossil human bones have ever been found (*id.*).

In *The origin* Darwin has to devote one chapter to this difficulty.

The affirmative reason are catastrophes.

Cuvier writes:

if there is one thing certain in geology, it is that the surface of the globe has undergone a great and sudden catastrophe whose date cannot go back beyond 6 000 years, that this catastrophe has overwhelmed the countries inhabited by men and the species of animals with which we are familiar today, that this catastrophe has caused the beds of the previously marine areas to dry up and thus form the land areas inhabited today, that the few beings which have escaped this catastrophe have since spread and propagated their kinds on the newly dried up lands, that these lands have been inhabited before this last catastrophe by terrestrial animals, if not by man, and therefore an earlier catastrophe has engulfed them beneath its waves; moreover, to judge by the different orders of animals of which remains have been found, there have been several of these marine irruptions (*id.*).

Darwin devotes to this difficulty the one sentence: as we do not see the cause, we invoke cataclysms.

Geoffroy Saint-Hilaire was one of the philosophers who accompanied Napoleon on his expedition to Egypt in 1798-9.

Cuvier writes:

my colleague Geoffroy Saint-Hilaire took pains to collect from tombs and temples as many animal mummies as possible; he brought back cats, ibis, birds of pray, dogs, monkeys, crocodiles, and an ox head; and one cannot detect a greater difference between any of these creatures and those which we see today than between the human mummies and the skeletons of men today (*id.*).

Lamarck writes: I do not deny that these animals are of the same species as those living today (*Philosophy*, I, 3); but this only proves that the species living today inhabited Egypt at that time (*id.*); men, in general, judge everything in relation to themselves (*id.*); and if the record of their observations reaches a few thousand years, this may seem infinite to them, but it is nothing in relation to the great changes which have occurred on the surface of the earth (*id.*).

The development of the earth.

Cuvier writes:

my most important concern, and the chief object of my researches, constituting also their connection with the theory of the earth, has been to ascertain the particular strata in which each species is found.

In the passage above, as he establishes the chronological series:

shellfish,
fishes,
oviparous quadrupeds,
viviparous sea animals,
viviparous land animals,

Cuvier has simultaneously established the chronological series:

formation of the primary rocks,
beginning of the production of secondary
formations,
before the formation of chalk strata,
during the formation of chalk strata,
deposition of the coarse shell limestone.

And fossils have been the chronological foundation of the investigation of the development of the earth.

Werner, in 1786, in the *Short classification and description of the different kinds of mountains*, established that the formations on the surface of the earth are primary and secondary. As Cuvier writes in the *Discourse on the revolutions*:

in everything concerning the nature of formations positive geology begins with Werner, and with him alone (*id.*).

Cuvier, in 1811, in the *Geological description of the region of Paris*, with Brongniart, established that the formations are also tertiary. As Lyell writes in *Principles of geology*:

after some progress had been made in classify-

ing the secondary, another division of sedimentary formations was established, called the tertiary (III, 2);

the first series of deposits belonging to it was described by Cuvier and Brongniart (*id.*).

From William Smith, in 1815, in *A delineation of the strata of England and Wales, with part of Scotland*, through Lyell, in 1833, in the third volume of the *Principles of geology*, the tertiary and the secondary formations were analyzed into their chronological series.

Sedgwick, in 1852, in *On the classification and nomenclature of the lower paleozoic rocks of England and Wales*, analyzed the primary formations into their chronological series.

As the classes of organisms have had to be analyzed into more general branches, or phyla, themselves comprising the more general kingdoms, so the chronological series of the formations on the surface of the earth has had to be analyzed into periods, comprising more general eras, comprising more general eons. The names of the eons and the eras reflect the fact that fossils have been the foundation of the dating of the formations.

After the tertiary, another division was established, the quaternary. The quaternary is the present, and it is still a period. The tertiary corresponds, next to the quaternary, to the rest of the present era, the cenozoic. The secondary corresponds to the mesozoic era. The primary corresponds to the paleozoic era and the earlier eons.

And these are, beginning with the present and back to the formation of the earth, the eons, eras, periods, and their chronology.

eons	eras	periods	beginning
phanerozoic			
cenozoic			
	quaternary	2.6 million years	
	neogene	23,	ago,
	paleogene	66,	
mesozoic			
	cretaceous	150,	
	jurassic	200,	
	triassic	250,	
paleozoic			
	permian	300,	
	carboniferous	360,	
	devonian	420,	

silurian	440,
ordovician	490,
cambrian	540,
proterozoic	2 500,
archean	4 000,
hadean	4 600.

cclxxviii

Geoffroy Saint-Hilaire proposed that there are no missing links in the modification of organs from one species to another.

He published the *Anatomical philosophy*, in 1818-22. He writes:

I have developed a new method to arrive at the real determination of organs;

this method consists in the intimate association of four rules, the theory of analogues, the principle of connections, the elective affinity of organic elements, and the balance of organs;

the theory of analogues is the demonstration that it is not always the organs in their entirety, but only the materials of which they are composed, that maintained their identity;

the principle of connections is that there is mutual, necessary, and consequently invariable dependence of the parts;

the elective affinity of organic elements is that materials are grouped to form an organ not arbitrarily, but always according to the necessity of position, which constrains adjacent elements into reciprocal suitability;

the balance of organs is that the volumes of the organic masses balance one another;

and the general and definite result of the method I call the unity of organic composition (II, pr.)

is it a question, then, of demonstrating that the leg of the horse corresponds to the human hand? no one wants to make such comparisons; but we no longer have to fear that intermediate links are missing;

there is no particular creation of the respiratory organs of the fishes, they resemble those of other vertebrates; and you no longer have to fear that intermediate links are missing;

consider man, a ruminant, a bird, and a fish; dare to compare them directly, and you arrive at what is the most general and the most philosophical in all anatomy (I, pr.).

Robert Brown discovered the nucleus of the cell.

He discovered also Brownian motion, in *A brief account of microscopic observations, made in the months of June, July, and August 1827, on the particles contained in the pollen of plants; and on the general existence of active molecules in organic and inorganic bodies*, in 1828. He reports the observation, under the microscope, of pollen grains suspended in a fluid in a rapid oscillatory motion, then the attempts to isolate the cause of the motion, and concludes:

the motion arose neither from the currents in the fluid nor from its gradual evaporation, but belonged to the particles themselves (*id.*).

Moreover:

this seeming vitality was retained by the molecules long after the death of the plant (*id.*).

Brownian motion is a general property of matter. It was explained by Einstein in 1905, as his title shows, too, in *On the motion of particles suspended in liquids at rest required by the molecular kinetic theory of heat*.

Robert Brown discovered the nucleus of the cell, in the *Observations on the organs and mode of fecundation in orchideae and asclepiadeae*, in 1831. He writes:

in each cell of the epidermis in the members of the family of orchideae, and especially those with membranous leaves, a single circular aureola, generally more opaque than the membrane of the cell, is observable;

only one aureola belongs to each cell;

this aureola, or nucleus of the cell, is not confined to the epidermis, it is found also in many cases in the parenchyma, or internal cells, of the tissue;

the nucleus of the cell is not confined to the orchideae either, but it is equally manifest in many other monocotyledonous families; I have even found it, although still in only a few cases, in the epidermis of dicotyledonous plants (*id.*).

Darwin knew Robert Brown. This is one of the anecdotes he recounts in his *Recollections*: I remember praising Whewell's *History of the inductive sciences* to him, and he answered, yes, I suppose he has read the prefaces of very many books (*id.*).

Von Baer discovered the mammalian egg.

In *On the mammalian egg and the origin of man*, in 1827, he writes:

every animal, which originates from the coitus of male and female, evolves from an egg, never from mere formative fluid (concl.).

Von Baer wrote also *On the developmental history of the animals, observation and reflection*, in 2 volumes, in 1828-37. This is the treatise that laid the foundations of embryology. And Darwin writes in *The origin*: embryology is one of the most important subjects in the whole round of natural history (14).

Von Baer showed that in the development of the animal from the egg, the egg divides into distinct layers of tissue. First an inner layer and an outer layer develop, from these layers then two intermediate layers develop, and from these then the various organs of the animal develop.

Epigenesis, not preformation.

The embryo of the animal when it is a two-layered structure is the gastrula, as Haeckel named it. And this stage in the development of the animal is preceded by the initial differentiation of the egg, or the blastula. The single cell of the egg develops into a multicellular blastula, and the blastula then develops into a two-layered gastrula.

Von Baer discovered also the notochord. This is the chord of cells which forms the axial supporting structure of the body of the lower chordates and of the embryos of the higher chordates, or true vertebrates.

And von Baer formulated:

the law of the development of the individual (I, 5.4).

The law of the development of the individual is:

1. the generic characters develop in the embryo earlier than the specific;
2. from the characters of the highest genus next those of the lower develop, and so forth, until finally those of the species appear;
3. instead of passing through various forms, the embryo steadily diverges from given forms;
4. and the embryo of a higher form is thus not like a lower form, but like its embryo (*id.*).

With Linnaeus: you must understand that a character does not make a genus, the genus makes the character.

Von Baer illustrates the law:

the embryos of mammals, birds, lizards, and snakes, and probably also turtles, are so exceedingly alike in their earliest stages, and both as a whole and in the mode of the development of their parts, that we can often distinguish the embryos only by their size; I own two little embryos in spirit, whose names I have omitted to attach, and at present I am quite unable to say to what class they belong; they may be lizards, small birds, or very young mammals, so complete is the similarity in the mode of formation of the head and the trunk in these animals; the extremities are still absent in these embryos; but even if they were present from the first stage of their development, we would have learned nothing, for the feet of lizards and mammals, the wings and feet of birds, and the hands and feet of man all develop from the same fundamental form (*id.*).

Darwin writes: on my theory we can understand the great leading facts in embryology; the embryos of distinct animals in the same class are often strikingly similar; a better proof of this cannot be given than the circumstance, mentioned by von Baer, that having forgotten to ticket the embryo of some vertebrate animal, he cannot now tell whether it be that of a mammal, bird, or reptile; and the leading facts of embryology are second in importance to none in natural history.

Schleiden and Schwann laid the foundations of cytology. And cytology became one of the most important subjects in the whole round of natural history, too. The problem of the origin and development of organisms was formulated with the development of embryology as the problem explicitly also of the origin and development of the individual organism. This is why the leading facts of embryology are second in importance to none in natural history. And with the development of cytology the problem was formulated as the problem explicitly also of the origin and development of the cell. As Schleiden formulates it: what is the origin of that peculiar little organism, the cell?

And the theory of evolution is the solution of the problem of the origin and development of organisms because it is simultaneously the solution of the problem of the origin and development of the individual organism and of the problem of the origin and development of the cell.

Schleiden wrote the *Contributions to phyto-genesis*, in 1838. He begins:

the fundamental general law of human reason, to strive toward unity in knowledge, has manifested itself, as in science generally, so in the investigation of organisms; many attempts have thus been made to establish the analogy between the two great kingdoms, animals and plants; but all these attempts have failed; and it has recently become possible to point at the cause of this phenomenon with clarity and distinction;

the cause is that the concept of the individual, such as it occurs in nature among the animals, has no application at all in the world of plants; this concept can be applied at most to the lowest plants, algae and fungi, which consist of a single cell each; developed to any degree higher than that, the plant is an aggregate of fully individualized enclosed in themselves single beings, namely, the cells; every cell leads a double life, one entirely independent, concerned only with its own development, and one incidental, to the degree to which it is an integral part of the plant;

it is thus easy to see that the first, indispensable, and fundamental ground on which not only plant physiology, but comparative physiology in general must be built, are the vital processes of the individual cell, and most particularly the question, what is the origin of that peculiar little organism, the cell (*id.*)?

Schleiden wrote also the *Botany as an inductive science*, in 1842.

Schleiden is also the first philosopher who appreciated the restoration of spirit to the substantiality and purity of its being and its consequences. In 1844 Schleiden wrote *Schelling's and Hegel's relation to natural science*. And in 1845, in the introduction to the second edition of the *Botany as an inductive science*, Schleiden recognizes with the greatest clarity and distinction that the restoration is waging a war, who the enemy against whom the restoration is waging the war is, and what the end of the war can only be.

Schleiden writes:

what Schelling and his school and Hegel and his school fight is the critical and inductive method;

what they fight is the method which was developed between the time of the discovery of

America and that of Newton; which was developed first and especially by Bacon, Galileo, and Kepler; which became the foundation of the immortal discoveries of Kant;

and in this battle there can be no compromise;

the battle can only end with the complete annihilation and destruction of those who are introducing their dogma into philosophy and natural science, and into the state and the church;

nothing else can ensure progress and make their violent transformation impossible (*id.*).

And so the battle eventually ended, with complete annihilation and destruction. But not of the side against which Schleiden fought.

In 1836 Schwann showed that fermentation is caused not by oxidation, but by an organism. Yeast.

And in the *Microscopic investigations on the agreement in the structure and growth of animals and plants*, in 1838, Schwann proposed, as his title indicates, too, to establish the analogy between the two great kingdoms, animals and plants, by applying to animals the concept of the individual demonstrated by Schleiden in the kingdom of the plants, on the first, indispensable, and fundamental ground of comparative physiology, and most particularly by answering the question, what is the origin of that peculiar little organism, the cell?

The *Microscopic investigations* is in three parts.

Part I is *On the structure and growth of the notochord and cartilage*. More particularly, Schwann investigates the cells in the notochord of the frog embryo and the cells in cartilage. And concludes:

the most important phenomena of the structure and development of these cells agree completely with the corresponding phenomena in plants; the great divide between the animal kingdom and the plant kingdom, the difference in ultimate structure, thus disappears; cells, cell membranes, cell contents, nuclei are all exactly analogous; the form of the cell of the animal is indeed of a round vesicle; yet flattening of the cells against one another, cellular substance between the cells, thickening of the cell walls, have all been observed in the animal cell as in the plant cell (2).

Part II is *On cells as the foundation of all*

tissues of the animal body. More particularly, Schwann generalizes the results of part I, showing that, as the cells of the notochord and cartilage, so the cells of lymph (1.1), blood (1.2), mucus (1.3), pus (1.4), epithelium (2.1), pigment (2.2), nail (2.3), hoof (2.4), feather (2.5), crystalline lens (2.6), bone (3.1), tooth (3.2), areolar tissue (4.1), fibrous tissue (4.2), elastic tissue (4.3), muscle (5.1), nerve (5.2), capillary (5.3).

And in part III, after a *Review*, Schwann formulates *The theory of the cell*.

He writes:

the various views concerning the fundamental power of the organic body can be reduced to two;

the first is that every organism begins with an inherent power which shapes it according to a predetermined idea; the fundamental power of the organism, or its soul, as Stahl describes it, is like an immaterial principle endowed with consciousness;

the other view is that the fundamental power of the organic body agrees essentially with the forces of inorganic nature; and even if this power occurs nowhere at all in inorganic nature, it cannot be essentially different from all other physical and chemical forces;

the first view can be called the teleological, the second the physical;

with respect to plants, the uniformity of their elementary parts has already led to the hypothesis that the organisms are, in fact, the cells, and the whole plant is only an aggregate of these organisms;

now, if the elementary parts of animals are exactly analogous to the elementary parts of plants, the individuality of the whole animal disappears; and it is precisely on the individuality of the whole animal that the supposition rests, that the animal possesses an inherent power which shapes it according to a predetermined idea;

we propose, therefore, the hypothesis that the organic body is not shaped by such an idea, but develops according to the blind laws of necessity;

the question about the fundamental power of the organic body thus resolves itself into the question about the fundamental power of the individual cell;

and the phenomena of the formation and

development of the cell can be arranged into two natural groups; first, the phenomena of the combination of molecules to form a cell; we can call these plastic; and second, the phenomena of the chemical changes in the component particles of the cell and in the medium surrounding these particles in the cell; we can call these metabolic (*id.*).

Schwann then considers in detail the plastic phenomena. And concludes:

we have seen that the laws regulating the deposition and formation of molecules into the tissues of organisms are the same for the cells of all tissues,

that there is a common principle in the development of all tissues, namely, the formation of cells,

that the power which induces the bonding of new molecules does not reside in the whole organism, but in the cells; this we have called the plastic power of the cells;

and we have seen, finally, that the laws according to which new molecules combine to form cells are the same as those by which substances capable of imbibition crystallize;

and the cells do, in fact, consist of substances capable of imbibition;

are we not justified, therefore, in affirming the proposition, the formation of the elementary parts of the organism is nothing but the crystallization of substances capable of imbibition, and the organism is nothing but an aggregate of crystals capable of imbibition?

to establish this proposition, two further points have to be proved, 1. that as the plastic phenomena of the cell, so the metabolic phenomena are the necessary consequence either of the capacity for imbibition or of some other capacity of the substances of the cells; and 2. that crystals capable of imbibition combine according to laws to form a systematic whole similar to the organism (*id.*).

So much for the soul.

The organism is nothing but an aggregate of crystals capable of imbibition.

And if, pace Schwann, the organism is not nothing but an aggregate of crystals capable of imbibition, very much with Schwann, the organism is nothing but an aggregate of proteins capa-

ble of replication.

celxxix

Boucher de Perthes dated man.

Darwin writes in *The descent of man*, in 1871: the high antiquity of man has recently been demonstrated by the labors of a host of eminent men, beginning with Boucher de Perthes; and this is the indispensable ground for understanding the origin of man (intr.).

Boucher de Perthes demonstrated that the antiquity of man is higher, in particular, by far, than the antiquity of creation. He demonstrated that human tools predate the end of the glaciations of the quaternary period, and are thus not less than some tens of thousands of years old.

So much suffices to explain why Boucher de Perthes's demonstration was vigorously resisted. Why also results anticipating the demonstration had been ignored. The excavation of antediluvian tools and the demonstration of fossil human bones both predate Boucher de Perthes's demonstration.

Frere published an *Account of flint weapons discovered at Hoxne in Suffolk* in 1800.

Schmerling demonstrated that excavated human bones are fossil by correlating them with the stratigraphic evidence in the *Researches on fossil bones discovered in the caves of the province of Liège* in 1833-4.

Lartet discovered an anthropomorphic fossil ape in 1836.

Boucher de Perthes correlated tools, the stratigraphic evidence, and fossil human bones with one another, and thus dated man.

Hence an additional reason for the resistance. The demonstration of the high antiquity of man – and this is the indispensable ground for understanding the origin of man – depended on the presumption that man is attested by tools. By his labor rather than spirituality.

Boucher de Perthes published *On primitive industries, or on the arts and their origin*, in 1846. He distinguished the excavated objects into two kinds, corresponding to the strata in which they were found. Axes of polished stone and horn tools were found in beds near the surface and peat bogs. Axes of hewn stone were found in the lower strata, mixed with the fossil bones of

extinct elephants and rhinoceroses. Boucher de Perthes ascribed the later tools to a Celtic people, immediate predecessors of the Gauls, the earlier tools to an antediluvian people, predating the last of the series of catastrophes which Cuvier had proposed to explain the fossils of extinct animals and plants.

In 1847 Boucher de Perthes expanded the treatise into the first volume of *On Celtic and antediluvian antiquities*. He added a second volume in 1857.

In 1860 Lartet published *On the geological antiquity of the human species in western Europe*, in which he classified ancient tools. This explains why most of the ancient industries are named after localities in France.

In 1864 Boucher de Perthes added a third volume to *On Celtic and antediluvian antiquities*. It attests his most important discovery. In 1863 he correlated hewn axes with fossil human teeth and jawbone. He was accused of fabrication.

The most ancient tools discovered by Boucher de Perthes are Acheulean.

Christian Thomsen, in 1820, proposed the chronological division of ancient tools into stone, bronze, and iron.

The stone age has since been divided into old and new, paleolithic and neolithic.

The paleolithic begins with the oldest tools some 2 600 000 years ago. Gradually, slowly, the tools become more complex, and some 10 000 years ago the neolithic begins.

Homo habilis began making tools, then *Homo erectus* continued making them, then *Homo sapiens* has continued making them.

The paleolithic divides into lower, middle, and upper.

The lower paleolithic begins with Oldowan tools, the oldest, some 2 600 000 years ago. Oldowan tools are correlated with *Homo habilis*.

The next major step are Acheulean tools, beginning some 1 600 000 years ago. Acheulean tools are correlated with *Homo erectus*.

The middle paleolithic begins with the next major step, Mousterian tools, some 150 000 years ago. Mousterian tools are correlated with neanderthals. Neanderthals are *Homo sapiens neanderthalensis*.

The upper paleolithic begins with the next

major step, Aurignacian tools, some 40 000 years ago. Aurignacian tools are correlated with cromagnons. Cromagnons are *Homo sapiens sapiens*. Lascaux and Altamira are Aurignacian.

The neolithic begins some 10 000 years ago. And the oldest city, at Çatal Hüyük, is 8 500 years old.

Tabulating, beginning with the present and back to the oldest tools, these are the divisions, major industries, and their chronology.

	industries	beginning
neolithic		10 000 years
paleolithic		ago,
upper	Aurignacian	40 000,
middle	Mousterian	150 000,
lower	Acheulean	1 600 000,
	Oldowan	2 600 000.

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Hooke, in 1665: there are 1 259 712 000 cells in 1 cubic inch of cork.

Redi, in 1668: no animal of any kind is ever generated in dead flesh unless there is in it a previously laid egg.

Swammerdam, in 1669: the development of insects is like the development of frogs and birds.

Steno, in 1669: the bones of animals and plants had become solid when the earth and the rock containing them were still fluid.

Malpighi, in 1675: to study the highest animals insects need to be studied, to study insects plants need to be studied.

Leeuwenhoek, in 1676: there are in one drop of water 1 000 animalcules 10 000 times smaller than the smallest animalcules previously observed.

Ray, in 1686-704: there are 18 655 species of plants.

Tournefort, in 1694: there are 698 genera of plants.

Camerarius, in 1694: the anther is the male part, the ovary with the style the female part of the plant.

Linnaeus, in 1735-58: 11 841 species of 1 417 genera of 146 orders of 31 classes of the 2 kingdoms belong in the system of nature.

Bonnet, in 1740: an isolated female aphid produces offspring without insemination.

Réaumur, in 1741: each part into which a

hydra is divided regenerates the whole animal.

Buffon, in 1749: all matter is attracted as the square of the distance inversely; and from this force, combined with that of heat, originate the living particles which give rise to, and sustain, all the various actions of organisms.

Also: the history of the earth is necessary for the study of animals and plants.

Haller, in 1757-66: muscles have the capacity for contraction with stimulus.

Buffon, in 1775: the earth is at least 75 000 years old.

In 1779: there are seven epochs in the development of the earth. The first inhabitants of the globe appeared in the 3rd epoch, some 30 000 to 40 000 years after the formation of the earth. They were of kinds different from those today. They continued for some 10 000 or 15 000 years, then perished. The first men appeared in the 7th epoch. Naked in mind and body, they began to sharpen hard stones, jades, and flints into the hand axes, which are the first relics of the art of man in the state of nature.

Ingen-Housz, in 1779: plants have respiration as animals do.

Lavoisier and Laplace, in 1783: respiration is combustion, perfectly similar to that of carbon.

Kant, in 1790: there is a real relationship in their production from a common parent of man, polyp, mosses, and lichens. It is a development with a view to the preservation of the race grounded in variation which becomes hereditary. Epigenesis finds peculiar favor with reason, because it regards nature as producing itself, not merely unfolding itself.

Galvani, in 1791: electricity is present in the animal itself.

Lamarck, in 1809: species have been formed one after another, have only a relative stability, and are invariable only temporarily. By the habits, mode of life, and all influences of the environment nature has, little by little, in the course of time, succeeded in building the body of the animal and its parts, and hence also its faculties. New needs imperceptibly give birth to new parts by working on the inner feelings of the animal. The animals form a branching series which, even if graded irregularly, is free from discontinuity.

Cuvier, in 1796: the fossil belongs to the family of sloths, armadillos, pangolins, anteaters, and armadillos. The genus I name *megatherium*, the species *megatherium americanum*.

In 1809: with the fossil of a flying reptile, which we name pterodactyl, we constitute the genus saurians.

In 1812: shellfish, fishes, oviparous quadrupeds, viviparous sea animals, and viviparous land animals appeared in this order.

Geoffroy Saint-Hilaire, in 1818-22: is it a question of demonstrating that the leg of the horse corresponds to the human hand? we no longer have to fear that intermediate links are missing.

Von Baer, in 1827: every animal which originates from the coitus of male and female evolves from an egg.

In 1828-37: the generic characters develop in the embryo earlier than the specific. The embryo of a higher form is thus not like a lower form, but like its embryo.

Robert Brown, in 1831: the nucleus of the cell is not confined to monocotyledonous families, it is found also in dicotyledonous plants.

Schleiden, in 1838: the ground on which physiology must be built are the processes of the individual cell.

Schwann, in 1838: if the elementary parts of animals are exactly analogous to the elementary parts of plants, the individuality of the whole animal disappears.

Also: the formation of cells is a common principle of the formation of all tissues. The phenomena of the formation and development of the cell are plastic and metabolic. The laws of the plastic are the same as those by which substances capable of imbibition crystalize. The cells consist of substances capable of imbibition. Are we not justified in affirming that the formation of the elementary parts of the organism is nothing but the crystallization of substances capable of imbibition, and the organism is nothing but an aggregate of crystals capable of imbibition?

Boucher de Perthes, in 1846: axes of polished stone and horn tools were made by a Celtic people, immediate predecessors of the Gauls, axes of hewn stone were made by an antediluvian people, predating the last of the catastrophes

which have caused extinctions.

The investigation itself showed them the way, and it showed them the way to Darwin.

Himself, Darwin added a *Historical sketch* to the third edition of *The origin*, and expanded it in the subsequent editions:

to give a brief sketch of the progress of opinion on the origin of species (*id.*).

Darwin notes 44 writers.

8 are important,

384 - 322^{BC} **Aristotle**,

1707 - 1788^{AD} **Buffon**,

1744 - 1829 **Lamarck**,

1749 - 1832 **Goethe**,

1772 - 1844 Geoffroy Saint-Hilaire,

1792 - 1876 **von Baer**,

1820 - 1903 Spencer,

1823 - 1913 Wallace.

5 of the 8 have just been presented. Goethe has been presented earlier.

Of Spencer Darwin writes in the sketch:

in 1852 Spencer argued that species have been modified (*id.*).

Spencer developed a doctrine of the progress of the universe, including the progress of the solar system, the progress of the totality of organisms, the progress of each individual, and of his physical and mental constitution as well as socialization, and – what is the main concern of the doctrine – the progress of society. Before Darwin Spencer proposed to ground the doctrine in Lamarck, after Darwin he proposed to ground it in Darwin.

Hegel established the vogue of the philosophy of nature, *die Naturphilosophie*, in place of the investigation of nature. Goethe was important in the dissemination of that vogue. Never interested in nature, that vogue was replaced in the next generation by the more human vogue of the philosophy of society. Spencer is a younger contemporary of Comte and of Marx. Spencer is important because his is the first of the doctrines arguing what came to be called social Darwinism.

Of Wallace Darwin writes in the sketch:

in 1858 Wallace promulgated the theory of natural selection with admirable force and clarity (*id.*).

Wallace wrote *On the tendency of varieties to depart indefinitely from the original type*. This

brief communication of Wallace to Darwin in 1858 spurred Darwin, after two-odd decades of research, to write and publish *The origin*. Wallace is important because he anticipates natural selection with some force and clarity. Wallace excluded man's mental faculties from the action of natural selection, and continued to exclude them in all his work in the following five decades.

Spencer indicates the breadth of the progress of opinion on the origin of species toward Darwin's, Wallace indicates the strength.

These are the 8 important.

The remaining 36 are not important. At least 16 are very very not important. In the order in which Darwin notes them, Mr Clair Grece, Dr Karl Meding, Mr Rowley, Mr Brace, Prof Grant, Mr Patrick Matthew, Prof Haldeman, M Isidore Geoffroy Saint-Hilaire, Dr Freke, Bronn, Dalton, Gordon, Poiret, Dr Schaaffhausen, M Lecoq, Prof Rudolf Wagner. Their inclusion in the *Sketch* shows the breadth of Darwin's reading.

The remaining 20 are,

1731 - 1802 Erasmus Darwin,
 1757 - 1817 William Wells,
 1774 - 1853 von Buch,
 1776 - 1847 Burdach,
 1778 - 1846 Bory de Saint-Vincent,
 1778 - 1847 William Herbert,
 1779 - 1851 Oken,
 1783 - 1840 Rafinesque,
 1783 - 1875 d'Omalius d'Halloy,
 1792 - 1871 John Herschel,
 1794 - 1865 Pander,
 1794 - 1878 Fries,
 1796 - 1860 Baden Powell,
 1800 - 1870 Unger,
 1802 - 1871 Chambers,
 1804 - 1892 Owen,
 1815 - 1891 Keyserling,
 1815 - 1899 Naudin,
 1817 - 1911 Hooker,
 1825 - 1895 Huxley.

Darwin writes:

in 1794 my grandfather, Dr Erasmus Darwin, anticipated the views and erroneous grounds of Lamarck;

in 1813 William Wells recognized distinctly the principle of natural selection, but applied it only to the races of man, and to certain characters

alone;

in 1822 William Herbert expressed his belief that single species of each genus were created in an originally highly plastic condition and these have produced all existing species;

in 1836 von Buch expressed clearly his belief that varieties slowly become changed into permanent species which are no longer capable of intercrossing;

in 1836 Rafinesque wrote that all species might have been varieties once;

in 1844 Chambers wrote that organization progresses by sudden leaps;

in 1846 d'Omalius d'Halloy published his opinion that it is more probable that new species have been produced by descent with modification;

Prof Owen I included in the fourth edition of *The Origin* among the paleontologists firmly convinced of the immutability of species; but it appears now that this was on my part a preposterous error; Prof Owen now claims to have promulgated the theory of natural selection before I had done so;

in 1852 Naudin expressly stated his belief that species are formed as varieties are formed under cultivation;

also in 1852 Unger published his belief that species undergo development and modification;

Pander has expressed a similar belief;

similar views have been maintained by Oken in his mystical *Naturphilosophie*;

it seems that Bory de Saint-Vincent, Burdach, and Fries have all admitted that new species are continually produced;

in 1853 Keyserling suggested that as new diseases have arisen and spread over the world, so at certain periods the germs of existing species may have been chemically affected by ambient molecules and thus given rise to new forms;

in 1855 Baden Powell wrote that the introduction of new species is a regular, not a casual phenomenon;

John Herschel wrote that it is a natural, not a miraculous process;

in 1859 Huxley wrote that the hypothesis, which supposes the species living at any one time to be the result of the gradual modification of preexisting species, is the only one to which

physiology lends any countenance;

also in 1859 Hooker admitted the descent and modification of species (*id.*).

Also in 1859 *The origin* was published.

Darwin was born in 1809. Counting generations with respect to that year, and ignoring the very very not important, this is how the number of the writers who admitted evolution evolved,

-1719	2 writers,
1720-1749	3,
1750-1779	7,
1780-1809	11.

The progress of opinion on the origin of species was rather inexorably toward Darwin's.

cclxxxi

Darwin writes in *The origin*:

it is a truly wonderful fact that all animals and all plants throughout all time and space should be related to each other in group subordinate to group; this great fact is explained through inheritance and the complex action of natural selection;

from the first dawn of life all organic beings can be classed in groups under groups; this grand fact of the natural subordination of organic beings is in my judgment explained;

all the great leading facts of paleontology seem to me simply to follow on the theory of descent with modification through natural selection;

all the grand leading facts of geographical distribution are explicable on the theory of migration with subsequent modification;

all the great facts in morphology become intelligible;

the leading facts of embryology, second in importance to none in natural history, are explained on the principle of slight modifications in the many descendants from some one ancient progenitor.

And the facts of cytology are explained.

And the facts of genetics.

All animals and all plants throughout all time and space.

All organic beings from the first dawn of life.

A truly wonderful fact.

cclxxxii

The problem of the soul is very much the same, what is it?

Beginning with Copernicus the investigation of the world began advancing very greatly once again, and as two of the three parts of the problem of the earth, its shape, and at rest or in motion, were solved conclusively, the third part of the problem, its position – the position of the third planet of the solar system now – became an increasingly smaller part of an increasingly larger investigation, a minute part of the problem of the position of the solar system, itself then a minute part of the problem of the position of the Milky Way.

Beginning with Vesalius the part of the investigation of the world, which the investigation of organisms is, began advancing very greatly once again as well, and the problem of the soul became part of an increasingly larger investigation, much like the problem of the earth. Unlike the problem of the earth, however, the problem of the soul did not become an increasingly smaller part of the increasingly larger investigation of organisms, it became an increasingly larger part of it.

Nor could it not become an increasingly larger part of the increasingly larger investigation of organisms. Not if at issue was the proposition, the soul is the principle of life. One and all results obtained in the course of the investigation could not not inform the question, true? or false? implicitly or explicitly. And one and all results did so, in fact, very much explicitly.

In the two centuries from Descartes to Darwin the problem of the soul was very much the fundamental problem of the investigation of organisms, and most particularly, the question, the proposition, the soul is the principle of life, true? or false? was.

In 1633, Descartes writes: it is not necessary to suppose that there is a soul in the body, or any other principle of life than its blood and its spirits, and the body is nothing but a machine made of earth, and all functions of this machine follow entirely naturally from the mere disposition of its organs.

In 1649: what is a passion in the soul is an action in the body; everything we experience in us must be attributed to our body; it is an error to believe that the soul gives motion and heat to the body.

Results mounted. Heavily on the side, not true,

false.

In 1697, Stahl reaffirmed: the soul is the principle of life.

Results continued to mount. Ever more heavily on the same side, not true, false.

In 1749, Buffon writes: the living particles which give rise to, and sustain, all the various actions of organisms originate from the force of attraction combined with that of heat.

It was still one of two views on nature. The other was: nature is an immense living power subordinated to the supreme being who animates the world by his command.

On this view, in 1762, Bonnet formulated the theory of preformation.

This view, however, was losing its life. Even the relative not importance of Bonnet attests this fact.

Results continued to mount. And conclusively now on the same side, not true, false.

In 1838, Schwann writes: the fundamental power of the organic body agrees essentially with the forces of inorganic nature, and even if this power occurs nowhere at all in inorganic nature, it cannot be essentially different from all other physical and chemical forces.

There still was the other view: the fundamental power of the organism, or its soul, is like an immaterial principle endowed with consciousness.

On this view, in 1834-40, Müller wrote the *Handbook of the physiology of man*, in 8 books. Book VI is *On the life of the soul*.

But this view was barely alive now. Müller is not only a contemporary of Schwann. He is a younger contemporary of Boucher de Perthes and von Baer and an older of Darwin.

In 1859, selection explained the facts of paleontology, the facts of geographical distribution, the facts of morphology, the facts of embryology, and the fact that all animals and all plants throughout all time and space and all organic beings from the first dawn of life are related to each other in group subordinate to group.

The other view was dead.

Darwin began also its burial.

He writes: it has been asserted that man alone is capable of progressive improvement; that he alone makes use of tools or fire, domesticates

other animals, or possesses property; that no animal has the power of abstraction, or of forming general concepts, is self-conscious and comprehends himself; that no animal employs language; that man alone has a sense of beauty, is liable to caprice, has the feeling of gratitude, or of mystery; believes in god, or is endowed with conscience.

And Darwin hazarded a few remarks on the more important of these points.

There was one attempt to resurrect that view.

In 1894, Driesch wrote the *Analytic theory of organic development*. Müller had argued the soul on physiological grounds. Two generations later, Driesch argued the soul – called the autonomy of living processes now – on cytological grounds.

But that view was dead. The not importance of Driesch attests this fact. And Boveri, in any case, closed it back into its tomb.

Boveri, in 1902, in *On multipolar mitosis as a means of analysis of the cell nucleus*, showed that: the chromosome is the carrier of heredity (*id.*). And Boveri writes also: Driesch says that only a complicated machine can achieve what we observe in ontogenesis, and we do not observe a machine, because a machine will not remain the same if random parts are removed or transposed; this contradiction leads him to postulate an autonomy of living processes; but such a contradiction does not seem to exist in reality; the structure we have described behaves exactly in the way Driesch demands from a machine (*id.*).

The body is nothing but a machine made of earth, and all its functions follow entirely naturally from the mere disposition of its organs. Everything we experience in us must be attributed to our body.

The living particles which give rise to, and sustain, all the various actions of organisms originate from the force of attraction combined with that of heat.

The fundamental power of the organic body agrees essentially with the forces of inorganic nature, and even if this power occurs nowhere at all in inorganic nature, it cannot be essentially different from all other physical and chemical forces.

Selection explains the facts of paleontology, the facts of geographical distribution, the facts of

morphology, the facts of embryology, and the fact that all animals and all plants throughout all time and space and all organic beings from the first dawn of life are related to each other in group subordinate to group.

Selection is the principle of life. Selection is the efficient principle, or the cause, of life.

Also, the chromosome behaves exactly like a machine.

The soul is not the principle of life. The soul is nothing.

cclxxxiii

In the two centuries from Descartes to Darwin the investigation of life advanced very greatly. In the first of these centuries, from small beginnings, compared with the investigation of the heavenly bodies, or the investigation of motion, the investigation of life accelerated very greatly, and in the second of these centuries, at long last, caught up with those investigations, as they continued to be very greatly successful themselves.

To tabulate the results, dividing them by generation,

Redi	there is no spontaneous generation,
Ray	described 18 655 species,
Malpighi	began microscopic observation, began the investigation of tissues,
Leeuwenhoek	discovered protozoa,
Hooke	discovered the cell,
Swammerdam	described the development of insects,
Steno	fossils are the remains of organisms more ancient than the rocks in which they are embedded,
Tournefort	described 698 genera.

The collection of the particulars accelerated greatly.

In this generation the investigation of life also became experimental. The secrets of nature reveal themselves not when nature is free and left to herself, but when she is constricted and vexed, when by the arts and services of man she is forced out of her state, and molded and shaped. And she was for the first time vexed into reveal-

ing deeply hidden secrets of life. The cell. Protozoa. Fossils.

Nineteen and a half centuries earlier, with Herophilus and Erasistratus and the development of anatomy and physiology, the investigation of life had penetrated inside the organism. With the investigation of tissues it took a major step deeper inside.

Numbers were attached to the objects of investigation. There are 1 259 712 000 cells in 1 cubic inch of cork. There are in 1 cubic inch of cork more cells than there are descendants of Adam on earth, more than twice more. There are animalcules more than 10 000 times smaller than the animalcules which Swammerdam named water fleas. More than 10 000 000 times smaller than the descendant of Adam.

Stahl	the soul is the principle of life,
Camerarius	discovered the sexes of plants,
Réaumur	demonstrated regeneration.

Observation and experiment continued to accumulate.

The discovery of the sexes of plants established conclusively the unity and uniformity of animals and plants.

Linnaeus	established the classification of organisms,
Buffon	the history of the earth is necessary for the study of animals and plants, the earth is at least 75 000 years old, there are seven epochs in its development, the first organisms appeared in the 3 rd epoch, the first men appeared in the 7 th epoch,
Haller	demonstrated irritability.

In this generation the investigation of life began reaching the most general principles.

With its classification life became a systematic unity.

The investigation of life was grounded in the investigation of its habitat.

The habitat was dated. The result for its age was improved by an order of magnitude. The age of the earth also, as the later labors to determine it better show not less than the lack of earlier

labors to determine it, is a much more difficult number to obtain than the distance of the sun.

The development of the habitat was described. Its first inhabitants appeared in the third epoch. Adam appeared in the seventh.

Bonnet	demonstrated parthenogenesis, formulated the theory of preformation,
Ingen-Housz	discovered photosynthesis,
Lamarck	formulated the theory of evolution of animals.

The discovery of photosynthesis established conclusively that there is a continuum from dephlogisticated air to plants to animals. From combustion to the respiration of plants to the respiration of animals.

With the theory of evolution of animals the investigation of life was grounded in a theory *par excellence*.

Cuvier	began the reconstruction of organisms from fossils, arranged extinct organisms in their series, correlated fossils and strata,
Geoffroy Saint-Hilaire	– there are no missing links in the modification of organs,
Robert Brown	discovered the nucleus of the cell,

With the description of the species *megatherium americanum* and the genus saurians nature was vexed into revealing secrets she had been hiding, it was to turn out, for tens of millions of years.

The development of organisms was endowed with a relative chronology.

So was their habitat.

Boucher de Perthes	– dated man,
von Baer	discovered the mammalian egg, formulated the law of the development of the individual,
Schleiden	physiology must be built on the processes of the cell,
Darwin	formulated the theory of evolution, selection is the principle of life,
Schwann	formulated the theory of the cell.

The generic characters of the descendant of Adam develop before the specific. In his early stages he is exceedingly like a bird, a lizard, a snake, and probably also a turtle.

With the theory of the cell the investigation of life penetrated yet deeper inside the organism.

Also, Adam is his tools and his fossils, and the tools he was making when he was his fossils are, it was to turn out, 2 600 000 years old.

And with the theory of evolution the advance in the investigation of life is second to none.

The steps of advance in the acquisition of knowledge are the 17, Redi, Malpighi, Leeuwenhoek, Hooke, Swammerdam, Steno, Camerarius, Linnaeus, Buffon, Ingen-Housz, Lamarck, Cuvier, Robert Brown, Boucher de Perthes, von Baer, Darwin, Schwann. In particular,

Redi	there is no spontaneous generation,
Malpighi	began microscopic observation, began the investigation of tissues,
Leeuwenhoek	discovered protozoa,
Hooke	discovered the cell,
Swammerdam	described the development of insects,
Steno	fossils are the remains of organisms more ancient than the rocks in which they are embedded,
Camerarius	discovered the sexes of plants,
Linnaeus	established the classification of organisms,
Buffon	the history of the earth is necessary for the study of animals and plants, the earth is at least 75 000 years old, there are seven epochs in its development, the first organisms appeared in the 3 rd epoch, the first men appeared in the 7 th epoch,
Ingen-Housz	discovered photosynthesis,
Lamarck	formulated the theory of evolution of animals,
Cuvier	began the reconstruction of organisms from fossils,

	arranged extinct organisms in their series,
	correlated fossils and strata,
Robert Brown	discovered the nucleus of the cell,
Boucher de Perthes – dated man,	
von Baer	discovered the mammalian egg, formulated the law of the development of the individual,
Darwin	formulated the theory of evolution, selection is the principle of life,
Schwann	formulated the theory of the cell.

cclxxxiv

And the ethical problem of the soul in the two centuries from Descartes to Darwin.

The relation of physics and ethics, in general, and of the physical problem of the soul and the ethical problem of the soul, in particular, is exclusion. In general, ethics is the subjugation of the search for truth to the search for action. In particular, it is the substance and the essence of the formulation and solution of the ethical problem to subjugate the physical to itself.

Earth and air and other such things are not the true causes of things, the true cause of things is what is for the best. Hence, have you realized that the soul is immortal?

And if ethics, in general, and the solution of the ethical problem, in particular, succeed, they will arrest the development of physics, in general, and of the physical problem of the soul, in particular.

Who, therefore, can be so completely mad not to be moved to ineffable gratitude for the mercy of god who saves whomever he wishes? Every care should be taken to hold fast to what is believed everywhere, always, by all.

But physics, in general, and the formulation and solution of the physical problem of the soul, in particular, can eventually extricate themselves from ethics, in general, and the ethical problem of the soul, in particular, and begin advancing once again.

In anatomy most of all the soul must be considered. They do not understand that the physi-

cian, just to perform his art properly, must examine these matters.

They understand.

And the more physics, in general, and the formulation and solution of the physical problem of the soul, in particular, advance, the more ethics, in general, and the ethical problem of the soul, in particular, will demand that they be subjugated once again.

Thus, not quite one century after Vesalius: it is not necessary to suppose that there is a soul in the body; the body is nothing but a machine made of earth; and all functions of this machine follow entirely naturally from the mere disposition of its organs.

After Descartes physics, in general, and the solution of the physical problem of the soul, in particular, continued advancing very greatly, and the evidence for the fact that the soul is nothing mounted, and became conclusive.

And then, and hence: spirit has lost its essential life; but it is conscious of the loss; turning away from the chaff, confessing that it lies in wickedness, and reviling itself for doing it, spirit demands from philosophy the restoration of the substantiality and purity of its being.

Spirit is the soul, and the soul is spirit.

In the two centuries from Descartes to Darwin there are thus, with respect to ethics, in general, and the ethical problem of the soul, in particular, the two developments.

On the one hand, first, the consequences of the fact that the soul is nothing were drawn.

On the other hand, next, spirit was restored to the substantiality and purity of its being.

In the former, the most important are

1632 - 1677 Spinoza,
1711 - 1776 Hume,
1724 - 1804 Kant.

In the latter, the most important are

1685 - 1753 Berkeley,
1770 - 1831 Hegel.

These philosophers have all been presented above in some detail as well.

It is not necessary to suppose that there is a soul.

Not in physics, and not in ethics either.

One generation-odd after Descartes, Spinoza is contemporary with Redi, Malpighi, Leeuwen-

hoek, Hooke, Swammerdam, and Steno.

In Spinoza philosophy is physics and mathematics.

In the *Theologico-political treatise* Spinoza shows that the method of history does not differ from the method of physics at all, but agrees with it entirely.

Hence he solves the problem of when and how the holy scriptures were authored. The holy scriptures were not authored supernaturally. They were authored naturally, by fabrication.

And concludes:

the whole goal of this treatise is to separate faith from philosophy;

philosophy has no end but truth, faith has no end but obedience and piety; the ground of philosophy are axioms obtained only from nature itself, the ground of faith are histories and words obtained only from scriptures and revelation;

between faith and philosophy there is no connection whatsoever, no relation whatsoever.

In the *Ethics demonstrated in geometrical order* Spinoza demonstrates ethics in geometrical order. And in the *Ethics demonstrated in geometrical order* Spinoza does not even mention the soul.

The demonstration is in five parts.

Part I is *On god*. God is nature. There is, and can only be, one substance. Nature.

Part II is *On the nature and origin of the mind*. The nature and origin of the mind is body. The mind is body.

Part III is *On the origin and nature of the emotions*. The origin and nature of the emotions is the same. The emotions are body.

Part IV is *On human bondage, or on the strength of the emotions*. And it is with respect to human bondage that Spinoza draws the ethical conclusion:

the most useful thing in life is to perfect as much as possible the intellect, or reason; in this alone the sum of man's happiness and blessedness consists.

Part V is thus *On the power of the understanding, or on human freedom*, and it follows further: understanding is freedom.

Not quite two generations later, Berkeley is a younger contemporary of Camerarius, an older contemporary of Linnaeus and Buffon.

In *The principles of human knowledge* Berkeley concludes:

I shall esteem my labors altogether useless and ineffectual, if by what I have said I cannot inspire my readers with a pious sense of the presence of god;

and having shown the falseness or vanity of those barren speculations which make the chief employment of learned men, the better dispose them to reverence and embrace the salutary truths of the gospel, which to know and to practice is the highest perfection of human nature.

On the one hand, the sum of man's happiness and blessedness consists in this alone, to perfect as much as possible the intellect, or reason. Understanding is freedom.

On the other hand, to reverence and embrace the salutary truths of the gospel, and to know and to practice them, is the highest perfection of human nature.

And to this end, on the other hand, Berkeley's is a great advance in the formulation and solution of the ethical problem of the soul.

First, with Berkeley the exclusion of physics and mathematics from philosophy begins. Berkeley not only shows the falseness or vanity of their barren speculations. To physics and mathematics Berkeley allots:

hypotheses and the apparent.

Hence:

you, gentlemen, have no right to decide.

Because:

there is a first philosophy, a certain transcendental science, superior to and more extensive than mathematics and physics, in which we deal with truth, and which it behooves you, gentlemen, to learn rather than despise.

Thus, next, where the holy scriptures used to be the source of truth, now the first philosophy, the certain transcendental science, superior to and more extensive than mathematics and physics, in which we deal with truth, is the source of truth.

And, next:

everything is my ideas in mind, spirit, soul, or myself;

existence of sensible objects without the mind, spirit, soul, or myself, marks out either a direct contradiction or else nothing at all.

This is what it most of all behooves you,

gentlemen, to learn rather than despise.

And hence, finally, Berkeley writes in *The principles*:

nothing can be plainer than that the motions, changes, decays, and dissolutions which we hourly see befall natural bodies cannot possibly affect an active, simple, un compounded substance; such a being, therefore, is indissoluble by the forces of nature, and that is, the soul of man is naturally immortal (1).

The problem is thus very much the same. It is very much about what is salutary for you. It is very much also about what is believed everywhere, always, by all, salutary are the truths of the gospel. It is very much also about the naturally immortal soul of man.

The solution has abandoned the urgency of the fathers, who, therefore, can be so completely mad? It is more like Plato's, have you realized that the soul is immortal? and even this is posed to you implicitly rather than explicitly.

But the formulation and solution of the problem has also been vastly generalized.

Rid of physics and mathematics, philosophy is ethics.

And the soul is the soul of philosophy.

In Descartes soul is mind. In Spinoza as well soul is mind. Because it is not necessary to suppose that there is a soul. In Berkeley mind is soul. Because it is necessary to suppose that there is a soul. Mind, or soul, is thus also spirit, like the holy spirit. Mind, soul, or spirit, is thus also myself, like yourself.

And everything is my ideas in mind, soul, spirit, or myself.

The transmigration of the soul into the subject has begun. As three generations later, when the transmigration is complete, Schopenhauer will write: Berkeley made the principle, there is no object without subject, the foundation stone of philosophy.

Everything is my ideas in mind, soul, spirit, or myself. There is nothing without my mind, soul, spirit, or myself. There is no object without subject.

And the first philosophy, the certain transcendental science, superior to and more extensive than mathematics and physics, in which we deal with truth, of which my mind, soul, spirit, or

myself is the mind, soul, spirit, or myself cannot not be ethics. Or synonymously, apparently, religion. His labors in the first philosophy, the certain transcendental science, superior to and more extensive than mathematics and physics, in which we deal with truth, Berkeley shall esteem altogether useless and ineffectual, if by what he has said he cannot inspire his readers with a pious sense of the presence of god. Or as Hegel, three generations later, when the restoration of mind, soul, spirit, or myself to the substantiality and purity of its being is complete, will write: there can be religion without philosophy, but there can be no philosophy without religion.

In the next generation, Hume is a younger contemporary of Linnaeus and Buffon.

Hume writes:

when I enter most intimately into what I call myself, I always stumble on some particular perception, of heat or cold, light or shade, love or hatred, pain or pleasure; I never catch myself without a perception and never observe anything but the perception; when my perceptions are removed for any time, as by sound sleep, I am insensible of myself and may be truly said not to exist; and were all my perceptions removed by death, and could I neither feel nor think nor see nor love nor hate after the dissolution of my body, I should be entirely annihilated, nor do I conceive what is further requisite to make me a perfect nonentity; the identity which we ascribe to the mind of man is a fictitious one, and of a like kind with that which we ascribe to vegetables and animal bodies.

Nor do I conceive what is further requisite to make me a perfect nonentity. Make me, spirit, soul, or mind a perfect nonentity. The identity which we ascribe to the mind of man is a fictitious one, and of a like kind with that which we ascribe to vegetables and animal bodies. The identity of the mind, soul, spirit, or myself.

Unlike Spinoza earlier, or Diogenes or Aristotle earlier still, or Kant later, Hume does not propose to teach you what is the best for you. To perfect reason, and that understanding is freedom. To prefer nothing to freedom. That the happiest is the life of search for truth. To have the courage to use your own understanding. Even if these are, one and all, the best for you.

In Hume there are no prayers, incantations, or exhortations at all.

Instead, Hume's is *A treatise of human nature, being an attempt to introduce the experimental method of reasoning into moral subjects*.

All our ideas are copied from our impressions.

The foundation of all reasonings is experience.

Experimental reasoning itself, which we possess in common with the beasts, and on which the whole conduct of life depends, is a species of instinct, or mechanical power.

All our ideas, including ethics.

All reasonings, including ethics.

Experimental reasoning itself, on which the whole conduct of life depends, including ethics.

The attempt to introduce the experimental method of reasoning into moral subjects is the first attempt, and a decisive attempt, to explain ethics. To explain the subjugation of the search for truth to the search for action, teaching you what is the best for you, relentlessly teaching you what is the best for you, and rewarding and punishing you for having learned and failed to learn the lesson for all eternity. To explain not how and why it is the best, or the good, or from god. To explain how and why it is a species of instinct, or mechanical power. And not a species of instinct, or mechanical power, in the pinnacle of creation which you are. A species of instinct, or mechanical power, in the beast which you are. And more generally, the organism which you are. Hume's is the first attempt, and a decisive attempt, to explain this fact:

let us suppose that by the dropping of its seed, an oak or elm produces a sapling below it, which, springing up by degrees, at last overtops and destroys the parent tree; I ask, if in this instance there be any relation wanting, which is discoverable in parricide or ingratitude?

Later in the same generation, Kant is an older contemporary of Ingen-Housz and Lamarck.

Kant writes:

as the goal of its investigation metaphysics has only three ideas, god, freedom, and immortality; metaphysics needs these ideas not for the sake of natural science, but to go beyond nature;

and reason can employ all methods and principles to go to the innermost of nature, but never to fly beyond the boundaries of nature, outside

which there is nothing for us but empty space.

The three ideas, god, freedom, and immortality, are all three in that empty space.

And when Kant ventures into that empty space, it is to teach you about freedom, never mind god and immortality. He writes:

to know an object requires that I can prove its possibility; but to think I can whatever I will, as long as I do not contradict myself;

and even if I cannot know freedom as a property of anything to which I can ascribe effects in the sensible world, I can think freedom, and that is, its representation contains no contradiction;

and if freedom of the will is presupposed, morality follows by the mere analysis of the concept; for freedom of the will can be nothing but autonomy.

There can be thus no morality without enlightenment. And:

enlightenment is man's emergence from his self-imposed minority; minority is the inability to use one's understanding without the direction of another; it is self-imposed when its cause is not lack of understanding but lack of resolution and courage; *sapere aude!* have the courage to use your own understanding! is thus the motto of enlightenment.

Have the courage to use your own understanding!

Your self-imposed minority, however, is not only also the best for you. It has been very much better for you.

Two generations later, Hegel is a younger contemporary of Cuvier, an older contemporary of Robert Brown, Boucher de Perthes, and von Baer.

Hegel writes:

spirit has lost its essential life, it lies in wickedness, and reviles itself for doing it; but spirit is conscious of the loss, it turns away from the chaff, and it demands from philosophy the restoration of the substantiality and purity of its being.

With Hegel the exclusion of physics and mathematics from philosophy is complete.

Physics and mathematics are chaff. Philosophy must revile them.

The first philosophy, the certain transcendental science, superior to and more extensive than mathematics and physics, in which we deal with

truth, is brought to perfection:

the idea of the good can find its completion only in the idea of the true;

this is the absolute idea;

the absolute idea is the identity of the theoretical and the practical; everything else is error, confusion, opinion, striving, faltering, and perishing; the absolute idea alone is being, imperishable life, truth knowing itself, and all truth;

it is the only object and content of philosophy.

Philosophy is ethics. Everything else is error, confusion, opinion, striving, faltering, and perishing. Ethics and religion. There can be no philosophy without religion.

The transmigration of the soul into the subject is complete.

Hegel writes:

spirit is what was commonly called in the past the soul (*Logic*, II, 3.2);

thought is spirit is self-consciousness (*id.*);

the I in my self-consciousness is the subject.

Spirit, or the soul, or thought, or self-consciousness, or the I, or reason, or the subject, is everything, it is all reality, it is truth, it is the world, it is absolute knowledge:

reason is spirit in that the certainty that it is all reality is raised to truth, and it is conscious of itself as its own world, and of the world as itself;

absolute knowledge is spirit knowing itself as spirit.

And spirit, or the soul, or the subject, is unconditionally the spirit, or the soul, or the subject, of philosophy:

the ideality of the finite is the fundamental proposition of philosophy; every true philosophy is for this reason idealism;

every philosophy is essentially idealism; and philosophy is idealism as much as religion is.

Needless to add:

spirit is immortal, spirit is never past, never not yet; spirit is now essentially.

Begun by Berkeley, the war for the restoration of spirit, or the soul, as it was commonly called in the past, or the subject, as it will be commonly called after Hegel, to the substantiality and purity of its being was won triumphantly by Hegel. In the next generation Schleiden writes: in this battle there can be no compromise; it can only end with the complete annihilation and destruction of those

who are introducing their dogma into philosophy and natural science, and into the state and the church. And so in the couple of generations after Hegel the war was concluded, with complete annihilation and destruction. Of those who had deprived spirit of its essential life, made it confess that it lies in wickedness, and revile itself for doing it.

The continuing attempts to teach you what is the best for you, even if it is, in fact, the best for you, cannot be part of the acquisition of knowledge. The war for the restoration of spirit to the substantiality and purity of its being, and the victory in the war, is part of the continuing reinforcement of the barriers before the acquisition of knowledge.

Hume's, however, is a step of advance in the acquisition of knowledge. Twenty-two centuries after the beginning of ethics, counting back to Socrates, Hume's is at long last the first attempt, rather than yet again foist it on you, to explain ethics.

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The two centuries since Darwin.

In the previous two centuries the physical problem of the soul was solved conclusively. The soul is nothing. Selection is the principle of life, the efficient principle, or the cause, of life.

As to the ethical problem of the soul, in the previous two centuries, in the course of the war for the restoration of spirit, or the soul, as it was commonly called in the past, to the substantiality and purity of its being, the soul, or spirit, transmigrated into the subject, as it was to be commonly called now.

Thus, where the two centuries since Darwin are concerned, with respect to the physical problem of the soul, there is no development, nor can there be a development. The advance in the formulation and solution of the problem of life, however, of which the problem of the soul had been a part, continued, and for the sake of completeness, the main steps in this development can be indicated.

With respect to the ethical problem of the soul, nothing needs to be said. About the consequences of the restoration of spirit to the substantiality and purity of its being enough has been said above.

Something can be said instead about the attempt, which had begun with Hume, to explain ethics.

The problem of life.

Darwin writes: our ignorance of the laws of variation is profound. And: unless favorable variations be inherited by some at least of the offspring, nothing can be effected by natural selection.

It was the only difficulty before the theory of evolution. The theory had to demonstrate hereditary variation. And this twofold.

First, the theory had to demonstrate hereditary variation such as will produce species, and species which, both, remain stable at any given time and change over time from one into another.

Second, Schwann had shown that: the question about the fundamental power of the organic body resolves itself into the question about the fundamental power of the individual cell. The theory had to demonstrate the hereditary variation of the individual cell.

And in the century after Darwin the difficulty was resolved.

In 1865, in the *Experiments on plant hybrids*, Mendel discovered the laws of heredity. The law of segregation, each hereditary trait is controlled by two factors which separate and pass into separate germ cells, and the law of independent assortment, the hereditary units are pairs of factors which segregate independently when germ cells are formed.

In 1908, in the *Mendelian proportions in a mixed population*, Hardy discovered the law of the equilibrium of a population. In a free-crossing population the distribution of Mendelian factors continues unchanged.

In 1926, in *On certain aspects of the evolutionary process from the point of view of contemporary genetics*, Chetverikov showed that Mendel's laws are part of Darwin's theory. Chetverikov writes: free crossing is by its very essence the conservative factor preserving the genotypic structure of the species in the condition in which it is at any given moment; natural selection is the direct antagonist of free crossing; if free crossing stabilizes a population, selection continuously displaces its equilibrium; it is the evolutionary principle, leading ceaselessly to modification of the species.

Selection is the cause, both, of the stability of species at any given time and of their change over time from one into another.

Darwin already had shown that selection explains the facts of morphology. And that it is the cause also of instinct.

In 1932, in the *Observations on the recognition of instinctive behavior characteristic of the species in birds*, and 1935, *The companion in the environment of the bird*, Lorenz demonstrated learning as closely associated with instinct as imprinting, the learned permanent response to an object, and as remotely associated with instinct as tradition, the transmission of learned behavior from one generation to the next, in corvids and anatids.

Selection is the cause also of behavior.

Darwin already had shown that selection explains the facts of embryology.

And the penetration deeper inside the organism continued.

Mendel's laws presume that the germ cells are different from the somatic. That the unit of heredity is a pair of factors. That the two factors are transmitted by the germ cells. And that each of the two factors passes into a different germ cell.

All this Mendel inferred from the observed traits of the organism, from the phenotype. In the following century all this was demonstrated experimentally in the genotype, in the structure which the units of heredity, Mendel's factors, or the genes, are.

In 1885, in *The continuity of the cell plasma as a foundation of a theory of inheritance*, Weismann established the fundamental difference between somatic cells and germ cells. The germ cells produce both the somatic cells and the germ cells. The germ cells are passed on from one generation to the next unchanged. The somatic cells perish with each generation. The collection of somatic cells in an individual body is but a vehicle for the transmission of the germ cells.

In 1902, in *On multipolar mitosis as a means of analysis of the cell nucleus*, Boveri demonstrated that: the chromosome is the carrier of heredity.

The chromosome is a structure in the cell nucleus.

Mitosis is the division of the somatic cell into two cells identical to itself and to one another in the number of chromosomes. Meiosis is the division of the germ cell into cells different from itself and from one another in that, as it became clear with Boveri's demonstration, the chromosome of the mother cell is divided between the daughter cells.

In 1903, in *The chromosomes in heredity*, Sutton showed that: the phenomena of heredity are Mendelian (*id.*).

The chromosome obeys Mendel's laws. It is the effect of selection.

In 1910, in a series of articles beginning with the *Chromosomes and heredity*, Morgan demonstrated mutations and the linkage of traits with sex. Hence, in 1913, in *The linear arrangement of six sex-linked factors in Drosophila, as shown by their mode of association*, Sturtevant deduced the first genetic map.

These results were still inferred from the phenotype. In the following generation-odd experiment reached the genotype, and began reducing it to its chemical composition.

In 1941, in the *Genetic control of biochemical reactions in neurospora*, Beadle and Tatum showed that: genes control or regulate specific reactions either by acting as enzymes or by determining the specificity of enzymes (*id.*).

Enzymes are proteins. Proteins are compounds of amino acids. Amino acids are compounds of carbon, hydrogen, oxygen, and nitrogen.

In 1953, in the *Molecular structure of nucleic acids*, Crick and Watson showed that replicating DNA is the carrier of heredity.

The chromosome is DNA in whole or in part. DNA, or deoxyribonucleic acid, is a nucleic acid. Nucleic acids are compounds of carbon, hydrogen, oxygen, nitrogen, and phosphorus.

The molecular structure which the unit of heredity, or the gene, is obeys Mendel's laws. It is the effect of selection.

In 1961, in the *Genetic regulatory mechanisms in the synthesis of proteins*, Jacob and Monod discovered a regulatory mechanism of the replication of DNA. Jacob and Monod conclude: the genome contains not only a series of blue-prints, but a coordinated program of protein synthesis and the means of controlling its execution (6).

The series of blue-prints which the genome contains and the coordinated program of protein synthesis and means of controlling its execution obey Mendel's laws, too. They are the effect of selection, too.

In 1936, in *The origin of life on earth*, Oparin formulated a theory of the origin of life. The atmosphere of the early earth was highly charged with electricity and rich in carbon and hydrogen. The gradual accumulation of hydrocarbons led to the formation of coacervate droplets, held together by electrostatic attraction and separated by a membrane from the surrounding medium. The coacervates accumulated chemical elements and catalysts from the medium. And the most stable grew in complexity, until selection began determining their interactions.

After the discovery of RNA it was proposed that life began with self-replicating RNA.

RNA, or ribonucleic acid, is a nucleic acid, substantially smaller than DNA. RNA is central to the mechanism of replication of DNA demonstrated by Jacob and Monod.

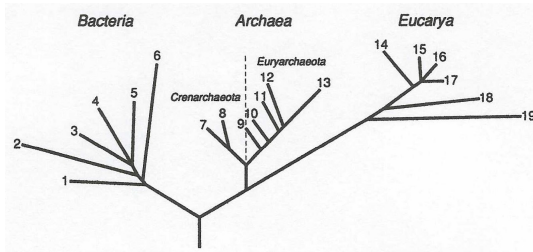
That was proposed, among others, by Woese in 1967.

The distinction between procaryotes and eucaryotes, cells without a nucleus and cells with a nucleus, was established at this time. Bacteria are procaryotes. In procaryotes DNA is localized in the middle of the cell. Jacob and Monod demonstrated the mechanism of replication of DNA with experiments on a bacterium, *Escherichia coli*.

In 1977, in the *Phylogenetic structure of the procaryotic domain; the primary kingdoms*, Woese, with Fox, established on molecular grounds another procaryotic domain, next to the bacteria, that of the archaea.

In 1990, in *Toward a natural system of organisms; proposal for the domains archaea, bacteria, and eucarya*, Woese, with Kandler and Wheelis, corrected on molecular grounds the natural system of organisms. They write: within the last decade it has become possible to trace evolutionary history back to the most recent common ancestor of all life, perhaps 3.5-4 billion years ago (*id.*).

Hence, the picture is of: a universal phylogenetic tree (*id.*).



The eucarya are: 14. the animals, 15. the ciliates, 16. the green plants, 17. the fungi, 18. the flagellates, 19. the microsporidia (*id.*).

Once again selection is the cause.

Life is replicating DNA, and mostly, and by far, procarya.

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Ethics.

Darwin writes:

it seems to me highly probable that any animal with well-marked social instincts, parental and filial affections included, would inevitably acquire a moral sense or consciousness as soon as its intellectual powers had become as well, or nearly as well, developed as in man;

this view of the origin and nature of the moral sense accords well with what we see of the early undeveloped condition of this faculty in mankind; the virtues which must be practiced are those which are still recognized as the most important; but they are practiced almost exclusively in relation to the men of the same tribe.

This is the origin of the moral sense.

And not the origin of ethics. Eager as ethics would be to be ubiquitous, to find itself everywhere, and positively in that animal with well-marked social instincts since its creation. Ethics begins one and a half centuries after the beginning of philosophy.

The intellectual powers of that animal with well-marked social instincts became as well, or nearly as well, developed as in man, not later than *Homo habilis*. This fact Oldowan tools attest some 2 600 000 years ago.

The animal thus inevitably acquired a moral sense not less than 2 600 000 years ago as well.

The tribe cannot be younger either, cannot be less than 2 600 000 years old.

The generalization from the tribe to a larger

tribe, and larger, and larger, and all the way to humanity, and hence the generalization of the practice of the virtues which must be practiced from the men of the tribe to the men of a larger tribe, and larger, and larger, and all the way to humanity, has been achieved by the princes. It has been achieved by the expansion of the state, and the mixing of peoples under the expanding state, and the development of the law under the expanding state.

The generalization begins with the first princes in the valley of the Nile and the valley of the two rivers.

The decisive steps are these.

The great expansion of the state under the princes of Akkad in the one and a half centuries beginning with

Sargon prince c. 2337-281.

The consolidation of the state under the princes of Ur in the one century beginning with

Ur-Nammu prince c. 2121-03,

and the first code of laws.

The consolidation of the state under the princes of Babylon in the two centuries beginning with

Hammurabi prince c. 1792-49,

and the greatest code of laws.

The expansion of the state under the princes of Egypt in the three centuries beginning with

Amenhotep prince c. 1545-21.

The great and continuous expansion of the state under the Assyrian princes in the seven centuries beginning with

Ashur-uballit prince c. 1363-27.

The very great expansion of the state under the great kings in the two centuries beginning with

Cyrus prince 559-30.

The very great and lasting expansion and consolidation of the state under the Roman consuls, generals, then emperors in the six centuries beginning with the second Punic war, in 220-02, including the development of jurisprudence beginning in the third of these six centuries.

The tribe and the moral sense predate ethics by not less than 26 000 centuries.

The generalization of the tribe and of the practice of the virtues which must be practiced predates ethics by 26 centuries.

Ethics is teaching you what is the best for you.

It begins with Socrates. Socrates discovered that Anaxagoras has nothing to teach him. Earth and air and other such things are not the true causes of things. The true cause of things is what is for the best.

Ethics is all about the animal with moral sense. There would be no ethics without the animal with moral sense. The animal with moral sense is a necessary condition of ethics. It is not a sufficient condition, however. There would be no ethics without the animal with moral sense, but not the other way around. The animal with moral sense developed, and its moral sense developed for not less than 26 000 centuries without the help, if it is help, of ethics.

Ethics is all about the generalization of the practice of the virtues which must be practiced, too. There would be no ethics without this generalization either. This generalization is a necessary condition of ethics, too. Again, not a sufficient condition, however. The labors of the princes, and, in particular, of mixing the peoples under the expanding state, are a development which leads to ethics. And again, there would be no ethics without the generalization of the practice of the virtues which must be practiced, but not the other way around. The state developed, and mixed the peoples under itself, generalizing the tribe and the practice of the virtues which must be practiced for 26 centuries without the help, if it is help, of ethics.

The necessary and sufficient condition of ethics is the development of philosophy, and, in particular, of physics. Ethics is substantially and essentially from the development of philosophy, and, in particular, of physics.

Ethics is teaching you what is the best for you.

It is the subjugation of the search for truth to the search for action. It teaches all of you. It teaches the best. If the best is the best, moreover, it follows that it is better that the search for truth, and it follows that it is the best for all of you.

And that ethics learned the substance and the essence of all this from physics, Socrates attests clearly and distinctly. Anaxagoras taught Socrates that there are the causes of things, that there are the true causes of things, and that earth and air and other such things may be the true causes of things.

Hence, those are not the true causes of things, the true cause of things is what is for the best.

And hence also the generalization from what is good to what is the best, and the best for all of you.

The search for truth, and the generalization from what is good to what is the best, and for all of you, are from physics.

The subjugation of the search for truth, and teaching you, are not from physics.

And the irresistible intensity of the lesson is not from physics either.

The intensity with which it has protected you from Anaxagoras. From the satisfaction of that sick curiosity for the sake of which freaks are put on show in the theater and men investigate nature and want knowledge for the sake of knowledge.

The intensity with which it has claimed not truth. All truth. Not knowledge. Absolute knowledge.

The intensity with which it has found its source in the maker of the world who made the world because he was good.

The intensity with which it has been eager to punish you. To flay you and lacerate you on thorn bushes. Not very severe and full of pain. Very severe, full of pain, and for all eternity. Chains, fetters, robes of fire, scalding water, melting your skin and belly, iron rods, and when you try to run away, you will be told, have now the taste of fire.

The intensity with which it has guarded you against your foes as well as friends. With which it has kept you in the flock, secure against the savage wolves, holding fast to what is believed everywhere, always, by all.

And, most important, the intensity of its difficult relation with the state.

On the one hand, clearly, if it is in possession of what is the best for you, it cannot abandon you to the misdeeds of the state. On the other hand, the state is a power contrary to our natural passions, keeping men in awe, reducing their wills unto one will. Hence, it has with the greatest intensity built a state of its own. Keeping men in awe, reducing their wills unto one will. The ideal state in which you would like to be a philosopher, but would most likely be a producer. And always the church, always your mother without whom you cannot have god for your father.

The intensity is not from physics.

Monod, in the *Chance and necessity, an essay on the natural philosophy of modern biology*, in 1970, writes:

during entire eons the lot of man was identical with that of the group, of the tribe to which he belonged and without which he could not survive; the tribe, for its part, could survive only through its cohesion; hence the extreme power of the laws which organized and guaranteed this cohesion; a man might perhaps infringe them; it is unlikely that any man ever dreamed of rejecting them;

given the immense selective importance which these social structures had to assume over such vast periods of time, it is difficult not to presume that they must have impressed themselves on the genetic evolution of the innate categories of the human brain; this evolution must not only have facilitated the acceptance of the tribal law, it must also have created the need for the myths which explained it, and grounded its sovereignty;

we are the descendants of such men (9).

Counting from Oldowan tools to Narmer, the lot of man was identical with that of the tribe for not less than 26 000 centuries, not less than 80 000 generations.

And the norms which guaranteed the cohesion of the tribe do not even have to have impressed themselves on the genome. They do not have to have become instinct. A tradition they cannot not have become. A tradition passed on for 80 000 generations. And it is difficult not to presume that, if not an instinct, this tradition must be as firm as instinct.

We are the descendants of such men, counting back to Narmer, not yet 180 generations removed from them.

Given its immense selective importance, the extreme power of the pressure toward cohesion is explained.

And the extreme power of the pressure toward cohesion can explain the intensity of ethics. Against the assimilation of the tribe into a larger tribe, and larger, and all the way to humanity, and the attendant loss of cohesion, or even only danger of loss of cohesion, ethics is keeping you in your tribe, larger, and larger, and as large as humanity, as your tribe is becoming.

It can explain also the resistance of ethics to

change.

Monod, for instance, does not approve of the restoration of spirit to the substantiality and purity of its being either. He writes:

in the last three centuries science, founded on the postulate of objectivity, has conquered its place in society; in the practice of men, but not in their hearts; modern society has accepted the treasures and power which science has laid in its lap, but it has not accepted, it has not even heard, its deeper message, the new and only source of truth which demands the thorough revision of the presuppositions of ethics, which demands a total break with the animist tradition; armed with all the powers, enjoying all the wealth, which it owes to science, society still lives by and teaches systems of values exploded by science at the root; for its moral basis society in the west still teaches a disgusting farrago of Judeo-Christian religiosity (*id.*).

It is, however, the traditional farrago.

Lorenz, in *The so-called evil, toward a natural history of aggression*, in 1963, writes: hardly ever did inspiration give rise to traditional norms and rites in human culture, the age-old process of natural selection did; historians will have to face the fact that natural selection determines the evolution of cultures in the same way in which it determines the evolution of the species (13).

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And the following conclusions can be drawn from the attempt to collect all particulars, and marshal them in their series and order, with respect to the physical problem of the soul.

In the course of the development of this problem, as in the course of the development of the problem of the earth, a large amount of knowledge has been acquired.

Very many propositions have been won, they have been brought in connection with one another in very many chains of inferences, and the chains have become ever more free of gaps.

From the thought and the sense to the reference and the true.

The indestructible spirit of Unas, my soul which will not talk to me, the thought distraught of Hector, the soul which comes in from the whole with every breath borne by the wind, is in

all animals, high and low, is the ratio of the elements which each of the parts of the body is, is in all plants and animals, is the principle of life, is not the principle of life, the forces of attraction and heat originate all actions of organisms, selection explains the facts of the classification of organisms, paleontology, geographical distribution, morphology, instinct, behavior, embryology, cytology, and genetics.

The soul is air. Respiration is the distinctive mark of life. There is no other principle of life than the blood and spirits of the body. The organism is nothing but an aggregate of crystals capable of imbibition. Life is replicating DNA.

Goats inspire through their ears. 600 kinds of animals and 550 kinds of plants. 11 841 species of 1 417 genera. Life is bacteria, archaea, and eucarya.

And it has been done in this case, the physical problem of the soul, as it has been done in that, the problem of the earth. The council of our predecessors has done it.

In order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

In order to collect the development of this problem, as of that, the predecessors of the philosophers and philosophers have been collected, all important among them, and all steps of advance in the acquisition of knowledge among them.

In particular, with respect to the problem of the earth, 108 important predecessors of the philosophers and philosophers have been collected, and 55 steps of advance in the acquisition of knowledge among them, from the thirty-seven centuries from the temple register from Lahun through Bessel.

With respect to the physical problem of the soul, 75 important philosophers have been collected, and 36 steps of advance in the acquisition of knowledge among them, from the twenty-four centuries from Anaximenes through Darwin. Counting centuries from the year of birth assigned to Anaximenes, their distribution is,

590- 491	8	5,
490- 391	11	1,
390- 291	8	5,

290- 191	—	—,
190- 91	1	—,
90 ^{BC} - 10 ^{AD}	1	—,
11- 110	2	—,
111- 210	1	1,
211- 310	—	—,
311- 410	—	—,
411- 510	—	—,
511- 610	—	—,
611- 710	—	—,
711- 810	—	—,
811- 910	1	—,
911-1010	1	—,
1011-1110	1	—,
1111-1210	2	1,
1211-1310	2	—,
1311-1410	—	—,
1411-1510	4	—,
1511-1610	7	6,
1611-1710	14	9,
1711-1810	11	8.

One of the main differences in the development of the two problems is in their beginnings.

The problem of the earth begins with the first philosopher. But the problem is from observation, and the beginning of the observation of the heavenly bodies predates the first philosopher by thirteen centuries. The beginning of observation is also slow and halting. There is one observation in the valley of the Nile, two-odd centuries later there is the discovery of Venus in the valley of the two rivers, there is in the valley of the Nile another observation three-odd centuries after the first, then there is nothing in this valley, and in the other valley there is nothing for nine centuries after the discovery of Venus. But then observation resumes, continues, becomes sustained, reaches dozens of the heavenly bodies, becomes accurate, continues, and the problem of the earth is discovered.

The physical problem of the soul begins with the third philosopher. The relation of earlier developments to this problem is very remote. This problem is from the development of philosophy. And its beginning is fast and, like the development of philosophy in general, accelerated from the first.

Thus, in the first three centuries the advance in the formulation and solution of the problem of

the soul is continuous and great. But not as great as the advance in the formulation and solution of the problem of the earth. Where the problem of the earth is concerned, by the last of these three centuries observation has been accumulating steadily, the theory of the motions of the heavenly bodies, of which the problem is a part, has been formulated, and results as substantial as the measurement of earth's radius have been obtained. Where the problem of the soul is concerned, observation begins with the collection of animals and plants in the last of these three centuries, and no theory, of which the problem would be a part, has yet been formulated.

The council is in evidence dramatically in these centuries. The problem of the soul is from the development of philosophy, and from the development of the problem of the principles of all things, in particular. From its beginning and through these three centuries of its development, the philosophers formulate and solve the problem in much the same way, as the principle of all things so the soul.

Also, the collection of the philosophers who are part of the development of the problem of the soul overlaps substantially with the collection of the philosophers who are part of the development of the problem of the earth. 10 important philosophers are part of both collections, Anaximenes, Pythagoras, Parmenides, Anaxagoras, Empedocles, Democritus, Plato, Ponticus, Aristotle, Dicaearchus.

Adopting the sound and avoiding the erroneous in what their predecessors have said, the philosophers do as their predecessors have done.

In the next five centuries the development of the problem of the soul decelerates. As does the development of the problem of the earth. It is the time of the schools, then of Roman dominion and the growth of teaching, then of the growth of the church. Even as it decelerates, however, the advance in the formulation and solution of the problem of the earth continues, the theory of the motions of the heavenly bodies acquires increasing mathematical sophistication, and the theory and the formulation and solution of the problem of the earth culminate in Ptolemy. In the formulation and solution of the problem of the soul the deceleration is decisive, and there is but the one

advance of Galen.

Doing as their predecessors have done, adopting the sound and avoiding the erroneous, the philosophers do much in the previous three centuries. Doing as their predecessors have done, they do less in these five centuries, and less.

Thus, in the next six centuries there is nothing. Doing as their predecessors have done, and having done less in the previous five centuries, and less, the philosophers, if they are still philosophers, do nothing in these six centuries. They take every care to hold fast to what is believed everywhere, always, by all.

In the next seven centuries there is a slow acceleration. Even as there is the one advance by the emperor, *stupor mundi*, the acceleration in the development of the problem of the soul is again more halting than it is in the development of the problem of the earth. The barrier which has been erected before the problem of the soul, too, is very much more massive than the barrier which has been erected before the problem of the earth. Where the problem of the earth is concerned, Aristotle and Ptolemy, and thus the theory of the motions of the heavenly bodies as well, and thus the formulation and solution of the problem of the earth as well, have been assimilated into Christian philosophy only in the thirteenth Christian century, and have thus not become part of the Christian creed. Even if god creates the world and the earth. Where the problem of the soul is concerned, the fathers of the church already, more than one millennium earlier, have solved the ethical problem of the soul, and the solution has at the time of the fathers already become most explicitly part of the Christian creed, and hence so have also, explicitly or implicitly, all questions about life. Doing as their predecessors have done, however, and having done more in the previous seven centuries, and more, the philosophers are at long last overcoming the barriers, first the barrier before the investigation of the world, in general, and the problem of the earth, then also the barrier before the investigation of life, in particular, and the problem of the soul.

Thus, in the last three of the twenty-four centuries there is again a great advance, and a great acceleration of the advance. Observation and experiment mount, a theory of life, of which

the problem of the soul is a part, is formulated, observation and experiment continue to mount, and in the last of the three centuries the theory of life is formulated conclusively. The problem of the soul is solved. The soul is nothing. And with the theory of evolution the advance in the investigation of life is second to none.

In these three centuries 4 important philosophers are part of both collections, of the development of the problem of the soul and of the development of the problem of the earth, Descartes, Hooke, Steno, Buffon.

In general, over all the twenty-four centuries, the physical problem of the soul is discovered, the attempts to solve it multiply, the collection of the particulars begins, and when the collection resumes, and accelerates, the theory of life is formulated. It is formulated in one major step, from Linnaeus and Buffon through Darwin.

The council is one and only one. As tightly bound as this, Aristotle first reviews what our predecessors have said about the soul, and Darwin's sketch of the progress of opinion on the origin of species begins with Aristotle. Aristotle also first reviews what our predecessors have said about the earth, and Darwin counts: of the 34 authors named in this sketch, who believe in the modification of species, or at least disbelieve in separate acts of creation, 27 have written on special branches of natural history or geology.

Thus, the reference of 'the soul' in the proposition, the soul is air, the proposition, the soul is the principle of life, the proposition, the divisions of the soul and their seats must be examined in anatomy, the proposition, it is not necessary to suppose that there is a soul in the body, and the proposition, the organic body is not shaped by a soul but develops according to the blind laws of necessity, is the same. The sense is different, but the reference is the same. The five propositions are bound inextricably in one and only one body of chains of inferences, and it is this fact, that they are bound inextricably in one and only one body of chains of inferences, that shows, and nothing else can show, that the reference of the first is the false, the reference of the second is the false, the reference of the third is the true, the reference of the fourth is the true, and the reference of the fifth is the true. From the sense to the

reference. The reference is discovered. Hence it is also these propositions in relation to one another that show, and nothing else can show, that 'the soul' has a reference. The reference of 'the soul' is nothing.

It may be objected that if the soul is nothing, it cannot be an object, and it cannot be investigated. And if it has been investigated, and it is an object, it cannot be nothing.

Not so.

The fact of the very many propositions brought in connection with one another in chains of inferences from Anaximenes through Darwin and further guarantees that 'the soul' has a reference. And the fact that the reference of many of these propositions is the false and the reference of many more of them is the true guarantees that the reference of 'the soul' is nothing. It has been discovered that the soul is nothing.

Descartes writes: it is not necessary to suppose that there is a soul in the body. Clausius writes: Carnot's assumption of caloric must be rejected (*Force*, 2). Einstein writes: the introduction of a luminiferous aether is proved to be superfluous (*Electrodynamics*, pr.).

The soul is an object like caloric and aether. And not only is it an object, it is a most important object. On the presumption of caloric Carnot discovered the cycle of reversibility of the heat engine, the cycle which became the foundation of thermodynamics. On the presumption of aether Maxwell discovered the laws of electrodynamics, the laws which became the foundation of relativity, too. And the investigation of the soul is the ground on which Descartes reformulated the problem of life. Nor is the object orbit of the sun around the earth less important. Or the object god.

If these are not objects, if 'the soul', 'god', 'the orbit of the sun around the earth', 'caloric', 'aether' do not have a reference each, and the same reference each, nothing, the acquisition of knowledge has been a series of miracles. Or rather a collection of fictions. Hence, in particular, neither does the proposition, the soul is nothing, have to have a reference, nor does the reference of this proposition have to be the true. Continuing as the soul in any case is, whether it is nothing or not, to be saturated with ideas and

feelings.

And the reference of 'life' in the propositions, the distinctive mark of life is respiration, the characteristic mark of life is nourishment, the heart is the first cause of life, every cell leads a double life, from the first dawn of life all organic beings can be classed in groups under groups, is the same. Bound in the same one and only one body of chains of inferences, these propositions in relation to one another also show, and nothing else can show, that 'life' has a reference, and the reference of 'life' has been fixed conclusively. The reference of 'life' is life. Replicating DNA, bacteria, archaea, and eucarya, inclusive of respiration, nourishment, every cell, and the heart, and its principle, its efficient principle, or cause, is selection.

The work of the council abounds in error, and the development of the problem of the soul more so and more so longer than the development of the problem of the earth. The main cause is the most formidable barrier before the acquisition of knowledge. Man has had to be a central part of the object of this problem, and for this reason, too, he has been vigorously protected by the development of ethics, and by the formulation and solution of the ethical problem of the soul, in particular, too.

But the work of the council abounds very much more in finality. And for the same reason, because man has been a central part of the object of this problem, and he has been vigorously protected by the development of ethics, and by the formulation and solution of the ethical problem of the soul, the finality obtained in the formulation and solution of this problem informs decisively man, his protection by the development of ethics, the development of ethics, and the ethical problem of the soul.

From the description of the development of the ethical problem of the soul the following conclusions can be drawn.

The problem begins with Plato, two centuries after the beginning of philosophy, one generation after the beginning of ethics.

Two earlier developments lead to this problem.

One is the discovery of the immortal soul of man nineteen and a half centuries before Plato, and its subsequent pervasive presence in the

Egyptian record.

The other is the generalization of the tribe and the moral sense with the expansion of the state and the mixing of peoples under the expanding state, continuing for twenty-six centuries by Plato's time.

Also, teaching is as old as organisms capable of transmitting newly acquired experience to the young.

One and a half centuries before Plato, immortality enters philosophy with Pythagoras who was Aethalides, Euphorbus, Hermotimus, and Pyrrhus before he became Pythagoras. Two generations later, two before Plato, Empedocles was once a boy and a girl and a bush and a bird and a fish, as well as an immortal god amongst you.

The ethical problem of the soul is nevertheless Plato's. The ethical problem of the soul, like ethics, in general, is from the development of philosophy. In particular, it is Plato who brings together the subjugation of the search for truth to the search for action, the immortal soul of man, the generalization from what is good to what is for the best, what is the best for you, for all of you, teaching you, and punishing you and rewarding you, all in connection with one another, in connection with one another in many propositions in many chains of inferences, and all these propositions and chains in connection with the appropriate prayers, incantations, and exhortations.

After Plato, the problem is ignored. Aristotle, a student of Plato's for two decades, ignores it. The Academy, attested for three centuries after Plato, ignores it. The other schools ignore it.

Immortality enters the Hebrew writings two and a half centuries after Plato. It is then attested by the Hebrew writings produced in Alexandria in Greek, then by the pharisees. It is with these that Plato's problem is no longer ignored. Then through st Paul, four-odd centuries after Plato, immortality enters the fathers of the church. St Paul and the fathers teach the resurrection of the body. All in all, for the first three and a half centuries of its development in the Hebrew and Christian writings immortality remains the resurrection of the body. Origen then, six centuries after Plato, restores the solution of the problem to its Platonic substantiality and purity. Origen also divests the solution of most of the propositions

which contaminate it in Plato. The balance between propositions, on one side, and prayers, incantations, and exhortations, on the other, shifts resolutely against the former.

For the following fourteen and a half centuries, through the seventeenth Christian century, the solution of the problem is everywhere, always, by all believed.

Thus, the problem is discovered by Plato, then it is for a long time ignored, then it conquers all.

And if the cause is in Unas, in Plato, in the Hebrew writings, and in st Paul and the fathers of the church the same, it can explain the vicissitudes of the problem.

It is in the second of the three brightest periods in the acquisition of knowledge that the problem is formulated and solved by Plato. The advance in the acquisition of knowledge continues. And the problem is ignored. It is when philosophy degenerates, and more, and more, that the philosophers do not scoff any longer, all listen. The problem is divested of all theoretical sophistication which it has in Plato. And it conquers all.

Thus also, when in the third of the three brightest periods in the acquisition of knowledge, in the seventeenth Christian century, the problem begins being ignored again, in the eighteenth Christian century the war for the restoration of spirit to the substantiality and purity of its being begins, and in the nineteenth Christian century the war is won. And the soul transmigrates into the subject. Even as the same propositions and chains of inferences, which show that the soul is nothing, show, and cannot not show, that the subject is nothing.

The development of the ethical problem of the soul is not part of the acquisition of knowledge.

From the reference and the true to the thought and the sense.

Finally, the problem of knowledge divides into two parts, the acquisition of knowledge and knowledge.

The parts of the problem of the acquisition of knowledge are the beginning of the acquisition, the collection of the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition.

The problem of the earth is from observation, and observation is from mathematics. The rela-

tion of the development of the description of the surface of the earth by the labors of the princes and the development of the problem of the earth is not remote either. On the evidence of the problem of the earth, the acquisition of knowledge has to be traced back to the beginning of mathematics, and to the beginning of the labors of the princes.

The physical problem of the soul is from the development of philosophy. Its relation to any earlier development is very remote. It thus does not inform the problem of the beginning of the acquisition directly. The relation of the ethical problem of the soul to earlier developments is not remote, however. Hence, even if the ethical problem of the soul is a development against the acquisition of knowledge, if it is from the acquisition of knowledge, and the labors of the princes of expansion of the state, and mixing of peoples under the expanding state, lead to the ethical problem of the soul, the ethical problem of the soul corroborates the problem of the earth in indicating that the beginning of the acquisition has to be traced back to the labors of the princes.

The problem of the earth has yielded 108 important predecessors of the philosophers and philosophers, 55 steps of advance in the acquisition of knowledge. The physical problem of the soul has yielded 75 important philosophers, 36 steps of advance in the acquisition of knowledge. The two collections overlap substantially. The total for the joint collection is 168 important predecessors of the philosophers and philosophers, 82 steps of advance in the acquisition of knowledge. And given the generality of the two problems, the one a part of the investigation of the world in its entirety, the other a part of the investigation of life, this collection of 168, and 82, has to be a substantial fraction of the acquisition in its entirety.

On the evidence of the problem of the earth, as soon as it begins, philosophy encompasses the acquisition of knowledge in its entirety. And so philosophy does on the evidence of the physical problem of the soul. The results that there is soul in all animals, high and low, or that the soul is the ratio of the elements which each of the parts of the body is, have not been obtained outside the one and only one development of philosophy

from Thales through Anaximenes through Darwin. Nor have any of the results built on these results. Nor could they have been.

The problem of the earth shows also that philosophy is an empirical generalization, and a vast generalization, over the earlier acquisition of knowledge. From philosophy, and thus postdating philosophy, the physical problem of the soul cannot corroborate this conclusion. The ethical problem of soul, however, if it is a generalization from what is good to what is for the best, and for all of you, corroborates it. It is an empirical generalization, and a vast generalization, over earlier developments, even if these developments are not part of the acquisition, and even if the problem itself is not part of the acquisition.

As soon as it begins, philosophy encompasses the acquisition of knowledge in its entirety. As it develops, however, philosophy encompasses more than the acquisition of knowledge. And on the evidence of the physical problem of the soul and the ethical problem of the soul in the twenty-four centuries from Anaximenes to Darwin, the acquisition of knowledge has not been a very large part of the development of philosophy. At first the acquisition of knowledge coincides with the development of philosophy, but it then becomes only a part, and a steadily smaller part, until it is arrested altogether by the other part. And when it becomes a part of the development of philosophy again, and a steadily larger part, the other part begins a war to exclude it from philosophy.

As to the kinds of the acquisition of knowledge, as the problem of the earth shows, so the physical problem of the soul shows affirmatively, the ethical problem of the soul shows negatively, and the two problems in relation to one another cannot have shown more clearly and distinctly, as well as dramatically, that the acquisition of knowledge is theory, exclusively. Search for truth uncontaminated not just by any concern for action, uncontaminated by any concern for the subjugation of the search for truth to the search for action.

On the evidence of the physical problem of the soul, as on the evidence of the problem of the earth, the acquisition of knowledge is physics.

It is also history not only because the collec-

tion of the particulars includes Aristotle's *History of animals*, Theophrastus's *History of plants*, Pliny's *Natural history*, Gesner's *History of animals*, Belon's *Natural history of strange marine fishes* and *Natural history of birds*, Ray's *History of fish*, *History of plants*, and *History of insects*, Swammerdam's *General history of insects*, Réaumur's *History of insects*, Buffon's *Natural history*, Lamarck's *Exposition of considerations on the natural history of animals* and *Natural history of invertebrate animals*, Cuvier's *Foundation of the natural history of animals* and *History of fishes*, von Baer's *On the developmental history of the animals*, and Lorenz's *Toward a natural history of aggression*.

If the standard is the performance of the indispensable duty, the examination of all original materials, Aristotle, Theophrastus, Pliny, Galen, Gesner, Linnaeus, Buffon, Cuvier, Darwin are historians *par excellence*. History is a part of physics.

The parts of the problem of knowledge are the relation of knowledge and truth, and the relation of knowledge and the world.

The development of the problem of the earth shows the acquisition of knowledge. And the development of the physical problem of the soul shows the acquisition of knowledge.

The development of the problem of the earth shows the accumulation of very many propositions whose reference is the false, very many more propositions whose reference is the true, and the sharp and conclusive separation of the true from the false. The development of the physical problem of the soul shows the same. Very many of the true propositions are shared between the two problems, too.

The development of the problem of the earth shows that the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world have been described, and all in relation to one another. The development of the physical problem of the soul shows the same. All organic beings from the first dawn of life have been described, all parts of the same system.

The relation of the accumulation of propositions whose reference is the true, the description of the system of the world, and the acquisition of

knowledge cannot be exclusion.

And the relation of truth, the world, and knowledge cannot be exclusion.

And these propositions cannot not entail one another.

**The best state,
and both, the well-governed state,
and the one conceived in speech**

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The task is to show that the acquisition of knowledge can be collected in its entirety.

The ground is the collection of the predecessors of the philosophers and philosophers, the separation of the important among them, and the separation of the steps of advance in the acquisition of knowledge among them.

And, in particular, the task is to show that the ground is firm enough to capture the acquisition of knowledge in its entirety by collecting the predecessors of the philosophers and philosophers, the important among them, and the steps of advance in the acquisition of knowledge among them, with respect to five problems. First the earth. Then the soul, and both the theoretical problem of the soul and the practical problem of the soul. And the best state, next, and both the theoretical problem of the best state and the practical problem of the best state, presents graver difficulties before the task. Which is all by itself a reason to undertake the task with respect to this problem.

Aristotle begins in book II of the *Politics*: in order to consider which is the best state, we must examine constitutions, and not just those of well-governed states, but those also conceived in speech and held in high esteem.

The well-governed states pose the theoretical problem, the search for truth. It is a historical problem, in particular, about the best state that there has been.

Those conceived in speech and held in high esteem pose the practical problem, the search for action. It is a political problem, in particular, about the best state that there should be.

All of book II of the 8 books of the *Politics* is a review, of well-governed states and of those

conceived in speech and held in high esteem.

And of the latter, those conceived in speech and held in high esteem, Aristotle's review is detailed, comprehensive, and nearly enough complete. Of the former, the well-governed states, the review is not detailed, not comprehensive, and not complete at all. Such as it is, it is also guided by the review of the states conceived in speech.

And the review is representative.

Aristotle collected the constitutions of 158 states, not less than one half of all contemporary states. *The constitution of Athens* attests a historian *par excellence*, too. But barely any of this historical research is in evidence anywhere in the 8 books of the *Politics*. The *Politics* is concerned, more or less exclusively, with the state conceived in speech.

And the *Politics* is representative.

Ever since they began concerning themselves with the state, the philosophers have with great determination ignored all real states and devoted themselves to conceiving the state in speech instead.

The true way begins with the collection of the particulars, duly and orderly verses itself in the particulars, and by continuous and gradual ascent only ultimately ends with the most general principles.

The false way flies to most general principles and, taking them to be settled, proceeds to judgment.

And in the investigation of the state, Plato already follows the false way with great determination, Aristotle does in this as in nothing else as Plato does, and Zeno of Citium, Cicero, Augustine, al-Farabi, Aquinas, Marsilius, Occam, More, Grotius, Hobbes, Spinoza, Locke, Montesquieu, Rousseau, Hamilton and Madison, Sieyès, Burke, Godwin, Hegel, Marx, one and all follow the false way with great determination. The investigation of the state is not grounded in the investigation of but 1 particular state in any of these philosophers. The only exception has been Polybius. Ibn Khaldun, Machiavelli, and Bodin are partial exceptions, and the part is not too large in any of the three.

Nor have the philosophers just ignored the particulars. They have condemned them.

Plato writes: until the philosophers become kings, the states will have no respite from evil. Aristotle agrees: none of the constitutions of exiting states is good (II, 1).

All evil, none good.

Nor have the philosophers just followed the false way. They have elevated it into a principle.

Hegel writes: we must not look at particular states, we must look at the idea of the state. It is with respect to the state, too, that Hegel writes: not more does one need to be a shoemaker in order to know whether his shoes fit than he needs to learn a trade in order to have knowledge of matters of general interest.

We must not look at particular states. One does not need to be a shoemaker to know whether his shoes fit.

Clearly, as his shoes so the state.

In the investigation of the state, if, indeed, it has been investigation, the subjugation of theory to practice, of the search for truth to the search for action, has been, more or less, total. Not only the city in heaven is about justice, and Plato has explained how and why you should be just, all cities on earth are about justice. Not only the heavenly city is about god's providence, and Augustine has explained how and why you should go to heaven, all earthly cities are about god's providence. Not only should the state begin with a covenant, or a compact, or a contract, and Hobbes, Locke, and Rousseau have explained how and why, the state begins with a covenant, or a compact, or a contract. Not only the state of the proletariat is an ideology, and Marx has explained how and why, all states are ideologies.

The investigation of the state has not been of the state that there has been. It has been more or less exclusively of the state that there should be. Its central concern has thus been the end of the state. Its final principle, what is it for. Taking it for granted that the state has a final principle, it is for something, like shoes.

And such as it has been, the investigation of the state that there should be has also more or less exclusively presumed itself to be the investigation of the state that there has been. It has thus also more or less exclusively presumed that the final principle of the state, taking it for granted that the state has a final principle, is its efficient principle,

the cause of its motion, such as the state, like anything in motion, cannot not have.

The end of the state is a central concern of Plato, Aristotle, Zeno of Citium, Polybius, Cicero, Augustine, al-Farabi, Aquinas, Marsilius, Occam, ibn Khaldun, Machiavelli, More, Bodin, Grotius, Hobbes, Spinoza, Locke, Montesquieu, Rousseau, Hamilton and Madison, Sieyès, Burke, Godwin, Hegel, Marx. And the exceptions are the same. Only in Polybius is the concern with the efficient principle of the state, its cause, rather than the final, what it is for, and to a degree, and not too large a degree, in ibn Khaldun, Machiavelli, and Bodin.

Thus, the investigation of the state that there should be has developed. The state has been conceived in speech, and again, and again. The investigation of the state that there has been has barely begun. And that is, counting back to Plato, twenty-four centuries of investigation of the state. Counting back to Narmer, fifty-odd centuries of development of the state.

And the state – the real state – is important. Ever since the beginning of the state, all practice, including politics, including ethics, never mind shoemaking, has been determined fundamentally, substantially, and essentially by the development of the state.

Also, ethics cannot be part of the acquisition of knowledge, but politics cannot not be. The subjugation of the search for truth to the search for action is essential to ethics, but, with all due respect to all the philosophers who have reduced politics to ethics, it is not to politics. Which the fact that the state predates ethics, and the reduction of politics to ethics, by two and a half millennia suffices to show. And if politics is concerned with the state, and the state has determined all practice, the generality of this fact cannot not be a theoretical problem.

All of which can, and should, be shown in particular.

It is a massive task. To collect the philosophers who have concerned themselves with the state is not difficult. The list above is, in fact, of well over one half of all important philosophers who have concerned themselves with the state. Difficult is to separate the sound in what they have said from the erroneous. For not all that they

have said is erroneous. Against their better judgment as it may have been, the investigation itself showed them the way in this, too. It is yet difficult to separate the sound from the erroneous because the state has been conceived in speech, and again, and again, but the investigation of the real state, the state such as the state has been, has barely begun.

The ground on which the sound can be separated from the erroneous can only be the real state, the state such as the state has been. What the real state, the state such as the state has been, has been can only be ascertained by the true way. The collection of the particulars and marshaling them in their series and order. And this now, the collection and marshaling in their series and order of the particulars who have been part of the development of the state, is the massive task. Temporally and spatially. The state has developed for fifty-odd centuries, and until it has brought under itself the entire surface of the earth. Nor has it mattered whether the shoes fit.

It is a massive task, and undertaking this task in its entirety is also best postponed.

Here, in order to show that the ground of the collection of the acquisition of knowledge in its entirety – the collection of the predecessors of the philosophers and philosophers, the separation of the important among them, and the separation of the steps of advance in the acquisition of knowledge among them – is firm enough to capture the acquisition in its entirety, one part of the task can be undertaken.

The problem is the best state, and both, the well-governed state, and the one conceived in speech.

Beginning once again with Aristotle's review.

Aristotle reviews the states conceived in speech first, and the earliest of these was conceived by Hippodamus, one century before Aristotle. Aristotle then turns to the real states, of those in the review the earliest to rise was Sparta, and the rise of Sparta can be traced back to Theopompus, three-odd centuries before Aristotle.

The rule is to divide difficulties into parts such as will lead to their better solution, and the real states, not those conceived in speech, can only determine the parts.

Thus, Aristotle is not an important step in the development of the state, but his younger contemporary Alexander, whose tutor Aristotle was, is. The great kings brought the state to its highest peak of development yet, and Alexander conquered the domain of the great kings. Alexander, prince 336-23, is one of the most important steps in the development of the state.

Theopompus, prince *c.* 710, is a very important step in the development of the state, but not as important as Alexander. Theopompus, however, is contemporary with Sargon II, prince 722-05, and Sargon II is not only a more important step, he points to a step as important as Alexander. Sargon II is one of the series of 6 Assyrian princes who, before the great kings, brought the state to its highest peak of development yet, and this series begins with Tiglath-pileser III. Tiglath-pileser III, prince 745-27, is one of the most important steps in the development of the state as well.

The development of the state is cumulative. In particular, the development of the state from that of the Assyrian princes through that of the great kings through those of the Greek cities through that of the Macedonian princes is one joint cumulative development.

The task, then, can be confined to these limits, the four-odd centuries from

Tiglath-pileser III prince 745-27
to

Alexander prince 336-23.

The temporal limits also entail the spatial.

And the collection of the particulars proceeds in this order. First the political problem, of the states conceived in speech, and of the best of them, is addressed. Beginning with Aristotle's review, the review is completed, and the particulars which are part of the development of the state conceived in speech, and all important among them, through the time of Alexander are collected. Conclusions are drawn from these states, the ones conceived in speech, to the real states. Then the historical problem, of the real states, and of the best of them, is addressed. Again beginning with Aristotle's review, the particulars which are part of the development of the real state, and all important among them, from the four-odd centuries from Tiglath-pileser III to

Alexander are collected. Conclusions are drawn from these states, the real, to both the real states and the ones conceived in speech. The steps of advance in the acquisition of knowledge are separated, and both, with respect to the real states and the ones conceived in speech. And how the particulars before Tiglath-pileser III, and after Alexander, can be collected is indicated.

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The first three fifths of Aristotle's review are devoted to states conceived in speech. The first three fifths of this are devoted to criticism of Plato's two states, the one in the *Republic* and the one in the *Laws*. Aristotle then criticizes Phaleas, then Hippodamus.

These are the states conceived in speech. Aristotle finds what Hippodamus, Phaleas, and Plato have said erroneous, and turns to the review of the real states.

Hippodamus is well attested independently of Aristotle's review. Phaleas is not attested independently of the review, but Aristotle's reports suffice to fix his date. Aristotle writes: Hippodamus first among those who were not leaders of a state spoke about the best state (II, 8). Hippodamus thus predates Phaleas. Also: Phaleas first said that the citizens of a state should have equal property (7). Phaleas thus predates Plato.

There are 3 particulars in this review. It is not many. In the review of what our predecessors have said about the principles of all things Aristotle has 20, in that of what they have said about the earth 8, in that of what they have said about the soul 14. And when the reviews are completed, the important philosophers who are part of the development of the problem of the earth in the two and a half centuries from Thales to Aristotle are 19. Those who are part of the development of the problem of the soul are 20. And not only the philosophers who concerned themselves with the best state are not many, the philosophers who concerned themselves with the state, the best or otherwise, are not many. Of the 3 in Aristotle's review 1 is not important. And all the important philosophers who concerned themselves with the state, the best or otherwise, in the centuries through Aristotle are 4. But 1 predecessor of Aristotle can be added to those in the review,

Protagoras. And 1 is Aristotle himself.

Through the time of Alexander – or what is the same, through Aristotle – these 4 are all important philosophers who concerned themselves with the state, the best or otherwise,

490 - 420 **Protagoras,**

484 - 403 Hippodamus,

427 - 347 **Plato,**

384 - 322 **Aristotle.**

The 1 not important,

455 Phaleas,

will do to show that the line between important and not important is drawn sharply.

The state, and whether the best or otherwise, did not attract the attention of the philosophers too much. The number 4 shows this. As the quantity so the quality. The more particular concerns of these 4 philosophers show it. And the concerns of the few other philosophers who came close to concern with the state, and it is not very close, also show it.

Not quite one and a half centuries after the beginning of philosophy, then, four-odd generations after Thales, concern with the state among the philosophers begins with Protagoras.

Laetius reports: Aristoxenus said that nearly all of Plato's *Republic* could be read in Protagoras's *Antiarguments* (III, 37). Among the treatises of Protagoras Laetius lists also *On the first order of things* (IX, 55). And what Plato has Protagoras say in the *Protagoras* about the first order of things has to be from *On the first order of things*.

Plato has Protagoras say:

once there were only gods; then, when the appointed time came, the gods made mortal beings out of earth and fire; they then appointed Prometheus and Epimetheus to give them powers and abilities; so Epimetheus gave to some swiftness, to some strength, to some wings, to some claws, to some hides, to some hooves, to some the ability to procreate fast; but Epimetheus was not very wise, and he forgot to give anything to the human race; so Prometheus, desperate to find something for men, stole arts and fire from Athena and Hephaestus, and gave them to men; sharing thus in some way in the divine, men set up shrines, discovered speech, began building houses and nourishing themselves from the fruits of the earth; but they were scattered, there were

no cities; and they were destroyed by the beasts because they were weaker in every way; they tried to band together, but they kept doing wrong to one another, because they did not have the art of government; and because the art of war is a part of the art of government, they could not fight the beasts; so, afraid that they will all die out, Zeus ordered Hermes to give them shame and justice, and to distribute these not like the arts, where one doctor, for instance, suffices for many, but to have them all have a share; for a city can never come to be if only a few have shame and justice (320d-2d).

Given arts and fire, men set up shrines, discovered speech, began building houses and nourishing themselves from the fruits of the earth. But they were scattered, there were no cities, and they were destroyed by the beasts. They tried to band together, but they kept doing wrong to one another, because they did not have the art of government. And because the art of war is a part of the art of government, they could not fight the beasts. So they were given shame and justice, and distributed not like the arts, but so that all have a share. For a city can never come to be if only a few have shame and justice.

Concern with the state begins among the philosophers with Protagoras.

And Protagoras discovered the fundamental problem which the state poses, its beginning.

The solution of the problem is, shame and justice, and distributed so that all have a share. For a city can never come to be if only a few have shame and justice.

Concern with the state begins among the philosophers with Protagoras. As his more particular concerns show, however, the investigation of the state does not begin with Protagoras. The investigation of the state is yet to begin.

Concern with the state begins among the philosophers also with the discovery of the fundamental problem which the state poses, its beginning. The solution of the problem, however, shame and justice, and distributed so that all have a share, shows also that among the philosophers the concern is from the first with the state that there should be, not the state that there has been.

Hippodamus is a younger contemporary of Protagoras.

Aristotle reports in the review:

Hippodamus discovered the art of planning cities; he laid out Piraeus;

and he first among those who were not leaders of a state spoke about the best state; his city would have 10 000 citizens, divided into 3 classes, artisans, farmers, and soldiers; the land would also be divided into 3 parts, sacred, public, and private; the law would also have 3 parts, dealing with insult, injury, and murder; there would be a single court of appeal; and all magistrates would be elected by the people, by all the 3 classes (8).

The art of planning cities begins with Hippodamus. Concern with the best state begins with Hippodamus as well. His is the first state conceived in speech. But the investigation of the state does not begin with Hippodamus either. The investigation of the state is yet to begin.

Two generations then separate Protagoras and Hippodamus from Plato.

The few other philosophers who came close to concern with the state are from this time,

475 - 411 Antiphon,
467 Thrasymachus,
455 Lycophron,
453 - 403 Critias.

Antiphon said: the law is by convention, not by nature (fr. 44a). Also: by nature there is no difference between Greek and barbarian (fr. 44b).

Thrasymachus said: justice is what is useful to the stronger; for, whether in a tyranny, or in a democracy, or in an aristocracy, the rulers make the laws, they make them useful to themselves, and proclaim what they have made just; so justice is the same in all cities, it is what is useful to the established rule; and since the established rule is the stronger, justice is the same everywhere, it is what is useful to the stronger.

Lycophron said: the law is only a convention, it is men's promise to one another to be just, and it has no power to make the citizens good or just (Aristotle, *Politics*, III, 9). Also: highborn is an empty word, in truth there is no difference between lowborn and highborn (fr. 91).

Critias said: at first men lived like beasts; next laws were established so that justice may rule mankind, and the laws held men back from doing evil in the open, but not from doing it in secret;

so a wise man discovered the fear of gods, to frighten evildoers even in their thoughts.

One generation after Protagoras, the philosophers have begun concerning themselves with the law. Their concerns are thus close to concern with the state. But not very close. The concern with the law is subjugated, and subjugated steadily more explicitly, to the concern with justice. The law is by convention, not by nature. The rulers make the laws, justice is thus what is useful to the stronger. The law is men's promise to one another to be just. Laws were established so that justice may rule mankind.

These philosophers anticipate Plato, most particularly in the subjugation of the concern with the law, and hence more generally with the state, to the concern with justice. In the state what should be is what is. Even if some of what these philosophers affirm, and Plato denies, is, or should be, both true and admirable. By nature there is no difference between Greek and barbarian. In truth there is no difference between low-born and highborn. Some of it also is, or should be, true, if not admirable. Justice is what is useful to the stronger. A wise man discovered the fear of gods, to frighten evildoers even in their thoughts.

Phaleas is contemporary with these philosophers.

Aristotle reports in the review:

Phaleas first said that the citizens of a state should have equal property; he said that in a new city this can easily be achieved from the beginning; in an already established city the easiest way to achieve it is for the rich to give dowries and not receive them, for the poor to receive dowries and not give them (7).

Phaleas's is the second state conceived in speech. It is also, or should be, admirable, if not true, that the citizens should have equal property.

With these philosophers the investigation of the state has not yet begun either.

Then there is Plato.

The *Republic* has been presented above in some detail.

In book I Plato asks: what is justice? And answers: justice is not to do good to your friends and harm to your foes; justice is to give everyone his due. In book I as well Plato reports what Thrasymachus said about justice, and has Socrates

object:

no one who rules, to the degree to which he is a ruler, seeks his own advantage, he seeks the advantage of those he rules.

Thrasymachus wonders, Socrates, do you still have your wet nurse? why? Socrates asks; to have her wipe your nose, Thrasymachus replies, you seem to think that shepherds and cowherds seek the advantage of the sheep and the cattle.

Next, in order to find out what justice is, and that is, to find out that justice is what he has found out it is in book I already, in books II-X, Plato builds the best state. He begins by addressing the problem of the beginning of the state. He writes: I think a city comes to be because none of us is self-sufficient, but we all need many things; men thus gather in one place to help each other (II, 369b-c). That is that. Plato has Socrates say: come now, let us build our city in speech; yes, let us, Adimantus says (*id.*), and so they do.

And not only: our state, having been built correctly, is completely good (IV, 427e). It is so completely good: you will not find a state as great as ours among the states of either the Greeks or the barbarians (423a).

The *Republic* is the third state conceived in speech.

The fourth is also Plato's, conceived in the 12 books of the *Laws*. This is how Aristotle described the difference between Plato's two states in the review: everything is the same, there is the same education in both, the citizens are free from work in both, they share their meals in both; the only difference is that in the *Laws* women are admitted to the common meals, and the soldiers are 5 000, while in the *Republic* they are only 1 000 (6).

The state conceived in speech and held in high esteem Plato has brought to perfection.

The investigation of the state, on the other hand, is yet to begin.

It begins in the next generation, with Aristotle.

It begins with the *Politics*. More particularly, it begins with book I of the *Politics*. More particularly still, it begins with the opening couple of pages of book I of the *Politics*. Most of the *Politics*, too, is not concerned with the state, the one that there has been, most of the *Politics*, too, is concerned with the state that there should be.

The beginning is the most difficult.

And the beginning of the investigation of the state is especially difficult.

The beginning of the investigation of the state is hampered severely by the great determination with which the false way is followed. Never mind Plato. Aristotle collected the constitutions of 158 states, but this historical research – later labors apparently – is not in evidence in the *Politics* at all.

Not hampered by the true way, the beginning of the investigation of the state is further hampered severely by the subjugation of politics to ethics. Again never mind Plato. Aristotle writes in the *Ethics*: politics encompasses even the most highly esteemed actions, warfare, economics, rhetoric; it legislates; and even when the end is the same for man and for the state, that of the state is greater and more complete. The end of the state is greater and more complete. But not in the *Politics*. In the *Politics*: to decide which is the best state we must decide which is the best life; for what is the best for a man and what is the best for the state is the life of virtue (VII, 1). The end is in any case what it is for, not the cause of its motion.

Hampered by ethics, the beginning of the investigation of the state is hampered most severely by the best state conceived in speech in place of the real state. Once again never mind Plato. The central concern of the *Politics*, too, is to build the best state in speech. Aristotle does so explicitly in the concluding books VII-VIII. Books II-VI are also dominated by that state, however. Book II is the review, three fifths are devoted to the 3 philosophers who have conceived states in speech, to all real states the remaining two fifths are devoted, and the former are the guide to the latter. In books III-VI Aristotle deduces the state, and the deduction begins with the definition: a citizen is who shares in the deliberative or judicial government of the state; and the state is a collection of such a number of citizens as is self-sufficient for life (1). An admirable definition of the state. The definition of the state has also precluded more or less all states that there have been from being states.

The beginning is also the most important.

Which can go a long way to explain why the

investigation of the state has not advanced much since Aristotle either.

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And this is how the state conceived in speech compares with the real state.

The best state of Hippodamus would have 10 000 citizens.

The best state of the *Republic* would have: 1 000 men to fight for it (IV, 423a), and, Plato adds: you will not find a state as great as ours among the states of either the Greeks or the barbarians, even if many of them are many times larger (*id.*). The best state of the *Laws* would have: 5 040 guardians (V, 737e-40d), and, Plato adds: the number 5 040 should not be increased even by 1 (XI, 929a).

Aristotle's best state would be: large enough to be sufficient for the good life of the citizens (VII, 4); and small enough for the citizens to know one another so as to be able to choose their magistrates well; the limits should be the smallest number sufficient for life and the largest number that can be encompassed in a glance (*id.*).

Aristotle's argument begins with the analogy: neither a boat 1 span long is a boat, nor a boat 2 stades long is a boat (*id.*).

And it is not that, if Hippodamus's, Plato's, and Aristotle's are boats, the aircraft carrier is not a boat. The trireme is not a boat either. Only the toy boat is a boat.

At the time when Hippodamus, Plato, and Aristotle conceived the state in speech, the state of the great kings was the state of some 25 000 000 people, one fifth of all people. It encompassed 77 countries and peoples large enough to pay annual tax of more than 10 talents each. The tax which the 77 large countries and peoples yielded could maintain 400 000 infantrymen and 40 000 cavalrymen for one year in the field. And this state was a state in fact for more than two centuries.

The most powerful Greek states in these two centuries were Sparta and Athens.

At the height of its power, when it stopped Xerxes, Sparta was a city of: about 8 000 men (Herodotus, VII, 234). In 479, at Plataea: Pausanias led 5 000 Spartans, accompanied by 7 helots each (IX, 10), for a total of 40 000. At this time

Sparta dominated the Peloponnese, not less than 500 000 people.

In 431 Athens entered the Peloponnesian war: with 16 000 infantrymen in garrisons abroad, 13 000 infantrymen guarding the city, 1 200 cavalrymen, 1 600 archers, and 300 triremes in active service (Thucydides, II, 13). A trireme was manned by 200 men. The total for 300 triremes is 60 000. At this time Athens dominated the Aegean, some 1 000 000 people.

1 span is about .25 m, 2 stades are about 350 m. A trireme was about 35 m long, an aircraft carrier can be more than 300 m long.

If the states of Hippodamus, Plato, and Aristotle are triremes, Sparta and Athens are aircraft carriers, and the state of the great kings is a boat so long it cannot be described.

If Sparta and Athens are triremes, the state of the great kings is an aircraft carrier, and the conclusion is unavoidable. Hippodamus, Plato, and Aristotle are not concerned with the boat, they are concerned with the toy boat.

The 158 states whose constitutions Aristotle collected are not less than one half of all contemporary states. The largest was the state of the great kings. Sparta and Athens were two of the largest half dozen. The remaining couple of hundred were smaller, and smaller, and smaller. And the state has always been large and small. The state appeared in the valley of the Nile as a large state. The state appeared in the valley of the two rivers as small cities. Today states are the United States and China, and Monaco and Nauru.

And it may be, unlikely as it has to be, that the best state is small. But it may not be that the large state is not a state. And it may not be that the small state will explain the state. Only the large state can explain the state.

Galileo writes: a large boat built the way a small boat is built will fall apart under its own weight; one cannot reason from the small to the large because many things which are successful in the small cannot exist in the large.

A large boat built the way a small boat is built will fall apart under its own weight. A small boat built the way a large boat is built will not fall apart under its own weight.

The state of the great kings is not less than 1 000 times larger than the best state, and whether

it is Hippodamus's best state, or either of Plato's best states, or Aristotle's best state. The states of Sparta and Athens are not less than 20 times larger each.

Thus, also, a toy boat built the way a trireme is built will be a fine toy boat. And a trireme built the way a toy boat is built will be a fine toy boat as well. Never mind an aircraft carrier built the way a toy boat is built.

And as quantitatively so qualitatively.

The state of the great kings explored the earth. From the northernmost shores of the Black Sea to the southernmost shores of the Red Sea, across 33° of latitude, and from the strait of Gibraltar to the mouth of the Indus, across 73° of longitude. The state of the great kings conquered most of this domain. In 3 generations, under Cyrus, Cambyses, and Darius. The state of the great kings administered the domain it conquered. From the Colchians in the north to the Ethiopians in the south, 3 200 km apart, and from the Greek city of Barca in the west to the Indians in the east, 4 900 km apart. The state of the great kings built. Herodotus approves: the royal road is safe along its entire length. The royal road was 2 700 km long, safe, and maintained safe, along its entire length. The state of the great kings mixed humanity. Plato disapproves: had the Persians won the war, we would have been all mixed together, Greek with Greek, Greek with barbarian, barbarian with Greek, as we now see the peoples ruled by the Persians all mixed together. 77 large peoples, and several times this number of small peoples. Darius approves: the peoples were in commotion, all peoples fighting each other; I brought an end to the fighting; they no longer fight each other at all, all are at peace; they fear the law; the strong do not fight or destroy the weak.

In Plato and Aristotle, on the other hand, the state does not explore the earth, does not conquer countries, does not administer peoples, does not build roads, does not mix humanity.

Plato's main concerns in the *Republic* are four.

Who should rule. In books VIII-IX Plato explains that in states one man rules, or a few men rule, or all men rule. States are thus tyrannies or kingdoms, aristocracies or oligarchies, or democracies, and such as they are they are all

bad. It follows, the philosopher should rule: until the philosophers become kings or the kings become true philosophers, the states will have no respite from evil. It is the main argument of the treatise, in the center of Plato's concerns throughout, books I through X.

Who should do what. Other than the philosophers, who should rule, the guardians should guard, the producers should produce. It is the foundation of the main argument, in the center of Plato's concerns in books II-VII.

Who should own what. The division of all land and all chattel, including husbands, wives, and children, is a central concern of books IV-V.

And education. The education of the philosophers, the education of the guardians, the education of the children, and at great length, and more than once at great length, what kind of poetry should and should not be read to the children. Education is a central concern of Plato's in books II-VII and X.

The conclusion is unavoidable. Plato is not concerned with the state. Plato is concerned with the household.

Aristotle's *Politics* is one of the three most important political treatises. Because of the opening pages of book I.

Book II is the review. In books III-VI Aristotle deduces the state. In books VII-VIII he builds the best state.

The building of the best state begins: to decide which is the best state we must decide which is the best life; for what is the best for a man and what is the best for the state is the life of virtue. Aristotle then elaborates on the life of virtue (VII, 2-3). He then assigns size and site to the best state (4-6), and character to its citizens: they should be both intelligent and courageous (7). And he then turns to who should do what (8-9), who should own what (10-2), who should rule (13), and education (14-VIII, 8). Well over one half of the building of the best state is devoted to education.

The deduction of the state in book III begins with the definition of the state: a citizen is who shares in the deliberative or judicial government of the state; and the state is a collection of such a number of citizens as is self-sufficient for life. What the state is is what is the best for the state,

and what is the best for the state is what is the best for a man.

The deduction proceeds: what makes a state one and the same is the same constitution, whether the name stays the same or not, and whether the people remain the same or change entirely (3), and: constitution is the arrangement of magistracies and especially the highest (6). Aristotle has thus deduced Plato's result, the state is first and foremost about who should rule.

Another definition follows: government is for the common good (7). What is the best for the state is the life of virtue, and the state should be ruled accordingly, whether it is so ruled or not.

The result is deduced: the forms of the state are kingship, aristocracy, and democracy; their perversions are tyranny, oligarchy, and mob rule (*id.*). What the forms of the state are depends on who rules. Even if: how many rule is an accident (8); the true end of the state is virtue (9). Aristotle considers the various accidents of who rules (10-8). And concludes with who should rule: the best state is the state governed by the best; for, clearly, in the same way in which a man becomes good he will make the state good, whether the state is an aristocracy or a monarchy (19).

Books IV-VI are devoted to the accidents of who rules, kingship, aristocracy, and democracy, and their perversions, tyranny, oligarchy, and mob rule.

In books II-VIII of the *Politics* theory is subjugated to practice as it is nowhere else in Aristotle's treatises. The true cause of things is what is for the best. These books are from Aristotle's early years at the Academy, presumably.

And the same conclusion is unavoidable. In these books Aristotle is not concerned with the state. He is concerned with the household.

Hippodamus, conceiving the first best state in speech, is concerned with who should do what.

Phileas, conceiving the second best state, is concerned with who should own what.

In the *Republic* the search is for justice. The search for justice leads to the concern with the state. The concern is then with who should do what, who should own what, education, and first and foremost who should rule.

In books II-VIII of the *Politics* the search is for the best state. The search for the best state

leads to the concern with the state. The concern is then with who should do what, who should own what, education, and first and foremost who should rule. In the review in book II as well, Aristotle's criticism of Plato's two best states is devoted to the single matter of whether the citizens should hold all their property, and especially their wives and children, in common (2-6). Of all that our predecessors have said about the state nothing demands more attention than who should own what.

And if all this is not enough to show that in both Plato and Aristotle the concern is with the household, and with the state only to the degree to which it is like the household, the following cannot not be enough. In both Plato and Aristotle the concern is in its entirety management of relations which are, implicitly or explicitly, the relations of master and slave, husband and wife, and father and children.

And book I of the *Politics*, where theory is not subjugated to practice, is explicit.

Aristotle begins in book I: whether it is the state or anything else, things are considered with the greatest clarity when they are considered from their origin and first growth; and first there has to be a union of those that cannot exist without one another, male and female, for in common with animals and plants men by nature strive to leave behind their progeny, and master and slave, for that which foresees by reason is master by nature and that which acts on the foresight by the body is slave by nature; from these two relations the family arises by nature to supply man's daily needs; when several families unite, and the union aims at more than the supply of daily needs, the first community is formed in the village; and when several villages unite in a single community large enough to be more or less self-sufficient, the state has come into being, originating in the bare needs of life, and continuing for the sake of the good life; it is thus evident that the state is by nature, and man is a political animal by nature.

He notes: it is evident also that man is a political animal more than bees and other gregarious animals; for he is the only animal that has speech; speech is for the sake of establishing what is useful and what is useless, and hence also what is just and what is unjust; man alone has a sense of

good and bad, just and unjust, and the like, and it is the association of animals with this sense that makes a family and a state.

And continues: we should begin the examination of everything with the smallest number of possible elements; and the state is composed of households; and the first and smallest number of possible elements of the household are master and slave, husband and wife, and father and children.

The rest of book I, and its bulk, is devoted to the management of the property of the household (4-12). And Aristotle concludes with the virtues of the master and the slave, of the husband and the wife, and of the father and the children (13).

The investigation of the state begins with book I of the *Politics*. Aristotle discovered that to explain the state is to begin with what is first by nature, to end with the state, and to bridge the gap between what is first by nature and the state. It is a great advance.

Aristotle discovered all three problems, but he did not solve any of the three very well.

First by nature is an animal. A political animal, however. An animal with a sense of just and unjust.

A citizen is who shares in the deliberative or judicial government of the state, and the state is a collection of such citizens as is self-sufficient for life. Hence, the state may have originated in the bare needs of life, but it continues for the sake of the good life.

And to bridge the gap between what is first by nature and the state there is nothing but the household. Master and slave, husband and wife, and father and children.

Hobbes improved on the solutions of two of the three problems.

First by nature there is no industry, no knowledge of the earth, no account of time, no arts, no letters, no society. Nature has made men equal, in body and in mind. Life is but a motion of limbs. For what is the heart but a spring, and the nerves but so many springs, and the joints but so many wheels giving motion to the whole body?

First by nature is an animal. But not an animal with a sense of just and unjust. An animal which is but a motion of limbs.

The state is a power contrary to our natural

passions, a power keeping men in awe, a power reducing the wills of men unto one will.

And this result of Hobbes's is conclusive. Which the fact shows that, while Aristotle's definition of the state will not accommodate even Plato's toy boat, Hobbes's definition will accommodate even the Persian aircraft carrier.

Rousseau improved on the solutions of two of the three problems.

Savage man, left by nature to instinct alone, begins with purely animal functions.

This result of Rousseau's is also conclusive. Which Darwin shows. First by nature is an animal left to instinct alone.

The problem is to bridge the gap between an animal left to instinct alone and a power contrary to our natural passions, keeping men in awe, reducing the wills of men unto one will.

As the human race spread, difficulties multiplied, differences in soils, climates, seasons, barren years, hard winters, hot summers, and in response to these difficulties metallurgy and agriculture were the two arts whose invention produced the great revolution of property, labor, and the first rules of justice.

And this result of Rousseau's is also conclusive. Which Çatal Hüyük shows. Discovered by Mellaart in 1958, excavated by him beginning in 1961, Çatal Hüyük attests, both, the oldest city and metallurgy and agriculture. To its lowest level Mellaart assigns the C-14 date 6486 ± 102^{BC} (Çatal, 3).

The problem is to bridge two gaps now, and neither of them quite as wide. From an animal left to instinct alone to metallurgy and agriculture, and from metallurgy and agriculture to a power contrary to our natural passions, keeping men in awe, reducing the wills of men unto one will.

And this problem remains to be solved.

The beginning of the investigation of the state with Aristotle is a great advance. But not nearly as great as the advance in the investigation of the earth or the advance in the investigation of the soul. The investigation of the state has advanced since Aristotle as well. But again not even remotely as much as the investigation of the earth or the investigation of the soul. Rousseau is contemporary with Linnaeus, Euler, and Buffon. And the advance in the investigation of the earth

and the investigation of the soul through Linnaeus, Euler, and Buffon, and since Linnaeus, Euler, and Buffon has been presented.

The investigation of the state has not advanced further than the investigation of the earth and of the soul had with Anaxagoras and Empedocles.

The reason is that the philosophers have continued to follow the false way, to subjugate politics to ethics, and to conceive the best state in speech in place of the real state.

More particularly, the *petitio principii* of a political animal, a rational animal, a free animal, a conscious animal, will not go away. In Hobbes: the laws of nature are rules of reason. In Rousseau: man is free, in the consciousness of freedom the spirituality of the soul shows itself, and about spiritual acts the laws of mechanics explain nothing. In Hegel: the state is rational in itself, it is the reality of the ethical idea. In Marx: in the history of society the actors are endowed with consciousness, nothing happens without conscious purpose, without willed goal. The end of the state has accordingly remained what it is for, not the cause of its motion.

And if they have not remained as staunchly committed to the toy boat as Plato and Aristotle are, the philosophers have remained as staunchly committed to the household as Plato and Aristotle are. The investigation of the state has remained unwaveringly concerned with the state only to the degree to which it is like the household. Who should rule is the central concern of Polybius, Cicero, Augustine, al-Farabi, Aquinas, Marsilius, Occam, ibn Khaldun, Machiavelli, More, Bodin, Grotius, Hobbes, Spinoza, Locke, Montesquieu, Rousseau, Hamilton and Madison, Sieyès, Burke, Godwin, Hegel, Marx. Who should own what became with Locke once again as prominent a concern as it is in Plato. Education became with Rousseau once again as prominent a concern as it is in Plato. In Marx not only is all concern with who should own what, who should do what, education, and first and foremost who should rule, the investigation is in its entirety, and rather explicitly, management of the relation of master and slave, if not so much of the relations of husband and wife and father and children.

Spinoza writes: statesmen have written far more happily about politics than the philosophers.

Because statesmen have not tended to steer, or write about, the household. Or to vary the analogy, Montesquieu writes: it is ridiculous to decide the rights of kingdoms, nations, and the world by the same maxims by which the right of men over a drainpipe is decided (*Laws*, XXVI, 16).

And the appeal of the household can be explained.

It is only secondarily in that it is more readily managed than the state. It is first and foremost in that you set up your own household and then you manage it, or delude yourself that you manage it, as you should.

Neither Plato nor Aristotle ever distinguishes city and state. This would be encouraged by the fact of the many small and smaller contemporary city states. And it would be encouraged greatly by the fact of colonization, whose outcome the many small and smaller contemporary city states also are.

Colonization has been a means of man's harvesting his habitat, as it has been of all organisms', from the first and through the scramble for Africa in the late nineteenth Christian century and its consequences in the following century. Human colonization has greatly accelerated, and been in turn greatly accelerated by, the development of boats and navigation. Hence, the largest waves have been two. Exploration of the surface of the earth resumed with prince Henry and the rounding of cape Bojador by Eanes in 1434. In 1492 Columbus crossed the Atlantic. Then the second of the two largest waves began and continued for four-odd centuries.

The first began twenty-three centuries earlier. Hesiod sings in the *Work and days*:

I have never been on the open sea,
I have sailed but once, from Aulis to Euboea
(650-1).

Clearly, having never been on the open sea, and sailed but once, to make the .2 km voyage from Aulis to Euboea, Hesiod is in an excellent position to offer advice on sailing. And so he does. After much advice on planting and harvesting, and before much advice on marriage, he offers much advice on sailing (618-94). With good reason. The first of the two largest waves of human colonization, and relative to population

the larger of the two, had begun one century earlier.

A colony of Tyre, Carthage was founded in 814 (Timaeus, fr. 60). The Phoenician cities continued sending out colonies, a couple of generations later the Greek cities began sending out colonies, the Phoenician cities continued sending out colonies, the Greek cities continued sending out colonies, the colonies began sending out colonies, and the colonies of the colonies began sending out colonies. The littoral was colonized from the Tyrian colony of Gades beyond the strait of Gibraltar in the west to the Milesian colony of Phasis on the eastern shore of the Black Sea in the east, 4 100 km along the great circle. Plato writes: we live around the sea from Phasis to the strait of Gibraltar like ants or frogs around a swamp (*Phaedo*, 109b).

By the time of Plato and Aristotle, four and a half centuries after the foundation of Carthage, the wave had abated. But the prospect of a new beginning which a colony offers, or seems to offer, had lost none of its allure. Not for the philosophers.

Hippodamus discovered the art of planning cities. What he proposed, Phaleas said, can easily be achieved in a new city from the beginning. Plato writes in the *Laws*: our mandate is to frame the laws of a new colony (III, 702a-d). And Aristotle's mandate in the *Politics* is not only to frame the laws of the best state, but to choose a site for it, choose a population for it, and to build its houses, and to adorn the houses, too (VII, 11). Needless to add, to manage also the drainpipes (*id.*). Moreover: it is best to lay out the streets in the manner of Hippodamus (*id.*).

Hence the delusion that the distance from the city in speech to the city in fact is the shortest of distances. Certainly it cannot be greater than the .2 km separating Aulis from Euboea.

One particular manifestation of the delusion is the older supposition that the state begins with a lawgiver. Another particular manifestation is the more recent supposition that the state begins with a covenant, in Hobbes (*Leviathan*, 14), with a compact, in Locke (*Government*, II, 8), with a contract, in Rousseau (*Contract*, I, 6).

The general manifestation is the supposition that a philosopher writes the best state, then all it

takes to practice it is will.

In 444 Thuri was founded. Pericles sent out the colonists (Plutarch, *Pericles*, 11). Hippodamus laid out the city (Hesychius, *s. v.*). Protagoras drew its laws (Laertius, IX, 50). Herodotus (Strabo, XIV, 2.16) and Lysias (Dionysius of Halicarnassus, *Orators*, *Lysias*, 1) were among the colonists. Empedocles came to live in the new city (Laertius, VIII, 52).

Then this new city of philosophers did what every other tiny city did. It fought the aboriginal population. It flourished, plundering its neighbors. It declined, plundered by its neighbors. It was overrun by the wars of the real states, first by the war of the state of Pyrrhus and the state of the Romans, then completely by the war of the state of the Carthaginians and the state of the Romans. In 193 the Roman senate sent out its own colony to settle on its site (Livy, XXXIV, 53).

In 1787, Alexander Hamilton writes: it seems to have been reserved for the people of this country to decide the important question whether societies of men are capable of establishing good government from reflection and choice.

Then the new state of good government established from reflection and choice did what the state of the great kings did twenty-three and a half centuries earlier, and what the state of Sargon and his successors did eighteen centuries earlier still. The new state of good government established from reflection and choice explored the continent, conquered much of it, administered the peoples, built, mixed humanity.

In 1920, Lenin writes: the proletariat has won political power in Russia (*Leftism*, 1); what has happened among us has the significance of historical inevitability for all countries (*id.*); Marxism is the only true revolutionary theory (2).

Then the new state of the proletariat and the only true revolutionary theory did what the state of Sargon and his successors did forty-two and a half centuries earlier. What its immediate predecessor, the truly reactionary state of the tsars, did, too. The new state of the proletariat and the only true revolutionary theory conquered, administered peoples, built, mixed humanity. There was by that time no surface of the earth left to explore. The new state of the proletariat and the only true revolutionary theory began exploring the space

around the earth. As did the new state of good government established from reflection and choice. Both for reasons which are very difficult to distinguish from Sargon's reason when he began exploring the valley of the two rivers.

The distance from the city in speech to the city in fact is greater. Say, if Aristotle covered the first .3 km, Hobbes covered the next .4 km, and Rousseau covered the next .2 km, the distance cannot be less than 1 000 km. Then, after the remaining not less than 999.1 km are covered, and the site is thus reached, building better may well be viable.

To borrow Hobbes's analogy, too, can you pull the leviathan by a fishhook?

For Tiglath-pileser, prince 1115-076, and a predecessor of Tiglath-pileser III, did pull it.

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There is, then, the political problem of the best state, of the states conceived in speech, and the best of them.

There is also the historical problem of the best state, of the real states, and the best of them.

And the attempt to collect the particulars and marshal them in their series and order with respect to this problem can also begin with Aristotle's review.

After the states conceived in speech Aristotle turns to the real states. He criticizes the constitutions of the Spartans (9), of the Cretans (10), and of the Carthaginians (11), appends a list of early lawgivers (12), and that is that.

It is not a successful review.

Aristotle writes: none of the constitutions of existing states is good.

None good, all bad.

Still, they are not all equally bad. The least bad are the constitutions of the Spartans, the Cretans, and the Carthaginians, and Aristotle discusses the respective household arrangements. His discussion of the Carthaginian constitution is not burdened with any particulars at all. His discussion of the Spartan constitution is burdened with 1 particular, Lycurgus. His discussion of the Cretan constitution also with 1 particular, Minos. Neither of whom, Minos or Lycurgus, can be burdened with the presumption of reality.

It is representative. Particularity is sparse in

the review, and severely beset by the difficulty, is it real? is it not?

All in all, Aristotle notes 25 particulars.

3 are the sources of states conceived in speech.

Among the remaining 22, there is the poet, quoted here as everywhere (7):

alike in honor share the bad and the good.

There is the father of Hippodamus: Euryphon (8).

There is an anecdote: Autophradates was about to besiege Atarneus; Eubulus, its ruler, told him, consider what the siege will cost you, for I will leave the city to you at once for less; Autophradates pondered, and desisted from the siege (*id.*).

The review ends with a list of early lawgivers.

Among them is: Philolaus of Corinth, who gave laws to the Thebans, whose lover was the Olympic victor Diocles, whose mother was Halcyone, who conceived an incestuous passion for him (12). Aristotle notes one household arrangement of Philolaus's: he gave the Thebans laws for the procreation of children (*id.*).

Among them is also: Androdamas of Rhegium who gave laws to the Chalcidians (*id.*). Aristotle notes two household arrangements of his: he gave the Chalcidians laws for murder and for heiresses (*id.*).

There is also what some say: some say that Onomacritus was the first to have a special skill in lawgiving, that although a Locrian by birth, he was trained in Crete and practiced prophecy there, Thales became his companion, and Lycurgus and Zaleucus were students of Thales, and Charondas was a student of Zaleucus; but all this confuses the times of these men (*id.*).

Euryphon, Autophradates, Eubulus, Philolaus of Corinth, Diocles, Halcyone, and Androdamas left no trace in the record of anything more substantial than what Aristotle reports. All 7 can be discarded out of hand.

Onomacritus left behind more. Herodotus reports: he was banished from Athens when he was caught inserting his own oracle among the oracles of Musaeus. Empiricus reports: he says in the Orphic books that earth, water, and fire are the principles. Onomacritus's special skill in lawgiving is as important, or real, as his solution of the material principles of all things.

The poet and the state need not be burdened

with one another.

The first philosopher and the state need not be confused with one another.

This leaves 12.

Among them is: king Charillus, whose guardian Lycurgus was (10).

Among them is also: Pheidon of Corinth, one of the most ancient lawgivers, who said that the number of families in the city should stay the same (6). Pheidon of Corinth left no other trace in the record. Pheidon of Argos, however, is well attested, including by Aristotle himself later in the *Politics*: Pheidon of Argos was at first a king, then he became a tyrant (V, 10). It can be presumed, given all the other errors in the review, that Pheidon of Corinth is Pheidon, one of the ancient lawgivers, the king of Argos.

Arranged, then, by state, beginning with Crete and Sparta, then in the chronological order of the first particular, the 12 are,

Crete	Minos,
Sparta	Lycurgus, Charillus,
Argos	Pheidon,
Locri	Zaleucus,
Athens	Draco, Solon, Ephialtes, Pericles, Diophantus,
Mytilene	Pittacus,
Catana	Charondas.

8 are important, Pheidon, Zaleucus, Draco, Pittacus, Solon, Charondas, Ephialtes, and Pericles. 2 are not important, Charillus and Diophantus. 2 would have been important, had they been real, Minos and Lycurgus.

Thus Aristotle's review.

Next, instead of beginning with the household arrangements, the particulars can be collected, and hence, on this ground, from the relations of the particulars, the historical problem can be approached, the problem of the real states, and the best of them. Including the problem of what its household arrangements have been.

The evidence is mountainous. Narmer has left behind a palette and a macehead. Three centuries later, Enmebaragesi has left behind an inscription. His successors have left behind more inscriptions, and steadily more. Six and a half centuries after Enmebaragesi, there is the Sumerian prince list. There are then more prince lists. The inscriptions of the princes mount. Fifteen and a half centuries

after the Sumerian prince list, Herodotus notes 804 predecessors of his, one generation later Thucydides notes more, one generation later Xenophon notes more, and by this time, the time of Plato, the number of historians is itself becoming mountainous. One generation later, by the time of Aristotle, it is mountainous. There are very many particulars.

The collection can begin anywhere, the particulars are in relation to one another, and will sort themselves out in any case. In particular, given Aristotle's review, the collection can begin with the states in the review. Because the particulars in the review include early lawgivers, and the supposition of dependence of the state on a lawgiver is general enough to be shared by the Hebrew scriptures. Because the least bad states are Crete, Sparta, and Carthage, and, excepting Carthage – he does not have much use for barbarians – Plato agrees. He writes: most people praise the constitutions of Crete and Sparta (*Republic*, VIII, 544c), and agrees with most people at least in that: philosophy had its first beginning among the Cretans and the Spartans, and the Spartans have the best education in philosophy, to be a Spartan is to be a philosopher. And because Lycurgus, Charondas, and Solon are in the review. Until the philosophers become kings, the states will have no respite from evil. Nevertheless: is any city governed well because of the laws which Homer gave it, as Sparta is because of the laws of Lycurgus, Sicily because of the laws of Charondas, Athens because of the laws of Solon?

Beginning with the states in the review, then, and aiming at completeness within the limits

Tiglath-pileser III prince 745-27
to

Alexander prince 336-23,
below are the particulars, and all important among them, who are part of the development of the state in the four-odd centuries from Tiglath-pileser III to Alexander.

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Crete.

Crete is the fifth largest island in the Mediterranean, southeast of the Peloponnese.

In Homer, in the *Iliad*, the Cretan is one of the four largest contingents in the Achaean army.

Idomeneus:

the king of Crete of the 100 cities (II, 645-52),
leads 80 ships (*id.*). Idomeneus reports:

I am the seed of Zeus;
Zeus sired Minos, who ruled the people,
and Minos sired Deucalion, who sired me
to rule the many people of large Crete
(XIII, 449-53).

In the *Odyssey*, Odysseus reports:

in the middle of the sea there is a land called
Crete;
and there is Cnossus there, the great city,
where Minos was the king and spoke to Zeus
(XIX, 172-9).

Odysseus reports also what he saw in the underworld:

I saw Minos, the son of Zeus, seating on a throne,
holding a golden scepter, judging the dead
(XI, 568-70).

In Herodotus there is some detail on Minos. Herodotus writes: there was a quarrel between the sons of Europa, Minos and Sarpedon; Minos won and drove Sarpedon out of Crete (I, 173); Minos vanquished the Carians; he was very successful in war and conquered much land (171); Polycrates first among the Greeks tried to master the sea; except for Minos and others before him who were lords of the sea; Polycrates was the first mortal to try to do so (III, 122); Minos went to Sicily to look for Daedalus, and died a violent death there (VII, 170). Other than that, in the couple of centuries of events in the *Histories*, involving some hundreds of states, Crete is very difficult to find. In 480, Herodotus reports: the Cretans refused to help the Greeks against the barbarians (169), and that is that.

In Thucydides, Minos is still there. Thucydides writes: the story is that Minos was the first to build a navy, he controlled a large part of the Aegean, founded colonies on the Cyclades, and set up his sons to rule them after he drove out the Carians (I, 4). Other than that, in the two-odd decades of events of the great war in the *History*, involving some hundreds of states, Crete is again very difficult to find. Once the Athenians plunder the Cretan shore (II, 85). In 415, in the Sicilian expedition of the Athenians, Nicias leads 134 triremes and 5 100 infantrymen, supported by light troops, among whom are 700 slingers and

480 archers, among whom are 80 archers from Crete (VI, 43). And that is that.

In Xenophon, there is nothing about Minos. As to Crete, in the five decades of events in the *Greek history*, involving some hundreds of states, in 394, the Spartan king Agesilaus leads an army of 6 000 infantrymen, among the support troops are 300 Cretan archers (IV, 2), and that is that.

As among the Sumerians, then, so among the Greeks princship descended from heaven. Among the Sumerians, it descended to Eridu, Alulim was prince, he reigned 28 800 years. Among the Greeks, it descended to Crete, Minos was prince, he became the judge of the dead. If conclusions will be drawn about the state as it descended from heaven, what the Sumerian princes did was smite one another. What Minos did was smite the Carians, build a navy, and control a large part of the Aegean.

As to the states in fact, it can be concluded that, whatever the household arrangements of the Cretans were, they produced archers. Not too many, though, and to the end, clearly, of supporting the armies of the Athenians and the Spartans.

Plato begins in the *Laws*: the Athenian asks, tell me, sir, who established your laws, a god, or a man? the Cretan replies, a god, good sir, a god (I, 624a).

As to the states in fact, the development of the state in Crete – inclusive of all the states of all the cities on the fifth largest island in the Mediterranean – has not left behind in the record a single particular of any importance whatsoever, not 1. The Cretan state is not important at all. The numbers in Thucydides and Xenophon can be taken as indicative, the Cretan state is not less than 20 times, and closer to 100 times, less important than the Spartan state and the Athenian state.

Sparta.

There is much about Sparta in Homer. Menelaus is the king of the Spartans, Helen is his wife, and both are prominent in both the *Iliad* and *Odyssey*. Although Lycurgus is not one of the Homeric heroes.

There is no reason to look for Sparta in Homer, however. The state of Sparta, unlike the state of Crete, is not difficult to find in the histo-

rians. The state of Sparta won battle after battle, war after war, and dominates the events in Herodotus, Thucydides, and Xenophon.

As there is the Sumerian prince list, so there is the Spartan prince list. Sparta had dual kings, the Agiads and the Eurypontids. In 480, Leonidas led the army which met the army of Xerxes at Thermopylae. Herodotus reports: Leonidas, he was the son of Anaxandridas, the son of Leon, the son of Eurycratidas, the son of Anaxandrus, the son of Eurycrates, the son of Polydorus, the son of Alcamenes, the son of Teleclus, the son of Archelaus, the son of Agesilaus, the son of Dorussus, the son of Leobotes, the son of Echestratus, the son of Agis, the son of Eurysthenes, the son of Aristodemus, the son of Aristomachus, the son of Cleodaeus, the son of Hyllus, the son of Heracles (VII, 204). In 479, Leotychidas led the army and navy which defeated conclusively the army and navy of Xerxes at Mycale. Herodotus reports: Leotychidas, he was the son of Menares, the son of Hegesilaus, the son of Hippocratides, the son of Leotychidas, the son of Anaxilas, the son of Archidamus, the son of Anaxandridas, the son of Theopompus, the son of Nicandrus, the son of Charillus, the son of Eunomus, the son of Polydectes, the son of Prytanis, the son of Eurypon, the son of Procles, the son of Aristodemus, the son of Aristomachus, the son of Cleodaeus, the son of Hyllus, the son of Heracles (VIII, 131). The Agiad Leonidas succeeded his brother Cleomenes. The Eurypontid Leotychidas succeeded his cousin Demaratus, the son of Ariston, the son of Agasicles, the son of Hippocratides.

And as the princes in the Sumerian prince list, so the ancestors of Leonidas and Leotychidas are of three kinds.

Heracles, his son Hyllus, his son Cleodaeus, his son Aristomachus, his son Aristodemus, and his sons Eurysthenes and Procles need not be burdened with the presumption of reality.

Beginning with the founders of the royal houses, Agis and Eurypon, the reality of the princes need not be doubted. Most of them, though – the Agiads Agis, Echestratus, Leobotes, Dorussus, Agesilaus, Archelaus, Teleclus, Alcamenes, Polydorus, Eurycrates, Anaxandrus, Eurycratidas, and Leon, the Eurypontids Eurypon, Prytanis, Polydectes, Eunomus, Charillus,

Nicandrus, Anaxandridas, Archidamus, Anaxilas, Leotychidas, Hippocratides, Agasicles, and Demaratus, as well as the not royal ancestors of Leotychidas Hegesilaus and Menares – are not important, most of them very not important.

Important are,

Theopompus	prince c. 710,
Anaxandridas	prince c. 560-20,
Ariston	prince c. 550-15,
Cleomenes	prince c. 520-490,

and when to these ancestors of Leonidas and Leotychidas,

Chilon ephor 556,

is added, these are all important particulars who are part of the development of the state of Sparta through Cleomenes.

As Enmebaragesi is the first prince in the Sumerian prince list to leave behind his inscription, so Theopompus is the first prince in the Spartan prince list to leave behind his deeds. Before Theopompus the city of Sparta dominated the surrounding country of Laconia. Under Theopompus it conquered Messenia to the west. Tyrtaeus sings:

our king Theopompus, dear to the gods, won
Messenia,
Messenia good for ploughing, good for planting,
for Messenia nineteen years incessantly fought
the fathers of our fathers (fr. 5).

The people of Laconia were already helots. With the conquest of Messenia good for ploughing, good for planting, its people became helots as well. The helots were, as Aristotle describes them: the producers of the Spartans (*Politics*, II, 9), or: the slaves of the Spartans (10).

One and a half centuries separate Theopompus from Anaxandridas and Ariston. In this time, in 669, Argos defeated Sparta at Hysiae. The rise of Sparta resumed under Anaxandridas and Ariston. Anaxandridas and Ariston conquered Arcadia to the north. In the next generation, Cleomenes defeated Argos, further north, intervened repeatedly in Athens, still further north, to remove and to install its rulers, campaigned in Boeotia, still further north. It is under Cleomenes that Sparta became the mistress of the entire Peloponnese, and the dominant power among the Greek states.

And before continuing with the particulars, it can be shown that Lycurgus, even if he is not one

of the Homeric heroes, cannot be burdened with the presumption of reality.

Singing of our king Theopompus, under whom the fathers of our fathers for nineteen years incessantly fought, Tyrtaeus sings also of this:

having heard the voice of Apollo,
they brought home the oracles of the god;
council should come from the kings
and the elders; according to their decrees,
the people should say and do what is right;
victory and power will then attend the people;
this is what Apollo said to the city (fr. 4).

Two generations after Theopompus, the laws of Sparta came from Apollo.

Two centuries later, they came from Lycurgus. Herodotus reports: the Spartans say that their laws were very bad at first; but then Lycurgus brought a new constitution from Crete; this was at the time when Leobotes was king, Lycurgus became his guardian, and changed all the laws; this is how their laws became good (I, 65-6). And this is how Lycurgus enters the record, two generations after Cleomenes, two and a half-odd centuries after Theopompus. Lycurgus brings a new constitution from Crete, becomes the guardian of Leobotes, four generations before Theopompus – or, according to Aristotle in the review, the guardian of Charillus, two generations before Theopompus – changes all the laws, and the laws become good.

This is what the Spartans said. It proved irresistible. In the *Spartan constitution*, Xenophon begins: I wondered why Sparta, although the least populous, is the most powerful of all Greek states; then I considered its constitution, and I wondered no longer; for Lycurgus gave the Spartans their laws, and he was the wisest (1). Plato writes: Sparta is governed well because of the laws of Lycurgus; and the Spartans have the best education in philosophy, to be a Spartan is to be a philosopher. Plutarch writes: for those who doubt that it is possible, Lycurgus showed that a whole city can practice philosophy (*Lycurgus*, 31).

That at first Apollo sufficed to give Sparta its laws suffices to preclude the possibility that the lawgiving of Lycurgus is real. Moreover, to judge by the facts, first victories and power came. Then the need to explain the good laws which brought

victory and power came. Then the need to ascribe the good laws to wisdom and philosophy came. The name of the fallacy is ὕστερον πρότερον, the posterior prior.

Apollo said to the city: council should come from the kings and the elders. In particular, it came from the two kings and five ephors. Aristotle reports: Theopompus established the ephorate (*Politics*, V, 11). Laertius reports: Chilon first yoked the ephors alongside the kings (I, 68). In 491 the Eurypontid Demaratus was deposed. He became an advisor of Xerxes. After Thermopylae he advised the great king to blockade the Peloponnese with his navy. He told the king: Chilon, the wisest among us, said that the Spartans should rather drown at sea than float on it. Following the martial wisdom of the wisest among us, the Spartans did not build a navy, and remained invincible on land for two centuries.

Continuing with the collection, these are all important particulars who are part of the development of the state of Sparta after Cleomenes and through the death of Alexander in 323,

Leotychidas	prince 491-69,
Leonidas	prince 490-80,
Pausanias	general in 479,
Archidamus	prince 469-27,
Brasidas	general in 424,
Agis	prince 427-00,
Gylippus	general in 414,
Lysander	general in 408,
Agésilas	prince 400-360.

Leonidas, Pausanias, and Leotychidas won the Persian war. In 480, Leonidas fell at Thermopylae. In 479, Pausanias won at Plataea, and Leotychidas won at Mycale. The city of 8 000 men vanquished the great king, the lord of one fifth of humanity. After the Persian war Athens rose, and the great war came, the war of Athens and Sparta, the Peloponnesian war. Archidamus, Brasidas, Agis, Gylippus, and Lysander won this war. Beginning in 431, Archidamus confined Athens on land to its walls. But Athens was the mistress of the sea. Beginning in 424, Brasidas won the north Aegean coast. In 415, Athens embarked on the expedition to conquer Sicily. The Syracusans asked Sparta for help. Sparta sent Gylippus to command them, and Gylippus vanquished the Athenian expedition. Beginning in

413, Agis resumed the land siege of Athens. Beginning in 408, Lysander won the coast of Asia Minor, won control of the Aegean, and in 404 forced the surrender of Athens.

Victory and power drew Sparta steadily further afield. After the Persian war, Pausanias campaigned in Asia Minor. After the Peloponnesian war, Lysander campaigned further. Agésilas campaigned further still.

In 394 Agésilas defeated Thebes at Coronea. In 371 at Leuctra, however, and in 362 at Mantinea, Thebes, under Epaminondas, vanquished Sparta. Epaminondas had discovered tactics. Sparta lost even Messenia, first won by Theopompus three and a half centuries earlier.

Sparta, then, is a very important state. Because of, with Tyrtaeus: victory and power. Victory after victory, the rise of Sparta continued for three and a half centuries.

Its power reached a peak in the victory in the Persian war. Sparta stopped the expansion of the domain of the great kings, and thus the mixing of peoples, Greek with Greek, Greek with barbarian, barbarian with Greek, and the consequences of this fact have affected humanity ever since.

Its power reached a higher peak in the victory in the Peloponnesian war. Thucydides writes: it was a great war; the greatest ever among the Greeks, it affected a large part of the barbarians as well, and indeed all of humanity (I, 1). The Peloponnesian was not a war of one battle, or two, one campaign, or two, fought in one place, or two, in one year, or two. The Peloponnesian was the first war of many dozens of battles, many dozens of campaigns, fought on land and on sea, from the Peloponnese to Thrace to Sicily to Asia Minor, over twenty-seven years. And the consequences of this fact are larger.

That this is what the real state, Sparta, is, victory and power, negatively the fact shows that, even if the question, real? or not? is ignored, but 1 lawgiver pondering household arrangements can be recovered from the record. Affirmatively the fact shows it that of the 14 important particulars 13 are victorious princes and generals and 1 is the wisest among us with his martial wisdom.

It has to be concluded that the Spartan constitution is not about who rules, who owns what,

who does what, and education. It is about victory and power.

Victory and power are won. This did not have to be discovered. That victory and power will maintain the men who win victory and power did not have to be discovered either. That the men who will win victory and power can be maintained so that they win victory and power Sargon discovered: 5 400 ate bread daily before Sargon, and: Sargon won 34 battles. That a population of helots, or producers, or slaves, will maintain the men who will win victory and power for three and a half centuries Sparta discovered. And the consequences of this fact are yet larger.

Victory and power are won in battles. Battles are won by numbers, and Sargon discovered this, too. Eighteen centuries later, the great kings still won their battles with numbers.

Sparta discovered that determination can vanquish numbers. And this fourfold.

First, for Messenia nineteen years the fathers of our fathers incessantly fought.

Second, Leonidas led 300 Spartans in the doomed defense of Thermopylae. 298, including Leonidas, fell. Of the 2 who returned home: one hanged himself in shame; to the other, called the coward, no one spoke (Herodotus, VII, 231-2).

Third, Pausanias led 5 000 Spartans at Plataea. Out of a city of 8 000 men.

Fourth, the 5 000 were accompanied by 7 helots each. After Thermopylae Xerxes asks Demaratus: how many are men like that? Demaratus replies: there are 8 000 men in Sparta and they are all men like that; the rest of the Lacedaemonians are not equal to them, but they are good men, too (234). Sparta levied a force of 35 000 good men from the population it ruled.

Wars are not just battles. And in the great war Sparta discovered that wars are won by lines of communication. It deprived Athens of the surrounding country, then deprived it of its bases in the north, then stopped its attempt to acquire new bases in the west, then deprived it of its bases in the east, then deprived it of its bases on the sea. Athens capitulated.

The state of Sparta is a step in the development of warfare. Who rules, who owns what, who does what, and education, are all parts of, and serve, the development of warfare.

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Argos.

The city of Argos is in the Peloponnese, north of Sparta.

In the *Iliad*, the king of Argos, Agamemnon, leads the Achaean army.

In fact, there is 1 important particular,

Pheidon prince c. 669.

Herodotus writes: Pheidon, the king of Argos, established the weights and measures of the Peloponnesians, and he showed the greatest insolence when he took the Olympic games from the Eleans and set them up himself (VI, 127). Strabo writes: Pheidon harassed all cities captured by the descendants of Heracles (VIII, 3.33). Pausanias of Magnesia writes: the Argives defeated the Spartans at Hysiae in 669 (II, 24.7). Hence, Pheidon defeated the Spartans at Hysiae.

Pheidon, who defeated the Spartans, and established the weights and measures of the Peloponnesians, is important. The state of Argos, whose development left no other particular of any importance in the record, is not important.

Locri.

The city of Locri is in Calabria, on its eastern coast.

There is again 1 important particular,

Zaleucus laws in 663.

Strabo writes: the first city to have written laws is believed to have been Locri; Ephorus reports that Zaleucus wrote its laws; before him judges decided the penalties in each case, Zaleucus first defined the penalties in the laws (VI, 1.8). Polybius reports a case decided on a law of Zaleucus: the quarrel of two young men over a slave (XII, 16).

Later on, ὕστερον πρότερον, Zaleucus became a student of Pythagoras (Siculus, XII, 20). Given the dates of the two, during one of the transmigrations of Pythagoras before he became Pythagoras. At the same time the laws of Zaleucus were equipped with a preamble: all citizens should believe that the gods exist (*id.*).

Zaleucus, who first committed the laws of a Greek city to writing, is important. The state of Locri is very not important.

Athens.

The important particulars are,

Draco	laws in 621,
Solon	laws in 594,
Pisistratus	tyrant 560-27,
Miltiades	archon 524,
Cleisthenes	laws in 508,
Themistocles	archon 493,
Cimon	leader after 479,
Ephialtes	leader after 465,
Pericles	leader after 463,
Cleon	leader after 431,
Nicias	leader after 424,
Alcibiades	leader after 420,
Demosthenes	leader after 354.

If a state would show that the state is about its household arrangements, and first of all about who should rule, it would be Athens. It was a turbulent state. In *The constitution of Athens* Aristotle counts 11 changes of its constitution (41). He writes in the *Politics*: constitution is the arrangement of magistracies and especially the highest. Athens changed the arrangement of its magistracies and especially the highest continuously for two and a half centuries. The changes also produced a democracy *par excellence*. And when Athens rose to rival the power of Sparta, it became a rivalry also of household arrangements, democracy against aristocracy.

Aristotle writes in *The constitution of Athens*: the first change of constitution was when Ion brought the people together in a city, divided them into 4 tribes, and set up kings to rule the tribes; Theseus then established a single kingship; the second was of Draco, who drew the first code of laws; the third was of Solon, when democracy began; the fourth was the tyranny of Pisistratus; the fifth was the constitution of Cleisthenes, which strengthened democracy; the sixth, after the Persian war, gave the rule to the Areopagus; the seventh was the constitution of Aristides, completed by Ephialtes, which overthrew the rule of the Areopagus; it was under this constitution that, under the sway of leaders of the people, the state committed the gravest errors in pursuit of mastery of the sea; the eighth was the oligarchy of the 400; the ninth restored democracy; the tenth was the tyranny of the 30; the eleventh overthrew the 30 and continues to this day; the people made themselves the masters of everything, ruling everything by their votes in the

popular assembly (41).

Ion and the Homeric hero Theseus need not be burdened with the presumption of reality.

Draco drew the first code of laws. In the review in the *Politics*, Aristotle writes: Draco accommodated his laws to the existing constitution, and there is nothing worth noting in them, except the severe punishments (II, 12). In *The constitution*, he writes: in 621 Draco enacted his laws; he gave the franchise to all men who armed themselves; the election of the 9 archons he gave to those who owned 100 minae or more and had legal children over 10 years of age; the Areopagus became the guardian of the laws (4). The Areopagus was the council of all former archons (3).

The beginning of democracy, then, can be traced back to Draco. To the precedents of changing the laws, and extending the franchise. In 594 Solon was appointed to change the laws, and extended the franchise. In 508 Cleisthenes was appointed to change the laws, and extended the franchise. Thereafter, to be part of Athenian politics was to vie for the leadership of the people, change the laws, and extend the franchise.

In 594 Solon: abolished debt bondage; established the right of claiming redress; and the right of appealing to a jury court (9). Also: he divided the people into four classes by property, five-hundred-bushel men, cavalrymen, infantrymen, and laborers; to the first three classes he gave all magistracies, to each class a number proportionate to the value of their property; to the fourth class he gave nothing but a place in the popular assembly and in the juries (7).

It is the radical step in the extension of the franchise. Draco gave the franchise to all men who armed themselves. Solon gave all magistracies to all men who armed themselves, the infantrymen, the cavalrymen, and the five-hundred-bushel men who equipped and maintained the ships.

Democracy fostered faction. In 560 the leader of a faction, Pisistratus, set himself up as a tyrant. Expelled by the opposite faction, Pisistratus returned, expelled again, he returned again in 546, and Athens remained a tyranny until 510. Aristotle writes: before becoming a tyrant, Pisistratus was known as an extreme democrat (14);

when he became a tyrant, his government was government by law rather than a tyranny; he was temperate in every respect; he gave money to the poor and helped them in their labors; he increased greatly the revenues of the city; this is why his tyranny is known as a golden age (16). Thucydides writes: Pisistratus and his sons ruled by the previously established laws (V, 54).

The laws of Solon established democracy. The tyranny of Pisistratus gave democracy permanence.

In 510 faction brought Cleomenes to Athens to expel the son of Pisistratus Hippias.

In 508 Cleisthenes was appointed to change the laws. Aristotle writes in *The constitution*: in order to mix the people and give the franchise to more of them, Cleisthenes divided the people into 10 new tribes in place of the 4 ancestral tribes; he divided the country into 30 municipalities, and assigned 3 of them to each tribe by lot; he enlarged the council to 500 members, each tribe contributing 50 (21). In the *Politics*, Aristotle adds: Cleisthenes enrolled in the tribes many foreigners, and both free and slaves (III, 2).

The laws of Cleisthenes established democracy *par excellence*. Residence, not arming oneself, became the qualification for citizenship. All magistracies were opened to all citizens. Foreigners, and both free and slaves, became citizens.

Thereafter, to be the leader of Athens was to be the leader of the people, to be the leader of the people was to be the leader of Athens. And to be the leader of the people was not to hold office, it was to hold sway in the popular assembly.

Thus, a democracy *par excellence*, Athens kept becoming more democratic. It became the law that: the generals leading the army be elected (22). It became the law that: the crews of the triremes be paid (23). It became the law that: service in the courts be paid (24). In 462 Ephialtes changed the law: he stripped the Areopagus of its powers, and divided them between the popular assembly, the council of 500, and the courts (25). By this time it had become the law that: 20 000 people be maintained by the state (24). A coveted benefit for some time, Athenian citizenship became too coveted a benefit. In 451 Pericles changed the law: to the franchise were admitted only those born to both a citizen father and a

natively born mother (26).

Athens, then, was a democracy *par excellence*. Whether Sparta was a monarchy or an aristocracy is harder to decide. Either way, Athens, the democracy *par excellence*, did what Sparta, and whether it was a monarchy or an aristocracy, did.

The rise of Athens to victory and power began under Solon. Athens won Salamis from Megara. A general in the war, Solon himself sings:

let us go to Salamis,

let us fight for the lovely island (fr. 3).

Salamis is off the coast of Athens. Pisistratus captured Naxos, in the middle of the Aegean, and Sigeum on the eastern Aegean coast. Miltiades was archon in 524. In the following years he established Athenian control in the Chersonesus, and extended it along the northern Aegean coast. The north was the main source of timber, ships need timber, and Athens now controlled the timber. Themistocles was archon in 493. In that year he began the expansion and fortification of the harbor at Piraeus. In 490 Darius sent a force to conquer the Greek cities. At Marathon, under Miltiades, the Athenians vanquished the Persians. In 483, Aristotle reports in *The constitution*: the silver mines in Thrace were discovered, and the state made a profit of 100 talents; some wanted to divide the money among the people; Themistocles took the money, and had 100 triremes built (22). Athens became the greatest naval force on earth. In 480 Xerxes invaded Greece. At Salamis, under Themistocles, the Athenian navy vanquished the Persian navy. After the Persian war Sparta was the mistress of the land. Athens was the mistress of the sea. Victory and power led to victory and power. After 479 Cimon was the leader of the people. Athens was fortified by walls. It fought Corinth, campaigned in Boeotia, and transformed the Aegean into an Athenian sea. In 478 it brought the cities on the islands and along the littoral into an alliance. Thucydides writes: the Athenians decided which city will contribute money, which city will contribute ships (I, 96). Some 200 cities were brought into the alliance. After 463 Pericles was the leader of the people. In 454 he transferred the treasury of the alliance from the sacred island of Delos to the secular city of Athens. Aristotle writes in *The constitution*: the money collected from the allies

maintained 20 000 people; there were 6 000 jurymen, 1 600 archers, 1 200 cavalymen, 500 members of the council, 500 guards of the docks, 50 guards of the Acropolis, 700 magistrates at home, 700 magistrates abroad, 2 500 infantrymen, 20 guard ships, other ships collecting the tribute, crews amounting to 2 000 men, indigent people, orphans, jailers, and others; it was in this way that the people earned their livelihood (24-5). The Spartans had helots. The Athenians had allies.

In 431 the great war began. Athens entered the war: collecting 600 talents annually from its allies, with 6 000 talents in its treasury, 16 000 infantrymen in garrisons abroad, 13 000 infantrymen guarding the city, 1 200 cavalymen, 1 600 archers, and 300 triremes in active service (Thucydides, II, 13). And Athens was a state of not more than 100 000 citizens in 431.

Cleon, Nicias, and Alcibiades were the leaders of the people during the war, and the generals who fought the war. Cleon fell in Thrace. Nicias fell in Sicily. Alcibiades fell in Asia Minor.

Athens lost the great war. Had it won it, the philosophers may have admired its constitution more than they did the Spartan.

Defeat in the great war proved irreversible. Thebes, not Athens, vanquished Sparta. After 354 Demosthenes was the leader of the people. At this time, in the north, Macedon rose under Philip. Philip developed greatly logistics and tactics. Demosthenes fought Philip with great speeches. In 338 at Chaeronea Philip vanquished the army of Thebes and Athens, and this was the end of the independence, and importance, of the states of the Greek cities.

It has to be concluded that the Athenian constitution is about victory and power, too.

Athens took a major step toward the discovery of tactics, at Marathon and at Salamis. Infantry had gradually become heavy infantry. Heavy infantry, arranged in lines, the best men in the front lines, shield in the left hand, sword in the right, pressed in battle steadily harder to the right, and won its battles on the right flank. Battles of determined foes began with the advance of each right flank into the left flank of its foe, and were won by the more determined wheeling of the more determined right flank. This is how the

Spartans won their battles. At Marathon, Miltiades put his best men on both flanks, and as his foe broke his center, his flanks fell on its flanks, and achieved the first victory by a pincer movement (Herodotus, VI, 110-3). Xerxes had 1 207 ships (VII, 89). At Salamis, Themistocles commanded 380 ships (VIII, 82), 180 of them Athenian triremes (44); he had the advantages of a more unified command, and of seasoned crews, experienced in ramming and boarding; but he faced vastly superior numbers; Themistocles tempted his foe into the narrow channel between Salamis and the mainland, where his foe lost the advantage of numbers, was congested and disordered, and Themistocles achieved the victory (75-91), for which all Greece thought him the wisest of all Greeks.

Athens is also the decisive step in the development of the navy. Herodotus writes: Polycrates was the first mortal to try to master the sea. The tyrant of Samos *c.* 538-22: Polycrates had 100 fifty-oared ships (III, 39). Thucydides writes: triremes were first built just before the Persian war (I, 14). The trireme was a one-hundred-and-seventy-oared weapon, armed with a bronze ram, of mass of 40 tons and speed, in the hands of seasoned oarsmen, such as the Athenian citizens became, of 9 knots. In 480, as he gathered 1 700 000 men from his domain, so Xerxes gathered 1 207 ships. Thucydides writes: Themistocles first told the Athenians that their fortune is on the sea (93). In 483 Themistocles had 100 triremes built. In 480 the 180 Athenian triremes won the victory at Salamis. In 431 Athens entered the great war with 300 triremes in active service. 300 triremes in active service required crews of 60 000.

Athens discovered also that a population of allies will maintain the men who will win victory and power just as well as a population of helots.

Also, at one time during the Persian war the Spartans came to suspect the Athenians of colluding with the barbarians. The Athenians sent ambassadors to reassure them. The ambassadors told the Spartans: we and you are Greeks, the same in blood and the same in language, our shrines to the gods and our sacrifices are shared, and our way of life is common (Herodotus, VIII, 144). So the Spartans and the Athenians, the

same in blood and the same in language, their shrines to the gods and their sacrifices shared, their way of life common, fought the barbarians. Two generations later, the same in blood and the same in language, their shrines to the gods and their sacrifices shared, their way of life common, they fought for twenty-seven years one another.

The state of Athens is very important. And the state of Athens, too, is a step in the development of warfare. Who rules, who owns what, who does what, and education, are all parts of, and serve, the development of warfare.

Mytilene.

Mytilene is the main city on Lesbos, one of the larger islands off the coast of Asia Minor.

There is 1 important particular,

Pittacus tyrant c. 590.

Aristotle writes: the Mytileneans elected Pittacus tyrant (*Politics*, III, 14), and in the review: Pittacus drew laws, not a constitution; and there is one law peculiar to him, that, if a drunken man commits a wrong, he should be punished more severely than if he were sober (II, 12).

Pittacus is, with Solon, Thales, and Bias, one of the four counted among the seven wise men in all the many lists of the seven. Also: Athens and Mytilene were at war; the general of the Athenians was the Olympic victor Phrynon; the general of the Mytileneans was the wise man Pittacus; the Olympic victor challenged the wise man to a single combat; the wise man accepted the challenge, and slew the Olympic victor (Strabo, XIII, 1.38).

The wise man Pittacus is important. The state of Mytilene is not.

Catana.

The city of Catana is in Sicily, north of Syracuse.

There is again 1 particular,

Charondas laws in the generation around c. 570.

Plato's list of the best lawgivers is: Lycurgus, Solon, and Charondas. Aristotle's is the same: Lycurgus, Solon, and Charondas (*Politics*, IV, 11). In the review he writes: there is nothing worth noting in the laws of Charondas, except the suits for perjury; he first instituted these; his laws are also more precise than even those of lawgivers today (II, 12).

The date of Charondas is not recorded. Aristotle reports: some say that he was a student of Zaleucus. Eusebius dates the laws of Zaleucus to 663 (*Chronicle*, II). The tyrant of Syracuse, Hiero, conquered Catana and drove out its citizens in 476. If these are the *termini post quem* and *ad quem*, and they are the first commitment of the laws of a Greek city to writing and the end of Catana, the date of the laws of Charondas is the middle of that time, the generation around c. 570. Hence, one of the earlier lawgivers, Charondas is important, just. The state of Catana is very not important, including because the state of Syracuse is important, which it is including because Hiero conquered Catana.

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7 states so far to which the particulars point.

And the particulars point to 20 more states in the four-odd centuries from Tiglath-pileser III to Alexander. These states are arranged in the chronological order of the first particular as well, except in 3 cases where the spatial continuity in the development of the state demands violating the temporal order. In general, the temporal order corresponds precisely to the spatial. The state developed from east to west.

The 20 states with the first important particular in each are,

Assyria	Tiglath-pileser III,
Urartu	Sarduri II,
Judah	Ahaz,
Babylon	Nabopolassar,
Egypt	Piyi,
Media	Cyaxares,
Phrygia	Midas,
Lydia	Gyges,
Corinth	Cypselus,
Megara	Theagenes,
Sicyon	Cleisthenes,
Samos	Polycrates,
Miletus	Thrasybulus,
Persia	Cyrus,
Acragas	Phalaris,
Syracuse	Gelon,
Carthage	Malchus,
Rome	Twelve tables,
Thebes	Epaminondas,
Macedon	Philip.

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Assyria.

The particulars are,

Tiglath-pileser III prince 745-27,
 Shalmaneser V prince 727-2,
Sargon II prince 722-05,
Sennacherib prince 705-680,
Esarhaddon prince 680-68,
Ashurbanipal prince 668-27.

Not quite sixteen centuries before Tiglath-pileser III, Sargon, prince *c.* 2337-281, first conquered the entire valley of the two rivers. And for one and a half centuries under Sargon and his successors, the princes of Akkad, the entire valley was one and only one state. Then Akkad fell. It took two-odd generations to restore the one and only one state. Ur-Nammu, prince *c.* 2121-03, conquered the valley. And for one century again under Ur-Nammu and his successors, the princes of Ur, the valley was one and only one state. Then Ur fell. It took this time longer, two-odd centuries, to restore the one and only one state. Hammurabi, prince *c.* 1792-49, conquered the valley. And for two centuries once again under Hammurabi and his successors, the princes of Babylon, the valley was one and only one state. Then Babylon fell. And it took this time much longer to restore the one and only one state. It took eight and a half centuries. But the princes of Ashur did restore it.

The rise of Ashur began with Ashur-uballit, prince *c.* 1363-27. And his successors continued for six centuries to labor to conquer the entire valley, and restore the one and only one state, and again, and again, and again, and succeeded. Adad-nirari, prince *c.* 1305-273, Shalmaneser, prince *c.* 1273-43, and Tukulti-Ninurta, prince *c.* 1243-05, conquered the entire valley. Then Ashur retreated. Tiglath-pileser, prince 1115-076, conquered nearly the entire valley. Then Ashur retreated again. Adad-nirari II, prince 912-891, Ashur-nasirpal II, prince 884-59, and Shalmaneser III, prince 859-24, conquered the entire valley. And again Ashur retreated.

Then Tiglath-pileser III conquered the entire valley. And this time Ashur did not retreat. This time the one and only one state was restored.

In 745 the state whose highest magistrate

Tiglath-pileser III became had retreated to the upper valley of the Tigris around the city of Ashur. After eighteen years of victories, by 727 Tiglath-pileser III had conquered the entire valley, and built Ashur into the most powerful state on earth. And in the following one century the victories of his 5 successors maintained it the most powerful state on earth, and built it into a steadily more powerful state.

Tiglath-pileser III campaigned north, west, and southeast, in the same directions, defined by the valley of the two rivers and the adjoining Mediterranean littoral, in which Sargon began campaigning sixteen centuries earlier to build the most powerful state on earth. More immediately, in the two-odd generations after Shalmaneser III and before Tiglath-pileser III the state of Ashur retreated. In the north, in these two-odd generations the state of Urartu rose. Tiglath-pileser III vanquished Urartu repeatedly. In the west, two and a half centuries earlier states appeared along the littoral, north to south, Byblos on the coast, Damascus inland, Tyre on the coast, the state of David in Jerusalem which after Solomon divided into the two states of Israel and Judah, Moab inland. These all became tributaries of Shalmaneser III. They recovered in the following two-odd generations. Tiglath-pileser III vanquished them all repeatedly. In the southeast, in the lower valley, after centuries of disorder the campaigns of Tiglath-pileser III restored order. He subdued the lower valley repeatedly, and in 734 installed himself prince of Babylon.

The entire valley became the domain of Tiglath-pileser III. But the valley had not been under one and only one state, counting back to the fall of Babylon, for eight and a half centuries. The successors of Tiglath-pileser III had to campaign continuously throughout the valley to maintain it under one and only one state. And they first also broke out of the valley and continued to expand the domain of the one and only one state.

Shalmaneser V campaigned in the west.

Sargon II continued campaigning in the west, and destroyed the states along the littoral. Only Judah, reduced to the city of Jerusalem, lingered on as his tributary. In the southeast, Sargon II subdued Babylon. In the north, he destroyed

Urtartu. In the northwest, he broke into Asia Minor. Phrygia, on the Black Sea coast, became his tributary. He ventured also on the sea. Cyprus became his tributary.

Sennacherib subdued the littoral in the west. In the southeast, he subdued Babylon again, and conquered Elam, further east.

Esarhaddon campaigned from Elam to the littoral, broke out of the valley in the west, and conquered Egypt.

Ashurbanipal subdued Elam, subdued Babylon, subdued the littoral, subdued Egypt.

Ashurbanipal reports:

Ashurbanipal, the great prince, the true prince, the lord of the earth, the prince of Ashur, the lord of the four quarters, the prince of princes, the prince without rival from the upper sea to the lower sea, who made all princes from Tyre in the upper sea to Dilmun in the lower sea prostrate themselves before his feet, and pull the straps of his yoke (ins. II, 969/73).

Assyria under Tiglath-pileser III and his 5 successors is a decisive advance in the development of the state. It is the third of the most important steps of advance in the development of the state. After the beginning of the state. And the decisive advance under Sargon and his 4 successors, Rimush, Manishtushu, Naram-Sin, and Sharkalisharri.

In the following ways.

Size of the state.

The domain of Tiglath-pileser III and his 5 successors reached from the source of the Euphrates in the north to Thebes in the valley of the Nile in the south and from the delta of the Nile in the Mediterranean in the west to Susa in Elam in the east, across 14° of latitude and 18° of longitude, 1 600 km along the meridian and 1 700 km along the parallel. It was the state of some 10 million people, not less than one tenth of humanity. It was the largest state ever, not less than twice larger in area and more than twice larger in population than the largest earlier states, that of Sargon and his 4 successors, that of Ur-Nammu and his 4 successors, and that of Hammurabi and his 5 successors.

Unity of the state.

The Sumerian prince list presumes that princship is one. It is an empirical generalization. As

soon as the state appeared in the lower valley of the two rivers, in the cities of Kish, Uruk, Ur, Lagash, Umma, the cities began smiting one another. Attempting, apparently, to establish that princship, wantonly divided among the cities, is one and only one. And Sargon succeeded. Under him and his 4 successors there was one and only one princship in the valley of the two rivers. On earth there was but one other state, that in the valley of the Nile.

Akkad fell, and princship was divided again. Then under Ur-Nammu and his 4 successors there was again one and only one princship in the valley of the two rivers. On earth there was again but one other state, that in the valley of the Nile.

Ur fell, and princship was divided again. Then under Hammurabi and his 5 successors there was again one and only one princship in the valley of the two rivers. On earth there was again but one other state, in Asia Minor this time, the Hittite state had just appeared. In the valley of the Nile the state had fallen apart.

Babylon fell, and once again princship was divided. Then the attempts of the princes of Ashur to restore it began. The attempts nearly succeeded with Tukulti-Ninurta, again with Tiglath-pileser, again with Ashur-nasirpal II.

And then the attempts succeeded. Under Tiglath-pileser III and his 5 successors princship was again one and only one. When their labors reached their peak, under Ashurbanipal, there was – very nearly – one and only one state on earth. Far to the east, the state had appeared in the valleys of the Chang Jiang and Huang He. This side of the valleys of the Chang Jiang and Huang He, however, in 664 Ashurbanipal subdued Egypt for the second time, and there remained not more than 3 states outside Ashurbanipal's domain. To the west, there was Lydia under Gyges in western Asia Minor, further west Argos had just defeated Sparta. All 3 were very small, all 3 were just becoming states.

Assyria, moreover, achieved the unity of princship, and of the state, against much greater resistance.

Ur-Nammu restored it after two-odd generations of disorder. Hammurabi restored it after two-odd centuries of greater disorder and divided authority. Tiglath-pileser III restored it after eight

and a half centuries of much greater disorder, repeatedly divided and subdivided authority, and hence increasingly vigorous resistance to the increasingly vigorous attempts to restore it.

Assyria, moreover, unified large and firmly established states. Tiglath-pileser III and his 5 successors had to conquer large and firmly established states. Urartu was a large state. The Assyrian princes had to campaign for two generations to destroy it. The states along the littoral were smaller but more firmly established. Tyre, Israel, and Judah had each continued for more than two centuries. The Assyrian princes had to campaign for two generations to vanquish them, and to continue to subdue them in every generation. After the fall of Babylon the Kassite and Hurrian hordes had brought great disorder to the valley, and in the lower valley the disorder continued for a very long time. But the state had deep roots in the lower valley. And the Assyrian princes had to campaign continuously in Babylon, and to continue to expel and to install its rulers. They had to campaign continuously against Elam further east. The state had the deepest roots in Egypt, and after several centuries of disorder, the state had become stronger in Egypt once again. In Egypt as well the Assyrian princes had to campaign repeatedly, and to continue to expel and to install its rulers.

Assyria under Tiglath-pileser III and his 5 successors is the first state to unify large and firmly established states into one state, and thus unify into one state many and large populations which had long been brought under states.

Permanence of the state.

The five millennia of the development of the state can be analyzed into two parts. First, in the first two millennia of its development the state is, so to say, stammering, babbling, and prattling. Second, the state then matures, and acquires permanence.

First, in the first two millennia of the development of the state, times of states and order keep alternating with times of no states and disorder. – In the valley of the Nile, the state begins with the first prince, Narmer, and there is for seven and a half centuries the state of the continuous series of princes from Narmer, prince *c.* 3000, to Pepi II, prince *c.* 2240. Then the state disappears in

disorder for two centuries. Then there is again for two and a half-odd centuries the state, and a larger and more powerful state, from Mentuhotep, prince *c.* 2069-18, to Sobeknefru, prince *c.* 1790-86. Then the state again disappears in disorder for two-odd centuries. Then there is again for four centuries the state, and again a larger and more powerful state, from Ahmose, prince *c.* 1570-45, to Ramesses III, prince *c.* 1191-59. Then the state again disappears in disorder. – In the valley of the two rivers, the state begins also with the first prince, Enmebaragesi, and for five centuries, from Enmebaragesi, prince *c.* 2680, to Sharkalisharri, prince *c.* 2220-195, there are at first states smiting one another, then the one state of Sargon and his 4 successors. Then the state disappears in the disorder brought by the Gutian hordes for two-odd generations. Then for five centuries, from Utuhegal, prince *c.* 2128-1, to Ammisaduqa, prince *c.* 1646-25, there are again states smiting one another, then the one state of Ur-Nammu and his 4 successors, then again states smiting one another, then the one state of Hammurabi and his 5 successors. Then in the valley of the two rivers the state disappears in the disorder brought by the Kassite and Hurrian hordes for two and a half centuries. – Further west, in Asia Minor, there is the Hittite state from Labarna, prince *c.* 1650, to Telipinu, prince *c.* 1510, then the Hittite state disappears in disorder, then there is again the Hittite state from Suppiluliuma, prince *c.* 1350-25, to Tudhaliya IV, prince *c.* 1245-15, then the Hittite state disappears altogether. – In the valley of the two rivers Ashur begins to rise under Ashur-uballit, prince *c.* 1363-27, rises greatly, declines, rises greatly again, declines again, rises greatly again, declines again.

Second, the state then acquires permanence. And this second part of the development of the state can itself be analyzed into two parts.

First, the Assyrian state from
 Ashur-uballit prince *c.* 1363-27,
 through
 Adad-nirari prince *c.* 1305-273,
 Shalmaneser prince *c.* 1273-43,
 Tukulti-Ninurta prince *c.* 1243-06,
 through
 Tiglath-pileser prince 1115-076,
 through

Adad-nirari II	prince 912-891,
Ashur-nasirpal II	prince 884-59,
Shalmaneser III	prince 859-24,
through	
Tiglath-pileser III	prince 745-27,
Shalmaneser V	prince 727-2,
Sargon II	prince 722-05,
Sennacherib	prince 705-680,
Esarhaddon	prince 680-68,
Ashurbanipal	prince 668-27,

is one and the same state. These princes and the lesser princes between them are one and the same continuous succession of 41 princes. These princes are the first not only to maintain a state one and the same for seven centuries, but to try to build a state into the one and only one on earth, and again, and again, and again, and succeed.

Second, there have since been no times of no states and disorder on earth. The three millennia since have been a time of states and order. And the state has steadily and irreversibly brought all human populations on earth under itself.

All three, the size of the state, its unity, and its permanence, are inseparable from the development of warfare.

Ashur-nasirpal II, prince 884-59, discovered cavalry. Chariots are first attested seven and a half centuries earlier, by the Hittite prince Hattusili, prince c. 1630. Ashur-nasirpal II first attests cavalry. Ashur-nasirpal II reports also: I mustered 50 000 men (ins. 40). Ashur-nasirpal II is a major step in the development of warfare.

And one century-odd later, Tiglath-pileser III and his 5 successors built, including now also cavalry, the most powerful army the earth had seen, by very far. Its strength can be assessed on the following grounds.

The general facts. The Assyrian princes vanquished many, large, firmly established states. They campaigned from Elam to Asia Minor to Egypt for more than one century annually.

More particularly, there are the reports of those who suffered the strength of the army. One of the writings collected in the Hebrew scriptures is *Kings*, the chronicle of Israel and Judah, and their destruction by the Assyrian princes. *Kings* reports:

Sennacherib, the king of Assyria, marched against Judah; he imposed on king Hezekiah a

tribute of 300 talents of silver and 30 talents of gold; Hezekiah cut down the doors and the doorposts of the temple of the lord, and gave them to the king of Assyria (II, 18).

Hezekiah then prayed to the lord, Isaiah told Hezekiah that the lord had heard his prayer, and, *Kings* continues:

that night an angel of the lord struck down 185 000 in the Assyrian camp (19).

Sennacherib corroborates the tribute, if not the angel of the lord. Sennacherib reports:

in the 3rd year of my reign Hezekiah did not submit to my yoke; I took 200 150 people from his cities; horses, mules, asses, camels, cattle, and sheep without number; I shut him in Jerusalem like a bird in a cage; and he gave me 30 talents of gold and 800 talents of silver; he gave me gems, antimony, and ivory; he gave me his daughters, his wives, and his musicians (ins. II, 232/54).

There are also the reports of the princes themselves. Ashur-nasirpal II mustered 50 000 men. Tiglath-pileser III and his 5 successors do not report how many men they mustered. Their inscriptions have no number for their entire army. They have many other numbers, however. Sargon II, for instance, in his annals, next to all the cities taken and other booty collected, reports also this:

in the 2nd year of my reign, I took 9 033 people;

in the 4th year of my reign, I took 7 350 people;

in the 5th year of my reign, I selected from the people I took 50 chariots, 200 cavalymen, and 300 infantrymen for my host;

in the 7th year of my reign, I took 4 200 people;

and I took 4 820 people;

and I took 2 530 people;

in the 8th year of my reign, I took 6 170 people;

in the 12th year of my reign, I took 16 490 people;

in the 13th year of my reign, I took 90 580 people;

and I selected 150 chariots, 1 500 cavalymen, 20 000 bowmen, and 10 000 shield-bearers for my host (ins. II, 4/51).

Sennacherib, also for instance, took 200 150 people in his 3rd year. In the report of his cam-

paings in his 1st year he concludes:

and I came back to Ashur with 208 000 people, 7 200 horses, 11 073 asses, 5 230 camels, 80 050 cattle, and 800 100 sheep (ins. II, 256/67).

In 721 – I took 9 033.

In 719 – I took 7 350.

In 718 – I selected 550 for my host.

In 716 – I took 11 550.

In 715 – I took 6 170.

In 711 – I took 16 490.

In 710 – I took 90 580, and I selected 31 650 for my host.

In 705 – I took 208 000.

In 703 – I took 200 150.

It has to be concluded that it became a host of not less than 100 000. Maintained in active service for more than one century. Deployed annually.

Tiglath-pileser III and his 5 successors are a major step in the development of logistics, in particular, and of warfare, in general.

The state of Assyria is very important. It is one of the most important states.

And the Assyrian constitution is also about victory and power. Its household arrangements are also all parts of, and serve, the development of warfare. Who should rule? The great prince, the true prince, the lord of the earth, the prince of Ashur, the lord of the four quarters, the prince of princes, the prince without rival from the upper sea to the lower sea, he made all princes from Tyre in the upper sea to Dilmun in the lower sea prostrate themselves before his feet, and pull the straps of his yoke. Who should own what? I took 200 150 people; horses, mules, asses, camels, cattle, and sheep without number; he gave me 30 talents of gold and 800 talents of silver; he gave me gems, antimony, and ivory; he gave me his daughters, his wives, and his musicians. He gave me also the doors and the doorposts of the temple of the lord. On but one occasion in but one small part of the state. Never mind who should do what, and education.

Assyria brought one tenth of humanity under one state. It became, with very minor qualifications, the only state on earth. It brought under itself all earlier states. The attempts to build it continued for seven centuries. In the last of these centuries it deployed a host of 100 000 annually.

The end of the state – the cause of its motion, never mind what it is for – is to vanquish all other states and bring them under itself, under the one and only one state which the state is.

This has to be concluded from this largest of the 8 boats considered so far, the Assyrian. It has to be concluded also from the 2 largest of the 7 Greek boats considered so far, the Spartan and the Athenian. The monarchy or aristocracy Sparta, which is a step in the development of warfare, and the democracy *par excellence* Athens, which is also a step in the development of warfare, which both first rose to dominate all other Greek states, then fought for twenty-seven years the greatest war ever to determine who among the two of them will dominate whom. Which explains also why the 5 small boats considered so far are not important, 1 produced archers to serve Sparta and Athens, 1 was dominated by Sparta, 1 was dominated by Athens, and the 2 in the west were dominated by Syracuse.

Assyria brought one tenth of humanity under one state. To do so it campaigned continually for more than one century from the source of the Euphrates in the north to Thebes in the valley of the Nile in the south and from the delta of the Nile in the Mediterranean in the west to Susa in Elam in the east.

And, hence, the state of Assyria is a decisive step also in the discovery and solution of the problem of peace.

In their labors to unify the valley the Assyrian princes first came close to success with Adad-nirari, Shalmaneser, and Tukulti-Ninurta. These 3 princes labored to that end for one century. Their inscriptions are abundant, they record their campaigns in detail, and their campaigns end uniformly:

I razed, destroyed, and burned (*passim*).

These princes failed to unify the valley.

Next Tiglath-pileser came close to success. This prince labored to that end for four decades. His inscriptions are more abundant, they record his campaigns in more detail, and his campaigns also end uniformly:

I razed, destroyed, and burned (*passim*),
but also:

and I counted them among the people of Ashur (*passim*).

This prince also failed to unify the valley. He counted them among the people of Ashur. But they will not be among the people of Ashur.

Next Adad-nirari II, Ashur-nasirpal II, and Shalmaneser III came close to success. These 3 princes labored to that end for nearly one century. Their inscriptions are yet more abundant, they record their campaigns in yet more detail, and their campaigns end in the same way:

I razed, destroyed, and burned (*passim*),
then also:
and I counted them among the people of Ashur (*passim*).

These princes also failed to unify the valley. They again counted them among the people of Ashur. But they again will not be among the people of Ashur.

And then Tiglath-pileser III and his 5 successors succeeded. These 6 princes labored to that end for one century-odd. These prince also counted them among the people of Ashur. And this time, whether they will be among the people of Ashur or not, they were among the people of Ashur.

The inscriptions of these 6 princes are yet more abundant, they record their campaigns in yet more detail, and their campaigns end uniformly as well, but this time differently. Sargon II reports his conquests and booty in the 1st year of his reign, and concludes:

and I set my official as a governor over them (*id.*),
his conquests and booty in his 2nd year, and concludes:

and I set my official as a governor over them (*id.*),
his conquests and booty in his 3rd year, and concludes:

and I set my official as a governor over them (*id.*),
and so through all the years of his reign, and all the years of the reigns of his successors. These princes succeeded in bringing the entire valley under administration, the administration of one tenth of humanity.

And Sargon II reports also:

I settled the tribes of Tamud, Haiapa, and Arabia in Samaria;

I made the Hilaki live in peace; and I settled

people of Ashur there;

I spared the people of Gurgum; and I counted them among the people of Ashur;

I captured Ashdod and Gath; I built their cities anew; I settled people from the lands I had conquered there; and I counted them among the people of Ashur (*id.*).

In 721 – 9 033 were taken, settled, made to live in peace, and counted among the people of Ashur.

In 716 – 11 550 were taken, settled, made to live in peace, and counted among the people of Ashur.

In 710 – 90 580 were taken, settled, made to live in peace, and counted among the people of Ashur.

In 703 – 200 150 were taken, settled, made to live in peace, and counted among the people of Ashur.

It is under the Assyrian princes, in their state, that the mixing of peoples accelerates greatly. Or what is the same, the mixing of humanity accelerates greatly. And thus also the taming of humanity.

Clausewitz writes: war is in its result never something absolute (*War*, I, 1.9).

Sargon and his 4 successors discovered this fact.

Hobbes writes: war is not battle only, but the tract of time when the will to battle is sufficiently known; war is not actual fighting, but the known disposition to it; all other time is peace (*Leviathan*, 13).

In the discovery of the fact that peace is the tract of time when the disposition to fighting is vanquished, the Assyrian princes are a decisive step.

Before *pax Romana* there was *pax Persica*, and before *pax Persica* there was the attempt, and a very successful attempt, at *pax Assyrica*.

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Urartu.

Beginning at the time of Tiglath-pileser III, the important particular is 1,

Sarduri II prince c. 740.

Urartu is an important state, if not very important. Assyria declined after Shalmaneser III, prince 859-24, and rose again under Tiglath-

pileser III, prince 745-27. In the two-odd generations between them Urartu rose in the north, built by the 4 princes, Sarduri, Ishpuini, Menua, and Argishti. Urartu was at its peak under Argishti. Sarduri II is his successor. Then Tiglath-pileser III and Sargon II destroyed Urartu.

Judah.

Two and a half centuries before Tiglath-pileser III states appeared along the Mediterranean littoral. Byblos, whose prince Ittobaal, prince c. 980, left behind the oldest alphabetic inscription. Further south Tyre, at its peak under Hiram, prince c. 969-35, which began the colonization of the Mediterranean. Inland Damascus, at its peak under Hadadezer, prince 885-45. Further south and also inland Moab, whose prince Mesha, prince c. 860, left behind the oldest inscription attesting independently the reports of the writings collected in the Hebrew scriptures. And south of Tyre, west of Moab, the state of David, prince c. 1003-963, and Solomon, prince c. 963-23, which after Solomon split into the two states of Israel and Judah. All these states became tributaries of Shalmaneser III. They recovered in the two-odd generations after him. Then Tiglath-pileser III, Shalmaneser V, and Sargon II destroyed them. Israel fell in 721. Judah alone was allowed to linger on as a tributary. Nebuchadnezzar, prince of Babylon 605-562, destroyed Judah in 587.

Had it not been for the development of Judaism, and hence the development of Christianity, and hence the development of Islam, the state of David and Solomon would have been important, if not very important, and the two successor states of Israel and Judah would have been very not important. As it is, the development of Judaism begins in the state of Judah, and the state of Judah is very important.

Beginning at the time of Tiglath-pileser III, the important princes of Judah are 3,

Ahaz	prince c. 738-26,
Hezekiah	prince 726-697,
Josiah	prince 640-09.

Kings is the chronicle of Solomon and the princes of Israel and Judah. Beginning with the last days of David, it relates the glory days of Solomon, and then the more somber days of the 19 princes of Israel through its fall in 721, and the 20 princes of Judah through its fall in 587.

Kings reports:

Ahaz did what was displeasing to the lord (II, 16);

Hezekiah did what was pleasing to the lord (18);

Josiah did what was pleasing to the lord (22).

And so much suffices to show that *Kings* is a new kind of chronicle. The chronicles of the Assyrian princes record – as Herodotus, too, will in the *Histories* – great and magnificent deeds. *Kings* does not record great and magnificent deeds. Unlike the chronicles of the Assyrian princes as well, *Kings* not only judges the deeds it records on the presumption that it knows, as the princes did not, what displeases and pleases the lord. It does so centuries after the deeds.

Like all the writings collected in the Hebrew scriptures, *Kings* incorporates in its final redaction older materials. In its final redaction it is the last of the four books of *Joshua*, *Judges*, *Samuel*, *Kings*, all four from a single hand.

The oldest of the writings collected in the Hebrew scriptures is the Yahwistic document. Next are the prophetic books. Next, and by far the most important, are 4 writings of which *Joshua*, *Judges*, *Samuel*, *Kings* is the second. It is with these 4 writings that Judaism was established. With their dates,

<i>Deuteronomy</i>	the law promulgated in	622,

<i>Joshua, Judges, Samuel, Kings</i>	– the two	generations after 587,

<i>Priestly code</i>	the century before 445/33,

<i>Chronicles, Ezra, Nehemiah</i>	– the century	after 445/33.

Deuteronomy is the second step, after the Yahwistic document, in the development of the law of the Hebrew scriptures. It introduces the law by relating the events of the lawgiving, how the lord gave the law to Moses, and Moses gave the law to the Hebrews. It is dated from the promulgation of the law under Josiah in 622.

Joshua, Judges, Samuel, Kings relates the events which befell the Hebrews after the Mosaic lawgiving and through the fall of Judah in 587. It is dated from the latest events it relates.

The Priestly code is the third step in the development of the law of the Hebrew scriptures. It incorporated the Yahwistic document into itself.

It is the final redaction of the law. And it introduces the law by relating the events which befell the Hebrews before the Mosaic lawgiving and beginning with the beginning of the world. Ezra convoked the great assembly in 445/33, and the Priestly code is dated from the great assembly.

The three books of *Chronicles*, *Ezra*, *Nehemiah* are from a single hand as well. *Chronicles*, *Ezra*, *Nehemiah* records the genealogies of the Hebrews from Adam to David, then relates the events which befell the Hebrews from David through the great assembly under Ezra. It is also dated from the great assembly.

Ezra great assembly in 445/33, was a priest, not a prince. And Ezra is the fourth and final step in the development of the law of the Hebrew scriptures. Ezra enforced the law.

Merging the enumerations, the series of the 8 particulars is this,

Ahaz	prince c. 738-26,
Hezekiah	prince 726-697,
Josiah	prince 640-09,
Deuteronomy	the law promulgated in 622,

Joshua, Judges, Samuel, Kings – the two generations after 587,

Priestly code the century before 445/33,

Ezra great assembly in 445/33,

Chronicles, Ezra, Nehemiah – the century after 445/33.

Ahaz is contemporary with Tiglath-pileser III. *Hosea*, accordingly, prophesies: the people will be taken to Assyria, tribute to its great king (11). As the people already had been. *Isaiah* prophesies: the lord will bring against you the king of Assyria and his multitude, he will overflow your land, he will sweep over you like a flood (8). As the lord already had, and the king of Assyria already had.

One and a half centuries later, *Kings* judges:

Ahaz did what was displeasing to the lord.

So did, lamentably, most of his predecessors, and in both Israel and Judah. 18 of the 19 princes of Israel one and all did what was displeasing to the lord. This explains the destruction of Israel. 5 of the 11 predecessors of Ahaz in Judah one and all also did what was displeasing to the lord. This explains the decline of Judah. Of the 8 successors of Ahaz 2, Hezekiah and Josiah, did not do what

was displeasing to the lord. The other 6, however, lamentably, one and all also did what was displeasing to the lord. And this explains the destruction of Judah. *Kings* is a paradigm of steadfast commitment to one and only one cause.

Ahaz did what was displeasing to the lord. In particular, *Kings* reports:

Ahaz set up shrines and made sacrifices and offerings in the ways of other peoples abhorrent to the lord (II, 16).

In the ways of other peoples. Attesting the success of the labors of the Assyrian princes at mixing the peoples under their one and only one state. Taking them, settling them, making them live in peace, and counting them among the people of Ashur.

The successor of Ahaz, Hezekiah is contemporary with Shalmaneser V, Sargon II, and Sennacherib. Israel fell in 721. Judah was reduced to the city of Jerusalem, allowed to linger on as a tributary.

Hezekiah did what was pleasing to the lord. In particular, *Kings* reports:

Hezekiah abolished the shrines, smashed the pillars, and cut down the sacred posts (18).

Hezekiah's is the first attempt to exclude the Hebrews from the mixing of the peoples.

Lamentably, his 2 successors reverted to doing what was displeasing to the lord. But then there was Josiah.

Three generations after Hezekiah, Josiah is contemporary with Ashurbanipal and the Babylonian king Nabopolassar, prince 626-05. Assyria declined catastrophically after Ashurbanipal, and in 612 Nabopolassar and the Mede king Cyaxares destroyed it. It brought no respite to Judah. The grip on it of the Babylonian princes now became even firmer.

Josiah did what was pleasing to the lord. In particular, *Kings* reports:

in the 18th year of Josiah, during repairs in the house of the lord, the high priest Hilkiah found the book of the law; Hilkiah gave the book to the scribe Shaphan, who read it; Shaphan then read the book to the king; and when the king heard what was in the book, he rent his clothes (22).

After that:

Josiah defiled and demolished the shrines; he killed the horses that the kings of Judah had

dedicated to the sun; he tore down the altars and scattered the rubble; he scattered the pillars and cut down the sacred posts; he dug the bones out of the graves and burned them; he slew the priests and burned human bones on them; so the king fulfilled the law recorded in the book found in the house of the lord (23).

And after that:

the king called the men of Judah and of Jerusalem, all the priests and all the prophets, all the people, young and old; he read the entire book to them; and he made a covenant before the lord that they will obey the lord and will keep his law (*id.*).

Thus the second attempt to exclude the Hebrews from the mixing of the peoples began, and this time the attempt succeeded. Not because of the labors of the princes of Judah. The 4 successors of Josiah one and all reverted to doing what was displeasing to the lord, and in 587 the Babylonian king Nebuchadnezzar destroyed Judah. Because of the labors of the priestly writers who build, in place of the state of Judah, the first church.

Contemporary with Josiah, who defiled and demolished the shrines, slew the priests and burned bones on them, called all the priests and all the prophets, all the people, young and old, and made a covenant before the lord that they will obey the lord and will keep his law, *Deuteronomy* is the book of the law which they will keep. The law is more elaborate than any earlier, a code of laws of greater number and complexity than any earlier. Nor is *Deuteronomy* just the book of the law. It is the word of the lawgiver. In *Deuteronomy* Moses himself speaks. The lawgiver himself gives the law to the Hebrews. Nor is indeed Moses the lawgiver. The lord gave Moses the law, Moses informs the Hebrews, to give the law to the Hebrews. Nor are the dangers of disobeying the law to be taken lightly. If they disobey the law, Moses informs the Hebrews:

cursed will be the issue of your womb and the produce of your land;

cursed will be your comings and goings;

the lord will let loose against you calamity, fear, and pain in everything that you do;

the lord will strike you with madness, blindness, and dismay;

you will pay the price for a wife, and another

man will enjoy her, you will build a house, and you will not live in it;

the lord will bring a people from the far end of the earth to swoop over you like an eagle;

the lord will inflict on you and your offspring plagues of which you have not even heard;

plagues and diseases, which are not even mentioned in this book of law, will wipe you out;

the lord will scatter you among other peoples from one end of the earth to the other (28).

In general:

if you forget the lord god, he will destroy you. The rewards for obeying the law are correspondingly great as well. Moses informs the Hebrews:

if you obey the law, you will be blessed above all other peoples, you will destroy all the peoples whom the lord god delivers to you.

Nor is *Deuteronomy* about what will happen. It is about what has happened. The lawgiving occurs on the eve of the conquest of the promised land.

Clearly, if the conquest occurred, the Hebrews obeyed the law, were blessed above all other peoples, and destroyed all the peoples whom the lord god delivered to them.

Ben Halafta calculates: 890 years from the exodus to the destruction of the first temple. If *Deuteronomy* is dated to the time of Josiah, the Mosaic lawgiving related by *Deuteronomy* predates its relation by not quite nine centuries.

Some two generations after *Deuteronomy*, *Joshua*, *Judges*, *Samuel*, *Kings* is contemporary with the rise of the kings of Babylon. Judah had been destroyed, the Hebrews deported to Babylon.

Joshua, *Judges*, *Samuel*, *Kings* begins where *Deuteronomy* ends, with the death of Moses, and relates the subsequent events, through the conquest of the promised land, to which *Joshua* is devoted, through the rise and reign of David, to which *Samuel* is devoted, through the fall of Israel and Judah. The nine centuries since the Mosaic lawgiving are filled with what was pleasing to the lord and what was displeasing to the lord.

All events on the near side of the Mosaic lawgiving are thus explained, when you obeyed the law, you were blessed above all other peoples, when you forgot the lord god, he destroyed you.

Some two generations later, the Priestly code is contemporary with the rise of the great kings. What had been the state of Judah was now a part of the domain of the great kings. A very small part, next to the 77 countries and peoples large enough to pay annual tax of more than 10 talents each.

The Priestly code constitutes parts of *Genesis*, *Exodus*, and *Numbers*. The remaining parts of these are the Yahwistic document, incorporated by the Priestly code into itself. The Priestly code constitutes also, all by itself, *Leviticus*. Wellhausen, in 1878, in the *Prolegomena to the history of Israel*, identified and named the Priestly code, as, Wellhausen writes: it is entitled to be called by both its content and its origin (intr.2).

The Priestly code is the final redaction of the law. The laws are now 613, as the amoraim counted them, 365 prohibitions, because this is the number of days in the year, and 248 injunctions, because this is the number of parts in the body.

The Priestly code also explains all events on the far side of the Mosaic lawgiving, the creation of the world, the first man and woman, their progeny, the flood, the lord's repeated promises to the patriarchs, the sojourn in Egypt and subsequent captivity, the miraculous deliverance, the forty years in the wilderness, the first approach to the promised land. Ben Halafta calculates: 2 448 years from creation to the exodus. In these two and a half millennia, too, when you obeyed the law, you were blessed above all other peoples, when you forgot the lord god, he destroyed you. Hence also, in general, from the beginning of the world to the present day, when you obeyed the law, you were blessed above all other peoples, when you forgot the lord god, he destroyed you.

The Priestly code also entrusts the guardianship of the law to the priesthood. One generation after *Deuteronomy*, the lord informs the Hebrews in *Ezekiel*: the levitical priests alone may enter my sanctuary, they alone may approach my table to minister to me, they alone shall keep my charge (44). In *Leviticus* the Priestly code has once again Moses inform the Hebrews of the law of offerings (1-7), consecration (8-10), purification (11-6), again offerings and celebrations (21-7), all law guarded by the levitical priests, they

alone may approach my table to minister to me, they alone shall keep my charge.

Ezra is contemporary with *pax Persica*. The mixing of the peoples under the great kings had continued and accelerated for one century.

Ezra convoked the great assembly. All foreign wives and all children born to them were expelled. And after twenty-four days of reading of the law, celebration, and prostration, the people bound themselves in writing to obedience to the law. Ezra was, *Ezra* reports: a priest, steeped in the law of the lord in heaven (7).

Some two generations later, *Chronicles*, *Ezra*, *Nehemiah* is still contemporary with *pax Persica*. The mixing of the peoples under the great kings continued and continued to accelerate.

Chronicles, *Ezra*, *Nehemiah* collects from the earlier Hebrew writings the genealogies leading from Adam to David, then revisits the events from David to the fall of Judah, adding the event of the great assembly. The concern is throughout liturgical. In the genealogies the concern is first and foremost with the Levites (*Chronicles*, I, 2-7). In the new relation of the events since David the concern is with the proper worship of the lord in Jerusalem, in his temple, under his priests. The proper worship of the lord in Jerusalem, in his temple, under his priests, was instituted by David (10-29), and continued uninterrupted through the building of the temple by Solomon (II, 1-9), through the kings of Judah (10-36), through the great assembly (*Ezra*, *Nehemiah*).

This is how Wellhausen describes this development. He writes: for Isaiah the kingdom of Yahweh is identical with the kingdom of David, the tasks he sets for it are political and similar, we may say, to the tasks one would set today for the Ottoman empire (XI, 1.3).

The Ottoman empire, in retreat for two centuries by that time, had just been defeated decisively in the Russo-Turkish war of 1877-8.

Wellhausen continues: but the tabernacle of David fell at last and no king was born to raise it again; the state suffered not a crisis but destruction; the result was that the religious hopes of the people turned away from the existing political conditions; earlier the goal had been the state as it had existed under David, now in place of the state a world empire was built in imagination; but

the extravagant hopes built on Yahweh were balanced by sober aims dictated by events; with the old community gone, the godly both had the opportunity and felt the obligation to lead the creation of a new Israel; the weaker the state grew the more powerful the priesthood became; thus arose the sacred constitution of Judaism; in the Priestly code we see it in detail; with the destruction of Samaria, then Judah, old Israel shrank to a religious congregation, public life was absorbed in the service of the sanctuary, the people became, as we read in *Exodus*, 19, the kingdom of priests and holy people (2.1); the Mosaic theocracy is the residue of the ruined state; it is itself not a state but a church, in its nature intimately connected with the Catholic church, which is in fact its child (*id.*).

Nor is the sacred constitution of Judaism about household arrangements. It, too, is about victory and power. Moses asks the Hebrews: is there a god in heaven or on earth who can match the deeds of the lord god? And answers the question: search the ages since the day god created man on the earth, from one end of the heaven to the other, has a people heard a god speak out of the fire and remained alive? has a god by his deeds, signs, portents, and war taken a people from amidst another people? you have before you clear proof that the lord alone is god, there is no other.

In heaven or on earth.

And the Mosaic theocracy, the residue of the ruined state, itself not a state but a church, manifests the end of the state – to vanquish all other states and bring them under itself – rather unambiguously.

When the sacred constitution of Judaism arose, and the state shrank to a religious congregation, itself not a state but a church, the church yet never lost sight of the end. The hopes built on Yahweh were balanced by sober aims dictated by events, yet a world empire was built in imagination. The prophet prophesies: thus said the lord to Cyrus, you do not know me, but it is I who has given you your strength, so that through you all men from the rising sun to the setting sun will know that there is no one but me (*Isaiah*, 45).

From the rising sun to the setting sun.

And if the first church built its world empire in imagination, its children built theirs in fact.

The first child, the Christian church, asserted the end not just explicitly, it asserted it with a vengeance. The pontiff writes to the emperor: two, august emperor, rule the world, the sacred authority of the pontiffs and the power of the princes; and of these the authority of the pontiffs is weightier than the power of the princes, because the pontiffs have to render an account to god even for the princes.

The sacred authority of the pontiffs rules the world. And when it reached its peak, the sacred authority of the pontiffs ruled, in fact, a substantial portion of the world.

The next child, Islam, never distinguished state and church, and asserted the end with an even greater vengeance. The koran teaches: fight those who do not believe in god and in the last day, who do not forbid what god and his apostle have forbidden, who do not embrace the true faith, until they pay tribute and are utterly subdued (9.29). The prophet himself wrote to the emperor of Rome and the king of Persia to advise them, embrace the true faith or pay tribute and utterly submit (ibn Ishaq, 1560-2, 1572).

Fight until they are utterly subdued. And when the fighting reached its peak, a substantial portion of the world had been, in fact, utterly subdued.

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Babylon.

By the time of Tiglath-pileser III the lower valley of the two rivers had been in disorder for centuries. Tiglath-pileser III restored order. But he and his successors had to campaign continuously in the lower valley to maintain order.

Tiglath-pileser III reduced it, then had to reduce it again, and again. He installed himself prince of Babylon. Sargon II had to reduce it again, and again. He installed himself prince of Babylon. Sennacherib had to reduce it again, and again. He installed his son prince of Babylon. His son was deposed, and Sennacherib had to reduce it again, and this time to raze Babylon. Esarhaddon rebuilt Babylon. Ashurbanipal installed his brother prince of Babylon. His brother rebelled, and Ashurbanipal had to reduce it again. And he reduced it.

Then Babylon rose powerfully.

Important are,

Nabopolassar prince 626-05,
Nebuchadnezzar prince 605-562,
 Neriglissar prince 560-56.

The reign of Ashurbanipal ended in 627. In the following years Assyria declined catastrophically and Babylon rose. In 612 Nabopolassar and the Mede prince Cyaxares sacked Nineveh, and this was the end of Assyria. Nebuchadnezzar built Babylon into the dominant power in the valley, and one of the four kingdoms vying to become the dominant power on earth.

In 539 Cyrus conquered Babylon, and Babylon became part of his domain.

Egypt.

By the time of Tiglath-pileser III the valley of the Nile had been in disorder for several centuries as well. Piyi restored order. The important particulars are,

Piyi prince c. 729-08,
 Shabaka prince c. 708-693,
 Taharqa prince 690-64,
 Psamtik prince 664-10,
Necho prince 610-595,
 Amasis prince 570-26.

Ashurbanipal removed Taharqa and installed Psamtik. After the fall of Assyria, Psamtik and Necho built Egypt into one of the four kingdoms vying to become the dominant power on earth. Under Necho also, twenty-one centuries before da Gama, Africa was circumnavigated. Herodotus reports: Necho dispatched a fleet of Phoenicians to circumnavigate Africa; they set out from the Red Sea sailing south; in the third year of the voyage they passed through the strait of Gibraltar, and came back to Egypt from the west; they said that as they rounded Africa the sun was on their right (II, 42).

In 526 Cambyses conquered Egypt, and Egypt became part of the domain of the great kings.

The states of Urartu, Judah, Babylon, and Egypt the Assyrian princes brought under themselves. They fared differently. One disappeared, one gave rise to a church, two rose after the fall of Assyria.

Media.

Media is the third of the kingdoms which after the fall of Assyria vied to become the dominant power on earth. The important particulars are,

Cyaxares prince c. 625-585,

Astyages prince c. 585-50.

The Assyrian princes campaigned continuously beyond the valley in the east and devastated Elam. The country of Media is northeast of Elam. With the decline of Assyria the princes of Media rose, and Cyaxares and Nabopolassar destroyed Assyria. The domain of the Mede princes extended to the east and north of the domain of the Babylonian princes. In the north, in Asia Minor, the Mede princes encountered the Lydian princes. It was the battle of Cyaxares and the Lydian prince Alyattes that was interrupted by the eclipse, in 585, which Thales had predicted.

The country of Persia is southeast of Media. Cyrus became the prince of Persia, under the Mede princes, in 559. In 550 he defeated Astyages, and became the prince of Persians and Medes.

Phrygia.

There is 1 particular,

Midas prince c. 710.

Attested by both Sargon II (ins. II, 4/51) and Herodotus (I, 14), Midas is important. The state of Phrygia, in northwestern Asia Minor, is not important. After Midas it was overrun by Cimmerian hordes.

Lydia.

Lydia is very important. The important particulars are,

Gyges prince c. 680-45,
Alyattes prince c. 615-560,
 Croesus prince 560-46.

The country of Lydia, around the city of Sardis, is in eastern Asia Minor. Alyattes built Lydia into a powerful state, dominating Asia Minor, and the fourth of the kingdoms which after the fall of Assyria vied to become the dominant power on earth.

In 546 Cyrus conquered Lydia, and Lydia became part of his domain.

The four kingdoms are an important step in the development of the state. They are the step from the state of the Assyrian princes into the state of the great kings.

The state acquired permanence with Tiglath-pileser III and his 5 successors. In the two millennia before them times of states and order keep alternating with times of no states and disorder. In the three millennia beginning with them all

times have been of states and order. And the state has steadily and irreversibly brought all human populations under itself.

After the fall of Assyria Babylon dominated the valley of the two rivers, first brought under one state by Sargon. Egypt dominated the valley of the Nile, first brought under a state by Narmer. Media brought under a state populations further east. Lydia brought under a state populations further west. And the contiguous domains of the four indicate clearly the one and only one domain of the great kings.

The century after Ashurbanipal also divides clearly into two parts. From 627 to 550 the four kingdoms rose and fought each other. From 550 to 526 they became one and only one kingdom.

Asia Minor, and the state of Lydia, in particular, is also the bridge between Mesopotamia and Greece. Miletus, the city of Thales, Anaximander, Anaximenes, and Hecataeus, Colophon, the city of Xenophanes, and Ephesus, the city of Heraclitus, are all three on the Aegean coast of Asia Minor, all three at the time of the first philosophers within the domain of the Lydian princes, then the domain of the great kings.

The *Histories*, too, begins with the Lydian princes: Gyges (I, 8-15), Ardys (15), Sadyattes (16), Alyattes (16-25), and Croesus (26-91).

Herodotus reports: the Lydians first cut gold and silver and used it for money; they were also the first shopkeepers; they also invented the games of dice, of bones, of ball, and all other games except draughts (94).

Herodotus reports also: at the time of Croesus the Lydians were the bravest and most valiant soldiers in all of Asia; they fought on horse, with long lances, and were excellent horsemen (79).

Corinth.

The important particulars are 2,

Cypselus tyrant c. 658-28,

Periander tyrant c. 628-588.

Among the Greek cities the state begins much as it begins two millennia earlier among the Sumerian cities. The state appears in quick succession in several cities, and as soon as it appears the cities begin smiting one another. In particular, the state begins in Sparta, with Theopompus, king c. 710, then in Argos, with Pheidon, king c. 669, then in Corinth, with Cypselus, tyrant c. 658-28.

The philosophers, later on, made tyrant a term of abuse, his life 729 times less happy than the life of the philosopher. Early on, the difference, if any, is that a king inherits the rule of a city, a tyrant takes it for himself.

Sparta, under Theopompus, conquered Messenia. Then Argos, under Pheidon, defeated Sparta. Then Corinth rose under Cypselus and Periander. Aristotle writes: Cypselus was a leader of the people, Periander was a warrior (*Politics*, V, 12). Periander built the harbor of Corinth. In the west he controlled Corcyra off the coast in the Ionian Sea, in the south he captured Epidaurus, in the east he arbitrated between Athens and Mytilene, advised the tyrant of Miletus Thrasybulus, maintained relations with the Lydian prince Alyattes. Corinth's, under Periander, is the first attempt to become the dominant power among the Greek states. The state of Corinth is important.

Megara.

The important particular is 1,

Theagenes tyrant c. 632.

Theagenes intervened in Athens, and established his control over Salamis, off the coast of Athens. Athens took Salamis under Solon, and thereafter dominated Megara. The state of Megara is not important.

Sicyon.

The important particular is again 1,

Cleisthenes tyrant c. 585.

Aristotle writes of him as well: Cleisthenes was a warrior (*id.*). Cleisthenes of Sicyon fought Argos in the south, and in the north won the first sacred war, which destroyed the city of Crisa and elevated Delphi with the sanctuary of Apollo into the most prominent shrine of the Greeks. Thereafter Sparta dominated Sicyon. The state of Sicyon is not important either.

The rise of Sparta begins under Anaxandridas, prince c. 560-20, and Ariston, prince c. 550-15. The rise of Athens begins under Pisistratus, tyrant 560-27.

And given the end of the state, why at first Corinth is important, and then only Sparta and Athens are important, distances alone can explain. The distances are, from Sparta to Argos 60 km, from Argos to Sicyon 40 km, from Sicyon to Corinth 20 km, from Corinth to Megara 30 km, from Megara to Athens 30 km. From Sparta, at

one end, to Athens, at the other, the distance is 150 km along the great circle.

The time of the rise of Sparta and Athens is the time also of Cyrus.

Of the Greek cities on the Aegean islands one, Mytilene, under the wise man Pittacus, tyrant c. 590, has been noted among the states reviewed by Aristotle. The particulars indicate one more.

Samos.

The important particular is 1,

Polycrates tyrant c. 538-22.

Polycrates was the first mortal to try to master the sea. He built a navy of 100 fifty-oared ships. An ally of the Egyptian prince Amasis, he was threatened at home by a Spartan intervention, sought the support of Cambyses, and in turn provided Cambyses with 40 ships in his conquest of Egypt. In 522 Polycrates was captured and crucified by Oroetes, the satrap of Sardis.

Polycrates, the first mortal to try to master the sea, is very important. The state of Samos, which begins and ends with Polycrates, is not.

After the Persian war the islands were dominated by Athens.

Miletus.

The important particulars are 2,

Thrasybulus tyrant c. 610,

Aristagoras leader after c. 505.

Neither the city of the first philosophers, nor any other Greek city in Asia Minor, developed into an important state. And the city of the first philosophers alone tried.

Miletus, under Thrasybulus, resisted Alyattes the longest. Thereafter the Greek cities in Asia Minor were ruled by the Lydian princes, then by the great kings.

In 499 Aristagoras led a revolt of the cities. In 497 Aristagoras was killed. In 494 the cities were defeated at sea. Miletus was razed, its inhabitants enslaved.

The Ionian revolt is among the causes of the Persian war.

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Persia.

The state of the great kings has been presented above in some detail.

There are two reports of Herodotus attesting concern with household arrangements.

Herodotus writes: after the death of Cambyses, seven Persians gathered to discuss what form of government to establish; Otanes proposed democracy, Megabyzus proposed aristocracy, Darius proposed monarchy; one man gave us freedom, Darius said, one man will preserve our freedom.

This was in 522.

Also: when Mardonius came to Ionia, a wonderful thing happened, wonderful, that is, for those who do not believe that Otanes proposed that Persia should be a democracy; Mardonius brought down the tyrants and established democracies in the cities.

This was in 480.

The discussion of the seven Persians affected the state, including its household arrangements, as much as the discussions of the philosophers did. Cyrus reports: I am the lord of the earth (cylinder), Darius reports: I am the lord of the earth (Susa ins.), Xerxes reports: I am the lord of the earth (Persepolis ins.), Artaxerxes reports: I am the lord of the earth (Persepolis ins.), Darius II reports: I am the lord of the earth (Susa ins.), Artaxerxes III reports: I am the lord of the earth (Persepolis ins.). Himself, Otanes, the democrat, supported Darius in 522, and served him thereafter.

Mardonius brought down the tyrants and established democracies in the cities. And then the democracies, too, sent their contingents to the army of the great king. Mardonius was the commander of the army defeated at Plataea, and himself fell in the battle.

The important particulars are the 9,

Cyrus prince 559-30,

Cambyses prince 530-22,

Darius prince 522-486,

Xerxes prince 486-65,

Artaxerxes prince 465-24,

Darius II prince 424-05,

Artaxerxes II prince 405-359,

Artaxerxes III prince 359-38,

Darius III prince 336-0.

The conquests of the great kings reached their peak with Darius. Their domain stretched from the city of Barca in the west to the valley of the Indus in the east, 4 900 km apart.

Xerxes told his council: Persians, you know all the peoples Cyrus, Cambyses, and Darius con-

quered; so when I succeeded to the throne, I considered how I can be worthy of my predecessors and how I can give the Persians as much power as they did; I decided to lead my army in Europe, against Greece; if we conquer the Greeks, we will make the limits of Persia the same as the limits of heaven are for Zeus; I will march to the end of Europe, and when the sun looks down, it will see no country bordering on ours, because I will make them all one country.

And he led 1 700 000 men and 1 207 ships.

He was defeated by Sparta and Athens. The consolidation, nevertheless, of the entire domain, Barca to the Indus, is the measure of the success of his reign.

The domain encompassed 77 countries and peoples large enough to pay annual tax of more than 10 talents each. 25 million people. One fifth of all people.

Darius writes: the peoples were in commotion, all peoples fighting each other; I brought an end to the fighting; they no longer fight each other at all, all are at peace; they fear the law; the strong do not fight or destroy the weak.

All are at peace. They fear the law. The strong do not fight or destroy the weak.

And the success of *pax Persica* in taming one fifth of humanity is attested by the fact of the lasting unity of the state. In the one and a half centuries from Xerxes through Darius III the great kings maintained their control over the entire domain conquered by Cyrus, Cambyses, and Darius. In Asia Minor they successfully resisted continuous pressure from Sparta and Athens. There was no revolt of the Greek cities again. There were revolts in Babylon and Egypt, the two countries where the state had the deepest roots. Revolt succeeded only once. Egypt seceded at the accession of Artaxerxes II and remained outside the control of the great kings for two generations. In 343 Artaxerxes III brought Egypt back into the domain of the great kings. And when Alexander defeated Darius III, the entire domain, Barca to the Indus, became his domain.

Dramatically, too, the success of *pax Persica* is attested by the fact of Herodotus and the *Histories*, under Artaxerxes. Herodotus described the domain of the great kings, and in order to do so, Herodotus traveled, from Olbia to Elephantine

and from Cyrene to Susa, the domain of the great kings.

Dramatically as well it is attested by the fact of Ezra and the great assembly, also under Artaxerxes. In order to exclude his people from the mixing of the peoples, Ezra convoked the great assembly under the auspices of the great king, organized by the cupbearer of the great king.

Dramatically as well it is attested by Plato's judgment, under Artaxerxes II and Artaxerxes III: had the Persians won the war, we would have been all mixed together, Greek with Greek, Greek with barbarian, barbarian with Greek, as we now see the peoples ruled by the Persians all mixed together.

After the success of Assyria, the greatest yet, in vanquishing all other states and bringing them under itself, beginning not even three generations later, and building on that earlier success, the success of Persia is greater, by far.

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Persia extended to the Indus in the east. To the west were Athens and Sparta. And at the time of the rise of the three, states appeared further west as well.

Of the Greek cities in the west two, Locri and Catana, have been noted among the states reviewed by Aristotle. The particulars indicate two more.

Acragas.

The important particular is 1,

Phalaris tyrant c. 550.

The cities in the west were colonies of the cities in the east and colonies of the colonies. They thus enter the record with lawgivers, Zaleucus in Locri, laws in 663, Charondas in Catana, laws in the generation around c. 570. As soon as the cities established themselves firmly against the aboriginal populations, however, they began doing what other cities did, smiting one another.

The city of Acragas is close to the southern coast of Sicily. It was the first Greek city in the west to rise, under Phalaris. Phalaris conquered Himera on the northern coast.

Polybius writes: there is the well-known story of the brazen bull; Phalaris had a brazen bull in which he shut his enemies; he would light a fire under the bull, and as the brass grew hot and the

man inside was roasted, the bull was so constructed that the cries of the man came out as the roar of a bull (XII, 25).

Acragas is not important. It was at its peak under Phalaris. Two generations later Syracuse rose, and thereafter Syracuse dominated Sicily.

Syracuse.
Through the time of Alexander the important particulars are 4,

Gelon	tyrant 491-78,
Hiero	tyrant 478-66,
Dionysius	tyrant 405-367,
Timoleon	leader from 344 to c. 335.

Syracuse was a turbulent state, more turbulent even than Athens.

Acragas was a colony of Gela, 60 km to the southeast, on the southern coast, itself a colony of Crete and Rhodes. Hippocrates became the tyrant of Gela c. 498, and campaigned over much of the east of the island. He was killed in 491. His cavalry commander Gelon became the tyrant of Gela. In 485 he seized Syracuse, 90 km to the east, on the eastern coast, moved much of the population of Gela to Syracuse, and made it the center of his domain. In 480 Gelon defeated Carthage at Himera, and became the master of the entire island. His brother Hiero succeeded him. Syracuse was at its peak under Hiero. He intervened in Italy, in Locri, Sybaris, and Croton, colonized Ischia, fought the Etruscans. After his death in 466 Syracuse became a democracy, and lost its dominion over the island. Under the Spartan Gylippus the Athenian expedition in Sicily was defeated in 413. In 405 Dionysius set himself up as a tyrant. He reestablished the control of Syracuse over the eastern coast. Carthage had returned to the island, in 409 Hannibal Mago had destroyed Himera, in 406 Himilco had destroyed Acragas. War with Carthage continued throughout Dionysius's reign, and divided the island, Carthage dominating the west, Syracuse the east. In 367 Dionysius was succeeded by his son Dionysius II. It was Dionysius II whom Plato tried to make into a philosopher king. In 357 civil strife began. One faction asked Corinth for help, and Corinth sent Timoleon. Timoleon arrived in 344 with a force of 700, seized Syracuse, and established his control over the eastern coast. In 341 he defeated the Carthaginians at the Crimi-

sus. He organized Syracuse under a constitution and the cities in the east into a league, and c. 335 resigned his office.

Civil strife followed Timoleon. Agathocles was tyrant 317-289. He campaigned against Carthage in Africa, campaigned in Italy as far as Tarentum, and captured Corcyra in the east. Civil strife followed him as well. Hiero II was tyrant 275-15. He was an ally of Rome in the first Punic war, and again in the second Punic war. His successor Hieronymus allied himself with Carthage. In 212 Marcellus sacked Syracuse. Archimedes was active in the defense of the city and was killed in its sack.

The state of Syracuse is important.

Carthage.

Through the time of Alexander the important particulars are 4,

Malchus	prince c. 550,
Mago	prince c. 520,
Hannibal Mago	general 410-06,
Himilco	general 406-396.

A colony of Tyre, the city of Carthage was founded in 814. Timaeus, in the generation after Aristotle, records the date. He records also the story of the foundation of the city by Elissa or, as she was called by the Libyans, Dido (fr. 82).

Under the great kings Tyre became the base of their navy, and Carthage acquired the dominion over the Phoenician colonies in the west, along the coasts of Africa and Spain, from the city of Leptis on the coast of Libya to the city of Gades beyond the strait of Gibraltar.

The rise of the state of Carthage begins with Malchus. Malchus campaigned in Sicily and Sardinia. He was succeeded by Mago. Trogus reports: Mago organized the army, and established the empire of the Carthaginians; he built also the power of the state (XIX, 1). Polybius records the first treaty of Carthage and Rome in 508, and comments: the treaty shows that the Carthaginians consider Africa and Sardinia to belong to them absolutely (III, 23). Defeated by Gelon in 480, Carthage returned to Sicily in 409. Hannibal Mago and Himilco conquered the west of the island. War with Syracuse continued throughout Dionysius's reign. The peace concluded after his death in 367 divided the island, and thereafter the Carthaginians considered

western Sicily to belong to them absolutely as well.

At this time the domain of Carthage extended from the northern coast of Sardinia in the north to Leptis in Libya in the south, and from Gades beyond the strait of Gibraltar in the west to Leptis in the east, 1 000 km along the meridian, 1 900 km along the parallel. Carthage was the greatest maritime power on earth.

To this time belongs also the *Description of the sea*, composed in Greek in the century before 269, beginning: it pleased the Carthaginians to send out Hanno to found cities beyond the strait of Gibraltar (*id.*), and recounting the voyage of Hanno down the Atlantic coast of Africa.

Carthage and Rome fought the first Punic war in 264-41, the second Punic war in 220-02, and the third Punic war in 149-6. In the first Punic war Rome learned naval warfare from Carthage, and defeated Carthage. In the following years Carthage rose to the peak of its power when Hamilcar began the conquest of Spain north to the Ebro. In the second Punic war Rome learned strategy from Hannibal, and defeated Carthage. After the third Punic war Rome razed Carthage.

Rome.

Through the time of Alexander the important particulars are 5,

Twelve tables	promulgated in 451-49,
Camillus	tr. mil. 403,
Sextius Lateranus	cos. 366,
Licinius Stolo	cos. 364,
Valerius Corvus	cos. 348.

The beginning of the state in Rome, too, poses the difficulty, what is real? what is not? As the beginning of the state poses it in Egypt, in the Sumerian cities, in Ashur, in Sparta and in Athens, in Carthage. As also, for a different reason, the beginning of Judaism poses it, and the beginning of Christianity poses it.

Timaeus dated the foundation of the city of Rome to the year of the foundation of Carthage, 814 (fr. 60). Two and a half centuries later, Varro fixed the date at 753 (Plutarch, *Romulus*, 12), and this remained the base of the chronology of Rome, *ab urbe condita*.

Two generations after Varro, Livy wrote *From the foundation of the city*, *Ab urbe condita*. Livy writes under Augustus, seven and a half centuries

after 753. And the early history of Rome is substantially and essentially Livy.

Livy superseded his sources Coelius Antipater, Quadrigarius, and Valerius Antias, who wrote in the century before him. They had in turn superseded the first Roman historian Fabius Pictor, who wrote, in Greek, two centuries before Livy, five and a half centuries after 753.

Of the Greek historians, Timaeus wrote three centuries before Livy, and before the rise of Rome, and although it is important that he notes Rome, little in his fragments is about Rome. Polybius writes one and a half centuries before Livy, at the time of the rise of Rome and about the rise of Rome, but – with the important exception of recording the treaties between Carthage and Rome – he does not concern himself with events before the Punic wars. Siculus writes in the generation before Livy, when Rome had already risen greatly, and much in him is about Rome, but it adds nothing substantial or essential to Livy. And Livy's contemporary Dionysius of Halicarnassus has left behind the *Roman antiquities*, but it adds nothing substantial or essential to Livy either.

The history of early Rome is Livy. To try and resolve the difficulty, what is real? what is not? the particulars in Livy have to be sorted out. And Livy himself provides the ground on which to resolve the difficulty.

In 753, then, the city of Rome was founded. In particular, Livy reports: begotten by Mars, suckled by a she-wolf, Romulus and Remus built a new city, and when Remus leapt over its wall, Romulus killed his brother to become the eponymous founder of the city, its first king, and upon his death, murdered and dismembered in one account, taken to heaven on a cloud in another, the god Quirinus (I, 4-16).

Romulus was the first of seven kings. The last was Tarquinius Superbus.

In 509 Rome became a republic, under two consuls and a senate. In particular, Livy reports: raped by the son of Tarquinius, Lucretia killed herself rather than bear the dishonor, incensed, the people expelled the Tarquinii, elected Brutus and Collatinus consuls, replaced Collatinus with Valerius Poplicola, and when two of Brutus's sons plotted the restoration of the Tarquinii,

Brutus executed his sons (57-II, 5).

The republic, unlike the kingdom, is well attested independently. The names of the two consuls were the name of the year, and lists of consuls, or years, reach back to the time to which Livy dates the beginning of the republic.

Beginning in 509 *From the foundation of the city* is annalistic. And beginning his account of the year 389, Livy writes: so far I have dealt with matters which are obscure not only because of their antiquity, like things far away which can hardly be described, but because in those days there was but scanty use of writing, the only true guardian of the memory of past events, and such records as there were nearly all perished in the great fire in the city; from this point on, however, a clearer and more accurate account can be given of both the civil and the military deeds of the city (VI, 1).

The difficulty concerns the civil and the military deeds of the city before 389.

To begin with the military deeds, Livy reports, with barely an exception, annual campaigns of the Romans against their neighbors, north, south, and east of the city. West was the sea.

North of the city their neighbors were the Etruscans.

And early on: the Etruscan king Porsenna besieged Rome, Mucius stole in the Etruscan camp to kill the king, but mistakenly killed his scribe instead, was caught, a fire was lighted to torture him, Mucius thrust his hand in the fire and told the king, this is what those who seek glory think of their bodies, Porsenna freed the prisoner, abandoned the siege, and made peace (II, 9-13). This was in 508.

Later on: after a ten-year siege, Camillus conquered the Etruscan city of Veii, the city was razed, its citizens enslaved, its gods removed to Rome (V, 21-3). This is in 396.

Rome to Veii, the distance is 15 km.

Also: a Gaul horde descended from the north and sacked Rome, *vae victis* and the Gauls were collecting the ransom of the Romans when Camillus, recalled from exile and appointed dictator, gathered his troops, vanquished the Gauls, and the people hailed him as a second founder of the city (41-9). This is in 390.

Rome was not to be sacked again for eight

centuries, until 410^{AD}.

It is beginning with the fall of Veii that the fights of the Romans in the north become conquest. Camillus not only has to be burdened with the presumption of reality, he can be hailed as the founder of the city.

South of the city the neighbors of the Romans were the Latini, the Hernici, and the Volsci, east of the city the Aequi, further east the Samnites.

And early on: the Aequi trapped the Roman army on mt Algidus, Cincinnatus was called from the plough and appointed dictator, assembled a new army, defeated the Aequi, celebrated a triumph, and resigned the dictatorship in sixteen days, to return to his field and continue ploughing (III, 25-9). This was in 458.

Rome to mt Algidus, the distance is 20 km.

Later on, the campaigns of the Romans continued, year after year after year. Livy reaches the campaign in 385, and writes: I do not doubt that those who are weary of reading about these endless wars will ask, as I ask on examining the older historians who record these wars, where did the Aequi and the Volsci find so many men for so many defeats (VI, 12)?

Later on still, turning to the year 343, he writes: from this point on wars of greater importance begin, for in that year the first war with the Samnites begins (VII, 29).

From this point on the Romans fought, and won, the first Samnite war in 343-1, the Latin war in 340-38, the second Samnite war in 326-04, the third Samnite war in 298-0. Wars of yet greater importance followed, the war with Pyrrhus, then the Punic wars.

It is with the Latin war that the dominion of the Romans over Latium in the south becomes unconditional. It is with the first Samnite war that their fights in the east become conquest as well.

Valerius Corvus was consul in 348, in 346, in 343, and in 335. Victory in the first Samnite war begins under him. Other generals in the two generations after Camillus can be burdened with the presumption of reality, Valerius Corvus first can be burdened with the presumption also of importance.

And the civil deeds of the city are like the military, obscure before 389, clearer after.

These involve the strife of patricians and

plebeians, and the law.

The patricians dominated the city, the plebeians demanded debt relief, land, and access to the government of the city.

And early on: the plebeians withdrew from the city to the Sacred mount, and fortified themselves there (II, 32). This was in 494.

Pacified, the plebeians returned to the city. Strife continued, and the plebeians withdrew from the city again.

This time: to end the quarrels of patricians and plebeians decemviri were appointed to write down and publish the laws of the city, until then the prerogative of the patricians; the decemviri wrote and published the laws on ten tables, and the laws were approved by the assembly of the people; the following year new decemviri were appointed, they wrote and published laws on two more tables, and these were approved by the assembly of the people as well; the following year the laws were engraved on bronze and displayed in the forum (III, 32-57). This was in 451-49.

Strife continued.

Eventually: the plebeian tribunes Licinius Stolo and Sextius Lateranus proposed the removal of all legal disabilities on the plebeians and the creation of the office of praetor of the city to supervise the implementation of the law; the assembly of the people approved the Licinian-Sextian laws; and the following year Sextius Lateranus was elected the first plebeian consul (VI, 34-42). This is in 367-6.

Sextius Lateranus and Licinius Stolo are very important.

And while the events surrounding the writing and enactment of the Twelve tables – including: the patricians sent to Athens three of their number to copy Solon's famous laws and study the institutions, customs, and laws of the other Greek cities (III, 31) – need not be burdened with the presumption of reality, the Twelve tables are real, and very important. Livy writes: to this day, when so many laws have been passed one upon the other, the Twelve tables remain the source of all public and private law (34). Two generations earlier, Cicero writes: if anyone is looking for the origin and source of the law, the small booklet of the Twelve tables surpasses all by itself all the

libraries of the philosophers in both authority and utility (*On the orator*, I, 195-6).

And the relation of the civil and the military deeds of the city can be described by the relation of the two goddesses, Concordia and Victoria. One century after Livy, Plutarch records an event which Livy does not: after the adoption of the Licinian-Sextian laws, the lifelong adversary of the plebeians Camillus dedicated a temple to Concordia (*Camillus*, 42). Livy reports that a temple to Victoria was dedicated by Postumius Megellus in 294 (X, 33). To judge by the precedence of her temple, Concordia was first, Victoria followed. To judge by the fact that Camillus predates Sextius Lateranus and Licinius Stolo, Victoria was first, Concordia followed.

These are the important particulars, then, through the time of Alexander, the Twelve tables, Camillus, Sextius Lateranus, Licinius Stolo, Valerius Corvus.

The state of Rome begins with the Twelve tables.

Victory follows with Camillus. Much like the Spartans, who for nineteen years incessantly fought for Messenia, the Romans for ten years incessantly fought for Veii.

Concord follows with Sextius Lateranus and Licinius Stolo.

Victory follows with Valerius Corvus.

And victory after victory after victory follows.

Turning to the year 294, Livy writes: 46 years and four books of Samnite wars so far and there are more to come; but can one tire of writing and reading about them if the men who fought them did not tire of fighting them (X, 31)?

They had not tired of fighting them by the time of writing, three centuries later. And one century-odd later still, under Trajan, prince 98-117^{AD}, the state of Rome extended from the Firth of Forth in Britain in the north to the fifth cataract of the Nile in the south and from the Atlantic coast of Spain and Africa in the west to the Persian Gulf in the east, across 38° of latitude, 59° of longitude, 4 400 km along the meridian, 5 600 km along the parallel.

Livy did not tire of writing about them either. He completed the 142 books of *From the foundation of the city*, ending with the year 9^{BC}.

Livy wrote under Augustus. Augustus reports:

500 000 Roman citizens served under my command. Also: I counted 4 937 000 citizens. Under Augustus as well the rise of jurisprudence began. Labeo is a contemporary of Livy, the rise of jurisprudence begins with Labeo, and the development of jurisprudence in the following two and a half centuries, through Julian, and through Paul and Ulpian, cemented *pax Romana*. And *pax Romana* is the decisive step in the taming of humanity.

To complete the review of the states in the four-odd centuries from Tiglath-pileser III through Alexander two states remain, Thebes and Macedon.

In 371 Sparta was the most powerful state west of the state of the great kings, and had been for one and a half centuries. Thebes had never been a powerful state. By 362 Thebes had destroyed Sparta.

In 359 the domain of the great kings extended from the Aegean to the Indus, and theirs had been the most powerful state on earth for two centuries. The domain of the Macedonian kings was the small and backward country around the city of Pella, and theirs had never been a powerful state. By 331 the Macedonian king was the master of the domain of the great kings.

These are the most meteoric rises and falls of states. And the discovery of tactics explains these most meteoric rises and falls of states. Epaminondas discovered tactics. Philip and Alexander greatly advanced logistics and tactics.

Thebes.

The important particular is 1,

Epaminondas leader 371-62.

In 386 Sparta imposed its peace on Thebes. In 382 it occupied Thebes. In 379 Thebes reestablished the confederacy of Boeotian cities under itself. Sparta demanded the dissolution of the confederacy, and in 371 Cleombrotus marched out the Spartan army to enforce the dissolution.

The sources for the battle of Leuctra are Xenophon (*History*, VI, 4), Siculus (XV, 51-6), and Plutarch (*Pelopidas*, 20-3).

Cleombrotus led 10 000 men, Epaminondas faced him with 6 000. Cleombrotus arranged his men in a line, 12 men deep, his best men on his right. Epaminondas arranged his men obliquely, his left close to his foe, his right further behind.

He thinned his right and massed his forces on his left, 50 men deep. He left his best men, the 300 of the sacred band under Pelopidas, in reserve. When the battle engaged, the mass of Epaminondas's left stopped the advance of Cleombrotus's right, prevented its wheeling, pinned it, Pelopidas with the 300 charged its flank, and Epaminondas won a crushing victory.

In 370 Epaminondas invaded the Peloponnese and detached Arcadia and Messenia from Sparta. In 362, at Mantinea, he defeated a coalition including both Sparta and Athens.

The sources for the battle of Mantinea are Xenophon (*History*, VII, 5), Polybius (IX, 8), and Siculus (XV, 84).

The army of the coalition was again arranged in a line. Epaminondas massed his forces on his left in a wedge, and, led by a cavalry charge supported by light troops, the wedge broke the line of his foe, turned it, and won a crushing victory.

This was the end of the importance of the state of Sparta.

Epaminondas fell at Mantinea, and this was the end of the importance of the state of Thebes as well. As Trogus writes: the glory of Thebes was born and died with one man (VI, 8).

Macedon.

The important particulars are 2,

Philip prince 359-36,

Alexander prince 336-23.

Polybius writes of Philip and Alexander: with their indefatigable energy they built a petty kingdom into the greatest and most glorious kingdom on earth (VIII, 10).

The petty kingdom is attested one and a half-odd centuries before Philip, when it became a tributary of Darius (Herodotus, V, 18). Philip's father, Amyntas, was the king of Macedon c. 393-70. In the following decade the kingdom was assailed by Thebes and Athens in the south, the Illyrians in the west, and was on the verge of destruction. In 359 Philip became the regent, and in 356 the king of Macedon.

Philip had been a hostage in Thebes, Trogus reports: living for three years in the house of Epaminondas, the great philosopher and general (VII, 5).

Before Epaminondas battles were fought by

heavy infantry, supported by light troops and cavalry. Epaminondas integrated heavy infantry, light troops, and cavalry. Before him battles were won by numbers and determination. Clausewitz writes: tactics is the theory of the use of troops in battle. Beginning with Epaminondas battles have been won by tactics.

Philip developed the troops demanded by tactics. He developed the mustering, training, and deployment into smaller units of the heavy infantry of the phalanx, of the light infantry to guard the flanks of the phalanx and to support the charges of the cavalry, of the cavalry to penetrate the phalanx of the foe and to destroy the foe pinned by the phalanx, and most important the integration of the three. It is with Philip that troops become a systematically built weapon of sophistication and complexity which makes possible the use of that weapon with both force and flexibility.

Hence, in 358 Philip defeated the Illyrians in the west. In 357 he captured Amphipolis and Pydna, and gained access to the Aegean coast. In 356 he captured Potidaea, in 354 Methone, and gained control of the north Aegean coast. In 352 he conquered Thessaly in the south. In 346 he concluded the second sacred war, and gained control over Delphi further south. Between 346 and 342 he defeated the Illyrians and the Epirotes, and gained access to the Adriatic coast. Between 343 and 341 he defeated the Thracians in the east, and reached the Black Sea coast. In 340 he besieged Byzantium further east. In 338 he won the battle of Chaeronea, and became the master of all Greece.

At Chaeronea Philip massed his phalanx on his right, thinned his center, and massed his cavalry, under Alexander, at 18, on his left. He feigned retreat of the phalanx, turned his foe, the cavalry charged and broke his line, the phalanx advanced, and won a crushing victory.

In 336 Philip was assassinated.

Alexander succeeded him. In 335 he pacified Thrace in the north and Greece in the south. In 334 he crossed to Asia with 35 000 men, 30 000 foot and 5 000 horse (Arrian, *Alexander*, 11). In 334 he defeated a Persian force at the Granicus, in northwest Asia Minor. In 333 he defeated Darius III at Issus, in southeast Asia Minor. In

331 he defeated Darius III at Gaugamela, across the Tigris, and became the master of the domain of the great kings. In 329 he marched east, and in 326 reached the Ganges. In 324 he was back in Babylon, and organized his state. In 323 he died.

At Gaugamela Darius III had vast superiority in numbers. He arranged his army in a line. Alexander arranged his army in a short oblique line, his right in advance, and his force massed on his right. When his foe advanced and engaged his left, Alexander charged with his right, with local superiority penetrated through the left of his foe, reached his rear, turned left, and attacked him in his rear. It is the paradigmatic battle of penetration. Napoleon said: read and reread the campaigns of Alexander, Hannibal, Caesar, Gustav II, Turenne, Eugene, and Frederick II; model yourself on them; this is the only way to become a great general and to discover the secrets of the art of war (maxims, 78).

And the end of the state can hardly be attested more unambiguously than it is by Alexander, too.

Arrian reports: Alexander said, when we march to the ocean in the east, all Asia will be ours; when we next sail south from the Persian Gulf and come back through the strait of Gibraltar, all Africa will be ours; and the limits of our empire will then be the same as the limits Zeus set on the earth (*Alexander*, V, 26). Arrian reports on the authority of Ptolemy (I, 1). Ptolemy was Alexander's general from his first campaign to the last, became, when Alexander's successors divided his domain, the king of Egypt, and wrote himself the history of Alexander.

In 324, back from the east, Alexander organized his state. He married Barsine, the daughter of Darius III (VII, 4). He married 80 of his officers and 10 000 of his soldiers to Persian and Mede wives (5). He had 30 000 Persian boys brought up and trained in Macedonian ways (6). He enlisted in his regiments Persians, Medes, Bactrians, Sogdians, Arachosians, Zarangians, Arians, Parthians, under their own officers (*id.*).

He founded 70 cities, from Alexandria at the mouth of the Nile to Alexandria at the mouth of the Indus.

In 323 the domain from the Adriatic coast in Europe and the coast of Libya in Africa to the valley of the Indus in Asia was – pacified, the

mixing of peoples, including the Greeks now, greatly accelerated – one and only one state. Alexander died at 33. Had he died at 77, as Augustus did, humanity would have fared differently, and better.

ccc

These are the states to which the collection of the particulars points in the four-odd centuries from Tiglath-pileser III through Alexander.

And west of the Indus.

The temporal limits entail the spatial.

Several centuries earlier the state had appeared in the valleys of the Chang Jiang and Huang He, too. The development of the state in the west, however, and the development of the state in the east remained independent of one another for a long time.

Darius conquered the valley of the Indus.

Two centuries later, Alexander reached the Ganges.

Thirteen and a half centuries later, Mahmud, prince of Ghazni 998-1030, advanced further east. Then the penetration of India began.

Two centuries later, Genghis Khan, prince 1206-27, campaigned from the Pacific in the east to the Black Sea in the west. Genghis Khan and his successors in the next two generations, Ögedei, Batu, Möngke, Hülegü, and Kublai, began unifying, first by overrunning and pillaging it, then by ruling it, the domain from the Dniester and the Euphrates in the west to the Pacific in the east. Kublai, prince 1260-94, became Yuan emperor. Polo reached China under Kublai, served Kublai, and wrote the *Description of the world*.

Two-odd centuries later, da Gama reached India in 1498, Albuquerque captured Malacca in 1511, and in 1517 a Portuguese trading station was established in China. And then development in the west and development in the east began converging, gradually, haltingly, to become eventually one joint development.

Two centuries later, Leibniz discovered the importance of China. Civilization is one. And the fundamental problem, which its independent development in the preceding several millennia in the west and in the east poses, is of its differential development. Including most particularly the

problem of the differential development of the state.

Similarly, the state first appeared in the valley of the Nile, three-odd centuries later it appeared in the valley of the two rivers, and the development of the state in the valley of the Nile and the development of the state in the valley of the two rivers remained independent of one another for one and a half millennia. It was after Thutmose, prince c. 1521-09, that development in the valley in the west and development in the valley in the east began converging, gradually, haltingly, to become eventually one joint development. The state is one, and the problem is the same, of its differential development.

And the collection of the particulars can only be the ground of the formulation and solution of these problems, too.

ccci

Here, to the end of showing that the ground is firm enough to capture the acquisition of knowledge in its entirety, the task is the best state. The theoretical problem of the best state, in particular, as distinct from the practical. The problem of the states that there have been, and the best of them, as distinct from the problem of the states conceived in speech, and the best of these.

And from the four-odd centuries from Tiglath-pileser III to Alexander 94 important particulars have been collected pointing to 27 states.

Ignoring Crete, to which in fact no particular points, the distribution of the 94 particulars among the other 26 states, in the order of the first particular in each, is,

Assyria	6,
Urartu	1,
Judah	8,
Egypt	6,
Phrygia	1,
Sparta	14,
Lydia	3,
Argos	1,
Locri	1,
Corinth	2,
Megara	1,
Babylon	3,
Media	2,
Athens	13,

Miletus	2,
Mytilene	1,
Sicyon	1,
Catana	1,
Carthage	4,
Persia	9,
Acragas	1,
Samos	1,
Syracuse	4,
Rome	5,
Thebes	1,
Macedon	2.

And the four-odd centuries divide clearly into three parts. 745 to 627, the 118 years from the beginning of the reign of Tiglath-pileser III to the end of the reign of Ashurbanipal. 627 to 559, the 68 years from the end of the reign of Ashurbanipal to the beginning of the reign of Cyrus. And 559 to 323, the 236 years from the beginning of the reign of Cyrus to the end of the reign of Alexander.

745 to 627.

This is the time of Assyria, and of great growth of the state.

The Assyrian princes build the most powerful state humanity has seen. The state extends 1 700 km east to west, from the Zagros to the Nile. It is the state of one tenth of humanity.

The most powerful state is the outcome of an advance in the development of warfare. The princes develop an army of 100 000, and deploy their army annually, for more than one century. Hence, one tenth of humanity is made to live in peace, and counted among the people of Ashur.

Urtu is brought under the Assyrian state.

Judah is one of the small principalities brought under the Assyrian state as well.

In Egypt, early in this time, after several centuries of disorder, the state is restored. Then it is also brought under the Assyrian state.

And at this time states appear west of the Assyrian state.

In Asia Minor, Phrygia and Lydia. Further west, in Greece, Sparta, Argos, Corinth, and Megara. Further west, in Italy, the outcome of colonization, Locri. These are all tiny, none of them attested at this time by more than 1 particular. The 1 particular attesting Sparta fights incessantly for nineteen years. The 1 attesting Argos

defeats Sparta. The 1 attesting the outcome of colonization is a lawgiver.

627 to 559.

This is the time of the four kingdoms, Babylon, Media, Lydia, and Egypt, and of expansion of the state.

The four kingdoms destroy Assyria, divide its domain, and under them the state reaches further east and west. Attested by 2 or more particulars each, all four are strong states, fighting one another.

In Judah the state is destroyed, and the building of the first church has begun. *Deuteronomy* is from this time. If you obey the law, you will destroy all the peoples whom the lord god delivers to you.

More tiny Greek states appear. Miletus in Asia Minor, Mytilene on the islands, further west Sicyon and Athens, further west Catana. These are attested with 1 particular each at this time. The 1 particular attesting Mytilene is a wise man who slays an Olympic victor in a single combat. The 1 attesting Sicyon is a warrior. The 1 attesting Athens is a lawgiver. The 1 attesting Catana is also a lawgiver. Catana is also the outcome of colonization. Corinth is the first Greek state attested by 2 particulars, and Corinth is the first Greek state to rise, with the second particular, who is a warrior.

Further west, Carthage is attested with 1 particular.

559 to 323.

This is the time of the state of the great kings, Sparta and Athens, Carthage, and Macedon, and of very great growth of the state.

The great kings build the most powerful state humanity has seen. The state extends 4 900 km east to west, from the Indus to Libya. It is the state of one fifth of humanity.

The most powerful state is the outcome of an advance in the development of warfare. In two generations the great kings campaign from deep in the deserts of central Asia to deep in the desert of the Sahara to deep in the wilderness north of the Black Sea. The great king leads 1 700 000 men and 1 207 ships. And, hence, one fifth of humanity no longer fights at all, all are at peace, they fear the law, the strong do not fight or destroy the weak.

Media, Lydia, Babylon, and Egypt are brought under the state of the great kings.

In Judah, within the domain of the great kings, the transformation of a state into a church is complete. This is the time of the Priestly code and Ezra. You do not know me, but from the rising sun to the setting sun there is no one but me.

West of the state of the great kings, on the islands, another state appears, Samos. The 1 particular is the first mortal to try to master the sea, he builds a navy of 100 fifty-oared ships.

Sparta and Athens rise. Sparta dominates the land, Athens dominates the sea. And Sparta and Athens stop the great kings.

Victory and power are the outcome of an advance in the development of warfare. Of 300 Spartans 298 fall, of all 8 000 Spartans 5 000 march out. Athens develops the trireme, and maintains 300 triremes in active service. And Sparta and Athens fight a twenty-seven-year war. Hence, also, the Spartans have helots, the Athenians have allies, helots and allies both pacified.

Argos, Sicyon, Corinth, Megara, Mytilene, and Samos are dominated by Sparta and Athens. Miletus is in the domain of the great kings.

Further west, two more Greek states appear, Acragas and Syracuse. Syracuse rises. All 4 Syracusan particulars fight, the first seizes Syracuse, the following 3 fight the Carthaginians, the second fights also as far north as the Etruscans.

Locri and Catana are dominated by Syracuse. Acragas is destroyed by Carthage.

Carthage becomes a maritime power. Its domain extends 1 900 km east to west, from the domain of the great kings to the Atlantic.

In the north the state begins in Rome. It begins with laws. Victory follows. Concord follows. Victory after victory after victory follows.

The last two states are Thebes and Macedon.

The 1 Theban particular discovers tactics, and destroys Sparta. The destruction of the most powerful Greek state is the outcome of an advance in the development of warfare.

The 2 Macedonian particulars develop logistics and tactics. The first conquers the Balkan peninsula. The second conquers the state of the great kings. The conquest of the Balkan peninsula is the outcome of an advance in the development

of warfare. The conquest of the most powerful state humanity has seen is the outcome of an advance in the development of warfare. The second particular becomes the highest magistrate of the same most powerful state humanity has seen. He marries the daughter of his predecessor. Hence, also, he enlists in his regiments Persians, Medes, Bactrians, Sogdians, Arachosians, Zargians, Arians, Parthians, under their own officers, marries 80 of his officers and 10 000 of his soldiers to Persian and Mede wives, has 30 000 Persian boys brought up and trained in Macedonian ways.

It is one joint cumulative development.

And it is not the outcome of concern with household arrangements. It is the outcome of the development of warfare.

The state expands extensively. It reaches much further in space. And it expands intensively. It becomes much more powerful.

By the time of Tiglath-pileser III the surface of the earth under the state does not extend beyond the 1 700 km east to west, from the Zagros to the Nile, which Tiglath-pileser III and his 5 successors will bring under their state. Outside this domain only a part of Asia Minor had also been under the state, under the Hittite state, and this state had disappeared in disorder four and a half centuries earlier. Four-odd centuries after Tiglath-pileser III, at the time of Alexander the surface of the earth under the state extends 6 900 km, from the Indus to the Atlantic.

At the time of Tiglath-pileser III and his 5 successors 10 million are under the state, one tenth of humanity. Four-odd centuries later, at the time of Alexander 40 million are under the state, more than one quarter of humanity.

Of the 26 states, 3 are older, 23 begin at this time. They can thus inform the question of the beginning of the state.

The beginning of the state is beset with difficulty.

This is how Livy describes the difficulty with respect to the beginning of Rome. Livy writes: the traditions before the foundation of the city are poetic lore rather than true deeds; it is the privilege of antiquity, in order to add dignity to the beginning of the city, to mix human things with divine (pr.).

And if antiquity has stopped exercising its privilege, the philosophers who have concerned themselves with the state have continued exercising it.

On the evidence of the particulars, then, Assyria, Urartu, and Egypt begin earlier. At this time, the Assyrian princes build the most powerful state on earth. Urartu is destroyed. Egypt is restored after two centuries of disorder, then it is brought under Assyria.

The remaining 23 begin at this time.

Babylon and Media are direct heirs of Assyria. They begin with the destruction of Assyria. And they continue to fight, until they are themselves destroyed.

Further west, the attempt of Midas to build a state in Phrygia is abortive, it succeeds only in enriching the poetic lore.

The attempt to build the state in Lydia is successful. Lydia is an indirect heir of Assyria. Lydia rises, fights, the Lydians become the bravest and most valiant soldiers in all of Asia, until Lydia is also destroyed.

Further west, the particulars point to 13 Greek states. These are the most successful among the many more attempts of the Greek cities to build the state, and the majority of these 13 attempts are abortive, too.

6 are on the mainland, Sparta, Argos, Athens, Corinth, Megara, and Sicyon. Sparta begins with the conquest of Messenia. Argos begins with a victory over Sparta. Megara begins fighting for Salamis. Sicyon begins with a warrior. Corinth rises to prominence under a warrior. Then Sparta and Athens rise. Having begun under a victorious prince, Sparta continues under victorious princes and generals, until it is destroyed. Athens begins with a lawgiver. Vigorous lawgiving continues, but the lawgivers become also generals. Lawgiving becomes even more vigorous, but Athens becomes the state of 300 triremes in active service. Then it is destroyed.

3 are in the east, in Asia Minor and on the islands, Miletus, Mytilene, and Samos. Miletus begins with resistance to the Lydian kings. Mytilene begins with a wise man. They begin and end with the resistance and the wise men, respectively, dominated by the states further east and west, respectively. Samos begins with the first

attempt at mastery over the sea. And ends with it.

4 are in the west, Locri, Catana, Acragas, and Syracuse. These are colonies. Locri and Catana begin with lawgivers, then are dominated by states which do not begin with lawgivers. Acragas begins with conquest, then it is destroyed. Syracuse begins by being conquered. It continues by conquest, and fighting.

If the state begins independently with the Greek cities, it does not begin differently with the Greek cities. The degree to which it begins independently with the Greek cities cannot be large either, given the spatial and temporal contiguity with Lydia. Then the expeditions of the great kings reach the Greek states and thereafter development is intensely joint. Achieved before in the large states in the east, the great advances in the development of warfare are thereafter achieved in the smaller states in the west.

Persia is the direct heir of Media. It begins with the conquest of Media. It then conquers much of the earth, and pacifies it, until it is conquered.

Carthage is also a colony. It rises when its metropolis is brought under the great kings. And continues to rise.

Rome begins with laws. And lawmaking much more vigorous than in Athens continues. The lawmakers are generals from the first, however, and ever better generals.

If the state begins independently in Rome, it does not begin differently in Rome either. Nor can the degree to which it begins independently in Rome be large either, given the spatial and temporal contiguity with Syracuse and Carthage.

Thebes is a direct heir of Spartan and Athenian warfare. It begins with the general who discovers tactics. And it ends with him.

Macedon is a direct heir of Theban warfare. It begins with the general who greatly advances logistics and tactics. And it rises spectacularly with the general who advances tactics spectacularly, and becomes the highest magistrate of the most powerful state humanity has seen.

Judaism, finally, is the direct heir of the destruction of Judah. The law is all-important. If you obey the law, you will destroy all the peoples whom the lord god delivers to you.

The state does not begin with a covenant, or a

compact, or a contract. On the evidence of the particulars, in the four-odd centuries from Tiglath-pileser III to Alexander, the state begins – to borrow the term from the Sumerian prince list – with smiting.

At the time of Tiglath-pileser III, prince 745-27, there is one state dominating all states, bringing them under itself, Assyria.

One century later, at the time of Ashurbanipal, prince 668-27, there is one state dominating all states, bringing them under itself, the same, Assyria.

One century-odd later, at the time of Darius, prince 522-486, there are three states, dominating all states, bringing them under themselves, the state of the great kings, Sparta, and Athens. Two more states are laboring at becoming states dominating all states, bringing them under themselves, Carthage and Syracuse.

Not quite one century later, at the time of the Peloponnesian war, 431-04, there are four states dominating all states, bringing them under themselves, the same, the state of the great kings, Sparta, Athens, and Carthage. There is also the first church, dominating all states, in imagination, bringing them under itself.

Not quite one century later, under Alexander, prince 336-23, there are two states dominating all states, bringing them under themselves, the state of the great kings, under Alexander now, and Carthage. There is also the church. Rome is also just beginning.

All in all, through the four-odd centuries, five states dominate all states, bringing them under themselves, Assyria, Sparta, Athens, the state of the great kings, and Carthage. Or, counting Macedon separately, six states. There is also the church, Judaism.

It is not the outcome of concern with household arrangements. It is the outcome of the development of warfare.

It is 422 years from the beginning of the reign of Tiglath-pileser III to the end of the reign of Alexander.

Counting only the times of great advance in the development of warfare – the 118 years from the beginning of the reign of Tiglath-pileser III in 745 to the end of the reign of Ashurbanipal in 627 – the 79 years from the beginning of the

reign of Cyrus in 559 to the expedition of Xerxes in 480 – the 76 years from the victory of Athens and Sparta at Salamis and Plataea in 480 to the end of the Peloponnesian war in 404 – and the 48 years from the victory of Epaminondas at Leuctra in 371 to the end of the reign of Alexander in 323 – the total is 321 years. 321 years, out of 422, of great advance in the development of warfare.

Hence, the state also discovers the problem of peace, and begins solving it.

Counting the 118 years of *pax Assyrica* from the beginning of the reign of Tiglath-pileser III in 745 to the end of the reign of Ashurbanipal in 627, when one tenth of humanity is made to live in peace, and counted among the peoples of Ashur – and the 236 years of *pax Persica* from the beginning of the reign of Cyrus in 559 to the end of the reign of Alexander in 323, when at first one fifth of humanity, and at length one quarter of humanity, no longer fights at all, all are at peace, they fear the law, the strong do not fight or destroy the weak – the total is 344 years.

War is not battle only, but the tract of time when the will to battle is sufficiently known; war is not actual fighting, but the known disposition to it; all other time is peace.

Of all the 422 years all 422 are a tract of time when the disposition to fighting is sufficiently known. In not less than one half of these years the disposition is manifested by actual fighting. But 344 of these 422 years are also the tract of time when the disposition to fighting begins being vanquished.

And given so much, which the best state is, in the four-odd centuries from Tiglath-pileser III to Alexander, is clear.

Polybius is the only philosopher who has concerned himself with the state after, rather than before, its examination.

Polybius writes: Plato's republic is praised highly by many philosophers; but we do not admit to the contests athletes who have not trained for them; and to bring Plato's republic to a discussion of the constitutions of Sparta, Rome, and Carthage is like bringing a statue to compete with living and breathing men; the craftsmanship may be praiseworthy, the comparison is yet ridiculous (VI, 47).

And what the contest has been, and who the

athletes have been who have trained for it, and who have won it, is clear.

Assyria is the best state.

At first. Then the great kings build a better state, and the state of the great kings from Cyrus through and including Alexander is, consequently, and overall, the best state.

Sparta and Athens are very good states.

Carthage is a very good state.

Considered separately, Macedon is a very good state.

Babylon, Media, Egypt, and Lydia are good states.

Corinth and Syracuse are good states.

Rome is just becoming a good state.

Urtu is, at this time, not a good state.

Phrygia, Argos, Locri, Megara, Miletus, Mytilene, Sicyon, Catana, Acragas, Samos, and Thebes are not good states, they are all abortive attempts at building a state, and only Samos and Thebes among them valiant attempts.

Crete is not even an attempt at building a state.

Judah, finally, is not a good state either, but Judaism is a very good state. The first church is tiny, and at the mercy of other states, yet showing itself from the first indestructible.

ccci

It remains to determine the relation of the state, and of the best state, to the acquisition of knowledge, and hence, to separate among the important among the particulars the steps of advance in the acquisition of knowledge.

Assyria.

Above, in the course of the collection of the particulars from the Sumerian prince list, the great advance in the acquisition of knowledge by the princes in the first six and a half centuries of development of the state in the valley of the two rivers, from Enmebaragesi to Ibbi-Sin, has been analyzed into the solutions of the four problems, of time reckoning, of the description of the surface of the earth, of the description of the past, and of the end of the state.

And the Assyrian princes from Tiglath-pileser III to Ashurbanipal attest great advance in the solutions of all four of these problems.

Time reckoning.

In the valley of the two rivers the explicit

distinction of one year from another begins with Sargon. Sargon is the first prince to leave behind names of the years of his reign. And reckoning years by the names bestowed on them by the deeds of a prince remained the way of reckoning years for the following two millennia and through the Assyrian princes.

Three centuries after Sargon, the reckoning of time across centuries begins with the Sumerian prince list. The list arranges the princes with their reign years in their succession in each state, arranges the states in their succession and totals the reign years for each state, and then totals the totals. From the beginning of the reign of Sargon to the end of the reign of Ibbi-Sin the total is 417 years, 6 months, and 47 days. And with this result the problem of reckoning time across centuries is for the first time solved. From the beginning of the reign of Sargon to the end of the reign of Ibbi-Sin the total is, in fact, 324 years $\pm$ 15 years.

Thirteen and a half centuries later, Esarhaddon reports:

Esarhaddon, the lord of the earth, the prince of Ashur, the son of Sennacherib, the lord of the earth, the prince of Ashur, the son of Sargon, the lord of the earth, the prince of Ashur;

my ancestor Shalmaneser, the prince of Ashur, the son of Adad-nirari, the prince of Ashur, built the temple of Ashur; the temple fell into ruin; 586 years passed and the temple fell into ruin; on the place where the temple stood I established its new foundation, gold, silver, and gems; I built it, I made it magnificent; for the wonder of the people, for life, for the length of days, for the firmness of my reign, for the well-being of my posterity, for the power of the princship, for the destruction of its foes, for the prosperity of Ashur, for the good life of Ashur, I built it (ins. II, 702).

And when Esarhaddon reports that from his ancestor Shalmaneser, the prince of Ashur, the son of Adad-nirari, the prince of Ashur, to himself, Esarhaddon, the lord of the earth, the son of Sennacherib, the lord of the earth, the son of Sargon II, the lord of the earth, 586 years have passed, Esarhaddon reports a fact. 586 years have passed. The problem of time reckoning has been solved conclusively.

The second most important list, the Assyrian prince list, shows this.

It begins with the names of:

17 princes who lived in tents (*id.*).

It has next the names of:

10 princes who were ancestors (*id.*).

It has next the names of:

6 princes whose names were written on bricks (*id.*).

The last 3 of these princes are:

Puzur-Ashur,
Shallim-ahhe,
Ilushuma (*id.*).

The list has next:

Erishu, the son of Ilushuma, he reigned 40 years (*id.*),
and beginning with Erishu it has the names, fathers, and number of reign years of the princes.

Beginning with Puzur-Ashur, the princes in the list are real. And these are, beginning with him, and assigning dates, the first 11 princes in the list, the important on the left, the not important on the right,

Puzur-Ashur	prince c. 1980,
Shallim-ahhe	prince c. 1960,
Ilushuma	prince c. 1940,
Erishu	prince c. 1920,
Ikunu	prince c. 1900,
Sargon	prince c. 1880,
Puzur-Ashur	prince c. 1860,
Naram-Sin	prince c. 1840,
Erishu II	prince c. 1820,
Shamshi-Adad	prince c. 1809-776,
Ishme-Dagan	prince c. 1776-59.

Puzur-Ashur left behind no inscriptions, but is attested by the inscriptions of his 3 successors. Shallim-ahhe left behind 1 inscription, Ilushuma 2 inscriptions, Erishu one dozen inscriptions, Ikunu 4 inscriptions, Sargon 2 inscriptions, the next 3 princes none, Shamshi-Adad two dozen inscriptions, Ishme-Dagan none.

Toward the end of that time Ashur fell to Hammurabi.

The list has next:

Ashur-dugul, the son of nobody (*id.*),
then:

6 princes, the sons of nobodies (*id.*),
then 11 princes of whom only 1 is:
the son of nobody (*id.*),
but who were all 11 nobodies themselves, to judge by the fact alone that they left no other

trace in the record.

This was the time when the valley of the two rivers was dominated by the princes of Babylon, the successors of Hammurabi.

Babylon fell c. 1594.

The next 4 princes in the list are,

Shamshi-Adad III	prince c. 1590,
Ashur-nirari	prince c. 1570,
Puzur-Ashur III	prince c. 1550,
Enlil-nasir	prince c. 1530.

They left behind between 2 inscriptions and one dozen inscriptions each.

This is the time also when the disorder brought by the Kassite and Hurrian hordes began.

The next 6 princes in the list left behind, between the 6 of them, 1 inscription.

All dates above rest on long chains of inferences. And the foundation of all chains is the following series of princes in the list. The list has next 41 princes through Shalmaneser V, under whom, given that he is the last prince in the list, the list was compiled. Adding his 4 successors, it is a continuous series of 45 princes, the highest magistrates of one and the same state for eight centuries,

Ashur-bel-nisheshu	prince c. 1417-08,
Ashur-rim-nisheshu	– prince c. 1408-0,
Ashur-nadin-ahhe II	– prince c. 1400-390,
Eriba-Adad	prince c. 1390-63,
Ashur-uballit	prince c. 1363-27,
Enlil-nirari	prince c. 1327-17,
Arik-den-ili	prince c. 1317-05,
Adad-nirari	prince c. 1305-273,
Shalmaneser	prince c. 1273-43,
Tukulti-Ninurta	prince c. 1243-06,
Ashur-nadin-apli	prince c. 1206-3,
Ashur-nirari III	prince c. 1203-197,
Enlil-kudurri-usur	prince c. 1197-2,
Ninurta-apil-Ekur	prince c. 1192-79,
Ashur-dan	prince 1179-33,
Ninurta-Tukulti-Ashur	– prince 1133,
Mutakil-Nusku	prince 1133,
Ashur-resh-ishi	prince 1133-15,
Tiglath-pileser	prince 1115-076,
Ashared-apil-Ekur	prince 1076-4,
Ashur-bel-kala	prince 1074-56,
Eriba-Adad II	prince 1056-4,
Shamshi-Adad IV	prince 1054-0,

Ashur-nasirpal	prince 1050-31,
Shalmaneser II	prince 1031-19,
Ashur-nirari IV	prince 1019-3,
Ashur-rabi II	prince 1013-972,
Ashur-resh-ishi II	prince 972-67,
Tiglath-pileser II	prince 967-35,
Ashur-dan II	prince 935-12,
Adad-nirari II	prince 912-891,
Tukulti-Ninurta II	prince 891-84,
Ashur-nasirpal II	prince 884-59,
Shalmaneser III	prince 859-24,
Shamshi-Adad V	prince 824-11,
Adad-nirari III	prince 811-783,
Shalmaneser IV	prince 783-73,
Ashur-dan III	prince 773-55,
Ashur-nirari V	prince 755-45,
Tiglath-pileser III	prince 745-27,
Shalmaneser V	prince 727-2,
Sargon II	prince 722-05,
Sennacherib	prince 705-680,
Esarhaddon	prince 680-68,
Ashurbanipal	prince 668-27.

The earliest of these dates are certain within ± 3 years. Beginning with Ashur-dan, prince 1179-33, the dates are certain without qualification.

The dates of the 45 princes are corroborated amply by the inscriptions of the princes. The inscriptions are steadily more numerous. Beginning with Adad-nirari, prince *c.* 1305-273, the inscriptions become chronicles, the systematic record of the deeds of the princes. Beginning with Tiglath-pileser, prince 1115-076, the inscriptions include annals, the record of the deeds of the princes year by year. Beginning with Adad-nirari II, prince 912-891, the princes have left behind the names of all years. Beginning with Ashur-nasirpal II, prince 884-59, the inscriptions become massive chronicles. And beginning not later than Sargon II, the inscriptions are rather too many than just many and inform posterity of rather too much than just much.

The dates of all contemporary events depend on the chronology of these 45 princes fixed in the Assyrian prince list. Including all early dates in Herodotus. Including all that the Hebrew scriptures report. And all earlier dates depend on the chronology of these 45 princes fixed in the Assyrian prince list. Back to Enmebaragesi in the

valley of the two rivers, back to Narmer in the valley of the Nile.

Ptolemy fixed chronology absolutely from the motions of the planets, and in particular, fixed absolutely the dates of Tiglath-pileser III, Shalmaneser V, and Sargon II. And, hence, from the Assyrian prince list, all chronology is fixed absolutely.

The Assyrian princes solved the problem of time reckoning conclusively.

The description of the surface of the earth.

Enmebaragesi attests 1 locality. One century later, Mesanepada attests 2. Two generations later, Ur-Nanshe attests 4. Two generations later, Eanatum attests 16.

One and a half centuries later, Sargon and his 4 successors attest 58.

Sixteen centuries later, Tiglath-pileser III and his 5 successors conquered a domain twice larger than the domain of Sargon and his 4 successors. And the description of the surface of the earth advanced greatly quantitatively and qualitatively.

An inscription of Tiglath-pileser III begins:

Tiglath-pileser, the great prince, the true prince, the lord of the earth, the prince of Ashur, the prince of Babylon, the prince of Sumer and Akkad, the lord of the four quarters, the prince who brought under his sway the lands from the sea of the rising sun to the sea of the setting sun (ins. I, 787/804).

And in this one inscription, in which he records the countries and cities which:

I brought under my sway from the 1st year of my reign to the 17th year of my reign (*id.*), Tiglath-pileser III notes, other than Ashur, Babylon, Sumer, Akkad, the sea of the rising sun, and the sea of the setting sun, the following localities:

Bit-Iakin, Bikni, Egypt, Itu, Rubu, Hamarani, Luhuat, Harilu, Rubbu, Rupiku, Hiranu, Rabilu, Nasiru, Gulusu, Nabatu, Rahiku, . . . , Rummulutu, Adile, Kipre, Ubudu, Gurumu, Hadadu, Hindiru, Damunu, Dunanu, Nilku, Rade, . . . , Ubulu, Karma, Amlatu, Rua, Kabi, Litau, Marusu, Amatu, Hagaranu, Dur-Kurigalzu, Adinni, Sarragiti, Labbanat, Kar-belmatati, Arameans, Tigris, Euphrates, Surappi, Uknu, Humut, Kar-Ashur, Sippar, Nippur, Borsippa, Kutha, Kish, Dilbat, Uruk, Babylon, Pukudu, Lahiru, Hilimmu, Pillutu,

Elam, Arrapha, Labdudu, Sarrabani, Tarbasu, Iabullu, Bit-Shaalli, . . . Amlilatu, Sapia, Bit-Shilani, Bit-Amukkani, Larak, Namri, Bit-Sangibuti, Bit-Hamban, Sumurzu, Bit-Barrua, Bit-Zualzash, Bit-Matti, Nuku, Tupliash, Bit-Taranzai, Parsua, Bit-Zatti, Bit-Abdadani, Bit-Kabsi, Bit-Sangi, Bit-Tazzakki, Bit-Ishtar, Zakruti, Gizinkissi, Nishsha, Sibur, Urumzan, Ruasan, Uparia, Bustus, Ariarmi, Saksukni, Arakuttu, Kar-Zibra, Gukinnana, Bit-Sakbat, Silhazi, Ushkakkana, Shikraki, Medes, Tarlugale, Ellipi, . . . , Urartu, Melid, Kummuhu, Kishtan, Halpi, Sinzi, Bira, . . . , Tema, Saba, Haiapa, Badana, . . . , Kue, . . . , Hamath, Samal, Gurgum, . . . , Kaska, Tabal, Tuna, Tuhana, Ishtunda, Hubishna, Arvad, Beth-Ammon, Moab, . . . , Ashkelon, Judah, Edom, . . . , Gaza, . . . , Tyre, Hittites, Kalhu, Amanos, Lebanon, Anti-Lebanon (*id.*).

149 localities, not counting those, some dozens of them, whose names are lost in lacunae.

And his 5 successors note steadily more.

Thus quantitatively.

Qualitatively, with the steadily more detailed descriptions of their campaigns the inscriptions of the princes have become steadily more elaborate chronicles, and have thus also begun steadily more clearly and distinctly to imply maps.

This is the beginning of the campaigns of Sargon II in his 8th year. Sargon II reports:

in the month of Dumuzi, I set out from Kalhu, the city of my princeship;

I boldly crossed the Upper Zab;

I led my army across the Lower Zab;

I entered the passes of the mountains of Kullar in the land of the Lullubeans which they call Zamua;

in the land of Sumbi I inspected my army, I counted my infantrymen, cavalrymen, and chariots;

I marched into the high mountains of Zikirtu and Andini;

I passed between the lands of Nikappa and Upa, where the cedar groves never see the sun;

in the gully between them I crossed the Puia;

in the mountains of Simirru, whose summit reaches the heaven, impenetrable for chariots and horses, I cut into the ravines;

I crossed seven mountains, crossed the Rapaa,

crossed the Aratta, and descended on Surikash on the border of Karalli and Allabria (ins. II, 140/78).

The description of the past.

As the description of space, so the description of time. Tiglath-pileser III and his 5 successors are the decisive step toward Herodotus, the father of history. The problem of time reckoning has been solved. The record has become chronicles. It has begun to imply maps.

And the record of Tiglath-pileser III and his 5 successors has unified the area from the Aegean Sea in the west to the Zagros mountains in the east and from the Black Sea in the north to the valley of the Nile in the south temporally as well as spatially. Within their domain, in the north, the history of Urartu is inseparable from their record. In the east, the history of Babylon is inseparable from their record. In the west, the history of Israel and Judah is – for all the millennia-long efforts to separate it – inseparable from their record. Further west, the history of Egypt has become intertwined with their record. In the far east, they first attest the Medes. In the far north, they first attest Phrygia. In the far west, they first attest Lydia. Sargon II writes about the Phrygian prince Midas (ins. II, 4/51). And so does Herodotus (I, 14). Ashurbanipal writes about the Lydian prince Gyges (ins. II, 765/840). And so does Herodotus (I, 8-15).

In each of the three, time reckoning, the description of the surface of the earth, and the description of the past, and the more so in all three jointly, the advance of the Assyrian princes from Tiglath-pileser III to Ashurbanipal over their predecessors is very great. In each of the three also, and the more so in all three jointly, before these Assyrian princes advance is halting, times of advance are followed by times of no advance at all, it is at this time, with these Assyrian princes, that advance becomes sustained and continuous.

Each of the three shows, and the more so all three jointly show, advances in the acquisition of knowledge as great and as many as have been achieved through this time. Each of the three shows, and the more so all three jointly show, that the labors of these princes are part of the acquisition of knowledge substantially and essen-

tially, and a large part of it, too.

The labors of the princes are first and foremost labors of building the state. The advance in the development of the state is part of the acquisition of knowledge substantially and essentially, and a large part of it, too.

The relation, moreover, is causal.

A cause is an object followed by another, and where all the objects similar to the first are followed by objects similar to the second, or, in other words, where, if the first object had not been, the second never had existed.

Had the labors of these Assyrian princes at building the state not been, the advance in time reckoning, the description of the surface of the earth, and the description of the past never had existed.

The advance in the development of the state under the Assyrian princes from Tiglath-pileser III to Ashurbanipal is a principal cause of the acquisition of knowledge.

And there is further evidence for this conclusion.

Two generations separate Ashurbanipal from the first philosopher.

The problem of the earth begins with the first philosopher. The problem of the earth is from observation. And the relation of the labors of these Assyrian princes to observation is not remote.

There is the Venus tablet of Ammisaduqa. Then there is nothing for two centuries. Then there is the map of Nippur. Then there is nothing for seven centuries. Then a series of observations of lunar eclipses begins in 747, the oldest accurate observations are of lunar eclipses from 721 to 720, another series of observations of lunar eclipses begins in 703, *The plough* records the morning risings of three dozen stars, *When Anu and Enlil* records more stars, from 647 to 633 Saturn is observed. All these observations are made in Babylon. The sustained observation of the heavenly bodies begins under the Assyrian princes from Tiglath-pileser III to Ashurbanipal. When time begins being reckoned accurately across 586 years, hundreds of localities are collected from a domain 1 700 km across, and this domain is unified temporally as well as spatially.

The physical problem of the soul is from the development of philosophy. The ethical problem of the soul is also from the development of philosophy. If ethics, however, is the generalization from what is good to what is the best for all of you, the relation of these Assyrian princes to ethics is not remote either.

Two and a half-odd centuries separate Tiglath-pileser III and Socrates. The first of these two and a half-odd centuries is the time when humanity, one tenth of it over a large part of its habitat, is vigorously mixed, made to live in peace, counted among the peoples of Ashur, by these Assyrian princes. The last of these two and a half-odd centuries is the time when humanity, one fifth of it now over a much larger part of its habitat, continues to be vigorously mixed, no longer to fight at all, be at peace, fear the law, the strong not to fight or destroy the weak, by the direct successors of these Assyrian princes, the great kings. Part of the mixing is the attempt, too, in the generation of Socrates's parents, to include the people of Socrates and their habitat in the mixing, and by actually bringing to them, for their inspection, a very substantial part of humanity, 1 700 000 of them.

The description of the past advanced greatly with the labors of these Assyrian princes, and so did the description of the future. It is in the century before Ashurbanipal that divination becomes prominent in the record. It is at this time, under these Assyrian princes, that the 7 000 omens of *When Anu and Enlil* are collected. At this time as well the prophecy collected in the Hebrew scriptures begins.

The dates of the oldest prophetic books – calculated from their own reports, fixed by synchronisms with the Assyrian princes – are,

Amos	prophecy between c. 791 and c. 750,
Hosea	prophecy between those of <i>Amos</i> and <i>Isaiah</i> ,
Isaiah	prophecy began c. 750,
Micah	prophecy between c. 750 and c. 697.

Amos is contemporary with the immediate predecessors of Tiglath-pileser III, when Israel has already been ruined. *Hosea*, *Isaiah*, and *Micah* are contemporary with Tiglath-pileser III, Shal-

maneser V, Sargon II, and Sennacherib, who destroy Israel and ruin Judah. And the beginning of Hebrew prophecy is not just contemporary with the Assyrian princes. The Assyrian princes are its cause. The effect, in particular, of the campaigns of the Assyrian princes against Israel and Judah, Hebrew prophecy is concerned with nothing but the destruction of Israel and Judah by the Assyrian princes. The prophets prophesy, too, what, to judge by what the princes report, has already happened.

Amos: thus said the lord, an enemy will fall upon your land, will strip you of your strength, will plunder your palaces (3).

Hosea: and the people will be taken to Assyria, tribute to its great king.

Isaiah: the lord will bring against you the king of Assyria and his multitude, he will overflow your land, he will sweep over you like a flood.

Micah: when the Assyrian comes and tramples on our fortresses, we will raise seven shepherds and eight princes, and they will shepherd the Assyrian by the sword (5).

Nor did they shepherd the Assyrian by the sword, the seven shepherds and eight princes.

There is the fact also of Ashurbanipal's library.

Nothing comparable is attested earlier.

There are archives earlier. The oldest archive, from Ur, fourteen centuries earlier, contains 23 documents. The most important archive, from Amarna, seven centuries earlier, contains some 380 documents.

Ashurbanipal's library contains more than 30 000 tablets. Not less than 1 000 distinct compositions. Including *Gilgamesh*, *When on high*, and *When Anu and Enlil*, including the Venus tablet of Ammisaduqa.

Time reckoning, the description of the surface of the earth, the description of the past, and observation attest the acquisition of knowledge. They all show the winning of propositions clearly and distinctly. From the thought and the sense to the reference and the true, and they all show the reference and the true clearly and distinctly.

The mixing of peoples, prophecy, and books do not attest the acquisition of knowledge. The generalization implicit in the mixing, however, all conquered peoples are people of Ashur, the

inference implicit in prophecy, as the past so the future, and the generalization implicit in the collection of 1 000 books attest the acquisition of knowledge.

And the advance in the development of warfare attests the acquisition of knowledge.

I took 7 350 people, and I selected from the people I took 50 chariots, 200 cavalrymen, and 300 infantrymen for my host. – I took 90 580 people, and I selected 150 chariots, 1 500 cavalrymen, 20 000 bowmen, and 10 000 shield-bearers for my host. – I set out from Kalhu, crossed the Upper Zab, led my army across the Lower Zab, entered the passes of the mountains of Kullar, counted my infantrymen, cavalrymen, and chariots in the land Sumbi, marched into the mountains of Zikirtu and Andini, passed between the lands of Nikappa and Upa, crossed the Puia in the gully between them, cut into the ravines in the mountains of Simirru, impenetrable for chariots and horses, crossed seven mountains, crossed the Rapaa, crossed the Aratta, and descended on Surikash on the border of Karalli and Allabria.

The winning of these propositions would have been an advance in the acquisition of knowledge even if the actions of the princes affirmed the propositions only implicitly. And the princes affirm the propositions explicitly.

Building a host of 100 000, and deploying the host annually for more than one century, the princes solved theoretical problems, irrespective of whether they thus solved also practical problems. Difficult theoretical problems.

And the advance in the development of the state attests the acquisition of knowledge.

Tiglath-pileser, the great prince, the true prince, the lord of the earth, the prince of Ashur, the prince of Babylon, the prince of Sumer and Akkad, the lord of the four quarters, the prince who brought under his sway the lands from the sea of the rising sun to the sea of the setting sun. – And I set my official as a governor over them. – And I counted them among the peoples of Ashur. – Ashurbanipal, the great prince, the true prince, the lord of the earth, the prince of Ashur, the lord of the four quarters, the prince of princes, the prince without rival from the upper sea to the lower sea, he made all princes from Tyre in the upper sea to Dilmun in the lower sea prostrate

themselves before his feet, and pull the straps of his yoke.

The princes won these propositions.

Building the most powerful state, bringing very nearly all states under this state, bringing one tenth of humanity under it, and mixing it, the princes solved theoretical problems, irrespective of whether they thus solved also practical problems. Difficult theoretical problems. Real problems, too, as distinct from those conceived in speech.

Shalmaneser V reigned but 5 years. The steps of advance in the acquisition of knowledge are the 5, Tiglath-pileser III, Sargon II, Sennacherib, Esarhaddon, and Ashurbanipal.

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Egypt.

Necho dispatched a fleet of Phoenicians to circumnavigate Africa.

Necho is a step of advance in the acquisition of knowledge.

The state of the great kings.

The expeditions of the great kings reached from the northernmost shores of the Black Sea in the north to the southernmost shores of the Red Sea in the south and from the strait of Gibraltar in the west to the mouth of the Indus in the Indian Ocean in the east, across 33° of latitude and 73° of longitude. The portion of the earth brought under exploration was increased by an order of magnitude, some 10 times.

The expeditions of the great kings are the decisive step toward the development of geography. Hecataeus, under Darius, wrote the first geographical treatise. He describes 300-odd localities, their distribution coinciding precisely with the reach of the expeditions of the great kings. Herodotus, under Artaxerxes, describes 1 012 localities, their distribution coinciding with the reach of the expeditions of the great kings in the east and west, reaching but a little further in the south and north.

The great kings advanced warfare. In two generations they led armies from deep in the deserts of central Asia to deep in the desert of the Sahara to deep in the wilderness north of the Black Sea. They led 1 700 000 men and 1 207 ships in a single campaign.

The great kings greatly advanced the state. Their domain reached from the southern slopes of the Caucasus, where they taxed the Colchians, in the north, to the southern shores of the Red Sea, where they taxed the Ethiopians, in the south, 3 200 km apart, and from the city of Barca, which they organized into satrapy VI, in the west, to the valley of the upper Indus, which they organized into satrapy XX, in the east, 4 900 km apart, 77 countries and peoples large enough to owe more than 10 talents each. All at peace, fearing the law, the strong not fighting or destroying the weak. Their domain also remained intact for 236 years.

The great kings solved theoretical problems, irrespective of whether they thus solved also practical problems.

The decisive are the labors of the first 5 great kings, of the first 3 in building the most powerful state, of the next 2 in giving the most powerful state permanence.

The steps of advance in the acquisition of knowledge are the 5, Cyrus, Cambyses, Darius, Xerxes, and Artaxerxes.

The Greek states.

Sparta and Athens greatly advanced warfare. They also greatly advanced the state. With the Assyrian princes and the great kings warfare is the means of building the state. With Sparta and Athens the state becomes a weapon. The soldiers are citizens, the citizens are soldiers. The dominated populations and states are the base of the maintenance of an army and a navy. In Athens legislation also develops with the state. Before the development of the navy in Athens, its building begins with Polycrates. Sparta and Athens fight a twenty-seven-year war. After the great advance of warfare in Sparta and Athens, Epaminondas discovers tactics. Philip and Alexander greatly advance logistics and tactics. Alexander is also a major step of advance in the development of the state.

And again, theoretical problems are thus solved, irrespective of whether practical problems are thus solved as well.

The steps of advance in the acquisition of knowledge are the 17, the 6 Spartans, Theopompus, Anaxandridas, Ariston, Cleomenes, Pausanias, and Lysander, the 7 Athenians, Draco, Solon, Pisistratus, Miltiades, Cleisthenes, The-

mistocles, and Pericles, and the 4, Polycrates, Epaminondas, Philip, and Alexander.

Carthage and Rome.

At this time Carthage builds a maritime empire. Rome just begins, auspiciously, however, with both legislation and victory. And in the following one and a half centuries Carthage and Rome will fight the wars to determine which will be the state to continue bringing all other states under itself.

The steps of advance in the acquisition of knowledge are the 5, the Carthaginian Mago, and the 4 Romans, the Twelve tables, Camillus, Sextius Lateranus, and Licinius Stolo.

Judaism.

The first church is the solution of theoretical problems, irrespective of whether it is the solution also of practical problems, more clearly and distinctly even than the other states. It is inseparably conceived in speech and built in deed.

The steps of advance in the acquisition of knowledge are the 4, *Deuteronomy*, *Joshua*, *Judges*, *Samuel*, *Kings*, the Priestly code, and Ezra.

All in all, 37 steps. Tabulating,

Tiglath-pileser III – Assyria becomes the most powerful state on earth,

Sargon II Assyria becomes more powerful,

Theopompus Sparta begins with conquest,

Sennacherib Assyria becomes more powerful,

Esarhaddon Assyria becomes more powerful,

Ashurbanipal Assyria becomes more powerful,

Deuteronomy the foundations of Judaism are laid with the law,

Draco Athens begins with legislation,

Necho Africa is circumnavigated,

Solon Athens continues with legislation and conquest,

Joshua, Judges, Samuel, Kings – the development of Judaism continues, as the law replaces the state,

Cyrus Persia becomes the most powerful state on earth,

Anaxandridas the rise of Sparta begins,

Ariston the rise of Sparta begins,

Polycrates the first navy is built,
Pisistratus the rise of Athens begins,
Mago the rise of Carthage begins,
Cambyses Persia becomes more powerful,
Darius Persia becomes more powerful,
Cleomenes Sparta becomes the most

powerful Greek state,
Cleisthenes legislation continues in Athens,
Miltiades Athens stops Persia,
Priestly code the development of Judaism continues, as the law is brought under a priesthood,

Themistocles Sparta and Athens stop Persia,
Pausanias Sparta and Athens stop Persia,
Xerxes the most powerful state on

earth is consolidated,
Pericles Athens becomes as powerful as Sparta,

Twelve tables Rome begins with legislation,
Artaxerxes the most powerful state on earth is consolidated,

Ezra the building of the first church is complete,

Lysander Sparta defeats Athens,
Camillus the rise of Rome begins,
Epaminondas Thebes defeats Sparta,
Sextius Lateranus – Rome continues with

legislation,
Licinius Stolo Rome continues with

legislation,
Philip Macedon conquers the Balkan peninsula,

Alexander Macedon conquers the most powerful state on earth.

The advance in the development of the state is part of the acquisition of knowledge, and in the four-odd centuries from Tiglath-pileser III to Alexander the advance in the development of the state is a principal cause of the acquisition.

This is the time also of the beginning of philosophy. Thales postdates Ashurbanipal by two generations, and predates Cyrus by one generation-odd. And the relation of continuity, and the causal relation of continuity, from the advance in the development of the state to the great advance in the acquisition of knowledge with the development of philosophy is indicated clearly and distinctly.

The development of geography is inseparable

from the advance in the development of the state. From the expeditions of the Assyrian princes and the expeditions of the great kings – to Anaximander, who drew the first map of the earth, Hecataeus, and Herodotus. And the expansion of the state has remained the principal cause of the advance of geography. Carthage opened the Atlantic. Rome opened northern Europe. And through the final stage in the advance with the opening of the interior of Africa in the nineteenth Christian century and of Antarctica in the twentieth.

The development of history is inseparable from the advance in the development of the state. Again, from the Assyrian princes and hence their chronicles – to the rise of the great kings, and of Sparta and Athens, and hence Herodotus – to the greatest war, and hence Thucydides. And the rise of the state has remained the principal cause of the development of history. The rise of Rome, and hence Polybius. The triumph of Rome, and hence Livy. The triumph of the Christian church, and hence Eusebius.

The advance in the development of the state first showed how large the earth is. The advance in the development of the state first showed how large humanity is also.

And the continuity in the advance in the development of warfare all by itself attests the acquisition of knowledge. From the capability of raising an army of 100 000 and the capability of deploying an army continuously for more than one century – through the capability of campaigning in two generations across 4 900 km and the capability of leading 1 700 000 men and 1 207 ships in a single campaign – through the capability of maintaining 100 fifty-oared ships – through the capability of sustaining 298 casualties out of 300 and of deploying 5 000 out of 8 000, the capability of maintaining 300 triremes in active service, and the capability of fighting a twenty-seven-year war – through the discovery and victory of tactics – through the further advance in logistics and tactics and the capability of conquering the domain from the Adriatic to the Indus in 32 years.

And the continuity in the advance in the development of the state all by itself attests the acquisition of knowledge as well. The Assyrian

princes build the largest and most powerful state, bringing under it very nearly all states, and one tenth of humanity, and mixing it. This is the base from which the great kings build a larger and more powerful state, bringing under it more states, and one fifth of humanity, and mixing it. On the western border of this state, Sparta and Athens build yet more powerful, if smaller, states. Alexander then unifies the Greek states and the state of the great kings into a yet larger and more powerful state, bringing under it yet more states, and one quarter of humanity, and mixing it.

Thus, then, the real states, and the best of them, in the four-odd centuries from Tiglath-pileser III to Alexander.

Assyria is the best state, then and overall the state of the great kings from Cyrus through Alexander is the best, Sparta, Athens, and Carthage, as well as Macedon considered separately are very good states, Babylon, Media, Egypt, Lydia, Corinth, and Syracuse are good states, Rome is just becoming a good state, and Judaism is a very good state, too.

And the real states are part and a principal cause of the acquisition of knowledge.

The states conceived in speech in those four-odd centuries are these of Hippodamus, Plato, and Aristotle.

And none of these is a state, never mind a good state, never mind the best state. To admit these statues to the contest of the athletes is ridiculous. Polybius concedes: the craftsmanship may be praiseworthy. With all due respect to Polybius, the craftsmanship may not be praiseworthy either.

Nor does any of these states have anything to do with the acquisition of knowledge.

Not quite two generations after Aristotle, Zeno of Citium next conceived a state in speech.

Zeno of Citium wrote the *Republic*. Plutarch reports: the argument of Zeno's *Republic* can be summed up in a single principle, the inhabitants of this earth should not live divided into different states and different peoples, each with its own rules of justice, all men should be one people and one state with a common life and common order, like a herd grazing together in a common field (*Fort. Alexander*, 6).

Zeno's is the first state conceived in speech

whose craftsmanship is praiseworthy. For it is the first statue drawn from life.

Plutarch adds: this was Zeno's idea of a well-governed philosophical state; Alexander, however, realized this idea (*id.*).

In the generation before Zeno, Alexander realized this idea. In the generation after Alexander, this was Zeno's idea of a well-governed philosophical state.

The inhabitants of this earth should not live divided into different states and different peoples, each with its own rules of justice, all men should be one people and one state with a common life and common order, like a herd grazing together in a common field.

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The task is to show that the acquisition of knowledge can be collected in its entirety. And to this end, in particular, to collect the predecessors of the philosophers and philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them, with respect to five problems. The earth. Then the soul, and both the physical problem of the soul and the ethical. Then the best state, and both the problem of the real best state and of that conceived in speech.

With respect to this last problem, however, the task is undertaken only in part. The great difficulty of the problem, and specifically in that the philosophers have conceived the state in speech, and again, and again, but the investigation of the state such as the state has been has barely begun, presents the problem as particularly well suited to show that the ground is firm enough to capture the acquisition of knowledge in its entirety. The problem is so massive, however, it is better to deal with only a part of it here.

The particulars have been collected and marshaled in their series and order with respect to the four-odd centuries from Tiglath-pileser III to Alexander.

4 important philosophers who have concerned themselves with the state have been collected. And to collect all philosophers who have concerned themselves with the state in the following centuries is not very difficult. These philosophers are not more than a few dozen. But the labors of

these philosophers should be considered in relation to the labors of building the real state. The statues should be considered in relation to the athletes. For there has been, next to the contest of the statues, the contest of the athletes.

And with respect to the real state, from these four-odd centuries 94 important particulars have been collected pointing to 26 states.

Counting back to the first prince, twenty-two and a half centuries of development of the state predate Tiglath-pileser III. Twenty-three and a half centuries of development of the state also postdate Alexander.

Those are four-odd out of the fifty-odd centuries of development of the state. If the numbers of the important particulars and the states to which they point are representative, the totals would be of well over 1 000 important particulars and well over 300 states. The problem of the real state is massive.

How to continue collecting the particulars before Tiglath-pileser III and after Alexander is clear. The philosophers who have concerned themselves with the state will not help. But the philosophers and their predecessors who have described the state, the historians, in particular, will. There is also all the evidence which the particulars themselves have left behind. Thus, for instance, above, in the course of the examination of the Sumerian prince list, all the particulars and all the states to which they point in the first six and a half centuries of the development of the state in the valley of the two rivers, from Enmebaragesi to Ibbi-Sin, have been collected.

Here, to the end of showing that the acquisition of knowledge can be captured in its entirety, those four-odd centuries should do.

In conclusion, then, the question of whether those four-odd centuries are representative can be addressed on general grounds.

On the authority of Aristotle: the state is for the sake of the good life. Of Hobbes: the final cause of the state is man's preservation and a more contented life. Of Locke: the chief end of the state is the preservation of property. Of Rousseau: the state tends toward equality and public utility. Of Hegel: the end of the state is the unity of the particular self-consciousness in the generality of the ethical spirit. Of Marx: the state

serves the need to hold class conflicts in check.

On the authority of the real states in the four-odd centuries from Tiglath-pileser III to Alexander, the end of the state is to vanquish all other states and bring them under itself, under the one and only one state which the state is.

Aristotle writes at the time when Philip vanquished the Greek states, then Alexander marched to the Ganges to conquer the state of the great kings.

Hobbes writes at the time when Cromwell won the civil war in England, then subdued Scotland and Ireland, then vanquished the United Provinces and Spain.

Locke writes at the time when Louis XIV waged war for two generations to build France, against England, into the dominant power on earth, and may well have succeeded had not William III – our great restorer, Locke writes – and his successors vanquished him.

Rousseau writes at the time when Britain vanquished France in the seven years' war, and expanded greatly in America and India, to become the overwhelmingly dominant power on earth.

Hegel writes at the time of the revolution and Napoleon, when the greatest war ever – until that time – was waged for twenty-three years first against, then by, then again against the revolution and Napoleon.

Marx writes at the time when Britain, France, Russia, and the United States were dividing the surface of the earth in its entirety, when Cavour built Italy, in war, Bismarck built Germany, in war, and as soon as they were built, Italy and Germany joined the division of what was left of the surface of the earth.

If this is the end of the state, to vanquish all other states and bring them under itself, it can explain why the development of the state has been inseparable from the development of warfare. And the more important the state the more important the step in the development of warfare.

It can explain the size of the state. Why the state has not been of the size of Plato's and Aristotle's preference. Assyria is large, the state of the great kings is larger, Rome is larger, and the state has continued to grow larger, and the churches have continued to grow larger, too.

It can explain the unity of the state. Sargon II made them live in peace, and counted them among the peoples of Ashur. And the state has continued to make them live in peace, and count them among its peoples.

It can explain the permanence of the state. Why at the time of Plato all states were evil, and at the time of Marx all states continued to be evil, and yet the state continues, evil as it is.

It can explain why the animal practicing virtues almost exclusively in relation to the men of the same tribe has been reluctantly forced to practice them in relation to a larger tribe, and larger, and all the way to humanity. For it is not that the animal has been reading the philosophers. The state has been the means of the expansion of the tribe into humanity.

It can explain the taming of humanity. For, again, it has not been by reading the philosophers that the disposition to fighting has been, to the degree to which it has been, vanquished. The state has been vanquishing it.

If this is the end of the state, it can also show that the definition of the state, with Hobbes: the state is a power contrary to our natural passions, a power keeping men in awe, a power reducing the wills of men unto one will, is an empirical generalization.

Kant writes: the philosopher cannot assume that men and their drama have a rational purpose, his only starting point can be to try to discover in the senseless course of human affairs a natural goal.

A power contrary to our natural passions, its end cannot be a rational purpose, it can only be a natural end.

The state has developed for fifty-odd centuries.

In 1958, in *Physics and philosophy*, Heisenberg concludes by identifying: a process which has fundamentally changed conditions of life on earth (11); whether one approves of it or disapproves, whether one considers it progress or a threat, one must realize that this process is beyond human control (*id.*); it is the general historical process toward unification (*id.*); this process, however, is accompanied by another process acting in the opposite direction; unification leads to the instigation of all forces in the existing cultural communities to try to ensure for their

traditional values the largest possible share in the final state; and the two processes are so closely linked together that every intensification of the process of unification intensifies also the struggle for influence in the final state thus adding to the instability of the transitional state (*id.*).

Heisenberg does not associate the process with the state. It is the state, however, exclusively, that attests the general historical process toward unification, a process beyond human control, a process which has fundamentally changed conditions of life on earth, from the first.

Forty-nine and a half centuries before Heisenberg, the state begins with the first prince, with Narmer. Narmer has left behind a macehead and a palette. A macehead. And on both sides of the palette the prince is depicted. On the obverse, the prince holds an upraised mace in his right hand ready to smite his kneeling foe whose forelock he holds in his left hand (*id.*). He wears the crown of upper Egypt (*id.*). On the reverse, the prince marches in a procession with his retinue, on the left, toward the corpses of his decapitated foes, on the right (*id.*). He wears the crown of lower Egypt (*id.*). The two crowns attesting the unification of upper Egypt and lower Egypt.

The state begins with smiting, and unification.

**The sum of the angles of the triangle,
and the assumptions on which
the sum depends**

cccv

Practical philosophy, politics and ethics, has rather generally been pursued on the supposition that: not more does one need to be a shoemaker in order to know whether his shoes fit than he needs to learn a trade in order to have knowledge of matters of general interest. It is the main reason practical philosophy has not advanced too much, not if the standard is the acquisition of knowledge, and practical philosophy is compared with theoretical philosophy. One does not need to learn a trade in order to have knowledge of matters of general interest. One has knowledge. The generality of the matters is the generality of men. All men wear shoes. Or in any case, all men need shoes. Or in any case, all men should need

shoes. Or, indeed, all men should wear shoes.

It is for the same reason that it is difficult, in practical philosophy, to find the sound in what our predecessors have said, and separate it from the erroneous.

Theoretical philosophy, physics and mathematics, and history, too, has rather generally not been pursued on that supposition. One has had to learn a trade. One has not had knowledge of these matters. These matters have not been of general interest, their generality has not been the generality of men, it has been the generality of the world. Whether all men, or any men, have ever heard of shoes, or will ever hear of shoes, has not mattered. It is a reason, too, if not in this case the main reason, theoretical philosophy has advanced, if the standard is the acquisition of knowledge, and advanced very greatly compared with practical philosophy.

Difficult, in this case, is the matter. Hence as well the need to learn a trade. If this is overcome, however, it is not difficult to find the sound in what our predecessors have said, and separate it from the erroneous.

Physics has also rather more often than mathematics displayed, or appeared to display, matters of general interest. Matters whose generality is, or appears to be – never mind the generality of the world – the generality of men. Copernicus, for instance. Does the earth fit? Or Darwin. Does life fit? Mathematics has been rather better protected than physics from displaying, or appearing to display, such matters by its greater remoteness from the actual experience of the generality of men. It has been more sophisticated, and increasingly more sophisticated, shoemaking.

The most dramatic advance in mathematics has been the development of non-Euclidean geometry. On this occasion more than on any other mathematics displayed, or appeared to display, matters of general interest. As its very name shows, it overthrew Euclid. It overthrew the standard of rigor, and of certainty, and of truth. Mathematics has always been suspected – as, clearly, what is mathematics? is a matter of general interest and one does not need to learn a trade in order to have knowledge of matters of general interest – of being man-made, made by man, such as, even if he may no doubt learn a

trade, does not need to learn a trade in order to have knowledge of matters of general interest. Non-Euclidean geometry showed conclusively that mathematics is man-made, now made by the man Euclid now made by the man non-Euclid, and thus positively that all knowledge is man-made.

The development of non-Euclidean geometry is a very great advance in the acquisition of knowledge. And that it is not man-made the fact suffices to show – even if nothing can suffice to show it to man such as he does not need to learn a trade in order to have knowledge of matters of general interest – that within two generations it became the foundation of the theory of relativity, and hence, in the next generation, the foundation of the big bang theory. Of the theory of the origin and development of the world.

The attempt to show that the acquisition of knowledge can be captured in its entirety should include a mathematical problem. Because the shoemaking has been, of all kinds of the acquisition of knowledge, the most sophisticated in mathematics. And non-Euclidean geometry can supply the problem. Because it is a very great advance in the acquisition of knowledge, but this advance has been also, or appeared to be also, a matter of general interest more than other advances in mathematics, and to examine the question of whether what has been, or appeared to be, a matter of general interest is, in fact, a matter of general interest may itself be, or appear to be, a matter of general interest.

cccvi

The formulation of this problem as well can be gathered from Aristotle. Aristotle does not formulate or solve the problem explicitly, or even concern himself with the problem specifically. He does rather show, however, that the problem can illustrate all and any matters.

Knowledge is acquired only by demonstration. Demonstration is the central problem of the *Prior analytics*. One of the kinds of demonstration is the *reductio ad absurdum*. And one of Aristotle's illustrations is: the same false result can follow from different false assumptions, that parallels intersect, for instance, follows from the assumption that the interior angle is greater than the

opposite exterior angle as well as from the assumption that the sum of the angles of the triangle is greater than two right angles (II, 17).

What is is truth, what is not is falsehood. Truth is the central problem of books VI-X of the *Metaphysics*. One of the problems which this problem poses is the relation of actual and potential. And one of Aristotle's illustrations is: why is the sum of the angles of the triangle equal to two right angles? because the sum of the angles on one side of a line at a point is equal to two right angles; and if through one of the vertices of the triangle a line parallel to the opposite side is drawn, this is evident at a glance (IX, 9).

Knowledge of the soul contributes greatly to the advance of truth, and most of all physics. Aristotle begins in *On the soul* with the difficulties which the problem of the soul poses. And one of his illustrations is: knowledge that the sum of the angles of the triangle is equal to two right angles depends on knowledge of what is straight and what is curved, and of line and plane (I, 1).

The happiest life is the theoretical. The spurious *Great ethics*, produced in the Lyceum shortly after Aristotle, epitomizes parts of the genuine ethical treatises. And among the little that is original is the yet greater fondness for the sum of the angles of the triangle, and the assumptions on which the sum depends. Thus: it would be absurd to demonstrate that the sum of the angles of the triangle is equal to two right angles from the assumption that the soul is immortal (I, 1). Also: when some things are assumed, assumed is also what follows from them; for instance, if the sum of the angles of the triangle is two right angles, the sum of the angles of the quadrilateral is four right angles, and if the sum of the angles of the triangle changes, so does the sum of the angles of the quadrilateral, and if the sum of the angles of the quadrilateral is not four right angles, the sum of the angles of the triangle is not two right angles either (10). And again: when it is affirmed that the sum of the angles of the quadrilateral is four right angles, and someone asks, why? the answer is, because the sum of the angles of the triangle is two right angles (17).

This is the problem, the sum of the angles of the triangle, and the assumptions on which the sum depends.

The six illustrations are clear that the sum of the angles of the triangle depends on assumptions. It is clear also that the immortality of the soul is not one of the assumptions. But which the assumptions are the illustrations are not clear.

In the generation after Aristotle, Euclid wrote the *Elements*. In the *Elements* the assumptions are formulated explicitly. They are distinguished also into definitions, postulates, and axioms. One of the postulates is the parallel postulate. And that the sum of the angles of the triangle is equal to two right angles depends unconditionally on the parallel postulate.

Not quite four and a half centuries later, Ptolemy made the first attempt to replace the parallel postulate with a theorem. To deduce what the postulate affirms from the other explicitly formulated assumptions, and thus exclude the postulate from these assumptions.

Not quite sixteen centuries later, Saccheri published *Euclid freed from all blemish, or a geometrical investigation in which are established the principles of universal geometry*, in 1733. The attempts to replace the parallel postulate with a theorem had accumulated. And the parallel postulate is the chief blemish, from which Saccheri proposes to free Euclid, again by proving what the postulate affirms. To do so, Saccheri proposes to prove that the assumptions that the sum of the angles of the triangle is greater than two right angles and that the sum is less than two right angles are both false. To prove that the second of these assumptions, that the sum is less than two right angles, is false, however, demands some dozens of theorems. And even if Saccheri concludes that this assumption as well is false, the theorems proved by Saccheri on this assumption, that the sum of the angles of the triangle is less than two right angles, are the beginning of non-Euclidean geometry. Saccheri is explicit also that the search is for the principles of universal geometry.

Two generations later, Lambert wrote *The theory of parallel lines*, published in 1786. Lambert proves more theorems on the assumption that the sum of the angles of the triangle is less than two right angles. Also, in Saccheri the assumption is a means of a *reductio*, in Lambert it is explicitly the ground on which results are built.

Lambert does not conclude, as Saccheri does, that the assumption is false either.

Two generations later, Lobachevsky published *On the principles of geometry*, in 1829. And with Lobachevsky the development of a non-Euclidean geometry is complete. That the sum of the angles of the triangle is less than two right angles is an assumption as sound as the assumption that the sum is two right angles. Also, if the sum of the angles of the triangle is less than two right angles, in a limiting case it is two right angles. Also, as from the set of Euclidean assumptions a vast structure of results has been deduced, so from the new set of assumptions a vast structure of results, corresponding to that deduced from the set of Euclidean assumptions, has begun being deduced. Also, the problem of the consistency of the new non-Euclidean geometry has been posed explicitly, and hence for the first time the problem of the consistency of the old Euclidean geometry has been posed explicitly as well. Also, the problem of which geometry obtains in nature, the old Euclidean or the new non-Euclidean, has been formulated explicitly. The new non-Euclidean geometry is, Lobachevsky proposes, a generalization of the old Euclidean.

One generation later, Riemann published *On the hypotheses which lie at the foundation of geometry*, in 1854. Riemann formulates explicitly and generalizes vastly the problem of the hypotheses which lie at the foundation of geometry. That the assumption that the sum of the angles of the triangle is greater than two right angles is not false either, is a trivial and very minor consequence from the generalization. Non-Euclidean and Euclidean geometries are unified into the vastly more general theory of manifolds and theory of metrics.

The problem of the assumptions, or hypotheses, on which the sum of the angles of the triangle depends is of the hypotheses, or assumptions, which lie at the foundation of geometry.

The Aristotelean illustrations are all concerned with assumptions. Because they are concerned with demonstration. With proof.

Two centuries before Aristotle, proof begins with Pythagoras.

Before that, twelve-odd centuries before that, the discovery of parallel lines can be traced back

to the mathematician who has left behind 14 problems, beginning with a triangle divided into 5 areas. The tablet is Strasbourg 364, from the century before *c.* 1694.

And before that, one century before that, the discovery of the triangle can be traced back to the book of 87 problems. The book of 87 problems was written under Amenemhet III, prince *c.* 1845-00.

The main steps in the development of the problem are thus these,

1885 the book of 87 problems,
570 - 495 Pythagoras,
346^{BC} Euclid,
87^{AD} Ptolemy,
1667 - 1733 Saccheri,
1826 - 1866 Riemann.

The steps divide the development into seven parts,

all the centuries before the book of 87 problems,
the thirteen-odd centuries from the book of 87 problems to Pythagoras,
the two-odd centuries from Pythagoras to Euclid,
the not quite four and a half centuries from Euclid to Ptolemy,
the not quite sixteen centuries from Ptolemy to Saccheri,
the one and a half centuries from Saccheri to Riemann,
the one and a half-odd centuries since Riemann.

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Above, with respect to the problems of the earth and of the soul, the collection of the particulars has proceeded in this order. Beginning in each case with Aristotle's review, first the particulars from the first centuries of the development of philosophy have been collected, then the earlier particulars have been collected, then the later particulars have been collected.

The rule is to begin with the simplest and easiest and ascend gradually to the more complex.

And where the attempt to collect the particulars is concerned, the first centuries of the development of philosophy are the simplest and easiest. Not only because Aristotle has begun the collection of the particulars from these centuries. These centuries present neither the complexity

and attendant difficulties of the earlier centuries nor the complexity and attendant difficulties of the later centuries. It is the first philosophers who began building on one another's labors explicitly, and vigorously, adopting the sound in what our predecessors have said and avoiding the erroneous. In these centuries the council of our predecessors is for the first time clearly and distinctly, and vigorously, in evidence. These centuries thus do not present the complexity and difficulties attending the investigation of the earlier centuries, when the sessions of the council are implicit rather than explicit, and punctuated by many apparent discontinuities in time and place. Nor do these centuries present the complexity and difficulties attending the investigation of the later centuries, when the sessions of the council increase in number, become divided, and are often tumultuous, and often enough in arms against one another. In the first centuries of the development of philosophy the council of our predecessors is for the first time explicitly in session, and it is also still small, united, and its various sessions are as amiable as they have ever been.

Thus where the attempt to collect the particulars is concerned.

Where the particulars themselves are concerned, the earliest centuries have to be the simplest, even if they are not the easiest to investigate. Their intrinsic complexity cannot not be the least.

Hence, with respect to this problem now, the sum of the angles of the triangle, and the assumptions on which the sum depends, the attempt can be made to collect the particulars in their chronological order. In this way the clarity and distinction in the development of the problem, in how and why the problem has begun, and how and why it has gradually acquired its complexity – the strictly mathematical character of the problem – should be easier to make explicit.

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All the centuries before the book of 87 problems.

In these centuries there is for a very long time no mathematics.

Then, twelve and a half centuries before the book of 87 problems, mathematics begins. While

the book of 87 problems is from the valley of the Nile, however, mathematics begins in the valley of the two rivers.

The beginning of mathematics can be traced back to the 3 oldest calculations of area.

The 3 oldest calculations of area are recorded on the two tablets W 20044/20 and W 19408/76, from the century around c. 3080.

On the one tablet are recorded these four numbers, in this arrangement:

$$\begin{array}{cc} 80 & 140 \\ 140 & 60 \text{ (id.)} \end{array}$$

On the other tablet are recorded eight numbers, these four, in the same arrangement:

$$\begin{array}{cc} 1\ 200 & 930 \\ 1\ 200 & 870 \text{ (id.)} \end{array}$$

and these four, in the same arrangement:

$$\begin{array}{cc} 980 & 1\ 280 \\ 1\ 420 & 520 \text{ (id.)} \end{array}$$

The arrangement implies, and the consistency of the arrangement implies unambiguously, the approximations for the areas of the three quadrilaterals,

$$\begin{aligned} \frac{80+140}{2} \times \frac{140+60}{2} &= 11\ 000, \\ \frac{1\ 200+1\ 200}{2} \times \frac{930+870}{2} &= 1\ 080\ 000, \\ \frac{980+1\ 420}{2} \times \frac{1\ 280+520}{2} &= 1\ 080\ 000. \end{aligned}$$

Those are the numbers in the decimal notation. The tablet employs the sexagesimal notation. And in units of 60, the first set of numbers is:

$$\begin{array}{cc} 1\ \frac{1}{3} & 2\ \frac{1}{3} \\ 2\ \frac{1}{3} & 1, \end{array}$$

the second set is:

$$\begin{array}{cc} 20 & 15\ \frac{1}{2} \\ 20 & 14\ \frac{1}{2}, \end{array}$$

and the third:

$$\begin{array}{cc} 16\ \frac{1}{3} & 21\ \frac{1}{3} \\ 23\ \frac{2}{3} & 8\ \frac{2}{3}. \end{array}$$

The implied areas are $183\ \frac{1}{3}$, 18 000, and 18 000.

The two tablets show numbers, including

fractions. The numbers are arranged, and different sets of numbers are arranged in the same way. This is all explicit.

A problem is solved. The same problem three times. The three solutions are of increasing difficulty. The problem is solved by calculation. The calculation includes multiplication, including the multiplication of fractions. Calculated is area. Several propositions are brought in connection with one another in chains of inferences. And the solutions of the problems are only approximations, but the method employed by the tablets is still, five-odd millennia later, the best to obtain quickly a good approximation for the area of an irregular quadrilateral. All this is implicit.

All of it, explicit and implicit, attests mathematics unambiguously. It attests the first mathematician, and a mathematician *par excellence*.

It is the beginning of mathematics, and it is, in general, the beginning of the acquisition of knowledge. There is nothing remotely comparable, earlier, or at that time, or for several centuries after that time, in terms of propositions, inferences, a problem, and its solution.

And given but this one mathematician no conclusions can be drawn as to why mathematics begins. To try and do so the relations of this mathematician to other earlier and contemporary developments have to be made explicit. Given that this is the first mathematician, however, conclusions can be drawn as to why mathematics begins the way it begins.

It begins with several numbers, their arrangement in several sets, a problem and its several solutions, calculation, the calculation of area, several propositions brought in connection with one another in several chains of inferences, and a conclusive result, too.

It can be concluded that mathematics begins with numbers and sets, because these objects manifest the principle of contradiction, and thus clarity and distinction, more immediately than any other objects. It begins with calculation, because objects manifest the principle of contradiction, and thus clarity and distinction, in calculation more immediately than in any other relation. It begins with the winning of propositions and bringing the propositions in connection with one another in chains of inferences, because this

is how knowledge is acquired. From the thought and the sense to the reference and the true. It thus also begins with the collection of the propositions and chains in a problem, and its solution. It thus also begins with a conclusive result.

It begins with the calculation of area, because it is all, numbers, sets, calculation, propositions, chains, an empirical generalization.

And it begins also with several numbers, several sets, several calculations, several propositions, several chains, and thus a problem and its several solutions, because the beginning is a massive advance. It is like Euclid later on. Ptolemy. Galileo. Newton. It can be concluded that the beginning cannot be small, the beginning has to be a massive advance.

As to the problem at issue, the sum of the angles of the triangle, and the assumptions on which the sum depends, it has not yet begun.

The triangle has not been discovered.

Mathematics begins with calculation, and implicitly the calculation of area. Given the distinction of arithmetic and geometry, mathematics begins with arithmetic, of which geometry is a part. But the distinction of arithmetic and geometry has not yet been discovered either.

Mathematics begins with chains of inferences of some length and complexity. Some of the propositions are conclusions, some are premises. But this has not been discovered either. That there are assumptions, and that results depend on assumptions, has not been discovered.

Five centuries later, there are the next two mathematicians.

The tablets are TSS 50 and VAT 12593, from the century around *c.* 2580.

The first calculates:

a granary;

1 man receives 7 sila of grain;

how many men receive grain?

164 571 men;

the remainder is 3;

1 152 000 sila of grain in the granary (*id.*).

1 sila $\approx$ 1 liter.

The second attests the oldest of the kind of tables, which constitutes the contents of the large majority of mathematical tablets. It is a table of squares, and thus also of roots, as well as a multiplication table, and a table for the conver-

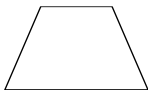
sion of measures. The mathematician calculates:

length in rods	length in rods	area in burs and sars 1 bur = 1 800 sars 1 sar = 1 rod ²
10×60	10×60	200
9×60	9×60	162
8×60	8×60	128
7×60	7×60	98
6×60	6×60	72
5×60	5×60	50
4×60	4×60	32
3×60	3×60	18
2×60	2×60	8
60	60	2
$5/6 \times 60$	$5/6 \times 60$	1 700
$4/6 \times 60$	$4/6 \times 60$	1 600
$3/6 \times 60$	$3/6 \times 60$	900
$2/6 \times 60$	$2/6 \times 60$	400
$1/6 \times 60$	$1/6 \times 60$	100
$1/12 \times 60$	$1/12 \times 60$	25 (<i>id.</i>).

Not quite four centuries later, there are the next two mathematicians.

The tablets are IM 58045 and HS 815, from the century before *c.* 2156.

The first attests the oldest mathematical figure:

 with the numbers 2, $1\frac{1}{6}$, 2, and $2\frac{5}{6}$ over the sides of the quadrilateral (*id.*).

The area is approximately 4. Not quite nine centuries after the first mathematician, the problem is the same, and the method of the implied solution is the same. But the problem is equipped now with a figure.

The second attests the oldest explicitly formulated problem:

the length is $67\frac{1}{2}$, the area is 100;

what is the width (*id.*)?

The solution is: $1\frac{13}{27}$ (*id.*). And the solution

shows that the mathematician assumes that the area is rectangular.

These two mathematicians attest the first inkling of geometry. The first figure and the assumption that an area is rectangular are the first inkling that there are problems in which spatial

relations are essential. The inkling is still very remote, however. For the calculation in neither of the two problems are the spatial relations essential.

Three and a half centuries later, twelve and a half centuries after the beginning of mathematics, the first of the three brightest periods in the development of mathematics begins.

Before that, those 5 are all the mathematicians attesting the development of mathematics in the first twelve and a half centuries of this development. The results of these 5 mathematicians attest the entire development of mathematics in these centuries.

There are other results in the valley of the two rivers attesting calculation. The inscriptions of the princes count battles won, men destroyed and captured. The Sumerian prince list counts the reign years of the princes and totals them. None of these results shows, or implies, development of mathematics other than that attested by the 5 mathematicians.

In the valley of the Nile there are the pyramids, including the greatest, those of Khufu and Khafre, and building implies calculation. There is the Palermo stone, recording the level of the high Nile, and this implies measurement. In the last of these twelve and a half centuries, under Senwosret, prince *c.* 1776-32, an account of work on a building was compiled, including dimensions of chambers and wages of workers (*id.*), and these also imply calculation. The account goes under the name Reisner papyrus. These results anticipate mathematics. None of them, however, shows, or implies, results comparable to the 3 oldest calculations of area. In the valley of the Nile mathematics begins with the book of 87 problems.

And there is nothing at all in any other valley.

The scarcity of mathematicians will be blamed on the deficiency of the record. Wantonly.

The 5 mathematicians show that in the first twelve and a half centuries of the development of mathematics the advance is slow and halting.

As the advance has been slow and halting in most of the thirty-nine centuries of the development of mathematics since, too. The first twelve and a half centuries are followed by the first of the three brightest periods in the development of

mathematics, from the century before *c.* 1794 to the century before *c.* 1594. Then that period comes to an end, and in the next ten and a half centuries, from the century before *c.* 1594 to the first philosophers, the advance is very slow and very halting. There are no mathematicians at all in these centuries. Then philosophy begins and for seven centuries mathematics advances greatly. Then that time comes to end as well, and in the next fourteen and a half-odd centuries, from Ptolemy to Galileo, the advance is again slow and halting, interrupted repeatedly by long periods of no advance at all. Nor are there many more than 5 mathematicians in these centuries.

In general, the record is suspected on the supposition that it, too, is a matter of general interest, one does not need to learn a trade in order to know that there is no knowing what is lost.

These 5, then – the 3 calculations of area, the division of grain among 164 571 men, the table of squares with sides from 10×60 to $\frac{1}{12} \times 60$, the

quadrilateral with sides $2, 1\frac{1}{6}, 2,$ and $2\frac{5}{6}$, and

the width of a rectangle with length $67\frac{1}{2}$ and

area 100 – are all the mathematicians attesting the development of mathematics in the first twelve centuries of this development. All 5 are very important, all 5 are steps of advance in the acquisition of knowledge.

But not with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends. Twelve centuries after the beginning of mathematics, this problem has still not begun. The first mathematical figure and the assumption that an area is rectangular attest the first remote inkling of geometry. But the triangle has not been discovered. That results depend on assumptions has not been discovered either.

cccix

The thirteen-odd centuries from the book of 87 problems to Pythagoras.

In the valley of the two rivers mathematics begins with a massive advance. The next 2 mathematicians follow five centuries later, the next

two not quite four centuries later. Three centuries later still, a very great advance begins, and there are in the next three centuries more than a dozen mathematicians. All in all, there are some two dozen mathematicians, and although punctuated by periods of no advance, the development of mathematics which the mathematicians attest is, from one mathematician to the next, substantially gradual.

In the valley of the Nile mathematics begins with a very massive advance. And the record attests but 3 mathematicians, all 3 also from the same century. The first has left behind the book of 87 problems, written under Amenemhet III, prince c. 1845-00. The second has left behind a book of 25 problems, from the century around c. 1786. The third has left behind a single problem, also from the century around c. 1786. The first goes under the name Rhind papyrus, the second under the name Moscow papyrus, the third under the name Berlin papyrus.

Mathematics begins in the valley of the Nile with the book of 87 problems, and the book of 87 problems is a very massive advance.

Again, the deficiency of the record will be suspected. Again, wantonly.

The book of 87 problems is a very massive advance. Like Euclid. Ptolemy. Galileo. Newton.

The book begins:

the rules of calculation, the gateway to the knowledge of all things and the uncovering of all secrets;

this book was copied in the thirty-third year of Apepi, the prince of all Egypt, from an old book written in the time of Amenemhet III, the prince of all Egypt;

the scribe Ahmose copied it.

Amenemhet III was the prince of all Egypt c. 1845-00. Apepi was the prince of all Egypt c. 1590. And the scribe Ahmose rather explicitly attests that at least in these two and a half centuries there was no other advance.

On the analogy of Euclid, Ptolemy, Galileo, Newton, it can be presumed that the mathematician who has left behind the book of 87 problems found the gateway to the knowledge of all things and uncovered all secrets all by himself.

The book divides into three parts, problems 1-23, problems 24-34, and problems 35-87.

Problems 1-23 introduce the rules of calculation, addition, subtraction, multiplication, division.

Problems 24-34 apply the rules of calculation to quantity, without reference to particular objects. Problem 31 is:

the sum of a quantity, $\frac{2}{3}$ of it, $\frac{1}{2}$ of it, and $\frac{1}{7}$

of it is 33;

what is the quantity (*id.*)?

Or what is the same,

$$x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 33.$$

The quantity is: $14 \frac{28}{97}$ (*id.*).

Problem 28 is:

from the sum of a quantity and $\frac{2}{3}$ of it $\frac{1}{3}$ of

this sum is subtracted and 10 remain;

what is the quantity (*id.*)?

Or what is the same,

$$x + \frac{2}{3}x - \frac{x + \frac{2}{3}x}{3} = 10.$$

The quantity is: 9 (*id.*).

And problems 35-87 apply the rules of calculation to many particular objects of many different kinds. Problem 39 deals with loaves, 64 with barley, 66 with fat, 67 with cattle, 71 with beer, 72 with small loaves exchanged for large loaves, 77 with beer exchanged for loaves, 82 with feed for geese, 84 with feed for oxen. Problem 40 is:

100 loaves are divided among 5 men;

$\frac{1}{7}$ of the three largest shares equals the two

smallest shares;

what is the difference between the shares (*id.*)?

The solution assumes that the shares are in arithmetical progression. In particular:

make the difference between the shares $5 \frac{1}{2}$;

the shares then are 1, $6 \frac{1}{2}$, 12, $17 \frac{1}{2}$, and 23;

the total is 60;

100 is $1 \frac{2}{3}$ times 60;

multiply the shares by $1 \frac{2}{3}$;

the shares are $1 \frac{2}{3}$, $10 \frac{5}{6}$, 20, $29 \frac{1}{6}$, and $38 \frac{1}{3}$;

the difference is $9\frac{1}{6}$ (*id.*).

Problem 79 attests the discovery of geometrical progression:

7 barns, 49 cats, 343 mice, 2 401 malt, 16 807 heqat;

the total is 19 607 (*id.*).

1 heqat $\approx$ 4.8 liters. Why the barns, cats, mice, malt, and heqat are totaled remains a secret.

Problems 41-6 deal with volumes of granaries.

Problem 44 is:

a granary of length 10, width 10, height 10;
how much grain goes in it (*id.*)?

The solution is: 7 500 heqat (*id.*). The solution shows that the mathematician assumes that 75 heqat go in 1 unit of volume.

And problems 48-60 attest the discovery of geometry. These are the oldest problems in which spatial relations are essential.

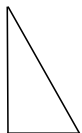
Problems 48-55 deal with areas. Problem 50 is:

a round field of diameter 9;
what is the area (*id.*)?

The solution is: 64 (*id.*). The solution implies the oldest result for $\pi = \frac{256}{81} = 3.16049\dots$

And problem 51 attests the discovery of the triangle. The problem is:

a triangle;
the side is 10, the base is 4;
what is the area (*id.*)?



The area is: 20 (*id.*). The solution shows that the mathematician assumes that the triangle is right. As the figure also indicates. And the mathematician has drawn this and the subsequent figures. Problem 52 is:

a truncated triangle;
the side is 20, the lower base is 6, the upper base is 4;
what is the area (*id.*)?



The area is: 100 (*id.*). Showing that the angle at the base is again assumed right.

Problems 56-60 deal with the slope of the side of a pyramid. Problem 56 is:

a pyramid;
the height is 250 cubits, the base is 360 cubits;
what is the slope (*id.*)?
The solution is:



divide 360 by 2, you have 180;

1 cubit is 7 palms;

multiply $\frac{180}{250}$ by 7;

and the slope is $5\frac{1}{25}$ palms (*id.*).

A rise of 7 over a run of $5\frac{1}{25}$ determines the angle at the base of the pyramid at $54^\circ 15'$. As does the tangent of the angle, $\frac{250}{180}$.

The rules of calculation. The gateway to the knowledge of all things and the uncovering of all secrets. The mathematician has uncovered the secrets of loaves, barley, fat, cattle, beer, small loaves exchanged for large loaves, beer exchanged for loaves, feed for geese, feed for oxen, barns, cats, mice, malt, heqat. Mathematics is an empirical generalization. Generalizing over the generalizations, the mathematician has uncovered the secrets of quantity, too. He has discovered also arithmetical progression, and geometrical progression. And he has discovered the triangle.

The arrangement of the problems in the book is all by itself a great advance. First the rules of calculation, then the more general problems of the calculation of quantity, then the more particular problems of the calculation of various quantities. And among these the geometrical problems are clearly separated.

In the Moscow papyrus the problems are similar, but not so well arranged. The most sophisticated is problem 14:

the height is 6, the lower base is 4,
the upper base is 2 (*id.*).



And the solution shows the calculation of the volume of the truncated square pyramid at: 56 (*id.*). Implying the formula for the volume, $V = \frac{h}{3}(a^2 + ab + b^2)$.

Problem 4 is the same as problem 51 in the book of 87 problems:

a triangle;
the side is 10, the base is 4;
what is the area (*id.*)?

Problem 7 is:

the area is 20;
the side is $2\frac{1}{2}$ times greater than the base

(*id.*).

And the solution obtains for the side and the base: 10 and 4 (*id.*).

The single problem on the Berlin papyrus is: a square;

another square whose side is $\frac{3}{4}$ of that of the

first;

the sum of their areas is 100 (*id.*).

Or what is the same,

$$x^2 + \left(\frac{3}{4}x\right)^2 = 100.$$

The side of the first square is: 8 (*id.*).

In the valley of the Nile, then, the triangle has been discovered. In general, geometry has been discovered.

That results depend on assumptions has not been discovered.

cccx

In the valley of the two rivers the first three of the thirteen-odd centuries from the book of 87 problems to Pythagoras attest the first of the three brightest periods in the development of mathematics.

Other than the half dozen tablets noted above, and the many tablets which postdate the first philosophers, all mathematical tablets are from these three centuries.

Dated from the stratification of finds in excavations, the development of language, including the development of writing, and the development of subject matter, all these in relation to one another, the tablets from these three centuries divide into early, from the century before *c.* 1794, middle, from the century before *c.* 1694, and late, from the century before *c.* 1594.

The early are two.

The tablets are AO 8862 and BM 13901.

The first of these mathematicians solves 8 problems. Problems 1-4 deal with rectangles and their sides, problems 5-8 deal with men carrying bricks and their wages. Problem 1 is:

a rectangle;

the sum of the area and the difference of the length and the width is 183;

the sum of the length and the width is 27;

what are the length, the width, and the area (*id.*)?

Or what is the same,

$$xy + (x - y) = 183,$$

$$x + y = 27.$$

The length, the width, and the area are: 15, 12, and 180 (*id.*). Problem 5 is:

a man carries 540 bricks and earns 20 sila of grain;

if 5 men carry the bricks, and the first man carries 1 part, the second man carries 2 parts, the third 3 parts, the fourth 4 parts, and the fifth 5 parts, how many bricks does each man carry and how much grain does he earn (*id.*)?

The parts are 15, 1 part is $\frac{540}{15}$ bricks and $\frac{20}{15}$

sila, and the men carry and earn, respectively: 36 bricks and $1\frac{1}{3}$ sila, 72 bricks and $2\frac{2}{3}$ sila, 108

bricks and 4 sila, 144 bricks and $5\frac{1}{3}$ sila, and

180 bricks and $6\frac{2}{3}$ sila (*id.*).

The second of these mathematicians solves 24 problems. The problems are of increasing difficulty. 1-7 deal with one square and its sides, 8-19 with two squares and their sides, 20-2 are lost in a lacuna, and 23 deals again with one square and its sides, but 24 then deals with three squares and their sides. Problem 1 is:

a square;

the sum of the area and the side is $\frac{3}{4}$;

what is the side (*id.*)?

Or what is the same,

$$x^2 + x = \frac{3}{4}.$$

The side is: $\frac{1}{2}$ (*id.*). Problem 24 is:

three squares;

the sum of their areas is 1 750;

the side of the second is equal to the sum of two thirds the side of the first and 5;

the side of the third is equal to the sum of one half the side of the second and $2\frac{1}{2}$;

what are the sides of the squares (*id.*)?

Or what is the same,

$$x^2 + y^2 + z^2 = 1750,$$

$$y = \frac{2}{3}x + 5,$$

$$z = \frac{1}{2}y + 2\frac{1}{2}.$$

The sides of the squares are: 30, 25, and 15 (*id.*).

The advance is very great. Calculation has reached more objects, and more kinds of objects. The complexity of operations has increased greatly. And the number of formulated and solved problems has increased greatly as well.

But the triangle has not been discovered. And the inkling that there is geometry remains remote. In all problems numerical relations are essential, and in none of the problems are spatial relations essential. The calculations are of numbers, and hence of areas. The problems deal with numbers and their sums and products, and hence with the sides of figures and their areas. That there are distinct geometrical problems has not been discovered. That results depend on assumptions has not been discovered either.

Among the middle tablets there are more than a dozen, and among them are these.

One mathematician calculates 15 Pythagorean triples.

The tablet is Plimpton 322.

The mathematician calculates:

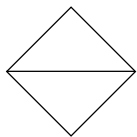
60	45	75
72	65	97
90	56	106
120	119	169
240	161	289
360	319	481
600	481	769
960	799	1 249
2 400	1 679	2 929
2 700	1 771	3 229
2 700	2 291	3 541
3 456	3 367	4 825
4 800	4 601	6 649
6 480	4 961	8 161
13 500	12 709	18 541 (<i>id.</i>).

The tablet has the series of numbers in the second and third columns. The series in the first column is a reconstruction, each number in it implied unambiguously by the pair in the second and third columns.

Another calculates the value of $\sqrt{2}$.

The tablet is YBC 7289.

The tablet has the figure, the



number 30 outside the square, and the numbers

$$1 \frac{8\,947}{21\,600} \text{ and } 42 \frac{307}{720} \text{ inside (*id.*)}.$$

The third number is the product of the first two.

$$\text{And } 1 \frac{8\,947}{21\,600} = 1.414\,212\,963 \dots$$

Another solves 286 problems.

The tablet is YBC 4668, of which YBC 4713 and YBC 4712 are also variants.

Problem 1 is:

a rectangle;

the area is 600;

multiplied by a number the length is 150;

multiplied by another number the breath is 80;

the sum of the two numbers is 9;

what are the length and the breath (*id.*)?

Or what is the same,

$$xy = 600,$$

$$ax = 150,$$

$$by = 80,$$

$$a + b = 9,$$

and in order to solve the system, the equation

$$2x^2 - 135x + 2250 = 0$$

has to be solved. The length and the breath are:

30 and 20 (*id.*). The following problems continue

to investigate the same rectangle. Problem 9 is:

a rectangle;

the area is 600;

the difference of the length and breath multiplied by a number is equal to the length;

the difference of the length and breath multiplied by another number is equal to the breath;

the sum of the first number, the second number, and their product is 11;

what are the length and the breath (*id.*)?

Or what is the same,

$$xy = 600,$$

$$a(x - y) = x,$$

$$b(x - y) = y,$$

$$a + b + ab = 11,$$

and in order to solve the system, again a quadratic equation has to be solved. The length and the breath are again: 30 and 20 (*id.*). Problem 42 is:

a rectangle;

the area is 600;

the difference of the length and the breath is multiplied by 150, and when the square of their sum is subtracted, 1 000 is lacking;

what are the length and the breath?

Or what is the same,

$$xy = 600,$$

$$150(x-y) - (x+y)^2 = -1000.$$

And if the quadratic equations, which have to be solved in order to solve the systems of equations, are recognized in the problems only implicitly, in problem 42 the negative magnitude is recognized explicitly. The length and the breadth are again: 30 and 20 (*id.*).

Another solves 44 problems.

The tablet is YBC 5037.

Problem 1 is:

an excavation in the shape of a prism;

the base is a square with a side $3\frac{3}{4}$, the depth is $2\frac{1}{2}$;

the volume assigned to one man to dig is $7\frac{1}{2}$;

the wage of one man is 6;

what are the volume, the number of the men, and the total of the wages (*id.*)?

The volume is: $\left(3\frac{3}{4}\right)^2 \times 2\frac{1}{2} = 35\frac{5}{32}$ (*id.*), the

men are: $\frac{35\frac{5}{32}}{7\frac{1}{2}} = 4\frac{11}{16}$ (*id.*), the wages are:

$4\frac{11}{16} \times 6 = 28\frac{1}{8}$ (*id.*). And by bravely dealing with

fractional men, this mathematician attests his commercial sophistication not less than the mathematical.

Another solves 43 problems.

The tablet is YBC 4714.

Problem 2 is:

the sum of four numbers in arithmetical progression is 140;

the sum of their squares is 5 400;

what are the numbers?

Or what is the same,

$$x + (x+y) + (x+2y) + (x+3y) = 140,$$

$$x^2 + (x+y)^2 + (x+2y)^2 + (x+3y)^2 = 5400.$$

The numbers are: 20, 30, 40, and 50 (*id.*).

Yet another is the mathematician who formulates 14 problems, beginning with a triangle divided into 5 areas.

The tablet is Strasbourg 364.

Problem 1 is:

a triangle;
it is divided
into 5 areas;

the first area
is 1 100, the
second area is 900;

the difference between one base and the next
is $13\frac{1}{3}$;

the fourth base is 40;

the last area is 100;

what is the third area (*id.*)?

And this mathematician only formulates the problems, leaving to others the solutions. As Descartes concludes in the *Geometry*: leaving to others the pleasure of discovery (3). Indulging the pleasure, the bases are in arithmetical progression, the fourth base is 40, and the series of the bases follows, $80, 66\frac{2}{3}, 53\frac{1}{3}, 40, 26\frac{2}{3}, 13\frac{1}{3}$.

The bases are also six, rather than five, as they are in the figure, because from the arithmetical progression of the bases and the numbers for the first, second, and last areas the arithmetical progression of the areas follows, 1 100, 900, 700, 500, 300, 100, and it is clear that one of the areas in the figure is assumed itself divided into two areas. The third area is thus 700.

The series of the bases and of the areas show that the lines dividing the triangle are assumed parallel. As the figure also indicates.

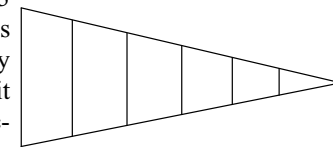
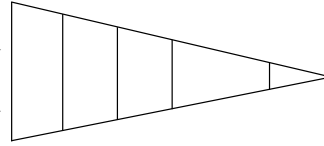
Problem 2 is the same as problem 1, with the same figure, the question now being: what are the bases (*id.*)?

In problem 3
the triangle is
divided explicitly
into 6 areas, as it
is also in the ac-
companying fig-
ure. The question is: what are the bases and the
areas (*id.*)?

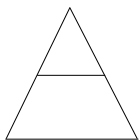
The lines dividing the triangle continue to be assumed parallel.

Problems 4 and 5 and a part of 6 are lost in a lacuna.

In problems 7-14 the triangle is divided into 2 areas, as it is also in the accompanying figure. Problem 7 is:



a triangle;
the area is 3 000;
a line divides it into 2 areas;
the height of the lower area is



$$33 \frac{1}{3};$$

the base of the upper area is 40;
what are the lower base of the lower area, and
the height of the upper area (*id.*)?

And while the previous problems can be solved without an appeal to the assumption that the lines dividing the triangle are parallel, this and the following problems cannot be solved without an appeal to the assumption that the line dividing the triangle is parallel to the base. The system has to be solved

$$\frac{x \left(33 \frac{1}{3} + y \right)}{2} = 3000,$$

$$\frac{40}{y} = \frac{x}{y + 33 \frac{1}{3}},$$

where x is the lower base of the lower area and y is the height of the upper area, and the second of these equations depends on the assumption that the sides of the triangles are proportional, and that is, that the triangles are similar, and that is, that the line dividing the triangle is parallel to its base.

The quadratic equation resulting from the system yields two solutions, $x = 60$, $y = 66 \frac{2}{3}$, and

$$x = 120, y = 16 \frac{2}{3}.$$

Problems 8-14 are the same as problem 7, varying the questions, and a couple of times the parameters.

This mathematician attests the discovery of geometry in this valley. More particularly, he attests also the discovery of the triangle. He attests also a very substantial advance in the investigation of the triangle. And his results are also the remotest beginning of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

The mathematician has discovered parallel lines. He has discovered similar triangles. He has discovered the proportionality of the sides of similar triangles. All three in their dependence on

one another. And all three in their Euclidean dependence on one another. In their dependence on the parallel postulate such as, fourteen and a half centuries later, Euclid will formulate explicitly. The parallel lines of the mathematician who solves the 14 problems, beginning with a triangle divided into 5 areas, are Euclidean parallel lines.

And there are from this time 3 more tablets, recording one problem each, which continue the investigation of the triangle and of parallel lines.

The tablet is Strasbourg 367.

The problem is:

a trapezoid is divided into two areas;

the lower area is 783, the upper area is 1377;

the height of the upper area is 3 times the height of the lower;

the sum of the difference of the lower base and the middle base and the difference of the middle base and the upper base is 36;

what are the bases and the heights (*id.*)?

If the bases are w , x , and y , and the heights are z and $3z$,

$$\frac{(w+x)}{2} z = 783$$

$$\frac{(x+y)}{2} 3z = 1377$$

$$(w-x) + (x-y) = 36$$

$$\frac{(w+y)}{2} 4z = 2160,$$

and $w = 48$, $x = 39$, $y = 12$, and $z = 18$.

The tablet is IM 55357.

The problem is:

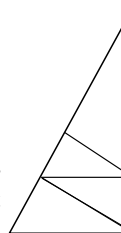
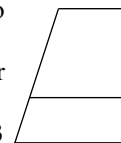
a triangle;

the sides are $\frac{3}{4}$, 1, and $1 \frac{1}{4}$;

the area is $\frac{3}{8}$;

the triangle is divided into 4 smaller triangles by 3 lines; the first line is dropped from the vertex to the opposite side, the second is dropped from the new point of intersection to the opposite side, and the third is dropped from the new point of intersection to the opposite side;

the area of the first of the 4 smaller triangles is $\frac{27}{200}$, the area of the second $\frac{54}{625}$, the area of the



third $\frac{864}{15625}$, and the area of the fourth $\frac{1536}{15625}$;

what are the sides of the smaller triangles (*id.*)?

The given triangle is assumed right. The lines dropped to the opposite sides are assumed perpendicular to these sides. And two sets of parallels are thus related to one another. Nor can the problem be solved without an appeal to proportionality.

Thus, for the first of the smaller triangles proportionality yields the system

$$\frac{1}{3/4} = \frac{x}{y}$$

$$\frac{xy}{2} = \frac{27}{200},$$

$$\text{and } x = \frac{3}{5}, y = \frac{9}{20}.$$

The tablet is VAT 8512.

The problem is:

a triangle is divided by a line;

the difference between the area of the trapezoid and the area of the small triangle is 420;

the difference between the height of the trapezoid and the height of the small triangle is 20;

the lower base of the trapezoid is 30;

what are the two heights and the base of the small triangle (*id.*)?

The line dividing the triangle is again assumed parallel to its base. Nor can this problem be solved without an appeal to proportionality.

Thus, if the height of the trapezoid is x , the height of the small triangle y , and the base of the small triangle z ,

$$\frac{30+x}{2}x - \frac{yz}{2} = 420$$

$$y - x = 20$$

$$\frac{x+y}{30} = \frac{y}{z},$$

$$\text{and } x = 40, y = 60, \text{ and } z = 18.$$

These tablets are all among the middle, from the century before c. 1684.

The late, from the century before c. 1594, are two.

One of these two mathematicians solves 18 problems.

The tablet is BM 85196.

Problem 9 is:

an upright beam of length 30 has slipped down a wall a distance of 6;

how far from the wall is its lower end (*id.*)?

The solution is: 18 (*id.*). The triangle is assumed right, and the solution shows that it employs the result of the Pythagorean theorem. The beam, or the hypotenuse, is 30. Its upper end touches the wall at $30 - 6 = 24$ above the ground. And the lower end is $\sqrt{30^2 - 24^2} = 18$ from the wall.

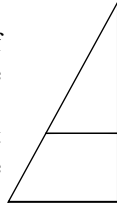
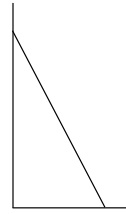
From this time is also the Venus tablet of Ammisaduqa.

Then the first of the three brightest periods in the development of mathematics comes to an end, and for a long time mathematics does not advance at all, in the valley of the two rivers, or in the valley of the Nile, or in any other valley.

Counting from the 3 oldest calculations of area to the latest tablet above, these are the first fourteen and a half centuries of the development of mathematics. Toward the end of that time, in the valley of the Nile twelve and a half centuries after the beginning of mathematics, in the valley of the two rivers thirteen and a half centuries after the beginning of mathematics, geometry is discovered, and conclusions can be drawn as to how and why.

Geometry is a small part of mathematics. Of all the problems formulated and solved those in which spatial relations are essential are a small fraction, and in the valley of the two rivers, where many more problems are formulated and solved, a smaller fraction. Geometry is also a part of arithmetic. All problems in which spatial relations are essential are still concerned, as all other problems are, with calculation, the large majority with the calculation of the areas and sides of figures, and 5 problems in the book of 87 problems with the calculation of the slope of the side of a pyramid.

Mathematics begins with the calculation of areas, and the calculation of areas continues to be the main problem which the mathematicians formulate and solve. Steadily more objects are brought under calculation, however, grain, men, bricks, wages, loaves, barley, fat, cattle, beer,



1000 In order to formulate the problems we are to solve we call the council of our predecessors

barns, cats, mice, malt. Quantity, in general, without reference to particular objects, is brought under calculation. The complexity of calculation has increased vastly, extending to arithmetical progressions, geometrical progressions, problems of four unknowns, quadratic equations, negative magnitude. The complexity of the calculation of areas, in particular, has increased accordingly.

And, hence, geometry begins.

In the valley of the Nile it begins with the discovery of the triangle, and with two kinds of problems concerning the triangle, the problem of its area, and the problem of the slope of its side. The first kind is routine, extending to the triangle the calculation of areas. The second kind is new, it is posed by the triangle.

In the valley of the two rivers it also begins with the discovery of the triangle. In this valley, however, it begins also with a great advance in the investigation of the triangle. The triangle is divided into parts. It is divided into parts by lines. And the lines are parallel. It is divided into parts which are triangles themselves. And the triangles are similar. Parallel lines and similar triangles depend on one another. The problems are also all concerned with the calculation of the areas and sides of the figures, but they are all concerned with the calculation of the areas and sides of the figures in relation to one another. The calculation depends on the relation of the lines, such as they are parallel, and the relation of the figures, such as they are similar. It depends on the proportionality of the sides of the figures. The sides of similar triangles are proportional. In one of the problems a right triangle is divided into parts by pairs of lines which are also perpendicular to the sides. Parallel and perpendicular depend on one another as well. In another of the problems the dependence of the length of the side of the right triangle on the lengths of its two other sides is established. And all these problems are posed by the triangle.

It can be concluded that geometry begins with the discovery of the triangle, because the triangle is the fundamental unit of the analysis of space.

It begins decisively with the division of the triangle into parts which are themselves triangles, because space is thus analyzed into sets of the fundamental unit. Just as number, whose analysis

has advanced substantially by this time, is sets of units.

In particular, it begins with the division of the triangle into parts by parallel lines. It can be concluded that being parallel is the fundamental relation of lines, which determines the sets into which space has to be analyzed.

Hence the dependence of parallel lines, similar triangles, and proportionality on one another.

The parallel lines are Euclidean. Or rather, lines are discovered to be parallel such as they will be defined, fourteen and a half centuries later, and after vigorous further investigation in the last two and a half of these centuries, by Euclid with the parallel postulate.

If the parallel lines are Euclidean, the sum of the angles of the triangle is two right angles. This has not yet been discovered, and more generally, the investigation of the angles of the triangle has not begun. The first steps toward the investigation of the angles, both remote, are the calculation of the slope of the side of the triangle and the perpendiculars dropped in the right triangle. Nevertheless, the discovery of parallel lines is the first inkling of the problem of the sum of the angles of the triangle.

The discovery of geometry is an empirical generalization over the calculation of areas, as calculation acquires increasing complexity, and the calculation of areas, in particular, acquires increasing complexity, and this calculation becomes the analysis of space, becomes the investigation, by calculation, of relations which are essentially spatial. Spatial relations are also analyzed just as numerical relations already are. And the analysis is Euclidean – the discovery of parallel lines is of Euclidean parallel lines – because it is the analysis of real space, and that Euclidean geometry is the continuation of the same analysis of real space, the fact of all the physics which rests unconditionally on Euclidean geometry shows.

The triangle has been discovered, then. Parallel lines have also been discovered. That results depend on assumptions has not yet been discovered.

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Ten and a half centuries separate the Venus

tablet of Ammisaduqa and Thales, the end of the first of the three brightest periods in the development of mathematics and the beginning of philosophy. The first nine of these centuries are among the darkest. There is but one substantial advance in the acquisition of knowledge, the map of Nippur, from the century before c. 1400. Then, one and a half centuries before Thales, Assyria rises. With the Assyrian princes the advance in the description of the surface of the earth resumes, the inscriptions of the princes become chronicles and begin to imply maps. Ashurbanipal collects 1 000 books. The observation of the heavenly bodies resumes, becomes sustained, becomes accurate, reaches dozens of the heavenly bodies.

Then philosophy begins.

The fourth philosopher, two generations after the first, is Pythagoras, and Pythagoras discovered that results depend on assumptions.

Pythagoras discovered proof. And this is what proof is – conclusions are shown to follow from premises, results from assumptions, and conversely, premises are shown to lead to conclusions, assumptions to results.

Pythagoras discovered the incommensurability of the side and diagonal of the square. And the incommensurability of the side and diagonal of the square is inaccessible to anything but proof. Aristotle writes: that the side and diagonal of the square are incommensurable is demonstrated by showing that, if they are supposed commensurable, even numbers turn out to be odd.

Thus,

if the side and diagonal of the square are commensurable, their ratio is rational,

let it be $\frac{p}{q}$ in least terms,

by the Pythagorean theorem $2p^2 = q^2$,

q^2 is thus even, q is thus even, and as $\frac{p}{q}$ is in

least terms, p is odd,

q is even, so let $q = 2r$,

then, $2p^2 = 4r^2$, and $p^2 = 2r^2$,

p^2 is thus even, and p is thus even,

but it is also odd.

If – then.

In general, the *reductio* depends on the assumption that even numbers are not odd, odd

numbers are not even, and *tertium non datur*.

And the development of philosophy through the fourth philosopher, in general, and the results of the fourth philosopher, in particular, can explain the discovery of proof.

The beginning of philosophy is a vast acceleration in the acquisition of knowledge. Thales, Anaximander, Anaximenes, and Pythagoras are but the first four philosophers, and the following are but their most important results. With the first philosopher the problem of the earth begins, the solution of the problem is advanced by the second philosopher, advanced further by the third philosopher, and advanced further by the fourth philosopher. With the third philosopher the problem of the soul begins, and the solution of the problem is advanced by the fourth philosopher. With the first philosopher also the problem of the principles of all things begins, and the solution of the problem is advanced by the third philosopher, and advanced further by the fourth philosopher. Quantitatively alone, the accumulation of the problems and their several solutions improving continuously on one another cannot not demand proof steadily more urgently. Cannot not demand steadily more urgently making explicit the assumptions on which the results depend. And the more so qualitatively, all things, their principles, the earth among all things, the soul among all things, and these in relation to one another, and in the relation of their differences and their unity, both, cannot not demand proof steadily more urgently.

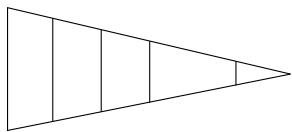
Pythagoras discussed the order of the planets. He said that the soul is number, it is immortal, and any soul can be in any body. He said that number is the principle of all things. These results all by themselves, and the more so in relation to the results of his predecessors, the earth floats on water, Anaximander discussed the sizes and distances of the planets, the earth is in equilibrium, the earth floats on the air below, the soul is air, water is the principle, air is the principle, cannot not demand proof.

And as more and more propositions are won, and brought in connection with one another in more and longer chains of inferences, the mathematical inferences will first succeed in closing the gaps in the chains sufficiently to become unam-

biguously proof, because the objects of these inferences, numbers rather than the earth, the soul, water, air, manifest the principle of contradiction, and thus clarity and distinction, more immediately than any other objects.

Eudemus wrote the first history of mathematics, and he writes: Thales traveled to Egypt, and first introduced mathematics among the Greeks.

In particular, the first philosopher measured the height of the pyramids, and the distance of ships from the shore. To borrow the figure from his predecessor, the mathematician who solved the 14 problems, beginning with a triangle divided into 5 areas, if the base and side of the largest triangle are the height of the pyramid and its shadow, and the base and side of the smallest triangle are ourselves and our shadows, proportionality and the measurement of ourselves, our shadows, and the shadow of the pyramid yield the height of the pyramid. If the ship at sea is at the vertex, and the two largest bases and the distance between them are measured on land, proportionality and these measurements yield the distance of the ship. Both results depend on the assumption of similar triangles, and that is, the assumption of parallel lines. Euclidean parallel lines.



Aristotle writes: the Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this science, and from its study they came to think that its principles are the principles of all things; and since among these principles numbers are by nature first, they saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water; so they said that numbers are the principles of all things, and the whole world is a harmony and number.

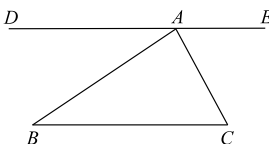
In particular, the fourth philosopher obtained these results also.

Proposition 47 in book I of the *Elements* is the Pythagorean theorem. And Proclus, in the commentary on book I of the *Elements*, commenting on this proposition, writes: one method for finding Pythagorean triples is attributed to Pythagoras; if a is odd, the other two numbers are given by $\frac{a^2 - 1}{2}$ and $\frac{a^2 + 1}{2}$ (428).

The method is general, it does not yield all Pythagorean triples, but it assigns a Pythagorean triple each to all odd numbers.

Proposition 32 in book I of the *Elements* is the proof that the sum of the angles of the triangle is equal to two right angles. Commenting on this proposition, Proclus writes: Eudemus attributes the discovery of this proposition, that the sum of the angles of the triangle is equal to two right angles, to the Pythagoreans; he says that they demonstrated it in this way;

let ABC be a triangle;
draw through A line DE parallel to BC ;



since BC and DE are parallel, angle BAD is equal to angle ABC , and angle CAE is equal to angle ACB ;

add the common angle BAC ;

the sum of the angles BAD , BAC , and CAE , which is equal to two right angles, is equal to the sum of the three angles of the triangle;

therefore, the sum of the three angles of the triangle is equal to two right angles (379).

Ascribed to Pythagoras, not to the Pythagoreans, as they are called, it is the oldest geometrical demonstration. The oldest geometrical demonstration is the proof that the sum of the angles of the triangle is equal to two right angles. And it, too, depends on the assumption of parallel lines, Euclidean parallel lines, and this assumption in some detail now, in that the alternate angles defined by a line intersecting a pair of parallel lines are equal.

The method for finding Pythagorean triples is arithmetical. The proof that the sum of the angles of the triangle is equal to two right angles is geometrical. The proof of the incommensurability of the side and diagonal of the square is also arithmetical.

This last proof can be presumed the most dramatic result on which the generalization, number is the principle of all things, rested.

This is how Aristotle describes the drama: all men are astonished when they first learn that there is something which cannot be measured even by the smallest unit, but nothing will astonish the geometer more than were the side and

diagonal of the square to turn out to be commensurable.

But the proof also poses the problem, what exactly is number? what exactly is the principle of all things?

When Pythagoras discovered the incommensurability of the side and diagonal of the square, he thus discovered the irrationality of $\sqrt{2}$. If there is a square of side 1, and there is, its diagonal is $\sqrt{2}$. The incommensurability of the side and diagonal of the square is inaccessible to anything but proof. And the proof depends on the assumption that even numbers are not odd, odd numbers are not even, and *tertium non datur*. It does not depend on the assumption that numbers are either odd, or even, and *tertium non datur*. Nevertheless, if numbers are either odd, or even, and *tertium non datur*, $\sqrt{2}$ is not a number. Or the other way around, if $\sqrt{2}$ is a number, numbers are not either odd, or even, and *tertium non datur*. In the first case, the side of the square is a number, but its diagonal is not a number. In the second case, the *reductio* on which the incommensurability of the side and diagonal depends is under some strain.

The proof thus poses dramatically also the problem of the relation of numbers and – of all things – lines. Of arithmetic and geometry.

The results of 16 mathematicians, then – the book of 87 problems, the 24 problems, beginning with the sum of the area and side of a square, the 8 problems, beginning with the sum of the area and difference of sides of a rectangle, the Moscow papyrus, the Berlin papyrus, the 286 problems, the 44 problems, beginning with an excavation in the shape of a prism, the 43 problems, including 4 numbers in arithmetical progression whose sum is 140, the 15 Pythagorean triples, the 14 problems, beginning with a triangle divided into 5 areas, the 1 problem, a trapezoid divided into 2 areas, the 1 problem, a triangle divided into 4 smaller triangles, the 1 problem, a triangle divided into 2 areas whose difference is 420, the calculation of the diagonal of the square, the 18 problems, including an upright beam of length 30, and Pythagoras – have been described, attesting the development of mathematics in the thirteen-odd centuries from the book of 87 problems to Pythagoras. All 16 are important, the

majority are very important, and the majority are steps of advance in the acquisition of knowledge.

But not all with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

These 7 – assigning for the sake of uniformity years of birth – are all important predecessors of the philosophers and philosophers, mathematicians, in particular, who are part of the development of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends,

1885	book of 87 problems,
1784	14 problems, beginning with a triangle divided into 5 areas,
1784	1 problem, a trapezoid divided into 2 areas,
1784	1 problem, a triangle divided into 4 smaller triangles,
1784	1 problem, a triangle divided into 2 areas whose difference is 420,
1684	18 problems, including an upright beam of length 30,
570 - 495	Pythagoras.

The steps of advance in the acquisition of knowledge are the 5, the book of 87 problems, the 14 problems, beginning with a triangle divided into 5 areas, the 1 problem, a triangle divided into 4 smaller triangles, the 18 problems, including an upright beam of length 30, and Pythagoras. In particular,

87 problems	discovered the triangle,
14 problems	discovered parallel lines,
1 problem	discovered sets of parallel lines,
18 problems	discovered the dependence of the lengths of the sides of the right triangle on one another,
Pythagoras	discovered proof,
	discovered that the sum of the angles of the triangle is two right angles.

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The two-odd centuries from Pythagoras to Euclid.

In these centuries the acquisition of knowledge advanced very greatly. Physics advanced very greatly. Mathematics advanced very greatly. Geometry advanced very greatly. Mathematics

also advanced more than physics. And geometry advanced more than mathematics.

The magnitude of the advance of physics Aristotle's treatises, toward the end of that time, show. The *Physics* investigates motion, *On the heaven* the world in its entirety, *On generation and corruption* the elements, the *Meteorology* comets, the Milky Way, rain, clouds, snow, hail, rivers, deserts, floods, the sea, winds, earthquakes, lightning, thunder, hurricanes, the rainbow, *On the soul* the principle of life, the five treatises on animals 600 kinds of animals. The magnitude of the advance the single result, the solution of the problem of the shape of the earth, shows better yet. Two generations after Pythagoras, Parmenides discovered that the earth is a sphere, one century later, Eudoxus calculated its radius, at 10 200 km, one generation later, toward the end of that time, Dicaearchus calculated it at 7 600 km. Better yet the fact that physics began becoming mathematical shows it. Eudoxus formulated the theory of the motions of the heavenly bodies in concentric spheres, and two generations later, at the end of that time, Autolycus, in *On the moving sphere* and *On the risings and settings*, and Euclid, in the *Phenomena*, defined the circles of the equator, ecliptic, horizon, and meridian, and proved 60 propositions about the heavenly bodies.

The magnitude of the advance of mathematics, and that it was greater than that of physics, the fact of the *Elements*, at the end of that time, shows. In the *Elements* from 5 axioms, 5 postulates, and 131 definitions 514 theorems, lemmas, and corollaries are deduced. In general, by the end of that time, mathematics, resting on the *Elements*, had become axiomatic, deductive, and hence a collection of proofs. Hence it had become the part of the acquisition of knowledge – and identified explicitly as a distinct part – of the greatest systematic unity, by far, a collection of results shown to depend on one another more clearly and distinctly, by far, than any other. Hence it had become also the part of the acquisition of knowledge of the greatest power, by very far, a collection of results leading to more results, and more, and more, more clearly and distinctly, by very far, than any other.

And the magnitude of the advance of geome-

try, and that it was greater than that of mathematics, the fact that the *Elements* is geometry shows. In general, the fact shows it that mathematics, resting on the *Elements*, had become fundamentally, substantially, and essentially geometry. Fundamentally, spatial relations had become the ground of proofs. Substantially, all objects had been reduced to spatial relations. Essentially, spatial relations had closed the gaps in the chains of inferences. If geometry is a part of mathematics, and had been a small part, and a part developed after the other part, arithmetic, by the end of that time that part, geometry, had come to coincide with the whole.

In the commentary on book I of the *Elements*, on the authority of Eudemus, Proclus reports: Hippocrates of Chios wrote the first *Elements* (66). Hippocrates of Chios is one century-odd after Pythagoras, one century-odd before Euclid. And the two-odd centuries from Pythagoras to Euclid divide into two parts. The one century-odd from Pythagoras to Hippocrates of Chios, and the one century-odd from Hippocrates of Chios to Euclid. The dates are,

570 -	495	Pythagoras,
460		Hippocrates of Chios,
346		Euclid.

In the one century-odd from Pythagoras to Hippocrates of Chios physics advanced very greatly, and as part of this advance important mathematical results were obtained. It was in the one century-odd from Hippocrates of Chios to Euclid, however, as physics continued to advance very greatly, that the advance of mathematics accelerated very much more than the advance of physics, and mathematics became geometry, and by far the most advanced part of the acquisition of knowledge. The philosophers who accomplished this feat accomplished it in four generations.

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In the one century-odd from Pythagoras to Hippocrates of Chios, then, physics advanced very greatly.

Parmenides discovered that the earth is a sphere. Anaxagoras discovered the cause of eclipses, discovered that the earth is below the moon, and said that the sun is a piece of red-hot

metal larger than the Peloponnese. Oenopides discovered the obliquity of the ecliptic. Meton measured the solstices, calculated the length of the year, and synchronized the periods of the moon and the sun.

Alcmaeon said that the ruling power is in the brain, excised eyes, and assigned cause to disease. Anaxagoras said that there is soul in all animals. Empedocles said that the parts of the body are ratios of the elements, and the soul is the ratio of the elements of the parts of the body.

Hecataeus described 300-odd localities, and drew the map of the earth, beginning the determination of distance and orientation. Herodotus described 1 012 localities, drew the map of the earth, improving substantially on the determination of distance and orientation, and measured the Black Sea, the Red Sea, and the Caspian Sea.

Heraclitus said that the principle is fire. Parmenides said that the principles are fire and earth. Anaxagoras said that the principles are an infinite number of particles, as each thing is generated from its own particles and destroyed into its own particles, and reason is the cause of the world and its order. Empedocles said that the principles are earth, water, air, and fire, and the sources of motion are love and strife. Leucippus said that the principles are atoms and void.

Early in that time, also, Xenophanes said that if oxen, horses, and lions had hands and could draw and make statues like men, horses would draw the gods like horses, oxen would draw them like oxen, and every kind like itself.

And given the great advance of physics in this one century-odd, the mathematical results are relatively few, and the strictly mathematical results are relatively very few and were all obtained at the end of that time.

Among the results above, Empedocles said that the parts of the body are ratios. In bones, for instance, the ratio of earth to water is 2:6, of earth to fire 4:4. It is the first attempt at quantitative analysis, even if not grounded in measurement or calculation yet. In the next generation, quantitative analysis was grounded in measurement and calculation, and increasingly sophisticated measurement and calculation. Oenopides discovered the obliquity of the ecliptic. The angle of inclination of the plane of the ecliptic to the plane of the

celestial equator is a little more than 23° . The discovery is impossible without the distinction of these objects, the ecliptic, the celestial equator, and the angle between them, and the measurement, at least approximately, of the angle. Herodotus measured the Black Sea, the Red Sea, and the Caspian Sea. He measured the length of the Black Sea at 11 000 stades. The length of Herodotus himself was $\frac{1}{100}$ of 1 stade. Meton observed

a summer solstice on 27 June 432 at dawn, determined the length of the solar year at $365 + \frac{1}{4} + \frac{1}{76}$ days, and discovered that 235 lunar months are very nearly equal to 19 solar years. Made continuously over not less than 19 years, these measurements are precise within $\frac{1}{4}$ of 1 hour per 1 year.

The investigation itself showed them the way, and showed them the way to measurement and calculation.

The great advance of physics showed them the way to measurement and calculation.

Among the philosophers above as well, Parmenides discovered the principle of contradiction. There are two ways, what is and cannot not be, which is the way of truth and conviction, and what is not is not and must not be, which you will never know or utter.

In the next generation, Zeno of Elea formulated the paradoxes of infinite division. Aristotle in the *Physics* describes the four paradoxes, dichotomy, Achilles, arrow, and stadium. This is the Achilles:

the quickest can never catch the slowest;

because the quickest must first reach the point where the slowest is when the pursuit begins, and the slowest has advanced, no matter how little;

the quickest must then reach the point to which the slowest has advanced, and the slowest has again advanced, no matter how little;

and so on to infinity (VI, 9).

It is the first attempt to analyze infinity. It is the first attempt also to follow unflinchingly the consequences of the principle of all principles wherever they lead. Aristotle adds: Zeno concluded that motion is impossible (*id.*).

In the same generation, Melissus followed the consequences just as unflinchingly. He said:

it always was and will be; otherwise it is nothing; and nothing comes out of nothing;

everlasting, it is unlimited; if it is unlimited, there is no void; if there is no void, it is full; if it is full, it does not move;

if it is divided, it moves; if it moves, it cannot be;

if it is, it is one.

In the next generation, Socrates discovered definitions.

The investigation itself showed them the way, and showed them the way to the principle of all principles, the imperative that its consequences be followed wherever they lead, and definitions.

The great advance of physics showed them the way to the assumptions on which results depend.

It showed them the way to measurement and calculation and the assumptions on which results depend. It showed them the way to mathematics.

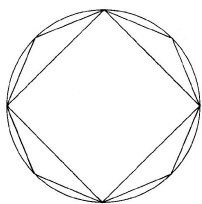
It showed them also the way to ethics. Socrates abandoned physics altogether.

The following are the strictly mathematical results.

Among the philosophers above, Anaxagoras first tried to square the circle.

In the next generation, Antiphon made the next attempt.

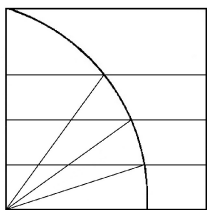
Simplicius reports: to square the circle Antiphon inscribed in it a square, then a regular octagon, then a regular sixteen-sided polygon, and so on (comm. *Physics*, 54).



Antiphon anticipates the method of exhaustion, Euclid will prove that the area of the circle is proportional to r^2 by the method of exhaustion, the method of exhaustion anticipates integration, and soon after integration is discovered, it will be proved that the area of the circle is πr^2 .

In the same generation, Hippias discovered the quadratrix.

Pappus describes the generation of the curve: given a square whose side is the radius of a circle, if, when the radius is drawn beginning at the upper vertex, the upper side si-



multaneously descends while remaining parallel to the lower side, the intersection of the radius and the descending side describes the quadratrix (*Collection*, IV, 30).

While it descends, the upper side remains parallel to the lower side. Euclidean parallel.

The quadratrix is the first curve, after the circle, which the philosophers discovered. Which is also one part of why the quadratrix was discovered. It is generated by the circle.

The other part is the trisection of the angle. Proclus reports: the geometers use the quadratrix of Hippias to trisect the angle (comm. *Elements* 272). The division of the angle into two equal parts, and hence into any number of equal parts which is a multiple of two, is easier. It is proposition 9 in book I of the *Elements*. The division of the angle into three equal parts, and into any number of equal parts which is not a multiple of two, is harder. Given the quadratrix, however, the division of the angle into three equal parts, and into any number of equal parts, is not hard any longer either. The upper side of the square descends with uniform motion, and *ipso facto*, if the side along which the upper side descends, or a part of this side, is divided into equal segments, and parallel lines are drawn through the ends of the segments, the angles at the vertex of the square, which is the center of the circle, defined by the intersections of the parallel lines and the quadratrix are equal.

The division of a line into a number of equal segments greater than two also depends on parallel lines, and hence also on proportionality. In the *Elements* this is established by proposition 9 in book VI. The trisection of the angle thus also depends on proportionality.

The two problems, to square the circle and to trisect the angle, the first two strictly mathematical problems which the philosophers formulated and began solving, are both analysis of the circle, both attempts to reduce the circle to measurement. Ptolemy writes: the ancients already saw that the heavenly bodies are carried from east to west in circles parallel to one another and always about the same center. The investigation of the heavenly bodies demanded the analysis of the circle.

The great advance of physics posed the two

problems.

The great advance of physics showed them the way to measurement and calculation, the assumptions on which results depend, and circles, angles, and parallel lines.

cccxiv

And in the following one century-odd mathematics entered the second of the three brightest periods in its development.

The great advance of physics continued.

Hicetas discovered the earth's rotation on its axis. Eudoxus formulated the theory of the motions of the heavenly bodies in concentric circles, and measured the radius of the earth. Ponticus said that Mercury and Venus revolve around the sun. Aristotle wrote *On the heaven* and the *Meteorology*. Dicaearchus improved substantially on the measurement of the radius of the earth. Autolycus wrote *On the moving sphere* and *On the risings and settings*, Euclid wrote the *Phenomena*.

Euclid wrote also the *Optics*.

Democritus said that respiration is the distinctive mark of life. Ponticus said that the body has all the faculties ascribed to the soul. Aristotle wrote *On the soul* and the five treatises on animals. Theophrastus investigated 550 kinds of plants. Herophilus identified the origin of the nerves in the brain, distinguished sensory from motor nerves, described the paths of cranial nerves, distinguished the cerebrum from the cerebellum.

Pytheas sailed the Atlantic north of the strait of Gibraltar, and measured Britain. Dicaearchus measured the Mediterranean. Hanno sailed the Atlantic south of the strait of Gibraltar.

Democritus said that the modification of all things is due to the shape, order, and position of atoms. Plato formulated the theory of the forms. Aristotle wrote the *Physics* and *On generation and corruption*.

Early in that time, also, Critias said that a wise man discovered the fear of gods to frighten evildoers in their thoughts.

Among the results above are the measurement of Britain, of the Mediterranean, and of the earth. The theory of the motions of the heavenly bodies is a mathematical theory. It is a theory of circles.

Autolycus defined the circles of the equator, ecliptic, horizon, and meridian. And he in *On the moving sphere* and *On the risings and settings* and Euclid in the *Phenomena* deduced from these definitions the first 60 physical theorems.

Among the philosophers above, Aristotle developed logic. And logic is a theory, and a mathematical theory, of the assumptions on which results depend.

The following are the strictly mathematical results.

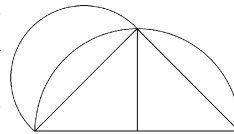
The second of the three brightest periods in the development of mathematics begins with Hippocrates of Chios.

Hippocrates of Chios solved the problem of squaring the circle.

Aristotle reports: Hippocrates squared lunes (*Refutations*, 11). Lune is the figure bounded by the arcs of two circles. And Simplicius records the different cases solved by Hippocrates. This is the simplest, and the fundamental:

circles are to one another as the squares on their diameters;

hence, so are semicircles;



hence, given a right isosceles triangle, the semicircle whose diameter is the side of the triangle is equal to one half the semicircle whose diameter is the base of the triangle;

and subtracting the common area from the semicircle whose diameter is the side and from the half of the semicircle whose diameter is the base, the remaining lune is equal to one half the isosceles triangle (comm. *Physics*, 56-7).

It is the first solution of the problem of squaring the circle. It is a partial solution, but a solution it is. A part of the circle, the lune, has been reduced to a triangle. The problem of the quadrature of a part of the circle has thus been solved. Shown equal to the area of a triangle, whose calculation is trivial, the area of a part of the circle has thus been calculated. In general, it is the first solution of the problem of quadrature, the reduction of a curvilinear figure to a rectilinear figure.

The solution first attests also the assumption on which it rests, circles are to one another as the squares on their diameters.

Hippocrates of Chios first attests also the problem of doubling the cube, as well as the method by which the problem will be solved.

Eutocius reports: Hippocrates of Chios discovered that to double the cube is to find two mean proportionals, thus reducing a great mystery to a smaller one (comm. *Sphere*, II).

$$\text{If } \frac{a}{b} = \frac{b}{c} \text{ and } \frac{b}{c} = \frac{c}{d}, \text{ then } \frac{a^3}{b^3} = \frac{a}{d},$$

$$\text{if } d = 2a, \text{ then } \frac{a^3}{b^3} = \frac{1}{2},$$

and given a cube with side a , if the two mean proportionals b and c between a and $2a$ are found, b is the side of the cube whose volume is double the volume of the cube with side a .

The smaller mystery to which the great one has been reduced involves quadratic equations of the kind $a\sqrt{bd} = b^2$.

Of the solutions of both problems, the squaring of the circle and the doubling of the cube, proportionality is a fundamental assumption. Eratosthenes said: proportion is the bond of mathematics. It is the bond of the three problems, the squaring of the circle, the trisection of the angle, and the doubling of the cube.

Proclus reports also: Hippocrates of Chios wrote the first *Elements*.

The first treatise in which theorems were deduced from explicitly stated axioms. Deductive since Pythagoras, for one century-odd by this time, with Hippocrates of Chios mathematics became axiomatic.

In the same generation, Theodorus of Cyrene proved that $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$ are irrational.

In the same generation, Democritus solved the problems of the volume of the cone and the volume of the pyramid. Archimedes reports: Democritus first said that the volume of the cone is $\frac{1}{3}$ of the volume of the cylinder of the same base and height, and the volume of the pyramid is $\frac{1}{3}$ of the volume of the prism of the same base and height (*Method*, pr.).

And among Democritus's many treatises Laertius lists: *On the division of the angle*, *Geometry*, *Numbers*, and *On irrational lines and solids* (IX, 47).

Democritus obtained very many physical

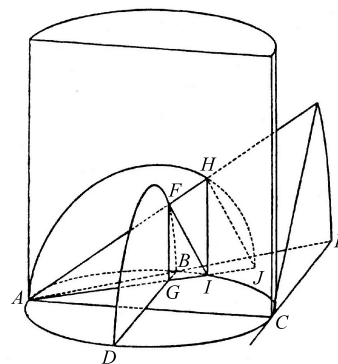
results, including solutions of the problems of the principles of all things, the earth, and the soul. Hippocrates of Chios obtained one physical result, he explained comets. Aristotle reports: Hippocrates of Chios said that comets are planets, the tail is only an appearance due to the reflection of the sun's light by the moisture which a comet attracts when it moves through the northern regions. Theodorus of Cyrene obtained no physical results.

Two-odd generations later Aristotle will affirm: philosophy is theoretical and practical, and theoretical philosophy is physics and mathematics. It is with the labors of Hippocrates of Chios, Theodorus of Cyrene, and Democritus that mathematics began becoming a distinct part of philosophy. Began becoming the body of problems concerned with circles, squaring, triangles, lunes, cubes, doubling, proportionals, axioms, irrationals, cones, cylinders, pyramids, prisms, angles, numbers, lines, solids.

And in the following two generations it was established which the assumptions are on which the formulations and solutions of the problems depend, and which among them are the fundamental.

One generation after Hippocrates, Theodorus, and Democritus, Archytas doubled the cube. He solved the great mystery, and by the smaller one to which the great had been reduced. It is a spectacular result.

Eutocius records the solution:



given length AB , construct $AC = 2AB$, and draw a circle with diameter AC and chord AB ; on semicircle ABC construct a right semicylinder; AB produced and tangent CE intersect at E ; revolve triangle ACE on axis AC ; the triangle

describes a right cone whose surface intersects the surface of the semicylinder;
 construct on AC as diameter a semicircle in the plane perpendicular to the plane ABC , and, with point A fixed, rotate the semicircle in the direction of point B ; the semicircle intersects the semicylinder and the cone at point H , and the position of the semicircle is now AHJ ;
 draw BD parallel to CE , and on BD as diameter construct a semicircle in the plane perpendicular to the plane ABC ; this semicircle lies on the surface of the cone and intersects AH at F ;
 perpendicular HI to AJ lies on the surface of the semicylinder and intersects circle ABC at I , perpendicular FG intersects AI at G ;
 from the semicircle BFD , $FG^2 = BG \cdot DG$;
 from the circle $ABCD$, $BG \cdot DG = AG \cdot GI$;
 hence $FG^2 = AG \cdot GI$;
 hence triangles AFG , FGI , and AFI are similar;
 hence angle AFI is right;
 hence triangles AFI , AHI , and AHJ are similar,
 and $\frac{AF}{AI} = \frac{AI}{AH} = \frac{AH}{AJ}$;
 from the triangle of revolution ACE , $AB = AF$,
 and from the semicircle of rotation AHJ ,
 $AC = AJ$;
 hence $\frac{AB}{AI} = \frac{AI}{AH} = \frac{AH}{AC}$;
 hence $\frac{AB^3}{AI^3} = \frac{AB}{AC}$;
 but $AC = 2AB$;
 hence $\frac{AB^3}{AI^3} = \frac{1}{2}$;
 and given a cube with side AB , the side of the cube double its size is AI (comm. *Sphere*, II).
 Archytas's is a spectacular result. It shows the very great advance of mathematics. It shows the very great advance of proof. It shows spectacularly also that proof depends on assumptions. Among the assumptions on which Archytas's proof depends is once again proportionality, and in particular similar triangles, and thus parallel lines. Among the assumptions are also in some detail the properties of the circle. Most of all, however, the assumptions are very many, nor is it clear which among them are the fundamental. Archytas's proof shows that, if the accumulation of results early in the development of philosophy demanded proof steadily more urgently, the

subsequent accumulation of proofs demanded steadily more urgently making explicit which the assumptions on which proof depends are, and which among them are the fundamental. The accumulation of strictly mathematical results demanded the *Elements*.

And later in the same generation Theaetetus took a major step toward the solution of this problem, and Eudoxus solved it. Eudoxus showed which the fundamental assumptions are.

Theaetetus formulated the theory of the irrationals. Eudoxus formulated the theory of proportion.

Proclus, on the authority of Eudemus, reports: when Euclid put together the *Elements*, he systematized the theorems of Eudoxus, and perfected the theorems of Theaetetus (comm. *Elements*, 68). More particularly, Pappus, also on the authority of Eudemus, ascribes the theory of irrational lines developed in book X of the *Elements* to Theaetetus (comm. *Elements*, I, 1). More particularly as well, an anonymous scholiast on book V of the *Elements* ascribes the theory of proportion developed in book V of the *Elements* to Eudoxus (*id.*). And on the authority of Archimedes the axiom on which the generality of the theory of proportion rests, demonstrated in proposition 1 of book X of the *Elements*, has to be ascribed to Eudoxus (*Sphere*, pr.; *Parabola*, pr.).

Two generations after Theaetetus and Eudoxus, then, Euclid perfected the theorems of Theaetetus, and systematized the theorems of Eudoxus. Two generations before Euclid, Theaetetus formulated the theory of irrational lines, perfected by Euclid in book X of the *Elements*, and Eudoxus formulated the theory of proportion, systematized by Euclid in book V of the *Elements*.

In book X of the *Elements* 135 theorems, lemmas, and corollaries are proved, more than one quarter of the 514 theorems, lemmas, and corollaries proved in all its thirteen books. The 135 theorems, lemmas, and corollaries of book X divide into two kinds, those concerned with magnitudes, and those concerned with lines.

And those concerned with lines are the theory of irrational lines developed by Theaetetus.

Propositions 1-8 are concerned with magnitudes. The first and the fundamental concerned

with lines is proposition 9. It shows:

given the straight lines a and b , and any positive integers m and n ,

if, whenever $\frac{a}{b} = \frac{m}{n}$, also $\frac{a^2}{b^2} = \frac{m^2}{n^2}$,

and whenever $\frac{a^2}{b^2} = \frac{m^2}{n^2}$, also $\frac{a}{b} = \frac{m}{n}$,

then a and b are rational;

otherwise they are irrational (*id.*).

The majority of the following propositions are concerned with lines. Among them, proposition 36 demonstrates the line

$a + \sqrt{b}$ (*id.*),

and proposition 42 shows that the division of a line into the segments a and $\sqrt{b}$ is unique (*id.*), that

if $a + \sqrt{b} = x + \sqrt{y}$,

then $a = x$ and $b = y$.

And hence, the central concern of the theorems, lemmas, and corollaries in the book concerned with lines is to demonstrate and classify the irrational lines of the form $\sqrt{a \pm \sqrt{b}}$, where a and b are rational lines.

If numbers are lines, the theory developed by Theaetetus shows, with the result of proposition 9, when numbers are rational, and when they are irrational, and hence investigates the irrational numbers of the form $\sqrt{a \pm \sqrt{b}}$, where a and b are rational numbers.

Numbers, however, are not lines.

The definition of number in the *Elements* is definition 2 in book VII:

number is a multitude of units (*id.*).

And that this definition was discovered in the generation of Theaetetus, the fact attests that Aristotle – in the generation between that of Theaetetus and that of Euclid – takes it for granted throughout his treatises. In the *Physics*: number is a multitude of units (III, 7). In book III of the *Metaphysics*: number is units (4). In book X of the *Metaphysics*: number is a multitude of units (1). In the *Categories*: number is a discrete quantity (6).

Thus, not only the irrationals are not numbers, the rationals are not numbers. It is a difficulty even whether the unit is a number, whether it is a multitude of units, a multitude of itself. Aristotle writes: strictly, the smallest number is 2 (*Physics*,

IV, 12). If 1 is not a number, number is the natural numbers beginning with 2, exclusively. Number is 2, 3, 4, ... If 1 is a number, number is the natural numbers, exclusively. Number is 1, 2, 3, ... Either way, the object discovered by Pythagoras, $\sqrt{2}$, the objects discovered by Theodorus of Cyrene, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$, $\sqrt{15}$, $\sqrt{17}$, and the objects discovered by Theaetetus, $\sqrt{2} + 3$, $2 + \sqrt{3}$, $\sqrt{2} + \sqrt{3}$, $\sqrt{\sqrt{2} + 3}$, $\sqrt{2 + \sqrt{3}}$, $\sqrt{\sqrt{2} + \sqrt{3}}$, and the differences of those as well as the sums, and of those also not only through $\sqrt{17}$, but trivially now also $\sqrt{18}$, $\sqrt{19}$, $\sqrt{20}$, and so on to infinity, all these objects are not numbers. Just as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, and so on, are not numbers.

And so number remained – the natural numbers, exclusively – for the following nineteen centuries, at least until Cardano.

In the centuries since Cardano number has been generalized. Number is still, to begin with, the natural numbers. Further, however, under subtraction the natural numbers yield the integers, negative as well as positive, and including 0, and hence, more generally, number is the integers. Further, under division, excluding division by 0, the integers yield the rationals, and hence, more generally, number is the rationals. Further, under extraction of roots the rationals yield irrational numbers and complex numbers. There are very many other irrationals, too, an infinite number of them, in fact, including π , e , π^2 , $\sqrt[3]{e}$, π^e , $\sqrt[e]{\pi}$. Hence, more generally, number is the real numbers, rational and irrational. Still more generally, number is the complex numbers, $a + ib$, where a and b are real numbers and i is $\sqrt{-1}$.

Yet, as this description shows, the foundation of the generalization is that first definition of number. The natural numbers yield all other numbers, integers, rational, real, complex.

And not only is that definition the first definition of number. And not only is it the foundation of the subsequent generalization of number. And not only is it an anticipation, a remote anticipation, yet unambiguously an anticipation, of the definition of number which has established itself after Cantor and Frege, number is a set of equivalent sets, and sets are equivalent when they can be

arranged in one-to-one correspondence.

In the one century-odd from Hippocrates of Chios to Euclid, decisively with the labors of Theaetetus and Eudoxus, and conclusively with the labors of Euclid, mathematics became geometry, and mathematics became by far the most advanced part of the acquisition of knowledge, both. These are also one another's cause and effect. Because it became geometry, mathematics became by far the most advanced part of the acquisition of knowledge. And the definition of number is an essential part of how and why mathematics became geometry.

Democritus wrote *On irrational lines and solids*.

And the theory developed by Theaetetus is not negative. It does show that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and so on, and $\sqrt{2} + 3$, $\sqrt{2} + \sqrt{3}$, $\sqrt{\sqrt{2} + 3}$, $\sqrt{\sqrt{2} + \sqrt{3}}$, and so on, are not numbers. It shows this, however, by showing, affirmatively, that they are lines. It shows this as clearly and distinctly, too, by proof as rigorous as any in at least the following twenty-four centuries.

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and so on, and $\sqrt{2} + 3$, $\sqrt{2} + \sqrt{3}$, $\sqrt{\sqrt{2} + 3}$, $\sqrt{\sqrt{2} + \sqrt{3}}$, and so on, are lines.

The definition of line in the *Elements* is definition 2 in book I:

line is a length without breadth (*id.*).

And that this definition as well was discovered in the generation of Theaetetus Aristotle again attests. In the *Topics* he criticizes: those who define line as a length without breadth (VI, 6). Because, Aristotle argues: things have to be defined *genus proximus differentia specifica*, and both genus and species affirmatively, for, if the species is negative, the genus becomes part of the species; so, in that definition, length is the genus, without breadth is the species, and the genus is part of the species (*id.*). One and a half centuries after Aristotle, one century after Euclid, Apollonius explained the definition. Proclus reports: Apollonius said that we conceive the line from the measurement of a length, of a wall or a road, when the breadth is ignored, because distance is measured in one dimension (comm. *Elements*, 100).

The irrationals are lines. It has been demonstrated that they are lines. It has thus been dem-

onstrated that they are of the same kind as everything else that is analyzed into lines – circles, triangles, lunes, squares, cubes, cones, cylinders, pyramids, prisms.

It has thus been demonstrated that line is very general. Number is not so general.

Given lines a and b , and any positive integers – and that is, any numbers – m and n , if, whenever $\frac{a}{b} = \frac{m}{n}$, also $\frac{a^2}{b^2} = \frac{m^2}{n^2}$, and whenever

$\frac{a^2}{b^2} = \frac{m^2}{n^2}$, also $\frac{a}{b} = \frac{m}{n}$, then a and b are rational; otherwise they are irrational.

And this demonstration, too, depends on proportion.

Later in the same generation, Eudoxus wrote a *Description of the earth*.

He catalogued 32 northern and 15 southern constellations.

He measured the radius of the earth.

He formulated the theory of the motions of the heavenly bodies in concentric spheres.

He developed the method of exhaustion.

And he formulated the theory of proportion.

The theory is developed in book V of the *Elements*. The foundation of the theory is the definition of proportion, definition 5. It affirms:

given magnitudes a , b , c , and d , and any positive integers m and n ,

if, whenever $ma > nb$, also $mc > nd$,

whenever $ma = nb$, also $mc = nd$,

and whenever $ma < nb$, also $mc < nd$,

then $\frac{a}{b} = \frac{c}{d}$ (*id.*).

The fundamental assumption on which all results which depend on proportion depend has thus been made explicit.

Among the results which depend on proportion are the measurement of the height of the pyramids and of the distance of ships from the shore, the proof that the sum of the angles of the triangle is equal to two right angles, the ratios which the parts of the body are, the measurement of the Black Sea, the Metonic cycle, the division of a line into equal parts, the trisection of the angle, the squaring of the circle, the doubling of the cube, the demonstration that the irrationals are lines, the measurement of the earth's radius.

One century-odd after Eudoxus, Eratosthenes

said: proportion is the bond of mathematics.

Nineteen and a half centuries after Eudoxus, in 1591, in *The analytic art*, Viète formulated the master rule: if four terms are such that the product of the extremes is equal to the product of the means, they are proportional, and conversely, if four terms are proportional, the product of the extremes is equal to the product of the means; thus a proportion is that from which an equation is composed and an equation is that into which a proportion is resolved.

Equation depends on proportion.

Twenty-two and a half centuries after Eudoxus, in 1872, in the *Continuity and irrational numbers*, Dedekind defined the irrational numbers. Dedekind reports: the conception of the irrational number as a ratio of two measurable quantities is most clearly expressed in the celebrated definition of equality of ratios in book V of the *Elements*, and this definition is the source of my theory as well (pr.).

The irrational numbers not less than irrational lines depend on proportion.

Given magnitudes a , b , c , and d , and any positive integers – and that is, any numbers – m and n , if, whenever $ma > nb$, also $mc > nd$, whenever $ma = nb$, also $mc = nd$, and whenever $ma < nb$, also $mc < nd$, then $\frac{a}{b} = \frac{c}{d}$.

The definition depends on multiplication, and thus on addition, on equality, and on greater and less. The assumption on which all results which depend on proportion depend itself depends on addition, equality, and greater and less.

And making explicit the assumption on which the consistency of these, addition, equality, and greater and less, depends also has to be ascribed to Eudoxus.

Proposition 1 of book X of the *Elements* is:

given two unequal magnitudes, if from the greater a magnitude greater than its half is subtracted, from the remainder a magnitude greater than its half is subtracted, and so on, a magnitude smaller than the less magnitude will ultimately remain (*id.*).

Two generations after Euclid, Archimedes writes in the *Quadrature of the parabola*: I assume the following lemma,

the excess by which the greater of two unequal

areas exceeds the less, by being added to itself, can be made to exceed any given finite area; and the earlier geometers have used this lemma; for they have proved that circles are to one another in the duplicate ratio of their diameters, that spheres are to one another in the triplicate ratio of their diameters, that the volume of the pyramid is $\frac{1}{3}$

of the volume of the prism of the same base and height, and that the volume of the cone is $\frac{1}{3}$ of

the volume of the cylinder of the same base and height; and none of these results can be proved without this lemma (pr.). In *On the method of mechanical theorems*: Democritus first said that the cone is $\frac{1}{3}$ of the cylinder of the same base

and height, and the pyramid is $\frac{1}{3}$ of the prism of

the same base and height, but he did not prove this (pr.). And in *On the sphere and cylinder*:

Eudoxus first proved that the pyramid is $\frac{1}{3}$ of the

prism of the same base and height, and the cone is $\frac{1}{3}$ of the cylinder of the same base and height (pr.).

The earlier geometers have used this lemma, and no one but Eudoxus used it first.

Aristotle, once again, corroborates this conclusion. A younger contemporary of Eudoxus, one generation-odd before Euclid, Aristotle attests the result. Discussing time in the *Physics*, Aristotle writes:

by the continuous addition to a finite magnitude a magnitude can be obtained which exceeds any given limit, and in the same way by continuous subtraction a magnitude can be obtained which falls short of any given limit (VIII, 10).

It is a physical argument in Aristotle, a proposition in Euclid, a lemma in Archimedes.

Twenty-two and a half centuries after Eudoxus, in 1899, in the *Foundations of geometry*, Hilbert has the theorem:

if $a > 0$ and $b > 0$ are any two numbers, then it is always possible to add a to itself a sufficient number of times so that the resulting sum is greater than b ;

symbolically,

$$a + a + \dots + a > b \text{ (13).}$$

The theorem is: Archimedes's theorem (*id.*), and the first of the two: theorems of continuity (*id.*). Earlier, Hilbert has the axiom:

if AB and CD are any segments, then there exists a number n such that n segments CD constructed contiguously from A along the ray from A through B will pass beyond B (8).

This is: the axiom of measure, or Archimedes's axiom (*id.*), and the first of the two: axioms of continuity (*id.*). The second is: the axiom of completeness (*id.*). And Hilbert writes: a completeness axiom which does not require the axiom of Archimedes entails a contradiction (*id.*).

By the continuous addition to a finite magnitude a magnitude can be obtained which exceeds any given limit.

The assumption has thus been made explicit on which the consistency of addition, equality, and greater and less depends. And on addition, equality, and greater and less not only all mathematics depends. There cannot be many propositions, if there be any, which do not depend, implicitly if not explicitly, on any of these, addition, equality, and greater and less.

Both, the definition of proportion and the assumption on which its consistency depends, are concerned with magnitude.

In Aristotle's argument attesting the assumption the magnitude is time, in Euclid's proposition attesting it the magnitude is line, in Archimedes's lemma the magnitude is area.

Magnitude is thus very general. It is more general than line. Line, area, and time are all particulars over which magnitude is the general.

Magnitude is introduced in the *Elements* with definition 1 in book V:

magnitude is a part of a magnitude when the less measures the greater (*id.*).

Aristotle does not attest this proposition. He is concerned with magnitude, however, throughout the *Physics*. Discussing time, he writes: all magnitudes are continuous (IV, 11). Discussing motion, again: all magnitudes are continuous (VI, 2), and: since magnitude is continuous, it is divisible into an infinite number of parts (8). And discussing the principles of all things: there is no smallest magnitude (I, 4).

Magnitude is continuous. There is no smallest

magnitude.

Number, on the other hand, is discrete. The smallest number is 2. Or 1.

Clearly, number is not a magnitude.

Aristotle is clear about the difficulty. He writes in the *Posterior analytics*: there can be no arithmetical demonstration of magnitude unless magnitude is number (I, 7).

In general, the resolution of the difficulty is this, Aristotle argues: demonstration cannot cross from one genus to another, there can be no demonstration of things of different genera, of arithmetic and geometry, for instance (*id.*), hence: demonstration demands that the kinds of knowledge be subordinated to one another, optics to geometry, mechanics to stereometry, harmonics to arithmetic, observation to astronomy (13).

In particular, this is how the difficulty is resolved in the *Elements*. Euclid deals with number in books VII-IX. Definition 2 in book VII is of number:

number is a multitude of units.

Definition 3 is:

number is a part of a number when the less measures the greater (*id.*).

Just as magnitude is a part of a magnitude when the less measures the greater. And then the 106 theorems and corollaries of books VII-IX demonstrate that number obeys proportion. Many of them also demonstrate of number exactly what those in other books demonstrate of magnitude. To that end, too, number is always represented by a line, sums are always represented by lines, products always by areas.

Great generality is thus achieved. Frege writes: the second stage in the development of functions is reached with the advance to general laws which hold for all numbers; in notation to this corresponds the transition to calculation with letters; in

$$(a + b).c = a.c + b.c$$

we have a theorem of this kind; with this the consideration of single functions is reached. The second stage in the development of functions is reached with the *Elements*. In notation this corresponds to calculation with lines. The theorem

$$(a + b).c = a.c + b.c$$

is proposition 1 in book V.

On the other hand, the tension has remained. Magnitude is continuous. Number is not.

And the tension will not be resolved until it is proved that the real numbers, rational and irrational, are continuous. Dedekind proved this in 1872, in the *Continuity and irrational numbers*.

In particular, then, even if discrete, number has been shown a magnitude.

In general, demonstration demands that the kinds of knowledge be subordinated to one another, and arithmetic has been subordinated to geometry.

Magnitude is thus very general. Time, line, and area as well as number are all particulars over which magnitude is the general.

Magnitude is a part of a magnitude when the less measures the greater.

Line is a length without breadth. We conceive the line from the measurement of a length, of a wall or a road, when the breadth is ignored, because distance is measured in one dimension.

Line is thus the fundamental magnitude.

Time Aristotle always represents by lines. Areas are in Archimedes as in Euclid the products of lines or represented directly by lines. Numbers are represented by lines. In general, everything which succumbs to proof is either analyzed into lines directly or represented by lines and thus analyzed into lines. All objects in the *Elements*, plane figures, rectilinear and curvilinear, solids, rectilinear and curvilinear, numbers, and irrationals are lines. The motions of the heavenly bodies in Autolycus and in Euclid's *Phenomena* are lines. Light in Euclid's *Optics* is lines. In Archimedes conoids, spheroids, parabolas, and spirals are lines, and weights and fluids are lines.

Magnitude, moreover, is continuous, and the consistency of addition, equality, and greater and less depends on continuity, too.

All objects are reduced to spatial relations, spatial relations are the ground of proof, and spatial relations close the gaps in the chains of inferences.

Mathematics is geometry.

That mathematics is by far the most advanced part of the acquisition of knowledge the fact of Euclid, Archimedes, and Apollonius suffices to show.

Decisive in the achievement of this feat are the labors of Eudoxus, the definition of proportion and the axiom on which its consistency, and

hence also its generality, depends.

And the relation of these facts is causal.

The results of Eudoxus, the explicit formulation of the fundamental assumptions on which proof depends, made possible the *Elements*. The *Elements* is axiomatic, the assumptions on which its 514 proofs depend are stated explicitly. Of this, that the *Elements* is axiomatic, the results of Eudoxus are the ground and the beginning. The *Elements* is also paradigmatically methodical, it includes in judgments nothing but what is so clear and distinct that it cannot be doubted, divides difficulties into parts such as will lead to their better solution, and proceeds in order, beginning with the simplest and easiest and ascending gradually to the more complex. Of this, the clarity and distinction, division of difficulties, and order of the *Elements*, the results of Eudoxus are the ground. The *Elements* is also geometry. And of this, that the *Elements* is geometry, again the results of Eudoxus are the ground. The results of Eudoxus are the fundamental assumptions, on whose ground in the *Elements* numerical relations are reduced to spatial relations.

The *Elements* in turn made possible the strictly mathematical results of Archimedes and Apollonius. It made possible the mathematical development of optics, from Euclid's *Optics* through Ptolemy's *Optics*. It made possible the mathematical development of statics and hydrostatics by Archimedes. It made possible the mathematical development of geography through Eratosthenes through Hipparchus through Ptolemy. It made possible the mathematical development of astronomy, from Euclid's *Phenomena* through Hipparchus through Ptolemy. All these remained unsurpassed until the sixteenth Christian century. Of all these the *Elements* is the ground. All these are also geometry.

Because it became geometry, mathematics became by far the most advanced part of the acquisition of knowledge.

And the cause of the cause, the cause of Eudoxus, can only be the great advance of physics.

Magnitude is continuous, and the consistency of addition, equality, and greater and less depends on continuity.

Magnitude is a part of a magnitude when the less measures the greater.

Number is also a part of a number when the less measures the greater.

We conceive the line from the measurement of a length.

The source of Dedekind's theory is the definition of the equality of measurable quantities.

And Hilbert's axiom is the axiom of measure.

The definition of proportion and the assumption on which its consistency, and hence also its generality, depends are an empirical generalization over measurement.

Eudoxus's predecessors measured the pyramids, the distance of ships from the shore, the obliquity of the ecliptic, the Black Sea, the Red Sea, the Caspian Sea, the summer solstice, the solar year, the period synchronizing moon and sun. Eudoxus measured the earth. Here are three more measurements of his.

Strabo reports: to describe the promontory of Attica, Eudoxus said that, if a straight line is drawn easterly from the Ceraunian mountains to cape Sunium, all the Peloponnese will be on the right, to the south, and all the sea coast of Attica on the left, to the north (IX, 1.1).

It is the first determination of the orientation of the promontory of Attica, or of any promontory. Eudoxus determined the direction from the Ceraunian mountains to cape Sunium as easterly. The distance from cape Glossa, where the Ceraunian mountains abut on the Adriatic, to cape Sunium, on the Aegean, is 510 km. The latitude of cape Glossa is $40^{\circ} 30' N$, that of cape Sunium $37^{\circ} 40' N$.

Archimedes reports: Eudoxus said that the sun is 9 times larger than the moon (*Sand-reckoner*).

It is the first number assigned to the sun. The result on which it improves is Anaxagoras's, the sun is larger than the Peloponnese. The sun is a little more than 400 times larger than the moon.

Simplicius reports that Eudoxus measured the periods of the planets (comm. *Heaven*, 495-6).

And of these, too, it is the first measurement. It improves on Plato: in the spindle of necessity, spinning around all things in the heaven, there are 8 whorls, the innermost, the eighth, spins the fastest, the seventh, sixth, and fifth spin together with the same speed, the fourth is slower, the third slower still, and the second yet slower. Eudoxus's numbers for the second and third

whorls, and for both the synodic period, the return to the same position with respect to the sun, and the sidereal period, the return to the same position with respect to the fixed stars, are:

	synodic	sidereal
Saturn	13 months	30 years,
Jupiter	13 months	12 years (<i>id.</i>).

To the nearest tenth, the synodic period of Saturn is 12.4 months, the sidereal 29.4 years, the synodic period of Jupiter 13.1 months, the sidereal 11.9 years.

It has been concluded above that it would have taken a miracle to prevent Eudoxus from calculating the earth's radius. It can be concluded here that it would have taken a miracle to prevent him from defining proportion.

Archytas, then, doubled the cube. Later in the same generation Theaetetus showed that the irrationals are lines. Later in the same generation Eudoxus defined proportion, and formulated the axiom of measure.

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Plato is contemporary with Archytas.

On the Academy's door was written: the not geometer is not to enter (Philoponus, comm. *Soul*, 117). Theaetetus, Eudoxus, Aristotle, and Menaechmus were all residents of the Academy. Later on, no geometer entered the Academy.

Plato first said what mathematics is, and is not.

In the *Republic*, for instance: mathematics leads the soul forcibly upward and compels it to study numbers themselves, never allowing it to attach them to the sensible things; numbers themselves can be grasped in thought alone and in no other way.

Plutarch reports: Plato disapproved of how Archytas doubled the cube; solving problems by mechanical means, Plato said, corrupts geometry and distracts it from rising to the immaterial and immortal (*Conversations*, VIII, 2), and: Eudoxus and Archytas were the originators of the art of mechanics; with great ingenuity they first demonstrated geometrical theorems by mechanical means, grounding in experience propositions too difficult to prove by word or figure; Plato was indignant at this, and assailed them for corrupting geometry and destroying its purity, for leaving the objects of thought and turning for help to the

sensible things and to means of base and manual labor (*Marcellus*, 14).

In the next generation, Aristotle said that philosophy is theoretical and practical. Theoretical philosophy is physics and mathematics. The physicist studies both form and matter, the mathematician separates form from matter. The objects investigated by the mathematician are abstracted from the sensible things.

Aristotle developed logic.

He distinguished terms, propositions, and inferences.

He developed the calculus of terms.

Proposition is a sentence which affirms what is true or false.

Inference is of four kinds, demonstration, dialectical argument, examination, and contentious argument.

Demonstration is deduction. Deduction always depends on principles. The principles must be true and general.

Knowledge is acquired only by demonstration.

And the principles are Aristotle's fundamental concern. Which the principles are, truth, which the true principles are, generality, which the general principles are, and which the true and general principles are.

It is a theory. It is the theory of the assumptions on which results depend.

The assumptions are as fundamental, and important, as truth and generality are fundamental, and important. Conversely, too, truth and generality are as fundamental, and important, as the assumptions are.

And that it is a mathematical theory the fact shows that nothing illustrates it better than, developed in the generation after Aristotle, the *Elements*.

The fact shows it also that the standing illustrations in the Aristotelean treatises are the mathematical.

And among the illustrations are the six from which the formulation of the problem is gathered, the sum of the angles of the triangle, and the assumptions on which the sum depends.

These are, again, the six illustrations.

In the *Great ethics*: it would be absurd to demonstrate that the sum of the angles of the triangle is equal to two right angles from the

assumption that the soul is immortal.

It would not be absurd in Plato. If it would yet be, this raises the question of which the assumptions are from which it would not be absurd to demonstrate that the sum of the angles of the triangle is equal to two right angles. It implies also that these assumptions may be as fundamental, and important, as, perhaps even more fundamental, and important, than, that the soul is immortal.

Again in the *Great ethics*: when some things are assumed, assumed is also what follows from them; for instance, if the sum of the angles of the triangle is two right angles, the sum of the angles of the quadrilateral is four right angles, and if the sum of the angles of the triangle changes, so does the sum of the angles of the quadrilateral, and if the sum of the angles of the quadrilateral is not four right angles, the sum of the angles of the triangle is not two right angles either.

This is why – when some things are assumed, assumed is also what follows from them – the assumptions are fundamental, and important. And that, if the sum of the angles of the triangle is two right angles, it follows that the sum of the angles of the quadrilateral is four right angles, is trivial. So much so, it does not merit a theorem of its own in the *Elements*. But it again suggests that the assumptions on which this, in particular, depends, that the sum of the angles of the triangle is equal to two right angles, are fundamental, and important.

The illustration also considers the possibility that the sum of the angles of the triangle is not equal to two right angles. Depending on the assumptions from which the sum follows.

Yet again in the *Great ethics*: when it is affirmed that the sum of the angles of the quadrilateral is four right angles, and someone asks, why? the answer is, because the sum of the angles of the triangle is two right angles.

The more fundamental, and important, is also the cause.

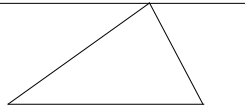
In *On the soul*: knowledge that the sum of the angles of the triangle is equal to two right angles depends on knowledge of what is straight and what is curved, and of line and plane.

In the *Elements* line, straight, and plane are defined at the very beginning, in book I. The

definition of line is definition 2, the definition of straight line definition 4, the definition of plane definition 7. What is curved is implied by the definitions of line and straight line. Curve is a line which is not straight.

Line, straight line, plane, and curve are the assumptions, or among the assumptions, on which that the sum of the angles of the triangle is equal to two right angles depends.

In book IX of the *Metaphysics*: why is the sum of the angles of the triangle equal to two right angles? because the sum of the angles on one side of a line at a point is equal to two right angles; and if through one of the vertices of the triangle a line parallel to the opposite side is drawn, this is evident at a glance.



The figure is that of Pythagoras's proof that the sum of the angles of the triangle is equal to two right angles.

That the sum of the angles of the triangle is equal to two right angles depends on line, straight line, plane, and curve. More particularly, it depends on that, that the sum of the angles on one side of a line at a point is equal to two right angles. Also, the sum of the angles of the quadrilateral is equal to four right angles because the sum of the angles of the triangle is equal to two right angles. The sum of the angles of the triangle is equal to two right angles because the sum of the angles on one side of a line at a point is equal to two right angles. And this last is evident at a glance if a parallel line is drawn. Parallel lines are the connection of the sum of the angles on one side of a line at a point and the sum of the angles of the triangle. The sum is in both cases two right angles.

And in the *Prior analytics*: the same false result can follow from different false assumptions, that parallels intersect, for instance, follows from the assumption that the interior angle is greater than the opposite exterior angle as well as from the assumption that the sum of the angles of the triangle is greater than two right angles.

The equivalence of the two assumptions, that the interior angle is greater than the opposite exterior angle and that the sum of the angles of

the triangle is greater than two right angles, is – given Pythagoras's proof that the sum of the angles is equal to two right angles – evident at a glance.

That parallels intersect is false. That the sum of the angles of the triangle is greater than two right angles is also false. And the former follows from the latter. Whether parallels intersect depends on the sum of the angles of the triangle. Parallel lines depend on the sum of the angles of the triangle.

The assumptions, then, are fundamental, and important. As fundamental, and important, as truth and generality are fundamental, and important.

They are fundamental, and important, because when some things are assumed, assumed is also what follows from them.

The assumptions on which the sum of the angles of the triangle depends may be among the most fundamental, and important.

The assumptions, or among the assumptions, on which the sum of the angles of the triangle depends are the most fundamental, and important, line, straight line, plane, and curve.

More particularly, that the sum of the angles of the triangle is equal to two right angles depends on that, that the sum of the angles on one side of a line at a point is equal to two right angles.

Parallel lines are the connection of the sum of the angles on one side of a line at a point and the sum of the angles of the triangle. The two sums are equal, and equal to two right angles.

That parallels intersect is false.

That the sum of the angles of the triangle is greater than two right angles is also false.

And whether parallels intersect depends on the sum of the angles of the triangle. Parallel lines depend on the sum of the angles of the triangle.

This can be gathered from Aristotle with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

In the generation of Aristotle, Menaechmus discovered the conic sections.

Proclus reports on the authority of Eudemus: Menaechmus was a student of Eudoxus (comm. *Elements*, 67), and on the authority of Eratosthenes: Menaechmus discovered the conic sections

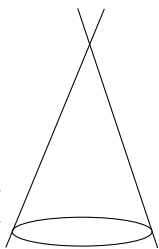
(111).

One generation after Menaechmus, Euclid wrote four books of *Conics*. Two generations after Euclid, Archimedes investigates the conics in *On conoids and spheroids*, the *Quadrature of the parabola*, *On the method of mechanical theorems*, *On the equilibrium of planes*, and *On floating bodies*. Two generations after Archimedes, Apollonius wrote the *Conics*. The second of the three brightest periods in the development of mathematics culminates in the investigation of the conics.

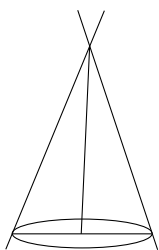
Eighteen centuries after Apollonius, Galileo showed that the path of a falling body is a parabola. Kepler showed that the paths of the planets are ellipses. The brightest period in the acquisition of knowledge, and the third of the three brightest periods in the development of mathematics, opens with the continuation of the investigation of the conics, and the discovery that nature is fond of the conics. And the acquisition of knowledge since has corroborated this result, the conics are everywhere, nature is very fond of the conics.

This is how Apollonius generates the conics in book I of the *Conics*:

given a fixed point, and a circle not in the same plane as the point, if a straight line through the point rotates about the circumference of the circle, the straight line traces the surface of a cone; the point is the vertex of the cone, the circle is the base of the cone (def. 1-2);



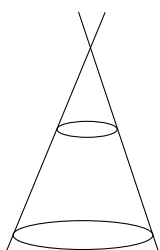
the straight line from the vertex to the center of the base is the axis; if the cone is cut by a plane through the vertex, the intersection of the plane with the surface and base of the cone is a triangle with its sides lying on the surface of the cone and its base lying on the base of the cone; if the cutting plane passes through the axis, the triangle is the axial triangle, its base is a diameter of the base of the cone (prop. 3);



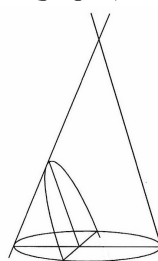
if the cone is cut by a plane parallel to its base, the intersection of the plane with the surface of

the cone is a circle (prop. 4);

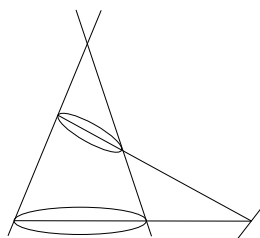
if, in the plane of the axial triangle, at a point on one side of the triangle a line parallel to the opposite side is



drawn, and at the point of intersection of this line and the base of the triangle, which is a diameter of the base of the cone, in the plane of the base of the cone a perpendicular to the diameter is drawn, this line and the perpendicular define another plane; and the intersection of this plane with the surface of the cone is the parabola (prop. 11);



if the line is drawn not parallel to the opposite side, it intersects the opposite side; the line and the perpendicular to the diameter of the base again define a plane;

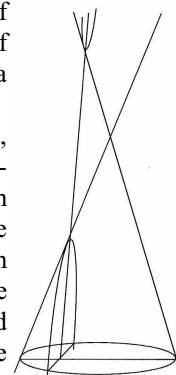


and if the line intersects the opposite side below the vertex, the intersection of the plane with the surface of the cone is the ellipse (prop. 13);

if the line intersects the opposite side above

the vertex, the intersection of the plane with the surface of the cone is the hyperbola (prop. 12).

The names parabola, ellipse, and hyperbola are from Apollonius. Before Apollonius, in Euclid and in Archimedes, the sections are called the section of the right-angled cone, the section of the acute-angled cone, and the section of the obtuse-angled cone, respectively, referring to the angle at the vertex of the cone. Hence the inference that the conics were first generated by Menaechmus by cutting with a plane a right-angled, an acute-angled, and an obtuse-angled cone, respectively. Their generation is taken for granted by Euclid already. In the



Phenomena, for instance: if a cone or a cylinder is cut by a plane not parallel to the base, the section is that of the acute-angled cone (pr.). The parabola can be generated by cutting a right-angled cone, the ellipse by cutting an acute-angled cone, and the hyperbola by cutting an obtuse-angled cone – except in that they are not generated uniformly – only in the way in which they are generated by Apollonius.

Thus, the assumptions on which the generation of the most important curves depends are very much the same. Line, straight line, plane, and curve. The circle is fundamental. The triangle is very important. Parallel lines are decisive. Parallel lines distinguish the curves, parabola, ellipse, and hyperbola. And parallel lines do not intersect.

And in the generation after Aristotle and Menaechmus, Euclid wrote the *Elements*, the *Data*, the *Phenomena*, and the *Optics*.

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The *Elements* is in thirteen books.

Book I is the foundation. It investigates the triangle, parallel lines, the triangle and parallel lines in their dependence on one another, and begins the investigation of area.

Book II is the first *tour de force*. It investigates the equality of areas, laying the foundations of algebra.

Book III investigates the circle.

Book IV inscribes and circumscribes rectilinear figures in and about circles.

Book V is the next *tour de force*. It develops the theory of proportion.

Book VI, the next *tour de force*, develops the theory of similar figures.

Books VII-IX are a unity, and develop the theory of numbers. Book VII lays the foundations of the theory, books VIII-IX, the next *tour de force*, investigate primes, progressions, and perfect numbers.

Book X, the next *tour de force*, develops the theory of irrational lines.

Books XI-XIII are also a unity. Book XI lays the foundations of solid geometry, book XII, the next *tour de force*, develops the method of exhaustion, and investigates volumes, and book XIII, the final *tour de force*, constructs the five regular solids and inscribes them in spheres.

Book I begins with 23 definitions, 5 postulates, and 5 axioms, or common notions, then 49 theorems and corollaries, or porisms, are deduced.

Book II begins with 2 definitions, then 14 theorems are deduced.

Book III begins with 11 definitions, then 39 theorems and corollaries are deduced.

Book IV begins with 7 definitions, then 19 theorems and corollaries are deduced.

Book V begins with 18 definitions, then 27 theorems and corollaries are deduced.

Book VI begins with 4 definitions, then 36 theorems and corollaries are deduced.

Book VII begins with 22 definitions, then in the following three books, VII-IX, 106 theorems and corollaries are deduced.

Book X has 16 definitions, and 135 theorems, lemmas, and corollaries are deduced.

Book XI begins with 28 definitions, then in the following three books, XI-XIII, 89 theorems, lemmas, and corollaries are deduced.

In all thirteen books, the theorems are 465, the lemmas 17, the corollaries 32, for a total of 514.

The definitions are 131, the postulates 5, the axioms 5, for a total of 141.

The 141 definitions, postulates, and axioms are the explicitly stated assumptions on which the subsequent results, the 514 theorems, lemmas, and corollaries, depend.

141 assumptions is many assumptions. Some of them are more fundamental, and important, than others, and only a few of them are the most fundamental, and important. And which the most fundamental, and important, assumptions are, and how, and why, only the generality of the assumptions, and in relation to the results which depend on them, can decide.

In particular, their division into the three kinds, definitions, postulates, and axioms, cannot decide. The postulates are 5, the axioms are 5, the definitions are 131, and this can be taken to indicate that the postulates and axioms are more fundamental, and important, than the definitions. More important, the postulates and axioms are in book I, indicating that all subsequent results, in all thirteen books, depend on them. But the 23 definitions in book I indicate the same, that all subsequent results, in all thirteen books, depend

on them. And 33 definitions, postulates, and axioms is still many definitions, postulates, and axioms. Some of them are more fundamental, and important, than others, and only a few of them are the most fundamental, and important. And in indicating that all subsequent results, in all thirteen books, depend on them, these 33 assumptions indicate rather that, that only generality, and in relation to the results which depend on them, can decide which among them are the most fundamental, and important, and how, and why. As it has to decide also whether all the most fundamental, and important, assumptions are among those in book I, or some of them are among the 108 definitions of the following books, and again also how, and why.

In fact, the attempt to determine which the most fundamental, and important, assumptions are, cannot be helped by their division into the three kinds, definitions, postulates, and axioms, at all. Nor will it be helped at all by opening the question about the genus of which these three are species.

Discussing the principles in the *Posterior analytics*, Aristotle writes: of the principles on which demonstration rests, some are peculiar to each kind of knowledge, others are common to all; for instance, peculiar is that line is of such and such a character, or that straight line is; common is that, if equal things are subtracted from equal things, the remainders are equal (I, 10). Later in the same discussion, Aristotle writes also: what is necessarily true is not a hypothesis or a postulate; a hypothesis or a postulate is what, although it can be proved, is assumed without proof (*id.*); and when it agrees with the opinions of the student, it is a hypothesis, when the student has no opinion on the matter or has the contrary opinion, it is a postulate (*id.*).

Ignoring the student, whose opinions are liable to make everything into a postulate, Aristotle's earlier distinction, particular and general, is the decisive. With Aristotle: the principles must be true and general. And since the principles had all better be either true, or false, and *tertium non datur*, lest truth admitted of degree, generality, which cannot not admit of degree, has to decide which among the principles are the most fundamental, and important, and how, and why.

And Aristotle's examples show that the generality of the assumptions, and in relation to the results which depend on them, has to decide which are the most fundamental, and important, whether the assumptions are axioms, or definitions, or, whatever the opinions of the student, postulates.

Aristotle's example of a principle common to all kinds of knowledge, if equal things are subtracted from equal things, the remainders are equal, is in the *Elements* axiom 3. And the axiom is very general. Propositions in all thirteen books depend on it. Aristotle's examples of principles peculiar to one kind of knowledge, that line is of such and such a character, or that straight line is, are in the *Elements* definitions 2 and 4 in book I. And these are very general, too. There is no proposition in the *Elements*, in any of its thirteen books, which does not depend on line, and on straight line.

Among Euclid's successors, Archimedes is very particular about stating explicitly the assumptions on which his proofs depend, but not very particular about distinguishing them into definitions, postulates, and axioms. In *On the sphere and cylinder* he begins with a discussion of earlier results, then has 6 axioms and 5 assumptions, or hypotheses, then deduces 58 theorems and lemmas. In *On conoids and spheroids* he begins with several pages in which he discusses and introduces the assumptions, including among them theorems proved earlier, without listing or numbering the assumptions, then deduces 34 theorems and lemmas. In *On spirals* he again begins with several pages in which he discusses earlier results and introduces the assumptions, again without listing or numbering them, then deduces 11 theorems, then lists and numbers 7 definitions, then deduces 17 more theorems. In the *Quadrature of the parabola* he begins with the discussion of the lemma, as he has it, or theorem, as Euclid has it, or axiom, as Hilbert has it, of measure, then deduces 17 theorems, then has 1 definition, then deduces 7 more theorems. In *On the equilibrium of planes* he begins with 7 postulates, deduces 16 theorems and lemmas, then has 1 definition, then deduces 10 more theorems and lemmas. In *On floating bodies* he begins with 1 postulate, then deduces

19 theorems. In the *Measurement of the circle* he deduces 3 theorems, without stating the assumptions, taking it for granted, clearly, that it is clear which they are. All in all, in the seven treatises, Archimedes has, listed and numbered, 9 definitions, 8 postulates, 6 axioms, and 5 hypotheses, from which he deduces 192 theorems and lemmas.

Also, in all his proofs Archimedes takes for granted the *Elements*, and in particular its definitions, postulates, and axioms, in a few very important cases correcting them.

Apollonius, in the *Conics*, has 11 definitions early in book I, then in books I-V deduces 303 theorems and lemmas, then has 6 more definitions early in book VI, then in books VI-VII deduces 84 more theorems and lemmas. Apollonius also takes Euclid for granted.

Later on, Galileo, in the third and fourth parts of the *Two new sciences*, has 4 axioms, 2 definitions, and 1 postulate, from which he deduces 71 theorems, lemmas, and corollaries. Galileo takes for granted Euclid, Archimedes, and Apollonius.

Huygens, in *The pendulum clock*, has 5 hypotheses and 17 definitions, from which he deduces 63 theorems. Huygens takes for granted Galileo as well.

Newton, in *The mathematical principles*, begins with 8 definitions and 3 axioms or laws, and later has 1 more definition and 3 hypotheses, from which he deduces 635 theorems, lemmas, and corollaries. And Newton also takes for granted his predecessors, including the *Elements*.

Later on still, Peano, in *The principles of arithmetic*, has 9 axioms and 31 definitions, from which he deduces 202 theorems.

Hilbert, in the *Foundations of geometry*, begins: geometry, like arithmetic, requires only a few and simple principles for its logical development; these principles are called the axioms of geometry; the establishment of the axioms of geometry and the investigation of their relations is a problem treated in many excellent works since Euclid; this problem is equivalent to the logical analysis of our perception of space. And Hilbert then has 20 axioms and 29 definitions, from which he deduces 68 theorems.

Peano's and Hilbert's are both explicitly attempts, and the most successful attempts, to

take nothing in particular for granted.

All in all, in these treatises Euclid, Archimedes, Apollonius, Galileo, Huygens, Newton, Peano, and Hilbert have 245 definitions, 14 postulates, 47 axioms, and 13 hypotheses, from which they deduce 2 132 theorems, lemmas, and corollaries.

2 132 results depending on 319 explicitly stated assumptions.

The assumptions have thus continued to be definitions, postulates, and axioms, as well as hypotheses. The development has yet been toward favoring definitions and axioms over postulates, or hypotheses. And toward favoring axioms over definitions. The foundation of this development, too, is the *Elements*.

Also, the assumptions are the elements, or the foundations, or most of all the principles. This is their genus.

They are the principles on which demonstration rests.

They are only a few and simple.

Or rather, the most fundamental, and important, are only a few. And simple.

And which these are, and how, and why, only their generality, and in relation to the results which depend on them, can decide.

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Here is, then, more particularly, what is in the *Elements*.

Book I.

First are the 23 definitions. Definitions 1-7 are:

1. point is what has no parts;
2. line is a length without breadth;
3. the limits of a line are points;
4. straight is a line which lies evenly with the points on itself;
5. surface is what has only a length and a breadth;
6. the limits of a surface are lines;
7. plane is a surface which lies evenly with the straight lines on itself (*id.*).

Euclid begins at the beginning. What has no parts – a length only – a length and a breadth only. The definitions make explicit what the objects are. Point – line – straight line – surface – plane. The definitions build on one another, and depend

on one another. Point – line – the relation of line and point – surface – the relation of surface and line. And in all these ways, by beginning at the beginning, by making explicit what these rather than any other objects are, and by building on one another, and depending on one another, the definitions point at the fundamental objects and their relations. Line – and the fundamental line is the straight – surface – and the fundamental surface, built out of straight lines, is the plane.

Definitions 8-12 are:

8. plane angle is the inclination to one another of two lines in the plane which intersect and do not lie in a straight line;

9. when the lines containing the angle are straight, the angle is rectilinear;

10. when a straight line intersecting a straight line makes the adjacent angles equal, each of the angles is right, and each of the straight lines is perpendicular to the other;

11. obtuse is the angle greater than right;

12. acute is the angle less than right (*id.*).

The definitions continue to make explicit what these objects rather than any other are, to build on one another, and depend on one another. And they thus continue to point at the fundamental objects and their relations. The more fundamental character of the straight line is reaffirmed. The fundamental relation of lines is that of two lines. The relation of the two lines is intersecting one another, the angle – the fundamental angle is the rectilinear – and the fundamental rectilinear angle is the right. The right angle is defined by straight lines, and equality. Hence also the obtuse and acute angles are defined by the relation of greater and less.

Definitions 13-5 are:

13. boundary is what is a limit of something;

14. figure is what is contained by a boundary or boundaries;

15. circle is a plane figure contained by one line such that all the straight lines falling on it from one point inside the figure are equal (*id.*).

The fundamental figure is the circle. It is contained by one line. It is defined by the relation of one line to one point. And the relation of the line and the point is again defined by straight lines, and equality.

Definitions 16 and 17 are of center and diameter of the circle, 18 of semicircle.

19 is of rectilinear figure, trilateral, quadrilateral, and multilateral. Rectilinear figure is: that contained by straight lines (*id.*), trilateral is: that contained by three straight lines (*id.*).

20 and 21 are of the kinds of the trilateral figure, of equilateral, isosceles, and scalene triangle, and of right, obtuse, and acute triangle.

22 is of the kinds of the quadrilateral figure, square, rectangle, rhombus, and rhomboid.

The fundamental rectilinear figure is the triangle. It is contained by three straight lines.

The more complex objects are built out of the simpler. The circle is one line. The angle is two lines. The triangle is three lines. The fundamental relation of lines is intersection. The fundamental relation of straight lines as well is intersection. And all figures are defined by straight lines, and equality.

And definition 23 is:

23. parallel are straight lines in the same plane which, produced indefinitely in both directions, do not intersect in either direction (*id.*).

Lines intersect, or do not intersect. Straight lines as well intersect, or do not intersect. Straight lines that intersect are an angle. Straight lines that do not intersect are parallel.

The 5 postulates follow:

1. let a straight line be drawn from any point to any point;

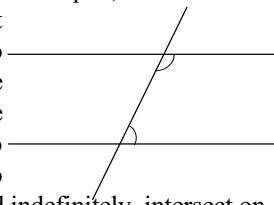
2. and a finite straight line be produced continuously in a straight line;

3. and a circle be drawn with any center and radius;

4. and all right angles be equal;

5. and, if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side on which are the interior angles less than two right angles (*id.*).

The 5 postulates are a unity, all 5 are subject to the introductory: let. Let a straight line be drawn – let a finite straight line be produced – let a circle be drawn – let all right angles be equal –



and let the two straight lines intersect.

The postulates generalize explicitly. From any point to any point – continuously – with any center and radius – all – indefinitely.

Postulates 1, 2, and 5 reaffirm the more fundamental character of the straight line, postulate 3 the more fundamental character of the circle, postulate 4 the more fundamental character of the right angle, postulate 5 the more fundamental character of the relation of intersection.

Postulate 5 points also to the more fundamental character of parallel lines. If a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines intersect. Thus, if a straight line intersecting two straight lines makes the interior angles on the same side not less than two right angles, it follows from definition 10 that the two interior angles are right, this implies that the two straight lines do not intersect, and it follows from definition 23 that the two straight lines are parallel.

The 5 axioms follow:

1. things equal to the same thing are equal;
2. if equal things are added to equal things, the wholes are equal;
3. if equal things are subtracted from equal things, the remainders are equal;
4. things which coincide are equal;
5. the whole is greater than the part (*id.*).

Equality is very fundamental.

Axioms 2, 3, and 5 in relation to axiom 1 make explicit also the relation of greater and less. By axiom 4 also coincidence is equality.

Equality is essential to the definition of the right angle, and this is reaffirmed by postulate 4. The relation of greater and less is essential to postulate 5, and hence to the definition of parallel lines. Equality, and hence the relation of greater and less, is thus essential to the relation of intersection. Equality is essential also to the definition of all figures, that of the circle as well as those of all rectilinear figures.

To begin with, then, the following can be concluded from the definitions, postulates, and axioms in book I.

The fundamental objects are the line and, built out of lines, the surface.

The fundamental line, and thus more funda-

mental than the line, is the straight line. The fundamental surface, and thus more fundamental than the surface, built out of straight lines, is the plane.

The fundamental relation of lines is intersection. The fundamental relation of straight lines, and thus again the more fundamental relation, is the intersection of straight lines. Straight lines intersect, or do not intersect.

The fundamental relations of lines are of one line, two lines, and three lines. The fundamental objects defined by these relations, and thus once again the more fundamental, are the circle, the angle and parallel lines, and the triangle. The object defined by the relation of one line is the circle. It is defined by the relation of one line to one point. The objects defined by the relation of two lines are the angle and parallel lines, when the two lines intersect and when the two lines do not intersect. The fundamental angle is the right. The object defined by the relation of three lines is the triangle. The circle and the triangle are figures, the angle and parallel lines are not. The angle and parallel lines and the triangle are rectilinear, the circle is not. They all, however, including the circle, are defined by straight lines. They all are defined also by equality.

Equality, and hence also the relation of greater and less, is very fundamental.

It can be concluded from the definitions, postulates, and axioms in book I alone, then, that the most fundamental objects and relations are the straight line, intersection, the right angle, the circle, the triangle, parallel lines, and equality.

The 49 proofs follow.

Proposition 1 is paradigmatic. The enunciation is:

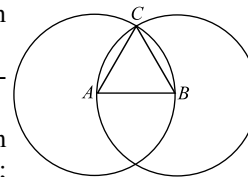
on a given finite straight line to construct an equilateral triangle (*id.*).

What a straight line is is clear by definition 4, what a finite straight line is by definition 3, what an equilateral triangle is by definition 20. The construction and proof proceed:

let AB be the given straight line (*id.*).

This is clear by postulate 1. Next:

draw a circle with center A and radius AB ;



and again, draw a circle with center B and radius AB (*id.*).

This is clear by postulate 3. Next:

from the point of intersection of the two circles C draw straight lines AC and BC (*id.*).

This is clear by postulate 1. Next:

since A is the center of a circle, $AB = AC$;

and since B is also the center of a circle, $AB = BC$ (*id.*).

This is clear by definition 15. Next:

therefore $AB = AC = BC$ (*id.*).

This is clear by axiom 1. And thus:

therefore triangle ABC is equilateral;

and it has been constructed on the given finite straight line AB ;

Q.E.F. (*id.*).

And all 48 subsequent proofs in book I, and all subsequent 465 proofs in the subsequent twelve books of the *Elements*, and all subsequent proofs of all of Euclid's successors do as proposition 1 in book I of the *Elements* does. What is to be proved, or constructed, is enunciated, and it is then proved, or constructed, step by step, as each step is either an assumption stated explicitly earlier, or, as the proofs accumulate, an earlier proof. Q.E.D., or Q.E.F. *Quod erat demonstrandum*, or *quod erat faciendum*.

The book divides into three parts. Propositions 1-26, whose central concern is the triangle. Propositions 27-32, whose central concern are parallel lines, and the dependence of the triangle and parallel lines on one another. And propositions 33-48, whose central concern is area, and the dependence of the triangle, parallel lines, and area on one another.

Proposition 2 constructs at a given point a straight line equal to a given straight line. 3 cuts from the greater of two given straight lines a straight line equal to the less. 4 proves that, if two sides and the angle contained by them of one triangle are equal to two sides and the angle contained by them of another triangle, the remaining side and angles are equal, and thus the two triangles are equal. 5 proves that in the isosceles triangle the interior angles at the base are equal, and so are the exterior angles. 6 is the converse of 5. 7 proves that, if at the limits of a given straight line two straight lines are constructed intersecting at a point, no other straight

lines can be constructed at the limits of the given straight line intersecting at the same point. 8 proves that, if the three sides of one triangle are equal to the three sides of another triangle, their angles are equal, and thus the triangles are equal. 9 bisects a given rectilinear angle. 10 bisects a given finite straight line. 11 erects a perpendicular at a point on a straight line. 12 drops a perpendicular from a point to a straight line. 13 proves that, if two lines intersect, the adjacent angles either are both right, or their sum is equal to two right angles. 14 proves that, if a straight line is intersected by one straight line on one of its sides, and by another straight line on its other side, and the sum of the angles of intersection is equal to two right angles, the two intersecting lines are in a straight line. 15 proves that the vertical angles of two intersecting straight lines are equal. A corollary draws the conclusion that the sum of the angles at the point of intersection of two straight lines is equal to four right angles. 16 proves that the exterior angle of the triangle is greater than either of the opposite interior angles. 17 proves that the sum of any two angles of the triangle is less than two right angles. 18 proves that the greater side of the triangle subtends the greater angle. 19 is the converse of 18. 20 proves that the sum of any two sides of the triangle is greater than the third side. 21 proves that, if at the limits of one side of the triangle two straight lines are constructed intersecting inside the triangle, the sum of these straight lines is less than the sum of the two opposite sides of the triangle. 22 constructs a triangle from three given straight lines. 23 constructs at a point on a given straight line a rectilinear angle equal to a given rectilinear angle. 24 proves that, if two sides of one triangle are equal to two sides of another triangle, and the angle contained by them in one of the triangles is greater than the angle contained by them in the other triangle, the side subtended by the greater angle is greater. 25 is the converse of 24. 26 proves that, if one side and two angles of one triangle are equal to one side and two angles of another triangle, the remaining sides and angle are equal, and thus the triangles are equal.

In the first part of book I, then, all 26 propositions are about straight lines. All 26 are also about equality, and greater and less, 1-15 about

equality, 16-21 about greater and less, 22-6 again about equality.

All but the preparatory 2-3 are not just about straight lines, but about intersecting straight lines. And about the equality, and greater and less, of these, intersecting straight lines. And this two-fold.

After the preparatory 1-10, the majority or the following propositions, 11-21, are about the right angle, sums of angles and sides, and the sums of angles in relation to the right angle.

And the triangle is the unwaveringly central concern of the 26 propositions. 1 is about the triangle, after the preparatory 2-3, 4-8 are about the triangle, after the preparatory 7-15, 16-22 are about the triangle, and after the preparatory 23, 24-6 are about the triangle. 16 of the 26 are about the triangle, the other 10 are preparatory to those 16. And the main concern is the equality of triangles. Equality is congruence. The equality of triangles is established by the equality of their parts – the equality of two sides and one angle in 4, the equality of three sides in 8, the equality of one side and two angles in 26. The equality of three angles is dramatically missing. The equality of triangles depends on the equality of their sides, not their angles.

The circle is not a concern of these propositions. It is employed, a means to an end, in 6 of the constructions, 1-3, 11-2, and 22.

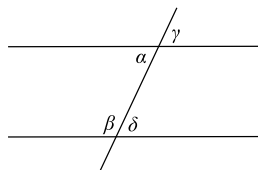
Dramatically, too, parallel lines are nowhere in the first 26 propositions.

The fundamental object is thus the triangle. The fundamental relation is the equality of triangles.

And fundamental in the relation of intersecting lines is the right angle. The right angle is the standard of equality, and greater and less. 17 has also begun establishing what the relation of the triangle and the right angle is. The sum of any two angle of the triangle is less than two right angles.

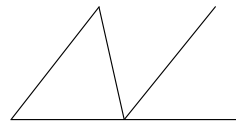
Parallel lines are nowhere in the first part of the book, and the second part, propositions 27-32, turns to parallel lines.

27 proves that, if a straight line intersect-



ing two straight lines makes the alternate angles equal, the two straight lines are parallel. If $\alpha = \delta$. 28 proves that, if a straight line intersecting two straight lines makes the exterior angle equal to the opposite interior angle, the two straight lines are parallel. If $\gamma = \delta$. 29 proves that, if a straight line intersects two parallel straight lines, the sum of the interior angles on the same side is equal to two right angles. $\alpha + \beta$ is equal to two right angles. 30 proves that straight lines parallel to the same straight line are parallel to one another. 31 constructs at a given point a straight line parallel to a given straight line. And 32 establishes the dependence of the triangle, the right angle, and parallel lines on one another.

32 proves that in any triangle, if one of the sides is produced, the exterior angle is equal to the two opposite interior angles, and the sum of the three interior angles is equal to two right angles.



32 depends on 29, which depends on postulate 5. 29 depends also on 27, which depends on definition 23.

The sum of the angles of the triangle is equal to two right angles.

This is a consequence of postulate 5: if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side, and definition 23: parallel are straight lines which do not intersect.

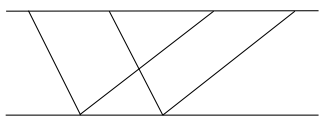
The equality of triangles depends on the equality of their sides, not their angles, because the sum of their angles is constant in all triangles, two right angles.

This is the fundamental property of the triangle, then. The sum of its angles is equal to two right angles. And the fundamental property of the fundamental object depends fundamentally on parallel lines.

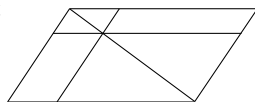
And hence, the third part of the book, propositions 33-48, begins drawing the consequences of this fact. Generalizing vastly the investigation.

33 proves that the straight lines joining equal and parallel straight lines are themselves equal and parallel. 34 proves that in the parallelogram

the opposite sides and angles are equal, and the diagonal bisects the parallelogram. 35 proves that parallelograms on the same base and in the same parallels are equal. 36



proves that parallelograms on equal bases and in the same parallels are equal. 37 proves that triangles on the same base and in the same parallels are equal. 38 proves that triangles on equal bases and in the same parallels are equal. 39 and 40 are partial converses of 38. 41 proves that, if a parallelogram and a triangle are on the same base and in the same parallels, the parallelogram is two times greater than the triangle. 42 constructs in a given rectilinear angle a parallelogram equal to a given triangle. 43 proves that in the parallelogram the complements about the diagonal are equal. 44



constructs on a given straight line and in a given rectilinear angle a parallelogram equal to a given triangle. 45 constructs in a given rectilinear angle a parallelogram equal to a given rectilinear figure. 46 constructs a square. 47 is the Pythagorean theorem, in the right triangle the square on the side subtending the right angle is equal to the sum of the squares on the sides containing the right angle. It is one of the many particularly elegant proofs in the *Elements*:

let the right triangle be ABC ;

construct on each side a square, AE , BG , and CI , respectively;

drop perpendicular CJ , dividing square AE into rectangles AJ and BJ ;

join CD and BI ;

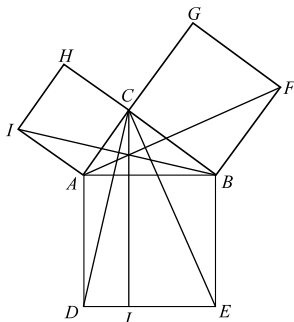
$AD = AB$, $AC = AI$, and $\angle DAC = \angle BAI$;

hence triangles ADC and ABI are equal;

triangle ADC is on same base and in the same parallels as rectangle AJ ;

triangle ABI is on same base and in the same parallels as square CI ;

hence rectangle AJ and square CI are equal;



join CE and AF ;

$BE = AB$, $BC = BF$, and $\angle EBC = \angle ABF$;

hence triangles BEC and BAF are equal;

triangle BEC is on same base and in the same parallels as rectangle BJ ;

triangle BAF is on same base and in the same parallels as square BG ;

hence rectangle BJ and square BG are equal;

and hence, adding rectangles AJ and BJ on one side, and squares CI and BG on the other, square AE is equal to the sum of squares CI and BG (*id.*).

48 is the converse of 47.

All these 16 propositions are about parallel lines. All 16 are about equality. The triangle is fundamental. After the preparatory 33-6, 37-45 are about the triangle, and after the preparatory 46, 47-8 are about the triangle. The central concern is area. After the preparatory 33-4, all the following 35-48 are about area. And the triangle is the standard of the measure of area. 41 establishes the relation of the area of the triangle and the parallelogram. And any rectilinear figure can be analyzed into triangles.

In the earlier propositions equality is congruence. The equality of triangles is established by the equality of their parts. Beginning with 35, the relation is generalized. Equality is equality of areas. Equality has thus become the measurement of the plane. The equality of areas is still established by triangles. And it can be established from one area to another, across the plane, because the sum of the angles of all triangles is constant. The area of the triangle depends on its sides. In particular, the area of the triangle is one half of the area of the parallelogram on the same base and in the same parallels. And that this, that the area of the triangle depends on its sides, depends on that, that the sum of the angles of all triangles is constant, two right angles, the fact that all proofs depend on parallel lines shows.

The straight line has thus become the measure of the plane. The straight line measures length only. The triangle measures length and breadth jointly. Straight lines in their relation in the triangle are the measure of the plane. And they are because the sum of the angles of the triangle is equal to two right angles.

The generalization is apparent in the fact, too,

that the proofs in this part of the book imply quadratic equations. 47, for instance, is a solution of $x^2 = a^2 + b^2$.

Book I is the foundation.

And it can be concluded from book I that the most fundamental, and important, object is the triangle. It is, because it is the foundation of the generalization of the measurement of the straight line into the measurement of the plane.

And it is the foundation of this generalization, because of its most fundamental, and important, property, that the sum of its angles is equal to two right angles.

And the most fundamental, and important, assumption on which this property depends is parallel lines.

cccxviii

Book II.

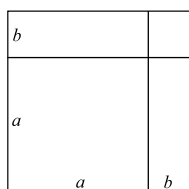
The book opens with 2 definitions, that the rectangle is contained by the straight lines containing the right angle, and of gnomon, or Γ -shaped area.

14 proofs follow.

All 14 are about the equality of areas. All 14 depend on parallel lines.

Establishing the equality of areas corresponds to calculation with straight lines. The book thus also lays the foundations of algebra.

4 proves that, if a straight line is cut at random, the square on the whole is equal to the sum of the squares on the parts and twice the rectangle contained by the parts.



If the parts are a and b , $(a+b)^2 = a^2 + 2ab + b^2$. 11 solves the problem, to cut a given straight line so that the rectangle contained by the whole and one of the parts is equal to the square on the other part. If the whole is a and one of the parts x , $a(a-x) = x^2$. 14 solves the problem, to construct a square equal to a given rectilinear figure. The figure can be of any number of sides, the rectangle equal to the figure can be constructed, and then the square equal to the rectangle. If the sides of the rectangle are a and b , $ab = x^2$, and conversely, the square root can be extracted, $\sqrt{ab} = x$.

All areas investigated in books I and II are

rectilinear. Book III thus turns to the circle.

Book III.

The book begins with 11 definitions. The fundamental is 1:

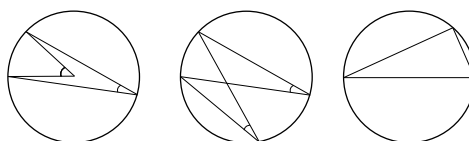
1. equal are circles whose diameters are equal (*id.*).

And all definitions make explicit the relation of the circle to straight lines and angles. 2 is of tangent, 4 of the distance of a chord from the center, 6 of segment.

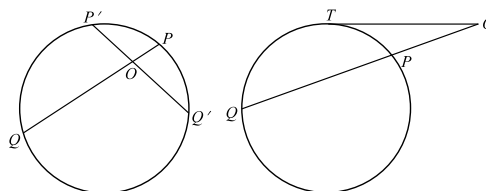
39 proofs follow.

All 39 are about the relation of the circle to straight lines and angles. In all 39 the straight lines intersecting one another and the circle define, implicitly when not explicitly, triangles.

20 proves that the angle at the center is two times greater than the angle at the circumference subtending the same arc. 22 proves that the angles at the circumference subtending the same arc are equal. 31 proves that the angle at the circumference subtending the diameter is right.



35 proves that, if two chords intersect, the product of the parts of one of the chords is equal to the product of the parts of the other. $OP \cdot OQ = OP' \cdot OQ'$. Or equivalently, for any point O inside the circle, $OP \cdot OQ$ is constant. 36 proves that, if a straight line from a point outside the circle cuts the circle, the product of the part outside and the whole is equal to the square on the tangent from the point. $OP \cdot OQ = OT^2$. Or equivalently, for any point O outside the circle, $OP \cdot OQ$ is also constant.



Book III investigates the relation of the circle to straight lines and angles. And book IV turns to the relation of the circle to figures.

Book IV.

The book begins with 7 definitions, of in-

scribed and circumscribed.

19 proofs follow.

4-5 inscribe and circumscribe the scalene triangle in and about a circle. 6-9 inscribe and circumscribe the square in a circle and the circle in a square. 11-4 inscribe and circumscribe the regular pentagon in and about a circle and the circle in a regular pentagon. 15 inscribes the regular hexagon in a circle. A corollary draws the conclusion that the side of the regular hexagon is equal to the radius of the circle. 16 inscribes the regular fifteen-sided figure in a circle.

In books II-IV, then, the investigation is vastly generalized. In book II the investigation of rectilinear area is generalized, in book III the fundamental curve is analyzed, the circle, and in book IV the investigation is generalized further by the analysis of the relation of the fundamental curve, and hence its area, to rectilinear area.

The foundation is everywhere the same, the most fundamental object with its most fundamental property depending on the most fundamental assumption. In book II all rectilinear areas depend on parallel lines. In book III, after the preparatory propositions 1-19, the analysis of the circle depends in all subsequent propositions 20-37 explicitly on the fact that the sum of the angles of the triangle is two right angles. In book IV, the analysis of the relation of the circle, and hence its area, to rectilinear area by inscription and circumscription is everywhere by the analysis of the rectilinear figures into triangles.

In general, books I-IV are the theory of area, and that is, the theory of the measurement of the plane. The standard of the measurement is the triangle. The sum of its angles. Depending on parallel lines.

And books XI-XIII will generalize the theory from two to three dimensions. From length and breadth jointly to length, breadth, and depth jointly.

Before that, however, in books V-X, the investigation is substantially formalized, and hence generalized vastly further. Book V develops the most formal and most general theory, the theory of proportion. In book VI the theory of the measurement of the plane developed in books I-IV is grounded in the theory of proportion. In books VII-IX, also grounded in the theory of

proportion, the theory of numbers is developed. In book X, also grounded in the theory of proportion, the theory of irrational lines is developed.

Book V.

The book begins with 18 definitions. Definitions 1-7 are:

1. magnitude is a part of a magnitude when the less measures the greater;
2. the greater is a multiple of the less when it is measured by the less;
3. ratio is the relation of two magnitudes of the same kind with respect to size;
4. magnitudes have a ratio when, if multiplied, they can exceed one another;
5. magnitudes have the same ratio, the first to the second and the third to the fourth, if, whenever any multiple of the first is greater than, or equal to, or less than, any multiple of the second, the same multiple of the third as of the first is respectively greater than, or equal to, or less than, the same multiple of the fourth as of the second;
6. magnitudes that have the same ratio are proportional;
7. whenever a multiple of the first magnitude exceeds a multiple of the second, and the same multiple of the third as of the first does not exceed the same multiple of the fourth as of the second, the first has a greater ratio to the second than the third to the fourth (*id.*).

The concern is magnitude. Magnitude, first of all, in 1 and 2, measures, and is measured. A ratio, in 3, magnitude is the relation with respect to size. And how it measures, and is measured, what the relation with respect to size more particularly is, the fundamental, 5, the definition of proportion, makes explicit. In letters,

if a , b , c , and d are magnitudes, and m and n indicate their multiplication,

if, whenever $ma > nb$, also $mc > nd$,

whenever $ma = nb$, also $mc = nd$,

and whenever $ma < nb$, also $mc < nd$,

then $\frac{a}{b} = \frac{c}{d}$.

This is how magnitude measures, and is measured. The relation with respect to size is equality. 7 is,

whenever $ma > nb$, and $mc \leq nd$,

then $\frac{a}{b} > \frac{c}{d}$.

This is also how, a consequence of 5, magnitude measures, and is measured. The relation with respect to size is greater and less.

All 7 definitions, and most particularly 5, depend on equality, and greater and less. Beginning with 2, and again most particularly 5, they depend also on multiplication. They thus all depend on axioms 1 and 2 in book I, things equal to the same thing are equal, and if equal things are added to equal things, the wholes are equal.

Thus, by making explicit proportion, the definition of proportion has made equality, greater and less, and addition as explicit as equality, greater and less, and addition have ever been made.

Again, on equality, greater and less, and addition not only all mathematics depends. There cannot be many propositions, if there are any, which do not depend on equality, greater and less, and addition.

Hence the generality of the theory of proportion. The theory of proportion is the theory of measurement. It is the theory of everything that has been measured, and everything that will be measured. Over everything that has been measured, the formulation of the theory is a generalization. Of everything that will be measured, the theory is, in the nineteen centuries after its formulation, the ground, and in the centuries since, after its further generalization, and generalization of generalization, the ground of these generalizations.

Magnitude is a part of a magnitude when the less measures the greater.

If it measures, it is a magnitude. And conversely, if it is measured, it is a magnitude.

And this explains why the fundamental magnitude is the line. Length only. With Apollonius, we conceive the line from the measurement of a length, of a wall or a road. And conversely, we conceive the measurement of a length, of a wall or a road, from the line.

It explains also why the fundamental line, and more fundamental than the line, is the straight line. The curve, as all propositions in book III show, too, has to be analyzed into length and breadth both. Strictly, only the straight line is length only. And the definition of the straight line is the one result of the *Elements* that had to be

corrected right away, two generations after Euclid, by Archimedes.

Definitions 8-18 are of transformations of ratios, alternation, $\frac{a}{b} = \frac{c}{d}$ into $\frac{a}{c} = \frac{b}{d}$, inversion, $\frac{a}{b}$ into $\frac{b}{a}$, composition, $\frac{a}{b}$ into $\frac{a+b}{b}$, separation, $\frac{a}{b}$ into $\frac{a-b}{b}$, conversion, $\frac{a}{b}$ into $\frac{a}{a-b}$.

27 proofs follow.

1 proves that $(a+b) \cdot c = a \cdot c + b \cdot c$. 4 proves that, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{ma}{nb} = \frac{mc}{nd}$. 11 proves that, if

$\frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$, then $\frac{a}{b} = \frac{e}{f}$. 17-8 prove that, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ and $\frac{a+b}{b} = \frac{c+d}{d}$. 25

proves that, if $\frac{a}{b} = \frac{c}{d}$, and a is the greatest of the four magnitudes and d the least, then $a + d > b + c$.

The theory developed in book V is more general than the theory developed in books I-IV. It is the theory of more objects. Magnitude, one kind of which the straight line is, is more general than the straight line. And the theory of measurement, one kind of which the measurement of the plane is, is more general than the theory of the measurement of the plane.

It is also more formal. The physicist studies both form and matter, the mathematician separates form from matter. The objects investigated by the mathematician are abstracted from the sensible things. Formal thus admits of degree. Less, or more, and more, of the matter can be abstracted from the form. And that the theory developed in book V is more formal than the theory developed in books I-IV the fact suffices to show that the former is readily reduced in its entirety to calculation with letters, the latter is not. If formal admits of degree, mathematical admits of degree. And more formal, the theory of measurement is thus also more mathematical than the theory of the measurement of the plane.

The *Elements* is the paradigm of deduction. Nevertheless, it can be analyzed into two steps, from books I-IV to book V, and from book V to books VI-XIII. And the first step, then, the generalization, from the measurement of the plane to

measurement, is the inductive. The second step, drawing the consequences of the generalization, is the deductive.

And the deductive step divides clearly into two kinds. Books VII-IX and X are more formal, more mathematical. These are the arithmetic of the *Elements*. Books VI and XI-XIII are less formal, less mathematical, more general, more physical. These, with the introductory books I-IV, are the geometry of the *Elements*. Book V, the theory of measurement, is the foundation of both, geometry and arithmetic.

Books I-IV are the theory of the measurement of the plane, book V is the theory of measurement, and book VI grounds the more particular theory, that of the measurement of the plane, in the more general, that of measurement.

Book VI.

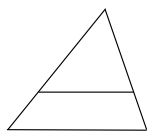
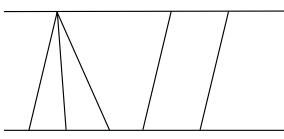
The book begins with 4 definitions. Definitions 1 and 4 are:

1. similar rectilinear figures are those, of which all angles are respectively equal to one another, and the sides about the equal angles are proportional;

4. height of a figure is the perpendicular from a vertex to a base (*id.*).

36 proofs follow.

1 proves that triangles and parallelograms under the same height are to one another as their bases. 2

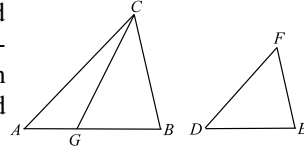


proves that a straight line parallel to one side of the triangle divides the other sides proportionally, and conversely. 4-7 prove that two triangles are similar when all angles

are equal, when all sides are proportional, and when one angle is equal and two sides are proportional. 11-3 solve the problems, to find a third proportional to two given straight lines, to find a fourth proportional to three given straight lines, and to find a mean proportional to two given straight lines. The last is equivalent to finding, if the lines are a and b , $\sqrt{ab}$. 15 proves that, if in two triangles one angle is equal to one angle and the sides about the equal angles are reciprocally proportional, the triangles are equal. 19 proves that similar triangles are to one another

as the squares on their sides. It is another of the many particularly elegant proofs in the *Elements*:

let $\triangle ABC$ and $\triangle DEF$ be the similar triangles, with $\angle CAB = \angle FDE$ and $\frac{AC}{AB} = \frac{DF}{DE}$;



construct the third proportional $\frac{AB}{DE} = \frac{DE}{AG}$;

join CG ;

$\frac{AC}{DF} = \frac{AB}{DE}$, and hence $\frac{AC}{DF} = \frac{DE}{AG}$;

therefore, in $\triangle ACG$ and $\triangle DEF$ the sides about the equal angles are reciprocally proportional, and $\triangle ACG = \triangle DEF$;

$\frac{AB}{DE} = \frac{DE}{AG}$, hence $\frac{AB}{AG} = \frac{AB^2}{DE^2}$;

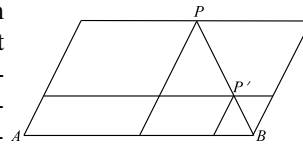
$\frac{\triangle ABC}{\triangle ACG} = \frac{AB}{AG}$, hence $\frac{\triangle ABC}{\triangle ACG} = \frac{AB^2}{DE^2}$;

and $\triangle ACG = \triangle DEF$;

therefore, $\frac{\triangle ABC}{\triangle DEF} = \frac{AB^2}{DE^2}$ (*id.*).

27-9 are the first solutions of problems of minima and maxima. 27 proves that of all parallelograms, which are drawn

on a given straight line so that a parallelogram is deficient by a paralle-



logram similar and similarly oriented to the parallelogram drawn on one half of the given straight line, that is the greatest, which is drawn on one half of the given straight line. If the given straight line is AB , and the parallelogram drawn on one half of AB is AP , AP is deficient by the parallelogram BP , which is similar and similarly oriented to AP ; another parallelogram drawn on AB is AP' , which is deficient by BP' , which is similar and similarly oriented to AP ; and of all such parallelograms the greatest is AP . 31 is a generalization of the Pythagorean theorem, in the right triangle the figure on the side subtending the right angle is equal to the sum of the similar and similarly oriented figures on the sides containing the right angle. 33 proves that in equal circles angles either at the circumference or at the center have the same ratio as the arcs which they sub-

tend.

After the formulation of the more formal and more general theory of measurement, then, the theory of the measurement of the plane is grounded in the more formal and more general theory, and thus itself vastly generalized. And as proposition 1 already shows, on which all subsequent propositions rest, the fundamental object continues to be the triangle, because the sum of its angles is two right angles, depending on parallel lines.

The main development is the generalization of the relation of equality, and greater and less, to include the relation of similarity. Similar figures are those of which the angles are equal and the sides proportional. Similarity is equality and proportionality. The relation of similarity has increased vastly both the precision and the complexity of the measurement of the plane. And as proposition 2 already shows, on which all subsequent propositions rest, similarity depends on parallel lines, and the sum of the angles of the triangle.

It can be concluded that book V is the means, book VI is the end. The formalization and generalization of measurement is the means to the end of generalization of the measurement of the plane. Magnitude is more general than the straight line, yet magnitudes are always represented by straight lines, straight lines, in particular, show what magnitudes, in general, are. And book VI reaffirms that the fundamental magnitude is the straight line. The theory of proportion is more formal and more general than the theory of similarity. But book VI shows proportion the ground of similarity, and thus reaffirms that straight lines in their relation in the triangle are the measure of the plane, and of the greater precision and greater complexity of this measurement now.

The most fundamental object, then, continues to be the triangle, because of the sum of its angles, depending on parallel lines.

And before generalizing the measurement of the plane, length and breadth, into the measurement of space, length, breadth, and depth, the investigation turns to number.

Books VII-IX.

The three books are a unity.

They begin with 22 definitions. Definitions 1-3 and 5 are:

1. unit is that by which any of the things that there are are collected into one;
2. number is a multitude of units;
3. number is a part of a number when the less measures the greater;
5. the greater number is a multiple of the less when it is measured by the less (*id.*).

3 and 5, affirming of number what definitions 1 and 2 in book V affirm of magnitude, establish that number is a magnitude. 2 establishes that it is a discrete magnitude. It is a multitude of 1's. What 1 is 1 establishes. It had been a question of some urgency, what 1 is, for two-odd centuries by this time. Pythagoras said that 1 is good. Parmenides said that all is 1. Plato said that the good is the 1. Euclid said that 1 is that by which any of the things that there are are collected into 1.

Definitions 4 and 6-22 are of kinds of numbers, even, odd, composite, square, cube. Prime is: a number measured by the unit alone (11), perfect is: a number equal to its parts (22). The first perfect number is $1 + 2 + 3 = 6$, the second $1 + 2 + 4 + 7 + 14 = 28$.

106 proofs follow.

The following are two more of the many particularly elegant proofs in the *Elements*.

20 in book IX:

given any number of primes, there are more primes;

let p_1, p_2, p_3 be the given primes;

take the number $p_1 \times p_2 \times p_3 + 1$;

this number is either prime or not;

if it is prime, there are more primes than the given number of primes;

if it is not prime, it is measured by some prime;

let this prime be q ;

and q cannot be any of the numbers p_1, p_2 , or p_3 ;

for let it be one of them;

each of them measures $p_1 \times p_2 \times p_3$;

hence q measures $p_1 \times p_2 \times p_3$; but it measures also $p_1 \times p_2 \times p_3 + 1$; it thus measures also the remainder, which is 1; and this is absurd;

and if q is not any of the numbers p_1, p_2 , or p_3 , it is by hypothesis prime, and there are more primes than the given number of primes (*id.*).

35 in book IX:

given any number of numbers in continued proportion, the difference of the last and the first to all those before it is as the difference of the second and the first to the first;

let the given numbers be $a_1, a_2, \dots, a_n$;

as they are in continued proportion,

$$\frac{a_n}{a_{n-1}} = \frac{a_{n-1}}{a_{n-2}} = \dots = \frac{a_2}{a_1};$$

by separation,

$$\frac{a_n - a_{n-1}}{a_{n-1}} = \frac{a_{n-1} - a_{n-2}}{a_{n-2}} = \dots = \frac{a_2 - a_1}{a_1};$$

and by composition, adding all antecedents and all consequents,

$$\frac{a_n - a_1}{a_{n-1} + a_{n-2} + \dots + a_1} = \frac{a_2 - a_1}{a_1} \text{ (id.)}.$$

The sum of a finite geometrical series thus is $S_n = \frac{(a_{n+1} - a_1)a_1}{a_2 - a_1}$. And book IX ends, as all

books in the *Elements* do, dramatically. The last proposition, 36, shows that, if $2^0 + 2^1 + \dots + 2^n$ is prime, $(2^0 + 2^1 + \dots + 2^n)2^n$ is perfect.

Of great formality, great generality, and great sophistication, the theory of numbers developed in books VII-IX stands firmly on its own.

It can nevertheless be concluded that it is a means to the same end, the further generalization of the theory of the measurement of the plane.

Both, the theory of numbers and the theory of the measurement of the plane, stand on the same ground, the theory of proportion, or the theory of measurement, developed in book V. Both are concerned with magnitudes, and beginning to end with the equality, greater and less, and addition of magnitudes. Magnitudes are always represented by straight lines, straight lines, in particular, show what magnitudes, in general, are. And numbers, in particular, are always represented by straight lines as well, straight lines, in particular, show what numbers, in particular, are as well.

Still, number is a discrete magnitude, the straight line is not.

And book X resolves this tension.

Book X.

The book begins with 4 definitions. Definitions 1-3 are:

1. commensurable are magnitudes measured by the same measure, incommensurable are

magnitudes that cannot have the same measure;

2. straight lines are commensurable in square when the squares on them are measured by the same area, incommensurable in square when the squares on them cannot have the same measure (*id.*).

a and $\sqrt{a}$ are commensurable in square, a and $a + \sqrt{a}$ are incommensurable in square.

3. there is an infinite number of straight lines, commensurable and incommensurable with a given straight line, respectively, and some in length only, others also in square; and let, if the given straight line is rational, those commensurable with it in either length or square also be rational, those incommensurable with it irrational (*id.*).

If a is rational, $\sqrt{a}$ is also rational, it is commensurable with a in square, $a + \sqrt{a}$ is irrational.

4 is of rational and irrational area.

Later in the book there are 12 more definitions, 6 after proposition 47, 6 after proposition 84, all 12 of particular irrational straight lines.

And the opening 4 definitions already indicate that, although the concern is, in general, in 1, with magnitude, it is, in particular, in 2-4, with the straight line, and area.

135 proofs follow.

Propositions 1-8 are concerned with magnitude.

1 is:

given two unequal magnitudes, if from the greater a magnitude greater than its half is subtracted, from the remainder a magnitude greater than its half is subtracted, and so on, a magnitude smaller than the less magnitude will ultimately remain.

This is the theorem which is in Hilbert the axiom: if AB and CD are any segments, then there exists a number n such that n segments CD constructed contiguously from A along the ray from A through B will pass beyond B . It is: the axiom of measure, or Archimedes's axiom, the first of the two: axioms of continuity, of which the second is: the axiom of completeness, and of which Hilbert notes: a completeness axiom which does not require the axiom of Archimedes entails a contradiction.

Euclid formulates the result in terms of magnitudes. Hilbert formulates it in terms of straight

lines. Archimedes formulates it in terms of areas: the excess by which the greater of two unequal areas exceeds the less, by being added to itself, can be made to exceed any given finite area.

5-6 prove that commensurable magnitudes have the ratio of numbers, and conversely. 7-8 prove that incommensurable magnitudes do not have the ratio of numbers, and conversely.

After the preparatory propositions 1-8, with 2 corollaries, the large majority of the following 125 proofs are concerned with straight lines.

9 proves that the squares on straight lines commensurable in length have the ratio of square numbers, and conversely, the squares on straight lines incommensurable in length do not have the ratio of square numbers, and conversely. In letters,

given the straight lines a and b , and any numbers m and n ,

$$\text{if, when } \frac{a}{b} = \frac{m}{n}, \text{ also } \frac{a^2}{b^2} = \frac{m^2}{n^2},$$

$$\text{and when } \frac{a^2}{b^2} = \frac{m^2}{n^2}, \text{ also } \frac{a}{b} = \frac{m}{n},$$

then a and b are commensurable,
otherwise they are incommensurable.

And by definition 3, if a and b are commensurable, they are rational, otherwise they are irrational.

And after the preparatory propositions 1-18, with 4 lemmas and 3 corollaries, the large majority of the following 110 proofs are concerned with areas.

19-21 prove that the rectangle contained by rational straight lines is rational, that rational rectangles are contained by straight lines commensurable in length, and that rectangles contained by lines commensurable in square only are irrational. 36 proves that, if two straight lines commensurable in square only are added, the sum is irrational. If the two straight lines are a and $\sqrt{b}$, the sum is $a + \sqrt{b}$. Euclid calls the irrational straight line $a + \sqrt{b}$ the first binomial. 54 proves that the side of the square equal to the area contained by a rational straight line and the first binomial is irrational. If the rational straight line is a and the first binomial $b + \sqrt{c}$, the area contained by them is $a(b + \sqrt{c})$, and the side of the square equal to this area is $\sqrt{ab + a\sqrt{c}}$. The central concern of the book is to demonstrate all

straight lines of the form $\sqrt{\sqrt{a} \pm \sqrt{b}}$, and the demonstration is everywhere by finding the side of the square equal to a rectilinear area.

And area everywhere depends on parallel lines.

Aristotle writes: all magnitudes are continuous, and: since magnitude is continuous, it is divisible into an infinite number of parts, and: there is no smallest magnitude.

And in book X Euclid establishes that magnitude is continuous.

On continuity the consistency of equality, greater and less, and addition depends, too.

Thus, on proposition 1 in book X not only all subsequent proofs depend. On proposition 1 in book X, implicitly if not explicitly, all proofs in the *Elements* depend, and all proofs which depend on the *Elements* depend.

Magnitude is continuous.

And this explains why the fundamental magnitude is the straight line.

Time in Aristotle is represented by straight lines. All objects in the *Elements*, areas, and curvilinear as well as rectilinear, volumes, and curvilinear as well as rectilinear, the irrationals, numbers, are represented by straight lines. And if time is not in Aristotle, all objects in the *Elements* are, not just represented by straight lines, they are, in fact, analyzed into straight lines. What holds for them holds, because it holds for straight lines, and conversely. In Archimedes conoids, spheroids, parabolas, and spirals are analyzed into straight lines, and weights and fluids are analyzed into straight lines. In Apollonius the conics are analyzed into straight lines. In Euclid's *Optics* light is analyzed into straight lines. In Autolycus and in Euclid's *Phenomena* already the motions of the heavenly bodies are analyzed into straight lines, and the analysis of the circle and the motions of the heavenly bodies, both, into straight lines will lead to their yet more explicit analysis into straight lines with the development of trigonometry by Hipparchus.

Aristotle writes: demonstration cannot cross from one genus to another, there can be no demonstration of things of different genera, of arithmetic and geometry, for instance, demonstration demands that the kinds of knowledge be subordinated to one another.

And in Euclid arithmetic is subordinated to geometry.

Number is a kind of straight line, the discrete straight line. If $a + \sqrt{b}$ is a straight line, a is a straight line.

And if the tension has remained, it will not be resolved until it is shown, twenty-one and a half-odd centuries later, that the other way around, if a is a number, $a + \sqrt{b}$ is a number.

The theory of irrational lines, then, is a further generalization of the theory of the measurement of the plane. It is a theory of numbers. With propositions 5-8, commensurable magnitudes have the ratio of numbers, incommensurable do not have the ratio of numbers. But it is fundamentally a theory of measurement. With definition 1, if it is measured, it is commensurable, if it is not measured, it is incommensurable. It is a theory of straight lines. With definition 2, straight lines are commensurable, or incommensurable. It is a theory of area. With definition 2 again, straight lines are commensurable in square when the squares on them are measured by the same area.

All straight lines $\sqrt{a} \pm \sqrt{b}$ are generated by area. And area everywhere depends on parallel lines, and the sum of the angles of the triangle.

Continuity, and consistency, demands the generalization of the theory of the measurement of the plane by the theory of irrational lines. And the fact that irrational lines are everywhere demands it, too. With definition 3, there is an infinite number of straight lines, some rational, others irrational, and books XI-XIII will show explicitly what the earlier books have shown only implicitly, that the irrational lines are, if not more, more important than the rational. All solids are built out of irrational lines.

And with books XI-XIII the *Elements* concludes. With the explicit generalization of the measurement of the plane into the measurement of space, of two dimensions into three.

Books XI-XIII.

These books are also a unity.

They begin with 28 definitions. Definitions 1-2 are:

1. solid is what has a length, a breadth, and a depth;

2. the limits of a solid are surfaces (*id.*).

The line is built out of points, the surface out

of lines, the solid out of surfaces. The limits of a line are points, the limits of a surface lines, the limits of a solid surfaces.

3-7 are of a straight line at right angles to a plane, a plane at right angles to a plane, the inclination of a straight line to a plane, the inclination of a plane to a plane. Next:

8. parallel are planes that do not intersect (*id.*).

Parallel planes are built out of parallel lines.

9-13 are of similar solids, equal solids, solid angle, pyramid, prism. Next:

14. if a semicircle, with its diameter fixed, is rotated until it returns to its original position, the comprehended figure is the sphere (*id.*).

15-24 are of the center and diameter of the sphere, cone, the axis and base of the cone, cylinder, the axis and base of the cylinder. All curvilinear solids are defined, like the sphere, by the rotation of figures. And 25-8 are of four of the five regular solids:

25. cube is the solid contained by six equal squares;

26. octahedron is the solid contained by eight equal and equilateral triangles;

27. icosahedron is the solid contained by twenty equal and equilateral triangles;

28. dodecahedron is the solid contained by twelve equal, equilateral, and equiangular pentagons (*id.*)

The definition of the fifth and simplest regular solid is implied by earlier definitions. Tetrahedron is the solid contained by four equal and equilateral triangles.

89 proofs follow.

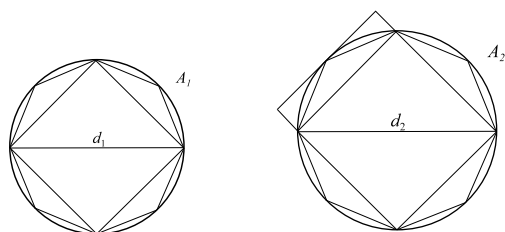
Propositions 1-24 in book XI are the preparatory, establishing the relations of intersecting straight lines and planes, straight lines and planes perpendicular to one another, straight lines and planes parallel to one another, solid angles and plane angles. The triangle enters with 2: if two straight lines intersect, they are in the same plane, and every triangle is in one plane (*id.*). 25-39 in book XI then investigate the fundamental solid, the parallelepiped.

As area depends on parallel lines, so volume depends on parallel lines. And in particular, all proofs in the following books XII and XIII depend on parallel lines, and the sum of the angles of the triangle.

So the 23 proofs in book XII turn to the more complex solids, the pyramid, and then the curvilinear, the cone, the cylinder, and the sphere. The proofs are by the method of exhaustion. And these are two more of the many particularly elegant proofs in the *Elements*.

Proposition 2:

circles are to one another as the squares on the diameters;



let the diameters of the circles be d_1 and d_2 , and the areas A_1 and A_2 ;

$$\text{then } \frac{A_1}{A_2} = \frac{d_1^2}{d_2^2};$$

for, if not, there is some other area S , such that $\frac{A_1}{S} = \frac{d_1^2}{d_2^2}$, where either $A_2 > S$ or $A_2 < S$;

let $A_2 > S$;

inscribe in the circle whose area is A_2 a square; the area of the square is greater than one half of the area of the circle, because it is equal to one half of the area of the square circumscribed about the circle, and the area of this square is greater than the area of the circle;

bisect the arc subtending one side of the square, and complete the isosceles triangle with a vertex at the midpoint of the arc; the equal sides of this triangle are sides of the regular octagon inscribed in the circle; draw a tangent at the midpoint of the arc, and complete the rectangle standing on the side of the square; the area of the isosceles triangle is greater than one half of the area of the segment of the circle, because it is equal to one half of the area of the rectangle, and this area is greater than the area of the segment;

in this way, by continually bisecting the remaining arc, and completing the isosceles triangle whose equal sides are sides of the regular sixteen-sided figure, thirty-two-sided figure, and so on, an area of the circle will be left not exhausted by

the regular polygons $< A_2 - S$;

for it is proved in proposition 1 of book X that, if from the greater of two magnitudes a magnitude greater than its half is subtracted, from the remainder a magnitude greater than its half, and so on, a magnitude smaller than the less will ultimately remain;

in this way, inscribe in the circle regular polygons of increasing number of sides until the area of the polygon $P > S$;

inscribe a similar polygon in the other circle; let its area be P_1 ;

hence, since similar polygons are as the squares on the diameters of the circles in which

$$\text{they are inscribed, } \frac{P_1}{P} = \frac{d_1^2}{d_2^2};$$

$$\text{but } \frac{A_1}{S} = \frac{d_1^2}{d_2^2}; \text{ and hence } \frac{A_1}{S} = \frac{P_1}{P};$$

$$\text{hence, } \frac{A_1}{P_1} = \frac{S}{P};$$

and as the area of the circle is greater than the area of the inscribed polygon, $A_1 > P_1$;

and $S > P$;

but also $P > S$;

which is absurd;

in the same way, nor can $A_2 < S$;

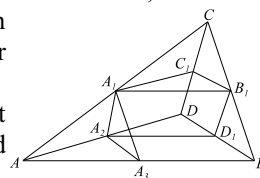
$$\text{therefore, } \frac{A_1}{A_2} = \frac{d_1^2}{d_2^2} \text{ (id.)}.$$

And proposition 5 employs the method in three dimensions:

pyramids of the same height and on triangular bases are to one another as the bases;

let $ABCD$ be a given pyramid on a triangular base;

bisect the sides at A_1, A_2, A_3, B_1, C_1 , and D_1 ;



the three planes, through A_1, A_2 , and A_3 , through A_1, B_1 , and C_1 , and through A_1, B_1, D_1 , and A_2 , divide the given pyramid into four parts;

these are two prisms, the prism with bases $A_1A_2A_3$ and B_1D_1B , and the prism with bases $A_1B_1C_1$ and A_2D_1D , and two pyramids, the pyramid with base $A_1A_2A_3$ and vertex A , and the pyramid with base CC_1B_1 and vertex A_1 ;

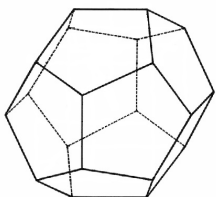
the two prisms are equal to one another, and jointly their volume is greater than the volume of one half of the given pyramid;

the two pyramids are similar to one another and to the given pyramid;

each of these pyramids can be divided in the same way into two equal prisms, whose volume is greater than one half of the volume of the pyramid, and two pyramids, which are similar to one another and to the given pyramid, each of which can be divided in the same way;

the *reductio* follows (*id.*).

Finally, the 24 proofs of book XIII relate curvilinear and rectilinear volumes to one another. The picture is of the dodecahedron. In propositions 13-7 the tetrahedron, the octahedron, the cube, the icosahedron, and the dodecahedron are each constructed and inscribed



in a sphere, and the relation of its side to the diameter of the sphere is established. In the concluding proposition 18 the relation of the sides of the five solids to one another and the diameter of the sphere is established. In terms of the radius of the sphere, if the radius is r , the sides of the solids inscribed in the sphere are,

respectively, of the tetrahedron $\frac{2\sqrt{2}}{\sqrt{3}}r$, the octa-

hedron $\sqrt{2}r$, the cube $\frac{2}{\sqrt{3}}r$, the icosahedron

$\frac{\sqrt{10(5-\sqrt{5})}}{5}r$, the dodecahedron $\frac{\sqrt{15}-\sqrt{3}}{3}r$. And

proposition 18 ends, and the *Elements* ends, with the proof that there can be no other regular solids than these five:

two planes cannot contain a solid angle; a solid angle thus cannot be constructed from less than three figures;

a solid angle cannot be greater than four right angles; the sum of the angles of the figures from which each angle of the solid is constructed thus cannot be equal to or greater than four right angles;

from three regular triangles the angle of the tetrahedron is constructed; from four the angle of the octahedron; from five the angle of the ico-

hedron; and from six a solid angle cannot be constructed because the angle of the regular triangle is equal to two thirds of one right angle, and the sum of six such angles is four right angles;

from three squares the angle of the cube is constructed; and the sum of four angles of squares is four right angles;

from three pentagons the angle of the dodecahedron is constructed; and the angle of the regular pentagon is equal to one right angle and one fifth, and the sum of four angles of such pentagons is greater than four right angles;

nor can a solid angle be constructed from any other regular polygons (*id.*).

There have been only two other treatises of such power, the *Almagest* and *The mathematical principles of natural philosophy*. Both are less formal, less mathematical. Both are more general, more physical.

cccix

The *Elements*, then, is the theory of space. More particularly, it is the theory of the measurement of space.

The theory is an empirical generalization. More particularly, it is a generalization over the earlier development of physics. More particularly still, it is a generalization over the great advance which has begun with Thales, and has been continuing for two and a half-odd centuries by the time of Euclid, in the investigation of the world.

The great advance in the formulation and solution of the problems of the principles of all things, of the earth, and of the soul has been described. The magnitude and scope of the advance Aristotle's treatises, one generation-odd before Euclid, show. From no inkling more sophisticated than Hesiod's *Theogony* before Thales – to the *Metaphysics*, *Physics*, *On the heaven*, *On generation and corruption*, *Meteorology*, *On the soul*, the treatises on animals. The quality of the advance the description of the world shows, from the earlier accounts of creation, to the degree to which these are a description of the world, in *When on high*, in the Yahwistic document, in Homer, and in Hesiod – to *On the heaven*.

And the advance of mathematics which culmi-

nates in the *Elements* is inseparable from that advance. The philosophers who are part of this advance are the philosophers who are part of that advance. In many cases dramatically so. Thales, Pythagoras, Parmenides, Anaxagoras, Democritus, Eudoxus, Aristotle. The results of the philosophers can all be divided into physical and mathematical, they all depend for their discovery, whether physical or mathematical, substantially and essentially on one another.

And it is not that one problem leads to another. This physical problem to that mathematical problem. The accumulation of the formulations and solutions of the problems into an investigation of the world leads to, and culminates in, the *Elements*. The accumulation of the solutions of the problems into a description of the world leads to, and culminates in, the *Elements*.

The investigation of the world leads to the investigation of space. The measurement of the world to the measurement of space.

All the three problems that have been described are concerned with the world. That of the earth explicitly with its position in the world. That of the principles, also explicitly, not just with all things in the world but with those also, should there be such things, outside the world. That of the soul, again explicitly, not less with events beyond the world, as well as in the nether-world, than in the world.

And the three problems are only a part of the systematic unity of all the problems which the philosophers formulated and solved in these two and a half-odd centuries. Concerned with the fixed stars, the planets beneath the fixed stars, the region between the planets and the earth, the earth, its position, whether it is at rest or in motion, and its shape, the localities on the surface of the earth, and the stones, the plants and the animals, the political animal, too, the states, and the generation and corruption of those things, their constitution, the elements, the causes, the principles.

And the *Elements* is an empirical generalization not over this problem or that, or these problems or those. It is an empirical generalization over their systematic unity, over the investigation and description of the world. And a most successful generalization, the *Elements* is the foundation

of their systematic unity. It established their systematic unity.

Proclus, in the commentary on book I, writing on proposition 8, notes that some philosophers have found the previous proposition useless. He writes: Euclid uses 7 to prove 8, but 8 can be proved without using 7, and if 7 is only for the sake of proving 8, and 8 can be proved without it, is 7 not useless? to this we should reply that the proof of 7 is very useful to the astronomers, by it they can show that three eclipses cannot occur at equal intervals, the third after the second as the second after the first (268); and in the same way many other theorems in the *Elements* are useful in astronomy; the last in book IV, for instance, which shows how to inscribe a fifteen-sided figure in a circle; what reason can there be for it but that it is useful in astronomy? for the angle between the celestial equator and the zodiac is subtended by the length of the side of the fifteen-sided figure inscribed in a circle; the writer of the *Elements* has thus given us proofs to prepare us for that science (269).

Proclus writes seven and a half centuries after Euclid. He is assigning a final cause, for the sake of what.

And it is the minor difficulty with final causes that there are no final causes. The major is that the supposition that there are is unlikely to confine itself to this proposition or that. For the sake of what is proof? For the sake of what is the *Elements*? For the sake of what is mathematics? For the sake of what is knowledge? Before Euclid already, in the *Republic*, for instance, it is for the sake of the good. After Euclid, but before Proclus, it is for the sake of nothing good at all. The futile curiosity masquerading under the name of knowledge scripture calls gratification of the eye; it is for the same reason, to satisfy that sick curiosity, that freaks are put on show in the theater and men investigate nature and want knowledge for the sake of knowledge. Proclus writes two generations after Augustine.

A cause can only be, redundantly, an efficient cause, a source of the motion. A cause, or synonymously an efficient cause, has to be assigned. And when Proclus writes: the writer of the *Elements* has given us proofs to prepare us for that science – even as ‘us’ can only refer to a very few

of us, and for several centuries after Proclus to none of us at all – this points to the relation of cause and effect.

After the *Elements*, the relation of the *Elements* and the advance of astronomy, and more generally the advance of physics, is the relation of cause and effect. The *Elements* is the foundation of this advance, and the foundation can be, and in this case is, a cause.

This is how Frege, in *The foundations of arithmetic*, describes the goal of proof: the goal of proof is not only to raise the truth of a proposition above all doubt, but also to give knowledge of the dependence of truths on one another; after one has convinced oneself by the futile attempts to move it that a boulder is unshakeable, there is still the question of what, then, supports it so securely.

The *Elements* not only raised the truth of very many propositions above all doubt, it also gave knowledge of the dependence of truths on one another. It answered the question of what, then, supports the boulder unshakably.

And, in general, knowledge of the dependence of truths on one another can be the ground of the discovery of other truths, and other, and other. The answer to the question of what supports the boulder unshakably can be the ground on which other questions are answered, and other, and other.

In particular, it is. The *Elements* is the explicit foundation of the advance of physics through Ptolemy, and of Galileo and Kepler still, and of *The mathematical principles of natural philosophy* still.

After the *Elements*, the *Elements* is a cause of the advance of physics. Before the *Elements*, the *Elements* is an effect of the advance of physics.

It is a generalization over the investigation and description of the world, and most particularly over the measurement of the world.

Thales measured the height of the pyramids and the distance of ships from the shore. These are, spectacular as they are, just measurements.

Oenopides measured the ecliptic. Herodotus measured the Black Sea and the Red Sea. Meton measured the summer solstice, the solar year, and the period synchronizing the motions of the moon and the sun. Eudoxus measured the orientation of

the promontory of Attica, the earth, the relative sizes of the sun and the moon, and the periods of the planets. Pytheas measured Britain. Dicaearchus measured the Mediterranean, and improved on the measurement of the earth. And these are not only very much more spectacular measurements each, jointly they are the measurement of the world. Of the heaven above, the earth below, the two in relation to one another, and in their systematic unity. After Euclid, too, astronomy and geography advanced more, by very far, than the other parts of physics.

The *Elements* is a generalization over the investigation and description of the world, and most particularly over the measurement of the world, because nothing but measurement has the clarity and distinction, and that is, the consistency, which can make any results, and which makes all these results, and all results which rest on these results, a systematic unity, and hence a description of the world.

The relation of physics and mathematics in the two and a half-odd centuries from Thales to Euclid, of great advance in the description of the world leading to the development of a mathematical theory of the greatest sophistication, has a strict analogy in the relation of physics and mathematics in the brightest period in the acquisition of knowledge, the three generations from Bacon, Galileo, and Kepler to Newton and Leibniz. Resting on Euclid, the great advance in the description of the world in these three generations led to, and was a cause of, the discovery of calculus, which in turn became the ground, and a cause, of the subsequent advance of physics.

The *Elements* is the theory of space, and more particularly, the theory of the measurement of space.

That the *Elements*, and more generally geometry, is a theory of space is trivial.

Riemann writes: it is well known that geometry assumes as given the concept of space as well as the first principles for constructions in space (*Hypotheses*, pr.). Hilbert writes: the problem of the establishment of the axioms of geometry and the investigation of their relations is equivalent to the logical analysis of our perception of space. Einstein writes: if geometry is conceived as the science of the laws governing the spatial relations

of rigid bodies, it is to be considered the oldest branch of physics.

Many philosophers have nevertheless argued, and many more have taken it for granted, that it is a theory not of space, but of a space, mathematical space distinct from physical space.

With Hume: it is usual with mathematicians to pretend that those ideas which are their objects do not fall under the conception of the fancy, but must be comprehended by pure and intellectual view of which the superior faculties of the soul alone are capable; but to destroy this artifice, we need but reflect on the principle that all our ideas are copied from our impressions.

In particular, if the *Elements* is a generalization over the investigation and description of the world, and most particularly over the measurement of the world, mathematical space distinct from physical space can only be fictional, mathematical space can only be physical space, and physical space can only be redundant. Space can only be space.

The investigation of the world led to the formulation of the theory of space, the measurement of the world led to the formulation of the theory of the measurement of space.

The *Elements* is a triumph of the acquisition of knowledge. More generally, Euclid – the 514 proofs of the *Elements*, the 94 proofs of the *Data*, the 18 proofs of the *Phenomena*, the 58 proofs of the *Optics*, and the treatises superseded by the treatises of his successors, the *Conics*, *Surface loci*, *Elements of music*, and *Fallacies* – is a triumph of the acquisition of knowledge.

In conformity with the goal of physics defined by Planck, also: the complete emancipation of the physical picture of the world from the individuality of the picturing mind, Euclid has left behind of his individuality nothing.

There are a couple of anecdotes, recorded seven and a half centuries after Euclid. In conformity with the goal of physics defined by Planck, too, the moral the anecdotes teach will not seem different, even if they teach it rather picturesquely, from the moral Planck teaches.

Proclus writes: it is reported that king Ptolemy once asked Euclid if there was a shorter road in geometry than through the *Elements*, and Euclid replied that there was no royal road in geometry

(comm. *Elements*, 68). Stobaeus, writing at the same time as Proclus, has instead Alexander ask Menaechmus to teach him geometry briefly, and Menaechmus reply: king, in the country there may be different roads for the multitude and for kings, but in geometry the road is the same for all (*Anthology*, II, 228). About Euclid Stobaeus has this: having just begun to study geometry, a student asked Euclid, what will I profit from learning these things? Euclid turned to his slave and told him, give him three obols, to have him profit (*id.*).

So.

The *Elements* is a unitary deductive axiomatic system of 131 definitions, postulates, and axioms and 514 theorems, lemmas, and corollaries.

It is the theory of the measurement of space.

It is a theory of measurement.

Thus, the fundamental concern is equality, greater and less, and addition.

Equality, greater and less, and addition are made explicit by the most formal and most general part of the theory, the theory of proportion. This theory was formulated by Eudoxus, perfected by Euclid.

A fundamental assumption of the theory is continuity. On this assumption the consistency of the theory depends. This assumption – the axiom of measure – was also formulated by Eudoxus, perfected by Euclid.

It is a theory of the measurement of space.

Thus, the fundamental magnitude is the straight line. The irrationals are also straight lines. This part of the theory was formulated by Theaetetus, perfected by Euclid. Numbers are also analyzed into straight lines.

The most fundamental, and important, object, the standard of the measurement of space, is the triangle.

Its most fundamental, and important, property, which makes it the standard of the measurement of space, is that the sum of its angles is equal to two right angles.

The most fundamental, and important, assumption on which the property depends are parallel lines. The postulate: if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely,

intersect on that side. And hence the definition: parallel are straight lines which do not intersect.

Also, equality, greater and less, and addition are generalized to include similarity. And similarity also depends on parallel lines, and the sum of the angles of the triangle.

cccxx

Counting back to the 3 calculations of area, mathematics has developed for fifty-one centuries. And those from the 3 calculations of area to Euclid are the first twenty-seven and a half-odd of the fifty-one.

The advance in these centuries is very great, but it is not evenly distributed over all the twenty-seven and a half-odd, not at all. In the first twelve and a half centuries, from the 3 calculations of area to the book of 87 problems, the advance is slow and halting. In the next two centuries, from the book of 87 problems to the Venus tablet of Ammisaduqa, the advance is very great, this is the first of the three brightest periods in the development of mathematics. In the next ten and a half centuries, from the Venus tablet of Ammisaduqa to Thales, there is hardly any advance at all. And in the next two and a half-odd centuries, from Thales to Euclid, the advance is very great again, accelerates, and in the last one century-odd of these two and a half-odd, from Hippocrates of Chios to Euclid, mathematics enters the second of the three brightest periods in its development.

Spatially, as the development of the state is for more than one and a half millennia independent in the valley of the Nile and in the valley of the two rivers, before in the following half a millennium it becomes unitary, so the development of mathematics is at first independent in each of the two valleys, before it becomes a part of the development of philosophy.

3 calculations of area to Euclid, it is one, and only one, development. Mathematics has the kinds Mesopotamian mathematics, Egyptian mathematics, and Greek mathematics not more, or less, than it has the kinds clay tablet mathematics, papyrus mathematics, and parchment mathematics. Where the last kind, parchment mathematics, would no doubt also be church mathematics, or Christian mathematics, for Eudoxus, Euclid, and Archimedes wrote, like Ahmose, on

papyrus, it was the fathers of the church who, having matters of greater permanence than earlier writers to communicate, began favoring parchment over papyrus.

3 calculations of area to Euclid, the council of our predecessors is one, and only one, including when its sessions are unaware of one another. The problems are the same, the solutions are the same, the accumulation of problems and solutions is the same. Which the problem at issue shows, too. The book of 87 problems attests the discovery of the triangle, in problem 51. The next problem, 52, truncates the triangle. By a line parallel to the base. In the other valley, the independent discovery of the triangle is attested by the 14 problems, beginning with a triangle divided into 5 areas. Divided into 5 areas by parallel lines. Then there is the 1 problem, a triangle divided into 4 smaller triangles. Divided into 4 smaller triangles by parallel lines. All are Euclidean parallel lines.

In the first two and a half-odd centuries of the development of philosophy, then, mathematics advanced very greatly. By the end of that time, in Euclid, mathematics had become a theory of very great scope, very great power, and very great unity and uniformity. Scope – the number of objects and kinds of objects brought under investigation. Power – the number of discoveries to which the investigation leads. Unity and uniformity – the small number of assumptions, on which it depends, and of modes of inference, which it employs.

The decisive steps in this advance are six.

Two generations after the first philosopher, Pythagoras discovered proof.

Mathematics became deductive.

Proof is results shown to follow from assumptions, and conversely, assumptions shown to lead to results.

The discovery of proof followed the accumulation of formulations and solutions of problems, of problems in relation to one another, of problems and their solutions recognized explicitly as problems and their solutions, of solutions of the problems improving on one another, and of solutions improving explicitly on one another, all this beginning with Thales, and only with Thales.

Two generations later, Parmenides discovered

the principle of contradiction.

The principle of all principles is the assumption on which all results depend, and all other assumptions, on which results depend, depend.

The discovery of the principle of contradiction followed four generations of accumulation of formulations and solutions of problems, in relations to one another, recognized explicitly as problems and their solutions, improving on one another, and improving explicitly on one another, and two generations of accumulation of proofs, results shown to follow from assumptions and assumptions shown to lead to results.

Two generations later, Hippocrates of Chios wrote the first *Elements*.

Mathematics became axiomatic.

Axiomatic is the system in which results are shown to follow from explicitly stated assumptions.

The first *Elements* followed six generations of accumulation of problems, solutions, and proofs.

The large majority of problems and solutions accumulated in these six generations are physical. And the large majority of problems and solutions accumulated in the following four generations, through that of Euclid, continues to be physical.

As the fact of the first *Elements* shows, however, the distinctly mathematical problems and solutions have begun to accumulate as well. Those are the more formal problems and solutions, those dependent unconditionally on proof, on showing explicitly the relation of assumptions and results of following from one another. Like squaring the circle and doubling the cube.

And mathematics is still not geometry more than it is arithmetic.

Pythagoras's proof that the sum of the angles of the triangle is two right angles is geometrical, his proof of incommensurability and his method for finding Pythagorean triples are arithmetical. Hippocrates of Chios squared the circle geometrically, doubled the cube algebraically. The results of his two contemporaries, Democritus and Theodorus of Cyrene, the volumes of the cone, cylinder, pyramid, and prism, and the irrationality of $\sqrt{3}$, $\sqrt{5}$, ..., $\sqrt{17}$, are one of them geometrical, the other not.

Not quite two generations after Hippocrates of

Chios, Eudoxus formulated the theory of proportion.

Mathematics became a theory of great formality and great generality.

And it is still not geometry more than it is arithmetic, or indeed algebra now.

Taking the theory of proportion as it is developed in book V of the *Elements*, and taking it by itself, the theory is not geometrical. It is the theory of magnitudes, which need not be objects in space. It is a theory of equality, greater and less, and addition, which need not be spatial relations. Nor is the axiom of measure, formulated by Eudoxus, geometrical. Taking it in its formulation in proposition 1 of book X of the *Elements*, the axiom concerns, like the theory of proportion, magnitudes, which need not be objects in space, and makes explicit equality, greater and less, and addition, which need not be spacial relations.

Also, Eudoxus's contemporary Theaetetus developed the theory of irrational lines, or of the irrationals. Taking this theory as it is developed in the early propositions of book X of the *Elements*, and taking it again by itself, this theory is not geometrical either. It is also the theory of magnitudes, and of equality, greater and less, and addition, which need not be spacial objects or relations.

In the generation after Eudoxus, Aristotle developed logic.

Logic became a theory of great formality and great generality. The formality of the predicate calculus is as great as that of the theory of proportion. The generality of the theory of terms, propositions, and inferences is greater than that of the theory of proportion, it is the greatest.

And logic is the theory of the assumptions on which results depend.

Finally, in the generation after Aristotle, Euclid wrote the *Elements*.

Mathematics became a unitary deductive axiomatic system of great formality and great generality.

It also became geometry.

Euclid collected, perfected, systematized, and unified the results of his predecessors. He obtained many new results himself. And most important, he integrated all these results into one

another.

He integrated the theory of numbers into the theory of proportion. Books VII-IX into book V. Into this more general theory he integrated the theory of irrational lines. Into books V and VII-IX book X. And this more general theory he integrated into the theory of the measurement of space. Books V and VII-X into books I-IV, VI, and XI-XIII.

He integrated arithmetic into algebra, and algebra into geometry.

Mathematics became fundamentally, substantially, and essentially geometry. Fundamentally, spatial relations became the ground of proofs. Substantially, all objects were reduced to spatial relations. Essentially, spatial relations closed the gaps in the chains of inferences.

Mathematical admits of degree, and physical admits of degree. And geometry is more physical than arithmetic, or algebra.

And, hence, the importance of the very great advance of mathematics which Euclid is, and most particularly in that mathematics becomes with Euclid's labors geometry, cannot be overstated. Its power is the consequence of this fact. In the following two millennia the ground of all advances of physics is geometry.

The development of optics which begins with Euclid himself is geometrical. The development of statics and hydrostatics by Archimedes is geometrical. The great advance of astronomy through Ptolemy is geometrical. The great advance of geography through Ptolemy is geometrical. Copernicus is geometrical. Galileo's two new sciences of mechanics and locomotion are geometrical. Kepler's laws are geometrical. The development of calculus, and of analysis, by Descartes, Cavalieri, and Fermat begins geometrically. Huygens's wave theory is geometrical. And after the development of calculus still, *The mathematical principles of natural philosophy* is geometrical. It is at this time, the time of Newton and Leibniz, that physics gradually begins becoming more analytical than geometrical.

And all this rests on the triangle, the sum of its angles, and parallel lines.

cccxxi

The task has been to collect the predecessors

of the philosophers and philosophers who are part of the development of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, and in particular, those who are part of it in the two-odd centuries from Pythagoras to Euclid.

Counting only the more mathematical results, those of 13 mathematicians – Parmenides, Anaxagoras, Antiphon, Hippias, Hippocrates of Chios, Theodorus of Cyrene, Democritus, Archytas, Theaetetus, Eudoxus, Aristotle, Menaechmus, and Euclid – have been described, attesting the development of mathematics in these two-odd centuries. All 13 are very important, and all 13 are steps of advance in the acquisition of knowledge.

But not all with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

These 5 are all important philosophers, mathematicians, in particular, who are part of the development of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends,

515 - 445 **Parmenides,**
460 **Hippocrates of Chios,**
408 - 356 **Eudoxus,**
384 - 322 **Aristotle,**
346 **Euclid.**

The steps of advance in the acquisition of knowledge are all 5. In particular,

Parmenides	discovered the principle of contradiction,
Hippocrates	wrote the first axiomatic treatise,
Eudoxus	formulated the theory of proportion,
	formulated the axiom of measure,
Aristotle	developed logic,
Euclid	wrote the <i>Elements</i> , the fundamental object is the triangle, its fundamental property is the sum of its angles, the sum depends on parallel lines, similarity as well depends on parallel lines,

parallel are straight lines that
do not intersect,
the assumption is that, if a
straight line intersecting two
straight lines makes the in-
terior angles on the same
side less than two right
angles, the two straight
lines, produced indef-
initely, intersect on
that side.

cccxxii

The not quite four and a half centuries from Euclid to Ptolemy.

They divide into two parts. The one and a half-odd centuries from Euclid to Hipparchus. And the two and a half-odd centuries from Hipparchus to Ptolemy. The dates are,

346	Euclid,
187 ^{BC}	Hipparchus,
87 ^{AD}	Ptolemy.

In the one and a half-odd centuries from Euclid to Hipparchus the great advance of physics and mathematics continued. The schools flourished. Outside the schools, the second of the three brightest periods in the development of mathematics culminated in Archimedes, Apollonius, and Hipparchus.

In the two and a half-odd centuries from Hipparchus to Ptolemy the advance of physics and mathematics began declining. Philosophy spread. The advances of physics and mathematics became fewer and less frequent. Physics and mathematics became, like all philosophy, increasingly derivative. There was yet at the end of this time Ptolemy. It was after that that the great decline accompanying the rise of the church began.

In these not quite four and a half centuries physics became mathematical. Euclid's contemporary Autolycus and Euclid himself began proving astronomical and optical theorems. Later in the same generation, Timocharis began the accurate observation of the planets and stars. Later in the same generation, Aristarchus measured the distances of the moon and the sun. In the next generation, Archimedes developed statics and hydrostatics mathematically. In the

same generation, Eratosthenes improved greatly on the measurement of the radius of the earth, and laid the foundations of mathematical geography. One generation-odd later, Apollonius formulated the eccentric and epicyclic hypotheses. One and a half generations later, Hipparchus continued the great advance of mathematical astronomy and mathematical geography. Two centuries later, Hero began the mathematical development of mechanics. Two generations later, the advance of mathematical astronomy, mathematical geography, and mathematical optics culminated in Ptolemy.

Most of this development has been presented in some detail above, the rest will be below.

Here two of the propositions with which this development begins can be presented.

Mathematical astronomy begins with Autolycus's *On the moving sphere* and *On the risings and settings* and Euclid's *Phenomena*. Autolycus and Euclid prove 60 propositions in the three treatises. This is proposition 1 in *On the moving sphere*:

if a sphere rotates uniformly about its axis, all points on the surface of the sphere, and not on the axis, describe parallel circles, about the same poles as the sphere, and perpendicular to the axis;

let A and B be the poles
and AB the axis of the
sphere;

take point C on the sur-
face of the sphere, and not
on the axis;

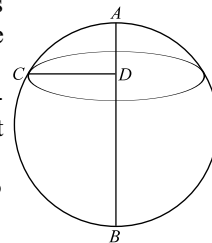
drop perpendicular CD
to AB ;

the intersection of the
plane through AB and CD and the surface of
the sphere is a circle;

let semicircle ACB , with AB fixed, rotate
about AB , carrying with it the straight line CD ,
until it return to its original position;

during the entire rotation CD remains perpen-
dicular to AB and describes on the surface of the
sphere a circle with center D on the axis and
about the poles A and B ;

in the same way all points on the surface of the
circle, and not on the axis, describe circles per-
pendicular to the axis with centers on the axis and
about the poles;



and circles about the same poles on the surface of a sphere are parallel (*id.*)

The points on the surface of the sphere, describing parallel circles about the poles, are the stars. Circles are parallel when the planes through them are parallel, and planes are parallel when they do not intersect.

Mathematical optics begins with Euclid's *Optics*. Euclid begins with 7 definitions. Among them: 2. let the space within sight be a cone with its vertex at the eye and its base at the limit of sight; 4. let things seen within a greater angle appear greater, things seen within a smaller angle appear smaller, and things seen within equal angles appear equal; 7. let things seen within more angles appear more clearly (*id.*). He then proves 58 propositions. This is proposition 2:

things at a smaller distance are seen more clearly than equal things at a greater distance;

let A be the eye; let BC and DE be the things seen; let them be equal and parallel; and let DE be at a greater distance than BC ;

the greatest angles from the eye to the two things are then BAC and DAE ;

for the straight lines from the eye to the ends of DE cannot pass through the points B and C ;

for, if they did, the straight line DE , which is the base of the triangle ADE , would be greater than the straight line BC , which is the base of the triangle ABC , and they are assumed equal;

seen through a greater angle than DE , BC is seen through more angles;

and it is therefore seen more clearly (*id.*).

The two things seen, BC and DE , are parallel, and had the straight lines from the eye to the ends of DE passed through the points B and C , the triangles ADE and ABC would have been similar.

Physics became mathematical, and, in particular, geometrical, and grounded in parallel lines.

The following are the most important mathematicians in these not quite four and a half centuries, and some of their most important, and elegant, more mathematical results.

Two generations after Euclid, Archimedes, next to the more physical *On the equilibrium of planes* and *On floating bodies*, wrote the more mathematical *On the sphere and cylinder*, *Mea-*

surement of the circle, *On conoids and spheroids*, *Quadrature of the parabola*, *On spirals*, *The cattle problem*, *On the method of mechanical theorems*, and *The sand-reckoner*.

He corrected the definition of the straight line in *On the sphere and cylinder*. He begins with 6 axioms and 5 assumptions, and assumption 1 is: of all lines that have the same limits the straight is the least (*id.*).

More explicit, quantitative, it superseded Euclid's definition.

In the *Measurement of the circle* Archimedes proves 3 propositions. This is proposition 3:

the ratio of the circumference and diameter of the circle is less than $3\frac{1}{7}$ and greater than $3\frac{10}{71}$;

let on the diameter AB , O be the center of the circle, and AC a tangent at A ;

let $\angle AOC = \frac{1}{3}$

of a right angle;

then, from the triangle AOC ,

$\frac{AO}{AC} > \frac{265}{153}$ and $\frac{CO}{AC} = \frac{306}{153}$ (*id.*).

In the right triangle AOC , in which $\angle AOC = \frac{1}{3}$

of a right angle, $\frac{AO}{AC} = \frac{\sqrt{3}}{1}$ and $\frac{CO}{AC} = \frac{2}{1}$. Archimedes

does not report how he obtained the value for $\sqrt{3}$. Hero, in the *Metrics*, has this method for approximating a square root indefinitely: we find the square root of 720 in this way; the nearest square is 729, its root is 27; $\frac{720}{27} = 26\frac{2}{3}$;

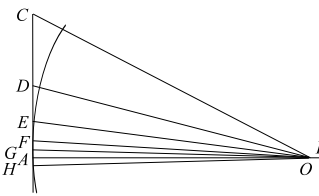
$\frac{26\frac{2}{3} + 27}{2} = 26\frac{5}{6}$; and $\sqrt{720} \approx 26\frac{5}{6}$, because

$\left(26\frac{5}{6}\right)^2 = 720\frac{1}{36}$; and as we now have the nearer

square $720\frac{1}{36}$, we can do the same again with it

instead of 729, and the difference will be much less than $\frac{1}{36}$ (I, 8). Archimedes continues:

bisect $\angle AOC$, the bisector OD intersecting AC at D ;



then, from the triangle AOC ,

$$\frac{CO}{AO} = \frac{CD}{AD} \text{ (id.)}.$$

Euclid, in proposition 3 of book VI of the *Elements*, proves that: if an angle of the triangle is bisected, the sides adjacent to the angle are in the ratio of the parts into which the bisector divides the opposite side; draw AP parallel to DO , and then $\frac{CD}{AD} = \frac{CO}{OP}$; but $OP = AO$; and

$$\frac{CD}{AD} = \frac{CO}{AO} \text{ (id.)}. \text{ Archimedes continues:}$$

$$\frac{CO + AO}{AO} = \frac{CD + AD}{AD} = \frac{AC}{AD};$$

$$\text{and } \frac{CO + AO}{AC} = \frac{AO}{AD};$$

$$\text{but } \frac{CO + AO}{AC} > \frac{306 + 265}{153};$$

$$\text{therefore } \frac{AO}{AD} > \frac{571}{153};$$

$$\frac{AO^2 + AD^2}{AD^2} > \frac{571^2 + 153^2}{153^2}$$

$$\text{but } AO^2 + AD^2 = DO^2;$$

$$\text{and } \frac{DO^2}{AD^2} > \frac{349450}{23409}$$

$$\text{and } \frac{DO}{AD} > \frac{591\frac{1}{8}}{153};$$

next, bisect $\angle AOD$, the bisector OE intersecting AC at E ;

and in the same way,

$$\frac{AO}{AE} > \frac{1162\frac{1}{8}}{153};$$

$$\text{and } \frac{EO}{AE} > \frac{1172\frac{1}{8}}{153};$$

next, bisect $\angle AOE$, the bisector OF intersecting AC at F ;

and in the same way again,

$$\frac{AO}{AF} > \frac{2334\frac{1}{4}}{153};$$

$$\text{and } \frac{FO}{AF} > \frac{2339\frac{1}{4}}{153};$$

next, bisect $\angle AOF$, the bisector OG intersect-

ing AC at G ;

$$\text{and } \frac{AO}{AG} > \frac{4673\frac{1}{2}}{153};$$

$$\angle AOC = \frac{1}{3} \text{ of a right angle, it has been bi-}$$

$$\text{sected four times, and } \angle AOG = \frac{1}{48} \text{ of a right}$$

angle;

draw $\angle AOH = \angle AOG$ on the other side of AO , OH intersecting AC produced at H ;

$$\text{thus, } \angle GOH = \frac{1}{24} \text{ of a right angle, and } GH \text{ is}$$

the side of the regular 96-sided polygon circumscribed about the circle;

$$\text{but } AO = \frac{1}{2}AB \text{ and } AG = \frac{1}{2}GH, \text{ and}$$

$$\frac{AB}{\text{perimeter of the 96-sided polygon}} > \frac{4673\frac{1}{2}}{153 \times 96};$$

$$\frac{4673\frac{1}{2}}{153 \times 96} = \frac{4673\frac{1}{2}}{14688}; \quad \frac{14688}{4673\frac{1}{2}} = 3 + \frac{667\frac{1}{2}}{4673\frac{1}{2}}; \text{ and}$$

$$3 + \frac{667\frac{1}{2}}{4673\frac{1}{2}} < 3\frac{1}{7};$$

and therefore, much more is the ratio of the circumference, which is less than the perimeter of the polygon, and the diameter of the circle less than $3\frac{1}{7}$ (id.).

And the lower limit Archimedes obtains as the upper, beginning by drawing a right triangle with an angle equal to $\frac{1}{3}$ of a right angle on the diame-

ter of the circle, with the value $\frac{\sqrt{3}}{1} < \frac{1351}{780}$ now,

bisecting the angle equal to $\frac{1}{3}$ of a right angle

four times, and showing that the ratio of the side of the 96-sided regular polygon inscribed in the circle and the diameter $> \frac{66}{2017\frac{1}{4}}$, and the ratio of

the perimeter of the polygon and the diameter $> \frac{6336}{2017\frac{1}{4}} > 3\frac{10}{71}$ (id.).

$$3\frac{10}{71} < \pi < 3\frac{1}{7}. \text{ It is clear also that in the same}$$

way the value of π can be calculated more precisely indefinitely, as it is clear also that it is not easy at all to calculate it more precisely than Archimedes. Nineteen centuries later, Huygens calculated it at $3.1415926533 < \pi < 3.1415926538$.

In the same generation, Chrysippus discovered the rules of inference.

In the next generation, Diocles wrote *On burning mirrors*. Diocles reports that Dositheus, in the previous generation, discovered that the mirror reflecting the sun's rays in a point is parabolic (pr.). The point is the focus of the parabola. And in propositions 4-5 Diocles constructs the parabolic mirror:

let PQ be the distance from the vertex to the focus;

draw on PQ square $PQRS$;

produce QR to P_5 as $QR = RP_5$;

join P_5S and produce it to intersect QP at U ;

take any points V and W on PQ , draw VV_1 and WW_1 parallel to QP_5 , intersecting P_5S at V_1 and W_1 ;

a circle with center Q and radius $QP_3 = VV_1$ intersects VV_1 at P_3 and P_4 ;

a circle with center Q and radius $QP_1 = WW_1$ intersects WW_1 at P_1 and P_2 ;

produce P_5Q to P_6 as $P_5Q = QP_6$;

then the points $P_6, P_4, P_2, P, P_1, P_3$, and P_5 lie on a parabola with focus Q (*id.*).

Diocles shows that the figure is a parabola by

showing that $\frac{WP_1^2}{VP_3^2} = \frac{PW}{PV}$ (*id.*). Archimedes

already takes it for granted that in the parabola the square on the ordinate is proportional to the abscissa. Diocles, however, shows also this:

complete the square on QP_5 ;

$QP_3 = VV_1$;

but $VV_1 = V_1U_v = P_3U_3$;

hence $QP_3 = P_3U_3$;

in the same way, $QP_1 = P_1U_1$ (*id.*).

Diocles has discovered the directrix.

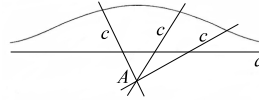
Five and a half centuries later, Pappus will prove that the locus of points whose distance from a given fixed point is in a given ratio to their distance from a given fixed straight line is a

conic, and the conic is an ellipse, a parabola, or a hyperbola, respectively, when the given ratio is less than, equal to, or greater than 1 (*Collection*, VII, 318). The fixed point is the focus, the fixed line is the directrix.

In particular, if Q is the focus, U_vU produced is the directrix, and $QP = PU$, $QP_1 = P_1U_1$, $QP_2 = P_2U_2$, and so on, the locus of P, P_1, P_2 , and so on, is a parabola.

In the same generation, Nicomedes discovered the conchoid (Pappus, *Collection*, IV, 22). Pappus describes the generation of the curve:

given a fixed point A , a fixed straight line a , another straight line rotating about A , and a fixed length c on the rotating line, then if one end of the length c traces the fixed line a , its other end traces the conchoid (*id.*).



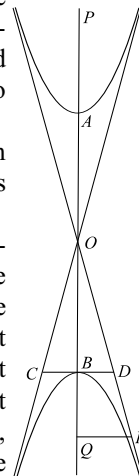
In the next generation, next to formulating the eccentric and epicyclic hypotheses, Apollonius wrote the more mathematical *Conics* and *Division of a ratio*. The *Conics*, in eight books, 387 proofs in books I-VII, is the most successful axiomatic treatise after the *Elements* and before *The mathematical principles of natural philosophy*. In book V Apollonius solves problems of minima and maxima.

This is proposition 1 in book II:

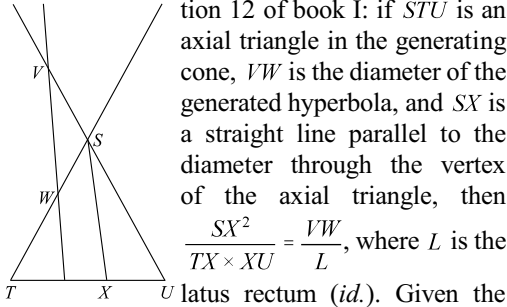
if a straight line is tangent to a hyperbola at its vertex, and on each side of the diameter a part of the tangent is cut, whose square is equal to one fourth of the product of the diameter and the latus rectum, and from the center to the ends of these two parts of the tangent straight lines are drawn and produced, these straight lines do not intersect the hyperbola;

let there be the hyperbola with center O , diameter AB , and latus rectum L (*id.*).

Apollonius has defined the diameter and the center in book I: the diameter of any curve is the straight line bisecting the straight lines parallel to a given straight line (4); the center is the midpoint of the diameter (9). In particular, the diameter PQ intersects the



hyperbola at the vertices A and B . The latus rectum Apollonius has defined from the generation of the conics. In the hyperbola, in proposition



12 of book I: if STU is an axial triangle in the generating cone, VW is the diameter of the generated hyperbola, and SX is a straight line parallel to the diameter through the vertex of the axial triangle, then $\frac{SX^2}{TX \times XU} = \frac{VW}{L}$, where L is the

latus rectum (*id.*). Given the focus of the conic, the latus rectum is the chord through the focus perpendicular to the major axis. Apollonius continues in proposition 1 of book II:

let CD be tangent at B ;

let $BC^2 = BD^2 = \frac{1}{4}AB \times L$;

let OC and OD be joined and produced;

then OC and OD do not intersect the hyperbola;

for, if possible, let OD intersect it at R ;

draw QR parallel to CD ;

$\frac{AB}{L} = \frac{AB^2}{AB \times L}$;

but $BO^2 = \frac{1}{4}AB^2$;

and $BD^2 = \frac{1}{4}AB \times L$;

hence, $\frac{AB}{L} = \frac{BO^2}{BD^2}$;

from the similar triangles OBD and OQR ,

$\frac{OB^2}{BD^2} = \frac{OQ^2}{QR^2}$;

and hence, $\frac{AB}{L} = \frac{OQ^2}{QR^2}$;

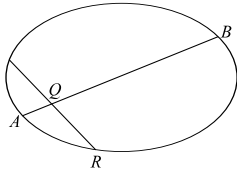
but $\frac{AB}{L} = \frac{AQ \times BQ}{QR^2}$ (*id.*).

This, Apollonius has proved in proposition 21 of book I, holds for all central conics, hyperbola, ellipse, and circle. He continues:

therefore, $OQ^2 = AQ \times BQ$;

and this is absurd (*id.*).

Proposition 6 of book II of the *Elements* is:



$(a + b)^2 = (2a + b)b + a^2$ (*id.*). And if $AB = 2a$ and $BQ = b$, $(a + b)^2 = (2a + b)b$, and this is absurd. Apollonius concludes:

therefore, OD does not intersect the hyperbola;

in the same way, OC does not intersect it;

therefore, OC and OD are asymptotes to the hyperbola (*id.*).

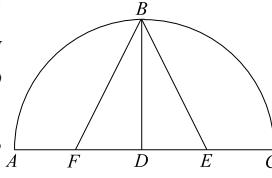
And in proposition 14 of book II he proves that: the asymptotes and the hyperbola produced indefinitely approach one another nearer than any given distance (*id.*).

Nor is it that the hyperbola and its asymptotes alone produced indefinitely approach one another nearer than any given distance. The conchoid, for instance, is also asymptotic to its base.

One and a half generations later, next to the formulation of the theory of the sun and of the moon, the determination of the distance of the moon, the discovery of the precession of the equinoxes, Hipparchus, among his more mathematical results, developed trigonometry. The results of the great lover of truth Hipparchus are the ground of Ptolemy's results in the *Almagest*. And this is how Ptolemy develops trigonometry in book I of the *Almagest*:

on semicircle ABC with center D draw BD perpendicular to AC ;

bisect CD at E , and make $EF = BE$;



$(2DE + DF)DF + DE^2 = (DF + DE)^2$ (10).

This is again proposition 6 of book II of the *Elements*, with $DE = a$ and $DF = b$. Ptolemy continues:

hence $CF \cdot DF + DE^2 = EF^2$;

but $EF^2 = BE^2 = BD^2 + DE^2$;

hence $CF \cdot DF + DE^2 = BD^2 + DE^2$;

and $CF \cdot DF = BD^2$;

and of the sides of the regular hexagon, decagon, and pentagon inscribed in the same circle the relations hold,

(decagon + hexagon) decagon = hexagon²,

pentagon² = decagon² + hexagon² (*id.*).

These are propositions 9 and 10 of book XIII of the *Elements*. Ptolemy continues:

thus, in the circle with radius BD , and thus side of the inscribed hexagon BD as well, the side of the inscribed decagon is DF , the side of the

inscribed pentagon is BF ;

the side of the hexagon subtends an arc of 60° , the side of the decagon an arc of 36° , the side of the pentagon an arc of 72° ;

thus, $(DF + BD)DF = BD^2$, and if BD , the chord of the arc of 60° is 60, DF , the chord of the arc of 36° , is $37\frac{55}{60}$;

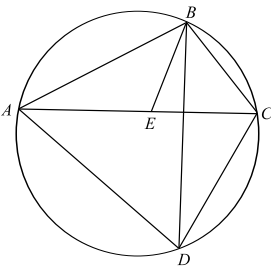
also, $BF^2 = DF^2 + BD^2$, and BF , the chord of the arc of 72° , is $70\frac{32}{60}$ (*id.*).

Next, Ptolemy obtains from the side of the inscribed square the chord of the arc of 90° . His

result is $84\frac{51}{60}$, where the radius is 60 (*id.*),

equivalent to $\sqrt{2} = 1.414\,214\,162\dots$. He obtains from the side of the inscribed equilateral triangle the chord of the arc of 120° (*id.*). The Pythagorean theorem gives the chords of the arcs supplementary to all those arcs (*id.*). Next:

inscribe quadrilateral $ABCD$ in a circle; join AC and BD ; draw $\angle ABE = \angle DBC$; so $\angle ABD = \angle EBC$; on the same arc, $\angle ADB = \angle ACB$; and triangles ABD and EBC are similar;



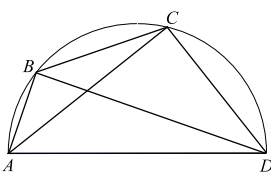
$\angle ABE = \angle DBC$;
 $\angle BAC = \angle BDC$;
 and triangles ABE and DBC are also similar;
 from the second pair of similar triangles $\frac{AB}{AE} = \frac{BD}{CD}$; from the first pair of similar triangles $\frac{BC}{EC} = \frac{BD}{AD}$;

hence $AB \cdot CD = AE \cdot BD$, and $AD \cdot BC = EC \cdot BD$;
 and hence $AB \cdot CD + AD \cdot BC = AC \cdot BD$ (*id.*).

Next:

given this preliminary lemma, draw a semicircle with diameter AD ;

draw the chords AB and AC , given in



length in terms of the radius;

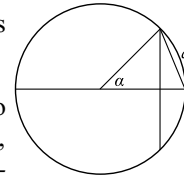
join BD and CD ;

clearly, BD and CD are also given, for they are supplementary to AB and AC ;

and therefore BC is also given (*id.*).

And hence, the addition, subtraction, multiplication, and division of the already obtained chords gives the chord of any arc with any degree of accuracy (*id.*).

And $\frac{1}{2}$ the chord on arc $2a$ is $\sin a$.



Two centuries later, next to the more physical *Mechanics*, *Dioptrics*, *Catoptrics*, *Pneumatics*, *On the making of automata*, and *On the making of catapults*, Hero wrote the more mathematical *Metrics*, *Definitions*, *Geometry*, *Stereometry*, and *On measures*. This is in the *Metrics*:

it is possible to find the area of the triangle, if the height is not given, but the three sides are given;

let the triangle be ABC , and let the three sides AB , BC , and AC be given;

to find the area;

inscribe in the triangle circle with center O , touching the sides at D , E , and F ;

join AO , BO , CO , DO , EO , and FO ;

$DO = EO = FO$;

and $AD = AF$, $BD = BE$, $CE = CF$;

so $AB \times DO = 2 \times \triangle ABO$;

$BC \times EO = 2 \times \triangle BCO$;

$AC \times FO = 2 \times \triangle ACO$;

and $(AB + BC + AC) \times DO = 2 \times \triangle ABC$;

produce BA to G , as $AG = CF$;

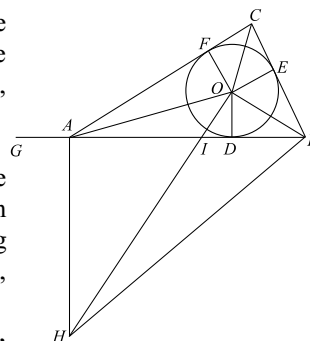
and since $AD = AF$, $BD = BE$, and $CE = CF$,

$BG = \frac{1}{2}(AB + BC + AC)$;

hence, $BG \times DO = \triangle ABC$;

draw OH perpendicular to BO , and AH perpendicular to AB , intersecting at H ; join BH ;

since $\angle HAB$ and $\angle HOB$ are both right, a circle with diameter HB can be circumscribed about



quadrilateral $HBOA$;

hence $\angle AOB + \angle AHB = 2$ right angles;

but $\angle AOB + \angle COF = 2$ right angles, because the angles at O are bisected by OA , OB , and OC , and $(\angle AOB + \angle COF) + (\angle BOC + \angle AOF) = 4$ right angles;

hence $\angle AHB = \angle COF$;

and $\angle HAB$ and $\angle CFO$ are right;

hence triangles HAB and OFC are similar;

hence $\frac{AB}{AH} = \frac{CF}{FO}$;

but $CF = AG$ and $FO = DO$;

so $\frac{AB}{AH} = \frac{AG}{DO}$;

and $\frac{AB}{AG} = \frac{AH}{DO}$;

but AH and DO are parallel, and hence the triangles HAI and ODI are similar;

so $\frac{AH}{DO} = \frac{AI}{DI}$;

and $\frac{AB}{AG} = \frac{AI}{DI}$;

$\frac{AB + AG}{AG} = \frac{AI + DI}{DI}$; or $\frac{BG}{AG} = \frac{AD}{DI}$;

then $\frac{BG^2}{AG \times BG} = \frac{AD \times BD}{DI \times BD}$;

but from the right triangle BOI , $DI \times BD = DO^2$;

$\frac{BG^2}{AG \times BG} = \frac{AD \times BD}{DO^2}$;

$BG^2 \times DO^2 = AG \times BG \times AD \times BD$;

but $BG \times DO = \triangle ABC$;

and $\triangle ABC = \sqrt{AG \times BG \times AD \times BD}$, where BG is one half the perimeter of the triangle, AG is one half the perimeter minus AB , AD is one half the perimeter minus BC , and BD is one half the perimeter minus AC (I, 8).

If A is the area, s one half the perimeter, and a , b , and c the sides, $A = \sqrt{s(s-a)(s-b)(s-c)}$.

One generation later, Menelaus wrote the *Spherics*. He begins with the definitions: a triangle on the surface of the sphere is the area enclosed by three intersecting arcs of great circles, these arcs always being smaller than semicircles (1); the angles of the spherical triangle are those contained by the arcs of the great circles (2); and they are equal when the inclinations of the planes containing the great circles are equal (3).

If the angles of the spherical triangle were

those contained by the tangents at its vertices, the sum of the angles of the spherical triangle would have been greater than two right angles.

In book III Menelaus develops spherical trigonometry. His results as well are the ground of Ptolemy's results in the *Almagest*. And this is how Ptolemy proves Menelaus's theorem in book I of the *Almagest*:

draw on the sphere the great circles AB and CD intersecting the great circles AE and CE , and intersecting one another at F ;

then

$\frac{\text{chord arc } 2BC}{\text{chord arc } 2BE} = \frac{\text{chord arc } 2CF}{\text{chord arc } 2DF} \times \frac{\text{chord arc } 2AD}{\text{chord arc } 2AE}$;

take the center of the sphere O ;

join OA , OF , and OB ;

join ED and produce it to intersect OA produced at K ; join CD intersecting OF at L ; and join CE intersecting OB at M ;

then the points K , L , and M lie on a straight line, for all three lie in two planes, the plane of the triangle CDE and the plane of the circle AFB , and must thus lie on the intersection of the two planes;

draw this line;

thus, the two straight lines KM and CD intersect the two straight lines KE and CE ,

and $\frac{CM}{EM} = \frac{CL}{DL} \times \frac{DK}{EK}$ (13).

Ptolemy has shown this earlier:

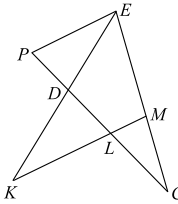
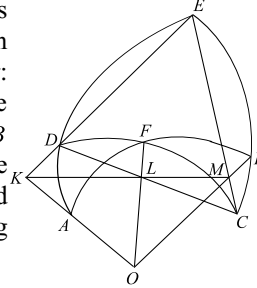
given the two straight lines KM and CD intersecting the two straight lines KE and CE , and intersecting one another at L ,

draw EP parallel to KM , and produce CD to intersect it at P ;

then $\frac{CM}{EM} = \frac{CL}{PL} = \frac{CL}{PL} \times \frac{DL}{DL}$;

but from the similar triangles DKL and DEP , $\frac{DL}{PL} = \frac{DK}{EK}$;

and $\frac{CM}{EM} = \frac{CL}{DL} \times \frac{DK}{EK}$ (id.).



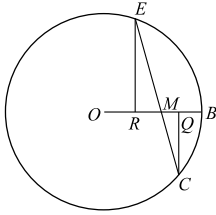
1050 In order to formulate the problems we are to solve we call the council of our predecessors

The proof of Menelaus's theorem continues:

$$\text{but } \frac{CM}{EM} = \frac{\text{chord arc } 2BC}{\text{chord arc } 2BE} \text{ (id.).}$$

This Ptolemy has shown earlier as well:

on the circumference of the circle with center O take any three points E , B , and C , the arcs EB and BC being each less than a semicircle;



join EC and OMB ;
drop perpendiculars CQ
and ER to the diameter;

$$\text{from the similar triangles, } \frac{CM}{EM} = \frac{CQ}{ER};$$

$$\text{but } CQ = \frac{1}{2} \text{ chord arc } 2BC;$$

$$\text{and } ER = \frac{1}{2} \text{ chord arc } 2BE;$$

$$\text{therefore, } \frac{CM}{EM} = \frac{\text{chord arc } 2BC}{\text{chord arc } 2BE} \text{ (id.).}$$

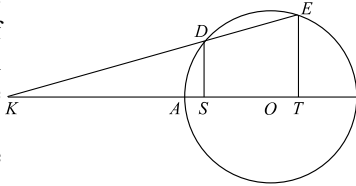
The proof of Menelaus's theorem continues:

$$\text{in the same way, } \frac{CL}{DL} = \frac{\text{chord arc } 2CF}{\text{chord arc } 2DF};$$

$$\text{and also } \frac{DK}{EK} = \frac{\text{chord arc } 2AD}{\text{chord arc } 2AE} \text{ (id.).}$$

And this, too, Ptolemy has shown earlier:

on the circumference of the circle with center O take any points A , D , and E , the arcs AD and



DE being each less than a semicircle;

join ED and OA and produce them to intersect at K ;

drop perpendiculars DS and ET to the diameter;

$$\text{and } \frac{DK}{EK} = \frac{DS}{ET};$$

$$\text{and as } DS = \frac{1}{2} \text{ chord arc } 2AD;$$

$$\text{and } ET = \frac{1}{2} \text{ chord arc } 2AE;$$

$$\frac{DK}{EK} = \frac{\text{chord arc } 2AD}{\text{chord arc } 2AE} \text{ (id.).}$$

The proof of Menelaus's theorem concludes:

thus,

$$\frac{\text{chord arc } 2BC}{\text{chord arc } 2BE} = \frac{\text{chord arc } 2CF}{\text{chord arc } 2DF} \times \frac{\text{chord arc } 2AD}{\text{chord arc } 2AE} \text{ (id.).}$$

$$\text{Equivalently, } \frac{\sin BC}{\sin BE} = \frac{\sin CF}{\sin DF} \times \frac{\sin AD}{\sin AE}.$$

And one generation after Menelaus is Ptolemy.

cccxxiii

The rules of inference do not depend on parallel lines, even if their discovery cannot not depend on the great advance of mathematics, Chrysippus discovered the rules two generations after Euclid, in the generation of Archimedes. The generation of the conchoid does not depend on parallel lines either, even if its discovery cannot not depend on the great advance of mathematics either, Nicomedes discovered the conchoid in the generation between Archimedes and Apollonius. All the other results above, the calculation of π , the demonstration of the directrix of the parabola, the demonstration of the asymptotes of the hyperbola, the calculation of the chords of the circle, the determination of the area of the triangle from its sides, the determination of the ratios of sines, depend on parallel lines. And all results which depend on these results, the results of Archimedes, of Diocles, of Apollonius, of Hipparchus, of Hero, of Menelaus, of Ptolemy, with barely an exception, depend on parallel lines. On Euclid's postulate. On the assumption that, if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side.

And two-odd centuries after Euclid, the examination of the assumption begins, implicitly, with Posidonius, and two-odd centuries later still, not quite four and a half centuries after Euclid, the examination of the assumption begins, explicitly, and decisively, with Ptolemy.

Posidonius proposed to improve on Euclid's definition of parallel lines. Ptolemy proposed to prove what Euclid's postulate affirms.

That these are the first attempts each, to improve on Euclid's definition, and to prove what Euclid's postulate affirms, the following facts show. Negatively, the fact of all the mathematicians in the intervening centuries, who advanced mathematics very greatly, advanced it on the

ground of explicitly stated assumptions, most particularly Euclid's assumptions, correcting these assumptions and improving on them, not often, given the perfection of the *Elements*, yet correcting them and improving on them, as Archimedes's definition of the straight line shows, in none of whose treatises is there such an attempt, and of none of whom is there a report of such an attempt. Positively, the fact that, seven and a half centuries after Euclid, three-odd after Ptolemy, in his commentary on book I of the *Elements*, Proclus writes in some detail on the earlier development of mathematics, in general, and on the earlier investigation of parallels, in particular, and Proclus reports of such attempts by no philosophers other than Posidonius and Ptolemy.

Euclid's definition of parallel lines is:

parallel are straight lines in the same plane which, produced indefinitely in both directions, do not intersect in either direction.

Posidonius, Proclus reports, proposed the definition:

parallel are straight lines in the same plane which neither converge nor diverge, but all perpendiculars from points on either of them to the other are equal (comm. *Elements*, 176).

Euclid's postulate is:

if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side on which are the interior angles less than two right angles.

Ptolemy, Proclus reports, wrote a treatise entitled *That, when a straight line intersects two straight lines, and the interior angles are less than two right angles, the two straight lines intersect* (365). And this is how Ptolemy proposed to prove what Euclid's postulate affirms:

first we prove that, if a straight line intersects two parallel straight lines, the interior angles on the same side are equal to two right angles;

let AB and CD be two parallel straight lines, and EF intersect them;

then $\alpha + \beta = 2$ right angles;

for one of the three, $\alpha + \beta > 2$ right angles,

or $\alpha + \beta = 2$ right angles, or $\alpha + \beta < 2$ right angles; and, if possible, let $\alpha + \beta > 2$ right angles;

then $\alpha_1 + \beta_1 > 2$ right angles as well, for AE and CF are not more parallel than EB and FD , and if the straight line intersecting AE and CF makes the interior angles greater than two right angles, the same straight line intersecting EB and FD makes the interior angles greater than two right angles;

but $(\alpha + \alpha_1) + (\beta + \beta_1) = 4$ right angles, and if $\alpha + \beta > 2$ right angles, then $\alpha_1 + \beta_1 < 2$ right angles;

$\alpha_1 + \beta_1 > 2$ right angles, and $\alpha_1 + \beta_1 < 2$ right angles;

and this is impossible;

in the same way, that $\alpha + \beta < 2$ right angles is impossible;

therefore, $\alpha + \beta = 2$ right angles;

and now we can prove that, if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side on which are the interior angles less than two right angles;

for, if the two straight lines do not intersect on the side on which the interior angles are less than two right angles, the less can they intersect on the side on which the interior angles are greater than two right angles;

the two straight lines thus do not intersect in either direction, and they are thus parallel;

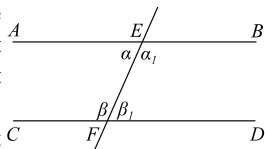
but it has just been proved that the interior angles of parallel lines are equal to two right angles;

the same two angles are equal to two right angles and less than two right angles;

and this is impossible (365-8).

Ptolemy's demonstration is a *petitio principii*.

The difficulty is in the assumption that, if a straight line intersects two parallel straight lines, the two straight lines are not more parallel in one direction than in the other, that as the interior angles on one side are greater than, equal to, or less than two right angles, so are, respectively, the interior angles on the other side. As Proclus notes: there is no reason why the interior angles on one side should not be greater than two right angles and the interior angles on the other side less than two right angles (368).



Proposition 31 in book I of the *Elements* is:
through a given point to draw a straight line
parallel to a given straight line;

let the given point be E
 A , and the given straight
line BC ;

take any point D on BC ;

join AD ;

on AD draw $\angle EAD = \angle ADC$;

and as the alternate angles are equal, the
straight line EA through A is parallel to BC (*id.*).

That, if a straight line intersecting two straight
lines makes the alternate angles equal, the two
straight lines are parallel, proposition 27 shows,
proposition 27 depends on the definition of
parallel lines, parallel are straight lines which,
produced indefinitely, do not intersect, and the
definition of parallel lines depends on the postu-
late, if a straight line intersecting two straight
lines makes the interior angles on the same side
less than two right angles, the two straight lines,
produced indefinitely, intersect on that side.

Proclus writes: proposition 31 shows that, as
there cannot be from one point two perpendicu-
lars to the same straight line, so there cannot be
through one point two straight lines parallel to the
same straight line; the straight line parallel to a
straight line through a point is unique (376).

Ptolemy's assumption that, if a straight line
intersects two parallel straight lines, as the inter-
ior angles on one side are greater than, equal to,
or less than two right angles, so are, respectively,
the interior angles on the other side, is equivalent
to the assumption that through one point there is
only one straight line parallel to a given straight
line. Which is equivalent to the assumption that,
if a straight line intersecting two straight lines
makes the interior angles on the same side less
than two right angles, the two straight lines,
produced indefinitely, intersect on that side.

Proclus comments on proposition 31 also: and
whatever distance separates the given point from
the given straight line is the distance between the
parallel through the point and the given straight
line (377).

Posidonius's definition is equivalent to Eu-
clid's postulate as well. The assumption that
parallel lines are equidistant is equivalent to the

assumption that through one point there is only
one straight line parallel to a given straight line,
which is equivalent to the assumption that, if a
straight line intersecting two straight lines makes
the interior angles on the same side less than two
right angles, the two straight lines, produced
indefinitely, intersect on that side.

Proclus writes on the postulate: this ought to
be struck from the postulates altogether, for it is
a theorem (191).

If it is a theorem, it ought to be struck from the
postulates altogether. Hence, after Ptolemy's, the
attempts to prove what it affirms, and strike it
from the postulates altogether, will continue,
beginning with Proclus himself, through and
including Saccheri. Nor will any of these attempts
succeed better than Ptolemy's.

It ought to be struck from the postulates alto-
gether, for it is a theorem. And Proclus has two
more particular reasons as well. First: Euclid
himself proves its converse as a theorem (192).
The converse is proposition 17 in book I, the sum
of any two angles of the triangle is less than two
right angles. Second: we should understand that
the absence of intersection does not make lines
parallel; concentric circles, for instance, do not
intersect; also, as Geminus says, there are lines in
the same plane, which do not enclose a figure,
and nevertheless, when produced indefinitely,
approach a straight line more and more, and yet
do not intersect it, like the hyperbola and the
conchoid; indeed, this is one of the most paradox-
ical theorems in geometry (176-7); that there are
lines which, when produced indefinitely, ap-
proach one another more and more, yet never
intersect, is implausible and paradoxical; it is
nevertheless true; may it not be, then, that straight
lines as well, when produced indefinitely, ap-
proach one another more and more, yet never
intersect? it is for these reasons that we should
seek a proof of this theorem (192).

It will turn out, in Saccheri, that it may be that
straight lines as well, when produced indefinitely,
approach one another more and more, yet never
intersect.

It is for these reasons that we should seek a
proof of this theorem.

The first is not a very good reason. That a
proposition is demonstrable does not entail that

its converse is demonstrable, and conversely.

But the second is a very good reason. At issue is what lines do when produced indefinitely, and that is, what they do at infinity. And what they do at infinity may be implausible and paradoxical, indeed, it may be the most paradoxical, at issue, however, is whether it is true. For this reason we should seek a proof of this theorem.

For it is this theorem – if it is a theorem – it is Euclid's postulate that decides what lines do at infinity.

Of all the assumptions on which the *Elements* rests some are more fundamental than others. And of the most fundamental, the postulate is the most fundamental specifically spatial assumption. The postulate decides what straight lines do at infinity, hence more generally what lines do at infinity, hence more generally what the properties of space at infinity are, and hence more generally what the properties of space are. The postulate is the most fundamental, and important, assumption on whose ground the *Elements*, and all that rests on the *Elements*, is the theory of space. Hence also it is the assumption on whose ground with the *Elements* mathematics became geometry. Hence also it is the assumption on whose ground, when mathematics became geometry, and that is, more physical, physics became mathematical, and that is, geometrical. On the postulate thus also depends the most fundamental, and important, property of the triangle, which makes it the standard of the measurement of space, that the sum of its angles is equal to two right angles.

Ptolemy's attempt to prove what the postulate affirms, to prove, rather than assume, what lines do at infinity, points at, both, its most fundamental, and important, character and its generality.

The failure of Ptolemy's attempt to prove what the postulate affirms, and the more so the failure of all subsequent attempts to prove it, points at Euclid's greatest perspicacity in, both, establishing its most fundamental, and important, character and recognizing that it is – as it will be eventually demonstrated, too – indemonstrable.

Posidonius, then, redefined parallel lines. Parallel are equidistant straight lines. The equidistance of parallel straight lines is equivalent to what Euclid's postulate affirms.

Posidonius is a very important philosopher. He

is also an unusual philosopher, and very important, because unusual. Posidonius did both history and physics. He wrote *Histories* in 52 books, continuing Polybius, of which fragments survive in Siculus and Plutarch. Posidonius was a Stoic at a time when the Stoa, like all schools, had long abandoned physics to devote itself to ethics. Posidonius wrote on the countries from the Atlantic to India, on their climate, flora, and fauna, investigated the ocean, earthquakes, volcanoes, and much of this work is preserved in Strabo's *Geography*, in epitomes by Geminus and Cleomedes, and in Laertius's exposition of the physics of the Stoa. Posidonius's solution of the problem of the tides and the dependence of their cycle on the motions of the moon (Strabo, III, 5.8) was not improved upon until Galileo. Posidonius's result for the distance of the sun at 13 100 earth radii, obtained as it is on unascertainable grounds, is better than the earlier result of Hipparchus and the later results of Ptolemy, Copernicus, Tycho, Galileo, and Kepler still. After two-odd centuries of mathematicians *par excellence*, too, counting back to Euclid, Posidonius first proposed to improve on the definition of parallel lines. And more explicit, quantitative, his definition is, like Archimedes's definition of the straight line, better than Euclid's. It can be concluded that the generality of his concerns led Posidonius to the attempt to improve on Euclid's definition.

Ptolemy first tried to prove what Euclid's postulate affirms. On the assumption that through one point there is only one straight line parallel to a given straight line. The assumption is equivalent to what Euclid's postulate affirms.

Ptolemy's attempt is the first of some dozens over more than one and a half millennia, and thus the beginning of what became a rather substantial part of mathematics.

Ptolemy also, among his very many very important results, measured the world. Or what is the same, measured space. It can be concluded that the two results led to one another, the measurement of the world and the attempt to prove, rather than assume, what lines do at the end of the world.

Counting only the more mathematical results, those of 10 philosophers – Archimedes, Chrysip-

pus, Diocles, Nicomedes, Apollonius, Hipparchus, Posidonius, Hero, Menelaus, and Ptolemy – have been described, attesting the development of mathematics in the not quite four and a half centuries from Euclid to Ptolemy. All 10 are important, the majority are very important, and the majority are steps of advance in the acquisition of knowledge.

But not all with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

These 2 are all important philosophers who are part of the development of this problem,

135 - 51^{BC} **Posidonius**,
87^{AD} **Ptolemy**.

And the steps of advance in the acquisition of knowledge are both. In particular,

Posidonius	improved on the definition of parallels,
Ptolemy	parallels are equidistant, first attempt to prove what Euclid's postulate affirms, there is at a point one parallel to a given straight line.

cccxxiv

The not quite sixteen centuries from Ptolemy to Saccheri.

They can be divided into four parts. The three-odd centuries from Ptolemy to Proclus. The not quite four centuries from Proclus to al-Khwarizmi. The seven centuries from al-Khwarizmi to Cardano. And the one and a half-odd centuries from Cardano to Saccheri. The dates are,

87	Ptolemy,
411 - 485	Proclus,
795 - 852	al-Khwarizmi,
1501 - 1576	Cardano,
1667 - 1733	Saccheri.

In the three-odd centuries from Ptolemy to Proclus the advance of physics and mathematics decelerated to a halt. Christianity rose. In the not quite four centuries from Proclus to al-Khwarizmi physics and mathematics did not just not advance, physics and mathematics were abandoned altogether. Christianity continued to rise, then Islam also rose. It was in the generation before al-Khwarizmi that the philosophers, the Muslim philosophers, in particular, began, cautiously,

concerning themselves with physics and mathematics again. In the seven centuries from al-Khwarizmi to Cardano physics and mathematics began, haltingly, slowly, advancing once again. As the Muslim dark age descended, the Christian dark age began coming to a close. Eventually, in the century before Cardano, physics and mathematics began advancing steadily again. And in the one and a half centuries from Cardano to Saccheri the advance of physics and mathematics accelerated very greatly, and more, and more.

In the three-odd centuries from Ptolemy to Proclus, the development of physics stopped. One generation after Ptolemy there is Galen. In the last of these centuries there are the commentaries of Pappus, Theon of Alexandria, and Hypatia. And there is nothing else.

In mathematics, before its development stopped as well, there are the two very important mathematicians, Diophantus and Pappus.

One and a half centuries after Ptolemy, Diophantus wrote the *Arithmetic*. It is the first treatise in which indeterminate equations are solved. For instance:

to divide a given square into two squares;

let the given square be 16;

let one of the required squares be x^2 ;

the other square is then $16 - x^2$;

take a square like this $(Ax - 4)^2$, where A is any integer and 4 is the square root of 16, and make it equal to $16 - x^2$;

take $(2x - 4)^2$, for instance;

then, $4x^2 - 16x + 16 = 16 - x^2$;

$5x^2 = 16x$,

$x = \frac{16}{5}$;

and the required squares are $\frac{256}{25}$ and $\frac{144}{25}$ (II,

8).

And $(Ax - \sqrt{B})^2 = B - x^2$, where A is any integer and B is any square, is a general solution of the problem, to divide a square into two squares.

It is in the margin of his copy of the *Arithmetic* next to this proposition that Fermat formulated the theorem, $x^n + y^n = z^n$ has no integer solutions for $n > 2$. Fermat writes: it is impossible to divide a cube into two cubes, or a biquadrate into two biquadrates, or in general any power greater than a square into two powers of the same degree; I

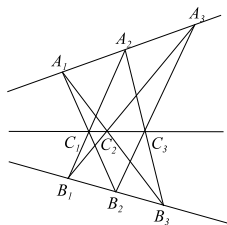
have discovered a truly marvelous proof of this, but this margin is too narrow to contain it (*id.*).

Diophantus greatly advanced algebraic notation. He writes x for the unknown, writes powers, x^2 , x^3 , x^4 , x^5 , and all the way to x^6 , writes equations explicitly, $4x^2 - 16x + 16 = 16 - x^2$. His solution of cubic equations, the first explicit solution of cubic equations, has been presented above.

Two generations later, Pappus wrote the *Collection*, in 8 books. It is, very unusually, a very important historical treatise and a very important mathematical treatise, both. On the first count, the *Collection* is the systematic collection of the results of Pappus's predecessors in mathematics. On the second count, the most important is book VII, in which alone Pappus proves 238 propositions. Among his results, he begins with the analysis of problems into analysis and synthesis. He formulates the locus problem, whose solution is the foundation of Descartes's development of analysis in the *Geometry*. He ends with the proof that the conics are defined by the relation of a line and a point, the directrix and the focus. These have been presented above. Here, another of Pappus's most important, and elegant, proofs can be presented.

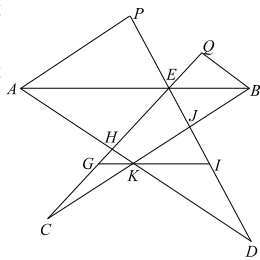
The theorem is:

if three points A_1 , A_2 , and A_3 on a given straight line and three points B_1 , B_2 , and B_3 on another given straight line are joined, A_1 to B_2 and B_3 , A_2 to B_1 and B_3 , and A_3 to B_1 and B_2 , then the three points of intersection C_1 , C_2 , and C_3 lie on a straight line.



And this is how Pappus proves it:

if the two straight lines CE and DE , intersecting one another at E , intersect the three straight lines CJ , DH , and GI , all three intersecting at K , then $\frac{CE \times GH}{CG \times EH} = \frac{DI \times EJ}{DE \times IJ}$;



at E draw AB parallel to GI , intersecting CJ

produced at B and DH produced at A ; and at A draw AP parallel to CJ , intersecting DE produced at P ;

from similar triangles $\frac{DI}{IK} = \frac{DE}{AE}$;

also from similar triangles $\frac{IK}{IJ} = \frac{AE}{EP}$;

hence $\frac{DI}{IJ} = \frac{DE}{EP}$,

and $DE \times IJ = DI \times EP$;

thus $\frac{DE \times IJ}{DI \times EJ} = \frac{DI \times EP}{DI \times EJ} = \frac{EP}{EJ}$;

again from similar triangles $\frac{EP}{EJ} = \frac{AE}{BE}$;

in the same way $\frac{BE}{AE} = \frac{CE \times GH}{CG \times EH}$ (196).

That is, $\frac{BE}{GK} = \frac{CE}{CG}$, also $\frac{GK}{AE} = \frac{GH}{EH}$, thus $GK = \frac{AE \times GH}{EH}$, and $\frac{BE}{AE} = \frac{CE \times GH}{CG \times EH}$. Pappus

continues:

but $\frac{BE}{AE} = \frac{DI \times EJ}{DE \times IJ}$;

and therefore $\frac{CE \times GH}{CG \times EH} = \frac{DI \times EJ}{DE \times IJ}$ (*id.*).

next, let the two straight lines CE and DE , intersecting one another at E , intersect the two straight lines CJ and DH , these two intersecting one another at K ;

and let CE and DE be so divided that $\frac{CE \times GH}{CG \times EH} = \frac{DI \times EJ}{DE \times IJ}$;

then the three points of intersection G , K , and I lie on a straight line;

join GK ; join also KI ;

at E draw AB parallel to GK , intersecting CJ produced at B and DH produced at A ;

at A draw AP parallel to CJ , intersecting DE produced at P ;

at B draw BQ parallel to DH , intersecting CE produced at Q ;

from similar triangles $\frac{CE}{EQ} = \frac{CG}{GH}$,

and $CE \times GH = CG \times EQ$;

thus $\frac{CE \times GH}{CG \times EH} = \frac{CG \times EQ}{CG \times EH} = \frac{EQ}{EH}$;

but $\frac{CE \times GH}{CG \times EH} = \frac{DI \times EJ}{DE \times IJ}$;

and again from similar triangles

1056 In order to formulate the problems we are to solve we call the council of our predecessors

$$\frac{EQ}{EH} = \frac{BE}{AE} = \frac{EJ}{EP} = \frac{EJ \times DI}{EP \times DI},$$

$$\text{hence } \frac{EJ \times DI}{DE \times IJ} = \frac{EJ \times DI}{EP \times DI},$$

$$\text{and } DE \times IJ = EP \times DI;$$

$$\text{so } \frac{EP}{DE} = \frac{IJ}{DI},$$

$$\frac{EP + DE}{DE} = \frac{IJ + DI}{DI},$$

$$\frac{DP}{DE} = \frac{DJ}{DI}, \text{ and } \frac{DP}{DJ} = \frac{DE}{DI};$$

$$\text{but again from similar triangles } \frac{AD}{DK} = \frac{DP}{DJ},$$

$$\text{and } \frac{DE}{DI} = \frac{AD}{DK},$$

therefore KI is parallel to AB ;

but GK is also parallel to AB ;

and therefore the three points of intersection G , K , and I lie on the same straight line (204);

next, given the triangle ADF , draw AE parallel to DF , and let EC intersect DF produced at C ;

$$\text{then } \frac{DF}{CF} = \frac{CE \times GH}{CG \times EH};$$

at D draw DR parallel to EC , intersecting AF produced at R ;

$$\text{from similar triangles } \frac{DR}{GH} = \frac{AD}{AH};$$

$$\text{also from similar triangles } \frac{AD}{AH} = \frac{CE}{EH};$$

$$\text{thus } \frac{DR}{GH} = \frac{CE}{EH},$$

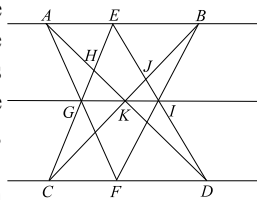
$$\text{and } CE \times GH = DR \times EH;$$

$$\frac{CE \times GH}{CG \times EH} = \frac{DR \times EH}{CG \times EH} = \frac{DR}{CG};$$

$$\text{again from similar triangles } \frac{DR}{CG} = \frac{DF}{CF};$$

$$\text{and } \frac{DF}{CF} = \frac{CE \times GH}{CG \times EH} \text{ (205).}$$

next, with these things proved, given the parallel straight lines AB and CD , and the straight lines AF , AD , BC , BF , EC , and ED intersecting them, then



the points G , K , and I lie on a straight line;

for there is the triangle AFD , AE is drawn parallel to FD , and EC intersects DF at C ;

$$\text{hence } \frac{DF}{CF} = \frac{CE \times GH}{CG \times EH};$$

again, there is the triangle BCF , BE is drawn parallel to CF , and ED intersects CF at D ;

$$\text{hence } \frac{CF}{DF} = \frac{DE \times IJ}{DI \times EJ};$$

$$\text{hence } \frac{CE \times GH}{CG \times EH} = \frac{DI \times EJ}{DE \times IJ};$$

therefore the points G , K , and I lie on a straight line (206);

and finally, let the lines AB and CD

not be parallel, but

intersect at N ;

again, the points G , K , and I lie on a straight line;

for there are the two straight lines CE and CD through the same point C , intersecting the three straight lines AN , AF , and AD ;

$$\text{hence } \frac{CE \times GH}{CG \times EH} = \frac{CN \times DF}{DN \times CF};$$

again, there are the two straight lines DE and DN through the same point D , intersecting the three straight lines BN , BC , and BF ;

$$\text{hence } \frac{CN \times DF}{DN \times CF} = \frac{DI \times EJ}{DE \times IJ};$$

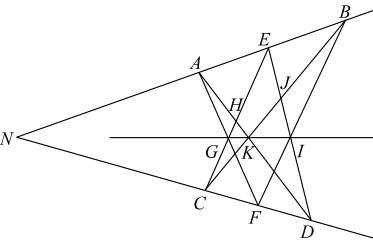
$$\text{hence } \frac{CE \times GH}{CG \times EH} = \frac{DI \times EJ}{DE \times IJ};$$

this case has been reduced to the previous;

and therefore, the points G , K , and I lie on a straight line (207).

This is Pappus's theorem.

Indeterminate equations do not depend on parallel lines. More generally, Diophantus developed arithmetic and laid the foundations of algebra substantially independently of geometry. Pappus's proof of his theorem depends on parallel lines throughout. With the development of projective geometry, after Desargues and Pascal, thirteen centuries later, it will be shown that Pappus's theorem is a fundamental result in projective geometry, and two centuries later still, after



Riemann, it will be shown, by Klein, that projective geometry can be developed independently of Euclid's postulate. Projective geometry is the investigation of invariance under projection from one surface to another.

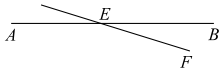
As to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, its development continues with Proclus. In the commentary on book I of the *Elements*, after showing that Ptolemy's proof of what Euclid's postulate affirms is a *petitio principii*, Proclus proposes to prove it himself. He writes:

first, we take as an axiom what Aristotle employs to show that the world is finite; namely, that, if from one point two straight lines making an angle are produced indefinitely, the distance between them becomes greater than any given magnitude (371).

This is in *On the heaven* (I, 5). It is equivalent to the axiom of measure. Proclus continues:

assuming this, we can prove that, if a straight line intersects one of two parallel lines, it intersects the other;

let AB and CD be two parallel lines and EF intersect AB ;

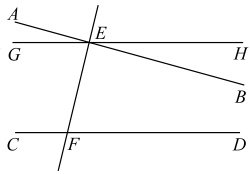


then, as AB and EF are two straight lines at one point making an angle, when they are produced indefinitely, the distance between them becomes greater than any given magnitude;

it thus becomes greater than the distance between the two parallel lines AB and CD , and EF thus intersects CD ;

and having proved this, we can prove what the postulate affirms;

let AB and CD be two straight lines and EF intersect them, making the interior angles on the same side less than two right angles;



draw at E the straight line GH parallel to CD ; and since AB intersects one of the two parallel lines GH , when produced indefinitely, it intersects the other CD ;

therefore, AB and CD intersect on that side on which are the angles less than two right angles

(371-3).

Proclus's demonstration is a *petitio principii* as well. The difficulty is in the assumption that the distance between two parallel lines is not greater than any given distance. The assumption that the distance between parallels is more generally finite is equivalent to the assumption that they are more particularly equidistant, which is equivalent to what the postulate affirms.

In the not quite four centuries from Proclus to al-Khwarizmi physics was abandoned.

So was mathematics.

There were philosophers, not many, fond of reckoning. To these philosophers, when there were such philosophers, Christianity posed the problem of calculating easter. Islam posed the problems of calculating the times of prayer, daybreak, midday, afternoon, sunset, and night-fall, and of orienting mosques to face the kaaba in Mecca.

Easter had to be celebrated on the first Sunday after the first full moon after spring equinox.

Bede solved this problem, in *On the reckoning of time*. He writes: when the lunar and the solar cycles are multiplied, the great paschal cycle of 532 years is completed; for whether you reckon 19×28 or 28×19 , it gives 532 (65); and $532 \times 365 \frac{1}{4}$ gives the days in the cycle, so that after 194 313 days the cycle repeats (*id.*).

The lunar and the solar cycles are multiplied. The lunar is the cycle of 19 solar years, after which the full moon will occur on the same day of the year. Meton discovered this cycle, seven and a half centuries before Pappus, and the Metonic cycle informs all subsequent investigation of the heaven through and including Pappus. Bede reports: Eusebius contrived the 19 year cycle (44). This, in turn, that Eusebius contrived the 19 year cycle, as Bede also reports, Jerome contrived, in *On illustrious men* (61). The solar is the cycle of 28 years, after which the Sunday will occur on the same day of the year. The day of the week advances 1 day in a regular year, 2 days in a leap year, thus 5 days in 4 years, and thus the cycle of 5×7 days of the week is complete in $4 \times 7 = 28$ years.

Bede writes four centuries after Pappus. More important, counting back to Pappus's contempo-

raries Eusebius and Athanasius, Bede is the successor of four centuries of triumphant Christianity. And that it took the Christian philosophers that long to find the great paschal cycle indicates well how thoroughly they had abandoned physics and mathematics. It indicates well also how many of them were fond of reckoning, as well as the level of mathematical sophistication of those that were. Bede begins in *On the reckoning of time*: before turning to the reckoning of time, we have decided to teach, with god's help, a few things about that very useful and easy skill of reckoning with fingers (1).

In the seven centuries from al-Khwarizmi to Cardano physics began advancing once again. Very slowly, and at long last. In the generation before al-Khwarizmi, al-Farghani wrote an elementary introduction to the *Almagest*. In the following two and a half centuries the treatises in preparation for the study of the *Almagest* multiplied, acquired mathematical sophistication, new observations were made, new tables of the motions of the heavenly bodies were prepared. Optical treatises were written. These labors of the Muslim philosophers, however, were neither very frequent nor very vigorous. Nor did any of them break new ground. Then the Christian dark age came to an end. Gerard translated 71 treatises. In the following three centuries the Christian philosophers did as the Muslim philosophers had done, wrote treatises in preparation for the study of the *Almagest*, made new observations, prepared new tables of the motions of the heavenly bodies, wrote optical treatises. Nor were these labors of the Christian philosophers more frequent or more vigorous than those of the Muslim philosophers had been. Nor did any of them break new ground. Throughout that time the philosophers concerned themselves with animals and plants less frequently or vigorously than they did with the heavenly bodies. Eventually, at long last, in the century before Cardano, physics began advancing once again. This is the century beginning with prince Henry and the rounding of cape Bojador in 1434. Three generations later the earth had been circumnavigated. Peurbach began, and in 1462 Regiomontanus completed, the *Epitome of the Almagest*, the first in the long series of introductions, epitomes, and commentaries of greater

mathematical sophistication than Pappus's. And the next generation, the one before Cardano's, is that of Copernicus. Copernicus wrote the *Little treatise* in 1514.

Most of this development as well has been presented in some detail above, the rest will be below.

The following are the more important mathematicians in these seven centuries, and some of their more important, and elegant, more mathematical results.

Five centuries after Pappus, Al-Khwarizmi wrote the *Algebra*. He solves quadratic equations. For instance:

the sum of a square and 10 of its roots is 39; what is the root (5)?

The solution is by completing the square:

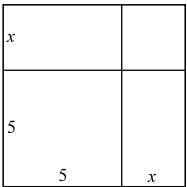
$$x^2 + 10x = 39;$$

$$x^2 + 10x + 25 = 39 + 25;$$

$$(x + 5)^2 = 64;$$

and the root is 3 (*id.*).

Al-Khwarizmi demonstrates the solutions geometrically. On the right is the demonstration of the solution above (11). Proposition 4 of book II of the *Elements*.



The equations solved in the *Algebra* are not as advanced as those solved in Diophantus's *Arithmetic*. Nor does al-Khwarizmi employ algebraic notation, he solves the equations verbally.

Two and a half centuries later, Khayyam wrote the *Demonstration of problems of algebra*. Khayyam does not employ algebraic notation either, he also solves equations verbally. He solves them geometrically, too. On the other hand, he solves them by finding the intersections of conics, anticipating Descartes with the implicit result that the equations of the conics are quadratic. Khayyam also solves cubic equations more generally than Diophantus. This has been presented above. The following is another instance:

a root; the sum of its cube, a number of its squares, and a number of itself equals a number; what is the root (*id.*)?

The equation is $x^3 + px^2 + qx = r$, where x is the root, and p , q , and r are the given numbers. Khayyam solves the equation by finding the

intersection of a circle and a hyperbola:

draw the straight

line $AB = p$;

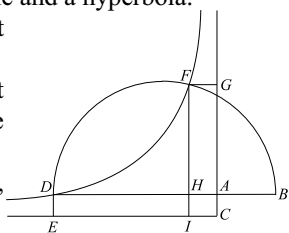
perpendicular to it

draw the straight line

$AC = \sqrt{q}$;

produce BA to D ,

so that $AD = \frac{r}{q}$;



on BD as a diameter draw a semicircle;

draw CE perpendicular to AC at C ;

through point D draw a hyperbola with CA and CE as asymptotes;

the hyperbola must intersect the semicircle at another point F ;

drop from F perpendiculars FI to CE and FG to CA produced;

as C is the center of a hyperbola and CE and CG its asymptotes, rectangle $CF =$ rectangle CD ;

thus also rectangle $AF =$ rectangle ID ;

$FH \times AH = DH \times HI$;

hence $\frac{FH}{DH} = \frac{HI}{AH}$;

but $AC = HI$, and $\frac{FH^2}{DH^2} = \frac{AC^2}{AH^2}$;

as DB is the diameter of a circle, $FH^2 = DH \times BH$;

so $\frac{FH^2}{DH^2} = \frac{BH}{DH}$;

and thus $\frac{AC^2}{AH^2} = \frac{BH}{DH}$;

or $AC^2 \times DH = AH^2 \times BH$;

but $BH = AH + AB$,

and $AH^2 \times BH = AH^3 + AH^2 \times AB$;

hence $AH^3 + AB \times AH^2 = AC^2 \times DH$;

adding $AC^2 \times AH$ to both sides,

$AH^3 + AB \times AH^2 + AC^2 \times AH = AC^2 \times AD$;

and the root is AH (*id.*).

Writing $x^3 + px^2 + qx = r$, $x = AH$, and the given numbers are, as constructed, $p = AB$, $q = AC^2$, $r = AC^2 \times AD$.

One century-odd later, Fibonacci wrote the *Book of calculation* and the *Book of squares*.

In the *Book of calculation*, he begins:

these are the nine figures of the Indians,

9, 8, 7, 6, 5, 4, 3, 2, 1;

and with these nine figures and the sign 0, which the Arabs call zephyr, any number can be written

(1).

In about one third of the *Book*, Fibonacci gives, and proves, rules for the addition, multiplication, subtraction, and division of integers and fractions (1-7), and for the extraction of square and cube roots and their addition, multiplication, subtraction, and division (14). In the rest of the *Book* (8-13, 15), about two thirds of it, he formulates and solves some hundreds of problems. This is one:

four men find a purse; with the purse the first has double what the second and the third have, with the purse the second has triple what the third and the fourth have, with the purse the third has quadruple what the fourth and the first have, with the purse the fourth has quintuple what the first and the second have; and this problem cannot be solved unless the first man has a debt (13).

Implying negative numbers. If a , b , c , and d are what each man has, and p is the purse,

$a + p = 2(b + c)$,

$b + p = 3(c + d)$,

$c + p = 4(d + a)$,

$d + p = 5(a + b)$,

and this problem cannot be solved unless a is negative. The solution is:

in least numbers, the second man has 4, the third man has 1, the fourth man has 4, the first man has a debt of 1, and the purse is 11 (*id.*).

Another problem is this:

a man has enclosed a pair of rabbits; we want to know how many pairs can be bred in one year, if the nature of these rabbits is such that they begin to breed in the second month after birth and each pair breeds one pair every month (12).

The solution is the Fibonacci sequence:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89,

144, 233, 377 (*id.*).

Adding 0, 1, $F_{n+1} = F_n + F_{n-1}$. Fibonacci adds:

and you can continue in this way for an infinite number of months (*id.*).

Given, clearly, the appropriate adjustment in the enclosure of the rabbits.

In the *Book of squares*, Fibonacci begins:

I thought about the origin of all square numbers, and discovered that they arise from the addition of the odd numbers; for

$1 = 1^2$,

$1 + 3 = 2^2$,

$1 + 3 + 5 = 3^2$,

1060 In order to formulate the problems we are to solve we call the council of our predecessors

and so the sequence of all squares arises from the addition of the sequence of the odd numbers (1).

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

Fibonacci continues:

I discovered also that every square exceeds the square preceding it by the sum of their roots (*id.*).

$$n^2 - (n - 1)^2 = n + (n - 1)$$

And further:

it can also be shown that every square exceeds every other square by the product of the sum and difference of their roots (*id.*).

$$n^2 - m^2 = (n + m)(n - m)$$

One and a half centuries later, Oresme, next to the more physical *Treatise on the configuration of qualities and motions* and *Book on the heaven and the world*, wrote the more mathematical *Questions on the geometry of Euclid*. With the *Questions* the investigation of infinite series begins. Oresme shows that:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots = 2 \quad (2),$$

and:

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{3^n} + \dots = \frac{3}{2} \quad (id.),$$

but:

the sum

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} + \dots$$

is infinite;

and this is proved in this way,

$$\frac{1}{3} + \frac{1}{4} > \frac{1}{2},$$

$$\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} > \frac{1}{2},$$

and so on to infinity (*id.*).

Khayyam, Fibonacci, and Oresme are the greatest advances of mathematics in the six and a half centuries from al-Khwarizmi to Regiomontanus, and more generally, in the eleven and a half centuries from Pappus to Regiomontanus.

One century-odd after Oresme, Regiomontanus, next to the more physical *Epitome of the Almagest*, with Peurbach, and *Ephemerides*, wrote the more mathematical *On triangles of all kinds*.

In *On triangles of all kinds* Regiomontanus defines the sine:

in every right triangle, if one of the acute vertices is the center of a circle and the hypothe-

nuse is its radius, the side subtending the acute angle is the sine of the arc; also, the other side is equal to the sine of the complementary arc;

let the right triangle be ABC , with the right angle at C , the center of the circle at A , and AB its radius;

BC is the sine of arc BD ;

also, the complementary arc is BE , and as $\angle ABC = \angle BAE$,

AC is equal to the sine of arc BE (I, 20).

Or what is the same, in the circle with radius AB , $\sin \angle BAC = BC$, and also, $\sin \angle ABC = AC$. Hence also, $\sin \angle ABC = \cos \angle BAC$.

The law of sines follows:

in any triangle, let A , B , and C be the angles, and a , b , and c the sides opposite the angles;

the height h divides the triangle into two right triangles;

hence, in the circle with center at A and radius

$$b, \sin A = \frac{h}{b}, \text{ and } b \sin A = h;$$

in the circle with center at B and radius a ,

$$\sin B = \frac{h}{a}, \text{ and } a \sin B = h;$$

$$\text{hence } \frac{a}{\sin A} = \frac{b}{\sin B};$$

in the same way the third side and angle;

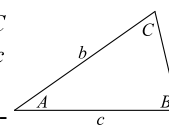
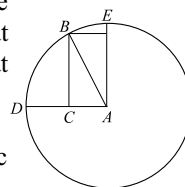
$$\text{and } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (\text{II, 1}).$$

And from Menelaus's theorem and the law of sines for plane triangles Regiomontanus obtains also the law of sines for spherical triangles:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C},$$

where a , b , and c are the sides of the spherical triangle, and A , B , and C are the angles opposite the sides (V, 1-2).

Two generations later, Cardano wrote *The great art*. *The great art* is the decisive step in the generalization of number to include explicitly 0 and negative numbers, in the extension of the operations of addition, multiplication, and their converses to all real numbers, and in the extension of exponentiation and its converse beyond the third power. Cardano writes: to go beyond the third power would be foolish; nature does not allow that; thus, in what follows all cases up to



the third power are demonstrated fully; but the others are included as well, either by necessity or out of curiosity (1). Either by necessity or out of curiosity, and, clearly, foolishly: fourth power, fifth power, sixth power, seventh power, and so on (*id.*). Cardano also discovered the complex numbers. He writes:

if it is said, divide 10 into two parts whose product is 40, it is clear that this is impossible;

we will nevertheless do it;

divide 10 into two equal parts of 5 each;

square these, obtaining 25;

subtract 40 from 25;

the remainder is -15 ;

and the square root of this, added to 5 and subtracted from it, gives the parts whose product is 40, namely, $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$ (37).

Having done the impossible, Cardano adds: thus advances mathematical subtlety, its end as elevated as it is useless (*id.*), then proceeds to show that the impossible is indeed routine:

the rule is the same, if it is said, divide 6 into two parts whose product is -40 ;

or if it is said, divide -6 into two parts whose product is -40 ;

or if it is said, divide -6 into two parts whose product is 24, in which case the parts are $-3 + \sqrt{-15}$ and $-3 - \sqrt{-15}$ (*id.*).

Cardano solved also all forms of the cubic equation, and formulated a general method for its solution. He wrote also the *Book on games of chance*, with which the investigation of probability begins.

These are not all mathematicians among the philosophers in the seven centuries from al-Khwarizmi to Cardano. There are more. But neither many more, nor nearly as important. There are many philosophers in these seven centuries, first many Muslim philosophers and but a very few Christian philosophers, then but a few Muslim philosophers and steadily more and more Christian philosophers, as the universities begin, spread, and flourish. There are many philosophers, but not many mathematicians among them at all. In particular, through the generation before that of Cardano, next to al-Khwarizmi, Khayyam, Fibonacci, Oresme, and Regiomontanus, in the two and a half centuries between al-Khwarizmi and Khayyam there are

also banu Musa, Abu Kamil, Thabit ibn Qurra, al-Battani, Abu'l-Wafa, ibn al-Haytham, and al-Karaji. In the one century-odd between Khayyam and Fibonacci there is also bar Hiyya. In the one and a half centuries between Fibonacci and Oresme there are also Nemorarius and al-Tusi. In the one century-odd between Oresme and Regiomontanus there is also Peurbach. In the generation after Regiomontanus there is also Pacioli. To show that the line between important and not important, here as always, is drawn among the particulars, al-Nayrizi, al-Jayyani, and Ferro will do. And counting centuries from the date of birth assigned to al-Khwarizmi, this is the distribution of the 17 mathematicians in the enumeration,

795- 894	5,
895- 994	3,
995-1094	2,
1095-1194	2,
1195-1294	2,
1295-1394	1,
1395-1494	3.

The year of birth of Cardano is 1501.

It can be concluded that after the four centuries when it was abandoned altogether, mathematics was rather vigorously restored in the century after al-Khwarizmi, but then the vigor rather waned, and continued to wane for the following not less than five centuries.

As to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, the attempts to prove Euclid's postulate continued.

One generation after al-Khwarizmi, Thabit ibn Qurra wrote the *Book in which Euclid's postulate is proved*. Proposition 1 is:

if a straight line intersects two straight lines and the alternate angles are equal, the two straight lines neither converge nor diverge in either direction (*id.*).

The proof is on the assumption that:

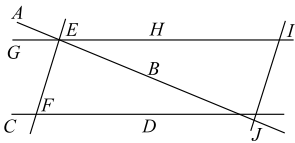
if two straight lines converge in one direction, they diverge in the other (*id.*).

This already is equivalent to Euclid's postulate. Proposition 3 is:

if the ends of two equal straight lines, which neither converge nor diverge, are joined by two straight lines, these two straight lines are also equal and neither converge nor diverge (*id.*).

This is Euclid's proposition 33 in book I. And Thabit ibn Qurra's proposition 5 is the proof of the postulate:

AB and *CD* are two straight lines, and the straight line *EF* intersects them, making the interior angles on the same side less than two right angles;



draw at E the straight line GH which neither converges nor diverges with CD ;

draw on the side on which the interior angles are less than two right angles straight lines which neither converge nor diverge with EF until the part of one of them between GH and AB produced is greater than EF ; let this part be IJ ;

and since the straight lines CD and GH neither converge nor diverge, and the straight lines EF and IJ neither converge nor diverge, CD intersects IJ ;

therefore, AB intersects CD (*id.*).

Other than that it is more labored, and on the assumption that parallels are not more generally finite but more particularly equidistant, the demonstration is like Proclus's.

Not quite one and a half centuries later, ibn al-Haytham wrote the *Book of commentaries on Euclid's Elements*. He begins:

Euclid's definition of parallel lines requires that they be extended indefinitely in both directions; and this cannot be conceived; for what can be conceived is always definite (*id.*).

He proposes the alternative definition:

When the perpendicular to a given straight line moves along the straight line and one of its ends traces it, its other end describes a straight line and this straight line is at the same distance from the given straight line in both directions (*id.*).

Other than that it introduces motion, the definition is like Posidonius's, parallels are equidistant. And ibn al-Haytham's demonstration of Euclid's postulate is again like Proclus's, again assuming that parallels are equidistant.

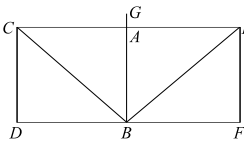
Along the way, however, ibn al-Haytham anticipates the Lambert quadrilateral, the quadrilateral constructed from a straight



line, two equal perpendiculars at its ends, and a perpendicular from one end of one of the perpendiculars to the other, in which thus only three of the four angles are assumed right.

In preparation for the demonstration of the postulate ibn al-Haytham has this proposition:

on the straight line AB , erect at its ends the equal perpendiculars AC and BD , and erect at



intersecting AC at C ;

then $CD = AB$:

for, if not, either $CD > AB$ or $CD < AB$;

let $CD > AB$:

produce CA to E as $CA = AE$;

produce DB and drop to it from E perpendicular EF ;

join BC and BE ;

$AC = AE$, AB is common, $\angle BAC = \angle BAE$, and hence $\triangle ABC = \triangle ABE$;

hence $BC = BE$ and $\angle CBD = \angle EBF$, and also $\angle BDC = \angle BFE$; hence $\triangle BCD = \triangle BEF$;

thus $CD = EF$;

and $EF > AB$;

then, when the perpendicular EF moves along the straight line FBD and one of its ends traces it, its other end describes a straight line which intersects BA produced at G beyond A and then passes through C ;

but CAE is also a straight line;

two straight lines enclose an area; and this is absurd;

in the same way, that $CD < AB$ is absurd;

therefore, $CD = AB$ (*id.*).

It follows that the fourth angle of the quadrilateral $DBAC$ is also right, and then the demonstration of Euclid's postulate readily follows.

Not quite one century later, Khayyam wrote the *Commentaries on the difficulties in the assumptions in Euclid's book*. Khayyam, too, assumes that parallels are equidistant, and his demonstration of Euclid's postulate is once again like Proclus's, on that assumption.

Along the way, however, Khayyam anticipates the Saccheri quadrilateral, the quadrilateral constructed from a straight



line, two equal perpendiculars at its ends, and the straight line joining the other ends of the perpendiculars, in which thus only the two angles at the base are assumed right.

In preparation for the demonstration of the postulate Khayyam has this proposition:

on the straight line AB the equal perpendiculars AD and BC are erected and their ends joined;

then angles ADC and BCD are both right;

for, if not, they are either both greater than a right angle, or both less than a right angle;

let them be both less than a right angle;

bisect AB , and at the midpoint E erect the perpendicular intersecting CD at F ; produce it to G , as $EF = FG$;

at G draw HGI perpendicular to EG ;

AD and BC produced intersect HGI at H and I ;

if now the quadrilateral CH is rotated on CD and superimposed on the quadrilateral CA , HI is greater than AB , for angle BCD is less than a right angle and hence angle ICD is greater than a right angle; the straight lines AD and BC thus diverge in the direction of D and C ;

but for the same reason the same straight lines diverge if produced in the opposite direction;

and that two straight lines diverge in both directions is absurd;

in the same way, it is absurd that angles ADC and BCD are both greater than a right angle;

therefore, angles ADC and BCD are both right (3).

The demonstration of Euclid's postulate readily follows.

The assumption that angles ADC and BCD are both right, or both greater than a right angle, or both less than a right angle, is also equivalent to what Euclid's postulate affirms.

This is not one of Khayyam's more elegant demonstrations. But it does anticipate the result also that the perpendicular bisecting the base of the Saccheri quadrilateral divides it into two Lambert quadrilaterals.

And one and a half centuries later, al-Tusi wrote the *Treatise which cures the doubts raised*

by parallel lines.

Al-Tusi's attempt to eliminate the eccentric and the equant from the theory of the planets has been presented above. In the *Revealing the mysteries of the secant figure* also al-Tusi formulated the law of sines, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

As to the doubts raised by parallel lines, al-Tusi cured them also on the assumption that parallels are equidistant, hence demonstrating, following Khayyam, that all angles in the quadrilateral constructed by Khayyam are right, and hence Euclid's postulate:

erect at E perpendicular HE to AB and drop from F perpendicular FG to AB ;

as angles BEF and EFD are less than two right angles, and angles HEF and EFG are alternate and equal, angle GFH in the quadrilateral $EGFH$ is less than a right angle, and CD intersects AB in the direction of D (7).

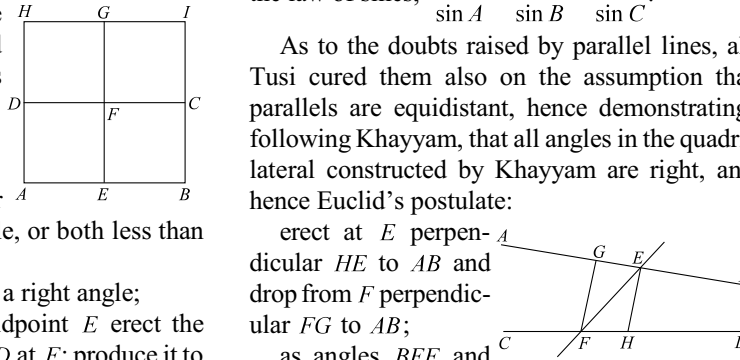
The attempts to demonstrate Euclid's postulate show that Posidonius's definition of parallels established itself in place of Euclid's, and that the demonstration of Proclus provided the paradigm for the subsequent demonstrations.

The advance is in the investigation of the quadrilateral of which it is assumed, still very provisionally, just *ad absurdum*, that not all four angles are right.

cccxv

In the one and a half-odd centuries from Cardano to Saccheri the acquisition of knowledge advanced very greatly, the advance accelerated, and more, and more, and the acquisition of knowledge advanced more than at any other time before or after.

Physics advanced very greatly. These are some five generations. The first is the generation of Vesalius, Gesner, Belon, and Cesalpino. The second is the generation of Gilbert, Tycho, and Stevin. The third is the generation of Galileo, Kepler, Harvey, and Snel. The fourth is the generation of Descartes, Guericke, Torricelli, Grimaldi, Picard, and Pascal. The fifth is the generation of Cassini, Redi, Boyle, Malpighi,



Huygens, Leeuwenhoek, Hooke, Swammerdam, Steno, Newton, Rømer, Leibniz, and Halley. Most of this development as well has been presented in some detail above, and the rest will be below.

And mathematics advanced very greatly. Mathematics is a part of physics, and at no other time has the development of the two, the part and the whole, been more explicitly inseparable, the philosophers advanced physics by advancing mathematics, and conversely. The first is the generation of Cardano. The second is the generation of Viète and Napier. The third is the generation of Galileo and Kepler. The fourth is the generation of Desargues, Descartes, Cavalieri, Fermat, Wallis, and Pascal. The fifth is the generation of Huygens, Newton, Leibniz, Jacob Bernoulli, Saccheri, and Johann Bernoulli. Most of this development, too, has been presented in some detail above, and the rest will be below. In particular, many of the results of Viète, Galileo, Kepler, Descartes, Cavalieri, Fermat, Wallis, Huygens, Newton, Leibniz, Jacob Bernoulli, and Johann Bernoulli have been presented. The following can be presented here.

This is how Viète, in the *Zetetics*, in 1593, solves the problem formulated by Diophantus, to divide a given square into two squares:

the problem is to find x and y such that

$$x^2 + y^2 = a^2;$$

let $x = q$, and $y = a - \frac{q}{r}$; the equation becomes

$$\left(\frac{2ars}{r^2 + s^2} \right)^2 + \left(\frac{a(r^2 - s^2)}{r^2 + s^2} \right)^2 = a^2;$$

let $a = r^2 + s^2$; the equation becomes

$$(2rs)^2 + (r^2 - s^2)^2 = (r^2 + s^2)^2;$$

and

$$\frac{2rs}{x} = \frac{r^2 - s^2}{y} = \frac{r^2 + s^2}{a};$$

a right triangle constructed from any numbers r and s thus solves the problem;

for instance, if $a = 100$, taking $r = 4$ and $s = 3$, $2rs = 24$, $r^2 - s^2 = 7$, $r^2 + s^2 = 25$,

$$\frac{24}{96} = \frac{7}{28} = \frac{25}{100},$$

and

$$96^2 + 28^2 = 100^2 \text{ (IV, 1).}$$

In *The analytic art*, Viète concludes: the

analytic art claims for itself the greatest problem of all, to solve all problems.

This is how Kepler, in the *Additions to Witelo*, in which the optical part of astronomy is treated, in 1604, generates the conic sections:

the conic sections are of five species, for the line on the surface of the cone defined by a section is a straight line, or a circle, or a parabola, or a hyperbola, or an ellipse;

and speaking analogically rather than geometrically, the order of the lines, determined by their properties, is this; the section passes from the straight line through an infinity of hyperbolas to the parabola, then from an infinity of ellipses to the circle; the most obtuse hyperbola is thus the straight line and the most acute is the parabola, the most acute ellipse is the parabola and the most obtuse is the circle; the parabola has on one side two things infinite in nature, the hyperbola and the straight line, on the other side two things finite, the ellipse and the circle; it is itself in the middle, of intermediate nature;

the opposite lines are thus the circle and the straight line, the purely curved and the purely straight; the hyperbola, parabola, and ellipse are in the middle partaking of both the straight and the curved, the parabola equally of both, the hyperbola more of the straight, the ellipse more of the curved; and this is why the hyperbola, when extended, becomes similar to the straight line, and that is, to its asymptotes, and why the ellipse emulates the circle (4.4).

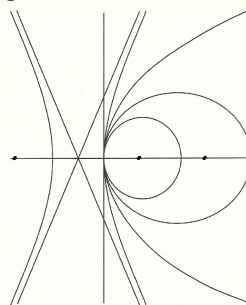
The straight line is a very obtuse hyperbola.

Kepler adds: I love analogies, they are my most faithful teachers, for they know all the deeply hidden secrets of nature (*id.*).

Kepler also established the fundamental character of the focus, and named it that, focus, hearth: for the sake of light (*id.*). In 1609 he showed that this is also where the sun is, in the hearth.

This is how Fermat, in 1640, in a letter to Frénicle, formulates Fermat's theorem:

every prime number is infallibly a factor of



one of the powers minus 1 of any progression whatsoever, and the exponent of this power is a divisor of the given prime number minus 1;

and after finding the first power that satisfies the proposition, all powers of which the exponents are multiples of the exponent of that power also satisfy the proposition;

for instance,

1	2	3	4	5	6
3	9	27	81	243	729 etc.,

with the exponents on top; take, for instance, the prime number 13; it is a factor of the third power minus 1, of which the exponent, 3, is a divisor of 12, which is 13 minus 1, and because the exponent of 729, which is 6, is a multiple of the first exponent, which is 3, it follows that 13 is a factor also of 729 minus 1; and this proposition is generally true for all progressions and all prime numbers (18 Oct).

If p is a prime, and a and p are relatively prime, $a^{p-1} - 1$ is a multiple of p . Or equivalently, if p is a prime, and a is a positive integer, $a^p - a$ is a multiple of p .

Desargues laid the foundations of projective geometry in the *Draft of a project for an attack on the results of the intersection of a cone with a plane*, in 1639. And this is how Desargues, in 1648, in an appendix to the *Universal method of Mr Desargues for the practice of perspective*, by Bosse, formulates Desargues's theorem:

if the straight lines OAa , OBb , OCc , ABa , aba , $AC\beta$, $ac\beta$, $BC\gamma$, $bc\gamma$, whether they lie in different planes or in the same plane, and whatever their order and direction, intersect in similar points, like O , α , β , γ , then the points α , β , γ lie on a straight line $a\beta\gamma$;

for in all cases, and whatever form the figure takes, the straight lines aba , $ac\beta$, $bc\gamma$ lie in one plane, the straight lines ABa , $AC\beta$, $BC\gamma$ lie in another plane, the points α , β , γ lie in each of the two planes, and therefore on the straight line $a\beta\gamma$;

and conversely, if the straight lines OAa , OBb , OCc , ABa , aba , $AC\beta$, $ac\beta$, $BC\gamma$, $bc\gamma$, whether in different planes or in the same plane, and what-

ever their order and direction, intersect in similar points, the straight lines aba and $bc\gamma$ tend toward the same goal b as OB ;

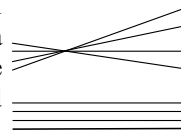
for if $aba\beta\gamma c$ is one plane, $OBbc\gamma C$ another plane, $OAabaB$ another plane, aba , $bc\gamma$, OBb are the intersections of the three planes, and therefore they all must tend toward the same goal b (*id.*).

Given two planes, and a point O in neither of the planes, if the triangles in the two planes ABC and abc are one another's projections from O , the intersections of their respective sides lie on the straight line $a\beta\gamma$.

Desargues proves the theorem by repeated applications of Menelaus's theorem.

Pascal, at 16, was introduced to Desargues's work. Pascal writes in the *Essay on the conics*: Mr Desargues of Lyons is one of the great minds of our time and one of the best in mathematics, and in particular in the conics, and his writings, although a few, give abundant proof of his knowledge (*id.*). This is how Pascal, still at 16, in the *Essay on the conics*, in 1640, formulates Pascal's theorem:

definition 1. when several straight lines intersect at a point, or are parallel to one another, all these lines are said to be of the same pencil (*id.*).

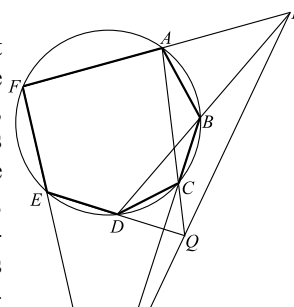


The definition is from Desargues. The assumption is that the straight line extended in either direction meets itself at a point at infinity. And as the straight lines intersecting at a point are a pencil with respect to that point, the straight lines parallel to one another are a pencil with respect to a point at infinity. Pascal continues:

definition 2. conic section is the circumference of the circle, ellipse, hyperbola, parabola, and rectilinear angle; for a cone cut through its vertex or otherwise generates these sections;

lemma 1.

if the two straight lines PA and PD are drawn from point P , the two straight lines QA and QD are drawn from point Q , the point of intersection of PA and QA is A , the point of inter-



section of PD and QD is D , and through A and D a circle is drawn, cutting the lines PD , QA , QD produced, and PA produced at points B , C , E , and F , then the lines PQ , BC , and FE are of the same pencil (*id.*).

The lines PQ , BC , and FE intersect at a point. If the lines are parallel, at a point at infinity. Pascal continues:

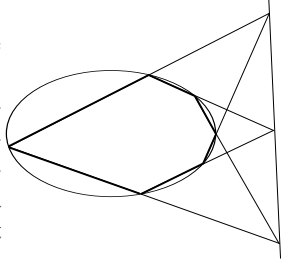
lemma 2.

if through the same line several planes pass, and are cut by another plane, all lines of intersection of these planes are of the same pencil as the line through which these planes pass;

and on the ground of these lemmas and several easy deductions from them, we can demonstrate that lemma 1. holds for all conics (*id.*).

Pascal does not go into the easy deductions.

The figure acquired the name the mystic hexagram. For, also, the intersections of the opposite sides of the hexagon inscribed in a conic lie on a straight line. If the opposite sides of the hexagon are parallel, the line is at infinity.



This is also how Pascal, in *The generation of conics*, in 1654, continuing the labors of Kepler, generates the conics:

the ellipse, if one focus is fixed, and along the straight line joining the foci the other focus moves further and further away, when that focus reaches infinity, the ellipse becomes the parabola;

when that focus reappears on the other side of the fixed focus, the parabola becomes the hyperbola;

when the two foci of the ellipse come together, the ellipse become the circle;

and when the two foci of the hyperbola move toward one another, the hyperbola become two parallel straight lines (1).

Parallel straight lines are a very obtuse hyperbola, too.

And this is how Leibniz, in *A supplement to dimensional geometry, or the most general production of all quadratures by motion*, in 1693, proves the fundamental theorem of calculus. He

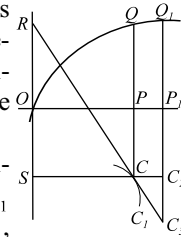
writes:

I will show that the general problem of quadrature can be reduced to finding a curve that has a given law of tangency (*id.*).

Cavalieri discovered integration in 1635. This is the general problem of quadrature. Fermat discovered differentiation in 1638. This is finding a curve that has a given law of tangency. Newton showed that integration and differentiation are one another's converses in 1671. This is the fundamental theorem of calculus. The reduction of the general problem of quadrature to finding a curve that has a given law of tangency. Leibniz continues:

let PQ_1 be the area that has to be squared, enclosed between the curve QQ_1 , the parallel lines QP and Q_1P_1 , and the axis PP_1 (*id.*).

Indices indicate the infinitesimals, as P_1 approaches P , Q_1 approaches Q , and C_1 , C_2 , and C_3 approach C . Leibniz continues:



let the fixed point on the axis be O , and perpendicular to it and parallel to PQ the conjugate axis RS be drawn;

we assume that through point C on QP , produced if necessary, the curve CC_1 passes, which has the property that, if from the point C both the ordinate CS , which is equal to PO , and the tangent CR to the conjugate axis RS are drawn, then $a \times RS = CS \times PQ$, where a is some constant and $CS \times PQ$, which is equal to $PO \times PQ$, is the area of the rectangle circumscribed about the curvilinear area $OPQO$;

and then the area $a \times C_2C_1$ is equal to the area PQ_1 (*id.*).

The property of the curve is the law of its tangency. From the property, $\frac{PQ}{a} = \frac{RS}{CS}$. From

similar triangles, $\frac{RS}{CS} = \frac{C_2C_3}{CC_2}$. Thus $\frac{PQ}{a} = \frac{C_2C_3}{CC_2}$,

and $a \times C_2C_3 = PQ \times CC_2$. And as C_2 and C_3 approach C , and P_1 and Q_1 approach P and Q , the areas $a \times C_2C_1$ and PQ_1 approach equality. Leibniz continues:

and this follows from our calculus immediately;

let $RS = t$, $PC = x$, $OP = y$, $PQ = z$;

then, by our assumption, $t = \frac{zy}{a}$;

but from the way of expressing tangents in our calculus, $t = y \frac{dx}{dy}$;

thus $a dx = z dy$;

therefore $ax = \int z dy$, and $\int z dx$ is the curvilinear area $OPQO$;

the relation of PQ to PO , which expresses the nature of the area to be squared, is given;

thus the relation of the sides of the triangle RSC is given;

and therefore, all that is needed, in order to perform the quadrature, is to describe the curve CC_1 when the relation of the sides of the characteristic triangle at C is given (*id.*).

The characteristic triangle at C is the right triangle whose hypotenuse is the inclination of the curve at C and sides are dx and dy .

In the one and a half-odd centuries from Cardano to Saccheri, then, mathematics advanced very greatly.

The most important development, by far, was calculus, and analysis. Projective geometry was developed. The theory of numbers was developed. The theory of probability was developed.

In the century after that the great advance of mathematics continued, increasingly analytically, decisively so after Euler. Geometry became but a part of mathematics. In the century after that the advance continued, increasingly analytically. New and newer parts of mathematics began being developed. Geometry began appearing but one of the all too many new parts of mathematics.

This subsequent development has been liable to obscure the fact that the very great advance in those one and a half-odd centuries – and the ground of all subsequent advance – is itself grounded substantially and essentially in geometry, and most particularly Euclid. That very great advance is a generalization of geometry, and most particularly on its Euclidean assumptions.

Like Cardano's generalization of number and of the operations of arithmetic, Viète's generalization of proportion into equation, and of numerical analysis into symbolic analysis, is a major step in the liberation of arithmetic, and algebra,

from their dependence on spatial relations. Proof in Viète is nevertheless always geometrical. The development of the theory of numbers by Fermat, and the development of the theory of probability by Fermat, Pascal, Huygens, and Jacob Bernoulli, are substantially independent of spatial relations. Nevertheless, proof, when there is proof, is again geometrical. And the development of calculus, and analysis, is geometrical through and through. From Cavalieri's *Geometry of indivisibles*, Descartes's *Geometry*, and Fermat's geometrical demonstration of differentiation, through Newton's *Method of fluxions*, which is *the geometry of curves*, and Leibniz's *Recondite geometry*. Projective geometry acquired its name after Poncelet's *Treatise on the projective properties of figures*, in 1822. In its first development by Desargues and Pascal, as in its anticipation by Pappus earlier, it is explicitly a generalization of Euclid and Apollonius. The straight line meeting itself at a point at infinity is an Euclidean straight line, as Archimedes defined it: of all lines that have the same limits the least. Its limits, more generally now, at infinity. Parallels are a very obtuse hyperbola. They are also a pencil. Either way, because, if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side. Produced, more explicitly now, to infinity.

Wallis writes: my design was not so much to show a method for demonstrating things already known as to show a way of investigation or finding out of things yet unknown.

Huygens writes: these demonstrations do not produce certainty as great as those of geometry, in which propositions are proved from certain and incontestable principles.

The design was not so much to demonstrate things already known as to find out things yet unknown. This can explain why, designing to find out things yet unknown, not so much to demonstrate things already known, these philosophers did not concern themselves with rigor nearly as much as later philosophers will. And conversely, why, not concerning themselves with rigor nearly as much, these philosophers found out so many things yet unknown.

Their demonstrations did not produce certainty as great as those of geometry. And this can explain why proof in these philosophers is always geometrical. Because in geometry propositions are proved from certain and incontestible principles.

And if so the part mathematics, the more so the whole physics. The whole physics advanced in these one and a half-odd centuries more even than its part mathematics, and advanced first and foremost geometrically. From Galileo, who writes: philosophy is written in the world in mathematical language. And in particular: the letters are triangles, circles, and other geometrical figures. Through Newton, who writes: this treatise proposes to perfect mathematics as it relates to philosophy. And in particular, this treatise is beginning to end geometrical.

The results of these philosophers are a vast generalization, and that the generalization is grounded in, and is a generalization of, geometry, and most particularly on its Euclidean assumptions, the fact shows that the philosophers did not only propose to prove their results geometrically. They succeeded. Indeed, the fact alone suffices to show it that this treatise of Newton's did not only propose to perfect mathematics as it relates to philosophy. It succeeded. Spectacularly, and geometrically.

Euclid is the theory of space.

And the great advance of mathematics and physics in these centuries posed the problem of space acutely, and explicitly.

Calculus posed the problem of the properties of space at a point, at the infinitely small. This was not new. This problem is implicit in Zeno's paradoxes already, and is central to Aristotle's discussion of space in book VI of the *Physics*. Calculus posed it for the first time explicitly.

Nor was infinity a new problem. It is the central problem of book III of the *Physics*. Projective geometry posed the problem of the properties of space at infinity, at the infinitely large, for the first time explicitly.

The world is also finite in Aristotle, and so it is in his successors in the following nineteen centuries. In the philosophers after Copernicus, and explicitly after Bruno, in the generation before Galileo and Kepler, the world is infinite.

As physics became unconditionally mathematical the problems of infinity, of the world, and of space began for the first time informing one another explicitly.

And space is space. Physical space is mathematical space, like the world, one and only one.

This is the solution of the problem of space in *The mathematical principles of natural philosophy*. Newton writes:

time, space, place, and motion are familiar to all; it must be noted, however, that these quantities are vulgarly conceived only in relation to objects of sense; and this is the source of prejudices; to eliminate them, those have to be distinguished into absolute and relative, true and apparent, mathematical and vulgar.

Relative is apparent is vulgar. Absolute is true is mathematical. In particular, absolute space is true space is mathematical space. Newton writes:

absolute space, by its nature in relation to nothing external, remains always the same and immovable.

And illustrates:

if that part of the earth, where a ship happens to be, moves absolutely eastward with a velocity of 10 010 units, and the ship is born westward by sails and wind with a velocity of 10 units, and a sailor walks on the ship toward the east with a velocity of 1 unit, then the sailor moves absolutely in motionless space toward the east with a velocity of 10 001 units, and relatively on the earth toward the west with a velocity of 9 units (I, sch. def.).

Moving absolutely in motionless space, the sailor moves in Euclidean space.

Three centuries after Newton, if the Local Supercluster moves in any direction, it does because the world, as it expands, moves it in that direction, or what is the same, if the Local Supercluster moves eastward with a velocity of 100 000 units, it moves absolutely eastward with a velocity of 100 000 units, and if the Milky Way is carried in it by gravity westward with a velocity of 110 000 units, and the solar system is carried in it by gravity eastward with a velocity of 1 000 000 units, and the sailor continues to walk as before, the sailor moves absolutely in motionless space toward the east with a velocity of 1 000 001 units.

Continuing to move absolutely in motionless space, the sailor moves in Riemannian space. That it is absolute space, with Newton's definition: by its nature in relation to nothing external, remaining always the same and immovable, the big bang, one and only one, guarantees. And the connection is as direct as the problem of space is the same, in Newton, who rests on Euclid, and in Einstein, who rests on Riemann, and the problem of the assumptions on which geometry rests is the same, and the problem in particular of Euclid's postulate, solved in the two-odd centuries between Newton and Einstein, by Saccheri, Lambert, Lobachevsky, Gauss, and Riemann.

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The great advance of physics and mathematics in the one and a half-odd centuries from Cardano to Saccheri is grounded in, and is a generalization of, geometry. If not all, nearly all resting on the triangle, the sum of its angles, and parallel lines.

And, in particular, as to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, in 1533 Proclus's commentary on book I of the *Elements* was published, in 1560 it was translated into Latin, and the attempts to prove Euclid's postulate began multiplying once again. These are neither more successful than the earlier, nor as important as the earlier, until Wallis's.

This is Wallis's, in the *Proof of Euclid's fifth postulate*, in 1663. Wallis writes:

I ground the proof in a few auxiliary propositions, as follows;

1. if a finite straight line, which lies on an infinite straight line, is extended in a straight line, the extension lies on the same infinite straight line;

2. if a finite straight line, which lies on an infinite straight line, is continued indefinitely in its own direction, it remains throughout this motion on the same infinite straight line;

3. if a finite straight line lies on an infinite straight line, and another straight line makes an angle with it, it makes the same angle with the infinite straight line;

4. if a finite straight line lies on an infinite straight line, and another straight line makes an angle with it, and it is moved in its own direction

together with the line making the angle, the angle remains throughout the same, or equal, with the infinite straight line;

5. if two straight lines intersected by a third make the interior angles on the same side less than two right angles, each of the exterior angles is greater than the opposite interior angle;

6. with these assumptions, if the straight lines AB and CD intersected by AC make the interior angles less than two right angles, and AC with the angle BAC is moved in its own direction to the position $\alpha\gamma$, so that α coincides with C , the straight line $\alpha\beta$, and that is AB , falls entirely outside CD ;

7. with the same assumptions, during its motion, before α reaches C , the straight line $\alpha\beta$, and that is AB , intersects the straight line CD ;

8. finally, I assume this fundamental proposition, to any figure there is always one similar to it of any size;

9. and with the help of these auxiliary propositions I now prove the proposition, if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side;

let AB and CD be the two straight lines intersected by the infinite straight line AC making the interior angles BAC and DCA less than two right angles;

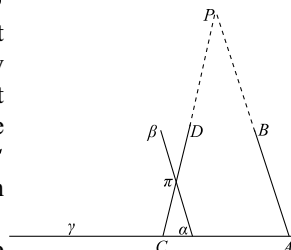
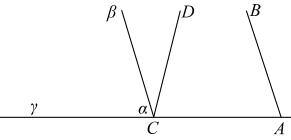
let AC with the angle BAC be moved along AC until $\alpha\beta$, and that is AB , intersects CD at point π ;

as $\alpha C\pi$ is a triangle, there is a triangle similar to it of any size;

thus on the straight line AC the triangle ACP similar to $\alpha C\pi$ can be drawn;

as the angles of similar figures are respectively equal, AP lies on AB produced and CP lies on CD produced;

and therefore, AB and CD intersect on the side on which the angles are less than two right angles



(*id.*).

Wallis's is a *petitio principii*, too. The fundamental proposition, to any figure there is always one similar to it of any size, is equivalent to what Euclid's postulate affirms. Wallis is clear about this fact. In favor of the assumption of similarity, in place of Euclid's postulate, Wallis proposes: it follows from the essence of the relation of magnitudes, namely, that they can be decreased and increased without limit (*id.*). Similarity follows from the axiom of measure. Wallis adds: all geometers, including Euclid, make this assumption; nor is it an objection against it that it introduces proportionality and similarity; instead of introducing these in books V and VI, respectively, Euclid could have introduced them both in book I already (*id.*).

More important, in the *Elements* the propositions that do not depend on the postulate are separated sharply from the propositions that depend on it, the first 28 propositions in book I do not depend on it, the following propositions depend on it. The subsequent attempts to prove the postulate are attempts to deduce what the postulate assumes from the other assumptions of the *Elements* and the propositions that do not depend on it. With Wallis – with the auxiliary propositions, all of which he proves independently of the assumption of similarity, or equivalently Euclid's postulate, with the appeal to the axiom of measure, and with the question of where in the *Elements* proportionality and similarity belong – it is for the first time explicit that the problem which the postulate poses is of the relation of the assumptions. The problem is of which the assumptions are on which that assumption depends, and from which it can therefore be deduced. Assuming that there are those assumptions from which it can be deduced. And Saccheri's, next, is the decisive step in that, the investigation of the assumptions on which that assumption depends. Assuming that there are those assumptions from which it can be deduced.

Two-odd generations later, in 1733, Saccheri published *Euclid freed from all blemish, or a geometrical investigation in which are established the principles of universal geometry*.

Saccheri begins: no one who has studied mathematics can not know how extraordinary the

merit of Euclid's *Elements* is (*pr.*); but this has not prevented some among the ancients as well as those more recently from finding blemishes in these most beautiful and never sufficiently praised *Elements* (*id.*).

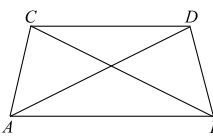
There are a couple of other blemishes. The chief, however, is what that postulate affirms. Saccheri writes: no one doubts the truth of this proposition; they only accuse Euclid of including it among the axioms (*id.*). Saccheri thus proposes: to demonstrate this controversial Euclidean axiom clearly, without any *petitio principii* (*id.*). And: no one can not see that thus the first principles of geometry are subjected to rigid examination (*id.*).

The first principles of geometry are subjected to rigid examination. Euclid is freed from all blemish. And the principles of universal geometry are established.

In particular, Saccheri takes for granted the other assumptions and the first 26 propositions of Euclid's book I. And, on this ground, in order to free Euclid from all blemish, he proves 39 propositions, 23 corollaries, and 5 lemmas. He introduces also 1 definition after proposition 4, and in 18 scholia comments on the propositions.

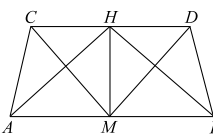
The argument proceeds:

proposition 1. if two equal straight lines AC and BD make equal angles with the straight line AB , and CD is joined, the angles at C and D are equal;



join AD and BC ; the triangles ABC and ABD are equal; and it follows that the triangles ACD and BDC are also equal;

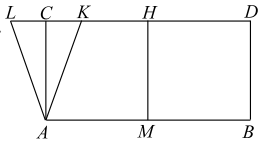
proposition 2. in the same quadrilateral $ABCD$, if the sides AB and CD are bisected at M and H , the angles at M and H are right;



join AH and BH , and CM and DM ; the triangles CAM and DBM are equal; hence the triangles ACH and BDH are equal; hence the triangles CHM and DHM are equal; and hence the triangles AMH and BMH are equal; therefore the angles at M and H are equal, and right (*id.*). And in proposition 3 Saccheri builds the Saccheri

quadrilateral:

proposition 3. in the same quadrilateral, if AC and BD are at right angles to AB , then CD is equal to, or less than, or greater than AB as the angles at C and D are right, or obtuse, or acute;



first part. the angles at C and D are right;

let, if possible, either AB or CD be greater than the other; let, for instance, CD be greater;

take in DC the part DK equal to BA ; join AK ; since the equal straight lines BA and DK are perpendicular to BD , the angles BAK and DKA are equal; but BAK is by construction less than the right angle BAC and DKA is by construction external and greater than the opposite interior right angle DCA ; and this is absurd;

therefore AB and CD are equal;

second part. the angles at C and D are obtuse;

bisect AB and CD and join the midpoints M and H ; as the two straight lines AM and CH are perpendicular to MH and on the straight line AC joining them the angle at A is right and the angle at C is not right, CH is not equal to AM ; nor is it greater, however; for, if it is, and KH is then equal to AM , the angles at A and K are equal; and this is absurd as above;

therefore CH is less than AM , and CD is less than AB ;

third part. the angles at C and D are acute;

since AM and CH are perpendicular to MH , and angle A on AC is right, CH is not equal to AM as above; nor is it less, however; for, if it is, and on HC produced HL is taken equal to AM , and AL is joined, the angles at A and L are equal as above; and this is absurd as above;

therefore, CD is greater than AB ;

therefore, in the quadrilateral $ABCD$, CD is equal to, or less than, or greater than AB as the angles at C and D are right, or obtuse, or acute (*id.*).

4 is the converse of 3. The definition follows:

definitions. in the quadrilateral with two equal sides and two right angles at the base, therefore, three hypotheses are possible, distinguished by the kind of the other two angles; I call the first the hypothesis of the right angle, the second the hypothesis of the obtuse angle, and the third the

hypothesis of the acute angle (*id.*).

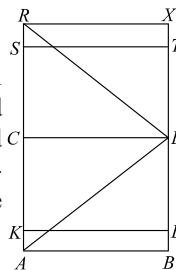
Thus, to free Euclid of all blemish is to show that the hypothesis of the obtuse angle and the hypothesis of the acute angle are both false, and hence the hypothesis of the right angle alone is true. It is what in the following propositions Saccheri proposes to do. Thus, proposition 14 is: the hypothesis of the obtuse angle is absolutely false, because it destroys itself (*id.*). Proposition 33 is: the hypothesis of the acute angle is absolutely false, because it is repugnant to the nature of the straight line (*id.*). And proposition 38 again: the hypothesis of the acute angle is absolutely false, because it destroys itself (*id.*). The numbers of the propositions show that, as Saccheri notes: the battle against the hypothesis of the acute angle is long (*ind.*). And it will turn out, pace Saccheri, that the battle is not successful in showing that the hypothesis of the obtuse angle destroys itself, or that the hypothesis of the acute angle either is repugnant to the nature of the straight line or destroys itself. It will turn out, however, with Saccheri, that the battle is most successful in developing geometry independently of Euclid's postulate, and in establishing the hypothesis of the acute angle as the foundation of non-Euclidean geometry such as Lobachevsky will develop it.

Saccheri continues:

proposition 5. if the hypothesis of the right angle is true in one case, it alone is true in every case;

CD is equal to AB ;

produce AC and BD to R and X as CR and DX are equal to them; join RX ; join AD and RD ; the triangles ACD and RCD are equal; hence the triangles ADB and RXD are equal; and hence the angles at R and X are right;



and since on the same base AB the perpendiculars can be increased infinitely, the hypothesis of the right angle is true for any increase of the sides;

for the case of decrease, take on AR and BX the equal perpendiculars AK and BL , and join KL ; if the angles at K and L are not right, they are nevertheless equal; let them be obtuse toward

AB and acute toward RX ; but the perpendiculars RK and XL are also equal; therefore KL is greater than RX and less than AB ; but RX and AB are equal; and this is absurd;

the hypothesis of the right angle is thus true also for any decrease of the sides; and it is true for any change of the base as well, for clearly any of the sides can be taken as a base;

therefore, if the hypothesis of the right angle is true in one case, it alone is true in every case (*id.*).

Saccheri, too, takes the axiom of measure for granted. He continues:

proposition 6. if the hypothesis of the obtuse angle is true in one case, it alone is true in every case;

CD is less than AB ;

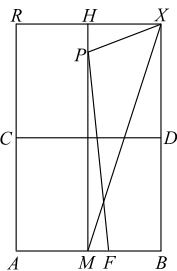
draw straight lines as above; if the angles at R and X are obtuse, the hypothesis is true; and they are not right, for then the hypothesis of the right angle holds in one case, and leaves no place for the hypothesis of the obtuse angle; nor are they acute; for RX is then greater than AB and still greater than CD ; and since AB is greater than CD and less than RX , there is in the series of straight lines, which cut off from CR and DX equal parts a straight line ST equal to AB ; thus the hypothesis of the right angle again holds in one case and leaves no place for the hypothesis of the obtuse; therefore the angles at R and X are obtuse;

similarly the angles at K and L cannot be either right or acute;

the hypothesis is thus true for any side;

and it is true for any base;

take the side BX as a base; the bisector HM of RX and AB is perpendicular to both; the angle at B is right, and the angle at X is obtuse as above; make then the right angle BXP , XP intersecting HM at P ; join XM ; since angle BXH is obtuse, angle BXM is acute; since the quadrilateral $XBMP$ contains three right angles and one obtuse at P , because it is exterior to the opposite interior right angle of the triangle PHX , the side XP is less than the opposite BM ; taking in BM the part BF equal to XP , and joining PF , the angles at P and F are equal, and



obtuse, because the angle at F is exterior to the opposite interior in the right triangle FMP ; and the hypothesis of the obtuse angle is thus true for any base;

therefore, if the hypothesis of the obtuse angle is true in one case, it alone is true in every case;

proposition 7. if the hypothesis of the acute angle is true in one case, it alone is true in every case;

the proof is easy; for if this hypothesis admits of one case of either of the other two hypotheses, no place is left for it;

therefore, if the hypothesis of the acute angle is true in one case, it alone is true in every case (*id.*).

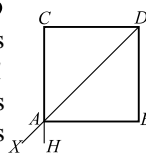
These propositions have established that the three hypotheses are each independent of the other two. Hence, there may be propositions that are true on any two of them or on all three of them. But there may not be a proposition true on any one of them that shows that either of the other two is false. Only internal inconsistency, a hypothesis destroying itself, can show that any of the three is false. And this is what Saccheri pursues next, to show that each of the two hypotheses alternative to Euclid's, of the obtuse angle and of the acute angle, contradicts itself. And in the pursuit of this goal he demonstrates more and more propositions, a few resting on the hypothesis of the obtuse angle, many resting on the hypothesis of the acute angle, independent of Euclid's postulate. Among them:

8. in any right triangle ABD with the right angle at B , if AD is produced to any point X , and at A the perpendicular HAC to AB is drawn, the exterior angle XAH is equal to, less than, or greater than the opposite interior angle ADB as the hypothesis of the right angle, the obtuse angle, or the acute angle is true; and conversely;

take on HC the part AC equal to BD and join CD ;

and first, on the hypothesis of the right angle, CD is equal to AB ; hence angle ADB is equal to angle DAC and to its equal the vertical angle XAH ;

second, on the hypothesis of the obtuse angle, CD is less than AB ; hence, from the two triangles ADB and DAC , angle DAC , or its vertical XAH ,



is less than angle ADB ;

finally, on the hypothesis of the acute angle, CD is greater than AB ; hence, from the same triangles, angle DAC , or its vertical XAH , is greater than angle ADB ;

and conversely;

9. in any right triangle the two acute angles are equal to, greater than, or less than one right angle as the hypothesis of the right angle, the obtuse angle, or the acute angle is true;

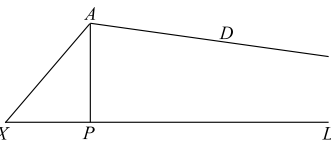
first, if angle XAH is equal to angle ADB , angles ADB and HAD are equal to two right angles, and subtracting the right angle HAB , angles ADB and BAD are equal to one right angle;

second, if XAH is less than ADB , ADB and HAD are greater than two right angles, and subtracting HAB , ADB and BAD are greater than one right angle;

finally, if XAH is greater than ADB , ADB and HAD are less than two right angles, and subtracting HAB , ADB and BAD are less than one right angle (*id.*).

And as any triangle divides into two right triangles, as the right triangle so any triangle. The sum of the angles of the triangle is equal to, greater than, or less than two right angles as the hypothesis of the right angle, the obtuse angle, or the acute angle is true. And conversely, any triangle of which the three angles are equal to, greater than, or less than two right angles establishes that the hypothesis of the right angle, the obtuse angle, or the acute angle is true. Proposition 15 is this converse.

In propositions 11 and 12 Saccheri proves that, if the straight line AP , as long as you please, is intersected by the straight lines PL at a right angle and AD at an acute angle, AD and PL intersect on the side of these angles, and at a finite distance, if either the hypothesis of the right angle or the hypothesis of the obtuse angle is true (*id.*). Hence: 13. if the straight line AX , as long as you please, intersecting the straight lines AD and XL , makes the interior angles on the same side AXL and XAD less than two right angles, AD and XL intersect



on that side, and at a finite distance, if either the hypothesis of the right angle or the hypothesis of the obtuse angle is true (*id.*).

Euclid's postulate is true, if either the hypothesis of the right angle or the hypothesis of the obtuse angle is true.

Saccheri notes in a scholium:

there is thus a notable difference between these two hypotheses, on one side, and the hypothesis of the acute angle, on the other; for on the hypothesis of the acute angle it cannot be demonstrated directly that, if a straight line is intersected by two straight lines, making the interior angles on one side less than two right angles, they intersect on that side (*id.*).

The battle against the hypothesis of the obtuse angle is then concluded:

14. the hypothesis of the obtuse angle is absolutely false, because it destroys itself;

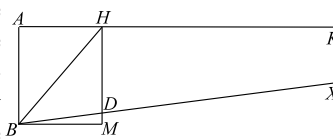
on this hypothesis Euclid's postulate is true; on this hypothesis, however, on LP produced to X , in the triangle APX , with the right angle at P , the two acute angles at A and X are greater than one right angle; the acute angle PAD may thus be such that with those two acute angles it makes two right angles; but the straight line AD must intersect PL ; and this is manifestly absurd against proposition 17 in Euclid's book I with respect to the straight line AX (*id.*).

Proposition 17 in Euclid's book I is that the sum of any two angles of the triangle is less than two right angles. Which, pace Saccheri, need not be true on the hypothesis of the obtuse angle.

The long battle against the hypothesis of the acute angle continues. Saccheri proves:

17. if the straight line AH is perpendicular to any straight line AB , as small as you please, it is not true on the hypothesis of the acute angle that every straight line BD making an acute angle toward AB intersects AH at a finite distance;

erect at B perpendicular BM ; from any point M on it drop perpendicular MH to AH ; join BH ; angle BHM is thus acute; angle BMH is also acute; therefore the perpendicular BDX to MH intersects it at some intermediate point D ; angle XBA



is thus acute; and the two straight lines AHK and BDX , however produced, cannot intersect because of the right angles at H and D ;

therefore, on this hypothesis it is not true that, if a straight line is perpendicular to any straight line AB , and another straight line intersects AB at any acute angle, the two straight lines intersect (*id.*).

In a scholium Saccheri concludes:

the hypothesis of the acute angle will thus be destroyed by the single assumption that two straight lines, intersected by a straight line making the interior angles less than two right angles, intersect on that side (*id.*).

By the single assumption of Euclid's postulate.

In scholia after proposition 21 Saccheri discusses the attempts of Proclus, al-Tusi, Borelli, and Wallis to prove Euclid's postulate.

The battle continues. Saccheri proves:

23. any two straight lines AX and BX in the same plane either have a common perpendicular or, produced in the same direction, either intersect at a finite distance or approach one another ever more;

from any point A on AX drop perpendicular AB to BX ;

if the angle at A is right, we have the case of the common perpendicular;

otherwise, the angle at A is acute in one direction or the other, let it be acute toward X ; drop from any points D, H, L between A and X perpendiculars DK, HK, LK to BX ; if any angle at D, H, L is acute toward A , there is between that point and A a common perpendicular to AX and BX ;

but if every one of these angles is greater than acute toward A , if any of them is right, we again have a common perpendicular; if they are all obtuse toward A and therefore acute toward X , in the quadrilateral $KDHL$ the angles at the points K are right and the angle at D acute, and DK is thus greater than HK ; similarly HK is greater than LK , and thus always;

therefore, AX and BX approach one another ever more;

therefore, any two straight lines AX and BX in

the same plane either have a common perpendicular or, produced in the same direction, either intersect at a finite distance or approach one another ever more (*id.*).

The two straight lines AX and BX have a common perpendicular, or intersect, or are asymptotic. Hence:

25. if two straight lines AX and BX in the same plane standing on the straight line AB , one at a right, the other at an acute angle, approach one another ever more, and their distance from one another is nevertheless always finite, the hypothesis of the acute angle is destroyed (*id.*).

This is how the hypothesis of the acute angle will contradict itself, if AX and BX are both asymptotic and at a finite distance from one another. The battle is thus now to that more particular end, to show that AX and BX are both asymptotic and at a finite distance from one another. And Saccheri does not attain that particular end, but attains the following:

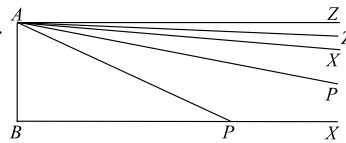
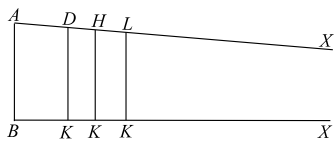
32. in the hypothesis of the acute angle there is a certain determinate angle BAX such that AX intersects BX only at infinity, and AX is thus a limit of all straight lines through A , which, on one side, making the acute angle BAX less, intersect BX at a finite distance, and, on the other side, making the acute angle BAX greater and all the way to right angle, have a common perpendicular with BX (*id.*).

AX is asymptotic to BX , and, on one side, all straight lines AP intersect BX , on the other side, all straight lines AZ are the upper bases of Saccheri quadrilaterals at whose midpoints are the perpendiculars common with BX . Next:

33. the hypothesis of the acute angle is absolutely false, because it is repugnant to the nature of the straight line;

the hypothesis is inimical to Euclidean geometry because on this hypothesis we must recognize the two straight lines AX and BX which, produced to infinity, run together into one and the same straight line and have at some infinite distance a common perpendicular (*id.*).

Lemmas after the proposition exhibit the repugnance:



two straight lines do not enclose a space;

two straight lines cannot have the same segment in common;

two straight line do not touch but intersect (*id.*).

The assumption, after all the previous propositions, that as straight lines at a finite distance so straight lines at infinity, is one of the few blemishes in *Euclid freed from all blemish*.

Inimical to Euclidean geometry though it is, the hypothesis of the acute angle has not yet destroyed itself, and the battle continues:

34. in which a certain curve arising from the hypothesis of the acute angle is investigated (*id.*).

In the quadrilateral $ABCD$, the angles at C and D are acute, and the sides AC and BD are thus greater than the perpendicular HM ; producing the perpendicular to K , as KM is equal to AC and BD , the ends of all perpendiculars FP to AB describe the curve $CFFFKD$, equidistant from AB (*id.*). And:

37. the curve $CFFFKD$ is equal to the base AB (*id.*).

The argument is by infinitesimals. As the infinitesimal tangent at any point F, F, F, K on the curve is equal to the infinitesimal arc of the curve at this point it is equal also to the infinitesimal at any point P, P, P, M on the base, from which Saccheri concludes that the curve is equal to the base (*id.*). This conclusion is another of the few blemishes in Saccheri's treatise. The proposition follows:

38. the hypothesis of the acute angle is absolutely false, because it destroys itself (*id.*).

The curve $CFFFKD$ is greater than CD which is greater than AB ; but it is also equal to AB (*id.*).

And Saccheri concludes with the demonstration of Euclid's postulate:

39. if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced to infinity, intersect on that side (*id.*).

The following are all major advances.

In the quadrilateral with two equal sides and two right angles at the base, three hypotheses are possible, that the other two angles are right, that

they are obtuse, and that they are acute.

If the hypothesis of the right angle is true in one case, it alone is true in every case, if the hypothesis of the obtuse angle is true in one case, it alone is true in every case, if the hypothesis of the acute angle is true in one case, it alone is true in every case.

The sum of the angles of the triangle is equal to, greater than, or less than two right angles as the hypothesis of the right angle, the obtuse angle, or the acute angle is true.

Euclid's postulate is true on the hypothesis of the right angle as well as on the hypothesis of the obtuse angle.

On the hypothesis of the acute angle straight lines in the same plane either have a common perpendicular or, produced in the same direction, either intersect or are asymptotic.

On this hypothesis through any point on the perpendicular to a straight line passes a straight line asymptotic to that straight line, and this asymptotic straight line divides all straight lines through the same point into two classes, those that intersect with that straight line, and those that have a common perpendicular with it.

On this hypothesis there is a curve equidistant from a straight line.

The first principles of geometry have been subjected to rigid examination. And the principles of universal geometry have been established.

Moreover, most beautiful and never sufficiently praised, is how Saccheri judges Euclid's treatise. Ignoring the question of how much it should be praised, Saccheri's is a most beautiful treatise itself.

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Two generations separate Wallis and Saccheri. And, on the one hand, Wallis's attempt to prove Euclid's postulate, for all its sophistication compared to the earlier attempts, is not too much more than that, another unsuccessful attempt to prove the postulate. Saccheri's, on the other hand, is very much more than yet another unsuccessful attempt to prove the postulate. With Wallis it is for the first time explicit that the problem is of the assumptions on which that assumption depends. With Saccheri the investigation of the assumptions on which that assumption depends

has succeeded in establishing geometry independently of Euclid's postulate. If the hypothesis of the right angle is true in one case, it alone is true in every case. But if the hypothesis of the obtuse angle is true in one case, it alone is true in every case. And if the hypothesis of the acute angle is true in one case, it alone is true in every case. And Saccheri has proved theorems on the hypothesis of the obtuse angle, and has proved many theorems on the hypothesis of the acute angle.

Three causes can be assigned to this development.

The development of projective geometry, and in particular Desargues. What straight lines do not just when produced indefinitely, what straight lines do at infinity is, for the first time, and explicitly, a central concern of Saccheri.

Descartes. The principles became a central concern of philosophy with Bacon. The principles are a central concern of Galileo. The world is written in mathematics, too. The principles are a central concern of Kepler. And the principles are not just a central concern of Descartes. Descartes formulated the rules of the method, and developed philosophy from its principles, in *The principles of philosophy*. More particularly, too, Descartes laid the foundations of analysis with the *Geometry, which is an essay in the method*, in which he developed mathematics, or geometry, from its principles. Saccheri, for the first time, and explicitly, addressed the problem of Euclid's postulate by developing geometry – universal geometry – from its principles.

The development of calculus, and both Newton and Leibniz. Calculus posed the problem of the infinitely small. More important, calculus posed the problem of the assumptions on which it was developed. And it was developed with dramatic rapidity. Cavalieri discovered integration, Fermat discovered differentiation, Newton discovered calculus. Newton discovered the whole of which his predecessors had discovered parts. Leibniz organized calculus into a discipline, collected its rules into a system, and gave it a crystal clear explanation. These are both very dramatic results, attesting the very great acceleration of the development of mathematics. The parts, very large parts, of the whole, a very very large whole, had been discovered but one and a

half generations earlier, and its organization into a system followed within two decades. Both results are a vast generalization each over earlier results. And these are the most dramatic features of Saccheri's treatise, too, the whole of the three hypotheses, of the right angle, the obtuse angle, and the acute angle, and their organization into a system. Both are a vast generalization over earlier results, too.

Again, then, counting only the more mathematical results, those of 37 philosophers – Diofantus, Pappus, Proclus, al-Khwarizmi, banu Musa, Abu Kamil, Thabit ibn Qurra, al-Battani, Abu'l-Wafa, ibn al-Haytham, al-Karaji, Khayyam, bar Hiyya, Fibonacci, Nemorarius, al-Tusi, Oresme, Peurbach, Regiomontanus, Pacioli, Cardano, Viète, Napier, Galileo, Kepler, Desargues, Descartes, Cavalieri, Fermat, Wallis, Pascal, Huygens, Newton, Leibniz, Jacob Bernoulli, Saccheri, Johann Bernoulli – have been noted, attesting the development of mathematics in the not quite sixteen centuries from Ptolemy to Saccheri. All 37 are important, the majority are very important, and the majority are steps of advance in the acquisition of knowledge.

Again, though, not all with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

These 11 are all important philosophers who are part of the development of this problem,

411 - 485	Proclus,
831 - 901	Thabit ibn Qurra,
965 - 1040	ibn al-Haytham,
1048 - 1127	Khayyam,
1201 - 1274	al-Tusi,
1591 - 1661	Desargues,
1596 - 1650	Descartes,
1616 - 1703	Wallis,
1642 - 1727	Newton,
1646 - 1716	Leibniz,
1667 - 1733	Saccheri.

And the steps of advance in the acquisition of knowledge are the 9, Proclus, ibn al-Haytham, Khayyam, Desargues, Descartes, Wallis, Newton, Leibniz, and Saccheri. In particular,

Proclus	second attempt to prove what
	Euclid's postulate affirms,
	straight lines may be
	asymptotic,

ibn al-Haytham	a quadrilateral of three right angles,
Khayyam	a quadrilateral of two right angles,
Desargues	developed projective geometry, parallels intersect at a point at infinity,
Descartes	developed mathematics from its principles,
Wallis	the problem of Euclid's postulate is of the assumptions on which it depends,
Newton	discovered calculus,
Leibniz	organized calculus into a system,
Saccheri	three hypotheses are possible, of the right, obtuse, and acute angle, the three hypotheses are independent of one another, Euclid's postulate is true on that of the right angle, on that of the acute angle there are asymptotic straight lines, asymptotic straight lines divide the straight lines through a point into two classes, those intersecting a given straight line and those that have a common perpendicular with it, there is a curve equidistant to a straight line.

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The one and a half-odd centuries from Saccheri to Riemann.

They divide into two parts, the one century-odd from Saccheri to Gauss, and the one half century from Gauss to Riemann. The dates are,
 1667 - 1733 Saccheri,
 1777 - 1855 Gauss,
 1826 - 1866 Riemann.

Division of that time by Gauss coincides rather precisely with division by Napoleon. The first part is the brighter time before and including the revolution, the second part is the more somber time after the revolution.

It coincides rather precisely with division also

by Hegel. The first part is the time of enlightenment, and of Hume and Kant. The second part is the time of restoration of spirit to the substantiality and purity of its being, and of catastrophic transformation of philosophy.

In the one century-odd from Saccheri to Gauss the great advance of physics continued. Mechanics was formulated analytically. This culminated in Lagrange and the *Analytic mechanics*, in 1788. The wave equation was discovered, by d'Alembert, in 1747, and generalized, by Euler, in 1753. The investigation of the heaven reached steadily further. Herschel discovered the proper motion of the solar system toward λ Herculis in 1783, mapped the Milky Way in 1785. In 1796 Laplace formulated the theory of the origin and development of the solar system. The investigation of the material principles of all things advanced very greatly, through Lavoisier and the first table of the elements, in 1789, and Dalton and the formulation of the atomic theory, in 1803. This is the time when the investigation of electricity and magnetism advanced from the stage of collection of the phenomena to the stage of attempts to formulate the theory of the phenomena. Coulomb discovered the first law of electricity in 1785. From 1749 through 1788 Buffon published the *Natural history*. Geology advanced. The investigation of organisms advanced very greatly. Linnaeus classified organisms, in 1735, Lamarck formulated the theory of the evolution of animals, in 1802, Cuvier laid the foundations of paleontology, in 1812. Most of this development as well has been presented in some detail above, and the rest will be below.

And the great advance of mathematics continued. Vast new ground had been broken in the previous century. In this one century-odd the new ground was brought under cultivation. Calculus had been the greatest advance, and its further development dominated this time. The investigation of functions advanced greatly, the techniques of differentiation and integration advanced greatly. Analysis expanded in breadth and in depth, problems and solutions were accumulated, and the following all became a massive domain each. Infinite series. Differential equations. Partial differential equations. Calculus of variations. The analytic development of the theory of

numbers began. The analytic development of geometry began. This is the time of Taylor, Daniel Bernoulli, Euler, d'Alembert, Lambert, Lagrange, Monge, Laplace, Legendre, Fourier. All mathematicians, all also physicists. The part and the whole, mathematics and physics, continued to advance inseparably, the philosophers continued to advance physics by advancing mathematics, and conversely. Those are all directly solutions of physical problems. Functions. Differentiation and integration. Infinite series. Differential equations. Partial differential equations – as ordinary differential equations involve the differentiation of functions of one variable, partial differential equations involve the differentiation of functions of two or more variables, partial being the derivatives of each of these variables as the others are held constant. Calculus of variations – as in differentiation the values maximizing and minimizing a function for these values are sought, in calculus of variations functions maximizing and minimizing a function of these functions are sought. Euler alone advanced greatly all of those, the investigation of functions, differentiation, integration, infinite series, differential equations, partial differential equations, calculus of variations, theory of numbers, analytic geometry, as well as mechanics, the investigation of the heavenly bodies, the investigation of fluids, optics, acoustics. He improved on the map of Russia, too. Some of this development has been presented above, and more of it will be below. Here one particularly elegant result can be presented, one of Euler's many particularly elegant results.

In the *Introduction to the analysis of infinities*, in 1748, Euler defines the logarithm:

if $a^x = y$, then $x = \log_a y$ (102),
defines the series:

$$a = 1 + \frac{k}{1} + \frac{k^2}{1 \cdot 2} + \frac{k^3}{1 \cdot 2 \cdot 3} + \dots \quad (116),$$

defines e :

$$\text{if } k = 1, \text{ then } a = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \dots,$$

its value is 2.71828182845904523536028..., for the sake of brevity we write for this number e , and we designate with it the base of the natural logarithm (122); the natural logarithm has the property that $\log_e (1 + \omega) = \omega$, where ω is an

infinitely small quantity (123),
defines π :

if the radius of the circle is 1, one half of its circumference is

3.141592653589793238462643383279502884197169399375

1058209749445923078164062862089986280348

253421170679821480865132723066470938446+,

for the sake of brevity we designate this number with the symbol π (126),

defines the trigonometric functions:

for the arc x we write $\sin x$ and $\cos x$, and since π is an arc of 180° , $\sin 0 = 0$, $\cos 0 = 1$,

$$\sin \frac{\pi}{2} = 1, \cos \frac{\pi}{2} = 0, \sin \pi = 0, \cos \pi = -1 \quad (127),$$

establishes their identities, including:

$\sin^2 x + \cos^2 x = 1$, from which by factoring we have $(\cos x + i \sin x)(\cos x - i \sin x) = 1$ (132).

Euler writes $\sqrt{-1}$ in the *Introduction*, but he first also writes i for it, in 1777, in *On the formulas of differential angles*:

for the expression $\sqrt{-1}$ we write i , so that $i^2 = -1$ (*id.*).

And, back in the *Introduction*, he shows that:

$$e^{ix} = \cos x + i \sin x \quad (138).$$

And if $x = \pi$,

$$e^{i\pi} = -1.$$

As to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, the next step in its development is, two generations after Saccheri, Lambert.

Lambert proved that π is irrational. He advanced greatly the theory of map projections.

And he wrote *The theory of parallel lines*, published posthumously in 1786.

Lambert begins: the question raised by Euclid's axiom does not concern its truth or thinkability; that the axiom is thought true is clear; its truth, however, is established by the consequences drawn from it, and indeed so completely, these consequences can be considered a complete induction (10); the difficulties concerning the axiom come down to this question, can it be derived from the other Euclidean postulates and axioms, or if these are not sufficient, are there others from which it can be derived (11)?

And after a few preliminary propositions, Lambert formulates the question explicitly in just the way Saccheri has formulated it. He writes:

I assume that in the quadrilateral $ABCD$ the

angles at A , B , and C are right; the question is then essentially about the angle at D ; and we must take as necessary three hypotheses,



1. $\angle BDC = 90^\circ$,
2. $\angle BDC > 90^\circ$,
3. $\angle BDC < 90^\circ$;

and it will be seen that all three hypotheses can be pursued quite long before arriving at *quod est absurdum* (39).

The first hypothesis, of the right angle, Lambert pursues briefly, to set up the constructions necessary for the pursuit of the other two. Then he pursues the second hypothesis, of the obtuse angle, until he arrives at *quod est absurdum*. The demonstration is grounded in the axiom of measure, or, as Lambert calls it: the law of continuity (53). He writes:

erect on AB the two perpendiculars AC and BD ; erect on AC produced the perpendiculars CD , EF , GH , etc., intersecting BD produced at D , F , H , etc.;

and then $AB > CD$;

for, if possible, let $AB = CD$;

bisect AC and erect at the midpoint perpendicular MN ; fold the figure along MN , A then coincides with C , AB with CD , NB with ND , and $\angle NBA = \angle NDC$, contrary to the hypothesis that one of these angles is right and the other obtuse;

in the same way, it cannot be that $AB < CD$;

therefore, $AB > CD$ (53);

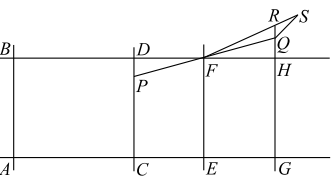
in the same way, $CD > EF > GH$, etc. (56);

moreover, make $CE = EG$;

and then $EF - GH > CD - EF$;

for on this hypothesis not only are $\angle BDC$, $\angle BFE$, $\angle BHG$ obtuse, $\angle BHG > \angle BFE > \angle BDC$;

draw at F the straight line PQ perpendicular to FE ; it intersects CD under D and GH over H , so that $CP = GQ$;



make $GR = CD$; draw FR ;

fold the figure along EF ; CE coincides with EG , CP with GQ , CD with GR , and so FP with FQ and DF with FR ;

hence $\angle DFP = \angle QFH = \angle RFQ$;

but $\angle BHG > \angle BDC$; and so $\angle CDF > \angle FHR$;

but $\angle CDF = \angle FRH$; and so $\angle FRH > \angle FHR$;

and therefore $FH > FR$;

produce FR to S so that $FS = FH$; draw QS ;

hence $\angle FQH = \angle FQS$; and $\angle FQH$ is obtuse;

thus $GQ = CP < EF$;

further, $\angle FRG = \angle FDC$ is acute;

thus $\angle QRS$ is obtuse; and thus $QS > QR$;

but $QS = QH$; thus $QH > QR$;

but $QR = DP$;

and therefore $QH > DP$;

or $GQ - GH > CD - CP$;

but $GQ = CP$;

$GQ - GH > CD - GQ$;

but $EF > GQ$; and $2EF > 2GQ$;

$GQ + 2EF - GH > CD + 2GQ - GQ$;

$2EF - GH > CD$;

and therefore, $EF - GH > CD - EF$ (63);

therefore, the further they are from AB , the perpendiculars CD , EF , GH , etc., not only decrease, they decrease ever more; and hence, however small the decrease may be, the sum of the decreases will eventually exceed AB ; and this cannot happen, because the straight line BH would then intersect the straight line AG , and this is prevented by the assumption that BH and AG are both at right angles to AB ;

the second hypothesis thus enters the realm of the impossible (64).

The pursuit of the third hypothesis, that of the acute angle, begins on the same ground, the law of continuity (68):

in the same construction as the last, on this hypothesis $AB < CD < EF < GH$, etc. (*id.*);

moreover, $GH - EF > EF - CD$ (70);

the further they are from AB , the perpendiculars CD , EF , GH , etc., not only increase, they increase ever more (*id.*).

The pursuit of this hypothesis has thus not arrived at *quod est absurdum*. The pursuit has to continue. Several propositions further on, Lambert concludes:

it is easily seen that the third hypothesis can be pursued further; I have searched for contradic-

tions; and I have seen that this hypothesis is not at all easy to overthrow; I will note some of the consequences I have drawn;

the most substantial is this, that if this hypothesis holds, we have an absolute measure of the length of every line and the content of every plane figure and every space enclosing a body (79);

let angles A, B, C, E , and F be right; on this hypothesis D, G, H , and I are acute, moreover, $D > H$ and $H > I$, and $D > G$ and $G > I$;

and then angle D is the measure of the quadrilateral $ABDC$ when $AB = AC$, and angle I is the measure of the quadrilateral $AEIF$ when $AE = AF$;

for, with $AB = AC$ and angles A, B , and C right, no other quadrilateral can pass through D ; taking the greater sides $AE = AF$ with right angles at E and F , $D > I$; and taking $AE = AF$ smaller, $I > D$;

angle D is thus the absolute measure of the quadrilateral $ABDC$;

if AB is 1 Paris foot and angle $D = 80^\circ$, as soon as the quadrilateral $ABDC$ is constructed with angle $D = 80^\circ$, AB is 1 Paris foot;

and this consequence is so enticing, it makes one wish the third hypothesis was true! indeed I so wish it, paying no heed to the countless inconveniences this will bring about; the trigonometric tables will have to be continued infinitely, the similarity and proportionality of figures will disappear, great calamity will befall astronomy, and so on (80).

Alas:

in geometry, as in all science, *argumenta ab amore et invidia ducta* have no place (81).

So Lambert continues with the consequences:

on this hypothesis not only is the sum of the angles of the triangle less than two right angles; the difference from two right angles grows with the area of the triangle; if the area of one triangle is greater than that of another, the sum of its angles is smaller than that of the other;

inscribe one triangle in another;

then $\alpha + \alpha_1 + \alpha_2 = 180^\circ - a$;

$\beta + \beta_1 + \beta_2 = 180^\circ - b$;

$\gamma + \gamma_1 + \gamma_2 = 180^\circ - c$;

$$\delta + \delta_1 + \delta_2 = 180^\circ - d;$$

$$\text{but } \alpha_1 + \delta_1 + \beta_1 = 180^\circ;$$

$$\beta_2 + \delta_2 + \gamma_2 = 180^\circ;$$

$$\alpha_2 + \delta + \gamma_1 = 180^\circ;$$

and subtracting,

$$\alpha + \beta + \gamma = 180^\circ - a - b - c - d;$$

thus, in the greater triangle the defects of all the smaller triangles are added, and the sum of its angles is that much smaller (*id.*);

on the third hypothesis the defect of the sum of the angles of the triangle with respect to two right angles is proportional to the area of the triangle; and I will note here that there is a similar proposition on the second hypothesis, the excess of the sum of the angles of the triangle with respect to two right angles is proportional to the area of the triangle;

it seems to me remarkable also that the second hypothesis holds if the plane triangles are assumed spherical, for in the spherical triangle the sum of the angles is greater than two right angles;

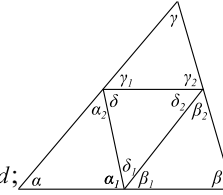
it seems to me even more remarkable that this hypothesis can be assumed to hold on a spherical surface without any consideration of the difficulties of parallel lines;

and it can be concluded from this also that the third hypothesis holds on an imaginary spherical surface (82).

On the hypothesis of the right angle, the sum of the angles of the triangle is $\alpha + \beta + \gamma = \pi$, and the sum is independent of the area of the triangle. On the hypothesis of the obtuse angle the sum of the angles of the triangle is $\alpha + \beta + \gamma > \pi$, and the excess $(\alpha + \beta + \gamma) - \pi$ is proportional to the area of the triangle. On the hypothesis of the acute angle the sum of the angles of the triangle is $\alpha + \beta + \gamma < \pi$, and the defect $\pi - (\alpha + \beta + \gamma)$ is proportional to the area of the triangle. The hypothesis of the obtuse angle holds on the surface of a sphere of radius r . And it can be concluded that the hypothesis of the acute angle holds on the surface of a sphere of radius $r\sqrt{-1}$.

Lambert concludes in *The theory of parallel lines*: it can be seen from all this that, although the second hypothesis is easily overthrown, this is not at all the case with the third hypothesis (88).

On the hypothesis of the acute angle there is an absolute measure of length.



The defect of the sum of the angles of the triangle is proportional to the area.

On the hypothesis of the obtuse angle the excess of the sum of the angles of the triangle is proportional to the area.

This hypothesis holds on the surface of the sphere.

The hypothesis of the acute angle holds on the surface of a sphere of radius $r\sqrt{-1}$.

These are all major advances.

Also, in geometry, as in all science, *argumenta ab amore et invidia* have no place, but not conversely. In Lambert's *argumentum ab amore* geometry, and all science, has found its place.

In the next generation, Legendre published the *Elements of geometry* in 1794, and expanded and revised it through its twelfth edition in 1823. Euclid's postulate is not one of Legendre's assumptions. Legendre demonstrates what Euclid's postulate affirms. And the demonstrations of the postulate in the different editions of the *Elements of geometry* Legendre collected in 1833 in the *Reflections on the different ways to demonstrate the theory of parallels or the theorem on the sum of the three angles of the triangle*.

This is from the first edition, in 1794:

in every rectilinear triangle the sum of the three angles is equal to two right angles;

two triangles are equal if one side and the two adjacent angles are respectively equal; call the side p , the adjacent angles A and B , and the third angle C ;

C is thus determined completely, if A , B , and p are given;

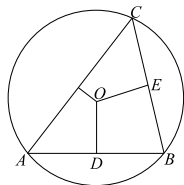
C would thus be a function of A , B , and p ; which I express in this way, $C = f(A, B, p)$;

but if so, given an equation of A , B , C , and p , the value of p could be obtained from A , B , and C ; from this it would follow that p is equal to a number; and this is absurd (*id.*).

And if C is a function of A and B , and not of p , that the sum of the angles of the triangle is two right angles follows, and hence what Euclid's postulate affirms follows.

In the second edition, in 1799, Legendre proves:

through three points A , B , and C not on the same straight



line a circle can always be drawn, and only one circle;

join AB and BC , and at the midpoints draw the perpendiculars DO and EO ;

these perpendiculars intersect; for, if they were parallel, the straight lines BA and BC drawn perpendicular to these parallels would be one and the same straight line contrary to the hypothesis;

the point of intersection O is equidistant from A , B , and C ; a circle can therefore be drawn with O as the center through A , B , and C ;

and no other circle can be drawn through these three points, for, if it could, its center would be on the straight lines DO and EO , and these can intersect in only one point (II, 7).

What Euclid's postulate affirms follows.

The first is *ab invidia*. As Lambert has shown. The second is a *petitio principii*. As Legendre's demonstration also shows, that through three points not on the same straight line only one circle can be drawn, too, is equivalent to what Euclid's postulate affirms.

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The one half century from Gauss to Riemann.

The great advance of physics continued. Bessel measured the distance of 61 Cygni in 1838. Thermodynamics was developed. In 1824 Carnot discovered the cycle of reversibility of the heat engine, in 1847 Helmholtz discovered the principle of conservation of energy, and in 1850 Clausius formulated the laws of thermodynamics. The investigation of electricity and magnetism entered its decisive stage. Ampère published the *Mathematical theory of electrodynamic phenomena* in 1827, Faraday began publishing the *Experimental researches in electricity* in 1839. By 1821 Young and Fresnel had established the wave theory of light. Chemistry advanced. Geology advanced. The investigation of organisms advanced very greatly. This is the time of von Baer, Schwann, and in 1859 *The origin of species*. Much of this development, too, has been presented in some detail above, and the rest will be below.

And the great advance of mathematics continued. Gauss towers over this time. Gauss advanced greatly differential equations, calculus of variations, complex functions and complex analysis,

algebra, the theory of numbers, differential geometry, mechanics, astronomy, optics, electricity and magnetism, geodesy. He improved on the telegraph, too. Cauchy and Weierstrass grounded calculus rigorously. Galois laid the foundations of the theory of groups. Poncelet developed projective geometry. Hamilton discovered quaternions. Boole began the mathematical formulation of logic. It is the time also of Poisson, Möbius, Abel, Jacobi, Dirichlet, Cayley. And the most important advance is the discovery of non-Euclidean geometry, and its generalization. Lobachevsky published *On the principles of geometry* in 1829, Riemann published *On the hypotheses which lie at the foundation of geometry* in 1854. Some of this development, too, has been presented above, and more of it will be below. And again, before turning to the discovery of non-Euclidean geometry, and its generalization, one other particularly elegant result can be presented, one of Gauss's many particularly elegant results.

The result is the *General solution of the problem, to represent the parts of one given surface on another given surface so that the representation be similar to the represented in its smallest parts*, in 1822. Gauss begins:

1. the nature of a curved surface is determined by an equation between the coordinates x, y, z of every point of the surface; by this equation each of these three variables can be considered a function of the other two; it is more general to introduce also two new variables t, u , and represent each x, y, z as a function of t and u , so that definite values of t and u always belong to a definite point of the surface, and conversely (*id.*). When each x, y, z is a function of t and u , so that definite values of t and u always belong to a definite point of the surface, and conversely, t and u describe the entire surface. Gauss continues:

2. let X, Y, Z, T, U have in relation to a second surface the meaning which x, y, z, t, u have in relation to the first;

3. to represent the first surface on the second is to formulate a law by which to every point of the first surface corresponds a definite point of the second; this will be accomplished by making T and U definite functions equal to the two variables t and u (*id.*).

Each X, Y, Z is a function of T and U . By making T and U functions of t and u , each X, Y, Z becomes a function of t and u , as each x, y, z is. And the law by which to every point of the first surface corresponds a definite point of the second surface is thus the function of t and u , by which each x, y, z is related to each X, Y, Z . Gauss continues:

as the representation has to satisfy certain conditions, these functions cannot be arbitrary; as by them X, Y, Z also become functions of t and u , these functions must, next to the condition which the nature of the second surface prescribes, satisfy also the demand which has to be fulfilled by the representation;

4. the problem prescribes that the representation be similar to the represented in its smallest parts; it comes to this, to express this condition analytically;

from the differentiation of the functions of t, u , by which x, y, z, X, Y, Z will be expressed, the following equations can result,

$$\begin{aligned} dx &= a dt + a' du, \\ dy &= b dt + b' du, \\ dz &= c dt + c' du, \\ dX &= A dt + A' du, \\ dY &= B dt + B' du, \\ dZ &= C dt + C' du; \end{aligned}$$

the prescribed condition demands, first, that all infinitely short lines extending from a point on the first surface be proportional to the lines corresponding to them on the second surface, and second, that these lines make between them the same angles as those lines;

one such line element on the first surface will be

$$\sqrt{(a^2 + b^2 + c^2)dt^2 + 2(aa' + bb' + cc')dt du + (a'^2 + b'^2 + c'^2)du^2} \quad (\textit{id.}).$$

By the Pythagorean theorem the line element extending from a point dx, dy is $\sqrt{dx^2 + dy^2}$, the line element extending from a point dx, dy, dz is $\sqrt{dx^2 + dy^2 + dz^2}$. Gauss continues:

the corresponding line element on the second surface will be

$$\sqrt{(A^2 + B^2 + C^2)dt^2 + 2(AA' + BB' + CC')dt du + (A'^2 + B'^2 + C'^2)du^2};$$

and if, independently of dt and du , the two line elements will be in a definite relation, obviously the three magnitudes

$a^2 + b^2 + c^2, aa' + bb' + cc', a'^2 + b'^2 + c'^2$ must be respectively proportional to the three magnitudes

$$A^2 + B^2 + C^2, AA' + BB' + CC', A'^2 + B'^2 + C'^2;$$

when to the end points of a second line element on the first surface the values

$$t, u \text{ and } t + \delta t, u + \delta u$$

correspond, the cosine of the angle which this element makes with the first line element is

$$\frac{(adt + a'du)(a\delta t + a'\delta u) + (bdt + b'du)(b\delta t + b'\delta u) + (cdt + c'du)(c\delta t + c'\delta u)}{\sqrt{[(adt + a'du)^2 + (bdt + b'du)^2 + (cdt + c'du)^2][(a\delta t + a'\delta u)^2 + (b\delta t + b'\delta u)^2 + (c\delta t + c'\delta u)^2]}}$$

and the expression for the cosine of the angle between the corresponding line elements on the second surface is completely similar with A, B, C, A', B', C' in place of a, b, c, a', b', c' ; obviously the two expressions become equal when the demanded proportionality occurs; the second condition is therefore contained in the first; which with a little reflection is also clear by itself;

the analytic expression of the condition of our problem is therefore this, it must be that

$$\frac{A^2 + B^2 + C^2}{a^2 + b^2 + c^2} = \frac{AA' + BB' + CC'}{aa' + bb' + cc'} = \frac{A'^2 + B'^2 + C'^2}{a'^2 + b'^2 + c'^2},$$

and this be a finite function of t and u , which we set $= m^2$;

m thus expresses the relation in which linear magnitudes on the first surface are increased or decreased in their representation on the second surface according as m is greater or less than 1; in general this relation will be different in different places; in the special case when m is constant, there will be complete similarity also in the finite parts; and when $m = 1$, there will be a complete equality, and the one surface can be developed on the other (*id.*).

Each x, y, z and X, Y, Z is a function of t and u . Thus,

$$\begin{aligned} dx &= a dt + a' du, \\ dy &= b dt + b' du, \\ dz &= c dt + c' du, \\ dX &= A dt + A' du, \\ dY &= B dt + B' du, \\ dZ &= C dt + C' du. \end{aligned}$$

And in order to be a solution of the problem, so that a line element on the first surface is proportional to the corresponding line elements on the second, the function of t and u , which Gauss now sets $= m^2$, has to be such that

$$\frac{A^2 + B^2 + C^2}{a^2 + b^2 + c^2} = \frac{AA' + BB' + CC'}{aa' + bb' + cc'} = \frac{A'^2 + B'^2 + C'^2}{a'^2 + b'^2 + c'^2}.$$

Gauss continues:

5. if for brevity we set

$$(a^2 + b^2 + c^2)dt^2 + 2(aa' + bb' + cc')dt du + (a'^2 + b'^2 + c'^2)du^2 = \omega,$$

we note that the differential equation $\omega = 0$ will allow two integrations, for, in relation to the linear dt and du , the trinomial ω resolves itself into two factors

and one or the other factor $= 0$, which yields two different integrations; to the one integration will correspond the equation

$$0 = (a^2 + b^2 + c^2)dt + [a'^2 + b'^2 + c'^2 + i\sqrt{(a^2 + b^2 + c^2)(a'^2 + b'^2 + c'^2) - (aa' + bb' + cc')^2}]du,$$

where i for brevity is written for $\sqrt{-1}$, as it is readily seen that the irrational part of the expression must be imaginary; to the other integration will correspond a completely similar equation with $-i$ in place of i ;

thus the integral of the first equation is

$$p + i q = \text{const.},$$

where p and q indicate real functions of t and u ; the other integral is

$$p - i q = \text{const.};$$

and it follows from the nature of the case that

$$(dp + i dq)(dp - i dq), \text{ or } dp^2 + dq^2$$

must be a factor of ω , and that is,

$$\omega = n(dp^2 + dq^2),$$

where n will be a finite function of t and u ;

now the trinomial, into which

$$dX^2 + dY^2 + dZ^2$$

changes when for dX, dY, dZ their values in T, U, dT, dU are substituted, we will indicate by Ω , and will suppose that, just as before, the two integrals of the equation $\Omega = 0$ are these

$$P + i Q = \text{const.},$$

$$P - i Q = \text{const.},$$

and

$$\Omega = N(dP^2 + dQ^2),$$

where P, Q, N indicate real functions of T and U ;

ignoring the general difficulties of integration, these integrations can obviously be performed before the solution of our main problem;

if now for T, U such functions of t, u are substituted by which the condition of our main problem is fulfilled, Ω passes over into $m^2\omega$, and

hence

$$\frac{(dP + i dQ)(dP - i dQ)}{(dp + i dq)(dp - i dq)} = \frac{m^2 n}{N} (id.).$$

m^2 is the function of t and u which will establish the proportionality of the line elements of the two surfaces and thus solve the problem, n and N establish the numerical relation of the line elements, p and q are real functions of t and u , P and Q are real functions of T and U , and to solve the problem entirely generally and completely it remains only to express P and Q in terms of p and q . Gauss continues:

it is easily seen that in the first part of this equation the numerator can be divisible by the denominator only when either $dP + i dQ$ is divisible by $dp + i dq$ and $dP - i dQ$ by $dp - i dq$, or $dP + i dQ$ is divisible by $dp - i dq$ and $dP - i dQ$ by $dp + i dq$;

in the first case, therefore, $dP + i dQ$ will vanish when $dp + i dq = 0$, or $dP + i dQ$ will be constant when $dp + i dq$ is assumed constant, and that is, $P + i Q$ will be a function only of $p + i q$, and $P - i Q$ a function of $p - i q$; in the second case, $P + i Q$ will be a function of $p - i q$ and $P - i Q$ a function of $p + i q$; it is easily seen that these consequences hold also conversely, namely, when $P + i Q$, $P - i Q$ are assumed functions of $p + i q$, $p - i q$, and either respectively or reversely, the finite divisibility of Ω by ω , and hence also the required proportionality, holds;

moreover, one easily sees that if, for instance, we set

$$\begin{aligned} P + i Q &= f(p + i q), \\ P - i Q &= f'(p - i q), \end{aligned}$$

the character of the function f' is determined by that of f ; namely, when the constant magnitudes which f may involve are all real, then f' must be completely identical with f , so that to all real values of p , q real values of P , Q correspond; in the opposite case f' will differ from f only in that in the imaginary elements of f everywhere $-i$ must be set in place of i ;

we therefore have

$$\begin{aligned} P &= \frac{1}{2}f(p + i q) + \frac{1}{2}f'(p - i q), \\ i Q &= \frac{1}{2}f(p + i q) - \frac{1}{2}f'(p - i q), \end{aligned}$$

or, what is the same, when the function f is assumed entirely arbitrary, including constant imaginary elements at will, P is set equal to the

real part and $i Q$, or in the second case $-i Q$, equal to the imaginary part of $f(p + i q)$, then after the elimination of T and U these will be functions of t and u ;

and thus the given problem is entirely generally and completely solved;

7. and if the functions which arise from the differentiation of the arbitrary functions f , f' are indicated by ϕ and ϕ' , respectively, so that

$$df(v) = \phi(v) dv, \quad df'(v) = \phi'(v) dv,$$

it follows from our general solution that

$$\frac{dP + i dQ}{dp + i dq} = \phi(p + i q), \quad \frac{dP - i dQ}{dp - i dq} = \phi'(p - i q);$$

therefore

$$\frac{m^2 n}{N} = \phi(p + i q) \cdot \phi'(p - i q);$$

and the relation of magnification is therefore determined by the formula

$$m = \sqrt{\frac{dp^2 + dq^2}{\omega} \cdot \frac{\Omega}{dP^2 + dQ^2} \cdot \phi(p + i q) \cdot \phi'(p - i q)} (id.).$$

The history of the discovery of non-Euclidean geometry, too, has been beset, to a degree proportionate to the importance of the discovery, by the distribution of credit and disgrace.

The great excitement of the mathematicians, in particular, about non-Euclidean geometry, and of the philosophers, in general, about the matter of general interest, the overthrow of Euclid, showing conclusively that mathematics is man-made, now made by the man Euclid now made by the man non-Euclid, began in the 1860's. The former can be ascribed to the quick and great impact of the labors of Riemann. The latter can be ascribed to the quick and by that time very much greater impact of the labors of Hegel.

In the meantime, Saccheri's *Euclid freed from all blemish*, published in 1733, remained in relative obscurity. Lambert's *Theory of parallel lines*, published posthumously in 1786, remained in relative obscurity. Lobachevsky's *On the principles of geometry*, in 1829, *New principles of geometry, with a complete theory of parallels*, in 1836-8, and *Geometrical investigations on the theory of parallels*, in 1840, were one after the other met with scepticism, and remained in relative obscurity. Bolyai's *True science of absolute space, independent of the truth or falsehood of Euclid's postulate, which can never*

be decided *a priori*, in 1832, was met with scepticism, and remained in relative obscurity. After the death of Gauss in 1855, the publication of his correspondence revealed that Gauss had been obtaining results in non-Euclidean geometry since 1799. A large number of earlier attempts to prove Euclid's postulate was brought out of obscurity. And two other anticipations of results in non-Euclidean geometry were brought out of obscurity, of Schweikart in 1818, and Taurinus in 1826.

Accordingly, the history of the discovery of non-Euclidean geometry, and rather, man-made as it is, the history of its invention, has been dominated by the question of who appropriated what from whom without, clearly, acknowledging the appropriation. There are many historians, Gibbon writes, who put us in mind of the admirable saying of the great Condé, *ces coquins nous font parler et agir comme ils auraient fait eux-mêmes à notre place*.

Below, the results of Lobachevsky are presented. Then the results of Bolyai, and Gauss. Then the question of when, and how and why non-Euclidean geometry was discovered is addressed.

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In 1826 Lobachevsky delivered a lecture, *A brief exposition of the principles of geometry with a rigorous demonstration of the theorem of parallels*. The lecture is lost.

In 1829 he published *On the principles of geometry*. He writes:

the sum of the angles of the spherical triangle is always $> \pi$;

the sum of the angles of the rectilinear triangle cannot be $> \pi$; it remains to assume that this sum is either $= \pi$, or $< \pi$; each of these can be assumed without leading to any contradiction in its consequences; and from this two geometries arise; one, customary to the present day, because of its simplicity, and in agreement in fact with all measurements; the other, imaginary, more general, and for this reason more difficult in its calculations, admits of the dependence of lines on angles (*id.*).

And in *On the principles of geometry* the other geometry, imaginary, more general, and for this reason more difficult in its calculations, admitting

of the dependence of lines on angles, is fully developed.

It is presented again in 1835, in the *Imaginary geometry*.

It is developed further in 1836-8, in the *New principles of geometry, with a complete theory of parallels*.

It is presented again in 1840, in the *Geometrical investigations on the theory of parallels*.

And it is presented yet again in 1856, in the *Pangeometry*.

Its exposition in 1840, in the *Geometrical investigations on the theory of parallels*, is brief. It is also of the greatest clarity and distinction, and elegance.

In particular, the exposition is in – as Lobachevsky calls them – 37 theorems. The first 15 theorems are the necessary definitions. Lobachevsky continues:

from here on follow the other theorems with their explanations and proofs;

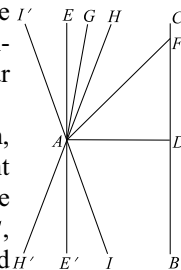
16. all straight lines in a plane through one point can, with respect to a given straight line in the same plane, be divided into two classes, intersecting and not intersecting;

the boundary lines separating the two classes are called parallel to the given line;

from point A drop to the given straight line BC perpendicular AD , and perpendicular to it again draw AE ;

in the right angle EAD , then, all straight lines through point A either intersect DC , like the line AF , or do not intersect DC , like the perpendicular AE ; and in the uncertainty whether the perpendicular AE is the only straight line which does not intersect DC , we assume that there may be other lines, like AG , which, however far they may be produced, do not intersect DC ; and passing from the intersecting lines, like AF , to the not intersecting lines, like AG , we must come on a line AH which is parallel to DC , a boundary line, on one side of which all straight lines AF intersect DC , on the other side all straight lines AG do not intersect DC ;

the angle HAD between the parallel HA and the perpendicular AD is called the angle of



parallelism, and is designated $\Pi(p)$ for $AD = p$;

if $\Pi(p) = \frac{1}{2}\pi$, the extension AE' of the perpendicular AE is also parallel to the extension DB of DC ; and, other than the parallel EE' , every straight line through A in either of the two right angles at A toward BC intersects BC ;

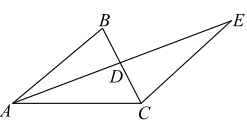
if $\Pi(p) < \frac{1}{2}\pi$, then on the other side of AD , making the same angle $DAI = \Pi(p)$, lies the straight line AI , parallel to the extension DB of DC ; on this assumption we must thus distinguish sides of parallelism; and, when the extensions AH' and AI' of the parallels HA and IA are drawn, all straight lines passing through A in the angles HAI or $H'AI'$ intersect BC , all straight lines in the angles HAI' or IAH' do not intersect BC ;

accordingly, on the assumption $\Pi(p) = \frac{1}{2}\pi$ the lines can be only intersecting or parallel; but on the assumption $\Pi(p) < \frac{1}{2}\pi$ there are two parallels, and the remaining lines must be distinguished into not intersecting and intersecting (*id.*).

Lobachevsky proves that a straight line remains parallel to a given straight line at all its points (17), that two lines are always mutually parallel (18), and continues:

19. in the rectilinear triangle the sum of the angles cannot be greater than two right angles;

for let the sum of the angles in the triangle ABC be $\pi + a$;

bisect the smallest side BC at D ; join AD and produce it to E so that $AD = DE$; join CE ; 

triangles ABD and CDE are congruent; the sum of the angles of triangle ACE is thus also $\pi + a$; but the smallest angle of the triangle ABC has been bisected; and continuing this, bisecting the side opposite the smallest angle in the new triangle, a triangle must be reached in which the sum of the angles is $\pi + a$, but two of the angles are each less than $\frac{1}{2}\pi$; and since the third angle cannot be less than π , a must be 0 or negative (*id.*).

Lobachevsky proves that, if the sum of the angles in any rectilinear triangle is $= \pi$, so is the sum in

all rectilinear triangles (20), and continues:

it follows that only two hypotheses are possible; either the sum of the angles in all rectilinear triangles is $= \pi$, or the sum of the angles in all rectilinear triangles is $< \pi$ (*id.*).

He proves that from a given point a straight line can be drawn making with a given straight line an angle smaller than any given angle (21), that, if two perpendiculars to the same straight line are parallel to one another, the sum of the angles in the rectilinear triangle is $= \pi$ (22), and continues:

it follows that either in all rectilinear triangles the sum of the angles is $= \pi$ and the angle of parallelism is $\Pi(p) = \frac{1}{2}\pi$ for every line p , or in all rectilinear triangles the sum of the angles is $< \pi$ and the angle of parallelism is $\Pi(p) < \frac{1}{2}\pi$;

the first assumption is the foundation of ordinary geometry and plane trigonometry;

the second assumption does not lead to any contradiction in its consequences either, and is the foundation of a new science of geometry, which I call imaginary geometry, and which I will develop now as far as the equations relating the sides and angles of the rectilinear and spherical triangle;

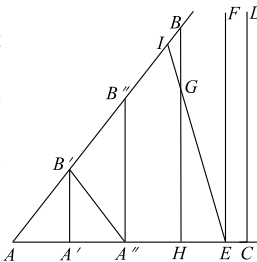
23. for every given angle α there is a line p such that $\Pi(p) = \alpha$;

let the straight lines AB and AC intersect at any acute angle α ;

take on AB any point B' , and drop perpendicular $B'A'$ to AC ; make $A'A'' = AA'$; erect at A'' perpendicular $A''B''$; and con-

tinue in this way until the perpendicular CD is reached, which no longer intersects AB ; this must happen, for, if in the triangle $AA'B'$ the sum of the angles is $\pi - \alpha$, in the triangle $AB'A''$ it is $\pi - 2\alpha$, in the triangle $AA''B''$ it is less than $\pi - 2\alpha$, and so on, until it becomes negative and it is thus no longer possible to construct a triangle;

thus, it may be that CD is the perpendicular, on one side of which all perpendiculars intersect AB , on the other side all perpendiculars do not intersect AB ; in any case, such a perpendicular EF



must exist;

draw from point E on AC toward A the straight line EG making with EF the acute angle FEG ; from any point G on EG drop perpendicular GH to AC , which produced must intersect AB somewhere at B , completing the triangle ABH ; EG thus enters this triangle and must intersect the hypotenuse somewhere at I ; but the angle FEG can be smaller than any given angle, and EF is therefore parallel to AB and $AE = p$;

and it is easy to see that as p decreases α increases, and as p approaches 0, α approaches $\frac{1}{2}\pi$; as p increases, α decreases, and as p approaches infinity, α approaches 0 (*id.*).

Two hypotheses are possible. The assumption that in all rectilinear triangles the sum of the angles is $=\pi$, and the angle of parallelism is $\Pi(p) = \frac{1}{2}\pi$ for every line p , which is the foundation of ordinary geometry. And the assumption that in all rectilinear triangles the sum of the angles is $<\pi$, and the angle of parallelism is $\Pi(p) < \frac{1}{2}\pi$, which is the foundation of a new science of geometry, which Lobachevsky has called imaginary geometry. And 8 theorems into the exposition of the new science of imaginary geometry, he has shown that the angle of parallelism is a function of length, $\Pi(p) = \alpha$. As p decreases α increases, as p increases, α decreases.

As p approaches infinity, α approaches 0. As α decreases, p reaches a point when it becomes no longer possible to construct a triangle. There is thus a greatest triangle.

And as p approaches 0, α approaches $\frac{1}{2}\pi$. 8 theorems into its exposition, Lobachevsky has shown that in the new science of imaginary geometry the old science of ordinary geometry is contained as a special case.

Next Lobachevsky proves that parallel lines are asymptotic on the side of their parallelism (24), and that two straight lines parallel to a third are parallel to one another (25). He defines the size of the spherical triangle (26-7), and continues:

28. if three planes intersect one another in parallel lines, the sum of the three surface angles

is equal to two right angles;

let three planes intersect in the parallel lines AA' , BB' , and CC' ;

let through any points A , B , and C on each of the straight lines a plane pass, which therefore intersects the three planes in the straight lines AB , AC , and BC ;

let another plane pass through any point D on BB' and the straight line AC , whose intersection with the plane of the parallels AA' and BB' is AD and intersection with the plane of the parallels BB' and CC' is CD ; let the inclination of this plane to the plane of the parallels AA' and CC' be w ;

let the angles between the three planes in which the parallels lie, at AA' , BB' , and CC' , respectively, be X , Y , and Z (*id.*).

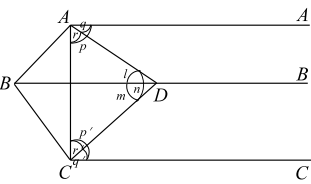
These are the three surface angles with which the theorem is concerned. Lobachevsky continues:

about A as a center describe a sphere; on the surface of this sphere the intersections of the straight lines AC , AD , and AA' determine a spherical triangle with sides p , q , and r ; let the size of this spherical triangle be α ; then opposite side q lies angle w , opposite side r angle X , and therefore opposite side p angle $\pi + 2\alpha - w - X$ (*id.*).

If the sum of the angles of the spherical triangle is S , the excess $S - \pi$ is proportional to the area of the triangle. In the previous theorem 27 Lobachevsky has defined the size of the spherical triangle as $\frac{1}{2}(S - \pi)$. Thus, in the spherical triangle of size α , if two of the angles are w and X , the third is $\pi + 2\alpha - w - X$. Lobachevsky continues:

in the same way, the intersections of the straight lines CA , CD , and CC' determine on the surface of a sphere with center C a spherical triangle of size β with sides p' , q' , and r' , and angles w opposite q' , Z opposite r' , and $\pi + 2\beta - w - Z$ opposite p' ;

and the intersections of the straight lines DA , DB , and DC determine on the surface of a sphere with center D a spherical triangle with sides l , m , and n and angles opposite them $w + Z - 2\beta$, $w + X - 2\alpha$, and Y , and its size is therefore



$$\delta = \frac{1}{2}(X + Y + Z - \pi) - \alpha - \beta + w;$$

decreasing w decreases also the size of the triangles α and β , and $\alpha + \beta - w$ can be made smaller than any given quantity;

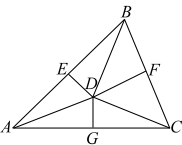
in the same way, in the triangle δ the sides l and m can be decreased more than any given quantity, therefore δ vanishes with w ; and therefore

$$X + Y + Z = \pi \text{ (id.)}.$$

The three surface angles X , Y , and Z at the three planes intersecting in parallel lines, whose sum is equal to two right angles, will in the following theorems turn out to be the angles between horocycles on the surface of a horosphere. Lobachevsky continues:

29. in the rectilinear triangle the perpendiculars erected at the midpoints of the sides either intersect in one point or do not intersect;

if it is assumed that in the triangle ABC the perpendiculars ED and FD erected at the midpoints of AB and BC intersect at D , join DA , DB , and DC ;



triangles ADE and BDE are congruent; so $AD = BD$; it follows that $BD = CD$; the triangle ADC is isosceles, and the perpendicular DG from D bisects AC at G ;

and if it is assumed that two of the perpendiculars do not intersect, the third cannot intersect them either;

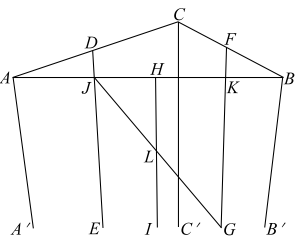
30. if it is assumed, in particular, that two of the perpendiculars are parallel to one another, all three are parallel to one another;

in the triangle ABC let DE , FG , and HI be the perpendicular at the midpoints of the sides AC , BC , and AB ;

let first DE and FG be parallel, intersecting AB at J and K with the perpendicular HI between them;

in the angle BJE draw from J the straight line JG ; it must intersect FG somewhere at G however small the angle GJE may be;

since in the tri-



angle JKG the perpendicular HI cannot intersect KG , it intersects JG somewhere at L , from which it follows that HI is parallel to FG and therefore also to DE ;

let $BC = 2a$, $AC = 2b$, and $AB = 2c$; and the angles of the triangle are then

$$A = \Pi(b) - \Pi(c),$$

$$B = \Pi(a) - \Pi(c),$$

$$C = \Pi(a) + \Pi(b),$$

which can easily be shown with the lines AA' , BB' , and CC' drawn from A , B , and C parallel to the perpendicular HI and therefore to the perpendiculars DE and FG as well;

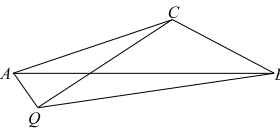
let now HI and FG be parallel;

DE cannot intersect them, and is either parallel to them, or intersects AA' ;

the assumption that it intersects AA' is no other than that $C > \Pi(a) + \Pi(b)$;

if then the angle

C is decreased to become equal to $\Pi(a) + \Pi(b)$, the line CA is thus



moved to the new position CQ ;

let $BQ = 2c'$; the angle at B has also been increased, and $\Pi(a) - \Pi(c') \neq \Pi(a) - \Pi(c)$; hence $c' > c$;

but in the triangle ACQ the angles at A and Q are equal, so in the triangle ABQ the angle at Q is greater than the angle at A ; hence $c > c'$;

31. the curve in a plane such that all perpendiculars erected at the midpoints of the chords are parallel to one another is called horocycle;

the curve can be generated in this way;

if at a point A on a given straight line

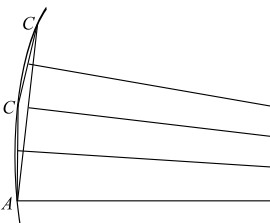
AB , chords $AC = 2p$ are drawn, making

angles $CAB = \Pi(p)$, the ends C of these

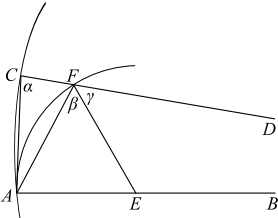
chords lie on a horocycle, and gradually determine it;

the perpendiculars at the midpoints of the chords AC and parallel to AB are called axes of the horocycle;

32. a circle with continually increasing radius merges into the horocycle;



given a chord AC of the horocycle, draw from its end-points A and C the axes AB and CD , making with the chord the equal angles $CAB = ACD = \alpha$;



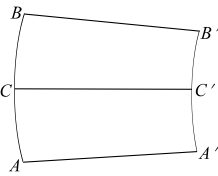
on one of the axes take any point E as the center of a circle, and with radius EA draw arc AF intersecting the other axis at F ;

the radius EF makes with the chord AF angle $AFE = \beta$ and with the axis CF angle $EFD = \gamma$; the angle between the two chords is thus $CAF = \alpha - \beta < \beta - \alpha$; and from this follows that $\alpha - \beta < \frac{1}{2}\gamma$;

and since, either when the center of the circle E moves in the direction AB or when F approaches C , angle γ approaches 0, angle $\alpha - \beta$ tends to vanish;

therefore, the horocycle can be called a circle with infinitely great radius;

33. let $AA' = BB' = x$ be two lines parallel in the direction AA' and axes of the two arcs of horocycles $AB = s$ and $A'B' = s'$;



then $s' = se^{-x}$;

to prove this, assume that the ratio of s to s' is equal to that of the whole numbers n and m ;

between AA' and BB' draw a third axis CC' , cutting off from AB the part $AC = t$ and from $A'B'$ the part $A'C' = t'$; assume the ratio of t to s to be equal to that of the whole numbers p and q , so that

$$s = \frac{n}{m} s', \quad t = \frac{p}{q} s;$$

divide s by axes into nq equal parts, and then there are mq such parts in s' and np in t ;

but to these equal parts in s and t equal parts in s' and t' correspond;

$$\text{therefore, } \frac{t'}{t} = \frac{s'}{s};$$

and however the two arcs t and t' between the axes AA' and BB' are taken, as long as the length x remains the same, the ratio of t to t' remains the same; and if, therefore, for $x = 1$, $s = es'$ is set, then for every x

$$s' = se^{-x};$$

and since e is subject only to the condition $e > 1$, and the linear unit for x is arbitrary, e can be taken to be the base of Napierian logarithms;

34. the surface arising from the revolution of the horocycle about one of its axes is called horosphere;

every axis can be considered the axis of revolution;

every plane which passes through an axis is called principal plane;

every principal plane cuts the horosphere in a horocycle, and every other plane cuts it in a circle;

and three principal planes cut one another in angles whose sum is π ;

these are angles between the arcs of horocycles, the intersections of three principal planes on the horosphere;

therefore, the same dependence of sides and angles obtains for triangles whose sides are horocycles as for rectilinear triangles in ordinary geometry (*id.*).

19 theorems into its exposition, Lobachevsky has shown that in the new science of imaginary geometry there is a surface, the horosphere, on which the old science of ordinary geometry holds.

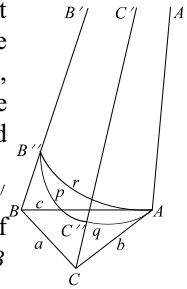
Finally:

35. let ' indicate a line, like x' , which is in this relation to another line, like x ,

$$\Pi(x) + \Pi(x') = \frac{1}{2}\pi;$$

let ABC be a rectilinear right triangle, with hypotenuse $AB = c$, and sides $AC = b$, $BC = a$, and angles opposite them $BAC = \Pi(\alpha)$ and $ABC = \Pi(\beta)$;

at point A erect the line AA' perpendicular to the plane of the triangle ABC , and at points B and C draw BB' and CC' parallel to AA' ;

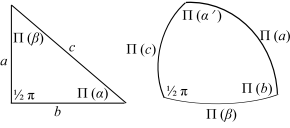


the planes in which these three parallels lie make with one another the angles $\Pi(\alpha)$ at AA' , $\frac{1}{2}\pi$ at CC' , and thus $\Pi(\alpha')$ at BB' ;

the intersections of the lines BA , BC , and BB' with a sphere about the center B determine a spherical triangle with sides $\Pi(c)$, $\Pi(\beta)$, and $\Pi(a)$, and opposite angles $\Pi(b)$, $\Pi(\alpha')$, and

$$\frac{1}{2}\pi;$$

therefore, the existence of a rectilinear triangle with sides a , b , and c and opposite angles



$\Pi(\alpha)$, $\Pi(\beta)$, and $\frac{1}{2}\pi$ implies the existence of a spherical triangle with sides $\Pi(c)$, $\Pi(\beta)$, and $\Pi(\alpha)$ and opposite angles $\Pi(b)$, $\Pi(\alpha')$, and $\frac{1}{2}\pi$;

and the existence of this spherical triangle in turn implies the existence of a rectilinear triangle with sides a , a' , and β and opposite angles $\Pi(b')$, $\Pi(c)$, and $\frac{1}{2}\pi$;

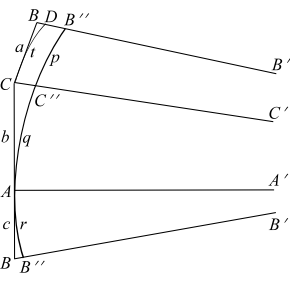
we can therefore pass from a , b , c , α , β to b , a , c , β , α as well as to a , a' , β , b' , c ;

draw through point A with AA' as axis a horosphere, which cuts the other axes BB' and CC' in B'' and C'' ;

its intersection with the planes of the three parallels determines a triangle of horocycles with sides $B''C'' = p$, $C''A = q$, and $B''A = r$ and opposite angles $\Pi(\alpha)$, $\Pi(\alpha')$, and $\frac{1}{2}\pi$, and in which therefore

$$p = r \sin \Pi(\alpha), \quad q = r \cos \Pi(\alpha);$$

break the connection of the three principal planes along the line BB' , and lay them down with all lines on them in one plane, so that therefore the arcs p , q , and r join in a single horocycle through point A with AA' as axis;



thus, on one side of axis AA' lie the arcs q and p , the side b of the triangle ABC perpendicular to AA' at A , the axis CC' from the endpoint of b parallel to AA' intersecting the horocycle at C'' , and from the endpoint of a the axis BB' parallel to AA' and intersecting the horocycle at B'' ;

on the other side of AA' lie the arc r , the side c perpendicular to AA' at A , and the axis BB' ;

the length of line CC'' depends on b , which we express thus $CC'' = f(b)$;

in the same way, $BB'' = f(c)$;

and if now, with CC' as axis another horocycle is drawn from point C to its intersection D with the axis BB' , and the arc $CD = t$, then $BD = f(a)$;

$$BB'' = BD + DB'' = BD + CC'';$$

$$\text{therefore } f(c) = f(a) + f(b);$$

moreover, we see that

$$t = p e^{f(b)} = r \sin \Pi(\alpha) e^{f(b)};$$

and if the perpendicular to the plane of the triangle ABC is erected at B instead of A , then the lines c and r remain the same, the arcs q and t become t and q , the straight lines a and b become b and a , angle $\Pi(\alpha)$ becomes $\Pi(\beta)$, and therefore

$$q = r \sin \Pi(\beta) e^{f(a)};$$

hence, by substitution,

$$\cos \Pi(\alpha) = \sin \Pi(\beta) e^{f(a)};$$

and if α and β are changed into b' and c ,

$$\sin \Pi(b) = \sin \Pi(c) e^{f(a)};$$

multiplying by $e^{f(b)}$,

$$\sin \Pi(b) e^{f(b)} = \sin \Pi(c) e^{f(c)};$$

from which it follows also

$$\sin \Pi(a) e^{f(a)} = \sin \Pi(b) e^{f(b)};$$

and since the straight lines a and b are independent of one another, and moreover, for $b = 0$, $f(b) = 0$, and $\Pi(b) = \frac{1}{2}\pi$, it follows that for every

straight line a

$$e^{-f(a)} = \sin \Pi(a);$$

therefore,

$$\sin \Pi(c) = \sin \Pi(a) \sin \Pi(b),$$

$$\sin \Pi(\beta) = \cos \Pi(\alpha) \sin \Pi(a);$$

and by changing the letters,

$$\sin \Pi(\alpha) = \cos \Pi(\beta) \sin \Pi(b),$$

$$\cos \Pi(b) = \cos \Pi(c) \cos \Pi(\alpha),$$

$$\cos \Pi(a) = \cos \Pi(c) \cos \Pi(\beta);$$

and if in the right spherical triangle the sides $\Pi(c)$, $\Pi(\beta)$, and $\Pi(a)$ with the opposite angles $\Pi(b)$ and $\Pi(\alpha')$, are indicated by a , b , c , A , and B , then the equations take the form known in spherical trigonometry for the right triangle, namely,

$$\sin a = \sin c \sin A,$$

$$\sin b = \sin c \sin B,$$

$$\cos A = \cos a \sin B,$$

$$\cos B = \cos b \sin A,$$

$$\cos c = \cos a \cos b;$$

and from these equations the equations for all spherical triangles in general can be obtained;

spherical trigonometry is therefore not depend-

ent of whether the sum of the angles of the triangle is equal to two right angles or not (*id.*).

In 36, next to the more general result $e^{-f(x)} = \sin \Pi(x)$, Lobachevsky obtains the explicit:

for every line x in general, whether positive or negative,

$$\tan \frac{1}{2} \Pi(x) = e^{-x} \text{ (id.)}.$$

In 37 he completes the equations relating the sides and angles of the rectilinear and spherical triangle. He has shown earlier that in $\Pi(p) = \alpha$, as p approaches 0, α approaches $\frac{1}{2}\pi$, and that on the surface of the horosphere ordinary geometry holds, and the equations here show that:

when the sides of the triangle are assumed very small imaginary geometry passes over into ordinary geometry (*id.*).

Lobachevsky concludes:

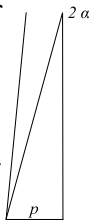
the above equations suffice to show that the assumption of imaginary geometry is possible;

and there is no way, other than astronomical observation, to judge the exactness of the calculations of ordinary geometry; and this exactness reaches very far, as I have shown in one of my investigations; in triangles accessible to our measurements, for instance, the sum of the three angles does not differ from two right angles by one hundredth of one second (*id.*).

Lobachevsky has first shown this in 1829, in *On the principles of geometry*. He writes:

in the theory which we have developed lines depend on angles; and whether this dependence obtains in nature or not, no one can prove; in any case, astronomical observations show that all lines accessible to our measurements are so small that the equations of ordinary geometry must hold without noticeable error;

thus, if the major axis of the orbit of the earth around the sun is p , and the largest annual parallax of a fixed star is 2α , the angle between p and the distance from one end of p to the star, assuming the distance perpendicular to the other end, is $\frac{\pi}{2} - 2\alpha$;



then it must be that $\Pi(p) > \frac{\pi}{2} - 2\alpha$ (*id.*).

The angle of parallelism must be greater than the acute angle at one end of the distance to the star.

Lobachevsky continues:

$$\Pi(p) > \frac{\pi}{2} - 2\alpha;$$

$$\text{hence, } e^p < \frac{1 + \tan \alpha}{1 - \tan \alpha};$$

$$\frac{1}{2}p < \tan \alpha + \frac{1}{3} \tan \alpha^3 + \frac{1}{5} \tan \alpha^5 + \dots$$

and so much more

$$p < \tan 2\alpha;$$

and assuming a parallax of Sirius of $1''.24$, $p < 0.00000602$;

and in such a triangle the sum of the angles cannot differ from two right angles by more than $0''.000372$ (*id.*).

In the theory which Lobachevsky has developed lines depend on angles. In general, $\Pi(p) = \alpha$, the angle of parallelism α is a function of the distance p of the point through which the parallel to a given straight line passes. In particular, $\tan \frac{1}{2} \Pi(p) = e^{-p}$.

As p approaches 0, α approaches $\frac{1}{2}\pi$. When the sides of the triangle are very small, imaginary geometry passes over into ordinary geometry. Thus, whether the dependence of lines on angles obtains in nature or not – whether imaginary geometry obtains in nature or ordinary geometry obtains in nature – astronomical observation has to decide. Triangles accessible to our measurements have to decide. If, then, p is the major axis of the orbit of the earth around the sun, and the angle of parallelism α is greater than the angle between p and the distance to a fixed star whose largest annual parallax is measured, assuming the distance perpendicular to the other end, $\tan \frac{1}{2} \Pi(p) = e^{-p}$ yields $p < 0.00000602$. And in such a triangle the sum of the angles cannot differ from two right angles by more than $0''.000372$. Such a triangle is very small.

In 1829 Lobachevsky assumes an annual parallax of Sirius of $1''.24$. In 1838 Bessel measured the annual parallax of 61 Cygni at $0''.3136$. Such a triangle is substantially smaller. In 1829, also, Lobachevsky infers that the distance of Sirius is very small from the theory which he has developed. In 1785, in order to fit the map of the Milky Way on a page, Herschel has to draw it on a scale where the distance of Sirius is not more than $\frac{1}{80}$ of an inch.

ccccxxxi

The greatest advance is the most general. The sum of the angles of the rectilinear triangle is either $= \pi$, or $< \pi$. Each of these can be assumed without any contradiction in its consequences. And from this two geometries arise. The first assumption is the foundation of ordinary geometry, customary to the present day, because of its simplicity. The second assumption is the foundation of the new science of imaginary geometry, more general, and for this reason more difficult in its calculations.

And all the following are a major advance each, on which the most general advance stands.

The angle of parallelism α is a function of the distance p of the point through which the parallel to a given straight line passes, $\Pi(p) = \alpha$.

On the assumption that in all rectilinear triangles the sum of the angles is $= \pi$, the angle of parallelism is $\Pi(p) = \frac{1}{2} \pi$ for every line p .

On the assumption that in all rectilinear triangles the sum of the angles is $< \pi$, the angle of parallelism is $\Pi(p) < \frac{1}{2} \pi$, and for every angle α there is a line p such that $\Pi(p) = \alpha$.

If $\Pi(p) = \frac{1}{2} \pi$, the straight lines through a point with respect to a given straight line are intersecting or parallel.

If $\Pi(p) < \frac{1}{2} \pi$, the straight lines through a point with respect to a given straight line are two parallels, and intersecting as well as not intersecting.

As p approaches infinity, α approaches 0, as p approaches 0, α approaches $\frac{1}{2} \pi$.

As α decreases, p reaches a point when it is no longer possible to construct a triangle. There is a greatest triangle.

There is a curve, the horocycle, such that all perpendiculars erected at the midpoints of the chords are parallel.

The horocycle is a circle of infinite radius.

The surface of revolution of the horocycle is the horosphere.

The geometry of the horosphere is ordinary geometry. In imaginary geometry a surface has thus been constructed on which ordinary geometry holds. Ordinary geometry is thus consistent

with imaginary geometry.

Spherical trigonometry is independent of the assumption that the sum of the angles of the triangle is equal to two right angles. Spherical trigonometry is independent of Euclid's postulate.

For every line x , $\tan \frac{1}{2} \Pi(x) = e^{-x}$.

When the sides of the triangle are very small imaginary geometry passes over into ordinary geometry.

Whether imaginary geometry or ordinary geometry obtains in nature astronomical observation has to decide.

The sum of the angles of the triangle determined by the major axis of the orbit of the earth and a fixed star whose parallax is measured cannot, on the assumption of a parallax of $1''.24$, differ from two right angles by more than $0''.000372$. Taking Bessel's measurement at $0''.3136$ with a mean error of $\pm 0''.0202$ as a standard, either the measured triangle has to be greater or the precision of its measurement has to be greater by not less than an order of magnitude.

All these results Lobachevsky obtains in *On the principles of geometry*, in 1829, barring some of the elegance of their exposition in the *Geometrical investigations on the theory of parallels*, in 1840.

Bolyai published the *True science of absolute space, independent of the truth or falsehood of Euclid's postulate, which can never be decided a priori*, in 1832.

And in the *True science of absolute space*, in 1832, Bolyai obtains independently the majority of the results which Lobachevsky has obtained in 1829.

Most important, the most general result. There are two geometries. Bolyai writes:

the system of geometry resting on the hypothesis of truth of Euclid's postulate we call Σ , the system founded on the contrary hypothesis S (15).

The system Σ in Bolyai is ordinary geometry in Lobachevsky, the system S in Bolyai is imaginary geometry in Lobachevsky. Bolyai adds:

all things, which are not expressly said to be either in Σ or in S , are enunciated absolutely, and that is, they are asserted true whether Σ or S is real (*id.*).

Bolyai draws this conclusion after considering

the straight lines through a point with respect to a given straight line, and distinguishing the two cases, when all lines are only parallels and intersecting, and when they are parallel, intersecting, and not intersecting (1-9).

He establishes the dependence of the angle of parallelism on the distance of the point through which the parallel to a given straight line passes.

He generates the line such that all perpendiculars to it are parallel: the L -line (10). The L -line in Bolyai is the horocycle in Lobachevsky.

The L -line is the circle of infinite radius, and by its revolution he generates: the F -surface (17). The F -surface in Bolyai is the horosphere in Lobachevsky.

He shows that: on the F -surface Euclid's postulate holds absolutely (21). As on the horosphere ordinary geometry holds.

He shows that spherical trigonometry is independent of Euclid's postulate (26).

Lobachevsky obtains for the length of every line x , $\tan \frac{1}{2} \Pi(x) = e^{-x}$, Bolyai obtains the equivalence of $\cot \frac{1}{2} \Pi(x) = e^x$ (29).

The fundamental differences are these. Lobachevsky's central concern is: to develop the new science of imaginary geometry. Bolyai's central concern are: the things that can be enunciated absolutely, independently of whether Σ or S is real.

Lobachevsky's development of imaginary geometry is analytic. It is grounded in the function $\Pi(p) = \alpha$. The problem of the consistency of two geometries, ordinary and imaginary, is an explicit concern of his. He addresses explicitly the problem of which of the two geometries obtains in nature. He is clear how this problem has to be solved. He also has a solution of this problem, negative as it is.

Bolyai concludes in the *True science of absolute space*:

it remains to set forth what the theory means;

1. whether Σ or some S is real remains undecided;

2. all things deduced from the hypothesis of falsehood of Euclid's postulate are true absolutely, and in this way they depend on no hypothesis;

there is therefore a plane trigonometry a

priori, in which it remains unknown which system is real, and the absolute magnitude in the expressions remains unknown, but a single known case would fix the entire system; spherical trigonometry, on the other hand, is established absolutely;

3. if it were agreed that Σ is real, nothing else would be unknown; if it were established that Σ is not real, angles would depend on length, and i would then be the natural unit of length as e is the base of the natural logarithm;

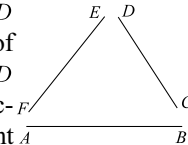
4. in this sense, it is clear that all things in space can be solved analytically;

5. finally, it may not be unacceptable to the friendly reader to show how a rectilinear figure equivalent to a circle can be constructed independently of whether Σ or S is real (33).

And in the course of this last demonstration, the one that may not be unacceptable to the friendly reader, Bolyai obtains the one very important result which Lobachevsky has not obtained.

Lobachevsky has shown that in imaginary geometry there is a greatest triangle. In the course of the construction of a rectilinear figure equivalent to a circle independently of Euclid's postulate, Bolyai constructs this figure:

the straight lines AB and CD are parallel in the direction of B and C , the straight lines CD and EF are parallel in the direction of D and E , the straight lines EF and AB are parallel in the direction of F and A ;



and this is manifestly the absolute limit of the area of the rectilinear triangles increasing beyond all bounds, and it is $= \pi i^2$ in F (43).

Gauss published the *Arithmetical disquisitions* in 1801, the first of the many treatises which established him as the prince of mathematicians.

The following is from his correspondence.

In 1799, Gauss writes to Wolfgang Bolyai:

I have advanced far in my work on this subject; and the way I have taken leads to making the truth of geometry doubtful;

if I could prove that there can be a rectilinear triangle whose area is greater than any given area, I could demonstrate from this the whole of geometry with full rigor; but it may well be that, how-

ever far apart in space the three vertices of the triangle are assumed to be, the area would be still below a certain limit (1 Dec).

In 1824, Gauss writes to Taurinus:

the assumption that the sum of the angles of the triangle is less than 180° leads to a curious geometry, quite different from ours, the Euclidean, yet self-consistent throughout, which I have developed to my entire satisfaction, and in which I can solve every problem, except the determination of a certain constant which cannot be ascertained *a priori*; the larger this constant is, the more nearly Euclidean geometry is approached, and an infinitely great value makes the two geometries coincide;

some of the theorems of this geometry appear paradoxical, but upon careful and calm reflection one finds that there is nothing impossible in them; thus, if the three angles of the triangle become as small as one wishes, and the three sides large enough, the area of the triangle can never exceed a certain definite limit;

all my efforts to find a contradiction in this non-Euclidean geometry have been in vain, and only this in it is repugnant to our intellect that, if it were true, there would exist in space a linear magnitude determinate in itself;

if this non-Euclidean geometry were the true one, and that constant were in some ratio to magnitudes within the scope of our measurements on earth or in heaven, it could be ascertained *a posteriori*; for this reason I have sometimes expressed in jest the wish that Euclidean geometry were not the true one, for we would then have an absolute standard of measurement *a priori* (8 Nov).

In 1831, Gauss writes to Schumacher:

in non-Euclidean geometry there are absolutely no similar figures;

in Euclidean geometry there is nothing of absolute size, but this is not so in non-Euclidean geometry; indeed, this is its essence; and those who reject this are *eo ipso* assuming Euclidean geometry, which is, in my opinion, a self-deception;

in non-Euclidean geometry one half of the circumference of the circle with radius r is

$$\frac{1}{2} \pi k \left(e^{\frac{r}{k}} - e^{-\frac{r}{k}} \right),$$

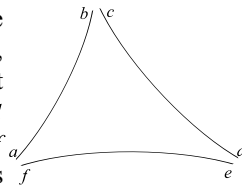
where k is a constant of which we know from experience that, compared to everything we can measure, it is extraordinarily large; in Euclidean geometry it is infinite (12 Jul).

If k is infinite, the exponential series yields for one half of the circumference of the circle πr .

In 1832, the correspondent is again Wolfgang Bolyai. The occasion is Wolfgang Bolyai's communication to Gauss of his son's *True science of absolute space*. Gauss writes:

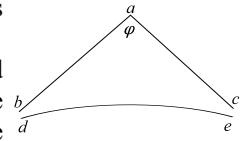
in several parts of the investigation I have taken a different path; as a specimen, I offer the outline of a purely geometrical proof of the theorem that the difference of the sum of the angles of the triangle from 180° is proportional to the area of the triangle;

1. the complex of the three straight lines ab , cd , and ef , arranged so that ab and dc are parallel, cd and fe are parallel, and ef and ba are parallel, forms a figure which I call T ; it can be proved that such a figure lies in one plane;



2. the portion of the plane between the three straight lines ab , cd , and ef has a definite finite area; I call this area t ;

3. if two straight lines ab and ac intersect at a in the angle φ , a third straight line de can be placed in the same plane so that ab is parallel to ed and ac is parallel to de ; then the plane area between these three lines is finite and depends only on the angle φ ;

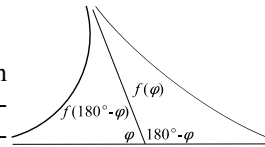


it is evident that when $\varphi = 180^\circ$, de and bac become a single straight line, and therefore the value of the area vanishes together with φ ; we therefore put, in general, the area $= f(180^\circ - \varphi)$, where f indicates a function;

4. theorem.

$$f(\varphi) + f(180^\circ - \varphi) = t;$$

the proof is given by the figure; the angles and areas are indicated;



5. theorem.

$$f(\varphi) + f(\psi) + f(180^\circ - \varphi - \psi) = t;$$

the proof is readily apparent from the

figure;

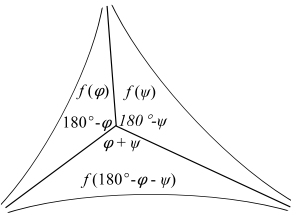
6. corollary.

therefore,

$$\begin{aligned} f(\varphi) + f(\psi) &= \\ &= t - f(180^\circ - \varphi - \psi) = \\ &= f(\varphi + \psi); \end{aligned}$$

from which it easily

follows that $\frac{f(\varphi)}{\varphi}$ is



constant, and equal to $\frac{t}{180^\circ}$ (6 Mar).

Saccheri has shown that on the hypothesis of the acute angle, if the sum of the angles of the triangle is φ , $\varphi < 180^\circ$. Lambert has shown that the defect $180^\circ - \varphi$ is proportional to the area of the triangle. Or what is the same, there is a constant of the increase of the defect. The function defined by Gauss $f(180^\circ - \varphi)$ is the area as proportional to the defect. From $f(\varphi) + f(\psi) = f(\varphi + \psi)$ it follows that $\frac{f(\varphi)}{\varphi}$ is constant, and as for $\varphi = 0$,

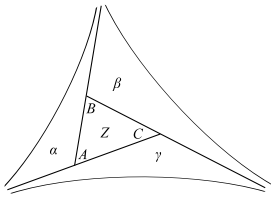
$$f(180^\circ) = t, \text{ the constant is } \frac{t}{180^\circ}.$$

Gauss concludes the specimen:

7. theorem.

the area of the triangle with angles A , B , and C is

$$\frac{180^\circ - (A + B + C)}{180^\circ} \times t;$$



the proof is given by the figure; the areas are

$$\alpha = f(A) = \frac{A}{180^\circ} \times t;$$

$$\beta = f(B) = \frac{B}{180^\circ} \times t;$$

$$\gamma = f(C) = \frac{C}{180^\circ} \times t;$$

$$t = \alpha + \beta + \gamma + Z = \frac{A + B + C}{180^\circ} \times t + Z \text{ (id.)}.$$

Finally, in 1818 Schweikart communicated to Gauss a one-page memorandum, in which he writes:

there are two kinds of geometry, geometry strictly, the Euclidean, and an astral theory of magnitude;

in the latter the sum of the angles of the triangle is less than two right angles;

the height of an isosceles triangle grows

continuously, but it can never become greater than a certain magnitude, which I call the constant;

Euclidean geometry holds only on the assumption that the constant is infinite (*id.*).

And in 1825 Taurinus published the *Theory of parallel lines*, and in 1826 *The first elements of geometry*. In 1825 he concludes:

a geometrical system in which the sum of the angles of the triangle is less than two right angles is indeterminate, because it requires a certain constant; and from this it follows that there can be for us no other geometry than the Euclidean, because such a constant can be assumed arbitrarily (*id.*).

Nevertheless, in 1826 he determines the constant. Lambert has shown that the hypothesis of the obtuse angle holds on the surface of the sphere, and has concluded that the hypothesis of the acute angle holds on the surface of a sphere of imaginary radius. And in 1826 Taurinus proposes to develop:

logarithmic spherical geometry, which is the opposite of spherical geometry (*id.*).

Replacing in the equations of spherical trigonometry the radius r with the imaginary radius $r\sqrt{-1}$, he proves several theorems. He concludes:

the question of whether logarithmic spherical geometry is possible or imaginary goes beyond the first elements (*id.*).

In a brief appendix, however, he obtains the equations:

$$R = \frac{P}{\log(1 + \sqrt{2})},$$

and hence,

$$\cot \frac{1}{2} \beta = e^{\sqrt{-1}} \text{ (id.)}.$$

In the first equation P is Schweikart's constant, the height of the isosceles triangle on the assumption that the sum of the angles of the triangle is less than two right angles, R is Gauss's k when he determines one half of the circumference of the circle in non-Euclidean geometry. Taurinus writes:

when $R = \infty$, logarithmic spherical geometry passes over into Euclidean (*id.*).

In the second equation β is an angle of parallelism, anticipating Lobachevsky's $\tan \frac{1}{2} \Pi(x) = e^{-x}$.

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Non-Euclidean geometry was discovered in 1829, by Lobachevsky.

The assumption that the sum of the angles of the rectilinear triangle is $< \pi$ is the foundation of the new science of imaginary geometry. The angle of parallelism is the function $\Pi(p) = \alpha$. For every angle α there is a line p such that $\Pi(p) = \alpha$. As p approaches infinity, α approaches 0, as p approaches 0, α approaches $\frac{1}{2}\pi$.

As α decreases, p reaches a point when it is no longer possible to construct a triangle. There is a curve, the circle of infinite radius, the horocycle, such that all perpendiculars at the midpoints of the chords are parallel. The surface of revolution of the horocycle is the horosphere. The geometry of the horosphere is ordinary geometry. Spherical trigonometry is independent of Euclid's postulate. For every line x , $\tan \frac{1}{2} \Pi(x) = e^{-x}$. When the triangle is very small imaginary geometry passes over into ordinary geometry. Whether imaginary geometry obtains in nature astronomical observation has to decide. On the assumption of stellar parallax of $1''.24$, the sum of the angles of the largest measured triangle would not differ from two right angles by more than $0''.000372$.

Ordinary geometry is consistent with imaginary geometry, too.

As to how and why it was discovered, first of all, there are all the attempts to prove Euclid's postulate. Counting back to Ptolemy, seventeen centuries' worth of attempts. Lambert's attention to the problem was drawn by a treatise, by Klügel, in 1763, in which more than 30 attempts were examined.

Next, the attempts have been unsuccessful, they have all failed to prove what Euclid's postulate affirms. They have been very successful, however, in showing its most fundamental character. The assumption that, if a straight line intersecting two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, produced indefinitely, intersect on that side is equivalent to the assumption that parallel lines are equidistant, equivalent to the assumption that through one point there is only one straight line parallel to a given straight line, equivalent to the assumption that the sum of

the angles of the triangle is two right angles, equivalent to the assumption that to any figure there is one similar to it of any size, equivalent to the assumption of a quadrilateral with four right angles, equivalent to the assumption that through three points not on the same straight line only one circle can be drawn. The attempts have shown that all geometry, or very nearly all, and all that rests on geometry, including the development of analysis, including all physics, rests on Euclid's postulate. The attempts have shown that the problem which Euclid's postulate poses is of the foundation. And Saccheri is explicit: no one can not see that thus the first principles of geometry are subjected to rigid examination.

Next, Saccheri is a major advance toward the discovery that there is a geometry which is not Euclidean. Absolutely false as they both are, the hypothesis of the obtuse angle and the hypothesis of the acute angle have both admitted of theorems, and the latter of many theorems. Lambert is a great further advance toward the discovery. The hypothesis of the obtuse angle can be assumed to hold on a spherical surface independently of Euclid's postulate. And it can be concluded that the hypothesis of the acute angle holds on an imaginary spherical surface.

Saccheri has formulated the problem in all its generality, the problem is of the foundation. Lambert has formulated the problem in all its particularity, the problem is of whether either a spherical surface or an imaginary spherical surface can be the foundation. And this is exactly the problem which Lobachevsky has solved, the hypothesis of the acute angle is the foundation of the new science of imaginary geometry, on an imaginary spherical surface, and for this reason itself imaginary, too.

Next, from Gauss's letters three more particular reasons can be extracted.

Gauss writes in 1829: it will probably be a long time before I prepare my very extensive researches on this subject for open publication, and perhaps not in my lifetime at all (to Bessel, 27 Jan). And in 1832: of my work on this subject, of which little is committed to paper, I intended to let nothing become public in my lifetime (to W. Bolyai, 6 Mar).

One reason is rigor. Gauss writes in 1799: I

have many things which would pass for proofs with most people; in my eyes, however, they as good as prove nothing (to W. Bolyai, 1 Dec).

Compared to Gauss's published results, Lobachevsky's are not very rigorous. Nor are Bolyai's.

Another reason are the Boeotians. Gauss explains in 1829: not in my lifetime, because I fear the outcry of the Boeotians (to Bessel, 27 Jan). And in 1832: not in my lifetime, because most people do not have the right sense at all for what matters in this subject (to W. Bolyai, 6 Mar).

And the Boeotians did, as they always do, meet Lobachevsky with scepticism. And Bolyai.

The third reason is metaphysics. Gauss writes in 1824: notwithstanding the verbal wisdom of the metaphysicians, we know, in fact, too little, or nothing at all, about the true nature of space to treat the apparently unnatural as the same as the absolutely impossible (to Taurinus, 8 Nov). In 1831: there is nothing contradictory in all this, if finite man does not presume to wish to contemplate something infinite as something given, and that is, something grasped by him by his ordinary sensibility; so you see that the matter at issue directly involves metaphysics (to Schumacher, 12 Jul). And in 1832: it is precisely the impossibility to decide between Σ and *S a priori* that affords the clearest proof that Kant was wrong to assert that space is only a form of our sensibility; in a little notice published last year I have indicated another reason, just as strong (to W. Bolyai, 6 Mar). In that notice Gauss writes: the distinction between right and left is in itself completely determined as soon as forward and backward are arbitrarily determined in the plane and above and below in relation to the two sides of the plane, even if we can communicate this to others only by reference to actually existing objects; Kant has made both these observations, and it is incomprehensible how this acute philosopher could have believed that the former proves his opinion that space is only the form of our outer sensibility; for the latter clearly proves the opposite, namely, that space must have a real significance independent of our mode of sensibility (*Notice*).

Gauss reads Kant as his older contemporary Hegel reads him.

Because space must have a real significance

independent of our mode of sensibility space is the form of our outer sensibility. Which explains the fact, too, that it is impossible to decide between Σ and *S a priori*, but, as Gauss writes in 1824: it could be ascertained *a posteriori*, and we would then have an absolute standard of measurement *a priori*.

The matter at issue directly involves metaphysics. And notwithstanding the verbal wisdom of the metaphysicians, including Gauss, Lobachevsky assumes that we know, in fact, enough about the true nature of space to treat the apparently unnatural not only as not absolutely impossible, but as necessary. So does Bolyai.

We know, for instance, that a triangle one of whose sides is 81 000 000 000 000 km is very small.

The problem whose solution non-Euclidean geometry is Saccheri formulated in all its generality. Lambert formulated it in all its particularity. And, pursuing more staunchly discovery than rigor, ignoring the Boeotians, and embracing metaphysics, Lobachevsky solved the problem. So did Bolyai.

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In the next generation, Riemann advanced greatly analysis, differential equations, complex functions, series, the theory of numbers. He wrote on gravity, heat, light, gases, magnetism, fluids, sound.

On the hypotheses which lie at the foundation of geometry, in 1854, is the foundation of all subsequent developments in geometry, analytic geometry, differential geometry, projective geometry, topology. Its impact on all physics has been very great, it is the foundation of general relativity. It is the first mathematical analysis of space not implicitly, by the analysis of what is in space, but explicitly. It is a dozen pages, unsurpassed in the number of fundamental problems formulated and solved in a dozen pages.

Riemann begins:

plan of the investigation;

it is well known that geometry assumes as something given the concept of space as well as the fundamental concepts of constructions in space; it gives only nominal definitions of these, and the essential determinations appear in the

form of axioms; the relations of these assumptions remain in the dark; one sees neither whether nor to what extent their connection is necessary, or *a priori*, or even possible;

this darkness, from Euclid to Lagrange, to note only the most famous recent investigator, has not been dispelled by either the mathematicians or the philosophers who have concerned themselves with the matter; undoubtedly, the reason is that the general concept of multiply extended magnitudes, which includes the spatial magnitudes, remains completely unexplored;

I have thus first set myself the problem to construct the concept of a multiply extended magnitude from the general concept of magnitude;

it will follow from this that a multiply extended magnitude is capable of various metric relations and space is only a special case of a triply extended magnitude;

from this it is a necessary consequence that the propositions of geometry cannot be derived from the general concept of magnitude; the properties which distinguish space from other thinkable triply extended magnitudes can only be obtained from experience;

from this arises the problem to search for the facts from which the metric relations of space can be determined; it is a problem which by the nature of the matter is not fully determined; for there may be several systems of simple facts which suffice for the determination of the metric relations of space; most important for the present purpose is the system established by Euclid; these facts are, like all facts, not necessary, but only of empirical certainty, they are hypotheses; their probability can be investigated, which is very great within the limits of observation, and hence their extension beyond the limits of observation can be judged, both in the direction of the immeasurably large and in the direction of the immeasurably small (*id.*).

This is the plan of the investigation.

From Euclid to Lagrange the concept of space has remained in the dark. The reason is that the general concept of multiply extended magnitudes, of which the spatial magnitudes are a particular case, has remained completely unexplored.

Hence the first problem, the most general,

from the general concept of magnitude to construct the more particular concept of a multiply extended magnitude.

Part of the solution of this problem is that metric relations, relations of measurement, are distinguished sharply. A multiply extended magnitude is capable of various metric relations. This shows that within the more particular concept of a multiply extended magnitude, which has been constructed from the general concept of magnitude, space is only a still more particular case of a triply extended magnitude.

It follows that the propositions of geometry cannot be derived from the general concept of magnitude. The properties which distinguish space from other thinkable triply extended magnitudes can only be obtained from experience.

Hence the next problem, to search for the facts from which the metric relations of space can be determined. There may be several such systems of simple facts. Most important is the system established by Euclid.

These facts are, like all facts, not necessary, but only of empirical certainty, they are hypotheses. Geometry is, like all facts, not necessary, but only of empirical certainty, it is a hypothesis.

Within the limits of observation its probability is very great. And from this its extension beyond the limits of observation can be judged, in the direction of what cannot be measured, and both the immeasurably large and the immeasurably small.

This is the plan of the investigation, and the investigation is accordingly in three parts:

I. concept of an n -fold extended magnitude;

II. metric relations, of which a manifold of n dimensions is capable, on the assumption that lines have length independent of their position, and that is, every line can be measured by every line;

III. application to space (*id.*).

Part I, concept of an n -fold extended magnitude, is itself in three parts, and as in the plan, so here Riemann begins with the greatest generality. He writes:

1. concepts of magnitude are possible only where there is already a general concept, which admits of different determinations; and according as there is in them continuous transition from one

to another or not, these determinations form a continuous or a discrete manifold;

concepts, whose determinations form a discrete manifold, are so common, that in the more developed languages a concept can always be found under which any collection of things can be comprehended; for this reason mathematicians consider in the theory of discrete magnitudes without hesitation any given things as of the same kind;

in opposition, concepts, whose determinations form a continuous manifold, are so rare in everyday life, that the places of sensible objects and colors are the only simple concepts, whose determinations form multiply extended manifolds; these concepts are more frequently introduced and developed first in higher mathematics;

comparison in quantity of different parts of a manifold, determined by a mark or a boundary, occurs in discrete magnitudes by counting, in continuous quantities by measurement; measurement consists in the superposition of the compared magnitudes upon one another; it thus requires a means of carrying one magnitude as a standard to another; barring that, two magnitudes can be compared only when one is a part of the other, and then only more or less, but not how much, can be decided;

the investigations which can be pursued in this case build a general and independent of measurement part of the theory of magnitude, where magnitudes are considered not as existing independently of their position, and not as expressible by a unit, but as regions in a manifold;

from this general part of the theory of extended magnitude, in which no more is assumed than what is already contained in the concept, for the present purpose it suffices to make clear two points concerning the essential characteristics of an n -fold extension, first, the introduction of the concept of a multiply extended manifold, and second, the reduction of the determination of position in a given extended manifold to determination of quantity (*id.*).

Concepts of magnitude are possible only where there is already a general concept, which admits of different determinations.

These determinations form a manifold. Manifolds are either discrete or continuous.

Discrete manifolds are so common, in the more developed languages a concept can always be found under which any collection of things can be comprehended.

Continuous manifolds are so rare, the places of sensible objects and colors are the only simple instances.

Comparison in quantity of different parts of a discrete manifold occurs by counting.

Comparison in quantity of different parts of a continuous manifold occurs by measurement. Measurement consists in the superposition of the compared magnitudes upon one another. It thus requires a means of carrying one magnitude as a standard to another.

In the general and independent of measurement part of the theory magnitudes are considered as regions in a manifold.

From this general part of the theory two points will make clear the essential characteristics of an n -fold extension. The concept of a multiply extended manifold. And the quantitative determination of position in a given extended manifold.

Parts 2 and 3 of part I thus make clear these two points. The concept of a multiply extended manifold:

2. if in a concept, whose determinations form a continuous manifold, one determination transitions to another in a determined way, these determinations form a singly extended manifold, whose essential characteristic is that continuous transition from one point is possible in only two directions, forward and backward;

if it is now thought that this manifold transitions to another completely different manifold in a determined way, and that is, so that every point of the one transitions to a determined point of the other, the joint determinations so contained form a doubly extended manifold;

similarly, a triply extended manifold is obtained when a doubly extended manifold transitions to a completely different manifold in a determined way; it is easy to see how this construction can continue;

and when, instead of considering the concept as determinate, its object is considered as variable, this construction can be characterized as a synthesis of a variability of $n + 1$ dimensions from a variability of n dimensions and a variabil-

ity of one dimension (*id.*).

When in a continuous manifold one determination transitions to another in a determined way, the determinations form a singly extended manifold, whose essential characteristic is that continuous transition from one point is possible in only two directions, forward and backward. When this manifold transitions to another completely different manifold in a determined way, and that is, so that every point of the one transitions to a determined point of the other, the joint determinations so contained form a doubly extended manifold. It is easy to see how this construction can continue to an n -fold extended manifold.

A line can instance a singly, or 1-fold, extended manifold. A surface can instance a doubly, or 2-fold, extended manifold. Space can instance a triply, or 3-fold, extended manifold. Depending on how many distinct magnitudes will describe it, color can instance a 1-fold, or 2-fold, or 3-fold extended manifold. To instance more-fold extended manifolds, higher mathematics, too, even as it has introduced them and developed them, has struggled.

The second point is the quantitative determination of position in a given extended manifold. Riemann shows:

how a variability, whose region is given, can be reduced to a variability of one dimension and a variability of dimensions fewer than the given (*id.*), and hence that:

the determination of position in an n -fold extended manifold is reduced to n determinations of magnitudes, and therefore the determination of position in a given manifold is reduced, whenever possible, to a finite number of determinations of magnitudes (*id.*).

Part II is: metric relations, of which a manifold of n dimensions is capable, on the assumption that lines have length independent of their position, and that is, every line can be measured by every line. And dramatic as the results obtained earlier are, these obtained in part II are substantially more dramatic. Riemann begins:

now, after the concept of an n -fold extended manifold has been constructed and it has been found that its essential characteristic is that the determination of position in it can be reduced to n

determinations of magnitudes, the second of the problems proposed above follows, the investigation of the metric relations of which such a manifold is capable, and of the conditions which suffice to determine these metric relations (*id.*).

The problem is solved in five steps, and part II is thus in five parts. Riemann continues:

1. measurement requires independence of the magnitude from position, which can occur in more than one way; the first assumption which suggests itself, and which I will pursue here, is just that, that the length of lines is independent of position, and that is, every line can be measured by every line;

if the determination of position is reduced to determinations of magnitudes, so that the position of a point in the given n -fold extended manifold is expressed by n variable magnitudes x_1, x_2, x_3 , and so on to x_n , the determination of a line is reduced to expressing the magnitudes x as functions of one variable; and the problem is then to formulate a mathematical expression for the length of a line, to which end the magnitudes x must be considered expressible in units;

I will treat this problem only under certain restrictions, and will restrict myself first of all to such lines in which the relations of the magnitudes dx – the changes in the magnitudes x – are continuous; the lines can thus be broken into elements, in which the relations of the magnitudes dx may be considered constant, and the problem is thus reduced to this, to formulate for every point a general expression for the line element ds , which comes out of the point, in which the magnitudes x and the magnitudes dx are contained;

I assume, second, that the length of the line element, when magnitudes of the second order are ignored, remains unchanged when all its points undergo the same infinitesimal change in position, and that is, that, if all magnitudes dx change in the same way, the line element changes in the same way as well;

on these assumptions the line element can be an arbitrary homogeneous function of the first degree of the magnitudes dx , which remains unchanged when all magnitudes dx change their signs, and in which the arbitrary constants are continuous functions of the magnitudes x ;

to find the simplest cases, I first seek an expression for these $(n - 1)$ -fold extended manifolds, which are everywhere equidistant from the origin of the line element, and that is, I seek a continuous function of position, which distinguishes the elements from one another; the function must thus either decrease or increase in all directions from the origin; I will assume that it increases in all directions, and there is therefore a minimum at the origin; when its first and second differentials are finite, then, the first order differential must vanish and the second order differential cannot be negative; I assume it remains always positive; this differential expression of the second order thus remains constant when ds remains constant, and increases in a quadratic relation when the magnitudes dx , and thus also ds , all increase in the same ratio; it is therefore $= \text{const.} \times ds^2$, and consequently $ds =$ the square root of an everywhere positive homogenous function of the second degree in the magnitudes dx , in which the coefficients are continuous functions of the magnitudes x ;

for space, when the position of a point is given in rectilinear coordinates, $ds = \sqrt{\sum (dx)^2}$; space is therefore included in this simplest case;

the investigation of more general cases does not, in fact, require essentially different principles;

I thus restrict myself to those manifolds where the line element can be expressed by the square root of a differential of the second degree;

the manifolds, in which, as in the plane and in space, the line element can be brought into the form $\sqrt{\sum dx^2}$, form only a special case of the manifolds to be investigated here; they deserve a special name, however, so I will call these manifolds, in which the square of the line element can be brought under the sum of the squares of complete differentials, flat;

to survey, now, the essential distinctions of the manifolds which can be represented in the assumed form, it is necessary to eliminate what depends on the mode of representation; and this is accomplished by choosing the variable magnitudes according to a definite principle (*id.*).

Measurement requires independence of the magnitude from position.

This can occur in more than one way. The way Riemann pursues is that the length of lines is independent of position. Every line can be measured by every line.

The problem is thus to formulate a mathematical expression for the length of a line, so that the magnitudes x , the positions of points in the given manifold, are expressible in units.

Riemann treats the problem under two restrictions. First, the lines are such that dx – the changes in x – are continuous. The problem is thus to formulate for every point a general expression for the line element ds , which comes out of the point, in which the magnitudes x and dx are contained. Second, all magnitudes dx change in the same way.

On these assumptions the line element can be an arbitrary homogeneous function of the first degree of the magnitudes dx , which remains unchanged when all magnitudes dx change their signs, and in which the arbitrary constants are continuous functions of the magnitudes x .

The simplest case is $ds =$ the square root of an everywhere positive homogenous function of the second degree in the magnitudes dx .

For space, when the position of a point is given in rectilinear coordinates, $ds = \sqrt{\sum (dx)^2}$.

Given the positions of two points in the plane in rectilinear coordinates, x_1, y_1 and x_2, y_2 , the Pythagorean theorem measures the length s between them, $s = \sqrt{\Delta x^2 + \Delta y^2}$. In three dimensions, $s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$.

And if the points of a line element near the origin are $x_1, x_2, \dots, x_n$, the measure of the length of the line element ds is $ds = \sqrt{dx_1^2 + dx_2^2 + \dots + dx_n^2}$, or $ds = \sqrt{\sum (dx)^2}$.

Space is therefore included in the simplest case.

The manifolds, in which, as in the plane and in space, the line element can be brought into the form $\sqrt{\sum dx^2}$, deserve a special name. Riemann calls them flat.

Flat manifolds are those, like the plane and space, in which measurement of the line element is Euclidean. In particular, measurement of the line element near the origin, in the infinitesimal neighborhood of any one point, is Euclidean. The

plane and space are Euclidean in the infinitesimally small. This will admit that they be non-Euclidean in the large.

To survey the essential characteristics of these manifolds it is necessary to eliminate what depends on the mode of representation. This is accomplished by choosing the variable magnitudes according to a definite principle.

This is the second step. Riemann begins:

2. to this end, the system of shortest lines coming out of any given point is constructed; the position of an undetermined point will then be determined by the initial direction of the shortest line in which the point lies and its distance in this line from the initial point (*id.*), he shows that:

the metric relations of these manifolds, in which the line element can be represented by the square root of a differential expression of the second degree, can be expressed in a way completely independent of the choice of the variable magnitudes (*id.*), and concludes:

the essential characteristic of these manifolds therefore can be called flatness in the smallest parts; for the present purpose, however, their most important characteristic, and the only reason they have been investigated here, is this, that the relations of the doubly extended manifolds can be represented geometrically by surfaces and the relations of the multiply extended manifolds can be reduced to the relations of the surfaces contained in them, which now requires a brief explanation (*id.*).

The metric relations of flat manifolds can be expressed in a way completely independent of the choice of the variable magnitudes.

Their essential characteristic is flatness in the smallest parts.

Their most important characteristic, however, is that the doubly extended manifolds can be represented geometrically by surfaces, and the multiply extended manifolds can be reduced to the doubly extended manifolds which can be represented geometrically by surfaces.

This requires a brief explanation, and this is the third step. Riemann writes:

3. in the conception of surfaces, the inner metric relations, in which only the length of the

paths in the surfaces is considered, are always mixed with their position with respect to points outside them; it is possible to abstract from these outer relations, however, by abstracting all changes in which the length of lines in the surfaces remains unchanged, and that is, by considering all surfaces which arise from one another by any bending – without stretching – equivalent;

thus, for instance, any cylindrical or conical surfaces count as equivalent to a plane, because they can be built from a plane by mere bending, under which the inner metric relations remain the same, and all theorems about them retain their validity; in opposition, they count as essentially different from the surface of the sphere, which cannot be transformed into a plane without stretching;

the inner metric relations at every point of a doubly extended magnitude, when the line element can be expressed as the square root of a differential expression of the second degree, as is the case with surfaces, are characterized by the curvature (*id.*).

This is defined earlier: the measure of the deviation from flatness at a point is the curvature (II, 2). Gauss has formulated the theory of curvature, in the *General investigations of curved surfaces*, in 1827. Riemann continues:

this magnitude can be given a sensible meaning for surfaces in this, that it is the product of the two curvatures of the surface at a point, or that the product of the two curvatures is equal to an infinitely small triangle built from shortest lines proportionally to the excess of the sum of its angles over two right angles (*id.*).

Metric relations are inner and outer, intrinsic and extrinsic. The intrinsic concern the length of the paths in a surface. The extrinsic concern position with respect to points outside the surface.

The extrinsic are abstracted when all surfaces arising from one another by any bending without stretching are considered equivalent. And all theorems retain their validity. Thus, intrinsically cylindrical and conical surfaces are equivalent to a plane. Spherical surfaces are essentially different.

The flatness in the smallest parts of the flat manifold is an intrinsic metric relation.

The intrinsic metric relations at a point of a doubly extended magnitude is characterized by the curvature.

And this magnitude, the doubly extended magnitude, can be given a sensible meaning for surfaces. It is the product of the two curvatures, those of the two extensions, of the surface at a point. It is also proportional to the excess of an infinitesimally small spherical triangle.

The next step is the generalization from flat manifolds to manifolds of constant curvature. Riemann writes:

4. before applying these results to space, it is necessary to make some general observations about flat manifolds;

in a flat n -fold extended manifold the curvature at every point in every direction is zero;

these manifolds can be considered a special case of the manifolds whose curvature is everywhere constant; the general character of these manifolds, whose curvature is constant, can be expressed in this, that the figures in them can be moved without stretching;

the metric properties of such a manifold, on the other hand, are determined completely by the curvature; they depend only on the value of the curvature (*id.*).

And hence the last step, before turning to the application to space, is the classification of manifolds of constant curvature. Riemann writes:

5. as a geometrical explanation the surfaces with constant curvature can be considered;

if these surfaces are considered as places of surface parts moving in them, as space can be considered as a place of bodies, then in all these surfaces surface parts can be moved without stretching;

the surfaces with positive curvature, namely spherical surfaces, can always be built so that the surface parts can be moved without bending, as surfaces with negative curvature cannot be; apart from this independence of the surface parts from position, in the surfaces with curvature zero there is also independence of direction from position, such as in the other surfaces there is not (*id.*).

Of surfaces with positive curvature the surface of the sphere is an instance. In such surfaces there is independence of the parts from position.

Of surfaces with zero curvature the Euclidean

plane is an instance. In such surfaces there is independence of the parts from position, and independence of the direction of the parts from position.

Of surfaces with negative curvature the surface on which the imaginary geometry developed by Lobachevsky holds is an instance. In such surfaces there is neither independence of the parts from position, nor independence of the direction of the parts from position.

The independence of the parts from position and of the direction of the parts from position in the Euclidean plane explains why Euclid's postulate is equivalent to the assumptions that parallel lines are equidistant, and through one point there is only parallel to a given straight line, and the sum of the angles of the triangle is two right angles, and similarity, and a quadrilateral with four right angles, and through three points only one circle can be drawn.

Part III is: application to space. And most dramatic as the results obtained earlier are, these obtained here are yet more dramatic. The application is to space, and to the world. This part is itself in three parts. Riemann writes:

1. after these investigations, now the conditions can be given which are sufficient and necessary for the determination of the metric relations of space, when the independence of the lines from position and the expression of the line element by the square root of a differential expression of the second order, and thus flatness in the smallest parts, is assumed;

they can be so expressed, first, that the curvature at every point in three surface directions = 0, and thus the metric relations of space are determined, when the sum of the angles of the triangle is always equal to two right angles;

if, however, second, with Euclid, the independence from position is assumed of the existence not only of lines, but also of bodies, it follows that the measure of the curvature is everywhere constant, and the sum of the angles of all triangles is determined when it is determined in one of them;

finally, third, instead of assuming the independence of the length of lines from place and direction, the independence also of their length and direction from place can be assumed; changes

in place or differences of place are then complex magnitudes expressible in three independent units (*id.*).

This, too, is explained. The sum of the angles of all triangles is determined as soon as it is determined in one of them, because the independence from position is assumed not only of lines, but of bodies, and thus that the measure of the curvature is everywhere constant.

Riemann continues:

2. in the course of the earlier observations first the relations of extension and metric relations were distinguished, and it was found that in the same relations of extension different metric relations are thinkable; simple systems of metric determinations were then sought by which the metric relations of space are completely determined and from which all theorems about them are a necessary consequence;

it remains to address the question, how, to what degree and to what extent, these assumptions are informed by experience;

in this connection there is an essential difference between the mere relations of extension and metric relations in that in the first, where the possible cases build a discrete manifold, the declarations of experience can indeed never be completely certain, yet are not inexact, while in the second, where the possible cases build a continuous manifold, every determination from experience always remains inexact, however great the probability that it is nearly correct may be; this circumstance becomes important when these empirical determinations are extended beyond the boundaries of observation to the immeasurably large and the immeasurably small; for the last can clearly become ever more inexact beyond the boundaries of observation, but the first cannot;

in the extension of constructions in space to the immeasurably large unboundedness and infinitude have to be distinguished; the first belongs to the relations of extension, the second to the metric relations; that space is an unbounded triply extended manifold is an assumption employed in every conception of the external world, through which the realm of actual perceptions is in every instant completed and the possible places of an object which is sought are constructed, and which is continually confirmed in

these applications; the unboundedness of space thus has a greater empirical certainty than any outer experience; but from this infinitude does not follow at all; moreover, space would be necessarily finite, if the independence of bodies from place is assumed, and thus a constant curvature is ascribed to it, as soon as this curvature has ever so small a positive value; an unbounded surface with constant positive curvature would take in a flat triply extended manifold the form of a spherical surface and would consequently be finite (*id.*).

The relations of extension and metric relations are distinguished. In the same relations of extension different metric relations are thinkable. Simple systems of metric determinations have been found by which the metric relations of space are completely determined and from which all theorems about them are a necessary consequence.

It remains to address the question, how, to what degree and to what extent, these assumptions are informed by experience.

In the mere relations of extension, where the possible cases build a discrete manifold, the declarations of experience are not inexact. Where the possible cases build a continuous manifold, however, every determination from experience always remains inexact, however great its probability. This becomes important when the empirical determinations are extended beyond the boundaries of observation to the immeasurably large and the immeasurably small. With respect to discrete manifolds empirical determinations extended beyond the boundaries of observation cannot become more inexact. With respect to continuous manifolds, however, empirical determinations extended beyond the boundaries of observation become ever more inexact.

It is a description of the greatest generality of the difference between arithmetic and geometry with respect to experience, and hence also of all the acquisition of knowledge that stands, and will stand, on arithmetic and on geometry.

The rest of Riemann's conclusions are concerned with the extension of the determinations of experience with respect to continuous manifolds to the immeasurably large and immeasurably small, and first the immeasurably large.

In the extension of constructions in space to the immeasurably large unboundedness and infinitude have to be distinguished. The first belongs to the relations of extension, the second to the metric relations.

That space is unbounded is an assumption employed in every conception of the external world. Through it the realm of actual perceptions is in every instant completed and the possible places of an object which is sought are constructed. The unboundedness of space is thus continually confirmed, and has a greater empirical certainty than any outer experience.

But from this infinitude does not follow at all. Moreover, space would be necessarily finite, if the independence of bodies from place is assumed, and thus a constant curvature, as soon as this curvature has ever so small a positive value. An unbounded surface with constant positive curvature would take the form of a spherical surface and would consequently be finite.

Unboundedness without infinitude, two generations later, general relativity will bring within the boundaries of observation.

Finally, Riemann turns to the immeasurably small. He writes:

3. for the explanation of nature questions about the immeasurably large are idle questions;

it is different, however, with questions about the immeasurably small; on the exactness, with which we pursue the phenomena in the infinitely small, knowledge of their causal connection rests essentially; the progress in the knowledge of mechanical nature in the last centuries has depended almost entirely on the exactness of the constructions, which have become possible with the discovery of the analysis of the infinite and the simple principles found by Archimedes, Galileo, and Newton, which physics today employs; while in the natural sciences in which the simple principles for such constructions are still lacking, in order to know causal connections, the phenomena are pursued into the spatially small as far as the microscope permits; the questions about the metric relations of space in the immeasurably small thus do not belong to the idle;

if it is assumed that bodies exist independently of place, the curvature is then everywhere constant, and it follows from the astronomical mea-

surements that it cannot be different from zero;

if such an independence of bodies from place is not the case, however, conclusions cannot be drawn from the metric relations in the large to the infinitely small; at every point the measure of the curvature in three directions can have an arbitrary value, if only the total curvature of every measurable part of space is not noticeably different from zero;

it is therefore thinkable that the metric relations of space in the infinitely small do not conform to the presuppositions of geometry, and this must in fact be assumed as soon as it allows a simpler way of explaining the phenomena;

the question about the validity of the presuppositions of geometry in the infinitely small is connected with the question of the inner ground of the metric relations of space; and in this question there is an application of the earlier consideration, that in a discrete manifold the principle of metric relations is contained already in the concept of the manifold, but in a continuous manifold it must come from something else; therefore, either a discrete manifold must build the reality which is the ground of space, or the ground of the metric relations must be sought outside it, in the binding forces acting on it;

the decision of these questions can be found only by starting from the conception of the phenomena which has so far been confirmed by experience, for which Newton laid the ground, and gradually reworking it under the pressure of the facts which cannot be explained by it; investigations like the present, which proceed from general concepts, can serve only this, that the work not be hindered by the limitations of the concepts and progress in the knowledge of the connections of things not be obstructed by traditional prejudices (*id.*).

Even if idle before him, that questions about the immeasurably large cannot be idle after him, Riemann's distinction of unboundedness and infinitude suffices to show.

As to the questions about the immeasurably small, the exactness of his answers is surpassed only by their importance.

On the exactness, with which we pursue the phenomena in the infinitely small, knowledge of their causal connection rests essentially.

Because the progress in the knowledge of nature has depended almost entirely on the discovery of the analysis of the infinite and the simple principles found by Archimedes, Galileo, and Newton.

And reaffirmed at Riemann's time by the development of electrodynamics, this conclusion will continue to be reaffirmed in the following three generations by the development of the kinetic theory, and of relativity, and of quantum theory.

If it is assumed that bodies exist independently of place, the curvature is then everywhere constant, and it follows from the astronomical measurements that space is Euclidean. It will quit following from the astronomical measurements three generations later.

If an independence of bodies from place is not the case, however, conclusions cannot be drawn from the metric relations in the large to the infinitely small; at every point the measure of the curvature in three directions can have an arbitrary value, if only the total curvature of every measurable part of space is not noticeably different from zero.

That an independence of bodies from place is not the case, general relativity will bring within the boundaries of observation.

That at every point the measure of the curvature in three directions can have an arbitrary value quantum theory will bring within the boundaries of observation.

The question about the validity of the presuppositions of geometry in the infinitely small is connected with the question of the inner ground of the metric relations of space.

In a discrete manifold the principle of metric relations is contained already in the concept of the manifold. This is also why with respect to discrete manifolds empirical determinations can be extended beyond the boundaries of observation without becoming more inexact.

But in a continuous manifold the principle of metric relations must come from something else. This is also why with respect to continuous manifolds empirical determinations cannot be extended beyond the boundaries of observation without becoming more inexact.

In a continuous manifold the principle of

metric relations must come from something else. Therefore, either a discrete manifold must build the reality which is the ground of space. Or the ground of the metric relations must be sought outside space, in the binding forces acting on it.

Extending beyond the boundaries of observation the empirical determinations of quantum theory, not quite three generations later, will appear to favor the former. Extending beyond the boundaries of observation the empirical determinations of the further development of quantum theory into the big bang theory, one generation later still, will appear to favor the latter.

The decision of these questions can be found only by starting from the conception of the phenomena so far confirmed by experience, the Newtonian, and gradually reworking it under the pressure of the facts which cannot be explained by it.

Just as electrodynamics, the kinetic theory, relativity, and quantum theory have all, implicitly as well as explicitly, decided these questions.

Finally, investigations like Riemann's, which proceed from general concepts, can serve only this, that the work not be hindered by the limitations of the concepts and progress in the knowledge of the connections of things not be obstructed by traditional prejudices.

This is the service of mathematics to physics. A very fine service, too, where Riemann's investigations, in particular, are concerned.

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These, then, are the hypotheses which lie at the foundation of geometry.

Concepts of magnitude are possible only where there is already a general concept, which admits of different determinations.

These determinations form a manifold. Manifolds are either discrete or continuous.

Comparison in quantity of different parts of a discrete manifold occurs by counting.

Comparison in quantity of different parts of a continuous manifold occurs by measurement. Measurement consists in the superposition of the compared magnitudes upon one another. It thus requires a means of carrying one magnitude as a standard to another.

When in a continuous manifold one determina-

tion transitions to another in a determined way, the determinations form a singly extended manifold, whose essential characteristic is that continuous transition from one point is possible in only two directions, forward and backward. When this manifold transitions to another completely different manifold in a determined way, and that is, so that every point of the one transitions to a determined point of the other, the joint determinations so contained form a doubly extended manifold. It is easy to see how this construction can continue to an n -fold extended manifold.

The essential characteristic of an n -fold extended manifold is that the determination of position in it can be reduced to n determinations of magnitudes.

Relations of extension are distinct from metric relations.

Measurement requires independence of the magnitude from position.

Metric relations are intrinsic and extrinsic. The intrinsic concern the length of the paths in a surface. The extrinsic concern position with respect to points outside the surface.

The extrinsic are abstracted when all surfaces arising from one another by any bending without stretching are considered equivalent, and all theorems retain their validity.

The intrinsic metric relations at a point of a doubly extended magnitude are characterized by the curvature.

In surfaces with positive curvature there is independence of the parts from position.

In surfaces with zero curvature there is independence of the parts from position, and of the direction of the parts from position.

In surfaces with negative curvature there is independence neither of the parts from position, nor of the direction of the parts from position.

In a discrete manifold the principle of metric relations is contained already in the concept of the manifold, and empirical determinations can be extended beyond the boundaries of observation without becoming more inexact.

But in a continuous manifold the principle of metric relations must come from something else, and empirical determinations cannot be extended beyond the boundaries of observation without becoming more inexact.

In the extension of constructions in space to the immeasurably large unboundedness and infinitude have to be distinguished. The first belongs to the relations of extension, the second to the metric relations.

On the exactness, with which we pursue the phenomena in the infinitely small, knowledge of their causal connection rests essentially.

And the question about the validity of the hypotheses of geometry in the infinitely small is connected with the question of the inner ground of the metric relations of space. In a continuous manifold the principle of metric relations must come from something else. Therefore, either a discrete manifold must build the reality which is the ground of space. Or the ground of the metric relations must be sought outside space, in the binding forces acting on it.

Thus, having remained in darkness, dispelled by neither the mathematicians nor the philosophers who had concerned themselves with the matter, the hypotheses which lie at the foundation of geometry have been brought to light.

And given the magnitude of the discovery, five causes can be assigned to it proportionate to its magnitude.

The development of analysis, and twofold.

First, the great advance of analysis as the investigation of the infinitely small. Riemann's investigation is of space in the infinitely small, and hence in the large.

Next, the great advance of analysis as independent of geometry. Hence the great advance in the investigation of the discrete and the continuous as distinct from one another. And hence the establishment of the discrete as the more fundamental. The distinction between the discrete and the continuous and the presumption that the discrete is the fundamental are central to Riemann's investigation.

In both, the great advance in the investigation of the infinitely small and the development of analysis independently of geometry and the establishment of the discrete as more fundamental than the continuous, the most important, by far, are the labors first of Euler then of Gauss.

Next, the development of non-Euclidean geometry. The conclusive demonstration, from Lambert to Lobachevsky, that geometry is not

Euclidean exclusively.

Next, the great advance of physics, in general, and of the investigation of electricity and magnetism, in particular. Riemann writes: the decision of these questions can be found only by starting from the conception of the phenomena which has so far been confirmed by experience, for which Newton laid the ground, and gradually reworking it under the pressure of the facts which cannot be explained by it. In precisely this way at Riemann's time the decision was being made in the investigation of electricity and magnetism. The Newtonian conception was being reworked under the pressure of the facts which could not be explained by it. Riemann's older contemporary Faraday first began reworking it in electricity and magnetism, in particular. Riemann's investigation reworks its mathematical foundations.

Finally, the advance in the investigation of knowledge, and in particular, Kant's. The importance of Kant's analysis of space is attested by Gauss's criticism. As to Riemann's investigation, it begins with concepts, and the deduction of particular concepts from general, and still more particular from the particular, and ends with experience, the assumption through which the realm of actual perceptions is completed, the assumption which has a greater empirical certainty than any outer experience, the knowledge of causal connections, the extension of empirical determinations beyond the boundaries of observation, and the validity of the presuppositions of geometry. No philosopher after Kant has, in fact, built on Kant either more decisively or more successfully than Riemann.

All five, the infinitely small, the discrete and the continuous, non-Euclidean geometry, the reworking of the Newtonian conception of the phenomena under the pressure of the facts, and Kant's investigation of the foundations of knowledge, each singly, and the more so all five jointly, pose the problem of the hypotheses which lie at the foundation of geometry, at the foundation of mathematics, at the foundation of physics, at the foundation of the acquisition of knowledge.

Riemann's investigation is a synthesis of the five, and a vast generalization, and an empirical generalization, over them.

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And again, counting only the more mathematical results, those of 26 philosophers – Taylor, Daniel Bernoulli, Euler, d'Alembert, Lambert, Lagrange, Monge, Laplace, Legendre, Fourier, Gauss, Poisson, Poncelet, Cauchy, Möbius, Lobachevsky, Abel, Bolyai, Jacobi, Hamilton, Dirichlet, Galois, Boole, Weierstrass, Cayley, Riemann – have been noted, attesting the development of mathematics in the one and a half-odd centuries from Saccheri to Riemann. All 26 are important, the majority are very important, and the majority are steps of advance in the acquisition of knowledge.

Again, however, not all with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

7 of the 26 are part of the development of this problem.

The results of Schweikart and Taurinus have also been noted.

On the authority of Gauss, and decisively on the authority of Riemann's investigation, Kant is also a part of the development of the problem. More generally, when the authority of Frege and Hilbert later on is added, Kant rather belongs among the mathematicians among the philosophers, too.

These 8, then, are all important philosophers who are part of the development of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends,

1707 - 1783 **Euler**,
1724 - 1804 **Kant**,
1728 - 1777 **Lambert**,
1752 - 1833 Legendre,
1777 - 1855 **Gauss**,
1792 - 1856 **Lobachevsky**,
1802 - 1860 **Bolyai**,
1826 - 1866 **Riemann**.

These are not important,

1780 - 1857 Schweikart,
1794 - 1874 Taurinus.

And the steps of advance in the acquisition of knowledge are the 7, Euler, Kant, Lambert, Gauss, Lobachevsky, Bolyai, and Riemann. In particular,

Euler	greatly advanced analysis,
Kant	investigated the foundations of

Lambert	mathematics, and of all knowledge, on the hypothesis of the acute angle there is an absolute measure of length, the defect of the sum of the angles of the triangle is pro- portional to the area, on the hypothesis of the obtuse angle the excess is pro- portional to the area, this hypothesis holds on the surface of the sphere, the hypothesis of the acute angle holds on the surface of a sphere of imaginary radius,	Bolyai	when the triangle is very small imaginary geometry passes over into Euclidean, whether imaginary or Euclidean geometry obtains in nature astronomical observa- tion has to decide, developed non-Euclidean geometry independently, concepts of magnitude pre- suppose a general concept admitting of different determinations, these determinations form a manifold, manifolds are discrete and continuous,
Gauss	greatly advanced analysis, formulated the theory of curvature, calculated the area of the triangle whose sides are parallel to one another,	Riemann	comparison of quantity in a discrete manifold occurs by counting, in a continuous manifold by measurement, continuous manifolds are singly extended, doubly extended, . . . , n-fold extended, relations of extension are distinct from metric relations, measurement requires inde- pendence of the magni- tude from position, metric relations are intrinsic and extrinsic, the intrinsic concern the length of the paths in a surface, the extrinsic are abstracted when all surfaces arising from one another by bending without stretching are considered equivalent, the intrinsic metric relations of a doubly extended magni- tude are characterized by the curvature, in surfaces with positive curvature there is indepen- dence of the parts from position, in those with zero curvature
Lobachevsky	discovered non-Euclidean geometry, the angle of parallelism is the function $\Pi(p) = \alpha$, on the assumption $\Pi(p) = \frac{1}{2}\pi$, the angle is always right, on the assumption $\Pi(p) < \frac{1}{2}\pi$, the angle depends on distance, as p approaches infinity, α approaches 0, as p approaches 0, α approaches $\frac{1}{2}\pi$, the horocycle is a circle of infinite radius, the surface of its revolution is the horosphere, on the horosphere Euclidean geometry holds, spherical trigonometry is independent of Euclid's postulate, for every line x, $\tan \frac{1}{2} \Pi(x) = e^{-x}$,		

there is independence of the
 parts and of the direction
 of the parts from
 position,
 in those with negative curvature
 there is independence neither
 of the parts nor of the
 direction of the parts
 from position,
 in a discrete manifold the
 principle of metric relations
 is contained in it,
 in a continuous manifold the
 principle must come from
 something else,
 in the immeasurably large
 unboundedness and infinitude
 have to be distinguished,
 on the immeasurably small
 knowledge of causal
 connection rests
 essentially,
 either a discrete manifold must
 build the reality which is
 the ground of space,
 or the ground of the metric
 relations must be sought
 in the binding forces
 acting on space.

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The one and a half-odd centuries since Riemann.

The discovery of non-Euclidean geometry, and hence the generalization of geometry, Euclidean and non-Euclidean, with the theory of manifolds and the theory of metrics, and the mathematical analysis of space, has been the foundation of all subsequent developments in geometry, hence more generally in mathematics, and hence more generally in physics, indirectly and more often most directly.

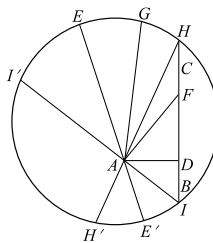
And the most important developments of which it has been the foundation most directly are the generalization of geometry into one explicitly formulated general problem, by Klein, the explicit solution of the problem of the foundations of geometry, by Hilbert, and the formulation of the theory of the world, which goes by the name

general relativity, by Einstein.

Klein published *On the so-called non-Euclidean geometry*, in 1871-3, and the *Comparative observations on the recent investigations in geometry*, in 1872.

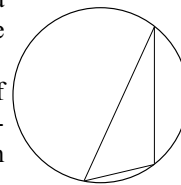
In *On the so-called non-Euclidean geometry* Klein demonstrated that Euclidean geometry and all non-Euclidean geometries are one another's projective transformations.

For instance. A conic section delimited by the surface of a cone is a part of a plane of projection from the vertex of the cone. And on a circle, for instance, which is a part of the Euclidean plane, the entire plane of the imaginary geometry developed by Lobachevsky can be projected. The transformation is of a line of imaginary geometry into a chord of the Euclidean circle, as points of the line at infinity in the plane of imaginary geometry correspond to the endpoints of the chord at the circumference of the Euclidean circle. Thus, this is a transformation of Lobachevsky's construction, in theorem 16, introducing the two asymptotic parallels. AD is perpendicular to BC at D , HH' and II' are the two parallels through A to BC and asymptotic to it toward H and I , AF intersects BC , AG does not intersect BC , EE' is perpendicular to AD at A . This is a transformation of a triangle whose three sides are parallel to one another.



With Klein's definition of the transformation – first defined in part by Beltrami in 1868 – the theorems of imaginary geometry have been shown to hold in the Euclidean circle. The transformation is thus the converse of Lobachevsky's construction of the horosphere, on whose surface he has shown Euclidean geometry to hold. Given the horosphere, if imaginary geometry is consistent, Euclidean geometry is consistent. Given Klein's transformation, if Euclidean geometry is consistent, imaginary geometry is consistent.

And Klein has shown that, in general, a geometry of a surface of positive curvature, a geometry



of a surface of zero curvature, and a geometry of a surface of negative curvature are all one another's projective transformations.

Hence, from the projective properties of the conics, Klein introduced also the names for these geometries elliptic geometry, parabolic geometry, and hyperbolic geometry. Riemann's geometry of a surface of positive curvature is elliptic. Euclidean geometry is parabolic. Lobachevsky's imaginary geometry is hyperbolic.

And hence, more generally still, in the *Comparative observations on the recent investigations in geometry* Klein formulated: the one comprehensive problem which arises as a generalization of geometry (1).

Klein writes in the *Comparative observations*: among the results of the last fifty years in the domain of geometry, first place occupies the development of projective geometry (pr.); the projective method now embraces all of geometry (*id.*);

as a generalization of geometry therefore arises the following comprehensive problem,

given a manifold and a group of transformations in it, to investigate the formations belonging to the manifold with respect to such properties as do not change by the transformations of the group;

or to use the modern way of expression,

given a manifold and a group of transformations in it, to develop the theory of invariants of the group (1).

This is the Erlangen program.

Groups became a central concern of mathematics with Galois, in the previous generation, invariance became a central concern of mathematics with Cayley, also in the previous generation.

Hilbert, beginning in 1900, formulated the theory of proof.

He published the *Foundations of geometry* in 1899, and continued to perfect it through its seventh edition in 1930.

He writes: the present investigation is a new attempt to establish for geometry a complete and as simple as possible system of axioms and to deduce from them the most important geometrical propositions in such a way that the meaning of the different groups of axioms and of the conse-

quences of the individual axioms is brought to light as clearly as possible (*intr.*).

The *Foundations* is in seven chapters:

1. the five groups of axioms;
2. the consistency and the mutual independence of the axioms;
3. theory of proportion;
4. theory of plane area;
5. Desargues's theorem;
6. Pascal's theorem;
7. geometrical constructions grounded in axioms I-IV (*id.*).

The consequences are brought to light as clearly as possible in the theorems which Hilbert proves after the formulation of each group of axioms in chapter 1, and then in the following chapters, the consistency and independence of the axioms, the theory of proportion, the theory of plane area, Desargues's theorem, Pascal's theorem, and the geometrical constructions.

The five groups of axioms are:

- I. axioms of incidence;
- II. axioms of order;
- III. axioms of congruence;
- IV. axiom of parallels;
- V. axioms of continuity (2-8).

The axioms of incidence are 8, the axioms of order 4, the axioms of congruence 5, the axiom of parallels is 1, the axioms of continuity are 2, for a total of 20.

A central concern of Hilbert is the consistency of the axioms:

the axioms in the five groups are not contradictory to one another, and that is, it is impossible to deduce from them by logical inference a result that contradicts one of them (9).

And Hilbert proves the consistency.

A central concern of his is also the independence, in particular, of each of the last 4 of the 20 axioms. The last of the axioms of congruence:

III.5. if for two triangles ABC and $A'B'C'$ the congruences

$$AB \equiv A'B', AC \equiv A'C', \angle BAC \equiv \angle B'A'C'$$

hold, then the congruence

$$\angle ABC \equiv \angle A'B'C'$$

also holds (5).

The axiom of parallels:

IV. Euclid's axiom. let a be any line and A a point not on it; then there is at most one line in

the plane determined by a and A that passes through A and does not intersect a (7).

And the 2 axioms of continuity:

V.1. axiom of measure or Archimedes's axiom. if AB and CD are any segments, then there exists a number n such that n segments CD constructed contiguously from A along the ray from A through B will pass beyond B ;

V.2. axiom of completeness. an extension of a set of points on a line with order and congruence relations which would preserve the relations among the original elements and the fundamental properties of line order and congruence following from axioms I-III and V.1 is impossible (8).

Hilbert proves that geometry can be developed, on the other 16 axioms, without each of these 4 axioms. Developed without Euclid's axiom, for instance, it is: non-Euclidean geometry (10). Developed without the axiom of measure or Archimedes's axiom, it is: non-Archimedean geometry (12).

Einstein formulated the special theory of relativity in 1905, the general theory of relativity in 1916.

In 1925, in the *Non-Euclidean geometry and physics*, he writes:

Riemann's service to the development of our ideas about the relation of geometry and physics is twofold;

first, he discovered the spherical-elliptic geometry, which is the counterpart to Lobachevsky's hyperbolic geometry; he thus first showed the possibility that geometrical space can be metrical of finite extension; this idea was understood right away and has led to the often considered question of whether physical space is not finite;

second, however, Riemann had also the daring thought to create a geometry which is incomparably more general than that of Euclid or the non-Euclidean geometries in the narrower sense; he created the Riemannian geometry which is, like the non-Euclidean geometries in the narrower sense, Euclidean only in the infinitely small; it is the extension of the Gaussian theory of surfaces to a continuum of any number of dimensions; according to this more general geometry the metric properties of space, with respect to the possible positions of infinitely many infinitely

small rigid bodies within finite regions, is not determined by the axioms of geometry alone; and instead of being discouraged by this discovery, or of concluding that his system is physically meaningless, Riemann had the daring thought that the geometrical behavior of bodies can depend on physical realities, or on forces; he thus came through purely mathematical speculations to the thought of the inseparability of geometry from physics, a thought which seventy years later was developed from facts in the general theory of relativity, through which geometry and the theory of gravity were jointed into a unity (*id.*).

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And the following conclusions can be drawn from the attempt to collect their predecessors and the philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them, with respect to the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends.

In the course of the development of this problem, as in the course of the development of the problem of the earth, and in the course of the development of the physical problem of the soul, a large amount of knowledge has been acquired.

Very many propositions have been won, have been brought in connection with one another in very many chains of inferences, the chains of inferences have become ever more free of gaps.

From the thought and the sense to the reference and the true.

From no inkling to the discovery of the triangle – the side is 10, the base is 4; what is the area? 20 – to the triangle the sum of whose angles cannot differ from two right angles by more than $0''.000372$. And in the immeasurably large unboundedness and infinitude have to be distinguished, on the immeasurably small knowledge of causal connection rests essentially.

It has been done in this case, the problem of the triangle, as it has been done in those, the problems of the earth and of the soul. The council of our predecessors has done it.

In order to formulate the problems we are to solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

In particular, from the thirty-seven centuries from the book of 87 problems through Riemann 33 important predecessors of the philosophers and philosophers have been collected, and 28 steps of advance in the acquisition of knowledge among them. Counting centuries from the year of birth assigned to the book of 87 problems, their distribution is,

1885-1786	1	1,
1785-1686	4	2,
1685-1586	1	1,
1585-1486	—	—,
1485-1386	—	—,
1385-1286	—	—,
1285-1186	—	—,
1185-1086	—	—,
1085- 986	—	—,
985- 886	—	—,
885- 786	—	—,
785- 686	—	—,
685- 586	—	—,
585- 486	2	2,
485- 386	2	2,
385- 286	2	2,
285- 186	—	—,
185- 86	1	1,
85 ^{BC} - 15 ^{AD}	—	—,
16- 115	1	1,
116- 215	—	—,
216- 315	—	—,
316- 415	1	1,
416- 515	—	—,
516- 615	—	—,
616- 715	—	—,
716- 815	—	—,
816- 915	1	—,
916-1015	1	1,
1016-1115	1	1,
1116-1215	1	—,
1216-1315	—	—,
1316-1415	—	—,
1416-1515	—	—,
1516-1615	2	2,
1616-1715	5	5,
1716-1815	6	5,
1815-1915	1	1.

As the enumeration has aimed at completeness through Riemann, whose date of birth is 1826, the count in the last line is not complete. The

count in the earlier lines should be complete.

The problem is gathered from Aristotle.

The sum of the angles of the triangle, and the assumptions on which the sum depends.

From Ptolemy to Saccheri there is the very clearly defined problem – and for this reason liable to appear a problem all by itself, as mathematical problems all too often are – to prove what Euclid's parallel postulate affirms. With Saccheri, however, and the formulation of the three hypotheses, of the right angle, of the obtuse angle, and of the acute angle, the vast, and explicit, generalization of that problem begins, and it is clear that that problem, to prove what Euclid's parallel postulate affirms, is about the first principles of geometry. It is about the angles, and everything that depends on the angles, and about the hypotheses on which the angles depend. The problem is, with Saccheri, about the principles of universal geometry.

It is thus also clear that the problem does not begin with the attempts to prove what Euclid's postulate affirms. Euclid is himself very much part of the development of the problem. Euclid is a perfect solution of the problem of the angles, and everything that depends on the angles, and the hypotheses on which the angles depend. Hence the attempts, too, to improve on the solution, on those most beautiful and never sufficiently praised *Elements*, and, in particular, by the proof of what the postulate affirms, the most fundamental assumption, as it has been gradually showing itself.

It is thus also clear that the problem begins before its solution by Euclid. As Aristotle, one generation-odd before Euclid, attests. And the problem gathered from Aristotle is very much the problem formulated explicitly by Saccheri which is very much the problem formulated explicitly by Riemann.

Before Aristotle, the problem can be traced back to Pythagoras, to the discovery of proof, and the proof that the sum of the angles of the triangle is two right angles.

And long before that, the remotest beginning of the problem is the discovery of the triangle.

The discovery of the triangle is attested by the book of 87 problems, in the valley of the Nile. One century later, in the valley of the two rivers,

the discovery of the triangle is accompanied by great further advance in its investigation. Parallel lines are discovered. Similar triangles are discovered. The proportionality of the sides of similar triangles is discovered. All these in their dependence on one another. All these in their Euclidean dependence on one another. Showing clearly and distinctly that geometry is first of all Euclidean, and will long remain Euclidean exclusively, not because of Euclid. The other way around. Euclid, because geometry is first of all Euclidean, and will long remain Euclidean exclusively. What Euclid established conclusively the council of our predecessors had begun establishing fifteen and a half centuries before Euclid.

In the valley of the Nile little predates the spectacular development of mathematics in the book of 87 problems. In the valley of the two rivers the contemporary development of mathematics succeeds twelve and a half centuries of slow and halting advance, and is itself a very great further advance. This is the first of the three brightest periods in the development of mathematics.

In both valleys geometry begins with the discovery of the triangle. In both valleys geometry is only a part of mathematics.

Thus in the first three of those thirty-seven centuries of development of the problem.

In the following ten centuries there is nothing.

In the following three centuries the great advance in the development of the problem resumes. These are the first centuries of the development of philosophy. Physics advances very greatly. The second half of this time is also the second of the three brightest periods in the development of mathematics. In particular, the development of the problem can be traced from Pythagoras, and the discovery of proof, to Hippocrates of Chios, and the first *Elements*, to Eudoxus, and the theory of proportion, to Aristotle, and the development of logic, to its culmination in Euclid, and the *Elements*.

Mathematics becomes geometry. Spatial relations are the ground of proofs, all objects are reduced to spatial relations, spatial relations close the gaps in the chains of inferences. The triangle is the fundamental object, the sum of its angles is its fundamental property, the fundamental as-

sumption on which this property depends are parallel lines, and parallel lines in turn depend on Euclid's postulate.

The council of our predecessors is in session most explicitly, as the philosophers adopt the sound and avoid the erroneous in what their predecessors have said. And mostly the former. As Euclid attests, too, systematizing and perfecting in the *Elements* the results of his predecessors.

In the following seventeen centuries the development of this problem, as the development of all problems, decelerates, and this problem is, as all problems are, for long periods of time abandoned altogether. In particular, there is no advance in this problem in thirteen of the seventeen centuries, there is nothing at all in eleven of the seventeen. Still, over all the seventeen centuries the advance is substantial. The attempts to prove what Euclid's postulate affirms begin with Ptolemy, and by the end of this time have accumulated in some numbers.

The council is in evidence throughout, as the few philosophers in its infrequent sessions do as their predecessors have done, keep trying to solve the problem posed by Euclid's postulate, and formulated explicitly by Ptolemy.

And in the last three of the thirty-seven centuries the great advance in the development of the problem resumes again. This is the time of the instauration of the sciences, and of the third of the three brightest periods in the development of mathematics. Physics advances very greatly. Calculus is discovered. Mathematics becomes analysis. In particular, the earlier accumulation of attempts to prove what Euclid's postulate affirms has shown the fundamental character of the postulate. The attempts have shown that all geometry, or very nearly all, and all that rests on geometry, including the development of analysis, including all physics, rests on Euclid's postulate. The attempts have shown that the problem is of the assumptions on which geometry rests. The postulate is the most fundamental specifically spatial assumption. And the vast generalization of the formulation and solution of the problem follows, from Saccheri to Lambert to Lobachevsky to Riemann.

Mathematics is fundamentally analysis. Geom-

etry is only a part of mathematics. And yet, the problem of the hypotheses which lie at the foundation of geometry has embraced the world in its entirety more explicitly than ever, from the immeasurably small to the immeasurably large. And in the next generation as a generalization of geometry will arise the one comprehensive problem, given a manifold and a group of transformations in it, to develop the theory of invariants of the group.

The triangle is not the fundamental object any longer. And yet, the doubly extended magnitude, to which all continuous manifolds can be reduced, is proportional to the excess of an infinitesimally small spherical triangle. Euclid's postulate is not the fundamental spatial assumption any longer. And yet, the Riemannian geometry is, like the non-Euclidean geometries in the narrower sense, Euclidean in the infinitely small.

The council cannot have been more explicitly in evidence either, as the most beautiful and never sufficiently praised *Elements* and the successive attempts to free them from all blemish are most explicitly the ground of every successful formulation and solution of the problem.

In general, over all the thirty-seven centuries, the triangle is discovered, proof is discovered, in one major step from Hippocrates of Chios through Euclid the theory of the measurement of space is formulated, the attempts to perfect the theory follow, and in one major step from Saccheri to Riemann the theory is generalized into the theory of manifolds and the theory of metrics.

The council is one and only one. As tightly bound in its unity and uniformity as this, fifteen and a half centuries before Euclid the beginning of geometry is Euclidean, and twenty-one and a half centuries after Euclid the non-Euclidean Riemannian geometry is Euclidean in the infinitely small.

The problem of the earth shows that the work of the council abounds in error, but abounds very much more in finality. The physical problem of the soul shows the same, the work of the council abounds in error, but abounds very much more in finality. The problem of the triangle shows barely any error in the work of the council, and shows finality very nearly exclusively. The formulations and solutions of the 87 problems in the book of

87 problems, the 14 problems, beginning with a triangle divided into 5 areas, the 1 problem, a triangle divided into 4 smaller triangles, the 18 problems, including an upright beam of length 30, are one and all sound. The structure of 141 definitions, postulates, and axioms and 514 theorems, lemmas, and corollaries of the *Elements* has revealed, for all the perseverance and vigor of the labors to find blemishes in it, hardly any blemishes at all. The attempts to prove Euclid's postulate, a *petitio principii* each, are yet not just errors, they have shown, soundly, proposition after proposition, the propositions to which the postulate is equivalent. As to the postulate itself, it is, with Riemann, a fact, and like all facts, not necessary, only of empirical certainty, a hypothesis. Only of empirical certainty, a hypothesis, its very great probability has continued to increase to become, twenty-one and a half centuries after Euclid, the ground of Lobachevsky's calculation of the very small triangle one of whose sides is 81 000 000 000 000 km. Later on still, itself only of empirical certainty, a hypothesis, Riemannian geometry is in the infinitely small Euclidean, and hence the general theory of relativity is in the infinitely small Euclidean, and hence the big bang theory is in the infinitely small Euclidean.

The cause is mathematics. This problem is more mathematical than those. The connection of propositions in chains of inferences in the course of the development of this problem has had to be substantially more free of gaps. That the chains of inferences be free of gaps has been a condition of the advance in the formulation and solution of this problem as uncompromising as it is in proof.

Finally, the problem of knowledge divides into two parts, the acquisition of knowledge and knowledge.

The parts of the problem of the acquisition of knowledge are the beginning of the acquisition, the collection of the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition.

This is a tabulation of the steps of advance in the acquisition of knowledge collected from the development of the three problems, the earth, the soul, and the triangle, counting centuries from the year of birth assigned to the temple register from

1116 In order to formulate the problems we are to solve we call the council of our predecessors

Lahun,

	earth	soul	triangle
1911-1812	1	—	1,
1811-1712	—	—	2,
1711-1612	1	—	1,
1611-1512	—	—	—,
1511-1412	—	—	—,
1411-1312	—	—	—,
1311-1212	—	—	—,
1211-1112	—	—	—,
1111-1012	—	—	—,
1011- 912	—	—	—,
911- 812	—	—	—,
811- 712	3	—	—,
711- 612	2	—	—,
611- 512	4	3	2,
511- 412	6	3	1,
411- 312	8	5	3,
311- 212	2	—	—,
211- 112	1	—	1,
111- 12	—	—	—,
11 ^{BC} - 89 ^{AD}	2	—	1,
90- 189	—	1	—,
190- 289	—	—	—,
290- 389	—	—	—,
390- 489	—	—	1,
490- 589	—	—	—,
590- 689	—	—	—,
690- 789	—	—	—,
790- 889	—	—	—,
890- 989	—	—	1,
990-1089	—	—	1,
1090-1189	—	—	—,
1190-1289	—	1	—,
1290-1389	—	—	—,
1390-1489	2	—	—,
1490-1589	4	5	—,
1590-1689	9	8	6,
1690-1789	10	7	4,
1790-1889	—	3	3.

Again, as the enumeration has aimed at completeness through Bessel, whose year of birth is 1784, Darwin, whose year of birth is 1809, and Riemann, respectively, the count in the last line is not complete. The count in the earlier lines should be complete.

The tabulation shows nothing more clearly and distinctly than the fact that, with Bacon: the times favorable to the advancement of knowledge have

been short. Ignoring the last line, of the 111 entries in the tabulation 76 are nothing.

The first three of the thirty-seven centuries are favorable. Two of the three problems begin, of the earth and of the triangle.

The following eight centuries are not favorable. There is nothing at all.

The following five centuries are very favorable. There is an advance in all three problems. First in the problem of the earth, then in the other two problems as well. This is the time of the beginning of philosophy.

The following seventeen centuries are not favorable. In the problem of the earth the advance slows down, then there is nothing for thirteen centuries. In the problem of the soul fifteen of the seventeen centuries are barren. In the problem of the triangle twelve of the seventeen. This is the time first of the rise of the schools, then of the rise of Christianity, then of the rise also of Islam.

The last four of the thirty-seven centuries are again very favorable. There is again an advance in all three problems. Again first in the problem of the earth, then also in the problem of the soul, then also in the problem of the triangle. This is the time of the instauration of the sciences.

In the first three of the thirty-seven centuries two of the three problems begin, of the earth and of the triangle.

The beginning of the problem of the earth is slow and halting. The beginning of the problem of the triangle is more decisive.

The problem of the earth is from observation. Observation begins with the temple register from Lahun and the Venus tablet of Ammisaduqa, in the valley of the Nile and in the valley of the two rivers. In both valleys observation succeeds great advance in mathematics. The relation of the development of mathematics and the beginning of observation is causal. The relation of the advance in the description of the surface of the earth by the labors of the princes and the beginning of observation is not remote either. On the evidence of the problem of the earth, the problem of the beginning of the acquisition of knowledge is of the beginning of mathematics, and in particular, the 3 oldest calculations of area, and the beginning of the labors of the princes, and in particular, Narmer. It is fundamentally of the differential

development in the two valleys, too.

The problem of the triangle begins with the discovery of the triangle in the book of 87 problems, in the valley of the Nile. In the valley of the two rivers, the discovery of the triangle in the 14 problems, beginning with a triangle divided into 5 areas, is followed by great further advance. It also succeeds the twelve and a half centuries of development of mathematics, slow and halting as it is, which has begun with the 3 oldest calculations of area. On the evidence of this problem, and the more so on the evidence of both problems, of the earth and of the triangle, the problem of the beginning of the acquisition of knowledge is first and foremost of the beginning of mathematics, and in particular, the 3 oldest calculations of area. It is fundamentally of the differential development in the two valleys, too.

All in all, with respect to the problem of the earth, collected have been 108 important particulars and 55 steps of advance distributed over all the thirty-seven centuries, with respect to the physical problem of the soul, 75 important particulars and 36 steps of advance distributed over twenty-four of the thirty-seven centuries, with respect to the problem of the triangle, 33 important particulars and 28 steps of advance distributed again over all the thirty-seven centuries. With respect to the last problem the numbers are smaller. Also, the ratio of steps of advance to important particulars is closer to 1 to 1, with respect to the other two problems it is closer to 1 to 2. Both differences can be ascribed to the fact that this problem is mathematical. If the demand that the chains of inferences be free of gaps has been a more uncompromising condition of advance in this problem, this can explain why advance in this problem has been overall slower, but more decisive, when there has been an advance.

The overlap between the three collections is very substantial. Of the 33 important particulars in the last collection, 4 are part of all three collections, Pythagoras, Parmenides, Aristotle, and Descartes. 12 more are part of another collection as well, Eudoxus, Euclid, Posidonius, Ptolemy, Thabit ibn Qurra, ibn al-Haytham, al-Tusi, Newton, Leibniz, Euler, Kant, and Gauss. The total for the joint collection is 185 important predeces-

sors of the philosophers and philosophers, and 98 steps of advance in the acquisition of knowledge.

Some of the 98 steps are smaller, others are larger, many are very large. Pythagoras, Parmenides, Anaxagoras, Empedocles, Democritus, Eudoxus, Aristotle, Euclid, Archimedes, Hipparchus, Ptolemy, Copernicus, Galileo, Kepler, Descartes, Huygens, Newton, Leibniz, Halley, Linnaeus, Euler, Buffon, Herschel, Laplace, Gauss, Lobachevsky, Darwin, Riemann are all 1 step each.

The 98 have to be a very substantial fraction of the acquisition of knowledge in its entirety.

On the evidence of the problem of the triangle, too, the acquisition of knowledge predates philosophy, but as soon as it begins, philosophy encompasses the acquisition of knowledge in its entirety. The discovery of proof, the formulation of the theory of proportion, the formulation of the theory of the measurement of space, are an empirical generalization over the great advance in the acquisition of knowledge which begins with the first philosopher. Beginning with the first philosopher, the physicists and the mathematicians are the philosophers, and conversely, the philosophers are the physicists and the mathematicians. The development of philosophy is a fundamental cause of the great acceleration of the acquisition of knowledge. Philosophy begins with the two problems, of the principles of all things and of the earth, and the accumulation of the formulations and solutions of problems, and in their dependence on one another, is a fundamental cause of the great acceleration of the acquisition. Hence, proof, the theory of proportion, the theory of the measurement of space formulated in the most beautiful and never sufficiently praised *Elements*, the attempts to prove what a postulate affirms, non-Euclidean geometry, the theory of manifolds and the theory of metrics, have not been obtained outside the one and only one development of philosophy from Thales through Pythagoras through Riemann. Nor could they have been.

The development of philosophy is a fundamental cause of the great acceleration of the acquisition of knowledge. The development of philosophy is also a fundamental cause of the great deceleration of the acquisition of knowledge. Philosophy is at first physics. Then, and in

response to physics, philosophy is also ethics. And when philosophy is dominated by physics, it is a fundamental cause of the great acceleration of the acquisition of knowledge. When it is dominated by ethics, and it has been dominated by ethics very much more than it has been by physics, it is a fundamental cause of the great deceleration of the acquisition of knowledge. And the tabulation of the steps of advance in the acquisition collected from the three problems shows this clearly and distinctly.

As to the kind of the acquisition, as the problem of the earth shows, and the physical problem of the soul shows affirmatively, and the ethical problem of the soul shows negatively, so the problem of the triangle shows that the acquisition of knowledge is theory, exclusively. Search for truth uncontaminated by any concern for action.

This informs also the question of whether non-Euclidean geometry, in particular, or theory, in general, has been, or can be, a matter of general interest.

Hegel writes: not more does one need to be a shoemaker in order to know whether his shoes fit than he needs to learn a trade in order to have knowledge of matters of general interest.

It is the perfect illustration. Whether his shoes fit is a matter of general interest. And to know whether his shoes fit is to have knowledge of matters of general interest.

And whether non-Euclidean geometry fits has been of general interest. Non-Euclidean geometry has been of general interest, however, when whether non-Euclidean geometry fits has been of general interest, not more than his shoes have been of interest in any of the theorems – some hundreds of them just presented – constituting non-Euclidean geometry.

Non-Euclidean geometry, in particular, or theory, in general, has not been, nor can it be, a matter of general interest. Which explains also the anecdote. What will I profit from learning these things? Give him three obols, to have him profit.

As to the kinds of the acquisition of knowledge, on the evidence of the problems of the earth and of the soul the acquisition of knowledge is physics. On the evidence of the problem of the triangle, it is mathematics. And mathematics is a

part of physics. From: a pyramid; the height is 250 cubits, the base is 360 cubits; what is the slope? the slope is $5\frac{1}{25}$ palms. Through the

theory of the measurement of space, the ground of the *Almagest*, and of *The mathematical principles of natural philosophy* still. Through the theory of n -fold extended manifolds, the ground of the theory of general relativity.

Also, if the standard is the performance of the indispensable duty, the examination of all original materials, many of the philosophers who are part of the development of the problems of the earth and of the soul are historians *par excellence*. In mathematics the original materials have been the earlier proofs. Because the chains of inferences have had to be substantially more free of gaps, the examination has had to be, explicitly or implicitly, more uncompromisingly of all earlier proofs. Earlier proofs cannot not be a part of all original materials, indeed the most valuable part. And the mathematicians have had to be, on this standard, historians *par excellence*. On this standard Euclid is a historian *par excellence*, Ptolemy is, Lambert is, Lobachevsky is, Riemann is. This has to be a matter of course, too. If experience is the foundation of the acquisition of knowledge, all earlier acquisition has to be part of the foundation of all later acquisition, and the more successful the earlier acquisition, the more it has to be the foundation of the later acquisition exclusively. If the standard is the examination of all original materials, history cannot not be a part of physics, and a part of the part of physics mathematics.

The parts of the problem of knowledge are the relation of knowledge and truth, and the relation of knowledge and the world.

The reference of all true propositions is the same, the true. And if the accumulation of very many true propositions in very many chains of inferences, ever more free of gaps, in the course of the development of the problem of the earth does not show this, that the reference of all true propositions is the same, the true, the accumulation of very many true propositions in very many chains of inferences, ever more free of gaps, in the course of the development of the two problems, of the earth and of the soul, in their depend-

ence on one another, the advance in the formulation and solution of both, as they share very many propositions and chains of inferences, and most particularly their generalizations depend on one another, the position of the earth among the planets, the position of the solar system, the position of the Milky Way, the soul of all animals and plants, the principle of life, the principle of all animals and all plants throughout all time and space, the theory of organisms grounded in the theory of the earth, the development of the two problems in their dependence on one another shows it. The reference of all true propositions is the same, the true. And the more the accumulation of very many true propositions in very many chains of inferences, ever more free of gaps, in the course of the development of the three problems, of the earth, the soul, and the triangle, in their dependence on one another, the advance in the formulation and solution of all three, as they share very many propositions and chains of inferences, and most particularly their generalizations depend on one another, the theory of the measurement of space is a generalization over the description of the world, and most particularly over the formulation and solution of the problem of the earth, its position, at rest or in motion, and its shape, and the formulation of the theory of the measurement of space is in turn the ground of all further advances in the description of the world, and most particularly in the formulation and solution of the problem of the earth, the development of the three problems in their dependence on one another shows it. The reference of all true propositions is the same, the true.

And knowledge cannot not be the reference of all true propositions, or what is the same, truth.

The world is the system of the world. The development of the problem of the earth shows that the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world have been described, and all in relation to one another. The earth is a sphere, of eccentricity 0.003, the third planet of the solar system, the solar system moves toward λ Herculis, on the map of the Milky Way Sirius is not more than $\frac{1}{80}$ of an inch from the earth, and 61 Cygni is 660 000 times further away

from the earth than the sun. The development of the physical problem of the soul shows the same. All organic beings from the first dawn of life throughout all time and space have been described, all parts of the same system. And the development of the problem of the triangle shows the same, the same system has been described, from the immeasurably small to the immeasurably large. It shows also clearly and distinctly how and why it has been described. Mathematics is the language of the world.

And the reference of all true propositions, or what is the same, truth, cannot not be the system of the world, or what is the same, the world. And knowledge cannot not be the reference of all true propositions, or what is the same, truth, and the reference of all true propositions cannot not be the system of the world, or what is the same, the world.

The relation of knowledge, truth, and the world cannot not be inclusion. Knowledge cannot not be truth cannot not be the world.

**The principles of all things,
and all three,
the material principles, or the elements,
of all things,
the efficient principles, or the causes,
of all things,
and the formal principles, or the principles,
of all things**

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The problem is knowledge. Its two parts, the acquisition of knowledge and knowledge.

Little is to be hoped for unless all particulars are collected and marshaled in their series and order.

The labors of their predecessors and the philosophers are the particulars.

Their predecessors and the philosophers have to be collected and marshaled in their series and order.

And to establish that they can be, their predecessors and the philosophers have been collected, all important among them separated, and all steps of advance in the acquisition of knowledge among them separated, with respect to these

problems.

The earth, its position, whether it is at rest or in motion, and its shape.

The soul, and both, what it is, and what you should do about it. Both the physical problem of the soul, and the ethical problem of the soul.

The best state, and both, the well-governed state, and the one conceived in speech. Both the historical problem of the best state, and the political problem of the best state.

The sum of the angles of the triangle, and the assumptions on which the sum depends.

And the principles of all things, and all three, the material principles, or the elements, of all things, the efficient principles, or the causes, of all things, and the formal principles, or the principles, of all things.

With respect to the earth, the earlier developments leading to the problem have been collected. The development of the problem in the thirty-seven centuries from its remote beginning with the temple register from Lahun through Bessel has been collected. And the development of the problem in the two centuries since Bessel has been described.

With respect to the soul, the earlier developments leading to the physical problem of the soul have been collected. The earlier developments leading to the ethical problem of the soul have been collected. The development of the physical problem in the twenty-four centuries from its beginning with Anaximenes through Darwin has been collected. The development of the ethical problem in these centuries from its beginning with Plato through the transmigration of the soul into the subject with the restoration of spirit to the substantiality and purity of its being has been collected. And the development of the two problems in the two centuries since Darwin has been described.

With respect to the best state, the development of the two problems, the historical and the political, in the four-odd centuries from Tiglath-pileser III through Alexander has been collected. The earlier and later developments have been indicated.

With respect to the triangle, the earlier developments leading to the problem have been collected. The development of the problem in the

thirty-seven centuries from the book of 87 problems through Riemann has been collected. And the development of the problem in the one and a half-odd centuries since Riemann has been described.

With respect to the principles of all things, the development of the problem in the two and a half centuries from Thales through Aristotle has been collected, as it has been, in fact, by Aristotle. That Aristotle's collection is complete has been shown.

It thus remains to continue and complete this last collection. To collect the earlier developments leading to the problem of the principles of all things. And to collect the development of the problem after Aristotle.

This is the task, to collect their predecessors and the philosophers, all important among them, and all steps of advance in the acquisition of knowledge among them, with respect to the problem of the principles of all things.

Before turning to the task, however, and in order to turn to the task, the difficulties attendant on all things, and on their principles, have to be addressed. For the difficulties attendant on all things, and on their principles, are quite proportionate to all things, and their principles.

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Thus, the main difficulty with all things is just that, that they are all things. They include the earth, and the soul, and the best state, and the triangle. They include the earth such as the sun revolves around it, and the earth such as it revolves around the sun, the soul such as it is the principle of life, and the soul such as it is nothing, what you should do, and what you do, whether you should or should not, the states of Tiglath-pileser III and of Alexander, and the states of Plato and of Aristotle, the triangle such as the sum of its angles is two right angles, and the triangle such as the sum of its angles is not two right angles. They include the planets, the epicycles on which the planets move, the angels whom god deputed to move the epicycles, and the ellipses on which the planets move. The nutritive, sensitive, and rational soul, its seat in the pineal gland, the cell, the chromosome, DNA, and carbon, hydrogen, oxygen, nitrogen, and phos-

phorus. The end of the state such as it is the good life, the end of the state such as it is the preservation of property, the end of the state such as it is to hold class conflicts in check, as well as the real end of the state. Male and female numbers, the irrationals, proportion, the base of the natural logarithm, manifolds, metric relations. They include the world, all that is in the world, and were there anything outside the world, they would have included that as well. They include not only all philosophy, all physics, all mathematics, all history, all politics, they include all ethics. They include not only the true way, they include the false way. They include their principles, too.

The main difficulty with the principles is thus similar. Descartes wrote *The principles of philosophy*. A list of thirty-two emulations, emulations of the emulations, and emulations of the emulations of the emulations, among only the more important, has been presented above. Among them *The mathematical principles*, *The principles of nature and grace*, the *Mechanico-physical principles*, *The principles of human knowledge*, *The principles of war*, *The principles of morals*, *The principles of the sensible and intelligible world*, *The principles of the theory of science*, *The principle of population*, *The principles of the evangelical church*, the *Principles of the theory of the shape of fluids*, *The principles of geometry*, the *Principles of geology*, *The principles of arithmetic*, *The principles of mechanics*, *The principles of ethics*, *The principle of relativity*, *The principles of quantum theory*. Nor are the material principles, the efficient principles, and the formal principles in this list of principles.

Which points to the main difficulty with the material principles, the efficient principles, and the formal principles. If there is the problem of the material principles of all things, the efficient principles of all things, and the formal principles of all things, no fundamental result can fail to inform this problem. But the many fundamental results obtained in the last centuries have tended to inform this problem only implicitly. The philosophers have tended to avoid concerning themselves explicitly with all things and their principles, material, efficient, and formal. And a reason can be assigned to the reticence. God was for a long time the principle of all things. Then

the instauration of the sciences began, and the relation of this principle of all things and other principles of all things came under examination, explicitly, too. But then spirit was restored to the substantiality and purity of its being, a catastrophic transformation befell philosophy, the physicists became not philosophers, the philosophers became not physicists, and all things, never mind their principles, became world-views and discourses.

The resolution of the difficulties has to be to make all things explicit. What the material principles of all things are. What the efficient principles of all things are. What the formal principles of all things are. What the relations of the material, the efficient, and the formal principles of all things are. What the principles of all things are. And what all things are.

At issue are propositions. The winning of propositions.

A general proposition is first surmised from an insufficient number of particular cases. It is more and more certainly established when through chains of inferences it is brought in connection with other propositions as either consequences are derived from it or conversely it is recognized as the consequence of already established propositions. This is the path by which a proposition is gradually won.

Because the gaps in the chains of inferences have been narrowed substantially it is possible to collect propositions into the formulation and solution of a problem. Because the narrowing of the gaps in the chains has been gradual it is possible to identify steps in the development of the formulation and solution of the problem. And both depend on the finality in the formulation and solution of the problem. At least some of the gaps in the chains of inferences have been eliminated completely, and the gradual ascent has reached that step.

Finality has to be the guide. In particular, the finality in the formulation and solution of the problem of the principles of all things has to be the guide in the attempt to resolve the difficulties. The problem of the material principles of all things, of the efficient principles of all things, and of the formal principles of all things has been formulated, and solved, and better, and better,

and better. This cannot not be a firm ground on which to try and make all things explicit.

Again, in this order. What the material principles of all things are. What the efficient principles of all things are. What the formal principles of all things are. What the relations of the material, the efficient, and the formal principles of all things are. What the principles of all things are. What all things are.

And then the attempt to collect the particulars and marshal them in their series and order can be made.

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The material principles of all things.

Thales said that water is the principle of all things.

Anaximenes said that air is prior to water and it is the principle.

Pythagoras said that number is prior and it is the principle.

Heraclitus said that fire is prior and it is the principle.

Thus in the first century of the development of philosophy already. The problem is formulated clearly and distinctly – even if it is not formulated explicitly yet – by the fact of accumulation of the attempts to solve it and improve on its solution, by the first philosopher, the third philosopher, the fourth philosopher, the eighth philosopher. And by the first philosopher, the third philosopher, the fourth philosopher, and the eighth philosopher the problem is solved, and better, and better, and better.

And to begin with, it is rather clear – very early in the development of philosophy – what all things are. All things are the world.

It is clear from the first that the problem of all things is about the world. Thales said that the earth floats on water. Anaximenes is explicit: he said that air encloses the world and holds it together. Pythagoras discussed the order of the planets. And Heraclitus is very explicit, he said: this world, the same for all, was made by neither god nor man, it always was and is and will be, an everlasting fire kindled in measures and extinguished in measures.

It is clear what all things are in general, they are the world, and it is rather clear as well what

they are in particular.

The philosophers will continue to fly to most general principles and, taking them to be settled, proceed to judgment. The true way is nevertheless in evidence from the first. Anaximenes discovered that to solve the problem in general is to solve it in particular. He said that air is infinite, air is always in motion, air is invisible and made visible by cold, hot, wet, and motion, all things come from air through rarefaction and condensation, rarefied air becomes fire, condensed it becomes wind, condensed further it becomes cloud, condensed further it becomes water, condensed further it becomes earth, and from these all other things come, and the soul is air, and the planets are moved by gusts of thick air, and the earth floats on the air below, like a lid. All things are infinite, in motion, cold, hot, wet, rarefaction, condensation, fire, wind, cloud, water, earth, all other things, the soul, the planets, gusts of thick air, lids floating on the air below.

It is thus clearer still what the principle of all things is. The principle of all things is what is prior.

Water is the principle. But air is prior to water. Condensed air becomes wind, condensed further it becomes cloud, condensed further it becomes water. Number, however, is prior to air. Pythagoras saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water, and said that numbers are the principles of all things, and all other things are by participation in numbers. And rather fire is prior. Fire always was and is and will be. Also, if, with Anaximenes, condensed air becomes wind, condensed further it becomes cloud, condensed further it becomes water, but rarefied it becomes fire, this rather implies that, with Heraclitus, fire is prior to air.

Hence it is clear also which the principle of all things is. The principle of all things is either number or fire. It is number, if number is prior to fire. Otherwise it is fire.

Thus, one and a half centuries after Heraclitus, Aristotle's definition of material principle: material principle is that out of which a thing comes to be and which persists, is an empirical generalization over the successes of his predecessors from the first. All other things come from air. All other

things are by participation in number. Fire always was and is and will be. The material principles of all things are that out of which all things come to be and which persists.

In the meantime, in the one and a half centuries from Heraclitus to Aristotle, it was discovered, by Parmenides, that the principles of all things are not of one kind. It became clear that those four, water, air, number, and fire, are of two kinds, water, air, and fire are solutions of the problem of the material principles of all things, number is a solution of the problem of the formal principles of all things. It was established also, where the material principles of all things are concerned, by Empedocles, that, not air is prior to water and fire is prior to air, but all three, water, air, and fire – and earth – are prior to all other things, are the material principles of all things, are that out of which all things come to be and which persists.

And so water, air, and fire remained for more than two millennia.

Lavoisier, in 1783, in the *Memoir whose object is to prove that water is not a simple substance, an element strictly, and is susceptible to decomposition and recomposition*, proved what the memoir proposes to prove, that water is not a simple substance, an element strictly, in the way it proposes to prove it, by decomposing water into, and recomposing it from, hydrogen and oxygen (*id.*). In 1789, in the *Elementary treatise on chemistry*, it is one of Lavoisier's fundamental experiments, the decomposition of water into hydrogen and oxygen (I, 8).

That air is not a simple substance, an element strictly, was proved in the previous generation. Van Helmont first produced gas from solid in 1648. In the generation before Lavoisier, Black, Cavendish, Priestley, and Scheele produced a dozen different gases. In the *Elementary treatise*, it is another of Lavoisier's fundamental experiments, the demonstration that atmospheric air is composed of oxygen and nitrogen in the ratio of 27 to 73 (3). Water was decomposed chemically, air was decomposed mechanically.

Bacon, not quite two centuries before Lavoisier, proposed that: the notion of fire is worthless, heat is a rapid motion of minute particles. It was yet two generations after Lavoisier when it was

proved that fire is not a simple substance, an element strictly. Lavoisier writes in the *Elementary treatise*: the cause of heat, or the exquisite elastic fluid which produces it, we distinguish by the name caloric; nor do we have to suppose it real matter, it is sufficient to consider it a certain repulsive cause which separates the particles of matter. Joule, in 1843, demonstrated the mechanical equivalent of heat. The decomposition of fire was thus also mechanical.

Lavoisier defines element in the preface to the *Elementary treatise*: if to the name elements, or principles of bodies, we attach the idea of the last terms which analysis has reached, then all substances which we have not yet been able to decompose by any means are elements.

And after all the decompositions by any means in the *Elementary treatise* Lavoisier compiles: a table of simple substances. The simple substances are 33. The first 5 are: simple substances which belong to the three kingdoms and which can be considered the elements of bodies. The 5 are: light, caloric, hydrogen, nitrogen, and oxygen.

Lavoisier's definition is an empirical generalization as well. It thus also explains Aristotle's definition. The elements, or principles of bodies, are all substances which we have not yet been able to decompose by any means. This, that we have not yet been able to decompose them by any means, shows that they are that out of which all things come to be and which persists.

It is clear – much later in the development of philosophy now – what all things are. They are bodies in particular, the three kingdoms in general. Minerals, plants, and animals.

It is very clear what the material principles of all things are. They are what has not been decomposed by any means. And most particularly, it is the decomposition of those, water, air, and fire – the last terms which analysis had reached before, and by whose decomposition it has reached the last terms which it has reached now – that points, or fails to point, to the material principles, or the simple substances, or the elements of all things. Hydrogen and oxygen are prior to water, water has been decomposed into hydrogen and oxygen. Nitrogen and oxygen are prior to air, air has been decomposed into nitrogen and oxygen. And whether the exquisite elastic fluid which produces

heat is real matter or a certain repulsive cause decomposition, or its failure, will decide.

Hence it is crystal clear which the material principles of all things, or the elements, are. They are the 33 simple substances in the table of simple substances.

After Lavoisier, also, the verbal infelicity which has been accruing to the problem since its first explicit formulation by Empedocles, the roots of all things, the material principles of all things, the material causes of all things, the principles of bodies, the simple substances, the elements, can be, and should be, avoided. They are the elements.

In the century after Lavoisier molecules and atoms were distinguished, by Avogadro, in 1811. It became clear that the decomposition of water is of the molecule of water into the atoms of hydrogen and oxygen. On the other hand, the molecules of air are separated into molecules of nitrogen and molecules of oxygen. And the exquisite elastic fluid which produces heat was shown to be a motion of molecules.

Thomson, one century-odd after Lavoisier, in 1897, in the *Cathode rays*, discovered the electron.

Thomson writes: as cathode rays carry a charge of negative electricity, and a magnetic force acts on them in just the way in which it would act on a negatively electrified body, I see no escape from the conclusion that cathode rays are charges carried by particles of matter; the question then arises, what are these particles? molecules, or atoms, or matter in a still finer state of subdivision (*id.*)?

Thomson determines the ratio of mass to charge for cathode rays, and continues: we see that the value of this quantity for cathode rays is 10^{-7} , compared to 10^{-4} for hydrogen ions in electrolysis, the smallest value of this quantity previously known (*id.*).

And concludes: in cathode rays matter is in a new state, this matter being the substance from which all chemical elements are built up (*id.*); the atoms of the different chemical elements are congregations of atoms of the same kind; in the form in which this hypothesis was enunciated by Prout the atoms of the different elements were hydrogen atoms, and in this form the hypothesis

is not tenable; but if we substitute for hydrogen some unknown primordial substance x , then nothing known is inconsistent with the hypothesis (*id.*).

Prout, for his part, had enunciated the hypothesis even with the relevant Greek. In 1816, Prout writes: we may consider hydrogen the πρώτη ὕλη of the ancients (*Correction*).

Thales said that water is the principle. Lavoisier said that hydrogen and oxygen are prior to water and they are the principles. Thomson said that the primordial substance x is prior to hydrogen and oxygen and it is the principle.

The material principles of all things are that out of which all things come to be and which persists. Or what is very much the same, they are the substance from which all chemical elements are built up.

Elements are all substances which we have not yet been able to decompose by any means. The new means of decomposition is the determination of the ratio of mass to charge.

It is clear – later still in the development of philosophy – what all things are. They are the chemical elements in particular, and everything built up from the chemical elements in general.

It is clear what the material principles of all things – the elements – are. They are the atoms of the same kind from which the atoms of the chemical elements are congregations. Where, when this new verbal infelicity is sorted out, too, the atoms of the chemical elements will be the atoms, the atoms of the same kind from which the atoms are congregations will be the elementary particles. And, again, most particularly those, the chemical elements – the last terms which analysis had reached before, and by whose decomposition it has reached the last terms which it has reached now – are shown congregations of elementary particles.

And which the elementary particles – the elements of the elements – are will become crystal clear in the two-odd generations after Thomson.

In the two-odd generations after Thomson quantum theory was developed, then the big bang theory. The primordial substance x is the electron, and the electron is one of the 6 leptons, electron, electron neutrino, muon, muon neutrino, tau, and

tau neutrino. The quarks are also 6, up, down, charm, strange, top, and bottom. The quarks and the leptons are the elementary particles. The elements of the elements.

Moreover, the substance x is primordial. Primordial has made the relation of, prior to, more explicit. And the big bang theory will make it as explicit as this relation, or any relation, can be made.

Quarks and leptons, including the primordial substance x , began being formed 10^{-6} seconds after the big bang. The chemical elements, beginning with hydrogen, began being built up from quarks and leptons 10^5 years after the big bang. Water began being formed on the surface of the earth 10^9 years after the big bang.

At first, then, there is no inkling that there are the elements.

Then it is discovered that there are the elements. And it is discovered that they are 4.

Thales said that water is the principle, Anaximenes said that air is the principle, Heraclitus said that fire is the principle, Empedocles added earth to these and said that all 4, earth, water, air, and fire, are the principles.

Then it is discovered that they are 76.

They are 33 in Lavoisier's table. In 1869, in the *Correlation of the properties and atomic weights of the elements*, Mendeleev discovered the periodic law and arranged 63 elements in: the general system of the elements. 23 of Lavoisier's 33 are among Mendeleev's 63. By 1897 discovery had brought the total in the general system to 76.

Then it is discovered that they are 12.

Thomson discovered the electron in 1897. Discovery then appeared to manifest ever more and more particles. Then in 1961 Gell-Mann published *The eightfold way, a theory of strong interaction symmetry*. In the following decade-odd the standard model was formulated, the unified theory of elementary particles and fundamental forces. All the ever more and more particles were reduced to order. The fundamental forces are 4. 6 bosons mediate the fundamental forces. And the elementary particles are the 12, the 6 quarks and 6 leptons.

From 0 to 4, from 4 to 76, and from 76 to 12.

Thus, had the principles of all things been just

material, and had the problem of the material principles of all things been just that, which the elements are, so much would have sufficed to make explicit what the problem is. And to collect the development of the problem in its entirety would have been just that, to collect the steps in the discovery of the 4 roots of all things, then the steps in their decomposition and the discovery of the 76 simple substances, and then the steps in their decomposition and the discovery of the 12 primordial substances.

As it is, which the elements are is only one part of the problem of the material principles of all things, which is itself only one part of the problem of the principles of all things.

ccccli

Parmenides said that all is one in form, only what necessarily is is and nothing else is. It yet appears as many and as such has two principles, fire and earth.

Anaxagoras said that an infinite number of particles are the principles, as each thing is generated from its own particles and destroyed into its own particles. And reason is the cause of the world and its order.

Empedocles said that earth, water, air, and fire are the principles. And love and strife are the sources of motion.

Thus early in the second century of the development of philosophy. When it was young, philosophy was stammering, babbling, and prattling. Then mature men appeared among the babblers. Parmenides, Anaxagoras, and Empedocles are the mature men. And considering how mature the babblers are, Thales, Anaximenes, Pythagoras, Heraclitus, the mature men are very mature.

The description of the world advanced dramatically. The earth is a sphere. The moon is beneath the sun and closest to us. The sun is a piece of red-hot metal larger than the Peloponnese. Lunar eclipses occur when the earth blocks the sun's light from falling on the moon, solar eclipses occur when the moon hides the sun from view. The evening star is the morning star.

That the true way is the way is substantially clearer. The discoveries of Parmenides and Anaxagoras imply observation. And Empedocles

attests observation on the verge of becoming experiment. He said that the earth is at rest because the very fast motion of the heaven prevents it from moving, like water in a cup, when the cup is spun fast, and the water is thus prevented from moving downward. He sings also:

when a girl plays with a bronze clepsydra,
and closes the neck with her beautiful hand,
and dips the other opening in bright water,
no water rushes in, for the air inside prevents it;
but when she opens the neck, the air gives way,
and the water rushes in (fr. 100).

In the *Physics*, while arguing that there is no void, Aristotle notes: those who play with clepsydras by cutting off the passage of air do not demonstrate void, they demonstrate the resistance of air, and thus only that air is a thing (IV, 6). This is what they demonstrate, the resistance of air. And to demonstrate it experimentally only to measure the resistance remains.

Parmenides discovered that the principles are not of one kind. All is one. But it appears as many and as such its principles are fire and earth. The kinds are thus real and apparent. Parmenides discovered also the principle of contradiction. The decisive step toward the discovery that the principles are also formal.

Anaxagoras discovered that wood does not make beds. The principles are efficient as well as material. All things have a portion of all things, only reason is mixed with nothing and is all by itself, and when reason begins motion, it separates things from all that is moved, and all things are moved and divided.

Empedocles formulated the theory of the four elements. The four roots of all things are fire, water, earth, and the immense height of air. Cursed strife separates them, love unites them. Cursed strife is equal in every way, love is equal in length and breadth, both are equally ageless. And there is nothing beside them, for can anything increase the all? where will it come from? and how will it perish? no, nothing can be void of them.

Parmenides said that the principle is one, but appears as many. And if the kinds of the principles are real and apparent, the principle is one, not really many.

But if the principles are really many, what the

many are such as they are one cannot not be a part of the problem. What they are such as they all are the principles cannot not be a part of the problem.

In particular, if the material principles of all things are many, it cannot not be a part of the problem what the many material principles are such as they all are of one and the same kind. Such as they all are the material principles.

Thus, if water is the element, or air is the element, or fire is the element, this is what the element is, water, or air, or fire. But if all four, earth, water, air, and fire, are the elements, it cannot not be a part of the problem what they all four are such as they all four are the elements. They are, Empedocles sings:

the roots of all things.

That the problem is of, both, which the elements are and what they are such as they are the elements, of, both, the species and their kind, the subsequent development of the problem will show clearly and distinctly. The chemical elements, in general, are hydrogen, helium, lithium, beryllium, in particular. Quarks and leptons, in general, are up, charm, electron, muon, in particular.

But both parts of the problem are anticipated by Anaxagoras and Empedocles already.

Empedocles said that earth, water, air, and fire, in particular, are the roots of all things, in general.

Anaxagoras said that an infinite number of particles are the principles.

And it is a more general and more formal solution of the problem of the elements than earth, water, air, and fire.

It is a more general solution. All things being all things, neither the problem of the principles of all things, in general, nor the problem of the elements, in particular, can be of generality lesser than the greatest. Lest there were more things than all things. Nevertheless, one part of the problem can be of greater generality than another. If the parts of the problem are the kind and its species, the former, the kind, is the more general part. If the solutions of the two parts are the roots of all things, and earth, water, air, and fire, the former is the more general. Earth, water, air, and fire are all four the roots. But the roots are only

part earth, part water, part air, part fire. And if the elements are an infinite number of particles, they are more like the former than like the latter, more like the roots than like earth, water, air, and fire.

It is also a more formal solution. Anaxagoras and Empedocles both attest the inkling that the solutions of problems have to be quantitative. Empedocles said that each of the parts of the body is a ratio of the elements. Bone is 2 parts water, 4 parts fire, 2 parts earth and air. And Anaxagoras attests the inkling that the solution of the problem of the elements has to be quantitative. He assigned a number to the elements, they are an infinite number. He said that there are as many seeds of all things as there are kinds of things. He said also that the particles are homogeneous.

And with Leucippus, in the next generation, and Democritus, in the generation after that, the distinction is clearer. Leucippus said that the elements are atoms and void. Democritus said that the modification of all things is due to the shape, order, and position of the elements, as A differs from N in shape, AN differs from NA in order, and N differs from Z in position. He distinguished also heavy and light by size, the lighter contains more void, the heavier less, and the larger and heavier an atom the hotter it is as well.

It is thus clearer also that the solution of the problem has to be quantitative. Aristotle writes: Leucippus and Democritus never explained the shape of the elements in detail, except to allot the sphere to fire, and to distinguish air, water, and the rest by the relative size of the atoms (*Heaven*, III, 4). Neither the shape of the atoms will be explained in more detail nor their relative size distinguished more explicitly until Dalton.

Atoms – indivisibles – is the kind of the elements. The species are distinguished by shape, order, position, and size.

With Aristotle, three generations later still, the distinction is still clearer. The elements are not earth, water, air, and fire, but simpler bodies similar to them. Simpler bodies similar to earth, water, air, and fire is the kind. The species are the simpler bodies similar to earth, those similar to water, those similar to air, and those similar to fire.

The problem of the elements is, both, of which

they are and of what kind they are. And the latter part is the more difficult. It is the general, its solution has to depend on the solution of the former, the particular. And it is its solution that has to make explicit what the relation of the two, particular and general, is.

Boyle, in 1661, in *The sceptical chemist*, destroyed the doctrine of the four elements, as well as the doctrine of the three elements, mercury, sulfur, and salt, proposed by Paracelsus one century-odd earlier. The destruction is by experiment. In general, physics was grounded in experiment two generations before Boyle, by Galileo, it was grounded in experiment exclusively in the generation of Boyle, and in this the quantity and quality of Boyle's experiments is among the most important. In particular, Boyle showed experimentally that nothing has been either immediately compounded or ultimately resolved into either earth, water, air, and fire or mercury, sulfur, and salt.

Boyle defines element: to prevent mistakes, I must advertise you that I mean by elements, as those chemists who speak the plainest do by their principles, certain primitive and simple or perfectly unmingled bodies, which, not made of any other bodies or of one another, are the ingredients of which all perfectly mixed bodies are immediately compounded and into which they are ultimately resolved (6).

Boyle's definition as well is an empirical generalization. Over the labors of those chemists who speak the plainest.

Elements are certain primitive and simple or perfectly unmingled bodies, not made of any other bodies or of one another. They are the ingredients of which all perfectly mixed bodies are immediately compounded and into which they are ultimately resolved.

Boyle's definition makes Aristotle's definition more explicit. The elements are that out of which all things come to be and which persists. They are that out of which all perfectly mixed bodies are immediately compounded and into which they are ultimately resolved. And Lavoisier's definition makes Boyle's definition in turn more explicit. The elements are all substances which we have not yet been able to decompose by any means.

Elements are certain primitive and simple or

perfectly unmingled bodies, not made of any other bodies or of one another. If this is a solution of the problem of the elements, it is a solution of the problem of their kind, not of the problem of which they are.

Descartes in the previous generation writes: all matter of which this visible world is composed was initially divided by god into particles as nearly equal as possible.

Newton in the next generation writes: it seems probable to me that god in the beginning formed matter in solid, massy, hard, impenetrable, movable particles.

Particles as nearly equal as possible. Primitive and simple or perfectly unmingled bodies. Solid, massy, hard, impenetrable, movable particles.

But the solution of this part of the problem, the more general, of the kind, cannot not depend on the solution of the other part, the more particular, of the species. And experiment had shown that the elements are neither Empedocles's four nor Paracelsus's three. But experiment remained to show which they are.

One century-odd after Boyle, in 1789, Lavoisier solved this part of the problem. The elements are the 33 in the table of simple substances.

The other part of the problem thus became explicit. Not what the elements are. What these 33 are such as they are the elements. And in the following three generations this part of the problem was solved as well. In three steps, by Dalton, Avogadro, and Mendeleev.

In 1808, in the *New system of chemical philosophy*, Dalton writes: all bodies of sensible magnitude, whether liquid or solid, are constituted of a vast number of extremely small particles, or atoms of matter, bound together by a force of attraction (I, 2.pr.); we may conclude that the ultimate particles of all homogeneous bodies are perfectly alike in weight, figure, etc.; in other words, every particle of water is like every other particle of water, every particle of hydrogen is like every other particle of hydrogen (*id.*); and it is one great object of this work to show the importance of ascertaining the relative weights of the ultimate particles, both of simple and compound bodies, the number of simple elementary particles which constitute one compound particle, and the number of less compound particles which

enter into the formation of one more compound particle (3.pr.).

Dalton also arranges the ultimate particles, or atoms of matter, into a table, and the table is more general and more formal. The atoms are: simple, binary, ternary, quaternary, quinquenary, etc. (*id.*). The simple atoms are 20. The first 4 of the 20, with their relative weights and signs, are:

hydrogen	1	⊙,
carbon	5	●,
nitrogen	5	⊕,
oxygen	7	○ (<i>id.</i>).

One of the binary atoms is:

water	8	⊙○ (<i>id.</i>).

One of the quaternary atoms is:

nitrate	26	⊙ ⊙ ⊙ ⊙ (<i>id.</i>).

Dalton reduces the atoms to equations, too. He writes for water:

1 atom of *A* + 1 atom of *B* = 1 atom of *C*, where *A* and *B* are simple, *C* is binary (*id.*). For nitrate:

1 atom of *A* + 3 atoms of *B* = 1 atom of *D*, where *A* and *B* are simple, *D* is quaternary (*id.*).

The ultimate particles of all homogeneous bodies are perfectly alike in weight, figure, etc. One part of the great object of this work is: to show the importance of ascertaining the number of simple elementary particles which constitute one compound particle. And the decisive step toward the solution of the problem of the kind of the elements is the other part of the great object: to show the importance of ascertaining the relative weights of the ultimate particles.

In 1811, in the *Essay on a means of determining the relative masses of the elementary molecules of bodies*, Avogadro distinguished the elementary molecules of bodies from the integral molecules (*id.*). Where – when this verbal infelicity is sorted out as well – the elementary molecules are the atoms, the integral molecules are the molecules.

Avogadro writes: Gay-Lussac has shown that gases always unite in a very simple proportion by volume (1); the only admissible hypothesis in this connection is that the number of integral molecules in all gases is always the same for equal volumes (*id.*); and from this hypothesis we have the means of determining very easily the relative

masses of the molecules of substances obtainable in gaseous state, as well as the relative number of these molecules in compounds, for the ratios of the masses of the molecules are the same as the densities of the different gases at equal temperature and pressure, and the relative number of molecules in a compound is given at once by the ratio of the volumes of the gases that form it; for instance, since 1.10359 and 0.07321 are the densities of oxygen and hydrogen with atmospheric air as the unity, this is on our hypothesis the ratio of the masses of their molecules; thus the mass of the molecule of oxygen is to that of hydrogen as 15.074 to 1; on the other hand, since the ratio of the volume of hydrogen and oxygen in the formation of water is 2 to 1, it follows that water results from the union of each molecule of oxygen with two molecules of hydrogen (*id.*).

Molecules are elementary and integral. Avogadro determined the masses of molecules, elementary and integral. And the decisive step toward the solution of the problem of the kind of the elements, again more general and more formal, is: the ratios of the masses of the molecules are the same as the densities.

And in 1869 Mendeleev solved the problem of the kind. Mendeleev discovered the periodic law.

In the *Correlation of the properties and atomic weights of the elements*, he writes: no matter how the properties of a simple body may change, something remains constant, and when the elements form compounds, this something has a material value and establishes the characteristics of the compound; we know of only one constant peculiar to each element, namely, the atomic weight; moreover, by the very nature of the matter the size of the atomic weight is a number related not to the state of division of the simple body, but to the material part common to the simple body and all its compounds; the atomic weight does not belong to the coal or the diamond, it belongs to carbon (*id.*); I have therefore based the system of the elements on the size of their atomic weights (*id.*); the size of the atomic weight determines the nature of the element (*id.*).

Mendeleev presents the system of the elements, again in a table, and concludes: 1. arranged according to the size of their atomic

weights, the elements show clear periodic properties; 3. comparisons of the elements and their groups in terms of the size of their atomic weights establish their atomicity; 5. the size of the atomic weight determines the character of the element (*id.*).

These are the first seventeen entries in the table:

H = 1		
	Be = 9.4	Mg = 24
	B = 11	Al = 27.4
	C = 12	Si = 28
	N = 14	P = 31
	O = 16	S = 32
	F = 19	Cl = 35.5
Li = 7	Na = 23	K = 39
		Ca = 40 (<i>id.</i>).

The elements = their atomic weights.

He = 4 was discovered in 1895, Ne = 20 in 1898, Ar = 40 in 1894.

Something remains constant. This something has a material value. It determines the nature of the element. It establishes the atomicity of the elements. It determines the character of the element.

This is the kind. The correlation of the properties and atomic weights. This is what the 63 in Mendeleev's table are such as they are the elements.

The periodic law shows which the elements are, what their kind is, and what the relation of the kind and its species is. And it is a law, it does so formally.

Finally, had the primordial substance x been the only primordial substance from which all chemical elements are built up, its discovery would have been the solution of both problems, of which the element is and of what kind it is. As it is, not only more primordial substances were discovered. It was discovered that they are waves. The particles are waves. Or are they? The question of what they are, those particles, or waves, or particle-waves, or neither particles nor waves, dominated the theory in the decade after the discovery of its law in 1926.

Then the standard model was developed. The standard model is not yet two generations old. It has nevertheless established which the elements are, quarks and leptons. What their kind is, the

fundamental properties of quarks and leptons are charge, $\frac{2}{3}$ and $-\frac{1}{3}$ of quarks, -1 and 0 of leptons, and spin, $\frac{1}{2}$ of quarks and leptons both. And what the relation of the kind and its species is as explicitly as to describe their generation in the big bang. All formally. The law is Schrödinger's equation.

The problem of the elements, or the material principles of all things, then, is both of which they are and of what kind they are. Of what kind they are is the more difficult part. And the solution of this part, and for this reason, because it is the more difficult, has made it clearer still which the elements are, what the elements are, and what all things are. Which the elements are – in tables. What the elements are – in laws. And what all things are – in the description of the world from its beginning 1.4×10^{10} years ago.

To collect the entire development of the problem of the elements, or the material principles, of all things cannot be too difficult.

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The efficient principles of all things.

Wood does not make beds.

So, when reason begins motion, it separates things from all that is moved, and all things are moved and divided.

Cursed strife separates them, love unites them, cursed strife equal in every way, love equal in length and breadth, both equally ageless.

In the next generation, Democritus said that the modification of all things is due to the shape, order, and position of the elements.

Two-odd generations later still, Aristotle said that the first mover is the efficient principle of all things. The world is spherical, at the circumference is the first mover, it moves the fixed stars, and thus communicates motion to everything inside, to Saturn, Jupiter, Mars, the sun, Venus, Mercury, the moon, and everything further inside, fire, air, water, and earth.

What all things are is clear. The philosophers are as explicit with respect to this problem, of the efficient principles, as they are with respect to that, of the material. Reason is the cause of the world. At the circumference of the world, the first mover moves everything in the world.

What the efficient principles are, or what the efficient principle is, on the other hand, is not so clear. Reason separates things. Strife separates them, love unites them. The modification of all things is due to the shape, order, and position of the elements. The first mover is the beginning of motion.

One century before Aristotle already the problem of the material principles had been solved as successfully as: earth, water, air, and fire, and: atoms and void. The problem of the efficient principles will not be solved nearly as successfully before Aristotle.

Aristotle's definition of efficient principle: efficient principle is the beginning of the change or rest of a thing, is thus also an empirical generalization over the successes of his predecessors. But these have not been as great with respect to the efficient principles as they have been with respect to the material. Aristotle complains: the problem of the beginning of change Leucippus and Democritus, like the others, lazily neglected. And this definition is not as successful as that either.

Which the efficient principles are, or which the efficient principle is, is still less clear. Reason. Or love and strife. Or the shape, order, and position of the elements. Or the first mover. These are not the neat series: water – air – fire – fire and earth – earth, water, air, and fire, and: number – an infinite number of particles – atoms and void.

The efficient principles are a substantially more difficult problem than the material.

Nevertheless, everything became clearer after Aristotle, then crystal clear.

One generation-odd after Aristotle, Zeno of Citium said that there are two principles of the whole, active and passive, the passive is matter, the active is the inherent in matter law, or god, which is everlasting and the producer of everything.

Later on, god, the active principle, the producer of everything, was released from the burden of having to be inherent in matter.

Five centuries after Zeno, early in the third Christian century, Empiricus generalizes: the best physicists have said that these two, god and undifferentiated matter, are the principles.

And if by that time there were no physicists

any longer, or more generally philosophers, the best or otherwise, to concern themselves with the passive principle, philosophers and otherwise concerned themselves very vigorously with the active principle.

Empiricus's contemporary Origen, in *On the principles*, begins: god is the principle of all things (I, 1).

Ten and a half centuries later, in the thirteenth Christian century, Aquinas assimilated Aristotle into Christian philosophy. He writes: the first mover everyone understands to be god.

The first mover everyone continued to understand to be god for three and a half more centuries, until the instauration of the sciences, in the seventeenth Christian century.

The instauration of the sciences culminated in *The mathematical principles*, in 1687.

And gravity is not the efficient principle of all things, but it is an efficient principle of all things.

There is gravity in bodies universally.

The lord god $\pi\alpha\nu\tau\omicron\kappa\rho\acute{\alpha}\tau\omicron\rho$ rules all things. It seems probable, however, that the lord god $\pi\alpha\nu\tau\omicron\kappa\rho\acute{\alpha}\tau\omicron\rho$ in the beginning formed matter in solid, massy, hard, impenetrable, movable particles. And thereafter, gravity moves the particles. Gravity gathers the particles into bodies. Gravity moves the bodies. Gravity gathers the particles into the earth, gravity shapes the earth, gravity moves the moon around the earth, gravity moves the earth around the sun, gravity moves all the other planets, and their satellites, and the comets. Gravity holds the solar system together. The fixed stars are the centers of similar systems. And to prevent these systems from falling on one another by their gravity the lord god $\pi\alpha\nu\tau\omicron\kappa\rho\acute{\alpha}\tau\omicron\rho$ placed them at immense distances from one another. Still, gravity has gathered them into similar systems, and gravity holds each such similar system together. There is gravity in bodies universally.

In 1796 Laplace released the lord god $\pi\alpha\nu\tau\omicron\kappa\rho\acute{\alpha}\tau\omicron\rho$ from the burden of creation. Gravity created the solar system. And if the solar system, then also the similar systems of which the fixed stars are the centers. Which explains also why they are at immense distances from one another.

And gravity has remained an efficient principle of all things, if not the efficient principle of all

things.

In the century after Laplace Faraday and Maxwell showed that there is another force, the electromagnetic.

But gravity has remained an efficient principle of all things.

In 1916, in *The foundation of the general theory of relativity*, Einstein writes: according to the general theory of relativity gravity plays an exceptional role in relation to the other, and in particular the electromagnetic forces, because the 10 functions representing the gravitational field define at the same time the metric properties of the four-dimensional space (4).

In 1934 Fermi discovered the weak force. In 1935 Yukawa discovered the strong force.

Gravity has yet remained an efficient principle of all things.

Indeed, gravity is there from the first.

The big bang theory begins at Planck time, 10^{-43} seconds after the big bang, with the separation of gravity from the still united strong and electroweak forces. The strong and electroweak forces decouple 10^{-32} seconds after the big bang. The electroweak force decouples into the weak and electromagnetic forces 10^{-12} seconds after the big bang.

Gravity, the strong force, the weak force, and the electromagnetic force are the efficient principles of all things.

And, again, had the problem of the efficient principles of all things been just that, which they are, so much would have sufficed to make explicit what the problem is. And to collect the development of the problem in its entirety would have been just that, to collect the steps leading to the discovery of the first mover, then the steps in the examination of the first mover, such as it was, then the steps leading to the discovery of gravity, then the steps leading to the discovery of the electromagnetic force, and then the steps leading to the discovery of the weak and the strong forces.

But which they are is only one part of this problem as well. The other part, what they are such as they are the efficient principles of all things, is again the more difficult. It is this part, too, that makes the problem of the efficient principles of all things substantially more diffi-

cult than the problem of the material.

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The anticipations of the problem Aristotle traces back to Hesiod. Hesiod spoke of love as a principle. Then Parmenides also spoke of love as a principle. Then Anaxagoras discovered that wood does not make beds. He said that reason is the principle. Then Empedocles said that love and strife are the principles.

Wood does not make beds. The carpenter does.

Love, reason, love and strife are all borrowed from the carpenter.

Which shows how great an advance Democritus's solution of the problem is. It borrows nothing from the carpenter. The shape, order, and position of the wood makes beds.

It shows also that it is still not too clear which the efficient principles are or the efficient principle is. The carpenter? and if so, does he make beds by reason or by love and strife? or the wood after all, its shape, order, and position?

Also, if the principles are many, next to the problem of which they are there will be the problem of their kind. But if the principle is one, the solution of the problem of which it is will be the solution also of the problem of its kind. The solitary principle of all things cannot not be *sui generis*.

And in the solution of the problem of the material principles of all things Aristotle's advance over his predecessors is not great. In the solution of the problem of the efficient principles of all things his advance is very great. He established which the efficient principle of all things is, and that it is solitary, too, and thus also of what kind it is. Nor is the advance in a return to borrowing from the carpenter.

Below is the argument that established this, which the efficient principle of all things is, that it is solitary, and thus also of what kind it is. In this case, as in all others, however, Aristotle does not distinguish principles, ἀρχαί, and causes, αἰτίαι, sharply. Bacon first distinguished principles and causes sharply, and principles and causes should be distinguished sharply. Principles should be distinguished by their generality, exclusively. Causes should be in relation to

effects, exclusively. And as the material principles of all things should be the elements, so the efficient principles of all things should be the causes. Taking this distinction as the guide, then, rather than the words, this is Aristotle's argument.

He begins in book III of the *Physics*: nature is the cause of motion and change.

Nature is the cause of motion and change. And rather, nature is the cause of motion, for change is motion. And rather, nature is the cause of locomotion, for motion is first of all locomotion.

In book VIII Aristotle concludes: there are three kinds of motion, motion with respect to size, motion with respect to quality, and motion with respect to position; and because the others are impossible without it, the last, locomotion, must be first; if there must always be motion, there must always be locomotion.

Nature is the cause of motion and change. So there must always be motion. And if there must always be motion, there must always be locomotion. Locomotion must be first.

Aristotle continues: locomotion is first in three ways; it is first in that, if it did not exist, other things would not exist, but if other things did not exist, it can exist; it is first in time; and it is first in being.

In the *Categories* he distinguishes five ways in which the first is first. Two of the five are: by being better, in which way it is least properly first, and by being first in knowledge. Ignoring these two ways, the other three are: by not implying the existence of another, by being the first in time, in which way it is also most properly first, and by being the cause of another. And these three ways have to be the three ways in which locomotion is first. By not implying the existence of another. In time, in which way it is also the most properly first. And first in being has to be first by being the cause of another.

Locomotion must be first. It must be the first of all things. Locomotion must be the first by not implying the existence of any other things, if it did not exist, no other things would exist, but if no other things existed, it can exist. Locomotion must be the first in time, it must precede all other things. And if first in being is first by being the cause of another, locomotion must be the first by

being the cause of all other things.

Aristotle continues: locomotion is first in time because it is the only kind of motion eternal things can have.

Eternal things are the stars. This is also why, because they are eternal, the stars cannot be presumed to be composed of earth, water, air, and fire, and: we conclude that there is another element beyond those we see on earth, aether. Locomotion is the only kind of motion the stars can have. Locomotion must be the first in time, it must precede all other things. And locomotion precedes all other things by being eternal.

Aristotle continues: and locomotion is first in being, because there must be motion continuously.

There must always be locomotion. There must also be motion continuously, and there thus must be locomotion continuously. If first in being is first by being the cause of another, there must be causes continuously. Locomotion is thus the continuous series of causes. Locomotion must be the first by being the cause of all other things. And locomotion is the cause of all other things by being the first cause in the continuous series of causes.

Aristotle reaffirms: no other kind of motion can be prior to locomotion; all other change, increase, decrease, perishing, must be posterior to locomotion.

And the concluding step is: everything in motion must be moved by something; and since an infinite series is impossible, there must be a first mover, itself unmoved.

There must be locomotion continuously. Everything in locomotion must be locomoved by something. The series of locomotions cannot be infinite. There must be a first locomover.

And, at the circumference of the world, the first locomover locomoves the stars, and thus communicates locomotion to everything inside.

Nature is the cause of motion and change. But change is motion, and motion is locomotion. For if there must always be motion, there must always be locomotion. Locomotion is thus eternal, it is thus also the continuous series of causes, and it is thus also the first cause. It is the first mover, and that is, the first locomover.

The solution of the problem has thus packed

the world spatially. The first in a series of continuous causes, the first mover moves, thing by thing, everything in the world, from its end to its center, planet by planet, element by element.

In the same way the solution has packed the world temporally. Thing by thing, in a series of continuous causes, for all eternity.

It has packed the world qualitatively, motion with respect to size and motion with respect to quality are motion with respect to position, all change, increase, decrease, perishing, is locomotion.

Being locomotion, it borrows little from the carpenter. Compared with the earlier attempts at solution, Aristotle's is more like Democritus's than like Anaxagoras's or Empedocles's. Even the difficulty in that the first mover moves things without itself moving is resolved without recourse to the carpenter. Aristotle writes in book XII of the *Metaphysics*: it thinks; it thinks of itself, its thinking is thinking of thinking (9). And in the *Physics*: it is like the lodestone, this is how it moves things without itself moving (VIII, 10). Thinking, positively thinking of thinking, is like the lodestone. Nor did everyone understand the first mover to be the first carpenter because it thinks. Never mind being like the lodestone.

And it is solitary. It is all by itself a great virtue of the solution, that the cause of all things is one. It has remained a beacon in the search for causes. Through Spinoza's: god is nature, and through the continuing search for the GUT, the grand unified theory.

Nineteen and a half centuries after Aristotle, Galileo writes: nothing in nature is older than motion, and about it volumes neither few nor small have been written by the philosophers; yet I find that its essential properties have not even been observed, let alone demonstrated.

And so the observations of its essential properties and their demonstrations began examining the cause of all things, too.

Galileo greatly advanced the observations and their demonstrations.

Kepler discovered the laws of the motions of the heavenly bodies. And concludes: no mind is necessary to move the planets, nor a soul to direct their revolutions; there is the one and only solar body in the center of the world and to it the

motion of the planets can be ascribed.

So the sun is the sign of the father, the fixed stars are the sign of the son, the planetary system in between is the sign of the holy spirit.

Descartes formulated the laws of nature. On this ground: by nature I understand not some goddess or other imaginary power, but matter itself; and from the single condition that god continues to conserve it as he created it follows necessarily that the changes in it cannot be attributed to the action of god; because the action of god does not change at all, they have to be attributed to nature; and the rules according to which these changes occur I call the laws of nature.

Nature is matter. The laws of nature are laws of conservation. The changes in matter follow from conservation.

And Descartes formulated the theory of vortices, packing the world with vortices.

Huygens discovered the conservation of momentum.

Newton then discovered gravity. There is gravity in bodies universally.

And gravity did everything that the first mover did very much better than the first mover did it. Except for one thing.

It packed the world very much better spatially. It explained the motion of the moon as the motion of a pendulum in Cayenne, the motion of Saturn as the motion of the moon, the motion of the solar system as the motion of Saturn, and the motion of the systems of the fixed stars as the motion of the solar system, one cause from the center of the world to infinity.

It packed the world very much better temporally. It seems probable that god in the beginning formed matter in solid, massy, hard, impenetrable, movable particles. After that, given the particles, gravity did everything, from gathering the particles into bodies to holding the systems of the fixed stars together.

It packed the world very much better qualitatively. All bodies seem to be composed of hard particles, the smallest particles may cohere by the strongest attractions and compose bigger particles of weaker virtue, these may cohere and compose bigger particles whose virtue is still weaker, and so on, until the progression ends in the biggest particles on which the operations of chemistry

and the colors of natural bodies depend.

And it not only borrowed nothing from the carpenter. And not only did everything from observation and experiment, and by demonstration. It did it quantitatively. And formally, by law.

$$F \propto \frac{m_1 m_2}{r^2}.$$

The one thing gravity did not do better than the first mover was being solitary.

And for this reason the problem of the causes of all things remained difficult.

Gravity had been shown a cause of all things, but not the cause of all things. The problem of which the causes are had been solved very much better, but only in part. This posed the problem of which the other causes, next to gravity, are. The problem thus also of their kind. But gravity continued to be a cause, and better, and better. This posed the problem of showing that gravity is, after all, the only cause. Another cause was discovered instead. Yet gravity continued to be a cause, and better than the other cause. Two other causes were discovered. Yet gravity continued to be a cause, and better. Continuing to pose the problem of their kind.

Newton is clear that gravity is a cause of all things. Rule 1 is: to admit no more causes of things than are true and suffice to explain the phenomena. And: there is gravity in bodies universally.

He is clear also that gravity is not the only cause. He has demonstrated that: the force of gravity is different in kind from the magnetic force, for magnetic attraction is not as the attracted matter.

He is clear also that this poses the problem of their kind. He concludes: bodies act on one another by gravity, magnetism, and electricity, and these show the tenor and course of nature, and make it not improbable that there may be more attractive powers than these. And more generally: it seems to me that the particles which god formed in the beginning had not only *vis inertiae*, with such passive laws of motion as naturally result from it, but that they also are moved by certain active principles, such as that of gravity, and that which causes fermentation, and the cohesion of bodies, and these principles I

consider not occult qualities, but general laws of nature.

He is clear also that the kind is forces.

He writes: the causes which distinguish true motion from relative are the forces impressed on bodies to generate motion.

And: the entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena.

And the law of gravity is a law of force, $F \propto \frac{m_1 m_2}{r^2}$, and the laws of motion, on which the law of gravity rests, are laws of forces, too, inertia, $F = ma$, and $F_{AB} = -F_{BA}$.

But which the other forces are – magnetism, and electricity, which show the tenor and course of nature, and make it not improbable that there may be more attractive powers, the certain active principles, such as that which causes fermentation, and the cohesion of bodies – remained to be demonstrated. And the solution of the problem of their kind could not not depend on the solution of the problem of which they are.

One century later, in 1796, Laplace writes: the perturbations in the motions of the planets, thought to be exceptions to the law of gravity, have become one of the striking examples of its truth; such has been the lot of Newton's brilliant discovery that every difficulty which it has encountered has provided it with an opportunity for a new triumph.

So much so, so much had the striking examples of the truth of Newton's brilliant discovery accumulated, that the problem of demonstrating which the other forces are had become the problem of demonstrating that there is, after all, but the one Newtonian force. The most striking of all the striking examples had been Coulomb's discovery of the fundamental law of electricity, in 1785. Coulomb writes: the fundamental law of electricity is, the repulsive force between two small spheres charged with the same kind of electricity is in the inverse ratio of the square of the distance between the centers of the spheres (*First*).

And so the problem remained, to demonstrate that there is but the one Newtonian force, until and including Faraday.

Faraday published the *Experimental researches in electricity* between 1839 and 1855.

In 1850 he writes: the long constant persuasion that all the forces of nature are mutually dependent, having one common origin, or rather being different manifestations of one fundamental power, has made me often think on the possibility of establishing, by experiment, a connection between gravity and electricity, and so introducing the former into the group, the chain of which, including also magnetism, chemical force, and heat, binds so many and such varied exhibitions of force together by common relations; though the researches I have made with this object in view have produced only negative results, yet I think a short statement of the results of the experiments may be useful (2702). He describes the experiments, and concludes: here end my trials for the present; the results are negative (2717).

All the forces of nature are mutually dependent, having one common origin, or rather being different manifestations of one fundamental power.

Faraday is one of the last philosophers, physicists, in particular, to concern himself explicitly with all the forces, the one common origin, the one fundamental power. Faraday is a younger contemporary of Hegel, and it was at that time that the philosophers, physicists, in particular, *en masse* abandoned the explicit concern with all such things, all the forces, the one common origin, the one fundamental power, to the philosophers, not physicists, in particular.

Faraday sought to establish, by experiment, a connection between gravity and electricity. The results were negative.

In the meantime, however, thermodynamics was developed. Clausius formulated its laws in 1850. And thermodynamics turned out to be yet another striking example of the truth of Newton's brilliant discovery. In 1843 Joule demonstrated the mechanical equivalent of heat. And in 1877 Boltzmann showed that if heat is the motion of particles, and the particles are centers of force, thermodynamics can be developed from the laws of mechanics.

In the meantime, however, electrodynamics was also developed. Maxwell formulated its laws in 1864. And if Faraday's negative results had

not, his affirmative results had indicated strongly that the electromagnetic is a force distinct from gravity. Maxwell's equations demonstrated it. Just as Newton, in fact, already had. The electromagnetic force is different in kind from gravity, for it is not as the attracted matter. Maxwell also showed that light is an electromagnetic wave.

Two forces, then, gravity and the electromagnetic.

Then Einstein developed the theory of relativity. The special in 1905, the general in 1916. The special is a theory of motion. And it showed that the laws of electrodynamics are consistent with the laws of mechanics. In 1911 Einstein deduced from the theory the deflection of light by the sun's gravitational field. The general is a theory of gravity. And: according to the general theory of relativity gravity plays an exceptional role in relation to the other, and in particular the electromagnetic forces. In 1919 Eddington observed the deflection of light by the sun's gravitational field. The first observational confirmation of relativity thus produced an affirmative result where Faraday's experiments had produced only negative, it showed the interaction of gravity and the electromagnetic force.

Two forces, but one plays an exceptional role in relation to the other.

In the meantime, in 1900, Planck discovered that thermal radiation imparts energy to matter in discontinuous quanta. In the next generation quantum theory was developed, Schrödinger discovered its law in 1926.

In the next generation-odd the big bang theory was developed.

The standard model was formulated.

10^{-43} seconds after the big bang gravity decoupled from the still united strong and electroweak forces. The strong and electroweak forces decoupled at 10^{-32} seconds. The electroweak force decoupled into the weak and the electromagnetic at 10^{-12} seconds.

The forces are mediated by six bosons. Gravity by one, the graviton. The strong force by one, the gluon. The weak force by three, the W^+ , the W^- , and the Z^0 . And the electromagnetic force by one, the photon. The bosons are also carriers of charge and spin. The charge of the graviton, gluon, Z^0 , and photon is 0, of W^+ +1, of W^-

-1, the spin of the graviton is 2, of the gluon, W^+ , W^- , Z^0 , and photon 1. The fundamental difference between these mediators of the forces, the bosons, on one side, and the elements, the quarks and leptons, on the other, is that the bosons do not obey the Pauli exclusion principle, the quarks and leptons obey the Pauli exclusion principle. The principle is that no two particles in the atom can be in the same quantum state. Pauli formulated it in 1925. The gluon, W^+ , W^- , Z^0 , and photon have been demonstrated experimentally, the graviton has not been.

Four forces, then, gravity, the strong, the weak, and the electromagnetic.

These are the causes of all things.

They have packed the world spatially from the point of observation to the limit of observation 4.4×10^{23} km away.

They have packed the world temporally from its beginning to its end.

They have packed the world qualitatively building it explicitly from its elements.

They have packed the world quantitatively. And formally, by law.

The problem of which the causes of all things are has been solved. And the more difficult problem, of what they are such as they are the causes of all things, has been solved, too, even if implicitly more than explicitly.

The theories of the four forces, gravity, the strong, the weak, and the electromagnetic, do everything that the Newtonian theory did very much better than it alone did it, and do very much more than it alone did. Except for that one thing. It did it alone.

As formulated by Newton already the Newtonian theory explained everything, more or less, that had ever been explained. And for the following one and a half centuries such was its lot that every difficulty which it encountered provided it with an opportunity for a new triumph. It remained the one theory into which all discoveries were either explicitly shown to belong or presumed to have to be shown to belong.

Then thermodynamics was developed with its laws. Electrodynamics with its laws. Relativity with its principles. Each of these was built on the presumption of continuity in the development of theory, and each was very much concerned, and

most explicitly, with maintaining the unity of theory. And it was shown that thermodynamics can be developed from the laws of mechanics, and relativity showed that the laws of electrodynamics are consistent with the laws of mechanics.

Then quantum theory was developed. And, as in the course of its development the supposition that this theory, now, is building everything anew, indeed *ex nihilo*, increasingly replaced the presumption of continuity in the development of theory – the supposition manifested rather spectacularly by the fact that the development of the theory has been accompanied by the coining of terms so eager as to have amounted to a dictionary of its own – quantum theory became an eager source of principles of its own. Moreover, one part of the standard model is the theory of electroweak force, of the unified weak and electromagnetic forces. But there is no theory of the unified strong and electroweak forces. And there is no GUT, the grand theory that would unify all four forces.

Also, if the standard is that of spirit, restored as spirit has been to the substantiality and purity of its being, the relations of quantum theory, on one side, and thermodynamics, electrodynamics, and relativity, on the other, are most explicit. So much so, they have all been gathered into the one big bang theory. If the standard is that of the theories themselves, however, the relations of quantum theory, including its part in the big bang theory, on one side, and thermodynamics, electrodynamics, and relativity, including their part in the big bang theory, on the other, have not been made as explicit as they have to be. It has been shown neither that quantum theory can be developed from their laws, nor that they can be developed from its law. Neither its law has been shown consistent with their laws, nor their laws with its law.

This is the difficulty. That quantum theory stands, or appears to stand, apart from the rest of theory. Or the other way around, that the rest of theory stands, or appears to stand, apart from quantum theory.

Thus, gravity, the strong force, the weak force, and the electromagnetic force are the causes of all things. What they are such as they all are the causes of all things is more difficult. But not too

difficult. Even if implicitly more than explicitly, this problem has been solved, too.

Clearly, they are all four forces. Just as clearly, they are all four forces because of their Newtonian pedigree. There is continuity in the development of theory. And the continuity in the development of theory indicates their kind more explicitly, and unambiguously.

Planck writes, in 1914: as long as there is a physical science its highest goal is the solution of the problem of embracing all observed and still to be observed natural phenomena in one simple principle from which all past and especially future occurrences can be calculated (*Action*).

And there is one simple principle from which all past and especially future occurrences have been calculated. The principle of conservation of energy.

In 1633 Descartes said that nature is matter, the laws of nature are laws of conservation, and the changes in matter follow from conservation.

In 1656 Huygens discovered the conservation of momentum.

In 1686 Leibniz discovered the conservation of motive force, as he calls it in 1686, or absolute force, or absolute living force, as he calls it in 1691, or living force, as he calls it in 1695, or, however it is called, twice the kinetic energy. He concludes: the conservation of the same force in the world is an inviolable truth.

In 1789 Lavoisier formulated the principle of conservation of matter.

In 1847 Helmholtz formulated the principle of conservation of energy in all its generality.

And from 1847 to 1926, from the discovery of the principle of conservation of energy to the discovery of the law of quantum theory, the laws of thermodynamics, of electrodynamics, of relativity, and of quantum theory were all discovered, all in but two and a half generations.

In 1850 Clausius formulated the laws of thermodynamics. The first is: the energy of the world is constant.

In 1864 Maxwell formulated the laws of electrodynamics. He writes: the doctrine of the conservation of energy is the one generalized statement which is found to be consistent with fact not in one physical science only, but in all; once comprehended it furnishes to the physical

inquirer a principle on which he may hang every known law relating to physical actions, and by which he may be put in a way to discover the relations of such actions in new branches of science (*Matter*, 73). As Maxwell himself hung on it the laws of electrodynamics, and by it his successors were put in that way.

Electrodynamics showed also the conservation of charge. The conservation of charge is a manifestation of the conservation of energy.

In 1900 quantum theory began with Planck's discovery that energy is quanta.

In 1905 Einstein formulated the special theory of relativity. $E = mc^2$. Einstein comments: the mass of a body is a measure of its energy content. The conservation of matter is also a manifestation of the conservation of energy.

In 1926 Schrödinger discovered the law of quantum theory:

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0,$$

where ψ is the wave function, m the mass of the particle, E is the total energy, V the potential energy, and h is Planck's constant (*Lectures*, 1). The wave function determines the position of the particle, and thus its energy level. In 1926, in the *Quantization as an eigenvalue problem*, Schrödinger develops the law inductively. In 1928, in the *Four lectures on wave mechanics*, he deduces it. The deduction is from: the principle of least action and the principle of conservation of energy (*id.*).

Quantum theory has shown also the conservation of lepton number and the conservation of baryon number. And these are also manifestations of the conservation of energy. Baryon is the particle of three quarks held together by gluons.

Thermodynamics, the theory of heat, is a theory of energy. Electrodynamics, the theory of the electromagnetic field, is a theory of energy. Special relativity, the theory of motion, is a theory of energy, hence $E = mc^2$. General relativity, the theory of the gravitational field, is a theory of energy. Most particularly, quantum theory, the theory of the particles and the forces, is a theory of energy. And the big bang theory begins at Planck time, 10^{-43} seconds after the big bang, with nothing but all the energy of the world.

The conservation of energy has to be the kind of the causes of all things.

To collect the entire development of the problem of the causes, or the efficient principles, of all things cannot be too difficult either.

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The formal principles of all things.

The efficient principles are more difficult than the material, and the formal are the most difficult.

The difficulties begin with the fact that the principles are many.

To illustrate.

It is in *The principle of least action*, in 1914, that Planck writes: as long as there is a physical science its highest goal is the solution of the problem of embracing all observed and still to be observed natural phenomena in one simple principle from which all past and especially future occurrences can be calculated.

And continues: among the more or less general laws, which show the success in the development of physics in the last centuries, the principle of least action has risen, in form and content, the closest to this ideal goal of theoretical investigation (*id.*).

In 1662 Fermat formulated the principle of least time.

In 1740 Maupertuis formulated the more general principle of least action.

In 1744 Euler formulatded Maupertuis's principle analytically.

In 1788 Lagrange formalized and generalized Maupertuis's principle, and deduced Newton's laws from it.

In 1834 Hamilton formalized and generalized the principle further, and deduced from it both the laws of mechanics and the laws of optics.

In 1915 Hilbert deduced the equations of relativity from Hamilton's principle, and in 1916 Einstein deduced the entire theory of general relativity from Hamilton's principle.

In 1926 Schrödinger deduced the law of quantum mechanics from Hamilton's principle.

Planck continues in *The principle of least action*: this central position of the principle of least action is not entirely undisputed; an especially keen competitor has been the principle of conservation of energy, which similarly reigns

over all of physics; it should thus be made clear what the relative position of these two principles is; the principle of conservation of energy can be derived from the principle of least action, and it is therefore contained in it; but the converse is not the case; the principle of conservation of energy is thus the special, the principle of least action the embracing law (*id.*).

The relative position of the two principles is determined by the relation of more or less general.

And the principle of conservation of energy cannot not be of the greatest generality. The energy of the world is constant.

But if the principle of conservation of energy can be derived from the principle of least action, and not conversely, and the less general is derived from the more general, the principle of least action has to be more general than the principle of conservation of energy. If the principle of conservation of energy is of the greatest generality, the principle of least action has to be of generality greater than the greatest. And:

can anything increase the all? where will it come from?

and how will it perish?

Fortunately, the relative position of the two principles is determined also by the relation of form and content.

Planck's concluding example is relativity. He writes: as the principle of least action so the principle of conservation of energy has its definite place in the theory of relativity; energy, however, is not invariant under the Lorentz transformation, just as it was not earlier under the Galilean transformation, because in energy time plays a privileged role; its correlate with respect to space is the principle of conservation of momentum; but over both these principles, even taken together, reigns the principle of least action, which thus appears to rule all reversible processes; yet it offers no explanation for irreversibility; because according to it every process, in space as well as in time, can develop in any direction, forward as well as backward (*id.*).

Clausius discovered irreversibility, in 1850, and in 1854 he discovered entropy. Hence the second law: the entropy of the world strives for a maximum.

And Planck writes: the changes in nature have one single direction; with every single irreversible process the world takes a step forward (*Physics*, 1); in nature there is not one single reversible process (*id.*); and in the domain of irreversible processes the principle of least action is no longer sufficient (*id.*).

The principle of least action rules all reversible processes. But in nature there is not one single reversible process. And the principle of conservation of energy rules all irreversible processes.

It can be concluded that the principle of least action is not more general than the principle of conservation of energy, but more formal. That it reigns over the principle of conservation of energy not because it is of greater generality, but because it is of greater formality.

Generality has to admit of degree, there has to be lesser and greater generality, as, given some particulars, a proposition describes fewer or more of these particulars. And there has to be a greatest generality, as a proposition describes all the particulars, or what is the same, describes the world. The danger is clear, were there not a greatest generality. Generality would abandon the world, or synonymously nature, there would be more than the world, there would be a super-nature, and worlds next to the world, and as many as you please, at will, man would be the measure of all things, the world would be world-views.

The same with formality. It has to admit of degree, there has to be lesser and greater formality, as, given some particulars, a proposition abstracts less or more from these particulars. There has to be also a greatest formality, as a proposition abstracts all that can be abstracted from all the particulars. And abstracts all that can be abstracted, but not all. For the danger is the same, if all could be abstracted. Formality would abandon the world, or synonymously nature, mathematics would be a free creation of the human mind, so would be logic, and so would be everything, at will, man would be the measure, and his measure would be world-views.

And if generality and formality do not coincide in their degrees, the danger disappears.

The difficulty is that generality has to be formality. To be general, to describe many, a proposition has to abstract from the particulars.

And formality has to be generality. To be formal, to abstract from even one, a proposition has to divide the particular into many. And any proposition that will distinguish generality, or formality, or the degrees of the one, or of the other, or compare them, has to be itself a generalization and a formalization both.

The distinction of form and content is the resolution of all these difficulties.

For, if generality and formality do not coincide in their degrees, the principle of least action and the principle of conservation of energy can be of the greatest generality both, as, in fact, they are, and the principle of least action can also be of greater formality than the principle of conservation of energy, as, in fact, it is.

In the domain of irreversible processes the principle of conservation of energy reigns. The domain of irreversible processes is the world. In nature there is not one single reversible process. Abstracting from irreversibility, the principle of least action reigns. The domain of reversible processes is also the world. Nature from which irreversibility is abstracted.

More and less content, less and more form.

And the generality and the formality of the problem, both, cannot be overstated. Both are the greatest.

Thus, the principle of conservation of energy cannot be derived from the principle of least action without taking the principle of contradiction for granted. The principle of all principles is of the greatest generality, too, it is a principle of the world, like the principle of least action. But it is also of greater formality than the principle of least action. It is, in fact, as the principle of least action is not, of the greatest formality.

From the principle of conservation of energy Schrödinger's equation is derived. Again, the principle and the equation are of the greatest generality both, but the principle is of greater formality than the equation.

The importance of the problem cannot be overstated either.

The principles, by being principles, cannot not be general. The principles of all things, by involving all things, cannot not be formal. And general and formal in relation to one another cannot not raise all those difficulties. That the general is

formal. The formal is general. Each has degrees. The degrees do not coincide. Any proposition concerned with any of those cannot not be a generalization and a formalization both. Yet, the principles being principles and all things being all things, the soundness of the solutions of the problems of the material principles of all things and of the efficient principles of all things cannot not depend on the solution of the problem of the formal principles of all things. And the principles being principles and all things being all things, the soundness of all solutions of all problems cannot not depend on the solutions of the problems of the material principles of all things and of the efficient principles of all things.

And philosophy being philosophy, the problem of the formal principles of all things, with all its generality, and formality, and difficulties, implicitly if not explicitly, is in the center of the development of philosophy from the first, if not with the first philosopher already, with the third and the fourth philosophers already.

And in this problem, too, as in all the others, the investigation itself showed them the way.

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Thales said that water is the principle. Anaximenes said that air is prior and it is the principle. Pythagoras said that number is the principle.

And number has remained the principle of all things.

It is also clear why number is the principle. Because of mathematics. Aristotle writes: Pythagoras advanced mathematics, and from its study came to think that its principles are the principles of all things.

Heraclitus said that fire is the principle. Parmenides said that all is one in form. He sings:

what is is and cannot not be,
and what is not is not and must not be.

And the principle of all principles has remained the principle of all principles.

It is also clear why what is is, and what is not is not. Because of truth and knowledge. Parmenides sings:

there are two roads,
what is is and cannot not be,
which is the road of truth and conviction,
and what is not is not and must not be,

which you will never know or utter.

Anaxagoras said that an infinite number of particles are the principles, and reason is the cause of the world and its order. Empedocles said that earth, water, air, and fire are the principles, and love and strife are the sources of motion. Leucippus said that atoms and void are the principles.

One and a half centuries into the development of philosophy the solution of the problem of the principles of all things has advanced very greatly. The solution of the problem of the material principles by Anaxagoras, Empedocles, and Leucippus will not be improved upon substantially or essentially for two millennia. That the problem is of both the material principles and the efficient principles has been discovered. Anaxagoras and Empedocles have also solved the problem of the efficient principles, and in the next generation Democritus's advance in the solution of this problem, too, will not be improved upon substantially or essentially for two millennia.

Socrates found the general in definitions. Democritus said that the modification of all things is due to the shape, order, and position of atoms and void.

Plato formulated the theory of the forms.

And two centuries into the development of philosophy, with the theory of the forms the problem of the formal principles of all things is formulated and solved.

It is again clear why.

Aristotle writes: in his youth Plato became acquainted with the Heraclitean doctrine that all sensible things are always in flux and there can be no knowledge of them, and Plato held this view in his later years as well.

All sensible things are always in flux.

Plato writes in the *Theaetetus*: to everything the Heracliteans say the Eleatics say the opposite.

All sensible things are always in flux. But what is is, and what is not is not.

Aristotle continues: in the meantime Socrates, while searching for the general in ethical matters, first concerned himself with definitions, and Plato followed Socrates.

All sensible things are always in flux. But what is is, and what is not is not. Aristotle continues: Plato presumed that definitions cannot be of

the sensible things, definitions have to be of entities of another kind, and these entities he called forms.

Finally: Plato agrees with the Pythagoreans that all other things are by participation in numbers.

And: the sensible things are named after the forms, and the many sensible things called by the same name as a given form are by participation in that form.

Plato continues the labors of Pythagoras, Heraclitus, Parmenides, Socrates. The theory of the forms is an empirical generalization over the successes of Plato's predecessors.

The investigation itself showed them the way.

And continued to show them the way.

Two and half centuries into the development of philosophy, Aristotle writes: when everything else is removed, nothing but matter remains; matter is what is by itself neither anything in particular nor a quantity or any other of the categories by which being is determined; we can thus speak of anything as form, or as having form, and of nothing as matter by itself.

Matter is no determination whatsoever. Form is any determination whatsoever.

And to solve the problem of form, or determination, Aristotle developed logic. And rather the other way around, Aristotle's solution of the problem of form, or determination, is logic.

A proposition is a sentence affirming what is true or false.

The elements of a proposition are terms, the subject and the predicate, is or is not relating the two.

The formal relation of propositions is the inference.

Inference is of several kinds, rhetoric, contentious argument, examination, dialectical argument, and demonstration.

And demonstration is an inference grounded in what is true.

Propositions are the formal principles of all things.

And the development of logic, and the solution of the problem of the formal principles of all things, is an empirical generalization.

Philosophy begins with all things.

All things are water. All things are number.

All things change. All things are what they are.

All things are the sensible things and the forms.

All things are matter and form.

Philosophy begins with all things, and thus with the search for the general, at first implicitly, and then explicitly.

Socrates found it in definitions.

Definitions are of entities of another kind, the forms. The many sensible things are by participation in a form.

To argue, while searching for the principles of the sensible things, that there is some other kind of essence, is prattle.

All things have principles. Material principles, efficient principles, and formal principles.

Aristotle's definition of formal principle is: formal principle is the form and paradigm of a thing, and that is, the definition of what it is and its genus. The first part: the form and paradigm of a thing, shows its pedigree. The second: the definition of what it is and its genus, is a definition of definition. Aristotle defines definition in the *Posterior analytics*: definition is a statement of what a thing is (II, 8), and in the *Topics*, better: a definition should state the near genus and the specific difference (I, 8).

All things change. There are thus the forms. But the addition of the forms to the sensible things has multiplied all things beyond all bounds. It has exposed them also to the danger of being thoughts. Plato writes: if what makes large things large is largeness itself, above this largeness and the things it makes large another form of largeness has to appear to make them both large, and above all these then another form has to appear to make them all large, and above these another, and another; perhaps, then, these forms are thoughts and are only in the mind; but then all things would be thoughts.

The forms are prattle.

All things do not change. Some of all things change, the rest do not change. On pain of contradiction, for instance, the principle of contradiction does not change.

The division of all things into matter and form, no determination and determination, has resolved the difficulty that all things both change and do not change without multiplying all things. Form

is determination, determination is propositions, and propositions are among all things. But all things are not propositions. That a proposition has elements, that it affirms what is true or false, that it is related to other propositions, all show that all things are not propositions. It has avoided the danger of making all things thoughts, too. What is true, what is not is false.

And if form is determination, and determination is propositions, all difficulties in the relations of general and formal are resolved.

General are propositions. And they are less and more general. Given a number of particulars, a proposition can describe none of them, or some of them, or more of them, or all of them.

But propositions are also formal. And they are also less or more formal. Determination admits of degree, from no determination to some determination to greater determination to complete determination. This resolves also the difficulty that, if matter is no determination whatsoever, and form is any determination whatsoever, form cannot be investigated in relation to matter, it can only be investigated in relation to itself, and matter cannot be investigated in relation to itself, it can only be investigated in relation to form. The lesser determination is less formal and more material, the greater determination more formal and less material. The lesser determination abstracts less from matter, the greater determination abstracts more. Given a number of particulars, a proposition can abstract none of what they all are, or some of it, or more of it, or all of it.

The degree of determination can itself be determined only by the relation of the elements in a proposition and the relation of propositions in inferences.

The elements of a proposition are the subject and the predicate, is or is not relating the two.

The formal relation of propositions is the inference.

If the degree of determination is determined by the relation of the elements in a proposition and the relation of propositions in inferences, both, the relation of the elements in a proposition is the lesser determination, less formal and more material, the relation of propositions in inferences is the greater determination, more formal and less material.

Propositions are particular and general. Less formal and more material, this is a relation of the elements in a proposition. This relation is content rather than form.

Inferences are conditional and unconditional. More formal and less material, this is a relation of propositions. This relation is form rather than content.

Both admit of degree. From particular to general to more general to most general. From a few things related by its elements in a proposition to more things to more things to all things. And from conditional to less conditional to less conditional to unconditional. From many gaps in the chains of inferences relating the same number of things in propositions to fewer gaps to fewer gaps to no gaps.

Of two equally general propositions one can be more formal than the other. Of two equally formal propositions one can be more general than the other.

Generalization, the addition of content from one proposition to another, is not formalization. Formalization, the removal of gaps in the chains of inferences connecting the propositions, is not generalization.

All by itself, the generalization of a proposition makes it less formal. All by itself, the formalization of a proposition makes it less general.

Deduction depends on formality rather than generality. Many more propositions in Plato are general than in Euclid.

And generality, not formality, is the distinguishing characteristic of the principles. Content rather than form makes the principles the principles. Form makes them true. Many more propositions in Euclid are true.

The development of logic by Aristotle is an advance over his predecessors so great, it has tended to appear a miracle. Aristotle himself reports: in this investigation it was not that something was done, something was not done, nothing at all was done. After Aristotle it has tended to encourage the supposition that man-made, in general, logic was made by the man Aristotle *ex nihilo*, in particular.

The development of logic is an empirical generalization. It is an empirical generalization over the very great advance of the acquisition of

knowledge in the two and a half centuries beginning with the first philosopher, and most particularly over the very great advance of mathematics in these centuries.

The advance culminated one generation-odd after Aristotle in the most beautiful and never sufficiently praised *Elements*.

The acquisition of knowledge is the winning of propositions. Mathematics is paradigmatically the winning of propositions. And rather the other way around, the most successfully won propositions are mathematics.

The acquisition of knowledge is generalization, formalization, and generalization by formalization. Mathematics is paradigmatically generalization by formalization. And again rather the other way around, the most successful generalizations by formalization are mathematics.

Physics investigates form as it is not separable from matter, mathematics separates form from matter for the sake of analysis. This is descriptive, Thales to Aristotle's physical treatises, Pythagoras to Eudoxus and the theory of proportion.

Inference is rhetoric, contentious argument, examination, dialectical argument, and demonstration. And this is also descriptive, from Protagoras and Gorgias, who gave them shoes rather than teach them how to make shoes, through Socrates and Plato, to Theaetetus and Eudoxus.

Aristotle developed logic at its most formal with the predicate calculus. As Łukasiewicz describes it: syllogistic is a strictly formal, mathematical theory, it is a system whose exactness surpasses even the exactness of a mathematical theory.

Logic is paradigmatically the winning of propositions. It is paradigmatically generalization by formalization. The development of logic is a step in the development of mathematics. In the separation of form from matter for the sake of analysis.

Propositions are the formal principles of all things.

Propositions are less and more general.

And the most general propositions are the principles.

Thus, not earth, water, air, and fire are the material principles of all things. The propositions

affirming that earth, water, air, and fire are the material principles of all things are principles. And very much better, earth, water, air, and fire are the elements of all things. The propositions affirming that earth, water, air, and fire are the elements of all things are principles.

Not the first mover is the efficient principle of all things. The propositions affirming that the first mover is the efficient principle of all things are principles. And very much better, the first mover is the cause of all things. The propositions affirming that the first mover is the cause of all things are principles.

Propositions are the formal principles of all things is then redundant. Propositions are the principles of all things. And the propositions affirming that propositions are the principles of all things are principles.

Propositions are also less and more formal.

And the principle of contradiction is the most formal most general principle. It is the principle of all principles.

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Twenty centuries after Aristotle, Descartes formulated: the project for a general science capable of raising our nature to its highest degree of perfection.

Not quite two generations later, Leibniz began realizing the project.

Two things have to be distinguished in all arguments, the form and the matter. A calculus which deals with the form can thus be separated from a calculus which deals with the matter. In a calculus which deals with the form, the quantity, quality, and truth of propositions will be expressed formally. Truth or falsehood will then be judged by form alone. By calculation.

He produced many, steadily more perfect specimens of the calculus dealing with the form. The philosophical calculus. The universal calculus. The general science. The fundamental arithmetic on which all knowledge rests. The method of examination of inferences by numbers.

He described how it will be completed.

All human thoughts can be resolved into a few primitives. If characters are assigned to these, characters can be formed for notions derivative from them. It will be then possible to find defini-

tions, or values, and to demonstrate the properties implied by the definitions. And it will be then possible to arrange propositions in their natural order. And to construct this miraculous language nothing more is necessary than to provide its grammar and to compile the dictionary of its most frequent terms, or what is the same, to give the characteristic numbers of all ideas.

He described the miracle the miraculous language will bring about.

There will be no more need for disputes between two philosophers than between two calculators, it will suffice to take their pens, sit down to their abaci, and say to one another, let us calculate.

He established that the miracle has, in fact, occurred.

I stumbled necessarily on this admirable observation, that an alphabet of human thought is possible, such that by the combination of its letters and the analysis of the words built from them alone all things can be both discovered and judged. The observation of mathematical analysis showed me the truest and most beautiful examples of this general analysis of human thought. In arithmetic and algebra all reasoning consists in the use of characters, and an error of the mind is the same as an error of calculation.

And he established the foundation of the miracle.

There are things that cannot be weighed, those that have no force or power. There are things that have no parts and cannot be measured. But there is nothing that cannot be numbered. Number is, as it were, the metaphysical form, and arithmetic is the statics of the world, by which the powers of things are explored. Men have known that the greatest mysteries lie hidden in number since Pythagoras.

Once again, it is an empirical generalization. It is an empirical generalization over the very great advance of the acquisition of knowledge in the three generations since the beginning of the instauration of the sciences with Bacon, Galileo, and Kepler, and most particularly over the very great advance of mathematics in these three generations. The world is written in mathematics. The laws of the heavenly bodies are discovered. The rules of the method. Calculus. The mathe-

mathematical principles.

It is thus the philosophical calculus. The universal calculus. The general science.

It deals with all things. All arguments. All human thoughts. All reasoning. With it all things can be both discovered and judged. It is the fundamental arithmetic on which all knowledge rests.

It deals with form. It separates the form from the matter. It is the calculus of the form. It judges truth and falsehood by form alone.

It is from the observation of mathematical analysis. The observation of mathematical analysis shows its truest and most beautiful examples.

It is arithmetic. It is the fundamental arithmetic on which all knowledge rests. Arithmetic is the statics of the world.

And its foundation is number. It gives all ideas their characteristic numbers. It is the method of examination of inferences by numbers. There is nothing that cannot be numbered. Number is the metaphysical form.

Propositions are the principles of all things.

Propositions are fundamentally mathematical propositions.

Mathematical propositions are fundamentally arithmetical propositions.

And the foundation of arithmetical propositions is number.

Number is the metaphysical form.

Leibniz discovered also that the principle of sufficient reason is the other, next to the principle of contradiction, principle of all principles.

Propositions are analytic and synthetic. The principle of contradiction is the principle of all analytic propositions, the principle of sufficient reason is the principle of all synthetic propositions. The principle of contradiction is the formal criterion of truth, the principle of sufficient reason is the material criterion of truth.

Two centuries after Leibniz, Frege completed the project.

The path by which a proposition is gradually won goes through stages of certainty. A proposition is surmised from an insufficient number of particular cases. It is then more and more certainly established. It is brought through chains of inferences in connection with other propositions as consequences are derived from it and it is

recognized as the consequence of other propositions. It is eventually most firmly grounded, and its inner nature is shown, when through chains of inferences it is brought in connection with all propositions which are its consequences and all propositions of which it is the consequence.

Proof is to bring a proposition in connection with the propositions which are its consequences and of which it is the consequence.

The stages of the path are stages of proof.

The end of the path is the firmest proof. Complete certainty. Chains of inferences free of gaps. The concept writing. The formula language of pure thinking modeled on that of arithmetic.

The signs of arithmetic, geometry, and chemistry are realizations of the philosophical calculus for particular areas. The concept writing adds a new one, the one in the middle, adjoining all others. From this, the work can begin, with the greatest prospect of success, at filling the gaps in the existing formula languages, connecting their separate areas into a single domain, and extending this domain to areas which have lacked such a language. It can be successfully employed to establish the foundations of the differential and integral calculus, it can extend its area to geometry, the transition to the pure theory of motion and then to mechanics and physics can follow, and if it is a task of philosophy to break the dominion of the word over the human mind, it can become an effective tool of the philosopher as well.

The path by which a proposition is gradually won is the path of all acquisition of knowledge. Its uniform path.

An idea is an image arising from memories of sense impressions and activities and saturated with feelings. Thought is the combination of subject and predicate. The relation of the thought to the true is that of the sense to the reference. And we thus recognize three stages of the difference of words, expressions, and whole propositions. The difference concerns either at most the ideas, or the sense but not the reference, or finally also the reference.

Reference connects propositions.

But how do you know that anything at all has a reference?

The thought gives no knowledge, the reference

gives knowledge. And the striving for truth forces us to advance from the sense to the reference.

From no knowledge to knowledge. From the thought and the sense to the reference and the true. It is the same path. The path by which a proposition is gradually won. The thought and the sense precedes the path, the reference and the true is the path.

All false propositions have the same reference, the false, and all true propositions have the same reference, the true. And a proposition is true when every one of its parts has a reference.

The false can need no winning. To win a proposition can only be to win a true proposition. There can thus only be one way to win a proposition. To discover that every one of its parts has a reference, and hence its reference is the true. And there can only be one way to do so. To discover the connection of the proposition with other propositions. There can be no other way because reference connects propositions, and the reference of all true propositions is the same. To discover a proposition can thus only be to discover its connection with all other propositions. To win one proposition explicitly can only be to win all other propositions implicitly. To win any one proposition explicitly can thus only be to make its connection with other propositions explicit. A proposition can thus only be won gradually. To win a proposition cannot not be a path. The path can only begin with the winning of several propositions, and it can only proceed from the winning of several propositions to the winning of more propositions, and the winning of more propositions, and more.

The end is reached when all propositions are won. Proof is then the firmest. Complete certainty is attained. The chains of inferences are free of gaps. No link is missing.

The end is knowledge. The path is its acquisition. And its acquisition is uniform. Every step of the path is the winning of a proposition. Every stage of the path is proof.

The reference is the object. The true is also an object.

Object is all that is not function.

Function is the unchangeable in an expression. The changeable is its argument. A function and its argument are a complete whole.

The elements of a proposition are function and argument.

Propositions are functions. Chains of inferences are chains of functions.

Indiscriminately propositions, hydrogen is lighter than carbon dioxide, $2 \cdot x^3 + x = 3$, and Caesar conquered Gaul, are indiscriminately functions and arguments.

And we must admit as arguments and values of functions objects without restriction.

Indiscriminately possible arguments and values, hydrogen, is, lighter than, carbon dioxide, $2 \cdot x^3 + x = 3$, Caesar, conquered, and Gaul, are indiscriminately objects.

Once again, it is an empirical generalization. Over the very great advance of the acquisition of knowledge in the not quite three centuries since the beginning of the instauration, and most particularly over the very great advance of mathematics in these centuries. Arithmetic. Frege writes two generations after Gauss. Geometry. One generation after Riemann. Chemistry. In the generation of Mendeleev. The differential and integral calculus. One generation after Weierstrass. The pure theory of motion. One generation-odd after Hamilton. Mechanics. In the generation of Boltzmann. Physics. One generation after Clausius, in the generation of Maxwell. Philosophy was reasserting the dominion of the word over the human mind, but it had been breaking it. Frege writes one century-odd after Kant.

It is thus the realization of the philosophical calculus. It adds the area in the middle, adjoining all others, connecting them into a single domain.

It deals with all things. All propositions. Ideas and thoughts. All references. All true propositions and all false propositions. Function. All that is not function. Objects without restriction.

It not only deals with form. It reduces existence to its form, generality to its form, and truth to its form.

And it deals with propositions in all their generality. The elements of a proposition. The reference of propositions. The connections of propositions. The path by which a proposition is won.

It is the formula language of pure thinking modeled on that of arithmetic.

Propositions are fundamentally mathematical

propositions.

Mathematical propositions are fundamentally arithmetical propositions.

And all difficulties in the relations of form and content are resolved.

If form is any determination whatsoever, form can only be investigated in relation to itself. Hence the problem of the separation of content from form.

Determination admits of degree. It is conditional, and unconditional. What is determined conditionally is content, what is determined unconditionally is form.

But what is determined conditionally can only be determined in relation to what is determined unconditionally. Content can only be content in relation to form. Hence the problem of the relation of content and form. How is unconditional determination possible? or what is the same, how is proof possible?

Aristotle solved this problem.

Inference is a relation of propositions. A proposition is a subject and a predicate, related by is or is not. And if given propositions affirm what is true, and their relation is such that another proposition follows necessarily, the inference is a proof.

But if content can only be content in relation to form, how is content as formal possible? How are synthetic *a priori* propositions possible?

Frege solved this problem.

The relation of the elements in the proposition is content rather than form. The relation of the propositions in the inference is form rather than content.

He showed that the elements of the proposition are function and argument. He showed how the relation of the propositions of following necessarily from one another can be expressed formally, and how the relation of the propositions to the true can be expressed formally. And he showed it all formally, by proof.

A chain of inferences connects propositions. A proposition is a function and its argument. Argument are objects without restriction. If all the objects in a proposition are in fact objects, or what is the same, every one of its parts has a reference, and that is, it has the same reference in this as in all other propositions, the reference of

the proposition is the true. And to the degree to which it is free of gaps, the chain of inferences is a proof.

In the chain of inferences the connection of the propositions is the unconditional, the propositions are the conditional. The relation of the propositions is the form, the relation of the elements of the proposition is the content. In the proposition itself the function is the form, its argument is the content. Objects without restriction are matter.

Object is all that is not function. Function is the unchangeable in an expression. A function and its argument are a proposition.

Propositions are the principles of all things.

cccxlvi

Pythagoras said that number is the principle of all things.

Leibniz said that number is the metaphysical form.

Frege writes: the laws of number are not actually applicable to the external things; they are not laws of nature; they are yet applicable to judgments which hold for the things of the external world, they are laws of the laws of nature.

And Frege defined number. Or what is the same, discovered what number is. For: the mathematician cannot create a thing at will any more than the geographer; he, too, can only discover what is there and name it; as the geographer does not create a sea when he draws boundary lines and says, the part of the water surface bounded by these lines I will name Yellow Sea, so the mathematician can create nothing by his definitions; that an oval drawn with ink on paper would acquire by a definition the property that, added to one, it gives one, I can only regard as a scientific superstition; one might as well make a lazy student diligent by definition.

In 1884, in *The foundations of arithmetic, a logico-mathematical investigation of the concept of number*, he writes:

we need a general criterion for the equality of numbers; so that when we have this means of grasping a definite number and of recognizing it again as the same, we can give it the name of a number as its proper name (62);

such a means Hume already notes; he writes,

when two numbers are so combined that the one always has a unit answering to every unit of the other, we pronounce them equal; and this judgment, that the equality of numbers must be defined by one-to-one correspondence, has recently gained widespread approval among mathematicians (63).

Hume writes this in the *Treatise of human nature* (I, 3.1). Earlier, Galileo, in the *Two new sciences*, recognizes explicitly one-to-one correspondence as the criterion for the equality of numbers. Earlier, when Euclid, in the *Elements*, defines number: number is a multitude of units, and his first axiom is: things equal to the same thing are equal, the two jointly imply one-to-one correspondence as the criterion for the equality of numbers.

Frege continues:

we thus have the condition, to order the objects which fall under the one and under the other of two concepts in one-to-one correspondence with respect to both; when this condition occurs, I say for brevity that the concept F is equinumeric to the concept G , but I must ask that this word be considered an arbitrarily chosen sign, whose reference is to be gained not from the coinage, but from the exposition;

I define therefore,

the number which belongs to the concept F is the extension of the concept 'equinumeric to the concept F ' (68);

and we go on now to the explanation of the individual numbers;

since nothing falls under the concept 'not equal to itself', I explain,

0 is the number which belongs to the concept 'not equal to itself' (74);

and now I explain the relation in which every two neighboring members of the series of the natural numbers stand; the proposition,

"there is a concept F and an object falling under it x such that the number which belongs to the concept F is n and that the number which belongs to the concept 'falling under F but not equal to x ' is m "

should have the same reference as

" n follows in the series of the natural numbers immediately after m " (76);

under the concept 'equal to 0' falls 0; on the

other hand, under the concept 'equal to 0 but not equal to 0' no object falls, so that 0 is the number which belongs to this concept; we thus have, both, a concept 'equal to 0' and an object falling under it 0, of which holds,

the number which belongs to the concept 'equal to 0' is equal to the number which belongs to the concept 'equal to 0';

the number which belongs to the concept 'equal to 0 but not equal to 0' is 0;

therefore, on our explanation the number which belongs to the concept 'equal to 0' follows in the series of the natural numbers immediately after 0; and when we now define,

1 is the number which belongs to the concept 'equal to 0',

we can express the last proposition thus,

1 follows in the series of natural numbers immediately after 0 (77).

The number which belongs to the concept F is the extension of the concept 'equinumeric to the concept F '. Equinumeric is the condition that the objects which fall under the one and under the other of two concepts can be ordered in one-to-one correspondence with respect to both.

In 1903, in *The principles of mathematics*, given the definition of a set, Russell reformulated Frege's definition: number is nothing but a set of similar sets (111). And: sets are similar when they can be arranged in one-to-one correspondence (109).

In 1878, in *A contribution to the theory of aggregates*, Cantor had established equivalent rather than equinumeric or similar for one-to-one correspondence. He begins: when two well-defined aggregates M and N can be ordered unambiguously and completely, element by element, the following way of expression can be adopted, that these aggregates have the same power, or are equivalent (*id.*).

In 1919, in the *Introduction to mathematical philosophy*, Russell writes: the question, what is a number? has often been asked, but has only been correctly answered in our own time; the answer was given by Frege in 1884, in his *Foundations of arithmetic* (2).

Yellow Sea is the part of the water surface bounded by the meridians at 117° and $127^\circ E$ and the parallels at 41° and $33^\circ N$.

Number is a set of equivalent sets.

Cantor defined set in 1895, in the *Contributions to the foundation of the transfinite theory of sets*: by set we understand every collection into a whole of definite well-distinguished objects of our sensibility or our thought.

That they are objects of our sensibility or our thought shows the Kantian pedigree of the objects. *Ipso facto* it shows also that, definite and well-distinguished, they are all objects. Objects without restriction.

Set is every collection of objects. Number is a collection of objects.

Proposition is a function and its argument. Arguments are objects without restriction. Set is every collection of objects. Number is a set.

The laws of number are laws of the laws of nature.

If propositions are the principles of all things, number is the principle of the principles.

If form is any determination whatsoever, it is because there is nothing that cannot be numbered.

If there is nothing that cannot be numbered, it is because proposition is a function and its argument, arguments are objects without restriction, set is every collection of objects, and number is a set.

Propositions are the principles of all things.

Number is the principle of the principles.

But the principle of contradiction and the principle of sufficient reason are the principles of the principles.

Number is the principle of the principles of the principles.

And no further. There is no principle of the principle of the principles of the principles.

Number is the form. Metaphysical, physical, mathematical, arithmetical, logical.

Which the principles are, and what they are, remains difficult. Not because of their generality, because of their formality. It remains difficult, but not impossible. It remains difficult, and it will forever remain difficult, because it is difficult. Not because it has not been made clear, and both, which the principles are, and what they are. From Pythagoras and Parmenides, through Aristotle, through Leibniz, through Kant, through Cantor and Frege, and further, which the principles are, and what they are, has been made clear, clear and

distinct.

To collect the development of this problem in its entirety cannot be impossible either.

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The relations of the elements, the causes, and the principles of all things.

Aristotle begins in the *Physics*: as the objects of investigation have principles or causes or elements, it is when we find these that we have knowledge; for we do not know what a thing is until we find its first causes and first principles and reach its elements; in physics, therefore, our task is to find the principles (I, 1).

In all of book I he then approaches the task, to find the principles, by solving the problem: whether the number of the principles is one or more than one (2). The solution is: the principles cannot be one in number (3), rather: they are two or three (6), and rather: they are three (7).

In book II he argues that they are rather four, material, efficient, formal, and final.

Then in books III-VIII he turns to the task, to find the principles. And he finds the first mover.

The principles cannot be one.

And this not only has to be implausible, it is impossible.

The principles can be one.

Thales said that water is the principle of all things.

With this one proposition of the first philosopher already all things are collected into one. Generalization has reached its limit. And so much suffices to establish, and to establish conclusively, and whether the proposition is true or false, that all things can be collected into one, and if they have principles, they can be reduced to their one principle. All things have been explicitly reduced to one principle, and the implication is irresistible, all things have one principle.

Pythagoras said that the whole world is a harmony and number.

With the fourth philosopher all things are collected explicitly into the world. Irresistible to begin with, the implication is stronger yet.

Parmenides said that all is one. On pain of contradiction now.

Wood does not make beds. Democritus yet reduced the beds to wood, change to atoms, the

causes to the elements.

Plato said that there are the forms. And above all the forms there is the one form, the good, the one.

For two and a half centuries Aristotle's predecessors have not just been vigorously searching for the principles of all things, they have been vigorously searching for the one principle of all things.

Alas, they have not found it.

The problem of whether the number of the principles is one or more than one Aristotle formulates and solves as he does all problems, by calling the council into session. Anaximander, Parmenides, Anaxagoras, Empedocles, Melissus, Antiphon, Democritus, Lycophron, and Plato participate in the session.

It can be concluded that Aristotle's conclusion: the principles cannot be one in number, is also an empirical generalization. Over the labors of his predecessors. They have searched for the one principle of all things, and the search has been unsuccessful.

Over his own labors, too. For they, too, imply one principle.

The fact implies it that as everywhere else so in book I of the *Physics* Aristotle not only does not distinguish sharply principles and causes, and elements, too, but rather assimilates them, all three, into one another. The fact also implies it that all four, material, efficient, formal, and final, are indiscriminately principles, and all four indiscriminately causes. Moreover, the efficient, formal, and final often coincide. Moreover, all things are matter and form, and any determination whatsoever is form.

The first mover implies it as well. Nature is motion, and the first mover is solitary. It moves the elements, too, aether, fire, air, water, and earth.

And Aristotle has not found one principle either, he has found many. In particular, he has found, just as his predecessors have, three. The first mover moves the elements, but does not constitute them. Nor, conversely, do the elements move the first mover. There are thus the material principles of all things next to the efficient principle of all things. Nor does the first mover imply propositions, nor do the elements imply proposi-

tions, nor, conversely, do propositions move the first mover, nor do they constitute the elements. There are thus the formal principles of all things next to the efficient principle of all things and the material principles of all things. Never mind the final principles of all things.

The investigation itself showed them the way. And it showed them the way this far, and not further. Three principles, not one.

The great success of the philosophers in the search for the principles of all things has thus been a failure in the search for the one principle of all things. This failure points to the great success of the true way. The true way ascends from the particulars to principles, descends from these principles to the particulars, ascends again to higher principles, descends again, ascends again, descends, ascends. And it has ascended this far, and not further. The elements, the causes, and the principles, and these have not been unified.

Thus the true way. But there is also the other way.

In Aristotle's review the one principle which Plato has found does not merit even a passing glance. In the *Republic* nevertheless the good, the one, is the cause of knowledge and truth. In the *Timaeus* the world has a maker. And why did he make the world? Because he was good.

The principles can be one.

It took some time to establish firmly that they are one. Five and a half centuries after Aristotle, Origen, in *On the principles*, begins: god is the principle of all things.

Then for a long time they remained incontrovertibly one.

But then the one principle began being assailed by the other principles.

Nine centuries after Origen, Abelard writes: god can do and do not only what he does and does not, and only in the way and at the time in which and at which he does it and does it not, and no other.

A trial of strength thus began between the one principle and the principle of all principles.

Two centuries after Abelard, Occam discovered the razor: what can be done with less is done in vain with more. And: you may object that producing infinitely many things is not repugnant

to god; I reply, it is repugnant.

The principle of all principles won. And not quite three centuries after Occam, the instauration of the sciences began.

The instauration of the sciences was first and foremost a search for the causes. Bacon: true knowledge is knowledge by causes. Galileo: to recognize the cause removes the marvel. Kepler: no other way can succeed but grounding the motions in their causes. Descartes: to be perfect knowledge must be deduced from first causes. Huygens: in the true philosophy the cause of all effects is sought. Spinoza: we understand god to the degree to which we know the causes of nature.

As early on, however, the investigation itself had showed them that the principles of all things are not only material, so it now, later on, showed them that the principles are not only efficient.

The search for the causes had to be a search also for the elements. God had to make particles. Particles as nearly equal as possible. Solid, massy, hard, impenetrable, movable particles.

The search had to be a search also for the form. True knowledge is knowledge by causes. But also: truth results from the discovery of the forms, and the forms are the law with its clauses. And the instauration culminated in, both, gravity, there is gravity in bodies universally, and the calculus dealing with the form, the universal calculus.

Once again, then, the elements, the causes, and the principles. Three, not one. Once again not unified either.

All in all, first, following the true way, three. Then, following the other way, one. Then, again following the true way, again three.

In the next one and a half centuries such was the lot of Newton's brilliant discovery that every difficulty which it encountered provided it with an opportunity for a new triumph.

Laplace, in 1814, writes: an intelligence, which at any one instant could know all the forces animating nature and the situation of the beings composing it, could embrace in the same formula the motions of the greatest bodies in the world as well as those of the lightest atoms; nothing will be uncertain for it, and the future will be just as present to its eyes as the past.

One formula. Embracing the motions. Of the atoms.

The next was a century of very great advance in the solutions of all three problems, the elements, the causes, and the principles. The three solutions remained three, not unified.

In the century after that, however, two of the three, the causes and the elements, were unified. For the first time, at long last, by following the true way. Not in one formula. But in one mathematical theory. Quantum theory, and its development the big bang theory, has unified all the forces animating nature, the situation of the beings composing it, the motions, of the greatest bodies in the world, those of the lightest atoms, the world itself, the future, the past, and most particularly the causes of all things and the elements of all things.

10^{-43} seconds after the big bang gravity decoupled, at 10^{-32} seconds the strong and electroweak forces decoupled, at 10^{-12} seconds the electromagnetic force decoupled, and at 10^{-6} seconds quarks and leptons began forming. Matter is energy, too. The causes are very much the causes of the elements.

On the other hand, the laws of number are laws of the laws of nature, number is a set, set is every collection of objects. But there is no theory, nothing obtained by following the true way, to unify this and that, the principles and the causes.

All in all, then, first three, then one, then three, then two.

cccxlix

The principles of all things.

The copperplate demands more etching.

Aristotle begins in the *Physics*: as the objects of investigation have principles or causes or elements, it is when we find these that we have knowledge; for we do not know what a thing is until we find its first causes and first principles and reach its elements; in physics, therefore, our task is to find the principles.

And continues: by nature investigation proceeds from what is more knowable and clearer to us toward what is clearer and more knowable by nature; and this path we must also follow, from what is more obscure by nature but clearer to us toward what is clearer and knowable by nature (I,

1).

So there is the more knowable and clearer to us. But the more knowable and clearer to us is not the more knowable and clearer by nature. In fact, we do not know what a thing is until we find its principles. Thus, from the clearer to us toward the clearer by nature. Our task is to find the principles. This is the path we must follow.

And in the *Posterior analytics* Aristotle describes the path more explicitly. He writes: things are first and known as such in two ways; for what is first by nature is different from what is first to us, and so is what is known as such by nature from what is known as such to us; first and known as such to us is the closest to perception, first and known as such by nature is the furthest removed from perception; and the closest are the particulars, the furthest removed is the general (I, 2).

There is the more knowable and clearer to us. But we do not know until we find the principles. Our task is thus to find the principles. From the particulars to the general. This is the path we must follow.

And himself Aristotle followed the path. As to the rest of us, the rest of us would presumably follow it, too. After all: all men by nature desire to know. Certainly the philosophers would follow it.

Philosophy was young.

Nineteen and a half centuries later, it was not so young any longer. Not many of us had followed the path, not many at all. Nor had the philosophers followed it. The philosophers had found the more knowable and clearer to us knowable and clearer enough. Never mind the rest of us.

Bacon thus does not assume that the philosophers desire to know. Never mind all men. Never mind by nature.

There was never the slightest hope that the errors that have flourished and will flourish forever would put themselves right by the native power of the mind or with the help of dialectic. So nothing remained but to try the whole thing anew. And from the very beginning not leave the mind to itself, but guide it continuously, so that the whole thing is done by machinery.

And so that the whole thing is done by ma-

chinery, Bacon described the way the philosophers follow, but must not follow, in all its particularity and generality, thus making also the most valiant attempt to close that way for good, and the way the philosophers do not follow, but must follow, in all its particularity and generality, too.

And in the next generation Descartes discovered the principles of the machinery. The rules of the method.

There is the more knowable and clearer to us. But there is also the more knowable and clearer by nature. There is thus also the acquisition of knowledge. From ignorance through the acquisition of knowledge to knowledge.

But the acquisition of knowledge is difficult. There was never the slightest hope that the errors that have flourished and will flourish forever would put themselves right by the native power of the mind. From the very beginning the mind is not left to itself, but guided continuously, the whole thing is done by machinery. The acquisition of knowledge is done by machinery.

The machinery is the method.

It is much better never to search for truth than to search without the method. Method is necessary to search for truth in things.

By method I understand the certain and easy rules, which, if followed exactly, will never allow the false to be taken for true.

The certain and easy rules are the principles of the acquisition of knowledge.

Descartes is the first philosopher to formulate explicitly the principles of the acquisition of knowledge, when he formulated the four rules of the method. He is the first philosopher to formulate explicitly the principles of things, when he formulated the three laws of nature. He is the first philosopher to write *The principles of philosophy*. And he is the first philosopher to distinguish explicitly the principles of the acquisition of knowledge and the principles of things. Part I of *The principles of philosophy* is *On the principles of human knowledge*, part II is *On the principles of the material things*.

And the importance of the facts, all three, that the principles of the acquisition of knowledge have been discovered, that the principles of the acquisition of knowledge are principles, and that the principles of the acquisition of knowledge are

not the principles of things, cannot be overstated. The generality of the first is not the greatest but it is very nearly the greatest, the generality of both the second and the third is the greatest. And as their generality so their importance.

The principles of the acquisition of knowledge have been discovered. The rules of the method are the principles of the acquisition of knowledge. The path, the acquisition of knowledge, reaches its end, knowledge. From the particulars to the general. The path reaches the general. It reaches the principles. It reaches the principles of things. The path reaches the principles of things. There is the general also in the path to the general. There are the principles also of the path to the principles. Descartes writes: the method is not new, for nothing is older than truth. And: the great minds of the past were aware of the method; arithmetic and geometry are spontaneous fruits of the innate principles of the method; in our time algebra is another attempt to revive the method. The acquisition of knowledge in its entirety is the fruit of the innate principles of the method, all spontaneous fruit before Descartes, mostly cultivated fruit after him. If the rules of the method are not the rules of the method, they are the way the chemical elements are not the elements. The chemical elements, and nothing else, has been decomposed into the elements of the elements. It may be that there are rules of the rules of the method. But it may not be that these are not the rules of the method. Never mind that there are no rules of the method.

The principles of the acquisition of knowledge are principles. They are not elements, not material principles. They are not causes, not efficient principles. They are principles, formal principles. They are form. They are a generalization over mathematics. Descartes writes: when I considered the matter carefully, I saw that mathematics is concerned with nothing but problems of order and measure, whether these involve numbers, or figures, or stars, or sounds, or anything else; and I saw that there must be general knowledge of order and measure. They are a particularization of the principles of all things. The rules of the method are consequences of the principle of contradiction and the principle of sufficient reason. To include in judgments nothing but what

is so clear and distinct that it cannot be doubted, to divide difficulties into parts such as will lead to their better solution, to proceed in order, beginning with the simplest and easiest and ascending gradually to the more complex, and always to make enumerations so complete and reviews so general as to omit nothing. The rules are of generality very nearly the greatest but not the greatest. They are the principles of the acquisition of the knowledge of all things, and not the principles of all things. They are consequences of the principles of all things.

And on this last, that the principles of the acquisition of knowledge are not the principles of things, the world depends. Such as it is not world-views. Had the principles of the acquisition of knowledge been the principles of things, the acquisition of knowledge would have made up things. It is in *The principles of human knowledge* that Berkeley will establish: everything is my ideas in mind, spirit, soul, or myself.

There are all things, then.

Among all things are the elements of all things, the causes of all things, and the principles of all things.

The principles of the acquisition of knowledge are also among all things. They are not the principles of all things, however, they are consequences of the principles of all things.

And everything is not my ideas in mind, spirit, soul, or myself.

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All things.

In 1885, in *On the different points of view with respect to the actual infinite*, Cantor writes: all so-called proofs against the possibility of actual infinite numbers are, as it can be shown in every particular case and can be concluded on general grounds as well, erroneous; their *πρῶτον ψεῦδος* lies in that they attribute to these numbers all the properties of the finite numbers; the infinite numbers, however, if they are to be thinkable in any form, must constitute by their opposition to the finite numbers a completely new species of number, whose essence depends entirely on the nature of things and is an object of investigation, not of our will or our prejudices (*id.*).

All of which – that all so-called proofs against

the possibility of actual infinite numbers are erroneous, that their $\pi\rho\acute{\omega}\tau\omicron\nu\ \psi\epsilon\acute{\upsilon}\delta\omicron\varsigma$ lies in that they attribute to these numbers all the properties of the finite numbers, that the infinite numbers constitute by their opposition to the finite numbers a completely new species of number, and that their essence depends entirely on the nature of things and is an object of investigation, not of our will or our prejudices – by 1885 Cantor had shown conclusively.

He had developed the theory of sets in *On a property of the totality of all real algebraic numbers*, in 1874, *A contribution to the theory of aggregates*, in 1878, and *On infinite linear aggregates of points*, in 1879-84. He continued the development of the theory in *On an elementary question in the theory of aggregates*, in 1891. And in 1895-7, in the *Contributions to the foundation of the transfinite theory of sets*, he completed the formulation of the theory.

The *Contributions to the foundation of the transfinite theory of sets* is under the Newtonian motto: *hypotheses non fingo* (*id.*).

Cantor writes:

by set we understand every collection into a whole M of definite well-distinguished objects m of our sensibility or our thought, which we call the members of M ; in signs

$$M = \{m\};$$

subset of a set M we call every other set M_1 whose members are also members of M ; if M_2 is a subset of M_1 , and M_1 is a subset of M , M_2 is a subset of M ;

to every set belongs a definite power, which we call its cardinal number; power, or cardinal number, of M we call the general concept which abstracts from the nature of its different members m and from the order in which they are given; the result of this double act of abstraction, the cardinal number or power of M , we indicate by $\overline{M}$;

the cardinal number is itself a definite set composed of units;

two sets M and N are equivalent, and we write

$$M \sim N \text{ or } N \sim M,$$

when it is possible to bring them by a law in such a relation that to every member of the one corresponds one and only one member of the other (1);

if for two sets M and N , with the cardinal

numbers a and b , the two conditions are fulfilled, 1. there is no subset of M which is equivalent to N , 2. there is a subset N_1 of N which is equivalent to M , a definite relation of the cardinal numbers a and b is thus expressed; and we say, a is less than b , or b is greater than a ; in signs

$$a < b \text{ or } b > a \text{ (2);}$$

every member m of a set M can be connected with every member n of a set N into a new member (m, n) ; for the set of all these connections (m, n) we write $(M \cdot N)$ and we call it the connection set of M and N ; thus,

$$(M \cdot N) = \{(m, n)\} \text{ (3);}$$

the sets with finite cardinal numbers are called finite sets, all others we call transfinite sets and the cardinal numbers belonging to them transfinite cardinal numbers;

the totality of all finite cardinal numbers v gives us the first example of a transfinite set; we call the cardinal number belonging to it aleph-null, we write $\aleph_0$, and therefore define

$$\aleph_0 = \overline{\{v\}};$$

that $\aleph_0$ is a transfinite number, and that is, is equal to no finite number μ , follows from the simple fact that, when to the set $\{v\}$ is added a new member e_0 , the union set $(\{v\}, e_0)$ is equivalent to the original set $\{v\}$; we thus have

$$\aleph_0 + 1 = \aleph_0;$$

and

$$\aleph_0 + \aleph_0 = \aleph_0;$$

and

$$\aleph_0 \cdot \aleph_0 = \aleph_0;$$

proof of the last proposition;

$\aleph_0 \cdot \aleph_0$ is the cardinal number belonging to the connection set

$$\{(\mu, v)\},$$

where μ and v are any independent of one another finite cardinal numbers; if λ also represents any finite cardinal number, so that $\{\lambda\}$, $\{\mu\}$, and $\{v\}$ are only different signs for the totality of all finite cardinal numbers, we have to show that

$$\{(\mu, v)\} \sim \{\lambda\};$$

if we indicate $\mu + v$ by ρ , ρ takes all values 2, 3, 4, . . . , and there are altogether $\rho - 1$ members (μ, v) for which $\mu + v = \rho$, namely,

$$(1, \rho - 1), (2, \rho - 2), \dots, (\rho - 1, 1);$$

in this series first the one member $(1, 1)$ for which $\rho = 2$ is thought, then the two members for which $\rho = 3$ are thought, then the three members

$$\mathfrak{c} = 2^{\aleph_0},$$

for, in fact, $2^{\aleph_0}$ is nothing other than the power of all representations

$$x = \frac{f(1)}{2} + \frac{f(2)}{2^2} + \dots + \frac{f(v)}{2^v} + \dots,$$

where $f(v) = 0$ or 1 , of the numbers x in the binary system (4).

Cantor has shown that

$$\aleph_0 < \mathfrak{c}.$$

He has shown that given any set, from it arises another set whose number is greater. Later in the *Contributions* he shows that given any set, the number of its subsets is greater than its number. He shows that, if the given set is $\aleph_0$, the number of its subsets is $2^{\aleph_0}$. He has shown that

$$\mathfrak{c} = 2^{\aleph_0}.$$

Later in the *Contributions* he defines ordinal number, on this ground shows how from any transfinite set arises by a definite law the next in order transfinite set, hence how the transfinite sets can be ordered, and hence that $\aleph_1$ is the next cardinal number after $\aleph_0$. Thus,

$$\aleph_1 \leq \mathfrak{c}.$$

That

$$\aleph_1 = \mathfrak{c},$$

or what is the same, that there is no number between $\aleph_0$ and $\mathfrak{c} = 2^{\aleph_0}$, is the continuum hypothesis. The continuum hypothesis is the first of Hilbert's twenty-three *Mathematical problems*, in 1900. In 1938, in *The consistency of the axiom of choice and of the generalized continuum hypothesis*, Gödel showed that if the axioms of set theory are consistent, the continuum hypothesis cannot be disproved from the axioms.

In the *Contributions*, after the proof that $\aleph_0$ is transfinite, Cantor continues:

every finite set F is of such a nature that it is equivalent to none of its subsets;

every transfinite set T is of such a nature that it has subsets T_1 equivalent to it;

after we have introduced the least transfinite cardinal number $\aleph_0$ and derived its most readily apparent properties, the question arises about the higher cardinal numbers and their origin from $\aleph_0$;

it will be shown that the transfinite cardinal numbers can be arranged by their magnitude, and that in this order they form, like the finite numbers, although in an extended sense, a well-ordered set; from $\aleph_0$ arises by a definite law the

next greater cardinal number $\aleph_1$, from it by the same law the next greater $\aleph_2$, and so on; but even the unlimited series of cardinal numbers

$$\aleph_0, \aleph_1, \aleph_2, \dots, \aleph_v, \dots$$

does not exhaust the concept of transfinite cardinal number; the existence of a cardinal number will be proved, which we indicate by $\aleph_\omega$ and which shows itself to be the next greater after all $\aleph_v$; from it arises in the same way as $\aleph_1$ arises from $\aleph_0$ a next greater $\aleph_{\omega+1}$, and so on without end (6).

And so on without end. Cantor has shown that infinity is a species of number, and has described in detail the varieties of the species. As to the genus of the species, there is nothing that cannot be numbered, including infinity.

Zermelo, in 1904, in the *Proof that every set can be well-ordered*, proved that every set can be well-ordered. In 1908, in the *Investigations on the foundations of the theory of sets*, he axiomatized the theory. Zermelo begins: the theory of sets is this branch of mathematics, to which falls the task to investigate the fundamental concepts of number, order, and function in their original simplicity and thus develop the logical foundations of all arithmetic and analysis.

Number, order, and function.

Clearly, the principles of all things.

cccli

In 1899, in a letter to Dedekind, Cantor writes:

you know that many years ago I arrived at a well-ordered series of powers, or transfinite cardinal numbers, which I call alephs,

$$\aleph_0, \aleph_1, \aleph_2, \dots, \aleph_{\omega_0}, \dots;$$

$\aleph_0$ indicates the power of the sets denumerable in the usual sense, $\aleph_1$ the next greater cardinal number, $\aleph_2$ the next greater, and so on; $\aleph_{\omega_0}$ is the one following all $\aleph_v$, and that is, the next greater, which equals

$$\lim_{v \rightarrow \omega_0} \aleph_v,$$

and so on;

the big question was whether there are still other powers of sets next to the alephs; for two years now I have been in possession of a proof that there are no other, so that, for instance, a definite aleph is the cardinal number of the arithmetic linear continuum, or the totality of all real numbers;

if we begin from the concept of a definite multiplicity, or a system, or a totality, of things, the necessity has forced itself on me to distinguish two kinds of multiplicities, and I always mean definite multiplicities;

namely, a multiplicity can be of such a nature that the assumption that all its members are together leads to a contradiction, so that it is impossible to comprehend the multiplicity as a unity, as a completed thing; such multiplicities I call absolutely infinite, or inconsistent, multiplicities;

as one can readily persuade oneself, the totality of all thinkable things, for instance, is such a multiplicity; later on other instances will present themselves (28 Jul).

Later on they did, abundantly.

Earlier on they also did, also abundantly.

Twenty-five centuries earlier, one generation before Thales, the Cretan Epimenides said: all Cretans are liars. Was he a liar?

One and a half centuries later, one generation after Parmenides, Zeno of Elea said: the quickest can never catch the slowest; for the quickest must first reach the point where the slowest is when the pursuit begins and the slowest has advanced no matter how little, the quickest must then reach the point to which the slowest has advanced and the slowest has again advanced no matter how little, and so on to infinity. Zeno concluded that motion is impossible.

Not quite one century later, in the generation before Aristotle, Eubulides proposed the heap (Laertius, II, 108): is 1 grain a heap? no; are 2 grains a heap? no; 3 grains? 4? . . . ? 10? 11? . . . ? so when did it become a heap (Galen, *Experience*, 17)?

Fifteen centuries later, Abelard asked: can god do more than what he does? and can he cease doing what he does? and replied: either way, whether we affirm or deny, troublesome difficulties follow.

Two centuries later, Occam asked: can an angel move? can an angel move through vacuum? can Christ's soul move Christ's body in the eucharist? can Christ's soul move Christ's body under the host both part by part and as a whole? is the extended material substance of Christ's body composed of substantial parts of the same

nature and distinct in position? can the three divine persons assume the same nature numerically? is his paternity distinct from the father?

Not quite two centuries later, Erasmus asked: can god have taken the likeness of a woman? the likeness of the devil? of an ass? of a gourd? and how would the gourd have preached the gospel? performed the miracles? been nailed to the cross?

Two and a half centuries later, one century-odd before Cantor, Kant proved in the *Critique*, both: that the world has a beginning in time and is bounded in space and that the world is infinite in time and infinite in space (I, 2.2.2.2).

What was different now, after Cantor and Frege, was that the totality of all things had been reduced to formal analysis, to number, order, and function, and that propositions and their truth or falsehood had been reduced to formal analysis, to number, order, and function. Clearly, propositions about the totality of all things and their truth or falsehood, too, had to be reduced to formal analysis, to number, order, and function. And so the contradictions, the aporias, the antinomies, the paradoxes, which presented themselves, were.

And they did present themselves.

In 1902, Russell writes to Frege: let w be the predicate, to be a predicate that cannot be predicated of itself; can w be predicated of itself? from each answer its opposite follows; it must be concluded that w is not a predicate; there is also no set, as a whole, of those sets which as a whole do not belong to themselves; from this I conclude that under certain circumstances a definable set does not constitute a whole.

Frege responds: it seems to me that the expression 'a predicate is predicated of itself' is not exact; a predicate is as a rule a first-level function, which demands as an argument an object and cannot therefore have itself as an argument; I would therefore rather say 'a concept is predicated of its own extension'; if the function $\Phi(\xi)$ is a concept, I indicate its extension, or the corresponding set, by ' $\epsilon \Phi(\epsilon)$ '; and in ' $\Phi(\epsilon \Phi(\epsilon))$ ', or ' $\epsilon \Phi(\epsilon) \cap \epsilon \Phi(\epsilon)$ ', we then have the predication of the concept $\Phi(\xi)$ of its own extension.

Frege's response is thus like Cantor's. The response of both is, to borrow Poincaré's simile, to separate the sheep from the wolves and erect a wall around the sheep to keep out the wolves.

Cantor's response is to distinguish two kinds of multiplicities, or systems, or totalities, of things; and a multiplicity of such a nature that the assumption that all its members are together leads to a contradiction, so that it is impossible to comprehend it as a unity, is an absolutely infinite, or inconsistent, multiplicity.

Frege's response is that the expression 'a predicate is predicated of itself' is not exact; a predicate is a first-level function, which demands as an argument an object and cannot have itself as an argument.

And so is Russell's response like Cantor's and like Frege's, when he formulated the theory of types, in 1903, in *The principles of mathematics*, and more explicitly, in 1908, in the *Mathematical logic as based on the theory of types*, and the theory rests on the rule: whatever involves all of a collection must not be one of the collection.

And so is Zermelo's response like Cantor's and like Frege's, in 1908, in the *Investigations on the foundations of the theory of sets*. Zermelo writes: in view of Russell's antinomy of the set of all sets which are not members of themselves it seems no longer permissible to assign to any logically definable concept a set as its extension; Cantor's original definition of a set as a collection into a whole of definite well-distinguished objects of our sensibility or our thought needs therefore a restriction; it has not, however, been replaced by another definition as simple as it is, which would not give rise to the same doubts; in these circumstances, then, nothing remains but to take the opposite path, and, starting from the historically developed theory of sets, to search for the principles necessary for the foundation of this mathematical discipline; and this problem must be so solved that the principles are sufficiently narrow to exclude all contradictions and at the same time sufficiently broad to retain all that is worthwhile in the theory (pr.).

And Zermelo's solution of the problem is conclusive. He found the principles necessary for the foundation of the theory sufficiently narrow to exclude all contradictions and at the same time sufficiently broad to retain all that is worthwhile in the theory. Zermelo's axioms are seven: 1. of definiteness, 2. of elementary sets, 3. of separation, 4. of power set, 5. of union, 6. of choice,

and 7. of infinity. Fraenkel, in 1922, in *On the foundations of the Cantor-Zermelo theory of sets*, added one more, of replacement, to extend the reach of the axiomatization.

It is on Zermelo's axiomatization that Poincaré comments: Zermelo has erected a wall to keep out the intruders who would come from without; but he does not ask whether there could not be intruders within his enclosure (*Thoughts*, 4); he has enclosed his sheepfold carefully, but I am not sure that he has not set the wolf to mind the sheep (*id.*).

Poincaré is not sure, and Zermelo cannot be sure, and no one can be sure. And not only no one can be sure. In 1931 Gödel proved that no one can be sure: in every consistent formal system that contains a certain amount of finitary number theory there exist undecidable propositions, and, moreover, the consistency of any such system cannot be proved in the system.

Moreover, Gödel's proof itself, is it a sheep? is it a wolf? The proof that the consistency of any consistent formal system cannot be proved in the system?

No one can be sure.

Everyone can be sure, however, what the probability that there are intruders within the enclosure is. What the probability that the wolf is set to mind the sheep is. Indistinguishable from 0. So much the fact that the theory of sets has become the foundation of arithmetic, and of all mathematics of which arithmetic is the foundation, and of everything of which mathematics is the foundation, and there are no mathematical objects which have been shown not reducible to sets, and there are no mathematical theorems which have been shown not demonstrable from the axioms of the theory of sets, shows.

Nor need anyone be troubled by anxiety that the sheep may be wolves. The sheep are not wolves. Nor is it difficult, after Zermelo, to separate the sheep from the wolves. The sheep have been separated from the wolves.

Nor need anyone be troubled by anxiety that all things may be wolves. All things are not wolves. All things are sheep.

Because fascination with the wolves has been an essential part of the completion of the catastrophic transformation of philosophy which

attended the restoration of spirit to the substantiality and purity of its being. Built on antinomies and paradoxes, can mathematics not be a free creation of the human mind? All about antinomies and paradoxes, can philosophy not be grammar? semantics? discourses?

Russell reports in 1959: throughout 1903 and 1904 my work was almost wholly devoted to the attempt to find a solution of the paradoxes; the conclusion to which I was led was that sets are merely a convenience in discourse; a set is only an expression.

Thus, in 1908, in the *Mathematical logic as based on the theory of types*, he writes:

I shall begin by stating some of the contradictions to be solved;

1) the oldest is the Epimenides;
2) the set of all sets which are not members of themselves;

3) the relation T which subsists between two relations R and S whenever R does not have the relation R to S ;

4) the name 'the least integer not nameable in fewer than nineteen syllables' is a name consisting of eighteen syllables;

5) the total number of definitions is $\aleph_0$, while the number of transfinite ordinals exceeds $\aleph_0$; hence there must be indefinable ordinals, among them there must be a least, and this ordinal is defined as 'the least indefinable ordinal';

6) Richard's paradox; the set E of all decimals that can be defined by a finite number of words has $\aleph_0$ members; so they can be ordered 1^{st} , 2^{nd} , . . .; let N be a number defined thus, if the n^{th} digit in the n^{th} decimal is p , let the n^{th} digit of N be $p + 1$, or 0 if $p = 9$; N is thus different from all members of E and we have defined it in a finite number of words;

7) Burali-Forti's contradiction; it can be shown that every well-ordered series has an ordinal number, that the series of ordinals up to and including any given ordinal exceeds the given ordinal by one, and that the series of all ordinals is well-ordered; it follows that the series of all ordinals has an ordinal number, say, Ω ; but in that case the series of all ordinals including Ω has the ordinal number $\Omega + 1$, which must be greater than Ω (1).

Russell adds: these contradictions are merely

a selection from an indefinite number (*id.*). And from the indefinite number of those that continued to accumulate after 1908 one has to be noted. It is also due to Russell, in 1918: you can define the barber as the one who shaves all those, and those only, who do not shave themselves; does the barber shave himself (*Ph. atomism*, 7)?

He continues in *The theory of types*:

in the above contradictions there is a common characteristic, which we may describe as self-reference or reflexiveness; in each contradiction something is said about all cases of some kind, and from what is said a new case seems to be generated, which both is and is not of the same kind as the cases of which all were concerned in what was said; all our contradictions have in common the assumption of a totality such that, if it were legitimate, it would at once be enlarged by new members defined in terms of itself; this leads to the rule,

whatever involves all of a collection must not be one of the collection (1).

This is the wall. While Zermelo finds it sufficient to exclude all wolves, however, Russell does not. He also finds that the wolves are not wolves. They are nonsense. He continues:

the above principle is, however, purely negative in its scope; it suffices to show that many theories are wrong, but it does not show how the errors are to be rectified; we cannot say, when I speak of all propositions, I mean all except those in which 'all propositions' are mentioned; for in this explanation we have mentioned the propositions in which all propositions are mentioned, which we cannot do significantly; it is impossible to avoid mentioning a thing by mentioning that we will not mention it; one might as well, in talking to a man with a long nose, say, when I speak of noses, I except such as are inordinately long, which would not be a very successful effort to avoid a painful topic; thus it is necessary, if we are not to sin against the above negative principle, to construct our logic without mentioning such things as all propositions or all properties, and without even having to say that we are excluding such things; the exclusion must result naturally and inevitably from our positive doctrines, which must make it plain that all propositions and all properties are meaningless phrases

(*id.*).

So the converse of the rule is: if, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total; and when I say that a collection has no total, I mean that statements about all its members are nonsense (*id.*).

Provided a collection had a total, it has no total. We cannot say, I mean all propositions except those in which 'all propositions' are mentioned. We cannot say it, and Russell has just said it. We cannot mention significantly propositions in which all propositions are mentioned. We cannot mention them significantly, and Russell has just mentioned them significantly.

The man with the long nose should nevertheless be reassured. Not only all that is said about inordinately long noses is nonsense. Inordinately long noses are nonsense.

Accordingly, Russell developed, in 1910-3, in the *Principia mathematica*, with Whitehead, a logic of relations. He reports: what is important in the logic of relations is what is different from the logic of sets.

Rather like Hegel, of whom Russell writes: Hegel criticizes traditional logic and professes to replace it by an improved logic of his own.

Accordingly, in the next generation, with Wittgenstein philosophy became grammar, with Carnap semantics. In the next generation it became discourses about discourses.

ccclii

All things have their difficulties.

All their difficulties, however, have one and only one source. Thinking.

Cantor's instance is the totality of all thinkable things. It leads to a contradiction.

All thinkable things, however, do not have to be the totality of all thinkable things in order to lead to a contradiction. All thinkable things all too readily, and all too often eagerly, lead to a contradiction in finite sets, too. They lead to a contradiction readily, and eagerly, in small finite sets, too. They lead to a contradiction readily, and eagerly, in sets of 1 member, too. Indeed, what is a thinkable thing? is it, being thinkable, a thinkable thing, and not a thing? is it, being a thing, a

thinkable thing, and not thinkable? or, being thinkable, and not a thing, it is a thinkable thing by being thinkable as a not thing? or, being a thing, and not thinkable, it is a thinkable thing by being not thinkable as a thing? is it a not thing thinkable as a thing? or a not thinkable thing thinkable as a thing? in any case, clearly, the unthinkable thing is a thinkable thing, and is then the unthinkable thinkable thing an unthinkable thing? or thinkable? and is the thinkable unthinkable thing a thinkable thing? or unthinkable? and so on without end? $\aleph_0, \aleph_1, \aleph_2, \dots, \aleph_{\omega_0}, \dots$?

Kant writes: to know an object requires that I can prove its possibility; but to think I can whatever I will, as long as I do not contradict myself.

As long as I do not contradict myself? With all due respect to Kant, the reference of 'I' in this proposition is the null set. To think I can whatever I will, and I will contradict myself, at will.

And all I's are such I's. The first self-proclaimed Kantian, Fichte, discovered: the principles of the entire theory of science, the I posits the I, the I posits the not I, and I posit in the I, against the divisible I, a divisible not I. Hegel discovered that: the necessity of contradiction is the nature of thought.

And philosophy did not have to wait for Fichte or Hegel to begin contradicting itself. Kant writes, and in reference to the *Critique*, too: apparent contradictions can be ferreted out of every piece of writing when propositions are separated from their connection (*pr.*). It will seem, in fact, a fair description of philosophy. Ferreting out contradictions, whether apparent or real, out of pieces of writing, either by separating propositions from their connection or by bringing propositions into their connection, and either, before Hegel, because the principle of contradiction will not tolerate contradiction or, after Hegel, either because the principle of contradiction will not tolerate contradiction or because the necessity of contradiction is the nature of thought.

And thinking has not had to be philosophy in order, to borrow a phrase from Freud: to exercise its inalienable right to inconsistency (*Dreams*, 2). Freud finds the inalienable right exercised in the interpretation of dreams. And either life is not a dream or, with Calderón, *Life is a dream*. Either way, can angels move through vacuum? can the

three divine persons assume the same nature numerically? is his paternity distinct from the father? And how did the gourd preach the gospel? was nailed to the cross?

Nor did thinking have to wait for the gospel. Parmenides reports:

they wander about two-headed, deaf, blind,
befuddled, hordes with no judgment,
believing that what is and what is not are the
same and not the same
and everything turns back on itself.

They include me. I wander about two-headed, deaf, blind, befuddled, all by myself a horde with no judgment. And what is and what is not are the same and not the same and everything turns back on itself is an excellent logically definable concept, not inferior in any way to any of the more recent contradictions, aporias, antinomies, and paradoxes.

What is is, however, and what is not is not.

The sense of a proposition is the thought. The reference of a proposition is the true or the false. The senses of the parts of a proposition are saturated with the ideas and feelings which thoughts arouse. The references of the parts of a proposition are the objects themselves whose relations in the proposition are the true or the false.

The reference and the true must be distinguished sharply from the thought and the sense. And they have been.

The striving for truth forces us to advance from the thought and the sense to the reference and the true. The thought gives no knowledge, the reference gives knowledge. And not irresistibly, far from it, but the striving for truth has been forcing us to advance from the thought and the sense to the reference and the true. For twenty-five centuries, counting since Parmenides, fifty-one centuries, counting since the 3 oldest calculations of area.

And on this ground the difficulties which all things pose can be resolved. Have been resolved.

With respect to the problem of the principles of all things all difficulties which all things pose reduce themselves to one. Should the concept 'all things' be abandoned? So that the facts that all things have principles, and there is the problem of the principles of all things, and the problem of the

principles of all things has been solved, all be abandoned, too. Because all thinkable things have their difficulties. And the resolution of the difficulty must be, on all counts, no. The concept 'all things' must not be abandoned.

The negative reason should all by itself be sufficient.

All things include all thinkable things. And not conversely. All thinkable things do not include all things. Everything is not my ideas in mind, spirit, soul, or myself.

But there are also the following four each by itself necessary and sufficient affirmative reasons.

First, all things are the world. The reference of 'all things' is the world.

And the world is the perfect multiplicity. The perfect system. The perfect totality. The perfect whole. And it is a collection into a whole M of definite well-distinguished objects m of our sensibility or our thought, in signs

$$M = \{ m \}.$$

From Empedocles:

can anything increase the all? where will it come
from?

and how will it perish?

through Kant: to speak of an intelligible world next to the sensible world is nothing but empty peddling of words; there is nothing other than the world; the world is the mathematical whole of all phenomena, and the very same world is called nature when considered a dynamical whole.

More recently, too.

Clausius, in 1865: the energy of the world is constant.

Planck, in 1897: it is not difficult to formulate Clausius's proposition so that it has, in fact, a reference; $\lim_{R \rightarrow \infty} \frac{1}{E} \frac{dE}{dt} = 0$, where E is the total energy in the very large space R .

Minkowski, in 1908: the multiplicity of all thinkable x, y, z, t systems of values is the world.

Friedmann, in 1922: since the radius of curvature of the world is constant with respect to space and dependent on time, it must decrease with the decrease of time to zero; the time of increase of the radius from zero we call the time from the creation of the world; and the period of the world is of the order of 10 milliard years.

The world is the set of 10^{11} plus galaxies, each galaxy itself a set, presuming the Milky Way average, of 2×10^{11} stars on average. These are the most important members of the set.

But subsets of not important members of the set can also be defined. The subset of all world-views, for instance. Dilthey writes: the types of world-views are religious, poetic, and metaphysical. The types are 3. Clearly, there is nothing that cannot be numbered. And as the types, so the world-views. Dilthey writes: the entities which act on one another in the wonderful whole of history and society are *individua*, each different from every other, each a world, for the world is nowhere but in the ideas of an individual, and the whole significance of each individual acting at some point of the spiritual cosmos is revealed by the proposition, *individuum est ineffabile*. Hardy writes: the set of all British subjects, past, present, and future, has a finite number n , though of course we cannot at present tell the actual value of n (*Course*, 52). As the set of all British subjects, past, present, and future, has a finite number n , so the set of all men, past, present, and future, has a finite number n . We can also tell the actual value of n . The past members of the set of all men are somewhat more than 10^{11} but substantially less than 10^{12} , counting not more than 300 000 years since the evolution of *erectus* into *sapiens*. The present members of the set are somewhat less than 10^{10} . As to the future, the world has been around for somewhat more than 10^{10} and substantially less than 10^{11} years. Packing the world with sand, like Archimedes, it will be packed today with 10^{96} grains of sand. Presuming that the world will not continue longer than it has continued, on the most optimistic, given philanthropy, or most pessimistic, given misanthropy, temporal hypothesis, there will not be men for more than 10^{11} years into the future. On the most optimistic, or pessimistic, spatial hypothesis, men will not pack the world more tightly in the future than grains of sand will pack it today. The number of men in any one year in the future thus cannot be greater than 10^{96} . Ignoring attrition, $10^{11} \cdot 10^{96} = 10^{107}$, the future members of the set cannot be more than 10^{107} . Past, present, and future, $10^{12} + 10^{10} + 10^{107} = 10^{107}$, very nearly, and the set of all men cannot have

more than 10^{107} members. And if *individua* and men are in one-to-one correspondence, and world-views and *individua* are in one-to-one correspondence, this is the number of the members of the set of all world-views, past, present, and future, less than 10^{107} . It may be, *individuum* being *ineffabile*, that *individua* are not in one-to-one correspondence with men, and world-views are not in one-to-one correspondence with *individua*. If so, yet *individua* are entities, each different from every other, each acting at some point of the spiritual cosmos, they cannot be more than the points of the continuum, or, on the continuum hypothesis, more than $\aleph_1$. And presuming, on the ground of philanthropy, that an *individuum* cannot be beset by a number of world-views greater than $\aleph_1$, too, $\aleph_1 \cdot \aleph_1 = \aleph_1$, and this is the number of the members of the set of all world-views, not more than $\aleph_1$.

Second, all things have principles. The world has principles.

And the principles of all things, or what is the same, the principles of the world, are not only perfectly formal, they are perfectly general.

The difficulties which all thinkable things have kept introducing among all things, on the other hand, are formal all right, but they are not general.

In general, formal difficulties have not been a barrier before generalization. Deduction has not had to precede induction. The other way around. Proof has not been a condition of discovery. The other way around. The development of mathematics in the twenty-seven and a half centuries before the *Elements* shows this. The development of calculus shows it. The development of non-Euclidean geometry shows it. The development of physics in its entirety shows it.

In particular, the resolution of formal difficulties has been the ground of generalization.

The liar had to wait for the development of set theory to be appreciated as a paradox. How it was appreciated earlier, its source, the *Letter to Titus* in the Pauline corpus, attests: it was a Cretan prophet who said, all Cretans are liars, vicious beasts, lazy gluttons.

First appreciated as a paradox – a formal difficulty, a contradiction – was incommensurability. Pythagoras discovered the incommensura-

bility of the side and diagonal of the square. Aristotle reports: all men are astonished when they first learn that there is something which cannot be measured even by the smallest unit. As all men well should be. For nothing but proof, nothing but propositions in their formal relation to one another in a chain of inferences, can show that the principle of contradiction, and nothing but the principle of contradiction, will not allow that there not be something which cannot be measured even by the smallest unit. If the side and diagonal are commensurable, their ratio is rational, $\frac{p}{q}$ in least terms; $2p^2 = q^2$, q^2 is thus even, q is even, and p is odd; q is even, so $q = 2r$, $2p^2 = 4r^2$, and $p^2 = 2r^2$; p^2 is thus even, and p is thus even; but it is also odd. Aristotle continues: but nothing will astonish the geometer more than were the side and the diagonal of the square to turn out to be commensurable. So the geometer, Eudoxus, developed the theory of proportion. And the geometer's, Ptolemy's, description of the world is impossible without the theory of proportion.

A formal difficulty was discovered, was resolved, became the ground of great generalization.

And incommensurability is representative.

Zeno of Elea discovered the paradoxes of motion, at length Galileo and Kepler began investigating indivisibles, calculus was developed, Newton described the system of the world.

Lobachevsky developed non-Euclidean geometry, Riemann generalized Euclidean and non-Euclidean geometries into the theories of manifolds and metrics, Riemann's generalization became the foundation of relativity with Einstein.

In the generation after Riemann, Cantor developed the theory of sets, in the next generation, that of Einstein, the paradoxes were discovered – not only are all Cretans vicious beasts and lazy gluttons, they are all liars – Zermelo axiomatized the theory of sets. In the generation of Cantor, Frege developed the concept writing, adding the area in the middle, adjoining all others, to the realizations of the philosophical calculus. In the next generation, that of Zermelo and Einstein, Hilbert developed the theory of proof. Its goal: to prove that the axioms are free of contradiction.

One of the problems of mathematics is: the axiomatization of physics. Because more generally, and in fact most generally, the goal is: to establish that in every domain of knowledge contradictions on the ground of the system of axioms on which it stands are impossible. It is at the same time, and after the discovery that energy is quanta, that Planck writes: as long as there is a physical science its highest goal is the solution of the problem of embracing all observed and still to be observed natural phenomena in one simple principle.

In general, formal difficulties have not been a barrier before generalization, and the resolution of formal difficulties has been the ground of generalization. The formal is general, but not the general, and the general is formal, but not the formal. And the perfectly formal is perfectly general, but the perfectly general is not perfectly formal.

In particular, where the principles of all things are concerned, the difficulties which all thinkable things have kept introducing among all things have beset, or appeared to beset, the principle of contradiction. They have not beset, or appeared to beset, the principle of conservation of energy, and not even the principle of conservation of matter, except by besetting the principle of contradiction. The difficulties have also been resolved, invariably, on the ground of the principle of contradiction. Indeed, even when the principle of contradiction has been abandoned, it has not been on formal grounds, it has been on general, Hegelian.

All thinkable things do not beset the world, they beset a part of the world only, and not a large part either. They do not beset the elements of all things, or the causes of all things, they beset the principles of all things only. And they do not beset the principles of all things as their equals either, they beset them as the less general besets the more general.

That their difficulties are not of all of all things, but of all of all thinkable things only, that they are formal, and not general, and that they have been resolved, the fact shows spectacularly, too – itself a paradox – that language is a barrier before the acquisition of knowledge and yet propositions are fundamentally mathematical propositions.

Leibniz writes: the vulgar languages are hampered by innumerable equivocations and cannot serve as a calculus. He yet developed the philosophical calculus. Frege writes: I found a barrier in the inadequacy of language; the more complex the relations, the less did language allow the precision which my goal demanded. He yet developed the concept writing. Thomson's primordial substance *x*, a particle at first, turned out to be a wave, and rather a particle-wave, and rather a thing, much like Dilthey's *individuum*, *ineffabile*. And the difficulty turned out to be the same. Bohr writes: the barriers before the gradually deepening knowledge of the laws of nature originate above all in the fact that every word in language refers to our ordinary perception (*Development*, 7). Yet Bohr, too, resolved the difficulty: when due regard is taken of the complementarity required by the quantum postulate, it is possible to build a consistent theory of the atomic phenomena which can be considered a rational generalization of the description of classical physics (*id.*).

From innumerable equivocations to the philosophical calculus, the concept writing, the quantum postulate and complementarity. It is the same path. From the thought and the sense to the reference and the true.

Third, there is the problem of the principles of all things.

And it is the perfect problem.

It is one and the same problem from Thales through the big bang theory, from the big bang theory back to Thales, and from Thales back to the big bang.

Fourth, the problem has been solved.

By Thales already, and better, and better, and better, and in all its three parts perfectly.

cccliii

Thus far the attempt to make explicit what the problem of the principles of all things is. What the material principles of all things are, what the efficient principles of all things are, what the formal principles of all things are, what the relations of the material, the efficient, and the formal principles of all things are, what the principles of all things are, and what all things are.

The attempt can now be made to collect the development of the problem.

It can be divided into eight parts. The dates are,

625	Thales,
384 - 322 ^{BC}	Aristotle,
87 ^{AD}	Ptolemy
1079 - 1142	Abelard,
1561 - 1626	Bacon,
1646 - 1716	Leibniz,
1822 - 1888	Clausius.

And the parts are,

all the centuries before Thales,

the two and a half centuries from Thales to Aristotle,

the four and a half centuries from Aristotle to Ptolemy,

the ten centuries from Ptolemy to Abelard,

the five centuries from Abelard to Bacon,

the three generations from Bacon to Leibniz,

the not quite two centuries from Leibniz to Clausius,

the one and a half-odd centuries since Clausius.

Proceeding in chronological order, first the problem of the beginning of the problem is addressed. Then the development of the problem from Thales to Aristotle is reviewed. Then the particulars are collected, all important among them, and all steps of advance in the acquisition of knowledge among them, from Aristotle to Clausius. Finally it is indicated how those since Clausius can be collected.

cccliv

All the centuries before Thales.

Thales discovered the problem of the principles of all things.

And like all things, this one among them, the problem of the principles of all things, is not *ex nihilo*. Hence the problem, whence the problem of the principles of all things. Which are the developments preceding the beginning of the problem of the principles of all things leading to the beginning of the problem of the principles of all things.

And first of all, the problem can be formulated more explicitly.

Thales said that water is the principle of all things.

Aristotle writes: perhaps Thales said so because he saw that the nourishment of everything is moist, or that heat is generated from moisture, or that the seeds of everything are moist; some even think that the ancients, who lived long before us and first spoke about the gods, held the same opinion about nature, for they made Ocean and Tethys the parents of creation and made the gods swear by the water called Styx.

Excellent reasons all.

The problem of the beginning of the problem of the principles of all things, however, is not how and why water.

The problem of the beginning of the problem of the principles of all things is how and why the principles of all things.

Anaximenes said that air is prior and it is the principle. Pythagoras said that number is the principle.

Again, it is clear why air, it is prior to water. It is clear also why number, Pythagoras saw in numbers many analogies with the things that are and become, and more so than in fire or earth or water. Philosophy was but two generations old. And not less than the discovery of the problem, the great advance in its formulation and solution in but two generations after its discovery shows that the problem of the beginning of the problem is not how and why water, or air, or number. It is how and why the principles of all things. Not that these are the principles. That they are the principles.

Anaximenes said that air encloses the world and holds it together. Pythagoras said that the whole world is a harmony and number. This is how the world enters the record. The world enters the record with the third philosopher and the fourth philosopher. No one before the third philosopher and the fourth philosopher spoke of the world. All things are the world. And the problem of the beginning of the problem of the principles of all things is of which are the developments preceding the discovery of the world leading to the discovery of the world. All things in all their generality, the greatest.

Thales said that water is the principle. Anaximenes said that all things come from air through rarefaction and condensation. Pythagoras said that number is the principle. The investigation

will not show them clearly and distinctly that the principles are not only material, they are also efficient, before Anaxagoras, and that they are not only material and efficient, they are also formal, before Plato, but the investigation was leading them clearly and distinctly toward all three, material, efficient, and formal, from the first. And the problem of the beginning of the problem of the principles of all things is of which are the developments preceding the discovery of the elements of the world, the causes of the world, and the principles of the world leading to the discovery of the elements of the world, the causes of the world, and the principles of the world. All things in all their generality, the greatest, and their principles in all their generality, the greatest.

And the problem is of propositions. The winning of propositions, and that is, connecting them in chains of inferences. Anaximenes said that air is invisible and made visible by cold, hot, wet, and motion, all things come from it through rarefaction and condensation, rarefied it becomes fire, condensed it becomes wind, condensed further it becomes cloud, condensed further it becomes water, condensed further it becomes earth, and from these all other things come, and the soul is air, and the planets are moved by gusts of thick air. Anaximenes is the third philosopher. The second is Anaximander. Anaximander said that the stars originated in a ball of fire, the earth was wet at first, then began to dry up, the first living creatures were born from the moisture evaporated by the heat of the sun, and man came from such a creature, and that is, from fish. The problem is of the winning of propositions, and that is, connecting them in chains of inferences, and connecting them in chains of inferences free of gaps, too. Pythagoras is the fourth philosopher. Pythagoras discovered proof, proved the irrationality of $\sqrt{2}$, proved that, if a is odd, a , $\frac{a^2-1}{2}$,

and $\frac{a^2+1}{2}$ are a right triangle, proved that the sum of the angles of the triangle is two right angles. The problem of the beginning of the problem of the principles of all things is of which are the developments preceding the winning of the propositions, with which the world and the principles of the world were first won, leading to

the winning of the propositions, with which the world and the principles of the world were first won.

The problem is of the world, its principles, and the propositions with which the world and its principles were first won.

That this is the problem, the two propositions also show which Thales connected in a chain of inferences. Not exactly free of gaps, but a single chain of inferences. Aristotle writes: Thales said that water is the principle of all things, and this is why he also said that the earth floats on water.

And the philosophers continued to labor at both problems, the principles of the world and the position of the earth in the world, and most particularly at winning the chains of inferences which will connect their solutions. Still in but the first two generations of the development of philosophy, Anaximander discussed the sizes and distances of the planets, said that the earth is at rest because of equilibrium, and it is shaped like a drum, and drew the map of the earth, Anaximenes said that the earth is at rest because it is flat, it floats on the air below, like a lid, Pythagoras discussed the order of the planets.

Thales said that water is the principle of all things, and this is why he also said that the earth floats on water. If so, Thales deduced the proposition about the earth from the proposition about all things.

It has to be more plausible the other way around. That Thales said that the earth floats on water, and this is why he also said that water is the principle of all things. That he induced the proposition about all things from the proposition about the earth. That the proposition about all things is a generalization over the proposition about the earth.

Either way, the relation of the two propositions is causal. One of them because the other. Either way, the connection of the two problems from the first, the earth and all things, has to be part of the problem of the beginning of the problem of the principles of all things.

The problem of the beginning of the problem of the principles of all things, then, is of the developments leading to the world, its principles, and the propositions with which the world and its principles were first won.

ccclv

And the solution of the problem is, first of all, Hesiod.

Hesiod is the one predecessor of the philosophers with whom earlier developments have come the closest to the beginning of philosophy, in general, and the problem of the principles of all things, in particular. And it is very close, Hesiod is on the verge of the discovery of the problem of the principles of all things, and the beginning of philosophy.

One century-odd before Thales, Hesiod wrote, and rather sang, the *Theogony* and *Work and days*. And it is with the *Theogony* that Hesiod is on the verge of the discovery of the problem of the principles of all things, and the beginning of philosophy.

The *Theogony* begins:

of the muses of Helicon let us begin to sing (1).
Of the muses, because the muses taught Hesiod the *Theogony*:

one day they taught Hesiod a beautiful song (21).
Before teaching Hesiod a beautiful song, the muses had taught Homer two very beautiful songs. Now, however, with Hesiod, there is from the first the danger that they may teach him not just a beautiful song:

one day they taught Hesiod a beautiful song;
he was tending his sheep, and they said,
savage shepherd, stupid oaf, nothing but a belly,
we know how to tell many lies as if they were
true,

but we know also how to tell the truth when we
want to (21-8).

They know how to tell the truth when they want to. There is thus the danger that they may want to, and instead of telling many lies as if they were true, they may teach Hesiod the truth. As Frege has it: when listening to an epic, next to the melodious sound of the language, we cling only to the sense of the propositions and the ideas and feelings these arouse; with the question of truth we leave the enjoyment of art and turn to scientific consideration. There is the danger that the muses may involve Hesiod, stupid oaf, in scientific consideration.

And it turns out, in fact, that the muses are

Cartesians. Beginning to end they teach Hesiod, in the *Theogony*, as he is tending his sheep, enumerations so complete as to omit nothing. So much so, before they teach him anything, the muses enumerate themselves:

the 9 daughters of the great Zeus,
Clio, Euterpe, Thalia, Melpomene, Terpsichore,
Erato, Polyhymnia, Urania, and Calliope (76-9).

Several centuries later the 9 daughters of the great Zeus will be assigned her offices each. And 7 of the 9 will be assigned the appropriate offices – given that it is songs that they teach – of flute playing, comedy, tragedy, choral poetry, lyric poetry, hymns, and epic poetry. 2 of the 9 will be assigned the less appropriate offices, Clio of history, Urania of astronomy.

Introductions over, Hesiod asks:

tell me children of Zeus,
how did gods and the earth first come to be,
and rivers, and the boundless raging sea,
and the shining stars, and the heaven wide above
(108-10).

Clearly, the savage shepherd, stupid oaf, nothing but a belly, is asking for scientific consideration. And the muses give him scientific consideration:

first of all Chaos came into being,
then big-bosomed Earth, the foundation of all
immortals,
and dark Tartarus below,
and Love, the fairest of the immortals;
from Chaos came Darkness and black Night;
Night lay with Darkness and bore Aether and
Day;
Earth bore starry Heaven, equal to herself,
and Mountains, and barren Sea;
then she lay with Heaven and bore Ocean (116-33),

she bore more, her progeny more, her progeny's progeny more, and until the enumeration is complete. Counting only the progeny given its name each, the total is 335.

Chaos through all the 335, it is a single chain of generation. Seven major lines of development can be distinguished in the chain.

One. First of all Chaos comes into being, then Earth, Tartarus, and Love. Generation is thus at first spontaneous. The sequence, first Chaos, then the other three, is only temporal.

Two. From Chaos come Darkness and Night. Night lays with Darkness and bears Aether and Day. Generation thus becomes of one from another, then it becomes procreation. Night will also continue to procreate (211-25), and her progeny will also procreate (226-32).

Three. Earth bears Heaven, Mountains, and Sea. Generation is now by parthenogenesis. With these births, however, the main line of development begins, whose progenitor is Earth, and thereafter all generation is by procreation.

Four. Earth lays with Heaven and bears 12 titans, Ocean, Coeus, Crius, Hyperion, Iapetus, Theia, Rhea, Themis, Mnemosyne, Phoebe, Tethys, and Cronus, 3 cyclopes, Thunder, Lightning, and Flash, and 3 one-hundred-handed creatures, Cottus, Briareus, and Gyges (133-53). The titans will themselves procreate abundantly, making their line of descent, more particularly, the main line of development.

Five. Earth lays with Sea and bears the Old Man of the Sea, Marvel, Phorcys, Whale, and Eurybia whose heart is of adamant (233-9). This is a line of many marvels. For instance, Phorcys lays with Whale and she bears 3 gorgons, one of whom is Medusa (274-6), among whose progeny in the third generation is the three-headed Geryoneus, whom later on Heracles will have to kill (282-90).

Six. Cronus, the last-born of Earth and Heaven, after castrating his father (154-210), lays with Rhea, the seventh-born of Earth and Heaven, and their last-born is Zeus (453-8), who, after his victory in the war against his father, the war of the titans, becomes: the king of mortals and immortals (459-506).

Seven. This is the line of development with which scientific consideration reaches its end. Ocean, the first-born of Earth and Heaven, lays with Tethys, their eighth-born, and she bears more daughters and sons than anyone:

for Ocean has 3 000 slender-ankled daughters,
glorious children of the gods, scattered far
and wide,
serving everywhere Earth and Sea;
and there are as many other rivers again,
noisy sons of Ocean, whom lady Tethys bore
(364-8).

The complete enumeration of the progeny of

Ocean and lady Tethys requires five steps.

First, Hesiod enumerates these 25:

Tethys bore to Ocean swirling rivers,
Nile, Alpheus, deep Eridanus,
Strymon, Menderes, fair Danube,
Rioni, Rhesus, silver Achelous,
Nestos, Rhodius, Haliacmon, Heptaporus,
Granicus, Aesepus, divine Simois,
Pineios, Gediz, fair Caicus,
great Sakarya, Ladon, Parthenius,
Euenus, Ardescus, divine Scamander

(337-45).

Next, he enumerates 41:

a holy company of daughters, who,
with lord Apollo and the rivers,
have youth in their keeping (346-8).

Among the holy company of 41 are Europe and Asia (357-9), as well as:

Styx, preeminent among them all (361).

Next, the 66 given her name each are far from all. Hesiod thus continues:

but there are many others,
for Ocean has 3 000 slender-ankled daughters
(363-4).

Next, the 3 000 slender-ankled daughters are not all either. Hesiod continues:

and there are as many other rivers again,
noisy sons of Ocean.

Finally, the 6 000 slender-ankled daughters and noisy sons are not all either. Thus, to make certain that nothing is omitted, Hesiod concludes:

and there are still more swirling rivers,
sons of Ocean whom mother Tethys bore;
all their names are hard for a man to tell,
but those who live by them know them

(367-70).

And the progeny of Ocean and mother Tethys divides sharply into two kinds. On one side, for a total of more than 5 975, are the holy company of daughters, the many other slender-ankled daughters, the as many noisy sons again, and the still more swirling rivers, whose names are hard for a man to tell, but those who live by them know them. On the other side, the 25 swirling rivers with which Hesiod begins are, in fact, rivers.

13 of 25 Herodotus describes in the *Histories*, the great Nile and Danube, the large Strymon, Menderes, Rioni, Achelous, Nestos, Haliacmon, Pineios, and Gediz, and the small Caicus, Parthe-

nus, and Scamander (I, 80; II, 34, 104; IV, 104; V, 118; VII, 42-3, 109, 113, 126-7, 129). 10 more of the 25 Strabo describes in the *Geography*, all small, the Alpheus, Rhesus, Rhodius, Heptaporus, Granicus, Aesepus, Simois, Sakarya, Ladon, and Euenus (VIII, 3.1, 3.12; XII, 3.7, 3.23, 3.27; XIII, 1.2, 1.31, 1.67). The remaining 2 of the 25 are errors. Of the Eridanus Herodotus writes: I do not think that there is a river Eridanus (III, 115). Of the Ardescus Hesiod's successors have nothing to say. 23 of the 25 are, in fact, rivers. 92 %.

Their distribution on the surface of the earth is within the following limits. The Danube issues into the Black Sea, in the northwest. The Rioni issues also into the Black Sea, in the east. The Nile issues into the Mediterranean, in the south-east. The Achelous issues into the Ionian Sea. And the mouth of the Danube in the north and the mouth of the Nile in the south, across 14° of latitude along the meridian, are 1 600 km apart, the mouth of the Achelous in the west and the mouth of the Rioni in the east, across 20° of longitude along the parallel, are 1 800 km apart.

Why scientific consideration has reached its end, of all objects of consideration, with the rivers, can be explained. There is no observation in the *Theogony*. If, however, the investigation itself was showing Hesiod the way, if it was showing him the way to observation, if his primary object of consideration is big-bosomed Earth, and if rivers are given to the senses as clearly and distinctly as any of the features on the surface of the earth, this can explain why it is with the rivers that the *Theogony* has come the closest to observation.

One way or another, the striving for truth forces us to advance from the thought and the sense to the reference and the true. And with the rivers the striving has forced Hesiod all the way to the reference and the true.

The problem is of the developments leading to the world, its principles, and the propositions with which the world and its principles were first won.

And there is much in the *Theogony* that strongly points that way.

Hesiod begins with the question: how did gods and the earth first come to be, and rivers, and the sea, and the stars, and the heaven? No one before

Hesiod has asked this question.

And Hesiod answers the question as explicitly as he has asked it.

First of all Chaos came into being.

Earth, however, is the foundation of all immortals. This, too, the foundation of all immortals, Earth is in three ways, by herself, by laying with Heaven, and by laying with Sea. And in all the three ways Earth is in some detail, the foundation of immortal after immortal after immortal.

Also, after some variation at first, all generation settles into the uniformity of procreation.

Also, Zeus is the king of mortals and immortals. How he won the kingship, too, is related in some detail, as is how his father before him won the kingship.

Also: Zeus is the father of gods and men (47). The father has a father, Cronus. The father of the father also has a father, Heaven. The father of the father of the father does not have a father, but he has a mother, Earth. The mother of the father of the father of the father does not have either a mother or a father, yet she follows, she follows Chaos. And there the regress stops. First of all Chaos came into being.

Nor are all these just sentences. They are propositions. And not only because the muses know how to tell the truth when they want to. Because the uniformity of procreation has connected them into a chain of inferences, connecting also explicitly with one another 335 named objects in the chains. Hesiod, stupid oaf, follows the fourth rule of the method staunchly beginning to end. So the reference of nearly all the propositions is the false. But not all. The 25 rivers are 2 of them errors, 23 of them rivers. And their distribution has extended the reach of the reference and the true, north to south, west to east, to a very substantial portion of the surface of the earth.

Huygens measured the solar system. He illustrates: I will use a good cannonball of a great gun which travels 100 toises in 1 second; this cannonball will spend 25 years in its passage from the earth to the sun. He borrowed the analogy, Huygens reports, from Hesiod. The passage is:

as far beneath Earth is Tartarus
as Heaven is above Earth;
for if a brazen anvil falls 9 nights and days from

Heaven,

it reaches Earth on the 10th;
and if it falls 9 nights and days from Earth,
it reaches Tartarus on the 10th (719-25).

Not yet from measurement, it is nevertheless the first number assigned to the spatial distribution of all things. Moreover, if the brazen anvil falls with the acceleration of gravity, which also Huygens first measured, 9.8 m/s^2 , it will spend 10.1 minutes in its fall from the mouth of the Achelous to the mouth of the Rioni.

Thales discovered also the problem of the earth. His date is,

625 Thales.

The earlier developments leading to the beginning of the problem of the earth are two. The observation of the heavenly bodies. And the description of the surface of the earth. Both advanced steadily and greatly in the one and a half centuries before the first philosophers.

In particular, the observation of the heavenly bodies culminates in these predecessors of the philosophers:

787	observation of lunar eclipses beginning in 747,
761	observation of lunar eclipses from 721 to 720,
743	observation of lunar eclipses beginning in 703,
727	<i>The plough,</i>
717	<i>When Anu and Enlil,</i>
687	observation of Saturn from 647 to 633,
661	observation of a lunar eclipse in 621,
627	observation of Venus from 587 to 586.

With the observation of lunar eclipses from 721 to 720 accurate observation begins, the foundation of the *Almagest*. *The plough* and *When Anu and Enlil* record the observations of three dozen-odd fixed stars.

The description of the surface of the earth culminates in the six Assyrian princes from

Tiglath-pileser III prince 745-27
through

Ashurbanipal prince 668-27.

In but one inscription, reporting his campaigns, Tiglath-pileser III records 149 localities, not

counting the dozens whose names are lost in lacunae. His successors record steadily more. And their steadily more explicit descriptions of their campaigns steadily more clearly and distinctly imply maps.

The convergence of the two developments, the great advance in the description of the surface of the earth and the great advance in the observation of the heavenly bodies, in the first philosophers is the beginning of the problem of the earth.

Hesiod first asks explicitly the question: how did gods and the earth first come to be, and rivers, and the sea, and the stars, and the heaven? And he answers the question explicitly. First of all Chaos came to be. Earth is the foundation of all immortals. The king of mortals and immortals, the father of gods and men, is also a descendant of Earth. Counting only the named, the immortals are 335. They are a single chain of generation, and beginning with Earth uniformly by procreation. All connected in a chain of inferences. All by enumerations so complete as to omit nothing, and including 23 rivers from the Danube to the Rioni to the Nile to the Achelous. His date is,

734 Hesiod.

And the convergence of the three, the great advance in the description of the surface of the earth, the great advance in the observation of the heavenly bodies, and Hesiod's explicit question and answer, in the first philosophers is the beginning of the problem of the principles of all things.

Also, the great advance in the description of the surface of the earth passes through Hesiod. The 25 rivers are an important step in this advance.

ccclvi

There is now the problem, whence Hesiod?

The solution is, first of all, Homer.

There is then the problem, whence Homer?

And Homer, now, has always been liable to appear *ex nihilo*. For three main reasons. The suppositions of discontinuity, of ineffability, and of spirituality. Nor has inconsistency tended to eliminate one or another of the three, it has tended to reinforce all three.

The Greek record begins with Homer. And to begin with, neither were the Greeks barbarians, nor the barbarians Greeks. Later on, when the

dark age came to an end, learning revived with a return to the ancients, and that is, the Greeks, and in opposition to the darkness in between, and as all knowledge of the true antiquity, in Egypt and in Mesopotamia, had been lost, the notion of a Greek miracle established itself. More recently, after the recovery of the true antiquity, the notion of cultures and civilizations *sui generis* each has entrenched itself. The notion that the Greek record, in general, and Homer, in particular, have their beginnings in the true antiquity has not been entertained. This is the supposition of discontinuity.

Homer is also the Poet. Indeed, Hesiod is a poet, too. And not only when listening to an epic, where poetry, in general, is concerned, next to the melodious sound of the language, we cling only to the sense of the propositions and the ideas and feelings these arouse. So where poetry, in general, is concerned, and Homer's epics, in particular, we have not suffered unduly from the striving which forces us to advance from the sense to the reference. We have clung to the ideas and feelings. To put it otherwise, this is how Schopenhauer defines the poet: the poet grasps the idea, the being of mankind, the adequate objectivity of the thing in itself in its highest degree outside all relation and all time (*World*, I, 51). And where poetry, in general, is concerned, and the Poet, in particular, we have remained outside all relation and all time. This is the supposition of ineffability.

Herodotus writes: Homer and Hesiod created for the Greeks their theogony, they distinguished the gods according to their descent from each other and described their honors, arts, and appearances (II, 53). The early philosophers, including Herodotus, began laboring at the formulation and solution of the problem, too, whence the creation of their theogony for the Greeks by Homer and Hesiod. But then god became the beginning of all things. The gods distinguished by Homer and Hesiod became demons. More recently, while god is no longer omnipresent, spirit is. In particular, belief in the omnipresence of belief in gods is omnipresent. Certainly coeval with man. That the Greek record, in general, and Homer, in particular, would begin with belief in gods is a matter of course. This is the supposition of spirituality.

In short, Clio and Urania know how to tell the truth when they want to. The other seven know rather better how to tell many lies as if they were true.

ccclvii

The problem is of the developments preceding Thales leading to the world, its principles, and the propositions with which the world and its principles were first won.

In the one and a half centuries before Thales, these developments are the great advance in the description of the surface of the earth, the great advance in the observation of the heavenly bodies, and Hesiod's explicit question and answer.

The problem is now of the developments preceding these advances leading to these advances, including most particularly to Hesiod.

To begin with, then, in all the centuries before Hesiod there is for a long time no writing. Not in the solar system. Then writing begins. In the valley of the two rivers. The oldest cuneiform inscriptions attesting a coherent system of notation appear two and a half-odd millennia before Hesiod. In the second half of the third millennium^{BC}. Some two to three centuries later writing appears also in the valley of the Nile. The oldest hieroglyphic inscriptions attesting a coherent system of notation postdate the oldest cuneiform inscriptions attesting a coherent system of notation by two to three centuries.

Writing then gradually accumulates. It accumulates more and faster in the valley of the two rivers, more and faster after Enmebaragesi, prince c. 2680, the first prince to leave his inscriptions behind, more and faster after Sargon, prince c. 2337-281, the first prince to conquer the entire valley.

The one state of Sargon and his 4 successors continues for one and a half centuries. Two-odd generations of disorder follow. Then order is restored. The 5 princes of Ur,

Ur-Nammu	prince c. 2121-03,
Shulgi	prince c. 2103-055,
Amar-Sin	prince c. 2055-46,
Shu-Sin	prince c. 2046-37,
Ibbi-Sin	prince c. 2037-13,

bringing the valley under one state again, and maintain it under one state for one century.

And this one century is the time when writing becomes massive. The bulk of the Sumerian record, by far, is from this century. In this century the inscriptions of the princes become a continuous series. Shulgi, Amar-Sin, Shu-Sin, and Ibbi-Sin have left behind the names of all 90 years of their reigns. The first code of laws is from this century, Ur-Nammu's code. The first archive is from this century, the Ur archive, 23 official documents collected under Ibbi-Sin.

In this century for the first time also longer compositions appear.

One is the Sumerian prince list.

The list begins: when princeship descended from heaven, it came to Kish. Princeship is one. Once it has descended from heaven, it is taken from one city to another by smiting. And the longer the smiting continues, the more powerfully the striving which forces us to advance from the thought and the sense to the reference and the true forces the list to the reference and the true. Counting the important, 33 of the 120 princes enumerated by the list are real. Counting the important, all princes in the valley through the time of the list are 53. All 10 cities enumerated by the list are real. Counting the important, all cities in the valley through the time of the list are 13.

The list has described a portion of the earth.

The magnitude of the advance the other longer compositions from this time show.

There is the oldest chronicle, *Gilgamesh and Aka*. There are the four other compositions about Gilgamesh, *Gilgamesh and Humbaba*, *Gilgamesh and the bull of heaven*, *Gilgamesh and the netherworld*, and *The death of Gilgamesh*. There are the compositions about Inanna, *The marriage of Dumuzi and Inanna*, *Inanna's descent to the netherworld*, *The death of Dumuzi*. These have all been presented above in some detail, too.

In all of these the thought and the sense dominates. Given the reference and the true attained by the list, the striving which forces us to advance to the reference and the true can be detected in the chronicle, and the chronicle has attained some of the reference and the true which the list has attained. In the four other Gilgamesh compositions and in the Inanna compositions this striving cannot be detected. In these compositions the

opposite striving is in evidence, the striving which forces us to remain with the thought and the sense and saturate them with the ideas and feelings these arouse.

The more dramatic the advance of the list is, too, in that all those nine compositions, including the list, are much the same in content. All nine are about princes, in all nine princes mingle with gods, in all nine princes and gods frequent heaven as they do cities, if not in all nine also the nether-world.

The list is a step, a remote step, yet the first decisive step, toward the world, its principles, and the propositions with which the world and its principles were first won.

And whence the list, and that is, whence the propositions connected in chains of inferences whose reference is the true, is clear. The prince list is from the princes.

The princes established that princeship descends from heaven. As their inscriptions affirm. The princes established that princeship is one. As their deeds as well as their inscriptions affirm. And the princes discovered the reference and the true. When they began collecting with their campaigns and with the record of their campaigns the localities on the surface of the earth. Enmebaragesi attests 1 locality, Mesanepada attests 2, Ur-Nanshe attests 4, Eanatum attests 16, Sargon attests 27, Naram-Sin attests 38, distributed from the source of the Euphrates to the Zagros to the Persian Gulf to the Mediterranean.

After the 5 princes of Ur the state loses its unity. Two-odd centuries of states smiting one another follow. Then the unity of the state is restored. Under the 6 princes of Babylon,

Hammurabi	prince c. 1792-49,
Samsuiluna	prince c. 1749-11
Abieshuh	prince c. 1711-683,
Ammiditana	prince c. 1683-46,
Ammisaduqa	prince c. 1646-25,
Samsudutana	prince c. 1625-594,

the valley of the two rivers is for two centuries again under one state.

And again these two centuries are a time of massive writing, and of massive growth of writing. This is the time of the first of the three brightest periods in the development of mathematics, thirteen centuries before the second,

thirty-three centuries before the third. From this time is the Venus tablet of Ammisaduqa, with which the striving which forces us to the reference and the true has advanced to Venus. At this time the older Gilgamesh compositions are reworked into the epic of the early Akkadian *Gilgamesh*.

And from this time is *When on high*.

The first account of creation begins:

when on high the heaven had no name,
and down below the earth had no name,
Apsu was the begetter, Tiamat bore all;
no hut had been built, no god had been born,
no one had a name, no destiny had been determined,

when these two mingled their waters;
within these two the gods were formed (I, 1-9).

The gods are formed in a single chain of generation, all in one short line of development. Within Apsu and Tiamat Lahmu and Lahumu are formed (10). Within Lahmu and Lahumu Anshar and Kishar are formed, greater than themselves (11-2). The firstborn of Anshar and Kishar is Anu, their equal (13-5). Anu begets Ea, great in wisdom, great in strength, greater than his grandfather Anshar (16-20). Ea slaughters his great-grandfather Apsu (21-80), and in the middle of the slaughtered Apsu Ea begets Marduk, the greatest of the gods, the wisest, the lord (81-108).

Generation is thus complete. The bulk of the composition is then concerned with the war of Marduk against Tiamat. In preparation for the war, the gods acknowledge Marduk:

you are the greatest of the great gods,
your destiny is unrivaled,
your command is supreme,
your command cannot be changed,
your command shall be truth,
your word shall never be wrong,
no god shall ever go beyond the limits you set
(IV, 1-10).

Marduk is victorious:

he split Tiamat in two, like a fish,
and set up one of her halves to be heaven (138-9).

After the victory Marduk arranges his domain.

First on high the heaven:

he set up the stations of the great gods, one star
for each;
he set up the year, and marked its limits,

twelve months of three stars each;
 he made the days of the year,
 fixed the paths of the stars, lest they go astray;
 he allotted their domains to Enlil and to Ea;
 in the belly of Tiamat he fixed the zenith;
 he made the moon, to shine in the night (V, 1-12),
 down below on earth:
 from the eyes of Tiamat he let the Euphrates and
 the Tigris flow (55),
 below the earth:
 he set up one of her halves as a roof, and made
 the netherworld (62),
 on earth again:

he built Babylon, abode of the great gods (129).
 The final arrangement is for the maintenance of
 the domain. The gods slaughter one of them-
 selves, from his blood Ea makes men, and
 charges them with the work of the gods (VI).

Creation concludes with an enumeration of the
 50 names of Marduk (VII).

The line of descent from *When on high* to
 Hesiod is indicated unambiguously. *When on
 high* is Apsu and Tiamat, Hesiod is Lahmu and
 Lahumu, or Anshar and Kishar, or most probably
 Ea. The line of descent is indicated by the con-
 tent. The primordial beginnings of the gods, the
 single chain of generation, the increasing great-
 ness of the gods from one generation to the next,
 the overthrow of the progenitor, the culmination
 of generation in the greatest of the great gods, the
 war and victory in the war of the greatest of the
 great gods. And the line of descent is indicated
 unambiguously by the fact that – anticipating in
 this that other of its descendants, *Genesis* – *When
 on high* is attested in copies as numerous and
 widely disseminated as any Akkadian composi-
 tion, and in particular in the century before
 Hesiod.

There is much thought and sense in *When on
 high*, little reference and true. On high in the
 heaven but the twelve months of three stars each,
 the paths of the stars, lest they go astray, the
 moon, to shine in the night. Down below on earth
 but the two rivers and the abode of the great gods.
 All saturated with ideas and feelings.

The first account of creation is nevertheless a
 very important step toward the world, its princi-
 ples, and the propositions with which the world
 and its principles were first won. Twofold.

First, with the attempt, and a very successful
 attempt, to begin at the beginning. When the
 heaven had no name, the earth had no name, no
 hut had been built, no god had been born, no one
 had a name, no destiny had been determined.

Second, with the collection of creation under
 Marduk. You are the greatest of the great gods,
 your destiny is unrivaled, your command is
 supreme, your command cannot be changed, your
 command shall be truth, your word shall never be
 wrong, no god shall ever go beyond the limits
 you set.

The first is without precedent. And it is an
 anticipation – and the clearest anticipation in the
 nine centuries between them – of Hesiod's ex-
 plicit question: how did gods and the earth first
 come to be, and rivers, and the sea, and the stars,
 and the heaven?

The second is from the princes.

This is how Samsuiluna, for one, describes
 himself:

Samsuiluna, mighty prince, prince of Babylon,
 prince of Kish, the prince who subdued the four
 corners, the prince who by the order of Anu and
 Enlil slew all who stood against him, the shep-
 herd to whom Inanna gave good omens and help,
 who bound the hands of all who disobeyed him,
 who chased from the land all evil, who brought
 daylight to his numerous people, the foremost
 heir of Hammurabi, the lord who conquered the
 land (ins. 8).

Moreover, the reference of the propositions –
 the prince of Babylon subdued the four corners,
 slew all who stood against him, bound the hands
 of all who disobeyed him, chased from the land
 all evil, brought daylight to his numerous people,
 conquered the land – is the true. If the proposi-
 tions – the destiny of the greatest of the great
 gods is unrivaled, his command is supreme,
 cannot be changed, shall be truth, his word shall
 never be wrong, no god shall ever go beyond the
 limits he sets – have a reference, and whether the
 reference is the true or the false, they have a
 reference only to the degree to which the greatest
 of the great gods is like the prince of Babylon.

After the 6 princes of Babylon the state loses
 its unity. In the valley of the two rivers two and a
 half centuries of disorder follow. This is the time
 of the greatest rise of the state in the valley of the

Nile. In Asia Minor it is the time of the Hittite state. The Hittite record attests another descendant of *When on high*, the *Princeship in heaven*. Then the rise of Assyria begins. The state rises greatly under

Adad-nirari	prince c. 1305-273,
Shalmaneser	prince c. 1273-43,
Tukulti-Ninurta	prince c. 1243-06,

again under

Tiglath-pileser	prince 1115-076,

again under

Adad-nirari II	prince 912-891,
Ashur-nasirpal II	prince 884-59,
Shalmaneser III	prince 859-24,

and under

Tiglath-pileser III	prince 745-27,
Shalmaneser V	prince 727-2,
Sargon II	prince 722-05,
Sennacherib	prince 705-680,
Esarhaddon	prince 680-68,
Ashurbanipal	prince 668-27,

the state is again very much one and only one.

The century-odd from Tiglath-pileser III through Ashurbanipal is the time of the greatest expansion and then also the greatest consolidation of the state since its beginning.

And it is once again a time of massive writing, of massive growth of writing, and on both counts very much more massive than before. The inscriptions of these 6 princes are larger in bulk than the inscriptions of all their predecessors combined. This is the time when the observation of the heavenly bodies becomes sustained, then also accurate. It is the time of compositions as large as the 70-tablet *When Anu and Enlil*. Ashurbanipal collects in his library more than 1 000 distinct compositions.

And it is the time when the Hebrew record and the Greek record begin. The Hebrew record begins in the century before Tiglath-pileser III, with the Yahwistic document. The Greek record begins in the century beginning with Tiglath-pileser III, with Homer. Both are dated from their *termini ad quem*. *Amos*, the oldest Hebrew prophetic book, dates its prophecy between c. 791 and c. 750. Hence, the Yahwistic document is from the century before c. 791/50. Terpander, the oldest Greek poet to sing Homeric verses, won a citharodic contest in 676/3. Hence, Homer is

from the century before 676/3. The relation of descent of the Hebrew record from the earlier and contemporary Akkadian, and in particular Assyrian, record is direct. It is attested explicitly by both, and it is as intensely direct as the relation of the Assyrian princes and the Hebrew principalities is, of the former conquering the latter. The relation of descent of the Greek record from the earlier and contemporary Akkadian, and in particular Assyrian, record is less direct. It is nevertheless attested implicitly, as of the *Theogony* from *When on high*, and as directly as this. Herodotus reports: the Phoenicians who came to Boeotia brought with them much learning, and in particular the alphabet, which, I think, the Greeks did not know before that (V, 58).

And in all of this writing, in all of its massive bulk, it is once again first and foremost the princes that the striving which forces us to advance to the reference and the true has forced to advance to the reference and the true, and once again first and foremost when it has forced them to collect with their campaigns and with the record of their campaigns the localities on the surface of the earth.

An inscription of Tiglath-pileser III begins:

Tiglath-pileser, the great prince, the true prince, the lord of the earth, the prince of Ashur, the prince of Babylon, the prince of Sumer and Akkad, the lord of the four quarters, the prince who brought under his sway the lands from the sea of the rising sun to the sea of the setting sun.

And Tiglath-pileser III then lists: the countries and cities which I brought under my sway from the 1st year of my reign to the 17th year of my reign. The total is several dozen more than 149.

Two generations later, in his annals, Sennacherib begins:

Sennacherib, the great prince, the true prince, the mighty, the lord of the earth, the prince of Ashur, the lord of the four quarters, the shepherd, the favorite of the gods, the guardian of the right, the lover of justice, who aids the weak, whose deeds are pious, mighty warrior, first among all princes, fire consuming the disobedient, thunderbolt striking the evil, to whom Ashur, from the high mountain, gave no rival from the sea of the setting sun to the sea of the rising sun, and brought all peoples in submission to his feet (ins.

II, 233/54).

And the following is only one part of only the 3rd year of Sennacherib. The rest of his 3rd year and the other 24 of his 25 years differ only in the direction in which Sennacherib set out. Sennacherib reports:

in my 3rd year I set out against the Hittite; the prince of Sidon, struck by fear, ran into the sea and disappeared; Sidon, Bit-Zitti, Zaribtu, Mahal-liba, Ushu, Akzib, Akku, all his cities with their pastures and watering places the terrible sword of Ashur overwhelmed, and they all bowed down to my feet; I set my governor over them, and imposed on them yearly tribute; the princes of Arvad, Byblos, Ashdod, Beth-Ammon, Moab, Edom, all of them brought me for the fourth time rich gifts and heavy tribute, and kissed my feet; but the prince of Ashkelon, who did not submit to my yoke, the gods of his fathers, himself, his wives, sons, daughters, brothers, the seed of his father's house I uprooted and brought to Ashur; I set up another prince in Ashkelon, I imposed tribute on him, and he pulled the straps of my yoke; the cities of Beth-Dagan, Joppa, Banai-barka, Asuru, whose prince did not bow down to my feet, I conquered and carried off the spoil; to the people of Ekron, who overthrew the prince pledged to Ashur and gave him to Hezekiah, the Judaeans, the princes of Egypt and of Nubia sent help, troops, archers, chariots, horses, a host without number; near Elteka, with the help of Ashur, I defeated them, I took the chariots and I took the princes alive in my hands; I besieged Elteka and Timnah, I took them, and I carried off the spoil; I came to Ekron, I killed the guilty princes, and I hung their carcasses from the walls; the guilty people I carried off as spoil, the innocent I restored in Jerusalem, and imposed my tribute on them (*id.*).

The reference of these propositions connected in chains of inferences is the true because, and to the degree to which, the localities on the surface of the earth have a reference each.

And because, and to the degree to which, the reference of these propositions is the true, those other propositions as well have a reference, and their reference is the true. The propositions: the prince of Ashur is the lord of the four quarters, who brought under his sway the lands from the

sea of the rising sun to the sea of the setting sun. And two generations later, claiming much more from the thought and the sense, the propositions: the prince of Ashur is the lord of the earth, the lord of the four quarters, the guardian of the right, the lover of justice, who aids the weak, whose deeds are pious, mighty warrior, first among all princes, fire consuming the disobedient, thunderbolt striking the evil, to whom Ashur gave no rival from the sea of the setting sun to the sea of the rising sun, and brought all peoples in submission to his feet.

Nothing else in the Assyrian record attests the reference and the true nearly as clearly and distinctly as the advance in the description of the surface of the earth. Little else attests it at all. Very much attests the thought and the sense.

And as the Assyrian record so the Hebrew record, and so the early Greek record, too.

Amos is the earliest Hebrew prophetic book. And not less than the earlier Yahwistic document – edited as resolutely as it has been by the much later Priestly code – *Amos* attests the remotest Hebrew antiquity. *Amos* reports:

he made the Pleiades and Orion,
he brightens darkness into dawn
and darkens day into night,
he summons the waters of the sea
and pours them down on the earth;
his name is the Lord;
and it is he who destroys the cities,
he who wipes them out (5).

The Lord thus does in part what Marduk and his retinue do, in part what the princes do. And the evidence that he does it is very much like that supplied by the princes, except that his weapon of choice is fire, and that what they did do he will do. *Amos* reports:

thus said the lord,
I will send down fire on Damascus (1);
I will send down fire on Gaza (*id.*);
I will wipe out Ashdod and Ashkelon (*id.*);
I will destroy Ekron, the Philistines will perish
to a man (*id.*);
I will send down fire on Tyre (*id.*);
I will send down fire on Teman, it will devour
Bozrah (*id.*);
I will send down fire on Rabbah (*id.*);
I will send down fire on Moab, it will devour

Kerioth (2);

I will send down fire on Judah, it will devour

Jerusalem (*id.*).

Except that they are rather fewer, and closer to the prophet's home – Amos was a breeder of sheep from Tekoa, *Amos* reports (1), 15 km south of Jerusalem – the localities are the same, too.

There is very little reference and true in *Amos*. But there is. The localities on the surface of the earth.

Genesis is a collaboration of the Yahwistic document and the Priestly code, achieved by the Priestly code all by itself, three-odd centuries after the Yahwistic document. The Yahwistic document predates Homer. The Priestly code is contemporary with the second century of the development of philosophy.

And again, there is very little reference and true in *Genesis*. But again, there is. Again, the localities on the surface of the earth.

The flood occurs for the first time in the Sumerian *Death of Gilgamesh*. On this first occasion: Zuisudra alone survived the flood (*id.*). Eight centuries later it recurs in Sin-liqe-unninni, in the late Akkadian *Gilgamesh*. On this occasion: Uta-napishti alone survived the flood (11). Four centuries later it recurs in the Yahwistic document. On this occasion: Noah alone with those in the ark survived the flood (*Genesis*, 7).

In the collaboration of the Yahwistic document and the Priestly code on the description of events after the flood, in particular, the document is allowed to supply much of the contents, the code takes upon itself the form. It does all the calculations, for instance. Thus, it is the code that reports: Noah was 500 years old when he begot Shem, Ham, and Japheth (5), and: Noah lived 350 years after the flood, and all his days came to 950 years (9).

And the code continues:

these are the descendants of Shem, Ham, and Japheth born to them after the flood;
the descendants of Japheth – Gomer, Magog, Madai, Javan, Tubal, Meshech, and Tiras;
the descendants of Gomer – Ashkenaz, Riphath, and Togarmah;
the descendants of Javan – Elishah, Tarshish, Kittim, and Dodanim; from these the sea peoples came, each into its own country, with

its own language, family by family, people by people;

the descendants of Ham – Cush, Mizraim, Put, and Canaan;

the descendants of Cush – Seba, Havilah, Sabtah, Raamah, and Sabteca;

the descendants of Raamah – Sheba and Dedan (10).

The code then lets the document, not altogether consistently, continue:

Cush begot Nimrod, he was the first man of might on earth, he was a mighty hunter, the mainstays of his kingdom were Babylon, Uruk, and Akkad in the land of Shinar; from that land Ashur came forth and built Nineveh, Rehoboth, Calah, and, between Nineveh and Calah, Resen, which is a great city;

Mizraim begot the Ludim, the Anamim, the Lehabim, the Naphtuhim, the Pathrusim, the Casluhim, and the Caphtorim, from whom the Philistines came forth;

Canaan begot Sidon, his firstborn, Heth, the Jebusites, the Amorites, the Girgashites, the Hivites, the Arkites, the Sinites, the Arvadites, the Zemarites, and the Hamathites; after that the Canaanites spread out from Sidon to Gerar, near Gaza, and as far as Sodom, Gomorrah, Admah, and Zeboiim, near Lasha;

these are the descendants of Ham, by their languages, countries, and peoples (*id.*).

The code resumes:

sons were born also to Shem, the ancestor of all descendants of Eber, and older brother of Japheth;

the descendants of Shem – Elam, Ashur, Arpachshad, Lud, and Aram;

the descendants of Aram – Uz, Hul, Gether, and Mash (*id.*).

The code lets the document continue again:

Arpachshad begot Shelah;

Shelah begot Eber;

Eber begot two sons; the first was called Peleg, because in his days the earth was divided; his brother was called Joktan;

Joktan begot Almodad, Sheleph, Hazarmaveth, Jerah, Hadoram, Uzal, Diklah, Obal, Abimael, Sheba, Ophir, Havilah, and Jobab; their lands extended from Mesha as far as Sephar, the hill country in the east;

these are the descendants of Shem, by their languages, countries, and peoples (*id.*).

The code concludes:

these are the descendants of Noah, by their peoples; and from them came forth all the peoples on earth after the flood (*id.*).

Then the code lets the document tell the story of the tower of Babel.

All in all, 71 descendants of Shem, Ham, and Japheth, from whom came forth all the peoples on earth after the flood, by their languages, countries, and peoples.

And not too many, but there are the references.

The ark made land on mt Ararat (8). Presuming that it then was where it now is, mt Ararat is at 39° 40' N 44° 20' E.

Thus, Ashur went south to Ashur. Canaan went further south, and west, to Canaan, on the coast of the promised land, where Sidon was Sidon and Gaza was Gaza. The Amorites and the Hivites also went south, in the general direction of the promised land, where the Amorites and the Hittites were. These references can be recovered from the Hebrew writings all by themselves.

Elam went south, and east, to Elam. Madai also went south, in the general direction of Elam, where the Medes were. Sheba went far to the south, to the shores of the Red Sea. Gomer went west, where the Cimmerians were. Javan went further west, to the Aegean, where the Ionians were. Kittim took to sea and went to Cyprus. Put went south, then far to the west to Libya. Cush went south, crossed the promised land, continued west, then went far to the south, to the upper valley of the Nile. Along the way, apparently, he begot Nimrod, he was the first man of might on earth, he was a mighty hunter, the mainstays of his kingdom were Babylon, Uruk, and Akkad, he build Nineveh and Calah. These references can be recovered with the help of the inscriptions of the princes.

Also, Ashkenaz went to Germany, to judge by the fact that Germans came to go by the name Ashkenaz.

Magog went to Mongolia. Polo reports: the country of Magog the natives call Mongol, and there the Tartars live (*Description*, 2). Or he passed Mongolia and reached the Pacific. Ibn Khaldun writes: Magog is furthest to the north-

east, it borders the sea surrounding the earth (*Introduction*, I, 144).

The Assyrian princes do not attest Ashkenaz and Magog. But they attest mt Ararat, Ashur, Canaan, Sidon, Gaza, the Amorites, the Hittites, Elam, the Medes, Sheba, the Cimmerians, the Ionians, Cyprus, Put, Cush, Nimrod, Babylon, Uruk, Akkad, Nineveh, and Calah. Hence it can be presumed also, ignoring the chronological difficulties, that Javan, when he went west, was well informed of the deeds of the princes and was carrying a copy of *When on high*.

As to creation, this is the second account of creation, that of the Yahwistic document: when god made the earth and the heaven, there was no shrub and no grass on the earth, because god had not yet sent rain; nor was there a man to till the earth; water welled up from below and covered the earth; and god made man from the dust of the earth.

The line of descent from the description of the surface of the earth by the Assyrian princes to the list of all the peoples on earth who came forth after the flood, by their languages, countries, and peoples, by the Yahwistic document and the Priestly code is indicated clearly and distinctly. The line of descent from the account of creation in *When on high* to that of the Yahwistic document and *Amos* is indicated clearly and distinctly as well. And if the advance in the description of the surface of the earth by the princes is a development leading to the world, its principles, and the propositions with which the world and its principles were first won, and the account of creation in *When on high* is a very important step toward the world, its principles, and the propositions with which these were first won, the early Hebrew writings are on neither count a further step. In general, the early Hebrew writings do not attest the striving which forces us to advance from the thought and the sense to the reference and the true. They attest the opposite striving.

The more dramatically they thus show the great advance which on both counts the early Greek writings are. On the second count, the account of creation, Hesiod. On the first count, the description of the surface of the earth, Homer.

The *Iliad* begins:

the wrath sing, goddess, of Peleus's son Achilles.

And having begun *in medias res*, with the wrath of Achilles in the tenth year of the Trojan war, in book I and the first half of book II the poet makes us cling to the thought and the sense as staunchly as we ever would, or should. Then, suddenly, he yields to the striving for truth, turns to scientific consideration, and asks the muses, who have sung to him, and will sing to him, so much thought and sense, to tell him also about the reference and the true:

tell me muses, you who live on mt Olympus,
you who are goddesses and know everything,
while we mortals know nothing until we are told,
who were the generals of the Danaans (484-7)?

And the muses not only tell him who the generals of the Danaans were, they arrange the generals in their series and order, contingent by contingent, count the ships in each contingent, and review the countries of the generals, listing cities, mountains, islands, and rivers in the countries.

These are the countries of the Danaans, with the number of their ships, and the number of the localities which the muses note in each country (494-761):

the Boeotians	50 ships	29 localities,
Orchomenus	30	2,
the Phocians	40	9,
the Locrians	40	9,
Euboea	40	7,
Athens	50	—,
Salamis	12	—,
Argos	80	8,
Mycenae	100	10,
Sparta	60	9,
Pylos	90	9,
Arcadia	60	10,
Elis	10	5,
islands across Elis	— 40	1,
Cephalonia	12	6,
Aetolians	40	5,
Cretans	80	7,
Rhodes	9	3,
Syme	3	—,
Cos	30	4,
Phthia	50	6,
Phylace	40	4,
Pherae	11	4,
Methone	7	4,
Ithome	30	2,

Ormenius	40	3,
Argissa	40	4,
Dodona	22	5,
the Magnetes	40	1.

29 countries, and 166 localities in them, for a total of 195.

And although there has thus been enough scientific consideration, and more than enough, and we are eager to go back to the thought and the sense, the muses go on to tell the poet who the generals of the Trojans were, too. They arrange them as well in their series and order, contingent by contingent, and while the Trojans have no ships to count, the muses yet review the countries of the generals, listing cities, mountains, islands, and rivers in them.

These are the countries of the Trojans, with the number of localities in each country (816-77):

the Trojans	—,
the Dardanians	1,
Zeleia	1,
Adrasteia	3,
Abydos	6,
Larissa	1,
the Thracians	1,
the Ciconians	—,
the Paeonians	1,
the Paphlagonians	7,
the Halizones	1,
the Mysians	—,
the Phrygians	1,
the Maeonians	2,
the Carians	4,
the Lycians	1.

16 countries, and 30 localities in them, for a total of 46.

The grand total is 241.

The muses then relent and go back to the thought and the sense.

There are difficulties.

The numbers for the localities in each country are maxima. For instance: Ascalaphus and Ialmenus led the men of Acpledon and Orchomenus of the Mynians (511-2). And if the Mynians indicate 1 locality, this locality coincides to one degree or another with the 2 localities indicated by the men of Acpledon and Orchomenus.

Also, among the localities are the Pelasgians: Hippothous led the Pelasgians who dwell in the

rich soils of Larissa (840-1). And in the *Odyssey* the Pelasgians dwell not in the rich soils of Larissa, but on Crete (XIX, 172-9). Elsewhere in the *Iliad* Zeus himself is Pelasgian (XVI, 233).

Worse, among the localities are the Halizones: Odius and Epistrophus led the Halizones (856). And Strabo has to sort out four centuries of controversy on whether, in order to find the Halizones on the surface of the earth, to emend one letter in the name Halizones, or two letters, or more letters, or to just read Amazons in place of Halizones (XII, 3.20-3).

The difficulties are the exceptions, however. Nearly all the localities are, in fact, localities on the surface of the earth.

Strabo begins in the *Geography* by affirming: Homer is the first philosopher (I, 1.1). He then shows, appealing to the example of a long list of predecessors of his, that the geographer is a philosopher. And draws the conclusion: being the first philosopher, Homer is the first geographer (1.11). Accordingly, as he describes in the seventeen books of the *Geography* 6 000 localities on the surface of the earth from the Atlantic to India, Strabo associates every locality he can with a Homeric passage, and identifies Homeric locality after Homeric locality. The identifications are of two kinds. Eratosthenes, as Strabo himself reports, said: you will find where Odysseus traveled when you find the cobbler who sewed the bag of winds (2.15). And Strabo finds where Odysseus traveled. The other kind of identifications are the localities of the catalogue in book II of the *Iliad*.

Thus, in the catalogue, Odysseus leads the men from Cephallenia (631), and the island of Cephallenia in the Ionian Sea is as far west as the reference has reached in the catalogue. Pylaemenes leads the Paphlagonians who live by the river Parthenius (851-4), and the mouth of the Parthenius in the Black Sea is as far east and as far north as the reference has reached. Idomeneus leads the Cretans (645), and the island of Crete in the Mediterranean is as far south as the reference has reached. The reference has reached from the Parthenius to Crete across 7° of latitude along the meridian, 800 km along the great circle, from Cephallenia to the Parthenius across 12° of longitude along the parallel, 1 100 km along the great circle. With not less than 200 real localities

within these limits arranged in their series and order.

Thus, pace Strabo, Homer is not the first geographer, and not the first philosopher. The first geographer is the seventh philosopher, Hecataeus. The first philosopher is the first philosopher, Thales.

Homer, however, is a great advance in the description of the surface of the earth. He is a decisive step toward the first geographer. And the advance in the description of the surface of the earth is from the princes.

One generation after Homer, Hesiod asks the question: how did gods and the earth first come to be, and rivers, and the sea, and the stars, and the heaven? And answers it. First of all Chaos came to be. Earth is the foundation of all immortals. The king of mortals and immortals, the father of gods and men, is also a descendant of Earth. The major immortals are 335. They are a single chain of generation, and beginning with Earth uniformly by procreation. All connected in a chain of inferences, all by enumerations so complete as to omit nothing. Including 23 rivers from the Danube to the Rioni to the Nile to the Achelous.

Hesiod's question and answer are also from the princes, and twofold. First, the princes have supplied the evidence. From the first prince through Hesiod's contemporary Sennacherib: the prince of Ashur is the lord of the earth, the lord of the four quarters, the guardian of the right, the lover of justice, who aids the weak, whose deeds are pious, mighty warrior, first among all princes, fire consuming the disobedient, thunderbolt striking the evil, to whom Ashur gave no rival from the sea of the setting sun to the sea of the rising sun, and brought all peoples in submission to his feet. Second, Hesiod is also from *When on high*, and *When on high* is also from the princes, also a generalization over the evidence supplied by the princes.

The observation of the heavenly bodies has become sustained, then also accurate.

And the convergence of the great advance in the description of the surface of the earth, the great advance in the observation of the heavenly bodies, and Hesiod's question and answer in the first philosophers is the beginning of the problem of the principles of all things.

ccclviii

The two and a half centuries from Thales to Aristotle.

The development of the problem in these centuries has been collected above.

The important particulars are 15. Dividing it into its three parts, and that of the elements into its two parts, which the elements are and what their kind is, the development of the problem is this.

The elements of all things are,
 Thales water,
 Anaximenes air,
 Heraclitus fire,
 Hippasus fire,
 Parmenides fire and earth,
 Empedocles earth, water, air, and fire,
 Diogenes of Apollonia – air,
 Aristotle earth, water, air, fire, and aether.

Their kind is,
 Anaxagoras an infinite number of particles,
 Melissus one,
 Leucippus atoms and void,
 Democritus atoms and void,
 Plato the one,
 Aristotle simpler bodies similar to the elements.

The causes of all things are,
 Parmenides love,
 Anaxagoras reason,
 Empedocles love and strife,
 Democritus the shape, order, and position of the elements,
 Aristotle the first mover.

The formal principles of all things are,
 Pythagoras number,
 Parmenides one,
 Anaxagoras reason,
 Socrates definitions,
 Plato the forms,
 Aristotle propositions.

The steps of advance in the acquisition of knowledge are 12. In chronological order,
 Thales water,
 Anaximenes air,
 Pythagoras number,
 Heraclitus fire,
 Parmenides fire and earth,

Anaxagoras one,
 an infinite number of particles,
 reason,
 Empedocles earth, water, air, and fire,
 love and strife,
 Leucippus atoms and void,
 Socrates definitions,
 Democritus the shape, order, and position of the elements,
 Plato the forms,
 Aristotle earth, water, air, fire, and aether,
 simpler bodies similar to the elements,
 the first mover,
 propositions.

Counting centuries from the year of birth assigned to Thales, the distribution is,

625- 526	4	4,
525- 426	10	7,
425- 326	1	1.

The count in the last line is not complete.

The advance is very great. From nothing to these results.

The principles of all things are material, efficient, and formal.

Earth, water, air, and fire are the elements.

The elements are atoms and void.

The first mover is the cause.

Propositions are the principles. And the principle of contradiction is the principle of all principles.

Aristotle has also integrated the cause and the elements into the systematic unity of the world. The first mover is at the circumference, moves the stars, hence the planets, and hence the elements, fire, air, water, and earth.

None of these general results will be improved upon for nineteen and a half centuries.

The advance in the formulation and solution of this problem, the principles of all things, can also be compared with the advance in the formulation and solution of the other problems whose development has been collected above. Not with the problem of the best state, only one part of which has been collected. Not with the ethical problem of the soul either. With the physical problem of the soul, with the problem of the earth, and with the problem of the sum of the angles of the triangle and the assumptions on which the sum

depends.

The soul is the principle of life. The ground of the formulation and solution of the problem is substantial. The ground is the investigation of animals, the investigation of plants, and theoretical medicine. And while the investigation of plants and theoretical medicine have not advanced much, the investigation of animals has advanced greatly. The ground is substantial. The chains of inferences, on the other hand, into which the formulation and solution of the problem is collected, are very far from being free of gaps. The results obtained in the investigation of animals and the general propositions affirming what the soul is are far removed from one another and tenuously connected with one another. No theory has been formulated.

The earth is a sphere, at rest, in the center. The ground is much more substantial and firmer. The observation of the heavenly bodies has advanced very greatly, has become increasingly accurate, increasingly mathematical. The radius of the earth has been measured. The chains of inferences are also much more free of gaps. The theory of the motions of the heavenly bodies in concentric spheres has been formulated.

The sum of the angles of the triangle is two right angles, this has been proved, proofs resting on this proof and proofs on which this proof rests have accumulated, and the assumptions on which the proofs rest have been collected in the first *Elements*. The theory of proportion has been formulated. The chains of inferences are so free of gaps, they have become their own ground of further advance, and the ground of the formulation and solution of other problems. Already of that of the earth. The advance in the formulation and solution of this problem is more substantial than the advance in that of the earth, far more substantial than the advance in that of the soul.

And on both counts, how firm the ground is and how free of gaps the chains of inferences are, the advance in the formulation and solution of the problem of the principles of all things is the least substantial of the four. And of the three parts of this problem, the advance in the formulation and solution of the problem of the causes is less substantial than the advance in that of the formal principles, and the advance in that of the elements

is less substantial than the advance in that of the causes.

In all three parts of the problem Aristotle attests the greatest advance.

The solution of the problem of the formal principles is part of the development of logic. Aristotle defined truth, distinguished term, proposition, and inference, classified inferences, developed the predicate calculus. It is the greatest advance in logic. But the solution of the problem of the formal principles of all things is implicit rather than explicit. That it is a solution of this problem, and what the relation of the solution of this problem and the solutions of the other two, of the elements and of the cause, is, depends on the general arguments concerned with the four kinds of the principles, and the yet more general arguments concerned with matter and form. Nothing in the greatest advance in logic is grounded explicitly in the advance of proof, nothing in the advance of observation. The chains of inferences connecting the greatest advance in logic with the proofs and observations accumulated in the advance of mathematics and physics are very far from being free of gaps. Which can explain also why in the twenty-two and a half centuries to Frege, with a few, and but a very few, exceptions, the philosophers will keep taking it for granted that logic is not logic, logic is rhetoric, or grammar.

The solution of the problem of the cause is explicit. It rests in part on the solution of the problem of the earth. A small part. The large part are the general arguments in the 8 books of the *Physics*. And there are no observations in the 8 books of the *Physics*.

The solution of the problem of the elements is also explicit. In this case, unlike the other two, Aristotle's is not a substantial advance over his predecessors, Empedocles and Democritus. In Aristotle the solution rests on the general arguments in the last 2 books of *On the heaven* and the 2 books of *On generation and corruption*. And there are no observations in Empedocles, or Democritus, or these 4 books of Aristotle either.

The great power of the solution of the problem of the principles of all things is in the integration of the two parts, the cause and the elements, into the systematic unity of the world, itself resting on

the solution of the problem of the earth.

Neither this nor any of the other general results obtained in the first two and a half centuries of the development of the problem will be improved upon in the following nineteen and a half centuries.

The particular results on which these general results stand, however, will be. The advance in these nineteen and a half centuries, when there will be an advance in these nineteen and a half centuries, will be in this, gathering the grounds on which the formulation and solution of the problem stands and filling the gaps in the chains of inferences into which the formulation and solution is collected.

ccclix

The four and a half centuries from Aristotle to Ptolemy.

They divide into two parts. The two centuries from Aristotle to Hipparchus, and the two and a half centuries from Hipparchus to Ptolemy. The dates are,

384 -	322	Aristotle,
187 ^{BC}		Hipparchus,
87 ^{AD}		Ptolemy.

The two centuries from Aristotle to Hipparchus are the time of the second of the three brightest periods in the development of mathematics. Mathematics advanced very greatly, and physics advanced very greatly.

This is the time also of the schools. The time also of the library.

In the generation after Aristotle, Theophrastus wrote the *History of plants* and *Causes of plants*.

He wrote the history of physics.

And he wrote *On fire* and *On stones*.

In *On fire*, Theophrastus writes: of the elements fire has the most peculiar powers; air, water, and earth cannot generate themselves, but fire can both generate itself and destroy itself; a small fire generates a large one, a large fire destroys a small one (1); fire also cannot persist without fuel; but if so, it seems foolish to speak of fire as a substance and an element (4). Also: if the sun is a form of fire, it is possible that the heat of our earthly fires arises from the action of the sun (5).

In the rest of the treatise Theophrastus de-

scribes the generation and destruction of fire and the conduction of heat (10-29), which things are combustible and which things conduct heat (30-49), the shape of the flame (50-6), the kinds of fire (57-62), the ignition of fire (63-4), how heat generates moisture (65-9), smoke (70-2), the ignition of fire by reflecting surfaces (73-4), charcoal (75-6).

In *On stones*, he writes: of the things in the earth, some are formed of water, some are formed of earth; of water are formed the metals, of earth are formed the stones (1); earth becomes pure and uniform by confluence or filtering (2); this pure uniform matter, when subjected to heat or cold, hardens and forms stones and earths, differing in color, density, smoothness, luster (3).

In the rest of the treatise Theophrastus describes some dozens of stones and earths, including pumice (19-22), the emerald (23-7), amber, the lodestone (29).

The investigation of plants begins with Theophrastus. The history of physics begins with him. And the investigation of fire and of stones, and thus the explicit collection of the particulars of which the elements of all things will be the elements, begins with him.

Theophrastus was the second scholarch of the Lyceum. The Academy was founded *c.* 385, the Lyceum in 335.

A younger contemporary of Theophrastus is Pyrrho. With Pyrrho scepticism begins. His followers, the sceptics, were one of the minor schools.

Another younger contemporary of Theophrastus is Diodorus Cronus. His followers were another of the minor schools, the dialecticians.

Diodorus Cronus investigated modality and meaning.

He formulated the master argument: every past truth is necessary, the impossible cannot follow from the possible, therefore nothing which is neither true nor will be true is possible (Epictetus, *Discourses*, II, 19.1). He said: a conditional is valid if it neither has nor can have a true antecedent and a false consequent (Empiricus, *Outlines*, II, 110). Presuming apparently that there is a difference between what is true and what will be true. He said: no word is ambiguous, no one says or thinks anything ambiguous; when you have

understood something other than what I mean, I should be held to have spoken obscurely rather than ambiguously; for an ambiguous word would mean two or more things, and no one says two or more things, if he thinks that he says one thing (Gellius, XI, 12). Presuming apparently that a word means what it means by itself, not in relation to other words. He also conducted an experiment: every utterance can signify, and to prove it, he named one of his slaves However and the others with other connectives (Ammonius, comm. *Interpretation*, 38).

Diodorus Cronus said also that the elements are minute indivisible bodies.

The next is the generation of Euclid.

Autolycus, in *On the moving sphere* and *On the risings and settings*, and Euclid, in the *Phenomena*, defined the circles of the equator, ecliptic, horizon, and meridian, and proved 60 propositions about the heavenly bodies. The first 60 physical theorems.

With these 60 theorems the mathematical investigation of the motions of the heavenly bodies, and thus the explicit collection of the particulars which the first mover will move, begins.

In the generation of Euclid, Epicurus founded the Garden, in 307, Zeno of Citium founded the Stoa, in 301. And the four major schools will flourish well into Christian times.

Epicurus writes: nothing comes out of nothing; nothing goes into nothing; the whole is always the same, and it is nothing but bodies and void; bodies are composed of the elements; the elements are atoms; atoms have no qualities, but shape, weight, and size; all bodies are configurations of atoms.

This is not new with Epicurus. New with him is the ethics he built on it.

In general, at the Academy under Plato mathematicians gathered. Then it became a school of sceptics. The Lyceum was at first the school of Aristotle and Theophrastus. Then it lost their books and it, too, became a school of ethicists. At the Garden Epicurus yet grounded ethics in physics. After him, as Lucretius describes them, bees sipped at the master's every blossom. The school of Zeno, then of Chrysippus, the Stoa became the school of Panaetius who taught that

the useful is honorable.

Zeno of Citium divided philosophy into logic, physics, and ethics.

Zeno said: nothing incorporeal is capable of action, everything that acts or is acted upon is body; all things are determined; determination is the chain of causes whereby things happen, or the law whereby the world develops; there are two principles of the whole, active and passive; the passive principle is matter; the elements are earth, water, air, and fire; the active principle is the inherent in matter law, or god, which is everlasting and the producer of everything.

Everything is body. The principles are two. And the solution of the problem of the passive principle is unambiguous. The passive principle is matter, the elements are earth, water, air, and fire. The solution of the problem of the active principle is not so unambiguous. The chain of causes is the law whereby the world develops, and the active principle is the inherent in matter law.

Then again, the inherent in matter law is god, the producer of everything.

In Plato the world has a maker, and he makes the world because he is good. In Aristotle the first mover locomoves the stars, and hence everything in the world, because there must always be motion. And if there must always be motion, he does not need to make the world because he is good. But if he makes the world because he is good, there need not always be motion. So also, is he the producer of everything, because the law is inherent in matter? or the law is inherent in matter, because he is the producer of everything?

Zeno was succeeded as scholarch at the Stoa by Cleanthes. Laertius reports: Cleanthes had no aptitude for physics, and his fellow students called him an ass, but he did not mind, telling them that he alone was strong enough to carry Zeno's burden.

Cleanthes begins in the *Hymn to Zeus*:

most glorious of gods, known by many names,
all-powerful, lord of all change in nature,
lawgiving pilot of the world, I hail you,

and ends:

bountiful Zeus, rescue mankind from wretched
ignorance,
give men the wisdom with which you steer all

things in justice;
thus honored, we will honor you,
for there is no greater glory for a man or god
than righteously to sing forever of the common
law.

Cicero writes: in one place Cleanthes says that the world is god, in another place that god is the mind and soul of the world, in yet another place that god is the fiery aether embracing the world, and in his books against hedonism he babbles like a madman, now saying that the gods have shape and form, now saying that the stars are divine, now saying that nothing but reason is divine (*Gods*, I, 37).

Which will seem to show unambiguously and clearly that, with all due respect to Diodorus Cronus, the ambiguous is obscure and the obscure ambiguous.

In the same generation, Ctesibius wrote a book on machines worked by air and water. Vitruvius writes: Ctesibius discovered air pressure and the principles of pneumatics (IX, 8.2). Vitruvius describes also the machines Ctesibius built: the first water pump (X, 7); a water clock (IX, 8); a water organ worked by air (X, 8). Ctesibius built also a singing cornucopia for a statue of queen Arsinoë (Athenaeus, XI, 497d-e).

The investigation of fluids begins with Ctesibius, experimentally, too, and thus the explicit collection of the particulars of which the elements will be the elements continues.

Still in the same generation, Aristarchus measured the distances of the moon and the sun.

Thus the determination of where exactly the first mover will be begins.

When Alexander's successors divided his domain, Ptolemy became the master of Egypt. And Ptolemy, prince 306-282, founded the Alexandrian library, the greatest and for a long time the greatest library.

The first head of the library was Zenodotus, the second was Apollonius of Rhodes, the third was Eratosthenes.

Callimachus, at the time of Ptolemy II, prince 282-46, catalogued the books in the library.

In the library, Tzetzes reports: there were at the time of Callimachus 532 800 books (*Prolegomena*, 11a). Not less than 200 000 distinct compositions.

In the library of Ashurbanipal, prince 668-27, there were more than 30 000 tablets, not less than 1 000 distinct compositions.

The two numbers, of the books collected by the kings of Assyria and of the books collected by the kings of Egypt, can be presumed a good measure of the quantity of writing accumulated by the time, respectively, of Ashurbanipal and of Ptolemy II. Hence a number can be attached to the fact that it was in the intervening four centuries that the writing of books became a major occupation of humanity. The quantity of books increased by a factor of not less than 200.

Of the quality of the books, the catalogue of Callimachus, the first catalogue, of the books in the greatest library, not less than 200 000 of them, can be presumed a good measure. Its title is recorded by the *Suda*: *Catalogue of those distinguished in all learning and their books* (s. v.). The catalogue was itself in: 120 books (*id.*). And Callimachus divided those distinguished in all learning and their books into two kinds, poetry and prose. Poetry in turn he divided into six kinds, epic, elegiac, iambic, lyric, tragic, and comic. And prose in turn he divided also into six kinds, history, rhetoric, philosophy, medicine, laws, and miscellaneous.

Euclid worked in Alexandria (Pappus, *Collection*, VII, 35), one generation before Callimachus. Prose rather than poetry, presumably, and not history, or rhetoric, or medicine, or laws, the *Elements* may have been philosophy. More likely it was miscellaneous.

The next is the generation of Archimedes, Chrysippus, and Eratosthenes.

With Archimedes the investigation of fluids became mathematical. The explicit collection of the particulars of which the elements will be the elements continued. Archimedes also packed the world with sand. If the first mover were to move sand, it was to move not less than 10^{51} grains of sand.

Chrysippus discovered the rules of inference. Aristotle developed syllogistic. The soundness of syllogistic, the predicate calculus, depends on the soundness of the rules of inference, of the propositional calculus. Aristotle takes the rules of inference implicitly for granted. Chrysippus formulated the rules explicitly. Chrysippus is the

greatest advance in logic in the twenty and a half centuries between Aristotle and Leibniz.

Laetius reports: Chrysippus recognizes five kinds of inference; in the first kind a conditional proposition and its antecedent are the premises, its consequent is the conclusion, thus,

if the first, then the second,
but the first,
therefore the second;

in the second kind, a conditional proposition and the contradictory of its consequent are the premises, the contradictory of its antecedent is the conclusion, thus,

if it is day, it is light,
but it is not light,
therefore it is not day;

in the third kind, the negation of a conjunctive proposition and one of its parts are the premises, the contradictory of its other part is the conclusion, thus,

not both Plato is dead and Plato is alive,
but Plato is dead,
therefore Plato is not alive;

in the fourth kind, a disjunctive proposition and one of its alternatives are the premises, the contradictory of the other is the conclusion, thus,

either the first or the second,
but the first,
therefore not the second;

in the fifth kind, a disjunctive proposition and the contradictory of one of its alternatives are the premises, the other is the conclusion, thus,

either it is day or it is night,
but it is not night,
therefore it is day (VII, 79-81).

Chrysippus succeeded Cleanthes as third scholarch of the Stoa. Laetius reports also: it was said that, if the gods take to logic, they will to no other but Chrysippus's. In the second kind of inference the minor premise is the contradictory of the consequent, thus,

if the gods take to logic, they will to no
other but Chrysippus's,
but they will not take to Chrysippus's,
therefore they will not take to any.

Eratosthenes measured the radius of the earth. The problem of the unit of measurement of the world was solved conclusively.

In the next generation, Philo of Byzantium

wrote the *Mechanical collection*. Book V is the *Pneumatics*, continuing the investigation of fluids. Philo describes the construction of siphons, clepsydras, hydraulic presses, pumps (4-65). He also explains how these machines work: the philosophers say that air consists of very minute particles (3); but void is a part of nature and mingled with air (*id.*); air and water thus combine so that there is no space between them, and this is why water can rise (*id.*).

To this time belongs Bolus. Bolus wrote *On sympathies and antipathies*. And with *On sympathies and antipathies* alchemy begins.

Alchemy being what it is, these propositions involve difficulties. Alchemy began flourishing some centuries later, in the early Christian centuries. That was a time of especial industry in the production of writings of the greatest authority. A corpus of the writings of Hermes Trismegistus was authored. A corpus of the writings of Pythagoras was authored. A corpus of the writings of Democritus was authored. Among others. Hermes Trismegistus and Democritus both revealed themselves as alchemists. And passages from Bolus's book were incorporated into the Democritean corpus.

Bolus is dated to the century after Callimachus from passages which appear to refer to Theophrastus and Callimachus.

The following is another passage: take mercury, fix it with the body of magnesia, or with the body of Italian antimony, or with native sulfur, or with selenite, or with burned limestone, or with alum of Milos, or with arsenic, or with whatever you will; cast this preparation of white earth on copper, and you will have copper without a shadow; add yellow electrum, and you will have gold (*id.*).

What exactly you will have, after you take whatever you will, is difficult to know. Perhaps you will have gold. Perhaps you will have gold coloring on copper with mercury or arsenic alloy. Whatever you will have, how you will have it, the description also leaves largely up to you, once you take whatever you will.

Whatever you will have, however, you also have the first anticipation of chemical experiment. Which the very fact of the distinction of mercury, magnesia, antimony, sulfur, selenite,

limestone, alum, arsenic, copper, and electrum shows. And if anything will show that the elements are the elements, if not yet with *On sympathies and antipathies*, and not for two millennia after Bolus, chemical experiment will show it.

One generation-odd after Philo of Byzantium, Apollonius developed the eccentric and the epicyclic models of planetary motion. Moving the stars in circles, the first mover will have to move the planets in circles upon circles.

One generation-odd later, Hipparchus vastly improved observation, and formulated the theory of the motions of the moon and the sun. Not in the lower regions of the world, and not yet in the highest, but in the middle regions of the world the first mover will now have to move things with exacting precision.

Then the second of the three brightest periods in the development of mathematics came to an end.

In the two and a half centuries from Hipparchus to Ptolemy the advance of mathematics and physics decelerated. The schools began declining. This is the time when Rome brought nearly the entire known earth under a single government. Philosophy spread through nearly the entire known earth. Not physics, or logic, ethics spread. The advances of mathematics and physics became fewer and less frequent. Like all philosophy, mathematics and physics became increasingly derivative. There was yet at the end of this time Ptolemy. The decisive decline occurred after that.

The gods did not take to logic. Nor did the philosophers. They assimilated logic into rhetoric and grammar instead.

The generation after Hipparchus is the generation of Hermagoras and Dionysius Thrax.

Hermagoras wrote the most important *Art of rhetoric*. Cicero, two generations later, learned rhetoric from Hermagoras. Quintilian, one and a half centuries later still, learned rhetoric from Hermagoras. Augustine, three-odd centuries later still, learned rhetoric from Hermagoras.

Protagoras and Gorgias began teaching rhetoric. As Aristotle describes it: they gave speeches to their students to learn by heart, like giving them shoes rather than teaching them how to make shoes. Hermagoras did not give them shoes, he taught them how to make shoes. Quintilian

reports: Hermagoras divided all questions into two kinds, logical and legal, questions of reasoning and questions of law (III, 5.4); and logical questions he divided into four kinds, conjecture, property, translation, and quality (6.56); is it the case? what is it? of what kind is it? and is the man involved good or bad (*id.*)? Cicero, for his part, writes: Aristotle prepared young men not for subtle philosophical discussions, but to be able with the fluency of orators to argue in rich and elegant language either side of any question (*Orator*, 46).

Dionysius Thrax wrote the most important *Art of grammar*. The philosophers continued to learn grammar from Dionysius Thrax longer than they did rhetoric from Hermagoras.

Prodicus began teaching that first of all the right use of the word should be learned. Chrysippus distinguished five parts of speech: proper name, common name, verb, conjunction, and article (Laertius, VII, 57). Three generations later, Antipater of Tarsus added: adverb (*id.*). One generation later, in *The art of grammar*, Dionysius Thrax distinguishes eight parts of speech: noun, verb, participle, article, pronoun, preposition, adverb, and conjunction (12-21). And this is the entire *Art of grammar*. There is not even syntax in *The art*. Dionysius Thrax begins by defining grammar: grammar is the expertise in all that involves the language of the poets and writers of prose (1).

In the next generation, Asclepiades said that the elements are seamless particles.

Empiricus, writing one century after Ptolemy, reviewed the problem of the principles of all things three times. In his reviews he collected 24 philosophers who concerned themselves with the principles of all things in the eight centuries from the first philosopher through the time of writing. 19 are from the two and a half centuries from the first philosopher through Aristotle. 4 are from the two generations after Aristotle – Diodorus Cronus, Epicurus, Zeno of Citium, and Strato. 1 is from the five centuries from Strato to the time of writing – Asclepiades, two centuries separating Strato and Asclepiades, three centuries separating Asclepiades and Empiricus.

The next century is the century of Roman learning, of Mucius, Varro, Cicero, Servius,

Caesar, Lucretius, Vitruvius, Sallust, Alfenus, Augustus, Livy, Labeo.

The century after that is the century of st Paul, of the first bishops, and of the commitment of the gospels to writing.

The development of philosophy and the development of Judaism were slow to begin informing one another explicitly. Three centuries after the beginning of philosophy, two centuries after the Priestly code – three centuries before st Paul – in Alexandria, in the library, the earlier Hebrew writings were rendered into Greek, into the septuagint. Two centuries after that – one century before st Paul – the *Wisdom of Solomon*, composed in Greek, in Alexandria, teaching the immortality of the soul, is the first Hebrew writing attesting clearly, yet only implicitly, acquaintance with Plato. In the following century – the century before st Paul – the pharisees taught the immortality of the soul and eternal rewards and punishments, also attesting clearly, yet also only implicitly, acquaintance with Plato. Toward the end of that century – one generation before st Paul – Philo Judaeus wrote, in Greek, in Alexandria, some two dozen commentaries on the Mosaic books. Steeped in the Hebrew scriptures, in their Greek rendition, in particular, Philo Judaeus is steeped also in the philosophers, in Plato, in particular, and most explicitly now. It is with Philo Judaeus that the development of Judaism becomes informed explicitly by the development of philosophy, and – much more important as it will turn out – the other way around, the development of philosophy becomes informed explicitly by the development of Judaism. St Paul was a pharisee before he became st Paul. Origen, later on, was steeped in Philo Judaeus.

The principles of all things are not the central concern of the apostle. His central concern is this. The *Letter to the Romans*: do you imagine that you will escape the judgment of god? all alike have sinned; all alike are justified by god alone through Jesus Christ; through one man sin entered the world, and through sin death, and death pervaded the human race; for all men have sinned; shall we all then persist in sin? no! no! baptized into Jesus Christ we are on the new path of life; confess that Jesus is the lord, and you will find salvation.

That the principles of all things are nevertheless very much a concern of the apostle, can be taken on the authority of the *Letter to the Colossians*, even if the *Letter to the Colossians* is not one of the genuine half of the letters in the corpus of the apostle.

The *Letter to the Colossians*: our lord Jesus Christ is the image of the invisible god, his is the primacy over all created things; in him was everything in heaven and on earth created, and not only the visible things, but the invisible order of thrones, dominions, powers, and authorities, too; the whole world was created through him and for him; he was before everything; in him are all things held together (1).

Nor is the apostle's solution of the problem of the principles of all things incidental to his central concern: moreover, he is the head of the body, he is the church; he is the origin, he is the first to return from the dead, to be alone in all things supreme (*id.*).

In the generation after st Paul, Hero wrote the *Pneumatics*. And the analysis of the elements will not advance beyond Hero in the following fifteen and a half centuries.

In the *Pneumatics* Hero describes the construction of 79 machines worked by air or water, and the first machine worked by steam (II, 6).

He introduces the machines with a discussion of the principles. He writes: when force is applied to it, air is compressed and, contrary to its nature, moves into the vacant spaces because of the pressure applied to its particles; when the force is removed, it returns to its position because of the elasticity of its particles (I, intr.). Also: that something is consumed by the action of fire is shown by coal cinders which have the same volume after combustion as before, but differ much in weight; the consumed particles pass with the smoke into the substance of fire, or air, or earth; the subtlest particles pass into the highest region where fire is; the coarser particles pass into air, and the coarsest particles, borne at first with the others some distance, descend into the lower regions to mingle with the earthly substances; water also, consumed by the action of fire, passes into air; for the vapor from a cauldron placed on a fire is nothing but liquid passing into the air; it is evident, therefore, that fire dissolves

and transforms all bodies coarser than itself (*id.*).

Two generations after Hero, Ptolemy formulated the theory of the motions of the planets, catalogued the stars, and measured the world. The stars are at least 19 865 earth radii away.

This is where the first mover will be. And if in the lower regions of the world, those of the elements, it will have to move things with precision not greater than that demanded by the principles of pneumatics, in the higher regions, and all the way to itself, at the circumference, it will have to move them with the mathematical precision demanded by the Ptolemaic theory.

ccclx

In the first two and a half centuries of the development of philosophy the problem of the principles of all things was very much a central concern of the philosophers.

In the four and a half centuries from Aristotle to Ptolemy it was not. The advance was nevertheless very substantial. It was in these centuries that the philosophers began committing the solution of the problem to the true way.

The elements of all things.

Diodorus Cronus said that the elements are minute indivisible bodies. Epicurus said that they are atoms. Zeno of Citium said that earth, water, air, and fire are the elements. Asclepiades said that the elements are seamless particles.

The general results obtained earlier were thus reaffirmed. These are not an advance.

The advance is the collection of the particulars. The description of fires and of earths by Theophrastus is the first step toward observation. The advance continues with the beginning of the investigation of fluids by Ctesibius. The first water pump and other machines worked by air and water are the beginning of experiment. The advance continues with the development of the mathematical theory of fluids by Archimedes. Philo of Byzantium builds siphons and clepsydras. The beginning of alchemy with Bolus is the beginning also of chemical experiment. Hero builds the first machine worked by steam.

The collection of the particulars leads to substantial improvements in the general results with Theophrastus already. Earth becomes pure and uniform by confluence or filtering. Subjected

to heat and cold, this pure uniform matter hardens and forms stones and earths. If the sun is a form of fire, it is possible that the heat of our earthly fires arises from the action of the sun. If fire cannot persist without fuel, it seems foolish to speak of fire as an element.

And with Hero the general results rest on a mathematical demonstration, and have begun cohering into a theory.

That something is consumed by the action of fire is shown by coal cinders which have the same volume after combustion as before, but differ much in weight. The consumed particles pass with the smoke into the substance of fire, or air, or earth. The subtlest particles pass into the highest region where fire is, the coarser particles pass into air, and the coarsest particles descend into the lower regions to mingle with the earthly substances. Consumed by the action of fire, water also passes into air. Fire dissolves and transforms all bodies coarser than itself.

Also, when force is applied to it, air is compressed, when the force is removed, it returns to its position because of the elasticity of its particles.

In general, the most substantial advance is in that Theophrastus, Ctesibius, Bolus, and Hero have begun establishing which of all things, and how, will show that the elements of all things are the elements of all things. They have begun establishing the ground on which the problem of the elements will be solved in particular, and hence in general. Theophrastus explicitly, Ctesibius and Bolus implicitly, Hero most explicitly.

Earth becomes pure and uniform by confluence or filtering. Subjected to heat and cold, this pure uniform matter hardens and forms stones and earths. Confluence, filtering, subjugation to heat and cold, hardening and forming will show that the elements are the elements. Machines worked by air and water will show the properties of air and water. A preparation of white earth cast on copper with electrum added will give you gold. And fire dissolves and transforms all bodies. The dissolution and transformation of bodies will show that the elements are the elements. By fire. Measured by volume and weight, too.

Tabulating the results,
Theophrastus described fires and earths,

	purity is from confluence or filtering,
	forming is from heat and cold,
Diodorus Cronus –	minute indivisible bodies,
Epicurus	atoms,
Zeno of Citium	earth, water, air, and fire,
Ctesibius	machines worked by air and water,
Archimedes	the theory of floating bodies,
Philo of Byzantium –	machines worked by air and water,
Bolus	recipes for the preparations of substances,
Asclepiades	seamless particles,
Hero	a machine worked by steam, air is compressed because of the elasticity of its particles, fire transforms all bodies, consumption by fire is measured by volume and weight.

The causes of all things.

And rather the cause. That the cause is one had been established firmly.

Zeno of Citium said that the active principle is the inherent in matter law, or god, which is everlasting and the producer of everything.

It can have been an advance, had it been grounded in the particulars, or had it been at least more explicit. Zeno said also that nothing incorporeal is capable of action, everything that acts or is acted upon is body, all things are determined, determination is the chain of causes whereby things happen, or the law whereby the world develops. The inherent in matter law, or god, which is everlasting and the producer of everything, is body. Zeno is more radical than Democritus or Aristotle. But not nearly as explicit. Never mind the particulars.

Cleanthes hailed the lawgiving pilot of the world. Ignoring the madman's babbling.

The great advance was the theory of the motions of the heavenly bodies.

From Autolycus and Euclid through Aristarchus through Apollonius through Hipparchus through Ptolemy the theory of the motions of the heavenly bodies became by far, by very far, the most sophisticated theory.

That these are the particulars, the motions of the heavenly bodies, which will show that the

cause of all things is, and how it is, the cause of all things, Anaxagoras already, when he discovered that wood does not make beds, is clear. Plato is clear. Aristotle is clear. And the theory of the motions of the heavenly bodies shows, in chains of inferences as free of gaps as any, that the first locomover, just as Aristotle describes it, is, and how it is, the cause of all things.

Autolycus, Euclid, Aristarchus, Apollonius, and Hipparchus do not address the problem of the first cause explicitly. Ptolemy does. With reference to Aristotle, too.

He writes: Aristotle is right to divide theoretical philosophy into physics, mathematics, and theology; theology is concerned with the first cause of the motion of the world, which can only be conceived as an invisible and motionless god, somewhere at the end of the world; mathematics is concerned with form and locomotion; and mathematics can deal with the object of theology.

And mathematics has, in fact, dealt with the object of theology. The theory of the motions of the heavenly bodies has filled the world with locomotion, in chains of inferences as free of gaps as any, from the invisible and motionless god, somewhere at the end of the world, circular locomotion by circular locomotion by circular locomotion, down to the moon.

Hero, moreover, has taken a major step, explicitly, too, toward the reduction of the lower regions, those below the moon, and again just as Aristotle describes them, to greater particularity.

Thus the true way.

By that time the other way had begun reasserting itself as well.

Our lord's is the primacy over all created things. In him was everything in heaven and on earth created, and not only the visible things, but the invisible order of thrones, dominions, powers, and authorities, too. He was before everything. In him are all things held together.

Thrones, dominions, powers, and authorities are the middle ranks of the invisible order. Four and a half centuries later, a convert of the apostle's, Dionysius the Areopagite, will be brought to life to write the *Celestial hierarchy* and establish all the ranks in the order: seraphim, cherubim, thrones, dominions, powers, authorities, principalities, archangels, angels (7-9).

1190 In order to formulate the problems we are to solve we call the council of our predecessors

Ptolemy by volume and weight,
the theory of the motions of the
heavenly bodies.

And continuing the tabulation of the distribution of the philosophers,

425- 326	8	4,
325- 226	7	6,
225- 126	2	1,
125- 26	—	—,
25 ^{BC} - 75 ^{AD}	2	1,
76- 175	1	1.

The count in the last line is not complete.

ccclxi

The ten centuries from Ptolemy to Abelard.

These centuries do not have to be divided into parts. Not with respect to the problem of the principles of all things.

Revelation was at first confined to the god of the Hebrews, on one side, and the Hebrew prophets, on the other. Revelation then came out of Judea. St Paul, in the *Letter to the Galatians*: I must make it clear to you, my friends, that the gospel you hear me preach is no human invention; I did not take it from some man; no man taught it to me; I received it by revelation from Jesus Christ (1). The *Letter to the Galatians* is one of the genuine half of the letters in the apostle's corpus. Thereafter revelation spread far and wide.

In the century after Ptolemy, the revelations of Hermes Trismegistus were collected in the Hermetic corpus. This is the beginning of the first revelation of Hermes Trismegistus: once, when thought came to me of the things that are, and my thinking soared, and my senses were restrained, as in someone heavy with sleep from too much food or too much toil, an enormous being appeared to me, completely unbounded in shape; he called me by my name, and said, what do you wish to hear and see? I said, I wish to learn about the things that are, to understand their nature, and to know god; he said, I will teach you; and saying this, he changed in appearance, and in an instant all was clear to me; all became light (1).

What do you wish to hear and see? I will teach you.

And what you wished to hear and see, and revelation very generally, that of Trismegistus as

well as all the others, taught you, was that you will live forever. This much all revelation taught you. Some of it, however, including that of Trismegistus, taught you more, both that you will live forever and that you will make gold. This is alchemy.

As Roger Bacon, in the thirteenth Christian century, after a long and fruitful development, describes it in *The great work*: the same medicine which would remove all impurities and corruptions of baser metals so that they become the purest gold is believed by the philosophers capable of removing also the corruptions of the human body and prolonging life for many ages (VI, 12).

The theoretical foundation of alchemy was the theory of the four elements. And it taught you that the same medicine would remove both all impurities of baser metals and the corruptions of the human body, because the theory of the four elements was the common foundation of both alchemy and medicine.

As, one generation-odd after Ptolemy, Galen writes: animal bodies are a mixture of hot, cold, wet, and dry, just as all bodies are mixtures of fire, air, water, and earth.

Alchemy is the beginning of chemical experiment. For a long time, it is its very remote beginning, however. The practical urgency of its ends hampered development of the theoretical aspects of the means to those ends.

Nor is it an accident that the promise of eternal life, the making of gold, revelation, and writings from the hereafter grew simultaneously and greatly. Whether they grew greatly because they grew simultaneously, or grew simultaneously because they grew greatly.

Thus, if some of the recipes that can be recovered from the Democritean corpus, those ascribed to Bolus, can be presumed to anticipate experiment, there are no such recipes in the Hermetic corpus. Or in the other corpora of alchemical writings, as they continued to accumulate, for more than half a millennium after the Hermetic.

The revelations of Hermes Trismegistus were not the most important. Those of Jesus Christ were the most important.

The accumulation of Christian writing began with the letters of the apostle. In the following two generations the gospels were committed to

writing, and the first fathers of the church began writing letters. Then the fathers began accumulating, they began writing books, and their books began accumulating.

These are the important fathers in the not quite two centuries after the apostle through Origen,

- 43 - 102 Clement of Rome,
- 56 Ignatius,
- 58 Polycarp,
- 98 - 166 Justin Martyr,
- 121 Irenaeus,
- 150 - 214 Clement of Alexandria,
- 160 - 240 Tertullian,
- 170 - 235 Hippolytus,
- 181 Africanus,
- 185 - 254 Origen.

Not quite two centuries after st Paul, Origen is the father with whom philosophy took the decisive step toward becoming Christian. And for a long time remaining – other than that it became also Muslim – Christian.

Origen's most important treatise is *On the principles*. And all the principles on which philosophy was to stand – with the same qualification – in the following millennium are in *On the principles*.

On the principles is in four books.

In book I Origen treats of the father, the son, the holy spirit, and the rational creatures.

He writes:

god is spirit (1); god is incomprehensible and immeasurable (*id.*); god is the principle of all things (*id.*).

Further, however:

god was always the father of his only begotten son, who was born of him and draws his being from him, but is yet without beginning (2); wisdom, therefore, must be believed begotten beyond the limits of any beginning of which we can speak or which we can understand (*id.*); the son is the truth and the life of all things that are (*id.*); the son is the word, and therefore nothing in him is perceptible; he is wisdom, and in wisdom there is nothing corporeal (*id.*);

thus, god the father is omnipotent because he holds dominion over all things, and that is, over heaven and earth, the sun, the moon, and the stars, and everything contained in the world; this dominion, however, he exercises through his

word, for it is written, in the name of Jesus every knee bows, of the things in heaven, of the things on earth, and of the things under the earth; therefore, if every knee bows to Jesus, undoubtedly it is Jesus to whom all things have been subjected, it is he who wields dominion over all things, and all things have been subjected to the father through him; for it is by wisdom, and that is, by word and reason, not by force and necessity, that all things are subjected (*id.*).

Still further, however:

it is necessary to inquire also why salvation cannot be obtained without the entire trinity, why is it impossible to partake of the father and the son without the holy spirit (3); it is because the action of the father and the son reaches both saints and sinners, both rational creatures and dumb animals, it reaches even lifeless things, it is in absolutely everything that is; but the action of the holy spirit does not reach lifeless things, or things that are alive but dumb, or creatures who, though rational, live in wickedness; only in those who are already walking in the path of Jesus Christ is the action of the holy spirit to be found (*id.*).

Thus the father, the son, and the holy spirit. As to the rational creatures, they are demons, angels, and men. Origen explains:

god made all souls pure and obedient to god (8); but some of them sinned badly and became demons (*id.*); others sinned more lightly and became angels (*id.*); still others sinned not as badly as the demons but not as lightly as the angels either; and for these god made the world and bound them in bodies for punishment (*id.*).

These are the principles.

God is the principle of all things. This result of Origen's stands on results of his predecessors. Seven and a half centuries earlier, Xenophanes had said: one god, greatest among gods and men, shakes all things without toil. Six centuries earlier, Plato had said: the world has a maker; and why did he make the world? because he was good. Five centuries earlier, Cleanthes had said: most glorious of gods, all-powerful, lord of all change in nature, lawgiving pilot of the world, I hail you. One generation before Origen, reviewing what his predecessors have said about the principles of all things, Empiricus concludes: the

best physicists have said that the efficient principles of all things are the gods.

With all the following results, however – all most explicitly concerned with the principles of all things – Origen is breaking new ground. God was always the father of his only begotten son. The son is the truth and the life of all things. The son is the word, and the father holds dominion over all things, but this dominion he exercises through his word, for if every knee bows to Jesus, undoubtedly it is Jesus to whom all things have been subjected, it is he who wields dominion over all things, and all things have been subjected to the father through him. And the action of the father and the son is in absolutely everything that is, but in those who are already walking in the path of Jesus Christ the action of the holy spirit is also to be found.

With all these results Origen is breaking new ground. Yet the most fundamental result with which Origen is breaking new ground, and the result on which all the other results with which he is breaking new ground stand, is the comprehension of the incomprehensible. God is incomprehensible. Origen has yet much to say about god. His only begotten son must be believed begotten beyond the limits of any beginning of which we can speak or which we can understand. Origen has yet much to say about that beginning.

In book II Origen treats of the world and the creatures in it.

He has explained in book I already: some souls sinned not as badly as the demons but not as lightly as the angels either, and these god bound in bodies for punishment and made the world for them. Hence, the world and the creatures in it do not demand that we should inquire into the world and the creatures in it. They demand that:

we should now inquire whether after this world there will be healing and improvement, very severe and full of pain for those who have refused to obey the word of god, but an instruction and training for those who in the present life have devoted themselves to the word of god, and having become purer in spirit and attained the capacity for divine wisdom, may advance to a richer understanding of the truth (3);

for there is no doubt that the savior promises something more glorious and more splendid than

the present world, and exhorts all who believe in him to direct their lives toward it (*id.*).

The inquiry yields these results:

there is resurrection of the dead, and there is punishment; but the punishment is not forever; for when the body is punished, the soul is purified, and it is thus restored to its original state; the punishment of the wicked, and of the demons, too, thus has an end, the wicked and the demons will be restored to their original state as well (10).

That punishment is very severe and full of pain Origen's successors, and both Christians and Muslims, will agree. That punishment has an end they will disagree. They will prefer punishment very severe, full of pain, and forever.

In book III Origen treats of the struggle of the soul, bound in a body for punishment, helped by angels, hindered by demons, to choose salvation.

In book IV he treats of the divinity of the holy scriptures.

The principles are, or the principle is, the entire trinity, father, son, and holy spirit. The rational creatures are demons, angels, and men. The world demands the inquiry into the punishments and rewards after the world.

It all stands on the comprehension of the incomprehensible.

Which in turn stands, and could only stand, on a new principle of all principles.

One generation after st Paul, and before all the gospels had been committed to writing, Clement of Rome writes: god sent forth Jesus, Jesus gave the gospel to the apostles, and the apostles appointed bishops, to be succeeded by other bishops. Origen begins in *On the principles*: the teaching of the church is handed down in an unbroken succession from the apostles; it is preserved and continues to exist in the church to the present day; and we hold that this alone which in no way disagrees with the church and the apostles is to be believed as the truth (I, pr.).

And this explains why the transformation of philosophy into Christian philosophy was an incessant war against heresy, and for orthodoxy. For dominion over the church, hence over the unbroken succession from the apostles, and hence over what alone is to be believed as the truth.

Parmenides discovered the principle of contradiction. Aristotle established that the principle of

contradiction is the principle of all principles.

Origen writes: this alone which in no way disagrees with the church and the apostles is to be believed as the truth. His younger contemporary Cyprian writes: he cannot have god for his father who does not have the church for his mother. The father holds dominion over all things, the mother holds dominion over the father. God is the principle, the church is the principle of the principle.

Not quite two generations after Origen, Porphyry wrote the *Introduction*.

It is an introduction to the *Categories*. Plato's forms are prattle. As the trinity, father, son, and holy spirit, became the principles, or the principle, however, and demons and angels populated the world, logic, if it was logic, returned to the deeper questions which Plato's forms posed. And to do so, logic, if it was logic, reduced itself to concern not with the one book of the *Categories*, but with an *Introduction* to the one book of the *Categories*. Later on, it reduced itself to concern not with an *Introduction* to the one book of the *Categories*, but with a *Commentary* on an *Introduction* to the one book of the *Categories*.

Porphyry writes in the *Introduction*: in order to teach Aristotle's *Categories* it is necessary to know what genus, difference, species, property, and accident are; and since reflection on these is useful for definitions, and in general for analysis and demonstration, I will treat of them briefly by way of an introduction; I will abstain, however, from the deeper questions, and will say nothing as to whether genera and species exist or are mere thoughts, whether they are corporeal or incorporeal, and whether they are separate from the sensible things or are in the sensible things (1).

These are the deeper questions: whether genera and species exist or are mere thoughts, whether they are corporeal or incorporeal, and whether they are separate from the sensible things or are in the sensible things.

Porphyry abstains from the deeper questions, and treats briefly by way of an introduction of genus, species, difference, property, and accident.

To the following century belongs Zosimus of Panopolis, in whose name the next corpus of alchemy was authored.

It is the century of the triumph of Christianity. In 325 the first ecumenical council of bishops

gathered at Nicaea. The bishops decreed Christianity a dogma, the Nicene creed. In 391 Theodosius decreed the closure of all pagan temples and interdiction of all pagan cults. This is the generation also of Augustine, and his 93 treatises, in 254 books. The next is the generation of Vincent of Lérins, and the culmination of the development, which had begun not quite four centuries earlier with st Paul, in the doctrine: every care should be taken to hold fast to what is believed everywhere, always, by all.

Two centuries separate Origen and Vincent of Lérins. In these two centuries the church, at the hands of the fathers, triumphed. It conquered the empire. As to the problem of the principles of all things, nothing substantial was added to it after Origen. Nor could it be. For the same reason. Philosophy, at the hands of the fathers, became a weapon in the war against heresy, and for orthodoxy.

These are the important fathers in the two centuries after Origen,

200 - 258	Cyprian,
250 - 325	Lactantius,
251 - 356	st Antony,
265 - 339	Eusebius,
295 - 373	Athanasius,
330 - 379	Basil of Caesarea,
330 - 390	Gregory of Nazianzus,
335 - 394	Gregory of Nyssa,
339 - 397	Ambrose,
340 - 420	Jerome,
354 - 430	Augustine,
384	Orosius,
394	Vincent of Lérins.

Any of them can illustrate both, that nothing substantial was added and why it was not. Basil of Caesarea, for instance.

The principle is, or the principles are, the trinity, father, son, and holy spirit. The council at Nicaea decreed the trinity. Heresy after the council, as before, concerned the son, is he of the same rank as the father? and the holy spirit, is he of the same rank as the father and the son? Clearly, the latter was a graver transgression. Basil of Caesarea's was a decisive victory over that transgression.

He wrote *On the holy spirit*.

These are the heads of some of the chapters in

the treatise: *On the origin of the heretics' examination of words* (2); *Their logic and splitting of words are from the wisdom of the pagans* (3); *The holy scriptures do not use words in this way* (4); *Against those who say that the holy spirit does not rank with the father and the son* (10); *That those who deny the holy spirit are transgressors* (11); *Against those who say that baptism in the lord suffices* (12); *Against those who say that the holy spirit is to be counted not with the father and the son, but under them* (17); *Against those who say that the holy spirit is not glorified* (19).

Against all those:

form is in matter; and as the holy spirit is the perfection of the rational creature, the holy spirit is its form (26); as the power of seeing is in the healthy eye, so the activity of the spirit is in the purified soul (*id.*); as the art is in him who practices it, so the spirit is in him who receives it; it is always present, but it is not always acting; it is always present in those who are worthy, but it is acting only when there is a need, in prophecy, in healing, or in some other act of power (*id.*); moreover, as health, or heat, or other changeable conditions are in the body, so the spirit is in the soul, yet it does not remain in those who drive away the grace they have received; as reason is in the soul, and whether as a thought in the heart or as an utterance on the tongue, so is also the holy spirit (*id.*); moreover, the holy spirit is in creatures, but not in the father and the son; as we say that heat is in the iron that is on the fire, but heat is with the fire, and that health is in the body, but life is with the soul (*id.*).

Origen. Salvation cannot be obtained without the entire trinity. It is impossible to partake of the father and the son without the holy spirit. Because the action of the father and the son reaches both saints and sinners, both rational creatures and dumb animals, it reaches even lifeless things, it is in absolutely everything that is. But the action of the holy spirit does not reach lifeless things, or things that are alive but dumb, or creatures who, though rational, live in wickedness. Only in those who are already walking in the path of Jesus Christ is the action of the holy spirit to be found.

Basil of Caesarea. As the power of seeing is in the healthy eye, so the activity of the spirit is in the purified soul. As the art is in him who practices it, so the spirit is in him who receives it. It is always present in those who are worthy, but it is acting only when there is a need, in prophecy, in healing, or in some other act of power. As health is in the body, so the spirit is in the soul, yet it does not remain in those who drive away the grace they have received. As health is in the body, but life is with the soul, so the holy spirit is in creatures, but with the father and the son.

And this is how Basil of Caesarea concludes the treatise:

to what, then, will we compare the present condition of the church? truly, it is like a battle at sea, which men, lovers of war, fight as their anger with one another continues to grow; one side rushes terribly to attack the other, and in desperation of unrestrained anger they meet and fight; a violent storm drives them, clouds cover everything, no one sees who is friend and who is foe any more; torrential rains fall from the clouds, terrible waves rise; jealousy for preeminence and desire to rule drive them to slaughter mercilessly one another; in the terrible confusion the voices of the commander and the pilot are not heard; they despair of living; they give themselves completely and fearlessly to sin; add to this the incurable sickness of vainglory, and you have soldiers who will not stop fighting over who is first even as the ship is already sinking (30).

Jealousy for preeminence and desire to rule. Merciless slaughter. Add vainglory, and you have soldiers who will not stop fighting over who is first.

Counting from Vincent of Lérins to Abelard, Christendom remained in the darkest of dark ages for seven centuries.

A last flicker of light, if it is a flicker of light, is Boethius, not quite one century after Vincent of Lérins.

Boethius wrote two commentaries on the *Introduction* to the *Categories*. And he did not abstain from the deeper questions.

In the *Second commentary on Porphyry's Introduction*, Boethius writes: genera and species cannot exist; this is clear from the following considerations; every thing common to several

things at one and the same time cannot be one thing; what is common belongs to many things when one and the same thing is in many things as a whole at one and the same time; for no matter how many species there are, their genus is one thing in all of them, and not in that each species takes away a part of it, but in that each of the species has the whole genus at one and the same time; it follows that the genus posited as a whole in several species at one and the same time cannot be one thing; for it cannot be that, although it is a whole in several things at one and the same time, it is nevertheless in itself one in number; and if a genus cannot be one thing, then it is nothing at all; for every thing that is is because it is one thing; and the same about species (I, 10).

Genera and species are nothing at all. Nevertheless: genera and species are thoughts (11). Moreover: genera and species exist in particulars, but they are thought as universals (*id.*). Also: genus is the thought gathered from the likeness of the species; and it is sensible when it is in particulars, but becomes intelligible when it is in universals (*id.*).

The *Digest* was compiled at this time. It is the time also of the last important commentators, Simplicius, Eutocius.

The centuries of the darkness of greatest intensity followed.

As every care had been taken to hold fast to what is believed everywhere, always, by all, the predilection for revelation had also been restrained in Christendom.

It yet occurred once again on the fringes of Christendom, and as consequentially as it had on the road to Damascus. An angel came to Muhammad and told him, you are the messenger of god.

Islam came out of the desert, and one century-odd after the hijra the caliphate reached from the Atlantic in the west to the Indus in the east.

Muslim rule flourished, the Muslims settled, became versed in the crafts and sciences, and began to desire to study philosophy. So al-Mansur sent envoys to the Roman emperor to ask for books, and the emperor sent him back Euclid's book and some writings on physics.

Al-Mansur was caliph 754-75.

And in the following centuries the Muslims studied philosophy.

They studied mathematics and physics.

They established the importance of Aristotle. Early on, al-Kindi and al-Farabi did. It is with their labors that Aristotle became: the philosopher (*passim*), as he also remained with their successors in the following seven centuries, until Bacon, Galileo, and Kepler, and the instauration of the sciences.

They advanced practical medicine. Al-Razi and ibn Sina were the greatest doctors after Galen.

They also vigorously studied alchemy.

The largest corpus of alchemical writings was authored in the name of Jabir ibn Hayyan. Its authority was unrivaled, among the Muslim philosophers as well as, later on, among the Christian. Writings continued to be added to the corpus in the thirteenth and fourteenth Christian centuries.

The beginnings of the corpus of Jabir ibn Hayyan, and hence Jabir ibn Hayyan, can be dated to the century before al-Razi. His date will thus be somewhat earlier than that of al-Khwarizmi and al-Kindi, just after the time when the Muslims began to desire to study philosophy and al-Mansur sent envoys to the Roman emperor to ask for books.

Never mind making the purest gold, never mind prolonging life for many ages, the corpus of Jabir ibn Hayyan was equipped with recipes for making men. In the appropriately named *Book of assembly*, for instance.

On the other hand, it is with the writings of Jabir ibn Hayyan, one millennium after its first beginnings, that alchemy has taken a decisive step toward chemical experiment.

This is from the *Book of the properties of things*: to make red stone out of mercury; take a round glass and pour some mercury in it; take a Syrian clay pot and drop a little powdered yellow sulfur in it; put the glass in the pot, and fill the pot around the glass with sulfur to the brim; close the mouth of the pot, then keep it over a gentle fire for one night; take it out, and you will see that the mercury has turned into a red stone the color of blood; cinnabar is what the philosophers call this stone (*id.*).

Also, the doctrine that all metals are composed of mercury and sulfur begins with Jabir ibn

Hayyan. This is from the *Book of explanations*: all metals are composed of mercury and sulfur; they only differ in their accidental properties; the differences are due to the quantity of sulfur in them, and this in turn is due to differences in the soils and the heat of the sun (*id.*).

Three generations after al-Mansur, al-Kindi is the first Muslim philosopher steeped in Aristotle. He is thus also the first to concern himself with the principles of all things.

In the *Explanation of the proximate efficient cause of generation and corruption*, al-Kindi writes: natural causes are material, formal, efficient, and final; material is the matter of the thing, like the gold of the coin; formal is the form of the coin, by which it is made one with the gold; efficient is the craftsman, who makes the form one with the gold; final is that for the sake of which the craftsman makes the form one with the gold, and that is, to gain something by the coin (8); if there were no matter, nothing would be generated or corrupted, for generation and corruption have to occur in something; as for the formal cause, a thing is generated because its form becomes one with its matter (9); as for the efficient cause, it is only when we find this cause that we find also the final; for the final either is above the efficient or is itself the efficient; if the efficient does not exist, the final cannot exist (10); the efficient cause is either proximate or remote; the one who shoots the arrow is the remote, the arrow that kills the animal is the proximate; and the remote cause of everything generated and corrupted, everything sensible and intelligible, is the first cause, and that is, god, praise be to him! the one who creates and completes the world, the cause of all causes, the one who creates everything (11).

He is the one who shoots the arrow. He is the one who makes the form one with the gold to gain something by the coin. Praise be to him!

Two generations after al-Kindi, al-Razi is the high point in the development of alchemy.

This is from the *Book of secrets*: take equal parts of alkali and lime and pour 4 times as much water over them; leave the mixture for 3 days; filter the mixture, then add one fourth as much alkali and lime; do this 7 times; pour the mixture into one half the volume of dissolved sal ammo-

niac; then keep it; for it is the strongest sharp water, it will dissolve mica instantly (III, 7).

With al-Razi alchemy has at long last discovered chemical experiment.

In the *Doubts concerning Galen*, al-Razi writes: all bodies are subject to generation and corruption, but the deterioration which will not happen to gold, gems, and glass in one hundred thousand days will happen to vegetables, fruits, and spices in a single day; the ratio of substance in the heavenly sphere to substance in gems is equal to the ratio of substance in gems to substance in vegetables; the corruption of the heavenly sphere can thus go on for thousands of years, and it may be perceptible only after a period so long that even the observations of one people cannot pass it to another as they can pass the observations of floods or plagues; and if the substance of the sun increased or decreased by as much as the greatest mountain, we would not be able to observe it (5).

All bodies are subject to corruption, including the heavenly sphere. Al-Razi draws this conclusion from the investigation of substances, gold, gems, vegetables, fruits, spices.

In the same generation, al-Farabi draws a different conclusion about the heavenly sphere. His is from the investigation of the first mover.

In *On the intellect* al-Farabi is expounding: master Aristotle (1). He writes: the heavenly bodies are the first efficient causes of all generated and corrupted bodies (25); every heavenly body is moved by a mover which is neither a body nor in any way in a body; this mover is the cause of the heavenly body; and the mover of the more perfect heavenly body is more perfect; so the most perfect is the mover of the first heaven; and the first mover of the first heaven is the principle by which two things exist; one is the substance of the first heaven, which is a body, the other is the mover of the sphere of the fixed stars, which is itself neither a body nor in a body; it thus has two natures, one of which is more perfect than the other (26); and since the mover of the first heaven is neither matter nor in matter, it follows that it is intellect, it thinks itself, and it is the principle of its own existence; clearly, of its two natures the nature of thinking itself is the more perfect (*id.*); the principle of the mover of

the first heaven is one in every way; it is absolutely impossible that there be anything more perfect than itself; it is the principle of all principles; this is the principle that Aristotle describes in book XII of the *Metaphysics* (27).

In the one and a half centuries after al-Mansur philosophy flourished in Islam. In the following two centuries there are fewer philosophers.

One century-odd after al-Razi, ibn Sina wrote the *Canon*. It is the most substantial contribution to therapy after Galen.

In *On healing*, ibn Sina writes: minerals can be divided into four kinds, stones, fusible substances, sulfurs, and salts; for some are weak, others are strong; of the strong, some are malleable, others are not; of the weak, some are salty and easily dissolved, like alum, vitriol, and sal ammoniac, others are oily and not easily dissolved, like sulfur and arsenic (II, 5.4).

Weak and strong, malleable and not, easily dissolved and not. It is the first step toward the division of substances according to their reactions with one another.

Ibn Sina writes also: as to the claims of the alchemists, it must be understood that it is not in their power to bring about any true change of species, even if they can produce excellent imitations, dyeing red metal white so that it looks like silver, for instance, or dyeing it yellow so that it looks like gold or copper (*id.*). On the other hand, these are some of the consequences ibn Sina draws from the doctrine that all metals are composed of mercury and sulfur: if pure mercury is mixed with white sulfur, which is not impure and does not induce combustion, the product is silver; if good mercury is solidified with sulfur, which is impure and induces combustion, the product is copper; if the mercury is corrupt and impure, and the sulfur is also impure, the product is iron; if the mercury is good, but the sulfur is corrupt, the product is tin (*id.*).

Not quite three generations after ibn Sina, al-Ghazali established that the errors of the philosophers are of twenty kinds, and on account of three of these they must be pronounced infidels, on account of the other seventeen they must be pronounced heretics.

Fortunately, or unfortunately, at that time the Christian dark age came to an end. The genera-

tion after al-Ghazali's is that of Abelard.

Porphyry had abstained from the three deeper questions. Boethius had answered them. In the *Logic for beginners, glosses on Porphyry*, Abelard writes: we add a fourth question, do genera and species, when they name, necessarily have a thing subject to themselves? or the universal is only what the intellect means, and thus even if the things named are destroyed? for instance, the name rose, even if there are no roses of which it is the name (*id.*)?

Hence, according as they answered the fourth deeper question, the philosophers divided themselves into two kinds, nominalists and realists. Into those, beginning with Abelard's first teacher, Roscelin, who said that the universals are names, and those, beginning with Abelard's second teacher, William of Champeaux, who said that the universals are things. Himself, Abelard was a nominalist. Later on, Albert and Aquinas were realists, Occam and Buridan were nominalists.

Very much more important, the importance of Abelard in bringing the darkest of dark ages to an end cannot be overstated.

He restored the principle of all principles, the one Aristotle had established is the principle of all principles, to its authority above that of god.

He writes in the *Theology*: god can do and do not only what he does and does not, and only in the way and at the time in which and at which he does it and does it not, and no other.

Even if: few or no one will agree with our conclusion, which seems to disagree with what many saints have said as well as with reason.

Even more important, Abelard showed that the two principles of all principles, the one Aristotle had established is the principle of all principles and the one the fathers of the church had established is the principle of all principles, do not exactly corroborate each other.

Abelard's most important treatise is the *Yes and no*. The *Yes and no* is a collection of 158 questions, and the answers of the fathers of the church, yes and no, both, to each of the 158 questions. The first question, for instance, is: must faith be completed by reason, or not (*id.*)? And Abelard collects 30 answers to the question, yes and no, both, from, in chronological order: Hilary, Ambrose, Jerome, Jerome, Augustine,

The *Decretals* decree: the forbidden books are condemned unto all eternity; those who read them are damned forever by an immovable anathema (*id.*); and anyone who disagrees with an iota of the approved books, or does not accept them with all veneration, is cursed (*id.*).

In Jabir ibn Hayyan the recipes for the preparation of substances are substantially more ex-

Among the Muslim philosophers, results in geometry and algebra accumulated. Results in astronomy accumulated. Results in alchemy accumulated. The formulation and solution of the problem of the efficient principles did not ad-

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vance beyond al-Kindi and al-Farabi. *Mutatis mutandis*, for the same reason among the Muslim philosophers as among the Christian.

Abelard pointed out that the incomprehensible is incomprehensible.

Tabulating, including the fathers who explicitly reaffirmed and reaffirmed and reaffirmed Origen's solution of the problem,

Origen	the trinity is the principle,
Cyprian	the trinity,
Lactantius	the trinity,
Eusebius	the trinity,
Athanasius	the trinity,
Basil of Caesarea	– the trinity,
Gregory of Nazianzus	– the trinity,
Gregory of Nyssa	– the trinity,
Ambrose	the trinity,
Jerome	the trinity,
Augustine	the trinity,
Vincent of Lérins	– the trinity,
al-Kindi	the first cause is god, praise be to him!
al-Farabi	the mover of the first heaven is the principle of all principles,
Abelard	god can do only what he does only in the way in which he does it.

The formal principles of all things.

Nor could there be an advance in this part of the problem.

Logic had been reduced to rhetoric and grammar. It was now reduced to the deeper questions posed by Plato's forms.

Tabulating,

Porphyry	do genera and species exist or they are mere thoughts?
Boethius	genera and species are nothing at all, they are thoughts,
Abelard	is the universal only what the intellect means?

So, in their series and order these 21 are all important philosophers who are part of the development of the problem of the principles of all things in the ten centuries from Ptolemy to Abelard,

120	Hermetic corpus,
185 - 254	Origen ,
200 - 258	Cyprian,
234 - 308	Porphyry,

250 - 325	Lactantius,
265 - 339	Eusebius ,
295 - 373	Athanasius ,
330 - 379	Basil of Caesarea,
330 - 390	Gregory of Nazianzus,
335 - 394	Gregory of Nyssa,
339 - 397	Ambrose,
340 - 420	Jerome,
354 - 430	Augustine ,
394	Vincent of Lérins ,
480 - 524	Boethius,
770	Jabir ibn Hayyan ,
801 - 866	al-Kindi ,
860 - 930	al-Razi ,
865 - 950	al-Farabi ,
980 - 1037	ibn Sina ,
1079 - 1142	Abelard .

The steps of advance in the acquisition of knowledge are the 3, Jabir ibn Hayyan, al-Razi, and ibn Sina. In particular,

Jabir ibn Hayyan	– explicit recipes for the preparation of substances, metals are composed of mercury and sulfur,
al-Razi	recipes describing experiments,
ibn Sina	minerals are weak and strong, easily dissolved and not.

Continuing also the tabulation of the distribution of the philosophers,

76- 175	2	1,
176- 275	5	–,
276- 375	7	–,
376- 475	1	–,
476- 575	1	–,
576- 675	–	–,
676- 775	1	1,
776- 875	3	1,
876- 975	–	–,
976-1075	1	1,
1076-1175	1	–.

The count in the last line is not complete.

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The not quite five centuries from Abelard to Bacon.

They can be divided into two parts. The three-odd centuries from Abelard to prince Henry. And the one and a half-odd centuries from prince Henry to Bacon. The dates are,

1079 - 1142 Abelard,
 1394 - 1460 prince Henry,
 1561 - 1626 Bacon.

In the three-odd centuries from Abelard to prince Henry philosophy was produced in the schools. In the one and a half-odd centuries from prince Henry to Bacon it was produced increasingly outside the schools and against them. And in these one and a half-odd centuries the acquisition of knowledge at long last accelerated, and in the second half of these one and a half-odd centuries accelerated very greatly.

The generation after Abelard's is the generation of Gerard. And the importance of Gerard in bringing the darkest of dark ages to an end cannot be overstated either. Gerard went to Toledo, learned Arabic, and translated 71 books, Aristotle, Autolycus, Euclid, Archimedes, Diocles, Menelaus, Ptolemy, Galen, Jabir ibn Hayyan, al-Farghani, al-Khwarizmi, al-Kindi, banu Musa, Thabit ibn Qurra, al-Razi, al-Farabi, ibn Sina.

It is the generation also of ibn Rushd.

Two generations earlier, al-Ghazali had written *The incoherence of the philosophers*. As he summarizes his results: in *The incoherence of the philosophers* I have established that the errors of the philosophers are of twenty kinds, and on account of three of these they must be pronounced infidels, on account of the other seventeen they must be pronounced heretics.

Ibn Rushd wrote *The incoherence of the incoherence*. He refutes al-Ghazali point by point, error by error.

Ibn Rushd's affirmative and general argument is in *The decisive treatise*.

He begins:

thus says the lawyer, imam, judge, and true scholar ibn Rushd;

praise be to god, and a prayer for Muhammad his chosen servant and apostle;

the goal of this treatise is to examine whether the law allows the study of philosophy, or prohibits it, or commands it, and this by way of a recommendation or by way of an obligation (1).

The argument is in three steps:

1. the law commands the study of philosophy by way of an obligation (2-17);

2. philosophy contains nothing opposed to Islam (18-52);

3. the philosophical interpretation of scripture should not be taught to the many, the law provides other ways of teaching them (53-75).

Among the evidence in support of the first step: that the law summons us to reflection and the pursuit of knowledge is clear from many verses in the book, blessed and exalted; for instance, have they not studied the kingdom of heaven and earth, and whatever god has created (3)? this is why the prophet, peace be on him, when he ordered a man to give honey to his brother for diarrhea, and the diarrhea increased, and the man complained, the prophet said, god spoke the truth, your brother's stomach lied (14). In support of the second step: truth does not oppose truth (18); and an error committed by a true judge is excused by god, an error committed by someone who is not a true judge is not excused (37); this is why the prophet, peace be on him, said, if a judge after exerting himself makes a right decision, he has two rewards, if he makes a wrong decision, he still has one reward (38).

Not that reward. The judgment of the other lawyer, imam, judge, and true scholar prevailed. The twenty errors of the philosophers are not excused by god.

In Christendom, one generation after Gerard, the philosophers discovered that not only the fathers have solved the problem of the principles of all things. Aristotle has also solved it.

Amalric said that god is the formal cause of all things.

David of Dinant said that god is the primary matter.

Two generation after Gerard, the philosophers discovered physics.

It is the generation of Albert. With Albert, having for a millennium described little but punishments and rewards after the world, philosophy returned to describing the creatures in the world.

Albert wrote the 26 books of *On animals* and 7 books of *On plants*.

He wrote also *On minerals*, in 5 books.

In *On minerals* Albert divides minerals into three kinds, stones, treated in books I-II, metals, treated in books III-IV, and intermediate, treated in book V. He writes: stones are mixtures of earth and water (I, 1.2). His authority on this is: Aris-

tote (*id.*). Also: metals are mixtures of mercury and sulfur, which are themselves mixtures of all four elements (III, 1.4). His authorities on this are: ibn Sina and Hermes Trismegistus (*id.*). In book II Albert deals with precious stones, dividing them into: those beginning with A (2.1), those beginning with B (2.2), and all the way to: those beginning with Z (2.20). In book IV he deals with metals, dividing them, more felicitously, into: mercury (2), lead (3), tin (4), silver (5), copper (6), gold (7), and iron (8).

Albert wrote also *On alchemy*. He argues: against ibn Sina, with Trismegistus, the transmutation of baser metals into gold is possible (3).

Albert also defines the operations of alchemy: sublimation is the volatilization of a dry substance by fire to make it cling to the sides of a vessel (30); calcination is the pulverization of a substance by fire to remove the moisture uniting its parts (31); coagulation is the reduction of a liquid substance to a solid to deprive it of its vapors (32); fixation is the tempering of a volatile substance by fire to fix its properties (33); solution is the resolution of a calcined substance into water to remove contamination (34); distillation is the rising of the vapors of a liquid substance to purify it (35).

The thirteenth Christian century is the century of the great growth of the schools, from Oxford to Salamanca to Bologna, with the greatest of them all at Paris.

Philosophy came to be produced on the model: question, for the affirmative, for the negative, distinction, conclusion. Both the greatest schoolmen, for instance, Aquinas and Occam, follow the model religiously. And philosophy had to be produced on this model.

With all due respect to the principle of all principles, the one Aristotle had established is the principle of all principles, the other principle of all principles, the one the fathers of the church had established is the principle of all principles, continued to be the principle of all principles. This principle of all principles demanded that philosophy ground itself in the fathers of the church. The fathers of the church, however, as Abelard had not too subtly shown, had answered all too many questions in the affirmative and in the negative, both. Hence the second and the third

parts of the model, for the affirmative and for the negative. Hence also the crucial part of the model was the fourth, distinction. In this part the contradictions were resolved. The means of resolving them was, as Erasmus describes it: to render the subtlest of subtleties still more subtle. And to hone this means, the philosophers discovered logic.

Albert wrote logic, Aquinas wrote logic, Henry of Ghent wrote logic, Giles of Rome wrote logic, Scotus wrote logic, Buridan wrote logic, Heytesbury wrote logic.

If it was logic. The high points in the development of scholastic logic are the doctrine of supposition and the doctrine of consequences.

Albert's younger contemporary William of Sherwood wrote the *Introduction to logic*, the first treatise informed by all of Aristotle's logical treatises. It is in 6 books, on the proposition, on the predicables, on the syllogism, on the dialectical topics, on the fallacies, and on the properties of terms. In book VI, on the properties of terms, the foundations of the doctrine of supposition are laid. The foundations of the doctrine of consequences are laid in book III, on the syllogism. In book III as well the mnemonic *barbara, celarent, darii, ferio*, appears for the first time. William of Sherwood wrote also the *Syncategorematics*, the first treatise on syncategorematic terms, some, all, is, not, if, only.

In the same generation, John XXI – as he became in 1276, Peter of Spain before that – wrote the *Little summa of logic*. It remained the textbook of logic in the following couple of centuries. In the *Little summa* the doctrine of supposition is developed further. John XXI writes: term is a word which signifies the universal or the singular, for instance, man or Socrates; the signification of a term is thus the representation of a thing by a word (7); and the supposition differs from the signification; the signification of a term exists by the imposition of a word onto a thing to signify it, the supposition of a term is its acceptance to stand for something; for instance, when one says, man runs, the term man stands for Socrates, and Plato, and the rest of men (*id.*).

In the same generation, Roger Bacon wrote *The great work*.

The great work is in seven parts. Part I is *On*

the causes of error. Part II is *On philosophy*. Part III is *On the study of languages*. Part IV is *On mathematics*. Part V is *On optical science*. Part VI is *On experimental science*. Part VII is *On moral philosophy*. Part IV, *On mathematics*, comprises more than one third of the treatise.

Roger Bacon obtained no results of great importance in either mathematics or experimental science. His most important results are in the investigation of the calendar, including the argument that the Julian calendar is wrong by 1 day in 125 years (III, 16), and in the investigation of light, including the argument that the visual ray travels with finite speed (V, 9.3). And they are not very important. Four centuries earlier already Thabit ibn Qurra had corrected the calendar better. In the investigation of light Grosseteste in the generation before Roger Bacon and Witelo in the generation after him obtained more and more important results. The great importance of Roger Bacon is in his generality.

He begins in *The great work*: the thorough investigation of knowledge consists in two things, the perception of what is necessary to obtain knowledge, and then the method to apply what is necessary to all matters (I, 1).

He is the first philosopher after Ptolemy, and the only one until Galileo, to argue that mathematics and experience, both, are the necessary foundation of all acquisition of knowledge.

He writes: the chief obstacles to grasping truth are four, the submission to false and unworthy authority, the influence of custom, vulgar prejudices, and the concealment of one's ignorance behind an ostentatious display of one's knowledge (*id.*); from these deadly banes all evils of the human race come; for the most useful, greatest, and most beautiful knowledge and all secrets of the sciences and arts remain unknown (*id.*).

The cure is mathematics and experience, both.

Roger Bacon writes: mathematics, which the saints discovered at the beginning of the world, is the gate and key to all sciences (IV, 1); who is ignorant of this can know neither the other sciences nor the affairs of the world; nor does he know his own ignorance, nor does he know to seek a cure (*id.*); every science requires mathematics because, first, in the other sciences mathematical examples make the matter clear (3);

second, mathematical truths are innate (*id.*); third, of all parts of philosophy mathematics was discovered first (*id.*); fourth, the natural path is from easy to difficult, and this science is the easiest (*id.*); fifth, we see that even the most ignorant priests, unable to attain the other sciences, grasp mathematical truths (*id.*); sixth, the natural path is to begin with what befits children, children are first taught to sing, and in the same way they can be taught figures and numbers (*id.*); seventh, the natural path is from the simple to us to the simple by nature, and in mathematics alone the same things are simple to us and by nature (*id.*); eighth, mathematics replaces doubt with certainty, it clears away every error and replaces it with unshakable truth (*id.*).

The encomium of experience is less comprehensive but not less exalted: without experience nothing can be known sufficiently (VI, 1); and experimental science is wholly unknown to the run of scholars (2); but it has three great prerogatives over the other sciences (*id.*); first, it proves their conclusions (*id.*); second, this mistress of the sciences alone is capable of discovering truths beyond the bounds of the other sciences (13); third, it uncovers by its own power the secrets of nature, and this twofold, it gives knowledge of the future, the past, and the present, and it gives knowledge of marvelous works beyond the knowledge of the heaven (14).

Who is ignorant of this can know neither the other sciences nor the affairs of the world. Nor does he know his own ignorance, nor does he know to seek a cure.

In 1278 Roger Bacon was accused of novelty, tried by his order – the Franciscan, just reformed by Bonaventure – found guilty, and confined to his cell.

It was in the previous year that the principle of all principles unconditionally reasserted itself.

In 1210, a council under the archbishop of Sens decreed: neither the books of Aristotle on natural philosophy nor their commentaries shall be read in Paris in public or in secret, this we forbid on pain of excommunication.

In 1215, on the authority of Innocent III rules for the university of Paris were laid down. Among them: they shall not read the books of Aristotle on metaphysics and on natural philoso-

phy nor commentaries on them.

In 1231, Gregory IX issued the bull *Mother of the sciences*. Among its provisions: the books on natural philosophy prohibited in provincial council for due cause we order examined and purged from all suspicion of error.

In 1277, under the logician, John XXI, and on his orders, the most sweeping condemnation of: obvious and abominable errors (condemnation, 7 Mar 1277) was issued. Stephen, bishop of Paris, writes: lest dangerous discourse drew the innocent into error, we strictly forbid and on pain of excommunication wholly condemn all those who shall have taught, or shall have defended, or shall have listened to, any of the following errors (*id.*). A list of 219 obvious and abominable errors follows.

Where the principle of all things is concerned, master Amalric said that god is the formal principle of all things. The principle of all principles decreed: the body of master Amalric is to be removed from the cemetery and cast into unconsecrated ground. David of Dinant said that god is the primary matter. The principle of all principles decreed: the books of David of Dinant are to be brought to the bishop of Paris and burned.

Aquinas is a younger contemporary of Roger Bacon.

To note only some of the more important doctors of theology, before Aquinas on god wrote Peter Damian, Anselm, William of Champeaux, Abelard, Bernard of Chartres, Gilbert of Poitiers, Clairvaux, Hugh of St Victor, Lombard, Joachim of Fiore, Grosseteste, William of Auvergne, Vincent of Beauvais, Albert. Aquinas's contemporaries Bonaventure, Roger Bacon, and Siger wrote on god. After Aquinas on god wrote Lull, Boethius of Dacia, Henry of Ghent, Giles of Rome, Eckhart, Scotus, Occam.

Aquinas wrote the book on god.

The *Theological summa* is in three parts: we shall treat of god, of the rational creature's advance toward god, and of Christ who, as man, is our way to god (I, 2). In all the three parts Aquinas asks and answers 512 general questions, divided into 2 669 particular questions.

The treatment of god is also in three parts: we shall consider whatever concerns the divine essence, whatever concerns the distinctions of

persons, and whatever concerns the procession of creatures from him (*id.*). In these three parts Aquinas asks and answers 118 general questions, divided into 574 particular questions. The questions concerning the divine essence are 25 general, divided into 149 particular (2-26), those concerning the distinctions of persons are 17 general, divided into 74 particular (27-43), those concerning the procession of creatures from him are 76 general, divided into 351 particular (44-119).

Concerning the divine essence the main general questions are: the existence of god (2), the simplicity of god (3), the perfection of god (4), the goodness of god (6), the infinity of god (7), the immutability of god (9), the eternity of god (10), the unity of god (11), the life of god (18), the will of god (19), the justice and mercy of god (21), the providence of god (22), the power of god (25).

The general question of the existence of god poses 3 particular questions. The first is: whether the existence of god is self-evident (2.1)? The answer is: the proposition, god exists, is not self-evident to us, it needs to be demonstrated (*id.*). The second is thus: whether it can be demonstrated that god exists (2.2)? The answer is: yes (*id.*). And the third is thus:

whether god exists (2.3)?

and the answer is:

the existence of god can be proved in five ways;

the first and most manifest way is from motion; a first mover is necessary; and this everyone understands to be god;

the second is from the nature of efficient cause; it is necessary that there be a first efficient cause; and this everyone calls god;

the third is from possibility and necessity; all beings cannot be merely possible, there must be something necessary; and this all men call god;

the fourth is from gradation; there must be a being which is of all beings the cause of their being, goodness, and every other perfection; and this we call god;

the fifth is from the government of the world; there must be an intelligent being directing all natural beings to their end; and this being we call god (*id.*).

The general question of the power of god poses 6 particular questions. The third is:

whether god is omnipotent (25.3)?

The answer is:

all confess that god is omnipotent;

yet it is difficult to explain in what precisely his omnipotence consists, for there may be doubt as to what, when we say that god can do all things, all means; but if we consider the matter carefully, power is with respect to possible things, and the proposition, god can do all things, is correctly understood to mean that god can do all things which are possible;

all things which do not imply a contradiction, therefore, are among all possible things these with respect to which god is called omnipotent; while whatever implies a contradiction does not come within the scope of his omnipotence because it cannot have the character of possibility; it is thus better to say that such things cannot be done, not that god cannot do them (*id.*).

Nevertheless, the fifth question is:

whether god can do what he does not (25.5)?

and the answer is:

yes, god can do what he does not (*id.*).

And the sixth question is:

whether god can do better than what he does (25.6)?

and the answer is:

yes, he can do better than what he does (*id.*).

Concerning the distinctions of persons the main general questions are: the procession of the divine persons (27), the plurality of the persons (30), what belongs to the unity and what belongs to the plurality in god (31), the person of the father (33), the person of the son (34), the person of the holy spirit (36).

The second particular question which the general question of what belongs to the unity and what belongs to the plurality in god poses is:

whether the son is different from the father (31.2)?

The answer is:

as Jerome writes, a heresy arises from words wrongly used; when we speak of the trinity, therefore, we must proceed with caution and befitting modesty;

we must beware of two opposite errors, namely, that of Arius, who made up a trinity of

substance next to the trinity of persons, and that of Sabellius, who made up a unity of persons next to the unity of essence;

to avoid the error of Arius, we must shun the words diversity and difference in god, lest we impair the unity of essence;

to avoid the heresy of Sabellius, we must shun the word singularity in god, lest we impair the communication of the divine essence (*id.*).

The consideration of whatever concerns the procession of creatures from him Aquinas begins with the general question of:

the first cause of all things (44).

The particular questions are 4:

whether god is the efficient cause of all beings?

whether the primary matter is created by god?

whether god is the exemplar cause of beings?

whether he is the final cause of things (*id.*)?

The answer is in each case: yes (*id.*). Earlier, when considering the simplicity of god, Aquinas has reported: Amalric said that god is the formal cause of all things; David of Dinant most absurdly said that god is the primary matter (3.8). As Jerome writes, a heresy arises from words wrongly used. Therefore, god is the efficient cause of all beings. And he is the final cause of things. But he is not the primary matter, he created the primary matter. Nor is he the formal cause of beings, he is the exemplar cause of beings.

Aquinas then considers the procession of creatures from him according to their three kinds: first, the purely spiritual creatures, which the holy scripture calls angels, second, the wholly corporeal creatures, and third, the composite creature, both corporeal and spiritual, namely man (50). The purely spiritual creatures pose 15 general questions (50-64), the wholly corporeal creatures pose 10 general questions (65-74), the composite creature poses 28 general questions (75-102). Aquinas then considers the divine government of things (103-19).

This concludes the treatment of god.

And the treatment of god is then completed by the treatment of the rational creature's advance toward god, in 303 general questions, 1 536 particular, and the treatment of Christ who, as man, is our way to god, in 90 general questions,

549 particular.

Still in the same generation, that of Roger Bacon and Aquinas, Peter Peregrinus wrote the *Letter on the magnet*.

Peter Peregrinus begins: I will tell you about the hidden virtues of the lodestone, because the philosophers have told us nothing about them, and it is so with good things, they remain hidden in darkness until they are brought to light for the use of men (I, 1).

It is so with good things, they remain hidden in darkness until they are brought to light. And it is with the *Letter on the magnet* that the investigation of the magnet begins.

The *Letter on the magnet* is in two parts. In the first part Peter Peregrinus describes the hidden virtues of the lodestone: it has poles, pointing at the poles of the world, north and south (5); and iron touched by it also turns toward the poles of the world (7); it also attracts iron (8). In the second part Peter Peregrinus describes three instruments for the use of men. One is: a wheel in perpetual motion (3). The other two are compasses. One of these is the first: azimuthal compass (2).

The azimuthal compass allowed azimuthal readings of the sun, the planets, and the stars, hence the determination of the position of the point of observation at sea as well as on land, and developed in the following couple of generations into the mariner's compass, was put to great use by men.

Two generations after Peter Peregrinus, Occam wrote the *Summa of all logic*.

The *Summa of all logic* is in three parts. Part I is *On terms*, in 77 chapters. Part II is *On propositions*, in 37 chapters. Part III is *On syllogisms*, in 173 chapters, and is itself divided into four parts. Part III-I is *On syllogisms*, in 68 chapters, part III-II is *On demonstration*, in 41 chapters, part III-III is *On consequences*, in 46 chapters, part III-IV is *On fallacies*, in 18 chapters.

In part I, *On terms*, Occam develops the doctrine of supposition to its highest point. He writes: supposition is a property of a term, and only when it is employed in a proposition (62); thus, if a terms stands in a proposition for a thing, and is true of the thing, it is the supposition of the thing (*id.*); the supposition is personal, or simple,

or material (63); the supposition is personal when a term stands for what it signifies, for instance, the term man in, man is an animal (*id.*); the supposition is simple when a term stands for a mental content, for instance, the term man in, man is a species (*id.*); the supposition is material when a term stands for a sound or a writing, for instance, the term man in, man is a noun (*id.*).

In part III-III, *On consequences* – itself a part of part III, *On syllogisms* – Occam develops the doctrine of consequences to its highest point. He writes:

there are many general rules of consequence;
one is, from truth falsehood never follows;
thus,

the antecedent is true,
therefore the consequent is false,

is not valid;

another is, from falsehood truth can follow;
thus,

the antecedent is false,
therefore the consequent is false,

is not valid either;

another is, from the opposite of the consequent
the opposite of the antecedent follows; thus,

all men are white,
Socrates is not while,
therefore Socrates is not a man;

is valid;

another is, what follows from the consequent
follows from the antecedent; thus, if,

all animals run,
therefore all men run,

is valid, then what follows from,

all men run,

follows from,

all animals run;

from this rule another follows, what is antecedent to the antecedent is antecedent to the consequent;

but the following is false, what follows from the antecedent follows from the consequent;

similarly, this is also false, what is antecedent to the consequent is antecedent to the antecedent;
and from these rules others follow;

what is consistent with the antecedent is consistent with the consequent; thus,

some asses do not run,

is consistent with,

all men run,
 but not with,
 all animals run;
 another rule is, what is inconsistent with the
 consequent is inconsistent with the antecedent;
 another is, from what is necessary what is
 contingent does not follow;
 another is, from what is possible what is
 impossible does not follow;
 and there are other rules, too, from what is
 impossible anything follows, and what is neces-
 sary follows from anything; thus,
 you are an ass,
 therefore you are a god;
 and,
 you are white,
 therefore god is three and one;
 but these are not formal and should not be used
 much, nor are they indeed used much (36).

Occam still, like all his predecessors in the
 fifteen and a half centuries since Chrysippus and
 all his successors in the three and a half centuries
 until Leibniz, does not distinguish clearly the
 terms in a proposition from words. Or proposi-
 tions from sentences. Or inferences from conse-
 quences. The predicate calculus of syllogistic is
 taken to be a propositional calculus.

Nevertheless, as the very division and order of
 the *Summa* shows, and Occam's development of
 the doctrine of consequences shows, with Occam
 logic has taken a step toward becoming again, as
 it had not been since Chrysippus and was not to
 be until Leibniz, the formal investigation of form.

Occam also not only discovered Occam's
 razor. He vigorously wielded it. You may object
 that producing infinitely many things is not
 repugnant to god. I reply, it is repugnant.

With Albert philosophy returned to describing
 the creatures in the world. But not too vigorously.

More vigorously the philosophers continued to
 search for the philosophers' stone. Albert wrote
 alchemy. Roger Bacon wrote alchemy. Vincent of
 Beauvais wrote alchemy. Lull wrote alchemy.
 Treatises continued to be added to the corpus of
 Jabir ibn Hayyan. In the generation of Occam,
 Peter Bonus wrote *The new most precious pearl,*
or on the most precious stone of the philosophers.
 In *The most precious pearl*, Peter Bonus marshals
 all objections to the search for the most precious

stone, then responds to them. The twenty-sixth
 objection, for instance, is: the alchemists cannot
 resolve metals into the elements (*id.*). The re-
 sponse is: it is impossible to generate man from
 the elements, we must have the elements in the
 form of a human seed; but where there is a human
 seed, man can be generated, and without first
 changing the seed back into the elements; in the
 same way, we cannot make metals from the four
 elements; we must have mercury; but from this
 substance, with the help of sulfur, gold is devel-
 oped, or common metals are developed from
 which gold is developed (*id.*).

Two centuries after Abelard, the investigation
 of propositions had not advanced beyond the
 ancients. The investigation of the elements had
 not advanced beyond Jabir ibn Hayyan and al-
 Razi.

The investigation of motion, however, began
 advancing substantially, and threefold. With the
 beginning of the investigation of magnetic motion
 by Peter Peregrinus. With the first steps in the
 quantitative investigation of locomotion by
 Bradwardine and Heytesbury, and then decisively
 by Oresme. And with the first steps toward the
 discovery of inertia by Buridan.

In the generation of Occam, Bradwardine
 wrote *On the proportions of velocities in motions.*
 He obtains the result: the velocity of motion is
 proportional to the ratio of the force of the mover
 to the force of the moved (3).

In the same generation, Heytesbury wrote the
Rules for the resolution of sophisms. He obtains
 the result: a body moving with uniform accelera-
 tion traverses in a given time the same distance
 which it traverses in the same time when moving
 uniformly with the mean speed (6).

The result of Bradwardine is erroneous. The
 result of Heytesbury is sound, it was proved by
 Oresme, and then by Galileo.

Swineshead continued the labors of Bradwar-
 dine and Heytesbury in the 16 books of the *Book*
of calculations.

More important, the generation of Bradwar-
 dine and Heytesbury is the generation also of
 Buridan.

Buridan formulated the theory of impetus in
 the *Questions on the Physics* and *Questions on*
On the heaven.

He writes: in moving a body, the mover imprints in it a certain impetus in the direction in which it moves it, and by this impetus a stone is moved after the thrower stops moving it (*Physics*, VIII, 12.4). Moreover: the natural gravity of the stone is always the same, before the motion, after the motion, and during the motion (*Heaven*, II, 12). The impetus can thus be determined quantitatively: the greater the amount of matter in a body, the more impetus it receives and the more intensely (*Physics*, VIII, 12.5). And Buridan draws the conclusion: as the holy scriptures do not say that intelligences move the heavenly bodies, it appears unnecessary to posit such intelligences; it can be presumed instead that god, when he created the world, moved each heavenly body as he pleased and impressed in it an impetus which continues to move it without he himself having to do so (*id.*).

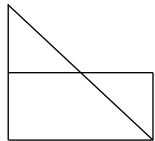
Buridan draws this very general conclusion as well: every general proposition in physics which has been proved by induction from experience, as many particulars have been established clearly and no instance to the contrary has been shown, must be accepted as a principle; for Aristotle well says that the principles must be gathered from sense, memory, and experience (IV, 8). Anticipating in this, too, Galileo.

In the next generation, Oresme wrote *The treatise on the configuration of qualities and motions*.

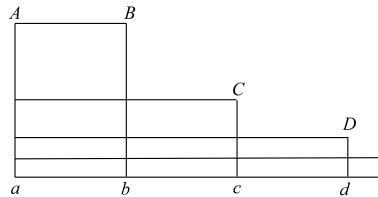
Oresme begins: every measurable thing is imagined as a continuous quantity; in order to measure it, therefore, points, lines, surfaces, and their properties have to be imagined (I, 1).

And hence, this is, for instance, the proof of Heytesbury's result: the height of the rectangle is one half the height of the right triangle on the same base; the base is time, the height is velocity; the area of the triangle is the distance traversed by a body moving with uniform acceleration; the area of the rectangle is the distance traversed by the same body moving uniformly with the mean speed; and the areas are equal (III, 7).

Also for instance, Oresme not only proposes this paradox, much like Zeno of Elea's paradoxes, he proves it:



the figure below is infinite; the bases ab , bc , cd , and so on, are equal; the height bB is twice the height cC , which is twice the height dD , and so on; thus the area aB is twice the area bC , which is twice the area cD , and so on; hence it is clear that a body moving with a certain velocity in one day, twice as slowly in the second day as in the first, twice as slowly in the third day as in the second, and so on, will not in all eternity traverse twice the distance it traverses in the first day (11).



The base is time, the height is velocity, the area is distance.

With Oresme also, in the *Questions on the geometry of Euclid*, the investigation of infinite series begins.

Oresme is two and a half centuries after Abelard.

And the advance in the acquisition of knowledge in these two and a half centuries can be compared with the advance in the acquisition of knowledge in the previous centuries.

In these two and a half centuries there was, toward the end of this time, the substantial advance in the investigation of motion. There was Fibonacci, one century after Abelard. And there was not too much else. Mathematics remained, other than Fibonacci, where Euclid had left it. Logic did not advance beyond Occam. Alchemy did not advance beyond al-Razi. The investigation of animals and plants remained where Aristotle and Theophrastus had left it. Medicine remained where Galen had left it. The investigation of the heavenly bodies did not advance beyond elementary introductions to the *Almagest* and revisions of its tables.

Where advance in the acquisition of knowledge is concerned, then, these two and a half centuries compare unfavorably with the two and a half centuries from Thales to Aristotle. They compare unfavorably as well with the two centuries from Aristotle to Hipparchus. But they do not

compare too unfavorably with the two and a half-odd centuries from Hipparchus to Ptolemy. They compare favorably with the two and a half centuries from Ptolemy to Augustine. They compare very favorably with the four and a half centuries from Augustine to al-Kindi. And they do not compare unfavorably with the two and a half centuries from al-Kindi to Abelard.

All in all, counting nineteen and a half centuries from Thales to Oresme, the two and a half centuries from Abelard to Oresme compare with rather more of the seventeen earlier centuries of development of philosophy favorably than unfavorably.

In general, the not acquisition of knowledge, not the acquisition of knowledge, is the norm.

Thus, the advance in the acquisition of knowledge in these two and a half centuries was not great. Yet the philosophers were successful in rebuilding the foundation on which their successors could begin advancing greatly once again.

And the more successful the philosophers were, considering how meager the beginnings, on which they had to begin rebuilding, were. The darkest of dark ages had been very dark. The beginnings had to be to sort out the fathers, yes and no, and to translate translations.

The more successful the philosophers were, considering also that throughout these two and a half centuries the principle of all principles, not the one Aristotle established is the principle of all principles, continued adamantly to maintain itself the principle of all principles.

Thus also the century after Oresme was barren, as barren as the centuries before Abelard. And this the general fact, that the not acquisition of knowledge is the norm, can explain.

But then the acquisition of knowledge accelerated very greatly. And this that general fact cannot explain.

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De Zurara, in 1453, in *The chronicle of the discovery and conquest of Guinea*, reports: prince Henry wanted to know the lands beyond the Canary Islands and cape Bojador; it seemed to him that if he or some other prince did not endeavor to gain this knowledge, sailors and merchants never will, because they never trouble

without the certain prospect of profit; and prince Henry began sending his ships out to those parts; and although he sent out many times some of his foremost men, no one dared to pass cape Bojador; beyond that cape, men said, there are no people, the land is sand, there is no water, no tree, no herb, and the currents in the sea are so violent no ship can ever return; for twelve years the prince sent out ships every year, and no one dared to pass cape Bojador; at last he sent out Eanes, one of his squires, whom he afterward knighted; Eanes went to the Canary Islands, and touched by the same terror, turned back; this was in 1433; but the next year the prince sent him out again, charging him to strain to the utmost; and on this voyage Eanes rounded cape Bojador.

Eanes rounded cape Bojador in 1434.

In 1456 Ca' da Mosto reached the Bissagos Islands. In 1482 Cão crossed the equator. In 1488 Dias rounded the cape of Good Hope.

In 1492 Columbus crossed the Atlantic.

Cape Bojador is at $26^{\circ} 10' N$ $14^{\circ} 30' W$.

The cape of Good Hope is at $34^{\circ} 20' S$ $18^{\circ} 30' E$.

The furthest point west which Columbus reached on his first voyage is on the northern coast of Cuba at about $23^{\circ} N$ $81^{\circ} W$.

North to south along the meridian, the distance from cape Bojador to the cape of Good Hope is $60^{\circ} 30'$.

East to west along the parallel, the distance from cape Bojador to the furthest point reached by Columbus on his first voyage is about $66^{\circ} 30'$.

South to north along the meridian, $60^{\circ} 30'$ from cape Bojador is well beyond the arctic circle.

West to east along the parallel, $66^{\circ} 30'$ from cape Bojador is in the Persian Gulf.

Prince Henry wanted to know the lands beyond cape Bojador.

In the two generations after prince Henry the surface of the earth brought under exploration was doubled.

And in the three generations after prince Henry more of the surface of the earth was mapped, and both qualitatively and quantitatively, than in the 84 generations of mapping the surface of the earth before and after – counting 67 generations from prince Henry back to Anaximander, and 17

generations since de Torenó – combined.

In 1462 Benincasa drew the first map of the African coast. In 1500 de la Cosa drew the first map of the earth showing the lands across the Atlantic. In 1507 Waldseemüller drew the first map of the earth showing America as a continent of its own. In 1526 de Torenó drew the first map of the earth showing the Atlantic, the Pacific, and the Indian oceans, and the continents between them, and the earth in its true proportions.

In 1543 Copernicus discovered the solar system.

It seemed to prince Henry that if he or some other prince did not endeavor to gain knowledge of the lands beyond cape Bojador, sailors and merchants never will. And sailors and merchants never will. Also, however, of all princes, but prince Henry did. Why prince Henry endeavored to gain knowledge of the lands beyond cape Bojador has to be explained.

On the other hand, given that prince Henry endeavored to gain knowledge of the lands beyond cape Bojador, and for twelve years sent out ships every year, and sent Eanes, and again, and continued to send his knights after Eanes, and for more than twenty-two years through Ca' da Mosto still, the very great advance in the investigation of the surface of the earth which begins with prince Henry is the main proximate cause of the subsequent acceleration of the acquisition of knowledge. And the acquisition of knowledge accelerated, and more, and more.

The scholars of the nineteenth century, once spirit was restored to the substantiality and purity of its being, christened this time renaissance and the men of this time humanists.

And humanists the men of this time, some of them anyway, were.

Pico, two generations after prince Henry, can illustrate.

In the *Oration on the dignity of man*, in 1486, Pico writes: and god put up man in the center of the world, and said to him; I have given you no fixed place, no form of your own, and no special gifts; you can have as your own the place, the form, and the gifts that you will; the nature of other creatures I have confined within the laws I have written; to you I have given free judgment and have confined you within no bounds; you can

fix the limits of your nature for yourself; I have placed you in the center of the world so that you can look around and see everything that is in the world; I have made you neither heavenly nor earthly, neither mortal nor immortal; you are the maker of yourself; you can sculpt yourself in any shape you will; you can lower yourself to the nature of the brutes; and with the reason of your soul you can raise yourself to the highest nature of the divine (*id.*).

And renaissance this time was. Art was reborn, education was reborn, scholarship was reborn.

Religion was reborn, too.

Luther writes: excluding all works, we go to the head of the beast called reason, which is the fountain and headspring of all mischief; for reason does not fear god, does not follow god, does not believe in god, but disdains god; this pestilential beast, reason I say, once slain, all outward and gross sins will be nothing.

Calvin writes: at this day the earth sustains in her bosom many monstrous minds, minds not afraid to use the seed, which god deposited in human nature, to suppress the very name of god; can anything be more detestable than this madness in man, who upon finding god a hundred times in his body and in his soul, makes his excellence a pretext to deny god? he will not say that chance made him different from the beasts; but he will, by making nature the maker of the world, suppress the very name of god (*Institutes*, I, 5.4).

In this way at least, religion was reborn very much as it had been before it was reborn.

Philosophy was also reborn. In two kinds, however. And in one of the kinds very much as it had been before it was reborn.

This is how Bacon, in 1620, in the *New organon*, describes this kind. He writes: the corruption of philosophy by superstition and theology is ubiquitous, and it does the greatest harm (I, 65); of this kind among the Greeks there is a striking example in Pythagoras, another, more dangerous and more subtle, in Plato (*id.*); but in this vanity the moderns have indulged with the greatest levity, too (*id.*); there is no deficiency of grandiloquent and phantasmagorical men who, in part from credulity, in part from imposture, have saddled the human race with promises of the

prolongation of life, the retardation of senescence, the alleviation of pain, the repair of natural defects and of the deficiency of the senses, the incitement of the affections, the exaltation of the intellectual faculties, the transmutation of substances, the multiplication of motions at will, the alteration of the air, the deduction and procurement of celestial influences, the divination of the future, the revelation of secrets, and many more; but there is in philosophy as much difference between these vanities and true art as there is in history between the deeds of Amadis of Gaul and Arthur of Britain and the deeds of Caesar and Alexander (87).

Also in the *Oration on the dignity of man*, Pico writes: first, I propose the concord of Plato and Aristotle (*id.*); second, I formulate seventy-two new physical and metaphysical doctrines, brought out of Plato and Aristotle, such that anyone who holds them will be able to answer every question about either the nature of things or god (*id.*); I also propose theorems about magic, of which one kind is the work of demons, but another kind is the absolute consummation of the philosophy of nature (*id.*); I also uncover the ancient mysteries of the Hebrews which confirm our holy Catholic faith (*id.*); and I also have opinions on the interpretation of the poems of Orpheus and Zoroaster (*id.*).

Bacon thus points out: I have no universal theory to propound (116).

Of this kind of philosophy are Pomponazzi, Pico, Fracastoro, Agrippa of Nettesheim, Paracelsus, Nostradamus, Telesio, Bruno.

These are deeds of Amadis of Gaul and Arthur of Britain.

Of this kind of philosophy prince Henry is not the cause. Of this kind of philosophy ubiquity – the ubiquity of the corruption of philosophy by superstition and theology – is the cause.

Deeds of Caesar and Alexander, on the other hand, there had not been for a long time.

Now rather suddenly of this other kind of philosophy are Peurbach, Regiomontanus, Machiavelli, Copernicus, Biringuccio, Brunfels, Agricola, Bock, Cardano, Vesalius, Gesner, Belon, Cesalpino, Bombelli, Bodin, Viète, Gilbert, Tycho, Stevin.

Rather suddenly the acquisition of knowledge

advanced far beyond Galen, far beyond Aristotle, far beyond Theophrastus, far beyond Euclid, far beyond Ptolemy.

Of this kind of philosophy prince Henry is the cause.

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And there was a great advance in the formulation and solution of the problem of the principles of all things, too. In all three parts of the problem still only implicit rather than explicit, yet in all three parts of the problem very substantial advance.

Two-odd generations after prince Henry, in the generation after Columbus, Copernicus discovered the solar system.

Aristotle's solution of the problem of the efficient principles, or the causes, of all things had been grounded firmly in the theory of the motions of the heavenly bodies. Grounded in observation as extensive as any, the theory of the motions of the heavenly bodies had filled the world with locomotion, in chains of inferences as free of gaps as any, from the first mover at the circumference of the world, circular locomotion by circular locomotion, down to the moon. A great virtue of the solution had thus also continued to be its uniformity. Below the moon locomotion was communicated to the elements in their spheres, fire, air, water, and earth, in the center.

Peurbach began and Regiomontanus completed the *Epitome of the Almagest*, in 1462, with new observations, corrections of Ptolemy's computations, critical notes, and mathematical sophistication equal to the *Almagest*.

Copernicus wrote the *Little treatise on the hypotheses concerning the heavenly motions* in 1514.

And the discovery of the solar system presented the first mover with some difficulties.

It is the minor difficulty that, if not the earth but the distance between the earth and the sun is imperceptible in relation to the sphere of the fixed stars, the circumference of the world has to recede so far from its center, if not to infinity, the first mover has to exert itself that much more. The major difficulty is the loss of uniformity. Communicating motion to Saturn, then Jupiter,

then Mars, once it reaches the earth, the first mover has to rotate it on its axis, revolve it around the sun, and revolve the moon around it, before continuing downward to move Venus, Mercury, and the sun. With the sun in the center, also, it is hard to know what to do with the spheres of the elements.

Buridan writes: it appears unnecessary to posit intelligences to move the heavenly bodies; it can be presumed instead that god, when he created the world, moved each heavenly body as he pleased and impressed in it an impetus which continues to move it without he himself having to do so.

The source of the motions of the heavenly bodies, that impetus was bound to recommend itself as a resolution of the difficulties.

That one way or another these are the particulars, the motions of the heavenly bodies, which will show which the cause of all things is, how it is, and that it is the cause of all things, the discovery of the solar system corroborated very powerfully.

In the course of the search for the philosophers' stone the solution of the problem of the material principles, or the elements, of all things had also begun being grounded more firmly in the collection of the particulars. The practical urgency of the end of that search, however, had not encouraged too much the development of the theoretical aspects of the means to the end.

The generation after Copernicus is the generation of Biringuccio, Paracelsus, and Agricola.

And Paracelsus continued the search for the philosophers' stone. But Biringuccio and Agricola did not.

It is the generation also of Vesalius, Gesner, and Belon.

These philosophers all abandoned the search for that stone. And as with Vesalius, Gesner, and Belon the investigation of organisms found the stone in observation, so with Biringuccio and Agricola the investigation of substances found it in experiment.

Paracelsus proposed that the elements are not the four, earth, water, air, and fire, the elements are these three, mercury, sulfur, and salt, of which those four are composed.

In *The book paraganum*, in 1530, he writes:

I expound the four pillars of the foundation of medicine on which I stand and will stand, namely, philosophy, namely, astronomy, namely, alchemy, namely, virtue; the first pillar is philosophy and it is the knowledge of earth and water; the second pillar is astronomy and it is the knowledge of air and fire; the third pillar is alchemy and it is the preparation of all four elements; the fourth pillar is virtue and remains with the doctor unto death for it completes and preserves the other three pillars (pr.).

In the *Philosophy of the generation and fruits of the four elements*, in 1529, Paracelsus writes: as to how god created the world, in the beginning, while the four elements were still being generated, he made one body; this body he fashioned in three pieces, mercury, sulfur, and salt, and so that these three things be one body; these three things make up everything that is and becomes the four elements (6). And in *The book paraganum*: man himself is nothing but mercury, sulfur, and salt (7); whatever burns is sulfur, whatever is humid is mercury, whatever is the balsam of the two is salt (*id.*).

Biringuccio wrote *On pyrotechnics, in which are fully treated not only all kinds and sorts of minerals, but also all that is necessary for the practice of the arts of smelting, casting, and related things*, in 1540.

Biringuccio writes: the more I look into this art of theirs so greatly praised and desired by men, the more it seems to me a vain wish and empty dream (I, 1); no one can purify the elements to perfection unless he is an angel (*id.*); it is better to turn to the natural gold and silver extracted from ores than to those of the alchemists which no one has ever seen (*id.*).

On pyrotechnics is in 10 books. In book I Biringuccio deals with metallic ores, in book II with mercury, sulfur, antimony, alum, vitriol, arsenic, pigments, gems, glass, in book III with the smelting of ores, in book IV with the extraction of gold and silver from ores with nitric acid, antimony sulfide, and sulfur, in book V with alloys of gold, silver, copper, lead, and tin, in book VI with the casting of statues, guns, and bells, in book VII with furnaces and melting, in book VIII with small castings, in book IX with pyrotechnical operations, including the distilla-

tion of acids and alcohol, in book X with the preparation of saltpeter and gunpowder. The 10 books are divided into 94 chapters, 39 of them under the heading: the method. From: the method of assaying ores (III, 1); the method of preparing ores for smelting (2); the method of smelting ores (4), through: the method of melting metals in the hearth (VII, 2); the method of melting metals in ladles (3); the method of melting metal in a wind furnace (4), to: the methods of loading guns and of firing them accurately (X, 3).

As qualitatively so quantitatively. Biringuccio writes: a superabundance of tests is necessary, to test and to try until the right way is found, and not only by using ordinary things, but by varying the quantities, adding now one half, now an equal portion, now twice the amount, now three times the amount (III, 2).

Agricola wrote *On the origin and causes of subterranean things; on the nature of things flowing out of the earth; on the nature of fossils; on the old and new metals*, in 1546.

The third treatise, *On the nature of fossils*, is in 10 books. Agricola describes them: book I considers the properties of minerals and discusses the origins of all minerals; book II treats of earth; book III of congealed fluids; book IV of congealed fluids, amber, and stones formed from bitumen; book V takes up stones; book VI gems; book VII marbles and rocks; book VIII metals; book IX metallic substances; book X mixed and compound substances (pr.).

Among the congealed fluids described in books III-IV are: salt, saltpeter, alum, vitriol, chrysocola, orpiment, realgar, sulfur, camphor, bitumen, coal, amber (*id.*). Among the stones described in book V: lodestone, bloodstone, gypsum, talc, asbestos, mica, calamine, fossils, geodes, emery, touchstones, pumice, fluorspar, quartz (*id.*). Among the metals described in book VIII: gold, silver, quicksilver, copper, lead, tin, antimony, bismuth, iron, electrum, brass (*id.*).

Agricola wrote also *On metals, in which the uses, instruments, machines, and all other things concerning metals are observed, and described not only in the clearest way, but with images in the appropriate places*, in 1556.

In *On metals* Agricola collects all the work of his predecessors, including Biringuccio. He

describes more minerals, more operations, more observations, more experiments. The images in the appropriate places are 292.

In *On metals* is in 12 books. Agricola describes them: book I contains the arguments used against the art of metals and mines, and the response to them; book II describes the miner, and teaches how veins are discovered; book III deals with veins, stringers, and seams in rocks; book IV explains the method for capturing veins, and the art of the foreman; book V describes how the ore is dug out, and the art of the surveyor; book VI describes the tools and the machines of the miner; book VII is on how ores are assayed; book VIII describes the rules for roasting, crushing, and washing ores; book IX explains the methods of smelting ores; book X teaches the studios in the arts of metals how to separate silver from gold, and lead from silver and gold; book XI teaches how to separate silver from copper; book XII teaches the rules for the preparation of salt, soda, alum, vitriol, sulfur, bitumen, and glass (ded.).

The methods of separation and preparation of substances, which the studios are taught in books X-XII, are not less than three dozen.

What, qualitatively, will show which the elements of all things are, how they are, and that they are the elements of all things, had been slowly becoming clearer since Theophrastus. Confluence, filtering, subjugation to heat and cold, hardening and forming. Machines worked by air and water. Preparations of substances. Dissolution and transformation by fire. Measurement of volume and weight. Sublimation, calcination, coagulation, fixation, solution, distillation.

It is now, with Biringuccio and Agricola, very much clearer.

It is clear now as well what, quantitatively, will show it. A superabundance of tests, until the right way is found, and not only by using ordinary things, but by varying the quantities, adding now one half, now an equal portion, now twice the amount, now three times the amount.

The generation of Biringuccio and Agricola, and of Vesalius, Gesner, and Belon, is the generation also of Cardano.

On pyrotechnics appeared in 1540, *On the revolutions of the heavenly spheres* in 1543, *On*

the structure of the human body in 1543, *The great art, or the rules of algebra* in 1545, the first volume of the *History of animals* in 1551, the *Natural history of birds* in 1555, *On metals* in 1556.

With Cardano mathematics began advancing greatly once again. And the great advance of mathematics, as it accelerates, and more, and more, will be the ground on which the formulation and solution of the problem of the formal principles of all things will at long last advance as well.

After Gerard the philosophers had rediscovered all of Aristotle's logical treatises. Much logic had been written. One and a half-odd centuries after Gerard, with Occam, logic had taken a step toward becoming again the formal investigation of form.

Then there logic remained. After Occam much logic continued to be written. Logic, if it was logic, was once again assimilated into rhetoric and grammar. The high point in this development – in that same generation, of Biringuccio, Agricola, Cardano, Vesalius, Gesner, and Belon – is Ramus.

Ramus wrote the *Dialectic*, in 1555.

He writes: dialectic is the art of disputing well; and it is called also logic, because both words are derived from logos (1); the parts of dialectic are two, invention and judgment; the first shows the parts of which every sentence is composed, the second teaches the ways of disposing these parts, just as grammar teaches the parts of speech, syntax teaches how to assemble the parts (*id.*). Beginning to end the *Dialectic* is provided with exemplars to emulate, beginning to end the foremost exemplar to emulate: Ronsard (*passim*).

Ramus acquired a following, and Ramus and his following practiced vigorously the art of disputing well.

On the disputes of the Ramists Bacon writes: where there is much controversy there is little inquiry (*Advancement*, II). On Ramus himself: I know not by what fatality it is that, as the poets say, the most precious things have the most pernicious keepers (*Growth*, VI, 2).

The next is the generation of Viète.

Viète writes: the analytic art claims for itself the greatest problem of all, to solve all problems.

In the following century – through Galileo, Kepler, Descartes, Cavalieri, Fermat, Wallis, Huygens, Newton, Leibniz – so the analytic art will, solve all problems.

And hence at long last logic will advance, and the formulation and solution of the problem of the formal principles of all things will advance.

The generation of Viète is the generation also of Gilbert, Tycho, and Stevin.

Tycho in November 1572 observed a new star: surpassing all others in brightness. By February 1573 it faded to first magnitude, by April to second, by August to third, by October to fourth, by January 1574 to fifth, by February to sixth, then it faded altogether. The fixed stars are not fixed. In November 1577 Tycho observed a new comet and measured its distance at 173 earth radii. In January 1578 he measured its distance at 1 733 earth radii. The comets are not atmospheric phenomena, they move deep among the planets.

With the discovery that the comets move among the planets the loss of uniformity in the communication of motion from the first mover down into the world continued. With the discovery that the fixed stars are not fixed the circumference of the world quit being the circumference of the world.

Stevin wrote *The principles of the art of weights*, in 1586. He writes: the fundamental properties of weight have remained hidden from our predecessors. Stevin then demonstrates: the force of an oblique weight is inversely proportional to the length of the oblique plane which supports it.

The investigation of force will decide which the causes of all things are, how, and that they are the causes of all things. The investigation of force begins with the discovery of the law of the inclined plane.

And Gilbert wrote *On the magnet, magnetic bodies, and the great magnet of the earth*, in 1600.

Galileo writes: I have the greatest praise and admiration for Gilbert who framed such a stupendous theory about an object which an infinite number of men of splendid intellect had handled without paying any attention to it.

On the magnet is in 6 books. In book I Gilbert establishes the foundation of the stupendous

theory with the discovery that: the terrestrial globe itself is a lodestone (17).

In book II he begins: there are five magnetic motions, coition, or the impulse to magnetic union, direction, or the inclination toward the earth's poles, variation, or the deflection from the meridian, declination, or the descent beneath the horizon, and revolution, or circular motion (1). He deals with coition in book II, direction in book III, variation in book IV, declination in book V, revolution in book VI.

And before turning to coition in book II, Gilbert distinguishes coition from attraction to amber. Magnetic motion from electric motion.

He writes:

men continue in ignorance imagining that attraction to amber is like magnetic coition;

to understand clearly by experience how attraction to amber takes place, make yourself a rotating needle of any kind of metal, 3 or 4 fingers long, very light, and poised on a sharp point; bring near one end of the needle a piece of amber or a gem, lightly rubbed, and the instrument will revolve at once;

the lodestone even when much rubbed has no electric attraction, while many bodies, which do not attract at all unless rubbed, attract when rubbed (2).

These bodies are:

electric bodies (*id.*).

And Gilbert shows that electric bodies are:

emerald, agate, carnelian, pearls, jasper, chalcedony, alabaster, porphyry, coral, marbles, flint, bone, ivory, the hardest woods, like ebony, other woods, like cedar, juniper, and cypress, and all metals, like silver, gold, copper, and iron (*id.*).

Gilbert concludes:

in all bodies there are two principles, matter and form; and electric motions are from matter, magnetic motions are from form;

the two differ greatly;

one is ennobled with many virtues and is of great potency, the other is lowly, of less potency, and confined in prisons from which its force has to be released by friction;

a lodestone attracts only magnetic bodies, an electric attracts everything; a lodestone lifts great weights, a strong one of 2 ounces lifts half an ounce; an electric lifts only small weights, a piece

of amber of 3 ounces lifts only one fourth of the weight of a grain of barley (*id.*).

Gilbert has discovered electricity, too.

And in the *New philosophy of our sublunar world*, in 1603, Gilbert writes:

the force of the moon extends to the earth, and the magnetic force of the earth reaches the moon; the two correspond to one another and agree in their action; the earth attracts and repels the moon, and the moon attracts and repels the earth (II, 19).

There are five magnetic motions, coition, direction, variation, declination, and revolution.

Electric motion differs greatly from magnetic motion.

Electric motion is confined in prisons and has to be released by friction.

Magnetic motion and electric motion differ in potency, a magnet of 2 ounces lifts half an ounce, an electric of 3 ounces lifts only one fourth of the weight of a grain of barley.

An electric attracts everything.

The earth is a magnet.

The force of the moon extends to the earth, the magnetic force of the earth reaches the moon.

The earth attracts and repels the moon, the moon attracts and repels the earth.

With Gilbert's stupendous theory the investigation of magnetism begins. The investigation of electricity begins. The analysis of motion into potency and attraction begins. The analysis of the motions of the earth and the moon into mutual attraction and repulsion begins. Their analysis into force begins.

With Gilbert's theory the ground on which the problem of the causes of all things will be solved has been extended stupendously, substantially and essentially.

ccclxvi

In the not quite five centuries from Abelard to the generation before that of Bacon, Galileo, and Kepler the advance in the formulation and solution of the problem of the principles of all things was in the first four of these centuries small, in the last not quite one century great.

The elements of all things.

David of Dinant said that god is the primary matter.

This attempt to make god the element failed. Aquinas said that god is not the primary matter, he is its creator.

Albert investigated minerals. His investigation of minerals, like that of animals and plants, is not an advance over his predecessors. But his classification of the operations of alchemy is an advance.

The operations had been described earlier. Albert systematized the earlier descriptions, and in explicit reference to the elements.

Sublimation is volatilization by fire. Calcination is pulverization by fire. Coagulation is reduction of liquid to solid. Fixation is tempering by fire. Solution is resolution into water. Distillation is rising of vapors of liquid.

The ground of alchemy remained such as Jabir ibn Hayyan had established it. Such as, five centuries after Jabir ibn Hayyan, Peter Bonus describes it. We cannot make metals from the four elements. We must have mercury. But from mercury, with the help of sulfur, gold is developed, or common metals are developed from which gold is developed.

Jabir ibn Hayyan said that all metals are composed of mercury and sulfur. Seven centuries after Jabir ibn Hayyan, Paracelsus added salt, and generalized the doctrine. Mercury, sulfur, and salt are the elements. It is the first attempt to establish that earth, water, air, and fire are not the elements. Earth, water, air, and fire are composed of the elements. Paracelsus said: mercury, sulfur, and salt make up everything that is and becomes earth, water, air, and fire.

On the other hand: whatever burns is sulfur, whatever is humid is mercury, whatever is the balsam of the two is salt.

Biringuccio described more minerals than his predecessors, many more operations, and experimentally. A superabundance of tests is necessary, and not only by using ordinary things, but by varying the quantities, adding now one half, now an equal portion, now twice the amount, now three times the amount.

Agricola described many more minerals, many more operations, and experimentally.

With Biringuccio and Agricola the ground on which the problem of the elements will be solved in particular, and hence in general, is at long last very firm.

Tabulating the important,	
David of Dinant	god is the primary matter,
Albert	classified the operations of alchemy,
Aquinas	god is the creator of the primary matter,
Biringuccio	described minerals, operations, and experimentally,
Paracelsus	the elements are mercury, sulfur, and salt,
Agricola	described minerals, operations, and experimentally.

The causes of all things.

Aquinas said that god is the efficient principle of all things.

He is also the final cause of all things. He is the father, the son, and the holy spirit. He is a unity and a trinity. He is simple, perfect, good, infinite, immutable, eternal, alive, just and merciful, provident, omnipotent.

It is not an advance in the solution of the problem of the cause of all things. The formulation and solution of this problem remained where the development of the theory of the motions of the heavenly bodies by Ptolemy had left it.

Aquinas, in general, however, and his investigation of god, in particular, is a very important step in the development of philosophy.

Philosophy is the search for truth, Aristotle had affirmed. And so philosophy had been, not exclusively, but for a long time substantially and essentially the search for truth. Then the fathers of the church had taken over philosophy. The darkest of dark ages had followed. But at length the darkest of dark ages came to an end. The doctors of theology were now the philosophers. And with the doctors of theology philosophy began becoming once again the search for truth. Once again not exclusively, far from it, and very slowly, yet decisively, and to become at length the ground on which the instauration of the sciences will begin.

In this development of philosophy, becoming slowly, yet decisively, once again substantially and essentially the search for truth, Aquinas, in general, and his investigation of god, in particular, is a very important step.

Nothing shows this more clearly and distinctly than the investigation of his omnipotence.

Peter Damian is a very early doctor, two-odd generations before Abelard, two-odd centuries before Aquinas.

Peter Damian wrote the *Letter on divine omnipotence*.

He writes: once, as I recall, the following passage of the blessed Jerome came up in our conversation; although god can do all things, he cannot raise up a virgin after her fall, he can free her from punishment, true, but he cannot give her back her crown once it has been corrupted; and I was terrified; and although I should not dare argue against the witness of so great a man, I said what I thought, this dishonors him who can do all things (2).

The fathers of the church, in general, and Jerome, in particular, had not been troubled by inconsistency. The doctors of theology, in general, beginning with Peter Damian, in particular, were troubled by inconsistency, and increasingly so.

Peter Damian concludes: god in his eternity can do so that what was done was not done (17). Never mind the virgin: god can do so that Rome which was founded in antiquity was never founded (*id.*).

Two-odd generations later, Abelard writes: god can do and do not only what he does and does not, and only in the way and at the time in which and at which he does it and does it not, and no other.

And he wrote the *Yes and no*.

One and a half centuries later, Aquinas writes: god can do what he does not, and: he can do better than what he does.

Nevertheless: whatever implies a contradiction does not come within the scope of his omnipotence.

One generation-odd later, Scotus argues: the first efficient cause has infinite power (*Ordination*, I, 2.2).

In the next generation, in the *Seven quodlibets*, Occam asks: can it be sufficiently proved by argument that god is intensively infinite (VII, 11)? and replies: the opinion of Scotus is yes (*id.*); the opinion of the author is no (*id.*). Again: can it be sufficiently proved from god's knowledge that god is intensively infinite (12)? and: the opinion of Scotus is yes (*id.*); the opinion of the

author is no (*id.*). Again: can it be sufficiently proved from his simplicity that god is intensively infinite (13)? and: the opinion of Scotus is yes (*id.*); the opinion of the author is no (*id.*). Again: can it be sufficiently proved from final causes that god is intensively infinite (14)? and: the opinion of Scotus is yes (*id.*); the opinion of the author is no (*id.*). Again: can it be clearly proved from his eminence that god is intensively infinite (15)? and: the opinion of Scotus is yes (*id.*); the opinion of the author is no (*id.*). Again: is the view of the philosopher and of the commentator that god is intensively infinite (16)? the answer is: no (*id.*). Again: do the philosopher's demonstrations in book VIII of the *Physics* and book XII of the *Metaphysics* show that god is intensively infinite (17)? and again: the opinion of Scotus is yes (*id.*); the opinion of the author is no (*id.*). Finally: does god in fact have intensively infinite power (18)? and the answer is: the affirmative answer is certain through faith (*id.*). Occam adds: this can also be argued plausibly from the fact that god can make a thing more perfect than any of the things he has already made (*id.*).

The division of power between god and the principle of contradiction was steadily shifting. The separation of proof from faith was becoming steadily sharper.

Thus, god is the efficient principle of all things.

And to show this Aquinas considers the procession of creatures from him, and the divine government of things.

And the consideration of the procession of creatures from the efficient principle of all things, and of its government of things, will show which the efficient principle of all things is, how it is, and that it is the efficient principle of all things.

With Peter Peregrinus the investigation of the magnet, and thus of magnetic motion, begins.

With Bradwardine, Buridan, Heytesbury, and Oresme the investigation of locomotion resumed. And both Buridan's and Oresme's are substantial advances. Buridan formulated the theory of impetus. Oresme proved the first theorems about locomotion.

With Copernicus the investigation of the heavenly bodies began informing the problem of

the first mover once again. The motions of the heavenly bodies are the particulars which will decide that the first mover is the first mover. And the motions of the heavenly bodies presented the first mover with serious difficulties.

With Tycho the difficulties increased.

With Tycho, Stevin, and Gilbert the ground on which the problem will be solved became observation and experiment, exclusively. The ground was also extended substantially and essentially. With Stevin the investigation of force begins. With Gilbert the investigation of magnetism, and of electricity, and the analysis of motion into potency and attraction, and into the mutual attraction and repulsion of the earth and the moon, begins.

The problem has thus become widely open. It will have to be reformulated, and the reformulated problem solved.

Tabulating the important philosophers, ignoring all doctors of theology, and all that they have said about the cause of all things, except Aquinas, Scotus, and Occam,

Aquinas	god is the efficient principle of all things,
Peter Peregrinus	the investigation of the magnet begins,
Scotus	the first efficient cause has infinite power,
Occam	that god is intensively infinite cannot be proved,
Bradwardine	investigated locomotion,
Buridan	the theory of impetus,
Heytesbury	investigated locomotion,
Oresme	proved theorems about locomotion,
Copernicus	discovered the solar system,
Gilbert	the investigation of magnetism begins,
	the investigation of electricity begins,
	motion is potency and attraction,
	the earth and the moon attract and repulse one another,
Tycho	the fixed stars are not fixed,
Stevin	demonstrated the force of an oblique weight.
The formal principles of all things.	

Amalric said that god is the formal cause of all things.

The attempt to make god this, too, also failed. God is not the formal cause, he is the exemplar cause.

After the darkest of dark ages logic as well revived. Logic yet did not fare well in these not quite five centuries, excepting Occam, not well at all.

Nor did mathematics for a long time fare well at all.

Mathematics, which the saints discovered at the beginning of the world, is the gate and key to all sciences. Who is ignorant of this can know neither the other sciences nor the affairs of the world. Nor does he know his own ignorance, nor does he know to seek a cure.

It was three centuries after Roger Bacon when at long last mathematics began advancing greatly once again with Cardano, very greatly with Viète.

And the great advance of mathematics will become the ground on which at length logic, too, will greatly advance, and the formulation and solution of the problem of the formal principles will greatly advance.

Cardano extended the operations of arithmetic to all real numbers. He discovered the complex numbers, too. If it is said, divide 10 into two parts whose product is 40, it is clear that this is impossible. We will nevertheless do it.

Number is the principle of all things, and 3 has long been a number. But 0 is also a number, and -3 is a number, and $\sqrt{9}$ is a number, and $\sqrt{-9}$ is a number, and $5 + \sqrt{-15}$ is a number, and $5 - \sqrt{-15}$ is a number.

A proportion is that from which an equation is composed and an equation is that into which a proportion is resolved. The analytic art uses not numbers, but symbols or signs for things, say, the letters of the alphabet. It no longer limits its reasoning to numbers, it employs the newly discovered logistic by species, which is far more fruitful and powerful than the old numerical logistic.

Mathematics is the investigation of form. The world is written in mathematics, and the conclusive demonstration that it is begins with Viète.

Hence: the analytic art claims for itself the greatest problem of all, to solve all problems.

Tabulating the important,	
Aquinas	god is the exemplar cause of beings,
Occam	the doctrine of consequences,
Cardano	generalized number,
Viète	analysis begins.

In their series and order these 19 are all important philosophers who are part of the development of the problem of the principles of all things in the not quite five centuries from Abelard to the generation before that of Bacon, Galileo, and Kepler,

1158 - 1220	David of Dinant,
1200 - 1280	Albert,
1225 - 1274	Aquinas,
1229	Peter Peregrinus,
1266 - 1308	Scotus,
1285 - 1349	Occam,
1295 - 1349	Bradwardine,
1295 - 1358	Buridan,
1295	Heytesbury,
1320 - 1382	Oresme,
1473 - 1543	Copernicus,
1480 - 1538	Biringuccio,
1493 - 1541	Paracelsus,
1494 - 1555	Agricola,
1501 - 1576	Cardano,
1540 - 1603	Viète,
1544 - 1603	Gilbert,
1546 - 1601	Tycho,
1548 - 1620	Stevin.

The steps of advance in the acquisition of knowledge are the 13, Albert, Peter Peregrinus, Buridan, Oresme, Copernicus, Biringuccio, Paracelsus, Agricola, Cardano, Viète, Gilbert, Tycho, and Stevin. In particular,

Albert	classified the operations of	alchemy,
Peter Peregrinus	the investigation of the magnet	begins,

Buridan	the theory of impetus,
Oresme	proved theorems about
	locomotion,

Copernicus	discovered the solar system,
Biringuccio	described minerals, operations,
	and experimentally,

Paracelsus the elements are mercury,
sulfur, and salt,

Agricola described minerals, operations,

	and experimentally,
Cardano	generalized number,
Viète	analysis begins,
Gilbert	the investigation of magnetism
	begins,
	the investigation of electricity
	begins,
	motion is potency and
	attraction,
	the earth and the moon attract
	and repulse one another,
Tycho	the fixed stars are not fixed,
Stevin	demonstrated the force of an
	oblique weight.

Continuing the tabulation of the distribution of
the philosophers,

1076-1175	2	—,
1176-1275	4	2,
1276-1375	5	2,
1376-1475	1	1,
1476-1575	8	8.

The count in the last line is not complete.

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The three generations from Bacon to Leibniz.

Counting 171 generations of acquisition of knowledge since its beginning on earth, with the 3 oldest calculations of area, in these three generations the acquisition of knowledge advanced more than in the other 168 combined.

Bacon explains.

He lists the causes of errors and of why errors have so long continued. The least diligence was given to natural philosophy. Natural philosophy scarcely ever possessed a man wholly, being always a means to something else. The end was not right, which is discoveries. The means was not right, which is experiment. The true way was not just deserted, it was barred and blocked.

In these three generations, the greatest diligence was given to natural philosophy. Natural philosophy possessed men wholly. The end was right, which is discoveries. The means was right, which is experiment. The true way was not just embraced, it was followed as staunchly as the thread of Theseus must be followed in the labyrinth.

The advance in the formulation and solution of the problem of the principles of all things, in

particular, was proportionately great.

Bacon, Galileo, and Kepler are contemporary. Their results, of each of the three, are a vast particularization, a vast generalization, and a vast systematization, all three, over the results of their predecessors.

Bacon's year of birth is 1561.

There are and can only be two ways to search for truth. One is to fly from the senses and the particulars to most general principles and, taking them to be settled, proceed to judgment. This way is now in use. The other is to obtain principles from the senses and the particulars by continuous and gradual ascent, and only ultimately end with the most general principles. And this is the true way even if it is yet untried.

There was never the slightest hope that the errors that have flourished and will flourish forever would put themselves right by the native power of the mind or with the help of dialectic. There remains but one remedy, to undertake the entire work of the understanding anew, and from the very beginning not leave the mind to itself, but guide it continuously, so that the whole thing is done by machinery.

Experience as it occurs is an accident, as it is pursued an experiment. The secrets of nature reveal themselves not when nature is free and left to herself, but when she is constricted and vexed, when by the arts and services of man she is forced out of her state, and molded and shaped.

Although there is nothing in nature but individual bodies performing individual acts, they do so by law, and this law and its investigation, discovery, and explanation is the foundation of knowledge and its application. It is this law with its clauses that is understood by the name forms. When we speak of forms we understand nothing but the laws and determinations which govern and constitute a simple nature. The form of heat and the form of light is the same as the law of heat and the law of light.

Bacon exemplifies with the investigation of the form, or the law, of heat.

First of all we have to prepare a natural and experimental history, sufficient and good, this is the foundation of the thing. And we have to begin with the collection of all known instances which agree in the nature under investigation. Next, we

have to make a presentation of instances in which the given nature is lacking. Next, we have to make a presentation of the instances in which the nature under investigation is found in greater or lesser degree. Presentation done, induction itself has to be set to work.

The natural and experimental history is prepared in four tables with 115 entries.

Bacon concludes: the notion of fire is worthless; heat is a rapid motion of minute particles, and not of the smallest, but of those a degree larger.

Galileo's year of birth is 1564.

He seems to think that philosophy is a book of man's fantasy, like the *Iliad* or *The furious Orlando*, books in which the least important thing is whether what is written in them is true. But it is not like that. Philosophy is written in that grandest book continually open before our eyes, the universe, which cannot be understood without learning to understand the language and to know the letters in which it is written. It is written in mathematical language and the letters are triangles, circles, and other geometrical figures, without which it is impossible to understand a single word, without which one wanders in vain in a dark labyrinth.

To try to treat questions about nature without geometry is to try to do the impossible.

In the mathematical sciences, geometry and arithmetic, the human intellect has absolute certainty, such as nature itself has. In them the human intellect is equal to the divine in objective certainty. The truth of knowledge by mathematical demonstration is the same as that known by divine wisdom.

Concerning the oldest object we set forth the entirely new science. For nothing in nature is older than motion, and about it volumes neither few nor small have been written by the philosophers. Yet I find that its essential properties have not even been observed, let alone demonstrated. It has been noted that in their natural motion falling bodies continually accelerate, but the proportion of the acceleration has not been found, no one has demonstrated that the distances traveled in equal times by a body descending from rest obey the same ratio as the odd numbers. It has been observed that projectiles describe a

curved line, but no one has discovered that this line is a parabola. These and other things neither few nor less worthy of being known I will demonstrate, and thus, what is more important still, the gate and the road to the largest and most excellent science will be opened, in which these labors of mine will be the elements.

Galileo discovered the satellites of Jupiter.

Proved that the earth revolves around the sun.

Discovered the law of falling bodies.

Discovered the law of the pendulum.

Discovered the principle of relativity.

Discovered inertia.

Showed what experiment is.

Showed that distance, time, and speed are functions.

In part I of the *Two new sciences* he investigates the constitution of bodies. He writes: the smallest particles of metal are joined firmly together, and yet fire goes through them and finally separates them; penetrating through the smallest voids in the metal, through which the smallest particles of air and other fluids cannot pass, the particles of fire free the particles of the metal from the force with which the voids attract the particles and prevent their separation.

Kepler's year of birth is 1571.

Body is what god created in the beginning. And it is clear why god created body and not something else. What god proposed is quantity. He needed everything that pertains to the essence of body, and quantity is the form of body which makes it body.

The proper movers of the planets are nothing but the properties of the planetary bodies themselves. All causes of the heavenly motions are purely bodily, in particular, magnetic.

All things are so interconnected with all things, that no other way can succeed but grounding the motions of the planets in their physical causes. And this is what I establish.

The orbits of the planets are ellipses with the sun in one of the foci. The areas defined by the arcs traversed by a planet in equal times and the respective radii to the sun are equal. And the ratio of the periodic times is the ratio of the $\frac{3}{2}$ powers of the distances.

I transferred all of astronomy from fictitious

circles to natural causes.

The position, order, and motion of the parts of the world is the system of the world.

Kepler writes also: it is by motion that light spreads through media, and is thrown against surfaces, and is in turn either absorbed by the medium or forced to rebound from it.

Harvey's year of birth is 1578.

Harvey discovered the circulation of the blood in 1628.

Van Helmont's year of birth is 1579.

Van Helmont's treatises were collected in the *Rise of medicine, or beginnings of a new physics*, in 1648.

Van Helmont writes: there are two primary elements, air and water, for they alone do not turn into one another; while earth is born from water and can be reduced back to water (19); and I deny that fire is an element or substance, it is rather, in the hands of the artificer, the death of substances (*id.*).

One of the experiments which will show that earth is born from water is this:

all plants are made of water only;

for I took a vessel and put in it 200 pounds of earth dried in a furnace;

I planted in it the stem of a willow tree weighing 5 pounds; and at the end of five years the tree weighed 169 pounds and 3 ounces;

I had watered it with rain water or distilled water; and it had become large; and I had covered it with an iron plate covered with tin, but with many holes, lest dust that flew about mingled with the earth;

I did not compute the weight of the leaves that fell off in the four autumns;

at the end of the five years I dried the earth again, and I found it to be the same 200 pounds;

therefore, 164 pounds of wood, bark, and roots were made of water only (*id.*).

In several experiments van Helmont produced from solids, by combustion and fermentation, a wild spirit, hitherto unknown, which he called gas. He concludes:

out of 62 pounds of oaken coal 1 pound is ashes, 61 pounds are wild spirit;

this spirit, hitherto unknown, I call by the new name of gas;

this spirit can neither be contained in vessels

nor reduced to a visible body, unless its seed is extinguished; but bodies contain this spirit, and they sometimes wholly depart into it, and not because it is in these bodies, but because it is a spirit coagulated like a body (*id.*).

One and a half-odd centuries later this wild spirit, or gas, will be called carbon dioxide.

When Boyle, in 1661, in the preface to *The sceptical chemist*, reports that: he has not received any intentional assistance from any of his predecessors (*id.*), he adds: except from some passages of the bold and ingenious van Helmont (*id.*).

Snel's year of birth is 1580.

Snel discovered the law of refraction, in 1621, in a now lost manuscript. In *On the nature and properties of light*, in 1662, Voss – upon inspection of Snel's manuscript, Voss reports (16) – records Snel's discovery:

AB is the surface of a body of water; the eye is in air at *E*;

and if the eye looks at an object at *C*, then the ray of light is refracted at *D*, and *E* sees *C* as if it were at *F*, at the intersection of the continuation of *ED* and the perpendicular from *C* to *AB*, and so that $\frac{CD}{DF}$ is constant;

and this holds for all media (*id.*).

Or what is the same, $\frac{\sin i}{\sin r}$ is constant, where *i*

and *r* are the angles of the incident and refracted rays to the perpendicular to the surface dividing the two media.

Mersenne's year of birth is 1588.

Mersenne measured the speed of sound, at 448 m/s, in 1644.

Hobbes's year of birth is 1588.

Hobbes published the *Leviathan, or the matter, form, and power of the commonwealth ecclesiastical and civil* in 1651.

Desargues's year of birth is 1591.

Desargues proved Desargues's theorem in 1648.

Gassendi's year of birth is 1592.

Kepler first calculated transits of Mercury and Venus, in 1629. Gassendi first observed one, in 1631. *Mercury seen in the sun, Venus not seen.*

Gassendi revived scepticism, in the *Paradoxical exercises against the Aristoteleans*, in 1624. Bayle, in 1697, in the *Dictionary*, in the article *Pyrrho*, comments: Gassendi opened our eyes (*id.*).

Gassendi revived also atomism, in several treatises on Epicurus, and in the *Philosophical collection, logic, physics, ethics*, published in 1658. Bayle, in the article *Leucippus*, comments: when the chimerical qualities of the scholastics were banished, one course remained, to admit insensible particles of matter, whose shape, angles, hooks, motion, and place constitute the essence of bodies; the moderns have rejected the eternity of atoms and their fortuitous motion, but by otherwise adopting the doctrine of Leucippus they have built a very beautiful theory; this is what Gassendi has done (*id.*); the doctrine of vacuum Gassendi has made into a fashion (*id.*). Leibniz writes: Gassendi is right to reject the demonstrations of Descartes concerning god and the soul, but I find the opinions of Descartes fundamentally better even when they are not scientifically demonstrated (to Remond, Jul 1714). Gassendi wrote also the fifth set of objections to the *Meditations*. Descartes replies: I have been delighted that such a celebrated writer has in the course of a long and careful essay not managed to produce a single reasoned objection to my arguments (*id.*). Atomism, too, even if against its name, and against the fashion of vacuum, Descartes revived fundamentally better.

Descartes's year of birth is 1596.

It is much better never to search for truth than to search without the method.

And Descartes formulated the rules of the method.

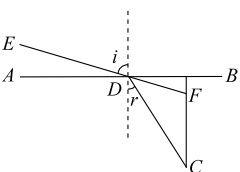
Mathematics is concerned with nothing but problems of order and measure, whether these involve numbers, or figures, or stars, or sounds, or anything else.

And Descartes greatly advanced analysis.

Galileo examines physical matters by mathematical arguments, and in this I agree with him entirely and hold that there is no other way to find truth.

All of corporeal nature is the object of pure mathematics.

By nature I understand not some goddess or



other imaginary power, but matter itself.

There are no secret forces, no miracles of sympathy or antipathy, nothing whatsoever in all of nature which cannot be attributed to purely corporeal causes entirely devoid of mind and thought.

God always conserves the quantity of motion in the world, and he always conserves the quantity of matter in the world.

From the single condition that god continues to conserve it as he created it follows that the changes in nature cannot be attributed to the action of god, they have to be attributed to nature. And the rules according to which these changes occur I call the laws of nature.

The laws of nature are laws of conservation.

And Descartes formulated the laws of nature.

Any thing by itself always perseveres in the same state, and hence, once moved, always continues to move. All motion is by itself in a straight line, and hence, a thing which moves in a circle always tends to recede from the center of the circle. A body, upon encountering a stronger body, loses nothing of its motion, upon encountering a weaker body, loses as much of its motion as it transfers to the weaker body.

And Descartes formulated the theory of vortices.

All matter of which this visible world is composed was initially divided by god into particles as nearly equal as possible. The particles had the same quantity of motion as there is in the world now and moved uniformly. As motion degenerated, however, the particles aggregated, composing two further kinds of particles. Some of the original particles constituted a second kind, larger and slower than the first, and some of the particles of the second kind constituted a third kind, larger and slower than the second. The stars are composed of particles of the first kind. The space between the stars is composed of particles of the second kind. The planets and comets are composed of particles of the third kind. There is no void in the world, all interstices of all bodies are filled by particles of the first kind.

As the particles had the same quantity of motion as there is in the world now and moved uniformly, they moved around its own center each, composing the fluid body which we pre-

sume the heaven is. Some moved together around certain points, others around other points. And so they composed as many vortices as there are now stars in the world.

The earth which we inhabit was formerly a star like the sun, in the center of its own vast vortex. But the vortex of the earth was smaller, and it ultimately descended into the larger vortex of the sun. And if we consider the earth a short time after its descent into the vortex of the sun, we can distinguish three regions, innermost, composed of matter of the first element, intermediate, composed of matter of the second element, and outermost, composed of matter of the third element; and this is the matter from which all bodies surrounding us originated.

Descartes writes also: light consists entirely in the force by which particles accumulate around the centers of the vortices of stars; and considered in itself, the force of light is nothing but a certain pressure along the straight lines from a star to the earth.

Nor is it necessary to suppose that there is in the body a soul, vegetative or sensitive, or any other principle of motion and life than its blood and its spirits agitated by the heat of the fire burning continuously in its heart, or to suppose that this fire is not of the same nature as all the fires in inanimate bodies.

Cavalieri's year of birth is 1598.

Cavalieri discovered integration in 1635.

Fermat's year of birth is 1601.

Fermat discovered differentiation in 1638.

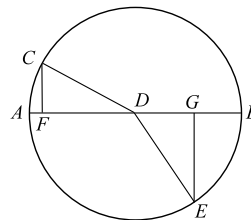
Fermat discovered also the principle of least time, in 1662, in the *Analysis and synthesis for refractions*.

This is the analysis:

let the diameter of a circle AB separate two media of different nature, the rarer above, the denser below; let D be the center of the circle, and CD an incident ray of light;

it is required to find the refracted ray DE , or the point E through which the ray passes after refraction;

drop the perpendiculars CF and EG ; since



point C is given as well as the diameter AB and the center D , point F is given;

let the ratio of the resistance of the denser medium to that of the rarer be the ratio of DF to some line x ;

it has to be that $x < DF$, because, by a most natural axiom, the resistance of the rarer medium has to be less than that of the denser;

we now have to measure the motions along CD and DE by means of DF and x ;

we can represent the entire motion along the two lines by the sum of the two products $CD \cdot x + DE \cdot DF$; and the problem has thus been reduced to dividing the diameter AB at point G such that, if the perpendicular GE is erected, and DE joined, the area $CD \cdot x + DE \cdot DF$ will be a minimum (*id.*).

Fermat solves the problem:

by our method which we have already distributed among the geometers (*id.*).

The method is differentiation. The solution is:

$$x = DG \text{ (} id. \text{)}.$$

Fermat has thus fixed the constant in Snel's law,

$$\frac{\sin i}{\sin r} = \frac{\rho_r}{\rho_i}, \text{ where } \rho_i \text{ and } \rho_r \text{ are the densities of}$$

the media. Fermat concludes the analysis:

derived from our principle, we have thus given to this theorem a rigorously exact demonstration (*id.*).

And this is the synthesis:

to establish the true law of refractions we need to know that the motion of light is easier and faster in rarer media than in denser;

and our demonstration rests on the single postulate that nature operates by the simplest and easiest ways and means; for this is how we believe it has to be enunciated, and not as it ordinarily is, when it is said that nature always operates by the shortest lines (*id.*).

Guericke's year of birth is 1602.

Guericke built the first air pump, and in 1657 demonstrated vacuum dramatically. The demonstration was reported in the same year by Schott. Boyle, in the *Experiments touching on the spring of air*, in 1660, comments: I was much delighted with this experiment since it rendered the great force of the external air more conspicuous than any other (*ded.*).

Guericke himself describes the experiment in

1672, in the *New experiments on vacuum in space*.

He writes:

I built a brass force pump, like those used to quicken a fire, with a piston and a plug worked very accurately so that air could not go in or out; in the pump I built two leather valves, the inner to draw water, the outer to force it out (III, 2).

Guericke filled a beer cask with water, and tried to evacuate it without allowing air to enter the cask. Air entered through the pores in the wood. He built a copper sphere, and this time tried to evacuate the air directly:

two strong men could barely pull the piston of the pump; and while they were still pulling it, the sphere suddenly collapsed like a cloth (3).

Guericke built a stronger sphere. The two strong men evacuated it, and:

this time vacuum was obtained; for no more air came out of the upper valve of the pump; and when the sphere was opened, air rushed in with such force as to pull in the man standing next to it (*id.*).

Guericke next built two hemispheres fitting tightly, with a diameter of about .5 m, and when this sphere was evacuated:

16 horses pulling in two teams of 8 could not separate the hemispheres (23).

Guericke also built the first electric machine. Also in the *New experiments on vacuum in space*, in book IV, *On the forces of the world*, he writes:

take a sphere of glass as large as a child's head; fill it with sulfur, and melt the sulfur over a fire; when it is cooled, break the glass, and take out the globe; mount it on an axle one palm high;

to show the attracting power in the globe, place under it little particles of gold, silver, paper, leaves, and other such things; rotate the globe, while rubbing it 2 or 3 times; and when it turns on its axle, it takes the particles with it;

to show the repelling power in the globe, hang a linen thread near it; and when you try to touch the thread with a finger, it moves away and cannot be touched (15).

Guericke has observed that electricity is positive and negative. Also:

when it is vigorously rubbed, the globe glows in the dark (*id.*).

Guericke has observed electroluminescence, too.

Torricelli's year of birth is 1608.

Torricelli first demonstrated vacuum, in 1644, and measured air pressure.

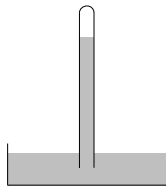
He writes:

we made philosophical experiments to demonstrate not only vacuum, but changes in the air, that it is now heavier and denser, now lighter and thinner;

we reasoned thus;

we live immersed at the bottom of a sea of elemental air which has been shown by experiment to have weight, and so much weight that the densest air close to the surface of the earth weighs $\frac{1}{400}$ of the weight of water;

we made a tube 2 cubits long, with one end closed, the other open; we filled it with mercury, closed the open end with a finger, and immersed it in a vessel filled with mercury; and we saw an empty space form at the upper end;



to explain this, some argue that the force which prevents the mercury from falling down is in the vessel; but we assert that it comes from without; for water in a similar tube, though much longer, rises to about 18 cubits, and that is, as much higher than the mercury as mercury is heavier than water; one and the same cause therefore acts on both; this makes it certain that the force comes from without;

and with this principle we explain all repugnances ascribed to vacuum (to M. Ricci, 11 Jun 1644).

1 cubit is about .5 m.

Kant writes in the *Critique*: when Torricelli made air bear a weight equal to that of a column of water, new light dawned on all investigators of nature.

Wallis's year of birth is 1616.

Wallis published the *Arithmetic of infinities* in 1656.

Grimaldi's year of birth is 1618.

Grimaldi discovered diffraction, in the *Physico-mathematical treatise on light, colors, and the rainbow, as well as related matters*, published in 1665.

Grimaldi begins with:

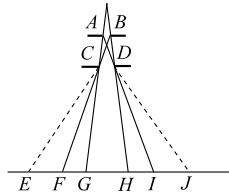
proposition 1. light is propagated and diffused not only directly, by reflection, and by refraction, but in a fourth way as well, by diffraction (I, 1).

He proves the proposition by:

two experiments (*id.*).

The second is:

the very small hole AB was made in the window shutter of a well darkened room, and through it sunlight was admitted, forming the cone of light with base GH ;



at some distance from AB a plate with the small hole CD was placed, through which the light of the first cone was admitted, and so that the base of its own cone FI would be considerably greater than the hole CD ;

and when the light was cut orthogonally by a white surface, it showed the illuminated base EJ much greater than the rays would have formed if they traveled in straight lines through the holes; this is proved by observing the base EJ and calculating how large the base FI would have been if it were formed by straight rays (*id.*).

In the first experiment Grimaldi had also placed an opaque body in the middle of the cone of light admitted through a very small hole, and:

in the field outside its shadow bands of colored light were seen, and such that in the middle of each the light was pure, but on the side nearer the shadow bluish, and on the other side reddish (*id.*).

Grimaldi has observed yet another of the ways of light, interference.

Before Galileo and Kepler optics is the investigation of perspective. The investigation of light begins with Galileo and Kepler. Kepler argued that light is a body. Descartes argued that light is a force.

The full title of Grimaldi's treatise, in 1665, is *A physico-mathematical treatise on light, colors, and the rainbow, as well as related matters, in two books, of which in the first new experiments are shown and reasons are deduced from them for the substantiality of light, but in the second the arguments adduced in the first are refuted and the probability of the Peripatetic doctrine of the accidentality of light is upheld.*

Newton showed that light is a body in 1672. Huygens showed that it is a wave in 1678. Not quite two and a half centuries later, quantum theory will show that it is both. In the meantime, first the doctrine of substantiality, or body, prevailed. Then, after Young and Fresnel, between 1801 and 1821, the doctrine of accidentality, or wave, prevailed. The strongest evidence for accidentality, or wave, and against substantiality, or body, were Grimaldi's discovery, diffraction, and interference, demonstrated conclusively by Young in 1807.

Picard's year of birth is 1620.

Picard measured the radius of the earth, at 6 372 km, in 1671.

Pascal's year of birth is 1623.

Pascal measured atmospheric pressure, in the *New experiments concerning vacuum*, in 1647, and *The grand experiment on the equilibrium of liquids performed on one of the highest mountains of Auvergne*, in 1648.

Pascal also correlated the phenomena of liquid equilibrium, and of pressure in liquids and in air, and discovered the law of pressure of confined fluids.

On the equilibrium of liquids and *On the weight of the mass of air* were published in 1663.

In *On the equilibrium of liquids*, Pascal writes: the force necessary to prevent water from escaping by an opening is proportional to the height of the water and not to its expanse, and the measure of the force is the weight of water in a column as high as the enclosed water and of the same area as the opening (1).

He demonstrates this experimentally, and concludes:

it is evident that a vessel full of water is a machine for the multiplication of force (2).

In *On the weight of the mass of air*, he lists:

all the effects commonly ascribed to nature's abhorrence of vacuum (1),

notes:

we will now show that they are all due to the weight of air (*id.*),

does so experimentally, and concludes:

every place is pressed by the weight of air;

places on the seashore are pressed by air as if they were by a column of water 32 feet and 2 inches high;

places 10 toises higher by 31 feet and 1 inch of water;

places 500 toises above sea level by 26 feet and 11 inches of water;

and so on (8).

1 toise = 1.949 m. And:

we see that the whole mass of air in the world can be treated as water 31 feet high rarefied and expanded until it changed into what we call air;

to determine this whole mass, if we assume that air presses the earth uniformly with the weight of 31 feet of water, the error will not be greater than $\frac{1}{2}$ foot of water; and then nothing is

easier than to calculate the weight of the whole mass, a child who knows addition and subtraction can do it;

1° of the circumference of the earth covers 50 000 toises;

there are 6 feet to a toise;

the surface of the earth is thus 3 711 420 000 000 000 square feet;

a cubic foot of water weighs 72 pounds;

a prism of water with a base 1 foot and height 31 feet thus weighs 2 232 pounds;

and therefore the whole mass of the sphere of air in existence weighs 8 283 889 440 000 000 000 pounds (9).

1 pound = 392.5 g.

Cassini's year of birth is 1625.

Cassini measured the distance of the sun, at 21 600 earth radii, in 1684.

Redi's year of birth is 1626.

Redi showed that there is no spontaneous generation in 1668.

Boyle's year of birth is 1627.

Boyle destroyed the doctrines of the four and of the three elements. In *The sceptical chemist, or chemico-physical doubts and paradoxes, touching the experiments whereby vulgar spagyrist are wont to endeavor to evince their salts, sulfur, and mercury, to be the true principles of things*, in 1661, he showed that no substance has been decomposed into earth, water, air, and fire, or mercury, sulfur, and salt. For instance:

there are no mixed bodies from which it has been shown that any degree of fire can separate salt, or sulfur, or mercury, much less all three; the most obvious instance is gold, which is a body so

fixed, and in which the elementary ingredients, if any, are so firmly united, that in no operation in which gold is exposed to fire, howsoever violent, it does so much as lose of its weight; so far is it from being dissipated into those principles (1).

In *The sceptical chemist* Boyle also defined element:

to prevent mistakes, I must advertise you that I mean by elements, as those chemists who speak the plainest do by their principles, certain primitive and simple or perfectly unmingled bodies, which, not made of any other bodies or of one another, are the ingredients of which all perfectly mixed bodies are immediately compounded and into which they are ultimately resolved.

Boyle began the attempts to ground the atomic theory in experiment. In 1661, in the *Certain physiological essays and other tracts*, the third of the eight essays is *Specimens of an attempt to make chemical experiments useful to illustrate the notions of the corpuscular philosophy*. In 1666 he wrote *The origin of forms and qualities according to the corpuscular philosophy, illustrated by considerations and experiments*.

Boyle showed that air is necessary for combustion and for the transmission of sound. In 1660, the *New physico-mechanical experiments, touching on the spring of air, and its effects, made for the most part in a new pneumatic engine* is a collection of 43 experiments. Boyle shows that:

in our evacuated receiver the attempt to fire the gunpowder in the pan of a pistol succeeded not (14), and:

when we tried to fire gunpowder with a pistol in our evacuated receiver, the noise made by the striking of the flint against the steel was exceedingly languid (27).

And in *The origin of forms* Boyle concluded that sound is a wave propagated in the air:

when a thing makes a percussion of the air, it puts that fluid body into an undulating motion; and if a man's ear chances to be in the way of these motions of the air forward and backward, it gives him the perception of them which he calls sound (I, 2).

The main object of the *New physico-mechanical experiments* is:

to show that air has elastic power (6).

And in *A defense of the doctrine touching the spring and weight of air*, in 1662, Boyle established that at constant temperature the volume of air is inversely proportional to its pressure.

He writes:

we took a glass tube crooked at the bottom with the two legs parallel to one another, and its length divided into inches;

we poured mercury until it was 29 inches higher in the long leg than in the short, this being the weight of the incumbent atmosphere, as Pascal has shown and as we are taught by Torricelli's experiment;

then more and more mercury was poured into the long leg, notice being taken watchfully how far the mercury had risen in the long leg when it had reached each division in the short leg; the observations afforded us the following table (5). It is:

a table of the condensation of air (*id.*).

In the table Boyle lists:

that height of mercury in the long leg which compressed the air in the short leg to each division;

the sum of that height and $29\frac{2}{16}$, which is the height of mercury counterbalanced by the pressure of the atmosphere;

and what the pressure should be on the hypothesis that the pressure and the expansion are inversely proportional (*id.*).

The first six of the twenty-five lines of the table are:

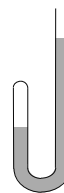
0	$29\frac{2}{16}$	$29\frac{2}{16}$,
$1\frac{7}{16}$	$30\frac{9}{16}$	$30\frac{6}{16}$,
$2\frac{13}{16}$	$31\frac{15}{16}$	$31\frac{12}{16}$,
$4\frac{6}{16}$	$33\frac{8}{16}$	$33\frac{1}{7}$,
$6\frac{3}{16}$	$35\frac{5}{16}$	35,
$7\frac{14}{16}$	37	$36\frac{15}{19}$ (<i>id.</i>).

On Boyle's law all subsequent investigation of fluids and of heat stands.

Ray's year of birth is 1627.

Ray described 18 655 species in the *History of plants*, published in 1710.

Malpighi's year of birth is 1628.



1228 In order to formulate the problems we are to solve we call the council of our predecessors

Malpighi published *On the lungs* in 1661.

Huygens's year of birth is 1629.

Huygens discovered the ring of Saturn.

Determined the true proportions of the solar system.

Discovered the isochronism of the cycloid.

Solved the problem of collisions.

Discovered the conservation of momentum.

Determined the acceleration of gravity.

Reformulated the laws.

Any body already in motion, if nothing obstructs it, continues to move perpetually with the same speed in a straight line. When two bodies equal to one another and with equal speed collide directly, each rebounds with the same speed with which it has arrived. And when two bodies collide, even if each of them is simultaneously involved in some other equal motion, they do not act on one another with respect to that common motion at all, but in all ways act as if that motion is absent.

Formulated the wave theory, discovered the constancy of the speed of light, and the principle of wave fronts.

Light consists in the motion of matter. But this motion cannot be the transport of matter, because this will violate the principal properties of light. Light is a motion impressed on matter. And if it takes time for its passage, it follows that this motion is successive, and that light spreads, like sound, in spherical waves.

If the motion of light were to become slower in proportion to the distance from its source, it could not conserve its great speed over great distances. But if the aethereal matter, which propagates light, is supposed elastic, its particles will have the property of equally rapid rebound whether they are struck strongly or feebly, and light will always propagate with equal speed.

Every point of a wave front is itself the source of a wave.

The geometers prove their propositions from certain and incontestable principles. Here the principles are verified by the conclusions drawn from them. The nature of things will not have it otherwise. It is nevertheless always possible to attain a degree of probability which falls short of complete evidence in nothing. For if the things demonstrated from the assumed principles corre-

spond perfectly to the phenomena which experience has manifested, the more so if the number of these phenomena is great, and the more so if new phenomena have been foreseen from the hypotheses, and effects have been found to correspond to the expectations, if there are all these proofs of probability, this must be a grand confirmation of the success of the investigation.

Richer's year of birth is 1630.

Richer discovered that the beat of a pendulum is slower at the equator than further north in 1679.

Spinoza's year of birth is 1632.

God is nature.

Philosophy has no end but truth, faith has no end but obedience and piety. The ground of philosophy are axioms obtained from nature itself, the ground of faith are histories and words obtained from scriptures and revelation. Between faith and philosophy there is no connection whatsoever, no relation whatsoever.

Whatever is contrary to nature is contrary to reason. Whatever is contrary to reason is absurd, and therefore to be rejected.

Spinoza showed when and how the Hebrew scriptures were authored.

The method of history does not differ from the method of interpretation of nature at all, but agrees with it entirely.

Locke's year of birth is 1632.

Locke wrote *An essay concerning human understanding*, in 1690.

He writes:

it is an established opinion amongst some men that there are in the understanding certain innate principles, primary notions, κοινὰ ἔννοιαι, characters, stamped as it were on the mind of man, which the soul receives in its very first being and brings with it into the world;

but to convince unprejudiced readers of the falseness of this supposition, it is sufficient to show how men, by the use of their natural faculties alone, may attain all the knowledge they have without the help of any innate impressions, and may arrive at certainty without any such original notions or principles (I, 2.1);

I shall begin with those magnified principles of demonstration, whatever is is, and, it is impossible for the same thing to be and not to be, which

of all others, I think, the most hold a title to innate (1.4);

it is evident that all children and idiots have not the least apprehension of them; and this want is enough to destroy that universal assent which must be the necessary concomitant of all innate truths, as it seems to me nearly a contradiction to say that there are truths imprinted on the soul which it does not perceive or understand (1.5);

let us then suppose the mind to be, as we say, white paper, void of all characters, without any ideas; how does it come to be furnished? whence does it come by that vast store which the busy and boundless fancy of man has painted on it with an almost endless variety? whence does it have all the materials of reason and knowledge?

to this I answer in one word, from experience;

in this all our knowledge is founded, and from this it ultimately derives itself; our observation employed either about external objects or about the internal operations of our minds, perceived and reflected upon by ourselves, is what supplies our understanding with all the materials of thinking; these two are the fountains of knowledge from which all ideas we have, or can naturally have, spring (II, 1.2).

Leeuwenhoek's year of birth is 1632.

Leeuwenhoek discovered protozoa in 1676.

Hooke's year of birth is 1635.

Hooke discovered the cell, in the *Micrographia*, in 1665.

In 1674, in the *Attempt to prove the motion of the earth from observations*, he formulated:

three suppositions;

first, all celestial bodies whatsoever have an attraction, or gravitating power, toward their own center by which they attract not only their own parts, and keep them from flying away, as we observe in the earth, but attract also all other celestial bodies that are within the sphere of their activity;

second, all bodies whatsoever put into a direct and simple motion will continue to move forward in a straight line until they are deflected and bent by some other powers into a motion describing a circle, ellipse, or some other curve;

third, these attractive powers are the more powerful in operation the nearer the body on which they act is to their center.

Hooke discovered also the law of elasticity.

In 1678, in *On the restoring force*, he writes: as the theory is very short, so the way of trying it is very easy;

take a quantity of evenly drawn wire, steel, or iron, or brass; coil it on an even cylinder into a helix of any length you please; turn the ends of the wire into loops; suspend this coil on a nail; at the other end suspend weights to extend it, and observe exactly to what length beyond the length to which its own weight stretches it each weight extends the wire;

you will find that, if 1 ounce extends it 1 line, 2 ounces extend it 2 lines, 3 ounces 3 lines, and so on;

and this is the rule or law of nature on which all restoring motion proceeds, whether of rarefaction, or extension, or condensation and compression (*id.*).

In the *Micrographia*, Hooke writes also: there is no such thing as an element of fire; that shining body which we call flame is nothing but a mixture of air and volatile sulfurous parts of dissoluble or combustible bodies (16).

Burnet's year of birth is 1635.

Burnet wrote *The sacred theory of the earth, containing an account of the origin of the earth, and of all the general changes which it has already undergone, or is to undergo, until the consummation of all things* in 1681.

Swammerdam's year of birth is 1637.

Swammerdam discovered the stages in the development of insects in 1669.

Steno's year of birth is 1638.

Steno showed that fossils are the remains of animals and plants more ancient than the rocks in which they are embedded, in 1669.

James Gregory's year of birth is 1638.

James Gregory calculated the distance of Sirius, at 83 190 distances of the sun, in 1668.

Newton's year of birth is 1642.

The entire difficulty in philosophy is this, from the phenomena of motion to discover the forces of nature, then from these forces to demonstrate the other phenomena.

All bodies are composed of particles. All the smallest particles are extended, hard, impenetrable, mobile, and endowed with inertia. The smallest particles may cohere by the strongest

attractions and compose bigger particles of weaker virtue, these may cohere and compose bigger particles whose virtue is still weaker, and so on, until the progression ends in the biggest particles on which the operations of chemistry and the colors of natural bodies depend, and which compose bodies of sensible magnitude.

I have shown that the revolutions of bodies in conic sections and the centripetal forces composing spherical bodies are effects of the same law. All particles in all bodies attract one another and all bodies attract one another.

The attractions of bodies must be ascertained by assigning the forces properly to their particles.

Fluid is every body whose parts yield to any force and are moved easily among themselves. Resistance in fluids arises in part from tenacity, in part from friction, in part from density. I have shown that the part from density is as the square of the velocity, the part from tenacity is uniform. I have demonstrated the properties of elastic fluids consisting of particles which repel one another.

Heat is proportional to rarefaction. And if the heat of air in winter when water begins to freeze is 0, and the heat by which water begins to boil 33, the heat by which tin is melted is 72, the heat by which bismuth is melted 81, the heat of live coals in a fire made of bituminous coals 192.

Light is a body. The analogy between the propagation of rays of light and the motion of bodies holds for reflection, refraction, the propagation of light, its speed, and diffraction. Light is a heterogenous mixture of differently refrangible rays. Color is not a qualification of light from refraction or reflection, but an original property, different in different rays.

Newton formulated the laws conclusively.

The laws of motion.

Every body perseveres in its state of rest or uniform motion in a straight line, unless it is compelled to change its state by forces impressed on it. Change in motion is proportional to the motive force impressed, and is in the straight line in which that force is impressed. And the actions of two bodies on one another are always equal and opposite in direction.

And the law of gravity.

$$F \propto \frac{m_1 m_2}{r^2}.$$

He deduced, among the 635 propositions, lemmas, and corollaries of *The principles*, the law of falling bodies, the law of the pendulum, the principle of relativity, Kepler's laws, Snel's law, diffraction, Boyle's law, the laws of collision, the conservation of momentum. Pascal's law and Hooke's law also follow from Newton's laws.

He deduced also, also among the 635 propositions, lemmas, and corollaries of *The principles*, the system of the world. The motion of a pendulum in Cayenne, the motion of the moon, the motion of Saturn, the motion of comets, the motion of the solar system, the motion of the systems of the fixed stars, one cause from the center of the world to infinity.

There is gravity in bodies universally.

Rule 1 is: to admit no more causes of things than are true and suffice to explain the phenomena.

And do not bodies and light act mutually on one another? do not all bodies, when heated, emit light? is not this emission performed by the vibrating motions of their parts? is not fire a body heated so hot as to emit light copiously? are not the sun and fixed stars great earths vehemently hot, whose heat is conserved by their greatness, action and reaction in them, and the weight and density of their atmospheres?

And do not black bodies conceive heat more easily from light than those of other colors, because the light falling on them is not reflected, but enters them, and is reflected and refracted in them until it is stifled and lost?

And are not bodies and light convertible into one another, and may not bodies receive much of their activity from the particles of light which enter into their composition?

Newton also discovered calculus.

Rømer's year of birth is 1644.

Rømer measured the velocity of light, at 2.14×10^8 m/s, in 1676.

Leibniz's year of birth is 1646.

I lay down the true elements of a new science of power and action, for the discovery of the admirable laws of nature of the forces of bodies and their actions on one another, and for their reduction to their causes, which can be called dynamics.

In corporeal things there is something prior to

extension, namely, force.

A force to act and react is the very substance of things.

The conservation of the same force in the world is an inviolable truth.

And I will now reduce the whole to three simple and beautiful equations. The linear equation expresses the conservation of relative speed,

$$v - y = z - x.$$

The plane equation expresses the conservation of progress,

$$av + by = ax + bz.$$

And the solid equation expresses the conservation of absolute force,

$$avv + byy = axx + bzz.$$

Absolute force is twice the kinetic energy.

Descartes formulated: the project for a general science capable of raising our nature to its highest degree of perfection.

Not quite two generations later, Leibniz developed the philosophical calculus.

Two things have to be distinguished in all arguments, the form and the matter. A calculus which deals with the form can thus be separated from a calculus which deals with the matter. In a calculus which deals with the form, the quantity, quality, and truth of propositions will be expressed formally. Truth or falsehood will then be judged by form alone. By calculation.

All human thoughts can be resolved into a few primitives. If characters are assigned to these, characters can be formed for notions derivative from them. It will be then possible to find definitions, or values, and to demonstrate the properties implied by the definitions. And it will be then possible to arrange propositions in their natural order.

The philosophical calculus will contain the true organon of the general science of all things under human reasoning, always being at the same time a calculus of evident demonstrations. It will exhibit also the order of the sciences, and it will become clear that the elements of arithmetic are prior and simpler than the elements of the logical calculus of the figures and modes of reasoning.

And it may seem to some that it will be of little use in civil history. I reply, as powerful as reason is in it, so powerful and much more powerful will be the philosophical calculus. It will

make it possible to reach a conclusion where without it no result at all can be obtained from the available data. It will make it possible either to attain an infinite approximation or to determine the degree of probability.

The observation of mathematical analysis showed me the truest and most beautiful examples of this general analysis of human thought. In arithmetic and algebra all reasoning consists in the use of characters, and an error of the mind is the same as an error of calculation.

There are things that cannot be weighed. There are things that cannot be measured. But there is nothing that cannot be numbered. Number is the metaphysical form, and arithmetic is the statics of the world, by which the powers of things are explored.

Leibniz discovered also that the principle of sufficient reason is the other, next to the principle of contradiction, principle of all principles.

Leibniz also organized calculus into a discipline, collected its rules into a system, and gave it a crystal clear explanation.

Halley writes:

we are now admitted to the banquets of the gods,
we know the laws of heaven, to which the earth
was blind before, and the immutable order of the
things,

hidden throughout the ages of the world.

Leibniz writes: we have achieved what the ancients could hardly envision in their prayers, we have reduced by geometrical analysis the primary phenomena of the world to the simplest and clearest principles.

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Number, the metaphysical form, can show best how great the advance in the acquisition of knowledge in these three generations is.

Eratosthenes measured the radius of the earth at 6 400 km. Ptolemy calculated the distance of the sun at 1 210 earth radii. And he calculated the distance of the sphere of the fixed stars at 17 distances of the sun.

Copernicus showed that the earth revolves around the sun. Tycho showed that the fixed stars are not fixed.

Then, in these three generations, Picard measured the radius of the earth at 6 372 km. The

radius is 6 378 km. Cassini measured the distance of the sun at 21 600 earth radii. The distance is 23 455 earth radii. James Gregory calculated the distance of Sirius at 83 190 distances of the sun, Newton calculated the distance of a star of the first magnitude at 950 000 distances of the sun, Huygens calculated the distance of Sirius at 28 000 distances of the sun. Proxima Centauri is 270 000 distances of the sun away.

In meters, Proxima Centauri is 10^{16} m away. James Gregory, Newton, and Huygens calculated the distance of the stars at 10^{15} to 10^{17} m.

The big bang occurred 1.4×10^{10} years ago, entailing the limit of observation at 4.6×10^{10} light years away. In meters, 10^{26} m away.

Of the distance up to the limit of observation not less than one half was covered in these three generations.

Thus in the large.

In the small, the ancients neglected the measurement of the small. Archimedes, in order to pack the world with sand, postulated that 10 000 grains of sand will pack a sphere with the diameter of a poppy seed, and measured a poppy seed at just over $\frac{1}{25}$ of a finger. In meters, 10^{-3} m.

Hooke counted 60 cells in the 18th part of 1 inch. One cell is thus 10^{-5} m. As cells are, 10^{-5} to 10^{-4} m. Leeuwenhoek counted not less than 1 000 animalcules in one drop of water. His lenses of magnifying power of 500 and resolving power of 1 μ imply that one animalcule is 1 μ . 10^{-6} m. As smaller protozoa are, 10^{-6} m.

The nucleus of the hydrogen atom is 10^{-15} m.

Of the distance down to the nucleus of the hydrogen atom not less than one quarter was covered in these three generations.

Further down, quarks and leptons are waves.

The theory of waves was formulated in these three generations.

In time, the advance is not as dramatic as it is in space quantitatively, but it is not less dramatic qualitatively.

Earlier, god made heaven and earth four millennia before he sent forth his only begotten son.

Descartes formulated a theory of the formation and development of the stellar systems, one of which the solar system is, and of the earth within

the solar system.

Huygens calculated that a cannonball traveling 100 toises in 1 second will spend 700 000 years in its journey to the nearest fixed star.

Newton calculated that a globe of incandescent iron equal to the earth will scarcely cool in 50 000 years.

Rates of cooling remained the method of determining the age of the earth until Rutherford.

In these three generations her deeply hidden secrets began being vexed out of nature.

Newton calculated the eccentricity of the earth at 0.004. The eccentricity is 0.003. Huygens measured the acceleration of gravity at 9.80 m/s^2 , Newton measured it at 9.81. The acceleration is 9.81. Newton measured the mass of Jupiter at $\frac{1}{1067}$ of that of the sun. The mass is $\frac{1}{1048}$ of

that of the sun. Mersenne measured the speed of sound at 448 m/s, Huygens measured it at 351. The speed is 343. Rømer measured the speed of light at $2.14 \times 10^8 \text{ m/s}$. The speed is 3.00×10^8 .

Truth results from the discovery of the law with its clauses.

The law with its clauses was discovered in these three generations. The law of falling bodies. The law of the pendulum. The laws of the motions of the heavenly bodies. The gas law. The law of elasticity. The principle of conservation of momentum. The laws of motion. The law of gravity. The conservation of absolute force.

The world is written in mathematics.

Analysis was developed, calculus was discovered, and functions entered the richest in consequences stage in their development in these three generations.

There is nothing comparable to *The mathematical principles of natural philosophy*, too, in either scope or power, before or after.

Three causes can be assigned.

First, more remotely, the great accumulation of the doctors of theology and their books which began five and a half centuries before Bacon, Galileo, and Kepler. Second, and decisively, the great acceleration in the exploration of the surface of the earth which began with prince Henry, one and a half-odd centuries before Bacon, Galileo, and Kepler. Third, immediately, the acquisition of knowledge in the century before Bacon, Gali-

leo, and Kepler, as it accelerated, and more, and more, and the acceleration encompassed the acquisition in its entire depth and breadth.

First.

The darkest of dark ages came to an end in the century-odd from Peter Damian to Gerard.

Peter Damian wrote the *Letter on divine omnipotence*. Daring to argue against the blessed father of the church.

In the next generation, Anselm proposed to prove the existence of god.

In the next generation, Abelard wrote the *Yes and no*.

In the next generation, Gerard translated Aristotle, Autolycus, Euclid, Archimedes, Diocles, Menelaus, Ptolemy, Galen, Jabir ibn Hayyan, al-Farghani, al-Khwarizmi, al-Kindi, banu Musa, Thabit ibn Qurra, al-Razi, al-Farabi, ibn Sina.

The dark age was over.

In the generation between Abelard and Gerard, Hugh of St Victor wrote the *Instruction, on the study of reading*. Hugh proposes: to teach the reader about the secular writings as well as the holy scriptures (pr.). And the *Instruction* is in six books, in the first three Hugh teaches the reader about the secular writings, in the last three he teaches him about the holy scriptures.

Hugh begins in the preface:

there are many, whose nature is so lacking in ability, they can hardly grasp the easiest things; but they are of two kinds; some, unaware of how dull they are, nevertheless struggle tirelessly after knowledge, and these deserve to obtain by their perseverance what they cannot otherwise obtain; others, who know they cannot reach the highest things, neglect even the lowest, and lose the light of truth altogether;

there are also some, whom nature has endowed with the full measure of ability and to whom she has given an easy path to truth; but among these, too, there is not always the will and virtue to cultivate what nature has given them; some, caught in the affairs of the world, bury god's gift in the earth; and these are utterly detestable; others lack the wealth needed to devote themselves to learning; but even these cannot be completely excused, for we have seen many who, although hungry and naked, have attained the

fruit of knowledge;

everyone acquires knowledge by two things, reading and thinking; of these two, reading is first; and it is reading that this book teaches, setting forth the rules for it; for three things are necessary to learn to read; first, every man ought to know what to read; second, he ought to know in what order to read it, what first, what next; third, he ought to know how to read it (pr.).

Many can hardly grasp the easiest things. Some of these nevertheless struggle tirelessly after knowledge, others lose the light of truth altogether. To some nature has given an easy path to truth. Some of these bury god's gift in the earth, others lack the wealth to devote themselves to learning.

Hugh does not presume that there are such as will follow the path of truth by themselves. And how wise Hugh is in this, the intensity and duration of the darkest of dark ages suffices to show.

Hugh proposes to teach all those kinds. The many who can hardly grasp the easiest things, to reward their perseverance or so that they do not lose the light of truth altogether. And those whom nature has endowed with ability, so that they do not bury god's gift in the earth, and those who lack the wealth needed to devote themselves to learning, too.

And the success of Hugh and his successors in teaching them all was spectacular. Three centuries after Hugh, in the fifteenth Christian century, there were some 100 universities, from St Andrews in the north to Catania in the south to Lisbon in the west to Kraków in the east.

Very many more than ever before were taught. The sheer number of universities attests to this fact. More important, very many more of all kinds were taught, the dull struggling tirelessly, those who would have otherwise lost the light of truth altogether, those who would have otherwise buried god's gift in the earth, and some of those who lacked the wealth to devote themselves to learning, too. The sheer number of universities attests to this fact, too.

Still more important, very many more than ever before taught them. The growth of universities implies tens of thousands. Yet more important, these, the ones who taught, were of all kinds as well, and of all the same kinds – the dull

struggling tirelessly, those who would have otherwise lost the light of truth altogether, those who would have otherwise buried god's gift in the earth, and some of those who lacked the wealth to devote themselves to learning. The sheer fact of the universities, in which the teachers had been students, attests to this fact.

And most important, they were all searching for truth. As Hugh describes them: devoting themselves to learning, struggling tirelessly after knowledge, searching for the fruit of knowledge, searching for the light of truth, searching for the path to truth.

To this fact the very many books they wrote, and left behind, attests.

These are the doctors of theology, and the doctors of theology wrote, and left behind, very many more books than had been written, and left behind, before.

The number of their books the number of teachers can indicate. The teachers, to become teachers, all wrote.

The magnitude of their books the most successful can indicate, the crowning achievements of the labors of the angelic doctor, of the subtle doctor, and of the singular and invincible doctor. The angelic doctor wrote the *Theological summa*, in 2 669 particular questions. The subtle doctor wrote the *Ordination*, in a prologue and 141 distinctions. The singular and invincible doctor wrote the *Summa of all logic*, in 283 chapters. Each of the three not less than a half dozen massive volumes.

The scope of their books those of the angelic doctor can indicate. The *Theological summa* remained incomplete when he died. Before it, he wrote commentary on the *Sentences*, *On the principles of nature*, *On being and essence*, *On truth*, *Quadlibetal questions*, *On Boethius on the seven days*, *On Boethius on the trinity*, *Summa against the gentiles*, *On power*, *Against the errors of the Greeks*, *On buying and selling*, *On the guidance of the principles*, commentary on the *Physics*, commentary on the *Metaphysics*, commentary on the *Nicomachean ethics*, commentary on *On the soul*, commentary on the *Politics*, *On the eternity of the world against the murmuring*, *On the unity of the intellect against the Averroists*, *On evil*, *On spiritual creatures*,

On the soul, *On the unity of the incarnate word*, commentary on *On causes*, commentary on the *Meteorology*, commentary on *On interpretation*, *On the mixtures of the elements*, *On the motion of the heart*, *On virtues*, commentary on *On the heaven*, commentary on *On generation and corruption*. He died not yet 50.

It was in these centuries that the search for truth became routine. That very many devoted themselves to learning, struggled tirelessly after knowledge, searched for the fruit of knowledge, the light of truth, the path to truth. More important, very many of all kinds. Most important, writing very many books. Increasing greatly, and steadily more as the accumulation of doctors and books continued, the odds that those whom nature has endowed with the full measure of ability and to whom she has given an easy path to truth will embark on the path.

The whole thing is done by machinery. Following the false way not less than following the true. And counting back to Abelard, Hugh of St Victor, and Gerard, by the generation of Bacon, Galileo, and Kepler the machinery had been under full steam for four and a half-odd centuries.

Second.

The relation of the great accumulation of the doctors of theology and their books and the great acceleration in the exploration of the surface of the earth which begins with prince Henry is causal.

Prince Henry endeavored to gain knowledge of the lands beyond cape Bojador. As no one before him did.

Prince Henry's knights endeavored to gain knowledge. Ca' da Mosto, who reached the Bissagos Islands in 1456, wrote *The first navigation of the ocean to the land of the blacks*. Ca' da Mosto reports: I heard that prince Henry was sending ships to navigate the seas which had never been sailed, and discovering many lands of strange peoples where marvels abounded (2); and I was seized by an ardent desire to go; I was young, ready to suffer any hardships, eager to see the world and things never before seen by anyone, hoping also for honor, and gain (*id.*).

237 embarked in 1519 with Magellan on the circumnavigation of the earth. 18 completed the circumnavigation in 1522 under El Cano. One of

the 18 is Pigafetta. Pigafetta wrote *The voyage around the earth*. Pigafetta reports: having learned many things from books and listened to those who spoke of the great marvels of the ocean sea, I decided to go and see those things for myself, and thus also win my renown among the later posterity (1).

One generation before prince Henry, d'Ailly is a doctor of theology *par excellence*. D'Ailly wrote on the soul and on the penitential psalms, was a chancellor of the university of Paris, developed the councilar doctrine – church councils have authority over the pope – was one of the bishops who led the council of Pisa in 1409, and one of the cardinals who led the council of Constance in 1414-8, which ended the great schism of the papacy, and condemned Hus, d'Ailly himself presiding over the trial of Hus.

D'Ailly also wrote the *Representation of the world*. Twelve and a half centuries after Ptolemy, d'Ailly's *Representation* is a decisive step in the restoration of geography. D'Ailly's *Representation* also led Columbus across the Atlantic.

Prince Henry's base was Lagos. Eanes rounded cape Bojador in 1434. Cape Bojador is 1 300 km along the great circle from Lagos. El Cano completed the circumnavigation of the earth in 1522. The circumference of the earth along the equator is 40 100 km.

And the earth was mapped.

The exploration of the surface of the earth in these three generations was the most massive success the search for truth had ever had.

The exploration of the earth became routine. Success in the exploration of the earth and mapping it became routine. Success in the search for truth became routine.

The whole thing is done by machinery. Prince Henry set the machinery on the path of success.

The accumulation of the doctors of theology and their books increased greatly, and steadily more, the odds that those whom nature has endowed with the full measure of ability and to whom she has given an easy path to truth will embark on the path. The acceleration in the exploration of the surface of the earth showed them success.

By the generation of Bacon, Galileo, and Kepler the machinery had been yielding success

for one and a half-odd centuries.

Third.

The generation of El Cano and Magellan is the generation also of Copernicus, Biringuccio, Paracelsus, Agricola, and Cardano.

The next is the generation of Vesalius, Gesner, Belon, and Cesalpino.

The next is the generation of Viète, Gilbert, Tycho, and Stevin.

And by the next generation, that of Bacon, Galileo, and Kepler, the acquisition of knowledge had been accelerating in its entire depth and breadth, and more, and more, for one century.

The whole thing is done by machinery. And in the next three generations the machinery was brought to perfection.

The greatest diligence was given to natural philosophy. Natural philosophy possessed men wholly. The end was right, which is discoveries. The means was right, which is experiment. The true way was not just embraced, it was followed as staunchly as the thread of Theseus must be followed in the labyrinth.

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The advance in the formulation and solution of the problem of the principles of all things was proportionately great.

The elements of all things.

There remained but one remedy, and Bacon undertook the entire work of the understanding anew.

He prepared a natural and experimental history, sufficient and good, the foundation of the thing. And concluded: the notion of fire is worthless, heat is a rapid motion of minute particles, and not of the smallest, but of those a degree larger.

Concerning the oldest object Galileo set forth the entirely new science.

In the new science he considered also the constitution of bodies. And concluded: penetrating through the smallest voids in the metal, through which the smallest particles of air and other fluids cannot pass, the particles of fire free the particles of the metal from the force with which the voids attract the particles and prevent their separation.

Kepler transferred all of astronomy from

fictitious circles to natural causes.

And concluded: body is what god created in the beginning, he needed everything that pertains to the essence of body, and quantity is the form of body which makes it body.

That the ground on which the problem of the elements will be solved is experiment Biringuccio and Agricola had shown. Bacon and Galileo established that this is the ground, exclusively.

The problem was reformulated. The problem is not of fire and air, but of their particles and the motions of these particles. Of the smallest particles, and those a degree larger, and their motion. Of the smallest particles of air and other fluids, the particles of fire, their penetration through voids, and the force attracting them and preventing their separation.

Quantity, moreover, is the form of body which makes it body.

With Galileo and Kepler also the investigation of light as distinct from sight begins. Galileo writes: does light travel instantaneously? or does it require time? an experiment occurs to me. Kepler writes: it is by motion that light spreads through media, and is thrown against surfaces, and is in turn either absorbed by the medium or forced to rebound from it.

Light, if it travels instantaneously, and if it requires time, if it is a body spreading by motion through media, thrown against surfaces, and absorbed or forced to rebound, cannot not inform the problem of the smallest particles and their motions.

Van Helmont solved the problem explicitly. The elements are air and water. These alone do not turn into one another. Earth is born from water and can be reduced back to water. Fire is not an element, it is the death of elements.

Van Helmont also produced gas from solid. Bodies contain this gas, and it is not in bodies, but it is coagulated like a body.

The elements alone do not turn into one another, and other substances are born from them and reduced back to them. Van Helmont also proposed to demonstrate this experimentally. 164 pounds of earth – wood, bark, and roots – were made of water only. Out of 62 pounds of coal 61 pounds are gas.

With van Helmont chemical experiment has

not yet become what Galileo showed experiment is. It will become that in the next generation with Boyle. With van Helmont, however, the object of chemical experiment is explicitly the problem of the elements.

Gassendi revived atomism.

The trials and tribulations of the atomic theory have been as severe as those of any theory. The physical theory developed by Leucippus and Democritus became the ground of an ethical doctrine in Epicurus, and remained so in Lucretius. The doctrine taught atheism. It was thus reviled by the doctors of theology. Nor did Fracastoro, when, one century before Gassendi, he first argued atomism again, restore the theory to repute as a physical theory. Nor did Gassendi. When Descartes, in the generation of Gassendi, restored the theory to physical repute, and developed it substantially further, he, too, spurned the name. Not until Dalton will the name be restored to repute.

Descartes established all the following.

Nature is matter. Nature, or matter, obeys laws. The laws are mathematical. They are laws of conservation. All variation of matter depends on motion. Motion is the transfer of matter. The laws of nature are laws of motion.

More particularly, Descartes analyzed matter into three kinds of particles.

All matter was initially divided by god into particles as nearly equal as possible. The particles had the same quantity of motion as there is in the world now and moved uniformly. As motion degenerated, some of the original particles constituted a second kind, larger and slower than the first, and some of the particles of the second kind constituted a third kind, larger and slower than the second. The stars are composed of particles of the first kind, the space between the stars of particles of the second kind, the planets and comets of particles of the third kind.

The three kinds of particles are the elements. The first kind is the heir of fire, the second of air, the third of earth. But the elements are also atoms *par excellence*. All properties of bodies are due to the size and speed of the elements. The second element, too – which fills the heaven, and is, either pure or intermingled with the other elements, aether as well as air as well as water – is a

major step in the generalization of water, air, and aether into fluid, such as it will be the foundation of all subsequent investigation of light, heat, and electricity. It is the foundation of the investigation of light and heat in Descartes already.

Descartes also writes: light consists entirely in the force by which particles accumulate around the centers of the vortices of stars; and the force of light is nothing but a certain pressure along the straight lines from a star to the earth.

Light began informing the problem of the elements explicitly with Descartes.

Torricelli measured air pressure.

Guericke built the first air pump.

Pascal showed that all effects ascribed to nature's abhorrence of vacuum are due to the weight of air. He showed that the force preventing water from escaping by an opening is proportional to the height of the water.

Air and water were reduced to measurement.

Boyle defined element. Elements are the primitive and simple bodies which, not made of any other bodies or of one another, are the ingredients of which all bodies are immediately compounded and into which they are ultimately resolved.

He showed that no substance has been decomposed into earth, water, air, and fire, or into mercury, sulfur, and salt.

He showed that air is necessary for combustion. He showed that air transmits sound.

He discovered the law, at constant temperature the volume and pressure of air are inversely proportional.

With Boyle the problem of the elements is formulated conclusively. It is to show the bodies of which all bodies are immediately compounded and into which they are ultimately resolved.

As to the kind of the elements, Boyle proposed that chemical experiments will illustrate the corpuscular philosophy.

The smallest particles, the corpuscles, the atoms, are the bodies of which all bodies are immediately compounded and into which they are ultimately resolved. Thus, also, as the experiments which will show which the corpuscles are began accumulating, air became central to the problem. Boyle's law, too, the fundamental law on whose ground these experiments will continue

to accumulate, is the law of air.

Grimaldi discovered diffraction.

He also argued explicitly that the new experiments show not the substantiality of light but its accidentality.

Huygens formulated the wave theory of light. Presuming the speed of light constant, he inferred the elasticity of the aethereal matter.

Newton resolved the entire difficulty in philosophy. He discovered from the phenomena of motion the forces of nature, and demonstrated from these forces the other phenomena.

The smallest particles are extended, hard, impenetrable, mobile. They obey the laws of motion and gravity. They cohere by the strongest attractions and compose bigger particles of weaker virtue.

Resistance in fluids arises in part from tenacity, in part from friction, in part from density. The part from density is as the square of the velocity, the part from tenacity is uniform.

Heat is proportional to rarefaction.

Light is a body, a heterogeneous mixture of differently refrangible rays.

The ground, then, on which the problem of the elements will be solved was established very firmly.

What the kind of the elements is was established very firmly as well. What the properties of the smallest particles must be was described in detail, experimentally, by laws.

Which the elements are remained unclear.

It was established, however, not only how it will be determined which they are.

Air became central to the problem.

Light became part of the problem.

The decisive character of fire was established.

Bacon said that the notion of fire is worthless, heat is a rapid motion of minute particles. Van Helmont said that fire is not an element, it is the death of elements. Hooke said that there is no such thing as an element of fire, flame is nothing but a mixture of air and volatile sulfurous parts of bodies.

The presumption that fire is a substance nevertheless remained the ground of chemical experiment in the following century-odd.

Becher wrote the *Chemical laboratory, or subterranean physics*, in 1669. Boyle had by that

time destroyed the doctrines of the four and of the three elements. Becher nevertheless proposed that the elements are these five: air, water, and three earths, vitreous earth, improperly called salt (I, 3.2); inflammable earth, improperly called sulfur (3.3); and fluid earth, improperly called mercury (3.4).

Stahl, in the next generation, developed Becher's inflammable earth into the theory of phlogiston, and the theory of phlogiston remained the ground of chemical experiment until Lavoisier.

Tabulating the important philosophers,	
Bacon	heat is a motion of minute particles,
Galileo	the particles of fire penetrate through the voids in metal,
Kepler	quantity is the form of body, light is a body absorbed by media or forced to rebound from them,
van Helmont	air and water are the elements, produced gas from solid,
Gassendi	revived atomism,
Descartes	three kinds of particles distinguished by size and speed are the elements, the quantity of motion of the particles is conserved, aether, air, and water are the same kind of particles,
Guericke	light is a force,
Torricelli	built the first air pump,
Grimaldi	measured air pressure, discovered diffraction,
Pascal	light is accidental, vacuum is due to the weight of air, discovered the law of pressure of confined fluid,
Boyle	defined element, air is necessary for combustion, air transmits sound, sound is a wave, discovered the gas law,
Huygens	light is a wave, aether is elastic,
Newton	the smallest particles obey the laws of motion and gravity, they cohere by the strongest

attractions and compose bigger particles of weaker virtue, resistance in fluids arises from tenacity, friction, and density, heat is proportional to rarefaction, light is a body.

The causes of all things.

The advance in this part of the problem was the greatest, by far.

The investigation of motion advanced very greatly.

Galileo demonstrated the essential properties of motion, and opened the gate and the road to the largest and most excellent science. He discovered the law of falling bodies, the law of the pendulum, the principle of relativity, and inertia.

Descartes formulated the laws of nature. The laws of nature are laws of conservation. And the laws of nature are the laws of motion.

Fermat discovered the principle of least time.

Huygens solved the problem of collisions, determined the acceleration of gravity, discovered the conservation of momentum, and the isochronism of the cycloid. He improved on the formulation of the laws.

Hooke discovered the law of elasticity. He improved on the formulation of the laws.

Newton formulated the laws conclusively. The laws of motion and the law of gravity.

Leibniz discovered the conservation of absolute force.

The investigation of the motions of the heavenly bodies advanced very greatly.

Galileo discovered the satellites of Jupiter, proved that the earth revolves around the sun.

Kepler discovered the laws of the motions of the heavenly bodies.

Huygens discovered Titan and the ring of Saturn, determined the true proportions of the solar system.

Newton calculated the masses of the earth, the moon, Jupiter, and Saturn.

The description of the system of the world advanced very greatly.

Kepler concluded: all causes of the heavenly motions are purely bodily, in particular, magnetic.

1240 In order to formulate the problems we are to solve we call the council of our predecessors

The method. There remained but one remedy, from the very beginning not to leave the mind to itself, but to guide it continuously, so that the whole thing is done by machinery. And it is most particularly on Descartes that Leibniz builds. Arithmetic and geometry are the spontaneous fruits of the method. In our time algebra is another attempt to revive it. I considered the matter carefully, and saw that there must be general knowledge of order and measure.

Tabulating,
Bacon the whole thing is done by machinery,
Descartes the method,
Leibniz the principle of sufficient reason,
the philosophical calculus.

In their series and order these 18 are all important philosophers who are part of the development of the problem of the principles of all things in the three generations from Bacon to Leibniz,

1561 - 1626 **Bacon**,
1564 - 1642 **Galileo**,
1571 - 1630 **Kepler**,
1579 - 1644 van Helmont,
1580 - 1626 **Snel**,
1592 - 1655 Gassendi,
1596 - 1650 **Descartes**,
1601 - 1665 **Fermat**,
1602 - 1686 **Guericke**,
1608 - 1647 **Torricelli**,
1618 - 1663 **Grimaldi**,
1623 - 1662 **Pascal**,
1627 - 1691 **Boyle**,
1629 - 1695 **Huygens**,
1635 - 1702 **Hooke**,
1642 - 1727 **Newton**,
1644 - 1710 **Rømer**,
1646 - 1716 **Leibniz**.

The steps of advance in the acquisition of knowledge are the 16, Bacon, Galileo, Kepler, Snel, Descartes, Fermat, Guericke, Torricelli, Grimaldi, Pascal, Boyle, Huygens, Hooke, Newton, Rømer, and Leibniz. In particular,

Bacon heat is a motion of minute particles,
the whole thing is done by machinery,
Galileo the particles of fire penetrate

Kepler

Snel
Descartes

Fermat
Guericke

Torricelli
Grimaldi

Pascal

Boyle

Huygens

through the voids in metal,
the satellites of Jupiter,
the law of falling bodies,
the law of the pendulum,
the principle of relativity,
inertia,
quantity is the form of body,
light is a body absorbed by media or forced to rebound from them,
the laws of the motions of the heavenly bodies,
all causes of the heavenly motions are magnetic,
the law of refraction,
three kinds of particles distinguished by size and speed are the elements,
aether, air, and water are the same kind of particles,
light is a force,
the laws of nature,
conservation,
the theory of vortices,
the method,
the principle of least time,
the air pump,
electricity attracts and repels,
electroluminescence,
air pressure,
diffraction,
light is accidental,
vacuum is due to the weight of air,
the law of pressure of confined fluid,
defined element,
air is necessary for combustion,
air transmits sound,
sound is a wave,
the gas law,
light is a wave,
aether is elastic,
the true proportions of the solar system,
collisions,
the acceleration of gravity,
the conservation of momentum,
the isochronism of the cycloid,

	reformulated the laws, the wave theory, the law of elasticity, reformulated the laws,
Hooke	
Newton	the smallest particles cohere by the strongest attractions and compose bigger particles of weaker virtue, resistance in fluids arises from tenacity, friction, and density, heat is proportional to rarefaction, light is a body, the masses of Jupiter and Saturn, the laws of motion, the law of gravity, there is gravity in bodies universally,
Rømer	the speed of light,
Leibniz	the conservation of absolute force, the principle of sufficient reason, the philosophical calculus.

Continuing the tabulation of the distribution of
the philosophers,

1476-1575	11	11,
1576-1675	15	13.

The count in the last line is not complete.

ccllxx

The not quite two centuries from Leibniz to
Clausius.

They divide into two parts. The one century
from Leibniz to Lavoisier, and the not quite one
century from Lavoisier to Clausius. The dates are,
1646 - 1716 Leibniz,
1743 - 1794 Lavoisier,
1822 - 1888 Clausius.

The one century from Leibniz to Lavoisier is
the time of Saccheri and the three hypotheses, of
the right angle, the obtuse angle, and the acute
angle, Linnaeus and the classification of organ-
isms, Buffon and the *Natural history*, Herschel
and the discovery of the proper motion of the
solar system. The advance culminates also in the
solution of the problem of the elements, with
Lavoisier.

Stahl formulated the theory of phlogiston,
between 1697, in the *General theory of fermenta-
tion*, and 1723, in the *Foundations of dogmatic
and experimental chemistry*.

Boyle destroyed the doctrines of the four and
of the three elements in 1661. He showed that no
substance had been decomposed into earth, water,
air, and fire, or mercury, sulfur, and salt. Lavo-
isier solved the problem of the elements, when he
compiled the table of 33 simple substances, in
1789. He showed that a great many substances
have been decomposed into the 33 simple sub-
stances. Lavoisier also, in 1783, destroyed the
theory of phlogiston. In the meantime, in the one
century-odd from Boyle to Lavoisier, the decom-
positions were gathered. On the ground of the
theory of phlogiston.

Phlogiston is a substance. It is the principle of
combustion. It is present in all three kingdoms of
nature, mineral, vegetable, and animal.

In vegetables and animals the expulsion of
phlogiston is irreversible, once its phlogiston is
lost, the original compound cannot be recom-
posed. The principle of life is soul.

In minerals, however, the expulsion and
reception of phlogiston are reversible. And Stahl
demonstrated the reversibility.

In the *General theory of fermentation*, Stahl
writes:

3 parts alkali salt and 1 part pulverized sulfur
fused in a crucible form liver of sulfur; and in a
quarter of an hour this can be converted by fire
alone into the salt obtained from oil of sulfur and
salt of tartar;

there is no more trace of alkali salt or sulfur; in
place of the red color of the liver of sulfur, this
salt is white, in place of the evil taste, it is bitter,
in place of the easy solution, it is most difficult to
dissolve, in place of the impossibility of crystalli-
zation, it forms octahedral crystals, in place of the
fusibility, it is devoid of all fusion;

and if this new salt, formed from alkali salt
and sulfur when the phlogiston is expelled, is
treated with charcoal, in a quarter of an hour the
original liver of sulfur reappears;

these conversions can be repeated a hundred
times;

and I can show by other experiments how the
phlogiston enters into metals and regenerates

them from the burned calx into their fusible, malleable state (12).

Of Stahl, too, next to Galileo and Torricelli, Kant writes in the *Critique*: when Stahl changed metal into calx and calx back into metal, new light dawned on all investigators of nature.

In the *Foundations of dogmatic and experimental chemistry*, Stahl begins:

universal chemistry is the art of resolving mixed, or compound, or aggregate bodies into their principles, and of composing such bodies from these principles (pr.).

Taylor, in 1713, in *On the motion of a tense string*, showed that: the acceleration of a particle on a tense string is proportional to the curvature of the string (*id.*).

The motion of a tense string is a wave. And beginning with Taylor's result, in the next generation, with d'Alembert, Daniel Bernoulli, and Euler, the investigation of waves reached the law.

Geoffroy compiled the first table of affinities, in 1718, in the *Table of different affinities observed in chemistry between different substances*.

Geoffroy writes:

whenever two substances disposed to unite unite, and a third which has a greater affinity for one of them is added, the third unites with one of the two separating it from the other;

this proposition is very general in chemistry, in which the effects of affinity are encountered at every step (*id.*).

The table has 15 columns. Geoffroy describes it:

in the first line are arranged the different substances used in chemistry, and in the column under each substance are arranged materials in the order of their affinity for the substance, so that the nearest has the greatest affinity and cannot be displaced by any of the materials under it;

thus, in the first column are the acid spirits, and under them the fixed alkali salts, the volatile alkali salts, the absorbent earths, and the metallic substances;

in the following three columns I have arranged the three mineral acid spirits, acid of common salt, nitric acid, and vitriolic acid;

the fifth column shows the affinities between the acid salts and the absorbent earths (*id.*), and so through the fifteenth column.

It is the decisive step in the quantification of the reactions of substances.

Gray discovered the conduction of electricity.

In 1731, in *A letter containing several experiments concerning electricity*, Gray reports:

a flint glass tube 3 feet 5 inches long was used; the first experiment was to see whether, when the tube was excited by rubbing, I could find any difference in its attraction when it was stopped at the ends by cork; but I perceived none, a feather was attracted and repelled by the cork; I was much surprised, and concluded that an electric virtue was certainly communicated by the excited tube;

I fixed an ivory ball on a fir stick 4 inches long, thrust the other end of the stick in the cork, and upon rubbing the tube, the ball attracted and repelled the feather;

I fixed the ball on a stick 8 inches long; then on a stick 24 inches long, and the attraction was the same;

and when the ball was hung by a packthread 3 feet long, the electricity was still carried, the ball attracted and repelled;

I went over to Mr Wheler to give a specimen of my experiments;

we hung a packthread from the window in the long gallery, 16 feet high;

then from the battlements of the house to the forecourt, 29 feet high;

then from the clock turret to the ground, 34 feet high;

as we had no greater height, Mr Wheler wanted to try if we could carry the electric virtue horizontally (*id.*).

And a packthread supported by silk lines carried the virtue across $80\frac{1}{2}$ feet, then 147 feet, then:

293 feet, to the end of the barn, and although the line was so much longer, we found no sensible difference in the attraction (*id.*).

At this point:

our silk lines broke, so we took iron and brass wire; but although the tube was very well rubbed, there was not the least attraction (*id.*).

Gray has discovered also the difference between conductors and nonconductors. Gray and Mr Wheler took packthread again, reinforced the silk lines, and carried the virtue across 666 feet, then:

765 feet, in the field north of the house, and the attraction was not less sensible (*id.*), and:

although the attraction grew weaker, across 886 feet (*id.*).

886 feet is 10 632 inches. Beginning with the fir stick 4 inches long, Gray, with Mr Wheler, has carried the electric virtue 2 658 times further.

Du Fay made the first attempt to formulate the theory of electricity. The two fluid theory.

In 1733, in *A letter concerning electricity*, du Fay writes:

I have discovered a very simple principle which accounts for a great part of the irregularities that seem to accompany most of the experiments on electricity; the principle is this, electric bodies attract all bodies which are not electric, and repel them as soon as they become electric by the vicinity or contact of an electric body;

and another principle, even more universal and remarkable, is this; there are two distinct electricities, which I call vitreous and resinous; the first is that of glass, crystal, precious stones, hair of animals, wool, and many other bodies; the second is that of amber, copal, silk, thread, paper, and many other bodies;

the characteristic of the two electricities is that a body of vitreous electricity repels all bodies of vitreous electricity and attracts all those of resinous electricity (*id.*).

Euler formulated mechanics analytically in the *Mechanics, or the science of motion presented analytically*, in 1736. He writes in the preface:

propositions obtained by the old synthetic method are solved more readily by my analytic method; I have also discovered many problems not found in the older treatises, and have enlarged the science by many new methods (*id.*).

Euler also formulated the principle of least action analytically, in 1744, in *A method for discovering curves with the properties of maxima and minima*.

Euler also discovered the general wave equation, in 1753, in the *Remarks on the preceding papers by Mr Bernoulli*.

Daniel Bernoulli published the *Hydrodynamics, or commentaries on the forces and motions of fluids*, in 1738. With the *Hydrodynamics* the investigation of fluids reached the law.

The law is Bernoulli's equation:

$$P \propto v^2,$$

where P is the pressure and v is the velocity of a fluid in a container (12.5-6).

The fundamental equation of fluids follows from Newton's laws. Bernoulli deduces it from these two rules:

I will give two rules for defining the motion of water anywhere;

1. assuming the velocity anywhere in a vessel known, one must discover the velocity in the other parts of the fluid;

2. one must discover at any instant how much of the living force, or the product of the potential ascent and the mass, does not contribute to the main flow, add this product to the product of the mass and the potential ascent of the main flow, and consider this sum equal to the product of the total mass of the water and the actual descent (8.1-3).

The rules follow from the conservation of living force, or absolute force.

In the *Hydrodynamics* Bernoulli has also taken a major step toward the formulation of the kinetic theory. He writes:

consider a vessel with a movable lid on which lies a weight; let the cavity under the lid contain very minute particles, infinite in number, in a very rapid motion this way and that, so that these particles support the lid by their continuous impacts, and form an elastic fluid which expands by itself, if the weight on the lid is removed or decreased, and is condensed, if the weight is increased;

the properties of this fluid explain all properties of air and other elastic fluids, and point at properties which have not yet been discovered (10.2-3).

Daniel Bernoulli also discovered the principle of superposition of waves, in 1753, in the *Reflections and clarifications on the new vibrations of strings*.

Maupertuis formulated the principle of least action.

In 1740, in the *Researches into the laws of rest*:

in all systems of elastic bodies in motion which act on one another the sum of the products of the masses and the squares of the velocities,

called living force, remains inalterably the same (*id.*).

In 1744, in the *Accord of different laws of nature which until now appeared incompatible*:

in the production of its effects nature always acts by the simplest means possible; whether a material body travels from one point to another without encountering an obstacle or encounters an impenetrable obstacle, nature always leads it along the path of the shortest distance and the briefest time (*id.*).

In 1746, in the *Researches into the laws of motion*:

in all changes in nature the quantity of action necessary for these changes is the least possible; the quantity of action is the product of the masses of the bodies, their speeds, and the distances they travel (*id.*).

Euler formulated the principle analytically in 1744. In *A method for discovering curves with the properties of maxima and minima*, he writes:

if m is the mass of a body, ds the distance it travels, and v its velocity, $m\sqrt{v}$ is the quantity of motion of the body, $m\sqrt{v} ds$ is its motion over the distance ds ;

and I say, the line which the body describes is, among all other lines, given by $\int m\sqrt{v} ds$, when m is constant and $\int \sqrt{v} ds$ is a minimum (app. II, 2).

D'Alembert formulated mechanics analytically in the *Treatise on dynamics* in 1743.

In part II, *General principle to find the motions of several bodies which act on one another in any way whatsoever*, he formulated d'Alembert's principle. Lagrange generalized Maupertuis's principle and d'Alembert's principle into the principle of virtual velocities. And this is how Lagrange, in the *Analytic mechanics*, in 1788, formulates d'Alembert's principle:

if a motion is impressed on several bodies so that they are forced to move consistently with their mutual inclinations, it is clear that these motions can be considered composed of those which the bodies actually follow and motions which are negated, from which it follows that these last motions must be such that the bodies following only these motions are in equilibrium (II, 1.10).

D'Alembert also discovered the wave equa-

tion, in 1747, in the *Researches on the curve which a tense string set in vibration forms*.

The investigation of waves reached the law in four steps.

In 1747, in the *Researches on the curve which a tense string set in vibration forms*, d'Alembert begins:

let t be the time since the beginning of the vibration of a string; let $y = \Phi(t, x)$ be an unknown function of t and x (2).

And d'Alembert proposes:

$$y = \Phi(t + x) + \Psi(t - x)$$

is the general equation of the curve (7);

and it is easy to see that this equation contains an infinity of curves (8).

In 1750, in *On the vibrations of strings*, Euler rewrites d'Alembert's equation:

$$y = \Phi(x + ct) + \Psi(x - ct),$$

where c is a constant (29).

And continues:

having thus found the general solution, let us consider some special cases (*id.*).

One case is the combination of two or more vibrations of a string:

and it is clear that, if a is the length of the string, the equation

$$y = \alpha \sin\left(\frac{\pi x}{a}\right) + \beta \sin\left(\frac{2\pi x}{a}\right) + \gamma \sin\left(\frac{3\pi x}{a}\right) + \dots,$$

where α , β , γ are constants, has the required form (30).

In 1753, in the *Reflections and clarifications on the new vibrations of strings*, Daniel Bernoulli formulates the principle of superposition of waves:

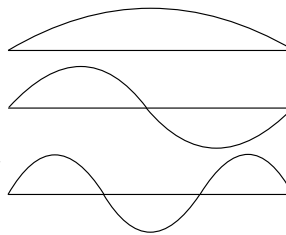
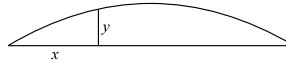
all sounding bodies contain potentially an infinity of tones and an infinity of corresponding ways of making their regular vibrations (4).

He deduces the equation:

$$y = \alpha \sin\left(\frac{\pi x}{a}\right) + \beta \sin\left(\frac{2\pi x}{a}\right) + \gamma \sin\left(\frac{3\pi x}{a}\right) + \dots$$

(12), and concludes:

in this general equation we have found the



infinity of curves; and our equation is the same as that of Mr Euler (13).

And also in 1753, in the *Remarks on the preceding papers by Mr Bernoulli*, Euler writes:

we owe to Mr Bernoulli the happy explanation why a string can produce several different tones at the same time (1);

I have given this equation, however, only as a particular solution of the general formula (5);

the principal question is thus whether all curves which fit a string set in motion are contained in this equation or not (*id.*)?

And Euler shows that they are not. They are contained in the equation:

$$\frac{\partial^2 y}{\partial t^2} = X \frac{\partial^2 y}{\partial x^2},$$

where X is a certain function of x which depends on the thickness of the string (45).

On the general wave equation the development of electrodynamics, and the development of relativity, and the development of quantum theory, all three, rests unconditionally.

Franklin made the next attempt to formulate the theory of electricity. The one fluid theory.

In 1747, Franklin writes:

a body is electrified positively or negatively, plus or minus; and to electrify plus or minus no more needs to be known than this, in the instant of friction the parts of the tube or sphere which are rubbed attract the electric fire and therefore take it from the thing which rubs;

the equality is never destroyed, the fire only circulates (to Collinson, 25 May).

In 1749, in the *Opinions and conjectures concerning the properties and effects of the electric matter, arising from experiments and observations made in Philadelphia*, he writes:

the electric matter consists of particles extremely subtle, since it can permeate common matter, even the densest metals (1);

electric matter differs from common matter in this that, while the particles of common matter attract one another, those of electric matter repel one another (3);

although repelling one another, however, the particles of electric matter are strongly attracted by all other matter (4);

hence, when a quantity of electric matter is applied to a mass of common matter it is immedi-

ately and equally diffused throughout the whole (5);

in general, in common matter there is as much electric matter as it can contain; if more is added, it lies on the surface and forms an electric atmosphere, and the body is then said to be electrified (7);

and we know that there is electric fluid in common matter because we can pump it out by rubbing a globe or a tube (9).

In his *Autobiography* Franklin reports: what gave my book sudden and general celebrity was the success of one of its experiments for drawing lightning from the clouds; this engaged the public attention everywhere (3).

Black established that air is not of one kind. That there are different gases. He discovered that earths contain air. He produced a quantity of gas and a quantity of solid from a quantity of solid, then produced the original quantity of the original solid from the resultant quantities of the resultant gas and the resultant solid. He thus discovered also that gases combine with solids. He discovered also latent heat, and specific heat.

With Black also chemical experiment began reaching the smallest particles. 1 ounce = 8 drams = 24 scruples = 480 grains. 1 ounce is 31.1 g, and 1 grain 0.065 g.

In the *Experiments on magnesia alba, quicklime, and some other alkaline substances*, in 1756, Black reports:

1 ounce of magnesia was exposed for about an hour to such heat as suffices to melt copper; when taken out, it weighed 3 drams and 1 scruple; it had thus lost $\frac{7}{12}$ of its weight (1).

Further experiments corroborate this result. Black continues:

observing magnesia to lose such a remarkable proportion of its weight, my next attempts were directed to the investigation of this volatile part (*id.*).

Further experiments show what the volatile part is:

2 drams of magnesia were calcined, and thus reduced to 2 scruples and 12 grains; the calcined magnesia was dissolved in spirit of vitriol, then separated from the acid by the addition of an alkali; washed and dried, it now weighed 1 dram

and 50 grains; it effervesced violently, emitting a large quantity of air; thrown into acids it formed a red powder when mixed with a solution of sublimate; it separated the calcareous earths from an acid; it sweetened lime water; and it had thus not only recovered all its original properties, it had acquired an addition of weight nearly equal to what had been lost in the fire; and part of this addition must certainly be air (*id.*).

Hence:

this led me to try experiments by which I might know what quantity of air is expelled from an alkali by acids (*id.*).

The outcome of the concluding experiment is:

therefore, 1 scruple and 8 grains were lost during the ebullition, of which a trifling portion may be water or something of this kind, but the rest is air (*id.*).

Black concludes:

it is clear that the calcareous earths contain a large quantity of fixed air (2).

Fixed air is the wild spirit which van Helmont called gas. Carbon dioxide.

Black discovered latent heat in 1761.

In his *Lectures on the elements of chemistry*, published in 1803, he reports:

water may be cooled to 6° , or 7° , or 8° below the freezing point without being frozen; if it is then disturbed, there is a sudden coagulation, and not of the whole, but of a part only; and the most remarkable fact is that when this happens, the mixture of ice and water suddenly becomes warmer, making a thermometer rise to the freezing point (I, 1.2).

After further experiments Black concludes:

heat in boiling is absorbed by water and enters into the composition of the produced vapors in the same manner as in melting it is absorbed by ice and enters into the composition of the produced water; the effect of the heat consists not in warming the surrounding bodies, but in making the ice fluid and making the water vapor; in both cases it is concealed, or latent, and I have given it the name latent heat (1.3).

He discovered specific heat in 1764.

He writes in the *Lectures*:

suppose water to be at 100° , and an equal measure of mercury at 150° to be suddenly mixed with it; the temperature of the mixture

turns out to be 120° instead of 125° ; the mercury, therefore, is less warm by 30° , the water is warmer by 20° , yet the quantity of heat which the water has gained is the very same quantity which the mercury has lost; this shows that the same quantity of matter of heat has more effect in heating mercury than in heating an equal measure of water;

mercury, therefore, has less capacity for the matter of heat than water, it requires a smaller quantity of it to raise its temperature by the same number of degrees (1.1).

Boscovich wrote *A theory of natural philosophy reduced to a single law of the forces existing in nature*, in 1758.

The single law stands on atoms.

This is how Faraday describes the atoms, in *A speculation touching electric conduction and the nature of matter*, in 1844: if we must assume at all, then the safest course appears to be to assume as little as possible, and in this way the atoms of Boscovich appear to me to have a great advantage over the more usual notion; his atoms are mere centers of forces, not particles of matter in which the forces reside; if, in the ordinary view of atoms, we call the particle of matter *a* and the system of forces in and around it *m*, then, while in the usual notion *a* is a little unchangeable piece of matter and *m* is an atmosphere of force around it, in Boscovich's theory *a* disappears, or is a mere mathematical point (*id.*).

Boscovich anticipates fields.

Watt, beginning in 1765, developed the steam engine.

Carnot, in 1824, in the *Reflections on the motive power of fire and on the proper engines to develop this power*, writes: we owe to Watt almost all the great improvements in the steam engine, Watt brought this engine to a state of perfection difficult even now to surpass (*id.*)

Cavendish discovered hydrogen, in 1766.

In the *Three papers containing experiments on factitious air*, he measured the specific gravities of inflammable air, common air, and fixed air, and thus showed that inflammable air is a distinct substance. Inflammable air is hydrogen.

In the first paper:

containing experiments on inflammable air (1), Cavendish begins:

I know of only three metallic substances, zinc, iron, and tin, which generate inflammable air by solution in acids (*id.*).

The outcome of the experiments is:

if the density of common air, at the time of the experiment, was 800 times less than that of water, inflammable air must be 5 490 times lighter than water, or nearly 7 times lighter than common air (*id.*).

In the second paper:

containing experiments on fixed air (2),

Cavendish reports 16 experiments. The outcome is:

if the outward air, at the time the experiment was tried, is supposed to have been 800 times lighter than water, fixed air is 511 times lighter than water, or 1.57 times heavier than common air (*id.*).

With Cavendish also the accurate measurement of electricity begins.

In 1775, in *An account of some attempts to imitate the effects of the torpedo by electricity*, he describes two methods of measuring the degree of electrification. One of the methods is very accurate:

the electrometer I used consisted of two straws, 10 inches long, hanging parallel to each other, and turning at one end on steel pins as centers, with cork balls about $\frac{1}{4}$ of an inch in

diameter fixed at the other end; I estimated the divergence of the balls by seeing whether they appeared to coincide with parallel lines placed behind them at a distance of about 10 inches, taking care to hold my eye always at the same distance from the balls, and not less than 30 inches off; to make the straws conduct the better, they were gilded;

this electrometer is very accurate (*id.*).

The other method is direct:

I employed an assistant to charge the Leyden jar, and when my hands were in the proper position, I received the shock myself (*id.*).

Cavendish also took the decisive step toward establishing what water is.

In 1784, in the *Experiments on air*, he reports:

in an experiment of Dr Priestley's, on firing a mixture of common air and inflammable air in a glass by electricity, the inside of the glass, clean

and dry before, became dewy (*id.*).

A series of experiments follows:

to better examine the nature of this dew (*id.*).

The outcome is:

it seemed pure water (*id.*).

Cavendish concludes:

we must allow that dephlogisticated air is in reality nothing but dephlogisticated water, or water deprived of its phlogiston; in other words, that water consists of dephlogisticated air united to phlogiston; and that inflammable air is either pure phlogiston, as Dr Priestley and Mr Kirwan suppose, or else water united to phlogiston;

I think the second of these explanations much the most likely (*id.*).

Dephlogisticated air is oxygen.

Priestley, in 1772, in the *Observations on different kinds of air*, distinguished 8 different kinds of air. Produced by heating substances, mixing them, and collecting residues from calcination, vegetation, and electric discharge, the kinds are:

fixed air (1);

air in which a candle, or brimstone, has burned out (2);

inflammable air (3);

air infected with animal respiration or putrefaction (4);

air in which a mixture of brimstone and iron filings has stood (5);

nitrous air (6);

air infected with the fumes of burning charcoal (7);

air extracted by means of spirit of salt (9).

In 1774 Priestley distinguished another kind, dephlogisticated air. He informed Lavoisier of this result in October of that year. He published the result in 1775, in *An account of further observations on air*.

Scheele discovered chlorine, in 1773, and oxygen and nitrogen, in 1774.

The discovery of chlorine Scheele describes first in a letter to Gahn (28 Mar 1773). All the experiments which led to the discovery he describes in 1774, in *On manganese, or magnesia, and its properties*. He writes:

I mixed manganese and acid of salt in a retort; in front of the neck of the retort I attached a bladder void of air, and I placed the retort in hot

sand; the bladder expanded by the effervescence in the retort; when the effervescence stopped, indicating saturation, I removed the bladder, and I found that the air in it was yellow, yet there was no trace of fixed air in it; it had a very strong suffocating smell, resembling that of warm aqua regia (23).

Scheele exposed water to the yellow air, thus obtained:

dephlogisticated acid of salt (25),

and in this:

litmus paper turned white;

oils became as tough as turpentine;

vitriol of iron turned red;

all metals were attacked, and gold was dissolved;

insects died instantly;

a flame was extinguished instantly (*id.*).

The yellow air is chlorine.

Scheele's discovery of oxygen is attested first by Bergman early in 1774. Later in that year Scheele describes it in a letter to Lavoisier (30 Sep).

In 1772 Daniel Rutherford had described yet another kind of air, mephitic air. Scheele distinguished it, quantitatively, from all the other kinds, and *ipso facto* discovered nitrogen. All the experiments which led to the discovery of oxygen and nitrogen Scheele describes in 1777, in the *Chemical treatise on air and fire*. He concludes:

air must be composed of elastic fluids of two kinds (8);

one is necessary for fire to start, and constitutes about one third of our common air; I have called it fire air;

the other is not in the least suitable for the production of fire, and constitutes about two thirds of our air; I have given it the already known name of vitiated air (29).

Fire air is oxygen, vitiated air is nitrogen. And Scheele determined quantitatively not only the composition of the atmosphere. He writes:

I found that vitiated air is lighter than common air; if so, does it not follow that fire air is heavier than common air? and, in fact, by weighing accurately as much fire air as occupied the space of 20 ounces of water, I found that it was almost 2 grains heavier than the same bulk of common air (49).

Ingen-Housz discovered photosynthesis, in 1779, in the *Experiments on vegetables, discovering their great power of purifying the common air in sunshine, and of injuring it in the shade and at night*. He writes:

dephlogisticated air oozes out of the leaves of plants;

the production of dephlogisticated air from the leaves is due not to the warmth of the sun, but to its light;

plants have respiration as animals do; leaves are its organs;

they have pores which absorb air, and others which throw it out like the excretory ducts of animals;

the secreted air, being dephlogisticated air, is thrown out as noxious to the plant.

Lavoisier's grand synthesis followed.

ccclxxi

The synthesis begins with a review of our predecessors, in 1774, in the *Physical and chemical essays*. Lavoisier reports: in the first part of this treatise I have confined myself to the character of the historian (*intr.*). And in order: to present everything that has been done on the combination of air with bodies (*id.*), he collects the labors of: Paracelsus, van Helmont (1), Boyle (2), Hales (3), Boerhaave (4), Stahl (5), Venel (6), Black (7), Saluzzo (8), Macbride (9), Cavendish (10), Johann Meyer (11), Jacquin (12), Crantz (13), van Smeth (14), Priestley (15), Duhamel de Monceau (16), Rouelle (17), Bucquet (18), and Baumé (19).

Lavoisier then wrote the *Memoir on the calcination of tin in closed vessels, and on the cause of the gain of weight which the metal acquires in this operation*, in 1774, the *Memoir on the nature of the principle which combines with metals during their calcination and augments their weight*, in 1775, the *Memoir on the existence of air in the nitrous acid, and on the means of decomposing and recomposing this acid*, in 1776, the *Memoir on the combustion of Kunckel's phosphorus and on the nature of the acid which results from this combustion*, the *Experiments on the respiration of animals and on the changes which happen to the air passing through their lungs*, *On the combination of the matter of fire*

with evaporable fluids, and on the formation of elastic aeriform fluids, and the *Memoir on combustion in general*, in 1777, the *General considerations on the nature of acids and on the principles of which they are composed*, in 1778, the *Experiments on the comparative effect of different combustibles* and the *Memoir on the formation of the acid named fixed air or acid of chalk, which I will henceforth designate by the name acid of coal*, in 1781, the *Memoir on the nature of the elastic aeriform fluids which are released from some animal matters in fermentation* and the *General considerations on the dissolution of metals in acids*, in 1782, the *Memoir whose object is to prove that water is not a simple substance, an element strictly, and is susceptible to decomposition and recomposition*, *On the action of fire animated by vital air on the most resistant mineral substances*, the *Memoir on the mutual precipitation of metallic substances*, the *Memoir on the affinity of the oxygen principle to the different substances with which it is susceptible to unite*, the *Memoir on the union of the oxygen principle with iron*, the *New reflections on the augmentation of weight which sulfur and phosphorus acquire by burning, and on the cause to which it must be attributed*, the *Reflections on phlogiston to serve as a continuation of the theory of combustion and calcination published in 1777*, and, with Laplace, the *Memoir on heat*, in 1783, the *Memoir on the combination of the oxygen principle with spirit of vine, oil, and different combustible bodies* and, with Meusnier, the *Memoir in which it is proved, by the decomposition of water, that this fluid is not a simple substance, and that there are several means of obtaining large quantities of the inflammable air which enters in it as a constituent principle*, in 1784, the *Reflections on the decomposition of water by vegetable and animal substances*, in 1786, the *Instruction on the establishment of nitreries and on the production of salpeter*, in 1787, the *General observations on the modern horizontal layers which have been deposited by the sea, and on the consequences which can be drawn from their disposition about the antiquity of the terrestrial globe*, in 1788.

Then he wrote the *Elementary treatise on chemistry*, presented in a new order and accord-

ing to the modern discoveries, in 1789.

The *Treatise* contains Lavoisier's final formulation of the theory of combustion and calcination. His final description of the decomposition of water. The first analysis of the composition of plants and animals into their constituent elements. The first formulation of the principle of conservation of matter. The first chemical equation. The first table of the elements. His one fundamental result which Lavoisier did not include in the *Treatise* is formulated conclusively in the *Memoir on heat*, with Laplace, in 1783:

respiration is combustion, moreover perfectly similar to that of carbon.

The *Treatise* also fixed the nomenclature of substances. For instance:

the species of air discovered almost simultaneously by Priestley, Scheele, and myself, Priestley called dephlogisticated air, Scheele called fire air, I called at first highly respirable air, then vital air (I, 1), but:

properly it ought to be called oxygen, from ὀξύς and γείνομαι, because, in fact, one of its most important properties is to form acids (4).

The *Treatise* opens with a *Preliminary discourse*. Part I is *On the formation of aeriform fluids and their decomposition, on the combination of simple bodies and the formation of acids*, in 17 chapters. Part II is *On the combination of acids with salifiable bases, and on the formation of neutral salts*, in 44 tables. Part III is the *Description of the instruments and operations of chemistry*, in 10 chapters. Appended are 8 *Tables of the usage of chemists*.

In the *Preliminary discourse*, Lavoisier writes:

we must subject reasoning continually to the test of experience; we must keep to nothing but the facts, which alone are given to us by nature and cannot mislead us, and search for truth only by following the natural chain of experiments and observations in the way in which the mathematicians arrive at the solution of a problem by the simple arrangement of the given and the reduction of reasoning to operations so simple and judgments so direct as never to lose sight of the evidence which guides them (*id.*).

In part I, chapter 1, *On the combinations of caloric and the formation of elastic aeriform*

fluids, he writes:

all bodies in nature are solid, liquid, or in an elastic aeriform state, according to the ratio of the attractive force of their particles and the repulsive force of heat, or what is the same, according to the degree of heat to which they are exposed;

it is difficult to conceive of these phenomena without admitting that they are the effect of a real material substance, of a very subtle fluid which insinuates itself among the particles of all bodies and separates them; and even if the existence of this fluid is only a hypothesis, it explains the phenomena of nature very happily;

this substance, whatever it is, if it is the cause of heat, and that is, of the sensation we call heat, which is the accumulation of this substance, cannot be rigorously called heat, for then the same name would express both cause and effect; for this reason we distinguish the cause of heat, or the exquisite elastic fluid which produces it, by the name caloric;

nor do we have to suppose it real matter, it is sufficient to consider it a certain repulsive cause which separates the particles of matter, and thus conceive of its effects in an abstract and mathematical manner;

the facts, then, authorize me to affirm as a general principle that nearly all bodies in nature are susceptible to existing in the three different states, solid, liquid, and aeriform, and that in which of these states a body is depends on the quantity of caloric combined with it (*id.*).

In chapters 2-4 Lavoisier shows that:

atmospheric air is composed of oxygen and nitrogen in the ratio of 27 to 73 (3).

Chapter 5, *On the decomposition of oxygen by sulfur, phosphorus, and carbon, and on the formation of acids in general*, begins with a principle:

a principle which must never be lost from view in the art of making experiments is to simplify them as much as possible and discard all circumstances which can complicate the effects (*id.*). In the art of experiment, and in any other. Later in the chapter, Lavoisier writes:

the convertibility of a combustible substance into an acid by the addition of oxygen is a common property of a great number of bodies; thus, the conversion of phosphorus into acid by oxy-

gen, and in general the combination of any combustible substance with oxygen, we name oxygenation;

it is clear that oxygen is the principle of acidity, and that acids differ from one another by the nature of the acidified substance (*id.*).

In chapter 7:

when metallic substances are heated, oxygen has a greater affinity for them than for caloric; in consequence, all metallic substances, with the excepting of gold, silver, and platinum, have the property of decomposing oxygen by engaging its base and disengaging the caloric (*id.*).

In chapter 8 Lavoisier shows that water is composed of hydrogen and oxygen. And notes:

fermentation, putrefaction, even vegetation are due to the decomposition of water;

and it is very extraordinary that this has escaped the attention of the physicists and chemists; it must be concluded that in the sciences as in morality it is difficult to vanquish prejudice and follow a path other than the customary (*id.*).

In chapter 9, *On the quantity of caloric disengaged in different kinds of combustion*, he writes:

when combustion is performed in a hollow sphere of ice supplied with air at temperature 0°, the quantity of ice melted in the sphere gives a measure of the relative quantity of caloric disengaged;

Mr Laplace and I have described such an apparatus; and with it we obtained the following results for the combustion of phosphorus, carbon, and hydrogen,

1 pound of phosphorus –
100 pounds of ice,
1 pound of carbon –
96 pounds, 8 ounces,
1 pound of hydrogen –
295 pounds, 9 ounces, $3\frac{1}{2}$ gros (*id.*).

In chapter 10, he writes:

as combustible substances in general are these that have a great affinity for oxygen, they must have affinity for one another; *quae sunt eadem uni tertio, sunt eadem inter se*; and this is, in fact, the case (*id.*).

Euclid's first axiom is, in fact, the case.

In chapter 12:

the true constituent elements of vegetables are

three, hydrogen, oxygen, and carbon (*id.*).

In chapter 14:

the composition of animal matter is not well known; it is known that it is composed of hydrogen, carbon, nitrogen, phosphorus, and sulphur (*id.*).

In chapter 17:

metals which have a weak affinity for oxygen are absolutely insoluble in acids which have a stronger affinity for it; it is for this reason that gold, silver, mercury, and lead are insoluble in muriatic acid (*id.*).

And in chapter 13 Lavoisier formulates the principle of conservation of matter. He writes:

nothing is created in the operations of either art or nature; and it can be laid down as a principle of all operations that the quantity of matter before and after an operation is the same; the quality and the quantity of the elements are the same, and nothing takes place but changes and modifications of the combinations of the elements;

on this principle the entire art of making experiments in chemistry is founded; in all experiments it must be supposed that a true equality, and that is, an equation, is examined;

thus, when grape must yields carbonic acid and alcohol, I can say,

grape must = carbonic acid + alcohol (*id.*).

In part II, the first of the 44 tables is of:

simple substances (*id.*).

Lavoisier notes:

chemistry marches toward its goal and toward perfection, by dividing, subdividing, and subdividing again, and we do not know what will be the end of its success; we can thus not be certain that what we regard as simple today is in fact such; all we can say is that a substance is the end which chemical analysis has reached and it cannot be subdivided further in the present state of our knowledge (*id.*).

There are 33 simple substances in the table, subdivided into 4 kinds:

5 simple substances which belong to the three kingdoms and which can be considered the elements of bodies,

6 nonmetallic oxidizable and acidifiable simple substances,

17 metallic oxidizable and acidifiable simple

substances,

and 5 salifiable simple earthly substances (*id.*).

2 of the first kind are:

light,

caloric (*id.*).

3 of the second kind are:

boracic radical,

fluoric radical,

muriatic radical (*id.*).

On the 5 of the fourth kind:

lime, magnesia, barite, argil, and silex (*id.*),

Lavoisier comments:

we can assume that the earths will soon cease to be considered simple bodies and will be considered compounds, perhaps oxidized metallic compounds (*id.*).

The remaining 23, in their order in the periodic table, are:

1. hydrogen,
6. carbon,
7. nitrogen,
8. oxygen,
15. phosphorus,
16. sulfur,
25. manganese,
26. iron,
27. cobalt,
28. nickel,
29. copper,
30. zinc,
33. arsenic,
42. molybdenum,
47. silver,
50. tin,
51. antimony,
74. tungsten,
78. platinum,
79. gold,
80. mercury,
82. lead,
83. bismuth.

Water Lavoisier decomposed into hydrogen and oxygen.

Airs had begun accumulating before Boyle already, and had continued to accumulate. In the generation through Lavoisier were produced fixed air, factitious air, inflammable air, dephlogisticated air, nitrous air, mephitic air, yellow air, fire air, vitiated air, vital air, among many others.

Lavoisier sorted them all out.

Earths had begun accumulating before Theophrastus already, and had continued to accumulate. Some earths men had handled, wittingly and unwittingly, since the beginnings of agriculture, pottery, and metallurgy. More earths the philosophers had isolated, again wittingly and unwittingly, since the beginnings of alchemy. Very many more earths the philosophers had isolated, and again wittingly and unwittingly, in the couple of generations through Lavoisier. Lavoisier sorted these all out, too.

He sorted them all out:

by following the natural chain of experiments and observations in the way in which the mathematicians arrive at the solution of a problem.

So he solved the problems of combustion, oxidation, and calcination.

The theory of combustion and calcination supplied the uniform criteria on which to sort out all substances, as the simple substances themselves:

oxidizable and acidifiable simple substances, attest, too.

So he solved the problem of affinity. 26 of the 44 tables in part II are tables of:

affinities in their order (*id.*), correlating between them the relative affinities in their order of every known substance to any known substance.

And so he solved the problem of the elements of all things. Lavoisier discovered that hydrogen, carbon, nitrogen, oxygen, phosphorus, sulfur, manganese, iron, cobalt, nickel, copper, zinc, arsenic, molybdenum, silver, tin, antimony, tungsten, platinum, gold, mercury, lead, and bismuth are elements. In 1869 Mendeleev will discover the periodic law and will arrange 63 elements in the periodic table. Of 23 of the 63 Lavoisier discovered that they are elements. Of 3 more, moreover, he anticipated that they are elements with their inclusion among the simple substances. With boracic radical that

5. boron,

with fluoric radical that

9. fluorine,

and with muriatic radical that

17. chlorine

are elements.

Part III is the *Description of the instruments and operations of chemistry*. Lavoisier describes the instruments and methods to determine absolute and specific gravities (1), to measure the weight and volume of a gas (2), to separate different gases from one another (4), to correct volume for atmospheric pressure (5), to correct it for variations in pressure and temperature (7).

The seventh of the 8 *Tables of the usage of chemists*, with which the *Treatise* concludes, is of:

weights of different gases at 28 inches of barometric pressure and 10° of temperature (*id.*). 1 French inch is 27 mm, 10° Réaumur is 12.5° Celsius, 1 French grain is 0.053 g.

The first 5 entries in the table, in grains for one cubic inch, are:

atmospheric air	0.46005,
nitrogen	0.44444,
oxygen	0.50694,
hydrogen	0.03539,
carbonic acid	0.68985 (<i>id.</i>).

The relative weights are thus, hydrogen 1, nitrogen 12.6, oxygen 14.3. The atomic weights, in fact, are, hydrogen 1, nitrogen 14, oxygen 16.

Lavoisier writes in the *Preliminary discourse*:

I will transcribe some observations of the abbé Condillac which, I think, describe truthfully the recent state of chemistry; when errors become prejudices and are adopted as principles, we only bewilder ourselves more and more, and then there is but one remedy, to forget all that we have learned, trace our ideas back to their origin, follow them as they arise, and, as my lord Bacon says, frame the human understanding anew (*id.*).

And Lavoisier, as my lord Bacon says, framed the human understanding anew.

ccclxxii

Kant wrote the *Critique of pure reason*, in 1781.

Whatever their source, and whatever their form, judgments are with respect to their content either analytic or synthetic, they either add nothing to the content of knowledge or enlarge knowledge.

Judgments from experience are always synthetic.

And since pure mathematical knowledge must

go beyond the concept, mathematical propositions can never arise through the analysis of concepts, and that is, analytically. They are therefore in their entirety synthetic.

Thus, the problem on which everything depends is, how are synthetic *a priori* propositions possible?

That they are possible the reality of pure mathematics and pure natural science shows.

And if synthetic *a priori* propositions are possible, there must be in perception already something *a priori*, bound inseparably with the concept before all actual experience, or distinct perception. But how is it possible to perceive *a priori*? How can perception of the object precede the object itself? There is only one way in which my perception can precede the actuality of the object and take place as knowledge *a priori*, namely, if it contains nothing but the form of sensibility, which in my subject precedes all actual impressions by which I am affected by objects.

Nor is it necessary to assume that the form of sensibility is limited to man, it may well be the form of sensibility of all rational beings.

All judgments are at first only judgments of perceptions. When a judgment agrees with an object, however, all judgments of the same object must agree with one another. The objective validity of a judgment of experience is thus nothing but its necessary general validity.

And here is the solution of the puzzle. No *a priori* knowledge is possible but of objects of possible experience. Synthetic *a priori* propositions are related to nothing but the possibility of experience.

The possibility of experience is what gives all knowledge *a priori* objective validity.

Perception has form. *Ipsos facto* experience, the synthesis of perceptions, has form. The form is the form not of actual experience, but of possible experience, inclusive of all actual experience, and hence inclusive of the possibility of extension of actual experience. Form thus gives experience its objective validity, or what is the same, its general validity.

The principle of contradiction is the principle of all analytic judgments. The possibility of experience is the principle of all synthetic judg-

ments. And the principle of sufficient reason is the reason of possible experience.

And the ground of the advance in the formulation and solution of the problem of the formal principles of all things is the same, in this case as in all earlier and later.

For instance, in *An attempt to introduce the concept of negative magnitudes in philosophy*, in 1763, Kant begins: the use made of mathematics in philosophy consists either in the imitation of its method or in the true application of its propositions to the objects of philosophy; from the first use one does not see that there has yet been any benefit (pr.); the second use, on the other hand, has been all the more beneficial to the parts of philosophy which, by employing the theories of mathematics to their advantage, have attained heights to which they could otherwise never aspire (*id.*).

ccclxxiii

The not quite one century from Lavoisier to Clausius is the time of Bessel and the measurement of the distance of 61 Cygni, Lobachevsky and non-Euclidean geometry, Schwann and the cell theory, Riemann and the theory of manifolds, Darwin and the theory of evolution. The advance culminates also in the discovery of the laws of thermodynamics, with Clausius.

Laplace proved, in 1783, that Uranus is the seventh planet of the solar system. He showed, between 1787 and 1790, that the current acceleration of Jupiter and deceleration of Saturn are an effect of their mutual gravitation with a period of 929 years, and that the acceleration of the moon is an effect of the eccentricity of the earth's orbit with a period of millions of years. He proposed, in 1796, that the comets are formed outside the planetary system. He formulated, also in 1796, the theory of the origin and development of the solar system from the cooling and contraction of a gaseous nebula. In general, Laplace reduced the solar system in its entirety to order, and spatially and temporally both. In 1799-825, in the *Treatise on celestial mechanics*, he systematized celestial mechanics analytically.

In 1785, in the *Theory of the attractions of spheroids and of the shape of the planets*, he discovered the Laplace equation.

Laplace writes:

consider the attractions of spheroids in general;

the expression for V as the sum of the quotients of the masses of the particles of a spheroid and their distances to an attracted point has the advantage of giving the attraction of the spheroid in any direction;

and if x, y, z are the coordinates of a particle of the spheroid, the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

is the foundation of the theory of the attraction of spheroids and of the shape of the planets (*id.*).

V is the potential.

Lagrange, in *On the secular equation of the moon*, in 1773, took the first step toward the definition of the potential (12). Laplace, in the formulation of the equation, took the second and conclusive step. The potential of a body is the sum of the quotients of the masses of every particle of the body and its distance to an attracted point, giving the attraction of the body in any direction. Gauss, in the next generation, established the name potential.

The Laplace equation is the foundation of the theory of the attraction of spheroids and of the shape of the planets.

More generally, the Laplace equation is the foundation of the investigation of the gravitational potential as well as the electromagnetic.

It is the foundation also of potential theory and of the investigation of harmonic functions.

On the Laplace equation as well the development of electrodynamics, of relativity, and of quantum theory, all three, rests unconditionally.

With Coulomb the investigation of electricity reached the law.

Coulomb discovered the first law of electricity, the law of electrostatics, in 1785, in the *First memoir on electricity and magnetism*. He constructs:

an electric balance based on the property of metallic wires to have a force of torsion proportional to the angle of torsion (*id.*), measures that force between two small charged spheres, and concludes:

the fundamental law of electricity, the repulsive force between two small spheres charged

with the same kind of electricity is in the inverse ratio of the square of the distance between the centers of the spheres (*id.*).

$$F \propto \frac{q_1 q_2}{r^2}.$$

Maxwell's first equation is a generalization of Coulomb's law.

Lagrange wrote the *Analytic mechanics*, in 1788.

The *Analytic mechanics* is the culmination of the analytic reformulation of mechanics. Lagrange deduces mechanics from a single principle, the principle of virtual velocities.

Euler formulated the principle of least action analytically in 1744. In 1762 Lagrange generalized the principle, in the *Essay on a new method to determine maxima and minima of indefinite integral formulas* and the *Application of the preceding method to the solution of the different problems of dynamics*. The principle of virtual velocities is a generalization of d'Alembert's principle and the principle of least action.

The *Analytic mechanics* is in two parts, *Statics* and *Dynamics*.

Lagrange writes in the preface:

I propose to condense the theory of mechanics and the method of solving the related problems to general formulas whose application gives all necessary equations for the solution of every problem (*id.*).

In part I, the *Statics*, he writes:

the principle of virtual velocities can be stated in this general way,

if an arbitrary system of any number of bodies or mass points, on each of which arbitrary forces act, is in equilibrium, and if an infinitesimal displacement is given to the system, in which each mass point traverses an infinitesimal distance expressing its virtual velocity, then the sum of the forces, each multiplied by the distance which the individual mass point traverses in the direction of the force, will always be equal to 0;

as far as I know, Johann Bernoulli is the first to have recognized the great generality of this principle, as can be seen from a letter he wrote to Varignon in 1717; the same principle is the foundation of the principle proposed by Maupertuis in 1740;

this principle is not only very simple and very

general; it has the unique advantage of expressing a general formula which covers all problems that may arise from the equilibrium of bodies (1.17).

And Lagrange shows the unique advantage:

for the equilibrium of an arbitrary number of forces $P, Q, R, \dots$, acting in the directions $p, q, r, \dots$,

$$P dp + Q dq + R dr + \dots = 0$$

is the general formula of statics for the equilibrium of an arbitrary system of forces (2.2).

In part II, the *Dynamics*, he writes:

in part I we reduced all principles of statics to one general formula which gives all laws of equilibrium (1.12);

one of the advantages of this formula is that it gives immediately the general equations which contain the principles known under the names conservation of living force, conservation of the motion of the center of gravity, conservation of the motion of rotation of areas, and least quantity of action (1.13).

And Lagrange again shows the unique advantage:

we use d to represent differentials with respect to time, and δ to express the variations which represent the virtual velocities;

thus, we have $m \left(\frac{d^2x}{dt^2} \right) \delta x$, $m \left(\frac{d^2y}{dt^2} \right) \delta y$, $m \left(\frac{d^2z}{dt^2} \right) \delta z$ for the moments of the forces $m \left(\frac{d^2x}{dt^2} \right)$, $m \left(\frac{d^2y}{dt^2} \right)$, $m \left(\frac{d^2z}{dt^2} \right)$, which act in the direction of the coordinates x, y, z and have a tendency to increase them; and the sum of their moments can then be represented by the formula

$$\int \left(\frac{d^2x}{dt^2} \delta x + \frac{d^2y}{dt^2} \delta y + \frac{d^2z}{dt^2} \delta z \right) m,$$

where the integral is assumed to extend to all bodies of the system (2.4);

now let $P, Q, R, \dots$ be given forces of acceleration which act on each body m of the system toward the centers to which these forces are assumed directed, and let $p, q, r, \dots$ be the rectilinear distances of each of these bodies from these centers;

the differentials $\delta p, \delta q, \delta r, \dots$ will represent the variations of the lines $p, q, r, \dots$ caused by

the variations $\delta x, \delta y, \delta z$ of the coordinates x, y, z of the body m ;

but because the forces $P, Q, R, \dots$ are assumed to have a tendency to shorten these lines, their virtual velocities must be represented by $-\delta p, -\delta q, -\delta r, \dots$;

therefore, the moments of the forces $mP, mQ, mR, \dots$ will be represented by $-mP\delta p, -mQ\delta q, -mR\delta r, \dots$, and the sum of the moments of these forces will be represented by $-\int (P\delta p + Q\delta q + R\delta r + \dots) m$;

and after equating this sum to the one above, we have

$$\int \left(\frac{d^2x}{dt^2} \delta x + \frac{d^2y}{dt^2} \delta y + \frac{d^2z}{dt^2} \delta z \right) m = - \int (P\delta p + Q\delta q + R\delta r + \dots) m,$$

and transposing,

$$\int \left(\frac{d^2x}{dt^2} \delta x + \frac{d^2y}{dt^2} \delta y + \frac{d^2z}{dt^2} \delta z \right) m + \int (P\delta p + Q\delta q + R\delta r + \dots) m = 0,$$

and this is the general formula of dynamics for the motion of an arbitrary system of bodies (2.5).

Hamilton, in 1834, in *On a general method in dynamics*, writes: among the successors of Galileo and Newton Lagrange has done perhaps more than any other analyst by showing that the most varied consequences respecting the motions of systems of bodies can be derived from one formula, the beauty of the method so suiting the dignity of the results as to make his great work a kind of scientific poem (intr.).

Galvani discovered that animals conduct electricity, and took a step toward the discovery of current electricity.

In 1791, in the *Commentary on the forces of electricity in muscular motion*, Galvani writes:

the discovery was made in this way;

I had dissected a frog, and had laid it on a table on which at some distance stood an electric machine; while I was attending to something else, an assistant touched accidentally the crural nerves of the frog with a scalpel, and all the muscles of the leg seemed to contract as if affected by powerful cramps; another assistant thought he had seen the action excited by a spark from the machine; astonished by this phenomenon, he called my attention to it; I was seized by an incredible zeal to test the result and bring to light what was concealed in it;

I touched the nerve with the scalpel while an assistant drew a spark; and the phenomenon was

always the same; there were contractions in every muscle of the leg the instant the spark jumped (1).

Galvani places the frog on a plate and attaches a hook to its spinal cord, and shows that contractions occur when metals are used for the plate and the hook, but not when glass or resin are used, and that the strength of the contractions depends on the kind of the metals. He concludes:

the electricity is present in the animal itself; and we are confirmed in this view by the assumption that during the occurrence of the phenomenon a very fine nervous fluid flows from the nerves to the muscles like the electric current from a Leyden jar (3).

Proust discovered the law of definite proportions.

In 1794, in the *Researches on Prussian blue*, Proust writes:

if iron, as it is thought, were susceptible of uniting with oxygen in all proportions between 27 to 100 and 48 to 100, must it not produce as many different combinations with the same acid as it can produce different oxides?

yet, on the contrary, a great number of facts shows that iron does not stabilize indifferently at all degrees of oxidation between those two ratios; for instance, only two sulfates of the metal are known;

I conclude from these experiments that it is a principle that iron is subject to the law of nature which rules all true combinations, that it is subject to two constant proportions of oxygen;

and in this way iron is not different from tin, mercury, lead, and so on, and in fact from all known combustibles (*id.*).

Proust also distinguished compound and mixture. In the *Mines of cobalt, nickel, and others*, in 1806, he writes:

what difference do you recognize between chemical combinations, which nature binds in a fixed proportion, and assemblages, which nature does not bind in a fixed proportion?

I reply, the power which makes a metal dissolve in sulfur is not at all like the power which makes one metallic sulfide dissolve in another; the solution of hydrogen in nitrogen which produces ammonia is not at all like that of ammonia in water (*id.*).

Volta discovered current electricity.

In 1800, in *On the electricity excited by the mere contact of conducting substances of different kinds*, Volta writes:

one striking result is that electricity is excited by the mere contact of metals of different kinds with other conductors, liquid or containing liquid, to which indeed they owe their conducting power;

and the most important result, which includes all the others, is the construction of an apparatus which produces effects resembling those of Leyden jars and similar batteries; its effects, however, are incessant; after each discharge the charge reestablishes itself; this apparatus, in a word, produces unlimited charge, or imposes on the electric fluid a perpetual action;

and it is essentially different from those batteries not only because its action is continuous, but because it is not made, as they are, of insulating plates thought to be electric bodies coated with conductors thought to be not electric bodies; this new apparatus is built entirely of these latter bodies, it is built, in fact, of the best conductors, and therefore of those bodies which are thought to be the furthest removed from having an electric nature;

yes! this apparatus, which will no doubt astonish you, is only an assemblage of a number of good conductors of different kinds arranged in a certain way; it consists of 30, or 40, or 60, or more pieces of copper, or better silver, each in contact with a piece of tin, or better zinc, and an equal number of layers of pure water, or another liquid which conducts better like salt water or lye, or pieces of cardboard or leather soaked with that liquid;

and when the layers are interposed between each pair of metals, the series of these three kinds of conductors always in the same order constitutes my new apparatus;

and it surpasses the power of the old batteries infinitely as it does not have to be charged by an external source and gives a shock no matter how many times it is touched (*id.*).

The discovery of the voltaic pile, and current electricity, accelerated the investigation of electricity very greatly. Its effects were incessant. After discharge the charge reestablished itself. In a word, it produced unlimited charge.

Nicholson discovered electrolysis.

Still in 1800, in *An account of the new electric or galvanic apparatus of Mr Volta, and experiments performed with it*, Nicholson reports:

very early in the course of the experiments Mr Carlisle observed the disengagement of gas around one of the plates; and this gas seemed to me to have the smell of hydrogen (*id.*).

Further experiments show the disengagement of two gases. And:

on measuring their quantities, the quantities of water displaced by each gas were, respectively, 72 grains by the gas from the zinc plate, and 142 grains by the gas from the silver plate;

and these are nearly the proportions in bulk of the component parts of water (*id.*).

Haüy established that minerals are crystals.

In 1801, in the *Treatise on mineralogy*, Haüy writes:

geometry has direct and necessary relations with mineralogy, by the description of crystalline forms, and even more by its numerous applications to the structure of crystals, which is nothing but the result of a natural geometry submitted to particular rules, and such that each solid has a figure determined by the combination of an infinite number of other little solids, which are like its primary elements;

these little solids, which are the elements of crystals, and which I call their integrant molecules, have an invariable form for each kind of mineral, in which the faces lie in the direction of the natural joints indicated by the mechanical division of these crystals;

these integrant molecules are the assemblages of the principal molecules, which chemistry will show us (*pr.*).

The primary crystalline forms are parallelepiped, rhombic dodecahedron, hexagonal dipyrmaid, right hexagonal prism, octahedron, and tetrahedron. Showing that geometry has direct and necessary relations with mineralogy.

Berthollet established that the reactions of substances depend on their masses.

In 1801, in the *Researches on the laws of affinity*, Berthollet writes:

if I can prove that a weaker degree of affinity can be compensated by an increase in quantity, it will follow that the action of a body is propor-

tional to such quantity of it as is necessary to produce a certain degree of saturation; this quantity, the measure of the capacity of bodies for saturation, I call mass;

and it follows that in estimating the comparative affinities of bodies their absolute weights must be considered, and must be equal;

but in comparing their actions, which depend on their affinities and mutual proportions, the mass of each body must be considered;

I shall prove, therefore, that in exposing body *A* to the combination *BC*, the combination *AC* can never take place, but body *C* will be divided between bodies *A* and *B* proportionally to the affinity as well as to the quantity of each, and that is, to their respective masses (1).

Dalton discovered Dalton's law. He measured atomic weights. And he formulated the atomic theory.

He discovered Dalton's law in 1801, in the *New theory of the constitution of mixed aeriform fluids*. He writes:

when two elastic fluids *A* and *B* are mixed together, there is no mutual repulsion among their particles, and that is, the particles of *A* do not repel those of *B* as they repel one another; consequently, the pressure, or the whole weight, on any one particle arises only from those of its own kind (*id.*).

In 1803, in *On the absorption of gases by water and other liquids*, Dalton concludes with the question: why does water not admit its bulk of every kind of gas alike (*id.*)?

Dalton's law has posed the question. And Dalton's answer is the atomic theory. He continues:

this question I have duly considered, and although I am not yet able to satisfy myself completely, I am nearly persuaded that the circumstance depends on the weight and number of the ultimate particles of the several gases;

the inquiry into the relative weights of the ultimate particles of bodies is a subject, as far as I know, entirely new; I have lately been prosecuting it with remarkable success; its principle cannot be entered upon in this paper; but I shall subjoin the results ascertained by my experiments (*id.*).

The results are subjoined in a:

table of the relative weights of the ultimate

particles of gaseous and other bodies (*id.*).

6 of its 21 entries, in their order in the periodic table, are:

1. hydrogen	1,
6. carbon	4.3,
7. nitrogen	4.2,
8. oxygen	5.5,
15. phosphorus	7.2,
16. sulphur	14.4 (<i>id.</i>).

The remaining 15 are compounds, including:

ammonia	5.2,
water	6.5,
nitrous gas	9.3,
carbonic acid	15.4 (<i>id.</i>).

Satisfied completely with the atomic theory, Dalton developed it in the *New system of chemical philosophy*, published from 1808 to 1827.

The atomic theory is the solution of the problem of the kind of the elements of all things.

In the *New system*, in 1808, Dalton writes:

all bodies of sensible magnitude, whether liquid or solid, are constituted of a vast number of extremely small particles, or atoms of matter, bound together by a force of attraction;

we may conclude that the ultimate particles of all homogeneous bodies are perfectly alike in weight, figure, etc.; in other words, every particle of water is like every other particle of water, every particle of hydrogen is like every other particle of hydrogen (I, 2.pr.);

we might as well attempt to introduce a new planet into the solar system or annihilate one already in existence as to create or destroy a particle of hydrogen;

and it is one great object of this work to show the importance of ascertaining the relative weights of the ultimate particles, both of simple and compound bodies, the number of simple elementary particles which constitute one compound particle, and the number of less compound particles which enter into the formation of one more compound particle (3.pr.).

The ultimate particles, or atoms of matter, Dalton analyzes into:

simple, binary, ternary, quaternary, quinquenary, etc. (*id.*).

The simple are 20. 14 of them, in their order in the periodic table, with the atomic weights determined by Dalton, and, on the right, the correc-

tions of the atomic weights, are:

1. hydrogen	1	1,
6. carbon	5	12,
7. nitrogen	5	14,
8. oxygen	7	16,
15. phosphorus	9	31,
16. sulphur	13	32,
26. iron	38	56,
29. copper	56	64,
30. zinc	56	65,
47. silver	100	108,
78. platinum	100	195,
79. gold	140	197,
80. mercury	167	201,
82. lead	95	207.

The inquiry into the relative weights of the ultimate particles of bodies was a subject entirely new. And Dalton prosecuted it with remarkable success.

The remaining 6 of the 20 simple ultimate particles, or atoms, are:

soda, magnesia, potash, lime, strontites, and barytes (*id.*),

of each of which Davy, in 1807 and 1808, isolated the element, respectively, 11. sodium, 12. magnesium, 19. potassium, 20. calcium, 38. strontium, and 56. barium.

Dalton also pictures the atoms:

hydrogen	
carbon	
nitrogen	
oxygen	

One of the binary atoms is:

water  (*id.*),

one of the quaternary is:

nitrate  (*id.*).

Dalton also writes, for water:

1 atom of *A* + 1 atom of *B* = 1 atom of *C*,
where *A* and *B* are simple, *C* is binary (*id.*),
for nitrate:

1 atom of *A* + 3 atoms of *B* = 1 atom of *D*,
where *A* and *B* are simple, *D* is quaternary (*id.*).

Dalton was satisfied completely with the atomic theory. To satisfy with it a substantial number of philosophers, even if those satisfied with the restored to the substantiality and purity of its being spirit are ignored, took one century. Resistance continued after the development of the

kinetic theory, too, two generations after Dalton, by Maxwell and Boltzmann. It was quantum theory that enforced satisfaction.

Davy isolated sodium and potassium, in 1807, and magnesium, calcium, strontium, and barium, in 1808, by electrolysis. In 1810 Davy also showed, also by electrolysis, that oxygen is not a constituent element of muriatic acid, and thus that, pace Lavoisier, oxygen is not the principle of acidity. That chlorine is an element as well.

Gay-Lussac discovered that gases expand in proportion to heat, in 1802, and discovered Gay-Lussac's law, in 1808.

In the *Researches on the expansion of gases by heat*, in 1802, Gay-Lussac writes:

the experiments which I have reported prove incontestably that atmospheric air and the gases oxygen, hydrogen, nitrogen, nitrous oxide, ammonia, muriatic acid, sulfurous acid, and carbonic acid all have the same expansion between the same degrees of heat, and consequently, that their greater or lesser density at the same pressure and temperature, their greater or lesser solubility in water, and their particular character have no influence on their expansion;

I conclude that under the same conditions all gases expand equally between the same degrees of heat (4).

In the *Memoir on the combination of gaseous substances with each other*, in 1808, he writes:

Davy, from analysis of the various compounds of nitrogen and oxygen, has found the following proportions by weight,

	nitrogen	oxygen
nitrous oxide	63.30	36.70,
nitrous gas	44.05	55.95,
nitrous acid	29.50	70.50;

reducing these proportions to volume, we find,

nitrous oxide	100	49.5,
nitrous gas	100	108.9,
nitrous acid	100	204.7;

the differences from 100 to 50, 100 to 100, and 100 to 200 are not very great;

and in experiments of the same kind I have assured myself that they are actually nil; we may therefore admit the following numbers for the proportions,

nitrous oxide	100	50,
nitrous gas	100	100,

nitrous acid 100 200 (*id.*).

Earlier in the *Memoir* Gay-Lussac has shown the proportion of combination by volume of nitrogen and hydrogen in ammonia, sulfurous gas and oxygen in sulfuric acid, carbonic oxide gas and oxygen in carbonic acid. He concludes:

it appears to me evident that when they act on one another gases always combine in the simplest proportions, 1 to 1, 1 to 2, 1 to 3;

it is very important to observe that there is no simple ratio between the elements of any compound when their weights are considered;

when volumes are considered, on the contrary, gases always form compounds from elements which are multiples of one another (*id.*).

And Gay-Lussac's law shows that nitrous oxide is N_2O , nitrous gas is NO , nitrous acid is NO_2 .

Proust, Berthollet, Dalton, and Davy presumed that reactions are in proportion to the weights of the reacting substances. Gay-Lussac showed that the reactions of gases are in proportion to the volumes of the reacting substances. Avogadro resolved the difficulty.

Avogadro distinguished atoms and molecules, and discovered Avogadro's law, in 1811.

In the *Essay on a means of determining the relative masses of the elementary molecules of bodies*, Avogadro writes:

Gay-Lussac has shown that gases always unite in a very simple proportion by volume;

the only admissible hypothesis in this connection is that the number of integral molecules in all gases is always the same for equal volumes.

This is the law. Avogadro continues:

and from this hypothesis we have the means of determining very easily the relative masses of the molecules of substances obtainable in gaseous state, as well as the relative number of these molecules in compounds, for the ratios of the masses of the molecules are the same as the densities of the different gases at equal temperature and pressure, and the relative number of molecules in a compound is given at once by the ratio of the volumes of the gases that form it;

for instance, since 1.10359 and 0.07321 are the densities of oxygen and hydrogen with atmospheric air as the unity, this is on our hypothesis the ratio of the masses of their molecules;

thus the mass of the molecule of oxygen is to that of hydrogen as 15.074 to 1;

on the other hand, since the ratio of the volume of hydrogen and oxygen in the formation of water is 2 to 1, it follows that water results from the union of each molecule of oxygen with two molecules of hydrogen.

The elementary molecules are the atoms, the integral molecules are the molecules.

And Avogadro has thus also distinguished atomic weight, the quantity of a substance, and equivalent weight, the quantity of it which reacts with a quantity of another substance.

Avogadro's number:

the number of molecules in all gases is always the same for equal volumes,

has been generalized. Avogadro's number is the number of atoms or molecules in a unit of substance. The number is 6.022×10^{23} .

Avogadro distinguished atoms and molecules, and atomic weight and equivalent weight, in 1811. The importance of this discovery, as of so many others, took time to appreciate. Until 1858, in particular, when Cannizzaro published the *Sketch of a course of chemical philosophy*. Given Avogadro's discovery, in 1869 Mendeleev discovered the periodic law.

Of the magnitude of the advance in the solution of the problem of the elements in the twenty-two years after Lavoisier's *Treatise* the following feat of Gay-Lussac's provides an analogy. The brothers Montgolfier built the first hot-air balloon in 1783. In 1804 Gay-Lussac ascended, as he records in the *Report of an aerostatic voyage*:

to a height of 6 977 m above Paris, or 7 016 m above the level of the sea (*id.*).

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Young and Fresnel, between 1801 and 1821, established the wave theory of light.

Newton had shown that light is particles, Huygens had shown that it is waves. And in the meantime, with Newton, against Huygens, the particle theory had prevailed.

In 1801, in *On the theory of light and colors*, with Huygens, against Newton, Young affirms:

light consists in undulations of the luminiferous aether (9).

And he formulates the principle of interference:

when two undulations from different origins coincide in direction, their joint effect is a combination of the motions of both (8).

In 1802, in *An account of some cases of the production of light not hitherto described*, he formulates the principle more explicitly:

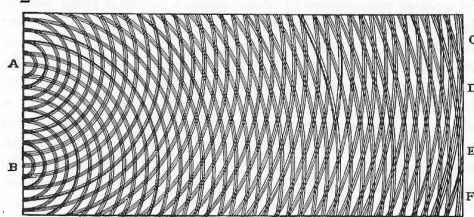
whenever two portions of light arrive at the eye by different paths in the same direction, light becomes most intense when the difference of the paths is any multiple of a certain length, and least intense in the intermediate regions of the interfering portions; and this length is different for light of different color (*id.*).

In 1807, in *A course of lectures on natural philosophy and the mechanical arts*, he demonstrates interference experimentally. He writes:

interference may be produced by diffraction, reflection, refraction, or any of these effects combined;

but the simplest case appears to be when a beam of homogeneous light falls on a screen in which there are two very small holes or slits, which may be considered centers of divergence, and from which light is diffracted in every direction; in this case, when the two newly formed beams are intercepted on a surface, their light is divided by dark stripes into positions nearly equal, wider when the surface is further from the apertures, subtending very nearly equal angles from the apertures at all distances, and wider in the same proportion as the apertures are closer to each other; the middle is always bright, and the bright stripes on each side are at such distances that light coming to them from one of the apertures must have passed through a space longer than that through which light coming from the other by an interval equal to the breadth of 1, 2, 3, . . . of the supposed undulations, while the intervening dark spaces correspond to $\frac{1}{2}$, $1\frac{1}{2}$,

$2\frac{1}{2}$, . . . (39).



A and B are the two very small holes or slits, C , D , E , and F are the dark spaces.

Hence Young also measures the wavelength of light of different colors:

and from the comparison of various experiments, it appears that the breadth of the undulations constituting extreme red light must be, in air, about $\frac{1}{36\,000}$ of an inch, of those of extreme violet about $\frac{1}{60\,000}$ (*id.*).

The wavelength of extreme red is thus about 700 nm, of extreme violet about 400 nm. The wavelength of extreme red is, in fact, 740 nm, of extreme violet 390 nm.

Young adds:

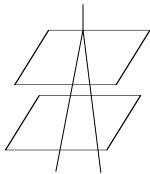
from these dimensions it follows, calculating from the known velocity of light, that almost 500 000 000 000 000 of the slowest of these undulations must enter the eye in a single second (*id.*).

The velocity of light Bradley had measured at 2.8×10^8 m/s in 1728.

In the century after Young, and decisively after Fresnel, who formulated the theory analytically, the wave theory of light prevailed.

Malus discovered the polarization of light in reflection, in 1808, in the *Memoir on a new phenomenon of optics*.

Huygens had discovered the polarization of light: a marvelous phenomenon of which I have not yet been able to find the cause (*Light*, 5); the phenomenon is this; if we place two blocks of Iceland crystal one over the other, a ray of light divides into two rays on passing through the first block, but each of these two rays passes through the second block without dividing again into two rays; it is wonderful that the two rays do not divide in the same way as the first ray; we can say that in passing through the first block the ray of light loses a certain property (*id.*).



In the *Memoir on a new phenomenon*, Malus writes:

this power of changing the character of light and giving it a new property is not peculiar to Iceland spar; I have found it in all known substances which produce double images;

for instance, light reflected by the surface of water at an angle of $52^\circ 45'$ with the vertical has all the properties of one of the rays produced by the double refraction of Iceland spar (*id.*).

In 1811, in the *Theory of the double refraction of light in crystal substances*, Malus explains the phenomenon:

to account for this phenomenon we must say that it depends on the form of the particles of light, which align themselves in reflection and refraction along their poles and remain afterward mutually parallel (II, 3).

He also formulates Malus's law:

the force of the polarized ray is represented by $k^2 \sin^2 \mu$, where k^2 is the living force of the incident ray, μ is the angle of the polarized ray with the vertical to the surface of the crystal (4).

Poisson, in 1813, in the *Remarks on an equation which presents itself in the theory of attraction of spheroids*, found this solution of the Laplace equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho,$$

where the spheroid is heterogenous, and ρ is the density of the attracted point (*id.*).

And on this ground Poisson formulated electrostatics analytically.

Maxwell writes: to find the distribution of electricity on the surface of a sphere, we need to expand the potential of the influenced system in a series of solid harmonics of positive degree with the center of the sphere as an origin, and to find a corresponding series of solid harmonics of negative degree expressing the potential due to the electrification of the sphere; by the use of this very powerful means of analysis, Poisson determined the electrification of a sphere under the influence of a given electric system, and solved also the more difficult problem of the distribution of electricity on two conducting spheres in presence of each other (*Treatise*, 155).

Fresnel formulated the wave theory of light analytically.

In 1815, in the *Complement to the memoir on the diffraction of light*, he writes:

analogy leads me to believe that light, as well as heat, is propagated not by the emission of caloric but by the vibrations of caloric (26).

By 1816 he has proposed that not a longitudi-

nal wave, vibrating in the direction of propagation, but a transverse wave, vibrating perpendicularly to the direction of propagation, can explain polarization. In the *Memoir on the influence of polarization on the action which rays of light exert on one another*, he concludes:

two systems of waves in which the longitudinal motion of the particles of the fluid are modified by a transverse motion, perpendicular and equal in intensity, can exert no action on one another when the discord of the transverse motions corresponds to the accord of the longitudinal motions, because the resultants of the forces in each system will be perpendicular to one another (14).

By 1818 he has proposed that light, in general, is a transverse wave, not, like sound, a longitudinal. In the *Note on the theory of colors in polarization developed by small slits*, he writes:

the oscillatory motions of the points of the aether in ordinary light are all directed perpendicularly to the waves or, if there are oblique motions, all take place around the normal to the same obliquity and with the same degree of force in all directions (*id.*).

And by 1819, in the *Memoir on the diffraction of light*, the analytic formulation of the theory complete, Fresnel has deduced reflection, refraction, diffraction, and polarization, as well as the principle of interference, as well as the rectilinear propagation of light, from Huygens's principle:

the vibrations of a luminous wave in each of its points can be considered the sum of the elementary motions which would be sent to this point simultaneously by each of the parts of the wave acting in isolation (43).

In 1821, in the *Note on the calculation of colors in polarization developed by crystal slits*, Fresnel writes:

light can be considered the reunion, or more exactly the rapid succession, of systems of waves polarized in all directions (13).

In 1864 Maxwell will show that light is an electromagnetic wave. In 1882 Kirchhoff will show that Huygens's principle follows from the wave equation.

Berzelius formulated the dualistic theory.

In 1819, in the *Essay on the theory of chemical proportions and on the chemical influence of*

electricity, Berzelius writes:

every chemical combination is entirely and solely dependent on two opposing forces, positive and negative electricity, and every chemical compound must be composed of two parts combined by the agency of their electrochemical reaction; for there is no third force;

it follows that every compound body, whatever the number of its constituents, can be divided into two parts, one of which is positively and the other negatively electric (3).

Berzelius isolated selenium in 1817, silicon in 1823, zirconium in 1824, titanium in 1825, thorium in 1828.

Ørsted discovered electromagnetism.

In 1820, in the *Experiments on the effect of electric conflict on a magnetic needle*, Ørsted showed that electric current deflects a magnetic needle. He writes:

let the opposite poles of a galvanic apparatus be joined by a wire;

let the wire be placed over a magnetic needle parallel to it;

when the wire receives electricity from the negative end of the apparatus the magnetic needle is deflected to the west;

when the wire is placed under the needle, the magnetic needle is deflected toward the east (*id.*).

Biot and Savart quantified Ørsted's discovery into Biot-Savart's law.

In 1820, in the *Note on the magnetism of Volta's battery*, they write:

the action of the voltaic current on a particle of magnetism is perpendicular to the axis of the wire, and its intensity is inversely proportional to its distance from the wire (*id.*).

Faraday discovered electromagnetic rotation, and thus the electromotor, in 1821.

Seebeck, in 1822, in the *Magnetic polarization of metals and ores by a difference in temperature*, showed that a current arises between metals at different temperature. He writes:

I encountered phenomena showing that two metals joined in a circuit can by themselves, without the additional action of a liquid conductor, become magnetic (*intr.*);

there is no action on the magnetic needle when glass is placed on one of the metals; but there is when a hand is placed on it; the conclusion can

be drawn that the heat communicated by the hand is the cause of the magnetism in the circuit (11).

And Seebeck corroborates the conclusion with several experiments (12-20).

Fourier wrote the *Analytic theory of heat*, in 1822.

Fourier writes:

in this work we have demonstrated all principles of the theory of heat, and solved all fundamental problems (pr.).

In particular:

the quantity of heat which a particle acquires is given by the general equation

$$\frac{\partial u}{\partial t} = \frac{K}{CD} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

where u is the temperature at time t of a particle with coordinates x , y , and z , K is the specific conductivity, C is the specific capacity, or the quantity of heat which raises a unit of weight from the temperature of 0 to that of 1, and D is the density of the solid;

this is the equation of the propagation of heat in the interior of all solid bodies (142).

The general solution of the equation is the Fourier series:

$$f(x) = A + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x \dots,$$

and one of the main elements of our analysis is that each of the coefficients is a definite integral,

$$2\pi A = \int_{-\pi}^{+\pi} dx f(x),$$

$$\pi a_i = \int_{-\pi}^{+\pi} dx f(x) \cos ix,$$

$$\pi b_i = \int_{-\pi}^{+\pi} dx f(x) \sin ix \quad (233).$$

Fourier concludes:

any function whatsoever can be developed in this form (230).

Maxwell writes: the methods developed by Fourier in *The theory of heat* belong to the general theory of the application of mathematics to physics; for in every branch of physics, when the investigation turns on the expression of arbitrary conditions, we have to follow the method which Fourier first pointed out in his *Theory of heat* (*Heat*, 18).

Riemann writes: with Fourier a new epoch in the development of this part of mathematics, the analytic representation of arbitrary functions, began, which soon became manifest also in the magnificent expansion of mathematical physics

(*Series*, 2).

Ampère distinguished electrostatic and electrodynamic phenomena, showed that magnetic phenomena are the effect of electric currents, and unified all those phenomena in the analytic theory of electrodynamics under a single law.

Ampère began developing the theory in the *Experiments on new electrodynamic phenomena* and the *Memoir on the determination of the formula which represents the mutual action of two infinitely small portions of voltaic conductors*, in 1822, and formulated it conclusively in the *Mathematical theory of electrodynamic phenomena, uniquely deduced from experience*, in 1827.

The *Mathematical theory of electrodynamic phenomena* is a magnificent expansion of mathematical physics.

In the *Experiments on new electrodynamic phenomena*, Ampère writes:

I use the term electrodynamic to unite under a common name all phenomena produced by the conducting wires of the voltaic pile and all those which I have myself observed between two voltaic conductors; the term expresses their true character, that of being produced by electricity in motion;

on the other hand, the electric attractions and repulsions, which have been known for a long time, are electrostatic phenomena produced by the unequal distribution of electricity at rest in the bodies in which they are observed (*id.*).

In the *Mathematical theory*, he writes:

the way in which I conceive magnetic phenomena is to regard magnets as assemblages of electric currents forming minute circuits around their particles (*id.*).

Ampère's law is:

$$F = \frac{i i' ds ds'}{r^2} (\sin \theta \sin \theta' \cos \omega - \frac{1}{2} \cos \theta \cos \theta'),$$

where F is the force between the two current elements $i ds$ and $i' ds'$ in two wires at a distance r , ω is the angle between the elements, and θ and θ' are the angles between the line joining the elements and the two planes containing this line (*id.*).

Maxwell's fourth equation is a generalization of Ampère's law.

Ampère concludes in the *Mathematical theory*:

my theory explains from a single principle three kinds of actions, which all phenomena associated with them prove are due to a single cause (*id.*).

Maxwell writes: the experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science; the whole, theory and experiment, seems as if it has leaped, full grown and full armed, from the brain of the Newton of electricity; it is perfect in form and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced and which must always remain the cardinal formula of electrodynamics (*Treatise*, 528).

Carnot discovered the Carnot cycle and the Carnot principle in 1824.

Ohm discovered Ohm's law, in 1827, in *The galvanic chain treated mathematically*.

This is how Maxwell describes the discovery: the relations of electromotive force, current, and resistance were first investigated by Ohm; the result of these investigations in the case of homogeneous conductors is Ohm's law; Ohm's law is, the electromotive force acting between the extremities of any part of a circuit is the product of the strength of the current and the resistance of that part of the current; here a new term is used, the resistance of a conductor, defined as the ratio of the electromotive force to the strength of the current which it produces; the introduction of this term would have been of no scientific value unless Ohm had shown, as he did experimentally, that it corresponds to a real physical quantity, and that is, it has a definite value altered only when the conductor is altered (*Treatise*, 241).

Robert Brown discovered Brownian motion in 1828.

Brownian motion is, as Einstein describes it: the motion of particles suspended in liquids at rest required by the molecular kinetic theory of heat.

Faraday discovered electromagnetic induction in 1831.

He discovered the laws of electrochemistry in 1834.

Lenz discovered Lenz's law also in 1834.

In *On the determination of the direction of the*

galvanic current excited by electrodynamic induction, Lenz writes:

the law according to which the magnetoelectric phenomena are reduced to electromagnetic phenomena is the following,

whenever a metallic conductor moves in the neighborhood of a galvanic current or a magnet, a galvanic current is induced in it, which has such a direction that it would have caused in the wire, if it were at rest, a motion in the exactly opposite direction to the one given to it (*id.*).

Helmholtz will show that Lenz's law is a particular case of the conservation of energy.

Hamilton generalized the principle of least action also in 1834.

Faraday discovered specific inductive capacity in 1837.

Gauss advanced the investigation of electricity and magnetism, too.

In 1832, in *The intensity of the magnetic force of the earth referred to an absolute measure*, Gauss measured magnetic force, and hence also electric force, in absolute units, distance, time, and mass.

In 1839 he discovered Gauss's law.

The potential entered theory with the investigation of the gravitational potential by Lagrange and Laplace. In 1839, in the *General theory of the magnetism of the earth*, Gauss applied the theory of the gravitational potential to the investigation of magnetic phenomena, and hence also electric phenomena. This application of the potential is Gauss's law. In 1840, in the *General theorems on the forces of attraction and repulsion acting inversely as the square of the distance*, Gauss extended the theory further, into the general mathematical potential theory.

Maxwell's third equation is a generalization of Gauss's law.

This is what Maxwell writes on the law:

the theorem, the potential due to a magnetic shell at any point is the product of its strength into the solid angle subtended by its edge at the given point, is due to Gauss in the *General theory of the magnetism of the earth* (*Treatise*, 409);

if we consider the solid angle as the potential of a magnetic shell of unit strength whose edge coincides with the closed curve, we must define it as the work done by a unit magnetic pole

against the magnetic force while it moves from an infinite distance to the point P ; hence, if σ is the path of the pole as it approaches the point P , the potential must be the result of line integration along this path; it must also be the result of a line integration along the closed surface s ; the proper form of the expression for the solid angle must therefore be that of a double integration with respect to the two curves s and σ (419);

if a closed curve is drawn so as to pass once through the shell, or in other words, if it is linked once with the edge of the shell, then the value of the integral $\iint \Pi ds d\sigma$ extended around both curves is 4π ;

it was Gauss's discovery of this very integral, expressing the work done on a magnetic pole while describing a closed curve in the presence of a closed electric current, and indicating the geometrical connection between the two closed curves, that led him to lament the small progress made in the geometry of position since the time of Leibniz, Euler, and Vandermonde; we have now, however, some progress to report, chiefly due to Riemann, Helmholtz, and Listing (421).

Joule demonstrated the mechanical equivalent of heat in 1843.

In 1841, in *On the heat evolved by metallic conductors of electricity and in the cells of a battery during electrolysis*, he discovered Joule's law:

when a current of voltaic electricity is propagated along a metallic conductor, the heat evolved in a given time is proportional to the resistance of the conductor multiplied by the square of the electric intensity (1).

In 1843, the discovery of the mechanical equivalent of heat is in *On the calorific effects of magneto-electricity, and on the mechanical value of heat*.

It is the report of a series of experiments. Joule measures the rise in temperature of the water in a brass vessel, caused by the friction produced by the rotation in the water of an axle with sixteen paddles, itself produced by an electromotor, over different periods of time.

This is how Helmholtz describes the experiments: it is to Mr Joule that we are indebted for the important and laborious experiments in that part of the theory, in which the applicability of

the law of conservation of force appeared the most doubtful and the gaps in knowledge were the greatest, namely, the production of work from heat and of heat from work (*Goal*).

Joule concludes from the experiments:

one degree of heat per pound of water from a mechanical force is capable of raising about 770 pounds to a height of one foot;

and I shall lose no time repeating and extending these experiments, being satisfied that the grand agents of nature are, by the creator's fiat, indestructible, and that wherever mechanical force is expended, an exact equivalent of heat is always obtained (*id.*).

The degree is Fahrenheit.

In 1843 Joule measures the mechanical equivalent of heat at about 770 foot-pounds. In 1845 he describes it picturesquely: the temperature of the river Niagara is raised by its fall of 160 feet about one fifth of a degree (*Ordinary*). In 1850 he measures it at: 772.692 foot-pounds (*Equivalent*). As it is, 772 foot-pounds.

Helmholtz writes: Joule established the important fact of the mechanical equivalent of heat; from this fact it follows necessarily that heat is not, as it was formerly believed, a fine imponderable substance, it is, like light, an undulating motion of the ultimate particles of bodies (*Interaction*).

Faraday discovered the magnetic rotation of the plane of polarization in 1845.

He discovered diamagnetism also in 1845.

Boole showed that logic is algebra in 1847.

Helmholtz discovered the principle of conservation of energy also in 1847.

Kelvin – as he became in 1892, William Thomson before that – discovered the absolute scale of temperature.

In 1848, in *On an absolute thermometric scale, founded on Carnot's theory of the motive power of heat, and calculated from the results of Regnault's experiments on steam*, Kelvin writes:

in the present state of physical science a question of extreme importance arises, is there any principle on which an absolute thermometric scale can be founded?

it appears to me that Carnot's theory of the motive power of heat enables us to give an affirmative answer;

the characteristic property of the scale which I propose is that all degrees have the same value, and that is, that a unit of heat descending from a body *A* at the temperature T° of this scale to a body *B* at the temperature $(T - 1)^\circ$ would give out the same mechanical effect whatever be the number *T*;

this may justly be termed an absolute scale, since its characteristic is quite independent of the physical properties of any specific substance (*id.*).

In *On the dynamical theory of heat, with numerical results deduced from Mr Joule's Equivalent of a thermal unit, and M Regnault's Observations on steam*, in 1851, Kelvin writes:

two fixed points of temperature being chosen according to Sir Newton's suggestion, the difference of these temperatures is to be called unity; the particular convention is that the difference of temperatures between the freezing and the boiling points of water under standard atmospheric pressure shall be called 100 degrees;

and the best result which we have yet been able to obtain with regard to the determination of the absolute temperature of these fixed points is that the temperature of freezing water is 273.7 on the absolute scale, that of boiling water being consequently 373.7 (100).

If, on the same convention, 0° is the freezing point of water, -273.15° is the absolute zero. This, the kinetic theory will show, is the temperature at which the kinetic energy of atoms and molecules is 0.

Clausius formulated the laws of thermodynamics in 1850.

And the results of Faraday, Carnot, Hamilton, Boole, Helmholtz, and Clausius should be presented more particularly.

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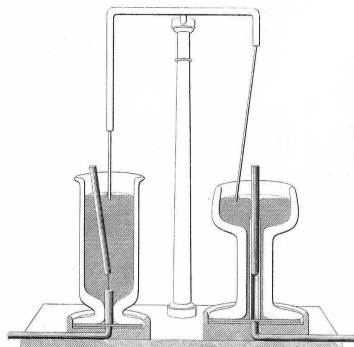
Faraday discovered electromagnetic rotation, and thus the electromotor, in 1821, electromagnetic induction in 1831, the laws of electrochemistry in 1834, specific inductive capacity in 1837, the magnetic rotation of the plane of polarization in 1845, diamagnetism also in 1845. These are only the most important of his experimental results. And in the course of developing the theory of the experimental results Faraday discovered that electromagnetism is a field.

Maxwell, in *On Faraday's lines of force*, in 1855, with which begins the formulation of the theory culminating in the discovery of the laws of electromagnetism in *A dynamical theory of the electromagnetic field*, in 1864, in *On Faraday's lines of force* Maxwell reports: I have hardly made a single experiment (*pr.*). He did not have to, Faraday had made them all. Maxwell continues: I am confining myself to the development of ideas originated by that philosopher (*id.*). That philosopher is Faraday, the ideas are the electromagnetic field.

Faraday published the three volumes of the *Experimental researches in electricity* from 1839 to 1855. The bulk of the *Experimental researches* is divided into 29 series of numbered paragraphs, for a total of 3 362 numbered paragraphs. Series of other publications are also appended in volumes II and III. In volume I Faraday collects his publications from 1831 to 1838. In volume II he collects his publications from 1838 to 1844, appending some of his earlier publications beginning in 1821. In volume III he collects his publications through 1855.

The discovery of electromagnetic rotation, and thus of the electromotor, in 1821, is in *On some new electro-magnetic motions, and on the theory of magnetism*. It is representative. From the first to the last, Faraday demonstrates the experimental results, then turns to the theory.

The picture is from the *Description of an electro-magnetic apparatus for the exhibition of rotary motion*, in 1822.



The two beakers are filled with mercury. The hanging wire dipped in the left beaker is fixed, the wire dipped in the right beaker hangs loosely. The cylindrical bar magnet in the left beaker is

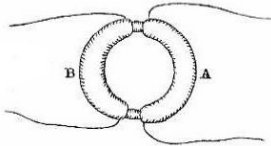
mounted loosely, that in the right beaker is fixed. The current passes through the mercury and wire on the left through the wire and mercury on the right, or the other way around. And when the current passes through the apparatus, the bar magnet in the left beaker rotates around the wire, the wire dipped in the right beaker rotates around the bar magnet.

As to the theory, Faraday concludes in *On some new electro-magnetic motions*:

it appears very probable that the iron or steel in the bar magnet is in the same state as the copper wire; and perhaps, as M Ampère maintains in his theory, by the same means, namely, currents of electricity (*id.*).

With the discovery of electromagnetic induction, in 1831, the *Experimental researches* opens, under the heads *On the evolution of electricity from magnetism*, then *New electric state or condition of matter*.

This is the first experiment in *On the evolution of electricity from magnetism*. Faraday writes:



a welded ring was made of soft round bar-iron, the metal being seven eighths of an inch in thickness, and the ring six inches in external diameter; three helices were put round one part of the ring, each containing about twenty-four feet of copper wire one twentieth of an inch thick; they were insulated from the iron and each other; this group may be distinguished by the letter *A*; on the other part of the ring about sixty feet of similar copper wire in two pieces were applied in the same manner, forming a helix *B*, which had the same common direction with the helices of *A*, but separated from it at each extremity by about half an inch of uncovered iron (27);

the helix *B* was connected by copper wires with a galvanometer three feet from the ring;

the helices of *A* were connected end to end so as to form one common helix, the extremities of which were connected with a battery of ten pairs of plates four inches square;

the galvanometer was affected immediately;

but although the contact was continued, the

effect was not permanent, the needle soon came to rest in its natural position, as if quite indifferent to the attached electro-magnetic arrangement;

upon breaking the contact with the battery, however, the needle was again powerfully deflected (28);

upon using the power of one hundred pairs of plates, the impulse on the galvanometer, when contact was completed or broken, was so powerful as to make the needle spin rapidly four or five times (31).

Although the contact was continued, the needle soon came to rest in its natural position, as if quite indifferent to the attached electro-magnetic arrangement.

Faraday concludes in the *New electric state or condition of matter*:

when subject to either volta-electric or magneto-electric induction, the wire appears to be in a peculiar state; for it resists the formation of an electric current in it, whereas, if in its common condition, such a current would be produced;

this electric condition of matter has not hitherto been recognized, but it probably exerts a very important influence in many if not most of the phenomena produced by currents of electricity; I have ventured to designate it the electro-tonic state (60);

this peculiar condition shows no known electric effects while it continues (61);

it shows no reaction by attractive or repulsive powers (62);

it exerts no retarding or accelerating power on electric currents passing through metal (65);

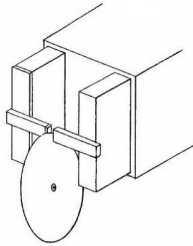
all metals take on this peculiar state (66);

this peculiar state appears to be a state of tension, and may be considered equivalent to a current of electricity; the current, however, first or last, is not to be considered a measure of the degree of tension (71);

all results favor the notion that the electro-tonic state relates to the particles, and not to the mass, of the wire or substance under induction;

and if so, the state may be assumed in liquids when no electric current is sensible, and even in non-conductors; the current itself, when it occurs, being as it were a contingency due to the momentary propulsive force exerted by the particles during their arrangement (73).

Still in 1831 Faraday built the first dynamo.
The picture is from the *Explication of Arago's magnetic phenomena* (84).



A copper disc is placed between the poles of a magnet. And if one end of a wire touches the center of the disc and its other end touches the edge of the disc, when the disc is rotated, a steady current runs through the wire.

In 1820 Ørsted converted electric force into magnetic, in 1831 Faraday converted magnetic force into electric. In 1821 Faraday converted electric force into mechanical, in 1831 he converted mechanical force into electric.

In 1833, in the *Identity of electricities derived from different sources*, Faraday writes:

the progress of my researches has brought me to a point where it is essential for their further prosecution that no doubt remains of the identity or distinction of electricities excited by different means;

many philosophers still draw distinctions between electricities of different sources, or at least doubt that their identity is proved (265).

Faraday tabulates his results:

	Physico-chemical Effects.	Magnetic Induction.	Magnetic fluids.	Sparks.	Heating Power.	True chemical Action.	Attraction and Repulsion.	Discharge by Air, etc.
1. Voltaic electricity ...	x	x	x	x	x	x	x	x
2. Common electricity...	x	x	x	x	x	x	x	x
3. Magneto-Electricity...	x	x	x	x	x	x	x	
4. Thermo-Electricity...	x	x	+	+	+	+		
5. Animal Electricity...	x	x	x	+	+	x		

In a note added in 1838 he explains:

many of the spaces originally left blank may now be filled; I have therefore filled up these spaces with crosses; there remain but five blank spaces; and though these effects have not yet been obtained, it is a necessary conclusion that they must be possible (360).

The discovery of the laws of electrochemistry,

in 1834, is in the seventh series of the *Experimental researches*.

In a *Preliminary*, Faraday introduces a series of new terms:

I propose the term electrode for that substance, or rather surface, whether of air, water, metal, or any other body, which bounds the extent of the decomposing matter in the direction of the electric current (662);

if the current is parallel to that supposed to exist in the earth, the electrode toward the east is called the anode, that toward the west the cathode (663);

the bodies decomposed directly by the electric current I propose to call electrolytes; water therefore is an electrolyte (664);

and I propose to distinguish such bodies by calling anions those that go to the anode of the decomposing body, cations those that go to the cathode, and when I have occasion to speak of them together, I shall call them ions;

thus, the chloride of lead is an electrolyte, and when electrolyzed, it evolves the two ions, chlorine and lead, the former being an anion, the latter a cation (665).

The laws Faraday formulates in *On the definite nature and extent of electro-chemical decomposition*:

the chemical power of a current of electricity is in direct proportion to the absolute quantity of electricity which passes (821);

and only single electrochemical equivalents of elementary ions can go to the electrodes, and not multiples (830); and electrochemical equivalents coincide, and are the same, with ordinary chemical equivalents (836).

The theory follows in *On the absolute quantity of electricity associated with the particles or atoms of matter*. Faraday writes:

the theory of definite electrolytic or electro-chemical action appears to me to touch immediately on the absolute quantity of electricity or electric power belonging to different bodies;

as soon as we perceive, through the teaching of Dalton, that chemical powers are definite for each body, we learn to estimate the relative degree of force residing in it, and when upon this knowledge comes the fact that the electricity, which we appear capable of loosening from its habitation

and conveying from place to place while it retains its chemical force, is as definite in its action as any of those bodies associated with the particles of matter, we seem to have found the link between the proportion of that, which we have evolved, and the proportion of that, which belongs to the particles in their natural state (852);

it seems probable, and almost a natural consequence, that the quantity of electricity which passes in chemical decomposition is the equivalent of, and therefore equal to, that of the separated particles (855);

in further proof I will describe an experiment of great simplicity and extreme beauty (862).

It is the electrolytic action of zinc in water. Water decomposes, zinc is oxidized, hydrogen escapes. Faraday weighs a piece of zinc before electrolysis and after, the difference is 8.45 grains, and this is thus the amount of zinc oxidized and dissolved in the water. He captures the escaping hydrogen, measures its volume, this gives its weight, and this gives the weight of the decomposed water at 2.3536 grains. And:

2.3536 to 8.45 is as 9 to 32.31;

and as 9 is the equivalent number of water, 32.5 is the equivalent number of zinc, a coincidence sufficient to show that for an equivalent of zinc to be oxidized an equivalent of water must be decomposed (866);

the equivalent weights of bodies are simply these quantities of the bodies which contain equal quantities of electricity or have naturally equal electric powers (869).

The two fluid theory, formulated first by du Fay in 1733, the one fluid theory, formulated first by Franklin in 1747, Coulomb's law in 1785, Ampère's law in 1827, and Ampère's theory of electric currents forming minute circuits around the particles of bodies, had all been formulated on the presumption of action at a distance, such as Newton had shown the action of gravity is. In 1837 Faraday began building a theory of electromagnetism explicitly against the presumption of action at a distance.

This is in the eleventh series of the *Experimental researches*.

In the *Induction an action of contiguous particles* he writes:

I believe ordinary induction to be in all cases

an action of contiguous particles, consisting in a species of polarity, not an action of either particles or masses at sensible distances (1165);

I have searched for an unexceptional test of my theory, whose consequences would be inconsistent with the theory of action at a distance;

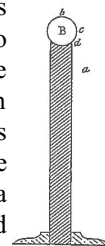
such a consequence seems to me the direction in which inductive action can be exerted; if in straight lines, though not perhaps decisive, it would be against my view; but if in curved lines also, this would be a natural result of the action of contiguous particles, but, I think, utterly incompatible with action at a distance (1166).

And in the *Induction in curved lines*:

I have endeavored to devise such experiments as should not include transfer, but relate entirely to the pure simple inductive action of static electricity (1216);

I will at once proceed to the simplest mode of proving the facts (1217);

a cylinder of solid shell-lac was made concave at its upper extremity so that a brass ball could stand on it; the upper half of the cylinder having been excited negatively by friction, a brass ball, *B*, was placed on the top; then the whole arrangement was examined by a carrier and an electrometer charged positively to about 360° ;



the electricity at the different positions *a*, *b*, *c*, *d*, observed in succession, was always positive and its force was as follows,

at <i>a</i>	above 1000° ,
at <i>b</i>	149° ,
at <i>c</i>	270° ,
at <i>d</i>	512° ,
at <i>b</i>	130° (1218);

to comprehend the full force of these results, it must be understood that all charges of the ball *B* and the carrier are charges by induction from the action of the excited surface of the shell-lac cylinder;

and the charges at *a*, *c*, and *d* are of such a nature as might be expected from an inductive action in straight lines; but that at *b* is not, it is clearly a charge by induction, and induction in a curved line (1219).

The discovery of specific inductive capacity, also in 1837, follows, in *On specific induction, or*

specific inductive capacity. Faraday writes:

I now proceed to examine the great general question of specific inductive capacity, i. e. whether different bodies actually possess any influence over the degree of induction which takes place through them; if such difference exists, it appears to me not only of high importance in the further comprehension of the laws and results of induction, but an additional and very powerful argument for the theory I have ventured to put forth (1252).

Faraday answers the question in the affirmative, here as everywhere experimentally, and proceeds to the theory, in the *General results as to induction*:

induction appears to consist in a certain polarized state of the particles, into which they are thrown by the electrified body sustaining the action (1298);

the principle of induction is of the utmost generality in electric action;

it constitutes charge in every ordinary case, and probably in every case;

it appears to be the cause of excitement, and to precede current;

the degree to which the particles are affected in their forced state, before discharge, appears to constitute what we call intensity (1299);

discharge is the return of these particles to their natural state from their state of tension (1300).

In 1840 Faraday has to write, in *An answer to Dr Hare's Letter on certain theoretical opinions*:

a man who advances what he thinks to be new truths, and develop principles which profess to be more consistent with the laws of nature than those already in the field, is liable to be charged first with self-contradiction; then with the contradiction of facts; or that he may be obscure in his expression and so subject to certain queries; or that he may be in disagreement with the opinions of others; the first and second points are very important, and every one subject to such charges must be anxious to be made aware of them; the third is also a fault to be removed if possible; the fourth is but of small consequence in comparison with the other three; for every man who has the courage, not to say rashness, of forming an opinion of his own, thinks it better than any from

which he differs; so it is only deeper investigation, and most generally future investigators, who can decide which is in the right (*id.*).

The development of the theory continues in *A speculation touching electric conduction and the nature of matter*, in 1844. Faraday writes:

if we must assume at all, then the safest course appears to be to assume as little as possible, and in this way the atoms of Boscovich appear to me to have a great advantage over the more usual notion;

his atoms are mere centers of forces, not particles of matter in which the forces reside;

if, in the ordinary view of atoms, we call the particle of matter *a* and the system of forces in and around it *m*, then, while in the usual notion *a* is a little unchangeable piece of matter and *m* is an atmosphere of force around it, in Boscovich's theory *a* disappears, or is a mere mathematical point;

with these atoms matter is everywhere present, and there is no intervening space;

matter is continuous throughout;

the atoms may be conceived highly elastic;

if an atom is conceived a center of power, what is ordinarily called shape would be the disposition and relative intensity of the forces;

matter and the atoms of matter would be mutually penetrable (*id.*).

The discovery of the magnetic rotation of the plane of polarization, in 1845, is in the nineteenth series of the *Experimental researches*.

In the *Action of magnets on light* Faraday begins:

I have long held the opinion, almost amounting to conviction, in common, I believe, with many other lovers of natural knowledge, that the various forms under which the forces of matter are made manifest have one common origin, or in other words, are so directly related and mutually dependent that they are convertible into one another and possess equivalents of power in their action;

in modern times the proofs of their convertibility have accumulated to a very considerable extent, and a commencement has been made of the determination of their equivalents (2146);

and I have recently at last succeeded in magnetizing and electrifying a ray of light, and in

illuminating a magnetic line of force (2148).

The experimental demonstrations follow. And in the *General considerations* Faraday concludes:

thus a true direct relation and dependance between light and the magnetic and electric forces is established, and thus a great addition is made to the facts and considerations which tend to prove that all natural forces are tied together and have one common origin (2221).

The discovery of diamagnetism, also in 1845, follows in the twentieth series.

In *On new magnetic actions, and on the magnetic condition of all matter*, Faraday begins by noting:

the effects to be described require magnetic apparatus of great power, and under perfect command (2243).

He introduces the definitions:

the chief directions of position across a magnetic field are two, one is that from pole to pole, or along the lines of magnetic force, and I call it the axial direction, the other is perpendicular to this, or across the lines of magnetic force, and I call it the equatorial direction (2252),

then proceeds to demonstrate that all matter exhibits the new magnetic actions, bodies in a magnetic field either align themselves axially or align themselves equatorially. He lists 56 substances which each aligns itself, either axially or equatorially, from rock crystal and sulphate of lime to wood and ivory, from water and alcohol to apple and bread (2280), and concludes:

it is strange to find a piece of wood, or beef, or apple obedient to or repelled by a magnet;

if a man could be suspended with sufficient delicacy and placed in a magnetic field, he would point equatorially, for all the substances of which he is formed, including the blood, posses this property (2281).

Substances aligning themselves along the lines of force Faraday calls paramagnetic, those aligning themselves across the lines of force diamagnetic (2790). Among metals, iron, for instance, is paramagnetic, bismuth diamagnetic (2424).

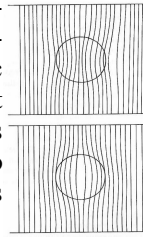
And Faraday continued to work on the theory.

In the *Magnetic conduction*, in 1850:

I will now endeavor to consider what the influence is, which paramagnetic and diamagnetic bodies, viewed as conductors, exert on the lines

of force in a magnetic field;

when a paramagnetic conductor, a sphere of oxygen, for instance, is introduced in a magnetic field, considered free of matter, it causes a concentration of the lines of force on and through itself, so that the space it occupies transmits more magnetic power around it;



if, on the other hand, a sphere of diamagnetic matter is placed in a similar field, it causes a divergence of the lines in the equatorial direction, and less magnetic power is transmitted through the space it occupies (2807).

In *On the lines of magnetic force*, in 1852:

the general conclusions are that the magnetic lines of force may be easily recognized and accounted for by the moving wire, both as to direction and intensity, within metals, iron or magnets, as well as in the space around;

that the wire sums up the action of many lines in one result, the lines of forces represent the nature, condition, direction, and amount of the magnetic forces;

that the effect is directly as the number of lines of force intersected, whether the intersection is direct or oblique;

that in a field of equal force, it is directly as the velocity, or as the length of the moving wire, or as the mass of the wire;

that the external power of the unchangeable magnet is definite yet unlimited in extent, and any section of the lines of force is equal to any other;

that the lines of force within the magnet are equal to those without, and also continuous with those without, the lines of force being closed curves (*id.*).

In *On the physical character of the lines of magnetic force*, also in 1852:

again and again the idea of an electro-tonic state has been forced on my mind; such a state would coincide and become identified with that which would then constitute the physical lines of force (3269);

as regards the wire, there must be a previous state, a state of tension or a static state, which, when motion is added, produces the dynamic state or current of electricity; this state is suffi-

cient to constitute and give a physical existence of the lines of magnetic force (3270);

all phenomena of the moving wire seem to me to show the physical existence of an atmosphere of power about a magnet (3271).

Maxwell wrote the *Dynamical theory of the electromagnetic field* in 1864.

Einstein writes in *Physics and reality*: the electric field theory of Faraday and Maxwell represents probably the most profound transformation in the foundations of physics since Newton; it is a step in the direction of constructive speculation which has increased the distance between the foundations of theory and what can be experienced by our five senses; the existence of the field manifests itself only when electrically charged bodies are introduced in it; the electric masses are nothing but places of continuing divergence of the electric field; light waves are undulatory electromagnetic field processes in space (3).

Einstein writes this in 1936. Two decades after the formulation of the theory of relativity, one decade after the formulation of quantum theory.

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Carnot discovered the Carnot cycle and the Carnot principle. The Carnot cycle is the complete cycle of operations of the most efficient reversible heat engine. The Carnot principle is the principle that the efficiency of any reversible heat engine depends only on the temperature range through which it works.

Carnot's discoveries are the synthesis of earlier results and a generalization over them. They are also the foundation of the theory of heat, or thermodynamics. The theory of heat, or thermodynamics, was developed in two major steps. The first is Carnot's synthesis and generalization in 1824. The second is Clausius's formulation of the laws of thermodynamics in 1850.

Beginning with Boyle, through Black, Lavoisier, Gay-Lussac – for one and a half centuries by Carnot's time – the great advance in the experimental investigation of heat had been an advance in the investigation of heat in its relation to the reactions of substances.

In the meantime the steam engine had also been developed.

Clausius writes in 1850: ever since heat was first used as a motive power in steam engines, thus showing practically that a quantity of work may be considered equivalent to the heat needed to produce it, it was not difficult to assume theoretically as well that there is a definite relation between the amount of heat and the work it can produce, and draw conclusions from this relation about the essence and the laws of heat itself; and, in fact, some attempts of this kind have already been made; the most important is that of Carnot (*Force*, intr.).

It was not difficult. Carnot's is nevertheless not only the most important attempt of this kind. It is – after four generations of development of the steam engine, counting back to the first steam engine, built by Papin in 1690 – the first attempt of this kind.

Carnot's is the synthesis of, and a generalization over, the investigation of substances, the development of the steam engine, and very much more generally the development of any heat engine.

He discovered how heat can be investigated generally, whatever the substances and whatever the machines. He discovered the fundamental character of reversibility. He discovered the fundamental character of the cycle. The reversible cycle. And the complete reversible cycle.

The foundation of Carnot's investigation is Boyle's law, $p \propto \frac{1}{V}$, where p is the pressure and V is the volume of a gas at constant temperature. Carnot discovered the dependance of heat, and hence pressure and volume, on work.

Carnot wrote the *Reflections on the motive power of fire and on the proper engines to develop this power*, in 1824.

He writes:

there is no one who does not know that heat is a cause of motion of great motive power; the steam engine, so widespread today, is a proof of this in front of everyone's eyes;

to heat are also due the great movements which take place on earth, it causes the agitations of the atmosphere, the rising of clouds, the fall of rain and of meteors, the currents of water, earthquakes and volcanic eruptions;

to develop this power is the object of heat

engines;

the study of these engines is of the highest interest, and of immense importance; their use is increasing constantly; they seem destined to produce a great revolution in the civilized world;

nevertheless, in spite of all the work of all kinds which has been accomplished by steam engines, in spite of the very satisfactory condition to which they have been brought, their theory is very little understood; their improvement has been guided entirely by chance;

the production of motion by heat has not been considered from a sufficiently general point of view;

in order to consider the principle of production of motion by heat in the most general way, it must be considered independently of any mechanism and of any particular agent; the principles must be established, which apply not only to steam engines, but to all imaginable heat engines, and to all substances, and to all methods by which they may be operated;

machines which are moved not by heat, those whose motor is the force of men, or animals, or a waterfall, or an air current, and so on, can be studied in the smallest detail in mechanics; all cases are foreseen, all imaginable motions are deduced from general principles, which are established firmly, and apply to all circumstances; this is the character of a complete theory;

clearly, a similar theory is needed for heat engines; and we will have such a theory only when the laws of physics are extended and generalized sufficiently to embrace all effects of heat acting in a determined way on any body (*id.*)

And Carnot proceeds to formulate this theory.

He continues:

we use the expression motive power to express the useful effect which a motor is capable of producing; this effect can always be compared to the elevation of a weight to a certain height (*id.*). The comparison will become quantitative with the measurement of the mechanical equivalent of heat. Carnot continues:

the production of motion in steam engines is always accompanied by a circumstance on which we must fix our attention; this is the reestablishment of equilibrium in the caloric, and that is, its passage from a body in which the temperature is

higher to another in which it is lower;

the production of motive power in heat engines is due not to the actual consumption of caloric, but to its transfer from a warm body to a cold body, and that is, to its reestablishment of equilibrium;

according to this principle, heat alone is not sufficient to give rise to motive power; cold is also necessary; without cold heat would be useless; indeed, if we found around us only bodies as hot as furnaces, how would we condense heat?

it is natural to ask the curious and important question, is the motive power of heat independent of the agent employed to realize it, or it varies with it?

and as soon as we reflect on the changes of volume caused by heat, it is clear that, wherever there is a difference of temperature, motive power can be produced (*id.*).

The last proposition is Carnot's first formulation of the Carnot principle.

The proposition, the production of motive power is due to the reestablishment of equilibrium in the caloric, is his first anticipation of the principle of conservation of energy, in general, and of the first law of thermodynamics, in particular, which is nothing but the principle of conservation of energy applied to the phenomena of heat.

Caloric is, on the one hand, a substance. It is one of the 5 simple substances, in Lavoisier's table: which belong to the three kingdoms and which can be considered the elements of bodies, next to light, hydrogen, nitrogen, and oxygen. On the other hand, Carnot writes: we speak of quantity of caloric and quantity of heat indifferently (*id.*).

He continues:

imagine two bodies *A* and *B*, each at constant temperature, that of *A* higher than that of *B*; suppose that we can give heat to each of these bodies and remove it from it without causing the temperature to vary; the two bodies function as unlimited reservoirs of caloric; we call them the furnace and the refrigerator;

and to produce motive power by transferring a certain quantity of heat from body *A* to body *B*, we proceed as follows,

1) in order to borrow caloric from body *A* to

produce steam we assume that the steam is produced at the same temperature as that of body A ;

2) the steam is received in a space capable of expansion, such as a cylinder with a piston, increasing the volume of the space, and consequently that of the steam; the steam is rarefied, and the temperature falls spontaneously, as it does in all elastic fluids; let the rarefaction continue to the point at which the temperature becomes precisely that of body B ;

3) to condense the steam we put it in contact with body B , exerting at the same time constant pressure on the steam until it is completely liquefied; body B plays here the role of the injection water in ordinary engines, with this difference that it condenses the vapor without mixing with it;

these operations can be performed in the inverse order; nothing prevents the formation of vapor with the caloric of body B , and at the temperature of this body, compressing the vapor until it acquires the temperature of body A , condensing it by contact with that body, and continuing the compression to complete liquefaction;

by the direct operations motive power is produced and caloric is transferred from body A to body B ; by the inverse operations motive power is expended and caloric is returned from body B to body A ;

and if we act in both cases on the same quantity of vapor, if there is no loss of either motive power or caloric, the quantity of motive power produced in the direct operations and the quantity of caloric transferred from body A to body B are equal to the motive power expended in the inverse operations and the quantity of caloric returned from body B to body A ; and then an indefinite number of alternative operations can be carried on without, in the end, either the production of motive power or the transfer of caloric from one body to the other;

now, if there existed any means of using heat preferable to that which we have just described, and that is, if there were any method of forcing the caloric to produce a quantity of motive power greater than that which we have forced it to produce by our series of direct operations, that method could be employed to divert a portion of

the motive power produced by our operations to force the caloric of body B to return to body A , from the refrigerator to the furnace, thus restore the initial conditions, and commence the operations exactly as they initially did; and again; and so on indefinitely;

and this would not only be perpetual motion, it would be unlimited creation of motive power without the consumption of either caloric or any other agent whatsoever; such a creation is entirely contrary to all ideas accepted today; it is contrary to the laws of mechanics and of sound physics; it is inadmissible (*id.*).

Perpetual motion, or unlimited creation of motive power, is contrary to the laws of mechanics and sound physics. Carnot anticipates the principle of conservation of energy, and the first law of thermodynamics, in all its generality.

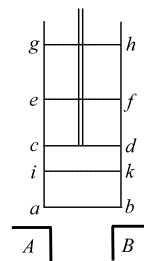
He continues:

we must conclude that the maximum of motive power produced by the use of steam is the maximum of motive power produced by any means whatsoever;

the necessary condition of the maximum is, therefore, that in the bodies employed to realize the motive power of heat no change of temperature can occur which is not due to a change of volume;

when a gaseous fluid is compressed its temperature rises, when it is rarefied its temperature falls; this is one of the facts most firmly established by experiment; we will take it as the ground of our demonstration;

let us imagine an elastic fluid, atmospheric air, for instance, enclosed in a cylindrical vessel $abcd$, provided with a movable piston cd ; the walls of the vessel we suppose to transmit caloric freely; let there be also two bodies A and B , each at a constant temperature, that of A higher than that of B ;



and let us now picture a series of operations;

1) the air in the space $abcd$ is brought in contact with body A , and the air attains by the contact the same temperature as body A ; the position of the piston is cd ;

2) the piston gradually rises to position ef ;

body A is in contact with the air which is thus kept at a constant temperature during the rarefaction, body A furnishes the caloric necessary to keep the temperature constant;

3) body A is removed, and the air is no longer in contact with any body furnishing it with caloric; the piston continues to rise from position ef to position gh ; the air is rarefied without receiving caloric and its temperature falls; let it fall until it becomes equal to that of body B ; at this instant the piston stops in position gh ;

4) the air is brought in contact with body B ; it is compressed by the return of the piston from position gh to position cd ; the air remains at a constant temperature because of its contact with body B , to which it yields caloric;

5) body B is removed; the compression of the air continues, and since it is isolated its temperature rises; the compression continues until the air acquires the temperature of body A ; the piston passes from position cd to position ik ;

6) the air is again brought in contact with body A ; the piston returns from position ik to position ef ; the temperature remains unchanged;

7) step 3) is repeated, then the successive steps 4), 5), 6), again 3), 4), 5), 6), and so on;

all these operations can be executed in the inverse direction and order; it is only necessary to execute in each step rarefaction instead of compression, and inversely;

the result of the direct operations is the production of a certain quantity of motive power and the transfer of caloric from body A to body B ; the result of the inverse operations is the consumption of the motive power produced and the return of the caloric from body B to body A ; the two series of operations annul one another;

we are thus led to establish the general proposition,

the motive power of heat is independent of the agents employed to realize it, its quantity depends only on the temperature of the bodies between which the transfer of caloric is effected (*id.*).

This is the conclusive formulation of the Carnot principle. The Carnot cycle is the demonstration of the principle.

And this is how Clausius, in 1850, in *On the moving force of heat and the laws which can be derived from it for the theory of heat*, describes

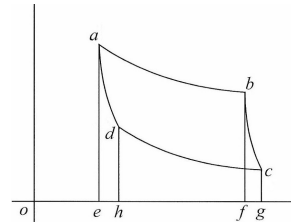
the importance of the cycle and the cycle itself:

if any body changes its volume, mechanical work is either produced or expended; in most cases, however, it is impossible to determine this exactly, because next to the external work there is an unknown amount of internal work; to resolve this difficulty, Carnot employed the ingenious method of allowing the body to undergo changes in succession, arranged so that it returns in the end exactly in its initial condition; thus, the internal work done in some of the changes is compensated exactly in the others, and we can be certain that the external work, which remains after the changes are completed, is all the work that has been done;

Clapeyron has represented this process in a very clear way, and we will follow his representation, with a slight modification required by our principle (1).

Clapeyron's representation is an application of the pressure-volume diagram, implied by Boyle's law, to the Carnot cycle. Clausius's principle requiring the slight modification is the second law of thermodynamics. Also, Clausius speaks of Mariotte's law. Mariotte's law is Boyle's law. Boyle demonstrated the law in 1662. Mariotte stated it in *On the nature of air*, in 1676.

Clausius continues:



let the abscissa oe represent the volume and the ordinate ea the pressure on a unit weight of gas, when its temperature = t ; we assume that the gas is contained in an expendable envelope which, however, cannot exchange heat with it;

if now the gas is allowed to expand in this envelope, its temperature will fall, if no heat is imparted to it; to avoid this, let it be put during the expansion in contact with a body A , which is kept at the constant temperature t , and which imparts just so much heat to the gas that its temperature also remains equal to t ; during this expansion at constant temperature the pressure of

the gas diminishes according to Mariotte's law, and can be represented by the ordinate of a curve, ab , which is a portion of an equilateral hyperbola;

when the volume of the gas has increased in this way from oe to of , body A is removed, and the expansion is allowed to continue without the introduction of more heat; the temperature will then fall, and the pressure will diminish more rapidly than before; the law followed in this part of the process can be represented by the curve bc ;

when the volume of the gas has increased in this way from of to og , and its temperature has fallen from t to τ , we begin to compress it, in order to restore it again to its original volume oe ; if it is left to itself, its temperature will again rise; this, however, we do not permit, but bring it in contact with a body B , at the constant temperature τ , to which it gives up the heat produced, so that it keeps the temperature τ ; and while it is in contact with this body we compress it by the amount gh , so that the remaining compression he , if during this last compression it gives up no heat, is exactly sufficient to raise its temperature from τ to t ;

during the former compression the pressure increases according to Mariotte's law and is represented by the portion cd of an equilateral hyperbola;

during the latter compression the increase is more rapid and is represented by the curve da ; and this curve must end exactly at a , because at the end of the operation the volume and the temperature have their original values, and the same must be true of the pressure, which is a function of them both; the gas is therefore in the same condition as it is at the beginning (*id.*).

Rankine introduced the term adiabatic for a change involving rise or fall in temperature. Thus, ab is isothermal expansion, bc adiabatic expansion, cd isothermal compression, da adiabatic compression.

In the *Reflections*, after defining the cycle and formulating the principle, Carnot shows, from recent experiments, that the motive power developed from the action of heat on atmospheric air, on vapor of water, and on vapor of alcohol is the same, reviews different designs of steam engines in operation, and concludes by calculating:

how far we are from realizing all the motive

power of combustion;

1 kg of carbon burned in a calorimeter furnishes a quantity of heat capable of raising 1000° centigrade about 7000 kg of water, and that is, it furnishes 7000 units of heat;

the greatest fall of caloric attainable is measured by the difference between the temperature of combustion and that of refrigerant bodies; of combustion it is difficult to perceive a limit other than the combination of oxygen with combustibles; if we assume that this limit is 1000° , we will certainly be below the truth; as to refrigerants, let us assume a temperature of 0° ;

we have estimated the quantity of motive power which 1000 units of heat develop between 100° and 99° at 1.112 units of power, each unit equal to the raising of 1 m^3 to a height of 1 m;

if the motive power is proportional to the fall of caloric and the same for each degree, from 1000° to 0° it is $1.112 \times 1000 = 1112$ units of power;

but this is only a rough estimate; we will thus reduce it by half and take it to be 560;

560 units of power correspond to 1000 units of heat; 1 kg of carbon furnishes 7000 units of heat; the motive power of 1 kg of carbon is thus capable of producing $7 \times 560 = 3920$ units of heat;

and to compare this theoretical result with experiment, we can consider the best results obtained by steam engines;

the best are the double-cylinder engines used in the drainage of tin and copper mines in Cornwall, and the best results which they have obtained are 65000000 pounds of water raised 1 English foot for 1 bushel of coal burned; this is equivalent to raising for 1 kg of coal burned 195 m^3 of water 1 m, thus producing 195 units of motive power for 1 kg of coal burned;

195 is one twentieth of 3920, the theoretical maximum; thus, only $\frac{1}{20}$ of the motive power of

the combustible has been utilized (*id.*).

Carnot died in 1832. His *Notes on mathematics, physics, and other subjects* were published in 1878. Carnot writes in the *Notes*:

is heat the result of an undulatory motion of molecules? if so, quantity of heat is simply quantity of motive power;

light is now considered the result of an undula-

tory motion of the aethereal fluid; light produces heat, or in any case accompanies radiant heat and moves with the same velocity; radiant heat is therefore an undulatory motion; and it would be ridiculous to suppose that heat is an emission of matter, if light, which accompanies it, can only be a motion;

can motion, radiant heat, produce matter, caloric? undoubtedly no; it can only produce motion; heat is then the result of motion (*id.*).

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Hamilton generalized the principle of least action. He also discovered quaternions.

Fermat formulated the principle of least time in 1662. Maupertuis formulated the more general principle of least action in 1740. Euler formulated it analytically in 1744. Lagrange generalized it in 1788. And in 1834 Hamilton's is the further and ultimate generalization. Hamilton deduced from it optics and dynamics. After Hamilton from it relativity has been deduced, and on it quantum theory has been built.

Hamilton's generalization is in two steps. The *Theory of systems of rays*, in 1827. And *On a general method in dynamics*, in 1834.

This is how Hamilton describes the results of the *Theory of systems of rays*. In *On a view of mathematical optics*, in 1831, he writes:

the general problem which I have proposed to myself in optics is to investigate the mathematical consequences of the law of least action;

the central idea from which my whole method flows is the idea of one radical or characteristic relation for each optical system of rays, and that is, for each combination of straight or bent or curved paths;

from this characteristic relation, different for different systems, all mathematical properties of the system can be deduced, in the same manner as in the method of Descartes for the algebraic solution of geometrical problems, whose central idea is of one radical relation for each plane curve or curved surface, in which all properties of the curve or surface are included;

in the radical relation contemplated by Descartes the related things are elements of position of a variable point, whose locus is a curve or surface, and the number of the related elements is

two or three;

in the relation contemplated by me, in my view of algebraic optics, the related things are eight; six are elements of position of two variable points of space considered connected, the seventh is an index of color, and the eighth, which I call the characteristic function, is the action between the two variable points (*id.*).

Action is a function of two variable points.

As Hamilton will describe the points in 1834: the initial and the final points. The function is thus, as Hamilton describes it from the first, radical, or the characteristic. If this function is defined, it will describe: any system of rays, any combination of straight or bent or curved paths. Hamilton has defined the function. And he draws the consequence:

to the study of this characteristic function, by means of this one fundamental formula, I consider reducible all problems of mathematical optics, respecting all imaginable combinations of mirrors, lenses, crystals, and atmospheres (*id.*).

This is the first step. The second and conclusive step is *On a general method in dynamics, by which the study of the motions of all free systems of attracting or repelling points is reduced to the search and differentiation of one general relation, or characteristic function*, in 1834.

On a general method in dynamics is in twelve brief sections.

The first section is the *Introductory remarks*. Hamilton begins:

the theoretical development of the laws of bodies is a problem of such interest and importance that it has engaged the attention of all the most eminent mathematicians since the invention of dynamics as a mathematical science by Galileo, and especially since the wonderful extension of this science by Newton;

among the successors of these illustrious men Lagrange has done perhaps more than any other analyst by showing that the most varied consequences respecting the motions of systems of bodies can be derived from one formula, the beauty of the method so suiting the dignity of the results as to make his great work a kind of scientific poem;

but the science of force, or of power acting by law in space and time, has already undergone

another revolution and become more dynamic, having almost dismissed the concepts of solidity, cohesion, and those other material ties, on which Lagrange so happily reasoned, and tending more and more to resolve all connections and actions of bodies into attractions and repulsions of points; and the method proposed in the present essay is for the deductive study of the motions of attracting and repelling systems of such points;

in the commonly employed methods the determination of the motion of a free point in space under the influence of accelerating forces depends on the integration of three equations in ordinary differentials of the second order; the determination of the motions of a system of free points attracting or repelling one another depends on the integration of a system of such equations, three times the number of the attracting or repelling points;

thus, in the solar system, when we consider only the mutual attractions of the sun and ten planets, the determination of the motions of the latter about the former is reduced by the usual methods to the integration of a system of thirty ordinary differential equations of the second order between the coordinates and the time; or by the transformation of Lagrange, to the integration of a system of sixty ordinary differential equations of the first order between the time and the elliptic elements;

in the method of the present essay this problem is reduced to the search and differentiation of a single function, which satisfies two partial differential equations of the first order and of the second degree;

and every other dynamical problem, respecting the motions of any system or attracting or repelling points, however numerous, is reduced to the study of one central function (intr.).

In the second section, the *Integration of the equations of motion of a system, characteristic function of such motion, and law of varying action*, Hamilton formulates the problem explicitly:

the known differential equations of motion of a system of free points repelling or attracting one another according to any function of their distance, and not disturbed by any foreign force, may be comprised in the following formula,

$$\Sigma m (x'' \delta x + y'' \delta y + z'' \delta z) = \delta U,$$

where the sign of summation Σ extends to all the points of the system, m is for any such point the constant called its mass, x'' , y'' , z'' are its component accelerations, or second differential coefficients of its rectilinear coordinates x , y , z with respect to the time; δx , δy , δz are any arbitrary infinitesimal displacements which the point can be imagined to receive in the same three rectangular directions, and δU is the infinitesimal variation corresponding to a function U of the masses and mutual distances of the several points of the system, whose form depends on the law of their mutual actions by the equation

$$U = \Sigma m m_1 f(r),$$

where r is the distance between any two points m , m_1 , and the function $f(r)$ is such that its derivative or differential coefficient $f'(r)$ expresses the law of their repulsion, being negative in the case of attraction;

the function U may be named the force function of a system; of great utility in theoretical mechanics, it was introduced by Lagrange (1).

It is Lagrange's generalization of Newton's second law. Hamilton continues:

if we now set

$$T = \frac{1}{2} \Sigma m (x'^2 + y'^2 + z'^2),$$

so that $2T$ denotes, as in the *Analytic mechanics*, the whole living force of the system, x' , y' , z' being the rectangular components of velocity of the point m , or the first differential coefficients of its coordinates with respect to time, then an easy well known combination of the differential equations of motion may be expressed in this way,

$$dT = dU,$$

and gives, by integration, the celebrated law of living force

$$T = U + H;$$

in this expression the quantity H is independent of the time, and does not change with the passage of the points of the system from one set of positions to another;

the increment of H is evidently connected with the analogous increments of the functions T and U , which may be conveniently written thus,

$$\delta T = \delta U + \delta H,$$

and this relation, when multiplied by δt , and integrated, leads to an important result;

for it becomes, by the earlier equations,

$$\int \Sigma m (dx \cdot \delta x' + dy \cdot \delta y' + dz \cdot \delta z') = \\ = \int \Sigma m (dx' \cdot \delta x + dy' \cdot \delta y + dz' \cdot \delta z) + \int \delta H \cdot dt,$$

and that is, by the principles of the calculus of variations,

$$\delta V = \Sigma m (x' \delta x + y' \delta y + z' \delta z) - \Sigma m (a' \delta a + b' \delta b + c' \delta c) + t \delta H,$$

if we denote by V the integral

$$V = \int \Sigma m (x' dx + y' dy + z' dz) = \int_0^t 2T dt,$$

namely, the accumulated living force, called also the action of the system, from its initial to its final position;

we consider, as it is easy to see that we may, the action V a function of the initial and final coordinates and the quantity H ;

and if this function V is known, we can eliminate H ;

and this would be the general solution of the general problem of dynamics;

we have therefore reduced that general problem to the search and differentiation of a single function V , which on this account we call the characteristic function of motion of a system;

and the above equation

$$\delta V = \Sigma m (x' \delta x + y' \delta y + z' \delta z) - \Sigma m (a' \delta a + b' \delta b + c' \delta c) + t \delta H,$$

expressing the fundamental law of its variation, we shall call the equation of the characteristic function, or the law of varying action (2).

The name has remained the principle of least action.

And Hamilton explains how and why the action or the accumulated living force of the system is the general solution of the general problem of dynamics:

if the points or bodies of a system be imagined to move from a given set of initial points to a given set of final points, not as they do, and not even as they could move consistently with the general dynamical laws or differential equations of motion, but so as not to violate any supposed geometrical connections, and not to violate that one dynamical relation between velocities and configurations which constitutes the law of living force, and this geometrically imaginable but dynamically impossible motion be made to differ infinitely little from the actual manner of motion of the system between the given extreme positions, then the varied value of the definite integral called the action or the accumulated living force of the system in the motion thus imagined will

differ infinitely less from the actual value of that integral;

and the principle introduced in the present paper under the name of the law of varying action, in which we pass from an actual motion to another dynamically possible motion by varying the extreme positions of the system and the quantity H , serves to express by means of a single function not the mere differential equations of motion but their intermediate and their final integral (3).

In the following nine sections Hamilton solves the problem. To facilitate the deduction, the equations are numbered in two series, numerical and alphabetical, for a total of 313 equations in the twelve brief sections.

And in the final twelfth section, the *General introduction of the time into the expression of the characteristic function in any dynamical problem*, the introduction of the time is the final generalization. Hamilton writes:

before we conclude this sketch of our general method in dynamics, it will be proper to notice briefly a transformation of the characteristic function which may be used in all applications;

this transformation consists in setting, generally,

$$V = tH + S,$$

and considering the part S , namely, the definite integral

$$S = \int_0^t (T + U) dt,$$

a function of the initial and final coordinates and of the time, varying by our law of varying action (23).

When the variation is 0,

$$\int_0^t (T + U) dt = 0.$$

And this is Hamilton's principle.

$2T$ is the whole living force of the system. After the discovery of the principle of conservation of energy in the next generation, it will be called the kinetic energy. U is the force function of the system. After the discovery of the principle of conservation of energy, it will be called the potential energy.

The beauty of the method so suits the dignity of the results as to make *On the general method in dynamics* a kind of scientific poem.

Hamilton's principle, for its part, may be used in all applications. And it was most dramatically used to deduce quantum mechanics. In 1926, Schrödinger deduced Schrödinger's equation from Hamilton's principle.

ccclxxviii

Boole showed that logic is algebra. On the presumption that everything, including nothing, including the universe, is classes, or equivalently sets. Propositions are relations of classes, or equivalently sets.

Boole did so in *The mathematical analysis of logic*, in 1847, and conclusively in *An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities*, in 1854.

In *The mathematical analysis of logic*, he writes:

assuming the notion of a class, we can, from any conceivable collection of objects, by a mental act, separate those which belong to the given class and contemplate them apart from the rest; such an act of election can be repeated; the group of individuals left under consideration can be limited further, by selecting those among them which belong to some other recognized class; and this process can be repeated with other elements of distinction, until we arrive at an individual possessing all the distinctive characters which we have taken into account, and a member, at the same time, of every class which we have enumerated; it is, in fact, a method similar to that which we employ whenever, in common language, we accumulate descriptive epithets for the sake of more precise definition (5).

The *Investigation of the laws of thought on which are founded the mathematical theories of logic and probability* is in 22 chapters.

In chapter 1, the *Nature and design of this work*, Boole begins by addressing:

all protests against the exhibition of logic in the form of a calculus (1.10), and affirming:

there is not only a close analogy between the operations of the mind in general reasoning and its operations in the particular science of algebra, but there is to a considerable extent an exact agreement in the laws by which the two classes of

operations are conducted (1.6);

the ultimate laws of logic are mathematical in their form (1.10).

And then Boole develops the mathematical theory of logic.

In chapter 2, *Of signs in general and of the signs appropriate to the science of logic in particular, also of the laws to which that class of signs are subject*, he formulates the laws, or axioms:

let us represent the class of individuals to which a particular name or description is applicable by a single letter, as x ; if the name is 'men', for instance, let x represent all men, or the class men;

usually by a class is meant a collection of individuals, to each of which a particular name or description may be applied;

in this work, however, the meaning of the term is extended to include the case, in which but a single individual exists answering to a given name or description, as well as the cases denoted by the terms 'nothing' and 'universe', which, as classes, should be understood to comprise, respectively, no beings and all beings;

also, if an adjective, as 'good', is employed as a term of description, let us represent by a letter, as y , all things to which the description 'good' is applicable, i. e. all good things, or the class good things;

further, let the combination xy represent that class of things to which the names or descriptions represented by x and y are simultaneously applicable; thus, if x stands for 'white things' and y for 'sheep', xy stands for 'white sheep';

and let us consider now the laws to which the symbols x , y , etc., are subject (2.6);

first, it is evident that the order in which two symbols are written is indifferent; the expressions xy and yx equally represent that class of things to which the names or descriptions x and y are together applicable; hence we have

$$xy = yx \quad (2.7);$$

the law thus expressed may be characterized by saying that the symbols x , y , etc., are commutative, like the symbols of algebra (2.8);

as the combination xy expresses the whole of that class of objects to which the names represented by x and y are together applicable, it

follows that the combination of the symbols expresses no more than either of the symbols alone; therefore

$$xy = x;$$

as y is supposed to have the same meaning as x , however, we may replace it by x , and we have

$$xx = x;$$

in algebra xx is represented by x^2 ; let us adopt the same notation, the equation becomes

$$x^2 = x,$$

and this expresses a second general law of these symbols (2.9);

strictly, the words 'and' and 'or', interposed between the terms descriptive of two or more classes of objects, imply that these classes are distinct, so that no member of one is a member of the other; in this respect and in all others the words 'and' and 'or' are analogous with the sign + in algebra, and their laws are identical; thus, the expression 'men and women' is – conventional meanings aside – equivalent with the expression 'women and men';

so, let x stand for 'men', y for 'women', and + for 'and' and 'or'; and we have

$$x + y = y + x;$$

let z stands for 'European'; as it is the same to say 'European men and women' as to say 'European men and European women', we have

$$z(x + y) = zx + zy;$$

and this equation exemplifies the general law that the symbols are distributive in their operation;

as we have expressed the operation of aggregation by the sign +, so we may express the operation of separation by –; thus, if x stands for 'states' and y for 'having a monarchical form', then 'all states except those that are monarchical' is expressed by $x - xy$ (2.11);

let us now take the proposition 'the stars are the suns and the planets', and represent stars by x , suns by y , planets by z ; we then have

$$x = y + z;$$

and if it be true that the stars are the suns and the planets, it will follow that the stars, except the planets, are the suns; this gives the equation

$$x - z = y,$$

which must therefore be a deduction from the previous equation; and this is in accordance with the algebraic rule of transposition;

but instead of dwelling on particular cases, we

may at once affirm the general axioms,

1. if equal things are added to equal things, the wholes are equal;

2. if equal things are subtracted from equal things, the remainders are equal;

and we may add or subtract equations just as in common algebra;

also, if two classes of things, x and y , are identical, we have the equation

$$x = y,$$

whatever class or property z may represent, we have also

$$zx = zy;$$

and this is formally the same as the algebraic law, if both members of an equation are multiplied by the same quantity, the products are equal (2.13);

but here the analogy of the present system with that of algebra appears to stop;

if the members of a class x , which possess a certain property z , are identical with the members of a class y , which possess the same property z , it does not follow that the members of the class x are universally identical with the members of the class y ; hence it cannot be inferred from the equation

$$zx = zy$$

that the equation

$$x = y$$

is also true;

but a little consideration will show that even in algebra this axiom does not have the generality of those other axioms; the deduction of the equation $x = y$ from the equation $zx = zy$ is valid only when z is not equal to 0 (2.14);

we have seen that the symbols of logic are subject to the special law

$$x^2 = x;$$

now, of the symbols of number there are but two, 0 and 1, which are subject to the same formal law; we know that $0^2 = 0$ and $1^2 = 1$; and the equation $x^2 = x$, considered as algebraic, has no other roots than 0 and 1;

hence, instead of determining the measure of formal agreement of the symbols of logic with those of number generally, it is more immediately suggested to compare them with symbols of quantity admitting only of the values 0 and 1;

let us conceive, then, of an algebra in which the symbols x , y , etc., admit indifferently of the

values 0 and 1, and of these values only;

and the laws, axioms, and processes of such an algebra will be identical in their whole extent with the laws, axioms, and processes of an algebra of logic (2.15).

The laws, or axioms, of the algebra of logic are thus formulated, and Boole proceeds to the propositions.

Chapter 3 is the *Derivation of the laws of the symbols of logic from the laws of the operations of the human mind*. Boole proves 4 propositions. These are the second, the third, and the fourth:

proposition 2. to determine the logical value and significance of the symbols 0 and 1;

the symbol 0 as used in algebra satisfies the formal law

$$0 \times y = 0, \text{ or } 0y = 0,$$

whatever the number y may represent;

in order that this formal law be obeyed in the system of logic, we must assign to the symbol 0 such an interpretation that the class represented by $0y$ be identical with the class represented by 0, whatever the class y may be;

a little consideration will show that this condition is satisfied if the symbol 0 represents nothing; and in accordance with a previous definition, we may term nothing a class;

in fact, nothing and universe are the two limits of class extension;

and whatever the class y may be, the individuals common to it and the class nothing are identical with the those comprised in the class nothing, for they are none;

the law is thus satisfied;

second, the symbol 1 satisfies in the system of number the following law

$$1 \times y = y, \text{ or } 1y = y,$$

whatever the number y may represent;

and assuming this formal equation equally valid in the system of this work, in which 1 and y represent classes, it appears that the symbol 1 must represent such a class that all the individuals in any proposed class y are also the individuals $1y$ common to that class y and the class represented by 1;

a little consideration will show that the class represented by 1 must be the universe, since it is the only class of all the individuals that exist in any class;

hence, the respective interpretations of the symbols 0 and 1 in the system of logic are nothing and universe (3.13);

proposition 3. if x represents any class of objects, then $1 - x$ represents the contrary or supplementary class of objects, i. e. the class including all objects which are not comprehended in the class x ;

let x represent the class men, for instance, and let us express the universe by 1; from the conception of the universe as consisting of men and not men, then, the class not men will be represented by $1 - x$;

proposition 4. that axiom of metaphysicians which is termed the principle of contradiction, and which affirms that it is impossible for any being to possess a quality and at the same time not possess it, is a consequence of the fundamental law of thought whose expression is $x^2 = x$;

let us write the equation in the form

$$x - x^2 = 0,$$

whence we have

$$x(1 - x) = 0;$$

if x is men, for instance, this expresses the principle that a class whose members are at the same time men and not men does not exist; in other words, that it is impossible for the same individual to be at the same time a man and not a man;

and this is identically the principle of contradiction which Aristotle has described as the fundamental axiom of philosophy (3.15).

Boole calls it:

the law of duality (3.16).

In chapters 4-14 Boole continues to deduce proposition after proposition. He applies the algebra of logic to mathematical propositions. For instance:

to develop any function $f(x)$ in which x is a logical symbol (5.10).

He subjects not mathematical propositions to the scrutiny of the algebra of logic. Among these:

Mr Senior's definition of wealth, viz., wealth consists of things transferable, limited in supply, and either productive of pleasure or preventive of pain (7.12);

Aristotle's presumption that virtue is either a passion or a faculty or a habit (9.6);

Plato's remarkable argument whose design it

is to prove the immutability of the divine nature (12.6);

Dr Clarke's demonstration of the being and attributes of god (13.3);

Spinoza's proof of the identity of god and the universe (13.4).

In chapter 15 Boole reduces syllogistic to the algebra of logic.

In chapters 16-21 he applies the algebra of logic to probability.

The final chapter 22 is *On the nature of science and the constitution of the intellect*. Boole writes:

the general object of this chapter suggests an instructive train of reflection, the comparison of the laws of thought in their scientific expression with the actual forms which physical speculation in the early ages and metaphysical speculation in all ages have tended to assume;

first, it has been shown that there is a scientific connection between the conceptions of unity in number and of the universe in logic; they occupy in their respective systems the same relative place and are subject to the same formal laws;

and to assign the nature of that unity of which all existence was thought to be a manifestation was the first aim of philosophy (22.7).

As these philosophers show:

Thales, Anaximenes, Diogenes of Apollonia, Hippasus, Heraclitus, Parmenides, Melissus, Anaxagoras, Xenophanes (*id.*).

Also:

second, in equally intimate alliance with the law of thought, termed in this treatise the law of duality, stands the tendency of ancient thought to those forms of philosophical speculation known under the name dualism (*id.*).

As these philosophers show:

Empedocles, Leucippus, Pythagoras, Alcmaeon, the Manicheans, Plato, the Alexandrian Platonists, Dionysius the Areopagite, Anselm, the friends and associates of Lorenzo the magnificent, like Pico, the recluses known as the Cambridge Platonists, like Henry More, Hegel (*id.*). Even if:

if the extremely arbitrary character of human opinion is considered, it will not be expected, nor is it here maintained, that the above are the only forms in which speculative men have shaped their

conjectural solutions of the problem of existence (22.8).

And Boole concludes by: reviewing the positive results to which we have been led (22.9).

The algebra in which the symbols x , y , etc., admit indifferently of the values 0 and 1, and of these values only, is Boolean algebra. Boolean algebra is the ground on which computers have been developed.

More generally, and in fact most generally, the laws of logic are mathematical. Logic is algebra. Everything is sets. Propositions are relations of sets.

The next is the generation of Cantor and Frege.

ccclxxix

Helmholtz discovered the principle of conservation of energy, in *On the conservation of force*, in 1847.

His most important other treatises are the *Handbook of physiological optics*, in 1856-67, *On the integrals of the hydrodynamic equations to which the motions of vortices conform*, in 1858, *The theory of the perception of tone as the foundation of the theory of music*, in 1863, *On the theory of electrodynamics*, in 1870-4, *The thermodynamics of chemical processes*, in 1882. These are important in the development of physiology, optics, hydrodynamics, potential theory, the kinetic theory, acoustics, electrodynamics, thermodynamics, chemistry.

Helmholtz is also the philosopher who laid to rest the last sustained attempt, by Müller, to revive vitalism. He is the philosopher who began the development of experimental psychology. He obtained important results in non-Euclidean geometry. He obtained important results in geology. He was a vigorous supporter of Darwin from the first. He grounded his results explicitly in Kant. He argued vigorously – and to no avail as it has turned out – against Hegel and the Hegelian transformation of philosophy.

His *On Goethe's natural-scientific works*, in 1853, *On the interaction of the forces of nature and the newest results in physics concerning it*, in 1854, *On the relation of the natural sciences to all science*, in 1862, *The new advances in the theory of vision*, in 1868, *On the goal and ad-*

vances of natural science, in 1869, *On the origin of the planetary system*, in 1871, have been quoted above.

In general, Helmholtz is the last philosopher who obtained important results in all parts of philosophy. It was at that time that this kind of philosopher, common in the previous two and a half centuries, disappeared.

On the conservation of force is in seven parts:

Introduction,

I. *The principle of conservation of living force*,

II. *The principle of conservation of force*,

III. *The application of the principle in the mechanical theorems*,

IV. *The force equivalent of heat*,

V. *The force equivalent of electric processes*,

VI. *The force equivalent of magnetism and electromagnetism*.

Helmholtz begins in the *Introduction*:

the present treatise must by its main contents be addressed chiefly to physicists; I have thus decided to state its grounds independently of their philosophical reasons purely in the form of a physical premise, to develop its consequences, and to compare these with the experimental laws of the phenomena in the different branches of physics (*id.*).

The first, the statement of the grounds independently of their philosophical reasons purely in the form of a physical premise, is the task of the *Introduction*. The second, the development of the consequences of the premise, is the task of parts I and II, *The principle of conservation of living force* and *The principle of conservation of force*. The third, the comparison of the consequences with the experimental laws of the phenomena in the different branches of physics, is the task of parts III-VI, *The application of the principle in the mechanical theorems*, *The force equivalent of heat*, *The force equivalent of electric processes*, and *The force equivalent of magnetism and electromagnetism*.

Helmholtz continues in the *Introduction*:

the development of the following propositions can be undertaken from two starting points, either the proposition that it is impossible by any combination of natural bodies to obtain unlimited amount of work, or the assumption that all actions in nature can be traced back to attractive

and repulsive forces whose intensity depends only on the distance between points acting on one another; that these two propositions are identical is shown at the beginning of the treatise itself;

in this introduction I will try to show that this has an essential bearing on the last and true goal of the physical sciences in general;

the task of these sciences is to seek the laws, by which the particular processes of nature can be traced back to general rules, and from which they can be further determined;

these rules, the law of reflection and refraction of light, for instance, or that of Mariotte and Gay-Lussac for the volume of gas, are clearly nothing but general concepts under which the phenomena belonging to them are collected; to search for these is the concern of the experimental part of our sciences; their theoretical part, on the other hand, seeks to find from their visible actions the unknown causes of the processes, it seeks to comprehend them according to the law of causality (*id.*).

In a note added in 1881, Helmholtz writes:

the philosophical discussion of the introduction is influenced by Kant's theory of knowledge more strongly than I should consider correct now; it became clear to me only later that the principle of causality is in fact nothing but the presumption that the phenomena are subject to law; the law recognized as objective power we call force (*id.*).

He continues in the *Introduction*:

and we are urged in this task and justified in it by the principle that every change in nature must have a sufficient cause;

the ultimate goal of the theoretical natural sciences, then, is to discover the unchangeable causes of the processes of nature; and this demand presents itself to us in this way, as ultimate causes in time unchangeable forces should be found;

matter with unchangeable forces we have called in science, in chemistry, elements; and if we now think of the world as decomposed into elements, into unchangeable qualities, the only possible changes in this system can be spatial, and that is, motions; the external relations of the elements, by which the actions of the forces are modified, can only be spatial;

the forces, then, can only be moving forces

dependant in their action only on spatial relations; motion is change of spatial relations; motion can thus appear in experience only as the spatial relations of at least two material bodies; motion is therefore defined as the endeavor of two masses to change their relative position; the force which two whole masses exert on one another, however, must be resolved into the forces which all their parts exert on one another; for this reason mechanics is concerned with the forces of material points; and points have to one another no other spatial relation than distance;

moving force can thus only be a cause of change of the distance between points, and that is, attractive or repulsive;

the task of the physical natural sciences, then, is to reduce the phenomena to unchangeable attractive and repulsive forces whose intensity depends only on distance;

the solvability of this problem is the condition of the complete comprehensibility of nature;

analytic mechanics has not yet undertaken this restriction of the concept of moving force; many of its general principles, however, on which the investigation of the motion of systems of masses stands, are valid only on the presumption that unchangeable attractive and repulsive forces act on one another, namely, the principle of virtual velocities, that of conservation of the motion of the center of gravity, that of conservation of the principal plane of rotation, that of the motion of rotation of free systems, that of conservation of living force;

only the first and the last have wide application; and the first, we will show, is only a special case of the last, which thus appears as the most general and most important consequence of the above argument (*id.*).

This is the statement of the grounds independently of their philosophical reasons purely in the form of a physical premise. The physical premise is influenced by Kant's theory of knowledge. And Helmholtz's note of 1881 is, pace his disclaimer, not less Kantian than the 1847 statement.

Helmholtz has affirmed what the goal of physics is. It is to discover the unchangeable causes of the processes of nature.

And he proposes to reach the goal by bringing all advances in physics to bear on one another.

In the two generations after Lavoisier the collection of the elements has proceeded apace. – Less that two decades later Mendeleev will formulate the periodic law. – And this is where Helmholtz begins the formulation of the task. With the world decomposed into the elements.

The analytic formulation of mechanics has culminated in its deduction by Lagrange from the single principle of virtual velocities. – There is in the previous decade Hamilton's further generalization, too. – The principle of conservation of living force has been generalized by d'Alembert and Daniel Bernoulli. And Helmholtz proposes to show that the principle of virtual velocities is a special case of the principle of conservation of living force. This is the most general and most important consequence of the premise.

The development of the consequences of the premise follows.

In part I, *The principle of conservation of living force*, Helmholtz writes:

we begin from the assumption that it is impossible by any combination of natural bodies to create moving force continually from nothing; from this proposition Carnot and Clapeyron have already deduced a series of laws, in part unknown, in part not yet proved by experiment, on the specific and latent heat of different natural bodies;

the goal of the present treatise is to extend this principle to all branches of physics;

the principle can be presented in this way,

we think of a system of natural bodies, which are in certain spatial relations to one another, and which are set in motion by mutual forces until they are in some other definite position; we can consider the velocities which they thus attain a certain mechanical work, and we can convert the velocities into work;

if we now want to let the same forces act a second time in order to obtain the same work, we must in some way, by some other forces at our disposal, bring the bodies back to their initial position; to do this, we have to use a certain quantity of work of these latter forces;

and our principle demands that the quantity of work, which the system of bodies gains in its transition from the first position to the second, and the quantity of work, which is lost in the

transition from the second position back to the first, be always the same, whatever the paths of the transitions and whatever the velocities; for if one of the two were greater than the other, we would have won an amount of mechanical force, we would have built a *perpetuum mobile*, which could not only set itself in motion, but could also give it to external bodies;

and if we seek the mathematical expression of this principle, we find it in the known law of conservation of living force;

the quantity of work, produced and used, can be expressed by the height h to which a weight m is raised, and it is then mgh , where g is the intensity of the force of gravity; to raise it perpendicularly to height h body m requires the velocity $v = \sqrt{2gh}$, and attains the same velocity by falling through the same height; hence, $\frac{1}{2}mv^2 = mgh$;

and hence, one half of the product mv^2 , known in mechanics as the quantity of the living force of body m , stands also for the quantity of work of the mass; and for the sake of better agreement with the established way of measuring the intensity of force, I propose to call the quantity $\frac{1}{2}mv^2$

the quantity of living force, which is also identical with the quantity of work;

if we suppose the living forces applied to raise one part of a system, or the equivalent masses, to a certain height, it follows from what has been shown that the quantities of work under the above conditions must be equal; this principle, however, is not valid for all kinds of forces; in mechanics it depends on the principle of virtual velocities, which can be proved only in the case of material points under attractive and repulsive forces; we will show that the principle of conservation of living force is all by itself valid in the case of forces between material points, which act in the direction of the lines connecting them, and whose intensity depends only on the distance; in mechanics such forces are called central forces;

and it follows, conversely, that in all actions of natural bodies on one another this principle can all by itself be applied to all the smallest particles of these bodies and the simplest fundamental forces can be assumed such central forces;

if we next consider a material point of mass m , which moves under the influence of the forces of

several bodies bound in the fixed system A , mechanics shows us how to determine the position and velocity of the point at any given time; we can therefore regard the time t as a primitive variable, and let depend on it, with respect to the fixed system A , the ordinates x, y, z of m , their tangential velocity q , whose components parallel to the axes are $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$, $w = \frac{dz}{dt}$, and

finally the components of the acting forces

$$X = m \frac{du}{dt}, Y = m \frac{dv}{dt}, Z = m \frac{dw}{dt};$$

our principle demands that $\frac{1}{2}mv^2$, and therefore also q^2 , be always the same when m has the same position in A , and it is therefore a function not only of the primitive variable t , but a function only of the coordinates x, y, z as well, and that is,

$$d(q^2) = \frac{d(q^2)}{dx} dx + \frac{d(q^2)}{dy} dy + \frac{d(q^2)}{dz} dz;$$

as $q^2 = u^2 + v^2 + w^2$, $d(q^2) = 2u du + 2v dv + 2w dw$; and if, from the corresponding values above, instead of u , we set $\frac{dx}{dt}$, and instead of du , $\frac{X dt}{m}$,

and in the same way v and w , we obtain

$$d(q^2) = \frac{2X}{m} dx + \frac{2Y}{m} dy + \frac{2Z}{m} dz;$$

as the equations must hold together for any dx, dy, dz , it follows also that in particular

$$\frac{d(q^2)}{dx} = \frac{2X}{m}, \frac{d(q^2)}{dy} = \frac{2Y}{m}, \text{ and } \frac{d(q^2)}{dz} = \frac{2Z}{m};$$

and if q^2 is a function only of x, y, z , it follows that X, Y , and Z , and that is, the direction and magnitude of the acting forces, are functions only of the position of m with respect to A ;

if now, instead of the system A , we consider a single material point a , it follows from what has been proved that the direction and magnitude of the force exerted by a on m is determined only by the position of m relative a ; and since the position of m relative a is determined only by the distance between them, the law has to be modified so that the direction and magnitude of the force be functions of that distance r ; and if we think of the coordinates of any system of axes with origin a , it must be that

$$m d(q^2) = 2X dx + 2Y dy + 2Z dz = 0$$

when

$$d(r^2) = 2x dx + 2y dy + 2z dz = 0$$

and that is, when

$$dz = -\frac{x dx + y dy}{z},$$

substituting this in the earlier equation gives

$$\left(X - \frac{x}{z}Z\right)dx + \left(Y - \frac{y}{z}Z\right)dy = 0$$

for any dx and dy , and hence also singly

$$X = \frac{x}{z}Z \text{ and } Y = \frac{y}{z}Z,$$

and that is, the resultant must be directed toward the origin of the coordinates, toward the point a ;

and it follows that in systems which yield the principle of conservation of living force in all its generality the elementary forces of material points must be central forces (*id.*).

Still more generally, the principle of conservation of energy follows. In part II, *The principle of conservation of force*, Helmholtz continues:

we will now give the law of the cases of central forces a still more general expression;

if ϕ is the intensity of the force, which acts in the direction of r , positive when it attracts, negative when it repels, then

$$X = -\frac{x}{r}\phi, \quad Y = -\frac{y}{r}\phi, \quad Z = -\frac{z}{r}\phi;$$

the equation of the previous section becomes

$$m d(q^2) = -2 \frac{\phi}{r} (x dx + y dy + z dz);$$

therefore

$$\frac{1}{2} m d(q^2) = -\phi dr;$$

and when Q and R , q and r represent the corresponding tangential velocities and distances,

$$\frac{1}{2} m Q^2 - \frac{1}{2} m q^2 = - \int_r^R \phi dr \text{ (id.)}.$$

This is the principle. Helmholtz continues:

if we consider this equation more closely, we find on the left side the difference of the living forces which m has at two different distances;

to find the meaning of the quantity $\int_r^R \phi dr$, if

we think of the intensities ϕ at different points of the line connecting m and a set as perpendicular ordinates, then this quantity indicates the surface which the curve between the ordinates from r to R and the abscissa encloses; and as this surface can be represented as the sum of all the infinite number of abscissas in it, this quantity indicates

all intensities of forces between r and R ; and if we call the forces, which strive to move the point m , before they have moved it, tensions, in opposition to what is called in mechanics living force, then the quantity $\int_r^R \phi dr$ is the sum of the ten-

sions between the distances R and r , and the law will be expressed thus,

the increase of living force of a mass point during its motion under the influence of a central force is equal to the sum of the tensions which correspond to the change of its distance (*id.*).

The final generalization follows:

to bring our law to complete generality, we think of any number of material points with masses m_1, m_2, m_3 , etc., expressed in general by m_a , whose coordinates are x_a, y_a, z_a , the components of the forces acting on them parallel to the axes are X_a, Y_a, Z_a , the components of the velocities along the axes u_a, v_a, w_a , the tangential velocity is q_a , the distance between m_a and m_b is r_{ab} , and the central force between them ϕ_{ab} (*id.*).

And:

$$- \sum \left[\int_{r_{ab}}^{R_{ab}} \phi_{ab} dr_{ab} \right] = \sum \left[\frac{1}{2} m_a Q_a^2 \right] - \sum \left[\frac{1}{2} m_a q_a^2 \right]$$

we have on the left again the sum of the tensions, on the right the living forces of the entire system, and we can now express the law in this way,

in all cases of motion of free material points under the influence of attractive and repulsive forces, whose intensity depends solely on the distance, the loss in tension is always equal to the gain in living force, and the gain in the former equal to the loss in the latter; therefore,

the sum of the living forces and the forces of tension is constant;

and in this most general form we can describe our law as the principle of conservation of force (2).

The sum of the living forces and the forces of tension is constant.

Or what is the same, the sum of the kinetic energy and the potential energy is constant.

Maxwell, in 1871, reports:

the use of the term energy to express the quantity of work which a body can do was introduced by Young;

the energy of a system of bodies acting on one

another with forces depending on their relative position is due partly to their motion, partly to their relative position;

the part due to their motion was called actual energy by Rankine, kinetic energy by Kelvin and Tait;

the part due to their relative position depends on the work which the forces would do if the bodies were to yield to their actions; this is called the sum of tensions by Helmholtz in his celebrated memoir on the conservation of force; Kelvin called it statical energy, Rankine introduced the term potential energy, a very felicitous name, because it not only signifies the energy which the system has not in possession but only has the power to acquire, it also indicates that this is to be found by what is called, on different grounds, the potential function (*Heat*, 4).

The derivation of the principle of conservation of energy in parts I-II – the development of the consequences of the premise – is a *tour de force*. And the application of the principle to all of physics in parts III-VI – the comparison of the consequences with the experimental laws of the phenomena in the different branches of physics – is another *tour de force*.

Helmholtz concludes in part VI:

I believe that in the foregoing I have proved that the law does not contradict any of the known facts of the natural sciences, it is rather corroborated by a great number of them in a striking way; I have tried to present as completely as possible the consequences which follow from its combination with the now known laws of the phenomena, and which must still await their confirmation by experiment (*id.*).

The consequences which follow from the combination of the principle with the laws of the phenomena include: motion under the force of gravity, motion of incompressible fluids, motion of elastic, solid, and fluid bodies, the phenomena of light (III), inelastic collisions, friction, the mechanical equivalent of heat (IV), the laws of static electricity, of current electricity, thermo-electricity (V), the laws of magnetism, of electro-magnetism, organic processes (VI). Except the last, to organic processes, all the applications are analytic.

The discovery of the principle of conservation

of energy is the greatest generalization, by very far, over the successes of the earlier acquisition of knowledge.

And the discovery can be explained.

Lagrange is the culmination of the development of analysis. Faraday is the culmination and synthesis of the investigation of electricity and magnetism. Carnot is the culmination and synthesis of the investigation of heat.

And Helmholtz is the synthesis of Lagrange, Faraday, and Carnot.

The goal is to discover the unchangeable causes of the processes of nature.

Faraday shares the goal.

I have long held the opinion, almost amounting to conviction, in common, I believe, with many other lovers of natural knowledge, that the various forms under which the forces of matter are made manifest have one common origin, or in other words, are so mutually dependent that they are convertible into one another. And the proofs of their convertibility have accumulated to a very considerable extent.

This is in 1845. In 1820 Ørsted had converted electric force into magnetic, Faraday had converted electric force into mechanical in 1821, magnetic force into electric, and mechanical force into electric in 1831. In 1845 he magnetized and electrified a ray of light. And: thus dependance between light and the magnetic and electric forces is established, and thus a great addition is made to the facts which tend to prove that all natural forces are tied together and have one common origin.

One third of *On the conservation of force*, in 1847, is concerned with electricity and magnetism. Most particularly with their conversion. Building on Faraday explicitly (V-VI).

The discovery of the principle of conservation of energy is a generalization over the facts of the conversion of force, demonstrated in the previous generation in the investigation of electricity and magnetism dramatically, and steadily more generally, by Faraday.

Carnot, in 1824, discovered the ground on which all natural forces will be tied together into one common origin.

Perpetual motion, unlimited creation of motive power without the consumption of either caloric

or any other agent whatsoever, is contrary to the laws of mechanics and of sound physics. It is inadmissible.

It is the fundamental assumption of *On the conservation of force*.

We begin from the assumption that it is impossible by any combination of natural bodies to create moving force continually from nothing. From this proposition Carnot and Clapeyron have already deduced a series of laws on the specific and latent heat of different natural bodies. The goal of the present treatise is to extend this principle to all branches of physics.

It is the first treatise to build on both Carnot and Faraday. The first treatise to reduce the facts of electricity and magnetism to Carnot's principle. The facts of the convertibility of magnetic force, electric force, and mechanical force into one another to the principle of convertibility of heat engines.

And Helmholtz brings all the power of analysis to bear on the reduction.

Not just its power. As Ampère has formulated the theory of electrodynamics analytically, and Maxwell will formulate the theory of the electromagnetic field analytically.

Helmholtz brings to bear on the reduction of mechanics, electromagnetism, and heat, all three, to Carnot's principle all the power accumulated by Lagrange in the deduction of mechanics from a single principle.

Lagrange showed that the most varied consequences respecting the motions of systems of bodies can be derived from one formula, the beauty of the method so suiting the dignity of the results as to make his great work a kind of scientific poem.

The power of the derivation of mechanics from one formula reduces the facts of the convertibility of electricity and magnetism to the principle of convertibility of heat engines.

And the discovery of the principle of conservation of energy is the greatest generalization.

Maxwell writes: the doctrine of the conservation of energy is the one generalized statement which is found to be consistent with fact not in one physical science only, but in all (*Matter*, 73); the discussion of the various forms of energy, gravitational, electromagnetic, molecular, ther-

mal, etc., with the conditions of its transfer from one to another, and the constant dissipation of the energy available for producing work, constitutes the whole of physical science as it has been developed in dynamical form under the various names of astronomy, electricity, magnetism, optics, theory of the physical states of bodies, thermodynamics, and chemistry (112).

The constant dissipation of the energy available for producing work is the second law of thermodynamics. The conditions of its transfer from one form to another is the first law of thermodynamics, and more generally, the principle of conservation of energy.

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Clausius discovered the second law in 1850. And discovered entropy in 1854.

Both are very great advances. And the latter is the greater of the two.

The discovery of the second law is the difficult but immediate consequence of earlier discoveries, and of a number of earlier discoveries. The discovery of the object, entropy, is the very difficult and not at all immediate consequence, and of but one earlier discovery, that of the second law.

This is how Clausius formulates the first and the second laws in 1850, and in 1865.

In 1850:

1. in all cases in which work is produced by heat, a quantity of heat is expended proportional to the work produced, and conversely, by the expenditure of a quantity of work an equal quantity of heat is produced;

2. Carnot assumed that the equivalent of the work produced by heat is in the mere transfer of heat from a hotter body to a colder body, while the quantity of heat remains undiminished; and the second part of Carnot's assumption, that the quantity of heat remains undiminished, must be rejected; but its first part then obtains in all cases.

In 1865:

1. the energy of the world is constant;
2. the entropy of the world strives for a maximum.

The discovery of the principle of conservation of energy is the final in a long series of steps leading directly to the discovery. Descartes

discovered that the laws of nature are laws of conservation in 1633. Huygens discovered the conservation of momentum in 1656. Newton's discovery of universal gravity, in 1687, implies conservation. In 1691 Leibniz formulated and solved explicitly the problem of what is conserved absolutely, living force is conserved absolutely. The formulation of the principle of least action by Maupertuis in 1740, and its subsequent generalizations, by Euler in 1744, by Lagrange in 1788, by Hamilton in 1834, imply conservation steadily more unambiguously. The accumulation of chemical conversions, beginning with Stahl's of metal into calx and calx back into metal, in 1697, their great accumulation after Black, in 1756, and their very great accumulation after Lavoisier, in 1784, imply conservation. The demonstrations of the conversion of the magnetic, electric, and mechanical forces into one another, by Ørsted and by Faraday, from 1820 to 1831, imply conservation. The magnetization and electrification of light, by Faraday, in 1845, implies conservation. And in Carnot's proposition, perpetual motion is inadmissible, in 1824, only the analytic formulation and the generality of the conclusions are lacking.

That the discovery of the principle of conservation of energy is the immediate consequence of a great number of earlier results is indicated by the fact, too, that after its discovery, by Helmholtz, in 1847, a half dozen claims to its prior discovery were immediately staked. In the ensuing distribution of credit and disgrace the number of claims rose to a dozen.

The principle implies the object, energy. And that the object as well is the consequence of a great number of earlier results is indicated by the amount of verbal infelicity that accrued on the object. Not force, energy. Not living force, and not actual energy either, kinetic energy. Not the sum of tensions, and not statical energy, potential energy. A very felicitous name. The same as that, very felicitous itself, if on different grounds, of the potential function. Both harking back to Aristotle's potentiality and actuality.

The discovery of the second law of thermodynamics is the final in a much shorter series of steps. The major are Carnot's, the Carnot cycle and the Carnot principle. The measurement of the

mechanical equivalent of heat is important as well. Clausius's formulation of the law followed.

The relative brevity of the series of steps is indicated by the fact, too, that the distribution of credit and disgrace involved in this case only Clausius and Kelvin. Kelvin inferred the law in 1851, in *On the dynamical theory of heat* (12).

The law implies the object, entropy. And the discovery of this object, entropy, is a battle Clausius fought and won by himself.

Bacon writes: the secrets of nature reveal themselves not when she is free and left to herself, but when she is constricted and vexed. Three centuries later, Einstein writes: the electric field has increased the distance between theory and what can be experienced by our five senses. The secrets which the constriction and vexation have been forcing her to reveal have not only been increasingly removed from what can be experienced directly by our five senses, they have increasingly violated what can be experienced directly by our five senses.

This can be traced back, before Bacon already, to Copernicus. The proposition that the sun is in the center of the world and devoid of local motion, is foolish and absurd, and formally heretical.

Kepler discovered the laws of the motions of the heavenly bodies. The sun captures the planets, it makes them revolve as it revolves, and by a bodily force, not by an animal force or a mental force. But how can the sun do this without hands? Instead of hands there is the force of the sun's body which is emitted in straight lines throughout the world. Can you make this clear by example? Yes, the lodestone. But is it not absurd to take this material force from the body of the sun and put it in the celestial aether? No, as the example of the lodestone shows.

Newton discovered gravity. Leibniz writes: I do not understand how Newton conceives of gravity; it seems that according to him gravity is an incorporeal and inexplicable force. Newton agrees: that gravity should be innate, inherent, and essential to matter so that one body may act on another at a distance through vacuum without the mediation of anything else, is to me so great an absurdity that I believe no man with a competent faculty of thinking in philosophical matters can ever fall into it.

One and a half centuries later, our five senses – if ‘our’ refers to those of some of the philosophers – had become more accustomed to the violation imposed by discovery. Water is two gases. Light is transverse waves polarized in all directions. Magnetism is electricity, electricity is magnetism, and electromagnetism is not even action at a distance, it is a field. It is yet entropy – the dissipation of the energy available for producing work – that increased the distance between the objects imposed on our five senses by discovery and what can be experienced by our five senses directly so much, our five senses began once again trusting what can be experienced by them directly more than the objects imposed on them by discovery.

Clausius developed thermodynamics in five very important treatises.

In 1850, in *On the moving force of heat and the laws which can be deduced from it for the theory of heat*, he established the fundamental character of irreversibility, and the fundamental character of the difference between internal and external work, and formulated the two laws.

In 1854, in *On a modified form of the second fundamental law of the mechanical theory of heat*, he formulated the laws more explicitly and more generally, and discovered entropy.

In 1857, in *On the kind of motion which we call heat*, he began the investigation of internal work. This is the investigation which, advanced greatly by Maxwell and Boltzmann, became the kinetic theory.

In 1862, in *On the application of the law of the equivalence of transformations to internal work*, Clausius formulated the second law yet more explicitly and more generally, and extended greatly its applications.

And in 1865, in *On different convenient for application forms of the main equations of the mechanical theory of heat*, he formulated the two laws yet more explicitly, in all their generality, and conclusively, and named entropy entropy.

The development is analytic from the first. In 1850 Clausius formulates the laws and demonstrates their corroboration by experiment analytically, in 1854 the analytic formulation and demonstration is improved substantially, in 1862 it is improved further, and in 1865 it is improved yet

further and conclusively.

Below are the two most important results. The argument on which the discovery of the second law stands in 1850. And the four consecutive statements of the two laws and their analytic formulation, in 1850, in 1854, in 1862, and in 1865. It is a spectacular instance of generalization and formalization, and the consequent clarity and distinction.

In 1850, in *On the moving force of heat and the laws which can be deduced from it for the theory of heat*, Clausius begins:

ever since heat was first used as a motive power in steam engines, thus showing practically that a quantity of work may be considered equivalent to the heat needed to produce it, it was not difficult to assume theoretically as well that there is a definite relation between the amount of heat and the work it can produce, and draw conclusions from this relation about the essence and the laws of heat itself; and, in fact, some attempts of this kind have already been made;

the most important is that of Carnot;

Carnot showed that whenever work is done by heat and no permanent change occurs in the condition of the working body, a certain quantity of heat passes from a hotter body to a colder body;

he proposed that no heat is lost in the process, but that the quantity of heat remains undiminished;

I do not believe, however, that it has been established sufficiently by experiment that no loss of heat occurs when work is done; and even if such a loss has not yet been proved directly, there are facts that make it very probable; for instance, it is impossible to explain the heat produced by friction except as an increase in the quantity of heat; to this can be added that recently many facts have become known suggesting that heat is not a substance, but consists in the motion of the smallest parts of bodies;

nothing, then, contradicts the assumption that in the production of work both take place at the same time, a certain portion of heat is consumed, and a further portion is transferred from a hot body to a cold body, while both portions stand in a definite relation to the quantity of work produced (intr.).

These are the introductory remarks.

Nothing contradicts the assumption that in the production of work both take place at the same time, a certain portion of heat is transferred from a hot body to a cold body, and a further portion is consumed, while both portions stand in a definite relation to the quantity of work produced.

The treatise is then in two parts, *Conclusions from the law of the equivalence of heat and work* and *Conclusions from Carnot's law in connection with the earlier law*.

In the first part Clausius formulates the first law:

we lay down the fundamental law, in all cases in which work is produced by heat, a quantity of heat is expended proportional to the work produced, and conversely, by the expenditure of a quantity of work an equal quantity of heat is produced (*id.*).

Distinguishes internal and external work:

the work produced is of twofold nature; to produce vapor, for instance, first, a certain quantity of work is necessary to overcome the mutual attraction of the particles, to separate them to the distance which they occupy in a state of vapor; second, in order to produce room for itself, the vapor must force back outward pressure;

the former is internal work, the latter external work (*id.*).

Introduces the Carnot cycle:

if any body changes its volume, mechanical work is either produced or expended; in most cases, however, it is impossible to determine this exactly, because next to the external work there is an unknown amount of internal work; to resolve this difficulty, Carnot employed the ingenious method of allowing the body to undergo changes in succession, arranged so that it returns in the end exactly in its initial condition; thus, the internal work done in some of the changes is compensated exactly in the others, and we can be certain that the external work, which remains after the changes are completed, is all the work that has been done (*id.*).

And begins in the second part by showing that in the production of work both take place at the same time, a certain portion of heat is transferred from a hot body to a cold body, and a further portion is consumed, while both portions stand in

a definite relation to the quantity of work produced.

He writes:

Carnot assumed that the equivalent of the work produced by heat is in the mere transfer of heat from a hotter body to a colder body, while the quantity of heat remains undiminished, and if we are to retain the fundamental law, the second part of Carnot's assumption, that the quantity of heat remains undiminished, must be rejected; but its first part then obtains in all cases (*id.*).

If we are to retain the first fundamental law, the second fundamental law is necessary:

when work is done by heat and the working substance is in the same state at the end as at the beginning, a transfer of heat from a hotter body to a colder body occurs;

in order to establish the relation between the heat transferred and the work done, however, a restriction is necessary; since a transfer of heat can take place if a hotter and a colder body are in contact and heat passes from the first to the second by conduction, in order to make the transfer of heat do the maximum work the process has to be carried out so that the bodies never come into contact;

and it is this maximum work that has to be compared with the transfer of heat;

if we assume that there are two substances, *A* and *B*, one of which can produce more work than the other by the transfer of a given quantity of heat, or what is the same, less heat needs to be transferred from *A* to *B* than needs to be transferred from *B* to *A* in order to produce a given quantity of work, we can use these two substances alternatively, to produce work with the first in Carnot's direct cycle and to expend work with the second in Carnot's reverse cycle;

at the end of the operations both bodies are in their initial condition; the work produced is counterbalanced exactly by the work done, and therefore, by the first fundamental law, the quantity of heat in the two bodies is neither increased nor decreased; the only change is in the distribution of the heat, as more heat is transferred from body *B* to body *A* than from body *A* to body *B*, and on the whole heat is transferred from body *B* to body *A*;

by repeating these two processes alternately it

would be possible, without any expenditure of force or any other change, to transfer as much heat as we please from a colder body to a hotter body; and this is not in accord with the other relations of heat, which always tends to equalize temperature differences, and therefore to pass from hotter bodies to colder bodies;

it is theoretically correct, therefore, to retain the first and really essential part of Carnot's assumption, and apply it as a second fundamental law next to the first; and the correctness of this approach is established, as we will see, in many cases by its success (*id.*).

This is the argument. Its correctness is established in all cases by its success.

And this is the statement of the two laws and their analytic formulation in 1850, 1854, 1862, and 1865.

In 1850:

1. assuming generally that heat is the measure of the living force of the particles of bodies, we lay down the fundamental law,

in all cases in which work is produced by heat, a quantity of heat is expended proportional to the work produced, and conversely, by the expenditure of a quantity of work an equal quantity of heat is produced;

2. Carnot assumed that the equivalent of the work produced by heat is in the mere transfer of heat from a hotter body to a colder body, while the quantity of heat remains undiminished,

and if we are to retain the fundamental law, the second part of Carnot's assumption, that the quantity of heat remains undiminished, must be rejected; but its first part then obtains in all cases.

The first law follows from the conservation of living force. The second follows from the application of the first to the Carnot principle.

Analytically, the first law:

$$dQ = dU + A \cdot R \frac{a+t}{v} dv,$$

where Q is the quantity of heat, and v , t , a , and R are from the ground of the derivation, the generalized law of Mariotte and Gay-Lussac, $pv = R(a+t)$, where p , v , and t are the pressure, volume, and temperature of a gas, a and R are the constants of proportionality, a the same for all gases, R depending on the specific weight of the gas (1).

The great advance is the definition of U and A and their reduction to the equation of Q . Clausius defines:

$$\frac{\text{the heat expended}}{\text{the work performed}} = A \text{ (id.)},$$

and:

U is an arbitrary function of v and t (*id.*).

The second law:

$$\frac{dQ}{dv} = \frac{R \cdot C}{v} \text{ (id.)}.$$

The great advance is the definition of C and its reduction to the equation of Q . Clausius defines:

we may represent the maximum work by $\frac{1}{C} dt$,

where C is a function of t only (2).

It is U , A , and C that Clausius will have to interpret as he formulates the laws in the following years in all their generality and all their elegance. And the second law will prove much the more difficult to formulate in all its generality and all its elegance.

In 1854:

1. the law of the equivalence of heat and work, which is only a particular case of the general relation of living force and mechanical work, can be stated thus,

when mechanical work is transformed into heat, and conversely heat into work, the quantity of the one is always proportional to the quantity of the other;

2. heat can never pass from a colder to a hotter body without some other change connected with it occurring at the same time (*id.*).

That the first law is only a particular case of the principle of conservation of energy is now explicit.

Analytically, the first law:

$$Q = U + A \cdot w \text{ (id.)}.$$

Clausius defines: the quantity of heat Q can be divided into three parts, the first increasing the heat of the body, the second producing the internal work, the third producing the external work; the first and the second can be represented by the one function U , which is determined completely by the initial and final states of the body; and if w is the external work and A the equivalent of heat for a unit of work, the third part is $A \cdot w$ (*id.*).

Very little thus remains to bring the first law to perfection.

The second law. Clausius writes:

$$\int \frac{dQ}{T} = 0$$

is the analytic expression of the second law for all reversible cyclical processes;

for all irreversible cyclical processes the sum of all transformations can only be positive (*id.*).

Clausius has written the analytic expression of the second law only for the extreme case of reversible processes. He has not written the analytic expression for irreversible processes, and that is, if the law holds, for all processes.

The quantity C is superseded by the quantity T . Clausius defines:

the transformation of heat into work and the transformation of work into heat, when they can mutually replace one another, are equivalent; the mathematical value of such a transformation we call its equivalence value, and write $f(t) = \frac{1}{T}$,

where T is an unknown function of the temperature involved in the equivalence value;

and if the quantity of heat Q of temperature t is produced from work, the equivalence value of this transformation is $\frac{Q}{T}$ (*id.*).

The second law is thus not formulated analytically generally. The quantity T also has to be interpreted. Clausius has discovered, however, that the analytic expression of the law cannot be an equation. He has discovered also the object:

a transformation which at the conclusion of a cyclical process remains without an opposite transformation we call for brevity an uncompensated transformation (*id.*).

An uncompensated transformation, implying a transformation content of the body, is entropy.

In 1862 Clausius focuses on the second law:

2. the algebraic sum of all transformations occurring during any change of condition whatsoever can only be positive, or as an extreme case equal to nothing; briefly,

uncompensated transformation can only be positive (*id.*).

Analytically:

$$\int \frac{dQ}{T} = 0$$

must be true for all reversible processes;

$$\int \frac{dQ}{T} \geq 0,$$

must be true for all processes whatsoever (*id.*).

The unknown function T is interpreted:

T is in all probability nothing but the absolute temperature (*id.*).

Thus, beginning with Galileo and Kepler, for more than two centuries the analytic expression of all laws, some hundreds of them, had demanded an equation. The second law, it turned out, demands an inequality.

And very little remains now to bring the second law as well to perfection.

In 1865, Clausius begins:

the entire mechanical theory of heat rests on two fundamental laws,

1. the law of the equivalence of heat and work,
2. the law of the equivalence of transformations (1).

Analytically, the first law:

$$dQ = dU + dW,$$

where I had defined U as the sum of the free heat in the body and that consumed by the interior work; Kelvin has proposed the term energy of the body for this magnitude, and I have now adopted this very appropriate name;

in the earlier expression $dQ = dU + A dw$, where w is the external work and A is the heat equivalent for a unit of work, we can express their product briefly $W = A \cdot w$, and write the above equation (*id.*).

The second law:

$$\int \frac{dQ}{T} \leq 0$$

where T is the temperature measured from the absolute zero, and the sign for equality holds for all reversible changes in cyclical processes, the sign $<$ for all irreversible changes (*id.*).

In 1862 the sign is $\geq$, because uncompensated transformation can only be positive. In 1865 its affirmative definition demands the sign $\leq$. And Clausius defines it affirmatively:

as the equation asserts, the integral $\int \frac{dQ}{T}$ is

equal to 0 whenever the body, beginning from any initial state, after passing through any other states, returns to its initial state, and the expression $\frac{dQ}{T}$ must therefore be the complete differential of a quantity which depends only on the momentary present state of the body, and not on

the way by which it has reached this state; indicating this quantity by S , we set

$$dS = \frac{dQ}{T},$$

and if we think of this equation as integrated for any reversible process, by which the body can pass from a given initial state to its present state, and indicate by S_0 the value of S in the initial state,

$$S = S_0 + \int \frac{dQ}{T}$$

is the equation by which we determine S ;

we may call S the transformation content of the body; but I hold that it is better to borrow the terms for important quantities from the ancient languages, so that they can be adopted in all modern languages unchanged, and I propose to name the quantity S – from the Greek word ἡ τροπή, the transformation – the entropy of the body; I have deliberately build the word entropy to be as similar as possible to the word energy, because the two quantities which should be named by these words are so closely related in their physical references (14).

For all reversible processes

$$S - S_0 = \int_{S_0}^S \frac{dQ}{T}.$$

For all irreversible processes

$$S - S_0 > \int_{S_0}^S \frac{dQ}{T},$$

and

$$S > S_0.$$

Entropy can only increase.

Clausius concludes:

the second law asserts that all transformations in nature can occur only in a certain direction, and that is, without compensation;

the application of this law to the entire world leads to a conclusion to which Kelvin first drew attention (17).

In 1852, in *On a universal tendency in nature to the dissipation of mechanical energy*, Kelvin draws from the second law the conclusion: there is at present in the material world a universal tendency to the dissipation of mechanical energy (*id.*).

And as the dissipation of mechanical energy, even if it is a universal tendency of the material world, may not be too exciting, Kelvin draws the

further conclusion: within a finite period of time past the earth must have been, and within a finite period of time to come the earth must again be, unfit for the habitation of man as at present constituted, unless operations have been, or are to be, performed, which are impossible under the laws to which the known operations going on at present in the material world are subject (*id.*).

As this conclusion may be too exciting, however, Kelvin has begun, before drawing the conclusions, by affirming: it is most certain that creative power alone can either call into existence or annihilate mechanical power (*id.*).

Clausius leaves both man and the creative power alone. He concludes:

the question is how this limiting condition can be characterized simply and nevertheless definitely;

and if what I have named entropy with respect to particular bodies is thought consistently with respect to all circumstances in the world and in relation to the other simpler concept of energy, then the two fundamental laws of the world corresponding to the two main laws of the mechanical theory of heat can be expressed in the following simpler form,

1. the energy of the world is constant;
2. the entropy of the world strives for a maximum (*id.*).

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And thus far the attempt to collect all important philosophers who are part of the development of the problem of the principles of all things in the not quite two centuries from Leibniz to Clausius, in particular, and in the development of the problem from its remotest beginning through Clausius, in general.

It remains to draw conclusions from the development of the problem in the not quite two centuries from Leibniz to Clausius, in particular.

To indicate how all important philosophers in the one and a half-odd centuries since Clausius can be collected.

To draw conclusions from the development of the problem, in general.

And to draw conclusions from the attempt to collect the particulars and marshal them in their series and order with respect to all five problems,

the earth, the soul, the best state, the triangle, and the principles.

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In the not quite two centuries from Leibniz to Clausius the advance in the formulation and solution of the problem, and in all its three parts, the elements of all things, the causes of all things, and the formal principles of all things, was very great.

Before turning to its explicit assessment, and the collection of the steps of advance in the acquisition of knowledge, however, it is better, alas, to undertake once again the task of cleaning the stables. The discovery of the principle of conservation of energy, as important as there has been a discovery, was mired in the distribution of credit and disgrace deeply from the first, and its history has since both greatly relished the mire and greatly added to it. *Ces coquins nous font parler et agir comme ils auraient fait eux-mêmes à notre place.*

Helmholtz discovered the principle in 1847.

Not so. Robert Mayer discovered it in 1842. Colding discovered it in 1843. Indeed, Mohr discovered it in 1837. Grove also discovered it in 1846.

Joule demonstrated the mechanical equivalent of heat in 1843.

As the demonstration was entangled with the discovery of the principle, it was entangled in the distribution of credit and disgrace as well.

Not Joule. Robert Mayer determined the equivalent in 1842. Indeed, Seguin determined it in 1839.

The distribution intensified as that between Kelvin and Clausius for the discovery of the second law joined those others. Indeed, neither Kelvin nor Clausius. Rankine discovered this law in 1849.

Joule conceded nothing. Clausius conceded nothing either.

Helmholtz, for his part, conceded.

Either way, more claims, and more, and more, have since been added on behalf of one philosopher, and another, and another, *ad hoc*, and at will.

Helmholtz discovered the principle of conservation of energy in *On the conservation of force*,

in 1847.

In 1854, in the lecture *On the interaction of the forces of nature and the newest results in physics concerning it*, Helmholtz says: in 1824 a Frenchman, S. Carnot, discovered a law which now bears his name; his work, however, remained unnoticed for a long time, and in 1842 different investigators in different countries, independently of one another and independently of Carnot, had the same thought; the first who conceived correctly the general law of the conservation of force was a German doctor, Robert Mayer of Heilbronn, in 1842; a little later, in 1843, a Dane, Colding, presented a memoir to the academy in Copenhagen, in which the same law found expression and some experiments were indicated for its further corroboration; in England Joule had begun at the same time to make experiments bearing on the same subject; we often find that with questions, to which the contemporary development of science points, several heads, quite independently of one another, are led to the same series of reflections; I myself, without knowing anything of Mayer and Colding, and having learned of Joule's experiments only at the end of my work, followed the same path; my investigation was published in 1847 in a small treatise under the title *On the conservation of force (id.)*.

We often find that with questions, to which the contemporary development of science points, several heads, quite independently of one another, are led to the same series of reflections. Hence that other series of reflections, to which of the several heads does the credit for the series of reflections belong?

The fallacy is ὅσπερον πρότερον.

The eagerness to distribute credit and disgrace is first. The supposition that several heads are led to the same series of reflections follows.

Discovery is discovery, precisely, because with questions to which the contemporary development of science points – and all theoretical questions, if not all practical, are such questions, questions to which the contemporary development of science points – the contemporary development of science points also to the answers of the questions, and the contemporary development of science *ipso facto* points also to the one, and only one, head that was led to the one, and only

one, series of reflections which are these answers. With all due respect to all the other heads and all the series of reflections to which they are led.

Thus in general, where the acquisition of knowledge is concerned, and thus in particular, where the discovery of the principle of conservation of energy is concerned.

Carnot's discovery of the reversible heat engine is another instance, in particular. It is a generalization over the development of the steam engine.

Again, Clausius writes: ever since heat was first used as a motive power in steam engines, thus showing practically that a quantity of work may be considered equivalent to the heat needed to produce it, it was not difficult to assume theoretically as well that there is a definite relation between the amount of heat and the work it can produce.

It was not difficult.

Hero, a younger contemporary of st Paul, built the first machine worked by steam.

Papin, in 1690, built the first steam engine.

In 1698 the first patent for a steam engine was issued to Savery.

In 1712 Newcomen built the first commercial steam engine, to pump water out of a tin mine.

Watt, beginning in 1765, developed greatly the steam engine, and developed its production into an industry. By 1800, when he retired from business, Watt had built more than 500 steam engines, working in tin mines, copper mines, coal mines, cotton mills, flour mills, paper mills, iron works, distilleries, canals.

In 1804 Trevithick mounted a steam engine on wheels, and the first steam locomotive hauled 10 tons of iron and 70 men for 10 miles at 5 miles per hour.

In 1807, built by Fulton, the first steamboat began ferrying goods and passengers the 300 nautical miles from New York to Albany and back.

In 1823 Stephenson began building the first railroad. Two years later, the Stockton and Darlington line was inaugurated, Stephenson's steam locomotive hauling 12 wagons of coal and 500 men for 25 miles at 10 miles per hour.

It was not difficult. All along the philosophers were reflecting on steam, heat, work, and motion.

Then, in 1824, seventeen and a half centuries after the first machine worked by steam, four-odd generations after the first steam engine, four generations after the first patent for a steam engine, three-odd generations after the first commercial steam engine, two generations after the steam engine became an industry, and as the steamboat and the steam locomotive were each becoming an industry, too, one, and only one, head was led to the one, and only one, series of reflections which are the discovery of the reversible heat engine.

And as Carnot's discovery, in particular, so all discoveries, in general, and so the discovery of the principle of conservation of energy, in particular.

Below, the results of Robert Mayer are presented. Then the results of the other philosophers who entangled themselves immediately in the distribution of credit and disgrace for the discovery are presented.

And it should then be superfluous to add, even as it is unlikely that it ever will be, that none of these results can be the discovery of the principle of conservation of energy. And that, pace Helmholtz, only his results can be.

Nor is his concession difficult to explain.

Nor is the distribution of credit and disgrace that matters. What matters is that neither this discovery, in particular, nor the acquisition of knowledge, in general, is chattel. Mine, and thine, and always better mine than thine.

Robert Mayer published in 1842 the *Remarks on the forces of inorganic nature*. In a half dozen pages Mayer has a general argument, an analytic formulation, and an experimental verification.

This is the general argument:

the goal of the following pages is to try to answer the question, what do we have to understand by 'forces'? and how are these related to one another?

forces are causes, and on them, therefore, the principle *causa aequat effectum* finds full application; if the cause c has the effect e , then $c = e$; if again e is the cause of another effect f , then $e = f$, and so on, $c = e = f \dots = c$; in a chain of causes and effects, by the nature of equation, a term or a part of a term can never become zero; this first property of all causes we call their

indestructibility;

if a given cause c has produced its equal effect e , c has thus ceased to be, c has become e ; if after the production of e , c still remained in whole or in part, this remaining cause would correspond to further effects, the general effect of c would be $> e$, contrary to the assumption $c = e$; therefore, c has become e , e has become f , and so on, and we must regard these magnitudes as different appearances of one and the same object; this ability to take different forms is the second essential property of all causes; and combining both properties, we say, causes are, quantitatively, indestructible and, qualitatively, convertible objects (*id.*).

This is the analytic formulation:

a cause which effects the raising of a weight is a force; its effect, the raised weight, is equally a force;

we call it the falling force;

falling force and fall, and in general falling force and motion, are two different appearances of one and the same object;

it is the task of mechanics to develop the equations holding between falling force and motion, and between motions;

the magnitude of the falling force v , when the earth's radius is set $= \infty$, is in direct relations to the magnitude of the mass m and its height d ; $v = m d$; if the height $d = 1$ of this mass m reaches in the motion of this mass the end velocity $c = 1$, then also $v = m c$; from the known relations between d and c , however, for other values of d and c , $m c^2$ results as the measure of the force v ; therefore, $v = m d = m c^2$; we thus find the law of the conservation of living force in the general law of the indestructibility of causes (*id.*).

And this is the experimental verification:

the natural connection between falling force, motion, and heat we can show in this way;

heat must be proportional to the magnitude of the weight and its distance;

and we find that the warming of a given weight of water from 0° to 1° centigrade corresponds to the fall of an equal weight from a height of about 365 m (*id.*).

The general argument would have been important, if not very important, in 1542. It would have been an embarrassment in 1642. It would have

been a very great embarrassment in 1742. And if it was not an embarrassment in 1842, it is because by 1842 the restoration of spirit to the substantiality and purity of its being was largely complete, and an essential part of the restoration was the new spiritual philosophy of nature, pioneered by Schelling with the *Ideas for a philosophy of nature* in 1797, furthered by Goethe with his optics better than Newton's in 1810, developed in all its all-embracing scope by Hegel in 1817, and entrenched far and wide in the next generation by, as Helmholtz describes them: enthusiastic supporters of Hegel's philosophy.

The analytic formulation should have been an embarrassment at any time. Never mind 1842^{AD}, the grasp on analytic formulation of the book of 87 problems, in 1842^{BC}, is firmer.

The experimental verification is different. But not too different.

The determination of the mechanical equivalent of heat is very important.

In 1842 Mayer writes: we find that the warming of a given weight of water from 0° to 1° centigrade corresponds to the fall of an equal weight from a height of about 365 m. He does not state what the given weight is. He does not state the grounds on which the result is obtained either. In 1845, in the *Organic motion in its connection to metabolism*, he states what the weight is, it is 1 kilogram, and he states the grounds on which calculation yields that result (3).

Then again, in 1843 Joule has not only obtained a better result, and a very nearly conclusive result. He has obtained it from the direct measurement of the equivalent. In 1824, too, Carnot's result from the indirect calculation of the equivalent is better than Mayer's in 1842.

Mayer's claim, on a comparison of his general argument with Helmholtz's general argument, should be most implausible. On a comparison of his analytic formulation with Helmholtz's analytic formulation, should be impossible. On a comparison of his experimental verification with Helmholtz's experimental verification, should be doubly impossible.

And impossible as it should be, Mayer's claim is much the most plausible of them all.

Mohr discovered the principle in 1837, Colding discovered it in 1843, Grove discovered it in

1846. Ignoring the rest of them all.

In none of the three is there an analytic formulation, wisely considering what an analytic formulation might be. In none of the three is there an experimental verification firmer than Mayer's. In each of the three there is a general argument. None of the three general arguments has the specificity of Mayer's general argument. In each of the three the vagueness has a great force, however, the force of the hammer in Mohr, that of spirit in Colding, that of god himself in Grove.

In 1837, in the *Views on the nature of heat*, Mohr writes: next to the known 54 chemical elements, there is in the nature of things only one other agent, and this is force; under appropriate relations it can appear as motion, chemical affinity, cohesion, electricity, light, heat, and magnetism, and from every one of these kinds of appearances we can bring out all the rest; the same force, which the hammer raises, can, when applied differently, bring out every one of the others (*id.*). A couple of pages of questions and answers follow: does light appear as electricity? does it appear as heat in the sun's rays? does it appear as magnetism? does it appear as affinity in the combination of chlorine and oxygen (*id.*)? and yes: magnetism appears as electricity; it appears as heat and light; it appears as affinity (*id.*). And that is that in the *View*.

In 1843, Colding presented the *Theses concerning force*. He writes: all known researches on the evolution of heat from moving force, as well as my own researches on the proportionality of the heat due to friction and the loss of force, seem to confirm the thesis that when a force seems to disappear it is merely transformed into a different kind; and I believe that this thesis holds not only for moving force; I believe it can be assumed valid universally for other forces, chemical force, for instance; to prove the impossibility of perpetual motion this thesis seems so necessary that without it every proof must be regarded as false (*id.*). Twenty years later, his claim still not sufficiently acknowledged, Colding wrote, in 1863, *On the history of the principle of conservation of energy*. To resolve the problem of acknowledgment, he describes the series of reflections which led him to the discovery. He reports: as the forces of nature are spiritual and immaterial entities, of

which we come to know only by their mastery over nature, these entities must of course be very superior to everything material in the world; and as it is obvious that the wisdom, which we perceive and admire in nature, expresses itself only through them, these powers must evidently be in relation to the spiritual, immaterial, and intellectual power itself which guides nature in its progress; but if so, it follows that it is quite impossible to conceive of these forces as anything mortal or perishable; surely, therefore, the forces ought to be regarded as absolutely imperishable; it was in the pursuit of this idea that twenty years ago I presented to the royal society of science, here in Copenhagen, a treatise in which I explained my idea that force is imperishable and immortal (*id.*).

In 1846, in *On the correlation of physical forces, being the substance of a course of lectures delivered in the London institution in the year 1843*, Grove writes: the position which I seek to establish in this essay is that the various imponderable agencies, or the affections of matter which constitute the main objects of experimental physics, viz., heat, light, electricity, magnetism, chemical affinity, and motion, are all correlative, or have a reciprocal dependence; that neither, taken abstractly, can be said to be the essential or proximate cause of the others, but either of them may, as a force, produce or be convertible into the other; thus, heat may mediate or immediately produce electricity, electricity may produce heat, and so the rest (*id.*). The position is then established with no more specificity than has already been attained in the couple of sentences which promise to establish it. In the final couple of sentences it becomes clear why. Grove concludes: in all phenomena, the more closely they are investigated, the more we are convinced that, humanly speaking, neither matter nor force can be created, and an essential cause is unattainable; causation is the will, creation, the act of god (*id.*).

Such are the series of reflections to which the several heads were led.

The discovery of the principle of conservation of energy, on the other hand, is the general argument in all its specificity, the following analytic formulation in all its generality and elegance, and the following experimental verification, itself analytic, in all its generality and power, in *On the*

conservation of force, in 1847, and nothing less.

Nor is it incidental that Helmholtz wrote also the *Handbook of physiological optics*, *On the integrals of the hydrodynamic equations to which the motions of vortices conform*, *The theory of the perception of tone as the foundation of the theory of music*, *On the theory of electrodynamics*, *The thermodynamics of chemical processes*.

Nor is it difficult to explain his concession.

In 1891, in his *Recollections*, Helmholtz writes: in those who have an inner urge toward science, as they continue to work, a higher conception of their relation to humanity comes to the fore; one comes to see his small contributions to the growth of science as serving an everlasting holy thing, to which he is attached by a bond of love; and through this, his work itself becomes holy; the world, which does not like to believe in ideal motives, calls this feeling lust for fame; but there is a decisive test to distinguish the two kinds of motivation; ask the question, is it the same to you whether the results you have won are acknowledged as belonging to you or not, when no considerations of external advantage are involved in the answer (*id.*).

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In the not quite two centuries from Leibniz to Clausius, then, the advance in the formulation and solution of the problem of the principles of all things was very great.

The elements of all things.

The advance in the formulation and solution of the problem of the elements can be analyzed into five parts.

One. Stahl writes: universal chemistry is the art of resolving compound bodies into their principles, and of composing such bodies from these principles. The foundation of the advance was the resolution and composition of bodies.

Two. Lavoisier: proved that water is not a simple substance, an element strictly, and is susceptible to decomposition and recombination. On that ground, the resolution and composition, which the elements are was demonstrated.

Three. Proust writes: oxidation is subject to a law of nature which rules all true combinations. On that ground, the resolution and composition, and hence the demonstration of the elements,

their laws were formulated.

Four. Dalton writes: we conclude that the ultimate particles of all homogeneous bodies are perfectly alike in weight, figure, etc. On that ground, the demonstration of the elements, and the formulation of their laws, of what kind the elements are was established.

Five. Faraday writes: the various forms under which the forces of matter are made manifest have one common origin. The relation of the elements and their laws and the forces and their laws began being established.

In 1697 Stahl changed metal into calx and calx back into metal.

With this result the advance in the resolution and composition of bodies begins, and the steady accumulation of resolutions and compositions.

Stahl also formulated the theory of phlogiston. The theory explained the resolution and combination of metal and calx into one another. In vegetables and animals the expulsion of phlogiston is irreversible, in minerals, however, it is reversible. And as metal and calx demonstrate reversibility, so all subsequent resolutions and combinations will demonstrate it. And reversibility implies law.

The accumulation of the demonstrations of reversibility will reinforce the presumption that it is fundamentally the reversibility of phlogiston, and hence the presumption of the elemental character of fire, phlogiston now, later on caloric.

In 1718 Geoffroy compiled the first table of affinities.

This is the major step toward the quantification of the resolutions and combinations. And hence toward the laws.

In 1756 Black resolved a quantity of magnesia alba into a quantity of fixed air and a quantity of calcined magnesia, then combined the resultant quantity of fixed air and resultant quantity of calcined magnesia into the original quantity of magnesia alba.

The demonstration of reversibility was greatly generalized. Earth had quit being elemental since the alchemists. Black demonstrated that air is not elemental. There are kinds of airs. And earths contain airs. And airs and earths are resolved and combined into each other.

Black's demonstrations are of the measurement of weights, in grains. And qualitatively, the

measurement of weight will remain the demonstration of resolutions and combinations. Quantitatively, with the precision attained by Black's results succumbed to weighing.

Black also discovered latent heat, and specific heat.

That the role of fire in resolution and combination is decisive had been clear since the alchemists, too. Black's results began quantifying this relation. They also reinforced the presumption of the elemental character of phlogiston.

And in the next generation the resolutions and combinations began yielding the elements.

In 1766 Cavendish discovered hydrogen.

He showed that inflammable air is 7 times lighter than common air.

In 1772 Priestley distinguished 8 kinds of airs.

In 1773 Scheele discovered chlorine, in 1774 oxygen and nitrogen.

He weighted yellow air, fire air, and vitiated air.

In 1779 Ingen-Housz discovered photosynthesis.

In 1789 Lavoisier published the *Elementary treatise on chemistry*.

He showed that water is hydrogen and oxygen. Formulated the theory of combustion and calcination. Formulated the principle of conservation of matter, and hence wrote the first chemical equation.

He showed that the constituent elements of vegetables are hydrogen, oxygen, and carbon. Animals are composed of hydrogen, carbon, nitrogen, phosphorus, and sulphur.

He showed that hydrogen, carbon, nitrogen, oxygen, phosphorus, sulfur, manganese, iron, cobalt, nickel, copper, zinc, arsenic, molybdenum, silver, tin, antimony, tungsten, platinum, gold, mercury, lead, and bismuth are elements.

With Lavoisier the problem of which the elements are is solved.

The solution embraces vegetables and animals explicitly.

It embraces also heat and light. One of the elements is caloric, another is light.

Caloric, earlier phlogiston, earlier fire, will quit being elemental in the next generation-odd, with the discovery of the mechanical equivalent of heat, and the formulation of the laws of ther-

modynamics. What the relation of heat and the elements is, in the generation after that the kinetic theory will explain.

Light as well will quit being elemental in the next generation, with the formulation of the wave theory. What the relation of light and the elements is, one century after that quantum theory will explain.

It was a glorious decade, the one culminating in the *Elementary treatise on chemistry* in 1789. Lagrange wrote the *Analytic mechanics*, in 1788. Laplace discovered the Laplace equation, and the potential, in 1785. Coulomb discovered the first law of electricity in 1785. Herschel discovered Uranus in 1781, the proper motion of the solar system in 1783, mapped the Milky Way in 1785. Kant wrote the *Critique of pure reason*, in 1781. The brothers Montgolfier built the first hot-air balloon in 1783.

In 1789 also the revolution began.

And the one and a half centuries culminating in 1789 are the time of enlightenment.

It is the time also of the beginning and growth of learned societies, and a time also of continued great growth of the universities.

The Royal Society was founded in 1660. The Academy of Sciences was founded in 1666. In 1789 the Royal Society had not less than 600 members. The Academy of Sciences, not as permissive, yet had more than 150 regular and corresponding members. Learned societies, royal and otherwise, of the repute of those two and of no repute at all, there were many dozens, spread far and wide. The universities, more than 100 of them three centuries earlier, had become many hundreds of them, spread very far and very wide, spreading steadily further and wider.

Philosophers there were, clearly, tens of thousands.

The growth of learned societies, the great growth of the universities, and the very great growth of the number of philosophers have been blamed on enlightenment. Wrongly. That those have nothing to do with enlightenment the fact suffices to show that the time of enlightenment came to an end, but those did not. On the contrary. As enlightenment came to an end, the growth of learned societies, the great growth of the universities, and the very great growth of the

1302 In order to formulate the problems we are to solve we call the council of our predecessors

number of philosophers, all three, accelerated, and more, and more, as they all three still do.

In the generation after Lavoisier the laws of resolution and composition were formulated, and the problem of the kind of the elements was solved. The isolation of the remaining elements became routine.

In 1794 Proust discovered the law of definite proportions.

Substances react in constant proportions by weight.

In 1801 Berthollet showed that the reactions of substances depend on their masses.

In 1801 Haüy showed that minerals are crystals.

He writes: solids are the combination of an infinite number of integrant molecules, which are the assemblages of the principal molecules, which chemistry will show us.

In 1801 Dalton discovered Dalton's law.

The pressure of a compound gas is equal to the sum of the pressures of the component gases.

Substances react in constant proportions by weight.

Lavoisier writes: all bodies in nature are solid, liquid, or in a gaseous state, according to the ratio of the attractive force of their particles and the repulsive force of heat.

If substances react in constant proportions by weight, the weight of their particles has to be measured.

Dalton formulated the atomic theory.

The problem of the kind of the elements is thus solved.

And Dalton began measuring the atomic weights.

If all bodies are solid, liquid, or in a gaseous state, and the weight of their atoms has to be measured, the weight of their atoms in their gaseous state will be measured. Gases, however, obey Boyle's law, $p \propto \frac{1}{V}$, where p is the pressure and V is the volume as the temperature, t , is constant. The constants of proportionality in Boyle's law have to be fixed, and in this Dalton's law is the first step, Gay-Lussac's law is the next step, Avogadro's law is the final step.

In 1808 Gay-Lussac discovered Gay-Lussac's law.

In reactions under constant temperature and pressure the volume of a compound gas is proportional to the volumes of the component gases.

In 1811 Avogadro distinguished atoms and molecules.

The problem of the kind of the elements is thus solved conclusively.

And Avogadro discovered Avogadro's law.

Under constant temperature and pressure, equal volumes of gas contain an equal number of molecules.

The constants of proportionality in Boyle's law are thus fixed. $pV = R(a + t)$, where a is the same for all gases, R depends on the specific weight of the gas.

Experimentally, the discovery of electrolysis, by Nicholson, in 1800, and hence decomposition by electrolysis, made the isolation of new elements routine.

In 1807 Davy isolated sodium and potassium, in 1808 magnesium, calcium, strontium, and barium.

In 1817 Berzelius isolated selenium, in 1823 silicon, in 1824 zirconium, in 1825 titanium, in 1828 thorium.

Mendeleev discovered the periodic law in 1869.

And in the two generations between Avogadro and Mendeleev the laws of electrochemistry were discovered, and organic chemistry was developed.

In the development of organic chemistry the most important are the labors of Dumas and of Liebig. Gay-Lussac in 1815, in the *Researches on prussic acid*, discovered the radical: a body which, although compound, acts the part of a simple substance in its combinations with hydrogen and metals (2). The radical which Gay-Lussac discovered is cyanogen, CN. In the next generation the general character of the radical was established. Dumas and Liebig established the fundamental character of the compound radical in the combinations and reactions of organic substances. In 1837, in the *Note on the present state of organic chemistry*, Dumas and Liebig write: in mineral chemistry the radicals are simple, in organic chemistry they are compound; and this is the whole difference; the laws of combination and the laws of reaction in the two

branches of chemistry are otherwise identical (*id.*). One of the compound radicals instanced in the *Note* is benzoyl, C_7H_5O .

In 1834 Faraday discovered the laws of electrochemistry.

The experimental ground is electrolysis. The laws are two. The quantity of chemical change is proportional to the quantity of electricity. And the quantity of chemical change produced in different substances by a given quantity of electricity is proportional to the chemical equivalents of the substances.

The relation of the elements and their laws and the forces and their laws thus began being established.

Faraday established what the relation of the elements and their laws and electricity and its laws is.

But light had quit being elemental. What the relation of the elements and their laws and light and its laws is will have to be established.

Heat quit being elemental in the following decade. And what the relation of the elements and their laws and heat and its laws is will have to be established.

And there was still very much the problem of what the relation of the elements and their laws and gravity and its laws is.

Tabulating the important philosophers with respect to the problem of the elements,

Stahl	resolved metal and calx into
	one another,
	in inorganic bodies phlogiston
	is reversible,
Geoffroy Black	the first table of affinities,
	resolved airs and earths into
	one another,
	airs are of different kinds,
	discovered latent heat and
	specific heat,
Cavendish Priestley	discovered hydrogen,
	distinguished several kinds of
	airs,
	discovered chlorine, oxygen,
Scheele	and nitrogen,
	discovered photosynthesis,
Ingen-Housz Lavoisier	water is hydrogen and oxygen,
	the theory of combustion and
	calcination,

the principle of conservation of
matter,
the first chemical equation,
the composition of vegetables
and animals,

23 of the elements,
the law of definite proportions,
reactions depend on the masses
of substances,

Proust
Berthollet
Haüy
Dalton
minerals are crystals,
the law of pressure of gases,
formulated the atomic theory,
the first measurements of atomic
weights,

Gay-Lussac
the expansion of gases is
proportional to heat,
the law of volume of gases,
discovered the radical,
distinguished atoms and
molecules,

Avogadro
the number of molecules in
gases is constant,

Davy
Berzelius
Faraday
isolated several elements,
isolated several elements,
the laws of electrochemistry.

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The causes of all things.

The advance in the formulation and solution of this problem can be analyzed into six parts. The causes, however, had been a more difficult problem than the elements from the first, and much the larger of the two. So the causes remained. And the parts into which this problem is analyzed are much larger than the parts into which that problem is analyzed.

One. The advance of mechanics. Mechanics was greatly formalized and generalized. This culminated in Lagrange, then in Hamilton. Lagrange wrote: the general formula of dynamics for the motion of an arbitrary system of bodies. Hamilton wrote: a single function expressing not the mere differential equations of motion but their intermediate and their final integral. The principles of greatest generality were formulated. Of least action. And of the conservation of energy. The potential was also defined. And on this ground, in the century after Hamilton, the advance of mechanics was unified with all other

advances.

Two. The formulation of the wave equation. This culminated in Euler. Euler wrote: the equation containing all curves which fit a string set in motion. It was the solution, however, principally of a problem in analysis, of the generalization of a function. And on this ground the wave equation became an essential part of the ground on which all advances were unified from the first.

Three. The attempts to formulate the theory of the motions of the smallest particles in their relation to the bodies which they constitute. It was not a central problem at this time, the not quite two centuries from Leibniz to Clausius. It became a central problem in the generation after Clausius, and the theory of the motions of molecules, the kinetic theory, became a part of the unification of all advances.

Four. The development of electrodynamics. This was by far the greatest advance. From nearly nothing to nearly everything. The accumulation of the phenomena began, and the first attempts to formulate their theory. The accumulation of the phenomena continued, and their laws began being discovered. The accumulation of the phenomena accelerated greatly, and the advance culminated in Ampère. Ampère's theory: is perfect in form and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced. The accumulation of the phenomena accelerated yet more, and the advance culminated in Faraday. Electromagnetism and mechanical motion, and electromagnetism and chemical action, were reduced to one another, and the dependance of electromagnetism and heat, and of electromagnetism and light, on one another was shown. Faraday's was: a great addition to the facts and considerations which tend to prove that all natural forces are tied together and have one common origin. And electromagnetism is a field. And the laws of the electromagnetic field were discovered in the generation after Faraday.

Five. The formulation of the wave theory of light. This culminated in Fresnel: light is a polarized wave. An electromagnetic polarized wave, it was shown in the generation after Fresnel.

Six. The development of thermodynamics. This was by far the most dramatic advance. From nothing to everything in twenty-six years. The

energy of the world is constant, the entropy of the world strives for a maximum.

And in the next one century-odd the unification of all advances followed, into the theory of relativity, quantum theory, and the big bang theory.

In 1713 Taylor showed that the acceleration of a particle on a tense string is proportional to the curvature of the string.

Taylor solves the problem, as Newton has solved all problems in *The principles*, synthetically. The analytic formulation and solution of the problem, one generation-odd later, will be the wave equation.

In 1731 Gray discovered the conduction of electricity.

He carried the electric virtue across 886 feet.

Gilbert discovered electricity. He writes: electric motion is lowly, of small potency, and confined in prisons from which it has to be released by friction. Four-odd generations later, Gray's discovery showed, when he led it far from its prisons, that electric motion is not lowly, and of great potency.

In 1733 du Fay made the first attempt to formulate the theory of electricity.

There are two distinct electricities, vitreous and resinous. A body of vitreous electricity repels all bodies of vitreous electricity and attracts all those of resinous electricity.

It is not a sophisticated theory. It is also, like that of gravity, a theory of attraction and repulsion. And the theory of electricity will continue to be developed on Newtonian grounds through Ampère.

In 1736 Euler developed mechanics analytically.

He writes: propositions obtained by the old synthetic method are solved more readily by my analytic method; I have also discovered many new problems, and have enlarged the science by many new methods.

This explains why physics will become analytic exclusively.

In 1738 Daniel Bernoulli discovered the law of fluids.

He derives the law from the principle of conservation of living force.

Daniel Bernoulli also took the first step toward

the kinetic theory.

Very minute particles, infinite in number, in a very rapid motion this way and that, forming by their continuous impacts an elastic fluid which expands by itself, can explain all properties of air and other elastic fluids.

In 1740 Maupertuis formulated the principle of least action.

In all systems of elastic bodies in motion which act on one another the sum of the products of the masses and the squares of the velocities, called living force, remains inalterably the same.

Leibniz discovered the conservation of living force. The law of fluids is a deduction from the conservation of living force. The principle of least action is a generalization over the conservation of living force.

Euler formulated the principle analytically.

If m is the mass of a body, ds the distance it travels, and v its velocity, the motion of the body is given by $\int m\sqrt{v} ds$, when m is constant and $\int \sqrt{v} ds$ is a minimum.

In 1743 d'Alembert developed mechanics analytically.

He formulated the principle: if a motion is impressed on several bodies so that they are forced to move consistently with their mutual inclinations, their motions can be considered composed of those which the bodies actually follow and motions which are negated.

In 1747 d'Alembert discovered the wave equation

$$y = \Phi(t + x) + \Psi(t - x),$$

where x and y are the coordinates of a point on a string, t is the time. y is a function of t and x .

Euler rewrote the equation

$$y = \Phi(x + ct) + \Psi(x - ct),$$

where c is a constant. And deduced the solution

$$y = \alpha \sin\left(\frac{\pi x}{a}\right) + \beta \sin\left(\frac{2\pi x}{a}\right) + \gamma \sin\left(\frac{3\pi x}{a}\right) + \dots,$$

where a is the length of the string, α , β , γ are constants.

Daniel Bernoulli discovered the principle of superposition of waves.

And Euler wrote the general equation

$$\frac{\partial^2 y}{\partial t^2} = X \frac{\partial^2 y}{\partial x^2},$$

where X is a function of x depending on the

thickness of the string.

This equation contains all curves which fit a string set in motion. Or a wave.

It contains also all the power of function.

It contains also, for the first time, all the power of the partial derivative.

All general equations, and analysis generally, will build on this equation.

In 1747 Franklin made the next attempt to formulate the theory of electricity.

It is a substantially more sophisticated theory.

Bodies are electrified positively and negatively. In the instant of friction the thing which is rubbed attracts the electric fire and therefore takes it from the thing which rubs. The equality is never destroyed, the fire only circulates. Electric matter consists of extremely subtle particles which permeate common matter. The particles of common matter attract one another, those of electric matter repel one another. Although repelling one another, however, they are strongly attracted by all other matter.

Franklin was a delegate to the second continental congress, in 1775-6, and served on the committee which drafted the declaration of independence. In 1776-85 he was in Paris. Paine writes: Dr Franklin was not the ambassador of a court, but of MAN (*Rights*). Mirabeau announced Franklin's death, in 1790, to the national assembly. He said: gentlemen, Franklin is dead; the genius who liberated America and threw the torrents of light upon Europe; the sage whom two worlds claim, the man over whom the history of the sciences and the history of empires quarrel (11 Jun).

In 1758 Boscovich proposed *A theory of natural philosophy reduced to a single law of the forces existing in nature*.

Atoms are not particles of matter in which forces reside, atoms are mere centers of forces.

Boscovich anticipates the field.

Beginning in 1765, Watt developed greatly the steam engine.

The development of the steam engine illustrates particularly well the fact that, if the acquisition of knowledge is a cause of the development of technology, it is neither a main nor a proximate cause. And the other way around, if the development of technology is a cause of the acquisition

of knowledge, it is neither a main nor a proximate cause.

Papin built the first steam engine, in *A new method to obtain the greatest motive power at little cost*, in 1690. And in 1690 the relation of cause and effect, from the acquisition of knowledge to the development of the steam engine, is readily apparent. Papin worked first with Huygens, then with Boyle, then with Leibniz.

Then the relation of cause and effect between the two is not apparent at all until 1824, when it is readily apparent again, the other way around this time, from the development of the steam engine to the acquisition of knowledge. Carnot's is explicitly a generalization over the development of the steam engine, too.

In the meantime, between Papin and Carnot, the development of the steam engine is very difficult to find in any of the philosophers who investigated steam, heat, work, and motion. The following is representative. Cagniard de la Tour discovered the critical temperature – the temperature above which a gas cannot be liquefied by the increase of pressure – in 1822. In the *Presentation of some results obtained by the combined action of heat and compression on certain liquids*, Cagniard de la Tour begins: it is well known that by the Papin digester the temperature of liquids can be raised far above the point at which they ordinarily boil (*id.*). And concludes: I presume that this note may be of interest particularly to those occupied in the application of steam engines (*id.*). The Papin digester is the first machine worked by steam, which Papin built in 1679. And nothing of the development of the steam engine subsequent to the Papin digester, in 1679, is noted, or needs to be noted, in Cagniard de la Tour's *Presentation*, in 1822.

In the meantime, between Papin and Carnot, between 1690 and 1824, the steam engine was developed very greatly, became an industry, became a major industry, became the foundation of all industry, and the foundation of the continuing great growth of industry.

All of which, between 1690 and 1824, the steam engine became in Britain.

Britain, for its part, between 1690 and 1824, became the dominant power on earth. It built the greatest fleet humanity had seen. It built the

greatest banking and credit system humanity had seen. Commerce flourished as never before. Wealth was accumulated as never before. In 1776 Adam Smith wrote *The wealth of nations*. He writes: there is a certain propensity in human nature to truck, barter, and exchange.

The great growth of commerce, and accumulation of wealth, grounded in the certain propensity in human nature to truck, barter, and exchange, itself grounded in opportunism, is a main and proximate cause of the development of the steam engine.

Watt built more than 500 steam engines, and amassed a fortune. Berthollet discovered a new bleaching technique. Watt proposed patenting the technique and a joint partnership. Berthollet replied: *quand on aime les sciences on a peu besoin de fortune* (to Watt, 15 Mar 1787). And the other way around, *quand on aime la fortune on a peu besoin des sciences*.

In 1775 Cavendish began the accurate measurement of electricity.

In 1785 Coulomb discovered the first law of electricity

$$F \propto \frac{q_1 q_2}{r^2},$$

where q_1 and q_2 are two small spheres charged with the same kind of electricity, r is the distance between their centers, and F is the repulsive force between them.

Coulomb's law shows clearly its descent from the law of gravity.

In 1785 Laplace discovered the potential.

He wrote the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

where x, y, z are the coordinates of a particle of a spheroid, and V is the sum of the quotients of the masses of the particles of the spheroid and their distances to an attracted point, or the attraction of the spheroid in any direction. Or the potential.

The Laplace equation shows clearly its descent from, both, the principle of least action and the wave equation.

In 1788 the *Analytic mechanics* appeared.

Lagrange generalized d'Alembert's principle and the principle of least action into the principle of virtual velocities

$$\int \left(\frac{d^2x}{dt^2} \delta x + \frac{d^2y}{dt^2} \delta y + \frac{d^2z}{dt^2} \delta z \right) m.$$

The principle of virtual velocities shows clearly its descent from, both, Euler's earlier formulation and the wave equation, too.

And Lagrange wrote the equation

$$\int \left(\frac{d^2x}{dt^2} \delta x + \frac{d^2y}{dt^2} \delta y + \frac{d^2z}{dt^2} \delta z \right) m + \int (P \delta p + Q \delta q + R \delta r + \dots) m = 0,$$

where $P, Q, R, \dots$ are forces acting in the directions $p, q, r, \dots$.

This is the general formula of dynamics for the motion of an arbitrary system of bodies.

The national assembly was succeeded by the legislative assembly, the legislative assembly was succeeded by the convention and the committee of public safety, the convention and the committee of public safety were succeeded by the directory, the directory was succeeded by the consulate, the consulate became the empire.

Along the way, incidentally, the revolution honored Lavoisier by sending him to the guillotine.

In 1791 Galvani discovered that animals conduct electricity.

In 1800 Volta discovered current electricity.

Yes! this apparatus will astonish you. It surpasses the power of the old batteries infinitely. Its effects are incessant. After discharge the charge reestablishes itself. In a word, it produces unlimited charge.

And yes! this apparatus accelerated the investigation of electricity astonishingly.

In 1800 Nicholson discovered elec-

$$f(x) = A + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x \dots,$$

trolsis. The decomposition of substances and isolation of elements became routine.

Beginning in 1801, Young developed the wave theory of light.

The descent is from Huygens.

Young demonstrated the generality of interference.

He measured the wavelength of light of different colors.

Young is also one of the philosophers, the other is Champollion, who deciphered hieroglyphic.

In 1808 Malus demonstrated the generality of polarization.

In 1813 Poisson wrote the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho,$$

where the spheroid is heterogenous, and ρ is the density of the attracted point.

It is a solution of the Laplace equation.

It shows the distribution of electricity between conducting spheres.

Beginning in 1815, Fresnel formulated the wave theory of light analytically.

Light is the rapid succession of systems of waves polarized in all directions.

In 1820 Ørsted discovered electromagnetism.

Electric force was converted into magnetic.

In 1820 Biot and Savart quantified the relation.

In 1821 Faraday discovered electromagnetic rotation.

Electric force was converted into mechanical.

In 1822 Seebeck showed that current arises from heat.

In 1822 the *Analytic theory of heat* appeared.

Fourier writes the equation

$$\frac{\partial u}{\partial t} = \frac{K}{CD} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

where u is the temperature at time t of a particle with coordinates x, y , and z , K is the specific conductivity, C the specific capacity, D the density of the solid.

The descent is the same.

The equation shows the propagation of heat in the interior of a solid.

The general solution of the equation is

as each of the coefficients is a definite integral,

$$2\pi A = \int_{-\pi}^{+\pi} dx f(x),$$

$$\pi a_i = \int_{-\pi}^{+\pi} dx f(x) \cos ix,$$

$$\pi b_i = \int_{-\pi}^{+\pi} dx f(x) \sin ix.$$

And any function whatsoever can be developed in this form.

A new epoch in the development of the analytic representation of arbitrary functions thus began, which soon became manifest in the magnificent expansion of mathematical physics.

Between 1822 and 1827 Ampère formulated the analytic theory of electrodynamics.

Electric currents form minute circuits around

the particles of bodies, magnetic phenomena are assemblages of electric currents.

The theory is summed up in a formula from which all phenomena can be deduced

$$F = \frac{i i' ds ds'}{r^2} (\sin \theta \sin \theta' \cos \omega - \frac{1}{2} \cos \theta \cos \theta'),$$

where F is the force between the two current elements $i ds$ and $i' ds'$ at a distance r , ω is the angle between the elements, θ and θ' the angles between the line joining the elements and the two planes containing this line.

And Ampère's law shows clearly its descent from the law of gravity. $\delta V = \Sigma m (x' \delta x + y' \delta y + z' \delta z) - \Sigma m (a' \delta a + b' \delta b + c' \delta c) + t \delta H$.

In 1824 Carnot discovered the Carnot principle and the Carnot cycle.

The motive power of heat is independent of the agents employed to realize it, its quantity depends only on the temperature of the bodies between which the transfer of caloric is effected.

In the cycle of the reversible heat engine the internal work done in some of the changes is compensated exactly in the others, and we can be certain that the external work, which remains after the changes are completed, is all the work that has been done.

In 1827 Ohm discovered the resistance of the conductor.

In 1828 Robert Brown discovered Brownian motion.

The motion arose neither from the currents in the fluid nor from its gradual evaporation, but belonged to the particles themselves.

In 1831 Faraday discovered electromagnetic induction, in 1834 the laws of electrochemistry, in 1837 specific inductive capacity, in 1845 the magnetic rotation of the plane of polarization, and diamagnetism.

He writes in 1837: I believe ordinary induction to be in all cases an action of contiguous particles, consisting in a species of polarity, not an action of either particles or masses at sensible distances. In 1844: if an atom is conceived a center of power, what is ordinarily called shape would be the disposition and relative intensity of the forces. In 1850: the long constant persuasion that all the forces of nature are mutually dependent, having one common origin, has made me often think on the possibility of establishing, by experiment, a connection between gravity and

electricity; the results are negative. In 1852: a state of tension constitutes the physical lines of force, this state produces the current of electricity, and it is sufficient to constitute and give a physical existence of the lines of force.

In 1834 Lenz quantified the relation of magnetoelectric and electromagnetic phenomena.

In 1834 Hamilton discovered Hamilton's principle.

The descent is the same.

Hamilton writes the equation

It is a generalization of the principle of Lagrange. V is the living force, H is the characteristic function, eliminated in the yet further generalization. Hamilton introduces time, and writes

$$\int_0^t (T + U) dt = 0,$$

where $2T$ is the living force and U is the force function of a system of free points.

And as it shows clearly its descent, so Hamilton's principle points clearly to the subsequent ascent. $2T$ is the kinetic energy, U the potential energy.

In 1839 Gauss discovered Gauss's law.

The law is the application of the gravitational potential to magnetism, and hence also electricity.

The descent is the same. Gauss's are: general theorems on the forces of attraction and repulsion acting inversely as the square of the distance.

In 1843 Joule demonstrated the mechanical equivalent of heat.

One degree of heat per pound of water from a mechanical force is capable of raising 772.692 pounds to a height of one foot.

In 1847 Helmholtz discovered the principle of conservation of energy.

$$\frac{1}{2} m Q^2 - \frac{1}{2} m q^2 = - \int_r^R \phi dr.$$

Q and q are the tangential velocities, R and r the distances, and ϕ is the intensity of the force in a system of material points.

And if we consider this equation more closely, we find on the left side the difference of the living forces, on the right side the sum of the tensions.

On the left side the kinetic energy, on the right side the potential energy.

In 1848 Kelvin discovered the absolute scale.

A unit of heat descending 1° gives out the

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du Fay	electricity is vitreous and resinous,	Ørsted	light analytically, light is a polarized wave,
Euler	developed mechanics analytically, formulated the principle of least action analytically, the general wave equation,	Biot Faraday	discovered electromagnetism, quantified electromagnetism, discovered electromagnetic rotation, electromagnetic induction, the laws of electrochemistry, specific inductive capacity, the magnetic rotation of the plane of polarization,
Daniel Bernoulli	– the law of fluids, very minute particles forming by their continuous impacts an elastic fluid, the principle of superposition of waves,	Seebeck Fourier	diamagnetism, electromagnetism is a field, current arises from heat, the equation of the propagation of heat in a solid, the Fourier series,
d’Alembert	developed mechanics analytically, formulated d’Alembert’s principle,	Ampère Carnot	developed electrodynamics, Ampère’s law, the Carnot principle, the Carnot cycle,
Franklin	discovered the wave equation, the one fluid theory of electricity,	Ohm Robert Brown	discovered resistance, Brownian motion,
Boscovich	atoms are mere centers of forces,	Lenz	quantified the relation of magnetic and electric phenomena,
Watt	developed the steam engine,	Hamilton	a single function expressing the differential equations of motion in their final integral,
Cavendish	the accurate measurement of electricity,		
Coulomb	Coulomb’s law,	Gauss	Hamilton’s principle, the measurement of electro- magnetism in absolute units, Gauss’s law, potential theory,
Laplace	the Laplace equation, the potential,		
Lagrange	the principle of virtual velocities, the general formula of dynamics,	Joule	the mechanical equivalent of heat,
Galvani	animals conduct electricity,	Helmholtz	the principle of conservation of energy,
Volta	discovered current electricity,		
Nicholson	discovered electrolysis,	Kelvin	the absolute scale,
Young	developed the wave theory of light, demonstrated the generality of interference, measured the wavelength of light of different colors,	Clausius	the laws of thermodynamics, entropy.
Malus	demonstrated the generality of polarization,	ccclxxxv The formal principles of all things.	
Poisson	the equation of the distribution of electricity,	With respect to their content, judgments are either analytic or synthetic, they either add noth- ing to the content of knowledge or enlarge knowl- edge.	
Fresnel	formulated the wave theory of		

And mathematical propositions are in their entirety synthetic.

As mathematical propositions had been showing themselves to be from the first, and had been showing themselves to be spectacularly in the one and a half-odd centuries since Galileo. The world is written in mathematics, and mathematics had enlarged knowledge spectacularly.

Mathematics had also become analysis, and most particularly analysis had enlarged knowledge spectacularly.

Analysis is in its entirety synthetic.

And this verbal infelicity, like that other one, Kant's idealism, has been the source of much, if not quite as much, confusion, and obfuscation.

Idealism? The very opposite of it!

Nor is the confusion in this case, as in that, or in any case, in Kant.

Kant writes: the analytic method, as opposed to the synthetic method, is something completely different from the analytic proposition; the analytic method indicates proceeding from what is sought as if it is given and ascending to the conditions under which alone it is possible; in this method often nothing but synthetic propositions are employed, of which mathematical analysis offers an example; and it may be better called the regressive method to distinguish it from the synthetic or progressive method; the name analytic appears also as one of the main divisions of logic, and it is there the logic of truth, in opposition to dialectic, without considering whether the knowledge belonging to it is analytic or synthetic (*Prolegomena*, 5).

Analytic is one of the main divisions of logic. The logic of truth, in opposition to dialectic. It has been since Aristotle, since the *Posterior analytics* and *Prior analytics*.

The analytic method, as opposed to the synthetic method, Pappus first distinguished explicitly. This is how Newton describes them: in analysis we proceed from effects to their causes, and from particular causes to more general, until the argument ends in the most general; this is the method of analysis; synthesis consists in assuming the causes discovered and established as principles, and explaining the phenomena from them, and proving the explanations.

Analytic judgments, in opposition to synthetic

judgments, Kant first distinguished. Analytic judgments add nothing to knowledge, synthetic judgments enlarge knowledge.

And mathematics, including most particularly analysis, is in its entirety synthetic.

It has enlarged knowledge spectacularly.

Hence the question, how are synthetic *a priori* propositions possible? That they are possible the reality of pure mathematics and pure natural science shows.

If synthetic *a priori* propositions are possible, there must be in perception already something *a priori*.

Perception has form. *Ipsa facto* experience, the synthesis of perceptions, has form. It is the form of possible experience, inclusive of all actual experience, and hence of the possibility of extension of actual experience.

Form gives experience its objective validity, or what is the same, its general validity.

The principle of contradiction is the principle of all analytic judgments.

The principle of sufficient reason is the principle of all synthetic judgments.

And mathematics continued to enlarge knowledge spectacularly. And most particularly analysis. Lagrange. Laplace. Fourier. Gauss. Hamilton.

And Boole showed that the laws of logic, too, are mathematical.

Logic is algebra.

Everything, including nothing and the universe, is sets.

0 is nothing, 1 is the universe, and if x is any set of objects,

$$x(1 - x) = 0.$$

Propositions are sets.

Tabulating,

Kant

experience has form,
form gives experience its objective validity, or what is the same, its general validity,
the principle of contradiction is the principle of all analytic judgments,
the principle of sufficient reason is the principle of all synthetic judgments,

Boole

logic is algebra,
everything, including nothing

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and the universe, is sets,
propositions are sets.

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In their series and order these 54 are all important philosophers who are part of the development of the problem of the principles of all things in the not quite two centuries from Leibniz to Clausius,

1660 - 1734 **Stahl**,
1666 - 1736 **Gray**,
1672 - 1731 **Geoffroy**,
1685 - 1731 **Taylor**,
1698 - 1739 du Fay,
1698 - 1759 **Maupertuis**,
1700 - 1782 **Daniel Bernoulli**,
1706 - 1790 **Franklin**,
1707 - 1783 **Euler**,
1711 - 1787 Boscovich,
1717 - 1783 **d'Alembert**,
1724 - 1804 **Kant**,
1728 - 1799 **Black**,
1730 - 1799 **Ingen-Housz**,
1731 - 1810 **Cavendish**,
1733 - 1804 Priestley,
1736 - 1806 **Coulomb**,
1736 - 1813 **Lagrange**,
1736 - 1819 Watt,
1737 - 1798 Galvani,
1742 - 1786 **Scheele**,
1743 - 1794 **Lavoisier**,
1743 - 1822 Haüy,
1745 - 1827 **Volta**,
1748 - 1822 Berthollet,
1749 - 1827 **Laplace**,
1753 - 1815 **Nicholson**,
1754 - 1826 **Proust**,
1766 - 1844 **Dalton**,
1768 - 1830 **Fourier**,
1770 - 1731 Seebeck,
1773 - 1829 **Young**,
1773 - 1858 **Robert Brown**,
1774 - 1862 Biot,
1775 - 1812 Malus,
1775 - 1836 **Ampère**,
1776 - 1856 **Avogadro**,
1777 - 1851 **Ørsted**,
1777 - 1855 **Gauss**,
1778 - 1829 Davy,

1778 - 1850 **Gay-Lussac**,
1779 - 1848 Berzelius,
1781 - 1840 Poisson,
1788 - 1827 **Fresnel**,
1789 - 1854 Ohm,
1791 - 1867 **Faraday**,
1796 - 1832 **Carnot**,
1804 - 1865 Lenz,
1805 - 1865 **Hamilton**,
1815 - 1864 **Boole**,
1818 - 1889 **Joule**,
1821 - 1894 **Helmholtz**,
1822 - 1888 **Clausius**,
1824 - 1907 **Kelvin**.

The steps of advance in the acquisition of knowledge are the 38, Stahl, Gray, Geoffroy, Taylor, Maupertuis, Daniel Bernoulli, Franklin, Euler, d'Alembert, Kant, Black, Ingen-Housz, Cavendish, Coulomb, Lagrange, Scheele, Lavoisier, Volta, Laplace, Nicholson, Proust, Dalton, Fourier, Young, Ampère, Avogadro, Ørsted, Gauss, Gay-Lussac, Fresnel, Faraday, Carnot, Hamilton, Boole, Joule, Helmholtz, Clausius, and Kelvin. In particular,

Stahl	resolved metal and calx into one another, in inorganic bodies phlogiston is reversible,
Gray	discovered the conduction of electricity,
Geoffroy	the first table of affinities,
Taylor	the acceleration of a particle on a string,
Maupertuis	discovered the principle of least action,
Daniel Bernoulli	– the law of fluids, very minute particles forming by their continuous impacts an elastic fluid, the principle of superposition of waves,
Franklin	the one fluid theory of electricity,
Euler	developed mechanics analytically, formulated the principle of least action analytically, the general wave equation,
d'Alembert	developed mechanics

Kant	analytically, formulated d'Alembert's principle, discovered the wave equation, experience has form, form gives experience its objec- tive validity, or what is the same, its general validity, the principle of contradiction is the principle of all analytic judgments, the principle of sufficient reason is the principle of all syn- thetic judgments,	Young	of heat in a solid, the Fourier series, developed the wave theory of light, demonstrated the generality of interference, measured the wavelength of light of different colors, developed electrodynamics, Ampère's law, distinguished atoms and molecules, the number of molecules in gases is constant,
	resolved airs and earths into one another, airs are of different kinds, discovered latent heat and specific heat,	Ørsted Gauss	discovered electromagnetism, the measurement of electro- magnetism in absolute units, Gauss's law, potential theory, the expansion of gases is proportional to heat, the law of volume of gases, discovered the radical, formulated the wave theory of light analytically, light is a polarized wave, discovered electromagnetic rotation,
	discovered photosynthesis, discovered hydrogen, the accurate measurement of electricity,	Gay-Lussac	electromagnetic induction, the laws of electrochemistry, specific inductive capacity, the magnetic rotation of the plane of polarization, diamagnetism, electromagnetism is a field, the Carnot principle, the Carnot cycle, a single function expressing the differential equations of motion in their final integral,
	Coulomb's law, the principle of virtual velocities, the general formula of dynamics,	Fresnel Faraday	Hamilton's principle, logic is algebra, everything, including nothing and the universe, is sets, propositions are sets, the mechanical equivalent of heat,
Scheele	discovered chlorine, oxygen, and nitrogen,		
Lavoisier	water is hydrogen and oxygen, the theory of combustion and calcination, the principle of conservation of matter, the first chemical equation, the composition of vegetables and animals,	Carnot Hamilton	
Volta Laplace	23 of the elements, discovered current electricity, the Laplace equation, the potential,		
Nicholson Proust Dalton	discovered electrolysis, the law of definite proportions, the law of pressure of gases, formulated the atomic theory, the first measurements of atomic weights,	Boole Joule	
Fourier	the equation of the propagation	Helmholtz	the principle of conservation of

Clausius	energy, the laws of thermodynamics, entropy,	
Kelvin	the absolute scale.	

Continuing the tabulation of the distribution of the philosophers,

1576-1675	18	16,
1676-1775	33	22,
1776-1875	18	13.

The count in the last line is not complete.

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The one and a half-odd centuries since Clausius.

Much has been said above about the development of the problem after Clausius. More than enough to indicate how the collection of the particulars can continue, and be completed through the present.

It can all be reviewed here in this way.

Counting generations, beginning with the year of birth of Clausius, 1822, these are five-odd generations.

The first is the generation, other than Clausius, of Maxwell, Mendelev, Boltzmann, Cantor, and Frege. The laws of thermodynamics were discovered. The laws of electrodynamics were discovered. The law of the elements was discovered. The kinetic theory was formulated. The theory of sets was formulated. What propositions are was discovered. Thermodynamics is the theory of the transformation of energy. The kinetic theory is the theory of the motions of particles, and the ground of thermodynamics. Electrodynamics is the theory of electromagnetism and light. The theory of sets is the foundation of mathematics, including logic, and hence also of physics.

The second is the generation of Planck, Hilbert, Minkowski, Rutherford, Zermelo, and Einstein. The theory of relativity was formulated. The foundations of quantum theory were laid. Set theory was axiomatized. The theory of proof was formulated. Relativity unified mechanics and electrodynamics. It is the theory of motion and gravity. Axiomatic set theory is the foundation of mathematics, logic, and physics. The theory of proof is the theory of the consistency of proof, mathematical, logical, physical.

The third is the generation of Bohr, Schrö-

ding, Friedmann, Hubble, and Heisenberg. Quantum theory was formulated. The foundations of the big bang theory were laid. Quantum theory unified electrodynamics and chemistry. It is the theory of particles, or waves, and forces.

In the next generation there are no philosophers of comparable importance. There are yet Turing and Gell-Mann. The mathematical development of logic was completed. Quantum theory was completed. The big bang theory was completed. The big bang theory unified relativity and quantum theory. It is the theory of the origin and development of the world, particles and forces through galaxies, beginning to end.

In the next generation-odd there are no philosophers of remotely comparable importance.

It is hard to avoid the conclusion that the problem has been solved.

It is hard to avoid the conclusion also that the present is not the highest.

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In the course of the development of this problem, then, the problem of the principles of all things, as in the course of the development of the problem of the earth, and in the course of the development of the physical problem of the soul, and in the course of the development of the problem of the triangle, a large amount of knowledge has been acquired.

Very many propositions have been won, brought in connection with one another in very many chains of inferences, ever more free of gaps.

From the thought and the sense to the reference and the true.

From no inkling to water is the principle, number is the principle, and reason is the principle, to god is the principle, to atoms and molecules, energy and entropy, and logic is algebra. The principle of all principles is $x(1-x) = 0$. -273.15° is the absolute zero. There are 602 200 000 000 000 000 000 000 atoms in 12 g of carbon-12.

It has been done in this case, the problem of the principles, as it has been done in those, the problems of the earth, the soul, and the triangle. The council of our predecessors has done it.

In order to formulate the problems we are to

solve we call the council of our predecessors so that we can adopt the sound in what they have said and avoid the erroneous.

In particular, from the twenty-four and a half centuries from Thales through Clausius 146 important philosophers have been collected, and 94 steps of advance in the acquisition of knowledge among them. Counting centuries from the year of birth assigned to Thales, their distribution is,

625- 526	4	4,
525- 426	10	7,
425- 326	8	4,
325- 226	7	6,
225- 126	2	1,
125- 26	—	—,
25 ^{BC} - 75 ^{AD}	2	1,
76- 175	2	1,
176- 275	5	—,
276- 375	7	—,
376- 475	1	—,
476- 575	1	—,
576- 675	—	—,
676- 775	1	1,
776- 875	3	1,
876- 975	—	—,
976-1075	1	1,
1076-1175	2	—,
1176-1275	4	2,
1276-1375	5	2,
1376-1475	1	1,
1476-1575	11	11,
1576-1675	18	16,
1676-1775	33	22,
1776-1875	18	13.

The count in the last line is not complete.

This problem begins with the first philosopher, and the other way around, philosophy begins with this problem.

In the first four centuries the advance is very great. In the following seventeen centuries the advance is not great. In the following four centuries the advance is again very great.

In the first two of the first four centuries the principles of all things are very much the central problem of philosophy. Philosophy begins with the problem of the principles of all things, and in the first two centuries of its development philosophy is fundamentally and substantially the ad-

vance in the formulation and solution of this problem. This advance is the ground and the consequence of the advance in the formulation and solution of all other problems. From the earth floats on water to the earth is in the center of the world, from the soul is air to the soul is the principle of life, from number is the principle to the theory of the measurement of space. In particular, the problem begins with the discovery of the problem of the elements, the advance in the formulation and solution of this part of the problem leads to the discovery of the problems of the causes and of the formal principles, and hence the continuing advance in the formulation and solution of all three parts of the problem.

In the next two centuries the advance continues, but the principles of all things are not the central problem of philosophy any longer. Early in these centuries the development of the problem, in all its three parts, culminates in Aristotle. Early in these centuries as well, however, before Aristotle already, in Plato, the central problem of philosophy is what is the best for you. This problem, what is the best for you, is from that problem, the principles of all things. What is the best for you is the true cause of things. Thus, after Aristotle the great advance of mathematics and physics continues. The theory of the motions of the heavenly bodies advances greatly. The grounds on which the solution of the problem of the principles will stand have become substantially firmer. This is, however, the time also of the growth of the schools, and the great growth of ethics. What is the best for you is the central problem of philosophy now. The solution of this problem is still from the solution of that problem, in Epicurus, in Zeno of Citium, ethics is still grounded, or presumed grounded, in physics.

The first four of the following seventeen centuries are thus a time of great deceleration of the acquisition of knowledge, in general, and of the advance in the problem of the principles, in particular. Increasingly infrequent, the advance of mathematics, physics, and the theory of the motions of the heavenly bodies culminates at the end of this time in Ptolemy. The grounds on which the solution of the problem of the principles will stand are yet firmer. Philosophy, however, has spread far and wide, and not mathemat-

ics and physics, rhetoric and grammar have spread far and wide, and ethics has spread very far and very wide. Varro shows that 288 schools on the supreme good are possible. And all too many of them are actual.

In the following five centuries there is thus no advance at all. Early in these centuries all schools, actual and possible, as well as impossible, are reduced to one. The best for you is the trinity. And Origen now assimilates that problem into this, the principle of all things into what is good for you. God is not just the first cause, he is his son, your salvation. Moreover, this alone which in no way disagrees with the church is to be believed as the truth. This is now the principle of all principles, the mother of the father. And in the next two centuries it all becomes orthodoxy.

The next eight centuries are thus dominated by god and his only begotten son, then also god and his messenger. Yet, slowly, haltingly, the advance resumes. The elements are investigated. Abelard shows that the principle of all principles is the principle of the father, and even of the mother. Motion is investigated. And the last of these centuries is the century of prince Henry, and then of Copernicus.

In the following four centuries the advance is thus again very great. The advance of mathematics and physics accelerates, and more, and more. In the first two of these centuries the development of the problem culminates in Newton and Leibniz. And in the next two centuries, the elements are discovered, that gravity is a cause of all things every observation and every calculation shows, the convertibility of motion, electricity, magnetism, light, and heat into one another is shown, the conservation of energy is discovered, the mathematical theory of the formal principles is formulated.

The council is one and only one. And the council is one and only one more resolutely and unwaveringly in the development of this problem, the problem of the principles, no less, of all things, no less, than in the development of any other problem, of the earth, or of the soul, or of the triangle.

Water is the principle, and water Lavoisier shows is not a simple substance, an element strictly. The first mover is the principle, the first

mover everyone understands to be god, and to treat of god from the phenomena pertains to natural philosophy. The principle of all principles, such as Parmenides discovers it, is, both, the principle of all analytic judgments and $x(1-x) = 0$, having in the meantime shown itself more omnipotent than the father, and even the mother.

And the whole thing is done by machinery. The philosophers all do at first as the first philosopher does. Then they all do what is best for you. Then they all become lovers of god, and fathers of the church. At length they all become doctors of theology. Then they all become lovers of truth again.

When spirit is restored to the substantiality and purity of its being, too, they will all become, including the lovers of truth, lovers of spirit.

In general, then, over all the twenty-four and a half centuries, the problem is discovered, and solved in all its three parts, but not on firm grounds, the observations are few, and the chains of inferences are very far from being free of gaps. As the grounds begin being gathered, however, the problem is assimilated into ethics, and so it long remains. Then the grounds on which the solution will stand are established very firmly, the observations become very many, the chains of inferences become very substantially free of gaps, and in two of its parts the problem is solved very nearly conclusively, in its third part there is at long last very great advance.

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Finally, conclusions can be drawn from all the five problems whose development has been collected.

Collected have been the development of the problem of the earth, its position, whether it is at rest or in motion, and its shape, from its remotest beginning through Bessel. The development of the problem of the soul, and both, what it is, and what you should do about it, the physical problem of the soul, and the ethical problem of the soul, from its remotest beginning through Darwin. The development of the problem of the best state, and both, the well-governed state, and the one conceived in speech, the historical problem of the best state, and the political problem of the best

state, in the four-odd centuries from Tiglath-pileser III through Alexander. The development of the problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, from its remotest beginning through Riemann. And the development of the problem of the principles of all things, and all three, the elements of all things, the causes of all things, and the formal principles of all things, from its remotest beginning through Clausius.

And to begin with, from the development of the ethical problem of the soul, what you should do about it, no steps of advance in the acquisition of knowledge have been collected. The development of this problem has not been a part of the acquisition of knowledge. From the development of the political problem of the best state, the one conceived in speech, no steps of advance in the acquisition of knowledge have been collected either. The development of this problem has not been a part of the acquisition of knowledge either. From the development of the historical problem, the well-governed state, 37 steps of advance in the acquisition of knowledge have been collected. Four-odd centuries, however, are only a part of the development of this problem, four-odd out of the fifty-odd since Narmer.

Ignoring here the well-governed state, then, this is a tabulation of all steps of advance in the acquisition of knowledge collected from the four problems, the earth, the soul, the triangle, and the principles, counting centuries from the year of birth assigned to the temple register from Lahun,

	earth	soul	triangle	principles
1911-1812	1	—	1	—,
1811-1712	—	—	2	—,
1711-1612	1	—	1	—,
1611-1512	—	—	—	—,
1511-1412	—	—	—	—,
1411-1312	—	—	—	—,
1311-1212	—	—	—	—,
1211-1112	—	—	—	—,
1111-1012	—	—	—	—,
1011- 912	—	—	—	—,
911- 812	—	—	—	—,
811- 712	3	—	—	—,
711- 612	2	—	—	1,
611- 512	4	3	2	4,
511- 412	6	3	1	6,

411- 312	8	5	3	6,
311- 212	2	—	—	4,
211- 112	1	—	1	1,
111- 12	—	—	—	—,
11 ^{BC} - 89 ^{AD}	2	—	1	2,
90- 189	—	1	—	—,
190- 289	—	—	—	—,
290- 389	—	—	—	—,
390- 489	—	—	1	—,
490- 589	—	—	—	—,
590- 689	—	—	—	—,
690- 789	—	—	—	1,
790- 889	—	—	—	1,
890- 989	—	—	1	1,
990-1089	—	—	1	—,
1090-1189	—	—	—	—,
1190-1289	—	1	—	2,
1290-1389	—	—	—	2,
1390-1489	2	—	—	2,
1490-1589	4	5	—	11,
1590-1689	9	8	6	16,
1690-1789	10	7	4	26,
1790-1889	—	3	3	8.

The count in the last line is not complete. The count in the earlier lines should be complete.

And the first conclusion has to be: the times favorable to the advancement of knowledge have been short.

Ignoring the last line, there are 148 entries in the tabulation. 97 of the 148 are nothing. Counting only the centuries after the beginning of philosophy, 611^{BC} through 1789^{AD}, the entries are 96. Of these 53 are nothing.

The steps of advance in the acquisition of knowledge collected from the development of the problem of the earth are 55, from that of the soul 36, that of the triangle 28, that of the principles 94. The total is 213. Subtracting those in the last line, the total is 199. And dividing the thirty-seven centuries from 1911^{BC} through 1789^{AD} into four parts, of thirteen centuries, three centuries, seventeen centuries, and three centuries, respectively, this is the distribution of the 199 steps among these parts, and the average per century in each part,

first thirteen	12	0.9,
next three	51	17.0,
next seventeen	30	1.7,
next three	106	35.3.

It is a fundamental fact, the times favorable to the advancement of knowledge have been short.

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The problem of knowledge divides into two parts, the acquisition of knowledge and knowledge.

The parts of the problem of the acquisition of knowledge are the beginning of the acquisition, the collection of the acquisition in its entirety, the relation of the acquisition and the development of philosophy, and the kinds of the acquisition.

The beginning of the acquisition of knowledge.

The problem of the earth the first philosopher first solved. Thales said that the earth floats on water, like wood.

But the problem is not *ex nihilo*. From the first philosopher through the big bang theory the problem is formulated and solved on the ground of observation, the observation of the heavenly bodies, in particular, and before the first philosopher the problem is from observation, the observation of the heavenly bodies, in particular.

Observation, and of the heavenly bodies, in particular, begins with the temple register from Lahun, in the valley of the Nile, and the Venus tablet of Ammisaduqa, in the valley of the two rivers, thirteen centuries and ten and a half centuries, respectively, before the first philosopher.

In the valley of the Nile, there is the first observation, of Sirius. Then there is three-odd centuries later another observation of Sirius. Then there is nothing.

In the valley of the two rivers, there is the discovery of Venus. Then there is for nine centuries nothing. Then there is the observation of a series of lunar eclipses, the accurate observation of another series of lunar eclipses, the observation of another series of lunar eclipses, the observation of three dozen constellations and stars, the observation of stars again, the observation of Saturn over 16 years, another accurate observation of a lunar eclipse, the observation of Venus again over 2 years.

And then there is the first solution of the problem of the earth.

Another development predating Thales and involving the earth is that of the accounts of

creation. These are three. In the first, when on high the heaven had no name, down below the earth had no name. Then Marduk split Tiamat in two, like a fish, and set up one of her halves to be heaven. In the second, when god made the earth and the heaven, there was no shrub and no grass on the earth. Then god made man from the dust of the earth. In the third, first of all Chaos came into being. Then big-bosomed Earth bore Heaven, then she lay with Heaven and bore Ocean, eleven other titans, three cyclopes, and three one-hundred-handed creatures.

In the accounts of creation no proposition has been won.

In the observation of the heavenly bodies many propositions have been won, brought in connection with one another in chains of inferences, increasingly free of gaps.

In the accounts of creation there is much thought and sense, saturated with ideas and feelings.

In the observation of the heavenly bodies there is the reference and the true.

The relation of the accounts of creation to the problem of the earth is remote.

The relation of the observation of the heavenly bodies to the problem is causal.

Another development predating Thales and involving the earth is that of the description of its surface.

The description of the surface of the earth can be traced back to Ur-Nanshe, nineteen and a half centuries before Thales. The inscriptions of Ur-Nanshe attest 4 localities. The expeditions of the princes continue, and the collection of localities grows. In the one and a half centuries before Thales the expeditions of the Assyrian princes reach from the Persian Gulf to Asia Minor to the lower valley of the Nile, and so does the largest collection of localities. In the century after Thales the expeditions of the great kings reach from the northern shores of the Black Sea to the Indus to the southern shores of the Red Sea to the gulf of Sidra. In the same century the second philosopher draws the first map of the earth, the seventh philosopher solves the problem of distance and orientation.

In the collection of localities, and hence the description of the surface of the earth, by the

princes many propositions have been won, brought in connection with one another in chains of inferences, increasingly free of gaps.

The relation of the description of the surface of the earth to the problem of the earth is not remote.

Observation is from mathematics. In the one valley the first observation is contemporary with, in the other valley it succeeds, great advance in mathematics. The relation of the development of mathematics to the beginning of observation is causal, too.

The description of the surface of the earth is from the princes.

The relation of the description of the surface of the earth to observation is not remote either. The relation of the description of steadily more localities across a steadily larger portion of the earth to the beginning of the observation of the heavenly bodies cannot be remote.

On the evidence of the problem of the earth, then, the problem of the beginning of the acquisition of knowledge is of the beginning of mathematics and the beginning of the labors of the princes.

In particular, mathematics begins with the 3 oldest calculations of area. The first prince is Narmer. The problem of the beginning of the acquisition of knowledge is, if not *ex nihilo*, what are the 3 oldest calculations of area and Narmer from, each singly, and both jointly.

The problem is also fundamentally of the differential development in the two valleys.

In the valley of the Nile, the first prince is Narmer, prince *c.* 3000. Mathematics cannot be traced back further than the Palermo stone, under Neferirkare, prince *c.* 2440. The remotest beginning of mathematics postdates the first prince by five and a half centuries.

In the valley of the two rivers, the first prince is Enmebaragesi, prince *c.* 2680. The 3 oldest calculations of area are from the century around *c.* 3080. The first prince postdates the beginning of mathematics by four centuries.

In the valley of the Nile, the labors of the princes accumulate slowly and less. The expansion of the state begins, and the expeditions of the princes begin reaching further afield, with Senwosret, prince *c.* 1976-32. The expansion of the

state reaches a peak with Amenhotep III, prince *c.* 1402-364, then stops. Mathematics attains a high peak with the book of 87 problems, under Amenemhet III, prince *c.* 1845-00. There are two other mathematicians in the following century, and then there is nothing. And there are but the two observations of Sirius.

In the valley of the two rivers, the labors of the princes accumulate rapidly and more. The expansion of the state, and the collection of localities, begins with Ur-Nanshe, prince *c.* 2535, one and a half centuries after the first prince, and his successors, and it accelerates very greatly with Sargon, prince *c.* 2337-281, and his successors. With Tiglath-pileser III, prince 745-27, and his successors, the expansion of the state, and the collection of localities, accelerates more than ever. Mathematics develops slowly, but more steadily. The first of the three brightest periods in the development of mathematics, part of which the Venus tablet of Ammisaduqa is, attests two dozen mathematicians. In the following nine centuries there is yet the first map, the map of Nippur. Then observation resumes, in 747, and, under the Assyrian princes, becomes sustained, becomes accurate, reaches dozens of the heavenly bodies.

The first solution of the problem of the earth follows, and the beginning of philosophy.

The problem of the soul the third philosopher first solved. Anaximenes said that the soul is air.

This problem is not *ex nihilo* either. And this problem, the physical problem of the soul, what it is, is from the development of philosophy.

Thales said that water is the principle. Anaximenes said that air is prior to water and it is the principle, and the soul is air and it holds us together, as air encloses the world and holds it together.

There is no earlier development which may lead to the problem.

From the third philosopher through the theory of evolution the problem is formulated and solved on the ground of the investigation of animals, the investigation of plants, and theoretical medicine. And the investigation of animals does not begin before Alcmaeon, two generations after Anaximenes, theoretical medicine begins with Alcmaeon, and the investigation of plants does not begin

before Empedocles, one generation later still. The beginnings of all three are small, too.

There are animals, plants, ailments and remedies in the earlier record. There is nothing which may suggest their examination, let alone observation, let alone investigation. No proposition has been won.

This problem, the physical problem of the soul, is all from the development of philosophy. It thus does not inform the problem of the beginning of the acquisition of knowledge directly.

The ethical problem of the soul, however, does. It is not a part of the acquisition of knowledge. It also begins with Plato, two centuries into the development of philosophy. Nevertheless, there are two earlier developments which lead to the problem, and one of these informs the problem of the beginning of the acquisition.

Have you realized that the soul is immortal? Are you sure? I would be wrong not to be, and so would you. You should consider the nature of the soul, you should conclude that the soul is immortal, you should believe that your immortal soul is yet altered by the life you choose, and you should choose accordingly, so that you avoid being flayed and lacerated on thorn bushes, earn your rewards, and here and in the hereafter be happy.

Both ethics, in general, and the ethical problem of the soul, in particular, are from the development of philosophy.

Ethics begins one and a half centuries into the development of philosophy. Socrates realizes that Anaxagoras has nothing to teach him, begins to study the causes of things by himself, and discovers that earth and air and other such things are not the true causes of things, the true cause of things is what is for the best.

And in the next generation Plato teaches you, in general, the most important thing to learn is the form of the good, if we do not know this, even the most perfect knowledge of anything else is of no use to us. In particular, have you realized that the soul is immortal?

Neither ethics, in general, nor the ethical problem of the soul, in particular, is a part of the acquisition of knowledge.

One of the two earlier developments leading to the problem is the immortal soul of man.

It enters the record with Unas, prince *c.* 2320,

nineteen and a half centuries before Plato, pervades the subsequent Egyptian record, enters philosophy with the fourth philosopher, the philosophers between Pythagoras and Plato mostly ignore it, and becomes your immortal soul with Plato. After Plato, the philosophers continue mostly to ignore the immortal soul of man, yours or otherwise, but then it enters belatedly the latest Hebrew writings, the early fathers of the church rediscover that it is, indeed, your immortal soul, and so it has since remained, with minor variations, and more minor hesitations, your immortal soul.

The immortal soul of man is not a part of the acquisition of knowledge either. No proposition has been won.

The other earlier development is the generalization of the tribe and the moral sense with the expansion of the state.

Ethics teaches you what is the best for you, all of you.

And the generalization of the tribe and the moral sense extensively, with the mixing of peoples under the expanding state, and intensively, with the development of the law under the expanding state, is a development which leads to ethics, in general, and to the ethical problem of the soul, in particular.

Counting back to the first prince, this development has continued for twenty-six centuries by Plato's time.

This development accelerates greatly in the valley of the two rivers.

The great expansion of the state, and attendant mixing of peoples extensively, begins with Sargon, prince *c.* 2337-281, and his successors, who maintain the entire valley under one state for one and a half centuries. The oldest code of laws is the code of Ur-Nammu, prince *c.* 2121-03, and the mixing of peoples intensively begins with Ur-Nammu and his successors, who maintain the valley under one state, and one code of laws, for one century. Under Hammurabi, prince *c.* 1792-49, and his successors the valley is again under one state, and one code of laws, for two centuries. In the two and a half centuries preceding the first philosophers the development accelerates very greatly. The Assyrian princes begin campaigning again over the entire valley with Adad-nirari,

prince c. 1305-273, and again, and again, and until Tiglath-pileser III, prince 745-27, and his successors build the greatest state ever. One tenth of humanity is mixed extensively, intensively, vigorously, and increasingly vigorously for two and a half centuries. By the time of Plato the great kings have built a greater state yet, and mixed one fifth of humanity extensively, intensively, vigorously, very vigorously, and for one and a half centuries.

Ethics is not a part of the acquisition of knowledge.

But the generalization of the tribe and the moral sense, by being a generalization, of the tribe into humanity, and of the moral sense into the law, is a part of the acquisition of knowledge. Propositions have been won.

Ethics, in general, and the ethical problem of the soul, in particular, is a development against the acquisition of knowledge. But it is from the acquisition of knowledge. Earth and air and other such things are not the true causes of things, the true cause of things is what is for the best.

The generalization of the tribe and the moral sense is a development which leads to the development of ethics and the ethical problem of the soul, such as this later development is against the acquisition of knowledge, but from the acquisition of knowledge.

The generalization of the tribe and the moral sense is also from the princes.

The generalization of the tribe and the moral sense and the advance in the description of the surface of the earth, with the expansion of the state both, are a joint development.

And this corroborates the conclusion drawn from the development of the problem of the earth. The problem of the beginning of the acquisition of knowledge is of the beginning of mathematics and the beginning of the labors of the princes.

From the development of the political problem of the best state, the one conceived in speech, the first three solutions have been collected. These are the first three of the statues which the philosophers have continued to bring to the contest of the athletes. These are not a part of the acquisition of knowledge. No proposition has been won.

The contest of the athletes is a part of the acquisition of knowledge. The development of

the historical problem of the best state, of the well-governed state, is a part of the acquisition. Propositions have been won.

Only the particulars and the states to which they point in the four-odd centuries from Tiglath-pileser III through Alexander have been collected. The state has developed for twenty-three and a half centuries since Alexander. And it begins twenty-two and a half centuries before Tiglath-pileser III. The collection thus does not inform the problem of the beginning of the acquisition of knowledge directly.

Nevertheless, the collection shows that the advance in the development of the state in these four-odd centuries is a part of the acquisition of knowledge and a principal cause of the acquisition of knowledge. The advance in the development of the state, in general, is. And the attendant advance in the development of warfare, description of the surface of the earth, generalization of the tribe and the moral sense, description of the past, time reckoning, are all inseparable from the labors of the princes.

The development of this problem, of the well-governed state, in these four-odd centuries thus lends strong, if indirect, support to the conclusion drawn from the development of the problems of the earth and of the soul. The problem of the beginning of the acquisition of knowledge is of the beginning of mathematics and the beginning of the labors of the princes.

The problem of the sum of the angles of the triangle, and the assumptions on which the sum depends, begins with the discovery of the triangle, in the book of 87 problems, in the valley of the Nile, and one century later, in the 14 problems, beginning with a triangle divided into 5 areas, in the valley of the two rivers.

This problem is from the earlier development of mathematics.

It thus corroborates the conclusion drawn from the development of the problems of the earth, the soul, and the best state. The problem of the beginning of the acquisition of knowledge is of the beginning of mathematics and the beginning of the labors of the princes.

And if it corroborates the first part of the conclusion, of the beginning of mathematics, explicitly, it corroborates the second part as well,

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of the beginning of the labors of the princes, implicitly. The problem is of the beginning of each of these singly. And of both jointly.

In the valley of the Nile mathematics begins with the book of 87 problems.

It is a very massive advance.

The book of 87 problems is in this way like Eudoxus, Euclid, Ptolemy, Galileo, Descartes, Newton, Leibniz, Euler, Gauss. The whole thing is done by machinery, and there have been those predecessors of the philosophers and philosophers who have singlehandedly brought the operation of the machinery to perfection.

Still, the book of 87 problems is not *ex nihilo*. And there is little in the valley of the Nile predating the book and indicating development of mathematics. But there is. The book of 87 problems was composed under Amenemhet III, prince c. 1845-00. One century earlier, there is, under Senwosret, prince c. 1976-32, the account of work on a building, including dimensions of chambers and wages of workers. This implies calculation. Five centuries earlier, there is the Palermo stone, under Neferirkare, prince c. 2440, recording the level of the high Nile. This is the first measurement. In the previous century there are the greatest pyramids, of Khufu, prince c. 2580, and of Khafre, prince c. 2540. The first pyramid is from the previous century, of Djoser, prince c. 2680. The princes begin leaving behind their tombs not later than the third prince, Djer, prince c. 2960, three centuries earlier. More remotely, these also imply calculation.

And the book of 87 problems follows not only the eleven and a half centuries of accumulation of the labors of the princes of building the state in the valley of the Nile. It follows the two centuries beginning with Mentuhotep, prince c. 2069-18, when at long last expansion of the state in this valley as well begins, and the expeditions of the princes begin reaching further afield. And Sirius is observed.

It can be concluded that the relation of the accumulation of the labors of the princes of building the state to the development of mathematics is not remote.

And the same conclusion can be drawn from the other valley.

The 14 problems, beginning with a triangle

divided into 5 areas, is one of the two dozen mathematicians whose labors are the first of the three brightest periods in the development of mathematics.

The first is in this way, being the brightest, like the second and the third.

It is the time of Hammurabi, prince c. 1792-49, and his successors.

In this valley the earlier development of mathematics is attested clearly and distinctly. There is the first mathematician. Five centuries later there are the next two mathematicians. Not quite four centuries later there are the next two mathematicians. And three and a half centuries later the first of the three brightest periods in the development of mathematics begins.

Just as clearly and distinctly that the earlier development of mathematics is not too vigorous is thus attested, too. Five mathematicians in twelve and a half centuries. And this is attested more clearly and distinctly still by the fact that there are a few more tablets copying these tablets.

And in this as well, that it is not too vigorous, the development of mathematics in its first twelve and a half centuries is like the development of mathematics in all too many of its later centuries.

In this valley the first mathematician predates the first prince, and by four centuries.

After the first prince, however, the labors of the princes accumulate in this valley rapidly and greatly. The state expands greatly, and again, and again. The law is codified, and again, and again, and again. The princes leave behind not their tombs, but their inscriptions, and ever more of them. They count the burial mounds they have made for their foe. They count the battles they have won. They count the thousands eating bread daily before them. They count the tens of thousands destroyed and tens of thousands captured. They collect the localities on the surface of the earth. They solve the problem of time reckoning. The first prince list, collecting the princes and their reign years, totaling the reign years, and totaling the totals, is composed. More prince lists are composed. The first chronicle, recording the deeds of the princes, is composed. More chronicles are composed.

The first of the three brightest periods in the development of mathematics follows the eight

and a half centuries of accumulation of the labors of the princes of building the state in this valley. It follows the five centuries of great expansion, and of accumulation of localities. It follows the three centuries of accumulation of codes of law. It is contemporary with the greatest intensive generalization of the tribe and the moral sense.

The relation of the accumulation of the labors of the princes of building the state to the development of mathematics cannot be remote. The problem is to make this relation explicit.

Finally, the problem of the principles of all things as well the first philosopher first solved. Thales said that water is the principle.

And this problem is also from the princes.

The convergence of the two developments, the advance in the description of the surface of the earth and the advance in the observation of the heavenly bodies, in the first philosopher is the beginning of the problem of the earth.

Hesiod asks the question: how did gods and the earth first come to be, and rivers, and the sea, and the stars, and the heaven? And answers it. First of all Chaos came to be. Earth is the foundation of all immortals. Counting only the named, the immortals are 335. They are a single chain of generation, and beginning with Earth uniformly by procreation. All connected in a chain of inferences. All by enumerations so complete as to omit nothing. And the king of mortals and immortals, the father of gods and men, is also a descendant of Earth. And he won the kingship from his father, as his father before him won the kingship from his father.

The convergence of the three developments, the advance in the description of the surface of the earth, the advance in the observation of the heavenly bodies, and Hesiod's question and answer, in the first philosopher is the beginning of the problem of the principles of all things.

This explains also why philosophy begins with both problems, of the earth and of the principles of all things. The convergence of the three developments in the first philosopher is the beginning of philosophy.

The advance in the description of the surface of the earth passes through Homer. Homer enumerates 241 localities on the surface of the earth, not less than 200 of them real, distributed from

Cephalonia to the Parthenius to Crete, across 7° of latitude, 800 km along the great circle, and 12° of longitude, 1 100 km along the great circle. It then passes through Hesiod. Hesiod enumerates 25 rivers, 23 of them real, distributed from the Danube to the Rioni to the Nile to the Achelous, across 14° of latitude, 1 600 km along the great circle, 20° of longitude, 1 800 km along the great circle.

The advance in the description of the surface of the earth is from the princes.

Hesiod's question and answer are also from the princes, and twofold.

First, the princes have supplied the evidence. From the first prince, who holds an upraised mace in his right hand ready to smite his kneeling foe whose forelock he holds in his left hand, through Hesiod's contemporary: the king of Ashur, the king of the earth, the king of the four quarters, the guardian of the right, the lover of justice, who aids the weak, whose deeds are pious, mighty warrior, first among all princes, fire consuming the disobedient, thunderbolt striking the evil, to whom Ashur gave no rival from the sea of the setting sun to the sea of the rising sun, and brought all peoples in submission to his feet.

Second, Hesiod is also from *When on high*, and *When on high* is also from the princes, also a generalization over the evidence supplied by the princes. The destiny of the greatest of the great gods is unrivaled, his command is supreme, his command cannot be changed, his command shall be truth, his word shall never be wrong, no god shall ever go beyond the limits he sets.

Hesiod predates the first philosopher by one century-odd.

In this one-century-odd observation resumes, becomes sustained, becomes accurate, reaches dozens of the heavenly bodies.

These are the dates of the Assyrian princes,
 785 - 727 Tiglath-pileser III,
 765 - 722 Shalmaneser V,
 760 - 705 Sargon II,
 740 - 680 Sennacherib,
 715 - 668 Esarhaddon,
 690 - 627 Ashurbanipal,

These are the dates of the poets,

764 Homer,
 734 Hesiod.

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These are the dates of the astronomers,
787 observation of lunar eclipses
beginning in 747,
761 observation of lunar eclipses from
721 to 720,
743 observation of lunar eclipses
beginning in 703,
727 *The plough,*
717 *When Anu and Enlil,*
687 observation of Saturn from 647 to
633,
661 observation of a lunar eclipse in
621,
627 observation of Venus from 587 to
586.

And this is the date of the first philosopher,
625 Thales.
Observation is from mathematics.
The relation of the accumulation of the labors
of the princes of building the state to the develop-
ment of mathematics cannot be remote.

And the relation of the description of steadily
more localities across a steadily larger portion of
the earth to the development of the observation of
the heavenly bodies cannot be remote.

The problem of the beginning of the acquisi-
tion of knowledge is of the beginning of mathe-
matics and the beginning of the labors of the
princes, singly, and jointly.

And this is either the whole problem, or a part
of the problem.

Given the generality of the five problems from
which the conclusion is drawn, it has to be rather
more likely that it is the whole problem than a
part of it, and if it is a part, much more likely that
it is a large part than a small part.

In any case, this has to be established. And
there is but one way to establish it, to solve
simultaneously the problem of the collection of
the acquisition of knowledge in its entirety.

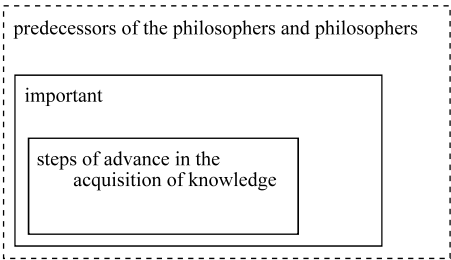
The negative test is always the same, if selec-
tion can produce it, it is not the acquisition of
knowledge. It is the acquisition of knowledge, if
selection cannot produce it.

And the affirmative test is always the same,
the council of our predecessors.

The centuries since the formation of the solar
system are some 46 000 000.
The centuries since the beginning of life on

earth are some 40 000 000.
The centuries since the beginning of man,
counting back to the oldest tools, the Oldowan
tools, are some 26 000.
The centuries since the oldest city, at Çatal
Hüyük, are 85.
The centuries since the first mathematician and
the first prince are 51.
40 000 000 centuries on earth alone indicate
the power of selection.
Çatal Hüyük, on one side, and the first mathe-
matician and the first prince, on the other, indi-
cate the time and place of the solution of the
problem.

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The collection of the acquisition of knowledge
in its entirety.
The ground is this.



The predecessors of the philosophers and philos-
ophers are collected, the important among them,
and the steps of advance in the acquisition of
knowledge among them. Counting all particulars,
and in particular also the princes, predecessors of
the philosophers and philosophers.

All three, who the predecessors of the philos-
ophers and philosophers are, who the important
among them are, and who the steps of advance in
the acquisition of knowledge among them are, the
council of our predecessors decides.

To collect the development of the problem of
the earth through Bessel, some 250 predecessors
of the philosophers and philosophers have been
collected. The important who are part of the
development of the problem through Bessel are
108. The steps of advance in the acquisition of
knowledge are 55.

To collect the development of the problem of
the soul, and both the physical problem and the
ethical problem, through Darwin, some 420

predecessors of the philosophers and philosophers have been collected. The development of the ethical problem has not been a part of the acquisition of knowledge. The important who are part of the development of the physical problem through Darwin are 75. The steps of advance in the acquisition of knowledge are 36.

To collect the development of the problem of the best state, and both the historical problem and the political problem, in the four-odd centuries from Tiglath-pileser III through Alexander some 430 predecessors of the philosophers and philosophers have been collected. The development of the political problem has not been a part of the acquisition of knowledge. The important who are part of the development of the historical problem in the four-odd centuries from Tiglath-pileser III through Alexander are 94. The steps of advance in the acquisition of knowledge are 35.

To collect the development of the problem of the triangle through Riemann, some 240 predecessors of the philosophers and philosophers have been collected. The important who are part of the development of the problem through Riemann are 33. The steps of advance in the acquisition of knowledge are 28.

And to collect the development of the problem of the principles through Clausius, some 660 predecessors of the philosophers and philosophers have been collected. The important who are part of the development of the problem through Clausius are 146. The steps of advance in the acquisition of knowledge are 94.

The overlap between the collections is very substantial. The totals for the joint collection are some 1 500 predecessors of the philosophers and philosophers, 377 important, and 195 steps of advance.

Some of the steps are smaller, others are larger, many are very large. Thales, Anaximenes, Pythagoras, Parmenides, Anaxagoras, Empedocles, Democritus, Eudoxus, Aristotle, Euclid, Archimedes, Hipparchus, Ptolemy, Copernicus, Viète, Bacon, Galileo, Kepler, Descartes, Boyle, Huygens, Newton, Leibniz, Halley, Linnaeus, Euler, Buffon, Kant, Herschel, Lagrange, Lavoisier, Laplace, Ampère, Gauss, Faraday, Carnot, Lobachevsky, Hamilton, Darwin, Helmholtz, Clausius, Riemann are all 1 step each.

On the ground of the generality of the five problems, on the ground of the substantial overlap of the steps of advance between the problems, and on the ground of the magnitude of many of the steps, it can be concluded that the joint collection of the five problems is a very substantial fraction of the acquisition of knowledge in its entirety.

All the predecessors of the philosophers and philosophers that have to be collected should not be more than 15 000. The important among them should not be more than 4 000. The steps of advance among them should not be more than 2 000. And in each case very probably substantially fewer.

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The relation of the acquisition of knowledge and the development of philosophy.

The problem of the earth begins with the first philosopher.

Philosophy is the formulation and solution of problems.

A problem is everything in which there is truth or falsehood.

Ipso facto philosophy is the search for truth.

The formulation and solution of a problem is the winning of propositions, and bringing them in connection with one another in chains of inferences.

Ipso facto the formulation and solution of a problem is a generalization.

Ipso facto philosophy is a vast generalization over the earlier acquisition of knowledge.

Philosophy is the formulation and solution of many problems, and in their dependance on one another, and the accumulation of the formulations and solutions of many problems, and in their dependance on one another.

All of which – that the problem of the earth begins with the first philosopher, that philosophy is the formulation and solution of problems, and of many problems, and in their dependance on one another, and their accumulation, and that it is a vast generalization – the formulation and solution of the problem of the earth by the first, the second, and the third philosophers suffices to show. Thales said that the earth is at rest because it floats on water, like wood. Anaximander said

that it is at rest because of equilibrium, and it is shaped like a drum, discussed the sizes and distances of the planets, drew the first map of the earth, said that the stars originated in a ball of fire, and the earth was wet at first, then began to dry up. Anaximenes said that the earth is at rest because it is flat, it floats on the air below, like a lid, said that air encloses the world and holds it together, the planets are moved by gusts of thick air, and the earth floats on the air.

And of the five problems Thales also first solved the problem of the principles, and Anaximenes improved on the solution of this problem. Anaximenes also first solved the problem of the soul.

Hence – a vast generalization, an accumulation of the formulation and solution of many problems in their dependance on one another – the development of philosophy has been a cause of great acceleration of the formulation and solution of the problem of the earth. As again the first, the second, and the third philosophers suffice to show.

Hence also – a cause of great acceleration – once it has begun the development of philosophy has encompassed the formulation and solution of the problem of the earth in its entirety.

The earth is a sphere. Its radius is 6 400 km. It rotates on its axis and revolves around the sun. It moves toward λ Herculis. Its distance from 61 Cygni is 657 000 times its mean distance from the sun. Light takes 10.3 years to traverse this distance.

None of these results has been obtained, or can have been obtained, outside the one and only one development of the problem – Thales – Parmenides – Eratosthenes – Copernicus – Herschel – Bessel.

Once it has begun the development of philosophy has encompassed the problem in its entirety, and it has been a cause of great acceleration of its formulation and solution.

Thus on one side.

On the other side, the observation of the heavenly bodies predates the first philosopher.

From the first philosopher through the big bang theory the problem of the earth is formulated and solved on the ground of the observation of the heavenly bodies. But the observation of the

heavenly bodies predates the first philosopher by thirteen centuries. It begins with the temple register from Lahun, in the valley of the Nile, and the Venus tablet of Ammisaduqa, in the valley of the two rivers.

Once philosophy has begun the observation of the heavenly bodies is unconditionally a part of the problem of the earth. But the part predates the whole by thirteen centuries.

Moreover, the advance in the description of the surface of the earth leads to the problem. This advance is from the princes. It is a part of the acquisition of knowledge. And it not only predates the development of philosophy more than observation, counting back to Ur-Nanshe, by nineteen and a half centuries. And it not only leads to observation, too. That the advance in the description of the earth is from the princes after the development of philosophy as before prince Henry suffices to show.

Also, the development of philosophy has been a cause of great acceleration of the formulation and solution of the problem, but not a very steady cause of its great acceleration.

It has been a cause of great acceleration from Thales to Hipparchus. And again from Regiomontanus to Bessel.

From Hipparchus to Hypatia there is no acceleration, there is deceleration. From Hypatia to al-Farghani there is nothing. From al-Farghani to Regiomontanus there is not too much either.

In the twenty-four centuries from Thales to Bessel, the development of philosophy has been a cause of great acceleration in four and a half. Then it has not been in sixteen. Then it has been in three and a half.

On one side, the acquisition of knowledge begins long before the development of philosophy. On the other side, once it begins the development of philosophy encompasses the acquisition of knowledge in its entirety. Then again, it does not, the labors of the princes are part of the acquisition of knowledge after the development of philosophy as before.

On one side, the development of philosophy is a cause of great acceleration of the acquisition of knowledge. On the other side, it is not. Then again, it is.

It can be concluded that the relation of the

acquisition of knowledge and the development of philosophy is difficult. And that the problem of their relation is very difficult.

The development of the problem of the soul corroborates amply the conclusion.

The problem of the soul begins with the third philosopher.

The first formulation and solution of the problem already is a vast generalization, and an accumulation of the formulation and solution of many problems in their dependance on one another. The soul is air and it holds us together, as air encloses the world and holds it together, the planets are moved by gusts of thick air, and the earth floats on the air.

This problem is from the development of philosophy and this suffices to show that the development of philosophy has been a cause of great acceleration of its formulation and solution. Air is prior to water and it is the principle, and the soul is air and it holds us together, as air encloses the world and holds it together.

And the development of philosophy has encompassed the formulation and solution of this problem in its entirety as well.

The soul is the principle of life. In anatomy most of all the soul has to be considered. The soul is not the principle of life, there is no other principle than the blood and springs of the body. The question of the fundamental power of the organic body is the question of the fundamental power of the cell. Selection is the principle of life.

None of these results has been obtained, or can have been obtained, outside the one and only one development of the problem – Anaximenes – Aristotle – Vesalius – Descartes – Schwann – Darwin.

Thus on one side.

On the other side, the development of philosophy has been a cause of great acceleration of the formulation and solution of the problem from Anaximenes to Erasistratus. And again from Vesalius to Darwin.

From Erasistratus to Galen there is deceleration. From Galen to al-Razi there is nothing. From al-Razi to Vesalius there is not too much either.

In the twenty-four centuries from Anaximenes

to Darwin, the development of philosophy has been a cause of great acceleration in three. Then it has not been in eighteen and a half. Then it has been in three.

And in this case there is the development of the ethical problem of the soul next to the physical.

In general, earth and air and other such things are not the true causes of things, the true cause of things is what is for the best. In particular, have you realized that the soul is immortal?

Philosophy is the formulation and solution, and accumulation, not just of many problems. One and a half centuries into its development philosophy has become the formulation and solution, and accumulation, of any and all problems. It is not just the search for truth, it has become also the search for action. It is not just physics, it has become also ethics.

The search for action, ethics, and the ethical problem of the soul are from the search for truth, physics, and the physical problem of the soul. Earth and air and other such things are not the true causes of things, the true cause of things is what is for the best. And have you realized that the soul is immortal?

And selection can explain why the search for truth will be subjugated to the search for action, physics to ethics, the solution of the physical problem of the soul to the solution of the ethical. Action, and is it good? or bad? beneficial? or noxious? is as old as organisms, and as powerful.

The more so if the means will be teaching. The transmission of experience, that it is good, or bad, beneficial, or noxious, is as old as organisms capable of the transmission of experience, and as powerful, too.

Ethics is against physics. The solution of the ethical problem of the soul is against the solution of the physical. Socrates and Plato already cannot have been more explicit.

Hence – with Socrates and Plato, the more so with the growth of ethics in the schools, the more so still with the spread of philosophy under the single government of Rome, and very much more so still with the fathers of the church – the development of philosophy has been a cause of great deceleration of the formulation and solution of the physical problem of the soul.

Had the development of philosophy arrested the formulation and solution of the physical problem altogether, the relation would have been simpler. As it is, at length the attempts at the formulation and solution of the problem resumed, and at length accelerated greatly once again.

When physics has dominated the development of philosophy, the development of philosophy has been a cause of acceleration of the acquisition of knowledge. When ethics has dominated the development of philosophy, the development of philosophy has been a cause of deceleration of the acquisition of knowledge.

This is one part of the difficulty.

The development of philosophy is not a cause of great acceleration of the acquisition of knowledge, even if now in operation now not in operation. It is not a cause of great deceleration of the acquisition of knowledge either, even if now in operation now not in operation. It is a cause of both, great acceleration, then great deceleration, then great acceleration again.

And the other part of the difficulty is the graver.

The ethical problem is from the development of philosophy.

Ethics, however, from Socrates and Plato through Hume and Kant and through Berkeley and Hegel and their successors has been built on the generalization of the tribe and the moral sense.

Ethics is not, and cannot be, a part of the acquisition of knowledge. Not if the true cause of things is what is for the best. But the generalization of the tribe and the moral sense is a part of the acquisition of knowledge.

The generalization of the tribe and the moral sense is also from the princes, and predates the development of philosophy, counting back to Sargon, by seventeen and a half centuries.

Nor has the development of philosophy, once ethics has begun, encompassed the generalization of the tribe and the moral sense in its entirety. Beginning with Socrates and Plato, the philosophers cannot have been more explicit in proposing to solve the ethical problem of the soul universally and absolutely. That nevertheless after the development of ethics as before the generalization has continued with the labors of the

princes the success of the Roman consuls and generals in bringing nearly the entire known earth under a single government suffices to show.

The labors of the princes are a part of the acquisition of knowledge after the development of philosophy as before.

On one side, the acquisition of knowledge begins before the development of philosophy. On the other, once it begins the development of philosophy encompasses the acquisition in its entirety. Then again, the labors of the princes are also a part of the acquisition.

On one side, the development of philosophy is a cause of great acceleration of the acquisition. On the other, it is a cause of great deceleration of the acquisition. Then again, it is again a cause of great acceleration.

The other three problems corroborate the conclusion. The relation of the acquisition of knowledge and the development of philosophy is difficult, and the problem of their relation is very difficult.

They show also the essential character of this problem. They show that to formulate and solve the problem of the acquisition of knowledge, and hence the problem of knowledge, too, it is essential to resolve the difficulty in the relation of the acquisition of knowledge and the development of philosophy. The difficulty that the acquisition has been, both, not a part of the development of philosophy and a part of the development of philosophy. The development of philosophy, for its part, has been a cause of, both, great acceleration of the acquisition and great deceleration of the acquisition.

The problem of the best state begins with Plato, two centuries into the development of philosophy. Plato first brings a statue to the contest of the athletes.

The contest of the athletes has by that time continued for twenty-six centuries. And the contest of the statues has not been a part of the acquisition of knowledge, but the contest of the athletes has been.

Only four-odd centuries of the development of the problem of the well-governed state and of the state conceived in speech have been collected. And in these four-odd centuries the development of philosophy has not been a cause of either

acceleration or deceleration of the formulation and solution of the problem of the well-governed state. The development of philosophy has been altogether irrelevant.

The formulation and solution of the problem of the well-governed state, however, shows rather spectacularly that the acquisition of knowledge has been, both, not a part and a part of the development of philosophy. On one side, the contest of the athletes has continued, paying no heed to the statues. On the other, the contest of the athletes has been essentially and substantially the ground of the development of history, Herodotus, Thucydides, Polybius, Livy. The contest of the athletes has been essentially and substantially the ground also of the contest of the statues. The first statue already informs the athletes: until the philosophers become kings or the kings become philosophers, the states will have no respite from evil. And so the statues have continued to inform the athletes.

The problem of the triangle begins with the book of 87 problems, in the valley of the Nile, twelve and a half centuries before the first philosopher. In the valley of the two rivers, from the 14 problems, beginning with a triangle divided into 5 areas, to the 18 problems, including an upright beam of length 30, the formulation and solution of the problem advances very greatly. These are the two centuries of the first of the three brightest periods in the development of mathematics. The first of the three brightest periods in the development of mathematics begins and ends, respectively, twelve and a half and ten and a half centuries before the development of philosophy.

Mathematics is a part of the acquisition of knowledge, indeed, with Kant, its most valuable province.

Once philosophy begins, mathematics is a part of philosophy. Its most valuable province, too.

The problem of the triangle thus show that the most valuable part of the acquisition of knowledge is, both, not a part of the development of philosophy and a part of it.

That it is a part of the development of philosophy, essentially and substantially a part of it, unconditionally a part of it, and its most valuable province, the subsequent development of the problem of the triangle shows.

The great acceleration of the formulation and solution of this problem occurs simultaneously with the great acceleration of the formulation and solution of the problems of the earth and of the soul.

In particular, it occurs in the two and a half centuries from Pythagoras to Euclid. When the mathematicians are the physicists, the physicists are the mathematicians, the physicists are the philosophers, and the philosophers are the physicists. And only the last part of this proposition, only after Socrates, needs a qualification, and still only a minor qualification.

It occurs again in the two and a half centuries from Desargues to Riemann. When again the mathematicians are the physicists, the physicists are the mathematicians, the physicists are the philosophers, and the philosophers are the physicists. And only the last part of this proposition needs a qualification, and only a minor qualification.

And the great deceleration of the formulation and solution of this problem occurs simultaneously with the great deceleration of the formulation and solution of the problems of the earth and of the soul. In the nineteen and a half centuries from Euclid to Desargues, culminating at the time when there are no mathematicians, physicists or otherwise, no physicists, mathematicians or otherwise, and once the labors of the fathers of the church have been completed, no philosophers at all, mathematicians, or physicists, or otherwise.

The problem of the principles begins with the first philosopher.

The development of philosophy has encompassed the formulation and solution of the problem in its entirety.

Yet the earlier expansion of the state with the labors of the princes, and the attendant advance in the description of the surface of the earth, and through Homer, and Hesiod, leads to this problem as well.

The great acceleration, then deceleration, then again acceleration of the formulation and solution of this problem as well occurs simultaneously with that of the others. In the four and a half centuries from Thales to Hipparchus, the sixteen and a half centuries from Hipparchus to Copernicus, the three and a half centuries from Coperni-

cus to Clausius.

The relation of the acquisition of knowledge and the development of philosophy is difficult.

To resolve the difficulty is essential.

The acquisition of knowledge predates the development of philosophy.

But the development of philosophy is from the earlier acquisition.

The acquisition of knowledge and the development of philosophy are one process. From the 3 oldest calculations of area and Narmer through Darwin, Clausius, and Riemann, and further.

The times favorable to the advancement of knowledge have been short.

But there have been these times.

If selection is not the cause of the acquisition of knowledge, given the power of selection, a very powerful cause has to produce the acquisition of knowledge.

And the development of philosophy – not when it has not encompassed the acquisition in its entirety, but when it has encompassed the acquisition in its entirety, and because, having not encompassed it in its entirety, it has encompassed it in its entirety, and not when it has been a cause of great deceleration of the acquisition, but when it has been a cause of great acceleration of the acquisition, and at long last has again been a cause of great acceleration of the acquisition, and because, having been a cause of its great deceleration, it has been a cause of its great acceleration – has to point to the cause.

If the whole thing is done by machinery, the development of philosophy has to point to the engine.

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The kinds of the acquisition of knowledge.

The genus of the acquisition of knowledge is theory. The search for truth. Uncontaminated by anything that is not the search for truth.

All five problems show this. All five dramatically.

The machinery has advanced against the most powerful vested interests. And never mind the church. Against all the philosophers, millennia's worth of them, who have built philosophy on the church. Foolishly, absurdly, and formally heretically, the earth revolves around the sun. Your

immortal soul will not be rewarded, it will not even be punished, your immortal soul is nothing. The principle of contradiction is more omnipotent than god, more omnipotent even than his mother. Gravity, too – with the strong, the weak, and the electromagnetic forces – created the world.

More dramatically still, the machinery has never advanced where the philosophers, millennia's worth of them, will have it advance. There is no medicine which would remove all corruptions of baser metals so that they become the purest gold and remove the corruptions of the human body and prolong life for many ages. Logic is not rhetoric, not even grammar, it is algebra. The true cause of things is not what is for the best, earth and air and other such things are the true causes of things. Nor is the best state a statue, evil as it is, the well-governed state is the best state.

More dramatically still, the machinery has advanced against the best theoretical intentions of the philosophers, millennia's worth of them, too. Euclid has been freed from all blemish into non-Euclidean geometry. As it has advanced against the most practical intentions of the princes, millennia's worth of these, too. The generalization of the tribe and the moral sense is the outcome of the development of warfare.

From the thought and the sense to the reference and the true, not once has the reference and the true yielded to the thought and the sense.

The genus is theory.

The species is physics. And it is the only species. Several as the species of the species are.

This as well the five problems show.

Aristotle writes in the *Physics*: physics is the investigation of nature.

Maxwell writes: physics is that department of knowledge which relates to the order of nature, or in other words, to the regular succession of events (*Matter*, 1).

Aristotle writes in the *Physics*: as the objects of investigation have principles or causes or elements, it is when we find these that we have knowledge; for we do not know what a thing is until we find its first causes and first principles and reach its elements; in physics, therefore, our task is to find the principles.

Planck writes: as long as there is a physical

science its highest goal is the solution of the problem of embracing all observed and still to be observed natural phenomena in one simple principle from which all past and especially future occurrences can be calculated.

Physics is the investigation of nature. It is knowledge of the order of nature. Its task is to find the first principles. Its goal is to embrace all natural phenomena in one simple principle.

Thus, Thales through quantum theory, the two parts of the problem of the principles, the problem of the elements and the problem of the causes, have been physics.

Mechanics, dynamics, chemistry, thermodynamics, electrodynamics are parts of physics.

Thales through the big bang theory, the problem of the earth has been physics.

The observation of the heavenly bodies has been a part of the problem of the earth. The description of the surface of the earth has been a part of the problem.

Astronomy and geography are parts of physics.

Anaximenes through the theory of evolution, the problem of the soul has been physics. Aristotle writes: knowledge of the soul contributes greatly to the advance of truth, and most of all physics; the investigation of the soul is a part of physics. Schwann writes: even if the fundamental power of the organic body occurs nowhere at all in inorganic nature, it cannot be essentially different from all other physical and chemical forces.

Biology is a part of physics.

The book of 87 problems through Riemann, the problem of the triangle has been geometry.

With Einstein: geometry is the oldest branch of physics.

More generally, mathematics is a part of physics.

It is the investigation of nature. It is knowledge of the order of nature. Its task is to find the first principles. Its goal is to embrace all natural phenomena in one simple principle.

Pythagoras through Frege, the problem of the formal principles has been mathematics. Aristotle through Frege, it has been logic.

Logic is a part of mathematics. And mathematics is a part of physics.

The laws of number are not laws of nature,

they are laws of the laws of nature.

As to the problem of the best state, it is a historical problem.

Hume writes: the records of wars, intrigues, factions, and revolutions are so many collections of experiments by which the politician or moral philosopher fixes the principles of his science in the same way as the physician or natural philosopher becomes acquainted with the nature of plants, minerals, and other external objects.

Kant writes: whatever concept one may have of the freedom of the will with a metaphysical purpose, its appearances, human actions, are, like all other natural events, determined in accordance with general laws of nature; the philosopher cannot assume that men and their drama have a rational purpose, his only starting point can be to try to discover in the senseless course of human affairs a natural goal.

And Polybius, the one philosopher to concern himself with the problem of the best state after, not instead of, examining the state, writes: the Roman state takes its foundation and growth entirely from natural causes (VI, 9), and: changes in constitutions occur inevitably by nature (10).

History is a part of physics, too.

Thus, if the standard is the performance of the indispensable duty, the examination of all original materials, too, Aristotle, Theophrastus, Euclid, Pliny, Ptolemy, Galen, Copernicus, Gesner, Bacon, Galileo, Kepler, Descartes, Boyle, Huygens, Newton, Leibniz, Halley, Linnaeus, Euler, Buffon, Lambert, Lagrange, Herschel, Lavoisier, Laplace, Cuvier, Ampère, Gauss, Faraday, Lobachevsky, Darwin, Helmholtz, Riemann, are all historians *par excellence*.

Nor can physics, if it is the investigation of nature, knowledge of the order of nature, its task to find the first principles, its goal to embrace all natural phenomena in one simple principle, not be the kind of the acquisition of knowledge, and the only kind of the acquisition of knowledge, of which all other kinds can only be parts.

Nature, or synonymously the world.

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Finally, the problem of knowledge is of the relation of knowledge and truth, and the relation of knowledge and the world.

1332 In order to formulate the problems we are to solve we call the council of our predecessors

In the course of the development of the problem of the earth very many propositions have been won, brought in connection with one another in very many chains of inferences, ever more free of gaps. In the course of the development of the problem of the soul very many propositions have been won, brought in connection with one another in very many chains of inferences, ever more free of gaps. In the course of the development of the state many propositions have been won, if not nearly as many, brought in connection with one another in many chains of inferences, if not nearly as many, or nearly as free of gaps. In the course of the development of the problem of the triangle very many propositions have been won, brought in connection with one another in very many chains of inferences, ever more free of gaps. In the course of the development of the problem of the principles very many propositions have been won, brought in connection with one another in very many chains of inferences, ever more free of gaps. All in connection with one another. True propositions.

The reference of all true propositions is the same, the true.

The relation of the acquisition of knowledge and the winning of propositions can only be identity.

And the relation of knowledge and truth can only be identity.

Knowledge is truth.

Also, in the course of the development of the problem of the earth the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world,

all in relation to one another. The earth revolves around the sun, moves toward λ Herculis, its distance from 61 Cygni 657 000 times its mean distance from the sun. In the course of the development of the problem of the soul the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world, all in relation to one another. All organic beings from the first dawn of life throughout all time and space. In the course of the development of the state the system of the world has been described, the position, order, and motion of the parts of the world, in relation to one another, even if in this case implicitly more than explicitly. The state takes its foundation and growth entirely from natural causes. In the course of the development of the problem of the triangle the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world, all in relation to one another. From the immeasurably small to the immeasurably large. In the course of the development of the problem of the principles the system of the world has been described ever better, the position, order, and motion of ever more of the parts of the world, all in relation to one another. The energy of the world is constant, the entropy of the world strives for a maximum.

The system of the world is the world.

And the relation of the acquisition of knowledge and the description of the world can only be identity.

And the relation of knowledge and the world can only be identity.

Knowledge is truth is the world.

enumeration and review

And below are the particulars collected in parts one, two, and three of the investigation.

important

3120 ^{BC}	3 calculations of area	the century around <i>c.</i> 3080 ^{BC}
3040	Narmer palette	prince <i>c.</i> 3000
3000	Djer	prince <i>c.</i> 2960
2720	Djoser	prince <i>c.</i> 2680
2720	Enmebaragesi inscriptions	prince <i>c.</i> 2680
2720	Imhotep	under Djoser, prince <i>c.</i> 2680
2690	Aka	prince <i>c.</i> 2650, son of Enmebaragesi
2690	Gilgamesh	prince <i>c.</i> 2650
2660	Ur-Nungal	prince <i>c.</i> 2620, son of Gilgamesh
2640	Meskalamdug inscriptions	prince <i>c.</i> 2600
2640	Sneferu	prince <i>c.</i> 2600
2620	Khufu	prince <i>c.</i> 2580, son of Sneferu
2620	division of grain among 164 571 men	– the century around <i>c.</i> 2580
2620	table of squares with sides from 10×60 to $\frac{1}{12} \times 60$	– the century around <i>c.</i> 2580
2610	Mesanepada inscriptions	prince <i>c.</i> 2570, son of Meskalamdug
2595	Mesalim inscriptions	prince <i>c.</i> 2555
2580	Aanepada inscriptions	prince <i>c.</i> 2540, son of Mesanepada
2580	Khafre	prince <i>c.</i> 2540, son of Khufu
2575	Ur-Nanshe inscriptions	prince <i>c.</i> 2535
2550	Meskiagnuna inscriptions	prince <i>c.</i> 2510, son of Aanepada
2545	Akurgal inscriptions	prince <i>c.</i> 2505, son of Ur-Nanshe
2535	Ush	prince <i>c.</i> 2495
2530	Elili inscriptions	prince <i>c.</i> 2490
2515	Eanatum inscriptions	prince <i>c.</i> 2475, son of Akurgal
2515	Enakale	prince <i>c.</i> 2475
2510	Enanatum inscriptions	prince <i>c.</i> 2470, son of Akurgal
2500	Enshakushana inscriptions	prince <i>c.</i> 2460, son of Elili
2485	Ur-Luma inscriptions	prince <i>c.</i> 2445, son of Enakale

2*		enumeration and review
2480	Enmetena inscriptions	prince <i>c.</i> 2440, son of Enanatum
2480	Lugalkignedudu inscriptions	prince <i>c.</i> 2440
2480	Neferirkare	prince <i>c.</i> 2440
2480	Palermo stone	under Neferirkare, prince <i>c.</i> 2440
2460	Puzur-Nirah	prince <i>c.</i> 2420
2455	II inscriptions	prince <i>c.</i> 2415, grandson of Enakale
2450	Enanatum II inscriptions	prince <i>c.</i> 2410, son of Enmetena
2450	Lugalkisalsi inscriptions	prince <i>c.</i> 2410, son of Lugalkignedudu
2440	Ku-Baba	prince <i>c.</i> 2400
2435	Gishakidu inscriptions	prince <i>c.</i> 2395
2430	Enentarzi inscriptions	prince <i>c.</i> 2390
2410	Puzur-Sin	prince <i>c.</i> 2370, son of Ku-Baba
2400	Lugalanda inscriptions	prince <i>c.</i> 2360, son of Enentarzi
2380	Lugalzagesi inscriptions	prince <i>c.</i> 2340
2380	Urukagina inscriptions	prince <i>c.</i> 2340
2380	Ur-Zababa	prince <i>c.</i> 2340, son of Puzur-Sin
2362 - 2281	Sargon inscriptions	prince <i>c.</i> 2337-281
2360	Unas tomb inscription	prince <i>c.</i> 2320
2340	Teti	prince <i>c.</i> 2300
2326 - 2257	Manishtushu inscriptions	prince <i>c.</i> 2272-57, son of Sargon, older brother of Rimush
2321 - 2272	Rimush inscriptions	prince <i>c.</i> 2281-72, son of Sargon
2320	Pepi	prince <i>c.</i> 2280, son of Teti
2300	Merenre	prince <i>c.</i> 2260, son of Pepi
2297 - 2220	Naram-Sin inscriptions	prince <i>c.</i> 2257-20, son of Manishtushu
2280	Pepi II	prince <i>c.</i> 2240, son of Pepi
2260 - 2195	Sharkalisharri inscriptions	prince <i>c.</i> 2220-195, son of Naram-Sin
2246	quadrilateral with sides 2, $1\frac{1}{6}$, 2, and $2\frac{5}{6}$ – the century before <i>c.</i> 2156	
2246	width of a rectangle with length $67\frac{1}{2}$ and area 100 – the century before <i>c.</i> 2156	
2232 - 2171	Dudu inscriptions	prince <i>c.</i> 2192-71
2211 - 2156	Shudurul inscriptions	prince <i>c.</i> 2171-56, son of Dudu
2205	Ur-Baba inscriptions	prince <i>c.</i> 2165
2198 - 2151	Ur-Nigin inscriptions	prince <i>c.</i> 2158-1
2192 - 2150	Laerab	prince <i>c.</i> 2152-0

	inscriptions	
2185	Gudea	prince <i>c.</i> 2145, younger brother of Ur-Baba
	inscriptions	
2185	drawing of the Eninnu	under Gudea, prince <i>c.</i> 2145
2178 - 2131	Yarlagan	prince <i>c.</i> 2138-1
	inscriptions	
2176 - 2145	Ur-Gigir	prince <i>c.</i> 2151-45, son of Ur-Nigin
	inscriptions	
2171 - 2124	Sium	prince <i>c.</i> 2131-24
	inscriptions	
2168 - 2121	Utuhegal	prince <i>c.</i> 2128-1
	inscriptions	
2161 - 2103	Ur-Nammu	prince <i>c.</i> 2121-03
	inscriptions	
	code	
2150	Puzur-Inshushinak	prince <i>c.</i> 2110
	inscriptions	
2128 - 2055	Shulgi	prince <i>c.</i> 2103-055, son of Ur-Nammu
	inscriptions	
2103	Gilgamesh and Aka	the century before <i>c.</i> 2013
2103	list of 15 remedies	the century before <i>c.</i> 2013
2103	Sumerian Gilgamesh	the century before <i>c.</i> 2013
	<i>Gilgamesh and Humbaba</i>	
	<i>Gilgamesh and the bull of heaven</i>	
	<i>Gilgamesh and the netherworld</i>	
	<i>The death of Gilgamesh</i>	
2102 - 2046	Amar-Sin	prince <i>c.</i> 2055-46, son of Shulgi
	inscriptions	
2094 - 2018	Mentuhotep	prince <i>c.</i> 2069-18
2082 - 2037	Shu-Sin	prince <i>c.</i> 2046-37, son of Amar-Sin
	inscriptions	
2077	Sumerian prince list	under Ibbi-Sin, prince <i>c.</i> 2037-13
2077	Ur archive	under Ibbi-Sin
2067 - 1994	Ishbi-Erra	prince <i>c.</i> 2027-1994
2067	Tummal chronicle	under Ishbi-Erra, prince <i>c.</i> 2027-1994
2064 - 2004	Naplanum	prince <i>c.</i> 2024-04
2062 - 2013	Ibbi-Sin	prince <i>c.</i> 2037-13, son of Shu-Sin
	inscriptions	
2046 - 1976	Amenemhet	prince <i>c.</i> 2006-1976
2034 - 1974	Shu-Ilishu	prince <i>c.</i> 1994-74, son of Ishbi-Erra
2020	Puzur-Ashur	prince <i>c.</i> 1980
2016 - 1932	Senwosret	prince <i>c.</i> 1976-32, son of Amenemhet
2016	account of work on a building	including dimensions of chambers – under Senwosret, prince <i>c.</i> 1976-32
2014 - 1953	Idin-Dagan	prince <i>c.</i> 1974-53, son of Shu-Ilishu
2000	Shallim-ahhe	prince <i>c.</i> 1960, son of Puzur-Ashur
	inscriptions	
1993 - 1933	Ishme-Dagan	prince <i>c.</i> 1953-33, son of Idin-Dagan
1980	Ilushuma	prince <i>c.</i> 1940, son of Shallim-ahhe
	inscriptions	
1973 - 1922	Lipit-Ishtar	prince <i>c.</i> 1933-22, son of Ishme-Dagan
	code	
1972 - 1897	Amenemhet II	prince <i>c.</i> 1932-897, son of Senwosret
1965	antediluvian prince list	between the Sumerian prince list and the Isin prince list
1960	Erishu	prince <i>c.</i> 1920, son of Ilushuma
	inscriptions	

1937 - 1878	Senwosret II	prince c. 1897-78, son of Amenemhet II
1920 - 1844	Sumulael	prince c. 1880-44
1920	Esagil chronicle	under Sumulael, prince c. 1880-44
1918 - 1845	Senwosret III	prince c. 1878-45, son of Senwosret II
1911	temple register from Lahun	– observation in 1871 ± 5
1885 - 1800	Amenemhet III	prince c. 1845-00, son of Senwosret III
1885	Kudur-Mabuk	prince c. 1845
1885	book of 87 problems	under Amenemhet III, prince c. 1845-00
1884	24 problems, beginning with the sum of the area and side of a square	– the century before c. 1794
1884	8 problems, beginning with the sum of the area and difference of sides of a rectangle	– the century before c. 1794
1865	Ipiq-Adad	prince c. 1825
1861	Larsa prince list	under Rim-Sin, prince c. 1821-761
1854	Isin prince list	under Damaqilishu, prince c. 1814-791
1849 - 1776	Shamshi-Adad inscriptions	prince c. 1809-776
1846 - 1761	Rim-Sin	prince c. 1821-761, son of Kudur-Mabuk
1839 - 1791	Damaqilishu	prince c. 1814-791
1837 - 1792	Sinmuballit	prince c. 1812-792
1830 - 1786	Sobeknefru	prince c. 1790-86
1826	Moscow papyrus	the century around c. 1786
1826	Berlin papyrus	the century around c. 1786
1820	Dadusha code	prince c. 1785, son of Ipiq-Adad
1817 - 1749	Hammurabi code	prince c. 1792-49, son of Sinmuballit
1789 - 1711	Samsuiluna inscriptions	prince c. 1749-11, son of Hammurabi
1784	286 problems	the century before c. 1694
1784	44 problems, beginning with an excavation in the shape of a prism	– the century before c. 1694
1784	43 problems, including 4 numbers in arithmetic progression whose sum is 140	– the century before c. 1694
1784	15 Pythagorean triples	the century before c. 1694
1784	14 problems, beginning with a triangle divided into 5 areas	– the century before c. 1694
1784	1 problem, a trapezoid divided into 2 areas	– the century before c. 1694
1784	1 problem, a triangle divided into 4 smaller triangles	– the century before c. 1694
1784	1 problem, a triangle divided into 2 areas whose difference is 420	– the century before c. 1694
1784	calculation of the diagonal of the square	– the century before c. 1694
1784	calculation of the area of the circle	– the century before c. 1694
1751 - 1683	Abieshuh	prince c. 1711-683, son of Samsuiluna
1723 - 1646	Ammiditana	prince c. 1683-46, son of Abieshuh
1690	Labarna	prince c. 1650
1686 - 1625	Ammisaduqa	prince c. 1646-25, son of Ammiditana
1686	Venus tablet of Ammisaduqa	– under Ammisaduqa, prince c. 1646-25
1686	<i>When the gods were men</i>	under Ammisaduqa
1684	18 problems, including an upright beam of length 30	– the century before c. 1594
1684	early Akkadian <i>Gilgamesh</i>	the century before c. 1594
1670	Hattusili	prince c. 1630
1665 - 1594	Samsuditana	prince c. 1625-594, son of Ammisaduqa
1665	Babylonian prince list	under Samsuditana, prince c. 1625-594
1634	When on high	the century around c. 1594
1630	Apepi	prince c. 1590
1630	Mursili	prince c. 1610, grandson of Hattusili
1610 - 1545	Ahmoose	prince c. 1570-45
1610	Ashur-nirari inscriptions	prince c. 1570

1590	Puzur-Ashur III inscriptions	prince c. 1550, son of Ashur-nirari
1585 - 1521	Amenhotep	prince c. 1545-21
1576	list of feasts beginning in the 9th year of Amenhotep	– observation in 1536 ± 10
1561 - 1509	Thutmose	prince c. 1521-09
1560	48 treatments of injuries	the century after c. 1570
1550	Telipinu	prince c. 1510
1534 - 1491	Thutmose II	prince c. 1509-491, son of Thutmose
1510	Karnak prince list	under Thutmose III, prince c. 1470-37
1496 - 1437	Thutmose III	child c. 1491 - prince c. 1470-37, son of Thutmose II
1490	map of Nippur	the century before c. 1400
1450 - 1408	Ashur-bel-nisheshu	prince c. 1417-08
1415 - 1363	Eriba-Adad	prince c. 1390-63, son of Ashur-bel-nisheshu
1390 - 1327	Ashur-uballit	prince c. 1363-27, son of Eriba-Adad
1387	Amarna archive	under Smenkhkare, prince c. 1347-4
1375 - 1325	Suppiluliuma	prince c. 1350-25
1372 - 1344	Smenkhkare	prince c. 1347-4
1366 - 1302	Ramesses	prince c. 1304-2, father of Seti
1360 - 1317	Enlil-nirari	prince c. 1327-17, son of Ashur-uballit
1346 - 1291	Seti	prince c. 1302-291, son of Ramesses, father of Ramesses II
1345 - 1300	Mursili II	prince c. 1325-00, son of Suppiluliuma
1342	Abydos prince list	under Seti, prince c. 1302-291
1335 - 1305	Arik-den-ili	prince c. 1317-05, son of Enlil-nirari
1331	Saqqara prince list	under Ramesses II, prince c. 1291-24
1330 - 1273	Adad-nirari inscriptions	prince c. 1305-273, son of Enlil-nirari
1330	<i>Princeship in heaven</i>	the century around c. 1290
1320 - 1245	Hattusili III	prince c. 1275-45, son of Mursili II
1316 - 1224	Ramesses II	prince c. 1291-24, son of Seti
1300 - 1243	Shalmaneser inscriptions	prince c. 1273-43, son of Adad-nirari
1296	Sin-liqe-unninni <i>Gilgamesh</i>	the century before c. 1206
1285 - 1215	Tudhaliya IV	prince c. 1245-15, son of Hattusili III
1281	Turin papyrus	the century after c. 1291
1270 - 1206	Tukulti-Ninurta inscriptions	prince c. 1243-06, son of Shalmaneser
1246 - 1191	Sethakhte	prince c. 1194-1, father of Ramesses III
1216 - 1159	Ramesses III	prince c. 1191-59, son of Sethakhte
1205 - 1133	Ashur-dan	prince 1179-33
1165	Nebuchadnezzar	prince c. 1125
1160 - 1115	Ashur-resh-ishi	prince 1133-15
1140 - 1076	Tiglath-pileser inscriptions	prince 1115-076, son of Ashur-resh-ishi
1095 - 1056	Ashur-bel-kala	prince 1074-56, son of Tiglath-pileser
1028 - 963	David	prince c. 1003-963
1020	Ittobaal	prince c. 980
988 - 935	Hiram	lived 53 years - prince c. 969-35
988 - 923	Solomon	prince c. 963-23, son of David
970 - 912	Ashur-dan II	prince 935-12
968 - 907	Sheshonq	prince c. 928-07
950 - 891	Adad-nirari II	prince 912-891, son of Ashur-dan II
930 - 884	Tukulti-Ninurta II	prince 891-84, son of Adad-nirari II
925 - 845	Hadadezer	prince c. 885-45
921 - 771	Omri	prince c. 881-71

910 - 859	Ashur-nasirpal II inscriptions	prince 884-59, son of Tukulti-Ninurta II
900	Mesha	prince <i>c.</i> 860
896 - 853	Ahab	prince <i>c.</i> 871-53, son of Omri
890 - 824	Shalmaneser III	prince 859-24, son of Ashur-nasirpal II
881	Li	deposed in 841
870 - 811	Shamshi-Adad V	prince 824-11, son of Shalmaneser III
870	Sarduri	prince <i>c.</i> 830
860	Yahwistic document parts of <i>Genesis</i> , <i>Exodus</i> , and <i>Numbers</i>	the century before <i>c.</i> 791/50
851	synchronistic chronicle	under Adad-nirari III, prince 811-783
850 - 783	Adad-nirari III	prince 811-783, son of Shamshi-Adad V
820	Menua	prince <i>c.</i> 790, father of Argishti
810	Amos	prophecy between <i>c.</i> 791 and <i>c.</i> 750
807 - 750	Uzziah	prince at 16 - prince <i>c.</i> 791-50
800	Hosea	prophecy between those of <i>Amos</i> and <i>Isaiah</i>
790	Argishti	prince <i>c.</i> 765, son of Menua
790	Isaiah	prophecy began <i>c.</i> 750
787	observation of lunar eclipses from 747 to 746	
785 - 727	Tiglath-pileser III inscriptions	prince 745-27
769 - 708	Piyi	prince <i>c.</i> 729-08
767	Assyrian prince list	under Shalmaneser V, prince 727-2
766 - 726	Ahaz	prince <i>c.</i> 738-26
765 - 722	Shalmaneser V	prince 727-2, son of Tiglath-pileser III
765	Sarduri II	prince <i>c.</i> 740, son of Argishti
764	Homer <i>Iliad</i> <i>Odyssey</i>	the century before 676/3
763	<i>Micah</i>	prophecy between <i>c.</i> 750 and <i>c.</i> 697
761	observation of lunar eclipses from 721 to 720	
760 - 705	Sargon II inscriptions	prince 722-05, younger son of Tiglath-pileser III
750	Midas	prince <i>c.</i> 710
750	Theopompus	prince <i>c.</i> 710
748 - 693	Shabaka	prince <i>c.</i> 708-693, brother of Piyi
746 - 697	Hezekiah	prince 726-697, son of Ahaz
743	observation of lunar eclipses	beginning in 703
740 - 680	Sennacherib inscriptions	prince 705-680, son of Sargon II
734	Hesiod <i>Theogony</i> <i>Work and days</i>	one generation after Homer
730 - 664	Taharqa	prince 690-64, son of Piyi
727	The plough	colophon in 687
717	<i>When Anu and Enlil</i>	the century before 627
715 - 668	Esarhaddon inscriptions	prince 680-68, son of Sennacherib
709	Pheidon	prince <i>c.</i> 669
705 - 645	Gyges	prince <i>c.</i> 680-45
703	Zaleucus	laws in 663
698 - 628	Cypselus	tyrant <i>c.</i> 658-28
694	Psalms	from the time of the Yahwistic document to that of the Priestly code
690 - 627	Ashurbanipal inscriptions	prince 668-27, younger son of Esarhaddon

689 - 610	Psamtik	prince 664-10
687	observation of Saturn from 647 to 633	
672	Theagenes	tyrant c. 632
667	<i>Jeremiah</i>	prophecy began in 627
666 - 605	Nabopolassar	prince 626-05
662	Deuteronomy	the law promulgated in 622
661	Draco	laws in 621
661	observation of a lunar eclipse	in 621
658 - 588	Periander	tyrant c. 628-588, son of Cypselus
653	Epimenides	purified Athens between c. 632 and 594
	fragments	
650 - 595	Necho	prince 610-595, son of Psamtik
650 - 585	Cyaxares	prince c. 625-585
650	Thrasybulus	tyrant c. 610
650	circumnavigation of Africa under Necho	– Necho, prince 610-595
649 - 578	Pittacus	died in 578 having lived more than 70 years
648 - 609	Josiah	prince at 8 - prince 640-09
640 - 560	Alyattes	prince c. 615-560
634	Solon	archon 594
	fragments	
633	<i>Ezekiel</i>	prophecy began in 593
630 - 562	Nebuchadnezzar	prince 605-562, son of Nabopolassar
627	observation of Venus from 587 to 586	
625 - 550	Astyages	prince c. 585-50, son of Cyaxares
625	Cleisthenes of Sicyon	tyrant c. 585
625	Thales	predicted an eclipse which occurred in 585
622	Anacharsis	between the time of Pittacus and that of Chilon
622	Bias	between Pittacus and Chilon
617 - 526	Amasis	general in 592 - 526
611 - 541	Anaximander	611 - the decade after 546
610	Charondas	laws in the generation around c. 570
601 - 527	Pisistratus	c. 601 - 527
600 - 556	Neriglissar	prince 560-56
600 - 546	Croesus	prince 560-46, son of Alyattes
599	Pherecydes	under Cyrus, prince 559-30
597	Joshua, Judges, Samuel, Kings	– the two generations after 587
596	Chilon	ephor 556
595	<i>Proverbs</i>	between <i>Deuteronomy</i> and the Priestly code
590	Anaximenes	between Anaximander and Xenophanes
590	Malchus	prince c. 550
590	Phalaris	tyrant c. 550
584 - 530	Cyrus	prince 559-30
	cylinder	
580 - 520	Anaxandridas	prince c. 560-20
578 - 522	Polycrates	tyrant c. 538-22
575 - 515	Ariston	prince c. 550-15
570 - 495	Pythagoras	c. 570 - the decade after c. 500
570 - 473	Xenophanes	c. 570 - the decade after 478
	fragments	
566	observation of Jupiter from 526 to 490	
565	Cleisthenes	archon 525
562	Democedes	under Darius, prince 522-486
562	Hecataeus	under Darius
	fragments	
560	<i>Haggai</i>	prophecy began in 520

560	Mago	prince <i>c.</i> 520
555 - 522	Cambyses	prince 530-22, son of Cyrus
555 - 489	Miltiades	<i>c.</i> 555 - 489
555	<i>Gathas</i>	the century before Artaxerxes, prince 465-21
551 - 479	Confucius	<i>c.</i> 551 - <i>c.</i> 479
	<i>Analects</i>	
550 - 490	Cleomenes	prince <i>c.</i> 520-490, son of Anaxandridas
550 - 486	Darius	<i>c.</i> 550 - 486
	Behistun inscription	
	Susa inscription	
	Naqsh-I Rostam inscription	
545 - 497	Aristagoras	leader of Miletus from <i>c.</i> 505 to 497
540 - 478	Gelon	<i>c.</i> 540 - 478
540	Heracitus	one generation after Pythagoras
	fragments	
535 - 466	Hiero	tyrant 478-66, brother of Gelon
532	Alcmaeon	between Pythagoras and Empedocles
531 - 469	Leotychidas	prince 491-69
530 - 480	Leonidas	prince 490-80, son of Anaxandridas
529	Priestly code	the century before 445/33
	parts of <i>Genesis</i> , <i>Exodus</i> , and <i>Numbers</i>	
	<i>Leviticus</i>	
528	<i>Job</i>	the century after 538
525 - 459	Themistocles	<i>c.</i> 525 - <i>c.</i> 459
520 - 465	Xerxes	<i>c.</i> 520 - 465
	Persepolis inscription	
520	Sataspes	nephew of Darius
519 - 470	Pausanias	general in 479 - <i>c.</i> 470
519	<i>Great learning, The mean</i>	the century around <i>c.</i> 479
519	Hippasus	between Pythagoras and Philolaus
515 - 445	Parmenides	<i>c.</i> 515 - the decade after <i>c.</i> 450
	fragments	
510 - 449	Cimon	<i>c.</i> 510 - 449
500 - 428	Anaxagoras	<i>c.</i> 500 - <i>c.</i> 428
	fragments	
497 - 459	Ephialtes	500/495 - 459
495 - 435	Empedocles	<i>c.</i> 495 - <i>c.</i> 435
	fragments	
495 - 429	Pericles	<i>c.</i> 495 - 429
494 - 427	Archidamus	prince 469-27, grandson of Leotychidas
491	Twelve tables	promulgated in 451-49
490 - 420	Protagoras	<i>c.</i> 490 - <i>c.</i> 420
490	Oenopides	a little younger than Anaxagoras
490	Zeno of Elea	born <i>c.</i> 490
485 - 380	Gorgias	<i>c.</i> 485 - <i>c.</i> 380
484 - 425	Herodotus	484 - the decade after 430
	<i>Histories</i>	
484 - 403	Hippodamus	planned Thurii in 444 - the decade after 408
483 - 424	Artaxerxes	
	Persepolis inscription	
481	Melissus	general in 441
	fragments	
480 - 402	Hellanicus	<i>c.</i> 480 - the decade after 407
480	Leucippus	between Anaxagoras and Democritus

479	Cleidemus	between Empedocles and Diogenes of Apollonia
479	Ezra	great assembly in 445/33
475 - 411	Antiphon fragments	a little younger than Gorgias - 411
472	Meton	observation in 432
471 - 422	Brasidas	general in 431 - 422
470 - 413	Nicias	c. 470 - 413
470 - 394	Hippias	younger than Protagoras - the decade after 399
470 - 394	Prodicus	younger than Protagoras - the decade after 399
470	Hippo fragments	parodied in the decade before 425
469 - 399	Socrates	
469	Hippocrates Hippocratic corpus, including <i>On ancient medicine</i> <i>Epidemics I</i> <i>Epidemics III</i> <i>On fractures</i> <i>Aphorisms</i> <i>Law</i> <i>On the art</i> <i>On the nature of man</i> <i>On decorum</i> <i>Precepts</i> <i>Letters</i>	contemporary with Socrates
469	Philolaus	contemporary with Socrates
467 - 400	Agis	prince 427-00, son of Archidamus
467	Thrasymachus	parodied in 427
464 - 399	Thucydides <i>History</i>	general in 424 - the decade after 404
463	Diogenes of Apollonia fragments	parodied in 423
462 - 422	Cleon	465/0 - 422
460 - 394	Theodorus of Cyrene	one generation after Oenopides - the decade after 399
460	Democritus	40 years younger than Anaxagoras
460	Hippocrates of Chios	one generation after Oenopides
459 - 380	Lysias speeches	459 - c. 380
455	Lycophron fragments	student of Gorgias
454	Gylippus	general in 414
453 - 405	Darius II Susa inscription	prince 424-05, son of Artaxerxes
453 - 403	Critias fragments	c. 453 - 403
451 - 404	Alcibiades	
450 - 406	Hannibal Mago	general 410-06
449	Cratylus	younger than Socrates
448 - 395	Lysander	general in 406 - 395
448	Hicetas	between Philolaus and Plato
447 - 364	Euclid of Megara	older than Plato - the decade after 369
446 - 396	Himilco	general 406-396
445	Ctesias	under Artaxerxes II, prince 405-359
444 - 361	Antisthenes	c. 444 - the decade after 366
444 - 360	Agesilaus	

10*		enumeration and review
443 - 365	Camillus	tr. mil. 403 - 365
437 - 361	Aristippus	a little older than Plato - the decade after 366
436 - 338	Isocrates	
	discourses	
433	Menestor	between Empedocles and Theophrastus
430 - 367	Dionysius	c. 430 - 367
430 - 359	Artaxerxes II	prince 405-359, son of Darius II
430 - 350	Xenophon	c. 430 - the decade after 355
	<i>Greek history</i>	
	<i>The march of Cyrus</i>	
	<i>The education of Cyrus</i>	
	<i>Agesilaus</i>	
	<i>Hiero</i>	
	<i>Apology</i>	
	<i>Memorabilia</i>	
	<i>Home management</i>	
	<i>Symposium</i>	
	<i>Spartan constitution</i>	
	<i>Revenues</i>	
	<i>Cavalry commander</i>	
	<i>On horsemanship</i>	
	<i>On hunting</i>	
429	Chronicles, Ezra, Nehemiah	– the century after 445/33
427 - 347	Plato	
	<i>Apology</i>	
	<i>Crito</i>	
	<i>Euthyphro</i>	
	<i>Laches</i>	
	<i>Lysis</i>	
	<i>Charmides</i>	
	<i>Greater Hippias</i>	
	<i>Lesser Hippias</i>	
	<i>Ion</i>	
	<i>Protagoras</i>	
	<i>Meno</i>	
	<i>Euthydemus</i>	
	<i>Gorgias</i>	
	<i>Menexenus</i>	
	<i>Phaedo</i>	
	<i>Symposium</i>	
	<i>Phaedrus</i>	
	<i>Republic</i>	
	<i>Cratylus</i>	
	<i>Parmenides</i>	
	<i>Theaetetus</i>	
	<i>Sophist</i>	
	<i>Statesman</i>	
	<i>Timaeus</i>	
	<i>Critias</i>	
	<i>Philebus</i>	
	<i>Laws</i>	
	[<i>Letters</i>]	
427	Archytas	contemporary with Plato
427	Ecphantus	between Democritus and Ponticus
414 - 369	Theaetetus	c. 414 - 369

410 - 362	Epaminondas	c. 410 - 362
410 - 322	Diogenes	c. 410 - 324/1
408 - 356	Eudoxus	c. 408 - c. 356
407 - 339	Speusippus	c. 407 - 339
407	Aeneas Tacticus	general in 367
	<i>On the defense of fortified positions</i>	
406	Sextius Lateranus	cos. 366
405 - 335	Ephorus	c. 405 - the decade after 340
404	Eubulides	older than Demosthenes
404	Licinius Stolo	cos. 364
399 - 338	Artaxerxes III	prince 359-38, son of Artaxerxes II
	Persepolis inscription	
396 - 324	Anaximenes of Lampsacus	under Philip, prince 356-36 - the decade after 329
395 - 317	Ponticus	contemporary with Xenocrates - the decade after 322
	fragments	
395 - 314	Xenocrates	
391	Metrodorus of Chios	between one and two generations after Isocrates
384 - 330	Timoleon	general in 344 - the decade after 335
384 - 322	Aristotle	
	book I of the <i>Metaphysics</i>	
	book XII of the <i>Metaphysics</i>	
	books XIII-XIV of the <i>Metaphysics</i>	
	<i>Physics</i>	
	<i>On the heaven</i>	
	<i>On generation and corruption</i>	
	<i>Meteorology</i>	
	<i>On the soul</i>	
	<i>History of animals</i>	
	<i>Parts of animals</i>	
	<i>Movement of animals</i>	
	<i>Progression of animals</i>	
	<i>Generation of animals</i>	
	<i>On the senses and the sensible</i>	
	<i>On memory and recollection</i>	
	<i>On sleep and waking</i>	
	<i>On dreams</i>	
	<i>On divination in sleep</i>	
	<i>On length and brevity of life</i>	
	<i>On youth, old age, life and death, and respiration</i>	
	book II of the <i>Metaphysics</i>	
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	<i>Posterior analytics</i>	
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	<i>Topics</i>	
	<i>Sophistical refutations</i>	
	<i>Politics</i>	
	<i>The constitution of Athens</i>	
	<i>Rhetoric</i>	
	<i>Poetics</i>	
	<i>Nicomachean ethics</i>	
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[On the world]
[On breath]
[On colors]
[On the audible]
[Physiognomy]
[On plants]
[On marvelous things heard]
[On indivisible lines]
[The situation and names of winds]
[On Melissus, Xenophanes, and Gorgias]
[book V of the Metaphysics]
[book XI of the Metaphysics]
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[Rhetoric to Alexander]
[Economics]

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384 - 322 Demosthenes

382 - 336 **Philip**

380 - 330 Darius III *c.* 380 - 330

378 **Menaechmus** student of Eudoxus

375 Anaxarchus between Metrodorus of Chios and Pyrrho

371 - 289 **Mencius** *c.* 371 - *c.* 289

Mencius

371 - 287 **Theophrastus**

History of plants

Causes of plants

On fire

On stones

On fish

On the senses

[On weather signs]

371 **Aristoxenus** contemporary with Theophrastus

Elements of harmonics

371 **Dicaearchus** contemporary with Theophrastus

371 **Eudemus** contemporary with Theophrastus

371 **Pytheas** contemporary with Dicaearchus

371 Valerius Corvus *cos.* 348 at 23

370 - 327 Callisthenes *c.* 370 - 327

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366 - 282 **Ptolemy**

361 - 289 Agathocles

360 - 270 **Pyrrho** *c.* 360 - *c.* 270

359 Hanno the century before 269

Description of the sea

358 - 281 **Seleucus**

356 - 323 **Alexander**

355 Diodorus Cronus older than Zeno of Citium

354 Callippus one generation after Aristotle

350 - 259 Timaeus *c.* 350 - the decade after 264

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346 **Autolycus** one generation before Arcesilaus

On the moving sphere

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- 346 **Euclid** under Ptolemy, prince 306-282
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- 346 **Herophilus** under Ptolemy
- 346 **Manetho** under Ptolemy
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- 341 - 270 **Epicurus**
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- 335 - 263 **Zeno of Citium**
- 335 **Timocharis** observations from 295 to 272
- 331 - 278 Metrodorus of Lampsacus
- 331 - 232 Cleanthes
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- 327 - 269 Strato scholar 287-69
- 324 - 261 Antiochus c. 324 - 261
- 322 **Ctesibius** under Ptolemy II, prince 282-46
- 322 Zenodotus under Ptolemy II
- 322 septuagint under Ptolemy II
- 321 Berossus under Antiochus, prince 281-61
- 320 - 230 Timon c. 320 - c. 230
- 320 **Aristarchus** observation in 280
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- 319 - 272 Pyrrhus
- 316 - 242 Arcesilaus
- 316 Ariston of Chios contemporary with Arcesilaus
- 316 **Erasistratus** one generation after Herophilus
- 312 - 241 **Callimachus** a little younger than Zenodotus - the decade after 246
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- 312 Dionysius observations from 272 to 241
- 309 *Ecclesiastes* between *Chronicles*, *Ezra*, *Nehemiah* and *Daniel*
- 308 - 246 Ptolemy II
- 305 - 215 Hiero II c. 305 - 215
- 287 - 212 **Archimedes** c. 287 - 212
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- 285 observation of Mercury and Saturn from 245 to 229
- 282 Hermippus student of Callimachus
- 279 - 206 **Chrysippus** 281/78 - 208/5
- 274 - 194 **Eratosthenes** 276/3 - 196/3
- 273 - 203 Fabius Maximus cos. 233 - 203
- 272 - 229 **Hamilcar** general in 247 - 229
- 272 Philo of Byzantium the century after 282
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14*		enumeration and review
262 - 208	Marcellus	cos. 222 - 208
262	Bolus	the century after Callimachus
	<i>On sympathies and antipathies</i>	
257	Diocles	one generation after Archimedes
	<i>On burning mirrors</i>	
256	Fabius Pictor	embassy in 216
253	Nicomedes	between Eratosthenes and Apollonius
247 - 183	Hannibal	
246	Demetrius	the generation after 221
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242 - 187	Antiochus III	c. 242 - 187
240 - 152	Diogenes the Babylonian	c. 240 - c. 152
240 - 151	Critolaus	contemporary with Diogenes the Babylonian - the decade after 156
236 - 183	Scipio	
234 - 149	Cato	
	<i>On agriculture</i>	
233	Apollonius	born 246/21
	<i>Conics</i>	
	<i>Division of a ratio</i>	
228 - 174	Flaminius	
228 - 160	Aemilius Paullus	c. 228 - 160
216 - 144	Aristarchus of Samothrace	c. 216 - c. 144
215 - 164	Antiochus IV	c. 215 - 164
214 - 129	Carneades	
203	Hypsicles	one generation after Apollonius
	<i>Rising times</i>	
200 - 118	Polybius	c. 200 - c. 118
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198 - 160	Judah Maccabee	revolt in 168 - 160
192 - 129	Antipater of Tarsus	scholarch c. 152-29
189	Daniel	the generation after 164
187	Hipparchus	observations from 147 to 127
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187	Theodosius of Bithynia	contemporary with Hipparchus
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	<i>On habitations</i>	
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186 - 114	Apollodorus of Athens	student of Aristarchus of Samothrace - the decade after 119
185 - 129	Scipio Aemilianus	
185 - 109	Panaetius	c. 185 - 109
170 - 90	Dionysius Thrax	c. 170 - c. 90
	<i>The art of grammar</i>	
165	Hermagoras	the two generations before Posidonius
164	<i>Second book of the Maccabees</i>	– in 124
150 - 74	Zeno of Sidon	c. 150 - the decade after 79
145 - 86	Sima Qian	c. 145 - the decade after 91
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140 - 82	Mucius	c. 140 - 82
138 - 78	Sulla	
135 - 51	Posidonius	c. 135 - c. 51
132 - 63	Mithridates	
130 - 63	Antiochus of Ascalon	c. 130 - the decade after 68
130	Asclepiades	contemporary with Antiochus of Ascalon
117	Geminus	the generation around 77
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- 116 - 27 **Varro**
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- 114 *Wisdom of Solomon* the century after 124
- 106 - 43 **Cicero**
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Laelius, on friendship
On duties
Letters to Atticus
- 106 - 43 Servius
- 100 - 44 **Caesar**
Commentaries on the Gallic war
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- 95 - 52 Lucretius 97/4 - 55/0
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- 95 - 46 Cato Uticensis
- 95 Andronicus one generation before Strabo
- 89 - 22 Vitruvius under Caesar, Rubicon 49 - the decade after 27
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- 89 - 16 **Siculus** under Caesar - the decade after 21
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- 86 - 35 Sallust
- 85 - 42 Brutus
- 84 - 42 Cassius pr. 44 - 42
- 83 - 43 Hirtius cos. 43 - 43
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- 82 - 30 **Antony**
- 79 Alfenus cos. 39
- 69 - 30^{BC} Cleopatra
- 67 Dionysius of Halicarnassus under Augustus, Augustus 27
Roman antiquities
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- 67 **Trogus** under Augustus
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- 65 - 29^{AD} **Strabo** 68/3 - the decade after 24^{AD}
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- 63 - 14 **Augustus**
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- 59 - 17 **Livy**
From the foundation of the city
- 58 - 21 **Labeo** pr. 18^{BC} - c. 21
- 52 Aenesidemus one generation before Philo Judaeus
- 42 - 37 **Tiberius**
- 27 Celsus under Tiberius, prince 14-37
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16*		enumeration and review
24	Cleomedes	between Posidonius and Ptolemy
	<i>The circular theory of the heavenly bodies</i>	
22	Philo Judaeus	old in 39 ^{AD}
10 - 54	Claudius	
4 ^{BC} - 65	Seneca	c. 4 ^{BC} - 65
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	<i>On anger</i>	
	<i>On the happy life</i>	
	<i>On the brevity of life</i>	
	<i>On clemency</i>	
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6 ^{AD} - 62	st Paul	on the road to Damascus c. 36 - c. 62
	<i>First letter to the Thessalonians</i>	
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22	Hero	observation in 62
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	<i>On the making of automata</i>	
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24 - 79	Pliny	
	<i>Natural history</i>	
25	<i>Gospel of Mark</i>	the decade before 70
35 - 98	Nerva	c. 35 - 98
35 - 100	Quintilian	c. 35 - the decade after 95
	<i>Training in oratory</i>	
37 - 68	Nero	
37 - 98	Josephus	37 - the decade after 93
	<i>The Jewish war</i>	
	<i>Against Apion, on the antiquity of the Jews</i>	
43 - 102	Clement of Rome	bishop 93-102
	<i>Letter to the Corinthians</i>	
43	Gospel of Matthew	between 75 and 90
48 - 125	Plutarch	45/50 - the decade after 120

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Whether war or art brought Athens more fame
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On tranquility
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Against the common notions of the Stoics
Theseus
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Marcellus
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- 50 - 135 Akiva c. 50 - 135
 50 **Gospel of Luke, Acts of the apostles** – between 80 and 100
 50 mishna from two generation before Akiva through two after
 53 - 117 **Trajan**
 55 - 135 **Epictetus** 50/60 - 130/40
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 55 **Gospel of John, Apocalypse** – between 90 and 100
 56 - 123 **Tacitus** c. 56 - the decade after 118
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 56 Ignatius under Nerva, prince 96-8
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 57 Marinus one generation before Ptolemy
 58 **Menelaus** observations in 98
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 58 Polycarp under Trajan, prince 98-117
 70 - 127 Suetonius c. 70 - the decade after 122
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 70 catholic letters between 70 and 150
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 76 - 138 **Hadrian**
 77 Dioscorides between Pliny and Galen
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 80 ben Halafta student of Akiva
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 85 - 155 Favorinus c. 85 - c. 155
 86 - 161 **Pius**
 87 - 177 Arrian c. 87 - 173/80
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 87 **Ptolemy** observations from 127 to 141

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87	Theon of Smyrna	observations from 127 to 132
	<i>Mathematical matters useful for the study of Plato</i>	
89	Aëtius	between Plutarch and Galen
	<i>Collection of doctrines</i>	
98 - 166	Justin Martyr	under Pius, prince 138-61 – the decade after 161
102 - 173	Julian	c. 102 - the decade after 168
102	Pomponius	contemporary with Julian
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120	Hermetic corpus	beginning in the generation before Clement of Alexandria
121 - 180	Marcus	
	<i>To myself</i>	
121 - 195	Irenaeus	under Marcus, prince 161-80 - the decade after 190
127	Gellius	born 125/8
	<i>Attic nights</i>	
129 - 208	Galen	129 - 199/216
	<i>On mixtures</i>	
	<i>On anatomical procedures</i>	
	<i>On the use of the parts</i>	
	<i>That the affections of the soul depend on the mixtures in the body</i>	
	<i>On the doctrines of Hippocrates and Plato</i>	
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133	Pausanias of Magnesia	wrote in 173
	<i>Description of Greece</i>	
135	ha-Nasi	born 135
146 - 211	Severus	
146 - 212	Papinian	contemporary with Severus - 212
150 - 214	Clement of Alexandria	c. 150 - 211/6
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159	Empiricus	one generation after Galen
	<i>Outlines of Pyrrhonism</i>	
	<i>Against the mathematicians</i>	
160 - 240	Tertullian	c. 160 - c. 240
	<i>On the soul</i>	
161 - 192	Commodus	
164 - 234	Cassius Dio	c. 164 - the decade after 229
	<i>Roman history</i>	
164	Alexander of Aphrodisias	wrote between 198 and 209
	commentary on the <i>Meteorology</i>	
170 - 235	Hippolytus	c. 170 - 235
	<i>Refutation of all heresies</i>	
176 - 223	Ulpian	contemporary with Paul - 223
	excerpts in the <i>Digest</i>	

176	Paul	one generation after Papinian
178	Philostratus	the generation after 193
	<i>Lives of the sophists</i>	
181	Africanus	wrote in 221
185 - 254	Origen	
	<i>On the principles</i>	
185	Modestin	wrote between 211 and 238
	excerpts in the <i>Digest</i>	
188 - 217	Caracalla	
200 - 258	Cyprian	c. 200 - 258
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200 - 258	Novatian	c. 200 - 258
205 - 270	Plotinus	
	<i>Enneads</i>	
216 - 276	Mani	216 - 274/7
219	Laertius	two generations after Empiricus
	<i>The lives and doctrines of the philosophers</i>	
224	Diophantus	the generation before 276-82
	<i>Arithmetic</i>	
234 - 308	Porphyry	234 - 305/10
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	<i>On the life of Plotinus and the arrangement of his works</i>	
	commentary on the <i>Harmonics</i>	
240 - 325	Iamblichus	c. 240 - c. 325
	<i>On the common mathematical knowledge</i>	
	<i>On the life of Pythagoras</i>	
244 - 313	Diocletian	
250 - 325	Lactantius	c. 250 - c. 325
	<i>Divine institutes</i>	
251 - 356	st Antony	
255 - 336	Arius	250/60 - 336
265 - 339	Eusebius	the decade after 260 - 337/40
	<i>Preparation for the gospel</i>	
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275 - 337	Constantine	c. 275 - 337
280	Pappus	observation in 320
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	commentary on book X of the <i>Elements</i>	
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295 - 373	Athanasius	c. 295 - 373
306	talmud	from two generations after ha-Nasi through two after Ashi
	<i>Sabbath</i>	
	<i>Lashes</i>	
324	Theon of Alexandria	observations in 364
	commentary on the <i>Almagest</i>	
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330 - 379	Basil of Caesarea	c. 330 - 379
	<i>On the holy spirit</i>	
330 - 390	Gregory of Nazianzus	c. 330 - 390
335 - 394	Gregory of Nyssa	c. 335 - c. 394
339 - 397	Ambrose	c. 339 - 397
340 - 420	Jerome	331/48 - 420
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347 - 395	Theodosius	

347	<i>Augustan history</i>	the century after 337
353 - 423	Pelagius	350/5 - the decade after 418
354 - 415	Hypatia	daughter of Theon of Alexandria - 415
354 - 430	Augustine	
	<i>On the Christian doctrine</i>	
	<i>Confessions</i>	
	<i>On the trinity</i>	
	<i>On the literal reading of Genesis</i>	
	<i>On nature and grace</i>	
	<i>The city of god against the pagans</i>	
	<i>Reconsiderations</i>	
357 - 427	Ashi	died in 427
380 - 461	Leo	pope 440-61
380	Capella	wrote between 410 and 429
	<i>Philology</i>	
384	Orosius	born c. 384
394	Vincent of Lérins	wrote in 434
	<i>Reminders</i>	
398	Theodosian code	promulgated in 438
401 - 450	Theodosius II	
411 - 485	Proclus	410/2 - 485
	commentary on book I of the <i>Elements</i>	
431 - 518	Anastasius	c. 431 - 518
432 - 496	Gelasius	pope 492-6
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	[<i>Decretals</i>]	
433 - 493	Odoacer	c. 433 - 493
443	Dionysius the Areopagite	the century before 533
	<i>Celestial hierarchy</i>	
480 - 524	Boethius	c. 480 - 524
	<i>Second commentary on Porphyry's Introduction</i>	
480 - 543	Tribonian	c. 480 - c. 543
	<i>Digest</i>	
483 - 565	Justinian	
488 - 585	Cassiodorus	485/90 - the decade after 580
489	Simplicius	left Athens in 529
	commentary on the <i>Physics</i>	
	commentary on <i>On the heaven</i>	
489	Eutocius	contemporary with Simplicius
	commentary on <i>On the sphere and cylinder</i>	
489	Zosimus	two generations after 408/50
	<i>New history</i>	
490 - 570	Philoponus	c. 490 - c. 570
	commentary on <i>On the soul</i>	
539 - 594	Gregory of Tours	
	<i>History of the Franks</i>	
540 - 604	Gregory	c. 540 - 604
	<i>Dialogues</i>	
560 - 636	Isidore	c. 560 - 636
	<i>Etymologies, or origins</i>	
570 - 632	Muhammad	c. 570 - 632
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581 - 656	Uthman	conversion c. 611 - 656
587 - 644	Umar	581/92 - 644
601 - 680	Muawiyah	595/607 - 680

- 671 Tariq ibn Ziyad general in 711
- 673 - 735 **Bede**
On the reckoning of time
- 704 - 767 **ibn Ishaq** c. 704 - 767
The life and campaigns of the prophet
- 711 - 775 **al-Mansur** 709/13 - 775
- 718 - 775 Constantine V
- 735 - 816 Leo III pope 795-816
- 742 - 814 **Charlemagne**
- 755 - 816 Syncellus in Constantinople in 784/806 - the decade after 811
Chronological collection
- 767 - 820 **al-Shafii**
Treatise on the sources of the law
- 770 **Jabir ibn Hayyan** the century before al-Razi
Book of the properties of things
Book of explanations
[*Book of assembly*]
- 773 - 866 **al-Farghani** under al-Mamun, caliph 813-33 - the decade after 861
Elements of the science of the motions of stars
- 778 - 840 Louis the Pious
- 786 - 833 **al-Mamun**
- 789 - 889 Habash al-Hasib observations in 829 and 864 - lived to be 100
Compendium
- 795 - 852 **al-Khwarizmi** 790/800 - the decade after 847
Algebra
- 801 - 866 **al-Kindi** c. 801 - c. 866
Explanation of the proximate efficient cause of generation and corruption
- 803 banu Musa Muhammad died in 873
- 810 - 870 al-Bukhari
- 820 - 867 **Nicholas** c. 820 - 867
- 825 Abu Kamil one generation after al-Khuwarizmi
- 831 - 901 Thabit ibn Qurra 826/36 - 901
On the motion of the eighth sphere
The things which need exposition before reading the Almagest
Book in which Euclid's postulate is proved
- 860 - 930 **al-Razi** died in 925/35
Doubts concerning Galen
Book of secrets
- 861 - 929 **al-Battani** one generation after Thabit ibn Qurra - 929
Astronomical treatise
- 865 - 950 **al-Farabi** 860/70 - c. 950
The views of the inhabitants of the ideal city
On the intellect
- 876 - 936 Henry c. 876 - 936
- 905 - 959 Constantine VII
- 912 - 973 **Otto**
- 935 *Suda* one generation after Constantine VII
- 940 - 998 Abu'l-Wafa
- 950 - 1009 ibn Yunus the decade around 950 - 1009
Astronomical handbook with tables
- 955 - 983 Otto II
- 958 - 1025 **Basil II**
- 965 - 1040 **ibn al-Haytham** 965 - c. 1040
On the configuration of the world
Optics

- Book of commentaries on Euclid's Elements*
- 971 - 1030 Mahmud
- 973 - 1020 Leif Eriksson 970/5 - c. 1020
- 973 - 1024 Henry II
- 973 - 1055 **al-Biruni** 973 - the decade after 1050
- The monuments of past generations*
- 980 - 1002 Otto III
- 980 - 1037 **ibn Sina**
- Canon*
- On healing*
- 990 - 1039 Conrad II c. 990 - 1039
- 994 al-Karaji between Abu'l-Wafa and Khayyam
- 1007 - 1072 Peter Damian
- Letter on divine omnipotence*
- 1017 - 1056 **Henry III**
- 1020 - 1085 **Gregory VII** c. 1020 - 1085
- The dictate of the pope*
- 1022 - 1092 al-Zarqali in Toledo in 1062 - the decade after 1087
- Tables*
- 1028 - 1087 **William**
- 1033 - 1109 **Anselm**
- Monologue*
- Prologue*
- 1035 - 1099 **Urban II** c. 1035 - 1099
- 1048 - 1123 Roscelin 1045/50 - 1120/5
- 1048 - 1127 **Khayyam** 1048 - 1123/31
- Demonstration of problems of algebra*
- Commentaries on the difficulties in the assumptions in Euclid's book*
- 1050 - 1106 Henry IV
- 1058 - 1111 **al-Ghazali**
- The incoherence of the philosophers*
- The revival of the religious sciences*
- Deliverance from error and union with the almighty*
- The alchemy of happiness*
- 1059 - 1128 **Fulcher** c. 1059 - 1127/9
- The deeds of the Franks on their pilgrimage to Jerusalem*
- 1070 - 1121 William of Champeaux c. 1070 - c. 1121
- 1071 - 1141 bar Hiyya died in the decade after 1136
- 1079 - 1142 **Abelard**
- Yes and no*
- Logic for beginners, glosses on Porphyry*
- Theology*
- History of calamities*
- 1080 - 1130 Bernard of Chartres 1075/85 - c. 1130
- 1085 - 1154 Gilbert of Poitiers 1080/90 - 1154
- 1086 - 1125 Henry V
- 1090 - 1153 Clairvaux
- 1093 - 1152 Conrad III
- 1096 - 1141 Hugh of St Victor c. 1096 - 1141
- Instruction, on the study of reading*
- 1098 - 1179 Hildegard
- On simple medicines*
- On compound medicines*
- 1100 - 1160 Lombard c. 1100 - 1160
- 1103 Tzetzes wrote in 1143

- Prolegomena to comedy*
 1113 - 1158 **Otto of Freising** 1111/5 - 1158
The history of the two cities
 1114 - 1187 **Gerard** c. 1114 - 1187
 1123 - 1190 **Frederick** c. 1123 - 1190
 1126 - 1198 **ibn Rushd**
The incoherence of the incoherence
The decisive treatise
 1133 - 1189 **Henry II**
 1133 - 1202 Joachim of Fiore 1130/5 - 1202
 1135 - 1204 Maimonides
The guide of the perplexed
 1138 - 1193 Salah al-Din
 1158 - 1220 David of Dinant under Innocent III, pope 1198-216 - the decade after 1215
 1161 - 1216 **Innocent III**
 letters to Acerbus
 1161 - 1227 **Genghis Khan** 1155/67 - 1227
 1165 - 1197 Henry VI
 1165 - 1223 **Philip II**
 1168 - 1241 Gregory IX 1165/70 - 1241
Mother of the sciences, 1231
 letters to William of Auxerre
 1169 - 1253 Grosseteste 1168/70 - 1253
On the motion of bodies and light
 1170 - 1245 **Fibonacci** c. 1170 - the decade after 1240
Book of calculation
Book of squares
 1181 - 1256 Sacrobosco in Paris in 1221 – 1256
On the sphere
 1185 - 1249 William of Auvergne 1180/90 - 1249
 1187 - 1241 Ögedei 1185/9 - 1241
 1190 - 1264 Vincent of Beauvais c. 1190 - c. 1264
 1194 - 1250 **Frederick II**
 constitutions of Melfi, 1231
On the art of hunting with birds
 1200 - 1280 **Albert** 1193/207 - 1280
On minerals
On plants
On animals
On alchemy
 1200 Nemorarius one generation after Fibonacci
 1201 - 1274 al-Tusi
Memorandum
Revealing the mysteries of the secant figure
Treatise which cures the doubts raised by parallel lines
 1206 - 1255 Batu 1205/7 - 1255
 1209 - 1259 Möngke
 1209 - 1271 William of Sherwood in Oxford in 1249 - the decade after 1266
Introduction to logic
Synecategorematics
 1214 - 1270 **Louis IX**
 1215 - 1277 John XXI 1210/20 - 1277
Little summa of logic
 1215 - 1286 William of Moerbeke c. 1215 - c. 1286
 1215 - 1294 **Kublai**

24* enumeration and review

1217 - 1265	Hülegü	c. 1217 - 1265
1217 - 1274	Bonaventure	c. 1217 - 1274
	<i>Discourses on the six days</i>	
1217 - 1292	Roger Bacon	1214/20 - 1292
	<i>The great work</i>	
1221 - 1284	Alfonso X	
	<i>Alfonsine tables</i>	
1225 - 1274	Aquinas	1224/6 - 1274
	commentary on the <i>Sentences</i>	
	<i>On the principles of nature</i>	
	<i>On being and essence</i>	
	<i>On truth</i>	
	<i>Quadriliteral questions</i>	
	<i>On Boethius on the seven days</i>	
	<i>On Boethius on the trinity</i>	
	<i>Summa against the gentiles</i>	
	<i>On power</i>	
	<i>Against the errors of the Greeks</i>	
	<i>On buying and selling</i>	
	<i>On the guidance of the principles</i>	
	commentary on the <i>Physics</i>	
	commentary on the <i>Metaphysics</i>	
	commentary on the <i>Nicomachean ethics</i>	
	commentary on <i>On the soul</i>	
	commentary on the <i>Politics</i>	
	<i>On the eternity of the world against the murmuring</i>	
	<i>On the unity of the intellect against the Averroists</i>	
	<i>On evil</i>	
	<i>On spiritual creatures</i>	
	<i>On the soul</i>	
	<i>On the unity of the incarnate word</i>	
	commentary on <i>On causes</i>	
	commentary on the <i>Meteorology</i>	
	commentary on <i>On interpretation</i>	
	<i>On the mixtures of the elements</i>	
	<i>On the motion of the heart</i>	
	<i>On virtues</i>	
	commentary on <i>On the heaven</i>	
	commentary on <i>On generation and corruption</i>	
	<i>Theological summa</i>	
1226 - 1281	Siger	in Paris in 1266 - 1281
1229	Peter Peregrinus	wrote in 1269
	<i>Letter on the magnet</i>	
1233	Witelo	born 1230/5
	<i>Perspective</i>	
1234 - 1316	Lull	1232/5 - 1316
1237	Boethius of Dacia	condemned in 1277
1238 - 1293	Henry of Ghent	1235/40 - 1293
1239 - 1307	Edward	
1244 - 1334	John XXII	c. 1244 - 1334
1247 - 1316	Giles of Rome	c. 1247 - 1316
1254 - 1324	Polo	c. 1254 - 1324
	<i>Description of the world</i>	
1258 - 1326	Osman	c. 1258 - 1326
1260 - 1328	Eckhart	c. 1260 - 1328

German sermons		
1266 - 1308	Scotus	c. 1266 - 1308
	<i>Ordination</i>	
1270 - 1326	Mondino	1265/75 - 1326
	<i>Anatomy</i>	
1280 - 1342	Marsilius	c. 1280 - c. 1342
	<i>Defender of the peace</i>	
1281 - 1360	Orhan	
1285 - 1349	Occam	1280/90 - 1349
	<i>Summa of all logic</i>	
	<i>Seven quodlibets</i>	
1286 - 1331	Odoric	c. 1286 - 1331
	<i>Description of the eastern part of the world</i>	
1295 - 1349	Bradwardine	1290/300 - 1349
	<i>On the proportions of velocities in motions</i>	
1295 - 1358	Buridan	c. 1295 - c. 1358
	<i>Questions on the Physics</i>	
	<i>Questions on On the heaven</i>	
	<i>Questions on the Metaphysics</i>	
1295	Heytesbury	wrote in 1335
	<i>Rules for the resolution of sophisms</i>	
1304 - 1373	ibn Battuta	1304 - the decade after 1368
	<i>Travels</i>	
1312 - 1377	Edward III	
1320 - 1382	Oresme	c. 1320 - 1382
	<i>Questions on the geometry of Euclid</i>	
	<i>Treatise on the configuration of qualities and motions</i>	
	<i>The book on the heaven and the world</i>	
1324	ibn al-Shatir	wrote in 1364
	<i>Final investigation concerning the correction of the planetary theory</i>	
1332 - 1406	ibn Khaldun	
	<i>Introduction to history</i>	
1350 - 1420	d'Ailly	
	<i>Representation of the world</i>	
1357 - 1433	John	
1370 - 1444	Bruni	
	<i>The history of the Florentine people</i>	
1371 - 1415	Hus	1369/73 - 1415
1387 - 1422	Henry V	
1389 - 1459	Fra Mauro	the decade around 1389 - 1459
	map, 1459	
1394 - 1460	prince Henry	
1395 - 1450	Eanes	c. 1395 - the decade after 1445
1403 - 1461	Charles VII	
1407 - 1457	Valla	
1410 - 1474	de Zurara	c. 1410 - 1474
	<i>The chronicle of the discovery and conquest of Guinea, 1453</i>	
1420 - 1482	Benincasa	
	atlas, 1462	
1423 - 1461	Peurbach	
	with Regiomontanus	
	<i>Epitome of the Almagest, 1462</i>	
1423 - 1483	Louis XI	
1432 - 1481	Mehmed II	
1432 - 1488	Ca' da Mosto	

The first navigation of the ocean to the land of the blacks, 1463/8

1433 - 1499 **Ficino**

1436 - 1476 **Regiomontanus**

On triangles of all kinds, 1463

Tables of directions, 1467

Ephemerides, 1474

with Peurbach, q. v.

1440 - 1505 **Ivan III**

1445 - 1517 Pacoli c. 1445 - 1517

1449 - 1492 Lorenzo

1450 - 1486 **Cão** c. 1450 - 1486

1450 - 1498 Cabot c. 1450 - 1498

1450 - 1500 **Dias** c. 1450 - 1500

1451 - 1504 **Isabella**

1451 - 1506 **Columbus**

report, 15 Feb 1493

letter to Isabella and Ferdinand V, 7 Jul 1503

1452 - 1516 **Ferdinand V**

1453 - 1515 **Albuquerque**

1454 - 1512 Vespucci

letter to Lorenzo il Popolano, Mar 1503

1457 - 1509 **Henry VII**

1460 - 1510 **de la Cosa**

map, 1500

1460 - 1524 **da Gama** c. 1460 - 1524

1462 - 1525 Pomponazzi

1463 - 1494 Pico

Oration on the dignity of man, 1486

1468 - 1520 Cabral 1468 - c. 1520

1468 - 1526 **de Toren** drew maps in 1508 - 1526

map, 1526

1468 - 1536 **Erasmus** 1466/9 - 1536

Encomium of custom, or the praise of folly, 1511

1469 - 1527 **Machiavelli**

The prince, 1513

Discourses on the first ten books of Livy, 1517

1470 - 1520 **Waldseemüller** c. 1470 - 1518/21

map, 1507

Introduction to cosmography, with certain principles of geometry and astronomy necessary for the matter, 1507

1473 - 1543 **Copernicus**

Little treatise on the hypotheses concerning the heavenly motions, 1514

Letter against Werner, 1524

On the revolutions of the heavenly spheres, 1543

1475 - 1541 **Pizarro** c. 1475 - 1541

1476 - 1526 **El Cano** c. 1476 - 1526

1478 - 1535 More

A truly golden little book, no less beneficial than entertaining, on the best state of the republic and the new island of Utopia, 1516

1478 - 1553 Fracastoro c. 1478 - 1553

Syphilis, or the Gallic disease, 1530

On the sympathy and antipathy of things, 1542

On contagion, contagious diseases, and their cure, 1546

1480 - 1521 **Magellan** c. 1480 - 1521

1480 - 1538 **Biringuccio** 1480 - c. 1538

On pyrotechnics, in which are fully treated not only all kinds and sorts of minerals, but also all that is necessary for the practice of the arts of smelting, casting, and related things, 1540

1483 - 1540 **Guicciardini**

The history of Italy, 1540

1483 - 1546 **Luther**

On the freedom of a Christian man, 1520

commentary on the Letter to the Galatians, 1531

table talk 1531-46

1485 - 1547 **Cortés**

1486 - 1535 Agrippa of Nettesheim

1488 - 1534 Brunfels c. 1488 - 1534

Images of living plants, drawn from nature with great accuracy and skill, as well as their effects, 1530-6

1489 Pigafetta sailed with Magellan in 1519

The voyage around the world, 1526

1491 - 1547 **Henry VIII**

1493 - 1541 **Paracelsus**

Philosophy of the generation and fruits of the four elements, 1529

The book paragranum, 1530

The books of the opus paramirum, 1531

1494 - 1555 **Agricola**

On the origin and causes of subterranean things; on the nature of things flowing out of the earth; on the nature of fossils; on the old and new metals, 1546

On metals, in which the uses, instruments, machines, and all other things concerning metals are observed, and described not only in the clearest way, but with images in the appropriate places, 1556

1495 - 1566 **Suleiman**

1498 - 1554 Bock

New book of plants, on the differences, effects, and names of the plants growing in the German lands, 1539

1499 - 1557 Tartaglia

1500 - 1558 **Charles V**

1501 - 1576 **Cardano**

The great art, or the rules of algebra, 1545

The great art of arithmetic, 1550

Book on games of chance, 1564

1503 - 1566 Nostradamus

1509 - 1564 **Calvin**

Institutes of the Christian religion, 1536

1509 - 1588 Telesio

1514 - 1564 **Vesalius**

On the structure of the human body, 1543

1515 - 1572 Ramus

Dialectic, 1555

1516 - 1565 **Gesner**

Universal library, or most complete catalogue of all books in the three languages, Latin, Greek, and Hebrew, extant and not extant, ancient, modern, and through this day, 1545-55

History of animals, 1551-87

Mithridates, or on the different languages, 1555

1517 - 1564 **Belon**

Natural history of strange marine fishes, with a true depiction and description of the dolphin, and of several others of its kind, 1551

Natural history of birds, with their description and portraits drawn from nature, 1555

1517 - 1590 Zarlino

1519 - 1603 **Cesalpino**

- On plants*, 1583
- 1527 - 1598 Philip II
- 1530 - 1596 **Bodin**
Method for the easy knowledge of history, 1566
- 1533 - 1584 William
- 1533 - 1603 **Elizabeth**
- 1540 - 1603 **Viète**
Mathematical canon, or on the triangle, 1579
Introduction in the analytic art, 1591
Preliminary notes on logistic by species, 1593
Zetetics, 1593
Two treatises on the examination and emendation of equations, 1593
Supplement to geometry, 1593
Responses to various mathematical matters, 1593
Celestial harmony, 1600
- 1540 - 1609 Scaliger
New work for the improvement of chronology, 1583
- 1544 - 1603 **Gilbert**
On the magnet, magnetic bodies, and the great magnet of the earth, new physiology demonstrated by many arguments and experiments, 1600
New philosophy of our sublunar world, 1603
- 1546 - 1601 **Tycho**
On the new and never in the memory of any age seen star, first observed in the month of November of the year of Christ 1572, a mathematical survey, 1573
On the recent phenomena in the aethereal world, 1588
Introductory exercises for the instauration of astronomy, 1602
- 1548 - 1600 **Bruno**
The Ash Wednesdays supper, described in five dialogues, by four interlocutors, with three reflections, on two subjects, 1584
On infinity, the universe, and the worlds, 1584
- 1548 - 1620 **Stevin**
On the decimal, teaching an unheard-of light, how all human calculation in need of fractions can be done with whole numbers, 1585
The principles of the art of weighs, 1586
The course of the heaven, 1608
- 1550 - 1617 **Napier**
- 1552 - 1610 Ricci
On the Christian expedition among the Chinese undertaken by the Society of Jesus, from the commentaries of father Ricci, in which the customs, laws, and institutions of the kingdom of China, as well as the most difficult beginnings of the new church there, are described accurately and with the greatest fidelity, 1615
- 1553 - 1610 **Henry IV**
- 1561 - 1626 **Bacon**
An advertisement touching controversies of the church of England, 1589
Essays or councils, civil and moral, 1597
Religious meditations, 1597
On the colors of good and evil, 1597
A declaration of the practices and treasons attempted and committed by Robert, late earl of Essex, and his accomplices, 1601
A brief discourse touching the happy union of the kingdoms of England and Scotland, 1603
Certain considerations touching the better pacification and edification of the church of England, 1604
Apology in certain imputations concerning the late earl of Essex, 1604
On the dignity and growth of the sciences, 1605
On the wisdom of the ancients, 1609

The charge touching duels, 1614
Second part of the great instauration, new organon, or directions for the interpretation of nature, 1620
The history of the reign of king Henry VII, 1622
Natural and experimental history to ground philosophy, or phenomena of the universe, which is the third part of the instauration of the sciences, part one, history of winds, 1622
History of life and death, 1623
Apophthegms new and old, 1625
New Atlantis, 1627
Forest of forests, or a natural history, 1627
Maxims of the law, 1630

1564 - 1642 **Galileo**

The little balance, 1586
On motion, 1592
Treatise on fortifications, 1593
On mechanics, 1600
Dialogue on the new star, 1605
Treatise on the sphere, or cosmography, 1606
The operations of the geometric and military compass, 1606
Defense against the calumnies and impostures of Baldesar Capra in his Astronomical consideration on the new star of 1604, as well as, and much more, in publishing as his invention the construction and uses of the geometric and military compass under the title Usus et fabrica circini proportionis, 1607
Starry messenger, unfolding great and most wonderful sights and displaying for the inspection of everyone, and especially of the philosophers and astronomers, what with a telescope built by himself Galileo Galilei observed about the face of the moon, the fixed stars, the Milky Way, the nebulous stars, and first of all about four planets circling Jupiter's star at unequal intervals and periods and with marvelous speed, which, unknown to anyone until now, he discovered and decided to name the Medicean stars, 1610
Discourse on bodies floating on water or moving in it, 1612
History and demonstrations concerning sunspots and their appearances, contained in three letters to Mark Welser, 1613
Letter to Mme Christina of Lorraine, grand duchess of Tuscany, 1615
Discourse on the ebbs and flows of the sea, 1616
Discourse on the comets, 1618
The assayer, in which on a delicate and precise scale are weighted the matters contained in The astronomical and philosophical balance of Lothario Sarsi, 1623
Letter to Francesco Ingoli in response to the Disputations on the location and rest of the earth, 1624
Report on flood control, 1630
Dialogue on the two main systems of the world, Ptolemaic and Copernican, 1632
Astronomical operations, 1637
Discourses and mathematical demonstrations in two new sciences dealing with mechanics and locomotion, 1638
Letter to prince Leopold of Tuscany, concerning chapter fifty of the Litheosphorus of Fortunio Liceti, 1641
On Euclid's definitions of proportion, 1642
 letters to Cesi; letters to Giuliano de' Medici; letters to Liceti

1567 - 1625 **Maurice**

1571 - 1630 **Kepler**

Harbinger of cosmographic treatises, containing the cosmographic mystery, on the marvelous proportion of the heavenly spheres, and on the true and proper causes of the number, magnitude, and periods of the heavens, demonstrated by the five regular geometric bodies, 1596
On the more certain foundations of astrology, new brief treatise toward a theory of the world, together with a physical forecast for the year from the birth of Christ 1602, 1602
Additions to Witelo, in which the optical part of astronomy is treated, and first of all artificial

- observation and estimation of the diameters and eclipses of the sun and the moon, with examples of important eclipses, 1604*
- Forecast for the year from the gracious birth of our lord and savior Jesus Christ 1605, together with a thorough report on the appearance of an unusual new star, 1604*
- On the new star in the foot of Serpent-bearer, book full of disquisitions astronomical, physical, metaphysical, meteorological, and astrological, ἐνδόξιος and παραδόξιος, 1606*
- On the star of third magnitude in Swan which was unknown until the year 1605 and has not yet been extinguished, 1606*
- On the true year of the birth of our savior Jesus Christ, examination of the recent proposal of Suslyga that the customary epoch is four years short, 1606*
- Exhaustive report on the new bearded star, or comet, which appeared in September and October of 1607, and what it means; together with an entirely new and peculiar yet well grounded discourse on what comets actually are, whence they come, by what their motion is governed, and with which of their shapes they have something to indicate for mankind, 1608*
- New astronomy ἀπιολογητός, or celestial physics derived from commentaries on the motions of the star of Mars, from the observations of Tycho, 1609*
- A singular phenomenon, or Mercury in the sun, 1609*
- Answer to Dr Röslin's Discourse on the constitution of the present, and how it develops into the future, 1609*
- Conversation with the Starry messenger sent recently to mankind by Galileo, 1610*
- Tertius interveniens, or warning to some theologians, physicians, and philosophers, and especially Dr Feselius, that by cheap condemnation of the stargazer's superstition they may throw out the child with the bath-water and thus act inadvertently against their profession, 1610*
- Dioptrics, or demonstration of the consequences for seeing and for what is seen of the newly discovered telescope, 1611*
- A new year's gift, or on the hexagonal snowflake, 1611*
- Report on observations of the four wandering satellites of Jupiter, which Galileo with the right of discoverer named the Medicean stars, 1611*
- Repeated more exhaustive report that our lord and savior Jesus Christ was born not just one year before the beginning of our present time-reckoning as Röslin, Bünting, and others hold, nor just two years as Scaliger and Calvisius together with many old church writers hold, but full five years; the time of the birth of Christ determined from correct harmony and comparison of heathen and Jewish histories, as well as from agreement with the course of the heaven, 1613*
- On the true year in which the eternal son of god assumed human nature in the womb of the blessed virgin Mary, 1614*
- New stereometry of wine casks, especially those most in shape like the Austrian, and the best use in them of a cubical gauging rod, with a supplement to Archimedean stereometry, 1615*
- Harmonics of the world, in five books, I. geometrical, II. architectonic, III. harmonic, IV. metaphysical, psychological, and astrological, V. astronomical and metaphysical, 1619*
- Epitome of Copernican astronomy, written in the form of questions and answers, 1618-21*
- On comets, in three books, I. astronomical, II. physical, III. astrological, 1619*
- New ephemerides of the heavenly motions beginning with the year of the common era 1617, from observations mainly of Tycho, physical hypotheses, and the Rudolphine tables, 1619-30*
- Kanones pueriles, or chronology from Adam to this current year of Christ 1620, divided in six parts and confirmed from the holy scripture of the Old and New testaments, with support from heathen antiquities and astronomical calculation, 1620*
- Astronomical report on two great and peculiar lunar eclipses seen in the past year 1620, and on the great solar eclipse which will occur in the current year 1621 on 11/21 May, with a brief recapitulation of all great solar eclipses seen in Europe in eighty years and what in each case the state of spiritual and secular affairs before and after has been, 1621*
- Defense of my Harmonics of the world against the Analytical demonstrations of Dr Fludd, 1622*
- Discourse on the great conjunction, or meeting, of Saturn and Jupiter in the fiery sign of Lion which occurs in July of the year 1623, as also on the many and multifarious prognostications about this year, 1623*
- One thousand logarithms of as many round numbers; legitimate premise of the demonstration of*

their origin and use; a new arithmetic or compendium, of which nothing is more admirable or useful for the solution of nearly all problems of calculation, especially those in the study of triangles, without multiplication, 1624

Supplement to the One thousand logarithms, containing precepts on their use, 1625

Shield bearer for the Dane Tycho against Chiaramonti's Antitycho, 1625

Rudolphine tables, with which the restoration of astronomical science, long collapsed, is continued, first conceived by that phoenix of astronomers Tycho in 1564, from most accurate observations of the stars especially after 1572, 1627

On the rare and marvelous phenomena of the year 1631, incursions of Venus and Mercury in the sun, an admonition to astronomers and students of the things in the heaven, 1629

On the letter of Dr Bartsch about the computation and edition of ephemerides, 1629

Dream, or on lunar astronomy, 1634

1575 - 1624 **Boehme**

1578 - 1657 **Harvey**

An anatomical essay on the movement of the heart and blood in animals, 1628

A second essay to Riolan in which many objections to the circulation of the blood are refuted, 1649

1579 - 1644 **van Helmont**

Rise of medicine, or beginnings of a new physics; new progress in medicine, for the destruction of disease, for long life, 1648

1580 - 1626 **Snel**

Batavian Eratosthenes, on the true measure of the circumference of the earth, recalled from the grave διὰ τῶν ἐξ ἀποστημάτων μετρούσων διοπτρῶν, 1617

1583 - 1645 **Grotius**

On the law of war and peace, 1625

1585 - 1630 **Cesi**

Apiarium, 1625

Pythosophical tables, 1630

1585 - 1642 **Richelieu**

1588 - 1648 **Mersenne**

Physico-mathematical reflections, in which the nature and art of admirable effects are explained by most certain demonstrations, 1644

1588 - 1679 **Hobbes**

Leviathan, or the matter, form, and power of the commonwealth ecclesiastical and civil, 1651

Six lessons to the professors of mathematics, one of geometry, the other of astronomy, in the university of Oxford, 1656

1591 - 1661 **Desargues**

Draft of a project for an attack on the results of the intersection of a cone with a plane, 1639

appendix to the Universal method of Mr Desargues for the practice of perspective, by small steps, like the geometrical, by Bosse, 1648

1592 - 1655 **Gassendi**

Paradoxical exercises against the Aristoteleans, in which all precepts of the Peripatetic doctrine and dialectics are examined, and true opinions either new or very old established, 1624

Mercury seen in the sun, Venus not seen, 1632

Philosophical collection, logic, physics, ethics, 1658

1594 - 1632 **Gustav II**

1596 - 1650 **Descartes**

A compendium of music, 1618

Treatise on clear and useful rules for the direction of the mind in the search for truth, 1628

The world, or a treatise on light and the other principal objects of sense, 1633

Discourse on the method to conduct one's reason well and search for truth in the sciences, as well as the Optics, Meteorology, and Geometry, which are essays in this method, 1637

Meditations on the first philosophy, in which the existence of god and the distinction between man's soul and body are demonstrated, 1641

Replies to objections raised by some men of learning against the Meditations, 1641

The principles of philosophy, 1644

The description of the human body and all its functions, those that do not depend on the soul and those that depend on it, as well as the principal cause of the formation of its members, 1648
Comments on a broadsheet published in Belgium at the end of 1647, entitled An account of the human mind, or rational soul, which explains what it is and what it can be, 1648

The passions of the soul, 1649

The search for truth by the natural light, 1650

letters to Mersenne; letters to Morin

1598 - 1647 **Cavalieri**

Geometry of indivisibles of continua developed by a new method, 1635

Six exercises in geometry, 1647

1599 - 1658 **Cromwell**

1601 - 1665 **Fermat**

Method for the investigation of maxima and minima, 1638

On the transformation and simplification of equations of location for the comparison of all forms of areas under curves, and on the use of geometrical progression for the quadrature of parabolas and hyperbolas to infinity, 1659

Analysis and synthesis for refractions, 1662

notes in the margins of the *Arithmetic*, 1670

letters to Frénicle

1602 - 1675 Roberval

1602 - 1686 **Guericke**

New, the so-called Magdeburgian, experiments on vacuum in space, 1672

1608 - 1647 **Torricelli**

letters to Michelangelo Ricci

1608 - 1679 Borelli

1611 - 1675 Turenne

1611 - 1687 Hevelius

1612 - 1694 Arnauld

1616 - 1703 **Wallis**

Arithmetic of infinities, or a new method of investigation of the quadrature of curves and other difficult mathematical problems, 1656

Proof of Euclid's fifth postulate, 1663

Treatise of algebra, both historical and practical, showing its origin, progress, and advancement, from time to time, and by what steps it has attained to the height at which it now is, 1685

1618 - 1663 **Grimaldi**

A physico-mathematical treatise on light, colors, and the rainbow, as well as related matters, in two books, of which in the first new experiments are shown and reasons are deduced from them for the substantiality of light, but in the second the arguments adduced in the first are refuted and the probability of the Peripatetic doctrine of the accidentality of light is upheld, 1665

1619 - 1683 Colbert

1620 - 1682 **Picard**

Measurement of the earth, 1671

1621 - 1686 Condé

1623 - 1662 **Pascal**

Essay on the conics, 1640

New experiments concerning vacuum, 1647

Account of the grand experiment on the equilibrium of liquids performed on one of the highest mountains of Auvergne, 1648

The generation of conics, tangents, and secants; or the projection of the circumference, tangents, and secants of the circle for every position of the eye and the plane of the figure, 1654

On the equilibrium of liquids, 1663

On the weight of the mass of air, 1663

1625 - 1712 **Cassini**

Astronomical observations made in different regions of the kingdom during the year 1672, 1673

The elements of astronomy, verified by Mr Cassini from the relation of his tables to the observations

made by Mr Richer on the island of Cayenne, 1684

1626 - 1697 **Redi**

Experiments on the generation of insects, 1668

1627 - 1691 **Boyle**

New physico-mechanical experiments, touching on the spring of air, and its effects, made for the most part in a new pneumatic engine, 1660

The sceptical chemist, or chemico-physical doubts and paradoxes, touching the experiments whereby vulgar spagyristes are wont to endeavor to evince their salts, sulphur, and mercury, to be the true principles of things, 1661

Certain physiological essays and other tracts written at distant times and on several occasions, 1661

A defense of the doctrine touching the spring and weight of air, 1662

The origin of forms and qualities according to the corpuscular philosophy, illustrated by considerations and experiments, 1666

1627 - 1704 Bossuet

1627 - 1705 Ray

Method of plants, 1682

History of plants, 1686-704

Synopsis of quadruped and serpentine animals, 1693

History of insects, 1710

with Willughby

Ornithology, 1678

History of fish, 1686

1628 - 1694 **Malpighi**

Anatomical observations on the lungs, 1661

The anatomy of plants, 1675

1629 - 1695 **Huygens**

Theorems on the quadrature of the hyperbola, ellipse, and circle when the center of gravity of segments is given, to which is appended 'Εξέτασις of the measurement of the circle of Gregory of St Vincent, 1651

On the magnitude of the circle, with an appendix on the constructions of some famous problems, 1654

On the motion of colliding bodies, 1656

Treatise on calculation in games of chance, 1656

New observation of a moon of Saturn, 1656

The clock, 1658

The system of Saturn, or on the causes of Saturn's marvelous phenomena and its satellite, a new planet, 1659

Short instruction on the use of the clock to find longitude, 1665

Rules for the motion of bodies in collision, 1669

The pendulum clock, or geometrical demonstrations concerning the motion of pendula applied to clocks, 1673

Treatise on light, in which are explained the causes of what occurs in reflection, in refraction, and particularly in the strange refraction of Iceland crystal, 1678

Compendium on star observation freed from difficulties by optical tubes, 1684

Memoirs on the grinding of lenses for telescopes, 1685

Discourse on the cause of gravity, 1690

Κοσμοθεωρὸς, or conjectures on the countries and inhabitants of the heavenly bodies, 1698

letters to the *Journal of scientists*

1630 - 1677 Barrow

1630 - 1696 Richer

Astronomical and physical observations made on the island of Cayenne, 1679

1632 - 1677 **Spinoza**

Treatise on the correction of the understanding, and on the way in which it is best directed toward true knowledge of things, 1662

Parts I and II of Descartes's Principles of philosophy demonstrated in the geometrical manner, 1663

Metaphysical thoughts, in which difficulties arising in the general as well as special parts of metaphysics are explained, 1663

Theologico-political treatise, 1670

Ethics demonstrated in geometrical order, 1677

Political treatise, in which it is demonstrated how the society in which monarchy rules, as well as that in which aristocracy rules, needs to be ordered so as not to lapse into tyranny and to preserve inviolate the peace and freedom of the citizens, 1677

Compendium of the grammar of the Hebrew language, 1677

letters to Oldenburg

1632 - 1694 Pufendorf

1632 - 1704 **Locke**

Two treatises of government, in the former the false principles and foundation of Sir Robert Filmer and his followers are detected and overthrown, the latter is an essay concerning the true origin, extent, and end of civil government, 1690

An essay concerning human understanding, 1690

1632 - 1707 Mabillon

On diplomatic matters, in which whatever pertains to the materials, writing, and style of old documents, whatever pertains to seals, monograms, subscripts, and chronological notes, whatever pertains to antiquities and forensic historical disciplines, is explained and illustrated, 1681

1632 - 1723 **Leeuwenhoek**

letters to Oldenburg

1633 - 1707 Vauban

1635 - 1702 **Hooke**

Micrographia, or some physiological descriptions of minute bodies made by magnifying glasses, with observations and inquiries thereupon, 1665

Attempt to prove the motion of the earth from observations, 1674

On the restoring force, 1678

1635 - 1715 Burnet

The sacred theory of the earth, containing an account of the origin of the earth, and of all the general changes which it has already undergone, or is to undergo, until the consummation of all things, 1681

1637 - 1680 **Swammerdam**

General history of insects, 1669

1638 - 1675 James Gregory

1638 - 1686 **Steno**

Harbinger of a dissertation on solids enclosed by nature in solids, 1669

1638 - 1715 **Louis XIV**

1638 - 1715 Malebranche

1639 - 1691 Louvois

1642 - 1727 **Newton**

Enumeration of lines of the third order, 1668

On analysis by equations of infinite number of terms, 1669

A treatise on the method of fluxions and infinite series, with its application to the geometry of curves, 1671

New theory of light and colors, 1672

An account of a new catadioptrical telescope, 1672

A series of queries propounded by Mr Newton, to be determined by experiments, positively and directly concluding his new theory of light and colors, and here recommended to the industry of the lovers of experimental philosophy, 1672

A treatise on the quadrature of curves, 1676

A treatise on the system of the world, 1685

The mathematical principles of natural philosophy, 1687

A historical account of two notable corruptions of scripture, 1691
On the nature of acids, 1692
The origin of monarchies, 1693
A scale of the degrees of heat; description and signs of heat, 1701
Optics, or a treatise on the reflection, refraction, inflection, and colors of light, 1704
Universal arithmetic, or a treatise on arithmetical composition and resolution, 1707
The chronology of ancient kingdoms amended, to which is prefixed a short chronicle from the first memory of things in Europe to the conquest of Persia by Alexander, 1728
Observations on the prophecies of Daniel and the Apocalypse of st John, 1733
 letters to Hooke; letters to Bentley

1644 - 1710 **Römer**

1646 - 1716 **Leibniz**

Dissertation on the art of combination, in which from the fundamental arithmetic new precepts are extracted for the theory of combinations and transpositions, and their uses for all sciences are shown; new seeds sown for the art of meditation or the logic of discovery, 1666
New method of teaching and learning jurisprudence, from the principles of the art of teaching and the light of experience, 1667
Elements of natural law, 1671
Project for the conquest of Egypt, presented to Louis XIV, 1671
A new physical hypothesis, by which most phenomena of nature are derived from one universal motion supposed about our globe, without disdaining either the Tychonians or the Copernicans, 1671
An addition to the arithmetic of infinities, in which it is shown that a greatest number or a number of all numbers is impossible or nothing, and it is shown by examples that what is taken to be axioms can be demonstrated, 1672
New method to investigate the tangents of curves, 1673
Second part of the analysis of quadrature, 1675
Examples of the inverse method of tangents, 1675
Jurisdiction of the emperor, on the supreme and delegated rights of the German princes, 1677
Beginning and specimens of a general science for the instauration and growth of the sciences, 1679
Calculus of consequences, 1679
 manuscript beginning *Cum animadverterem plerosque*, 1679
Elements of the universal characteristic, 1679
Elements of calculus, 1679
Elements of the universal calculus, 1679
Investigations into the universal calculus, 1679
Method of examination of inferences by numbers, 1679
Rules with which the validity of inferences and the forms and moods of categorical syllogisms can be judged by numbers, 1679
On dyadic numbers, 1679
On the analysis of situation, 1679
On the true proportion of the circle to the circumscribed square, expressed in rational numbers, 1682
Perpetual irony, a most Christian Mars, or an apology for the wars of the most Christian king against the Christians, 1683
On discovering the dimensions of figures, 1684
A new method for maxima and minima, as well as tangents, impeded by neither fractional nor irrational quantities, and a singular kind of calculus for it, 1684
Meditations on knowledge, truth, and ideas, 1684
A specimen of the universal calculus, 1685
An arithmetical machine, which performs not only addition and subtraction, but also multiplication without any and division nearly without any animate labor, 1685
 manuscript beginning *Primae veritates sunt*, 1685
General investigations into the analysis of concepts and truths, 1686
A new meditation on the nature of the angles of contact and osculation, and on their use in practical

- mathematics for substituting easier figures for more difficult ones*, 1686
- On a recondite geometry and the analysis of indivisibles and infinities*, 1686
- A brief demonstration of a memorable error of Descartes and others concerning a natural law, according to which god would always conserve the same quantity of motion, misused also in mechanics*, 1686
- Discourse on metaphysics*, 1686
- On a general principle useful for the explanation of the laws of nature through the divine wisdom*, 1687
- On optical lines, and other things*, 1689
- On the isochronic line in which a heavy body without acceleration descends*, 1689
- An examination of the causes of the heavenly motions*, 1689
- manuscript beginning *Cum geometrias demonstrationibus*, 1689
- On the line in which a body curves by its own weight, and its remarkable use in discovering any number of mean proportionals and logarithms*, 1691
- Preliminary specimen of dynamics, on the law of nature relating to the power of bodies*, 1691
- Essay on dynamics concerning the laws of motion, in which it is demonstrated that conserved is not the same quantity of motion but the same absolute force, or the same quantity of moving action*, 1691
- On the line formed by an infinite number of lines drawn ordinatewise which meet each other and are all tangent to it, and on the consequent new use of the analysis of infinities*, 1692
- Animadversions on the general parts of Descartes's Principles*, 1692
- A supplement to practical geometry extending to transcendental problems with the help of a new very general method of infinite series*, 1693
- A supplement to dimensional geometry, or the most general production of all quadratures by motion, as well as the construction of many lines from given conditions of the tangents*, 1693
- Protoearth, or treatise on the first state of the earth and the vestiges of its ancient history in the monuments of nature*, 1693
- preface to the *Code of the law of nations*, 1693
- A new application of the differential calculus and its use for multiple constructions of lines from a given condition of tangents*, 1694
- The proper construction of the problem of the paracentric isochronic curve, as well as some general facts about the nature and differential calculus of osculates, and on the constructions of transcendental lines, one entirely geometrical, another mechanical but very general; with a supplement on a way to render the discovery of transcendental lines universal, so that they include any case whatever and pass through a given point*, 1694
- A mathematics of reason*, 1695
- On the mathematical determination of the forms of the syllogism*, 1695
- A not inelegant specimen of demonstration in abstracts*, 1695
- manuscript beginning *Defin. 1. Eadem seu*, 1695
- A specimen of dynamics for the discovery of the admirable laws of nature of the forces of bodies and their actions on one another, and for their reduction to their causes*, 1695
- New system of the nature and of the communication of substances, as well as of the union which there is of the soul and the body*, 1695
- An anagogical essay in the research of causes*, 1695
- Letter on the connection of the houses of Brunswick and Este*, 1695
- Communication of his own solution of the problem of the curve of fastest descent, proposed to the geometers by Mr Johann Bernoulli, and of two other solutions, communicated to him first by Mr Johann Bernoulli, then by marquis de l'Hôpital, as well as his own solution of another problem proposed by the same*, 1697
- On the radical origination of things*, 1697
- preface to *The latest on China*, 1697
- Dissertation on the origin of the Germans*, 1697
- On nature itself, or on the inherent force and actions of created things, to confirm and illustrate their dynamics*, 1698
- Clarification of the difficulties which Mr Bayle has found in the new system of the union of soul and*

- body, 1698
 preface to the *Additions to history, writers on German matters collected from manuscripts*, 1698
Justification of the calculus of infinitesimals by that of the ordinary algebra, 1701
New specimen of analysis for the science of the infinite concerning sums and quadratures, 1702
Reply to the reflections contained in the second edition of the Critical dictionary of Bayle, article
Rorarius, on the system of preestablished harmony, 1702
 manuscript beginning *Nullum quidem librum*, 1702
Explication of binary arithmetic, which employs only the characters 0 and 1, with remarks on its
utility, and on how it makes sense of the ancient Chinese figures of Fu Xi, 1703
Manifesto for the defense of the rights of Charles III, king of Spain, 1703
New essays on human understanding, 1704
Opinion on the principles of Pufendorf, 1706
Essays of theodicy on the goodness of god, the freedom of man, and the origin of evil, 1710
The antibarbarian physicist, for real philosophy, against the revival of the qualities of the
scholastics and chimerical intelligences, 1710
Principles of nature and grace founded on reason, 1714
Principles of philosophy, or monadology, 1714
History and origin of the differential calculus, 1714
The metaphysical foundations of mathematics, 1715
Discourse on the natural theology of the Chinese, 1716
Brunswick annals of the western empire, 1716
 letters to Huygens; letters to Clarke; letters to Remond; letters to Wagner
- 1646 - 1719 Flamsteed
 letters to Cassini
- 1647 - 1706 Bayle
A historical and critical dictionary, 1697
- 1647 - 1713 Papin
A new method to obtain the greatest motive power at little cost, 1690
- 1650 - 1702 **William III**
- 1655 - 1705 **Jakob Bernoulli**
The art of conjecture, to which are added a treatise on infinite series and a letter written in French
on the game of tennis, 1713
- 1656 - 1708 Tournefort
Elements of botany, or method to know the plants, 1694
- 1656 - 1742 **Halley**
To that glory of our time and people, the mathematico-physical treatise of the most excellent Isaac
Newton, 1687
 review of *The mathematical principles of natural philosophy*, 1687
An estimate of the degree of the mortality of mankind, drawn from curious tables of the births and
funerals at the city of Breslau, with an attempt to ascertain the price of annuities upon lives,
 1693
A synopsis of the astronomy of comets, 1705
A singular method to determine the parallax of the sun, or its distance from the earth, by observing
Venus in the sun, 1716
Considerations on the change of the latitudes of some of the principal fixed stars, 1717
- 1660 - 1727 George
- 1660 - 1734 **Stahl**
Fundamental zymotechnic, or general theory of fermentation, in which the causes and effects of this
noblest art and most useful and most subtle part of chemistry are presented from their
mechanico-physical principles, 1697
Foundations of dogmatic and experimental chemistry, as well as physical, mechanical, pharmaceu-
tical, and medical, as well as the so-called hermetic and alchemical, 1723
- 1663 - 1736 Eugene
- 1665 - 1721 **Camerarius**
On the sex of plants, 1694

1666 - 1736 **Gray**

A letter containing several experiments concerning electricity, 1731

1667 - 1733 **Saccheri**

Euclid freed from all blemish, or a geometrical investigation in which are established the principles of universal geometry, 1733

1667 - 1748 **Johann Bernoulli**

The curvature of a ray of light in a nonuniform medium, the solution of the problem, proposed in these Acta in 1696, of finding the brachistochrone, or the line in which a heavy particle descends in the shortest time, and on the construction of the synchrone, or the wave front of rays of light, 1697

1668 - 1744 **Vico**

Principles of a new science dealing with the common nature of nations, 1725

1672 - 1725 **Peter**

1672 - 1731 **Geoffroy**

Table of different affinities observed in chemistry between different substances, 1718

1676 - 1745 **Walpole**

1679 - 1754 **Wolff**

1683 - 1757 **Réaumur**

Memoirs to serve as a history of insects, 1734-42

1685 - 1731 **Taylor**

On the motion of a tense string, 1713

1685 - 1740 **Charles VI**

1685 - 1753 **Berkeley**

Philosophical commentaries, 1708

An essay toward a new theory of vision, 1709

A treatise concerning the principles of human knowledge, wherein the chief causes of error and difficulty in the sciences, with the grounds of scepticism, atheism, and irreligion, are inquired into, 1710

Three dialogues between Hylas and Philonous, the design of which is plainly to demonstrate the reality and perfection of human knowledge, the incorporeal nature of the soul, and the immediate providence of a deity, in opposition to sceptics and atheists, 1713

On motion, or the principle and nature of motion, and on the cause of the communication of motions, 1721

The theory of vision, or visual language showing the immediate presence and providence of a deity, vindicated and explained, 1733

The analyst, or a discourse addressed to an infidel mathematician, wherein it is examined whether the object, principles, and inferences of the modern analysis are more distinctly conceived, or more evidently deduced, than religious mysteries, 1734

Siris, a chain of philosophical reflections and inquiries concerning the virtues of tar-water, and divers other subjects connected together and arising one from another, 1744

letters to Prior

1686 - 1736 **Fahrenheit**

1688 - 1740 **Frederick William**

1689 - 1755 **Montesquieu**

On the spirit of the laws, or on the relation which the laws must have with the constitution of every government, with customs, climate, religion, commerce, etc., 1748

1693 - 1762 **Bradley**

An account of a newly discovered motion of the fixed stars, 1728

1694 - 1768 **Reimarus**

Apology, or treatise in defense of the rational worshiper of god, 1767

1694 - 1778 **Voltaire**

The century of Louis XIV, 1751

The philosophy of history, or a historical dissertation on the origin, manners, customs, and religions of the different nations of antiquity, with a concise exposition of their religious superstitions, 1765

- 1698 - 1739 du Fay
A letter concerning electricity, 1733
- 1698 - 1746 Maclaurin
- 1698 - 1759 **Maupertuis**
On the measure of a degree of the meridian at the polar circle, 1737
Researches into the laws of rest, 1740
Accord of different laws of nature which until now appeared incompatible, 1744
Researches into the laws of motion, 1746
- 1700 - 1782 **Daniel Bernoulli**
Hydrodynamics, or commentaries on the forces and motions of fluids, 1738
Reflections and clarifications on the new vibrations of strings, 1753
- 1706 - 1790 **Franklin**
Opinions and conjectures concerning the properties and effects of the electric matter, arising from experiments and observations made in Philadelphia, 1749
Autobiography, 1791
 letters to Collinson
- 1707 - 1778 **Linnaeus**
The system of nature, according to the three kingdoms of nature, by classes, orders, genera, and species, 1735
The genera of plants, as well as their natural characters, by number, shape, place, and proportion of all fruit bearing parts, 1737
Botanical philosophy, in which the foundations of botany are explained, with definitions of parts, examples of terms, and observations of rarities, 1751
The species of plants, showing the properly recognized plants, related by genera, with specific differences, common names, selected synonyms, and native locations, arranged by the sexual system, 1753
- 1707 - 1783 **Euler**
Mechanics, or the science of motion presented analytically, 1736
A method for discovering curves with the properties of maxima and minima, or solution of isoperimetric problems in the broadest accepted sense, 1744
Theory of the motions of planets and comets, containing an easy method of determining the orbits of planets and comets from a few observations, 1744
Introduction to the analysis of infinities, 1748
On the vibrations of strings, 1750
Theory of the motion of the moon, accounting for all inequalities, 1753
Remarks on the preceding papers by Mr Bernoulli, 1753
Instruction in the differential calculus, with its application in finite analysis and the doctrine of series, 1755
Theory of the motion of solid or rigid bodies, established from the first principles of our knowledge and adapted to all motions of bodies of this kind, 1765
Instruction in the integral calculus, 1768-70
Letters to a princess of Germany on different subjects of physics and philosophy, 1768-72
Theory of the motions of the moon, treated by a new method, 1772
On the formulas of differential angles, 1777
- 1707 - 1788 **Buffon**
Natural history, general and particular, with a description of the cabinet of the king, 1749-89, including
On the manner of study and treatment of natural history, 1749
History and theory of the earth, 1749
Proofs of the theory of the earth, 1749
Experiments on the development of heat in bodies, 1774
Further experiments on the development of heat in different mineral substances, 1774
Researches on the cooling of the earth and the planets, 1775
Epochs of nature, 1779
- 1708 - 1777 Haller

Elements of the physiology of the human body, 1757-66

Practical medical library, 1776-88

1708 - 1778 **Pitt**

1711 - 1776 **Hume**

A treatise of human nature, being an attempt to introduce the experimental method of reasoning into moral subjects, 1739

An abstract of a book lately published, entitled A treatise of human nature, etc., wherein the chief argument of that book is farther illustrated and explained, 1740

Essays, moral, political, and literary, 1741

Of the middle station of life, 1742

A letter from a gentleman to his friend in Edinburgh, containing some observations on a specimen of the principles concerning religion and morality, said to be maintained in a book lately published, entitled A treatise of human nature, etc., 1745

An inquiry concerning human understanding, 1748

An inquiry concerning the principles of morals, 1751

The history of England, 1754-62

Four dissertations, I. The natural history of religion, II. Of the passions, III. Of tragedy, IV. Of the standard of taste, 1757

A concise and genuine account of the dispute between Mr. Hume and Mr. Rousseau, with the letters that passed between them during their controversy, 1766

My own life, 1776

Dialogues concerning natural religion, 1779

1711 - 1787 **Boscovich**

A theory of natural philosophy reduced to a single law of the forces existing in nature, 1758

1712 - 1778 **Rousseau**

Discourse on the question, whether the restoration of the sciences and the arts has contributed to the purification of the morals, 1750

Discourse on the origin and foundations of inequality among men, 1755

Julie, or the new Héloïse, letters of two lovers, inhabitants of a small town at the foot of the Alps, 1761

On the social contract, or principle of political right, 1762

Émile, or on education, 1762

Rousseau judge of Jean-Jacques, a dialogue, 1780-2

Reveries of a solitary walker, 1782

The confessions, 1782-9

1712 - 1786 **Frederick II**

The general principles of war, applied to the tactics and discipline of the Prussian troops, 1747

1713 - 1784 **Diderot**

Encyclopedia, or analytic dictionary of the sciences, arts, and trades, by a society of men of letters, 1751-72

Detailed explanation of the system of human knowledge, in the Encyclopedia, 1751

Encyclopedia, in the Encyclopedia, 1755

1715 - 1780 **Condillac**

1717 - 1768 **Winckelmann**

1717 - 1780 **Maria Theresa**

1717 - 1783 **d'Alembert**

Treatise on dynamics, 1743

Researches on the curve which a tense string set in vibration forms, 1747

Preliminary discourse of the editors, in the Encyclopedia, 1751

Limit, in the Encyclopedia, 1765

1718 - 1783 **Morelly** 1718 - the decade after 1778

The code of nature, or the true spirit of the laws neglected or unknown in all times, 1755

1720 - 1793 **Bonnet**

Considerations on organized bodies, 1762

The philosophical palingenesis, 1769

1723 - 1780 Blackstone

Commentaries on the laws of England, 1765-9

1723 - 1790 **Adam Smith**

An inquiry into the nature and causes of the wealth of nations, 1776
letters to Strahan

1724 - 1804 **Kant**

Thoughts on the true estimation of living forces and assessment of the proofs used on this controversial matter by Leibniz and other mechanics, together with some preliminary observations on the force of bodies in general, 1747

Investigation of the question whether the earth in its rotation on its axis, by which it brings about the alteration of day and night, has undergone any change since the times of its origin, and how this can be established, 1754

The question whether the earth ages, considered physically, 1754

General natural history and theory of the heaven, or an attempt to treat the constitution and mechanical origin of the entire structure of the world according to Newton's laws, 1755

A new elucidation of the first principles of metaphysical knowledge, 1755

Brief exposition of some reflections on fire, 1755

History and natural description of the most remarkable occurrences of the earthquakes which struck a large part of the earth at the end of the year 1755, 1756

On the causes of the earthquakes on the occasion of the disaster which befell the western countries of Europe toward the end of last year, 1756

Further observations on the earthquakes experienced for some time, 1756

The use of metaphysics combined with geometry in natural philosophy, whose first test is the physical monadology, 1756

New remarks toward an elucidation of the theory of winds, 1756

A brief consideration of the question whether the west winds in our regions are humid because they pass over a large sea, 1757

New doctrine of motion and rest, and its consequences for the first principles of natural science, 1758

An attempt at some observations on optimism, 1759

The false subtlety of the four syllogistic figures, 1762

The only possible argument for demonstration of the existence of god, 1763

An attempt to introduce the concept of negative magnitudes in philosophy, 1763

Investigation on the clarity of the principles of natural theology and morality, 1764

Observations on the feeling of the beautiful and sublime, 1764

Essay on the diseases of the head, 1764

Dreams of a seer of spirits, explained by dreams of metaphysics, 1766

On the first ground of the difference of directions in space, 1768

On the form and principles of the sensible and intelligible world, 1770

On the different races of men, 1775

Critique of pure reason, 1781

Prolegomena to any future metaphysics which will be able to stand as science, 1783

Idea for a general history with a cosmopolitan purpose, 1784

Answer to the question, what is enlightenment? 1784

Groundwork for the metaphysics of morals, 1785

review of Herder's *Ideas for the Philosophy of the history of humanity*, 1785

On the volcanoes on the moon, 1785

On the wrongfulness of the unauthorized publication of books, 1785

Determination of the concept of a human race, 1785

The presumed beginning of human history, 1786

Metaphysical elements of natural science, 1786

What does it mean to orient oneself in thinking? 1786

On the philosophers' medicine of the body, 1786

Critique of practical reason, 1788

On the use of teleological principles in philosophy, 1788

Critique of judgment, 1790

On a discovery by which any new critique of pure reason will be made dispensable by an older one, 1790

On the failure of all philosophical attempts at theodicy, 1791

Religion within the boundaries of reason alone, 1793

On the common saying, this may be correct in theory but it is of no use for practice, 1793

Something on the influence of the moon on the weather, 1794

The end of all things, 1794

Toward perpetual peace, a philosophical draft, 1795

On a recently arisen superior tone in philosophy, 1796

Proclamation of the near conclusion of a treaty of perpetual peace in philosophy, 1796

The metaphysics of morals, 1797

On a supposed right to lie from philanthropy, 1797

The quarrel of the faculties, 1798

Anthropology considered in pragmatic respect, 1798

On the power of the mind to be master of its feeling sick by sheer resolution, 1798

On turning out books, 1798

Declaration concerning Fichte's theory of science, 1799

Logic, a manual for lectures, 1800

Physical geography, edited from manuscripts, 1802

Which is the true progress metaphysics has made in Germany since the time of Leibniz and Wolff?
1804

letters to Beck

1726 - 1797 Hutton

Abstract of a dissertation concerning the system of the earth, its duration, and stability, 1785

The theory of the earth, with proofs and illustrations, 1795

1728 - 1777 **Lambert**

Cosmological letters on the arrangement of the structure of the world, 1761

Preliminary advice for those who seek to square and rectify the circle, 1766

Memoir on some remarkable properties of circular and logarithmic transcendental quantities, 1768

Notes and comments on the drawing of maps of the earth and heaven, 1772

The theory of parallel lines, 1786

1728 - 1779 **Cook**

1728 - 1799 **Black**

Experiments on magnesia alba, quicklime, and some other alkaline substances, 1756

Lectures on the elements of chemistry, 1803

1729 - 1786 Mendelssohn

1729 - 1797 Burke

1730 - 1799 **Ingen-Housz**

Experiments on vegetables, discovering their great power of purifying the common air in sunshine, and of injuring it in the shade and at night, 1779

1731 - 1810 **Cavendish**

Three papers containing experiments on factitious air, 1766

An account of some attempts to imitate the effects of the torpedo by electricity, 1775

Experiments on air, 1784

1732 - 1799 **Washington**

1732 - 1807 Lalande

Memoir on the parallax of the sun, deduced from observations in the South Sea, in the kingdom of Astrakhan, and in China, 1772

1733 - 1804 Priestley

Observations on different kinds of air, 1772

An account of further observations on air, 1775

1735 - 1826 John Adams

1736 - 1793 Bailly

1736 - 1806 **Coulomb**

First memoir on electricity and magnetism, 1785

1736 - 1813 **Lagrange**

Essay on a new method to determine maxima and minima of indefinite integral formulas, 1762

Application of the preceding method to the solution of the different problems of dynamics, 1762

On the secular equation of the moon, 1773

Analytic mechanics, 1788

1736 - 1819 Watt

1737 - 1794 **Gibbon**

The history of the decline and fall of the Roman empire, 1776-88

Antiquities of the house of Brunswick, 1796

1737 - 1798 Galvani

Commentary on the forces of electricity in muscular motion, 1791

1737 - 1809 Paine

Rights of man, being an answer to Mr. Burke's attack on the French revolution, 1791

1738 - 1822 **Herschel**

On the proper motion of the sun and solar system, with an account of several changes that have happened among the fixed stars since the time of Mr Flamsteed, 1783

On the construction of the heavens, 1785

Catalogue of one thousand nebulae and clusters of stars, 1786

On nebulous stars properly so called, 1791

On the nature and construction of the sun and fixed stars, 1794

1740 - 1784 Lexell

1740 Montgolfier

Joseph 1740 - 1810; Étienne 1745 - 1799

1741 - 1790 **Joseph II**

1742 - 1786 **Scheele**

On manganese, or magnesia, and its properties, 1774

Chemical treatise on air and fire, 1777

letters to Lavoisier; letters to Gahn

1743 - 1794 **Lavoisier**

Physical and chemical essays, 1774

Memoir on the calcination of tin in closed vessels, and on the cause of the gain of weight which the metal acquires in this operation, 1774

Memoir on the nature of the principle which combines with metals during their calcination and augments their weight, 1775

Memoir on the existence of air in the nitrous acid, and on the means of decomposing and recomposing this acid, 1776

Memoir on the combustion of Kunckel's phosphorus and on the nature of the acid which results from this combustion, 1777

Experiments on the respiration of animals and on the changes which happen to the air passing through their lungs, 1777

On the combination of the matter of fire with evaporable fluids, and on the formation of elastic aeriform fluids, 1777

Memoir on combustion in general, 1777

General considerations on the nature of acids and on the principles of which they are composed, 1778

Experiments on the comparative effect of different combustibles, 1781

Memoir on the formation of the acid named fixed air or acid of chalk, which I will henceforth designate by the name acid of coal, 1781

Memoir on the nature of the elastic aeriform fluids which are released from some animal matters in fermentation, 1782

General considerations on the dissolution of metals in acids, 1782

Memoir whose object is to prove that water is not a simple substance, an element strictly, and is susceptible to decomposition and recombination, 1783

On the action of fire animated by vital air on the most resistant mineral substances, 1783

Memoir on the mutual precipitation of metallic substances, 1783

Memoir on the affinity of the oxygen principle to the different substances with which it is susceptible to unite, 1783

Memoir on the union of the oxygen principle with iron, 1783

New reflections on the augmentation of weight which sulfur and phosphorus acquire by burning, and on the cause to which it must be attributed, 1783

Reflections on phlogiston to serve as a continuation of the theory of combustion and calcination published in 1777, 1783

Memoir on the combination of the oxygen principle with spirit of vine, oil, and different combustible bodies, 1784

Reflections on the decomposition of water by vegetable and animal substances, 1786

Instruction on the establishment of nitreries and on the production of salpeter, 1787

General observations on the modern horizontal layers which have been deposited by the sea, and on the consequences which can be drawn from their disposition about the antiquity of the terrestrial globe, 1788

Elementary treatise on chemistry, presented in a new order and according to the modern discoveries, 1789

with Laplace

Memoir on heat, 1783

with Meusnier

Memoir in which it is proved, by the decomposition of water, that this fluid is not a simple substance, and that there are several means of obtaining large quantities of the inflammable air which enters in it as a constituent principle, 1784

1743 - 1822 Haüy

Treatise on mineralogy, 1801

1744 - 1803 Herder

Extract from an exchange of letters about Ossian and the songs of ancient peoples, 1773

Also a philosophy of history, for the education of mankind, 1774

Ideas for the philosophy of the history of mankind, 1784-91

with Goethe and Möser

On the German kind and art, 1773

1744 - 1829 Lamarck

Hydrogeology, or researches on the influences which waters have on the surface of the terrestrial globe; on the causes of the existence of the basins of the seas, their shift and their successive movement to different points on the surface of the earth, as well as the changes which living bodies exert on that surface, 1802

Researches on the organization of living bodies and particularly on their origin, on the cause of their development and progress in their composition, and on that which, tending continually to destroy it in each individual, brings about their death, 1802

Zoological philosophy, or exposition of considerations on the natural history of animals, on the diversity of their organization and the faculties they obtain from it, on the physical causes which maintain life in them and give rise to the movements they make, as well as on what produces feeling in some of them and intelligence in those endowed with it, 1809

Natural history of invertebrate animals, presenting the general and particular characters of these animals, their distribution, their classes, their families, their orders, and citation of the principal reported species; preceded by an introduction offering to determine the essential characters of the animal, its distinction from vegetable and other natural bodies, as well as exposition of the fundamental principles of zoology, 1815-22

1745 - 1827 Volta

On the electricity excited by the mere contact of conducting substances of different kinds, 1800

1746 - 1818 Monge

1748 - 1822 Berthollet

Researches on the laws of affinity, 1801

letters to Watt

1748 - 1836 Sieyès

What is the third estate? 1789

- 1749 - 1791 Mirabeau
speeches
- 1749 - 1817 Werner
On the external signs of fossils, 1774
Short classification and description of the different kinds of mountains, 1786
An attempt at an explanation of the origin of volcanoes by the kindling of great strata of coal, as a contribution to the natural history of basalt, 1789
New theory of the origin of veins, with an application to the constitution of mountains, 1791
- 1749 - 1827 **Laplace**
Theory of the attractions of spheroids and of the shape of the planets, 1785
Memoir on the secular inequalities of the planets and satellites, 1787
Theory of Jupiter and of Saturn, 1788
On the secular equation of the moon, 1788
Memoir on the theory of the ring of Saturn, 1789
Memoir on the secular variations of the orbits of the planets, 1789
On the theory of the satellites of Jupiter, 1790
Exposition of the system of the world, 1796
Treatise on celestial mechanics, 1799-825
Philosophical essay on probabilities, 1814
with Lavoisier, q. v.
- 1749 - 1832 **Goethe**
Götz von Berlichingen, 1773
The sufferings of young Werther, 1774
An intermaxillary bone in the upper jaw is to be attributed to man as well as animals, 1784
An attempt to explain the metamorphosis of plants, 1790
Contributions to optics, 1791-2
Faust, 1808-32
Toward the theory of colors, 1810
letters to Lavater
with Herder and Möser, q. v.
- 1751 - 1836 Madison
with Hamilton and Jay, q. v.
- 1752 - 1833 Legendre
Elements of geometry, with notes, 1794
Reflections on different ways to demonstrate the theory of parallels or the theorem on the sum of the three angles of the triangle, 1833
- 1753 - 1815 **Nicholson**
An account of the new electric or galvanic apparatus of Mr Volta, and experiments performed with it, 1800
- 1753 - 1823 Lazare Carnot
- 1754 - 1826 **Proust**
Researches on Prussian blue, 1794
Mines of cobalt, nickel, and others, 1806
- 1756 - 1804 Alexander Hamilton 1755 or 1757 - 1804
with Madison and Jay
The federalist, a collection of essays written in favor of the new constitution as agreed by the federal convention on 17 Sep 1787, 1788
- 1756 - 1836 **Godwin**
An inquiry concerning political justice and its influence on morals and happiness, 1793
- 1758 - 1794 **Robespierre**
- 1758 - 1805 Nelson
- 1759 - 1805 **Schiller**
The robbers, 1781
The stage considered as a moral institution, 1784
History of the separation of the United Netherlands from the Spanish government, 1788

What is called and to what end is studied universal history? 1789

History of the thirty years' war, 1791-3

On the tragic art, 1792

On the esthetic education of man, in a series of letters, 1795

On the sublime, 1801

Wallenstein's camp, The Piccolomini, and Wallenstein's death, 1801-4

Wilhelm Tell, 1804

1762 - 1814 **Fichte**

Foundation of the entire theory of science, 1794

First introduction in the theory of science, 1797

Second introduction in the theory of science, for readers who already have a philosophical system, 1797

The system of the theory of morals according to the principles of the theory of science, 1798

1766 - 1817 **Mme de Staël**

On Germany, 1810

1766 - 1834 **Malthus**

An essay on the principle of population, or a view of its past and present effects on human happiness, with an inquiry into the prospects respecting the future removal or mitigation of the evils which it occasions, 1798

1766 - 1844 **Dalton**

New theory of the constitution of mixed aeriform fluids, and particularly the atmosphere, 1801

On the absorption of gases by water and other liquids, 1803

New system of chemical philosophy, 1808-27

1767 - 1794 **Saint-Just**

On nature, on the civil state, on the city, or the rules of independence, of government, 1792

Republican institutions, 1794

1768 - 1830 **Fourier**

Analytic theory of heat, 1822

1768 - 1834 **Schleiermacher**

The Christian faith, presented in connection with the principles of the evangelical church, 1821-2

1769 - 1821 **Napoleon**

Napoleonic code, 1804

maxims, 1820

1769 - 1832 **Cuvier**

A skeleton found in Paraguay, 1796

Species of quadrupeds, 1801

Memoir on the fossil skeleton of a flying reptile from the region of Eichstadt, which some naturalists have taken to be a bird, with which we constitute the genus saurians, and which we name pterodactyl, 1809

Researches on the fossil bones of quadrupeds, in which are established the characters of the several species of animals which seem to have been destroyed in the revolutions of the globe, 1812

Discourse on the revolutions of the surface of the globe, and on the changes which they have produced in the animal kingdom, 1812

The kingdom of animals according to its organization, to serve as a foundation of the natural history of animals and as an introduction to comparative anatomy, 1817

History of fishes, 1828-49

with Brongniart

Geological description of the region of Paris, 1811

1769 - 1839 **William Smith**

A delineation of the strata of England and Wales, with part of Scotland, 1815

1769 - 1852 **Wellington**

1770 - 1831 **Hegel**

The system of science, part one, the phenomenology of spirit, 1807

Announcement, 1807

The science of logic, 1812-6

- Encyclopedia of the philosophical sciences in outline*, 1817
Natural right and political science in outline, elements of the philosophy of right, 1821
Lectures on the history of philosophy, 1833-6
Lectures on the philosophy of history, 1840
- 1770 - 1731 Seebeck
Magnetic polarization of metals and ores by a difference in temperature, 1822
- 1772 - 1844 Geoffroy Saint-Hilaire
Anatomical philosophy, 1818-22
- 1773 - 1829 **Young**
On the theory of light and colors, 1801
An account of some cases of the production of light not hitherto described, 1802
A course of lectures on natural philosophy and the mechanical arts, 1807
- 1773 - 1858 **Robert Brown**
A brief account of microscopic observations, made in the months of June, July, and August 1827, on the particles contained in the pollen of plants; and on the general existence of active molecules in organic and inorganic bodies, 1828
Observations on the organs and mode of fecundation in orchideae and asclepiadeae, 1831
- 1774 - 1862 Biot
 with Savart
Note on the magnetism of Volta's battery, 1820
- 1775 - 1812 Malus
Memoir on a new phenomenon of optics, 1808
Theory of the double refraction of light in crystal substances, 1811
- 1775 - 1836 **Ampère**
Experiments on new electrodynamic phenomena, 1822
Memoir on the determination of the formula which represents the mutual action of two infinitely small portions of voltaic conductors, 1822
Mathematical theory of electrodynamic phenomena, uniquely deduced from experience, 1827
Essay on the philosophy of the sciences, or analytic exposition of a natural classification of all human knowledge, 1834-43
- 1775 - 1854 Schelling
Ideas for a philosophy of nature, as an introduction to the study of this science, 1797
- 1776 - 1856 **Avogadro**
Essay on a means of determining the relative masses of the elementary molecules of bodies, and the proportions in which they enter into these compounds, 1811
- 1777 - 1851 **Ørsted**
Experiments on the effect of electric conflict on a magnetic needle, 1820
- 1777 - 1855 **Gauss**
Arithmetical disquisitions, 1801
Theory of the motion of the heavenly bodies moving in conic sections around the sun, 1809
Determination of the attraction when, in place of a planet at a given point, its mass is distributed through its orbit in proportion to the time it takes to describe any one part, 1818
General solution of the problem, to represent the parts of one given surface on another given surface so that the representation be similar to the represented in its smallest parts, 1822
General investigations of curved surfaces, 1827
General principles of the theory of the shape of fluids in the state of equilibrium, 1829
Notice on the theory of biquadratic residues, 1831
The intensity of the magnetic force of the earth referred to an absolute measure, 1832
General theory of the magnetism of the earth, 1839
General theorems on the forces of attraction and repulsion acting inversely as the square of the distance, 1840
 letters to Bessel; letters to Wolfgang Bolyai; letters to Schumacher; letters to Taurinus
- 1778 - 1829 Davy
- 1778 - 1850 **Gay-Lussac**
Researches on the expansion of gases by heat, 1802

- Report of an aerostatic voyage*, 1804
Memoir on the combination of gaseous substances with each other, 1808
Researches on prussic acid, 1815
- 1779 - 1848 **Berzelius**
Essay on the theory of chemical proportions and on the chemical influence of electricity, 1819
- 1780 - 1831 **Clausewitz**
On war, 1832
- 1781 - 1840 **Poisson**
Remarks on an equation which presents itself in the theory of attraction of spheroids, 1813
- 1784 - 1846 **Bessel**
Determination of the axes of the elliptic spheroid of revolution corresponding best to the available measurements of meridian arcs, 1837
Determination of the distance of the 61st star of Swan, 1838
 letters to John Herschel
- 1786 - 1853 **Arago**
- 1787 - 1826 **Fraunhofer**
Determination of the refractive and color-dispersing power of different types of glass, in relation to the improvement of achromatic telescopes, 1817
- 1788 - 1827 **Fresnel**
Complement to the memoir on the diffraction of light, 1815
Memoir on the influence of polarization on the action which rays of light exert on one another, 1816
Note on the theory of colors in polarization developed by small slits, 1818
Memoir on the diffraction of light, 1819
Note on the calculation of colors in polarization developed by crystal slits, 1821
- 1788 - 1860 **Schopenhauer**
The world as will and representation, 1819
On the freedom of man's will, 1839
- 1788 - 1867 **Poncelet**
Treatise on the projective properties of figures, 1822
- 1788 - 1868 **Boucher de Perthes**
On primitive industries, or on the arts and their origin, 1846
On Celtic and antediluvian antiquities, 1847-64
- 1789 - 1854 **Ohm**
The galvanic chain treated mathematically, 1827
- 1789 - 1857 **Cauchy**
Course in analysis at the royal polytechnic school, part I, algebraic analysis, 1821
Summary of the lessons given at the royal polytechnic school on the infinitesimal calculus, 1823
- 1790 - 1832 **Champollion**
- 1790 - 1868 **Möbius**
- 1791 - 1867 **Faraday**
Experimental researches in electricity, 1839-55, including
On some new electro-magnetic motions, and on the theory of magnetism, 1821
Description of an electro-magnetic apparatus for the exhibition of rotary motion, 1822
On the evolution of electricity from magnetism, 1831
New electric state or condition of matter, 1831
Explication of Arago's magnetic phenomena, 1831
Identity of electricities derived from different sources, 1833
Preliminary, 1834
On the definite nature and extent of electro-chemical decomposition, 1834
On the absolute quantity of electricity associated with the particles or atoms of matter, 1834
Induction an action of contiguous particles, 1837
Induction in curved lines, 1837
On specific induction, or specific inductive capacity, 1837
General results as to induction, 1837
An answer to Dr Hare's Letter on certain theoretical opinions, 1840

A speculation touching electric conduction and the nature of matter, 1844
Action of magnets on light, 1845
General considerations, 1845
On new magnetic actions, and on the magnetic condition of all matter, 1845
Magnetic conduction, 1850
On the lines of magnetic force, 1852
On the physical character of the lines of magnetic force, 1852
Some points of magnetic philosophy, 1855

1792 - 1856 **Lobachevsky**

On the principles of geometry, 1829
Imaginary geometry, 1835
New principles of geometry, with a complete theory of parallels, 1836-8
Geometrical investigations on the theory of parallels, 1840
Pangeometry, 1856

1792 - 1866 Hincks

1792 - 1876 **von Baer**

On the mammalian egg and the origin of man, 1827
On the developmental history of the animals, observation and reflection, 1828-37

1796 - 1832 **Carnot**

Reflections on the motive power of fire and on the proper engines to develop this power, 1824
Notes on mathematics, physics, and other subjects, 1832

1797 - 1875 Lyell

Principles of geology, or the modern changes of the earth and its inhabitants considered as illustrative of geology, 1830-3

1798 - 1857 Comte

Course in positive philosophy, 1830-42
System of positive politics, or treatise on sociology, instituting the religion of humanity, 1851-4
Positive catechism, or summary exposition of the universal religion, in thirteen systematic conversations between a woman and a priest of humanity, 1852

1800 - 1873 Dumas

with Liebig

Note on the present state of organic chemistry, 1837

1802 - 1829 Abel

1802 - 1860 **Bolyai**

True science of absolute space, independent of the truth or falsehood of Euclid's postulate, which can never be decided a priori, 1832

1803 - 1873 Liebig

Chemistry in its application to agriculture and physiology, 1840

with Dumas, q. v.

1804 - 1851 Jacobi

1804 - 1865 Lenz

On the determination of the direction of the galvanic current excited by electrodynamic induction, 1834

1804 - 1881 Schleiden

Contributions to phytogenesis, 1838
Botany as an inductive science, 1842
Schelling's and Hegel's relation to natural science, 1844

1805 - 1859 Dirichlet

1805 - 1865 **Hamilton**

Theory of systems of rays, 1827
On a view of mathematical optics, 1831
On a general method in dynamics, by which the study of the motions of all free systems of attracting or repelling points is reduced to the search and differentiation of one general relation, or characteristic function, 1834

1807 - 1873 Agassiz

1808 - 1873 Napoleon III

1809 - 1882 **Darwin**

On the formation of mold, 1837

On the connection of certain volcanic phenomena in South America; and on the formation of mountain chains and volcanoes as an effect of the same power by which continents are elevated, 1838

Journal of researches into the geology and natural history of the various countries visited by H.M.S. Beagle, under the command of captain FitzRoy, from 1832 to 1836, 1839

Observations on the parallel roads of Glen Roy, and of other parts of Lochaber in Scotland, with an attempt to prove that they are of marine origin, 1839

On the distribution of the erratic boulders and on the contemporaneous unstratified deposits of South America, 1841

The structure and distribution of coral reefs, 1842

Notes on the effects produced by the ancient glaciers of Caernarvonshire, and on the boulders transported by floating ice, 1842

Geological observations on the volcanic islands visited during the voyage of H.M.S. Beagle, together with some brief notices of the geology of Australia and the cape of Good Hope, 1844

Geological observations on South America, 1846

On the geology of the Falkland Island, 1846

Geology, in John Herschel's *Manual of scientific inquiry, prepared for the use of Her Majesty's navy, and adapted for travelers in general*, 1849

A monograph of the sub-class cirripedia, with figures of all species, 1851-4

Extract from an unpublished work on species, consisting of a portion of a chapter entitled, On the variation of organic beings in a state of nature; on the natural means of selection; on the comparison of domestic races and true species, 1858

The origin of species by means of natural selection, or the preservation of favored races in the struggle for life, 1859

On the two forms, or dimorphic condition, in the species of primula, and on their remarkable sexual relations, 1861

The various contrivances by which orchids are fertilized by insects, 1862

On the sexual relations of the three forms of lythrum salicaria, 1864

The power of movement in plants, 1865

The variation of animals and plants under domestication, 1868

The descent of man, and selection in relation to sex, 1871

The expression of emotions in man and animals, 1872

Insectivorous plants, 1875

The effects of cross- and self-fertilization in the vegetable kingdom, 1876

Recollections of the development of my mind and character, 1876

The different forms of flowers on plants of the same species, 1877

Biographical sketch of an infant, 1877

The formation of vegetable mold, through the action of worms, with observations on their habits, 1881

The action of carbonate of ammonia on the roots of certain plants, 1882

The action of carbonate of ammonia on chlorophyl-bodies, 1882

notes in the *Athenaeum*

letters to Nägeli

1810 - 1861 **Cavour**

1810 - 1882 **Schwann**

Microscopic investigations on the agreement in the structure and growth of animals and plants, 1838

1810 - 1895 Rawlinson

The Persian cuneiform inscription at Behistun, deciphered and translated; with a memoir on Persian cuneiform inscriptions in general, and on that of Behistun in particular, 1846-51

The cuneiform inscriptions of western Asia, 1861-75

1811 - 1832 **Galois**

- 1811 - 1899 **Bunsen**
with Kirchhoff, *q. v.*
- 1814 - 1876 **Bakunin**
- 1815 - 1864 **Boole**
The mathematical analysis of logic, 1847
An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities, 1854
- 1815 - 1897 **Weierstrass**
Introduction in the theory of analytic functions, Berlin lectures, 1878
- 1815 - 1898 **Bismarck**
- 1818 - 1883 **Marx**
Economic-philosophical manuscripts from 1844 ad Feuerbach, 1845
The class struggles in France from 1848 to 1850, 1850
The eighteenth Brumaire of Louis Bonaparte, 1852
Contribution to the critique of political economy, 1859
The capital, critique of political economy, 1867
letters to Engels; letters to Lassalle; letters to Weydemeyer
with Engels
The German ideology, critique of the newest German philosophy in its representatives Feuerbach, B. Bauer, and Stirner, and of the German socialism in its different prophets, 1846
Manifesto of the communist party, 1848
- 1818 - 1889 **Joule**
On the heat evolved by metallic conductors of electricity and in the cells of a battery during electrolysis, 1841
On the calorific effects of magneto-electricity, and on the mechanical value of heat, 1843
On the existence of equivalent relation between heat and the ordinary forms of mechanical power, 1845
On the mechanical equivalent of heat, 1850
- 1819 - 1868 **Foucault**
A physical demonstration of the motion of rotation of the earth by means of a pendulum, 1851
Experimental determination of the speed of light, and the parallax of the sun, 1862
- 1820 - 1895 **Engels**
Herr Eugen Dühring's upheaval in science, 1878
Dialectic of nature, 1882
The origin of the family, private property, and the state, in connection with the investigations of Lewis H. Morgan, 1884
Ludwig Feuerbach and the end of classical German philosophy, 1886
Contribution to the history of early Christianity, 1895
letters to Mehring
with Marx, *q. v.*
- 1820 - 1903 **Spencer**
- 1821 - 1894 **Helmholtz**
On the conservation of force, 1847
On Goethe's natural-scientific works, 1853
On the interaction of the forces of nature and the newest results in physics concerning it, 1854
Handbook of physiological optics, 1856-67
On the integrals of the hydrodynamic equations to which the motions of vortices conform, 1858
On the relation of the natural sciences to all science, 1862
The theory of the perception of tone as the foundation of the theory of music, 1863
The new advances in the theory of vision, 1868
On the goal and advances of natural science, 1869
On the theory of electrodynamics, 1870-4
On the origin of the planetary system, 1871
The thermodynamics of chemical processes, 1882

- Recollections*, 1891
- 1821 - 1895 Cayley
- 1821 - 1902 Virchow
Cellular pathology grounded in physiological and pathological histology, 1858
- 1822 - 1884 **Mendel**
Experiments on plant hybrids, 1865
letters to Nägeli
- 1822 - 1888 **Clausius**
On the moving force of heat and the laws which can be derived from it for the theory of heat, 1850
On a modified form of the second fundamental law of the mechanical theory of heat, 1854
On the kind of motion which we call heat, 1857
On the application of the law of the equivalence of transformations to internal work, 1862
On different convenient for application forms of the main equations of the mechanical theory of heat, 1865
- 1822 - 1895 Pasteur
- 1823 - 1913 Wallace
On the tendency of varieties to depart indefinitely from the original type, 1858
- 1824 - 1887 Kirchhoff
with Bunsen
Chemical analysis by the observation of spectra, 1860
- 1824 - 1907 **Kelvin**
On an absolute thermometric scale, founded on Carnot's theory of the motive power of heat, and calculated from the results of Regnault's experiments on steam, 1848
On the dynamical theory of heat, with numerical results deduced from Mr Joule's Equivalent of a thermal unit, and M Regnault's Observations on steam, 1851
On a universal tendency in nature to the dissipation of mechanical energy, 1852
On the secular cooling of the earth, 1862
The age of the earth as an abode fitted for life, 1897
- 1825 - 1905 Oppert
Sumerian or Akkadian? 1875
- 1826 - 1866 **Riemann**
On the representation of a function by a trigonometric series, 1853
On the hypotheses which lie at the foundation of geometry, 1854
- 1831 - 1879 **Maxwell**
On Faraday's lines of force, 1855
On physical lines of force, 1861
A dynamical theory of the electromagnetic field, 1864
Address to the mathematical and physical sections of the British association, 1870
Theory of heat, 1871
On the mathematical classification of physical quantities, 1871
A treatise on electricity and magnetism, 1873
Matter and motion, 1877
- 1831 - 1916 **Dedekind**
Continuity and irrational numbers, 1872
What are and what should the numbers? 1888
- 1832 - 1920 Wundt
- 1833 - 1911 Dilthey
Introduction to the spiritual sciences, 1883
The types of world-view and their formation in the metaphysical systems, 1911
- 1834 - 1907 **Mendeleev**
Correlation of the properties and atomic weights of the elements, 1869
- 1834 - 1914 Weismann
The continuity of the cell plasm as a foundation of a theory of inheritance, 1885
- 1843 - 1910 Koch
- 1844 - 1900 **Nietzsche**

Untimely observations, second part; on the usefulness and disadvantage of history for life, 1874
Beyond good and evil, prelude to a philosophy of the future, 1886
Toward the genealogy of morality, a polemic, 1887

1844 - 1906 **Boltzmann**

1844 - 1918 Wellhausen

Prolegomena to the history of Israel, 1878

1845 - 1918 **Cantor**

On a property of the totality of all real algebraic numbers, 1874

A contribution to the theory of aggregates, 1878

On infinite linear aggregates of points, 1879-84

On the different points of view with respect to the actual infinite, 1885

On an elementary question in the theory of aggregates, 1891

Contributions to the foundation of the transfinite theory of sets, 1895-7
 letters to Dedekind

1848 - 1925 **Frege**

On a geometrical representation of the imaginary forms in the plane, 1873

Methods of calculation grounded in an extension of the concept of quantity, 1874

Concept writing, a formula language of pure thinking modeled on that of arithmetic, 1879

On applications of the concept writing, 1879

On the goal of the concept writing, 1882

On the scientific justification of the concept writing, 1882

Geometry of pairs of points in the plane, 1883

The foundations of arithmetic, a logico-mathematical investigation of the concept of number, 1884

On formal theories in arithmetic, 1886

Function and concept, 1891

On the law of inertia, 1891

On concept and object, 1892

On sense and reference, 1892

The laws of arithmetic, derived by the concept writing, 1893-903

The whole number, 1895

On the concept writing of Mr Peano and my own, 1896

On the foundations of geometry, 1903-6

What is a function? 1904

The thought, a logical investigation, 1919

Negation, a logical investigation, 1919

letters to Russell

1849 - 1925 **Klein**

On the so-called non-Euclidean geometry, 1871-3

Comparative observations on the recent investigations in geometry, 1872

Elementary mathematics from a higher standpoint, 1908

1852 - 1912 **Meiji**

1853 - 1928 Lorentz

1854 - 1912 Poincaré

Analysis situs, 1895

Last thoughts, 1913

1856 - 1939 **Freud**

The interpretation of dreams, 1900

1856 - 1940 **Thomson**

Cathode rays, 1897

1857 - 1894 Hertz

On the fundamental equations of electromagnetics for bodies at rest, 1890

On the fundamental equations of electromagnetics for bodies in motion, 1890

The principles of mechanics, in new interdependence, 1894

1858 - 1932 **Peano**

The principles of arithmetic presented by a new method, 1889

1858 - 1947 **Planck**

Lectures on thermodynamics, 1897
On the theory of the law of the distribution of energy in the normal spectrum, 1900
The unity of the physical picture of the world, 1908
Eight lectures on theoretical physics, 1909
The principle of least action, 1914

1861 - 1947 Whitehead

Process and reality, an essay in cosmology, 1929
Adventures of ideas, 1933
 with Russell, q. v.

1862 - 1915 Boveri

On multipolar mitosis as a means of analysis of the cell nucleus, 1902

1862 - 1943 **Hilbert**

Foundations of geometry, 1899
Mathematical problems, 1900
Axiomatic thinking, 1918
The new grounding of mathematics, first statement, 1922
The logical foundations of mathematics, 1923
On the infinite, 1925
Knowledge of nature and logic, 1930

1864 - 1909 **Minkowski**

Space and time, 1908

1864 - 1920 Max Weber

The Protestant ethics and the spirit of capitalism, 1905

1866 - 1925 **Sun Yat-sen**

1866 - 1945 Morgan

Chromosomes and heredity, 1910

1866 - 1952 Croce

History reduced to the general concept of art, 1893

1867 Wright

Wilbur 1867 - 1912; Orville 1871 - 1948

1868 - 1942 Hausdorff

1870 - 1924 **Lenin**

The infantile disorder of leftism in communism, 1920

1871 - 1937 **Rutherford**

The radiation and emanation of radium, 1904
Radium – the cause of the earth's heat, 1905
Some cosmical aspects of radioactivity, 1907

1871 - 1953 **Zermelo**

Proof that every set can be well-ordered, 1904
Investigations on the foundations of the theory of sets, 1908

1872 - 1970 **Russell**

The principles of mathematics, 1903
On denoting, 1905
Mathematical logic as based on the theory of types, 1908
Knowledge by acquaintance and knowledge by description, 1911
The problems of philosophy, 1912
Our knowledge of the external world as a field for scientific method in philosophy, 1914
The philosophy of logical atomism, 1918
Introduction to mathematical philosophy, 1919
How to read and understand history, the past as the key to the future, 1943
My mental development, 1944
A history of western philosophy, 1945
Is there a god? 1952
My philosophical development, 1959
 letters to Frege

- with Whitehead
Principia mathematica, 1910-3
- 1873 - 1967 **Hertzprung**
On the radiation of stars, 1905-7
- 1877 - 1916 Sutton
The chromosomes in heredity, 1903
- 1877 - 1947 **Hardy**
A course of pure mathematics, 1908
Mendelian proportions in a mixed population, 1908
On the zeroes of Riemann's $\zeta(s)$ function, 1914
A mathematician's apology, 1940
- 1878 - 1956 Łukasiewicz
Aristotle's syllogistic from the standpoint of modern formal logic, 1951
- 1879 - 1955 **Einstein**
On the motion of particles suspended in liquids at rest required by the molecular kinetic theory of heat, 1905
On the electrodynamics of moving bodies, 1905
Does the inertia of a body depend on its energy content? 1905
On the inertia of energy required by the principle of relativity, 1907
On the principle of relativity and the conclusions drawn from it, 1907
The principle of relativity and its consequences in modern physics, 1910
On the influence of gravitation on the propagation of light, 1911
The speed of light and the statics of the gravitational field, 1912
On the theory of the static gravitational field, 1912
On the present state of the problem of gravitation, 1913
The formal foundation of the general theory of relativity, 1914
On the general theory of relativity, 1915
The field equations of gravitation, 1915
The foundation of the general theory of relativity, 1916
Hamilton's principle and the general theory of relativity, 1916
Cosmological considerations on the general theory of relativity, 1917
On the special and the general theory of relativity, a popular account, 1917
Geometry and experience, 1921
Non-Euclidean geometry and physics, 1925
The problem of space, aether, and the field in physics, 1930
Physics and reality, 1936
- with Grossmann
Outline of a generalized theory of relativity and of a theory of gravitation, 1913
- 1880 - 1930 Wegener
- 1880 - 1959 **Chetverikov**
On certain aspects of the evolutionary process from the point of view of contemporary genetics, 1926
- 1881 - 1966 Brouwer
Intuitionism and formalism, 1912
- 1882 - 1944 Eddington
The internal constitution of the stars, 1926
- with Dyson and Charles Davidson
A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of 29 May 1919, 1920
- 1883 - 1946 Keynes
- 1885 - 1962 **Bohr**
The quantum postulate and the recent development of atomic theory, 1927
Discussion with Einstein on epistemological problems in atomic physics, 1949
- 1887 - 1961 **Schrödinger**
Quantization as an eigenvalue problem, 1926
Four lectures on wave mechanics, 1928

- 1888 - 1925 **Friedmann**
On the curvature of space, 1922
- 1889 - 1945 **Hitler**
My struggle, 1925-7
- 1889 - 1951 Wittgenstein
Notebooks 1914-6
Logico-philosophical treatise, 1921
Lecture on ethics, 1929
Philosophical investigations, 1953
- 1889 - 1953 **Hubble**
Cepheids in spiral nebulae, 1925
Extragalactic nebulae, 1926
A relation between distance and radial velocity among extragalactic nebulae, 1929
The realm of the nebulae, 1936
- 1889 - 1976 Heidegger
Being and time, 1927
What is metaphysics? 1929
Introduction to metaphysics, 1935
The question about the thing, contribution to Kant's theory of the transcendental principles, 1935
What is it – philosophy? 1955
- 1891 - 1970 Carnap
The logical structure of the world, 1928
Pseudoproblems in philosophy, the extrapsychological and the realism controversy, 1928
Elimination of metaphysics through logical analysis of language, 1932
- 1891 - 1970 Sturtevant
The linear arrangement of six sex-linked factors in Drosophila, as shown by their mode of association, 1913
- 1892 - 1987 de Broglie
- 1894 - 1966 Lemaître
A homogenous universe of constant mass and increasing radius accounting for the radial velocity of extragalactic nebulae, 1927
The primeval atom hypothesis and the problem of the clusters of galaxies, 1958
- 1894 - 1980 Oparin
The origin of life on earth, 1936
- 1900 - 1958 Pauli
- 1901 - 1954 Fermi
- 1901 - 1976 **Heisenberg**
On the visual content of quantum theoretical kinematics and mechanics, 1927
The physical principles of quantum theory, 1930
On the history of the physical explanation of nature, 1932
Physics and philosophy, the revolution in modern science, 1958
- 1901 - 1983 Tarski
The semantic conception of truth and the foundations of semantics, 1944
- 1902 - 1984 Dirac
The principles of quantum mechanics, 1930
- 1902 - 1985 Braudel
The Mediterranean and the Mediterranean world in the age of Philip II, 1949
Material civilization, economy, and capitalism, 15th-18th century, 1968-79
- 1902 - 1994 Popper
The logic of scientific discovery, 1934
On the sources of knowledge and of ignorance, 1960
Epistemology without a knowing subject, 1967
- 1903- 1989 Beadle
 with Tatum
Genetic control of biochemical reactions in neurospora, 1941

- 1903 - 1989 Lorenz
Observations on the recognition of instinctive behavior characteristic of the species in birds, 1932
The companion in the environment of the bird, 1935
Kant's theory of the a priori in the light of contemporary biology, 1941
The so-called evil, toward a natural history of aggression, 1963
- 1904 - 1968 Gamow
The creation of the universe, 1952
My world line, 1970
- 1906 - 1978 Gödel
Some metamathematical results on completeness and consistency, 1930
On formally undecidable propositions of Principia mathematica and related systems, 1931
The consistency of the axiom of choice and of the generalized continuum hypothesis, 1938
Russell's mathematical logic, 1944
- 1907 - 1981 Yukawa
- 1909 - 1975 Tatum
 with Beadle, q. v.
- 1910 - 1976 Monod
Chance and necessity, an essay on the natural philosophy of modern biology, 1970
 with Jacob
Genetic regulatory mechanisms in the synthesis of proteins, 1961
- 1912 - 1954 Turing
- 1916 - 2004 Crick
 with Watson
Molecular structure of nucleic acids, a structure for deoxyribose nucleic acid, 1953
- 1918 - 1988 Feynman
Lectures on physics, 1963-5
QED, the strange theory of light and matter, 1985
- 1920 - 2013 Jacob
 with Monod, q. v.
- 1925 - 2012 Mellaart
Çatal Hüyük, a Neolithic town in Anatolia, 1967
- 1928 - 2012 Woese
 with Fox
Phylogenetic structure of the procaryotic domain; the primary kingdoms, 1977
 with Kandler and Wheelis
Toward a natural system of organisms; proposal for the domains archaea, bacteria, and eucarya, 1990
- 1928 - 2025 Watson
 with Crick, q. v.
- 1929 - 2019 Gell-Mann
The eightfold way, a theory of strong interaction symmetry, 1961

poets

- 2103 Inanna compositions the century before c. 2013
The marriage of Dumuzi and Inanna
Inanna's descent to the netherworld
The death of Dumuzi
- 2022 poem beginning, I opened my mouth to my soul – the century before c. 1932
- 1684 *Etana* the century before c. 1594
- 714 Terpander victory at the Carnea in 676/3
- 688 Archilochus sings of an eclipse which occurred in 648
- 688 Tyrtaeus contemporary with Archilochus
 fragments
- 684 Alcman the two generations after 676/3

58*

enumeration and review

619	Alcaeus	one generation after Pittacus
610	Aesop	under Amasis, prince 570-26
610	Sappho	contemporary with Aesop
578	Anacreon	contemporary with Polycrates
556 - 466	Simonides	c. 556 - c. 466
556	Epicharmus	the two generations before 486
525 - 456	Aeschylus	c. 525 - 456
518 - 441	Pindar	518 - the decade after 446
496 - 406	Sophocles	497/5 - 406
494 - 428	Phidias	under Pericles, general in 454 - the decade after 433
485 - 406	Euripides	c. 485 - 406
	<i>Hecuba</i>	
	<i>Orestes</i>	
450 - 386	Aristophanes	c. 450 - 386
	<i>Peace</i>	
402	Praxiteles	worked in 364/1
370	Apelles	worked in 332/29
70 - 19	Virgil	
65 - 8 ^{BC}	Horace	
43 ^{BC} - 17 ^{AD}	Ovid	
120 ^{AD} - 190	Lucian	115/25 - c. 190
	<i>Eunuch</i>	
	<i>Dialogues of the dead</i>	
1452 - 1519	Leonardo	
1474 - 1533	Ariosto	
	<i>The furious Orlando</i> , 1516	
1475 - 1564	Michelangelo	
1489 - 1576	Titian	1488/90 - 1576
1508 - 1580	Palladio	
1524 - 1585	Ronsard	
1547 - 1616	Cervantes	
	<i>The ingenious hidalgo Don Quixote de la Mancha</i> , 1605-15	
1600 - 1681	Calderón	
	<i>Life is a dream</i> , 1635	
1660 - 1731	Defoe	
	<i>The life and strange surprising adventures of Robinson Crusoe, of York, a mariner</i> , 1719	
1729 - 1781	Lessing	
	<i>Laocoön, or on the limits of painting and poetry</i> , 1766	
1756 - 1791	Mozart	
1770 - 1827	Beethoven	
1788 - 1824	Byron	

not important

2605	Gunidu	father of Ur-Nanshe
2515	Zuzu	prince c. 2475
2500	Enbi-Eshtar	prince c. 2460
2455	Barairnum	daughter of Ur-Luma
2410	Uu	prince c. 2370, father of Lugalzagesi
2255 - 2212	Imta	prince c. 2215-2
2252 - 2206	Inkishush	prince c. 2212-06
2246 - 2200	Sarlagab	prince c. 2206-0
2240 - 2194	Shulme	prince c. 2200-194
2234 - 2188	Silulumesh	prince c. 2194-88
2232 - 2192	Eluldan	prince c. 2192

	inscriptions	
2232 - 2192	Igigi	prince <i>c.</i> 2192
2232 - 2192	Imi	prince <i>c.</i> 2192
2232 - 2192	Nanum	prince <i>c.</i> 2192
2228 - 2183	Inimabakesh	prince <i>c.</i> 2188-3
2223 - 2177	Igeshaush	prince <i>c.</i> 2183-77
2217 - 2162	Yarlagab	prince <i>c.</i> 2177-62
2202 - 2159	Ibate	prince <i>c.</i> 2162-59
2199 - 2156	Yarla	prince <i>c.</i> 2159-6
2196 - 2155	Kurum	prince <i>c.</i> 2156-5
2195 - 2152	Apilkin	prince <i>c.</i> 2155-2
2190 - 2148	Irarum	prince <i>c.</i> 2150-48
2188 - 2147	Ibranum	prince <i>c.</i> 2148-7
2187 - 2145	Hablum	prince <i>c.</i> 2147-5
2185 - 2139	Kuda	prince <i>c.</i> 2145-39
2180	Shimpishuk	father of Puzur-Inshushinak
2179 - 2134	Puzur-Ili	prince <i>c.</i> 2139-4
2174 - 2128	Ur-Utu	prince <i>c.</i> 2134-28
2170 - 2138	Puzur-Sin	prince <i>c.</i> 2145-38
2164 - 2124	Tirigan	prince <i>c.</i> 2124
2103	Anubanini	the century before Ibbi-Sin
	inscriptions	
2103	Barahenidu	the century before Ibbi-Sin
	inscriptions	
2103	Eiginimpae	the century before Ibbi-Sin
	inscriptions	
2103	Lugalatu	the century before Ibbi-Sin
	inscriptions	
2103	Lugaldalu	the century before Ibbi-Sin
	inscriptions	
2103	Lugalsilasi	the century before Ibbi-Sin
	inscriptions	
2103	Medurba	the century before Ibbi-Sin
	inscriptions	
2103	Uhub	the century before Ibbi-Sin
	inscriptions	
1940	Ikunu	prince <i>c.</i> 1900, son of Erishu
	inscriptions	
1934 - 1880	Sumuabum	prince <i>c.</i> 1894-80
1920	Sargon	prince <i>c.</i> 1880
	inscriptions	
1900	Puzur-Ashur II	prince <i>c.</i> 1860
1880	Naram-Sin	prince <i>c.</i> 1840
1860	Erishu II	prince <i>c.</i> 1820
1819 - 1759	Ishme-Dagan	prince <i>c.</i> 1776-59, son of Shamshi-Adad
1690	Horemkhauf	the century around <i>c.</i> 1690
	funeral stele	
1630	Ahmose	under Apepi, prince <i>c.</i> 1590
1630	Shamshi-Adad III	prince <i>c.</i> 1590
	inscriptions	
1570	Enlil-nasir	prince <i>c.</i> 1530
	inscriptions	
1430 - 1400	Ashur-rim-nisheshu	prince <i>c.</i> 1408-0, son of Ashur-bel-nisheshu
1410 - 1390	Ashur-nadin-ahhe II	prince <i>c.</i> 1400-390
1250 - 1203	Ashur-nadin-apli	prince <i>c.</i> 1206-3, son of Tukulti-Ninurta

1245 - 1192	Enlil-kudurri-usur	prince <i>c.</i> 1197-2, son of Tukulti-Ninurta
1230 - 1197	Ashur-nirari III	prince <i>c.</i> 1203-97
1230 - 1179	Ninurta-apil-Ekur	prince <i>c.</i> 1192-79, father of Ashur-dan
1185 - 1133	Ninurta-Tukulti-Ashur	prince 1133, son of Ashur-dan
1180 - 1133	Mutakil-Nusku	prince 1133, son of Ashur-dan, father of Ashur-resh-ishi
1100 - 1074	Ashared-apil-Ekur	prince 1076-4, son of Tiglath-pileser
1090 - 1050	Shamshi-Adad IV	prince 1054-0, son of Tiglath-pileser
1075 - 1054	Eriba-Adad II	prince 1056-4, son of Ashur-bel-kala
1070 - 1031	Ashur-nasirpal	prince 1050-31
1050 - 1019	Shalmaneser II	prince 1031-19
1045 - 972	Ashur-rabi II	prince 1013-972
1030 - 1013	Ashur-nirari IV	prince 1019-3
1010 - 967	Ashur-resh-ishi II	prince 972-67
990 - 935	Tiglath-pileser II	prince 967-35, father of Ashur-dan II
938	Nebneteru	one generation after Sheshonq
	funeral statue	
850	Ishpuini	prince <i>c.</i> 810, son of Sarduri
830 - 773	Shalmaneser IV	prince 783-73, son of Adad-nirari III
825 - 755	Ashur-dan III	prince 773-55
805 - 745	Ashur-nirari V	prince 755-45
788 - 734	Nabonassar	prince 748-34
730	Alcarnenes	prince <i>c.</i> 690
725	Anaxandridas	prince <i>c.</i> 685, son of Theopompus
705	Polydorus	prince <i>c.</i> 665
700	Archidamus	prince <i>c.</i> 660
687	observation of lunar eclipses	beginning in 647
680	Ardys	prince <i>c.</i> 635, son of Gyges
680	Eurycrates	prince <i>c.</i> 640
680	Hilkiah	priest under Josiah, prince 640-09
680	Shaphan	scribe under Josiah
678	<i>Nahum</i>	prophecy between 664 and 612
675	Anaxilas	prince <i>c.</i> 635
672	Megacles	archon <i>c.</i> 632
666	100-odd prognoses	the two centuries around 626
666	30-odd prognoses	the two centuries around 626
660	Sadyattes	prince <i>c.</i> 625, father of Alyattes
652	Teutames	father of Bias
650	Leotychidas	prince <i>c.</i> 610
649	Phrynon	contemporary with Pittacus
634	Cleobulus	contemporary with Solon
630	Eurycratidas	prince <i>c.</i> 590
628	<i>Ruth</i>	the century before 538
625	Hippocratides	prince <i>c.</i> 585
622	Myson	contemporary with Anacharsis
620	Orphic poems	the century before Pythagoras
616	Jehoiachin	prince at 18 in 598
601	Megacles	contemporary with Pisistratus
600	Leon	prince <i>c.</i> 565, father of Anaxandridas
600	Mnesarchus	father of Pythagoras
595	Agariste	daughter of Cleisthenes of Sicyon
590	Agasicles	prince <i>c.</i> 560, father of Ariston
570	Leon of Phlius	contemporary with Pythagoras
565 - 485	Hippias	<i>c.</i> 565 - the decade after 490
563	observation of lunar eclipses	from 523 to 491
560	<i>Zechariah</i>	prophecy began in 520

558	Onomacritus	expelled from Athens between 527 and 510
555	Milo	born <i>c.</i> 555
554	Atossa	daughter of Cyrus
550	Megabyzus	contemporary with Darius
550	Otanes	contemporary with Darius
541	Cleostratus	between Anaximander and Meton
540 - 475	Demaratus	prince <i>c.</i> 515-491, son of Ariston - the decade after 480
540	Hermodorus	contemporary with Heraclitus
538 - 491	Hippocrates	tyrant <i>c.</i> 498-1
529 - 467	Aristides	archon 489 - <i>c.</i> 467
523	Acusilaus of Argos	between Hecataeus and Herodotus
523	Cadmus of Miletus	between Hecataeus and Herodotus
520 - 479	Mardonius	general <i>c.</i> 490 - 479
520	Eurybiades	general in 480
514	Manlius Vulso	cos. 474
506	Postumius Albus	cos. 466
501	Sulpicius Camerinus	cos. 461
489 - 418	Laches	older than Socrates - 418
485	Herodicus	brother of Gorgias
485	Nehemiah	governor of Judea from 445 to 433
485	Phrynus	won a contest <i>c.</i> 445
484	Archelaus	between Anaxagoras and Socrates
479 - 413	Perdiccas	prince <i>c.</i> 454-13
472	Euctemon	contemporary with Meton
463	observation of Mars and Saturn	from 423 to 400
455	<i>Esther</i>	the century after 465
455	Phaleas	between Hippodamus and Plato
451 - 361	Timotheus	died at 90 in 365/57
450	Idaeus	the century after Anaxagoras
450	Pherecydes of Athens	one generation after Hellanicus
449 - 406	Phaedrus	younger than Socrates - the decade before 401
440 - 403	Charmides	<i>c.</i> 440 - 403
439	Polybus	son-in-law of Hippocrates
425 - 401	Cyrus	younger son of Darius II - 401
423	observation of lunar eclipses	from 383 to 382
421 - 400	Meno	very young in 401 - 400
418 - 370	Amyntas	prince <i>c.</i> 393-70
417	Phaedo	born <i>c.</i> 417
408 - 353	Dion	<i>c.</i> 408 - 353
405 - 371	Cleombrotus	<i>c.</i> 405 - 371
405 - 364	Pelopidas	<i>c.</i> 405 - 364
396 - 333	Dionysius II	<i>c.</i> 396 - the decade after 338
384	Diophantus	contemporary with Demosthenes
378	Praxagoras	student of Eudoxus
376	Chrysippus of Cnidus	taught Herophilus
371	Diocles of Carystus	contemporary with Theophrastus
361 - 321	Perdiccas	366/56 - 321
354 - 270	Polemon	scholarch 314-270
354	Meno	student of Aristotle
350	Barsine	daughter of Darius III
346 - 261	Philochorus	priest in 306 - 261
345	Postumius Megellus	cos. 305
336 - 265	Crates of Athens	older than Arcesilaus - the decade after 270
335	Aristyllus	contemporary with Timocharis
316 - 270	Arsinoë	<i>c.</i> 316 - 270

311	Hermarchus	student of Epicurus
299 - 225	Lyc0	
282 - 215	Lacydes	scholarch 242-15
281	Polystatus	two generations after Epicurus
274	Telecles	two generations before Carneades
259	Euandrus	one and a half generations before Carneades
246	Zeno of Tarsus	scholarch from 208/5
244	Hegesinus	one generation before Carneades
241	observation of lunar eclipses	from 201 to 174
236	Mago	the century before 146
236	Rosetta stone	decree 196
232 - 214	Hieronymus	235/0 - 214
202	observation of autumn equinoxes	from 162 to 158
194	Carneades the younger	younger than Carneades
194	Crates of Tarsus	younger than Carneades
191	Cassius Dionysius	two to three generations before Varro
190 - 124	Laelius	c. 190 - the decade after 129
187 - 110	Clitomachus	
171 - 133	Attalus III	
166	Coelius Antipater	two generations before Cicero
162	Fannius	cos. 122
161	Diophanes	one to two generations before Varro
160	Tubero	tr. c. 130
159 - 84	Philo of Larisa	
159 - 73	Rutilius	c. 159 - the decade after 78
158 - 87	Scaevola Augur	c. 158 - 87
155	Hecaton	student of Panaetius
149	Dardanus	scholarch from c. 109
149	Mnesarchus	scholarch from c. 109
121	Quadrigarius	the generation before Cicero
121	Valerius Antias	the generation before Cicero
112	Alexander Polyhistor	freedman of Sulla in 82
110 - 37	Philodemus	c. 110 - 40/35
	<i>On signs</i>	
	<i>On rhetoric</i>	
	<i>On piety</i>	
110 - 32	Atticus	
110 - 25	Tyrannio	c. 110 - the decade after 30
106 - 24	Athenodorus Calvus	contemporary with Cicero - died at 82
83 - 4	Arius Didymus	teacher of Augustus - the decade after 9 ^{BC}
82 - 46 ^{BC}	Vercingetorix	led revolt in 52 - 46
27 - 36 ^{AD}	Thrasyllus	under Tiberius, prince 14-37
27	Apion	under Tiberius
27 ^{BC}	hymn to Imhotep in the temple of Ptah at Karnak	– under Tiberius
6 ^{AD}	st Peter	contemporary with st Paul
52	Agrippa	observation in 92
58	Papias	under Trajan, prince 98-117
58	Rufus of Ephesus	under Trajan
	<i>On the names of the parts of the human body</i>	
77	Basilides	under Hadrian, prince 117-38
85	observation of a lunar eclipse	in 125
91	Tatian	one generation before Irenaeus
98	Hegesippus	under Pius, prince 138-61
98	Valentinus	under Pius
102 - 178	Herodes Atticus	c. 102 - c. 178

104	Marcion	excommunicated in 144
121	Herodian	under Marcus, prince 161-80
	<i>General prosody</i>	
121	Montanus	under Marcus
142 - 218	Zephyrinus	bishop 202-18
154 - 222	Bardaisan	
157	Athenaeus	wrote in the decade after 192
	<i>The sophists' dinner</i>	
158 - 223	Callistus	bishop 218-23
170	Sabellius	contemporary with Hippolytus
171	Beryllus	under Caracalla, prince 211-7
187 - 265	Dionysius of Alexandria	bishop 247-265
198	Censorinus	wrote in 238
	<i>On the birthday</i>	
200	Paul of Samosata	bishop c. 260-70
255 - 326	Alexander of Alexandria	contemporary with Arius - 326
271	Calcidius	under Constantine, prince 311-37
	commentary on the <i>Timaeus</i>	
273 - 355	Donatus	consecrated bishop in 313 - 355
295 - 367	Victorinus	290/300 - the decade after 362
299	Zosimus of Panopolis	the century before 389
313 - 398	Didymus the Blind	c. 313 - 398
315 - 368	Hilary	c. 315 - 368
315 - 403	Epiphanius	310/20 - 403
346	scholia in the <i>Elements</i>	reach back to the time between Pappus and Proclus
351 - 407	Chrysostom	347/54 - 407
359 - 383	Gratian	
372	Hesychius of Jerusalem	priest c. 412
376 - 444	Cyril of Alexandria	c. 376 - 444
384	Socrates Scholasticus	wrote in the generation through 439
	<i>History of the church</i>	
393 - 466	Theodore	c. 393 - 466
404	Mamertus	died in 474
	<i>On the state of the soul</i>	
405	Hesychius	the century after 395
	<i>Alphabetically arranged collection of all words in Aristarchus, Apion, and Heliodorus</i>	
405	Stobaeus	the century after 395
	<i>Anthology</i>	
419 - 455	Valentinian III	
441	Ammonius son of Hermias	student of Proclus
	commentary on <i>On interpretation</i>	
465 - 533	Fulgentius	462/8 - 533
487	Stephen of Byzantium	under Justinian, prince 527-65
	<i>Peoples</i>	
597	ibn Abi Wakkas	general in 637
722	Mashaallah	horoscopes from 762 to 809
	<i>On the science of the motion of the orb</i>	
773 - 868	al-Jahiz	under al-Mamun - 868
	<i>On animals</i>	
786 - 863	Paschasius	786 - 860/5
827 - 869	Cyril	
857	al-Nayrizi	wrote in 892/902
990 - 1084	al-Jayyani	990 - the decade after 1079
	<i>Tables</i>	
	<i>On the dawn</i>	

- 1095 - 1152 Geoffrey of Monmouth 1090/100 - c. 1152
 1096 - 1164 Héloïse 1092/100 - 1164
 1100 - 1170 al-Bayhaqi died in 1170
Addition to The keepers of wisdom
 1105 Jabir ibn Aflah
Correction of the Almagest
 1130 - 1195 Henry the Lion 1129/31 - 1195
 1130 - 1202 Eugenius of Sicily c. 1130 - 1202
 1135 - 1205 Amalric died in the decade before 1210
 1150 - 1231 William of Auxerre c. 1150 - 1231
 1161 Acerbus contemporary with Innocent III
 1170 decree of the archbishop of Sens, the bishop of Paris, and other bishops, gathered in Paris, on the
 expurgation of heresy and on the different books of Aristotle, 1210
 1175 decree prescribing the way of reading in the arts and in theology, indicating which books the scholars
 are not allowed to read, arranging the disciplines of the sciences and in general the university, Aug
 1215
 1210 - 1279 Tempier c. 1210 - 1279
 condemnation of 219 obvious and abominable errors, by Stephen, bishop of Paris, 7 Mar 1277
 1287 - 1360 John of Saxony
 1290 Peter Bonus wrote in 1330
The new most precious pearl, or on the most precious stone of the philosophers
 1305 Swineshead wrote in 1340/50
Book of calculations
 1430 - 1504 Walther
 1436 - 1517 Jiménez de Cisneros
 1444 - 1522 de Nebrija
 1449 - 1515 Manutius
 1450 - 1505 Montalvo c. 1450 - c. 1505
 1453 - 1536 Lefèvre d'Étaples 1450/5 - 1536
 1463 - 1503 Lorenzo il Popolano
 1465 - 1526 Ferro
 1468 - 1522 Johann Werner
 1488 - 1539 Ferdinand Columbus
The life of the admiral Christopher Columbus by his son Ferdinand, 1539
 1490 - 1573 de Sepúlveda
 1495 - 1552 Apian
 1498 - 1536 Hans Weiditz the younger 1495/500 - 1536
 1499 Fiore contemporary with Tartaglia
 1500 - 1547 Bunel
 1501 - 1566 Fuchs
On the history of plants, 1542
 1501 - 1577 Mattioli
 1502 - 1585 Gregory XIII
 1503 - 1576 de Carranza
 1504 - 1575 Bullinger
 1507 - 1566 Rondelet
On marine fishes, with their images, and universal history of aquatic animals, 1555
 1507 - 1577 Viglius
 1510 - 1589 Palissy c. 1510 - 1589
 1510 - 1590 Paré
The method of treating wounds caused byarquebuses and other guns, 1545
 1517 - 1586 Granvelle
 1519 - 1605 Beza
 1522 - 1568 count of Egmont
 1523 - 1562 Fallopius

Anatomical observations, 1561

1523 - 1576 Hoppers

1524 - 1568 count of Horn

1531 - 1598 Mocenigo

deposition against Bruno, 23 Mar 1592

1533 - 1619 Fabricius c. 1533 - 1619

On the formation of the foetus, 1604

1535 - 1612 van Meteren

1541 - 1593 Mercati

1545 - 1606 Bunting

1545 - 1616 Röslin

1550 - 1602 van Reyd

1552 - 1612 Rudolph II

1553 - 1617 de Thou

1556 - 1615 Calvisius

1558 - 1614 Welser

1560 - 1613 Rittershausen

1561 - 1636 Sartorio

On the art of measurement in medicine, 1614

1562 - 1647 Longomontanus

1565 - 1637 Christina of Lorraine

1565 - 1652 Chiaramonti

1567 - 1601 earl of Essex

1570 - 1640 Suslyga

1572 - 1649 Strada

1574 - 1636 Giuliano de' Medici

1574 - 1637 Fludd

1576 - 1649 Schopp

letter to Rittershausen, 17 Feb 1600

1576 report of the committee appointed to examine the propositions, the sun is in the center of the world and does not move, and the earth is not in the center and moves, to the holy office, 24 Feb 1616

1577 - 1657 Liceti

1578 - 1621 Philip III

1578 - 1649 Ingoli

1579 - 1639 Meursius

1579 - 1644 Bentivoglio

1580 - 1626 Capra

1580 - 1657 Riolan

1580 - 1667 Wendelin

Lunar eclipses observed from the year 1573 to the year 1643, based on Atlantic tables, their idea explained, 1644

1581 - 1656 Ussher

The annals of the old testament deduced from the first origin of the world, with a chronicle of Asian and Egyptian affairs, 1650

1583 - 1634 Wallenstein

1583 - 1654 Grassi

1583 - 1656 Morin

1584 - 1653 Dinet

1584 - 1667 Saint-Vincent

1585 - 1619 Vanini

1586 - 1649 Burgundius

1588 - 1653 Filmer

1590 - 1621 Cosimo II

1593 sentence in the trial of Galileo, 22 Jun 1633

1598 - 1671 Riccioli

New Almagest, embracing the old and the new astronomy, from the observations of others and one's own, proposing new theorems, problems, and tables, 1651

1599 - 1656 Ottavio Piccolomini

1600 - 1633 Bartsch

1600 - 1675 van Langren c. 1600 - 1675

1601 - 1652 de Beaune

1602 - 1676 Bosse

Universal method of Mr Desargues for the practice of perspective, by small steps, like the geometrical, 1648

1605 - 1665 Philip IV

1605 - 1675 Frénicle

1605 - 1694 Boulliau

1608 - 1666 Schott

1614 - 1687 Henry More

1615 - 1660 van Schooten

1617 - 1675 Leopoldo de' Medici

1618 - 1677 Oldenburg c. 1618 - 1677

1618 - 1689 Voss

On the nature and properties of light, 1662

1619 - 1682 Michelangelo Ricci

1620 - 1684 Mariotte c. 1620 - 1684

On the nature of air, 1676

1620 - 1687 Nicolas Mercator

1622 - 1685 de Sluse

1622 - 1689 Streete

1625 - 1679 Johann Friedrich

1628 - 1704 Hudde

1629 - 1698 Ernst August

1629 - 1707 Towneley

1630 - 1702 Kunckel

1632 - 1723 Wren

1632 Cassegrain wrote in 1672

1633 - 1701 James II

1635 - 1672 Willughby

with Ray, *q. v.*

1635 - 1682 Becher

Chemical laboratory, or subterranean physics, 1669

1641 - 1712 Grew

The anatomy of plants, with an idea of a philosophical history of plants, 1682

1650 - 1715 Savery c. 1650 - 1715

1654 - 1722 Varignon

1657 - 1757 Fontenelle

Conversations on the plurality of worlds, 1686

1660 - 1717 Gabriel Wagner c. 1660 - 1717

1661 - 1704 l'Hôpital

1661 - 1719 Garth

1661 - 1742 de Polignac

1662 - 1694 Mary II

1662 - 1742 Bentley

1663 - 1729 Newcomen

1664 - 1730 Johann von Eckhart

1668 - 1705 Sophie Charlotte

1668 - 1738 Boerhaave

1669 - 1724 Pound

1672 - 1719 Addison

- 1675 - 1729 Clarke
 1676 - 1725 Remond
 1677 - 1761 Hales
 1680 - 1751 Thomas Prior
 1680 - 1753 de Boze
 Eulogy for the cardinal de Polignac, 1742
 1682 - 1716 Cotes
 preface to the second edition of *The mathematical principles of natural philosophy*, 1713
 1683 - 1746 Philip V
 1683 - 1760 George II
 1686 - 1748 Gruber
 1688 - 1737 Conduitt
 Keynes manuscripts
 1693 - 1768 Collinson
 1700 - 1782 Duhamel de Monceau
 1701 - 1744 Celsius
 1705 - 1765 Johann Meyer
 1709 - 1761 Scheidt
 1711 - 1786 Thomas Wright
 1715 - 1785 Strahan
 1718 - 1779 Rouelle
 1720 - 1794 Möser
 History of Osnabrück, a general introduction, 1768
 with Herder and Goethe, q. v.
 1722 - 1797 Crantz
 1723 - 1775 Venel
 1726 - 1778 Macbride
 1727 - 1806 Collini
 1727 - 1817 Jacquin
 1728 - 1804 Baumé
 1730 - 1817 Messier
 1731 - 1802 Erasmus Darwin
 1732 - 1811 Maskelyne
 1733 - 1812 Kirwan
 1734 - 1794 Caspar Wolff
 The theory of generation, 1759
 1735 - 1784 Bergman
 1735 - 1788 John Brown
 1735 - 1796 Vandermonde
 1736 - 1796 Macpherson
 1739 - 1812 Klügel
 1740 - 1807 Frere
 Account of flint weapons discovered at Hoxne in Suffolk, 1800
 1740 - 1813 Stock
 An account of the life of George Berkeley, late bishop of Cloyne in Ireland, 1776
 1741 - 1801 Lavater
 1743 - 1819 Friedrich Jacobi
 1745 - 1792 Hearne
 1745 - 1818 Gahn
 1745 - 1829 Jay
 with Hamilton and Madison, q. v.
 1746 - 1780 Bucquet
 1746 - 1834 Piazzi
 1747 - 1826 Bode
 Introduction to the knowledge of the starry heaven, 1772

- 1749 - 1819 Daniel Rutherford
 1752 - 1840 Blumenbach
 On the genesis of the native varieties of man, 1776
 1754 - 1793 Meusnier
 with Lavoisier, *q. v.*
 1755 - 1830 Soemmerring
 1756 - 1825 Lacépède
 1757 - 1817 William Wells
 1758 - 1840 Olbers
 1761 - 1840 Beck
 1765 - 1815 Fulton
 1768 - 1840 Carlisle
 1770 - 1847 Brongniart
 with Cuvier, *q. v.*
 1771 - 1833 Trevithick
 1774 - 1853 von Buch
 1775 - 1856 Wolfgang Bolyai
 1776 - 1841 Herbart
 1776 - 1847 Burdach
 1777 - 1859 Cagniard de la Tour
 Presentation of some results obtained by the combined action of heat and compression on certain liquids, such as water, alcohol, sulfuric acid, and distilled petroleum spirit, 1822
 1778 - 1846 Bory de Saint-Vincent
 1778 - 1847 William Herbert
 1779 - 1851 Oken
 1780 - 1838 Antommarchi
 Memoirs, or the last days of Napoleon, 1825
 1780 - 1850 Schumacher
 1780 - 1857 Schweikart
 memorandum
 1781 - 1848 Stephenson
 1781 - 1858 Hare
 1783 - 1840 Rafinesque
 1783 - 1875 d'Omalius d'Halloy
 1785 - 1850 Prout
 Correction of a mistake in the essay on the relation between the specific gravities of bodies in their gaseous state and the weights of their atoms, 1816
 1785 - 1871 Bekker
 1785 - 1873 Sedgwick
 On the classification and nomenclature of the lower paleozoic rocks of England and Wales, 1852
 1786 - 1875 Seguin
 1788 - 1865 Christian Thomsen
 1790 - 1864 Senior
 1791 - 1836 Schmerling
 Researches on fossil bones discovered in the caves of the province of Liège, 1833-4
 1791 - 1841 Savart
 with Biot, *q. v.*
 1791 - 1865 Encke
 1792 - 1871 John Herschel
 1794 - 1865 Pander
 1794 - 1866 Whewell
 History of the inductive sciences, from the earliest to the present times, 1837
 1794 - 1874 Taurinus
 Theory of parallel lines, 1825
 The first elements of geometry, 1826

- 1794 - 1878 Fries
 1796 - 1860 Baden Powell
 1799 - 1864 Clapeyron
 1800 - 1870 Unger
 1801 - 1858 Müller
 Handbook of the physiology of man, 1834-40
 1801 - 1867 Lartet
 On the geological antiquity of the human species in western Europe, 1860
 1802 - 1870 Botta
 1802 - 1871 Chambers
 1804 - 1872 Feuerbach
 1804 - 1892 Owen
 1805 - 1865 FitzRoy
 1806 - 1856 Stirner
 1806 - 1879 Mohr
 Views on the nature of heat, 1837
 1807 - 1853 Laurent
 1808 - 1882 Listing
 1809 - 1882 Bruno Bauer
 1810 - 1878 Regnault
 1811 - 1896 Grove
 On the correlation of physical forces, being the substance of a course of lectures delivered in the London institution in the year 1843, 1846
 1812 - 1876 Kuhlmann
 1814 - 1878 Robert Mayer
 Remarks on the forces of inorganic nature, 1842
 Organic motion in its connection to metabolism, 1845
 1815 - 1888 Colding
 Theses concerning force, 1843
 On the history of the principle of conservation of energy, 1863
 1815 - 1891 Keyserling
 1815 - 1899 Naudin
 1816 - 1856 Charles Gerhardt
 1816 - 1899 Carl Gerhardt
 1817 - 1887 Grün
 1817 - 1891 Nägeli
 1817 - 1911 Hooker
 1818 - 1866 Weydemeyer
 1818 - 1881 Lewis Morgan
 1820 - 1872 Rankine
 1825 - 1864 Lassalle
 1825 - 1895 Huxley
 1826 - 1910 Cannizzaro
 Sketch of a course of chemical philosophy, 1858
 1826 - 1910 Rassam
 1831 - 1901 Tait
 1833 - 1921 Dühring
 1834 - 1919 Haeckel
 1835 - 1900 Beltrami
 1835 - 1910 Schiaparelli
 1835 - 1924 Hensen
 1842 - 1910 William James
 1843 - 1904 Tannery
 1846 - 1919 Mehring
 1847 - 1922 Favaro

- 1851 - 1916 Silvanus Thompson
The life of William Thomson, baron Kelvin of Largs, 1910
- 1857 - 1940 Adam
- 1859 - 1925 Hilprecht
- 1859 - 1941 Bergson
- 1859 - 1952 Dewey
- 1861 - 1931 Burali-Forti
- 1862 - 1929 Kugler
- 1862 - 1956 Jules Richard
- 1867 - 1941 Driesch
Analytic theory of organic development, 1894
- 1868 - 1914 Couturat
- 1868 - 1939 Dyson
 with Eddington, *q. v.*
- 1873 - 1958 Moore
Principia ethica, 1903
- 1875 - 1970 Charles Davidson
 with Eddington, *q. v.*
- 1878 - 1936 Grossmann
 with Einstein, *q. v.*
- 1880 - 1936 Spengler
The decline of the west, outlines of a morphology of world history, 1918
- 1889 - 1975 Toynbee
A study of history, 1934-61
- 1891 - 1965 Fraenkel
On the foundations of the Cantor-Zermelo theory of sets, 1922
- 1900 Bourbaki began publishing in 1940
Elements of the history of mathematics, 1960
- 1903 - 1950 Orwell
1984, 1949
- 1904 - 1993 Jacobsen
The Sumerian king list, 1939
- 1920 - 2017 Kandler
 with Woese, *q. v.*
- 1922 - 1996 Kuhn
The structure of scientific revolutions, 1962
- 1926 - 1984 Michel Foucault
The archeology of knowledge, 1969
History of sexuality, 1976-84
- 1930 *Dictionary of scientific biography*, 1970-90
- 1949 - 1984 Laporte
History of shit, 1978

not dated (with their earliest source in the text)

Lugalkalama (Mesanepada)
 Ninhursag (Aanepada)
 Ningirsu, Nanshe (Ur-Nanshe)
 Enlil, Inanna (Eanatum)
 Re (Unas)
 An, Nanna (Ur-Nammu)
 Humbaba, Utu, Ereshkigal, Dumuzi, Zuisudra (Sumerian *Gilgamesh*)
 Gashur, Kullassinabel, Nangishlishma, Endaraana, Babum, Puannum, Kalibum, Kalumum, Zuqaqip,
 Atab, Mashda, Arwium, Etana, Balih, Enmenuna, Melamkish, Barsalnuna, Samug, Tizqar, Ilku,
 Iltasadum, Meskiaggasher, Enmerkar, Lugalbanda, Utulkalama, Labashir, Enundarana, Meshe,

Melamana, Lugalkigin, Balulu, . . . , . . . , . . . , Susuda, Dadasig, Mamagal, Kalbum, Tuge, Menuna, Lugalmu, Hatanish, Argandea, . . . , Lugalanemundu, Anbu, Anba, Bazi, Zizi, Limer, Sharumiter, Unzi, Undalulu, Urur, Ishuil, Shu-Sin, Simudar, Usiwatar, Eshtarmuti, Ishme-Shamash, Nannia (Sumerian prince list)

Alulim, Alalgar, Enmeluana, Enmegalana, Dumuzi, Ensipaziana, Enmedurana, Ubartutu (antediluvian prince list)

Marduk (Esagil chronicle)

Ishtar (Venus tablet of Ammisaduqa)

Apsu, Tiamat, Lahmu, Lahumu, Anshar, Kishar, Anu, Ea, Qingu (*When on high*)

Osiris, Isis (48 treatments of injuries)

Uta-Napishti (Sin-liqe-uninni)

Yahweh, Adam, Cain, Enoch, Seth, Enosh, Kenan, Mahalalel, Jared, Enoch, Methuselah, Lamech, Noah, Shem, Ham, Japheth, Gomer, Magog, Madai, Javan, Tubal, Meshech, Tiras, Ashkenaz, Riphath, Togarmah, Elishah, Tarshish, Kittim, Dodanim, Cush, Mizraim, Put, Canaan, Seba, Havilah, Sabtah, Raamah, Sabteca, Sheba, Dedan, Nimrod, the Ludim, the Anamim, the Lehabim, the Naphtuhim, the Pathrusim, the Casluhim, the Caphtorim, the Philistines, Sidon, Heth, the Jebusites, the Amorites, the Girgashites, the Hivites, the Arkites, the Sinites, the Arvadites, the Zemarites, the Hamathites, Elam, Ashur, Arpachshad, Lud, Aram, Uz, Hul, Gether, Mash, Shelah, Eber, Peleg, Joktan, Almodad, Sheleph, Hazarmaveth, Jerah, Hadoram, Uzal, Diklah, Obal, Abimael, Sheba, Ophir, Havilah, Jobab, Abraham, Moses, Joshua (Yahwistic document)

Amos (*Amos*)

Hosea (*Hosea*)

Isaiah (*Isaiah*)

Ashur-dugul (Assyrian prince list)

Peleus, Achilles, Zeus, Apollo, Agamemnon, Hera, Odysseus, Idomeneus, Menelaus, Athena, Hector, Nestor, Theseus, Tartarus, Thetis, Ocean, Hephaestus, Hermes, Helen, Ascalaphus, Ialmenus, Machaon, Hippothous, Pylaemenes, Odius, Epistrophus, Styx, Iris, Aphrodite, Sarpedon, Hades, Orestes, Dionysus, Hecuba, Helios, Minos, Deucalion, Tethys, Heracles, Europa, Euphorbus, Daedalus, Ariadne, Oedipus, Proteus, Eidothea, Themis, Scylla, Melampus (Homer)

Micah (*Micah*)

Ashur (Sennacherib)

Clio, Euterpe, Thalia, Melpomene, Terpsichore, Erato, Polyhymnia, Urania, Calliope, Chaos, Earth, Love, Darkness, Night, Aether, Day, Heaven, Mountains, Sea, Coeus, Crius, Hyperion, Iapetus, Theia, Rhea, Mnemosyne, Phoebe, Cronus, Thunder, Lightning, Flash, Cottus, Briareus, Gyges, Hesperus, Old Man of the Sea, Marvel, Phorcys, Whale, Eurybia, Medusa, Geryoneus, Europe, Asia, Phosphorus, Dike, Prometheus, Epimetheus, Danaus, Aegyptus (Hesiod)

Adad (*When Anu and Enlil*)

Jeremiah (*Jeremiah*)

Ezekiel (*Ezekiel*)

Time (Pherecydes)

Gideon, Samuel, Saul, Abigail, Elisha (*Joshua, Judges, Samuel, Kings*)

Haggai (*Haggai*)

Zarathustra (*Gathas*)

Ahura Mazda (Darius)

Sibyl (Heraclitus)

Job (*Job*)

Nestis (Empedocles)

Tellus, Lycurgus, Oebares, Oroetes, Zopyrus, Aristas, Zalmoxis, Aristodemus, Eurysthenes, Procles, Hippoclidus, Titormus, Amyris, Hyllus, Cleodaeus, Aristomachus, Menares, Musaeus, Agis, Echestratus, Leobotes, Dorussus, Agesilaus, Archelaus, Teleclus, Anaxandrus, Eurypon, Prytanis, Polydectes, Eunomus, Charillus, Nicandrus, Hegesilaus (Herodotus)

Critobulus (Xenophon)

Chaerephon, Crito, Euthyphro, Agathocles, Lysis, Ion, Iccus, Pythoclidus, Euthydemus, Dionysodorus, Herodicus, Menexenus, Polemarchus, Glaucon, Adimantus, Ismenias, Er, Philebus, Timaeus (Plato)

Aethalides, Hermotimus, Pyrrhus (Ponticus)

Euryphon, Autophradates, Eubulus, Philolaus of Corinth, Diocles, Halcyone, Androdamas, Ion,
 Nicomachus (Aristotle)
 Matricetas, Phaeinos (Theophrastus)
 Dido (Timaëus)
 Herodotus, Menoeceus (Epicurus)
 Daniel, Gabriel (*Daniel*)
 Fu Xi (Sima Qian)
 Mochus (Posidonius)
 Jupiter, Mercury, Mars, Minerva, Venus, Hercules (Varro)
 Diodotus (Cicero)
 Neleus (Strabo)
 Romulus, Remus, Numa, Tarquinius Superbus, Lucretia, Brutus, Collatinus, Valerius Poplicola,
 Porsenna, Mucius, Cincinnatus (Livy)
 Marcia, Helvia, Polybius (Seneca)
 Jesus, Antichrist, Timothy, Titus (st Paul)
 Cleombrotus, Critobulus (Pliny)
 Mark, Mary (*Gospel of Mark*)
 Matthew (*Gospel of Matthew*)
 Concordia, Victoria, Athenodorus Cordylion (Plutarch)
 Luke, Dionysius the Areopagite (*Gospel of Luke, Acts of the apostles*)
 John (*Gospel of John, Apocalypse*)
 Hermes Trismegistus (Hermetic corpus)
 Noetus (Hippolytus)
 Simon, Menander, Cerinthus, Nicolaus, Saturninus, Carpocrates, Cerdo, Marcus, Thebuthis, Simon,
 Cleobius, Dositheus, Gorthaeus, Severus, Apelles, Potitus, Basilicus, Syneros, Priscilla, Maximilla,
 Florinus, Blastus, Miltiades, Themiso, Alexander, Alcibiades, Theodotus, Artemon, Theodotus,
 Asclepiodotus, Theodotus, Natalius, Symmachus, Ambrose, Proclus, Nepos, Coracion (Eusebius)
 Lucifer (Augustine)
 Philology, Rhetoric, Geometry (Capella)
 Sifilo (Fracastoro)
 Feselius (Kepler)
 Phoebadius, Faustinus, Gregory of Elvira, Philastrius, Idacius, Eucherius, Arnobius Junior, Eugenius
 of Carthage, Cerealis, Vigilus of Thapsus, Victor Vitensis, Facundus, Oecumenius, Nicetas, de
 Zúñiga, de Colines, Edward Lee, Hessels, Montano, Canter, Franciscus Lucas, Caryophilus,
 Chiffletius, Brian Walton, Richard Simon, Gilbert Burnet (Newton)
 Theophilus, Philalethes (Leibniz)
 Wheler (Gray)
 Hylas, Philonous (Berkeley)
 Julie, Émile, Sophie (Rousseau)
 Saluzzo, van Smeth (Lavoisier)
 Götz von Berlichingen, Werther, Lotte, Faust (Goethe)
 Bor, Hoof, Brandt, Le Clerc, Dinot, Spittler, Watson, Wilhelm Tell, Max Piccolomini (Schiller)
 Clair Grece, Karl Meding, Rowley, Brace, Grant, Patrick Matthew, Haldeman, Isidore Geoffroy Saint-
 Hilaire, Freke, Bronn, Dalton, Gordon, Poiret, Schaaffhausen, Lecoq, Rudolf Wagner (Darwin)
 George Fox, Wheelis (Woese)

Enki (Inanna compositions)
 Charon (Lucian)
 Orlando (Ariosto)
 Don Quixote, Sancho Panza (Cervantes)
 Robinson Crusoe (Defoe)
 Laocoön (Lessing)

Amun (Nebneteru)
 Nahum (*Nahum*)

Ruth (*Ruth*)
 Orpheus (Orphic poems)
 Nergal (100-odd prognoses)
 Zechariah (*Zechariah*)
 Esther (*Esther*)
 Ptah (hymn to Imhotep)
 scholiast on the *Iliad*
 scholiast on the *Hecuba*
 scholiast on the *Orestes*
 scholiast on the *Peace*
 Peter (Socrates Scholasticus)
 Heliodorus (Hesychius)
 Arthur (Geoffrey of Monmouth)
 Amadis (Montalvo)
 Ossian (Macpherson)

Reisner (account of work on a building including dimensions of chambers)
 Rhind (book of 87 problems)

the numbering of inscriptions and fragments follows these editions

Enmebaragesi, Meskalamdug, Mesanepada, Mesalim, Aanepada, Ur-Nanshe, Meskiagnuna, Akurgal, Elili, Eanatum, Enanatum, Enshakushana, Ur-Luma, Enmetena, Lugalkiginedudu, II, Enanatum II, Lugalkisalsi, Gishakidu, Enentarzi, Lugalanda, Lugalzagesi, Urukagina – Cooper
 Sargon, Manishtushu, Rimush, Naram-Sin, Sharkalisharri, Dudu, Shudurul, Laerab, Yarlagan, Sium, Puzur-Inshushinak – Gelb/Kienast
 Ur-Baba, Gudea – Edzard
 Ur-Nigin, Ur-Gigir, Utuhegal, Ur-Nammu, Shulgi, Amar-Sin, Shu-Sin, Ibbi-Sin, Samsuiluna – Frayne
 Shallim-ahhe, Ilushuma, Erishu, Shamshi-Adad, Ashur-nirari, Puzur-Ashur III, Adad-nirari, Shalmaneser, Tukulti-Ninurta, Tiglath-pileser, Ashur-nasirpal II – Grayson
 Tiglath-pileser III, Sargon II, Sennacherib, Esarhaddon, Ashurbanipal – Luckenbill
 Epimenides, Xenophanes, Heraclitus, Parmenides, Anaxagoras, Empedocles, Melissus, Antiphon, Hippo, Diogenes of Apollonia, Lycophron, Critias – Diels
 Solon – West
 Hecataeus, Callisthenes, Timaeus, Manetho, Demetrius – Felix Jacoby
 Ponticus – Wehrli
 Aristotle – Rose

Tyrtaeus – West

Eluldan – Gelb/Kienast
 Anubanini – Sollberger
 Barahenidu, Eginimpae, Lugalatu, Lugaldalu, Lugalsilasi, Medurba, Uhub – Cooper
 Ikunu, Sargon, Shamshi-Adad III, Enlil-nasir – Grayson

And counting centuries from the year of birth assigned to the 3 calculations of area, these are the distribution of the particulars and their totals,

	important and writings		poets and writings		not important and writings	
3120-3021	2	1	–	–	–	–
3020-2921	1	–	–	–	–	–
2920-2821	–	–	–	–	–	–
2820-2721	–	–	–	–	–	–
2720-2621	8	2	–	–	–	–
2620-2521	12	7	–	–	1	–
2520-2421	16	11	–	–	3	–

2420-2321	10	7	—	—	1	—
2320-2221	8	3	—	—	11	1
2220-2121	13	13	—	—	14	—
2120-2021	14	7	2	3	8	8
2020-1921	12	4	—	—	2	1
1920-1821	19	1	—	—	4	1
1820-1721	15	3	—	—	1	—
1720-1621	12	—	1	—	3	2
1620-1521	9	2	—	—	1	1
1520-1421	4	—	—	—	1	—
1420-1321	14	1	—	—	1	—
1320-1221	8	3	—	—	4	—
1220-1121	5	1	—	—	2	—
1120-1021	2	—	—	—	7	—
1020- 921	9	—	—	—	3	1
920- 821	10	2	—	—	3	—
820- 721	26	7	—	—	4	—
720- 621	34	4	4	1	21	—
620- 521	41	10	7	—	23	—
520- 421	69	70	5	3	24	—
420- 321	54	80	2	—	18	—
320- 221	34	19	—	—	13	—
220- 121	24	8	—	—	19	—
120- 21	29	31	3	—	10	3
20 ^{BC} - 80 ^{AD}	29	73	—	—	5	1
81- 180	28	35	1	2	14	2
181- 280	19	18	—	—	6	2
281- 380	16	14	—	—	10	—
381- 480	11	7	—	—	8	5
481- 580	10	9	—	—	1	1
581- 680	5	1	—	—	1	—
681- 780	10	7	—	—	2	2
781- 880	13	11	—	—	3	—
881- 980	14	7	—	—	—	—
981-1080	18	16	—	—	1	2
1081-1180	24	14	—	—	11	2
1181-1280	35	55	—	—	1	1
1281-1380	15	16	—	—	3	2
1381-1480	42	27	3	1	9	—
1481-1580	44	125	4	1	52	10
1581-1680	70	213	2	2	62	9
1681-1780	107	326	3	1	63	11
1781-1880	98	280	1	—	77	19
1881-1980	37	60	—	—	10	9
	1159	1611	38	14	541	96

and the not dated are 570.